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A Generalized Design Method for Learning-Based Disturbance Observer

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***Abstract*—This paper presents a generalized disturbance observer (DOB) design framework that is applicable to both multi-input-multi-output (MIMO) and/or non-minimum phase systems. The design framework removes conventional DOB's structure constraint, which allows minimizing the H-infinity norm of the dynamics from disturbance to its estimation error over a larger feasible set. The design procedure does not require explicit plant inverse, which is usually challenging to obtain for MIMO or non-minimum phase systems. Furthermore, the generalized DOB is augmented by a learning scheme, which is motivated by iterative learning control, to further enhance the estimation and suppression of the disturbance when it has repetitive components. Both numerical and experimental studies are performed to validate the proposed learning-based DOB design framework.**

***Index Terms*—Disturbance Observer, H-infinity Synthesis, Iterative Learning Control.**

I. Introduction

Disturbance reconstruction is a powerful control technique that estimates and compensates external disturbance without additional sensors. The existing methods in current literature usually fall into three categories: (1) the disturbance observer (DOB) that is designed in frequency domain based on the transfer functions of systems and has been widely applied to linear-time invariant (LTI) systems (e.g., [1]); (2) the extended state observer that is designed in time domain based on the state-space models of systems and has been applied to both LTI and nonlinear systems (e.g., [2], [3]); and (3) the data-driven technique that is designed based on the input-output data without explicitly considering system models and their inverse (e.g., [4]). In this paper, we explore the first one, the DOB method, which has been widely used in many applications including hard disk drives [5], quad-rotors [6], vehicles [7], and manipulators [8]. The DOB design methods were comprehensively reviewed in [9].

In general, the traditional DOB design procedure involves two key steps: a stable plant inverse design and a Q filter design (i.e., [10], [11]). For multi-input-multi-output (MIMO) and/or non-minimum phase systems, conventional DOB design would encounter lots of tuning effort. The non-minimum phase systems have unstable zeros, which bring challenges to both the plant inverse and the Q filter design. Several techniques including approximation (e.g. [12]) of plant and robust control theory (e.g., [13]–[17]) have been proposed. Most of these techniques for single-input-single-output (SISO) systems. The DOB design for MIMO systems is not an easy process either. DOBs for MIMO systems have been designed either by (1) ignoring the coupling effect of different input-output channels of the plant (e.g., [18]), or by (2) decoupling the plant using the nominal model and then following the standard DOB design procedure for SISO systems (e.g., [19]). Most of these DOB design techniques for MIMO systems are only applicable to the systems with the same input and output dimensions (i.e., *square systems*). The application of these DOB techniques to non-square systems, especially the ones with the inputs of higher dimension than that of the outputs, is very challenging or even impossible. To unnecessitate the design of the plant inverse for the non-minimum phase and the non-square MIMO systems, the DOBs without explicitly assuming any structure have been designed in [5] for high-precision systems and in [6] for UAVs. The design for these DOBs has been formulated into an optimization problem based on H-infinity synthesis.

In spite of the optimization formulation proposed in [5], the DOB's disturbance suppression performance is still fundamentally limited by the non-existence of an explicit plant inverse. This paper is an extended paper based on our previous conference papers [5], [6]. We first summarize the disturbance observer design method presented in [5], [6] and then augment it into a learning-based scheme by adding an learning component that is motivated by iterative learning control (ILC). The main purpose of the learning component is to further enhance disturbance estimate when the main components in the disturbance are repetitive and the plant inverse does not exist or not stable. This learning-based DOB framework enhances the DOB's performance and in turn extends the traditional ILC into an estimation problem. One challenge of such learning-based DOB is the design of the learning filter in a systematic way with guaranteed stability and convergence. In this paper, the DOB's learning filter is designed via solving an optimal feedback control problem, which can be further transformed into a convex optimization problem.

It is worth mentioning that, though there are many works on the combination of iterative learning control and disturbance observer (i.e., [20]–[24]), most of them use them independently. That is, in the state-of-the art methods,

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disturbance observer is usually to reject the non-repetitive components and the iterative learning control is usually to reject the repetitive components (eg., [20]). In this paper, the iterative learning component is to further enhance the DOB's estimation performance when the main components of the disturbance are repetitive. This is motivated by the fact that DOB may not work well even for repetitive disturbance because of the modeling uncertainty and the non-minimum phase property. Especially in real applications, DOB cannot be designed too aggressively otherwise it might be easy to become unstable because of modeling uncertainties and non-minimum phase property. Adding such a learning component allows a "mild" design of the DOB while further enhancement is done by the feedforward ILC component which does not affect the stability.

The following contributions are briefly summarized: the DOB framework presented in this paper mitigates the design efforts of the plant inverse and the Q filter, removes the conventional DOB's structure constraint, and minimizes the norm of the dynamics from disturbance to its estimation error. Furthermore, the proposed DOB is augmented into a learning scheme, which further enhances the compensation of disturbance with competitive components via learning from historical information. The design framework is proposed in a general form such that it can be readily applied to non-square or non-minimum phase systems.

The remainder of this paper is organized as follows. Section II briefly revisits conventional DOB basics and design considerations. Section III reformulates the DOB design into an optimization problem and generalizes the design method by removing of the conventional structure constraint of the DOB. Section IV extends the generalized DOB into a learning-based scheme such that the DOB could improve the performance via learning from historical information. Section V provides numerical and experimental studies to validate the proposed generalized DOB design framework as well as the self-enhancement capacity of the learning-based DOB. Section VI concludes the paper.

II. Conventional DOB and Design Considerations

The structure of a conventional DOB is illustrated in Figure 1, in which z is the Z-transform variable, $G(z)$ is the plant transfer function in discrete time and $G_n(z)$ is its nominal model, $C(z)$ is a well-designed baseline feedback controller, and $Q(z)$ is a to-be-designed filter that maintains the causality of the DOB. The conventional DOB design is to utilize the plant inverse to reconstruct the plant's input signal: u_e+d. As the control signal u_e is known, the disturbance d can be estimated as the difference between the constructed input and the known control signal. The conventional DOB includes $G_n^{-1}(z)$ and $Q(z)$ (in the dotted box in Figure 1) which will be designed such that $\hat{d}$ is close to d and the DOB is causal.

The following is to find the relationship between the disturbance and its estimation error. For simplicity, we only write the mathematical derivation for the case when the plant is a SISO system and the reference r is zero. The MIMO case is more complex using the traditional structure of the DOB.

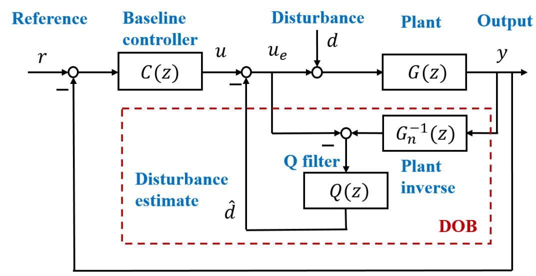

Fig. 1. A general system with the conventional DOB. In this system, r denotes the reference; y denotes the output; u denotes the controller signal from the baseline controller $C(z)$; d denotes the external disturbance and $\hat{d}$ denotes its estimate; u_e denotes the augmented control signal combined by u and $\hat{d}$.

Define $T_f(z)$ as the transfer function from d to $d-\hat{d}$. It is obtained as follows

$$\begin{aligned} & d-\hat{d} \\ & = \left[1-\frac{Q[G_n^{-1}G+GC][1+GC]^{-1}}{(1-Q)+Q[GG_n^{-1}+GC][1+GC]^{-1}}\right]d \quad (1) \\ & \triangleq T_f(z)d \end{aligned}$$

Here z is omitted for simplicity. The characteristic equation of the closed-loop system with DOB is derived from T_f in Eq. (1) by setting

$$(1-Q)+Q(G_n^{-1}G+GC)(1+GC)^{-1}=0 \quad (2)$$

By multiplying $(1+GC)$ to each term, we have

$$(1+GC)(1-Q)+Q\left(GG_n^{-1}+GC\right)=0 \quad (3)$$

It is worth noting that if (1) $Q(z)$=1 and (2) $G_n^{-1}(z)G(z)$=1, then d=$\hat{d}$. The stability of the closed-loop system would not be affected by the DOB: plugging $Q(z)$=1 and $G_n^{-1}(z)G(z)$=1 into Equation (3) results in 1+$G(z^{-1})C(z^{-1})$=0, which is the closed-loop characteristic equation without DOB.

However, it is normally difficult to design the plant inverse and guarantee the stability when $G(z)$ is a non-minimum phase plant system, i.e., when it has unstable zeros. The main design challenge comes from the simultaneous design of an approximate stable plant inverse and a Q-filter to guarantee the stability without re-modifying the original baseline feedback controller. Consider a SISO plant,

$$G(z^{-1})=\frac{z^{-q}B(z^{-1})}{A(z^{-1})}=\frac{z^{-q}B^s(z^{-1})B^u(z^{-1})}{A(z^{-1})} \quad (4)$$

where we use z^{-1} as the variable instead of z without changing the transfer functions (e.g., $G(z)$ is identical to $G(z^{-1})$) to explicitly include a system's delay q. $B^u(z^{-1})$ consists of the unstable zeros and $B^s(z^{-1})$ consists of the stable ones. With a non-minimum plant (4) with unstable zeros in $B^u(z^{-1})$ and the DOB structure in Figure 1, the following conventional design for $G_n^{-1}(z^{-1})$ in Equation (5) with $Q(z^{-1})=z^{-q}$ cannot always preserve the stability of the original closed-loop system without DOB,

$$G_n^{-1}(z^{-1})=z^q\frac{A(z^{-1})}{B^s(z^{-1})[B^u(z^{-1})]^{\#}} \quad (5)$$

where $[B^u(z^{-1})]^{\#}$ is an approximation of $B^u(z^{-1})$ with stable zeros.

The explanation is provided as follows. Considering that characteristic equation of the closed-loop system with DOB is Equation (3), plugging Equation (5) into Equation (3) results in the closed-loop system's characteristic equation as follows

$$\begin{aligned}&(1+GC)(1-Q)[B^u(z^{-1})]^{\#}\\&\quad+Q([B^u(z^{-1})]+GC[B^u(z^{-1})]^{\#})=0\end{aligned} \tag{6}$$

which implies the design of $[B^u(z^{-1})]^{\#}$ can affect the stability. In this case we need a coupled design method for $G^{-1}(z^{-1})$, $Q(z^{-1})$ and the baseline controller $C(z^{-1})$, which makes it difficult to design DOB as an add-on algorithm.

As stated above, in traditional DOB, the design effort for $G_n^{-1}(z^{-1})$ and $Q(z^{-1})$ are not trivial for non-minimum phase systems. The design effort would be significantly increased in the MIMO case: when $G(z)$ is a MIMO plant system that has more inputs than the outputs, i.e., $G(z) \in \mathbb{C}^{m\times n}$ where $m < n$, there does not exist an explicit plant inverse such that $G_n^{-1}(z)G(z) = I_{nn} \in \mathbb{R}^{n\times n}$. To reduce the design effort for $G_n^{-1}(z^{-1})$ and $Q(z^{-1})$, in Section III, we systematically formulate the DOB design into an H-infinity optimization problem, which is formulated directly for MIMO case and does not exclude the non-minimum phase system..

III. Reformulation and Generalization of DOB

A. Reformulation

This subsection reformulates the DOB design into an optimization problem. We assume that $T_f(z)$ has the minimum state-space realization of (A_c, B_c, C_c, D_c), where A_c, B_c, C_c and D_c are algebra matrices with compatible dimension. Since $T_f(z)$ is the transfer function from the disturbance to its estimation error, it is natural to quantify the DOB performance via the norm of $T_f(z)$. Therefore, we formulate the DOB design into the following optimization problem

$$\begin{aligned}\min_{G_n^{-1}(z),\ Q(z)} \quad & \|T_f(z)\|_\infty\\ \text{s.t.} \quad & Q(z)G_n^{-1}(z) \text{ is causal}\\ & \lambda_i(A_c)\le 1\ \forall i\end{aligned} \tag{7}$$

where $\lambda_i(A_c)$ is the i^{th} eigenvalue of A_c.

In this paper, we particularly study the closed-loop transfer function from the disturbance to its estimate error, i.e., $T_f(z)$. It is worth mentioning that, studying the closed-loop transfer function from the disturbance to the error is an alternative solution and design our method is applicable as well. One reason that we consider the closed-loop transfer function from disturbance to disturbance estimation error instead of to the system error is that, the purpose of this method is not only to suppress the disturbance, but also to estimate the disturbance. Such estimate can be used for further studies of the properties of the disturbance.

It is challenging to solve the non-convex optimization problem (7). To simplify this problem, here we introduce a new variable $D(z)$ as follows,

$$\begin{aligned}D(z) &= [D_1(z)\quad D_2(z)]\\ &= [-Q(z)\quad Q(z)G_n^{-1}(z)]\end{aligned} \tag{8}$$

The variable transformation leads to a new expression of $T_f(z)$ with respect to $D(z)$, as described in the following remark.

With the definition of $T_f(z)$ in Equation (1) and the definition of $D(z)$ in Equation (8), T_f can be rewritten as

$$\begin{aligned}T_f(z) &= F_l\left(M(z), D(z)\right)\\ &= M_{11}(z)\\ &\quad + M_{12}(z)D(z)\left(I-\begin{bmatrix}M_{22}(z)\\M_{32}(z)\end{bmatrix}D(z)\right)^{-1}\begin{bmatrix}M_{21}(z)\\M_{31}(z)\end{bmatrix}\end{aligned} \tag{9}$$

where F_l stands for the linear fractional transformation (LFT), and

$$\begin{aligned}M_{11} &= I,\ \ M_{12}=-I\\ M_{21} &= -C(z)\left[I+G(z)C(z)\right]^{-1}G(z)\\ M_{22} &= I-C(z)\left[I+G(z)C(z)\right]^{-1}G(z)\\ M_{31} &= \left[I+G(z)C(z)\right]^{-1}G(z)\\ M_{32} &= \left[I+G(z)C(z)\right]^{-1}G(z)\end{aligned} \tag{10}$$

and $M(z)$ is the transfer function from $[d,\ \hat{d}]^T$ to $[d-\hat{d},\ u_e,\ y]^T$. Then the optimization problem (7) is reformulated as

$$\begin{aligned}\min_{D(z),\ \text{causal}} \quad & \|F_l(M(z),D(z))\|_\infty\\ \text{s.t.} \quad & \lambda_i(A_c)\le 1\ \forall i\\ & D_2(z)=-D_1(z)G_n^{-1}(z)\end{aligned} \tag{11}$$

in which the constraint $D_2(z){=}{-}D_1(z)G_n^{-1}(z)$ is required by the conventional structure of the DOB.

B. Generalization

This subsection proposes a generalized DOB with a systematic design framework based on H-infinity synthesis. This generalized DOB aims to handle the challenging design problems of the plant inverse for non-square MIMO or non-minimum phase systems in a systematic design framework, which returns an optimal disturbance observer with guaranteed stability and causality. Specifically, we remove the explicit structure constraint of conventional DOB, and alternatively treat the DOB as a black box from u_e and y to $\hat{d}$. With this idea, the generalized DOB structure is illustrated in Figure 2.

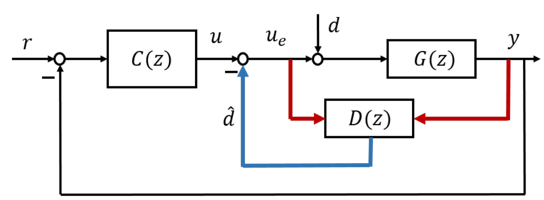

Fig. 2. Generalized DOB in a standard feedback system

By removing the structure constraint, the DOB design problem becomes as follows in which $D(z)$ is the design variable:

$$\begin{aligned}\min_{D(z),\ \text{causal}} \quad & \gamma\\ \text{s.t.} \quad & \lambda_i(A_c)\le 1\ \forall i\\ & \|T_f(z)\|_\infty\le\gamma\end{aligned} \tag{12}$$

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This optimization problem will return the best DOB $D(z)$ in terms of the smallest $\|T_f(z)\|_\infty$, i.e., the minimum amplification for the disturbance. It is easy to reformulate (12) into a convex optimization problem; details can be found in an analogous way in [25] and thus be omitted in this paper.

The requirement that $D(z)$ is causal in the optimization problem (12) guarantees the causality and thus the realizability of the designed DOB. The constraint that A_c is Schur in (12) guarantees the stability of the closed-loop system. The resulting $D(z)$ is a MIMO system, of which the input consists of the control signal and the measurement, and the output consists of the disturbance estimate. Via designing $D(z)$ in this way, the causality of $D(z)$ and the stability of the closed-loop system are guaranteed.

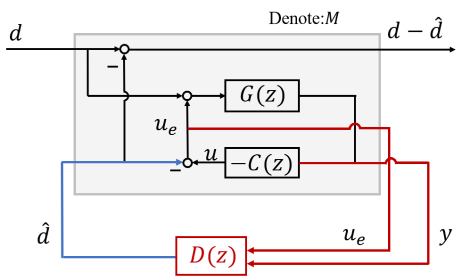

Fig. 3. A general system with the generalized DOB.

It is noted that the conventional DOB shown in Figure 1 is a special case of the proposed DOB: when $D(z)$ is designed to satisfy (8), the proposed DOB is equivalent to the one shown in Figure 1. The conventional DOB essentially adds the constraint (i.e., $D_2(z){=}{-}D_1(z)G_n^{-1}(z)$) to the optimization problem in Equation (7). Removing the constraint can benefit the DOB design in the following two aspects: (1) the newly formulated DOB design optimization has a larger feasible set and consequently a smaller γ; (2) the newly formulated optimization problem is a standard H-infinity optimal control design problem which can be designed easily.

It is noted that, while we use a SISO system when we introduce the traditional DOB structure in Section II, the DOB design method presented in this section is directly derived based on a general MIMO system. The reason for this is that (1) the traditional DOB introduced for a SISO system in Section II is only a brief introduction and is to motivate our generalized DOB that does not require an explicit plant inverse; and (2) the MIMO case would be complex if we design the DOB using the traditional way. In addition, the generalized DOB presented in this section does not exclude the non-minimum phase system. In brief, The generalized DOB formulation removes the structure constraints and allows the formulation of an optimization problem in a compact and easy way, which significantly simplify the design process that otherwise would involve lots of efforts for MIMO and/or non-minimum phase systems.

IV. Learning-based Disturbance Observer

This section extends the generalized DOB into a learning-based scheme. Since there does not exist an explicit plant inverse for the non-square MIMO or non-minimum phase systems, the DOB's performance would be severely degraded. The proposed learning-based DOB is motivated by the iterative learning control (ILC) and is promising to further enhance the estimation of the disturbance with repetitive components. The idea is to iteratively refine the disturbance estimate based on the historical data of the system.

It is worth mentioning that, even if the disturbance is purely repetitive, neither pure DOB or ILC can handle the disturbance suppression perfectly if the plant inverse does not exist or is unstable. Because of the plant inverse issue, DOB without the ILC component cannot fully recover the disturbance. ILC without the DOB can only start to suppress the disturbance after the first iteration, and the performance of the first iteration may be affected severely by the disturbance. In this paper, the iterative learning component is added to the disturbance estimate, aiming to further enhance the DOB's performance when accurate plant inverse is not available and the main components in the disturbance are repetitive.

The scheme of the learning-based DOB is proposed in Figure 4, in which the output of the DOB is denoted as $\hat{d}^o$. A recursive disturbance estimate modification $\hat{d}^f$ is generated by the learning component and added to $\hat{d}^o$ to enhance the disturbance estimate over iterations. Note that the $\hat{d}^f$ of current iteration is generated based on the one and the tracking error from previous iteration, i.e.,

$$\hat{d}^f_{j+1} = \hat{d}^f_j + L(z)e_j \tag{13}$$

in which j is the index iteration, $e_j{=}r_j{-}y_j$, and $L(z)$ is the to-be-designed learning filter. Define $H_r(z)$, $H_f(z)$ and $H_d(z)$ respectively as the closed-loop transfer functions, including the disturbance observer D, from r to y, d_f to y and d to y, i.e.,

$$y = H_r(z)r + H_f(z)\hat{d}^f + H_d(z)d \tag{14}$$

That is, the j^{th} iteration's closed-loop dynamics can be neatly written as

$$y_j = H_r(z)r_j + H_f(z)\hat{d}^f_j + H_d(z)d_j \tag{15}$$

In the following, we will analyze the dynamics between e_j and e_{j+1}. If the norm of e_{j+1} is less than the norm of e_j, we can conclude that the learning-based DOB (as illustrated in Figure 4) with the learning law (13) will further enhance the disturbance compensation. The step-by-step analysis is given as follows.

Plugging Equation (13) into the closed-loop system dynamics (14) in the $(j{+}1)^{th}$ iteration, we have

$$\begin{aligned} y_{j+1} &= H_r(z)r_{j+1} + H_f(z)\hat{d}^f_{j+1} + H_d(z)d_{j+1} \\ &= H_r(z)r_{j+1} + H_f(z)[\hat{d}^f_j + L(z)(r_j - y_j)] \\ &\quad + H_d(z)d_{j+1} \end{aligned} \tag{16}$$

Considering (15), we have

$$H_f(z)\hat{d}^f_j = -H_r(z)r_j - H_d(z)d_j + y_j \tag{17}$$

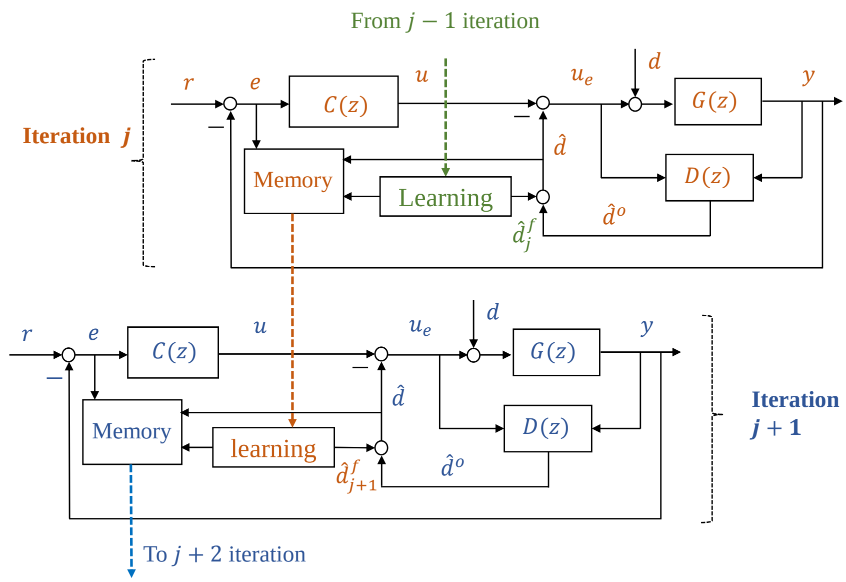

Fig. 4. Scheme for the proposed learning-based DOB

which further implies

$$\begin{aligned} y_{j+1} &= H_r(z)r_{j+1} + H_f(z)\hat{d}_j^f + H_f(z)L(r_j - y_j) \\ &\quad + H_d(z)d_{j+1} \\ &= (1 - H_f(z)L(z))y_j + H_r(z)(r_{j+1} - r_j) \\ &\quad + H_d(z)(d_{j+1} - d_j) + H_f(z)L(z)r_j \end{aligned} \tag{18}$$

Therefore,

$$\begin{aligned} & r_{j+1} - y_{j+1} \\ &= r_{j+1} + r_j - (I - H_f(z)L(z))y_j - H_r(z)(r_{j+1} - r_j) \\ &\quad - H_d(z)(d_{j+1} - d_j) - H_f(z)L(z)r_j - r_j \\ &= (I - H_f(z)L(z))(r_j - y_j) \\ &\quad + (I - H_r(z))(r_{j+1} - r_j) - H_d(z)(d_{j+1} - d_j) \end{aligned} \tag{19}$$

Assuming that r and d are consistent over iterations, i.e., $r_{j+1}{=}r_j$ and $d_{j+1}{=}d_j$, we have

$$r_{j+1} - y_{j+1} = (I - H_f(z)L(z))(r_j - y_j) \tag{20}$$

i.e.,

$$e_{j+1} = (I - H_f(z)L(z))e_j \tag{21}$$

The stability of the closed-loop system with the proposed learning-based DOB and the learning convergence are guaranteed. The explanation is provided as follows. Since the learning signal that is added to the DOB is in the feedforward loop, as shown in Fig. 4, the closed-loop system's stability is not affected and has been guaranteed by $D(z)$ designed in Section III. The learning convergence (i.e., the estimation error is reduced over iterations) can be achieved by designing a learning filter $L(z)$ such that $\|I{-}H_f(z)L(z)\|_\infty$ is less than 1. Thus, the tracking error e will be reduced over iterations and the disturbance estimate will be iteratively refined.

To enhance the robustness to modeling uncertainties at high frequencies, the learning signal $\hat{d}^f$ is usually processed by a low-pass filter first before being injected into the next iteration, i.e.,

$$\hat{d}_{j+1}^f = W(z)[\hat{d}_j^f + L(z)e_j] \tag{22}$$

where $W(z)$ is a frequency-dependent weighting filter to provide robustness to modeling uncertainties and additional flexibilities to the design of $L(z)$. As such, there exist a residual term in steady state and the tracking error will reduce into a bound. Here we consider the reference as zero ($r = 0$) and the weighting filter matrix as $W(z) = w(z)I$ where $w(z)$ is a scaler filter and I is the identify matrix with compatible dimension,

$$\begin{aligned} e_{j+1} &= W(z)(I - H_f(z)L(z))e_j \\ &\quad + H_d(z)(d - W(z)d) \end{aligned} \tag{23}$$

Denoting $e_0{=}H_d(z)d$ as the tracking error when implementing the DOB without learning. In the steady state in iteration domain, $e_\infty{=}e_{j+1}{=}e_j$ yields

$$\begin{aligned} e_\infty &= [I - W(z)(I - H_f(z)L(z))]^{-1}[I - W(z)]e_0 \\ &\triangleq R(z)e_0 \end{aligned} \tag{24}$$

As such, compared with the DOB without learning (proposed in Section III), the disturbance attenuation by the learning-based DOB (as illustrated in Figure 4 with the updating law 22) can be quantified by the norm of $R(z)$.

Since convergence rate of the learning-based DOB can be quantified by $W(z)[I{-}H_f(z)L(z)]$ [26], [27], learning filter $L(z)$ can be designed through solving the following optimization problem,

$$\min_{L(z)} \|W(z)(I - H_f(z)L(z))\|_\infty \tag{25}$$

Here we briefly explain that the optimization problem (25) can be into an H_∞ optimal control design problem. Figure 5 illustrates an equivalent dynamics from e_j to e_{j+1}, in which

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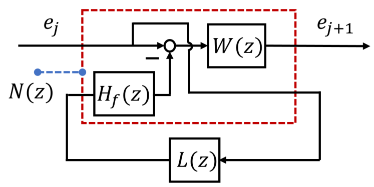

Fig. 5. Optimal learning filter design scheme

the learning filter $L(z)$ is in the feedback loop. Therefore, we have

$$F_l(N(z), L(z)) = W(z)(I - H_f(z)L(z)) \tag{26}$$

where

$$N(z) = \begin{bmatrix} W(z) & -W(z)H_f(z) \\ I & 0 \end{bmatrix} \tag{27}$$

As such, the learning filter design can be formulated into an optimal feedback control design which can be transformed to a convex optimization problem and solved readily [25].

Remarks: The learning-based DOB presented in this section is to further improve disturbance estimation performance when (1) the DOB cannot work perfectly well (e.g., the plant is a non-minimum phase system whose accurate plant inverse does not exist), and (2) the disturbance is approximately periodical. It is also wroth noting that, the learning filter design using H-infinity theory does not explicitly consider quantified modeling uncertainties. It works well in general when the modeling uncertainties are within a reasonable range, as demonstrated in the numerical studies in Section V, Part A. When the modeling uncertainties are relatively large, μ synthesis instead of H-infinity synthesis in robust control theory can be utilized to explicitly include modeling uncertainties in the learning filter design.

V. Study Cases

We present two study cases in this paper: (1) a non-square MIMO system and (2) a non-minimum phase system. The first example originally comes from a dual-stage hard disk drive (HDD) with dual inputs and single output. The second example comes from a tail-sitter vertical take-off and landing unmanned aerial vehicle (UAV) whose attitude dynamics is a non-minimum phase system.

A. Non-square MIMO System

This subsection applies the DOB design for the non-square dual-input-single-output plant, whose inverse does not exist. The plant comes from a dual-stage HDD, as shown in Figure 6 [28], in which $P_v(z)$ and $P_m(z)$ denote the two sub-plants (i.e., VCM and PZT) with two baseline controllers $C_v(z)$ and $C_m(z)$ respectively. The $\hat{P}_m(z)$ denotes the nominal plant of $P_m(z)$. The parameters are provided in Table 1.

To apply the proposed design framework, the control scheme in Figure 6 is transformed to Figure 7 by separating D from the known dynamics, in which

$$C(z) = \begin{bmatrix} (C_m(z)\hat{P}_m(z) + 1)C_v(z) \\ C_m(z) \end{bmatrix} \tag{28}$$
$$P(z) = \begin{bmatrix} P_v(z) & P_m(z) \end{bmatrix}$$

TABLE I
Parameters in Plant and Controller

System	Parameters
$P_v(z)$	$\frac{0.001335z^3+0.0233z^2+0.01665z-0.03664}{z^4-3.201z^3+3.664z^2-1.724z+0.2611}$
$P_m(z)$	$\frac{0.5332z^2+0.07978z+0.4815}{z^3-0.1508z^2+0.8646z-0.2729}$
$C_v(z)$	$\frac{1.62z^3-4.78z^2+4.7z-1.541}{z^3-2.275z^2+1.556z-0.2818}$
$C_m(z)$	$\frac{1.227z-0.09939}{z-0.9158}$

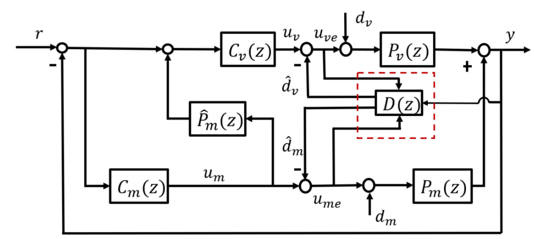

Fig. 6. The control scheme of a dual-stage HDD with the proposed DOB. The signals are defined as follows: reference r, output y, disturbances d_v and d_m, control signals u_v and u_m, and the position error signal (PES) e=r−y.

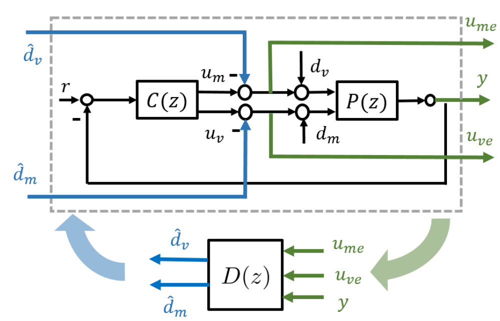

Fig. 7. H_∞-based DOB design scheme

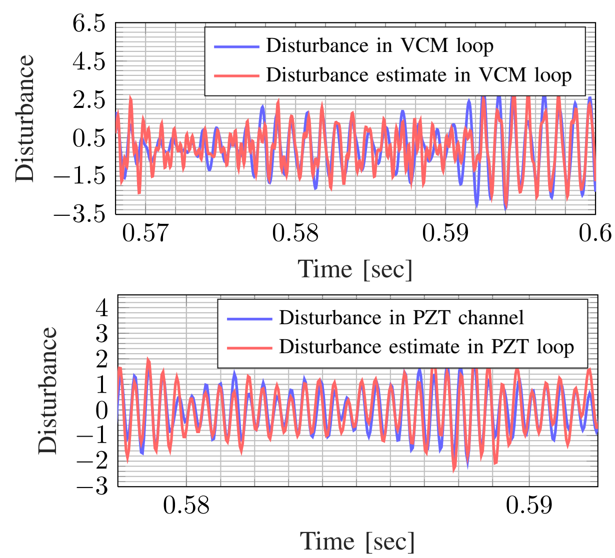

Fig. 8. Disturbances and the estimates (upper: VCM loop; Lower: PZT loop.)

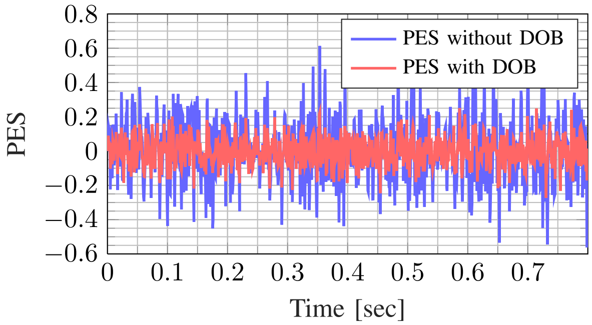

Fig. 9. Position error signals (PESs) with and without DOB

Figures 8 and 9 respectively illustrate the disturbance estimation in both two input channels and the position error signal (PES) reduction. Particularly, we add the following metrics (Table II) to compare the performance with and without DOB quantitatively. Though there is no explicit plant inverse for such a non-square system, the disturbances have been mostly recovered (Figure 8) and the PES has been significantly reduced (Figure 9).

TABLE II
COMPARISON OF PES WITH AND WITHOUT DOB

	2-norm of PES	Maximum magnitude of PES
Without DOB	5.897	0.6153
With DOB	2.675	0.2732

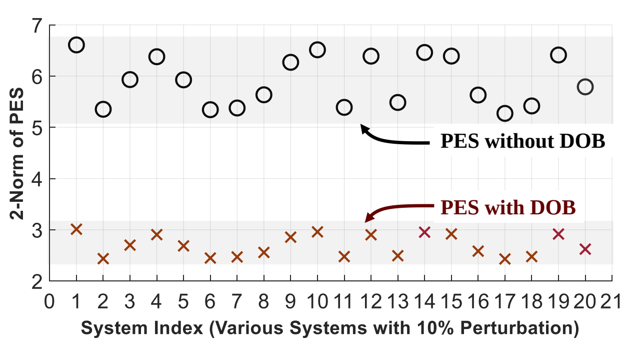

Fig. 10. Comparison of the 2-norm of the PES with and without DOB across various systems randomly sampled from the nominal system with 10% perturbation.

Additional numerical studies have been conducted to evaluate the robustness of the proposed DOB, in which we purposely perturbed the systems P_m and P_v with 10% uncertainties in the following way:

$$\begin{aligned} P_{m,\text{purturbed}} &= P_m(1+0.1\Delta_m) \\ P_{v,\text{purturbed}} &= P_v(1+0.1\Delta_m) \end{aligned} \tag{29}$$

where Δ_m and Δ_v are positive-real uncertain linear dynamic systems with the bound of 1. We used 20 random samples from the perturbed systems, $P_{m,\text{purturbed}}$ and $P_{v,\text{purturbed}}$, and run the simulations on these systems with and without DOB respectively. We compared the 2-norms of the PES from these systems and the results are provided in Fig. 10. It shows that, the PES without DOB lies in the range of 5∼7 and the PES with DOB lies in the range of 2∼3.

B. Non-minimum Phase System

In this subsection, the proposed DOB is implemented on the attitude control of a tail-sitter vertical take-off and landing (VTOL) UAV [29]. This UAV inherits both the maneuverability of the rotary-wing UAV and the efficiency of the fix-wing UAV. However, it has the drawback of being very sensitive to cross-wind during hovering flight [30] in which the DOB has the potential to enhance the robustness to external disturbance. Its attitude dynamics is a non-minimum phase system. As discussed in [6], the attitude dynamics can be represented by the following transfer function,

$$G(z) = \frac{0.14131\,(z+1.716)}{(z-1)\,(z-0.9447)} \tag{30}$$

where $z=-1.716$ is an unstable zero. The baseline feedback PID controller has been designed as

$$C(z) = \frac{0.9002\,(z-0.8343)\,(z-0.9987)}{z\,(z-1)} \tag{31}$$

It is noted that in the UAV experimental test, the disturbance was purposely added to validate the proposed learning-based algorithm for a non-minimum phase system whose plant inverse is not stable. Therefore, the purposely added disturbance mainly includes repetitive components and random noise. In addition, this experimental study based on the non-minimum phase system is to validate (1) the effectiveness of the proposed disturbance observer for non-minimum phase system and (2) the learning-based DOB can be used to further improve the performance when the disturbance is close to periodical. The UAV is only an experimental test bed to test this algorithm. Therefore in the experimental test, the UAV is not flying and is given near-repetitive disturbances purposely.

We design the DOB based on the proposed framework and apply it to the attitude control of the VTOL UAV. The bode plots of the designed DOB are provided in Figure 11 and the experimental test results are provided in Figures 13-15.

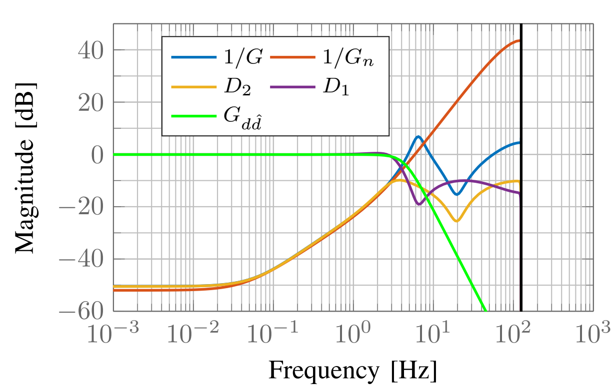

Fig. 11. Bode plots of the designed DOB

Figure 11 shows that $D_1(z)$ is approximately a low-pass filter with the DC gain of 1, which plays a similar role of the Q

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filter to the one in conventional DOB. $D_2(z)$ is approximately the plant inverse. Figure 11 also provides the bode plot of the system from d to $\hat{d}$ (i.e., $G_{d\hat{d}}$), which is close to 1 at low frequencies.

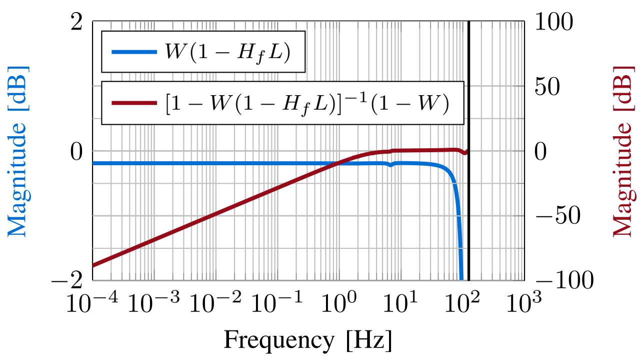

Fig. 12. Bode plots for the designed learning-based DOB

We further leverage the proposed DOB into the learning scheme (as illustrated in Figure 4) with the learning algorithm in (13). With the following designed weighting filter,

$$W(z) = \frac{0.53z^2 + 1.06z + 0.53}{z^2 - 0.8252z + 0.2946} \tag{32}$$

we obtain the learning filter by solving the optimization problem in (25). Figure 12 provides the bode plots of the resulting $W(z)(1 - H_f(z)L(z))$. It shows that $W(z)(1 - H_f(z)L(z))$ is below $0dB$, which grantees the stability and convergence of the learning-based disturbance observer.

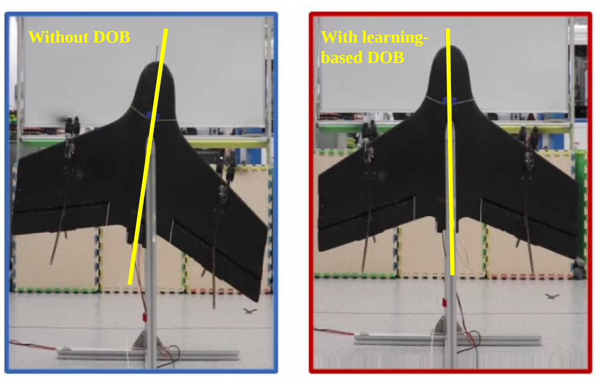

Fig. 13. Experimental setup and attitude control comparison. This figure captures the largest attitude error with (right) and without (left) the proposed learning-based DOB.

With the proposed DOB (related filters are provided in Figure 11) and the learning scheme (related filters are provided in Figure 12), we test the DOB with and without learning on a VTOL UAV platform. In the experiment, we intentionally add some disturbance signal to emulate a periodic disturbance. The added disturbance is band-limited noise within $1Hz$. To verify the effectiveness of the proposed learning-based DOB method, we test three cases: (1) the DOB is off (i.e., only the baseline controller is running); (2) the DOB is turned on without learning; and (3) the full DOB with learning is turned on.

It is noted that the experimental UAV testbed is built to test the proposed learning-based DOB for a SISO non-minimum phase system whose plant inverse does not exist (i.e., is not stable) and the approximate plant inverse is not trivial to get. In the experimental test, the UAV is not actually flying. Instead it is mounted to a fixed frame. We purposely add near-repetitive disturbances (i.e., periodical disturbances plus random noises) in the experiments to test the learning capability of the proposed algorithm.

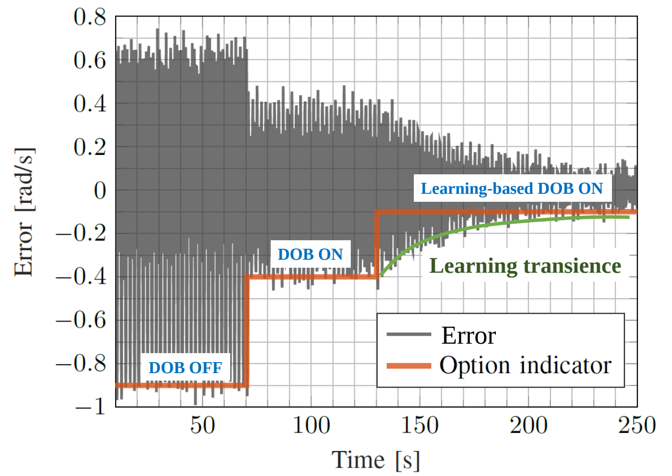

Fig. 14. Error reduction by turning on DOB and learning-based DOB

Figure 13 shows the experimental setup and video captures. We captured the largest attitude error, which shows that with the proposed learning-based DOB, the attitude error is considerably reduced. Figure 14 shows the attitude error in the three tests. It implies that the attitude error is reduced when the DOB is turned on, and is further reduced after the learning scheme is turned on. The decay of the attitude error with respect to the learning iterations is provided in Figure 15.

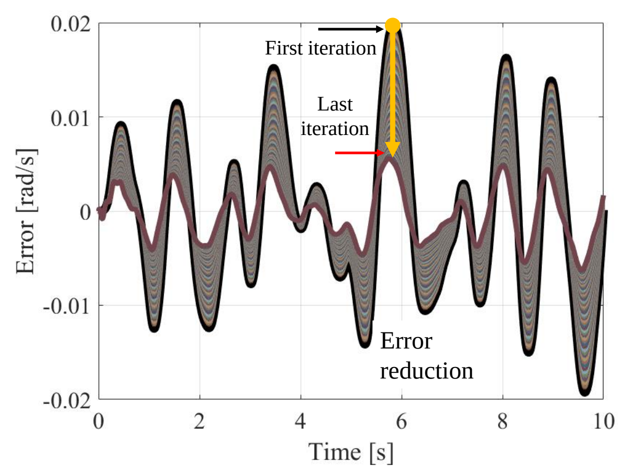

Fig. 15. Error reduction via learning-based DOB

From Figure 14, it is noted the learn transience starts at 130 seconds and approximately ends at 230 seconds, which implies that after 10th iteration the system goes into the steady state process. In Figure 15, we compared the attitude error of the first iteration and the tenth iteration. For the given validation example, 10 iterations are needed to enhance the DOB's performance. For a general system, the number of required iterations depends on several factors including the model accuracy, the requirement of tracking accuracy, and the baseline DOB.

VI. Conclusions

This paper has proposed a generalized design method for the disturbance observer (DOB) and augmented it into a learning scheme. This method is applicable to various types of systems including non-minimum phase and non-square multiple-input-multiple-output systems, for which the DOB design is traditionally challenging. The proposed DOB method has removed the internal structure constraint in the conventional DOB. The design problem has been formulated into an H-infinity optimal control problem to minimize the norm of the dynamics from the disturbance to its estimate. The proposed learning scheme for the DOB has further enhanced the estimation of the disturbance with repetitive components. Both numerical and experimental studies have validated the proposed DOB design framework and the effectiveness of the proposed DOBs with and without learning.

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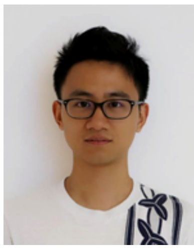

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