Taylor & Francis
Taylor & Francis Group

Vehicle System Dynamics
International Journal of Vehicle Mechanics and Mobility

ISSN: 0042-3114 (Print) 1744-5159 (Online) Journal homepage: www.tandfonline.com/journals/nvsd20

State of the art survey: active and semi-active suspension control

H. Eric Tseng & Davor Hrovat

To cite this article: H. Eric Tseng & Davor Hrovat (2015) State of the art survey: active and semi-active suspension control, Vehicle System Dynamics, 53:7, 1034-1062, DOI: 10.1080/00423114.2015.1037313

To link to this article: https://doi.org/10.1080/00423114.2015.1037313

Published online: 14 May 2015.

Vehicle System Dynamics, 2015
Vol. 53, No. 7, 1034–1062, http://dx.doi.org/10.1080/00423114.2015.1037313

Taylor & Francis
Taylor & Francis Group

State of the art survey: active and semi-active suspension control

H. Eric Tseng* and Davor Hrovat

Ford Motor Company, 2101 Village Rd, Dearborn, MI 48121, USA

(*Received 18 February 2015; accepted 31 March 2015*)

This survey paper aims to provide some insight into the design of suspension control system within the context of existing literature and share observations on current hardware implementation of active and semi-active suspension systems. It reviews the performance envelop of active, semi-active, and passive suspensions with a focus on linear quadratic-based optimisation including a specific example. The paper further discusses various design aspects including other design techniques, the decoupling of load and road disturbances, the decoupling of pitch and heave modes, the use of an inerter as an additional design element, and the application of preview. Various production and near production suspension systems were examined and described according to the features they offer, including self-levelling, variable damping, variable geometry, and anti-roll damping and stiffness. The lessons learned from these analytical insights and related hardware implementations are valuable and can be applied towards future active or semi-active suspension design.

Keywords: active suspension; optimal control; semi-active system; suspension control

1. Introduction

There are numerous important requirements that a typical automotive suspension must satisfy [1,2] including

- maintaining proper vehicle posture when subject to various inertial and external forces and moments caused by braking, turning, wind gust, and other events;
- providing ride comfort in view of the road roughness inputs which act as a major disturbance to the vehicle;
- securing good road handling and overall vehicle agility;
- avoiding excessive suspension stroke or related hard stop/impact.

Additional opportunities exist with the more advanced active and semi-active suspensions regarding various safety functions [3] and other new exciting functionalities.[4] Although most current suspensions are of the conventional passive type, this survey will focus on active suspensions where additional power sources – such as pumps and compressors – are needed to accomplish the desired functionality (for a more precise definition of passive and active

*Corresponding author. Email: htseng@ford.com

suspensions in terms of the well-established mathematical notion of passive operators, see [5–7] and references therein).

While advanced active suspensions offer opportunities for substantial improvements in ride and handling as well as overall vehicle posture, stability, and added functionality, they also typically imply higher energy consumption, cost, complexity, packaging, and additional operational requirements. The latter include smooth start-up and shutdown, robust operations and diagnostics, fault containment and management, as well as the ability to effectively adapt to different operating conditions including reconfiguration in case of sensor/actuator/mechatronic system (partial) failure.

As a compromise between passive and active suspension performance, complexity, and installation and operational costs, one sees more and more semi-active suspensions, which are marketed under names such as continuously controlled damping, MagneRide, and others. Semi-active suspensions typically include an adjustable damper or shock absorber [8] where the damping parameter is modulated through a relatively small computer-controlled actuator. Since it does not require much energy, the semi-active damper can be considered an essentially passive device.

Complete development of an advanced suspension must consider all of the above requirements. This requires consideration of practically all vehicle degrees of freedom (DOF) within the bandwidth of main interest, that is, in the case of vehicle main body mass or sprung mass this would include all 6 DOF (vertical, longitudinal, lateral, pitch, roll, and yaw). However, the main emphasis of the present survey is on ride quality, which is a traditional main attribute of interest when addressing advanced passenger vehicle suspensions. This would typically lead to designs using a reduced vehicle sprung mass model with motion limited to only vertical (or heave), pitch, and roll components.

Moreover, within the above somewhat limited context, the main focus will be on securing superior ride comfort in view of the always present road-induced disturbances caused by road roughness and irregularities such as potholes and bumps. Thus, we will for the most part separate the load-induced disturbance discussion – such as pitch and roll changes due to (sudden) braking, acceleration, turning, and similar external actions – from the ride performance optimisation analysis. This, of course, does not mean that we are ignoring their importance, but rather that the engineering task at hand has been 'modularised' or divided into more manageable and insightful subtasks. In the case of road-induced disturbances, lots of insight can be gained based on simple models which can later be integrated with other subsystems and subtasks including the aforementioned load-induced dynamic effects.

2. Optimal suspension control with H2/linear quadratic synthesis/design

As stated in the introduction and as established from many suspension studies,[7,9–11] the most significant and insightful conclusions for vehicle suspensions can be observed from a simple quarter car model with an undamped tyre and a road disturbance described by Gaussian white noise ground velocities or, equivalently,[7] a step in road displacement (or an impulse in road velocity). Therefore, in this paper, we will focus our discussion on control analysis and comparison using the quarter-car model.

The performance indices relating ride comfort and handling are often measured by the root-mean-square (RMS) values of sprung mass acceleration and tyre deflection, respectively.[7] Additionally, the RMS value of suspension displacement can be utilised to enforce the rattle space (or available suspension displacement) constraint. As a result, the optimisation of the suspension controller using the quarter-car model often lends itself to a well-defined H2 or linear quadratic (LQ) optimisation problem.

2.1. *Performance index*

The ground vehicle suspension design is influenced by a number of often conflicting factors and attributes. These factors will be discussed and then combined into an overall performance index for the H2 optimisation problem.

2.1.1. *Performance index of ride comfort*

One of the most popular and fitting ride measures is based on the RMS value of vertical acceleration of the sprung mass, typically measured or projected at the driver's or passengers' seat locations. In a field study in 1978,[12] the authors involved 78 passengers in two different vehicles and 18 different road sections to conclude that 'excellent correlation was found to exist between the subjective ride ratings and simple RMS acceleration measurement at either the vehicle floorboard or the passenger/seat interface'.

Further refinements of the RMS ride measure are possible through adding the RMS value of the sprung mass jerk (the derivative of acceleration) to the RMS value of sprung mass acceleration. Some literature [13–15] advocates the inclusion of jerk as an added measure that amplifies the contribution from high-frequency disturbances, which are important for NVH (noise, vibration, and harshness).

To account for the frequency dependency of human sensitivity to vibrations and the length of time of human exposure, a standard has been developed by the International Organization for Standardization as ISO 2631.[16,17] It noted that the region of greatest human sensitivity to vertical vibration lies between 4 and 8 Hz, which roughly includes the various resonances of human internal organs. The ISO-based metric can be easily incorporated into the H2 optimisation formulation.

National Aeronautics and Space Administration (NASA) developed a more comprehensive measure that takes into account interdependency of various modes/direction of vibrations.[18] A similar measure has been adapted and extended to automotive applications,[19] considering the interdependence between heave, pitch, roll, and other factors.

The ride metric of choice will depend on the context of its usage. While the more complex ones can reflect more details and nuances, the simple one can better focus on the system level and major benefits. This paper will utilise the simple RMS of sprung mass acceleration.

2.1.2. *Performance index for suspension displacement constraint*

In practice, the available suspension displacement or so-called rattle space is limited. One can include this constraint by adding the suspension displacement in the performance index. With the attempt to minimise the corresponding cost, the H2 optimisation problem effectively soft constrains the suspension displacement. Similar to the ride comfort metric, the suspension displacement-related cost can be reflected on the RMS value of suspension displacement, especially for random/stochastic road input.

2.1.3. *Performance index for handling/road holding*

As there exists a convex relationship between tyre cornering forces and normal forces (reflected by tyre deflection), the variation of tyre deflection leads to the variation of cornering forces. It is conceivable that a very large variation of tyre deflection may even lead to a loss of contact with the ground. Therefore, the RMS value of tyre deflection variation can be an explicit metric for handling characteristics. Note the direct relationship between tyre deflection and cornering capabilities have been experimentally studied.[7]

Based on the aforementioned discussion, it is no surprise that the most widely used performance index for active suspension study is the combination of the three previously discussed RMS values. That is, using a weighted combination of RMS acceleration of sprung mass, RMS suspension displacement, and RMS tyre deflection variation to represent the performance index for optimisation. This performance index lends itself to an H2 norm of the weighted states and input of the quarter-car model. Interestingly, the deterministic and the stochastic interpretations of this H2 norm lend themselves to the road disturbance of a step in road displacement, and that of Gaussian white noise ground velocities, correspondingly. The equivalence of these two interpretations was described in more detail in [20].

2.2. *H2 optimal control for 2 DOF quarter-car model*

The mathematical state space representation of a 2 DOF quarter-car model (Figure 1) can be found in [9]. Here, we choose the states as tyre deflection, unsprung mass velocity, suspension deflection, and sprung mass velocity (Figure 1). With this representation, we have the performance index as

$$\mathrm{PI} = E(r_1 x_1^2 + r_2 x_3^2 + u^2), \tag{1}$$

where x_1 and x_3 represent the tyre deflection and suspension stroke as illustrated in Figure 1(a), u represents the sprung mass acceleration ($u = U/m_s$), and the expectation E represents steady-state mean-square values. Figure 1(b) shows the corresponding limiting case when unsprung mass is negligible, and Figure 1(c) illustrates a passive quarter car.

Figure 2 shows a quarter car equipped with various suspension elements including adjustable spring and damper and different configurations such as springs in parallel to the actuator and in series to the actuator.

The earlier four-state LQ optimisation was solved numerically,[7,9] and the results of the corresponding global study for various weighting in the combination of performance index are shown in Figures 3 and 4. The figures illustrate how the weighting factors serve as tuning knobs, controlling the trade-off between acceleration, suspension stroke (design constraint), and tyre deflection (handling performance constraint). These plots were generated with the ratio of sprung-to-unsprung mass set to 10 and the unsprung natural frequency (or wheel-hop) as 10 Hz. The choice in optimisation with respect to different combinations of

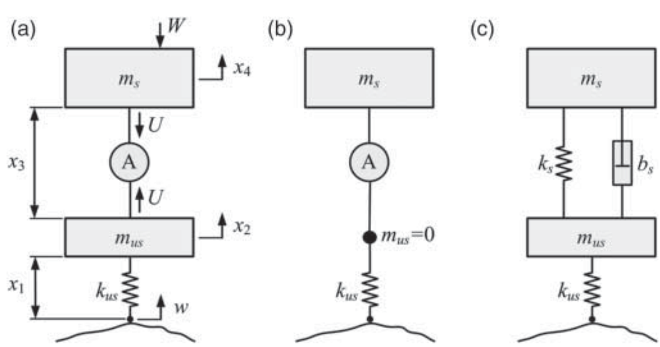

Figure 1. (a) Two DOF quarter-car model, its states x, road velocity w, and actuator input U; (b) one DOF quarter-car model; and (c) two DOF quarter-car model with passive spring and damper.

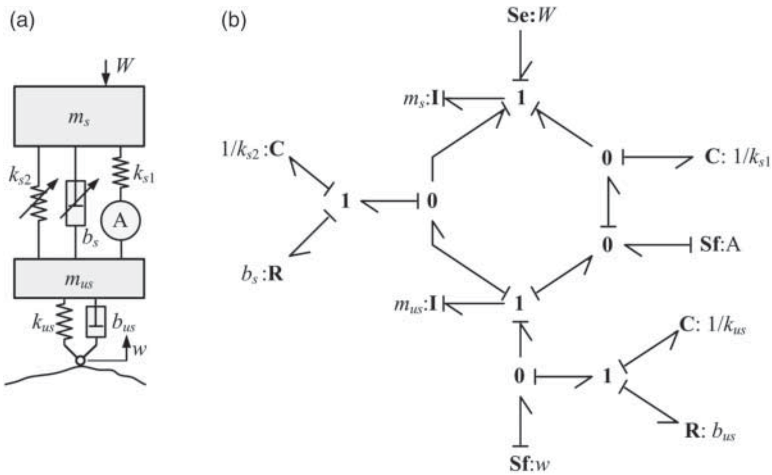

Figure 2. (a) Two DOF quarter-car model equipped with actuator, serial spring, and parallel adjustable spring and damper and (b) the corresponding bond graph model.

weighting in the performance index can be predetermined as the vehicle design suspension characteristic or adapted dynamically during on-road operation to enable the best driving experience.

We see from Figures 3 and 4 that reducing r_1, the weighting factor on tyre deflection, while maintaining r_2, the weighting factor on suspension deflection, results in increased tyre deflection and decreased sprung mass acceleration, that is, better ride at the expense of somewhat worse handling. On the other hand, keeping r_1 constant while reducing r_2 leads to increased suspension deflection and decreased sprung mass acceleration. In the extreme case of assigning r_2 to zero, the problem becomes a degenerative one where extremely large suspension displacement would result. The corresponding optimal force of this degenerated case would not attempt to support the static weight of the vehicle since there is no cost associated with suspension stroke. As a rule of thumb, choosing r_1/r_2 to about 10 leads to reasonable results.[7] The shaded area in Figures 3 and 4 illustrates where most optimal designs for automotive applications will reside due to the above-mentioned practical considerations. An important observation is that no suspension control design, linear or nonlinear, can exhibit behaviour in the area falling below the r_1 near zero line (represented by $r_1 = 10^{-2}$) or the r_2 near zero line (represented by $r_2 = 10^{-3}$).

Similar solutions can be obtained using linear matrix inequality (LMI)-based approaches.[21–23] A desirable feature of the LMI-type controllers is that they can incorporate the slowly time-varying parameters, such as changes in vehicle sprung mass. Related methods here include the linear parameter-varying techniques [22,24] that explicitly include possible slowly varying system parameters, such as road roughness and vehicle speed variations with results similar to gain scheduling investigated in [25].

Various observations about the suspension performance limitation from different approaches as well as limited experimental work [7] have affirmed the performance limits observed in Figures 3 and 4. For example, any improvement in one area (e.g. ride) would be reflected through corresponding loss of performance in other complementary areas (e.g. handling in wheel-hop mode and/or rattle space requirement). Essentially this is a consequence of the fact that a single actuator is required to fulfil two conflicting requirements. This is also reflected through the invariant properties reported in [26].

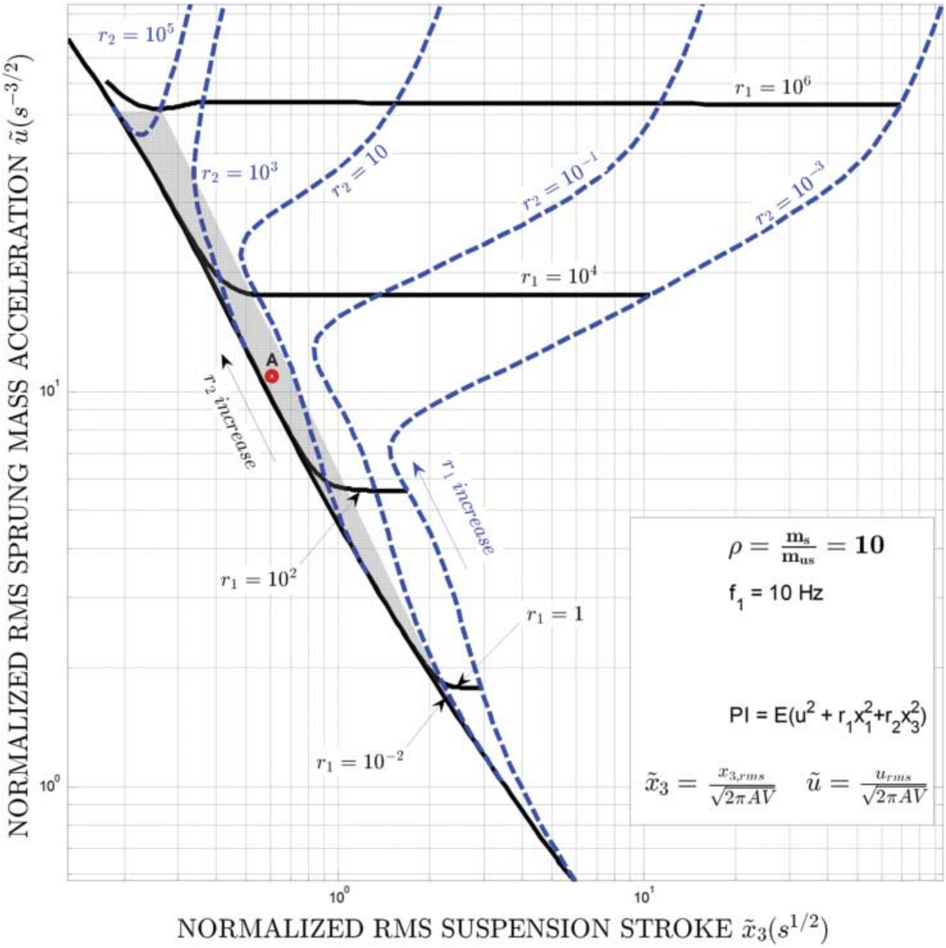

Figure 3. Normalised ride comfort versus suspension stroke (*A* as road roughness coefficient and *V* as vehicle speed).

2.3. *Illustrative example*

To illustrate the performance in frequency response as well as where it resides on the global 'carpet' plot of Figures 3 and 4, a practical example of H_2 optimal design active suspension is described in the following.

Assume that for a particular vehicle $\rho = 10$ and $\omega_{us} = 2\pi 10$ rad/s. The road is described by a road roughness coefficient $A = 1.6 \times 10^{-5}$ ft (4.9×10^{-6} m) which corresponds to a medium quality road. The vehicle is traversing this road at a speed of $V = 80$ ft/s (88.5 km/h). It is reasonable and desirable to design a suspension such that the tyre deflection from equilibrium under this road excitation would be maintained within 1 inch (2.54 cm) 99.7% of the time. This would translate to a requirement on the normalised tyre deflection, $\tilde{x}_1$, to be maintained within 0.3 $s^{1/2}$. Note that we use the accent $\sim$ to denote the normalised RMS value with respect to the road velocity excitation, for example, $\tilde{x}_1 := x_{1,\text{rms}}/\sqrt{2\pi AV}$, where *A* is the road roughness coefficient and *V* is the vehicle travelling speed.[27]

From Figure 4 and a normalised RMS tyre deflection value of 0.3 ($s^{1/2}$) in this illustrative example, we see the minimum possible normalised RMS acceleration $\tilde{u}$ is 10 ($s^{-3/2}$). If we accept the normalised value of 10.9 (which reflects to a 0.03 g sprung mass acceleration RMS,

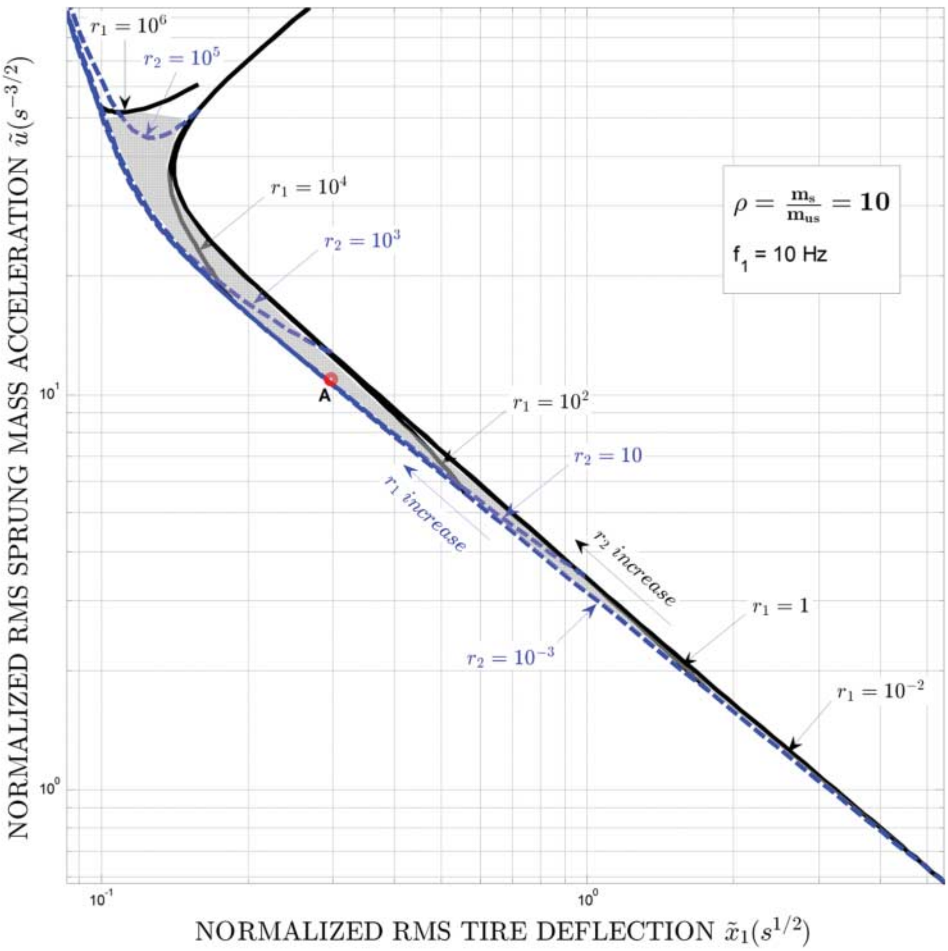

Figure 4. Normalised ride comfort versus handling.

or the sprung mass acceleration under 0.1 g 99.7% of the time), we can arrive at this optimal control law by picking the tuning factors $r_1 = 1100$ and $r_2 = 100$. This is represented as design point A in Figures 3 and 4. Using these values for r_1 and r_2 in Figure 3 reveals a normalised RMS secondary suspension deflection $x_{3,\mathrm{rms}} = 0.605\,\mathrm{s}^{1/2}$ which ensures that the suspension deflection will remain within ± 2 inch (5.08 cm) of the static value 99.7% of the time. With $r_1 = 1000$ and $r_2 = 100$, the optimal control gains for full state feedback are $k_1 = -6.084$, $k_2 = 0.548$, $k_3 = -10.0$, and $k_4 = -4.438$ for tyre deflection x_1, unsprung mass velocity x_2, suspension deflection x_3, and sprung mass velocity x_4, respectively. The closed-loop eigenvalues are $e_{1,2} = -2.20 \pm j2.26$ and $e_{3,4} = -2.75 \pm j62.9$. The first set of oscillatory eigenvalues corresponds to the vehicle heave mode with a natural frequency of 0.5 Hz and damping ratio of ≈ 0.7. The second set corresponds to the wheel-hop mode with a resonant frequency of 10 Hz and relatively small damping ratio of 4.4%. Whether or not this small amount of damping is of consequence will depend on the particular hardware implementation, which may include a combination of active and passive means to improve closed-loop robustness. Also in practice, an excessively soft suspension setting may require load levelling to keep the suspension static or steady-state deflection in the neighbourhood of zero or some nominal value.

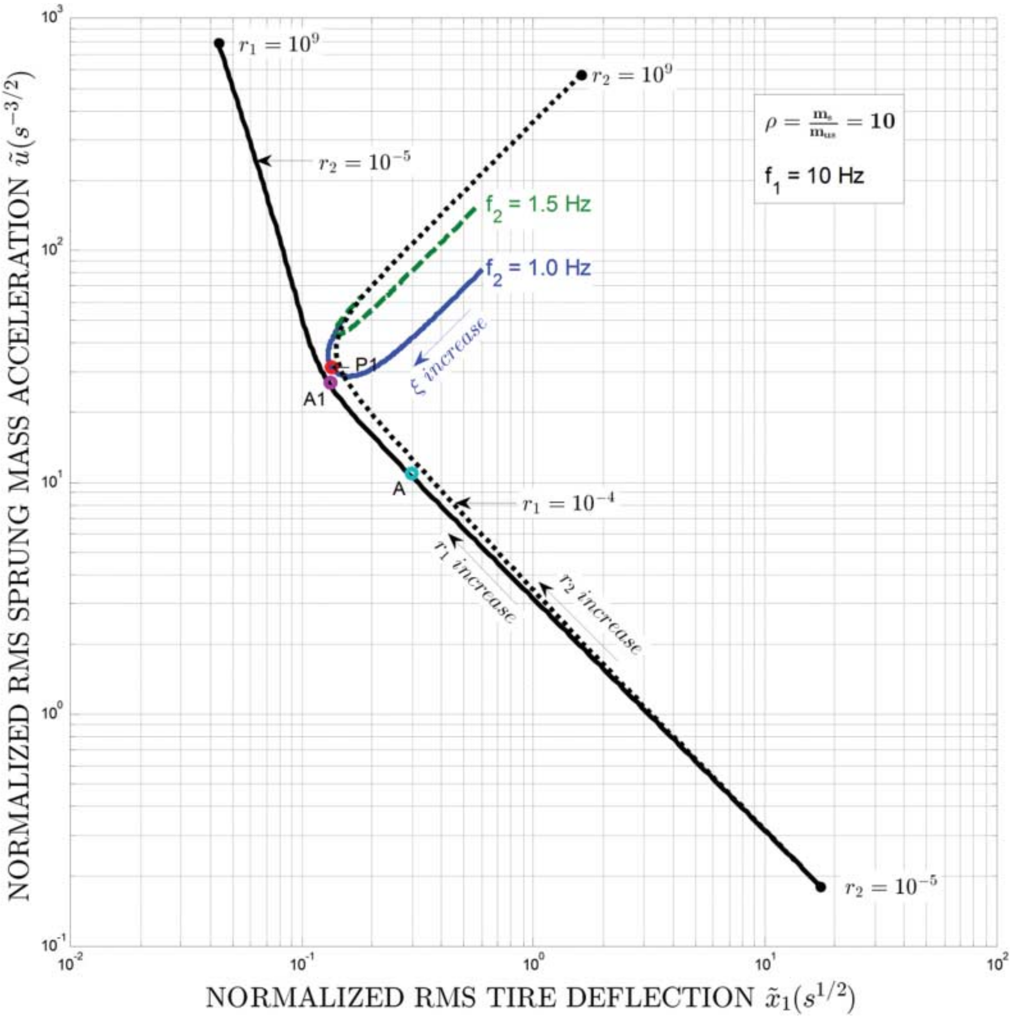

Figure 5. Normalised ride comfort versus handling – passive and active suspensions.

At this stage, it is instructive to compare the above active suspension results with a corresponding passive case with the structure shown in Figure 1(c). For this purpose consider Figure 5 which displays normalised RMS acceleration versus tyre deflection for active and passive suspensions. For simplicity, the optimal active suspension performance is shown via the limiting curves $r_1 \sim 0$ and $r_2 \sim 0$ only, which are representative of the main trends. Here, the design points with lower accelerations and higher tyre deflections are characterised by softer, lower frequency body modes and less damped wheel-hop modes.

The added curves in Figure 5 correspond to the passive performance trade-offs with sprung mass mode resonant frequencies f_2 at 1 or 1.5 Hz (which corresponds to different passive spring stiffness in Figure 1(c)), and damping ratios ς varying between 0.02 and 1 (which corresponds to varying passive damping in Figure 1(c)). A typical passive suspension setting is shown as design point P_1, where $f_2 = 1$ Hz and $\varsigma = 0.3$. As can be seen from Figure 5, this amount of damping is near optimal in terms of the ride and handling compromise for the passive case. These values are within the range typically found in most of today's vehicles which have evolved through many iterations primarily based on intuition and decades of experience.

The performance of an optimal active suspension with the same amount of wheel-hop as the optimal passive is shown in Figure 5 as design point A_1. Comparing design points P_1 and A_1, it can be seen that in this case the active suspension results in only 11% lower RMS acceleration levels with respect to P_1. However, it can also be seen that for some other road/speed operating conditions (to the right of A_1), there is a potential for substantial ride improvement with active suspensions compared with their passive counterparts. Indeed, as shown in Figure 5, for passive suspensions almost any deviation in operating conditions from P_1 will result in degradation of performance. On the other hand, for active suspensions, either handling or ride can be improved by choosing the tuning parameters r_1 and r_2 such that the resulting operation settles either to the left or to the right of A_1, respectively. For example, if we pick a different active suspension design point A (Figure 5) by relaxing the tyre deflection constraints, we see that the RMS acceleration at point A is as much as 67% smaller than that at Point P_1.

Based on the above results and observations, it can be concluded that the full advantage of active suspensions stems from possible adaptive tuning (or gain scheduling) of controller parameters, depending on the driving condition.[7,25] For example, if the steering wheel position or lateral acceleration sensors (possibly augmented with other road information) indicate operation on a straight section of a road where handling is less critical, then it may be possible to relax the wheel-hop constraint. Similarly, by knowing which sections of the road the vehicle is or will be travelling on and their corresponding road roughness, the tyre deflection constraints can be enforced differently, allowing more substantial improvement in RMS acceleration. Note that the exact amount of wheel-hop that could be tolerated under different driving conditions can be predetermined through appropriate vehicle tests.

The frequency responses of the three metrics with respect to road disturbance velocity offer insightful information between the trade-offs in suspension design. Different attributes of the performance at three operating points of interest (A, A_1, and P_1) are illustrated in Figure 6 in their corresponding Bode plots. From the sprung mass acceleration frequency plot, it can be seen that the main difference between the active suspension A_1 and corresponding passive counterpart P_1 occurs at the sprung mass mode around 1 Hz, where the active suspension brings much more damping through the damping on sprung mass velocity, the so-called 'skyhook' damping. The active suspension A results in further significant reduction of the acceleration levels (except at the 'invariant point' at the wheel-hop frequency [26]), but at the same time it deteriorates handling and rattle space performance due to a large, poorly damped resonant peak around the wheel-hop frequency. Note that the 'invariant point' arises from the structure of the suspension and results in the performance limit of sprung mass acceleration at the wheel-hop frequency regardless of control design, assuming negligible tyre damping. The reduction of acceleration at other frequency regions causing the rise of suspension and tyre deflection can also be interpreted as the 'waterbed' effect, a term coined in H infinity control analysis, that is, the improvement in one area (e.g. ride with respect to sprung mass frequency) of the frequency domain would be reflected through corresponding loss of performance in other complementary areas (e.g. handling in tyre-hop mode and/or rattle space requirement).

The suspension deflection Bode plot of Figure 6(b) indicates that, unlike their passive counterparts, the LQ optimal suspensions can result in a non-zero gain in very low frequency between the rattle space and road velocity input.[7] However, in practice this may not be detrimental since it is usually a standard practice to use appropriate signal processes, for example, high-pass filters to eliminate the very low-frequency components of the ground input signals resulting from hills and extended road grade.

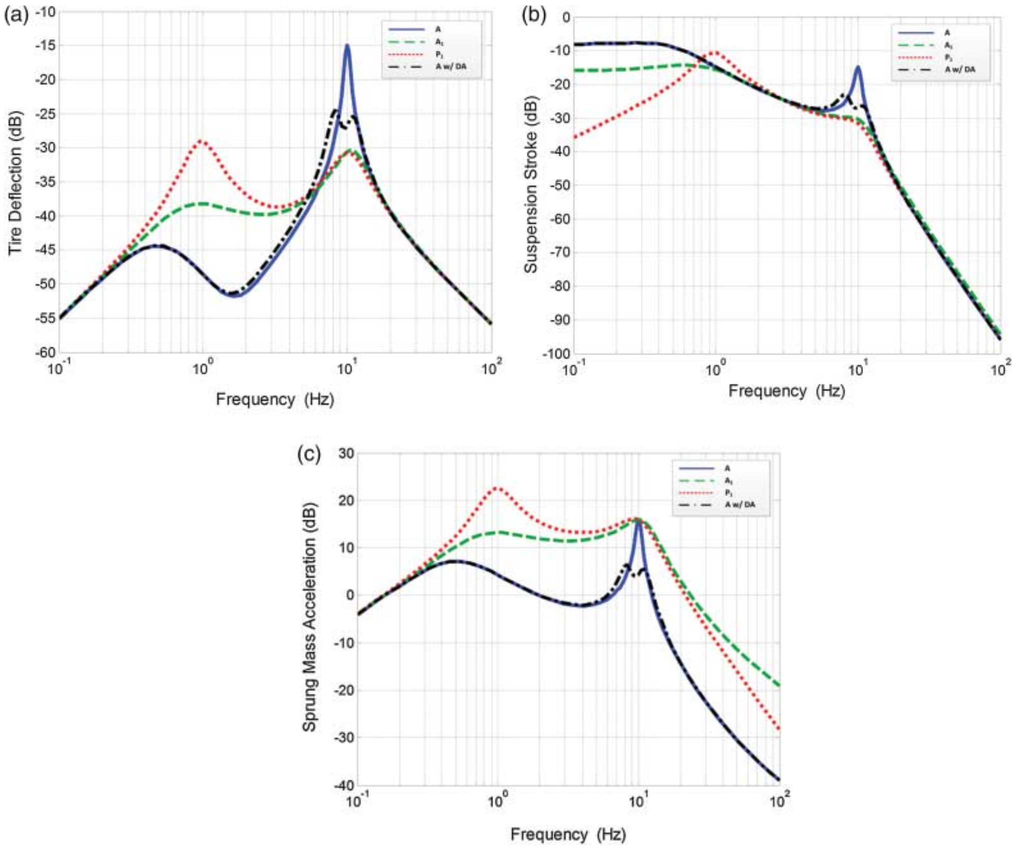

Figure 6. (a) Tyre deflection Bode plot of points on Figure 5 and point A with dynamic absorber, (b) suspension Bode plot of points on Figure 5 and point A with dynamic absorber; and (c) acceleration Bode plot of points on Figure 5 and point A with dynamic absorber.

Moreover, Figure 6 shows the Bode plots of an active suspension control (designed with the same weighting used in operating point A) combined with a dynamic vibration absorber. We see that the sprung mass acceleration can be further improved in the wheel-hop frequency since the introduction of the dynamic absorber (which interacts with only the unsprung mass) presents a new structure.

2.4. *Extension to semi-active optimal case*

In semi-active suspension systems, the suspension force can be modulated through a very small amount of supplied energy. In this case the suspension force cannot track an arbitrary desired force from the unconstrained LQ optimisation-derived control law. Rather, the problem becomes a constrained LQ optimisation. As a compromise, the semi-active control design typically follows its unconstrained active counterpart when it can, and operates along the passivity envelope when it cannot. For example, the damping force is adjusted to follow the desired suspension force derived from the optimal control law, and set to zero when a negative damping force is required. This control is therefore commonly referred to as 'clipped optimal'. This may result in 20% lower performance in ride comfort when compared to its active suspension counterpart.[11]

The optimal control law for the semi-active system has been posed as a constrained LQ optimisation and solved numerically in [28,29], involving the iterative solution of a time-varying force constraint. A specific example [29] shows that a 10% advantage with respect to clipped optimal can be achieved. However, it also finds that the amount of improvement depends on driven scenarios and is usually very limited. A later work [30] leveraged the explicit hybrid MPC to confirm analytically the previously numerical finding that clipped optimal is not the optimal control for semi-active suspensions in general.

2.5. *Extension to preview optimal control*

There are numerous investigations of the potential performance of the suspension if the road disturbance profile can be known in advance.[31–35] The solution consists of state feedback terms that have the same gains as the non-preview case, and feedforward terms that consider the road disturbance of the upcoming known duration.

It is worth noting that most of the papers place the origin of time (i.e. start of preview time for future disturbances) at the quarter-car location [31,35,36] or at the location of the front wheels.[32,37,38] In [34], this origin is simply moved to the location of preview, thus transforming the preview problem into an equivalent delay-time problem, where an extensive literature base already exists.[39] This approach is also used in [40] where the authors developed a method for direct and efficient calculation of the related performance index.

Comprehensive studies of the benefit of preview compared to its non-preview counterpart have been demonstrated for full-bandwidth active suspensions [34] and limited bandwidth ones.[41,42] The benefit for full-bandwidth active suspensions was illustrated on a plot similar to Figures 3 and 4 in [34], relating to its non-preview counterpart. And the benefit for limited bandwidth active suspensions in ride comfort and suspension stroke improvement was shown in [42].

3. Special consideration of handling, loading, and suspension travel limit

Although not always explicitly mentioned, most of the studies with traditional simple ride models (e.g. quarter-car, half-car, and full-car) focus on road roughness disturbances only, neglecting external (inertia) forces and loads due to, for example, braking and turning. The control damping for inertia load-induced excitations can be included later in a separate and independent design procedure. In practice, this is accomplished using longitudinal and lateral acceleration measurements as feedforward signals [43] indicating braking or turning actions. This typically leads to appropriate ‘stiffening’ of the active or semi-active suspension to prevent or counteract excessive pitch or roll. This stiffening action may reduce the ride quality, but in practice this may be of secondary importance, especially during hard braking and/or handling manoeuvres.

For the case when complete decoupling of the above road and load disturbance effects is desired, Smith,[44] Smith and Wang,[45] and Wang and Smith [46] present an interesting approach based on Youla parameterisation and LFT (linear fractional transformation). The resulting controller can be designed independently – first for the road-induced disturbance and then for the load counterpart. For example, the suspension can be made very stiff for load inputs in order to hold a desired attitude, while at the same time it could be made very soft for the ground inputs (of course, the latter has to be within some practical limits since any prolonged ramp-like ground input could lead to suspension bottoming and undesirable jerks and possibly even structural damage). This is somewhat similar to the above-mentioned sequential design approach used in practice with the exception of total decoupling. In this context, the 'practical' design uses longitudinal and lateral acceleration signals (in addition to other heave-related measurements) as a feedforward signal or proxy for gas/brake pedal and/or steering wheel applications.

A comprehensive and direct approach to simultaneous inclusion of road and load disturbances has been recently presented in [47] based on the above more conventional LQ stochastic optimisation. In addition to the usual road disturbance, which is assumed unmeasured, Brezas and Smith [47] also introduce the load disturbance but as a measured, constant disturbance. An advantage of this approach is that it directly addresses optimisation of the main attribute of interest, that is, ride comfort, which is introduced through an appropriately formulated LQ performance index.

In the case of longitudinal dynamics, further extensions of this approach are possible by introducing the force disturbance through a delay, which effectively presents a load preview from the standpoint of the suspension controller similar to the preview modelling used in [34]. This could be further refined by using a (simple) model of vehicle power train (brake) paths, from gas (brake) pedal to actual wheel torque production. Similar comments apply for load prediction due to steering inputs where ultimately some appropriate models of vehicle handling can be used too.

A simple approach to load disturbance containment is through load levelling, that is, applying an integral control to regulate the rattle space between sprung and unsprung masses.[48–51] In particular, Hrovat and Hubbard [48] use jerk-optimal LQ control to end up with an effective and systematic way of obtaining relatively fast load levelling with some – mostly minor – sacrifice in sprung mass acceleration performance. It should be pointed out that in practice a slow-acting load levelling is often used to centre the suspension against slow-changing loads. Davis and Thompson [49] use both integral and derivative controls in a ride quality- and handling capability-focused 7 DOF full-car model to regulate as well as maximise usable suspension deflection as the derivative action helps to decrease overshoot.

As the suspension stroke is limited, practical design of all suspensions may introduce additional feedback elements to stiffen up the suspensions as the rattle space approaches its travel limits. In addition, actuator dynamics may contain delays, nonlinearities (which is inherent in the semi-active case), or lags. Various control designs have been developed to address these issues including backstepping design,[52] adaptive robust control,[53] learning automata,[54,55] and Pareto optimisation.[56] In [52], a nonlinear filter whose effective bandwidth depends on the magnitude of the suspension travel was introduced to facilitate smooth shifting of the control objective between ride comfort and suspension stroke utilisation. In [53], a two-loop controller with adaptive robust control was introduced to ensure that the nonlinear actuator delivers the force requested from the LQ top (outer) loop design. Gordon et al. [54] and Howell et al. [55] used an approach combining concepts from stochastic optimal control with those of learning automata. And Valasek and Kortum [56] used the Pareto optimisation procedure to determine the state-dependent feedback gains in a predefined control structure.

It is worth noting that Gordon and Best [57] included non-quadratic terms for the optimisation of semi-active suspensions in their optimisation and observed the performance with the conclusion that the standard 'clipped optimal' technique is close to the one resulted from a complex optimisation. In [58], the optimisation of semi-active suspension with preview is numerically optimised and it was observed that only small improvement can be achieved compared to the simple 'clipped optimal' preview control. These conclusions are corroborated in [30] where analytical explicit model predictive control was used.

4. Other approaches and considerations

In addition to the above LQ optimisation-based approaches that are the main focus of the present survey, there are many other possible approaches to advanced active and semi-active suspension design and analysis. These include Youla parameterisation and LFT,[44–46] pole placement,[59] LMI,[60] H2/H-infinity, and H-infinity.[61–65]

Furthermore, multi-objective optimisation (symbolically or numerically) has been widely used to systematically manage nonlinearities and constraints.[66–68] It is particularly prevalent for semi-active suspension applications.[69,70] As the suspension performance index often includes conflicting terms, the solution of a multi-objective optimisation finds the best trade-off among the various predefined control terms. This is also known as Pareto optimisation used in systematic design procedures (note that a similar result for the linear system was obtained using the previously mentioned global optimisation in Section 2 with varying weights).

While ride comfort is the main focus for passenger vehicle suspensions, minimising road damage is important for truck and heavy vehicle suspensions.[71–74] An approach that pays particular attention to tyre force variation is an interesting extension of the well-known skyhook concept,[8] proposed in [75] as the so-called ground-hook control. Specifically, it amounts to augmenting the (semi-)active actuator desired force calculation with an additional, virtual damping term proportional to the relative velocity between the unsprung mass and ground (Figure 7), emulating a fictitious damper between the unsprung mass and road.

The typical objective of such a suspension optimisation was to minimise the integral square of the tyre force [76,77] by using a simple quarter-car vehicle model for the control synthesis, which is similar to previously mentioned LQ-based RMS tyre deflection minimisation except that the latter would typically be part of a performance index that includes additional terms due to ride comfort and rattle space (design) constraint. The integral square sprung mass acceleration term as a measure of ride comfort was later included in [78,79] where the ground hook and other coefficients were determined from numerical optimisation with specified cost function. Note that the above optimisation for the case of semi-active actuator amounted to essentially nonlinear system parameter optimisation since the extended ground-hook optimisation structure was fixed a priori.[76,77] Since the main nonlinearity is in the semi-active actuator, state dependence of the related parameters was also introduced in the process of optimisation.[56]

The ground-hook concept was found useful when minimising road damage [56,77] especially due to heavy trucks where the damage to pavement can be substantial and up to 10,000 times greater than due to passenger cars.[78] This leads to so-called road-friendly suspension,

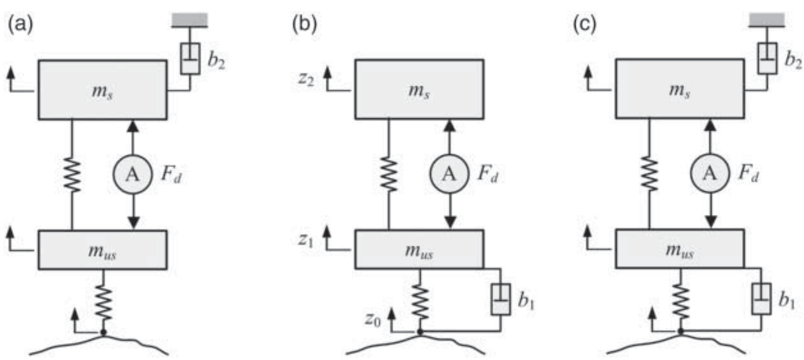

Figure 7. (a) Skyhook, (b) ground hook, and (c) combination of skyhook and ground hook.

which was further extended to bridge-friendly [79–81] and ESC,[82] that is, Electronic Stability Control-friendly, applications. The concept was further tested through experimental vehicle implementation [56,77] using a SKODA-LIAZ prototype truck.

The main emphasis in Section 2 was on advanced active and semi-active suspensions using simplified (minimal component count) system models. In practice, however, any implementation of these advanced concepts will also include additional conventional, passive components. For example, an active actuator may be positioned in parallel with a spring for static support and a damper to dissipate energy without an active element. Thus, one must eventually consider designing the whole suspension system consisting of both active and passive elements.

In addition, as is typically the practice when implementing the LQ or any other linear algorithm, one should first attempt (static) linearisation of any possible nonlinearities in actuators, plants, and/or sensors. The combined controller-actuator subsystem should then be further verified and as necessary refined through additional simulations based on a more detailed performance-oriented model. The latter should include a detailed model of actuator dynamics, which is often the most challenging and most important component of an advanced, active, or semi-active vehicle suspension system design.

A reasonable starting point in an active or advanced suspension system design would be a consideration of the passive system alone in order to establish the best passive benchmark against which any advanced mixed passive–active system could then be compared. At this stage there is a need to more precisely define passivity so that the whole passive suspension optimisation problem can in turn be properly defined. One of the first attempts at this was in [28] where the notions of passive operators from mathematics and electrical circuits are used. In addition, Hrovat [28] also posed the corresponding optimal passive suspension problem. However, at the time of its writing, much of the necessary mathematical Computer Aided Engineering (CAE) tools and computer optimisation capability were missing. This class of problems were effectively treated more than a quarter-century later [83] using LMIs and other modern CAE tools. In [83] problems with both H2 and Hinf optimisation cost are considered for the case of a quarter-car model with both ground and load inputs included. This approach could likely be further extended to hybrid configurations where both passive and active suspensions are combined.

Once the optimal passive transfer function is determined, there remains the problem of synthesising or actually implementing it via standard passive elements such as springs, dampers, and inertias. In the process, there is a need for an inertia element – called ‘inerter’ in [84] – that acts on a relative velocity rather than the customary absolute velocity of relevant masses or inertias. Perhaps the easiest way to see this is via bond graphs where one could imagine a hydraulic piston–cylinder combination between sprung and unsprung masses [85] (inerter-like structure was described in its third edition with more examples exhibited in its later editions). Another example can be found in modelling hydraulic mounts.[86,87] An effective and practical mechanical implementation of an inerter was shown in [83,84] consisting of multiple gear sets with large gear ratios resulting in a large reflected inertia. The latter facilitates relatively small overall weight for the device, which is an important practical consideration. In addition to possible suspension applications, the device was also used to mitigate steering instabilities of motorcycles.[88]

Another more constrained approach to optimisation was considered in [89] where the underlying passive structure was first fixed and then the optimisation was performed for each free design parameter and compared with the best tuning of the conventional passive suspension structure consisting of a parallel spring and damper. The results indicate around 10% and more improvements in the comfort RMS acceleration metric, although it is not clear if this included the same level of other constraints (tyre and rattle space).

Using the above-mentioned definition of passivity it is now possible to ask the following question. For a given optimal suspension control law can we determine if it could be implemented through a passive device? If the answer is positive this would have significant practical implications since such a device could then be realised using only passive components that are typically less expensive, less complex, and require no external power sources and consumption to operate. It was first shown in [6,28] that for the quarter-car LQ optimal control the resulting control law is necessarily active. A more comprehensive study was subsequently undertaken in [5] including additional control laws applied to half- and full-car cases.

In addition, Smith [44] and Smith and Walker [5] consider some other interesting aspects of active and passive suspensions. These include the derivation of a complete set of constraints for the road disturbance transfer functions for typical choices of measured outputs as well as performance limitations and constraints for active and passive suspensions for quarter-, half-, and full-car models. This complements the prior well-known results on the invariance of certain transfer functions at special frequency points.[26]

While the quarter-car model largely describes the vehicle response to the road disturbance, the heave and pitch experienced at various observed points along the vehicle longitudinal axis can vary due to the 'multi-input' disturbance (coming through both front and rear wheels). For passive vehicles, due to the fact that the excitation at the rear wheel is a time-delayed version of the front wheel, there is a 'wheelbase filtering' effect [90] that changes the transfer function to heave and/or pitch at particular frequencies as a function of vehicle speed and wheel base. This effect is more perceptible for trucks with a long wheel base. The quarter-car suspension design can be expanded to full car. One attractive approach to address the full-car and half-car suspension design and leverage the extensive knowledge from the quarter-car study is to first apply control design decoupling modes. The importance of decoupling between sprung mass pitch and heave mode as well as between the equivalent front and rear quarter-car models is described in [7,91]. In [91], an active suspension was designed to decouple sprung mass heave and pitch mode through zeroing the coupling terms between pitch and heave relative (suspension) deflections, and similarly those between pitch and heave relative (suspension) velocities. This structurally decoupled active suspension minimises body heave motion to road pitch disturbance, and similarly body pitch to road heave, to a secondary level. It was exhibited [91] that a superior performance can be achieved by simply adding the skyhook (or 'absolute') damping to the 'decoupled' active suspension design. This approach is further verified in [92] through mathematical manipulation and state space transformation where the heave and pitch modes and corresponding disturbances are decoupled.

Important aspects of state estimation are addressed in [93,94]. Analytical techniques for fault detection and facilitating safe start-up and shutdown of active system are described in [95]. Some recent attempts to introduce risk sensitivity into the LQ optimal suspension problem were addressed in [47]. The approach has some practical appeal since it facilitates tuning the balance between performance and associated risk due to plant uncertainties, modelling limitations, and possible unmodelled dynamics, thus providing system designers with additional flexibility through an added degree of tuning freedom.

5. Production and near production features and systems

5.1. *Self-levelling feature*

Various self-levelling systems have been introduced to the market including the Electronic Air Suspension systems developed by Citroen's hydropneumatic systems [96] and Ford

Motor Company's Electronically Controlled Air Suspension on Lincoln,[97] allowing these vehicles to maintain proper ride height and suspension stiffness over a wider range of vehicle loading. This feature adjusts the vehicle ride height, usually at low frequencies, in order to balance among (1) soft and comfortable ride stiffness from the passive spring, (2) proper vehicle attitude/stance, and (3) ample rattle space for anticipated or unknown road disturbance ahead. For example, the system in the 1984 Lincoln Continental allows its spring to be 32–38% softer than a corresponding steel spring suspension while maintaining vehicle stance and suspension travel.[97] In addition, this active suspension feature can enable the lowering of the vehicle at highway speeds to improve aerodynamics. This feature is implemented in the Lincoln Mark VIII [97] and most recently the Tesla Motor Model S,[98,99] providing improved aerodynamics, more consistent handling, better fuel economy, and extended range.

Load levelling suspensions are often constructed using pneumatic springs whose equilibrium length is adjusted by changing the volume of air in the spring. In the Lincoln Continental, height sensors are used to trigger the adjustment of the air volume, and signals from the ignition, doors, and brake systems are also used to ensure levelling only at appropriate times.

5.2. *Variable suspension damping (semi-active suspensions) feature*

Production semi-active suspension systems are generally constructed using an adjustable damper in parallel with the primary suspension spring. These are typically constructed from pneumatic and/or hydraulic piston/cylinder combinations with electromechanical control of an orifice. Actuator bandwidth is primarily determined by the reaction time of the controlling valve and associated pressure/force production dynamics. Semi-active suspension systems have been introduced to the market since the 1980s' Toyota Soarer's TEMS (Toyota Electronic Modulated Suspension).[100] A more recent system can be found in Lincoln's Continuous Controlled Damping system introduced in 2006. The Lincoln system uses a suite of sensors that constantly monitor suspension motion, body movement, steering, and braking inputs and adjusts the suspension in milliseconds, helping keep the car smoothly on track. Specifically, it monitors up to 46 inputs and reacts on average within 20 milliseconds [101,102] to reduce roll, pitch, and heave motions, to enhance driving comfort and dynamics, and to isolate road harshness.

Another implementation of adjustable damping is through magneto-rheological (MR) fluids. MR fluid characteristics can be changed electronically, allowing the force across the actuator to change quickly. This method benefits from faster response time, although limited fluid life can contribute to service concerns. One MR damper application is found in the 2002 Cadillac Seville STS and the 2003 Chevrolet Corvette whose MR fluid is co-developed by Delphi and Lord Corporation. It is composed of suspended iron particles in a base fluid of synthetic hydrocarbon. Its continuously variable damping is adjusted in milliseconds. The fast response time of the MR damper enables closed-loop vehicle stability control through the control of lateral and longitudinal load transfer characteristics of the suspension during transient movements.[103,104]

Another method to increase the damping rate adjustment frequency when employing limited bandwidth orifice control valves is to employ the resistance control semi-active damping method [105] demonstrated experimentally in [106] where a dual-acting damper with two control valves is used. Each valve sets the damping rate in one direction (jounce or rebound).

5.3. *Variable suspension geometry (low-power low-bandwidth active suspensions) feature*

A variable geometry active suspension adjusts the ratio of wheel movement to the deflection of the suspension spring in real time. By changing the leverage of the passive suspension spring with wheel motion, it essentially controls the wheel rate or effective spring stiffness.

Various systems and hardware configurations that provide variable suspension geometry have been proposed in the literature [107–110] including the ‘Delft Active Suspension’ concept that was implemented as a prototype vehicle and demonstrated experimentally.[109]

Ideally, one would like to adjust the variable suspension geometry without requiring much power. One way to do this is to adjust the suspension leverage by changing the suspension spring mounting point which requires only actuation perpendicular to the (vertical) suspension load or vehicle weight (Figure 8).[108] Ideally, with the perpendicular configuration, the mechanism would require very low power and low energy. In practice, however, the precisely perpendicular arrangement would be compromised by suspension motion and deflection. There may be additional trade-offs in mechanical design forcing some deviation from the ideal geometry. The trade-offs may include vehicle stance/self-levelling, range of mechanism motion (and resulting range of leverage ratio), packaging, effect of jounce/rebound motion, and effects on wheel turning. A system design study with the trade-offs in mind is discussed in [107].

Another form of leverage adjustment concept (Figure 9), the previously mentioned ‘Delft Active Suspension’, is described in [107,111], and its practical realisation with a cone mechanism is illustrated in [112,113]. This mechanism connects the spring to the car body on one end and to a rotatable crank on the other end. The crank is joint-connected to the suspension/wheel control arm and can be rotated at the joint around the base of the imaginary cone. The cone mechanism serves two purposes: (1) the length of the spring remains the same as the crank rotates and (2) the ratio of movement between the wheel/tyre control arm and the crank changes as the crank rotates (Figure 10).

Due to suspension deflections and motions when experiencing road disturbances, the alignment of the ideal position of the cone mechanism will be affected in practice. In particular, the base plane of the cone mechanism, that is, the plane of the crank motion, will rotate. Nevertheless, this is a promising technology since it could provide some active suspension characteristics with only a ‘semi-active’ type of actuator.

5.4. *Roll control (semi-active or active) feature*

A number of automobile manufacturers have been introducing active and semi-active suspensions that focus on roll control to improve ride and handling with moderate cost and

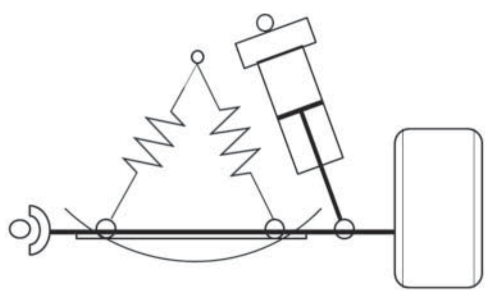

Figure 8. Illustration of a variable geometry suspension concept.

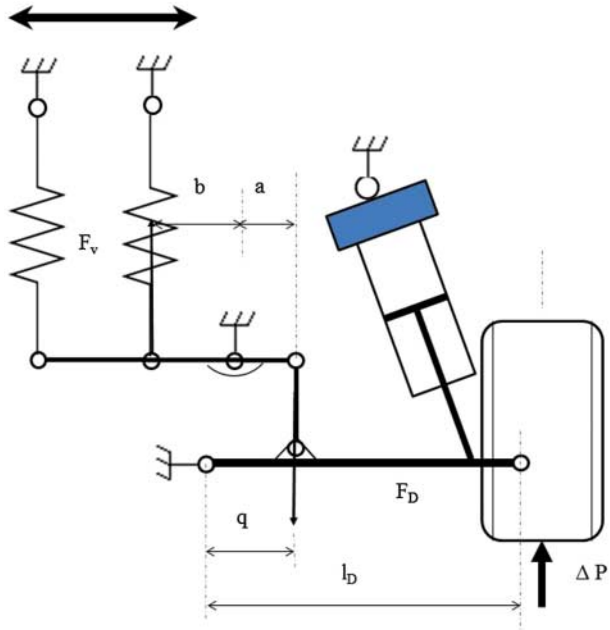

Figure 9. Illustration of delft active suspension concept.

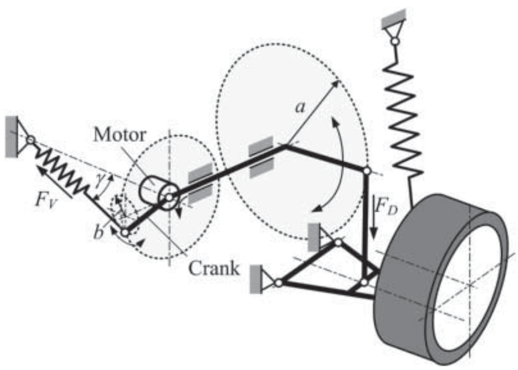

Figure 10. Illustration of delft active suspension realisation $F_V \cos(\gamma) * b = F_D * a$.

complexity. It is well known that good ride comfort can be achieved with softly tuned passive or semi-active suspensions combined with soft roll bar settings. However, this will result in large pitch and roll motions during more aggressive braking and cornering. The active (or semi-active) roll control is then used to satisfy these conflicting constraints. In practice, the concept is typically realised by inserting a small hydraulic jack and a gas spring accumulator with a control solenoid, all placed in series with the anti-roll bar. The anti-roll bar can be as stiff as in street race cars, without negatively impacting straight-line ride since the solenoid connecting the hydraulic jack and the accumulator is then kept open, thus lowering the effective stiffness of the roll bar/accumulator combination.

The performance of anti-roll/roll control devices varies as it may be achieved by a mechanical regulator that depends on suspension motion [114,115] or by an electronic controller that monitors additional non-suspension-related vehicle signals. It can be actuated with a semi-active device that manipulates hydraulic valves or the like, or with an additional active device such as a high-capacity pump. An idea similar to active tilt control has also been explored in [116,117].

Citroen introduced hydropneumatic suspensions in 1955 with its model DS. The hydropneumatic spring-absorber system incorporates the benefits of both the compressibility of gas (nitrogen) and responsiveness of (incompressible) hydraulic fluid. As the load increases, it compresses the suspension pistons and causes the gas volume to decrease and pressure to rise; the gas spring becomes stiffer but keeps the sprung mass frequency roughly the same. As the fluid passes through a two-way valve, damping is also provided. In addition to its self-levelling feature that regulates ride height and resonance frequency (through stiffer spring with increased load), the Citroen's hydropneumatic system can be applied towards anti-roll or roll control by manipulating the flow of hydraulic fluid between left and right corners.

Various improvements have been made in Citroen's hydropneumatic suspensions. Most notable are the changes from mechanical regulation to electronic control, the improved capability of the pump, and the associated increase in actuation bandwidth. For the roll control feature in particular, there have been several incarnations ranging from semi-active (Hydractive I and II) to active (Activa). Citroen's semi-active roll control (Hydractive) limits the roll angle to about 3° in soft mode or 2.5° in hard mode for a 0.6 g constant turn, while its active roll control (Activa) can eliminate roll angle completely due to the active and electronic manipulation of hydraulic fluid.[96]

Roll control can be achieved with the help of a hydraulic rotary actuator inserted around the middle of the stabiliser bar, as stated above, or it can be implemented through a rotary actuator fitted with the anti-roll bar as in the BMW 'Dynamic Drive' concept.[118] The BMW system monitors primarily the steering angle and lateral acceleration, among other signals, and distributes the front and rear axle anti-roll torques. It controls the front and rear actuators by means of two electronically regulated pressure control valves to minimise the roll motion in corners and road input roll disturbances, thus improving response to vehicle steering for vehicle dynamics handling and agility. Note that the roll angle will gradually increase at high lateral acceleration as the vehicle approaches its stability limit.

It is understood that the benefits of active roll control in overall vehicle stability and active safety can be further explored through coordinated action with other systems such as active brakes, active steering, and electronically controlled all-wheel drive. This can be particularly useful in emergency manoeuvres such as a sudden need for vehicle stabilisation and control during evasive actions on slippery road surfaces.

5.5. *Active suspensions with full features*

In general, active suspensions can be categorised into narrow bandwidth and broad bandwidth, referring to the control bandwidth of the actuation systems. This typically refers to actuator force generation bandwidth for all possible motions across the actuator mounting points. The narrow bandwidth systems typically exhibit actuator control bandwidth around or slightly above sprung mass frequency and would use the passive spring, positioned in series to the actuator, to address high-frequency disturbances and vibrations. As such, their bandwidth is typically between 3 and 7 Hz.[119] Broad bandwidth systems, on the other hand, attempt to address oscillations beyond the sprung mass frequency using high-bandwidth actuators typically above 10 Hz.[119]

Since the broad bandwidth active suspensions (BBASs) attempt to control both the slower body modes and the fast wheel-hop modes, it is generally expected that the broad bandwidth systems will be more challenging to implement. It is expected that such suspensions will incur higher costs, complexities, and possible fuel consumption.[120] However, one should also observe that narrow bandwidth systems may experience similar fuel consumption due to either their higher compliance and the corresponding increased pump flow/displacement requirement [119] or higher control effort associated with the mismatch between the desired and actual control bandwidth.[43]

5.5.1. *Narrow bandwidth active suspensions*

Some narrow bandwidth systems have been brought to the market with limited success. An example is the Nissan Infinity Q45a [121] whose hardware included: (1) an accumulator (for bandwidth limitation and NVH containment); (2) a mechanical supporting spring; and (3) a relatively inexpensive pressure control valve. It had a single-acting actuator configuration which is simple and inexpensive. It does, however, limit the system controllability on the rebound stroke, which is in good part determined by the spring/accumulator settings.

Production hydraulic active suspension systems typically use pressure control valves to regulate the desired force. Considering that a force-generating actuator is accompanied by corresponding displacements and velocities of the in-series compliance in the total system, one may consider the flow control valves to regulate the desired displacements/velocities. Benefits and limitations of both force and displacement control approaches have been discussed in [119,122–124].

DaimlerChrysler introduced the 'Active Body Control' (ABC) system in the 2000 CL sedan. It consists of four low-bandwidth (5 Hz) hydraulic actuators providing active load levelling, heave, pitch, and roll control. The ABC suspension system consists of [125,126] hydraulic cylinders in series with steel coil springs (Figure 11). This cylinder sets the spring preload and the series arrangement leads to the low-bandwidth system behaviour. In parallel with the spring/cylinder is a twin-pipe gas shock absorber (damper). Hydraulic fluid is supplied to each corner via a pump regulated to 200 bars, and an accumulator at each axle provides pressure levelling during peak loads. High-pressure fluid is carried by two valve blocks – one at each axle. Each block contains two 3-way proportional valves – one for each corner – which are used to maintain plunger pressure and a 2-way shut-off valve for each corner to lock the cylinder when the vehicle is stopped and to provide failsafe operation. Return fluid is damped through a return accumulator.

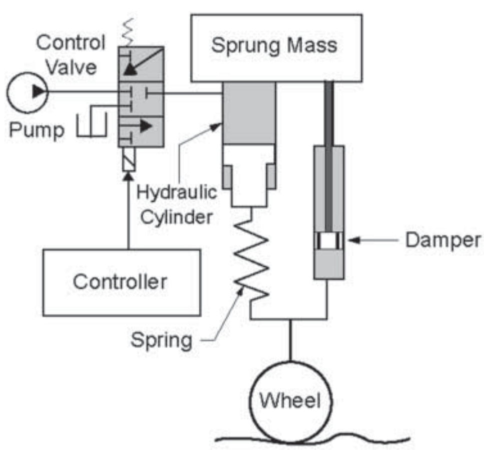

Figure 11. Schematic diagram of the Mercedes 'Active Body Control' actuator.

The ABC system utilises multiple sensors for mechanical state measurements. Sensors include longitudinal and lateral accelerometers, three vertical accelerometers (for bump/pitch/roll detections), as well as a level sensor and a plunger position sensor at each wheel. Additionally, several hydraulic pressure sensors are used for pressure control and monitoring.

The ABC system employs various algorithm modules for vehicle dynamics control depending on the vehicle states. Functions provided include speed-based load levelling and ride height adjustment, skyhook control of the vehicle body heave mode, roll control for roll angle response to vehicle lateral acceleration, and pitch control for pitch angle response to longitudinal acceleration. The algorithms are further controlled by driver input via instrument panel height and performance mode switches. ABC has continued to evolve since its first introduction in 1998. For example, the wheel damping is now continuously adjustable and the efficiency of the pump has been increased. In addition, computational processing power is more than double that of the previous system.[127]

5.5.2. *BBAS concept implementation at Ford*

A broad bandwidth system was developed at Ford Research Laboratory in the early 1990s and was demonstrated by fitting the suspension concept hardware onto a 1989 Ford Thunderbird.[128] The concept hardware and software verified the previous theoretical study in ride quality improvement for rough road and for going over a speed bump or crossing a crowned road. The effort also identified the shortcomings of the implemented hardware structure to facilitate next generation active suspension hardware design.

This concept car development provided first-hand experience and useful answers to many practical concerns such as actual power consumption, secondary ride harshness in the range above 5 Hz, and actuator noise. It also enabled the identification of specific next steps to address these issues. For example, some of the key items that have been identified to be addressed in the future include the parasitic losses with the parallel spring structure and the need for further refinement in the secondary ride quality.

The Ford Thunderbird BBAS research vehicle (Figure 12) used four high-fidelity electro-hydraulic servo actuators, one at each corner of the vehicle. While a typical narrow bandwidth suspension is controlled through three-way proportional pressure control valves, the BBAS

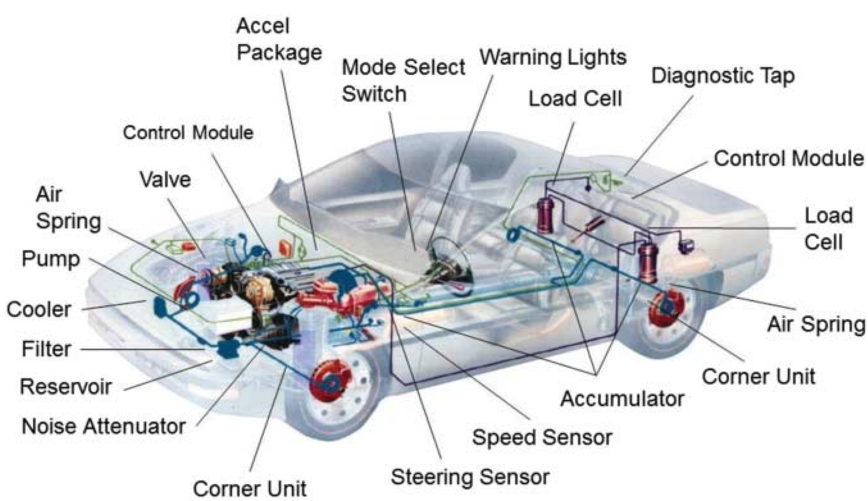

Figure 12. Ford BBAS concept implementation.

was controlled through four-way servo valves, which have much higher precision and speed of response. In addition, the BBAS actuators were based on double-acting cylinders capable of equally fast rebound and jounce strokes. The vehicle also had one central and four corner-unit microprocessors for fast signal/control processing; four actuator displacement sensors and four load cells for internal (force) loop calculations; and four air springs – one at each corner placed in parallel with the BBAS actuators. The air springs support and self-centre the vehicle sprung mass as a typical load levelling set would do and at the same time provide lower sprung mass natural frequency for more comfortable basic ride, which is then appropriately dynamically modified through the BBAS actuators. The system incorporated 26 various sensors, including accelerometers, pressure sensors, vehicle speed sensors, and others.

The BBAS control strategy was based on coordinated individual wheel control and consisted of two hierarchical levels.[43,128] The outer loop level operated at a 20 ms rate. It calculated the desired corner forces for the four BBAS actuators, desired operating modes (handling or ride dominated), and checked the overall system integrity. The ride-related calculations were based on quarter-car vehicle models aimed at emulating skyhook damping at each corner, which is often very close to the optimal possible ride benefit.[20] Different effective spring and damping rates were used depending on prevailing operating modes, that is, ride or handling. Table 1 summarises typical parameters used.[128]

The calculations also included roll and pitch attitude control during turning, braking, accelerating, and the like; this was based on lateral and longitudinal acceleration signals and steering sensors. The commanded or desired levels of corner forces were communicated to corner computers, which operated at higher sampling rates of 1 ms to implement the inner-loop force control based on feedback provided by load cell sensors. The corner units distributed the force requests between different servo valves. For example, the conventional (relative) suspension damping and the passive segments of skyhook force were provided by bypass or cross-port two-way servo valves that provided damping only when called for in a manner similar to semi-active damping control.

The BBAS R&D vehicle accomplished many of its key objectives, especially with respect to primary ride (up to circa 5 Hz) and handling. The typical results in terms of NASA ride metrics are summarised in [128] where a smaller number on the scale of 10 implies better ride. From this reference it can be seen that, on average, the active Thunderbird provided the best ride among a dozen different vehicles under consideration. The only areas where the BBAS had somewhat worse performance were on smooth roads, where Jaguar XJ6 and Lexus LS 400 (both well known for their smooth ride) had an advantage. This may be attributable to still less refined NVH (higher frequency) levels and related secondary ride quality that are some of the remaining challenges for BBAS concepts. Based on their summarised results, it should also be noted that the BBAS Thunderbird significantly outperformed both the baseline passive suspension Thunderbird and its narrow bandwidth (NBAS) counterparts as exemplified by production Infiniti Q45a and Toyota Soarer.

Table 1. Typical parameters used in Ford BBAS.

		Ride mode		Manoeuvring mode	
Parameter	Units	Front	Rear	Front	Rear
Skyhook damping	Lb-s/in	23.5	10.0	23.5	10.0
	N-s/m	4115.3	1751.2	4115.3	1751.2
Conventional damping	Lb-s/in	3.0	3.0	8.0	4.0
	N-s/m	525.4	525.4	1400.9	700.5
Suspension spring	Lb/in	15.0	15.0	120.0	120.0
	N/m	2626.8	2626.8	21,014	21,014

In the same paper, the handling characteristics of the active Thunderbird are compared with its passive (production) version. The results of back-to-back tests show that the active vehicle results in more agile handling with faster yaw response, less roll and pitch, and significantly larger maximum lateral acceleration (0.895 vs. 0.81 g). Thus, in a direct comparison between the two vehicles, the active car displayed significant improvements in both vehicle ride and handling.

While achieving the full potential of a BBAS still remains an elusive goal, significant progress has been achieved with the above early efforts. This holds particularly true for the case of primary ride and handling. The challenging areas in need of further development include secondary ride, NVH, and system power consumption, complexity, and cost. Possible approaches here have been outlined in many of the previously mentioned references (e.g. [7,11,88,107,119]).

In closing, it should also be mentioned that in addition to the above electro-hydraulic implementation of the BBAS there was an additional BBAS effort focused on Electrical Active Suspension (EAS), which was capable of power/energy regeneration.[129] The EAS concept was also successfully implemented in a prototype vehicle with similar results and challenges as for the electro-hydraulic counterpart.

5.6. *Road preview feature – using road information for semi-active and active control*

The concept of utilising the road profile information prior to road disturbances hitting the wheel, investigated in [31–35], has been partially applied to production vehicles. The early attempt includes the Nissan vehicles in 1990 with the Super Sonic Suspension [130] feature. More recently, the preview feature can be found in the 2013 Mercedes S-Class with the Magic Body Control feature.[131,132] The early Super Sonic Suspension was applied to a semi-active suspension system and the recent Magic Body Control is equipped with an active suspension.

The early Nissan Super Sonic Suspension had only three levels of damping that are automatically selected to provide comfort and handling based on the road surface profile scanned with ultrasonic sensors, as well as steering angle, vehicle speed, brake on/off status, and measured accelerations.

The Mercedes S-Class detects road surface undulations in advance using a stereo camera and adjusts the suspension to deal with them accordingly. The suspension system adjusts its shock absorber damping in advance, making it stiffer or softer for each individual wheel, and uses active hydraulic pistons to control the desired wheel loads.

The preview information for a three-dimensional road profile is obtained through a stereo camera that monitors an area up to 15 m ahead during good visibility – preferably in the day light with suitable road surface structures and at speeds up to 130 kph. The stereo camera has an update rate of about 60 ms. Each section of the road can be measured several times at various angles during vehicle approach, and statistical algorithms are used to provide the final estimate of road height profile. It has been demonstrated that, under good visibility, the system performs well, providing superior ride comfort [133] especially when going through speed bumps and excellent handling during dynamic manoeuvres.

6. Conclusions

A 2 DOF quarter-car model, equipped with an actuator that generates force between the car body/sprung mass and wheel/unsprung mass, has been widely studied in the literature and in the process facilitated many useful insights and new concepts (sky hook, ground hook, invariant points to name a few). Thus, this simple model served well as a base for the comparison study of various suspension concepts, controller designs, benefits, and limitations. The present survey first discusses the various performance index/cost functions that are important for the design of a passenger vehicle. It then reviews controller design limitations through the application of optimal control theory to a quarter-car model with an LQ-based cost function. Figures are generated to illustrate the design limitations and trade-offs in RMS measures of various vehicle attributes and their associated frequency contents. These plots should help suspension designers to visualise the ability of active suspensions to behave differently depending on the current (and forthcoming) road excitations and vehicle manoeuvre-induced loading (e.g. braking, accelerating, and turning).

Based on more than 40 years of theoretical and practical developments in the industry and academia, it can be concluded that one of the main challenges for widespread usage of active suspensions still lies in the area of actuator design and implementation. It is expected that superior outcome will eventually result from a synergetic interplay between control software intelligence and ingenious hardware design, all led by model-based system engineering and further facilitated by relentless developments in electronics, microcontrollers and associated sensors, and vehicle-to-vehicle and vehicle-to-infrastructure connectivity. Indeed, an important part of such an actuator development should include realisation of the desired actuator force or, alternatively, displacement. As mentioned before, producing and tracking the desired force (or displacement) is by no means a trivial task, especially when combined with practical constraints on cost, packaging, weight, and energy consumption. This important aspect of advanced suspension design should be carefully evaluated through computer simulations, bench testing, and eventually full vehicle on-road verification.

The above review of current state of the art reveals that the next advances in active and semi-active suspension design will mainly come from two thrust areas. The first is the increased efficiency in actuator design and implementation such as the usage of systematic control software algorithm design combined with the previously mentioned innovative hardware design measures such as inclusion of possibly variable in-series/parallel compliances and fast load levelling. The second is more comprehensive usage of preview information from camera, Global Positioning System (GPS), and electronic horizon such as vehicle-to-vehicle communication, vehicle-to-infrastructure communication, vehicle localisation and real-time accessing of cloud information, and crowd sourcing leading to up-to-date road profile maps.

Acknowledgement

The authors would like to thank their colleagues Li Xu, Vladimir Ivanovic, and Michael Fodor for their valuable input and feedback on this survey.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

[1] Galip UA, Peng H, Melih C. Automotive control systems. New York (NY): Cambridge University Press; 2012.
[2] Cao D, Song X, Ahmadian M. Editors' perspectives: road vehicle suspension design, dynamics, and control. Veh Syst Dyn. 2011;49(1–2):3–28.
[3] Lu J, Hrovat D, Thomas P, Engleman J, Tseng HE, Filev D. Adaptive crash height adjustment using active suspensions. US patent 13/736,397. 2013.

[4] Tseng EH, Hrovat D, Lu J, McConnell M, Dehmel M, Seemann M, Owens FV. Suspension control system to facilitate wheel motions during parking. US Patent 8,825,292. 2014 Sep 2.
[5] Smith MC, Walker GW. Performance limitations and constraints for active and passive suspension: a mechanical multi-port approach. Veh Syst Dyn. 2000;33(3):137–168.
[6] Hrovat D. A class of active LQG optimal actuators. Automatica. 1982;18(1):117–119.
[7] Hrovat D. Survey of advanced suspension developments and related optimal control applications. Automatica. 1997;33(10):1781–1817.
[8] Karnopp D, Crosby M, Harwood R. Vibration control using semi-active force generators. J Manuf Sci Eng. 1974;96(2):619–626.
[9] Wilson D, Sharp R, Hassan S. The application of linear optimal control theory to the design of active automobile suspensions. Veh Syst Dyn. 1986;15(2):105–118.
[10] Yue C, Butsuen T, Hedrick JK. Alternative control laws for automotive active suspensions. In American Control Conference; 1988 June 15–17; Atlanta, GA.
[11] Sharp RS, Peng H. Vehicle dynamics applications of optimal control theory. Veh Syst Dyn. 2011;49(7):1073–1111.
[12] Smith CC, McGehee DY, Healey AJ. The prediction of passenger riding comfort from acceleration data. ASME J Dyn Syst Meas Control. 1978;100(1):34–41.
[13] Hedrick JK, Ravera RK, Amderes JR. The effect of elevated guideway construction tolerances on vehicle ride quality. ASME J Dyn Syst Meas Control. 1975;97(4):408–416.
[14] Fearnsides JJ, Hedrick JK, Firouztash H. Specification of ride quality criteria for transportation systems: the state of the art and a new approach. High Speed Ground Transp J. 1974;8(2):125–132.
[15] Hoberock LL. A survey of longitudinal acceleration comfort studies in ground transportation vehicles. ASME J Dyn Syst Meas Control. 1977;99(2):76–84.
[16] Anonymous. ISO: Mechanical vibration and shock: evaluation of human exposure to whole-body vibration. ISO 2631, International Standard Organization; 1997.
[17] Smith CC. On using the ISO standard to evaluate the ride quality of broad-band vibration spectra in transportation vehicles. ASME J Dyn Syst Meas Control. 1976;98(4):440–443.
[18] Stephens GD. Comparative vibration environments of transportation vehicles. In: Proceedings of the Design Engineering Technical Conference. Passenger Vibration in Transportation Vehicles; 1977 Sept 26–28; Chicago, IL. p. 59–72.
[19] Hrovat D. Applications of optimal control to advanced automotive suspension design. ASME J Dyn Syst Meas Control. 1993;115(2B):328–342.
[20] Hrovat D. Chapter 31: Active and semi-active suspension control. In: Mastinu G, Plöchl M, editors. Road and off-road vehicle system dynamics handbook. Boca Raton (FL): CRC Press, Taylor and Francis Group; 2014. p. 1179–1226.
[21] Boyd S, El Ghaoui L, Feron E, Balakishnan V. Linear matrix inequalities in system and control theory. Philadelphia (PA): SIAM; 1994.
[22] Fialho I, Balas GJ. Road adaptive active suspension design using linear parameter-varying gain-scheduling. IEEE Trans Control Syst Technol. 2002;10(1):43–54.
[23] Becker G, Packard A. Robust performance of linear parametrically varying systems using parametrically-dependent linear feedback. Syst Control Lett. 1994;23(3):205–215.
[24] Chen H, Guo K. An LMI approach to multiobjective RMS gain control for active suspensions. In: Proceedings of the 2001 American Control Conference; 2001 June 25–27; Arlington, VA. p. 2646–2651.
[25] Tran M, Hrovat D. Application of gain-scheduling to design of active suspensions. In: Proceedings of the 1993 IEEE Conference on Decision and Control. Vol. 2; 1993 Dec 15–17; San Antonio, TX. p. 1030–1035.
[26] Hedrick JK, Butsuen T. Invariant properties of automotive suspensions. Proc Inst Mech Eng D: J Automob Eng. 1990;204(1):21–27.
[27] Hrovat D, Margolis D. Realistic road-track systems simulation using digital computers. In: Proceedings of the Winter Simulation Conference; 1975 Dec; Sacramento, CA. p. 481–489.
[28] Hrovat D. Optimal passive vehicle suspensions [Ph.D. thesis]. Davis (CA): University of California; 1979.
[29] Tseng HE, Hedrick JK. Semi-active control laws-optimal and sub-optimal. Veh Syst Dyn. 1994;23(1):545–569.
[30] Giorgetti N, Bemporad A, Tseng HE, Hrovat D. Hybrid model predictive control application towards optimal semi-active suspension. Int J Control. 2006;79(5):521–533.
[31] Tomizuka M. Optimum linear preview control with application to vehicle suspension – revisited. ASME J Dyn Syst Meas Control. 1976;98:309–315.
[32] Louam N, Wilson DA, Sharp RS. Optimal control of a vehicle suspension incorporating the time delay between front and rear wheel inputs. Veh Syst Dyn. 1988;17(6):317–336.
[33] Louam N, Wilson DA, Sharp RS. Optimization and performance enhancement of active suspensions for automobiles under preview of the road. Veh Syst Dyn. 1992;21(1):39–63.
[34] Hrovat D. Optimal suspension performance for 2D vehicle models. J Sound Vib. 1991;146(1): 93–110.
[35] Hac A. Optimal linear preview control of active vehicle suspension. Veh Syst Dyn. 1992;21:167–195.
[36] Hac A, Youn I. Optimal semi-active suspension with preview based on a quarter car model. J Vib Acoust. 1992;114(1):84–92.

[37] Hac A, Youn I. Optimal design of active and semi-active suspensions including time delays and preview. J Vib Acoust. 1993;115(4):498–508.

[38] Van Auken RM. Active suspension control synthesis using reduced order models that account for the time delay between road inputs. In: ASME 8th Biennial Conference on Engineering Systems Design and Analysis; 2006 July 4–7; Torino, Italy. p. 451–462.

[39] Jankovic M, Kolmanovsky I. Developments in control of time-delay systems for automotive powertrain applications. In: Balachandran B, Kalmár-Nagy T, Gilsinn DE, editors. Delay Differential Equations. Berlin: Springer; 2009. p. 55–92.

[40] Thompson AG, Pearce C, Millers E. Direct computation of the performance index for an optimally controlled active suspension with preview applied to a half-car model. Veh Syst Dyn. 2001;35(2):121–137.

[41] Pilbeam C, Sharp RS. On the preview control of limited bandwidth vehicle suspensions. Proc Inst Mech Eng D: J Automob Eng. 1993;207(3):185–193.

[42] Prokop G, Sharp RS. Performance enhancement of limited-bandwidth active automotive suspensions by road preview. IEE Proc Control Theory Appl. 1995;142(2):140–148.

[43] Goran MB, Bachrach BI, Smith RE. The design and development of a broad bandwidth active suspension concept car. In: XXIV FISITA Congress. Total Vehicle Dynamics. Vol. 2, no. IMECHE C389/418; 1992. p. 231–252.

[44] Smith MC. Achievable dynamic response for automotive active suspension. Veh Syst Dyn. 1995;24(1): 1–33.

[45] Smith MC, Wang FC. Controller parameterization for disturbance response decoupling: application to vehicle active suspension control. IEEE Trans Control Syst Technol. 2002;10(3):393–407.

[46] Wang FC, Smith MC. Disturbance response decoupling and achievable performance with application to vehicle active suspension. Int J Control. 2002;75(12):946–953.

[47] Brezas P, Smith MC. Linear quadratic optimal and risk-sensitive control for vehicle active suspensions. IEEE Trans Control Syst Technol. 2014;22(2):543–556.

[48] Hrovat D, Hubbard M. Optimum vehicle suspensions minimizing RMS rattlespace, sprung-mass acceleration and jerk. J Dyn Syst Meas Control. 1981;103(3):228–236.

[49] Davis BR, Thompson AG. Optimal linear active suspensions with integral constraint. Veh Syst Dyn. 1988;17(6):357–366.

[50] Youn I, Im J, Tomizuka M. Level and attitude control of the active suspension system with integral and derivative action. Veh Syst Dyn. 2006;44(9):659–674.

[51] Karnopp D. Force generation in semi-active suspensions using modulated dissipative elements. Veh Syst Dyn. 1987;16(5–6):333–343.

[52] Lin J, Kanellakopoulos I. Nonlinear design of active suspensions. IEEE Control Syst. 1997;17(3): 45–59.

[53] Chantranuwathana S, Peng H. Adaptive robust force control for vehicle active suspensions. Int J Adapt Control Signal Process. 2004;18(2):83–102.

[54] Gordon TJ, Marsh C, Wu QH. A learning automaton methodology for control system design in active vehicle suspensions. In: International Conference on Control ’94. Vol. 1; 1994 Mar 21–24; Coventry, UK. p. 326–331.

[55] Howell MN, Frost GP, Gordon TJ, Wu QH. Continuous action reinforcement learning applied to vehicle suspension control. Mechatronics. 1997;7(3):263–276.

[56] Valasek M, Kortum W. Nonlinear control of semi-active road-friendly truck suspension. In: Proceedings of AVEC. Vol. 98; 1998 Sep 14–18; Nagoya, Japan. p. 275–280.

[57] Gordon TJ, Best MC. Dynamic optimization of nonlinear semi-active suspension controllers. In: International Conference on Control ’94. Vol. 1; 1994 March 21–24, Coventry, UK. p. 332–337.

[58] Gordon TJ, Sharp RS. On improving the performance of automotive semi-active suspension systems through road preview. J Sound Vib. 1998;217(1):163–182.

[59] Wang J, Wilson DA. Mixed GL2/H2/GH2 control with pole placement and its application to vehicle suspension system. Int J Control. 2001;74(13):1353–1369.

[60] Abdellahi E, Mehdi D, M’Saad M. On the design of active suspension system by Hinf and mixed H2/Hinf: an LMI approach. In: Proceedings of the 2000 American Control Conference; 2000 June 28–30; Chicago, IL. p. 4041–4045.

[61] Lu J, DePoyster J. Multi-objective optimal suspension control to achieve integrated ride and handling performance. IEEE Trans Control Syst Technol. 2002;10(6):807–821.

[62] De Jager AG. Comparison of two methods for the design of active suspension systems. Optim Control Appl Methods. 1991;12(3):173–188.

[63] Yamashita M, Fujimoti K, Uhlik C, Kawatani R, Kumura H. Hinf control of an automotive active suspension. In: Proceedings of the 29th IEEE Conference on Decision and Control; 1990 Dec 5–7; Honolulu, HI. p. 2244–2250.

[64] Balas GJ, Doyle JC, Glover K, Packard A, Smith R. Mu-analysis and synthesis toolbox, Matlab user’s guide. Boston (MA): MathWorks Inc; 1993.

[65] Doyle J, Packard A, Zhou K. Review of LFTs, LMIs, and Mu. In: Proceedings of the 30th IEEE Conference on Decision and Control. Vol. 2; 1991 Dec 11–13; Brighton, UK. p. 1227–1232.

[66] Gobbi M, Mastinu G. Analytical description and optimization of the dynamic behaviour of passively suspended road vehicles. J Sound Vib. 2001;245(3):457–481.

[67] Chatillon M, Jezequel L, Coutant P, Baggio P. Hierarchical optimization of the design parameters of a vehicle suspension system. Veh Syst Dyn. 2006;44(11):817–839.
[68] Chen H, Sun P, Guo K. A multi-objective control design for active suspensions with hard constraints. Proc Am Control Conf. 2003;5:4371–4376.
[69] Georgiou G, Verros G, Natsiavas S. Multi-objective optimization of quarter-car models with a passive or semi-active suspension system 45. Veh Syst Dyn. 2007;45(1):77–92.
[70] Crews J, Mattson M, Buckner G. Multi-objective control optimization for semi-active vehicle suspensions. J Sound Vib. 2011;330(23):5502–5516.
[71] Yi K, Hedrick J. Active and semi-active heavy truck suspensions to reduce pavement damage. SAE Technical Paper 892486; 1989.
[72] Cole DJ, Cebon D. Truck suspension design to minimize road damage. Proc Inst Mech Eng D: J Automob Eng. 1996;210(2):95–107.
[73] Kitching KJ, Cole DJ, Cebon D. Theoretical investigation into the use of controllable suspensions to minimize road damage. Proc Inst Mech Eng D: J Automob Eng. 2000;214(1):13–31.
[74] Cole DJ. Fundamental issues in suspension design for heavy road vehicles. Veh Syst Dyn. 2001;35(4–5): 319–360.
[75] Novak M, Valasek M. A new concept of semi-active control of truck's suspension. In: Proceedings of AVEC'96; 1996 June 24–28; Aachen, Germany. p. 141–152.
[76] Valášek M, Novak M, Šika Z, Vaculin O. Extended ground-hook-new concept of semi-active control of truck's suspension. Veh Syst Dyn. 1997;27(5–6):289–303.
[77] Valášek M, Kortüm W, Šika Z, Magdolen L, Vaculın O. Development of semi-active road-friendly truck suspensions. Control Eng Pract. 1998;6(6):735–744.
[78] Vaculın O, Valášek M, Kortüm W. Multi-objective control of semi-active truck suspensions. In: Proceedings of AVEC'2000; 2000 August 22–24; Ann Arbor, MI. p. 405–412.
[79] Valasek M, Sveda J, Sika Z. Soil-friendly off-road suspension. Veh Syst Dyn. 1998;44(Suppl 1): 479–488.
[80] Valasek M, Kejval J, Maca J, Smilauer V. Bridge-friendly truck suspension. In: Proceedings of the 18th IAVSD Symposium. Vol. 41; 2003 Aug 25–29; Atsugi, Japan. p. 13–22.
[81] MVO, Valášek KJ. Global chassis control: integration synergy of brake and suspension control for active safety. In: Proceedings of the International Symposium on Advanced Vehicle Control; 2004 Aug 23–27; Arnhem, The Netherlands. p. 495–500.
[82] van Zanten A. Control of horizontal vehicle motion. In: Mastinu G, Plöchl M, editors. Road and off-road vehicle system dynamics handbook. Boca Raton (FL): CRC Press, Taylor and Francis Group; 2014. p. 1093–1178.
[83] Papageorgiou C, Smith MC. Positive real synthesis using matrix inequalities for mechanical networks:application to vehicle suspension. IEEE Trans Control Syst Technol. 2006;14(3):423–435.
[84] Smith MC. Synthesis of mechanical networks: the inerter. IEEE Trans Automat Control. 2002;47(10): 1648–1662.
[85] Karnopp DC, Margolis DL, Rosenberg RC. System dynamics: a unified approach. 3rd/8th ed. New York: Wiley; 2000 (3rd)/ 2012 (8th).
[86] Margolis D, Wilson R. Bond graph modeling of fluid filled engine mounts. Simulation Series 31: Proceedings of the 1999 International Conference on Bond Graph Modeling and Simulation (ICBGM '99): [held as part of] 1999 Western Multi Conference; 1999 January 17–20; San Francisco, CA. p. 211–216.
[87] Singh R, Adiguna H, Tiwari M, Tseng H, Hrovat D. Transient response evaluation of a hydraulic engine mount. J Sound Vibration. 2002;36(7):30–35.
[88] Evangelou S, Limebeer DJN, Sharp RS, Smith MC. Control of motorcycle steering instabilities. IEEE Control Syst Mag. 2006;26(5):78–88.
[89] Smith MC, Wang FC. Performance benefits in passive vehicle suspensions employing inerters. Veh Syst Dyn. 2004;42(4):235–257.
[90] Gillespie TD. Fundamentals of vehicle dynamics. Warrendale (PA): Society of Automotive Engineers; 1992.
[91] Malek KM, Hedrick JK. Decoupled active suspension design for improved automotive ride quality/handling performance. Veh Syst Dyn. 1986;15(Suppl 1):383–398.
[92] Hayakawa K, Matsumoto K, Yamashita M, Suzuki Y, Fujimori K, Kimura H. Robust H∞-output feedback control of decoupled automobile active suspension systems. IEEE Trans Automat Control. 1999;44(2): 392–396.
[93] Venhovens PJT. Optimal control of vehicle suspensions [Ph.D. Thesis]. Delft: Delft University of Technology; 1993.
[94] Best M, Gordon T, Dixon P. An extended adaptive Kalman filter for real-time state estimation of vehicle handling. Veh Syst Dyn. 2000;34:57–75.
[95] Jeppesen BP, Cebon D. Analytical redundancy techniques for fault detection in an active heavy vehicle suspension. Veh Syst Dyn. 2004;42(1–2):75–88.
[96] Nastasić Ž, Deák Jahn G. Citroen technical guide [Internet]. Available from: http://www.citroenkerho.fi/xantia/pdf/tekniikka/Tekniikkaopas.pdf
[97] Chance BK. 1984 Continental mark VII/Lincoln continental electronically-controlled air suspension (EAS) System. SAE Technical Paper 840342; 1984.

[98] Edmunds D. 2012 Tesla Model S signature performance suspension walkaround [Internet]. 2012 Sep 26 [cited 2015 Mar]. Available from: http://www.edmunds.com/car-reviews/track-tests/2012-tesla-model-s-signature-performance-suspension-walkaround.html
[99] Korosec K. Potholes and Tesla's model S: never the twain shall meet [Internet]. 2014 Sep 19 [cited 2015 Mar]. Available from: http://fortune.com/2014/09/19/tesla-model-s-suspension-upgrade/
[100] Kojima H, Nakano J, Nakayama H, Kawashima N, Fujimoto H. Development of new Toyota electronic modulated suspension-two concepts for semi-active suspension control. SAE Technical Paper 911900; 1991.
[101] Nicolas R. Intelligent suspension system of Lincoln MKZ help mitigating pothole damage [Internet]. 2014 Mar 20. Available from: http://www.car-engineer.com/intelligent-suspension-system-lincoln-mkz-help-mitigating-pothole-damage/
[102] Lincoln Drive Control: CCD. Lincoln Motor Company [Internet]. Available from: http://www.lincoln.com/cars/mks/features/?featID = 1#page = Feature1
[103] Bodie MO, Hac A. Closed loop yaw control of vehicles using magneto-rheological dampers. SAE Technical Paper 2000-01-0107; 2000.
[104] Hac A, Bodie MO. Improvements in vehicle handling through integrated control of chassis systems. Int J Veh Autonomous Syst. 2002;1(1):83–110.
[105] Fodor M, Redfield RC. Resistance control, semi-active damping performance. In: ASME Dynamic Systems and Control Division, International Mechanical Engineering Congress and Exposition; 1995 Nov. 12–17; San Francisco, CA. p. 161–169.
[106] Fodor M, Redfield RC. Experimental verification of resistance control, semi-active damping. Veh Syst Dyn. 1996;26(2):143–159.
[107] Sharp RS. Variable geometry active suspension for cars. Comput Control Eng J. 1998;9(5):217–222.
[108] Watanabe Y, Sharp RS. Mechanical and control design of a variable geometry active suspension system. Veh Syst Dyn. 1999;32(2–3):217–235.
[109] Tumova G. Delft active suspension (DAS-II) [Internet]. Research report OND1281145. Available from: http://www.narcis.nl/research/RecordID/OND1281145
[110] Venhovens PT, van der Knaap AM. Delft active suspension (DAS) background theory and physical realization. In: Pauwelussen JP, Pacejka HB, editors. Smart vehicles. Amsterdam: Swets & Zeitlinger Publishers; 1995. p. 139–165.
[111] van der Knapp A. Design of a low power anti-roll/pitch system for a passenger car. Delft University of Technology, Vehicle Research Laboratory, Report 89.3VT.2628.
[112] Venhovens P, van der Knaap A, Pacejka H. Semiactive vibration and attitude control. In: Proceedings of the International Symposium on Advanced Vehicle Control, AVEC '92; 1992 Sept 14–17; Yokohama, Japan. p. 170–175.
[113] Evers WJ, Besselink IJM, van der Knaap ACM, Nijmeijer H. Analysis of a variable geometry active suspension. In: Proceeding of International Symposium on Advanced Vehicle Control; 2008 Oct 6–9; Kobe International Conference Center, Kobe, Japan. p. 350–355.
[114] Packer MB. Active ride control – a logical step from static vehicle attitude control. SAE Technical Paper 780050; 1978.
[115] Crolla DA, Horton DNL, Pitcher RH, Lines JA. Active suspension control for an off-road vehicle. Proc Inst Mech Eng D: J Automob Eng. 1987;201(1):1–10.
[116] Raad J, Oliver J, Norton R, Kokotovic V. Method of improving response time in a vehicle active tilt control system. US Patent 5,948,028. 1999.
[117] Raad J, Oliver J, Rhode D, Norton R, Hermann S, Watson J. Method of controlling a vehicle active tilt control system. US Patent 6,076,027. 2000.
[118] Konik D, Bartz R, Barnthol F, Bruns H, Wimmer M. Dynamic drive – the new active roll stabilization system from BMW group – system description and functional improvements. In: Proceeding of International Symposium on Advanced Vehicle Control; 2000 Aug 22–24; Ann Arbor, MI. p. 413–420.
[119] Hrovat D, Tseng E, Fodor M, Asgari J. Chapter 22: Active and semiactive suspension systems. In: Mastinu G, Plöchl M, editors. Road and off-road vehicle system dynamics handbook. Boca Raton (FL): CRC Press, Taylor and Francis Group; 2014. p. 769–796.
[120] Hillebrecht P, Konik D, Pfeil D, Wallentowitz H, Zieglmeier F. The active suspension between customer benefit and technological competition. In: Proceedings of XXlV FISITA Congress. Total Vehicle Dynamics. Vol. 2; 1992 June 7–11; London. p. 221–230.
[121] Akatsu Y, Fukushima N, Talcahashj K, Satch M, Kawaranki Y. An active suspension employing an electrohydraulic pressure control system. SAE Technical Paper 905123; 1990.
[122] Alleyne A, Liu R. On the limitations of force tracking control of hydraulic servosystems. ASME J Dyn Syst Meas Control. 1999;121(2):184–190.
[123] Zhang Y, Alleyne A. A practical and effective approach to active suspension control. In: Proceedings of the 6th International Symposium on Advanced Vehicle Control; 2002 Sep 9–13; Hiroshima, Japan. p. 153–158.
[124] Peng H, Strathearn R, Ulsoy A. A novel active suspension design technique – simulation and experimental results. In: Proceedings of the 1997 American Control Conference. Vol. 1; 1997 June 4–6; Albuquerque, NM. p. 709–713.

[125] Merker T, Gaston G, Olaf T. Active Body Control (ABC) the DaimlerChrysler active suspension and damping system. SAE Technical Paper 2002-21-0054; 2002.
[126] Merker T, Wirtz J, Hill M, Jeglitzka M. Das SL Fahrwerk-Dynamik und Komfort vereint (The SL chassis-dynamics and comfort combined). ATZ/MTZ Magazine Special Issue; 2001. p. 84–91.
[127] Kruse J. Mercedes-Benz NEXT online magazine [Internet]. 2014 June 4. Available from: http://next.mercedes-benz.com/en/mbc-eng/
[128] Goran M, Smith R. Insights gained from active suspension development. In: Proceedings of the 26th FISITA Congress; 1996 June 16–23; Prague, Czech Republic. p. 2486–2514.
[129] Davis RI, Patil PB. Electrically powered active suspension for a vehicle. US Patent 5,060,959; 1991 Oct 29.
[130] Sugasawa F, Kobayashi H, Kakimoto T, Shiraishi Y, Tateishi Y. Electronically controlled shock absorber system used as a road sensor which utilizes supersonic waves. SAE Technical Paper 851652; 1985.
[131] Streiter IR. Active preview suspension system. ATZ Worldwide. 2008;110(5):4–11.
[132] Rauh J, Ammon D. System dynamics of electrified vehicles: some facts, thoughts, and challenges. Veh Syst Dyn. 2011;49(7):1005–1020.
[133] Mercedes-Benz S550. Consumer reports; 2014 Sep. p. 53.