Mixed Sky-Hook and ADD: Approaching the Filtering Limits of a Semi-Active Suspension

Sergio M. Savaresi[1]

Cristiano Spelta

Dipartimento di Elettronica e Informazione,
Politecnico di Milano,
Piazza L. da Vinci, 32,
20133, Milano, Italy

The problem considered in this paper is the design and analysis of control strategies for semiactive suspensions in road vehicles. The most commonly used control algorithm is the well-known sky-hook (SH) damping. Recently, a new control approach named acceleration driven damping (ADD) has been developed, using optimal-control theory. It has been shown that SH and ADD have complementary characteristics: SH provides large benefits around the body resonance; otherwise performs similarly to a passive suspension; instead, ADD provides large benefits beyond the body resonance. The first goal of this paper is to show that—in their specific frequency domains—SH and ADD provide quasi-optimal performances, namely, that it is impossible to achieve (with the same semi-active shock-absorber) better performances. This result has been obtained using the framework of the optimal predictive control, assuming full knowledge of the disturbance. This result is very interesting since it provides a lower-bound to semi-active suspension performances. The second goal of the paper is to develop a control algorithm which is able to mix the SH and ADD performances. This algorithm is surprisingly simple and provides quasi-optimal performances. [DOI: 10.1115/1.2745846]

Keywords: semi-active suspensions, sky-hook damping control, ADD damping control, optimal control, nonlinear systems, predictive control, two-state damper

1 Introduction

Among the many different types of controlled suspensions (see e.g. [1–25]), semi-active suspensions have received a lot of attention in the last two decades, since they seem to provide the best compromise between cost (energy-consumption and actuators/sensors hardware) and performance. Their advantage is that they only require shock-absorbers with controllable damping coefficient, which are known to be comparatively economic and to absorb a negligible amount of power for control purposes (as a matter of fact the modulation of the damping coefficient is made using low-voltage and low-current electric circuits). On the other hand, the main limit of semi-active suspensions is the well-known "passivity-constraint;" it is impossible to deliver vertical forces on the body having the same direction of the suspension relative elongation speed.

The concept of semi-active suspensions can be applied over a wide range of application domains: road vehicles suspensions, cabin suspensions in trucks or agricultural tractors, seat suspensions, lateral suspensions in high-speed trains, suspensions of appliances (e.g., washing machines), architectural suspensions (buildings, bridges, etc.) etc. This work focuses on road vehicle suspensions.

Many newly-designed cars are already equipped with semi-active suspensions, and a lot of academic and industrial research activities have been undergoing in this area (see e.g. [2,4,8,9,11–17,24–36,60]). The research activity is being developed along two mainstreams: the development of reliable, high-performance, and cost-effective semi-active controllable shock-absorbers (Electro-Hydraulic or Magneto-Rheologic, see e.g. [37,6,38,34]), and the development of control strategies and algorithms which can fully exploit the potential advantages of controllable shock-absorbers. This work is focused on the control-design issue.

The design of control algorithms for semi-active suspensions typically is made according to the sky-hook (SH) framework. Sky-hook damping is an ideal concept, which assumes that the shock absorber can deliver a force proportional to the chassis speed only (and not proportional to the chassis-wheel relative speed). Even if an ideal SH damping cannot be implemented with semi-active suspensions, the SH behavior is typically approximated using two-state (switching) or continuously-variable shock-absorbers. Linear and two-state SH algorithms have a strongly empirical flavor, but they are widely used since they are simple and are known to provide good results.

Recently, a new control approach named acceleration driven damping (ADD) has been developed ([17,33]). ADD has been developed on a theoretical basis, rooted on optimal-control theory.

It has been shown in [17] that SH and ADD have complementary characteristics: SH provides large benefits around the first (body) resonance; otherwise performs similarly to a passive suspension. On the other hand, ADD provides large benefits beyond the first resonance; around the first resonance performs similarly to a passive suspension.

This paper has two major goals.

- The first goal is to develop an optimality-analysis of SH and ADD; in particular, it will be shown that—in their specific frequency domains—SH and ADD provide quasi-optimal performances, namely, it is impossible to achieve (with the same semi-active shock-absorber) better performances. This result has been obtained using the framework of the optimal predictive control, assuming full knowledge of the disturbance ([39]). This result is interesting since it provides a lower-bound to the filtering capabilities of a controllable semi-active suspension.
- The second goal of the paper is to develop an algorithm which is able to mix the SH and the ADD performances. The algorithm proposed herein is surprisingly simple and

[1] Corresponding author.

Contributed by the Dynamic Systems, Measurement, and Control Division of ASME for publication in the Journal of Dynamic Systems, Measurement, and Control. Manuscript received June 1, 2005; final manuscript received November 20, 2006. Review conducted by Hemant M. Sardar.

provides an optimal mix of SH and ADD. Given the results of the optimality analysis, this algorithm can be considered a very close approximation of the best possible algorithm for semi-active suspensions, when comfort is the control goal.

The analysis presented herein is developed in the following setting:

- The control objective is the minimization of the vertical acceleration of the car; this is the so-called "comfort-objective" (most of the semi-active suspensions are designed to achieve primarily this control goal); however, notice that this analysis framework can be extended also to mixed control objectives.
- A single-suspension (quarter-car) control problem is considered. Focusing on a single suspension allows a much better understanding of the limit of performances of a semi-active suspension; the full-body control system then can be designed following the same rationale, by a suitable coordination of the four suspensions (some results obtained on a half-car model are briefly presented in Sec. 5).

In order to assess and to compare the closed-loop performance of the three control strategies discussed in this paper, both a theoretical and a numerical (simulation-based) approach are used. For the numerical approach, a frequency-domain method is proposed, but also the more classical time-domain analysis is used. The performance assessment is made both on a simple semi-active shock-absorber model, and on a real-like sophisticated model, validated on real data.

In the literature, the problem of searching an optimal solution for the design of semi-active control algorithms has been already addressed. In this respect, the work [35] (which is rooted in [11]) is particularly significant. These papers have been already considered and discussed in [17], which is the starting point of this work.

The outline of this paper is the following: in Sec. 2 the quarter-car model of a suspension equipped with a controllable shock-absorber is briefly described and modeled, and the SH and ADD algorithms are recalled. In Sec. 3 the attenuation limit of a semi-active suspension is computed, using a numerical procedure based on Optimal-Predictive control. In Sec. 4 the mixed SH-ADD control algorithm is developed and discussed. An extensive numerical analysis is presented in Sec. 5.

2 Previous Work and Problem Statement

The dynamic model of a quarter-car system equipped with semi-active suspensions can be described with the following set of differential equations (see e.g. [40–43,17]):

$$M\ddot{z}(t) = -c(t)(\dot{z}(t)-\dot{z}_t(t)) - k(z(t)-z_t(t)-\Delta_s) - Mg$$

$$m\ddot{z}_t(t) = +c(t)(\dot{z}(t)-\dot{z}_t(t)) + k(z(t)-z_t(t)-\Delta_s) - k_t(z_t(t)-z_r(t) - \Delta_t) - mg \quad [z_t(t)-z_r(t)<\Delta_t] \tag{1}$$

$$\dot{c}(t) = -\beta c(t) + \beta c_{in}(t) \quad c_{\min} \le c_{in}(t) \le c_{\max}$$

The symbols used in (1) have the following meaning: $z(t)$, $z_t(t)$, $z_r(t)$ are the vertical positions of the body, the unsprung mass, and the road profile, respectively; M is the quarter-car body mass; m is the unsprung mass (tire, wheel, brake caliper, suspension links, etc.); β is the bandwidth of the active shock-absorber; k and k_t are the stiffness of the suspension spring and of the tire, respectively; Δ_s and Δ_t are the length of the unloaded suspension spring and tire, respectively; the limit $[z_t(t)-z_r(t)<\Delta_t]$ means that the tire can be modeled as a linear symmetric spring as long as it keeps the contact with the road surface; $c(t)$ and $c_{in}(t)$ are the actual and the requested damping coefficients of the shock-absorber, respectively. Note that, thanks to the assumption of first-order dynamics of the damping-coefficient variations, if the control variable is limited ($c_{\min} \le c_{in}(t) \le c_{\max}$), also the actual damping coefficient (which is a state variable of (1)) remains in that interval ($c_{\min} \le c(t) \le c_{\max}$). This limitation is the so-called "passivity-constraint" of a shock-absorber. $c_{\min}$ and $c_{\max}$ are design parameters of the semi-active shock-absorbers ($c_{\max}$ can be almost freely designed; a very low value of $c_{\min}$ is more difficult to achieve, and represents a typical design goal for shock-absorbers manufacturers).

The model (1) represents a fifth-order dynamic system. It is constituted of three differential equations: a second order equation which models the dynamics of the body-mass; a second order equation which models the dynamics of the unsprung mass; a first order equation which models the dynamics of the damping-coefficient variations. Model (1) is standard, and provides an accurate description of the quarter-car vertical dynamics.

With reference to this model, the following remarks are due:

- Model (1) is nonlinear; the nonlinearity is due to the fact that the damping coefficient $c(t)$ is a state variable. The model of the passive suspension, characterized by a fixed nominal damping coefficient $\bar{c}$, can be obtained from (1) by simply setting: $\beta \to \infty$ and $c_{in}(t)=\bar{c}$. In this case the quarter-car model is linear and time-invariant.
- The model adopted for the active shock-absorber differs from the ideal behavior ($c(t) \equiv c_{in}(t)$) since it is assumed that the actual damping coefficient $c(t)$ follows the requested damping coefficient $c_{in}(t)$ with first-order low-pass dynamics (see Remark at the end of this section). The bandwidth of the low-pass-dynamics is β. Even if some nonlinear effects are neglected, this model is a simple but realistic approximation of the behavior of a semi-active damper. A much more detailed model of the damper will be used, for performance assessment in a real setting, at the end of Sec. 5 ([29]).

The general high-level structure of a comfort-oriented control architecture for a semi-active suspension device is the following: the control variable is the requested damping coefficient $c_{in}(t)$; the measured output signals are the vertical acceleration $\ddot{z}(t)$ of the car body, and the suspension displacement $z(t)-z_t(t)$; the controlled variable is the chassis vertical acceleration $\ddot{z}(t)$. Usually it is assumed that the road profile is a nonmeasurable and unpredictable disturbance (no road preview by sonar, laser, or video camera is available).

The design of a control algorithm delivering the best possible performance is a nontrivial issue, since (1) is nonlinear with respect to the control variable $c_{in}(t)$, and since $c_{in}(t)$ is limited by strongly asymmetric thresholds ([44–48]).

The starting point for this work are two control strategies: the sky-hook (SH) and the acceleration-driven-damping (ADD). These two algorithms are now briefly recalled.

(1) **SH control** The two-state approximation of the sky-hook control algorithm requires a two-state damper; the control law is given by

$$\begin{aligned} c_{in}(t) &= c_{\max} \quad \text{if } \dot{z}(\dot{z}-\dot{z}_t) \ge 0 \\ c_{in}(t) &= c_{\min} \quad \text{if } \dot{z}(\dot{z}-\dot{z}_t) < 0 \end{aligned} \tag{2}$$

SH control is the semi-active heuristic approximation of the ideal concept of sky-hook damping (see e.g. [22,23]); it is the most widely used control strategy in semi-active suspension systems. If a continuous modulation of the damping coefficient is possible, a slightly more sophisticated expression of the SH algorithm can be implemented (see e.g. [17,22]). It is easy to understand that the difference between the two-state and the continuous SH algorithms is remarkable only if the switching time of a two-state damper is comparative large. For fast-switching components, the two algorithms tend to provide similar results.

(2) **ADD control** The implementation of ADD control requires a two-state damper; the control law is given by

$$\begin{aligned} c_{in}(t) &= c_{\max} \quad \text{if } \ddot{z}(\dot{z}-\dot{z}_t) \geq 0 \\ c_{in}(t) &= c_{\min} \quad \text{if } \ddot{z}(\dot{z}-\dot{z}_t) < 0 \end{aligned} \tag{3}$$

ADD control has been recently developed in [17], using optimal-control theory; it has been proven to be the optimal control strategy, when the goal objective is the minimization of the body vertical acceleration, the road surface is completely unpredictable, and the optimization is made on a single-step horizon only. Interestingly enough, the SH and the ADD algorithms have a very simple (and similar) structure.

In practice, the switching rules (2) or (3) are implemented in a digital-control setting; they are implemented by updating the value of $c_{in}(t)$ at the sampling time $t=k\Delta T$ ($k \in Z$), and by keeping its value constant in the ΔT time-window.

The SH and ADD algorithms have been compared in [17]. Here the main result of that analysis is recalled.

A simple and intuitive way of analyzing the closed-loop performance of a comfort-oriented semi-active suspension is to compute and to inspect the magnitude of the frequency response (FR) $F_{\text{acc}}(j\omega)$ from the road-disturbance input z_r to the body-acceleration output $\ddot{z}$ ($F_{\text{acc}}(s)$ being the corresponding transfer function). This transfer function provides a clear frequency-domain picture of the attenuation (filtering) capabilities of the suspension.

This FR can be computed easily for a passive suspension, since the relationship between the disturbance z_r and the output $\ddot{z}$ is linear and time-invariant. Unfortunately, all the semi-active control strategies are strongly nonlinear, since they are based on a switching rationale. Henceforth, the notion of frequency response is not well-defined for such closed-loop systems ([49]).

An approximate FR for the nonlinear semi-active control systems can be computed using the notion of *variance gain* (see e.g., [50–55]). In order to compute numerically this function, the following procedure has been used: The closed-loop system is fed with a finite set of N pure tones. $z_{r_i}(t)=A\sin(\omega_i t)$; $i=1,2,\ldots,N$, $t\in[0,T]$. The corresponding output signals are recorded: $\ddot{z}_i$, $i=1,2,\ldots,N$, $t\in[0,T]$. The approximate ***variance gain*** is computed in the N available points,

$$\hat{F}_{\text{acc}}(j\omega_i) = \sqrt{\frac{\frac{1}{T}\int_0^T (\ddot{z}_i(t))^2 dt}{\frac{1}{T}\int_0^T (z_{r_i}(t))^2 dt}} \quad i=1,2,\ldots,N \tag{4}$$

The *variance gain* is a simple analysis tool which provides an approximation of the FR. Note that it is the square-root of the average power amplification experienced by a pure-tone signal when passing through the nonlinear plant. If the system is linear and time-invariant, the variance gain coincides with the magnitude of the frequency response of the plant. When the system is nonlinear, it provides a measure of the energy-attenuation provided by the suspension, when it is excited by a pure-tone input. Note that the notion of variance gain is very similar to that of *describing function* (see e.g. [56]); the variance gain however is more suited to the analysis of systems (like a suspension), when energy-filtering is the main goal. As a matter of fact, notice that the variance gain (4) takes into account all the energy of the output acceleration, and not only the energy at the fundamental frequency (as the describing function does).

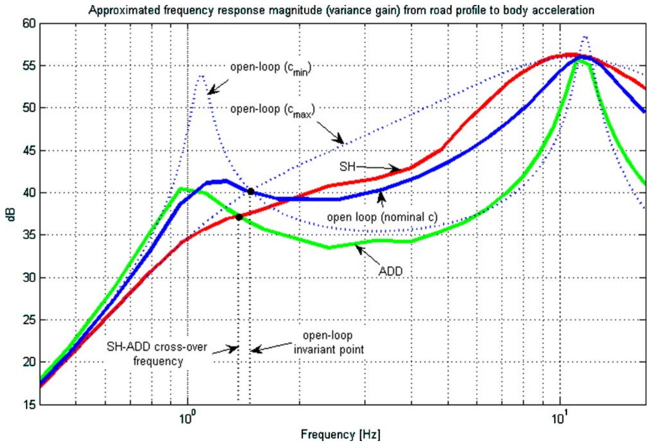

Fig. 1 ADD and SH performance

The numerical evaluation of the control algorithms has been done by computing *variance gain* of the *closed-loop control system* from the disturbance z_r to the output $\ddot{z}$. The following set of parameters has been used, throughout the paper: $M=400$ kg, $m=50$ kg, $k=20$ kN/m, $k_t=250$ kN/m, $\bar{c}=1500$ N s/m, $c_{\min}=300$ N s/m, $c_{\max}=4000$ N s/m, $\beta=30\pi$, $T=100$ s, $A=0.05$ m, $N=30$, $\omega_i/(2\pi)$ are distributed in the range 0.2–20 Hz. The control algorithms have been digitally implemented using a sampling time $\Delta T=2$ ms.

The comparison of the SH and the ADD control algorithms is displayed in Fig. 1, where the approximated FRs from the road profile to the body acceleration are shown. This figure provides a very clear picture of the performances of SH and ADD: SH provides a remarkable attenuation benefit around the first (body) resonance frequency (located at about 1.5 Hz); around that frequency ADD provides little or no benefits, if compared with a passive suspension; ADD provides a remarkable attenuation benefit beyond the first resonance frequency; beyond that frequency the filtering performances of the SH strategy are similar to those of the standard passive suspension.

This analysis shows some peculiar complementarities of SH and ADD; there is a frequency (it will be called SH-ADD crossover frequency throughout the paper), located just beyond the first resonance frequency, which separates the frequency ranges where SH and ADD provide their best results. Before the crossover frequency SH is best; beyond it, ADD outperforms SH.

This result has been already extensively commented and discussed in [17]. However, two major issues and questions have been left unanswered:

(1) How far are SH and ADD from the best possible filtering performances achievable by a semi-active suspension?
(2) Is it possible to find a simple control strategy which behaves as SH before the cross-over frequency and as ADD beyond it?

The goal of the next two sections is to provide an answer to the above key issues.

Remark (Passive Suspension Open-Loop Baseline). When assessing the performance of a control algorithm for semi-active suspensions, it is always useful to consider a passive suspension as "baseline." Throughout this paper, the open-loop passive suspension used as baseline is characterized by a nominal damping ratio $\bar{c}=1500$ N s/m. This value provides a reasonable compromise (see e.g. [10]) between the damping of the resonances and the disturbance attenuation at intermediate and high frequencies.

Considering this open-loop nominal suspension as baseline, it has been observed that the SH algorithm provides very good re-

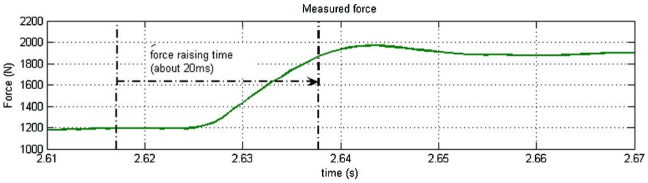

Fig. 2 Response of an electrohydraulic semi-active suspension to a switching command on the damping ratio

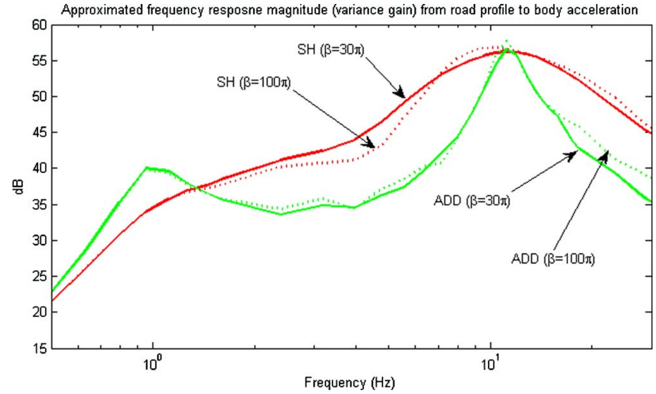

Fig. 3 ADD and SH performance, for two different switching-times of the shock-absorber

sults around the body resonance frequency, whereas there is little or no improvement beyond that frequency. Dual conclusions can be drawn for the ADD algorithm.

It is interesting to note that, by taking a "hard" tuning of the damper as baseline (see in Fig. 1 the curve characterized by $c_{\max}=4000$ N s/m), the SH algorithm seems to take a completely different "flavor": around the body resonance provides little or no improvements (the passive suspension is already very well-damped), but it provides superior performance in terms of noise attenuation at intermediate and high frequencies. Similarly, if we compare ADD with a passive baseline characterized by a very "soft" damper tuning (see in Fig. 2 the curve characterized by $c_{\min}=300$ N s/m), it provides little or no improvements at intermediate and high frequencies, but guarantees superior damping around the first resonance.

Obviously, ADD and SH performances are always the same, whatever is the passive baseline. The passive baseline simply provides a perspective and a point of view, which is useful for understanding the key advantage of a semi-active suspension: improving the performance of the suspension somewhere, without paying a bill in complementary circumstances or in complementary frequencies ranges (as is typical for passive suspensions).

Remark (Response Time of Semi-Active Shock Absorbers). In model (1) it is assumed that the relationship between the requested and the actual damping ratio ($c_{in}(t)$ and $c(t)$, respectively), can be modeled as a first order unitary-gain low-pass filter with bandwidth β. The choice of this bandwidth is critical for the overall performance of the semi-active control algorithms (the higher the bandwidth, the better the expected performance).

In order to get a feeling of the typical bandwidth of commercially available semi-active shock absorbers, in Fig. 2 the measured response of a fast-switching semi-active damper is presented (these measurements have been done on a test-bench, at constant compression speed). The effect on the actual damping ratio can be observed on the measured force delivered by the damper. The raising time of the force can be estimated in about 20 ms. This corresponds to a value of about $\beta=50\pi$ in model (1). In the rest of the paper a slightly slower component has been considered ($\beta=30\pi$, corresponding to a raising time of about 35 ms). In order to understand the effect of β on the performance of a semi-active control algorithms, in Fig. 3 the performance of SH and ADD for the standard value of β ($\beta=30\pi$) and for a very fast-reacting shock-absorber ($\beta=100\pi$) are displayed. As expected, the faster damper provides better performance (notice that this is particularly true for SH at intermediate frequencies). The differences in performance however are comparatively small. This is mainly due to the fact that 35 ms can be already considered a remarkably short raising time.

Remark (Sensor Noise). A possibly critical issue in the implementation of semi-active control algorithms is the noise affecting the measured variables used by the algorithms: the body acceleration $\ddot{z}(t)$, the body speed $\dot{z}(t)$, and the relative speed of the suspension $\dot{z}(t)-\dot{z}_t(t)$. To this end, in Fig. 4, a snapshot of real measurements collected on a motorbike are displayed in the time domain over a 1 s time window. All the signals are measured with 14 bit 1 kHz A/D converters, and preprocessed with low-pass filtering at 50 Hz. By inspecting these signals, it is clear that the acceleration signal is more noisy than the velocity signals. However, the noise level on the acceleration is sufficiently low to avoid high-frequency zero crossings.

From the signals in Fig. 4 it is apparent that, for mild road excitation conditions, all the signals are characterized by a lot of zero-crossings, since they are centered around zero. This may cause some "chattering" phenomena on the control strategies (high-frequency state switching of the damper). With respect to this issue, the following comments are due:

- The chattering phenomenon may occur also in velocity-based algorithms like SH (it is not a peculiar feature of acceleration-based algorithms).
- Since the control variable is simply the damping ratio of a passive component, and since at low speed (around zero) the force delivered by the shock absorber are very low, the chattering phenomenon has no practical effect on the performance of the control algorithm.
- In order to avoid the chattering effect (to decrease the stress on the control variable in a semi-active damper), a "dead-zone" can be added. The use of a dead-zone has the effect of strongly decreasing the number of commutations of the damping ratio, without reducing, in practice, the performance of the control algorithms. Alternatively, the same aim can be achieved by undersampling one of the control signals (for instance the acceleration), as presented and proposed in [57].

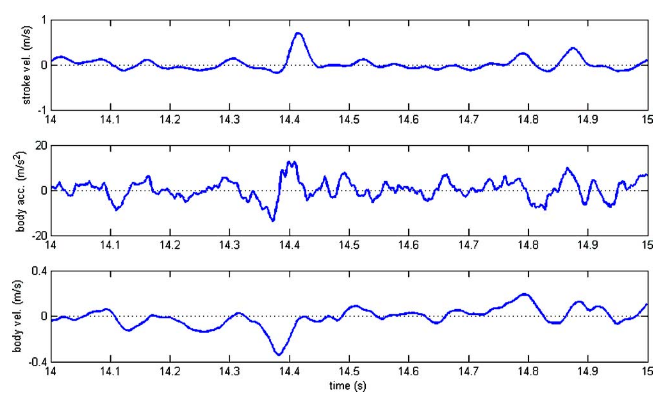

Fig. 4 Measured acceleration and velocities (real data, taken from a motorbike negotiating a standard road surface)

3 Optimal-Predictive Control: The Benchmark

Assume a pure-comfort control goal, and assume that the main characteristics of the semiactive shock-absorber ($c_{\min}$, $c_{\max}$, and the bandwidth β) are fixed. Moreover, assume that the controllable shock-absorber is a two-state (on-off) damper, namely that $c_{in}(t)=\{c_{\min},c_{\max}\}$ (note that in [17] it has been theoretically proven that this is a nonrestrictive assumption, provided that the sampling time ΔT of the digital control algorithm is sufficiently small). The design problem of the control algorithm hence can be reduced to the following: consider a time window $[0,T]$, a fixed initial condition, and a given road profile $\bar{z}_r(t), t\in[0,T]$; consider the global performance index $J=\frac{1}{T}\int_0^T \cdot z(t)^2dt$; find the sequence of digital control inputs $c_{in}(1\Delta T), c_{in}(2\Delta T)\cdots c_{in}(k\Delta T),\ldots c_{in}(H\Delta T)$, $H=T/\Delta T$, which minimizes J. The solution of the above control problem provides the best possible control strategy for the semi-active suspension.

In practice, it is impossible to implement in real-time such a globally-optimal control strategy on a real suspension; the main reason is that the road profile is unknown, and only a "blind" (one-step-ahead) optimal solution can be obtained. Moreover, even if an a priori complete knowledge of $\bar{z}_r(t)$, $t\in[0,T]$ is assumed, the optimization task is formidable: the optimal solution must be searched among 2^H possible sequences of digital control inputs (if $T=10$ s and $\Delta T=10$ ms, this means that $H=1000$, which makes the optimization task almost impossible to be dealt with).

Notice that, given this very general point of view, SH and ADD can be regarded as suboptimal solutions of the general optimal-control problem; however, the suboptimality drawback is compensated by the fact that they do not require any knowledge of the disturbance in advance, and that they are computationally very low-demanding.

Even if the optimal-control problem stated above cannot be solved in a real-time implementation, it can be solved numerically offline if we make the simplifying assumption that the road profile is a pure sinusoid $z_{r_i}(t)=A\sin(\omega_i t)$ $i=1,2,\ldots,N$, $t\in[0,T]$.

As a matter of fact, under this assumption the optimization task can be restricted to a semiperiod of the sinusoid, namely $H=N$, where $N=(\pi/\omega_i)/\Delta T$. Unfortunately, if the frequency of the road profile ranges from 0.2 to 20 Hz and ΔT is kept constant, at low frequencies the optimization task remains formidable. In order to solve this numerical-optimization issue, the key idea is to choose a fixed value of the number of switches in a half-period (say N), and to change the sampling (switching) time according to the road frequency. It can be shown that $N=8$ provides a quasi-optimal result (namely, the additional filtering benefit of using $N>8$ is negligible).

Using $N=8$, the global-optimization task is very tough, but it becomes tractable and the approximate frequency response of the optimal-predictive-control strategy can be computed offline. The results of this numerical analysis are condensed in Fig. 5.

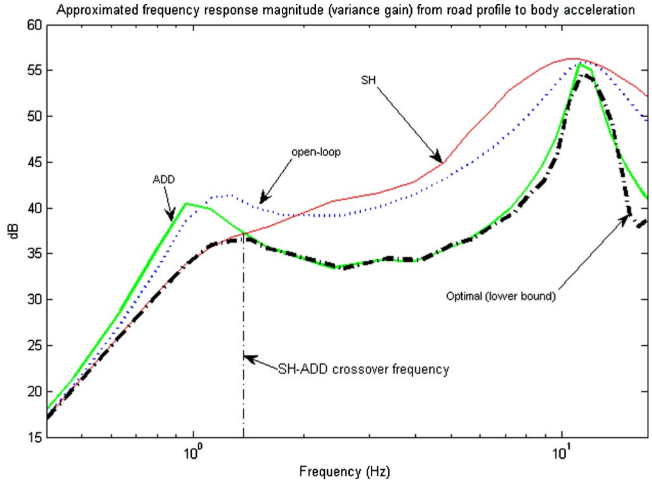

Fig. 5 Comparison of the filtering performance of SH, ADD, and the numerically-computed optimal lower bound

The results displayed in Fig. 5 can be easily interpreted; they are somewhat surprising: before the SH-ADD crossover frequency the SH provides the best possible filtering performances; beyond the crossover frequency the ADD provides the best possible filtering performances.

Notice that this result is very surprising since—as already pointed out—ADD and SH do not assume any knowledge of the future road profile (whereas the optimal-predictive controller assumes full knowledge of the disturbance), and their control law is extremely simple.

Given this result, a natural question arises: Is it possible to build a simple real-time control algorithm which inherits the simplicity of ADD and SH, and provides the performance of SH and ADD in their respective domains of "best" behavior? Such an algorithm would be a very close approximation to the intrinsic filtering limit of a semi-active suspension. This problem will be considered in the following section.

4 Mixed SH-ADD Control: The Algorithm

At the end of the previous section, it has been shown that SH and ADD provide the best possible filtering performance, below and beyond the crossover frequency, respectively. The problem is to develop a simple control algorithm capable of mixing their features and performances. The mixing algorithm proposed in this paper is as follows.

Mixed SH-ADD Control. Given a two-state damper, the control law is given by

$$\begin{cases} c_{in}(t)=c_{\max} \text{ if } [(\ddot{z}^2-\alpha^2\dot{z}^2)\le 0 \wedge \dot{z}(\dot{z}-\dot{z}_t)>0] \quad \vee \quad [(\ddot{z}^2-\alpha^2\dot{z}^2)>0 \quad \wedge \quad \ddot{z}(\dot{z}-\dot{z}_t)>0] \\ c_{in}(t)=c_{\min} \text{ if } [(\ddot{z}^2-\alpha^2\dot{z}^2)\le 0 \quad \wedge \quad \dot{z}(\dot{z}-\dot{z}_t)\le 0] \quad \vee \quad [(\ddot{z}^2-\alpha^2\dot{z}^2)>0 \quad \wedge \quad \ddot{z}(\dot{z}-\dot{z}_t)\le 0] \end{cases} \tag{5}$$

The control law proposed above is extremely simple, since—similarly to SH and ADD—it is based on a simple static rule, which makes use of $\dot{z},\ddot{z},(\dot{z}-\dot{z}_t)$ only. Equation (5) is characterized by a design parameter α; the value of α represents the desired crossover frequency (in rad/s) between SH and ADD. Hence, for a standard automotive suspension, α must be chosen around 11 rad/s (1.8 Hz).

It is interesting to observe that (5) selects, at the end of every sampling interval, the SH or the ADD rule, according to the current value of $(\ddot{z}^2-\alpha^2\dot{z}^2)$. If $(\ddot{z}^2-\alpha^2\dot{z}^2)>0$, then the ADD strategy is selected; otherwise the SH strategy is used. The amount $(\ddot{z}^2-\alpha^2\dot{z}^2)$ hence can be considered as a simple "frequency-range selector."

In Fig. 6 the mixed SH-ADD algorithm is compared with the

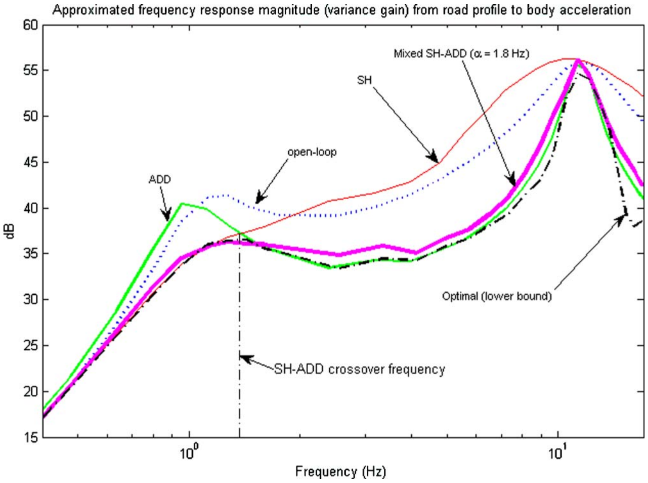

Fig. 6 Comparison of the filtering performance of SH, ADD, and mixed SH-ADD

other results previously obtained. The performances of this algorithm are almost optimal; it is able to provide an almost-perfect mix of the best performance of SH and ADD. As such, notice that its performances are very close to the lower bound: it is impossible to find another control algorithm which performs significantly better, in terms of body-acceleration variance.

Notice that the key idea of (5) is condensed in $(\ddot{z}^2-\alpha^2\dot{z}^2)$. In practice, this function turns out to be a simple but very effective frequency-range selector. Its effectiveness however is not based on heuristic reasoning, but it can be given a rational explanation. Two simple but very insightful interpretations of the effectiveness of $(\ddot{z}^2-\alpha^2\dot{z}^2)$ as frequency-range selector are now proposed.

First Interpretation: Single-Tone Disturbance. Consider the single-tone periodic signal $\dot{z}(t)=A\sin(\omega t)$. If the function $f(t)$ is defined as $f(t)=\ddot{z}(t)^2-\alpha^2\dot{z}(t)^2$, $\alpha\in\Re_+$, by plugging $\dot{z}(t)=A\sin(\omega t)$ into $f(t)$ we obtain:

$$f(t)=A^2\omega^2-A^2\sin^2(\omega t)(\alpha^2+\omega^2) \tag{6}$$

Consider now the problem of studying the positivity of $f(t)$ over one period of this function (the period being $T(\omega)=\pi/\omega$). It is easy to see that

$$f(t)>0 \quad \Rightarrow \quad \sin^2(\omega t)<\frac{\omega^2}{\omega^2+\alpha^2} \tag{7}$$

If we call $D_+(\omega)=\{t:f(t)>0,0\le t\le T\}$ the domain where $f(t)>0$, it is easy to see that the measure of this "positivity-domain," say $|D_+(\omega)|$, is given by (see also Fig. 7(a))

$$|D_+(\omega)|=\frac{2T}{\pi}\arcsin\left(\sqrt{\frac{\omega^2}{\omega^2+\alpha^2}}\right) \tag{8}$$

Hence, according to (8), the following holds:

$$\frac{|D_+(\omega)|}{T}\to 0 \text{ if } \omega\ll\alpha$$

$$\frac{|D_+(\omega)|}{T}\to 1 \text{ if } \omega\gg\alpha$$

$$\frac{|D_+(\omega)|}{T}=\frac{1}{2} \text{ if } \omega=\alpha$$

This means that, over a period T, $f(t)>0$ for more that $T/2$ if $\omega>\alpha$; $f(t)<0$ for more that $T/2$ if $\omega<\alpha$. Hence, $f(t)$ can be considered as a simple frequency-selector, centered around the frequency $\omega=\alpha$: when $f(t)>0$ we can assume that $\omega>\alpha$; otherwise $\omega<\alpha$. Since the function $|D_+(\omega)|/T$ rapidly saturates towards 0 or 1, when $\omega\neq\alpha$ (see Fig. 7(b)), a good frequency-selection quality is guaranteed.

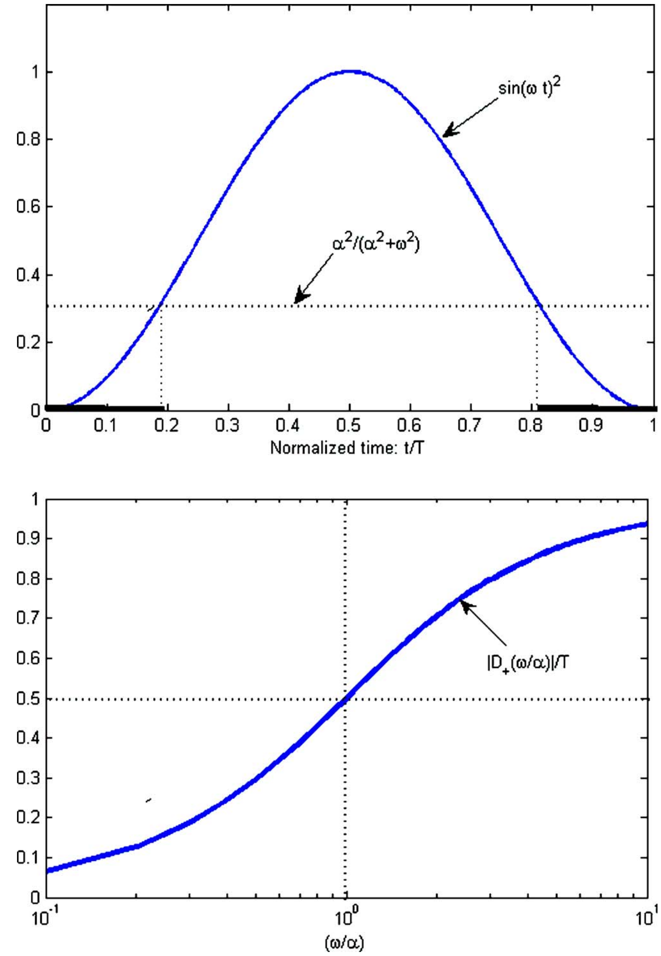

Fig. 7 Pictorial analysis of the inequality (8); (b) function $|D_+(\omega)|/T$ (in the normalized frequency)

Second Interpretation: Broadband Disturbance. Consider now a generic signal $\dot{z}(t)$. Note that the function $f(t)=\ddot{z}(t)^2-\alpha^2\dot{z}(t)^2$, $\alpha\in\Re_+$, can be rewritten as follows:

$$f(t)=(\ddot{z}(t)-\alpha\dot{z}(t))(\ddot{z}(t)+\alpha\dot{z}(t)) \tag{9}$$

From (9), it is easy to see that

$$\begin{aligned}
f(t)<0 &\Rightarrow \begin{cases}\ddot{z}(t)<\alpha\dot{z}(t)\\ \ddot{z}(t)>-\alpha\dot{z}(t)\end{cases} \text{ or } \begin{cases}\ddot{z}(t)>\alpha\dot{z}(t)\\ \ddot{z}(t)<-\alpha\dot{z}(t)\end{cases}\\
f(t)>0 &\Rightarrow \begin{cases}\ddot{z}(t)>\alpha\dot{z}(t)\\ \ddot{z}(t)>-\alpha\dot{z}(t)\end{cases} \text{ or } \begin{cases}\ddot{z}(t)<\alpha\dot{z}(t)\\ \ddot{z}(t)<-\alpha\dot{z}(t)\end{cases}
\end{aligned} \tag{10}$$

Consider now the following differential equations:

$$\begin{cases}\ddot{z}(t)=\alpha\dot{z}(t)\\ \ddot{z}(t)=-\alpha\dot{z}(t)\end{cases} \tag{11}$$

Note that the equations in (11) represent a first-order autonomous linear dynamic system; since $\alpha\in\Re_+$, $\ddot{z}(t)=\alpha\dot{z}(t)$ is unstable, and $\ddot{z}(t)=-\alpha\dot{z}(t)$ is asymptotically stable. Note that the position of the pole of this system is at α(rad/s) (which can be considered the

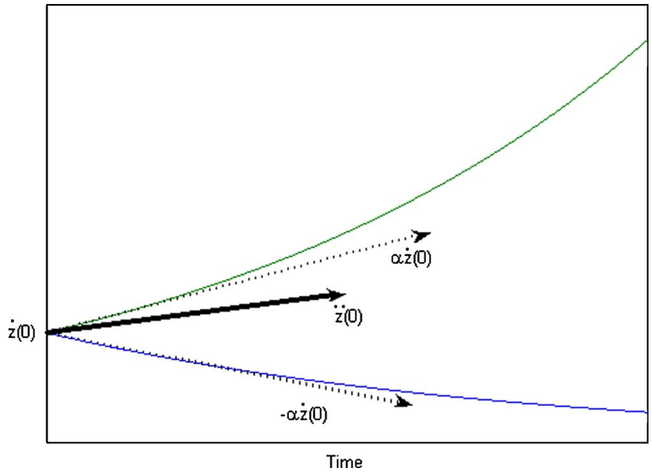

Fig. 8 Example of evolution of the autonomous systems $\ddot{z}(t) = \alpha\dot{z}(t)$ and $\ddot{z}(t) = -\alpha\dot{z}(t)$ (starting from $\dot{z}(0) > 0$)

cutoff frequency of this dynamic system). The evolution in the time-domain of this linear system, starting from a positive initial condition, is displayed in Fig. 8 ([58]).

Assume that the initial condition $\dot{z}(0) > 0$ (the same reasoning can be done also in the case $\dot{z}(0) < 0$). In this case, note that (10) can be simplified as

$$f(t) < 0 \Rightarrow -\alpha\dot{z}(t) < \ddot{z}(t) < \alpha\dot{z}(t)$$
$$f(t) > 0 \Rightarrow \ddot{z}(t) > \alpha\dot{z}(t) \quad or \quad \ddot{z}(t) < -\alpha\dot{z}(t) \tag{12}$$

The expression (12) can be easily interpreted. Note that when $f(t) < 0$ the system evolves with dynamics slower than the cutoff frequency of (11); when $f(t) > 0$ the system evolves with dynamics faster than the cutoff frequency of (11). Hence, $f(t)$ can be regarded as a simple frequency-separator, even for the case of generic signals.

5 Mixed SH-ADD Control: Numerical Analysis

In this section a numerical analysis of the mixed SH-ADD algorithm proposed in the previous section is developed. The starting point of this analysis is the plot of Fig. 6, where a comparison of the filtering performance of SH, ADD, and mixed SH-ADD is displayed, in the case of a single-tone disturbances. Many additional analysis results will be proposed in this section: time domain analysis; analysis of performance of the control algorithms, when the road input is a broad-band signal; analysis of performance of the control algorithms, in terms of road-tire contact force variations (handling goal); half-car model analysis; analysis of performance of the control algorithms, using a realistic shock-absorber.

Time-Domain Analysis. In Secs. 2–4, the semi-active control algorithms have been evaluated using the tool of the *variance gain*. This analysis tool provides a very clean and condensed picture of the performance of the algorithms. It is interesting, however, to complement the variance gain analysis with a time-domain analysis.

In Fig. 9, the time-domain responses of the body acceleration when the road profile is a pure-tone signal are displayed. In particular, two single-tone disturbances have been selected: 1.25 Hz (corresponding to the body resonance frequency), and 4 Hz (representing the intermediate frequency-range). Each figure is complemented with the variances of the body-acceleration, for each control algorithm. These results essentially confirm the conclusions drawn from the frequency-domain analysis. More specifically, by inspecting Fig. 9 the following comments can be done:

- The time domain behavior of the acceleration when a semi-active control algorithm is used reveals a very strong non-linearity of the closed loop control systems. This confirms that higher harmonics must be taken into account: henceforth—as already said in Sec. 2—the variance gain tool is more appropriate than the more classical describing function.
- The SH algorithm shows superior performance (as expected) at the body resonance frequency. It is clear how SH outperforms ADD at that frequency. Notice, however, that the mixed-SH-ADD algorithm is capable to slightly outperform SH even at that frequency. It is interesting to notice that both ADD and the mixed-SH-ADD algorithms at this frequency show a fast-switching behavior. This chattering phenomenon however is not particularly obnoxious, since it does not affect the acceleration peaks or the acceleration energy.
- At intermediate and high frequencies the superior performances of ADD are confirmed; it is also interesting to notice that when the frequency increases the behavior of the closed-loop algorithms tends to become more linear.

The second time-domain analysis refers to the classical response to a step in the road profile. In Fig. 10 the response of the body acceleration is displayed. Also in this case, the conclusions which can be drawn are consistent with the previous frequency-domain analysis:

- The SH algorithm provides very good damping, but it shows the worst acceleration peak.
- ADD has little acceleration peaks, but the low-frequency damping is very similar to that of the open-loop system.
- The Mixed-SH-ADD algorithm has the same ADD acceleration peaks, but a more efficient damping at low frequencies.

Analysis of Performance of the Control Algorithms, When the Road Input is a Broad-Band Signal. In Fig. 6 it has been shown, in the frequency domain, the attenuation properties of the control algorithms, when single-tone road disturbances are used (see the procedure for the computation of the variance-gain, explained in Sec. 2). However, it is very important to understand the behavior of the three control algorithms, when a generic broad-band road disturbance is considered. More specifically, the behavior of the control algorithms has been tested assuming that the first derivative of the road profile, $\dot{z}_r(t)$, is a band-limited white noise (its bandwidth being 50 Hz). This assumption on the spectral profile of the road is standard and represents a realistic road profile (see e.g. [13,59,22,23]).

The filtering performance of the three control algorithms have been computed. These performance are condensed in Fig. 11, where the approximated frequency-responses from the input $z_r(t)$ to the output $\ddot{z}(t)$ are displayed. Note that the results displayed in Fig. 11 do not represent the variance gain, but the more classical approximated frequency response (namely, the ratio between the spectrum of the output and the input signals).

Interestingly enough, also in the case of broad-band disturbance, the main conclusions are the same:

- SH provides a remarkable attenuation benefit around the first (body) resonance frequency (placed at about 1.5 Hz); around that frequency ADD provides no benefits, if compared with a passive suspension.
- ADD provides a remarkable attenuation benefit beyond the first resonance frequency; beyond that frequency the filtering performances of the SH strategy are similar to those of a standard passive suspension.
- The mixed SH-ADD algorithm is capable of providing the performances of SH and ADD in their respective range of best behavior.

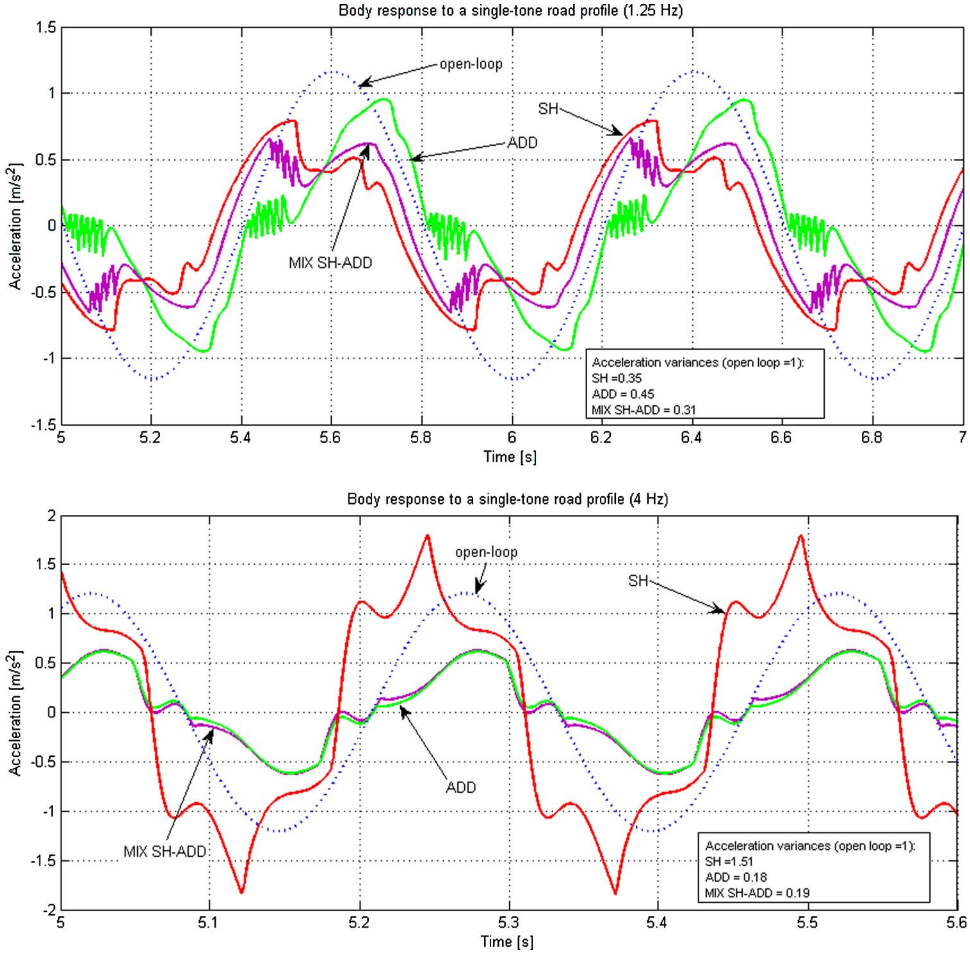

Fig. 9 Time responses of SH, ADD, and Mixed-SH-ADD to two pure-tone road disturbances (at 1.25 Hz and at 4 Hz)

Analysis of Performance of the Control Algorithms, in Terms of Road-Tire Contact Force Variations (Handling Goal). In the second numerical analysis the approximate frequency responses (FRs) from the road disturbance to the *road-tire contact force* have been computed. These FRs show the performance of the three control algorithms with respect to the so called "handling-goal," namely the goal of keeping the variations of the road-tire contact force as low as possible. The comfort goal and the handling goal are known to be conflicting; henceforth, for any comfort-oriented control algorithm, it is wise to check that the handling goal does not deteriorate too much. The handling-performance of the three control algorithms are presented in the frequency-domain in Fig. 12. Note that, the ADD algorithm outperforms the SH algorithm in the middle frequency range,

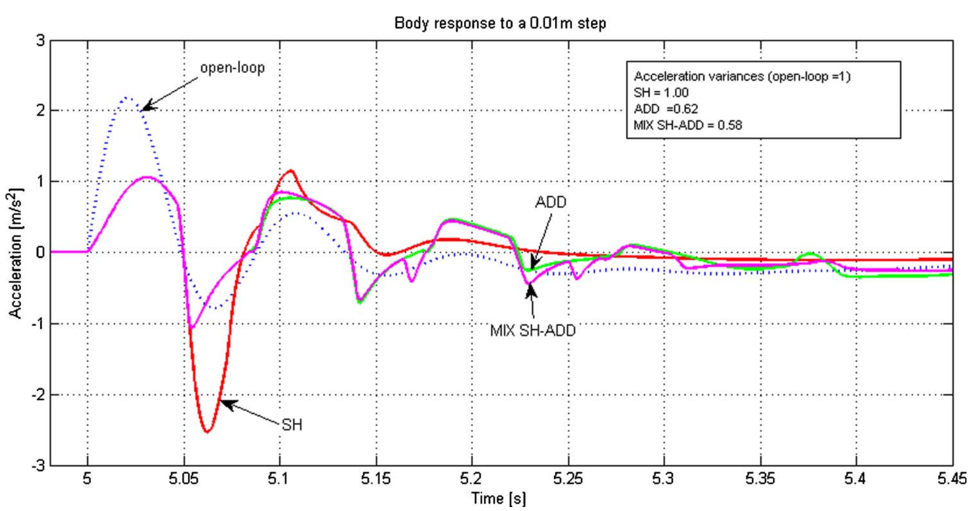

Fig. 10 Acceleration responses to a step on the road profile

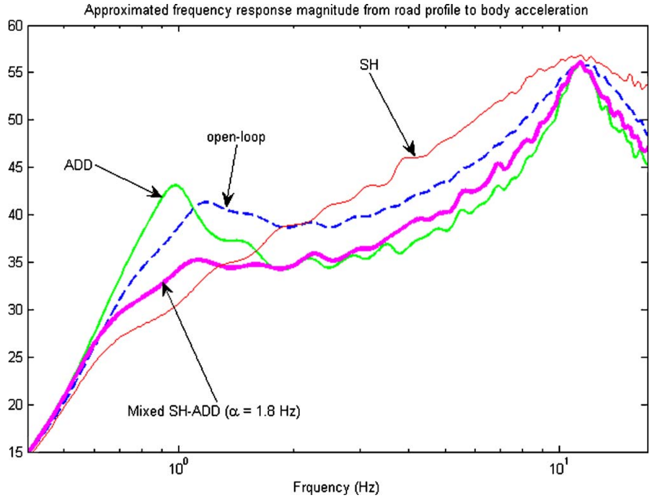

Fig. 11 Performance of the three control algorithms, when the road profile is a broad-band signal

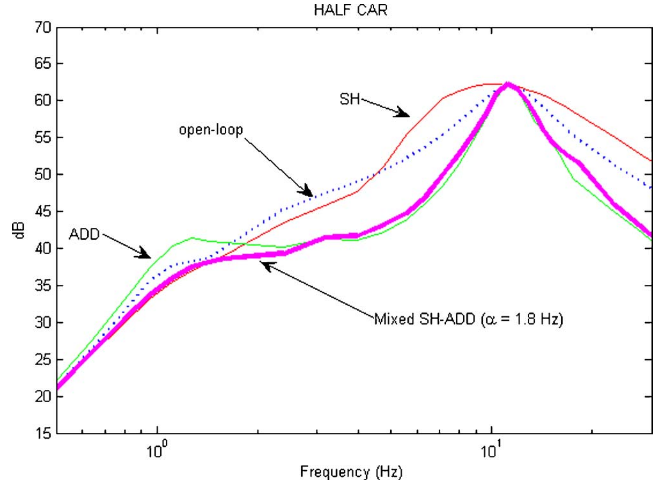

Fig. 13 Comparison of the filtering performance of SH, ADD, and mixed SH-ADD, for a half-car model

whereas it shows a worse damping around the wheel resonance. The handling performance however can be considered acceptable. As expected, the Mixed SH-ADD algorithm inherits the behavior of the SH algorithm before the crossover frequency α, and those of the ADD beyond α.

Half-Car Model Analysis. Throughout the paper, the quarter-car setting has been considered. It is well known that the quarter-car approach is very useful for theoretical design and deep understanding of semi-active control strategies. However, it is interesting to understand the behavior of a complete car, when using the three algorithms described in this paper.

In Fig. 13 the variance gain from the front-wheel road profile to the front body acceleration has been displayed. These results have been obtained using a half-car model (characterized by 4 degree-of-freedom instead of the 2 degree-of-freedom of the quarter-car model). The parameters used in the half car are those of the quarter car; the pitch momentum of inertia is $J=1000\ \mathrm{K\,gm}^2$. The three control algorithms have been applied independently and simultaneously at the front and at the rear wheel. Note that this simulation experiment represents a car placed on a 4-poster test-bench, where the excitation on the two wheels of the same axle is identical; in this way only the pitch movement is excited, whereas the roll movement is completely unexcited.

The results presented in Fig. 13 are very similar to those of the quarter car. It is interesting to observe that the pitch inertia has an additional damping effect on the vertical movement, and that the pitch resonant frequency is just beyond the body vertical frequency (the two resonances are hardly distinguishable).

Analysis of Performance of the Control Algorithms, Using a Realistic Shock-Absorber. We conclude this numerical analysis by presenting the performance of the three control algorithms, in the frequency domain, *in a real setting*. As already pointed-out, the major limit of (1) is that the model of the controllable damper is simplified. In Fig. 14 the results obtained using a high-accuracy model of the damper are displayed. The damper model used for the computation of these FRs refers to an electro-hydraulic orifice-controlled damper, which has been extensively validated using real experiments (see [29,42,38] for a complete description of the damper model). This model embodies all the nonlinear effects of a nonideal damper: nonzero soft limit; different dynamics in the compression and extension phases; hysteretic behavior; dead-zones around zero speed; saturations. The results in Fig. 14

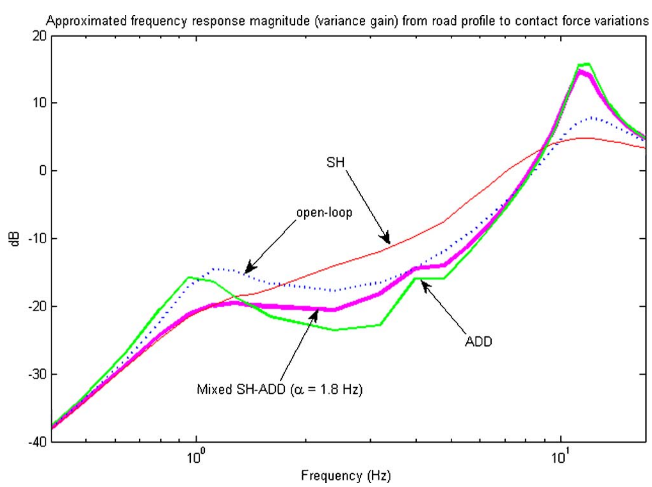

Fig. 12 Estimated frequency responses from road-disturbance to contact-force variations

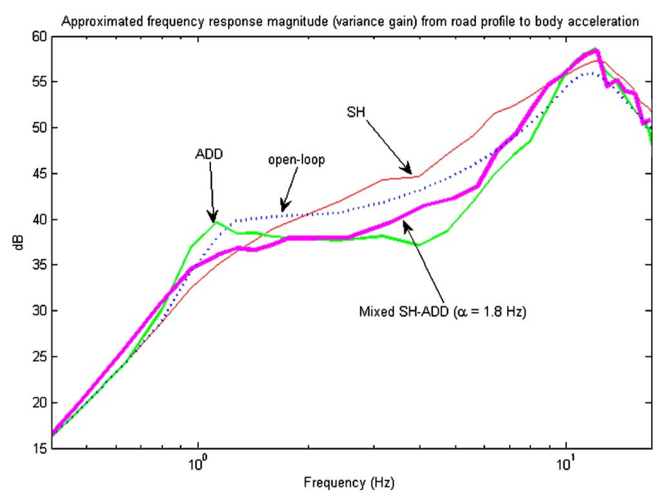

Fig. 14 Estimated frequency responses from road-disturbance to body acceleration, using a detailed damper model

henceforth can be considered a very close approximation to real experiments.

It is interesting to compare the results of Fig. 6 (ideal damper) with those of Fig. 14 (real setting). The following simple observations can be made:

- for all the control algorithms, the performance are slightly uniformly worse using the real damper (as expected);
- the performance gap between the ADD and the SH remains almost unchanged;
- the mixed SH-ADD inherits the performance of SH and ADD, before and beyond the cross-over frequency α, respectively.

6 Conclusions and Future Work

In this work a new control algorithm for semi-active suspensions have been developed, proposed, and analyzed: the mixed SH-ADD algorithm. This algorithm has the major feature of inheriting the behavior of SH and ADD, in their ranges of best behavior (namely around and beyond the first resonance, respectively).

Interestingly enough, it has also been shown that the performances of this algorithm are very close to those of an ideal globally-optimal control algorithm, which has been tested numerically using the framework of predictive-optimal control. This is somehow surprising, since the mixed SH-ADD algorithm is very simple, computationally low demanding, and does not require any future knowledge of the road profile.

Given these features, this algorithm can be regarded as a very close approximation to the best possible control algorithm which can be worked out for a semi-active suspension.

Finally, it is interesting to observe that the rationale used herein can be—in principle—extended also to mixed (comfort-handling) control objectives. This theoretical development and the practical testing of the mixed SH-ADD will be the object of future research activity.

Acknowledgment

This work has been supported by the MIUR project, "New methods for Identification and Adaptive Control for Industrial Systems." Thanks are due to Enrico Silani of CEFRIEL-Politecnico di Milano, Nicola Porciani of Ferrari, to Mauro Montiglio, Andrea Stefanini, and Pandeli Borodani of Centro Ricerche Fiat, R. Marjoram and J.D. Carlson of LORD Corp., Luca Fabbri, Mario Santucci, and Onorino di Tanna of Piaggio Group, Luca Tascedda, Francesco Taroni, Giovanni Esposito, and Massimo Ribaldone of SAME-Deutz-Fahr, Richard Käsler of Integral Accumulator KG, for enlightening discussions on the topic of semi-active suspensions.

References

[1] Campi, M. C., Lecchini, A., and Savaresi, S. M., 2003, "An Application of the Virtual Reference Feedback Tuning (VRFT) Method to a Benchmark Active Suspension System," Eur. J. Control, **9**, pp. 66–76.
[2] Caponetto, R., Diamante, O., Fargione, G., Risitano, A., and Tringali, D., 2003, "A Soft Computing Approach to Fuzzy Sky-Hook Control of Semi-Active Suspension," IEEE Trans. Control Syst. Technol., **11**(6), pp. 786–798.
[3] Choi, S. B., Choi, J. H., Lee, Y. S., and Han, M. S., 2003, "Vibration Control of an ER Seat Suspension for a Commercial Vehicle," ASME J. Dyn. Syst., Meas., Control, **125**(1), pp. 60–68.
[4] Choi, S. B., Park, D. W., and Suh, M. S., 2002, "Fuzzy Sky-Ground Hook Control of a Tracked Vehicle Featuring Semi-Active Electrorheological Suspension Units," ASME J. Dyn. Syst., Meas., Control, **124**(1), pp. 150–157.
[5] Fialho, I., and Balas, G. J., 2002, "Road Adaptive Active Suspension Design Using Linear Parameter-Varying Gain-Scheduling," IEEE Trans. Control Syst. Technol., **10**, pp. 43–54.
[6] Foo, E., and Goodall, R. M., 2000, "Active Suspension Control of Flexible-Bodied Railway Vehicles Using Electro-Hydraulic and Electro-Magnetic Actuators," Control Eng. Pract., **8**, pp. 507–518.
[7] Fujimori, K., Hayakawa, K., Kimura, H., Matsumoto, K., Suzuki, Y., and Yamashita, M., 1999, "Robust H∞ Output Feedback Control of Decoupled Automobile Active Suspension Systems," IEEE Trans. Autom. Control, **44**(2), pp. 392–396.
[8] Giuia, A., Seatzu, C., and Usai, G., 1999, "Semiactive Suspension Design With an Optimal Gain Switching Target," Veh. Syst. Dyn., **31**(4), pp. 213–232.
[9] Hong, K. S., Sohn, H. C., and Hedrick, J. K., 2003, "Modified Skyhook Control of Semi-Active Suspensions: A New Model, Gain Scheduling, and Hardware-in-the-Loop Tuning," ASME J. Dyn. Syst., Meas., Control, **124**(1), pp. 158–167.
[10] Hrovat, D., 1997, "Survey of Advanced Suspension Developments and Related Optimal Control Applications, Automatica, **33**(10), pp. 1781–1817.
[11] Hrovat, D., Margolis, D. L., and Hubbard, M., 1988, "An Approach Toward the Optimal Semi-Active Suspension," ASME J. Dyn. Syst., Meas., Control, **110**, pp. 288–296.
[12] Kawabe, T., Isobe, O., Watanabe, Y., Hanba, S., and Miyasato, Y., 1998, "New Semi-Active Suspension Controller Design Using Quasi-Linearization and Frequency Shaping," Control Eng. Pract., **6**(10), pp. 1183–1191.
[13] Kitching, K. J., Cole, D. J., and Cebon, D., 2000, "Performance of a Semi-Active Damper for Heavy Vehicles," ASME J. Dyn. Syst., Meas., Control, **122**(3), pp. 498–506.
[14] Nakai, H., Oosaku, S., and Motozono, Y., 2000, "Application of Practical Observer to Semi-Active Suspensions," ASME J. Dyn. Syst., Meas., Control, **122**(2), pp. 284–289.
[15] Sammier, D., Sename, O., and Dugard, L., 2001, "Comparison of Skyhook and H∞ Control Applied on a Quarter-Car Suspension Model," *3rd IFAC Workshop on Advances in Automotive Control*, pp. 107–112.
[16] Sammier, D., Sename, O., and Dugard, L., 2003, "Skyhook and H∞ Control of Semi-Active Suspensions: Some Practical Aspects," Veh. Syst. Dyn., **39**(4), pp. 279–308.
[17] Savaresi, S. M., Silani, E., and Bittanti, S., 2005, "Acceleration-Driven-Damper (ADD): An Optimal Control Algorithm for Comfort-Oriented Semi-Active Suspensions," ASME J. Dyn. Syst., Meas., Control, **127**(2), pp. 218–229.
[18] Silani, E., Fischer, D., Savaresi, S. M., Kaus, E., and Isermann, R., 2004, "Fault-Tolerant Filtering in Active Vehicle Suspensions," FISITA World Automotive Congress, Barcelona, Spain.
[19] Silani, E., Savaresi, S. M., Bittanti, S., Fischer, D., and Isermann, R., 2004, "Managing Information Redundancy for the Design of Fault-Tolerant Slow-Active Controlled Suspension," Tire Technology International, **2004**, pp. 128–133.
[20] Silani, E., Savaresi, S. M., Bittanti, S., Visconti, A., and Farachi, F., 2003, "The Concept of Performance-Oriented Yaw-Control Systems: Vehicle Model and Analysis," SAE Trans., **2002**, pp. 1808–1818.
[21] Valtolina, E., Savaresi, S. M., Bittanti, S., Visconti, A., and Longhi, A., 2001, "A Co-ordinate Approach for the Control of Road Vehicles," *6th European Control Conference*, Porto, Portugal, pp. 629–634.
[22] Williams, R. A., 1997, "Automotive Active Suspensions Part 1: Basic Principles," IMechE Conf. Trans., **211**, Part D, pp. 415–426.
[23] Williams, R. A., 1997, "Automotive Active Suspensions Part 2: Practical Considerations," IMechE Conf. Trans., **211**, Part D, pp. 427–444.
[24] Yoshida, K., and Okamoto, B., 1999, "Bilinear Disturbance-Accommodating Optimal Control of Semi-Active Suspension for Automobiles," *IEEE International Conference on Control Applications*, pp. 1496–1501.
[25] Active Shock Inc., 2005, Semi-Active Shock Absorber Control System, U.S. Patent No. 6,904,344.
[26] Ahmadian, M., Reichert, B. A., and Song, X., 2001, "System Nonlinearities Induced by Skyhook Dampers," Shock Vib., **8**(2), pp. 95–104.
[27] Atsugi Unisia Corporation, 1992, "Semi-Active Suspension Control System With Reduced Switching Frequency in Hard and Soft Suspension Characteristics," U.S. Patent No. 5,088,760.
[28] Bieber, M., Mackool, S. M., and Rhode, D. S., 1999, "Vehicle Suspension Control Systems, U.S. Patent No. 5,864,768.
[29] Fischer, D., and Isermann, R., 2003, "Mechatronic Semi-Active and Active Vehicle Suspensions," Control Eng. Pract., **12**(11), pp. 1353–1367.
[30] General Motors Corporation, 1991, "On/Off Semi-Active Suspension Control," U.S. Patent No. 5,062,657.
[31] Lord Corporation, 2001, "Adaptive Off-State Control Method, U.S. Patent No. 6,311,110.
[32] Lord Corporation, 2000, "No-Jerk Semi-Active Skyhook Control Method and Apparatus," U.S. Patent No. 6,115,658.
[33] Savaresi, S. M., Silani, E., Bittanti, S., and Porciani, N., 2003, "On Performance Evaluation Methods and Control Strategies for Semi-Active Suspension Systems," *42nd Control and Decision Conference*, Maui, Hawaii, pp. 2264–2269.
[34] Spencer, B. F. Jr., Dyke, S. J., Sain, M. K., and Carlson, J. D., 2003, "Phenomenological Model of a Magnetorheological Damper," ASCE J. Eng. Mech., **123**(3), pp. 230–238.
[35] Tseng, H. E., and Hedrick, J. K., 1994, "Semi-Active Control Laws—Optimal and Sub-Optimal," Veh. Syst. Dyn., **23**, pp. 545–569.
[36] Valasek, M., Kortum, W., Sika, Z., Magdolen, L., and Vaculin, O., 1998, "Development of Semi-Active Road-Friendly Truck Suspensions," Control Eng. Pract., **6**, pp. 735–744.
[37] Ahmadian, M., and Song, X., 1999, "A Non-Parametric Model for Magneto-Rheological Dampers," *Proceedings of 1999 ASME Design Engineering Technical Conference*, Las Vegas, Nevada.
[38] Savaresi, S. M., Bittanti, S., and Montiglio, M., 2005, "Identification of Semi-Physical and Black-Box Non-Linear Models: The Case of MR-Dampers for Vehicles Control," Automatica, **41**, pp. 113–117.

[39] Bertsekas, D. P., 2000, *Dynamic Programming and Optimal Control*, 2nd ed., Athena Scientific.
[40] Bosch, , 2000, *Automotive Handbook*, 5th ed., BOSCH Gmbh.
[41] Gillespie, T. D., 1992, "Fundamentals of Vehicle Dynamics, Society of Automotive Engineers Inc.
[42] Isermann, R., 2003, *Mechatronic Systems: Fundamentals*, Springer Verlag, UK.
[43] Kiencke, U., and Nielsen, L., 2000, *Automotive Control Systems for Engine, Driveline, and Vehicle*, Springer Verlag.
[44] Guardabassi, G. O., and Savaresi, S. M., 1997, "Approximate Feedback Linearization of Discrete-Time Non-Linear Systems Using Virtual Input Direct Design," Syst. Control Lett., **32**, pp. 63–74.
[45] Guzzella, L., and Isidori, A., 1993, "On Approximate Linearization of Nonlinear Control Systems, Int. J. Robust Nonlinear Control, **3**, pp. 261–276.
[46] Nijmeijer, H., and Savaresi, S. M., 1998, "On Approximate Model-Reference Control of SISO Discrete-Time Nonlinear Systems," Automatica, **34**(10), pp. 1261–1266.
[47] Savaresi, S. M., and Guardabassi, G. O., 1998, "Approximate I/O Feedback Linearization of Discrete-Time Non-Linear Systems via Virtual Input Direct Design," Automatica, **34**(6), pp. 715–722.
[48] Savkin, A. V., and Evans, R. J., 2002, *Hybrid Dynamical Systems*, Birkhauser.
[49] Guardabassi, G. O., and Savaresi, S. M., 2001, "Approximate Linearization via Feedback—an Overview," Automatica, **27**, pp. 1–15.
[50] Bittanti, S., and Savaresi, S. M., 2000, "On the Parametrization and Design of an Extended Kalman Filter Frequency Tracker," IEEE Trans. Autom. Control, **45**(9), pp. 1718–1724.
[51] Pintelon, R., and Schoukens, J., 2001, *System Identification: A Frequency Domain Approach*, IEEE, New York.
[52] Savaresi, S. M., Bitmead, R., and Dunstan, W., 2001, "Nonlinear System Identification Using Closed-Loop Data With no External Excitation: The Case of a Lean Combustion Process," Int. J. Control, **74**, pp. 1796–1806.
[53] Savaresi, S. M., Taroni, F., Previdi, F., and Bittanti, S., 2004, "Control System Design on a Power-Split CVT for High-Power Agricultural Tractors," IEEE/ASME Trans. Mechatron., **9**(3), pp. 569–579.
[54] Schoukens, J., Pintelon, R., Rolain, Y., and Dobrowiecki, T., 2001, "Frequency Response Function Measurements in the Presence of Nonlinear Distortions," Automatica, **37**, pp. 939–946.
[55] Stack, A. J., and Doyle, F. J. III, 1995, "A Measure for Control Relevant Nonlinearity," *American Control Conference*, Seattle, pp. 2200–2204.
[56] Gelb, A., and Van der Velde, W. E., 1968, *Multiple-Input Describing Functions and Nonlinear System Design*, McGraw-Hill, New York.
[57] Reed, D. H., and Radcliffe, S. T., 1991, "Output Filter and Method for On/Off Semi-Active Suspension System," U.S. Patent No. 5,024,460.
[58] Levine, W. S., 1996, *The Control Handbook*, IEEE.
[59] Robson, J. D., and Dodds, C. J., 1970, "The Response of Vehicle Component to Random Road-Surface Undulations," 13th FISITA Congress, Brussels, Belgium.
[60] Fu-Cheng, W., and Smith, M. C., 2002, "Controller Parameterization for Disturbance Response Decoupling: Application to Vehicle Active Suspension Control," IEEE Trans. Control Syst. Technol., **10**(3), pp. 393–407.