Discretionary Lane Change Model for Intelligent Connected Vehicles on Expressway

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Abstract

Intelligent connected vehicles (ICVs) are expected to improve expressway traffic mobility, safety, and green energy through sensing local environment, sharing information, and applying appropriate control measures. This paper proposes a model to express the decision of lane change for ICVs based on the advantages of information sensing, decision making, and driving operation. The model of the process of expected lane change decision is studied, and it is an improvement of the MOBIL (minimize overall braking induced by lane change), considering both transverse acceleration and longitudinal acceleration. The model of cooperative lane change decision of ICVs is based on the game theory. A numerical simulation case study is designed and exploited by MATLAB. The simulation results show that the proposed model can improve operation efficiency of traffic flow and reduce traffic jams caused by disorderly lane changing. Finally, the conclusion and future studies are presented.

INTRODUCTION

With the rapid development of Vehicle-to-Vehicle (V2V) technology, computer processing abilities and sensing technologies, ICVs have become the research hotspots and trends of Intelligent Transportation System technology. Lane change is one of the basic driving behaviors of vehicles, and research on lane change behavior is beneficial to the management of traffic flow, reducing traffic jams and ensuring traffic safety. The lane change behavior is divided into discretionary and mandatory lane change behavior. The current study on lane-changing behavior mainly focuses on mandatory lane change behavior and pays less attention to discretionary lane change behavior. This paper aims at constructing the discretionary lane change model for ICVs on expressway.

Kesting (2015) proposed a discretionary lane change model in which the acceleration is used as the basis for the judgement of generation and feasibility of lane change. The model also combines incentive criterion and safety criterion. It considers the change of acceleration of FV and PFV due to lane change behavior, and introduces a courtesy coefficient as the weight of the change of the FVs' acceleration, which avoids low efficiency caused by lane change behavior and large disturbance to other vehicles.

Bin Ran analyzed the initial and terminal phases of vehicle lane change decision preparation based on the NGSIM vehicle track data. According to the relative movement relationship between SV, and LV, PLV and PFV, the lane change decision model is constructed.

The lane change behavior is a behavior of moving the vehicle to an adjacent lane after analyzing a series of traffic environments such as the speed of the surrounding vehicle, the use of the road, and the traffic management. During the lane change process, the vehicle needs to complete the lateral movement required for the lane change on the one hand, and on the other hand, in the longitudinal movement, it is necessary to consider the following relationship with the preceding vehicle on the two lanes of the original lane and the target lane, and the influence of the lane and the target lane after the car. In addition, radical and unreasonable lane changes have a major impact on driving delays and driving safety.

This paper proposes a model to express the decision of lane change for ICVs according to information sensing, decision making and driving operation, and then construct a process for the discretionary lane change of ICVs. The model of the process of the expected lane change decision is studied, and shows it is an improvement of the MOBIL, considering both transverse acceleration and longitudinal acceleration. The model of cooperative lane change decision of ICVs is based on the game theory.

ANALYSIS OF DECISION-MAKING MECHANISM OF ICVs

Assumptions

(1) Not consider communication delays - The information exchange between components of ICVs is real-time and does not consider communication delay between vehicles, which means that within the communication range, vehicles can send and receive information in real time.

(2) Not consider errors of measurement and information transmission -Information acquired and sent by vehicles is accurate.

(3) The communication range of ICVs is 200m - The maximum communication distance of DSRC is 1000m without being disrupted. In the general 802.11p communication protocol, the DSRC communication range is 200m that can meet the need of vehicles to stay in touch with surrounding vehicles when ICVs are running at a high speed.

Lane Change Decision Process

The process of discretionary lane change decision for ICVs can be divided into three parts: whether to expect a change lane, whether there is a need for a cooperative lane change, or to make final driving decisions (Tehrani et al. 2015; Kelsch et al. 2015)

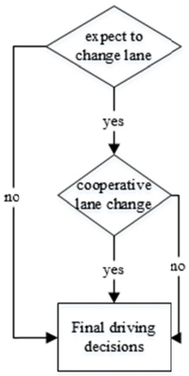

Figure 1. Lane change decision process.

MODEL OF THE PROCESS OF EXPECTED LANE CHANGE DECISION

The Improved MOBIL

MOBIL only considers the influence of SV on FV and PFV (Schubert 2011). However, the change of FV and PFV cannot reflect the influence of lane change of SV on traffic flow. This paper proposes an improved MOBIL considering both transverse acceleration and longitudinal acceleration of FV and PFV, which will reduce the influence of lane change on traffic flow (Hou et al. 2015). MOBIL considers two aspects from earnings and safety of lane change and proposes two rules.

Safety Criterion Model

The traditional lane change decision model uses the acceptance of gap to judge the safety of a lane change and outlines the elements as follows:

(1) Position restriction

When SV changes lanes, it cannot cross the road boundary and should stay between the center lines of the two lanes.

$$0 < \mathrm{y}(\mathrm{t}) < w, \forall t \in \left[0, t_f\right] \tag{1}$$

Where $y(t)$ is the lateral position of vehicle, w is the width of vehicle, it is usually is 3.75m, and t_f is the time used by SV when it changes lane.

(2) Speed restriction

When SV changes lanes, its transverse speed and longitudinal speed cannot exceed the maximum allowable speed of the current road, and it is always more than 0.

$$\begin{aligned} 0 \le v_x(t) \le v_{x\max} \\ 0 \le v_y(t) \le v_{y\max} \end{aligned} \tag{2}$$

Where $v_x(t)$ is the longitudinal speed of SV at time t, $v_{x,\max}$ is the maximum allowable speed of the current road; $v_y(t)$ is the transverse speed of SV at time t, and $v_{y,\max}$ is the maximum allowable speed of current road.

(3) Gap restriction

When SV changes lanes, its surrounding vehicles should keep a sufficient distance to avoid the collision and provide enough driving space for lane change.

$$\begin{aligned} D(M.L_d) &= x_{Ld}(t) - x_M(t) \\ D(M, F_d) &= x_M(t) - x_{Fd}(t) \\ D(M, L_0) &= x_{L0}(t) - x_M(t) \\ D(M, F_0) &= x_M(t) - x_{F0}(t) \end{aligned} \qquad \forall t \in [0, t_f] \tag{3}$$

Where $x(t)$ is the longitudinal position of vehicle, t_f is the time used by SV when it changes lane, $D(M, L_d)$ is the distance between vehicles, $D(M,i) > d_0$, in this paper d_0 is 5m.

(4)Acceleration restriction

When SV changes lane, it is longitudinal and transverse acceleration should not be more than the maximum allowable longitudinal and transverse acceleration.

$$\begin{aligned} 0 \le a_x(t) \le a_{x.\max} \\ 0 \le a_y(t) \le a_{y,\max} \end{aligned} \tag{4}$$

Where $a_x(t)$ is the longitudinal acceleration of SV at time t, $a_{x,\max}$ is the maximum allowable longitudinal acceleration, $a_y(t)$ is the transverse acceleration of SV at time t, and ,$a_{y,\max}$ is the maximum allowable transverse acceleration.

Incentive Criterion Model

The incentive criterion model is used to analyze the lane change earnings of SV, judging whether the lane change can gain a higher speed or more driving space. In the improved MOBIL proposed in this paper, the total earnings of SV are divided into the earnings and influence of SV on traffic flow on the original and target lane. The incentive criterion model of improved MOBIL is (Nie 2017):

$$\mathrm{u}_{\mathrm{sv}} = \tilde{a}_{sv} - a_{sv} + p\left\{\sum_{i=1}^{n_c}\left(\tilde{a}_{FVi} - a_{FVi}\right) + \sum_{j=1}^{n_t}\left(\tilde{a}_{PFVj} - a_{PFVj}\right)\right\} \tag{5}$$

a_{FVi} is the current acceleration of FV i (m/s^2), $\tilde{a}_{FVi}$ is the acceleration of FV i after lane change (m/s^2), a_{PFVj} is the current acceleration of FV j (m/s^2).

The first part of the right side of the equation, $\tilde{a}_{SV}$- a_{SV}, is the increment of acceleration of SV after SV has changed the lane, indicating that SV can increase the driving environment through changing lanes.

The second part of the right side of the equation is the effect of lane change of SV on FV and PFV. The courtesy coefficient p reflects the degree of altruism of SV. P=0 represents a completely egoistic lane change vehicle, and the vehicle only considers its own income. 0<p<1 means that both the self-interest and influence on FV are taken into account. p>1 means a completely altruistic lane change vehicle. If changing lanes causes the deterioration of the entire traffic state, SV will not change lanes. If changing lanes does not improve driving state, or even worsens its own driving state, but it can significantly improve driving state of FV, SV will also change lanes. While meeting the safety criterion, when the following formula is established, SV can change lanes. If SV can obtain a higher speed, it will generate the need to change lanes. Δa_{th} is a given lane change threshold.

$$u_{sv} \geq \Delta a_{th}$$

Follow-up Model

Treiber et al. (2000) proposed the IDM（Intelligent Driver Model) model composed of two parts: the acceleration trend of vehicles of free-flow state and the deceleration trend of SV that prevents collision with LV. The formula of IDM model is as follows:

$$a_a = a\left[1 - \left(\frac{v_a}{v_0}\right)^{\delta} - \left(\frac{s^*(v_a, \Delta v_a)}{s_a}\right)^2\right] \tag{6}$$

Where a_a is the acceleration of any vehicle α (m/s^2), v_a is the speed of any vehicle α (m/s), s_a is the following distance of any vehicle α (m), Δv_a is the speed difference between any vehicle α and its LV (m/s), *a* is the maximum acceleration of SV (m/s^2), δ is the acceleration index, v_0 is the expected speed of SV (m/s).

$a\left[1-(v_a-v_0)^\delta\right]$ is the acceleration of free flow, $a(s*(v_a,\Delta v_a)/s_a)$^2 is the brake deceleration and depends the expected following distance $s*(v_a,\Delta v_a)$ and real following distance s_α of SV. The formula of expected following distance $s*(v_a,\Delta v_a)$ is as follows:

$$s*(v_a,\Delta v_a)=s_0+v_aT+\frac{v_a\Delta v_a}{2\sqrt{ab}} \tag{7}$$

Where s_0 is the index of static safety distance, *T* is the safety headway (s), *b* is the comfortable deceleration of vehicle (m/s^2).The formula of s_a and Δv_α is as follows:

$$\begin{aligned} s_a &= \left\|x_{a-1}-x_a\right\|-l_a \\ \Delta v_a &= v_a - v_{a-1} \end{aligned} \tag{8}$$

Where x_a is the longitudinal position of any vehicle α(m), x_{a-1} is the longitudinal position of LV of any vehicle α(m), l_a is the length of any vehicle α(m), v_{a-1} is the speed of the LV of any vehicle α(m/s).

Maxime Guériau proposed an improved IDM model, considering the influence of three leading vehicles on SV.

$$a_a=f_{IDM}\left(v_a,\sum_{j=0}^{2}m_js_{a-j},\sum_{j=0}^{2}m_j\Delta v_{a-j}\right) \tag{9}$$

In this paper, the distance between SV and its leading vehicle is $s_{sv,}$ the relative speed is Δv_{SV}, the distance between SV-1 and its leading vehicle SV-2 is $s_{sv-1,}$ the relative speed is $\Delta v_{\mathrm{SV}-1}$,$s_{\mathrm{SV}-j}$、$\Delta v_{\mathrm{SV}-j}$ are the distance and relative speed of SV-j.

The weight coefficient is composed of two parts: the proximity of the vehicle and the movement of the front guide, and the reliability of the information.

$$m_j = p_j T_j \tag{10}$$

Where, p_j is the sports proximity between SV-j and its leading vehicle, T_j is the information reliability between SV-j and its leading vehicle, and T_j is 1 regardless of mistake of measurement and communication transmission.

$$\sigma_j = \frac{\left|\Delta v_{sv-j}\right|}{\sqrt{s_{sv-j}}}, \ \ p_j = \frac{\sigma_j}{\sum_{k=0}^{2} \sigma_k} \tag{11}$$

MODEL OF COOPERATIVE LANE CHANGE DECISION

This paper proposes the model of cooperative lane change decision based on game theory when SV and other vehicles want to change lanes at the same time.

Classification and Model of Cooperative Lane Change

There are four types of situations of the cooperative lane change between two vehicles: SV and LV have the same target gap; the target gap of SV is adjacent to that of LV; SV makes a cross lane change with LV; SV makes a symmetric collinear lane change with LV. When it comes to cooperative lane change of ICVs, we usually use three game theory elements: participants, strategies and payment to describe. The set strategies are {change lane, not change lane}. We use the double matrix method to study the four types of lane change situations. This paper gives an example of the first type: SV and LV have the same target clearance.

Type 1: SV and LV Have The Same Target Clearance And Target Lane.

Type 1 is that SV and PV are expected to have the same target gap and target lane. The analysis of the basic elements of participants, strategies, and payments in the game is as follows:

(1) Participants: SV and LV on the original lane.

(2) Strategy: The policy set of SV and LV are both {changing lanes, not changing lanes}, and are respectively recorded as θSV and θLV .

(3) Payment: We use the acceleration change value of SV and surrounding affected vehicles to calculate the earnings of SV and LV under different strategic combinations.

The matrix of SV and LV is shown in Table 1 below. We classify the lane change and the lane change strategy of SV as θSV1 and θSV2, and the lane change and lane change strategies of LV are recorded as θLV1 and θLV2. Therefore, in the cooperative lane change game between SV and LV, there are four sets of strategy combinations (lane change, lane change), (lane change, no lane change), (no lane change, lane change), and (no lane change, no lane change). The first element in each set of policy combinations represents the strategy of SV and the second element represents the strategy of LV. The payment combinations corresponding to the four policy combinations are (uSV1, uLV1), (uSV2, uLV2), (uSV3, uLV3), and (uSV4, uLV4). The first element in each set of payment combinations represents the earnings of SV under the corresponding strategy combination, and the second element represents the earnings of LV under the corresponding strategy combination. The payment function of SV and LV under each group of strategy combinations is studied in turn below.

Table 1.Cooperative Lane Change Game Theory Matrix of SV and LV

Behavior		LV	
		Lane change θ_{LV1}	No lane change θ_{LV2}
V	Lane change θ_{SV1}	(u_{SV1},u_{LV1})	(u_{SV2},u_{LV2})
	No lane change θ_{SV2}	(u_{SV3},u_{LV3})	(u_{SV4},u_{LV4})

SV and LV Change Lane at the Same Time.

SV adjusts its relative movement with LV to maintain the same speed and acceleration with LV. The headway is the minimum safe distance S_{safe}. We can treat SV and LV as a whole, an ICV with a speed of V_{1v}, acceleration of A_{1v}, and length of L_w, which means we can use the expected lane change model to analyze its safety and earnings.

$$l_w = l_a + s_{safe}, s_{safe} = v_{LV} T + s_0 \qquad (12)$$

First, we judge the safety of lane change wholly according to the safety criterion model. If those formulas are false, SV and LV cannot change lanes at the same time. The payment is: $u_{SV1}=-\infty$、$u_{LV1}=-\infty$. If those formulas are true, then judge whether the earnings of LV and SV are more than the assumed lane change threshold based on the incentive criterion model. If the earnings are not more than the threshold, SV and LV cannot change lanes, the payment is: u_{SV1}=-∞、u_{LV1}=-∞. If the incentive formula is true, the payment is like the following:

$$\begin{aligned} u_{sv1} &= \tilde{a}_{sv} - a_{sv} + p\left[\sum_{i=1}^{n_c}(\tilde{a}_{FVi} - a_{FVi}) + \sum_{j=1}^{n_T}(\tilde{a}_{PFVj} - a_{PFVj})\right] \\ u_{LV1} &= \tilde{a}_{LV} - a_{LV} + P\left[\sum_{i=2}^{n_{CLV}}(\tilde{a}_{LFVi} - a_{LFVi}) + (\tilde{a}_{SV} - a_{SV}) + \sum_{j=1}^{n_{TLV}}(\tilde{a}_{LPFVj} - a_{LPFVj})\right] \end{aligned} \quad (13)$$

SV Changes Lane and LV Does Not Change Lane.

The situation equals to the expected lane change decision model. We do not need to judge the safety and earnings of SV, since SV has already meet the need of safety and incentive criterion. The payment of SV is:

$$u_{SV2} = \tilde{a}_{SV} - a_{SV} + p\left[\sum_{i=1}^{n_C}(\tilde{a}_{FVi} - a_{FVi}) + \sum_{j=1}^{n_T}(\tilde{a}_{PFVj} - a_{PFVj})\right] \quad (14)$$

LV does not change lanes and the payment is its change of acceleration:

$$u_{LV2} = \tilde{a}_{LV} - a_{LV} \quad (15)$$

SV Does Not Change Lane and LV Changes Lane.

We do not need to judge the safety and earnings of LV since LV has already meet the need of safety and incentive criterion. The payment of LV is:

$$u_{LV3} = \tilde{a}_{LV} - a_{LV} + p\left[\sum_{i=1}^{n_{CLV}}(\tilde{a}_{LFVi} - a_{LFVi}) + \sum_{j=1}^{n_{TLV}}(\tilde{a}_{LPFVj} - a_{LPFVJ})\right] \quad (16)$$

SV does not change lanes and the payment is it's change of acceleration:

$$u_{SV3} = \tilde{a}_{SV} - a_{SV} \quad (17)$$

SV and LV Do Not Change Lanes.

If SV and LV do not change lanes together, we can calculate their acceleration for the next time according to the improved IDM model. The payment of SV and LV are their change of acceleration:

$$u_{SV4} = \tilde{a}_{SV} - a_{SV}, u_{LV4} = \tilde{a}_{LV} - a_{LV} \quad \textit{(18)}$$

It should be noted that u_{SV3} and u_{SV4} are not equal. Their leading vehicles are different which means the acceleration calculated through the improved IDM is not equal too.

Calculation of Model

Nash Equilibrium and Pareto Advantage.

Nash equilibrium is an equilibrium solution of non-cooperation, complete information and static games proposed by John Forbes Nash in 1950. When no one can change strategy to increase profits, this strategy combination of all participants is called Nash Equilibrium and defined by mathematical language is as the follows:

In a game theory of N participants, if there is a strategic combination consisting of all participants $\theta^*=(\theta^*_1,\theta^*_2,\ldots,\theta^*_i,\ldots,\theta^*_n)$ which make strategy θ^*_i of any participants are the optimal strategy $\theta^*_{-i}=(\theta^*_1,\theta^*_2,\ldots,\theta^*_{i-1},\theta^*_{i+1},\ldots,\theta^*_n)$ for any other participants, then θ^* is a Nash Equilibrium of N participants.

There is not only one Nash Equilibrium in a game. In the cooperative lane change decision model proposed in this paper, we need to choose one and there is only one optimal strategic combination. Therefore, we can choose an optimal strategic combination with the most Pareto advantage from many Nash Equilibrium's according to Pareto efficiency criterion.

Pareto efficiency was proposed by Italian economist Vilfredo Pareto more than a hundred years ago. If all the resources that can be allocated have been fully utilized, and there is no other state that can increase the interest of at least one party without harming the interests of any other parties, then in this state, Pareto efficiency is realized. Correspondingly, if we change the current state, we can improve the interests of at least one party without harming the interests of either party. Then we call it Pareto improvement.

The principle of Pareto efficiency can be expressed in the following mathematical way: Assume that in a two-person, non-cooperative, complete information static game, participants are recorded as P1 and P2, respectively, and each participant has two alternative strategies. If there are two Nash Equilibrium in a game, they are recorded as strategic combination (θ_a,θ_b) and (θ_c,θ_d), and their payment are (u_a,u_b) and $(u_c.u_d)$. According to Pareto efficiency criterion, the following formula is established:

$$\begin{aligned} &u_a \geq u_c and u_b \geq u_d \\ &u_b \geq u_d and u_a \geq u_d \end{aligned} \tag{19}$$

Strategic combination from (θ_a, θ_b) to (θ_c, θ_d) is a Pareto improvement and we choose (θ_c, θ_d) with Pareto advantage as the most optimal strategic combination.

Calculation Steps of Nash Equilibrium.

The model of cooperative lane change decision constructed in this paper is a 2×2 double matrix game theory model. According to the algorithm idea proposed by Yu (1999), the model of cooperative lane change decision in this paper steps of game theory are as follows:

Step1: Construct a payment matrix U_i for each participant i=1, 2, ..., n in the game theory.

$$U_i = \begin{array}{c} u_i \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{array} \begin{bmatrix} u_{i1} & u_{i2} & \cdots & u_{iT} \\ \theta_{11} & \theta_{12} & \cdots & \theta_{1T} \\ \theta_{21} & \theta_{22} & \cdots & \theta_{2T} \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{n1} & \theta_{n2} & \cdots & \theta_{nT} \end{bmatrix}, i = 1.2.\cdots.n \tag{20}$$

T means the total number of strategic combinations of all participants. The first element of the k column in U_i : $(u_{ik}, \theta_{s1k}, \theta_{2k}, \cdots, \theta_{ik}, \cdots, \theta_{nk})^T$ means the payment of participant I in strategic combination k: $(s_{1k}, s_{2k}, \cdots, s_{ik}, \cdots s_{nk})^T$; the second element means the strategy the first participant has chosen; the third element means the second strategy the second participant has chosen, and so on. The i+1 element means the strategy the participant *i* has chosen. Matrix U_i, U_{ik} is sorted in descending order of its value:

$$u_{i1} \geq u_{i2} \geq \cdots \geq u_{in}, i = 1,2,\cdots, \quad n \tag{21}$$

Step2: The rows labeled u and θ_i in the matrix U_i are removed to obtain a matrix B_i:

$$B_{\mathrm{i}} = \begin{array}{c} \theta_{1} \\ \vdots \\ \theta_{i-1} \\ \theta_{i+1} \\ \vdots \\ \theta_{n} \end{array} \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1T} \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{i-11} & \theta_{i-12} & \cdots & \theta_{i-1T} \\ \theta_{i+11} & \theta_{i+12} & \cdots & \theta_{i+1T} \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{n1} & \theta_{n2} & \cdots & \theta_{nT} \end{bmatrix} \tag{22}$$

Step3: In all the same columns of matrix B_i, the strategic combination with the smallest column label is selected into matrix R_i to get R_i.

Step4: A set of Nash equalizations is obtained according to the following equation. If set R only includes one set of strategic combination, set R is the optimal strategic combination. If set R includes many sets of strategic combinations, proceed into step 5.

$$R = \prod_{i=1}^{n} R_i \tag{23}$$

Step5: According to the Pareto efficiency principle, compare the payment combinations corresponding to their strategic combinations in set R and choose the optimal strategic combination with the Pareto advantage.

CONCLUSION

This paper studies the lane change decision of ICVs and constructs a discretionary lane change decision model. The main research results are as follows:

The paper assumes and analyzes the lane change decision system of ICVs and improves the MOBIL model based on the advantages of ICVs. The improved MOBIL considers the influence of SV's lane change on FV and PFV in order to reduce SV's influence on FV and PFV. This paper considers the cooperation and contradiction between two ICVs, and uses a double matrix to construct the cooperative lane change model and expounds the solving method to model.

The paper proposes a discretionary lane change decision model that can increase efficiency of traffic flow and reduce unreasonable lane change behavior. However, there are still some shortcomings in this paper that need further research. The model does not consider the communication delay, information measurement error and data transmission error and does not consider the more complicated lane change conflict and cooperation between the three surrounding vehicles. There are more types of coordinated lane changes between the three vehicles which are more complicated.

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