

```

knitr::opts_chunk$set(echo = TRUE)
library(tidyverse)

## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr     1.1.1     v readr     2.1.4
## vforcats   1.0.0     v stringr   1.5.0
## v ggplot2   3.4.2     v tibble    3.2.1
## v lubridate 1.9.2     v tidyr    1.3.0
## v purrr    1.0.1
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()   masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
library(RColorBrewer)
library(rstan)

## Loading required package: StanHeaders
## rstan (Version 2.21.8, GitRev: 2e1f913d3ca3)
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
##
## Attaching package: 'rstan'
##
## The following object is masked from 'package:tidyverse':
## extract
library(bayesplot)

## This is bayesplot version 1.10.0
## - Online documentation and vignettes at mc-stan.org/bayesplot
## - bayesplot theme set to bayesplot::theme_default()
##   * Does _not_ affect other ggplot2 plots
##   * See ?bayesplot_theme_set for details on theme setting
library(modelr)
library(tidybayes)
library(ggstance)

##
## Attaching package: 'ggstance'
##
## The following objects are masked from 'package:ggplot2':
## geom_errorbarh, GeomErrorbarh
library(brms)

## Loading required package: Rcpp
## Loading 'brms' package (version 2.19.0). Useful instructions
## can be found by typing help('brms'). A more detailed introduction
## to the package is available through vignette('brms_overview').
##
## Attaching package: 'brms'

```

```

##  

## The following objects are masked from 'package:tidybayes':  

##  

##      dstudent_t, pstudent_t, qstudent_t, rstudent_t  

##  

## The following object is masked from 'package:bayesplot':  

##  

##      rhat  

##  

## The following object is masked from 'package:rstan':  

##  

##      loo  

##  

## The following object is masked from 'package:stats':  

##  

##      ar  

library(loo)

## This is loo version 2.6.0
## - Online documentation and vignettes at mc-stan.org/loo
## - As of v2.0.0 loo defaults to 1 core but we recommend using as many as possible. Use the 'cores' arg
##  

## Attaching package: 'loo'
##  

## The following object is masked from 'package:rstan':  

##  

##      loo  

library(DescTools)

##  

## Attaching package: 'DescTools'  

##  

## The following object is masked from 'package:tidybayes':  

##  

##      Mode  

library(lme4)

## Loading required package: Matrix
##  

## Attaching package: 'Matrix'  

##  

## The following objects are masked from 'package:tidyr':  

##  

##      expand, pack, unpack  

##  

## Attaching package: 'lme4'
##  

## The following object is masked from 'package:brms':  

##  

##      ngrps

```

```

library(gamlss)

## Loading required package: splines
## Loading required package: gamlss.data
##
## Attaching package: 'gamlss.data'
##
## The following object is masked from 'package:datasets':
##
##      sleep
##
## Loading required package: gammLSS.dist
## Loading required package: MASS
##
## Attaching package: 'MASS'
##
## The following object is masked from 'package:dplyr':
##
##      select
##
## Loading required package: nlme
##
## Attaching package: 'nlme'
##
## The following object is masked from 'package:lme4':
##
##      lmList
##
## The following object is masked from 'package:dplyr':
##
##      collapse
##
## Loading required package: parallel
## **** GAMLSS Version 5.4-12 ****
## For more on GAMLSS look at https://www.gamlss.com/
## Type gammLSSNews() to see new features/changes/bug fixes.
##
##
## Attaching package: 'gamlss'
##
## The following object is masked from 'package:lme4':
##
##      refit
##
## The following object is masked from 'package:brms':
##
##      cs

library(dplyr)
library(progress)
library(ggplot2)
library(cowplot)

##

```

```

## Attaching package: 'cowplot'
##
## The following object is masked from 'package:lubridate':
##
##     stamp

theme_set(theme_gray())

rstan_options(auto_write = TRUE)
options(mc.cores = parallel::detectCores())
devAskNewPage(ask = FALSE)

source("model_check.R")

##
## Attaching package: 'ggdist'
##
## The following object is masked from 'package:DescTools':
##
##     Mode

##
## The following objects are masked from 'package:brms':
##
##     dstudent_t, pstudent_t, qstudent_t, rstudent_t

##
## Attaching package: 'ggpubr'
##
## The following object is masked from 'package:cowplot':
##
##     get_legend

##
## Loading required package: rngWELL
## This is randtoolbox. For an overview, type 'help("randtoolbox")'.
## Linking to ImageMagick 6.9.12.3
## Enabled features: cairo, fontconfig, freetype, heic, lcms, pango, raw, rsvg, webp
## Disabled features: fftw, ghostscript, x11
## Loading required package: proto

## Warning in doTryCatch(return(expr), name, parentenv, handler): unable to load shared object '/Library
##   dlopen(/Library/Frameworks/R.framework/Resources/modules//R_X11.so, 0x0006): Library not loaded: /
##   Referenced from: <5D128DE5-8E3C-3CF8-9A4F-8D762BB79C4E> /Library/Frameworks/R.framework/Versions/4
##   Reason: tried: '/opt/X11/lib/libSM.6.dylib' (no such file), '/System/Volumes/Preboot/Cryptexes/OS/OS

## tcltk DLL is linked to '/opt/X11/lib/libX11.6.dylib'
## Could not load tcltk. Will use slower R code instead.

```

In this document, we run the rational agent analysis for transit decision task and its corresponding visualizations in Fernandes et al. We will first use a statistical model to predict behavioral agents' response distributions to different trials given different assigned visualizations and arrival time distributions.

The framework can be thought of as a function that estimates four parameters under the experimental design of Fernandes et al. (including the realized sample size):

1. The *rational baseline (Rprior)*: the performance of the rational agent without access to the visualization signals, i.e., with prior beliefs only.
2. The *rational benchmark (R)*: the performance of the rational agent with access to the signal (provided

- by the visualization), i.e., with posterior beliefs.
3. The *behavioral score* ( $B$ ): the expected score of the behavioral agent predicted by a generative statistical model.
  4. The *calibrated behavioral score* ( $C$ ): the score of a rational agent on information structure  $\pi^B$ .

## Generate Behavioral Response

### Load and clean data

We start by loading the experiment data of Fernandes et al. We remove duplicated data and responses for Scenario 4, which was not analyzed in the original paper. We then do data transformations.

```
df = read.csv("data/final_trials.csv") %>%
  as_tibble()

df = df[!duplicated(df), ]

max_trial = max(df$trial)

df = df %>%
  mutate(
    trial_normalized = ((trial - max_trial) / max_trial) + 0.5
  )

df = df %>%
  filter(scenario != "s4") %>%
  mutate(scenario = factor(scenario))
```

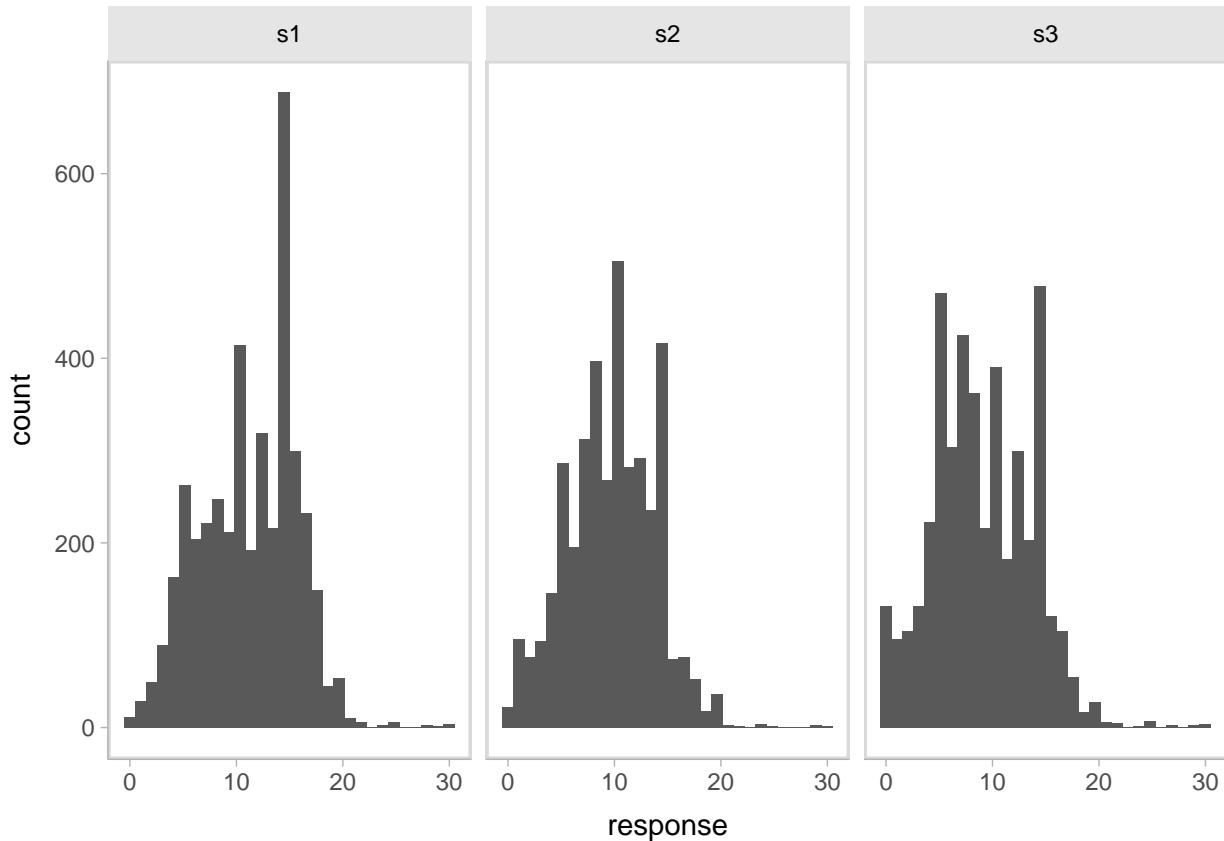
### Linear Model of Response

We start as simply as possible by just modeling the distribution of response.

Before we fit the model to our data, let's check that our priors seem reasonable. We'll use a weakly informative prior for the intercept parameter since we want the population-level centered intercept to be flexible. We set the expected value of the prior on the intercept equal to the mean value of the response that we sampled.

```
df %>%
  ggplot() +
  geom_histogram(aes(response)) +
  facet_grid(. ~ scenario)

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



```
df %>% group_by(scenario) %>% summarise(response = mean(response))
```

```
## # A tibble: 3 x 2
##   scenario response
##   <fct>     <dbl>
## 1 s1        11.2
## 2 s2        9.55
## 3 s3        8.93
```

We start modeling by specifying the simplest possible model regressing response on scenario id.

```
# get_prior(ddata = df, family = "gaussian", bf(response ~ scenario))
# starting as simple as possible: learn the distribution of response over scenario
prior.response <- brm(data = df, family = "gaussian",
                       bf(response ~ scenario),
                       prior = c(prior(normal(10, 1), class = Intercept),
                                 prior(normal(1, 0.5), class = b),
                                 prior(normal(0, 0.5), class = sigma)),
                       sample_prior = "only",
                       iter = 3000, warmup = 500, chains = 2, cores = 2)
```

```
## Compiling Stan program...
## Trying to compile a simple C file
## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/StanHeaders/inc
```

```

## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/src/Core/util/MatrixBase.h:18,
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/src/Core/util/VectorBase.h:18,
## /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/src/Core/util/VectorSpace.h:18,
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/src/Core/util/VectorSpace.h:18,
## namespace Eigen {
## ^
## ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/StanHeaders/include StanHeaders.h:18,
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/Core:96:10:
## /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/Core:96:10: fatal error: complex.h: No such file or directory
## #include <complex>
## ^~~~~~~
## 3 errors generated.
## make: *** [foo.o] Error 1

## Start sampling

```

Let's take a look at our prior predictive distribution compared with observed data. To evaluate the fit of these and subsequent models, we use a custom visual grammar we developed for model checking to compare the model's posterior predictive distribution with the distribution in data. The red lines represent prediction from model and the blue line is the distribution of the observed data.

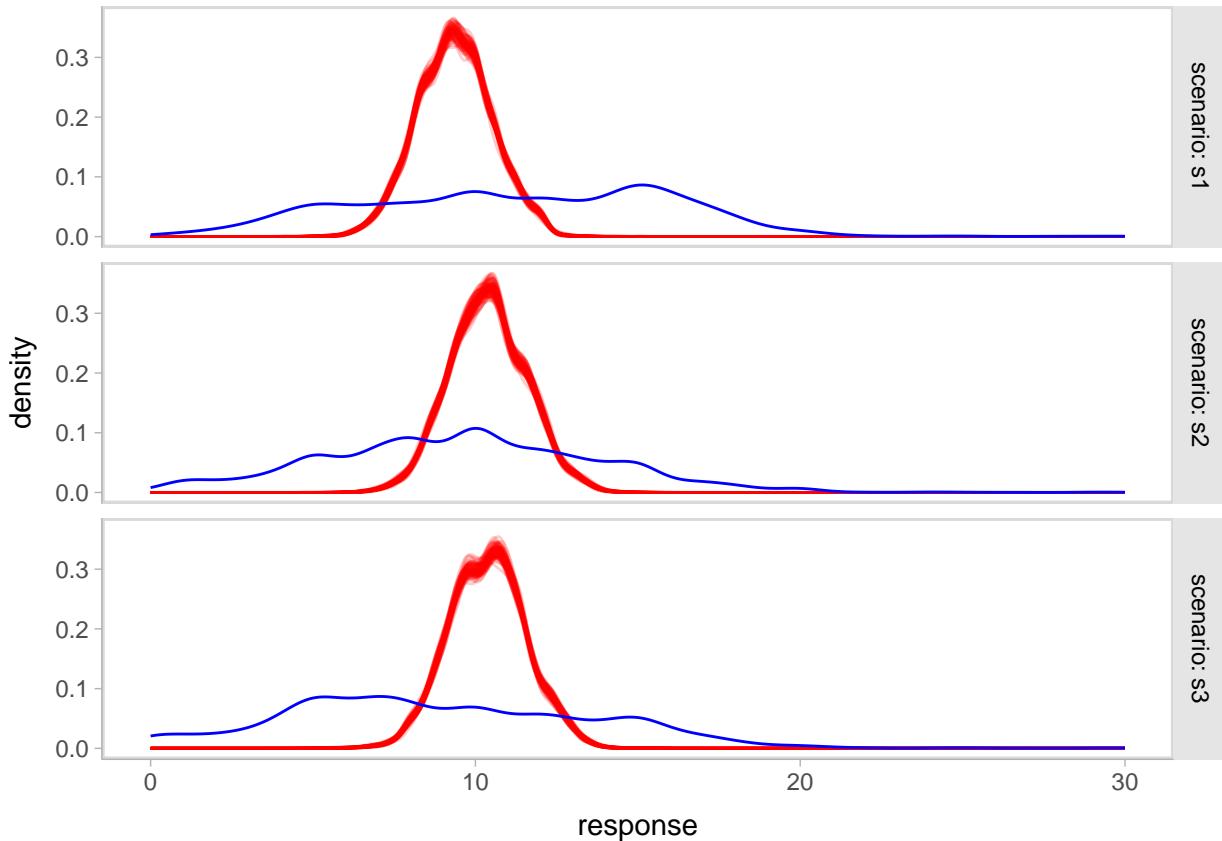
```

prior.response %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row'. You can override
## using the `groups` argument.

```



Now, let's fit the model to data.

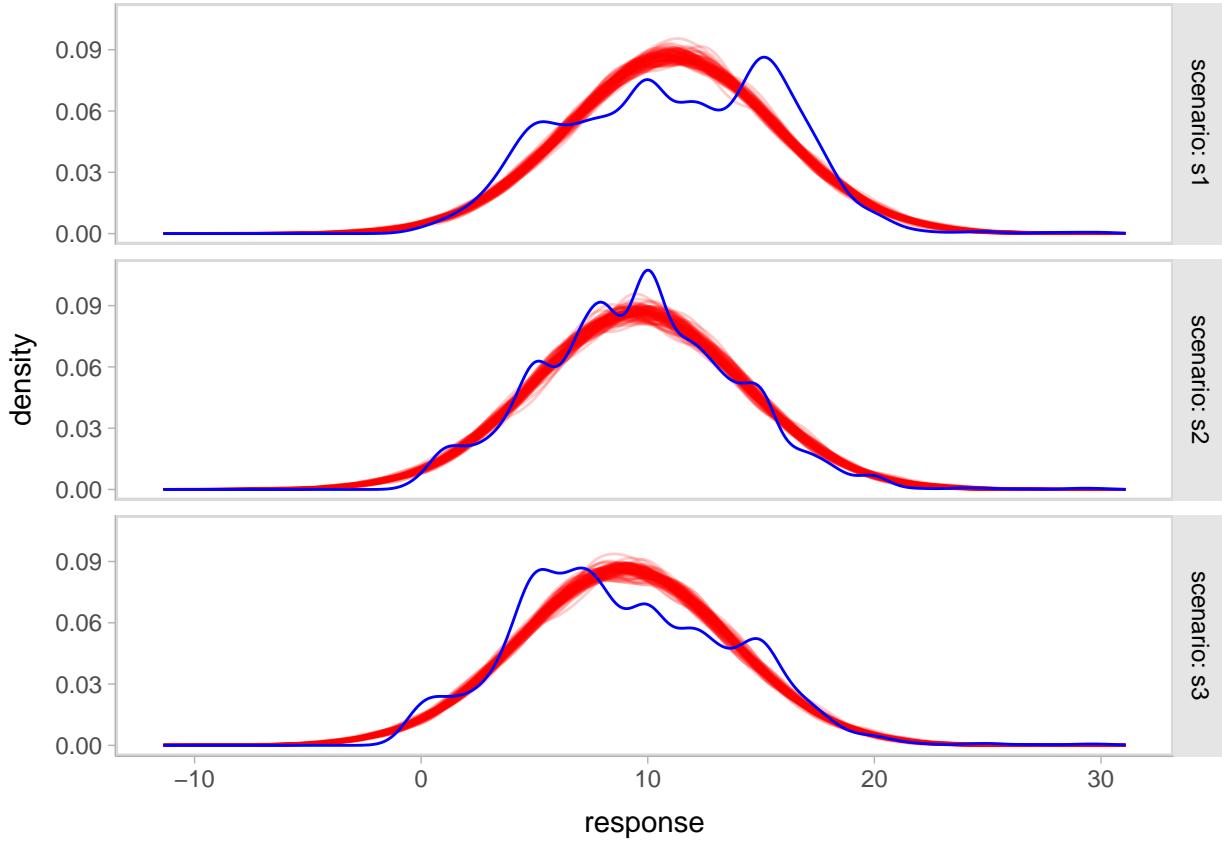
```
m.response <- brm(data = df, family = "gaussian",
bf(response ~ scenario),
prior = c(prior(normal(10, 1), class = Intercept),
prior(normal(1, 0.5), class = b),
prior(normal(0, 0.5), class = sigma)),
iter = 3000, warmup = 500, chains = 2, cores = 2,
file = "models/response_scenario_mdl")
```

Let's check our posterior predictive distribution.

```
m.response %>%
mcplot() +
mc_distribution("predictive") +
mc_uncertainty_representation(c("static")) +
mc_comparative_layout("superposition") +
mc_conditional_variables(row_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row'. You can override
## using the `groups` argument.
```



Our model is not sensitive to visualization condition and trial number, so we expect to see mismatches here.

Now we gradually add predictors for the visualization condition and the normalized trial number into the model.

Before we fit the model to our data, let's check that our priors seem reasonable. Since we are now including more slope parameters for visualization condition and the normalized trial number in our model, we can dial down the width of our prior for sigma to avoid over-dispersion of predicted responses.

```
# get_prior(data = df, family = "gaussian", bf(response ~ scenario + vis + trial_normalized))
# model with scenario, vis condition, and normalized trial number
prior.response_scenario_vis_trial <- brm(data = df, family = "gaussian",
                                         bf(response ~ scenario + vis + trial_normalized),
                                         prior = c(prior(normal(10, 1), class = Intercept),
                                                   prior(normal(1, 0.5), class = b),
                                                   prior(normal(0, 0.3), class = sigma)),
                                         sample_prior = "only",
                                         iter = 3000, warmup = 500, chains = 2, cores = 2)

## Compiling Stan program...
## Trying to compile a simple C file
## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -fPIC -I/Library/Frameworks/R.framework/Resources/include" -DNDEBUG -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/StanHeaders/include StanHeaders.h
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include RcppEigen.h
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include RcppEigen.h
## /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/src/Core/util
```

```

## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
## ^
## ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/StanHeaders/inc
## In file included from /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/inclu
## /Library/Frameworks/R.framework/Versions/4.2/Resources/library/RcppEigen/include/Eigen/Core:96:10: f
## #include <complex>
## ^
## 3 errors generated.
## make: *** [foo.o] Error 1

## Start sampling

```

Let's look at our prior predictive distribution.

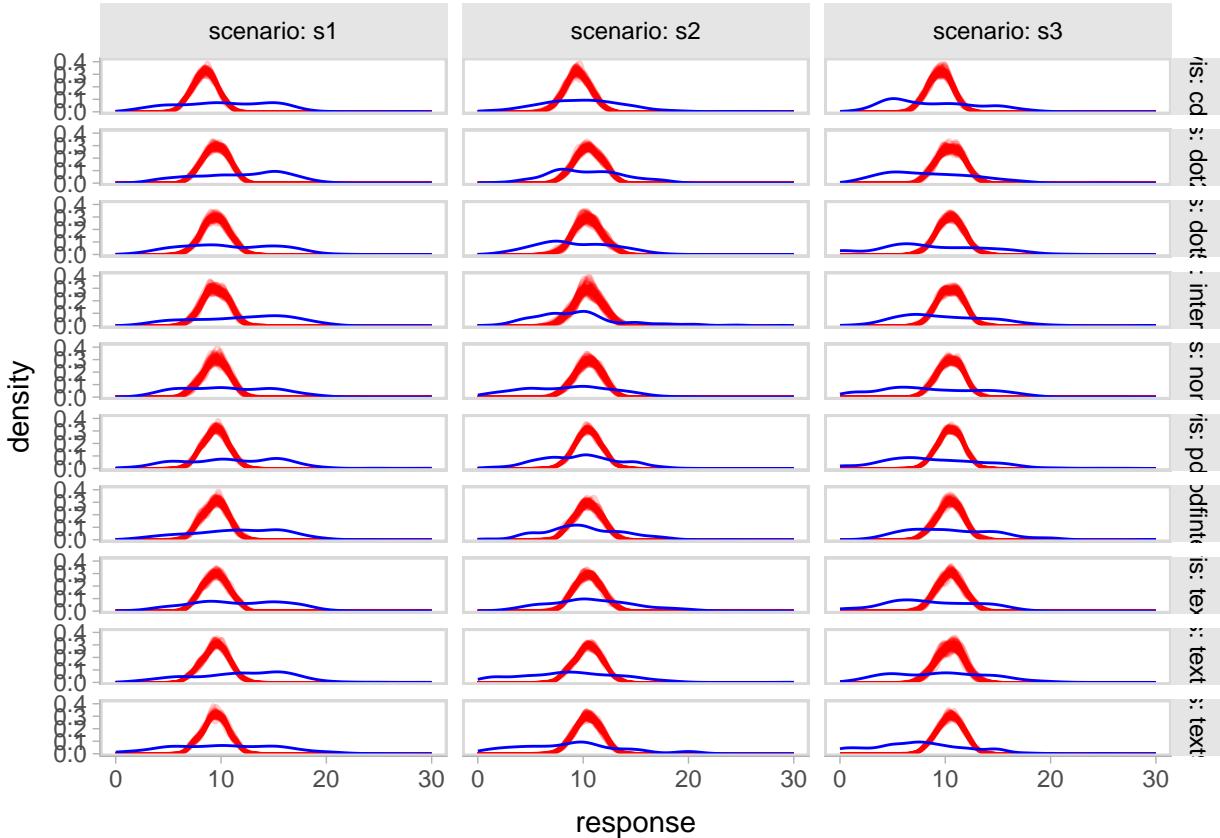
```

prior.response_scenario_vis_trial %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the `.groups` argument.

```



Now, let's fit the model to data.

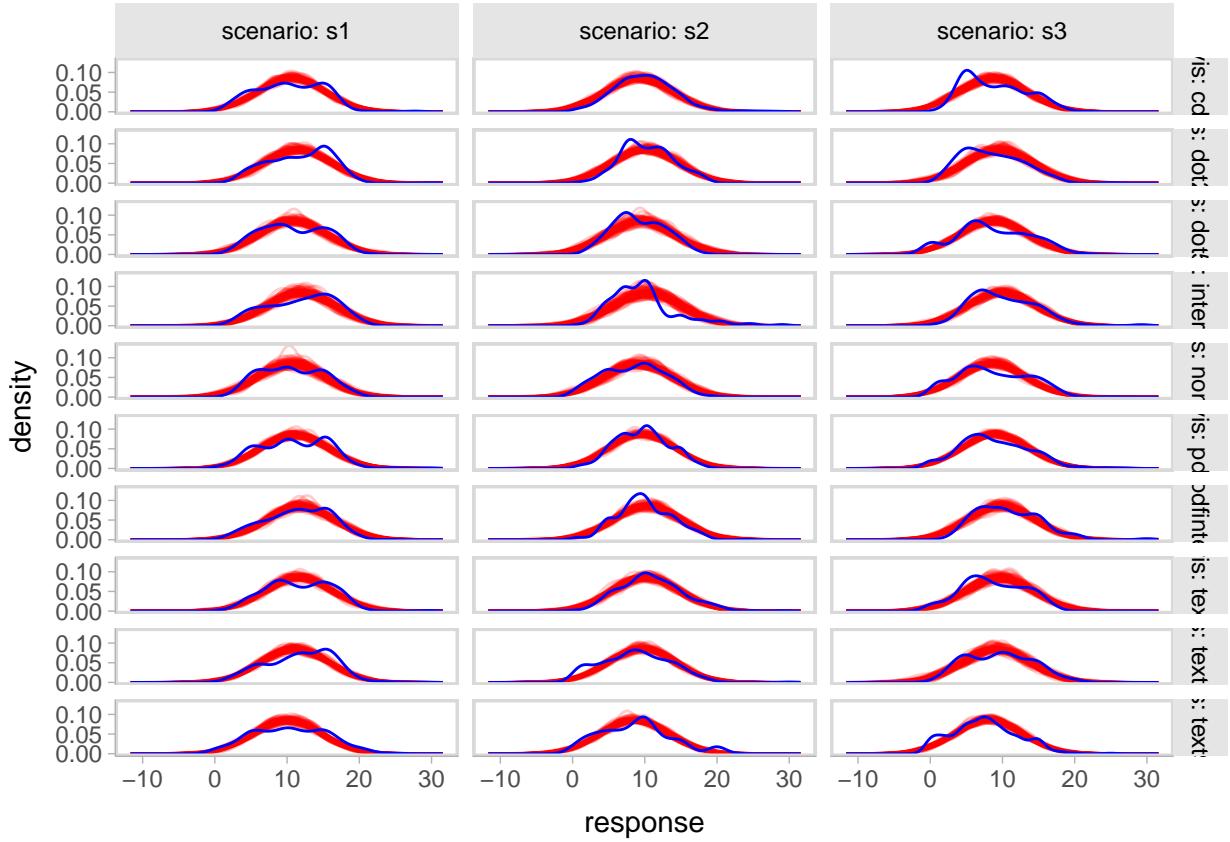
```
m.response_scenario_vis_trial <- brm(data = df, family = "gaussian",
  bf(response ~ scenario + vis + trial_normalized),
  prior = c(prior(normal(10, 1), class = Intercept),
            prior(normal(1, 0.5), class = b),
            prior(normal(0, 0.3), class = sigma)),
  iter = 3000, warmup = 500, chains = 2, cores = 2,
  file = "models/response_scenario_vis_trial_mdl")
```

Let's check our posterior predictive distribution.

```
m.response_scenario_vis_trial %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the `groups` argument.
```



These don't look great. We can see that the pattern in each visualization condition and scenario combination is more complex than our model predicts. We then add interactions between predictors to see whether our model can fit better.

### Add Interactions

We next add interactions between the scenario and visualization condition, to allow different visualization conditions to be affected by scenarios differently, as well as different effects of trial number depending on both visualization and scenario.

We use the same priors as we did for the previous model. Now, let's fit the model to our data.

```
m.response_scenario_vis_trial_interaction <- brm(data = df, family = "gaussian",
                                                 bf(response ~ scenario * vis + trial_normalized),
                                                 prior = c(prior(normal(10, 1), class = Intercept),
                                                           prior(normal(1, 0.5), class = b),
                                                           prior(normal(0, 0.3), class = sigma)),
                                                 iter = 3000, warmup = 500, chains = 2, cores = 2,
                                                 file = "models/response_scenario_vis_trial_interaction_mdl")
```

Let's check our posterior predictive distribution.

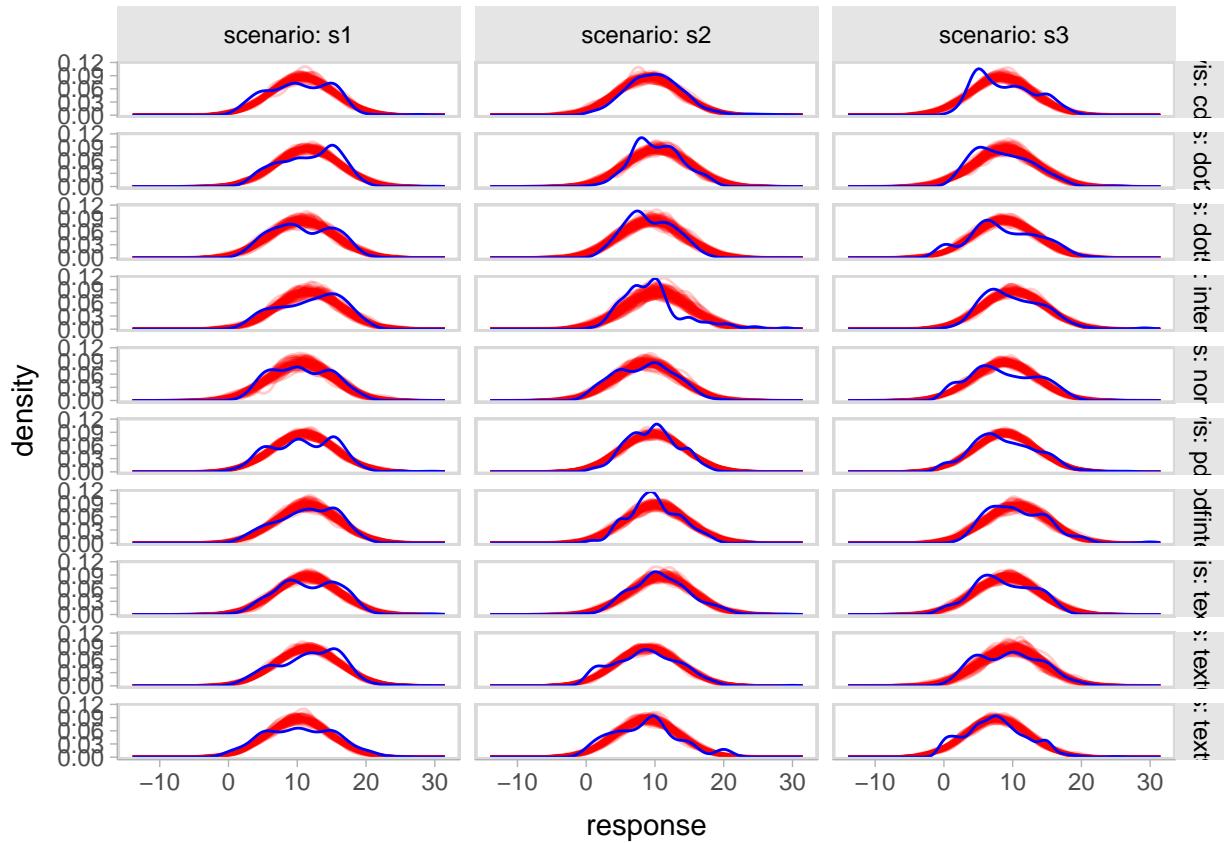
```
m.response_scenario_vis_trial_interaction %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))
```

```

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the `groups` argument.

```



We continue to add interaction with the normalized trial number.

```

m.response_scenario_vis_trial_interaction_all <- brm(data = df, family = "gaussian",
           bf(response ~ scenario * vis * trial_normalized),
           prior = c(prior(normal(10, 1), class = Intercept),
                     prior(normal(1, 0.5), class = b),
                     prior(normal(0, 0.3), class = sigma)),
           iter = 3000, warmup = 500, chains = 2, cores = 2,
           file = "models/response_scenario_vis_trial_interaction_all")

```

Let's check our posterior predictive distribution.

```

m.response_scenario_vis_trial_interaction_all %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."

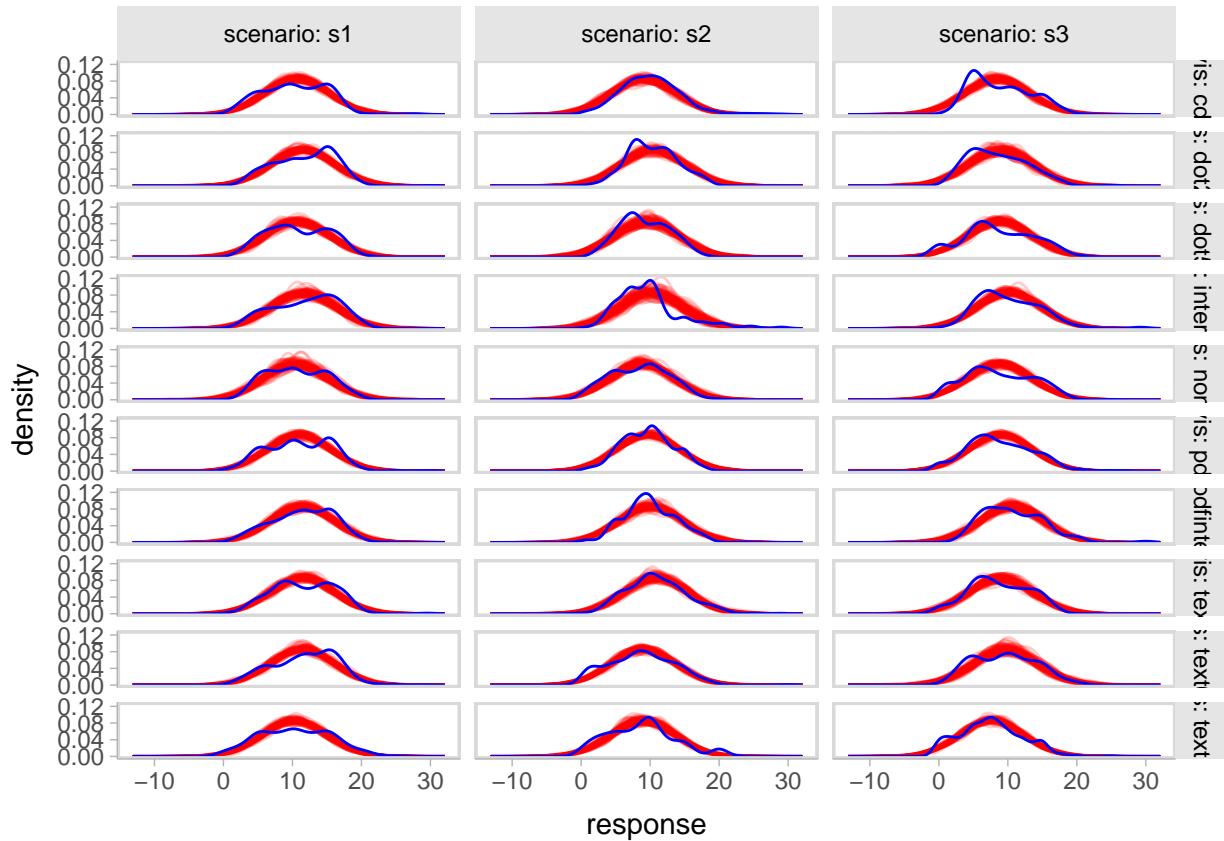
```

```

## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the ``.groups` argument.

```



This looks better, but it is still having trouble fitting the data. It may be that the model is not capturing the individual variability in response patterns. Next we'll add hierarchy to our model.

### Add Hierarchy for Intercepts

We next random intercepts to account for differences based on participant id.

```
m.response_scenario_vis_trial_interaction_all_mix <- brm(data = df, family = "gaussian",
  formula = bf(response ~ scenario * vis * trial_no),
  prior = c(prior(normal(10, 1), class = Intercept),
            prior(normal(1, 0.5), class = b),
            prior(normal(0, 0.3), class = sigma)),
  iter = 4000, warmup = 1000, chains = 2, cores = 2,
  file = "models/response_scenario_vis_trial_interaction_all_mix.RData")
```

Let's check our posterior predictive distribution.

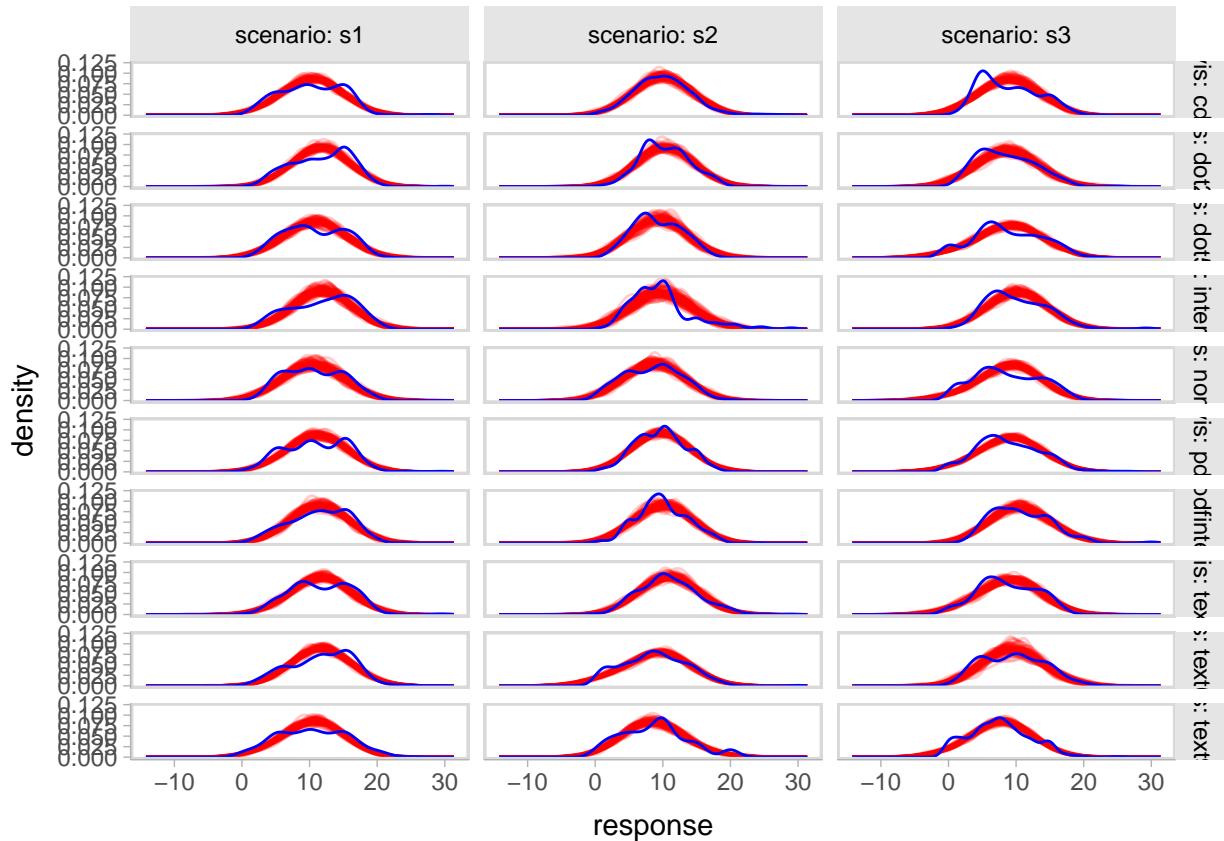
```
m.response_scenario_vis_trial_interaction_all_mix %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))
```

```

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the `groups` argument.

```



### Add Sigma

We next fit a submodel to predict the standard deviation  $\sigma$ , starting with visualization condition, scenario, and their interaction, then gradually adding random effects. We also add a random slope to allow for different learning effects by participant to both submodels.

```

m.response_scenario_vis_trial_interaction_all_mix_sigma <- brm(data = df, family = "gaussian",
  formula = bf(response ~ scenario * vis * trial_normal +
    sigma ~ vis * scenario),
  prior = c(prior(normal(10, 1), class = Intercept),
            prior(normal(1, 0.5), class = b),
            prior(normal(0, 0.3), class = b, dpar = sigma)),
  iter = 10000, warmup = 4000, chains = 4, cores = 4,
  file = "models/response_scenario_vis_trial_interaction_all_mix_sigma.brm")

m.response_scenario_vis_trial_interaction_all_mix_sigma %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +

```

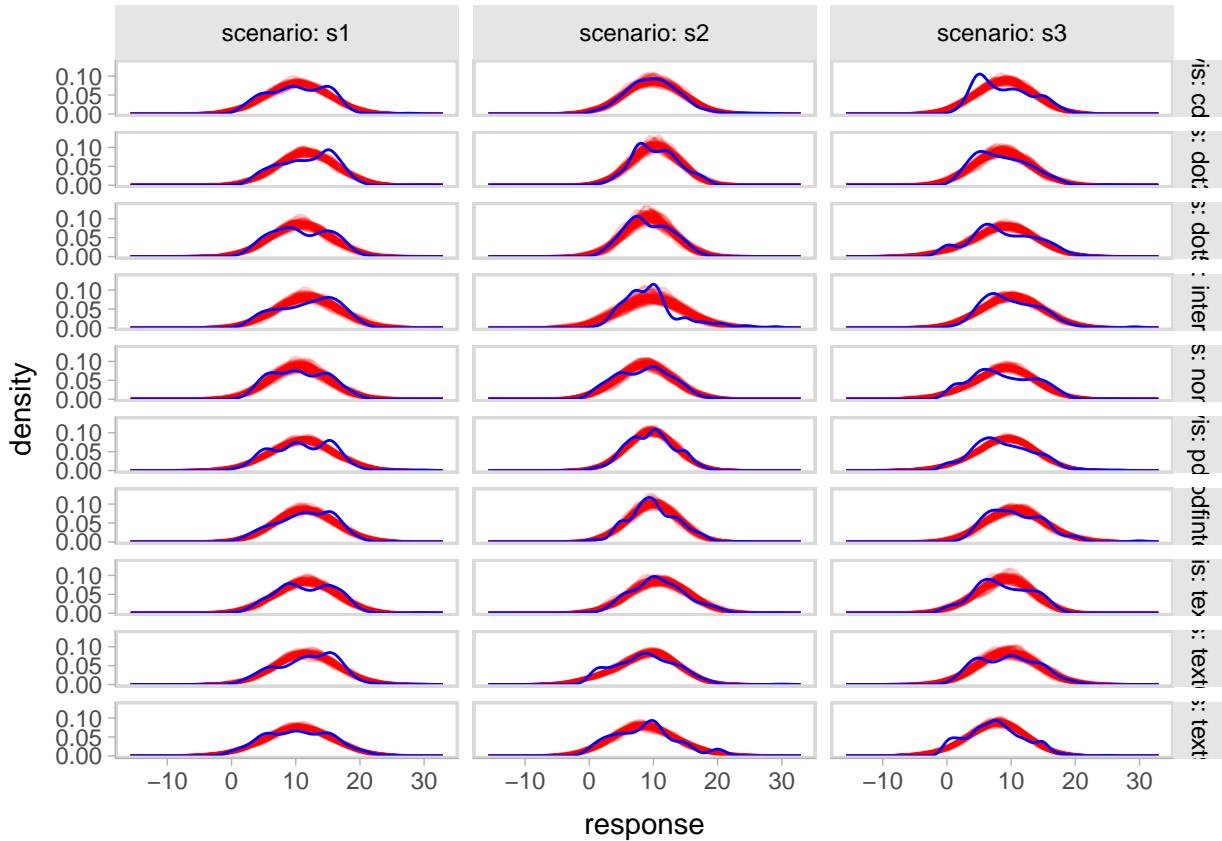
```

mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the `groups` argument.

```



```

m.response_scenario_vis_trial_interaction_all_mix_sigma_trial <- brm(data = df, family = "gaussian",
  formula = bf(response ~ scenario * vis * trial,
    sigma ~ vis * scenario * trial),
  prior = c(prior(normal(10, 1), class = Intercept),
    prior(normal(1, 0.5), class = b),
    prior(normal(0, 0.3), class = b),
    prior(0.01, class = sigma)),
  iter = 10000, warmup = 4000, chains = 4, cores = 4,
  file = "models/response_scenario_vis_trial_interaction_all_mix_sigma_trial")

m.response_scenario_vis_trial_interaction_all_mix_sigma_trial %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."

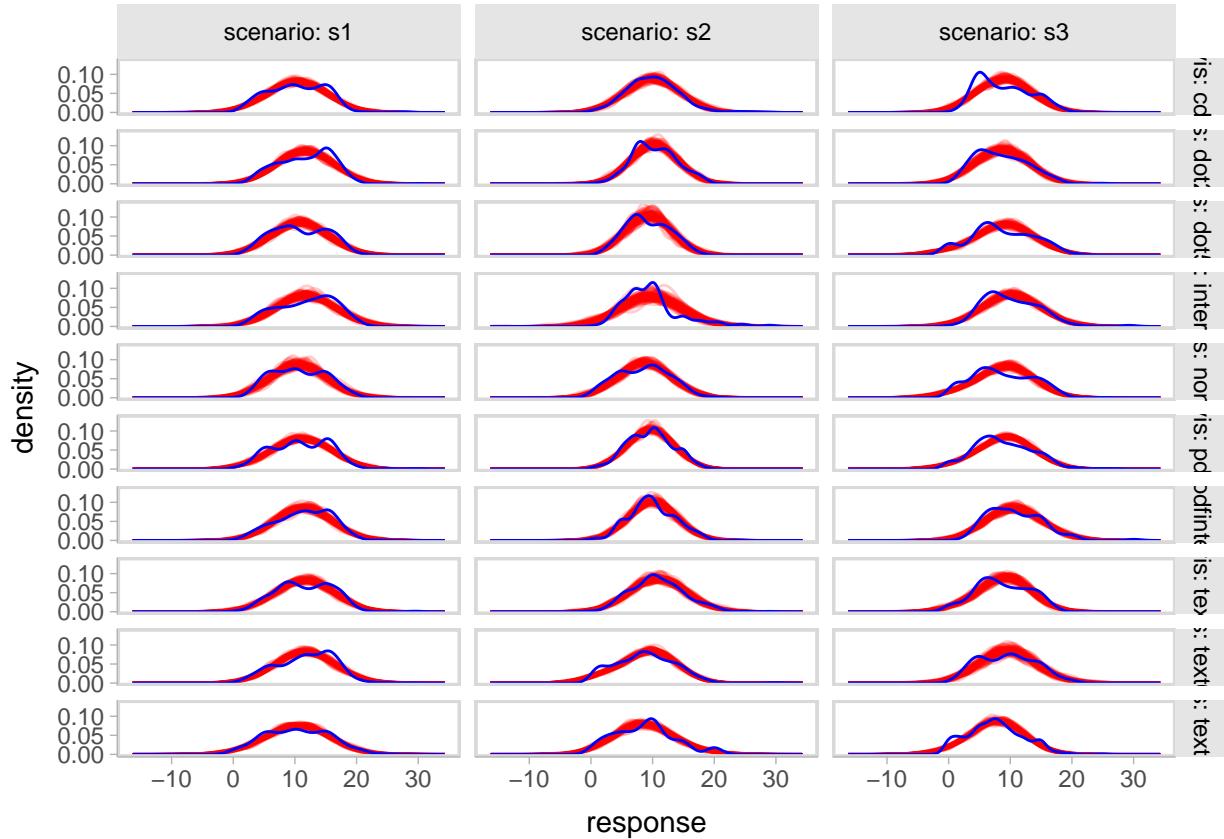
```

```

## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the ``.groups` argument.

```



```

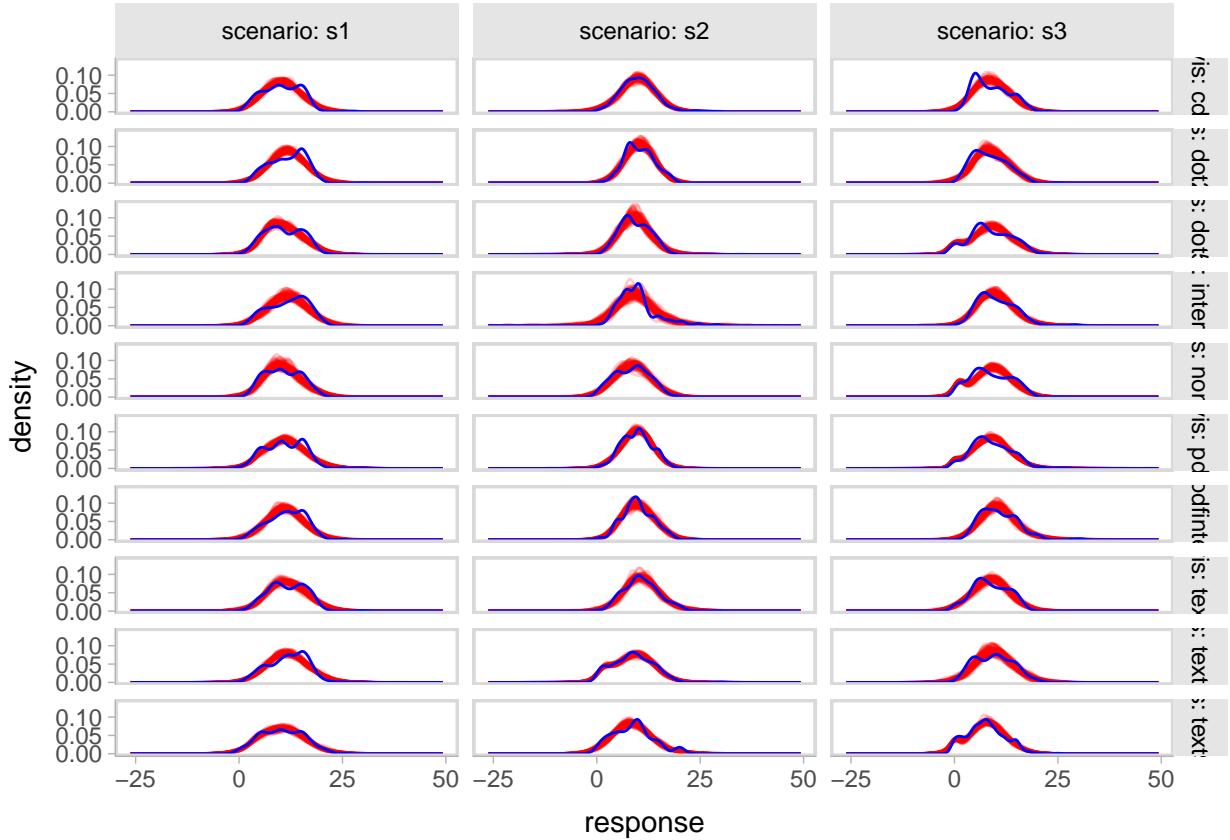
m.response_scenario_vis_trial_interaction_all_mix_sigma_trial_participant <- brm(data = df, family = "gaussian",
  formula = bf(response ~ scenario * vis),
  prior = c(prior(normal ~ 0, sd = 1), prior(normal ~ 0, sd = 1),
            prior(normal ~ 0, sd = 1)),
  iter = 10000, warmup = 5000,
  file = "models/response_bayes")

m.response_scenario_vis_trial_interaction_all_mix_sigma_trial_participant %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
## override using the ``.groups` argument.

```



## Final model

Next, we add two additional predictors to capture two critical parameters of the arrival time distributions that are used to generate the trial stimuli. We confirm via model checking that the model fits the distribution well.

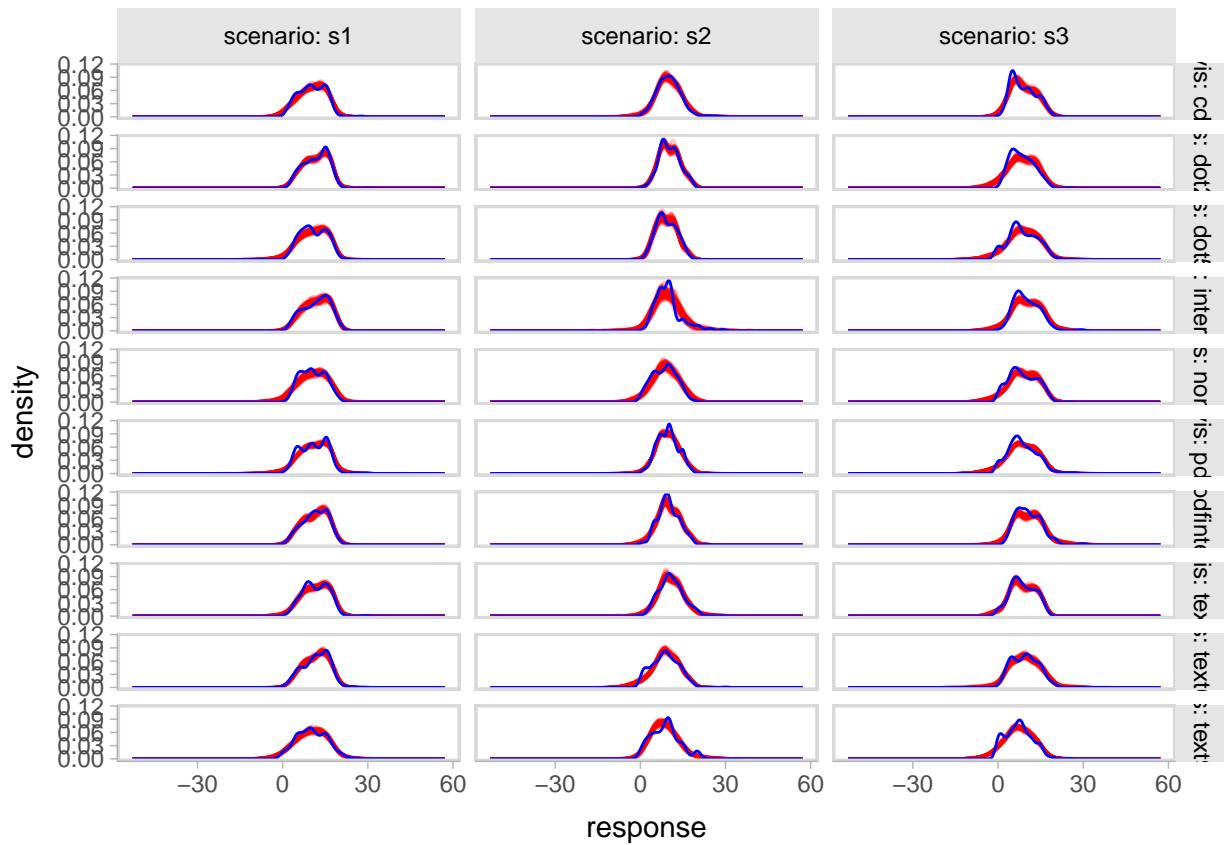
```
m.final_response <- brm(data = df, family = "gaussian",
  formula = bf(response ~ scenario * vis * trial_normalized + mu + sigma + (trial_normalized | participant),
    sigma ~ vis * scenario * trial_normalized + (trial_normalized | participant),
  prior = c(prior(normal(10, 1), class = Intercept),
            prior(normal(1, 0.5), class = b),
            prior(normal(0, 0.3), class = b, dpar = sigma)),
  iter = 10000, warmup = 4000, chains = 4, cores = 4,
  file = "models/response_final_mdl")

m.final_response %>%
  mcplot() +
  mc_distribution("predictive") +
  mc_uncertainty_representation(c("static")) +
  mc_comparative_layout("superposition") +
  mc_conditional_variables(row_vars = vars(vis), col_vars = vars(scenario))

## [1] "Getting distributions..."
## [1] "Operating distributions..."
## [1] "Visualizing..."
## [1] "Generating comparative layouts..."

## `summarise()` has grouped output by 'observation', '.row', 'vis'. You can
```

```
## override using the `groups` argument.
```



### Generating behavioral agents' responses

We now simulate behavioral agents' responses using the posterior predictive distribution of the model, separately for each combination of trial parameters (including scenario, visualization condition, trial number, and stimuli distribution parameters). Because the response variable is discrete (numbers of minutes as integer) we calculate a discrete version of the distributions. We also generate the quantile intervals of 60%, 85%, and 99% that are shown in the text conditions as we need these to simulate behavioral agent scores in those conditions.

```
distribution_df = df %>%
  dplyr::select(distribution, mu, sigma, nu, tau) %>%
  unique() %>%
  rowwise() %>%
  mutate(discrete = list(dBCT((-14:30) + 15, mu, sigma, nu, tau))) %>%
  unnest_wider(col = discrete, names_sep = '_') %>%
  mutate(
    text = round(gamlss.dist::qBCT(1 - .85, mu, sigma, nu, tau) - 15),
    text60 = round(gamlss.dist::qBCT(1 - .60, mu, sigma, nu, tau) - 15),
    text99 = round(gamlss.dist::qBCT(1 - .99, mu, sigma, nu, tau) - 15)
  )

meta.df = df %>%
  group_by(scenario) %>%
  data_grid(vis, trial_normalized, distribution) %>%
  merge(distribution_df %>% dplyr::select(distribution, mu, sigma, nu, tau), by="distribution")
```

```

pred.df = meta.df %>%
  add_predicted_draws(m.final_response, ndraws = 500, value="pred_response", re_formula = NA) %>%
  ungroup()

```

## Rational Agent Framework

### Scoring rule

First we define the scoring rules for each analyzed scenario in the transit decision problem. The rules allow us to calculate the expected payoff over a bus arriving distribution given a participant's stated arrival time at the stop.

```

scoring_rule = function(rider_at_stop, distribution, scenario) {
  all_scenario_params = list(
    # delay_bonus, wait_penalty, destination_reward, and max_destination_time
    s1 = c(8, 14, 14, 90),
    s2 = c(14, 14, 14, 60),
    s3 = c(8, 17, 17, 120)
  )
  scenario_params = all_scenario_params[[scenario]]

  payoff <- function(rider_at_stop, bus_at_stop, second_bus_at_stop, wait_penalty,
                       delay_bonus, destination_reward, max_destination_time){

    arrived_before_bus <- as.integer(rider_at_stop <= bus_at_stop)
    arrived_after_bus <- as.integer(rider_at_stop > bus_at_stop)
    time_waited <- (bus_at_stop * arrived_before_bus) +
      (second_bus_at_stop * arrived_after_bus) - rider_at_stop
    time_at_destination <- max(max_destination_time -
      # (bus_at_stop * arrived_before_bus) -
      ((second_bus_at_stop - bus_at_stop) * arrived_after_bus), 0)

    reward <- (delay_bonus * rider_at_stop) -
      (time_waited * wait_penalty) +
      (time_at_destination * destination_reward)
    return(reward)
  }

  pointwise.payoff <- lapply(0:(45 * 45 - 1),
    function(m,
              wait_penalty,
              delay_bonus,
              destination_reward,
              max_destination_time){
      first_bus = (m %% 45) - 14
      second_bus = (m %% 45) - 14
      first_prob <- distribution[first_bus + 15]
      second_prob <- distribution[second_bus + 15]
      payoff <- payoff(rider_at_stop, first_bus, second_bus + 30, wait_penalty,
                      delay_bonus, destination_reward,
                      max_destination_time)
      return(payoff * first_prob * second_prob)
    }, wait_penalty = scenario_params[2],
    delay_bonus = scenario_params[1],
    max_destination_time = max_destination_time)
}

```

```

        destination_reward = scenario_params[3],
        max_destination_time = scenario_params[4])

    Reduce('+', pointwise.payoff)
}

```

## Setup

We prepare by calculating and transforming some variables.

```

scenario_ids = unique(df$scenario)
distribution_ids = unique(df$distribution)
vis_ids = unique(df$vis)

# round the response predicted by model into integer and then bound them in [1, 30]
pred_df$pred_response = round(pmax(pmin(pred_df$pred_response, 30), 1))

```

We create a dictionary of payoffs for all scenario, stimuli distributions, and rider arrival times at the stop to accelerate later calculations. This may take a few minutes.

```

payoff_mapping = list()
for (scenario_id in scenario_ids) {
  distribution_seq = list()
  for (distribution_id in distribution_ids) {
    response_seq = sapply(0:30,
      function(response, distribution_id, scenario_id) {
        scoring_rule(response,
          distribution_df %>%
            filter(distribution == distribution_id) %>%
            dplyr::select("discrete_1":"discrete_45") %>%
            as.vector() %>%
            as.numeric(),
          scenario_id)
      },
      distribution_id = distribution_id,
      scenario_id = scenario_id)
    distribution_seq[[distribution_id]] = response_seq
  }
  payoff_mapping[[scenario_id]] = distribution_seq
}

```

We define some functions to calculate the four parameters of interest in the rational agent framework.

```

get_rational_posterior = function(sample_df, distribution_df) {

  # for visualization with full information, calculate the optimal payoff in 0:30 response for each sce
  posterior_optimal_payoff = list()
  for (scenario_id in scenario_ids) {
    distribution_seq = list()
    for (distribution_id in distribution_ids) {
      optimal_payoff = max(payoff_mapping[[scenario_id]][[distribution_id]])
      distribution_seq[[distribution_id]] = optimal_payoff
    }
    posterior_optimal_payoff[[scenario_id]] = distribution_seq
  }
}

```

```

sample_with_distribution = sample_df %>% merge(distribution_df,
                                                by="distribution")

# for partial information (text showing 60% interval, 80% interval, 99% interval),
# calculate the optimal payoff for each scenario and the interval value
vis_value_distribution_mapping = list()
for (scenario_id in scenario_ids) {
  case_data = sample_with_distribution %>% filter(scenario == scenario_id)
  vis_names = c("text", "text60", "text99")
  vis_seq = list()
  for (vis_name in vis_names) {
    values = unique(case_data[[vis_name]])
    value_seq = list()
    for (value in values) {
      distributions = case_data[case_data[vis_name] == value, ] %>% dplyr::select("discrete_1":"discrete_30")
      distribution = colSums(distributions) / nrow(distributions)
      payoff = Reduce("max", lapply(0:30,
                                    function(m, distribution, scenario) scoring_rule(m, distribution, scenario),
                                    distribution = distribution,
                                    scenario = scenario_id))
      value_seq[[as.character(value)]] = payoff
    }
    vis_seq[[vis_name]] = value_seq
  }
  vis_value_distribution_mapping[[scenario_id]] = vis_seq
}

# function used to retrieve optimal payoff from precalculated arrays
get_payoff = function(scenario_id, vis, data) {
  scenario_id = as.character(scenario_id)
  if (!startsWith(vis, "text")) {
    return (posterior_optimal_payoff[[scenario_id]][[data[["distribution"]]]])
  } else {
    return (vis_value_distribution_mapping[[scenario_id]][[vis]][[as.character(data[[vis]])]]))
  }
}

# simulate rational agent for every line in experiment data
posterior_action_payoff = c()
for (i in 1:nrow(sample_with_distribution)) {
  new_payoff = get_payoff(sample_with_distribution[i, "scenario"],
                         sample_with_distribution[i, "vis"],
                         sample_with_distribution[i, ])
  posterior_action_payoff = c(posterior_action_payoff,
                               new_payoff)
}
sample_with_distribution %>%
  mutate(posterior_payoff = posterior_action_payoff) %>%
  group_by(scenario) %>%
  summarise(payoff = mean(posterior_payoff))
}

```

```

get_rational_prior = function(sample_df, distribution_df) {
  prior_belief = sample_df %>%
    merge(distribution_df, by="distribution")
  prior_action_payoff = data.frame(matrix(ncol=2,nrow=0,
                                            dimnames=list(NULL, c("scenario", "payoff"))))
  for (scenario_id in scenario_ids) {
    # the distributions in the scenario
    distributions = prior_belief %>%
      filter(scenario == scenario_id) %>%
      dplyr::select("discrete_1":"discrete_45") %>%
      data.matrix()

    # the expected distribution in the scenario
    distribution = colSums(distributions) / nrow(distributions)

    # optimal payoff of 0:30 response
    payoff = Reduce("max", lapply(0:30,
                                  function(m, distribution, scenario) scoring_rule(m, distribution, scenario)))
    distribution = distribution,
    scenario = scenario_id)
    prior_action_payoff[nrow(prior_action_payoff) + 1,] = c(scenario_id, payoff)
  }
  prior_action_payoff = prior_action_payoff %>% transform(payoff = as.numeric(payoff))
  prior_action_payoff
}

get_behavioral = function(sample_df, distribution_df) {
  # for behavioral agent's payoff, we retrieve the payoff for the scenario, the distribution, and the response
  sample_df %>%
    ungroup() %>%
    mutate(scenario = as.character(scenario)) %>%
    rowwise() %>%
    mutate(behavioral_payoff = payoff_mapping[[scenario]][[distribution]][[pred_response + 1]]) %>%
    group_by(scenario, vis) %>%
    summarise(payoff = mean(behavioral_payoff))
}

get_calibrated_behavioral = function(sample_df, distribution_df) {
  calibrated_behavioral_payoff = tibble(
    "scenario" = sample_df$scenario,
    "vis" = sample_df$vis,
    "response" = sample_df$pred_response,
    "payoff" = -1
  )
  sample_with_distribution = sample_df %>% merge(distribution_df,
                                                   by="distribution")
  for (scenario_id in scenario_ids) {
    for (vis_id in vis_ids) {
      case_data = sample_with_distribution %>% filter(scenario == scenario_id,
                                                       vis == vis_id)
      responses = unique(case_data$pred_response)
      for (response_id in responses) {
        # the distributions in the scenario, the visualization condition, and the behavioral response
      }
    }
  }
}

```

```

distributions = case_data %>%
  filter(pred_response == response_id) %>%
  dplyr::select("discrete_1":"discrete_45") %>%
  data.matrix()
# the expected distribution
distribution = colSums(distributions) / nrow(distributions)
# the optimal payoff in the expected distribution
payoff = Reduce("max", lapply(0:30,
  function(m, distribution, scenario) scoring_rule(m, distribution, scenario),
  distribution = distribution,
  scenario = scenario_id))
calibrated_behavioral_payoff[calibrated_behavioral_payoff$scenario == scenario_id &
  calibrated_behavioral_payoff$vis == vis_id &
  calibrated_behavioral_payoff$response == response_id, "payoff"] =
}
}
}
calibrated_behavioral_payoff %>%
  group_by(scenario, vis) %>%
  summarise(payoff = mean(payoff))
}

```

## Pre-experiment analysis

We calculate the rational baseline and benchmark. This may take a few minutes.

```

all_rational_posterior = get_rational_posterior(meta.df, distribution_df)
all_rational_posterior

```

```

## # A tibble: 3 x 2
##   scenario payoff
##   <fct>     <dbl>
## 1 s1        1171.
## 2 s2        852.
## 3 s3        1919.

all_rational_prior = get_rational_prior(meta.df, distribution_df)
all_rational_prior

##   scenario    payoff
## 1         s3 1850.1801
## 2         s2  767.4534
## 3         s1 1078.7470

```

## Post-experiment analysis

We calculate the behavioral agents' expected score and the expected calibrated behavioral score. To capture uncertainty, we simulate the experiment a hundred times, drawing a new sample of participant arrival time decisions in each round. This provides us with a distribution comprised of 100 simulated scores. This will take a few hours.

```

n_round = 100
sample_size = 10

all_behavioral = tibble()
all_calibrated_behavioral = tibble()

```

```

pb <- progress_bar$new(
  format = " behavioral [:bar] :percent eta: :eta",
  total = n_round, clear = FALSE, width= 60)
# we run n_round rounds to generate n_round samples and calculate behavioral agent's score and calibration
for (round_id in 1:n_round) {
  pb$tick()
  # sample
  sample_df = pred_df %>% group_by(.row) %>% sample_n(sample_size)

  #behavioral score
  behavioral = get_behavioral(sample_df, distribution_df)
  behavioral$round = round_id
  all_behavioral = rbind(all_behavioral, behavioral)

  # calibrated score
  calibrated_behavioral = get_calibrated_behavioral(sample_df, distribution_df)
  calibrated_behavioral$round = round_id
  all_calibrated_behavioral = rbind(all_calibrated_behavioral, calibrated_behavioral)
}

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```

## Results

We show the results by visualizations in each scenario.

```
score_summary = merge(all_behavioral %>%
  dplyr::select(behavioral_score = payoff, everything()) %>%
  group_by(scenario, vis) %>%
  dplyr::summarise(behavioral_score = mean(behavioral_score)),
all_calibrated_behavioral %>%
  dplyr::select(calibrated_score = payoff, everything()) %>%
  group_by(scenario, vis) %>%
```

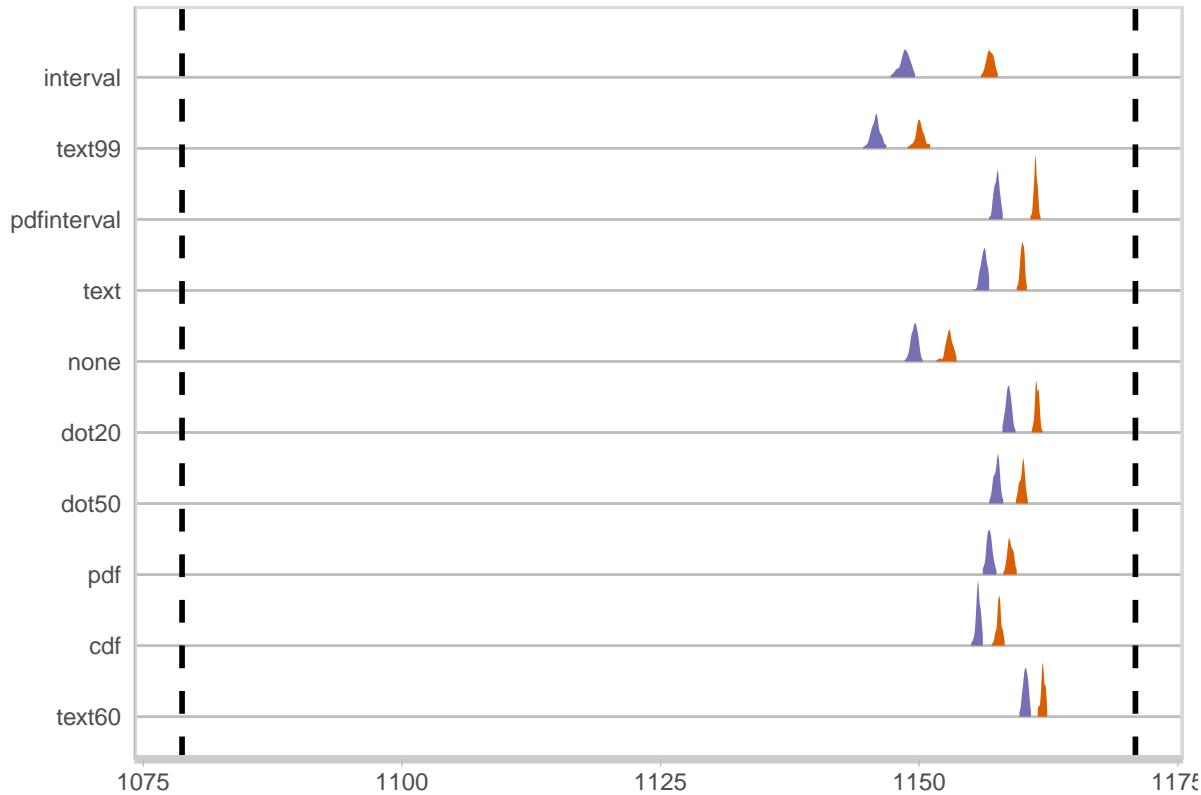
```

dplyr::summarise(calibrated_score = mean(calibrated_score)), by = c("scenario", "vis")) %>%
merge(all_rational_posterior %>%
      dplyr::select(rational_posterior_score = payoff, everything()), by = c("scenario")) %>%
merge(all_rational_prior %>%
      dplyr::select(rational_prior_score = payoff, everything()), by = c("scenario")) %>%
mutate(decision_loss = calibrated_score - behavioral_score,
       differentiation_loss = rational_posterior_score - calibrated_score,
       delta = rational_posterior_score - rational_prior_score) %>%
arrange(decision_loss) %>%
mutate(r1 = round(differentiation_loss / delta * 100),
       r2 = round(decision_loss / delta * 100))

## `summarise()` has grouped output by 'scenario'. You can override using the
## `.` argument.
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## `.` argument.

ggplot() +
  stat_slab(data = all_behavioral %>% filter(scenario == "s1"), aes(y = vis, x = payoff), fill = "#757070",
            stat_slab(data = all_calibrated_behavioral %>% filter(scenario == "s1"), aes(y = vis, x = payoff), fill = "#757070",
            geom_vline(xintercept = (all_rational_posterior %>% filter(scenario == "s1"))$payoff, linetype = "dashdot"),
            geom_vline(xintercept = (all_rational_prior %>% filter(scenario == "s1"))$payoff, linetype = "dashed"),
            labs(x = "", y = "") +
            ylim((score_summary %>% filter(scenario == "s1"))$vis) +
            theme(panel.grid.major.x = element_blank(),
                  panel.grid.minor.x = element_blank(),
                  panel.grid.major = element_line(colour = "grey"),
                  axis.line.x = element_line(linewidth = 1.5, colour = "grey80"),
                  panel.background = element_rect(fill = "white", color = "white"),
                  axis.ticks.y = element_blank(),
                  axis.ticks.x = element_line(colour = "grey")))

```

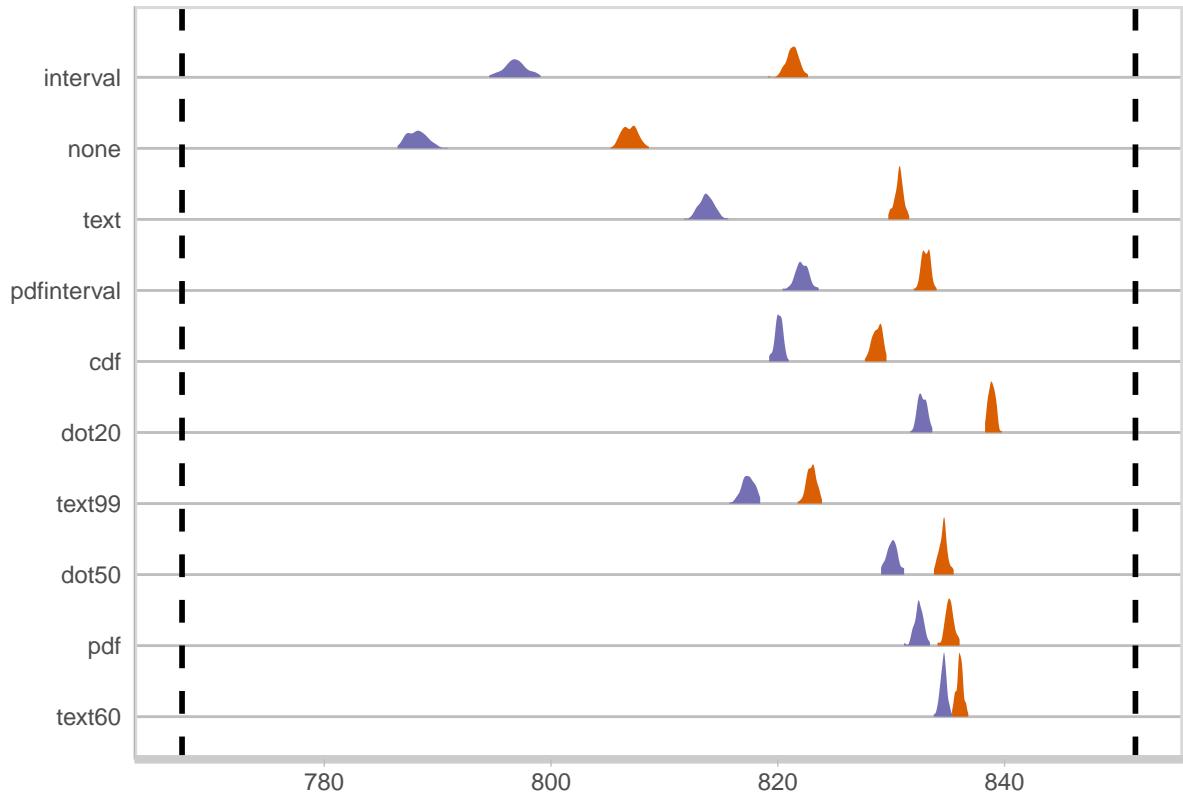


```

ggplot() +
  stat_slab(data = all_behavioral %>% filter(scenario == "s2"), aes(y = vis, x = payoff), fill = "#7570B1") +
  stat_slab(data = all_calibrated_behavioral %>% filter(scenario == "s2"), aes(y = vis, x = payoff), fill = "#E69138") +
  geom_vline(xintercept = (all_rational_posterior %>% filter(scenario == "s2"))$payoff, linetype = "dashed") +
  geom_vline(xintercept = (all_rational_prior %>% filter(scenario == "s2"))$payoff, linetype = "dashed") +
  labs(x = "", y = "") +
  ylim((score_summary %>% filter(scenario == "s2"))$vis) +
  theme(panel.grid.major.x = element_blank(),
        panel.grid.minor.x = element_blank(),
        panel.grid.major.y = element_line(colour = "grey"),
        axis.line.x = element_line(linewidth = 1.5, colour = "grey80"),
        panel.background = element_rect(fill = "white", color = "white"),
        axis.ticks.y = element_blank(),
        axis.ticks.x = element_line(colour = "grey"))

```

## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.  
## i Please use `linewidth` instead.  
## This warning is displayed once every 8 hours.  
## Call `lifecycle::last\_lifecycle\_warnings()` to see where this warning was  
## generated.



```

ggplot() +
  stat_slab(data = all_behavioral %>% filter(scenario == "s3"), aes(y = vis, x = payoff), fill = "#7570B1") +
  stat_slab(data = all_calibrated_behavioral %>% filter(scenario == "s3"), aes(y = vis, x = payoff), fill = "#E69138") +
  geom_vline(xintercept = (all_rational_posterior %>% filter(scenario == "s3"))$payoff, linetype = "dashed") +
  geom_vline(xintercept = (all_rational_prior %>% filter(scenario == "s3"))$payoff, linetype = "dashed") +
  labs(x = "", y = "") +
  ylim((score_summary %>% filter(scenario == "s3"))$vis) +
  theme(panel.grid.major.x = element_blank(),
        panel.grid.minor.x = element_blank(),
        panel.grid.major.y = element_line(colour = "grey"),
        axis.line.x = element_line(lineWidth = 1.5, colour = "grey80"),
        panel.background = element_rect(fill = "white", color = "white"),
        axis.ticks.y = element_blank(),
        axis.ticks.x = element_line(colour = "grey"))
  
```

