Detailed derivation of small-signal model for the LLC based on time-domain analysis

Zhiqiang Guo, Zhijie Huang
Beijing Institute of Technology

The topology of the full-bridge LLC resonant converter is shown in Fig.1, where v_{in} and v_o represent the input and output voltages. C_o denotes the output capacitor, and R_L is the load resistance. The primary stage is composed of Q_1-Q_4, and the rectifier stage is composed of D_{r1}-D_{r4}. The resonant tank consists of resonant inductor L_r, resonant capacitor C_r, and magnetizing inductor L_m of the transformer. For the ZVS of the switches, the LLC converter is suggested to work in PO mode for $f_s<f_r$ and NP mode for $f_s>f_r$ [25]. Therefore, the PO mode and NP mode will be analyzed below.

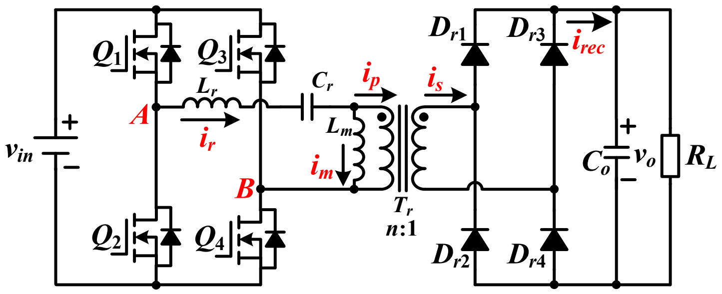

Fig.1 Topology of the LLC resonant converter

To facilitate the subsequent theoretical analysis, the time when the resonant current is equal to the magnetizing current is selected as t_0. The voltage across the resonant tank is v_{AB}. i_r and i_m represent the resonant current and magnetizing current. The transformer secondary current i_s is rectified to i_{rec}.

Variables with the subscript N are normalized in this article, where voltages are normalized with the voltage factor V_{in} and currents are normalized with the current factor $I_N=V_{in}/Z_0$. Z_0 is the characteristic impedance, expressed as $\sqrt{L_r/C_r}$, and the voltage gain M is defined as $M=nV_o/V_{in}$.

Section I. Time-domain expressions for PO mode

Typical waveforms and planar trajectory of the LLC converter for PO mode are shown in Fig.2 and Fig.3.

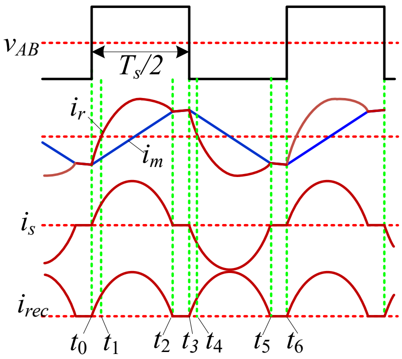

Fig.2 Typical waveforms of the LLC converter for PO mode.

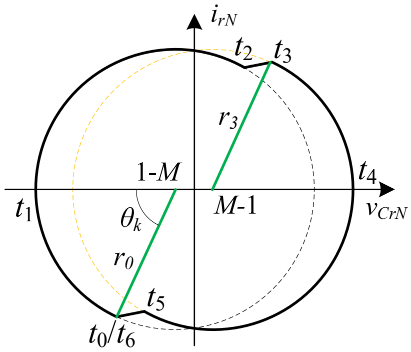

Fig.3 Planar trajectory of the LLC converter for PO mode

[t_0, t_2]

The converter operates in P mode. Setting $t_0 = 0$, only the resonant inductor and resonant capacitor are involved in resonance, and the magnetizing inductor is clamped by the output voltage. ω_{r0} is the resonant angular frequency of both. i_{r0} and v_{cr0} are the values of resonant current and resonant capacitor voltage at t_0, respectively. They can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r0} Z_0 \sin\left(\omega_{r0} t\right) + \left[v_{cr0} - \left(V_{in} - nV_o\right)\right]\cos\left(\omega_{r0} t\right) + \left(V_{in} - nV_o\right) \\ i_r &= i_{r0} \cos\left(\omega_{r0} t\right) - \frac{v_{cr0} - \left(V_{in} - nV_o\right)}{Z_0} \sin\left(\omega_{r0} t\right) \end{aligned} \tag{1}$$

After normalization

$$\begin{aligned} v_{crN} &= i_{r0N} \sin\left(\omega_{r0} t\right) + \left[v_{cr0N} - \left(1 - M\right)\right]\cos\left(\omega_{r0} t\right) + \left(1 - M\right) \\ i_{rN} &= i_{r0N} \cos\left(\omega_{r0} t\right) - \left[v_{cr0N} - \left(1 - M\right)\right]\sin\left(\omega_{r0} t\right) \end{aligned} \tag{2}$$

where $i_{r0N} = \dfrac{i_{r0} Z_0}{V_{in}}$, $v_{cr0N} = \dfrac{v_{cr0}}{V_{in}}$

Eq.(2) can be rewritten as

$$\begin{aligned} i_{rN} &= \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - \left(1 - M\right)\right]^2} \sin\left(\omega_{r0} t + \theta_0\right) \\ v_{crN} &= -\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - \left(1 - M\right)\right]^2} \cos\left(\omega_{r0} t + \theta_0\right) + \left(1 - M\right) \end{aligned} \tag{3}$$

where

$$\cos\theta_0 = -\frac{\left[v_{cr0N} - \left(1 - M\right)\right]}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - \left(1 - M\right)\right]^2}}, \sin\theta_0 = \frac{i_{r0N}}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - \left(1 - M\right)\right]^2}}$$

$$\theta_0 = \arctan\left(\frac{-i_{r0N}}{v_{cr0N} - \left(1 - M\right)}\right)$$

Let $r_0 = \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - \left(1 - M\right)\right]^2}$, then

$$\begin{aligned} i_{rN} &= r_0 \sin\left(\omega_{r0} t + \theta_0\right) \\ v_{crN} &= -r_0 \cos\left(\omega_{r0} t + \theta_0\right) + \left(1 - M\right) \end{aligned} \tag{4}$$

i_{r0N}, v_{cr0N}, i_{r2N}, and v_{cr2N} can be expressed in (5), where $\varphi_0 = \omega_{r0}(t_2 - t_1)$

$$\begin{aligned} i_{r0N} &= r_0 \sin\left(\theta_0\right) \\ v_{cr0N} &= -r_0 \cos\left(\theta_0\right) + \left(1 - M\right) \\ i_{r2N} &= r_0 \sin\left(\varphi_0 + \theta_0\right) \\ v_{cr2N} &= -r_0 \cos\left(\varphi_0 + \theta_0\right) + \left(1 - M\right) \end{aligned} \tag{5}$$

The expression of the magnetizing current i_m is shown in (6).

$$i_m = i_{r0} + \frac{nV_o}{L_m} t \tag{6}$$

(6) is normalized to (7).

$$i_{mN} = \frac{i_{r0} Z_0}{V_{in}} + \frac{nV_o Z_0}{V_{in} L_m} t = i_{r0N} + M \sqrt{\frac{L_r}{C_r}} \frac{1}{L_m} t = r_0 \sin\left(\theta_0\right) + \frac{M}{L_n} \omega_{r0} t \tag{7}$$

The output current of the rectifier bridge is expressed as

$$i_{rec1} = nI_n\left(i_{rN} - i_{mN}\right) = nI_n\left(r_0\sin\left(\omega_{r0}t+\theta_0\right) - r_0\sin\left(\theta_0\right) - \frac{M}{L_n}\omega_{r0}t\right) \tag{8}$$

Since $i_{rec} = 0$ from t_2 to t_3, the average value of i_{rec} over half a switching cycle can be expressed as (9).

$$\begin{aligned}\overline{i}_{rec1} &= \frac{2}{T_s}\int_0^{t_2} i_{rec}dt = \frac{2nI_n}{T_s}\int_{t_0}^{t_2}\left(r_0\sin\left(\omega_{r0}t+\theta_0\right) - r_0\sin\left(\theta_0\right) - \frac{M}{L_n}\omega_{r0}t\right)dt \\ &= \frac{2nI_n}{T_s}\left(-\frac{r_0}{\omega_{r0}}\cos\left(\omega_{r0}t+\theta_0\right) - r_0\sin\left(\theta_0\right)t - \frac{M}{2L_n}\omega_{r0}t^2\right)\Bigg|_0^{t_2} \\ &= \frac{2nI_n}{T_s}\left(\frac{r_0}{\omega_{r0}}\cos\left(\theta_0\right) - \frac{r_0}{\omega_{r0}}\cos\left(\omega_{r0}t_2+\theta_0\right) - r_0\sin\left(\theta_0\right)t_2 - \frac{M}{2L_n}\omega_{r0}t_2^{\,2}\right)\end{aligned} \tag{9}$$

[t_2, t_3]

The converter operates in O mode. The resonant inductor, the resonant capacitor, and the magnetizing inductor are involved in resonance. ω_{r1} is the resonant angular frequency of them. Z_1 is expressed as $\sqrt{\left(L_r + L_m\right)/C_r}$. v_{cr} and i_r can be expressed as follows.

$$\begin{aligned}v_{cr} &= i_{r2}Z_1\sin\left(\omega_{r1}\left(t-t_2\right)\right) + \left(v_{cr2} - V_{in}\right)\cos\left(\omega_{r1}\left(t-t_2\right)\right) + V_{in} \\ i_r &= i_{r2}\cos\left(\omega_{r1}\left(t-t_2\right)\right) - \frac{1}{Z_1}\left(v_{cr2} - V_{in}\right)\sin\left(\omega_{r1}\left(t-t_2\right)\right)\end{aligned} \tag{10}$$

After normalization

$$\begin{aligned}v_{crN} &= \frac{i_{r2N}}{Z_0/Z_1}\sin\left(\omega_{r1}\left(t-t_2\right)\right) + \left(v_{cr2N} - 1\right)\cos\left(\omega_{r1}\left(t-t_2\right)\right) + 1 \\ i_{rN} &= i_{r2N}\cos\left(\omega_{r1}\left(t-t_2\right)\right) - \frac{Z_0}{Z_1}\left(v_{cr2N} - 1\right)\sin\left(\omega_{r1}\left(t-t_2\right)\right)\end{aligned} \tag{11}$$

where $i_{r2N} = \dfrac{i_{r2}Z_0}{V_{in}}, v_{cr2N} = \dfrac{v_{cr2}}{V_{in}}, L_n = \dfrac{L_m}{L_r}, \dfrac{Z_0}{Z_1} = \sqrt{\dfrac{1}{1+L_n}}$

Eq.(11) can be rewritten as

$$\begin{aligned}v_{crN} &= -\sqrt{\left(1+L_n\right)i_{r2N}^{\,2} + \left(v_{cr2N} - 1\right)^2}\cos\left(\omega_{r1}\left(t-t_2\right) + \theta_1\right) + 1 \\ i_{rN} &= \frac{\sqrt{\left(1+L_n\right)i_{r2N}^{\,2} + \left(v_{cr2N} - 1\right)^2}}{\sqrt{1+L_n}}\sin\left(\omega_{r1}\left(t-t_2\right) + \theta_1\right)\end{aligned} \tag{12}$$

where

$$\cos\theta_1 = -\frac{\left(v_{cr2N} - 1\right)}{\sqrt{\left(1+L_n\right)i_{r2N}^{\,2} + \left(v_{cr2N} - 1\right)^2}}, \sin\theta_1 = \frac{\sqrt{1+L_n}\,i_{r2N}}{\sqrt{\left(1+L_n\right)i_{r2N}^{\,2} + \left(v_{cr2N} - 1\right)^2}}$$

$$\theta_1 = \arctan\left(-\frac{\sqrt{1+L_n}\,i_{r2N}}{v_{cr2N} - 1}\right)$$

Let $r_1 = \sqrt{\left(1+L_n\right)i_{r2N}^{\,2} + \left(v_{cr2N} - 1\right)^2}$, then

$$
\begin{aligned}
v_{crN} &= -r_1\cos\left(\omega_{r1}\left(t-t_2\right)+\theta_1\right)+1 \\
i_{rN} = i_{mN} &= \frac{r_1}{\sqrt{1+L_n}}\sin\left(\omega_{r1}\left(t-t_2\right)+\theta_1\right)
\end{aligned} \tag{13}
$$

i_{r2N}, v_{cr2N}, i_{r3N}, and v_{cr3N} can be expressed in (14), where $\varphi_1=\omega_{r1}(t_3-t_2)$

$$
\begin{aligned}
v_{cr2N} &= -r_1\cos\left(\theta_1\right)+1 \\
i_{r2N} &= \frac{r_1}{\sqrt{1+L_n}}\sin\left(\theta_1\right) \\
v_{cr3N} &= -r_1\cos\left(\varphi_1+\theta_1\right)+1 \\
i_{r3N} &= \frac{r_1}{\sqrt{1+L_n}}\sin\left(\varphi_1+\theta_1\right)
\end{aligned} \tag{14}
$$

[t_3, t_5]

Similar to t_0 to t_2, v_{cr} and i_r can be expressed as follows.

$$
\begin{aligned}
v_{cr} &= i_{r3}Z_0\sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3}+\left(V_{in}-nV_o\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(V_{in}-nV_o\right) \\
i_r &= i_{r3}\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\frac{v_{cr3}+\left(V_{in}-nV_o\right)}{Z_0}\sin\left(\omega_{r0}\left(t-t_3\right)\right)
\end{aligned} \tag{15}
$$

After normalization

$$
\begin{aligned}
v_{crN} &= i_{r3N}\sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3N}+\left(1-M\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(1-M\right) \\
i_{rN} &= i_{r3N}\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left[v_{cr3N}+\left(1-M\right)\right]\sin\left(\omega_{r0}\left(t-t_3\right)\right)
\end{aligned} \tag{16}
$$

where $i_{r3N}=\dfrac{i_{r3}Z_0}{V_{in}}, v_{cr3N}=\dfrac{v_{cr3}}{V_{in}}$

Eq.(16) can be rewritten as

$$
\begin{aligned}
i_{rN} &= \sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\sin\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right) \\
v_{crN} &= -\sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\cos\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right)-\left(1-M\right)
\end{aligned} \tag{17}
$$

where

$$
\cos\theta_2 = -\frac{\left[v_{cr3N}+\left(1-M\right)\right]}{\sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}}, \sin\theta_2 = \frac{i_{r3N}}{\sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}}
$$

$$
\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{cr3N}+\left(1-M\right)}\right)
$$

Let $r_2=\sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}$, then

$$
\begin{aligned}
i_{rN} &= r_2\sin\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right) \\
v_{crN} &= -r_2\cos\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right)-\left(1-M\right)
\end{aligned} \tag{18}
$$

i_{r3N}, v_{cr3N}, i_{r5N}, and v_{cr5N} can be expressed in (19), where $\varphi_2=\omega_{r0}(t_5-t_3)$

$$\begin{aligned} i_{r3N} &= r_2 \sin(\theta_2) \\ v_{cr3N} &= -r_2 \cos(\theta_2) - (1-M) \\ i_{r5N} &= r_2 \sin(\varphi_2 + \theta_2) \\ v_{cr5N} &= -r_2 \cos(\varphi_2 + \theta_2) - (1-M) \end{aligned} \tag{19}$$

The expression of the magnetizing current i_m is shown in (20).

$$i_m = i_{r3} - \frac{nV_o}{L_m}(t-t_3) \tag{20}$$

After normalization

$$i_{mN} = \frac{i_{r3}Z_0}{V_{in}} - \frac{nV_oZ_0}{V_{in}L_m}(t-t_3) = i_{r3N} - M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}(t-t_3) = r_2\sin(\theta_2) - \frac{M}{L_n}\omega_{r0}(t-t_3) \tag{21}$$

The output current of the rectifier bridge is expressed as

$$i_{rec2} = nI_n(i_{rN} - i_{mN}) = nI_n\left(r_2\sin(\omega_{r0}(t-t_3)+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\omega_{r0}(t-t_3)\right) \tag{22}$$

Since $i_{rec} = 0$ from t_5 to t_6, the average value of i_{rec} over half a switching cycle can be expressed as (23).

$$\begin{aligned} \overline{i}_{rec2} &= \frac{2}{T_s}\int_{t_3}^{t_5} i_{rec}dt = \frac{2nI_n}{T_s}\int_{t_3}^{t_5}\left(r_2\sin(\omega_{r0}(t-t_3)+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\omega_{r0}(t-t_3)\right)dt \\ &= \frac{2nI_n}{T_s}\left(-\frac{r_2}{\omega_{r0}}\cos(\omega_{r0}(t-t_3)+\theta_2) - r_2\sin(\theta_2)t + \frac{M}{2L_n}\omega_{r0}(t-t_3)^2\right)\Bigg|_{t_3}^{t_5} \\ &= \frac{2nI_n}{T_s}\left(\frac{r_2}{\omega_{r0}}\cos(\theta_2) - \frac{r_2}{\omega_{r0}}\cos(\omega_{r0}(t_5-t_3)+\theta_2) - r_2\sin(\theta_2)(t_5-t_3) + \frac{M}{2L_n}\omega_{r0}(t_5-t_3)^2\right) \end{aligned} \tag{23}$$

The output voltage can be calculated by (24).

$$V_o = \frac{\overline{i}_{rec1} + \overline{i}_{rec2}}{2}R \tag{24}$$

[t_5, t_6]

Similar to t_5 to t_6, v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r5}Z_1\sin(\omega_{r1}t) + [v_{cr5} + V_{in}]\cos(\omega_{r1}t) - V_{in} \\ i_r &= i_{r5}\cos(\omega_{r1}t) - \frac{v_{cr5}+V_{in}}{Z_1}\sin(\omega_{r1}t) \end{aligned} \tag{25}$$

After normalization

$$\begin{aligned} v_{crN} &= \frac{i_{r5N}}{Z_0/Z_1}\sin(\omega_{r1}(t-t_5)) + (v_{cr5N}+1)\cos(\omega_{r1}(t-t_5)) - 1 \\ i_{rN} &= i_{r5N}\cos(\omega_{r1}(t-t_5)) - \frac{Z_0}{Z_1}(v_{cr5N}+1)\sin(\omega_{r1}(t-t_5)) \end{aligned} \tag{26}$$

where $i_{r5N} = \frac{i_{r5}Z_0}{V_{in}}, v_{cr5N} = \frac{v_{cr5}}{V_{in}},, L_n = \frac{L_m}{L_r}, \frac{Z_0}{Z_1} = \sqrt{\frac{1}{1+L_n}}$

Eq.(26) can be rewritten as

$$
\begin{aligned}
v_{crN} &= -\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}\cos\left(\omega_{r1}(t-t_5)+\theta_3\right)-1 \\
i_{rN} &= \frac{\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}}{\sqrt{1+L_n}}\sin\left(\omega_{r1}(t-t_5)+\theta_3\right)
\end{aligned} \tag{27}
$$

where

$$
\begin{aligned}
&\cos\theta_3 = -\frac{(v_{cr5N}+1)}{\sqrt{(1+L_n)i_{r2N}{}^2+(v_{cr2N}+1)^2}}, \sin\theta_3 = \frac{\sqrt{1+L_n}\,i_{r5N}}{\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}} \\
&\theta_3 = \pi + \arctan\left(-\frac{\sqrt{1+L_n}\,i_{r5N}}{v_{cr5N}+1}\right)
\end{aligned}
$$

Let $r_3 = \sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}$, then

$$
\begin{aligned}
v_{crN} &= -r_3\cos\left(\omega_{r1}(t-t_5)+\theta_3\right)-1 \\
i_{rN} &= i_{mN} = \frac{r_3}{\sqrt{1+L_n}}\sin\left(\omega_{r1}(t-t_5)+\theta_3\right)
\end{aligned} \tag{28}
$$

i_{r5N}, v_{cr5N}, i_{r6N}, and v_{cr6N} can be expressed in (29), where $\varphi_3=\omega_{r1}(t_6-t_5)$

$$
\begin{aligned}
v_{cr5N} &= -r_3\cos(\theta_3)-1 \\
i_{r5N} &= i_{mN} = \frac{r_3}{\sqrt{1+L_n}}\sin(\theta_3) \\
v_{cr6N} &= -r_3\cos(\varphi_3+\theta_3)-1 \\
i_{r6N} &= i_{mN} = \frac{r_3}{\sqrt{1+L_n}}\sin(\varphi_3+\theta_3)
\end{aligned} \tag{29}
$$

Section II. Calculation of steady-state operating point for PO mode

Because of the semi-period symmetry, the i_{r0N} and v_{cr0N} at t_0 are equal to the negative of i_{r3N} and v_{cr3N} respectively. Therefore, (30) can be obtained.

$$\begin{aligned} i_{r3N} &= \frac{r_1}{\sqrt{1+L_n}}\sin(\varphi_1+\theta_1) = -i_{r0N} = -r_0\sin(\theta_0) \\ v_{cr3N} &= -r_1\cos(\varphi_1+\theta_1)+1 = -v_{cr0N} = r_0\cos(\theta_0)-(1-M) \end{aligned} \tag{30}$$

Mode P transitions to Mode O at t_2, with the resonant current i_{rN} equal to the magnetizing current i_{mN}, (31) can be obtain.

$$\begin{aligned} i_{r2N} &= r_0\sin(\varphi_0+\theta_0) = \frac{r_1}{\sqrt{1+L_n}}\sin(\theta_1) \\ v_{cr2N} &= -r_0\cos(\varphi_0+\theta_0)+(1-M) = -r_1\cos(\theta_1)+1 \\ i_{rec}(t_2) &= nI_n\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)=0 \end{aligned} \tag{31}$$

At steady state, $\overline{i}_{rec1}=\overline{i}_{rec2}$, $M=\overline{i}_{rec1}R$ .According to the definition of M, φ_0 and φ_1, (32) can be obtained.

$$\begin{aligned} M &= \frac{nV_o}{V_{in}} = \frac{2n^2RI_n}{T_sV_{in}}\left(\frac{r_0}{\omega_{r0}}\cos(\theta_0)-\frac{r_0}{\omega_{r0}}\cos(\omega_{r0}t_2+\theta_0)-r_0\sin(\theta_0)t_2-\frac{M}{2\omega_{r0}L_n}(\omega_{r0}t_2)^2\right) \\ &\frac{\varphi_2}{\omega_{r0}}+\frac{\varphi_2}{\omega_{r1}}=\frac{T_s}{2} \end{aligned} \tag{32}$$

Therefore, the following system of equations can be obtained

$$\begin{cases} r_0\sin(\varphi_0+\theta_0)-\dfrac{r_1}{\sqrt{1+L_n}}\sin(\theta_1)=0 \\ -r_0\cos(\varphi_0+\theta_0)-M+r_1\cos(\theta_1)=0 \\ \dfrac{r_1}{\sqrt{1+L_n}}\sin(\varphi_1+\theta_1)+r_0\sin(\theta_0)=0 \\ -r_1\cos(\varphi_1+\theta_1)-r_0\cos(\theta_0)+(2-M)=0 \\ r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\dfrac{M}{L_n}\varphi_0=0 \\ M-\dfrac{2n^2RI_n}{T_sV_{in}\omega_{r0}}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\dfrac{M}{2L_n}{\varphi_0}^2\right)=0 \\ \dfrac{\varphi_0}{\omega_{r0}}+\dfrac{\varphi_1}{\omega_{r1}}-\dfrac{T_s}{2}=0 \end{cases} \tag{33}$$

$[r_0 \quad \theta_0 \quad \varphi_0 \quad r_1 \quad \theta_1 \quad \varphi_1 \quad M]$ is defined as the variables to be solved under the steady state. By using the Newton-Raphson iteration method, the solution of the equations can be calculated, so the steady-state operating point of the system will be obtained, and then steady-state current and voltage values I_{r0N}, I_{r2N}, I_{r3N}, I_{r5N}, I_{r6N}, V_{r0N}, V_{r2N}, V_{r3N}, V_{r5N}, and V_{r6N} at different moments can be obtained.

Section III. Small-signal model of the LLC converter for PO mode

Set $x=[i_{r0N}, v_{cr0N}, v_o]^T$ as state variables, $u=[v_{in}, t_s]^T$ as input variables, and $y=v_o$ as output variable. The state-space expression for the system can be expressed as (34), where C=[0, 0, 1].

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{34}$$

The large-signal model of the LLC converter over one switching cycle is expressed as follows:

$$\begin{cases} \dot{i}_{r0N} = \dfrac{i_{r6N} - i_{r0N}}{t_s} \\ \dot{v}_{cr0N} = \dfrac{v_{cr6N} - v_{cr0N}}{t_s} \\ \dot{v}_o = \dfrac{1}{C_o}\left(\Delta \overline{i}_{rec} - \dfrac{v_o}{R}\right) \end{cases} \tag{35}$$

In this derivation for the small-signal model of the LLC converter, $\boldsymbol{g}$, $\boldsymbol{h}$, $\boldsymbol{k}$, $\boldsymbol{l}$, $\boldsymbol{m}$ represent the partial derivatives of the $\boldsymbol{\theta}$, $\boldsymbol{r}$, $\boldsymbol{i_{rN}}$, $\boldsymbol{v_{crN}}$, and $\boldsymbol{\varphi}$ to the corresponding variables. Add perturbations to the input and state variables at the quiescent-state operating point as follows:

$$\begin{cases} v_{in} = V_{in} + \hat{v}_{in} \\ v_o = V_o + \hat{v}_o \\ t_s = T_s + \hat{t}_s \\ i_{r0N} = I_{r0N} + \hat{i}_{r0N} \\ v_{cr0N} = V_{cr0N} + \hat{v}_{cr0N} \end{cases} \tag{36}$$

From t_0 to t_3 with half a switch period, time-domain expressions are as follows:

$$\begin{cases} i_{r0N} = r_0 \sin(\theta_0) \\ v_{r0N} = -r_0 \cos(\theta_0) + (1-M) \\ i_{r2N} = r_0 \sin(\varphi_0 + \theta_0) = r_1 \sin(\theta_1) \\ v_{r2N} = -r_0 \cos(\varphi_0 + \theta_0) + (1-M) = -r_1 \cos[\varphi_1 + \theta_1] + 1 \\ i_{r3N} = \dfrac{r_1}{\sqrt{1+L_n}} \sin(\varphi_1 + \theta_1) \\ v_{r3N} = -r_1 \cos(\varphi_1 + \theta_1) + 1 \\ i_{rec}(t_2) = nI_n\left(r_0 \sin(\varphi_0 + \theta_0) - r_0 \sin(\theta_0) - \dfrac{M}{L_n}\varphi_0 \right) = 0 \\ \overline{i}_{rec1} = \dfrac{nI_n}{\omega_{r0} T_s}\left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0 + \theta_0) - r_0 \sin(\theta_0)\varphi_0 - \dfrac{M}{2L_n}\varphi_0^{\,2} \right) \end{cases} \tag{37}$$

At time t_0, the converter starts to operates in the P mode. θ_0 and r_0 can be calculated by

$$\theta_0 = \arctan\left(-\frac{i_{r0N}}{v_{r0N} - (1-M)} \right) \qquad r_0 = \sqrt{i_{r0N}^{\;2} + \left[v_{r0N} - (1-M) \right]^2} \tag{38}$$

After first-order linearization:

$$
\begin{aligned}
&\theta_0+\Delta\theta_0=\theta_0+\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\\
&=\theta_0-\frac{v_{r0N}-(1-M)}{\left[v_{r0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{\left[v_{r0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{v}_{r0N}-\frac{i_{r0N}M/v_{in}}{r_0{}^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0{}^2}\hat{v}_o\\
&=\theta_0-\frac{v_{r0N}-(1-M)}{r_0{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{r_0{}^2}\hat{v}_{r0N}-\frac{i_{r0N}M/v_{in}}{r_0{}^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0{}^2}\hat{v}_o\\
&=\theta_0+g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{r0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\\
&r_0+\Delta r_0=r_0+\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\\
&=r_0+\frac{i_{r0N}}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{i}_{r0N}+\frac{v_{r0N}-(1-M)}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{v}_{r0N}\\
&-\frac{\left[v_{r0N}-(1-M)\right]M/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{v}_{in}+\frac{\left[v_{r0N}-(1-M)\right]n/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{v}_o\\
&=r_0+\frac{i_{r0N}}{r_0}\hat{i}_{r0N}+\frac{v_{r0N}-(1-M)}{r_0}\hat{v}_{r0N}-\frac{\left[v_{r0N}-(1-M)\right]M/v_{in}}{r_0}\hat{v}_{in}+\frac{\left[v_{r0N}-(1-M)\right]n/v_{in}}{r_0}\hat{v}_o\\
&=r_0+h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{r0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o
\end{aligned}
\tag{39}
$$

At time t_2, $i_{recN}(t_2)$=0, and $i_{recN}(t_2+\Delta t_2)$=0 after the perturbations are added. (40) can be obtained.

$$
\begin{aligned}
&i_{recN}(t_2+\Delta t_2)=n\left((r_0+\Delta r_0)\sin(\varphi_0+\Delta\varphi_0+\theta_0+\Delta\theta_0)-(r_0+\Delta r_0)\sin(\theta_0+\Delta\theta_0)-\frac{n(v_o+\Delta v_o)}{(v_{in}+\Delta v_{in})L_n}(\varphi_0+\Delta\varphi_0)\right)\\
&\approx n\left(r_0\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)-\frac{M}{L_n}\varphi_0\right)+\frac{\partial i_{recN}(t_2)}{\partial r_0}\Delta r_0+\frac{\partial i_{recN}(t_2)}{\partial\varphi_0}\Delta\varphi_0+\frac{\partial i_{recN}(t_2)}{\partial\theta_0}\Delta\theta_0+\frac{\partial i_{recN}(t_2)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}(t_2)}{\partial v_o}\hat{v}_o\\
&=i_{recN}(t_2)+\frac{\partial i_{recN}(t_2)}{\partial r_0}\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{recN}(t_2)}{\partial\varphi_0}\Delta\varphi_0\\
&+\frac{\partial i_{recN}(t_2)}{\partial\theta_0}\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{recN}(t_2)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}(t_2)}{\partial v_o}\hat{v}_o\\
&=i_{recN}(t_2)+n\left[\begin{array}{l}\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right)+r_0\cos(\varphi_0+\theta_0)\Delta\varphi_0-\frac{M}{L_n}\Delta\varphi_0+\\ r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right)+\frac{\varphi_0M/v_{in}}{L_n}\hat{v}_{in}-\frac{\varphi_0n/v_{in}}{L_n}\hat{v}_o\end{array}\right]\\
&=i_{recN}(t_2)+n\left[\begin{array}{l}\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{r0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right)+r_0\cos(\varphi_0+\theta_0)\Delta\varphi_0+\\ r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{r0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right)-\frac{M}{L_n}\Delta\varphi_0+\frac{\varphi_0M/v_{in}}{L_n}\hat{v}_{in}-\frac{\varphi_0n/v_{in}}{L_n}\hat{v}_o\end{array}\right]\\
&=i_{recN}(t_2)+n\left[\begin{array}{l}\left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0i}+r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0i}\right]\hat{i}_{r0N}+\\ \left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0v}+r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0v}\right]\hat{v}_{r0N}+\\ \left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0in}+r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0in}+\frac{\varphi_0M/v_{in}}{L_n}\right]\hat{v}_{in}\\ +\left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0o}+r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0o}-\frac{\varphi_0n/v_{in}}{L_n}\right]\hat{v}_o\\ +\left[r_0\cos(\varphi_0+\theta_0)-\frac{M}{L_n}\right]\Delta\varphi_0\end{array}\right]=0
\end{aligned}
\tag{40}
$$

$\Delta\varphi_0$ can be calculated as follows:

$$\Delta\varphi_0 = \frac{1}{\frac{M}{L_n} - r_0\cos(\varphi_0+\theta_0)} \begin{bmatrix} \left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0i} + r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0i}\right]\hat{i}_{r0N} + \\ \left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0v} + r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0v}\right]\hat{v}_{r0N} + \\ \left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0in} + r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0in} + \frac{\varphi_0 M/v_{in}}{L_n}\right]\hat{v}_{in} \\ + \left[\left(\sin(\varphi_0+\theta_0)-\sin(\theta_0)\right)h_{0o} + r_0\left(\cos(\varphi_0+\theta_0)-\cos(\theta_0)\right)g_{0o} - \frac{\varphi_0 n/v_{in}}{L_n}\right]\hat{v}_o \end{bmatrix}$$
$$= m_{0i}\hat{i}_{r0N} + m_{0v}\hat{v}_{r0N} + m_{0in}\hat{v}_{in} + m_{0o}\hat{v}_o \tag{41}$$

After $\Delta\theta_0$, Δr_0, and $\Delta\varphi_0$ are known, Δi_{r2N} and Δv_{r2N} can be calculated as follows:

$$\begin{aligned}
i_{r2N} + \Delta i_{r2N} &= r_0\sin(\varphi_0+\theta_0) + \frac{\partial i_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial i_{r2N}}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial i_{r2N}}{\partial v_{in}}\hat{v}_{in} + \frac{\partial i_{r2N}}{\partial v_o}\hat{v}_o \\
&= i_{r2N} + \left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}} + \frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}} + \frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial i_{r0N}}\right)\hat{i}_{r0N} + \left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{r0N}} + \frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{r0N}} + \frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
&+ \left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}} + \frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}} + \frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{in}}\right)\hat{v}_{in} + \left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o} + \frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o} + \frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_o}\right)\hat{v}_o \\
&= i_{r2N} + \left[\sin(\varphi_0+\theta_0)h_{0i} + r_0\cos(\varphi_0+\theta_0)(g_{0i}+m_{0i})\right]\hat{i}_{r0N} \\
&+ \left[\sin(\varphi_0+\theta_0)h_{0v} + r_0\cos(\varphi_0+\theta_0)(g_{0v}+m_{0v})\right]\hat{v}_{r0N} \\
&+ \left[\sin(\varphi_0+\theta_0)h_{0in} + r_0\cos(\varphi_0+\theta_0)(g_{0in}+m_{0in})\right]\hat{v}_{in} \\
&+ \left[\sin(\varphi_0+\theta_0)h_{0o} + r_0\cos(\varphi_0+\theta_0)(g_{0o}+m_{0o})\right]\hat{v}_o \\
&= i_{r2N} + k_{2i}\hat{i}_{r0N} + k_{2v}\hat{v}_{r0N} + k_{2in}\hat{v}_{in} + k_{2o}\hat{v}_o \\
v_{r2N} + \Delta v_{r2N} &= -r_0\cos(\varphi_0+\theta_0) + (1-M) + \frac{\partial v_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial v_{r2N}}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial v_{r2N}}{\partial v_{in}}\hat{v}_{in} + \frac{\partial v_{r2N}}{\partial v_o}\hat{v}_o \\
&= v_{r2N} + \left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}} + \frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}} + \frac{\partial v_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial i_{r0N}}\right)\hat{i}_{r0N} + \left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{r0N}} + \frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{r0N}} + \frac{\partial v_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
&+ \left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}} + \frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}} + \frac{\partial v_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{in}}\right)\hat{v}_{in} + \left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o} + \frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o} + \frac{\partial v_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_o}\right)\hat{v}_o \\
&= v_{r2N} + \left[-\cos(\varphi_0+\theta_0)h_{i0} + r_0\sin(\varphi_0+\theta_0)(g_{i0}+m_{0i})\right]\hat{i}_{r0N} \\
&+ \left[-\cos(\varphi_0+\theta_0)h_{0v} + r_0\sin(\varphi_0+\theta_0)(g_{0v}+m_{0v})\right]\hat{v}_{r0N} \\
&+ \left[-\cos(\varphi_0+\theta_0)h_{0in} + r_0\sin(\varphi_0+\theta_0)(g_{0in}+m_{0in}) + \frac{M}{v_{in}}\right]\hat{v}_{in} \\
&+ \left[-\cos(\varphi_0+\theta_0)h_{0o} + r_0\sin(\varphi_0+\theta_0)(g_{0o}+m_{0o}) - \frac{n}{v_{in}}\right]\hat{v}_o \\
&= v_{r2N} + l_{2i}\hat{i}_{r0N} + l_{2v}\hat{v}_{r0N} + l_{2in}\hat{v}_{in} + l_{2o}\hat{v}_o
\end{aligned} \tag{42}$$

At time t_2, the converter starts to work in O mode, θ_1 and r_1 can be expressed as follows:

$$\theta_1 = \arctan\left(-\frac{\sqrt{1+L_n}\,i_{r2N}}{v_{r2N}-1}\right) \qquad r_1 = \sqrt{(1+L_n)i_{r2N}^{\ 2} + (v_{r2N}-1)^2} \tag{43}$$

Therefore, $\Delta\theta_1$ and Δr_1 can be calculated by:

$$
\begin{aligned}
&\theta_1+\Delta\theta_1=\theta_1+\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o\\
&=\theta_1+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{r0N}}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_o}\right)\hat{v}_o\\
&=\theta_1+\left[-\frac{\sqrt{1+L_n}\left(v_{r2N}-1\right)}{r_1^2}k_{2i}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2i}\right]\hat{i}_{r0N}+\left[-\frac{\sqrt{1+L_n}\left(v_{r2N}-1\right)}{r_1^2}k_{2v}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2v}\right]\hat{v}_{r0N}\\
&+\left[-\frac{\sqrt{1+L_n}\left(v_{r2N}-1\right)}{r_1^2}k_{2in}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2in}\right]\hat{v}_{in}+\left[-\frac{\sqrt{1+L_n}\left(v_{r2N}-1\right)}{r_1^2}k_{2o}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2o}\right]\hat{v}_o\\
&=\theta_1+g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o\\
&r_1+\Delta r_1=r_1+\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o\\
&=r_1+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{r0N}}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_o}\right)\hat{v}_o\\
&=r_1+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2i}+\frac{v_{r2N}-1}{r_1}l_{2i}\right)\hat{i}_{r0N}+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2v}+\frac{v_{r2N}-1}{r_1}l_{2v}\right)\hat{v}_{r0N}\\
&+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2in}+\frac{v_{r2N}-1}{r_1}l_{2in}\right)\hat{v}_{in}+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2o}+\frac{v_{r2N}-1}{r_1}l_{2o}\right)\hat{v}_o\\
&=r_1+h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o
\end{aligned}
\tag{44}
$$

$\Delta\varphi_1$ can be calculated by

$$
\begin{aligned}
&\varphi_1=\frac{\omega_{r1}t_s}{2}-\frac{\omega_{r1}}{\omega_{r0}}\varphi_0\\
&\Delta\varphi_1=\frac{\omega_{r1}}{2}\hat{t}_s-\frac{\omega_{r1}}{\omega_{r0}}\Delta\varphi_0=m_{1t}\hat{t}_s-\frac{\omega_{r1}}{\omega_{r0}}m_{0i}\hat{i}_{r0N}-\frac{\omega_{r1}}{\omega_{r0}}m_{0v}\hat{v}_{r0N}-\frac{\omega_{r1}}{\omega_{r0}}m_{0in}\hat{v}_{in}-\frac{\omega_{r1}}{\omega_{r0}}m_{0o}\hat{v}_o\\
&=m_{1t}\hat{t}_s+m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{r0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o
\end{aligned}
\tag{45}
$$

After $\Delta\theta_1$, Δr_1, and $\Delta\varphi_1$ are known, Δi_{r3N} and Δv_{r3N} can be calculated as follows:

$$
\begin{aligned}
&i_{r3N}+\Delta i_{r3N}=\frac{r_1}{\sqrt{1+L_n}}\sin\left(\varphi_1+\theta_1\right)+\frac{\partial i_{r3N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r3N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial i_{r3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r3N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r3N}}{\partial t_s}\hat{t}_s\\
&=i_{r3N}+\left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial i_{r0N}}+\frac{\partial i_{r3N}}{\partial\theta_1}\frac{\partial\theta_1}{\partial i_{r0N}}+\frac{\partial i_{r3N}}{\partial\varphi_1}\frac{\partial\varphi_1}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{r0N}}+\frac{\partial i_{r3N}}{\partial\theta_1}\frac{\partial\theta_1}{\partial v_{r0N}}+\frac{\partial i_{r3N}}{\partial\varphi_1}\frac{\partial\varphi_1}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{in}}+\frac{\partial i_{r3N}}{\partial\theta_1}\frac{\partial\theta_1}{\partial v_{in}}+\frac{\partial i_{r3N}}{\partial\varphi_1}\frac{\partial\varphi_1}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_o}+\frac{\partial i_{r3N}}{\partial\theta_1}\frac{\partial\theta_1}{\partial v_o}+\frac{\partial i_{r3N}}{\partial\varphi_1}\frac{\partial\varphi_1}{\partial v_o}\right)\hat{v}_o+\frac{\partial i_{r3N}}{\partial\varphi_1}\frac{\partial\varphi_1}{\partial t_s}\hat{t}_s\\
&=i_{r3N}+\left(\frac{\sin\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}h_{1i}+\frac{r_1\cos\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}\left(g_{1i}+m_{1i}\right)\right)\hat{i}_{r0N}+\left(\frac{\sin\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}h_{1v}+\frac{r_1\cos\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}\left(g_{1v}+m_{1v}\right)\right)\hat{v}_{r0N}\\
&+\left(\frac{\sin\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}h_{1in}+\frac{r_1\cos\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}\left(g_{1in}+m_{1in}\right)\right)\hat{v}_{in}+\left(\frac{\sin\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}h_{1o}+\frac{r_1\cos\left(\varphi_1+\theta_1\right)}{\sqrt{1+L_n}}\left(g_{1o}+m_{1o}\right)\right)\hat{v}_o\\
&+\frac{r_1}{\sqrt{1+L_n}}\cos\left(\varphi_1+\theta_1\right)m_{1t}\hat{t}_s=i_{r3N}+k_{3i}\hat{i}_{r0N}+k_{3v}\hat{v}_{r0N}+k_{3in}\hat{v}_{in}+k_{3o}\hat{v}_o+k_{3t}\hat{t}_s
\end{aligned}
$$

$$
\begin{aligned}
v_{r3N}+\Delta v_{r3N} &= \left(-r_1\cos\left(\varphi_1+\theta_1\right)+1\right)+\frac{\partial v_{r3N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{r3N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial v_{r3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r3N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{r3N}}{\partial t_s}\hat{t}_s \\
&= v_{r3N}+\left(\frac{\partial v_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial i_{r0N}}+\frac{\partial v_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial i_{r0N}}+\frac{\partial v_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{r0N}}+\frac{\partial v_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{r0N}}+\frac{\partial v_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
&+\left(\frac{\partial v_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{in}}+\frac{\partial v_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{in}}+\frac{\partial v_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_o}+\frac{\partial v_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_o}+\frac{\partial v_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_o}\right)\hat{v}_o+\frac{\partial v_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial t_s}\hat{t}_s \\
&= v_{r3N}+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1i}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}+m_{1i}\right)\right)\hat{i}_{r0N}+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1v}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1v}+m_{1v}\right)\right)\hat{v}_{r0N} \\
&+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1in}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1in}+m_{1in}\right)\right)\hat{v}_{in} \\
&+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1o}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1o}+m_{1o}\right)\right)\hat{v}_o+r_1\sin\left(\varphi_1+\theta_1\right)m_{1t}\hat{t}_s \\
&= v_{r3N}+l_{3i}\hat{i}_{r0N}+l_{3v}\hat{v}_{r0N}+l_{3in}\hat{v}_{in}+l_{3o}\hat{v}_o+l_{3t}\hat{t}_s
\end{aligned}
\tag{46}
$$

The average output current of the rectifier from t_0 to t_3 can be expressed as

$$
\begin{aligned}
&\left\langle i_{rec1}\right\rangle+\Delta\left\langle i_{rec1}\right\rangle=\frac{nI_n}{\omega_{r0}t_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)\varphi_0-\frac{M}{2L_n}\varphi_0^{\,2}\right) \\
&+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial v_o}\hat{v}_o+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial t_s}\hat{t}_s \\
&=\left\langle i_{rec1}\right\rangle+\left(\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial r_0}\frac{\partial r_0}{\partial v_{r0N}}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{r0N}}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{r0N}}\right)\hat{v}_{r0N}+ \\
&\left(\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{in}}+\frac{nI_n}{\omega_{r0}t_s}\frac{M}{2v_{in}L_n}\varphi_0^{\,2}\right)\hat{v}_{in}+\frac{nC_r}{t_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)\varphi_0-\frac{M}{2L_n}\varphi_0^{\,2}\right)\hat{v}_{in} \\
&+\left(\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_o}-\frac{nI_n}{\omega_{r0}t_s}\frac{n}{2v_{in}L_n}\varphi_0^{\,2}\right)\hat{v}_o+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial t_s}\hat{t}_s \\
&=\left\langle i_{rec1}\right\rangle+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l}\left(\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\varphi_0\right)h_{0i}+ \\ \left(-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)-r_0\cos\left(\theta_0\right)\varphi_0\right)g_{0i} \\ +\left(r_0\sin\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)-\dfrac{M}{L_n}\varphi_0\right)m_{0i}\end{array}\right]\hat{i}_{r0N}+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l}\left(\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\varphi_0\right)h_{0v}+ \\ \left(-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)-r_0\cos\left(\theta_0\right)\varphi_0\right)g_{0v} \\ +\left(r_0\sin\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)-\dfrac{M}{L_n}\varphi_0\right)m_{0v}\end{array}\right]\hat{v}_{r0N} \\
&+\left\{\begin{array}{l}\dfrac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l}\left(\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\varphi_0\right)h_{0in}+ \\ \left(-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)-r_0\cos\left(\theta_0\right)\varphi_0\right)g_{0in} \\ +\left(r_0\sin\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)-\dfrac{M}{L_n}\varphi_0\right)m_{0in}+\dfrac{M}{2v_{in}L_n}\varphi_0^{\,2}\end{array}\right] \\ +\dfrac{nC_r}{t_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)\varphi_0-\dfrac{M}{2L_n}\varphi_0^{\,2}\right)\end{array}\right\}\hat{v}_{in}+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l}\left(\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\varphi_0\right)h_{0o}+ \\ \left(-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)-r_0\cos\left(\theta_0\right)\varphi_0\right)g_{0o} \\ +\left(r_0\sin\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)-\dfrac{M}{L_n}\varphi_0\right)m_{0o}-\dfrac{n}{2v_{in}L_n}\varphi_0^{\,2}\end{array}\right]\hat{v}_o \\
&-\frac{nI_n}{\omega_{r0}t_s^{\,2}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)-r_0\sin\left(\theta_0\right)\varphi_0-\frac{M}{2L_n}\varphi_0^{\,2}\right)\hat{t}_s=\left\langle i_{rec1}\right\rangle+k_{rec1i}\hat{i}_{r0N}+k_{rec1v}\hat{v}_{r0N}+k_{rec1in}\hat{v}_{in}+k_{rec1o}\hat{v}_o+k_{rec1t}\hat{t}_s
\end{aligned}
\tag{47}
$$

From t_3 to t_6

From t_3 to t_6 with half a switch period, time-domain expressions are as follows:

$$
\begin{cases}
i_{r3N} = r_2 \sin\left(\theta_2\right) \\
v_{r3N} = -r_2 \cos\left(\theta_2\right) - \left(1-M\right) \\
i_{r5N} = r_2 \sin\left(\varphi_2 + \theta_2\right) = \dfrac{r_3}{\sqrt{1+L_n}} \sin\left(\theta_3\right) \\
v_{r5N} = -r_2 \cos\left(\varphi_2 + \theta_2\right) - \left(1-M\right) = -r_3 \cos\left(\theta_3\right) - 1 \\
i_{r6N} = \dfrac{r_3}{\sqrt{1+L_n}} \sin\left(\varphi_3 + \theta_3\right) \\
v_{r6N} = -r_3 \cos\left(\varphi_3 + \theta_3\right) - 1 \\
i_{rec}\left(t_5\right) = nI_n \left(r_2 \sin\left(\varphi_2 + \theta_2\right) - r_2 \sin\left(\theta_2\right) + \dfrac{M}{L_n}\varphi_2 \right) = 0 \\
\overline{i}_{rec2} = \dfrac{nI_n}{\omega_{r0} T_s} \left(r_2 \cos\left(\theta_2\right) - r_2 \cos\left(\varphi_2 + \theta_2\right) - \sin\left(\theta_2\right)\varphi_2 + \dfrac{M}{2L_n}{\varphi_2}^2 \right)
\end{cases}
\tag{48}
$$

At time t_3, the converter starts to operates in the P mode.θ_2 and r_2 can be expressed as follows:

$$
\theta_2 = \pi + \arctan\left(-\frac{i_{r3N}}{v_{r3N} + \left(1-M\right)} \right), \quad r_2 = \sqrt{{i_{r3N}}^2 + \left[v_{r3N} + \left(1-M\right) \right]^2}
\tag{49}
$$

After first-order linearization:

$$
\begin{aligned}
&\theta_2 + \Delta\theta_2 = \pi + \arctan\left(-\frac{i_{r3N}}{v_{r3N} + \left(1-M\right)} \right) + \frac{\partial\theta_2}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial\theta_2}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial\theta_2}{\partial v_{in}}\hat{v}_{in} + \frac{\partial\theta_2}{\partial v_o}\hat{v}_o + \frac{\partial\theta_2}{\partial t_s}\hat{t}_s \\
&= \theta_2 + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}} + \frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial i_{r0N}} \right)\hat{i}_{r0N} + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{r0N}} + \frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{r0N}} \right)\hat{v}_{r0N} + \\
&\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}} + \frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{in}} - \frac{i_{r3N}M/v_{in}}{{r_2}^2} \right)\hat{v}_{in} + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o} + \frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_o} + \frac{i_{r3N}n/v_{in}}{{r_2}^2} \right)\hat{v}_o + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s} + \frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial t_s} \right)\hat{t}_s \\
&= \theta_2 + \left[-\frac{v_{r3N} + \left(1-M\right)}{{r_2}^2}k_{3i} + \frac{i_{r3N}}{{r_2}^2}l_{3i} \right]\hat{i}_{r0N} + \left[-\frac{v_{r3N} + \left(1-M\right)}{{r_2}^2}k_{3v} + \frac{i_{r3N}}{{r_2}^2}l_{3v} \right]\hat{v}_{r0N} \\
&+ \left[-\frac{v_{r3N} + \left(1-M\right)}{{r_2}^2}k_{3in} + \frac{i_{r3N}}{{r_2}^2}l_{3in} + \frac{i_{r3N}M/v_{in}}{{r_2}^2} \right]\hat{v}_{in} + \left[-\frac{v_{r3N} + \left(1-M\right)}{{r_2}^2}k_{3o} + \frac{i_{r3N}}{{r_2}^2}l_{3o} - \frac{ni_{r3N}/v_{in}}{{r_2}^2} \right]\hat{v}_o \\
&+ \left[-\frac{v_{r3N} + \left(1-M\right)}{{r_2}^2}k_{3t} + \frac{i_{r3N}}{{r_2}^2}l_{3t} \right]\hat{t}_s \\
&= \theta_2 + g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{r0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s \\
&r_2 + \Delta r_2 = \sqrt{{i_{r3N}}^2 + \left[v_{r3N} + \left(1-M\right) \right]^2} + \frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial r_2}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial r_2}{\partial v_{in}}\hat{v}_{in} + \frac{\partial r_2}{\partial v_o}\hat{v}_o + \frac{\partial r_2}{\partial t_s}\hat{t}_s s \\
&= r_2 + \left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}} + \frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial i_{r0N}} \right)\hat{i}_{r0N} + \left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{r0N}} + \frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{r0N}} \right)\hat{v}_{r0N} + \left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}} + \frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{in}} + \frac{\left[v_{r3N} + \left(1-M\right) \right]M/v_{in}}{r_1} \right)\hat{v}_{in} \\
&+ \left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o} + \frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_o} - \frac{\left[v_{r3N} + \left(1-M\right) \right]n/v_{in}}{r_1} \right)\hat{v}_o + \left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s} + \frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial t_s} \right)\hat{t}_s \\
&= r_2 + \left(\frac{i_{r3N}}{r_2}k_{3i} + \frac{v_{r3N} + \left(1-M\right)}{r_2}l_{3i} \right)\hat{i}_{r0N} + \left(\frac{i_{r3N}}{r_2}k_{3v} + \frac{v_{r3N} + \left(1-M\right)}{r_2}l_{3v} \right)\hat{v}_{r0N} \\
&+ \left(\frac{i_{r3N}}{r_2}k_{3in} + \frac{v_{r3N} + \left(1-M\right)}{r_2}l_{3in} + \frac{\left[v_{r3N} + \left(1-M\right) \right]M/v_{in}}{r_2} \right)\hat{v}_{in} + \left(\frac{i_{r3N}}{r_2}k_{3o} + \frac{v_{r3N} + \left(1-M\right)}{r_2}l_{3o} - \frac{\left[v_{r3N} + \left(1-M\right) \right]n/v_{in}}{r_2} \right)\hat{v}_o \\
&+ \left(\frac{i_{r3N}}{r_2}k_{3t} + \frac{v_{r3N} + \left(1-M\right)}{r_2}l_{3t} \right)\hat{t}_s = r_2 + h_{2i}\hat{i}_{r0N} + h_{2v}\hat{v}_{r0N} + h_{2in}\hat{v}_{in} + h_{2o}\hat{v}_o + h_{2t}\hat{t}_s
\end{aligned}
\tag{50}
$$

At time t_5, $i_{recN}(t_5)$=0, and $i_{recN}(t_5+\Delta t_5)$=0 after the perturbations are added. The following equation can be obtained.

$$\begin{aligned}
& i_{recN}\left(t_5+\Delta t_5\right)=n\left(\left(r_2+\Delta r_2\right)\sin\left(\varphi_2+\Delta\varphi_2+\theta_2+\Delta\theta_2\right)-\left(r_2+\Delta r_2\right)\sin\left(\theta_2+\Delta\theta_2\right)+\frac{n\left(v_o+\Delta v_o\right)}{\left(v_{in}+\Delta v_{in}\right)L_n}\left(\varphi_2+\Delta\varphi_2\right)\right) \\
& \approx n\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)+\frac{\partial i_{recN}\left(t_5\right)}{\partial r_2}\Delta r_2+\frac{\partial i_{recN}\left(t_2\right)}{\partial\varphi_2}\Delta\varphi_2+\frac{\partial i_{recN}\left(t_2\right)}{\partial\theta_2}\Delta\theta_2+\frac{\partial i_{recN}\left(t_2\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}\left(t_2\right)}{\partial v_o}\hat{v}_o \\
& =i_{recN}\left(t_5\right)+n\left[\begin{array}{l}\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)\Delta r_2+\left(r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}\right)\Delta\varphi_2 \\ +\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)\hat{\theta}_2-\frac{M}{v_{in}L_n}\varphi_2\hat{v}_{in}+\frac{n/v_{in}}{L_n}\varphi_2\hat{v}_o\end{array}\right] \\
& =i_{recN}\left(t_5\right)+n\left[\begin{array}{l}\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)\left(h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{r0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s\right)+\left(r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}\right)\Delta\varphi_2 \\ +\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)\left(g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{r0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s\right)-\frac{M}{v_{in}L_n}\varphi_2\hat{v}_{in}+\frac{n/v_{in}}{L_n}\varphi_2\hat{v}_o\end{array}\right] \\
& =i_{recN}\left(t_5\right)+n\left[\begin{array}{l}\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2i}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2i}\right]\hat{i}_{r0N}+ \\ \left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2v}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2v}\right]\hat{v}_{r0N}+ \\ \left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2in}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2in}-\frac{M}{v_{in}L_n}\varphi_2\right]\hat{v}_{in} \\ +\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2o}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2o}+\frac{n/v_{in}}{L_n}\varphi_2\right]\hat{v}_o \\ +\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2t}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2t}\right]\hat{t}_s \\ +\left(r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}\right)\Delta\varphi_2\end{array}\right]=0
\end{aligned} \tag{51}$$

$\Delta\varphi_2$ can be calculated as follows:

$$\begin{aligned}
\Delta\varphi_2 &= \frac{-1}{r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}}\left[\begin{array}{l}\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2i}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2i}\right]\hat{i}_{r0N}+ \\ \left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2v}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2v}\right]\hat{v}_{r0N}+ \\ \left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2in}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2in}-\frac{M}{v_{in}L_n}\varphi_2\right]\hat{v}_{in} \\ +\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2o}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2o}+\frac{n/v_{in}}{L_n}\varphi_2\right]\hat{v}_o \\ +\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2t}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2t}\right]\hat{t}_s\end{array}\right] \\
&= m_{2i}\hat{i}_{r0N}+m_{2v}\hat{v}_{r0N}+m_{2in}\hat{v}_{in}+m_{2o}\hat{v}_o+m_{2t}\hat{t}_s
\end{aligned} \tag{52}$$

After $\Delta\theta_2$, Δr_2, and $\Delta\varphi_2$ are known, Δi_{r5N} and Δv_{r5N} can be calculated as follows:

$$
\begin{aligned}
& i_{r5N}+\Delta i_{r5N}=r_2\sin\left(\varphi_2+\theta_2\right)+\frac{\partial i_{r5N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r5N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial i_{r5N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r5N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r5N}}{\partial t_s}\hat{t}_s \\
& =i_{r5N}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}}+\frac{\partial i_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{r0N}}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{r0N}}+\frac{\partial i_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
& +\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}}+\frac{\partial i_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_o}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_o}+\frac{\partial i_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_o}\right)\hat{v}_o \\
& +\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial t_s}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_s}+\frac{\partial i_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial t_s}\right)\hat{t}_s \\
& =i_{r5N}+\left(\sin\left(\varphi_2+\theta_2\right)h_{2i}+r_2\cos\left(\varphi_2+\theta_2\right)\left(g_{2i}+m_{2i}\right)\right)\hat{i}_{r0N}+\left(\sin\left(\varphi_2+\theta_2\right)h_{2v}+r_2\cos\left(\varphi_2+\theta_2\right)\left(g_{2v}+m_{2v}\right)\right)\hat{v}_{r0N} \\
& +\left(\sin\left(\varphi_2+\theta_2\right)h_{2in}+r_2\cos\left(\varphi_2+\theta_2\right)\left(g_{2in}+m_{2in}\right)\right)\hat{v}_{in}+\left(\sin\left(\varphi_2+\theta_2\right)h_{2o}+r_2\cos\left(\varphi_2+\theta_2\right)\left(g_{2o}+m_{2o}\right)\right)\hat{v}_o \\
& +\left(\sin\left(\varphi_2+\theta_2\right)h_{2t}+r_2\cos\left(\varphi_2+\theta_2\right)\left(g_{2t}+m_{2t}\right)\right)\hat{t}_s=i_{r5N}+k_{5i}\hat{i}_{r0N}+k_{5v}\hat{v}_{r0N}+k_{5in}\hat{v}_{in}+k_{5o}\hat{v}_o+k_{5t}\hat{t}_s \\
& v_{r5N}+\Delta v_{r5N}=-r_2\cos\left(\varphi_2+\theta_2\right)-\left(1-M\right)+\frac{\partial v_{r5N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{r5N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial v_{r5N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r5N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{r5N}}{\partial t_s}\hat{t}_s \\
& =v_{r5N}+\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}}+\frac{\partial v_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{r0N}}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{r0N}}+\frac{\partial v_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
& +\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}}+\frac{\partial v_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{in}}-\frac{M}{v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_o}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_o}+\frac{\partial v_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_o}+\frac{n}{v_{in}}\right)\hat{v}_o \\
& +\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial t_s}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_s}+\frac{\partial v_{r5N}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial t_s}\right)\hat{t}_s \\
& =v_{r5N}+\left(-\cos\left(\varphi_2+\theta_2\right)h_{2i}+r_2\sin\left(\varphi_2+\theta_2\right)\left(g_{2i}+m_{2i}\right)\right)\hat{i}_{r0N}+\left(-\cos\left(\varphi_2+\theta_2\right)h_{2v}+r_2\sin\left(\varphi_2+\theta_2\right)\left(g_{2v}+m_{2v}\right)\right)\hat{v}_{r0N} \\
& +\left(-\cos\left(\varphi_2+\theta_2\right)h_{2in}+r_2\sin\left(\varphi_2+\theta_2\right)\left(g_{2in}+m_{2in}\right)-\frac{M}{v_{in}}\right)\hat{v}_{in}+\left(-\cos\left(\varphi_2+\theta_2\right)h_{2o}+r_2\sin\left(\varphi_2+\theta_2\right)\left(g_{2o}+m_{2o}\right)+\frac{n}{v_{in}}\right)\hat{v}_o \\
& +\left(-\cos\left(\varphi_2+\theta_2\right)h_{2t}+r_2\sin\left(\varphi_2+\theta_2\right)\left(g_{2t}+m_{2t}\right)\right)\hat{t}_s=v_{r5N}+l_{5i}\hat{i}_{r0N}+l_{5v}\hat{v}_{r0N}+l_{5in}\hat{v}_{in}+l_{5o}\hat{v}_o+l_{5t}\hat{t}_s
\end{aligned}
\tag{53}
$$

At time t_5, the converter starts to operates in the O mode. r_3 and θ_3 can be expressed as follows:

$$
r_3=\sqrt{\left(1+L_n\right)i_{r5N}{}^2+\left(v_{r5N}+1\right)^2}\ ,\quad \theta_3=\pi+\arctan\left(-\frac{\sqrt{1+L_n}\,i_{r5N}}{v_{r5N}+1}\right) \tag{54}
$$

After first-order linearization:

$$
\begin{aligned}
& r_3+\Delta r_3\approx\sqrt{\left(1+L_n\right)i_{r5N}{}^2+\left(v_{r5N}+1\right)^2}+\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s \\
& =r_3+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{r0N}}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}+ \\
& \left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_o}\right)\hat{v}_o+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial t_s}\right)\hat{t}_s \\
& =r_3+\left(\frac{\left(1+L_n\right)i_{r5N}}{r_3}k_{5i}+\frac{v_{r5N}+1}{r_3}l_{5i}\right)\hat{i}_{r0N}+\left(\frac{\left(1+L_n\right)i_{r5N}}{r_3}k_{5v}+\frac{v_{r5N}+1}{r_3}l_{5v}\right)\hat{v}_{r0N}+ \\
& \left(\frac{\left(1+L_n\right)i_{r5N}}{r_3}k_{5in}+\frac{v_{r5N}+1}{r_3}l_{5in}\right)\hat{v}_{in}+\left(\frac{\left(1+L_n\right)i_{r5N}}{r_3}k_{5i}+\frac{v_{r5N}+1}{r_3}l_{5i}\right)\hat{v}_o+\left(\frac{\left(1+L_n\right)i_{r5N}}{r_3}k_{5i}+\frac{v_{r5N}+1}{r_3}l_{5i}\right)\hat{t}_s \\
& =r_3+h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s
\end{aligned}
$$

$$
\begin{aligned}
&\theta_3+\Delta\theta_3=\pi+\arctan\left(-\frac{\sqrt{1+L_n}\,i_{r5N}}{v_{r5N}+1}\right)+\frac{\partial\theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_3}{\partial v_o}\hat{v}_o+\frac{\partial\theta_3}{\partial t_s}\hat{t}_s \\
&=\theta_3+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{r0N}}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}+ \\
&\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_o}\right)\hat{v}_o+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial t_s}\right)\hat{t}_s \\
&=\theta_3+\left[-\frac{\sqrt{1+L_n}\,(v_{r5N}+1)}{r_3^{\,2}}k_{5i}+\frac{\sqrt{1+L_n}\,i_{r5N}}{r_3^{\,2}}l_{5i}\right]\hat{i}_{r0N}+\left[-\frac{\sqrt{1+L_n}\,(v_{r5N}+1)}{r_3^{\,2}}k_{5v}+\frac{\sqrt{1+L_n}\,i_{r5N}}{r_3^{\,2}}l_{5v}\right]\hat{v}_{r0N} \\
&+\left[-\frac{\sqrt{1+L_n}\,(v_{r5N}+1)}{r_3^{\,2}}k_{5in}+\frac{\sqrt{1+L_n}\,i_{r5N}}{r_3^{\,2}}l_{5in}\right]\hat{v}_{in}+\left[-\frac{\sqrt{1+L_n}\,(v_{r5N}+1)}{r_3^{\,2}}k_{5o}+\frac{\sqrt{1+L_n}\,i_{r5N}}{r_3^{\,2}}l_{5o}\right]\hat{v}_o \\
&+\left[-\frac{\sqrt{1+L_n}\,(v_{r5N}+1)}{r_3^{\,2}}k_{5t}+\frac{\sqrt{1+L_n}\,i_{r5N}}{r_3^{\,2}}l_{5t}\right]\hat{t}_s \\
&=\theta_2+g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{r0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s
\end{aligned}
\tag{55}
$$

$\Delta\varphi_3$ can be calculated by

$$
\begin{aligned}
&\varphi_3=\frac{\omega_{r1}t_s}{2}-\frac{\omega_{r1}}{\omega_{r0}}\varphi_2 \\
&\Delta\varphi_3=\frac{\omega_{r1}}{2}\hat{t}_s-\frac{\omega_{r1}}{\omega_{r0}}\hat{\varphi}_2 \\
&=\left(\frac{\omega_{r1}}{2}-\frac{\omega_{r1}}{\omega_{r0}}m_{2t}\right)\hat{t}_s-\frac{\omega_{r1}}{\omega_{r0}}m_{2i}\hat{i}_{r0N}-\frac{\omega_{r1}}{\omega_{r0}}m_{2v}\hat{v}_{r0N}-\frac{\omega_{r1}}{\omega_{r0}}m_{2in}\hat{v}_{in}-\frac{\omega_{r1}}{\omega_{r0}}m_{2o}\hat{v}_o \\
&=m_{3t}\hat{t}_s+m_{3i}\hat{i}_{r0N}+m_{3v}\hat{v}_{r0N}+m_{3in}\hat{v}_{in}+m_{3o}\hat{v}_o
\end{aligned}
\tag{56}
$$

At time t_6, Δi_{r6N} and Δv_{r6N} can be calculated as follows:

$$
\begin{aligned}
&i_{r6N}+\Delta i_{r6N} \\
&=\frac{r_3}{\sqrt{1+L_n}}\sin\left(\varphi_3+\theta_3\right)+\frac{\partial i_{r6N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r6N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial i_{r6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r6N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r6N}}{\partial t_s}\hat{t}_s \\
&=i_{r6N}+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial i_{r0N}}+\frac{\partial i_{r6N}}{\partial\theta_3}\frac{\partial\theta_3}{\partial i_{r0N}}+\frac{\partial i_{r6N}}{\partial\varphi_3}\frac{\partial\varphi_3}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{r0N}}+\frac{\partial i_{r6N}}{\partial\theta_3}\frac{\partial\theta_3}{\partial v_{r0N}}+\frac{\partial i_{r6N}}{\partial\varphi_3}\frac{\partial\varphi_3}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
&+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{in}}+\frac{\partial i_{r6N}}{\partial\theta_3}\frac{\partial\theta_3}{\partial v_{in}}+\frac{\partial i_{r6N}}{\partial\varphi_3}\frac{\partial\varphi_3}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_o}+\frac{\partial i_{r6N}}{\partial\theta_3}\frac{\partial\theta_3}{\partial v_o}+\frac{\partial i_{r6N}}{\partial\varphi_3}\frac{\partial\varphi_3}{\partial v_o}\right)\hat{v}_o \\
&+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial t_s}+\frac{\partial i_{r6N}}{\partial\theta_3}\frac{\partial\theta_3}{\partial t_s}+\frac{\partial i_{r6N}}{\partial\varphi_3}\frac{\partial\varphi_3}{\partial t_s}\right)\hat{t}_s \\
&=i_{r6N}+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3i}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3i}+m_{3i}\right)\right)\hat{i}_{r0N}+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3v}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3i}+m_{3v}\right)\right)\hat{v}_{r0N} \\
&+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3in}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3in}+m_{3in}\right)\right)\hat{v}_{in}+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3o}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3o}+m_{3o}\right)\right)\hat{v}_o \\
&+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3t}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3t}+m_{3t}\right)\right)\hat{t}_s=i_{r6N}+k_{6i}\hat{i}_{r0N}+k_{6v}\hat{v}_{r0N}+k_{6in}\hat{v}_{in}+k_{6o}\hat{v}_o+k_{6t}\hat{t}_s
\end{aligned}
$$

$$
\begin{aligned}
v_{r6N}+\Delta v_{r6N}&=-r_3\cos\left(\varphi_3+\theta_3\right)-1+\frac{\partial v_{r6N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{r6N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial v_{r6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r6N}}{\partial v_{o}}\hat{v}_{o}+\frac{\partial v_{r6N}}{\partial t_{s}}\hat{t}_{s}\\
&=v_{r6N}+\left(\frac{\partial v_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial i_{r0N}}+\frac{\partial v_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial i_{r0N}}+\frac{\partial v_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{r0N}}+\frac{\partial v_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{r0N}}+\frac{\partial v_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial v_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{in}}+\frac{\partial v_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{in}}+\frac{\partial v_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{o}}+\frac{\partial v_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{o}}+\frac{\partial v_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{o}}\right)\hat{v}_{o}\\
&+\left(\frac{\partial v_{r5N}}{\partial r_3}\frac{\partial r_3}{\partial t_{s}}+\frac{\partial v_{r5N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial t_{s}}+\frac{\partial v_{r5N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial t_{s}}\right)\hat{t}_{s}\\
&=v_{r6N}+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3i}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}+m_{3i}\right)\right)\hat{i}_{r0N}+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3v}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3v}+m_{3v}\right)\right)\hat{v}_{r0N}\\
&+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3in}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3in}+m_{3in}\right)\right)\hat{v}_{in}+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3o}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3o}+m_{3o}\right)\right)\hat{v}_{o}\\
&+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3t}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3t}+m_{3t}\right)\right)\hat{t}_{s}=v_{r6N}+l_{6i}\hat{i}_{r0N}+l_{6v}\hat{v}_{r0N}+l_{6in}\hat{v}_{in}+l_{6o}\hat{v}_{o}+l_{6t}\hat{t}_{s}
\end{aligned}
\tag{57}
$$

The average output current of the rectifier from t_3 to t_6 can be expressed as

$$
\begin{aligned}
&\left\langle i_{rec2}\right\rangle+\Delta\left\langle i_{rec2}\right\rangle=\frac{nI_n}{\omega_{r0}t_s}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)\varphi_2+\frac{M}{2L_n}{\varphi_2}^2\right)\\
&+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial v_{o}}\hat{v}_{o}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial t_{s}}\hat{t}_{s}\\
&=\left\langle i_{rec2}\right\rangle+\left(\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial r_2}\frac{\partial r_2}{\partial v_{r0N}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{r0N}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{r0N}}\right)\hat{v}_{r0N}+\\
&\left(\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{in}}-\frac{nI_n}{\omega_{r0}t_s}\frac{M}{2v_{in}L_n}{\varphi_2}^2\right)\hat{v}_{in}+\frac{nC_r}{t_s}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2+\frac{M}{2L_n}{\varphi_2}^2\right)\hat{v}_{in}\\
&+\left(\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial r_2}\frac{\partial r_2}{\partial v_{o}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{o}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{o}}+\frac{nI_n}{\omega_{r0}t_s}\frac{n}{2v_{in}L_n}{\varphi_2}^2\right)\hat{v}_{o}+\left(\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial r_2}\frac{\partial r_2}{\partial t_{s}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_{s}}+\frac{\partial\left\langle i_{rec2}\right\rangle}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial t_{s}}\right)\hat{t}_{s}\\
&-\frac{nI_n}{\omega_{r0}{t_s}^2}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2+\frac{M}{2L_n}{\varphi_2}^2\right)\hat{t}_{s}\\
&=\left\langle i_{rec2}\right\rangle+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{aligned}&\left(\cos\left(\theta_2\right)-\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2\right)h_{2i}+\\&\left(-r_2\sin\left(\theta_2\right)+r_2\sin\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\varphi_2\right)g_{2i}\\&+\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)m_{2i}\end{aligned}\right]\hat{i}_{r0N}+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{aligned}&\left(\cos\left(\theta_2\right)-\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2\right)h_{2v}+\\&\left(-r_2\sin\left(\theta_2\right)+r_2\sin\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\varphi_2\right)g_{2v}\\&+\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)m_{2v}\end{aligned}\right]\hat{v}_{r0N}\\
&+\left\{\begin{aligned}&\frac{nI_n}{\omega_{r0}t_s}\left[\begin{aligned}&\left(\cos\left(\theta_2\right)-\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2\right)h_{2in}+\\&\left(-r_2\sin\left(\theta_2\right)+r_2\sin\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\varphi_2\right)g_{2in}\\&+\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)m_{2in}-\frac{M}{2v_{in}L_n}{\varphi_2}^2\end{aligned}\right]\\&+\frac{nC_r}{t_s}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)\varphi_2+\frac{M}{2L_n}{\varphi_2}^2\right)\end{aligned}\right\}\hat{v}_{in}+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{aligned}&\left(\cos\left(\theta_2\right)-\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2\right)h_{2o}+\\&\left(-r_2\sin\left(\theta_2\right)+r_2\sin\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\varphi_2\right)g_{2o}\\&+\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)m_{2o}+\frac{n}{2v_{in}L_n}{\varphi_2}^2\end{aligned}\right]\hat{v}_{o}\\
&+\left\{\begin{aligned}&\frac{nI_n}{\omega_{r0}t_s}\left[\begin{aligned}&\left(\cos\left(\theta_2\right)-\cos\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\varphi_2\right)h_{2t}+\\&\left(-r_2\sin\left(\theta_2\right)+r_2\sin\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\varphi_2\right)g_{2t}\\&+\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)m_{2t}\end{aligned}\right]\\&-\frac{nI_n}{\omega_{r0}{t_s}^2}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)\varphi_2+\frac{M}{2L_n}{\varphi_2}^2\right)\end{aligned}\right\}\hat{t}_{s}=\left\langle i_{rec2}\right\rangle+k_{rec2i}\hat{i}_{r0N}+k_{rec2v}\hat{v}_{r0N}+k_{rec2in}\hat{v}_{in}+k_{rec2o}\hat{v}_{o}+k_{rec2t}\hat{t}_{s}
\end{aligned}
\tag{58}
$$

The change in output current of the rectifier bridge during one switching cycle is expressed as

$$\begin{aligned}\Delta\overline{i}_{rec} &= \Delta\langle i_{rec1}\rangle - \Delta\langle i_{rec2}\rangle \\ &= \left(k_{rec1i}-k_{rec2i}\right)\hat{i}_{r0N}+\left(k_{rec1v}-k_{rec2v}\right)\hat{v}_{r0N}+\left(k_{rec1o}-k_{rec2o}\right)\hat{v}_{o}+\left(k_{rec1v}-k_{rec2in}\right)\hat{v}_{in}+\left(k_{rec1t}-k_{rec2t}\right)\hat{t}_{s}\end{aligned} \tag{59}$$

According to the large signal model, the state space expression of the LLC converter can be expressed as

$$\begin{aligned}\dot{\hat{i}}_{r0N} &= \frac{i_{r6N}+\hat{i}_{r6N}-i_{r0N}-\hat{i}_{r0N}}{t_s+\hat{t}_s} \approx \frac{\hat{i}_{r6N}-\hat{i}_{r0N}}{T_s} = \frac{1}{T_s}\left[\left(k_{6i}-1\right)\hat{i}_{r0N}+k_{6v}\hat{v}_{r0N}+k_{6in}\hat{v}_{in}+k_{6o}\hat{v}_{o}+k_{6t}\hat{t}_{s}\right] \\ \dot{\hat{v}}_{r0N} &= \frac{v_{r6N}+\hat{v}_{r6N}-v_{r0N}-\hat{v}_{r0N}}{t_s+\hat{t}_s} \approx \frac{v_{r6N}-v_{r0N}}{T_s} = \frac{1}{T_s}\left[l_{6i}\hat{i}_{r0N}+\left(l_{6v}-1\right)\hat{v}_{r0N}+l_{6in}\hat{v}_{in}+l_{6o}\hat{v}_{o}+l_{6t}\hat{t}_{s}\right] \\ \dot{\hat{v}}_{o} &= \frac{1}{C_o}\left(\Delta\overline{i}_{rec}-\frac{\hat{v}_o}{R}\right) = \frac{1}{C_o}\begin{bmatrix}\left(k_{rec1i}-k_{rec2i}\right)\hat{i}_{r0N}+\left(k_{rec1v}-k_{rec2v}\right)\hat{v}_{r0N}+\left(k_{rec1o}-k_{rec2o}-\dfrac{1}{R}\right)\hat{v}_{o} \\ +\left(k_{rec1v}-k_{rec2in}\right)\hat{v}_{in}+\left(k_{rec1o}-k_{rec2t}\right)\hat{t}_{s}\end{bmatrix}\end{aligned} \tag{60}$$

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x}+B\hat{u} \\ \hat{y} &= C\hat{x} \\ A &= \begin{bmatrix}\dfrac{k_{6i}-1}{T_s} & \dfrac{k_{6v}}{T_s} & \dfrac{k_{6o}}{T_s} \\ \dfrac{l_{6i}}{T_s} & \dfrac{l_{6v}-1}{T_s} & \dfrac{l_{6o}}{T_s} \\ \dfrac{k_{rec1i}-k_{rec2i}}{C_o} & \dfrac{k_{rec1v}-k_{rec2v}}{C_o} & \dfrac{k_{rec1o}-k_{rec2o}-1/R}{C_o}\end{bmatrix} \\ B &= \begin{bmatrix}\dfrac{k_{6in}}{T_s} & \dfrac{k_{6t}}{T_s} \\ \dfrac{l_{6in}}{T_s} & \dfrac{l_{6t}}{T_s} \\ \dfrac{k_{rec1in}-k_{rec2in}}{C_o} & \dfrac{k_{rec1t}-k_{rec2t}}{C_o}\end{bmatrix} \\ C &= \begin{bmatrix}0 & 0 & 1\end{bmatrix}\end{aligned} \tag{61}$$

The transfer function of the LLC converter for PO mode can be expressed as

$$G(s)=C\left(sI-A\right)^{-1}B=\left[G_{vin}(s)\quad G_t(s)\right] \tag{62}$$

where

$$\begin{aligned}G_{vin}(s) &= \frac{\hat{v}_o}{\hat{v}_{in}} \\ G_t(s) &= \frac{\hat{v}_o}{\hat{t}_s}\end{aligned}$$

The disturbance is implemented after the half of the switching period delay. The following equation can be obtained.

$$\begin{aligned}G_{ts}(s) &= e^{-\frac{T_s}{2}s}G_t(s) \\ G_{vins}(s) &= e^{-\frac{T_s}{2}s}G_{vin}(s)\end{aligned} \tag{63}$$

Section IV. Small-signal model for PO mode with TSC

The definitions of t_{Z1}, t_{Z2} and t_{cs} are shown below.

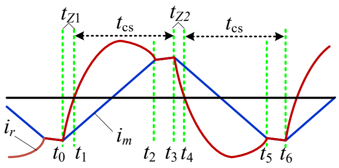

Fig.4 The analysis of control time under TSC for PO mode.

Δt_{Z1} and Δt_{Z2} can be expressed as follows:

$$\begin{aligned} t_{Z1} &= -\frac{\theta_0}{\omega_{r0}} \\ t_{Z2} &= \frac{\pi-\theta_2}{\omega_{r0}} \\ \Delta t_{Z1} &= -\frac{\Delta\theta_0}{\omega_{r0}} = -\frac{1}{\omega_{r0}}\left(g_{0i}\hat{i}_{r0N} + g_{0v}\hat{v}_{r0N} + g_{0in}\hat{v}_{in} + g_{0o}\hat{v}_o\right) \\ \Delta t_{Z2} &= -\frac{\Delta\theta_2}{\omega_{r0}} = -\frac{1}{\omega_{r0}}\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{r0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right) \end{aligned} \tag{64}$$

The relationship between $\hat{t}_{cs}$ and $\hat{t}_s$ can be shown below.

$$\begin{aligned} \hat{t}_s &= \Delta t_{Z1} + \Delta t_{Z2} + 2\hat{t}_{cs} \\ &= \frac{1}{\omega_{r0}}\left(-g_{0i}\hat{i}_{r0N} - g_{0v}\hat{v}_{r0N} - g_{0in}\hat{v}_{in} - g_{0o}\hat{v}_o\right) - \frac{1}{\omega_{r0}}\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{r0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right) + 2\hat{t}_{cs} \\ &= \frac{1}{\omega_{r0}}\left[\left(-g_{0i}-g_{2i}\right)\hat{i}_{r0N} + \left(-g_{0v}-g_{2v}\right)\hat{v}_{r0N} + \left(-g_{0in}-g_{2in}\right)\hat{v}_{in} + \left(-g_{0o}-g_{2o}\right)\hat{v}_o\right] - \frac{g_{2t}}{\omega_{r0}}\hat{t}_s + 2\hat{t}_{cs} \end{aligned} \tag{65}$$

The above equation can be rewritten as

$$\begin{aligned} \hat{t}_s &= \frac{\omega_{r0}}{\omega_{r0}+g_{2t}} \bullet \frac{1}{\omega_{r0}}\left[\left(-g_{0i}-g_{2i}\right)\hat{i}_{r0N} + \left(-g_{0v}-g_{2v}\right)\hat{v}_{r0N} + \left(-g_{0in}-g_{2in}\right)\hat{v}_{in} + \left(-g_{0o}-g_{2o}\right)\hat{v}_o\right] + \frac{2\omega_{r0}}{\omega_{r0}+g_{2t}}\hat{t}_{cs} \\ &= \frac{1}{\omega_{r0}+g_{2t}}\left[\left(-g_{0i}-g_{2i}\right)\hat{i}_{r0N} + \left(-g_{0v}-g_{2v}\right)\hat{v}_{r0N} + \left(-g_{0in}-g_{2in}\right)\hat{v}_{in} + \left(-g_{0o}-g_{2o}\right)\hat{v}_o\right] + \frac{2\omega_{r0}}{\omega_{r0}+g_{2t}}\hat{t}_{cs} \\ &= \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix} -g_{0i}-g_{2i} & -g_{0v}-g_{2v} & -g_{0o}-g_{2o} \end{bmatrix}\begin{bmatrix} \hat{i}_{r0N} \\ \hat{v}_{r0N} \\ \hat{v}_o \end{bmatrix} + \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix} -g_{0in}-g_{2in} & 2\omega_{r0} \end{bmatrix}\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix} \\ &= A_Z\hat{x} + B_Z\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix} \end{aligned} \tag{66}$$

where

$$A_Z = \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix} -g_{0i}-g_{2i} & -g_{0v}-g_{2v} & -g_{0o}-g_{2o} \end{bmatrix}$$

$$B_Z = \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix} -g_{0in}-g_{2in} & 2\omega_{r0} \end{bmatrix}$$

Replace $\hat{t}_s$ in the state space expression with $t_s = A_Z\hat{x} + B_Z\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix}$

$$
\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + B\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = \hat{x} + \begin{bmatrix}B_1 & B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = A\hat{x} + B_1\hat{v}_{in} + B_2\hat{t}_s \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2\left[A_Z\hat{x} + B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}\right] \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2A_Z\hat{x} + B_2B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= \left(A + B_2A_Z\right)\hat{x} + B_1\hat{v}_{in} + \frac{1}{\omega_{r0} + g_{2t}}B_2\left(\left(-g_{0in} - g_{2in}\right)\hat{v}_{in} + 2\omega_{r0}\hat{t}_{cs}\right) \\
&= \left(A + B_2A_Z\right)\hat{x} + \left(B_1 + B_2\frac{-g_{0in} - g_{2in}}{\omega_{r0} + g_{2t}}\right)\hat{v}_{in} + \frac{2\omega_{r0}}{\omega_{r0} + g_{2t}}B_2\hat{t}_{cs} \\
&= \left(A + B_2A_Z\right)\hat{x} + \begin{bmatrix}B_1 + B_2\dfrac{-g_{0in} - g_{2in}}{\omega_{r0} + g_{2t}} & \dfrac{2\omega_{r0}}{\omega_{r0} + g_{2t}}B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= A_c\hat{x} + B_c\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}
\end{aligned} \tag{67}
$$

Therefore, the small-signal model of the LLC converter for PO mode with TSC can be expressed as follows with $e^{-\frac{t_s}{2}}$ correction.

$$
\begin{aligned}
G_{cs}(s) &= C\left(sI - A_c\right)^{-1}B_c = \begin{bmatrix}G_{vin\_tc}(s) & G_{tc}(s)\end{bmatrix} \\
G_{tcs}(s) &= e^{-\frac{T_s}{2}s}G_{tc}(s) \\
G_{vin\_tcs}(s) &= e^{-\frac{T_s}{2}s}G_{vin\_tc}(s)
\end{aligned} \tag{68}
$$

Section V. Time-domain expressions for NP mode

Typical waveforms and planar trajectory of the LLC converter for NP mode are shown in Fig.5 and Fig.6.

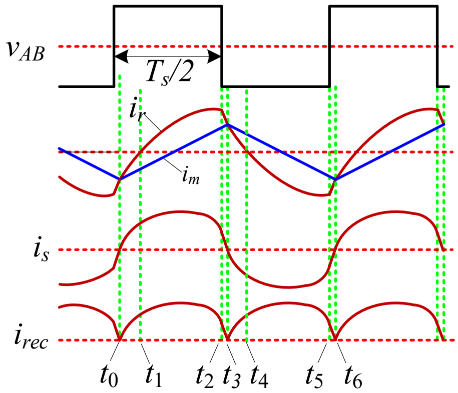

Fig.5 Typical waveforms of the LLC converter for NP mode.

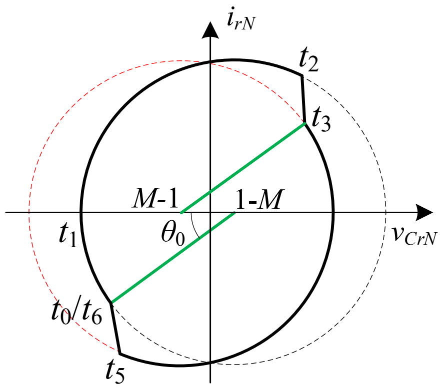

Fig.6 Planar trajectory of the LLC converter for NP mode

[t_0, t_2]

As in the case of PO mode from t_0 to t_2, resonant current and resonant capacitor voltage can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r0} Z_0 \sin(\omega_{r0} t) + \left[v_{cr0} - (V_{in} - nV_o)\right]\cos(\omega_{r0}t) + (V_{in} - nV_o) \\ i_r &= i_{r0}\cos(\omega_{r0}t) - \frac{v_{cr0} - (V_{in} - nV_o)}{Z_0}\sin(\omega_{r0}t) \end{aligned} \tag{69}$$

After normalization

$$\begin{aligned} v_{crN} &= i_{r0N}\sin(\omega_{r0}t) + \left[v_{cr0N} - (1-M)\right]\cos(\omega_{r0}t) + (1-M) \\ i_{rN} &= i_{r0N}\cos(\omega_{r0}t) - \left[v_{cr0N} - (1-M)\right]\sin(\omega_{r0}t) \end{aligned} \tag{70}$$

where $i_{r0N} = \dfrac{i_{r0} Z_0}{V_{in}}$, $v_{cr0N} = \dfrac{v_{cr0}}{V_{in}}$

Eq.(70) can be rewritten as

$$\begin{aligned} i_{rN} &= \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}\sin(\omega_{r0}t + \theta_0) \\ v_{crN} &= -\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}\cos(\omega_{r0}t + \theta_0) + (1-M) \end{aligned} \tag{71}$$

where

$$\cos\theta_0 = -\frac{\left[v_{cr0N} - (1-M)\right]}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}}, \sin\theta_0 = \frac{i_{r0N}}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}}$$

$$\theta_0 = \arctan\left(\frac{-i_{r0N}}{v_{cr0N} - (1-M)}\right)$$

Let $r_0 = \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}$, then

$$\begin{aligned} i_{rN} &= r_0 \sin(\omega_{r0}t + \theta_0) \\ v_{crN} &= -r_0\cos(\omega_{r0}t + \theta_0) + (1-M) \end{aligned} \tag{72}$$

i_{r0N}, v_{cr0N}, i_{r2N}, and v_{cr2N} can be expressed in (73), where $\varphi_0=\omega_{r0}(t_2-t_1)$

$$\begin{aligned} i_{r0N} &= r_0 \sin\left(\theta_0\right) \\ v_{cr0N} &= -r_0 \cos\left(\theta_0\right)+\left(1-M\right) \\ i_{r2N} &= r_0 \sin\left(\varphi_0+\theta_0\right) \\ v_{cr2N} &= -r_0 \cos\left(\varphi_0+\theta_0\right)+\left(1-M\right) \end{aligned} \tag{73}$$

The expression of the magnetizing current i_m is shown in (74).

$$i_m = i_{r0} + \frac{nV_o}{L_m}t \tag{74}$$

(74) is normalised to (75).

$$i_{mN} = \frac{I_{r0}Z_0}{V_{in}} + \frac{nV_oZ_0}{V_{in}L_m}t = I_{r0N} + M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}t = r_0 \sin\left(\theta_0\right) + \frac{M}{L_n}\omega_{r0}t \tag{75}$$

The output current of the rectifier bridge is expressed as

$$i_{rec} = nI_n\left(i_{rN}-i_{mN}\right) = nI_n\left(r_0 \sin\left(\omega_{r0}t+\theta_0\right) - r_0 \sin\left(\theta_0\right) - \frac{M}{L_n}\omega_{r0}t\right) \tag{76}$$

[t_2, t_3]

The converter operates in N mode from t_2 to t_3, and the voltage across the resonant tank is changed to $-v_{in}$. v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r2}Z_0 \sin\left(\omega_{r0}t\right) + \left[v_{cr2}+\left(V_{in}+nV_o\right)\right]\cos\left(\omega_{r0}t\right) - \left(V_{in}+nV_o\right) \\ i_r &= i_{r2}\cos\left(\omega_{r0}t\right) - \frac{v_{cr2}+\left(V_{in}+nV_o\right)}{Z_0}\sin\left(\omega_{r0}t\right) \end{aligned} \tag{77}$$

After normalization

$$\begin{aligned} v_{crN} &= i_{r2N}\sin\left(\omega_{r0}t\right) + \left[v_{cr2N}+\left(1+M\right)\right]\cos\left(\omega_{r0}t\right) - \left(1+M\right) \\ i_{rN} &= i_{r2N}\cos\left(\omega_{r0}t\right) - \left[v_{cr2N}+\left(1+M\right)\right]\sin\left(\omega_{r0}t\right) \end{aligned} \tag{78}$$

where $i_{r2N} = \dfrac{i_{r2}Z_0}{V_{in}}$, $v_{cr2N} = \dfrac{v_{cr2}}{V_{in}}$

Eq.(78) can be rewritten as

$$\begin{aligned} v_{crN} &= \sqrt{{i_{r2N}}^2+\left[v_{cr2N}+\left(1+M\right)\right]^2}\cos\left[\omega_{r0}\left(t-t_2\right)+\theta_1\right] - \left(1+M\right) \\ i_{rN} &= \sqrt{{i_{r2N}}^2+\left[v_{cr2N}+\left(1+M\right)\right]^2}\sin\left[\omega_{r0}\left(t-t_2\right)+\theta_1\right] \end{aligned} \tag{79}$$

where

$$\cos\theta_1 = -\frac{\left[v_{cr2N}+\left(1+M\right)\right]}{\sqrt{{i_{r2N}}^2+\left[v_{cr2N}-\left(1+M\right)\right]^2}}, \sin\theta_1 = \frac{i_{r2N}}{\sqrt{{i_{r2N}}^2+\left[v_{cr2N}+\left(1+M\right)\right]^2}}$$

$$\theta_1 = \pi + \arctan\left(-\frac{i_{r2N}}{\left[v_{cr2N}+\left(1+M\right)\right]}\right)$$

Let $r_1 = \sqrt{{i_{r2N}}^2+\left[v_{cr2N}+\left(1+M\right)\right]^2}$, then

$$\begin{aligned} v_{crN} &= r_1 \cos\left[\omega_{r0}\left(t-t_2\right)+\theta_1\right]-(1+M) \\ i_{rN} &= r_1 \sin\left[\omega_{r0}\left(t-t_2\right)+\theta_1\right] \end{aligned} \tag{80}$$

i_{r2N}, v_{cr2N}, i_{r3N}, and v_{cr3N} can be expressed in (81).

$$\begin{aligned} i_{r2N} &= r_1 \sin\left(\theta_1\right) \\ v_{cr2N} &= -r_1 \cos\left(\theta_1\right)-\left(1+M\right) \\ i_{r3N} &= r_1 \sin\left(\varphi_1+\theta_1\right) \\ v_{cr3N} &= -r_1 \cos\left(\varphi_1+\theta_1\right)-\left(1+M\right) \end{aligned} \tag{81}$$

The magnetizing current i_m can still be referred to Eq.(75), and the output current of the rectifier bridge is expressed as

$$i_{rec} = nI_n\left(i_{rN}-i_{mN}\right) = nI_n\left(r_1 \sin\left(\omega_{r0}\left(t-t_2\right)+\theta_1\right)-r_0 \sin\left(\theta_0\right)-\frac{M}{L_n}\omega_{r0}t\right) \tag{82}$$

From t_0 to t_3, the average value of i_{rec} over half a switching cycle can be expressed as follows.

$$\begin{aligned} \overline{i}_{rec1} &= \frac{2}{T_s}\int_0^{t_2} i_{rec}dt = \frac{2nI_n}{T_s}\int_0^{T_s/2}\left(i_{rN}-i_{mN}\right)dt = \frac{2nI_n}{T_s}\int_0^{T_s/2} i_{rN}dt-\int_0^{T_s/2} i_{mN}dt \\ &= \frac{2nI_n}{T_s}\int_0^{t_2} r_0 \sin\left(\omega_{r0}t+\theta_0\right)dt+\frac{2nI_n}{T_s}\int_{t_2}^{T_s/2} r_1 \sin\left(\omega_{r0}t+\theta_1\right)dt-0 \\ &= \frac{2nI_n}{T_s\omega_{r0}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right) \end{aligned} \tag{83}$$

[t_3, t_5]

Similar to t_0 to t_2, v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r3}Z_0 \sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3}+\left(V_{in}-nV_o\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(V_{in}-nV_o\right) \\ i_r &= i_{r3}\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\frac{v_{cr3}+\left(V_{in}-nV_o\right)}{Z_0}\sin\left(\omega_{r0}\left(t-t_3\right)\right) \end{aligned} \tag{84}$$

After normalization

$$\begin{aligned} v_{crN} &= i_{r3N}\sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3N}+\left(1-M\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(1-M\right) \\ i_{rN} &= i_{r3N}\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left[v_{cr3N}+\left(1-M\right)\right]\sin\left(\omega_{r0}\left(t-t_3\right)\right) \end{aligned} \tag{85}$$

where $i_{r3N} = \dfrac{i_{r3}Z_0}{V_{in}}$, $v_{cr3N} = \dfrac{v_{cr3}}{V_{in}}$

The above equation can be rewritten as

$$\begin{aligned} v_{crN} &= -\sqrt{i_{r3N}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\cos\left[\omega_{r0}\left(t-t_3\right)+\theta_2\right]-\left(1-M\right) \\ i_{rN} &= \sqrt{i_{r3N}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\sin\left[\omega_{r0}\left(t-t_3\right)+\theta_2\right] \end{aligned} \tag{86}$$

where

$$\cos\theta_2 = -\frac{\left[v_{cr3N}+\left(1-M\right)\right]}{\sqrt{i_{r3N}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}}, \sin\theta_2 = \frac{i_{r3N}}{\sqrt{i_{r3N}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}}$$

$$\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{cr3N}+\left(1-M\right)}\right)$$

Let $r_2 = \sqrt{i_{r3N}^{\ 2} + \left[v_{cr3N} + (1-M)\right]^2}$, then

$$
\begin{aligned}
v_{crN} &= -r_2 \cos\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right)-(1-M) \\
i_{rN} &= r_2 \sin\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right)
\end{aligned}
\tag{87}
$$

i_{r3N}, v_{cr3N}, i_{r5N}, and v_{cr5N} can be expressed as

$$
\begin{aligned}
i_{r3N} &= r_2 \sin\left(\theta_2\right) \\
v_{cr3N} &= -r_2 \cos\left(\theta_2\right)-(1-M) \\
i_{r5N} &= r_2 \sin\left(\varphi_2+\theta_2\right) \\
v_{cr5N} &= -r_2 \cos\left(\varphi_2+\theta_2\right)-(1-M)
\end{aligned}
\tag{88}
$$

The expression of the magnetizing current i_m is shown as follows

$$
i_m = i_{r3} - \frac{nV_o}{L_m}\left(t-t_3\right)
\tag{89}
$$

After normalization

$$
i_{mN} = i_{r3N} - M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}\left(t-t_3\right) = r_2 \sin\left(\theta_2\right) - \frac{M}{L_n}\omega_{r0}\left(t-t_3\right)
\tag{90}
$$

The output current of the rectifier bridge is expressed as

$$
i_{rec} = nI_n\left(i_{rN}-i_{mN}\right) = nI_n\left(r_2 \sin\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right) - r_2\sin\left(\theta_2\right) + \frac{M}{L_n}\omega_{r0}\left(t-t_3\right)\right)
\tag{91}
$$

[t_5, t_6]

Similar to t_0 to t_2, v_{cr} and i_r can be expressed as follows.

$$
\begin{aligned}
v_{cr} &= i_{r5}Z_0 \sin\left(\omega_{r0}t\right) + \left[v_{cr5} - \left(V_{in}+nV_o\right)\right]\cos\left(\omega_{r0}t\right) + \left(V_{in}+nV_o\right) \\
i_r &= i_{r5}\cos\left(\omega_{r0}t\right) - \frac{v_{cr5}-\left(V_{in}+nV_o\right)}{Z_0}\sin\left(\omega_{r0}t\right)
\end{aligned}
\tag{92}
$$

After normalization

$$
\begin{aligned}
v_{crN} &= i_{r5N}\sin\left(\omega_{r0}t\right) + \left[v_{cr5N}-(1+M)\right]\cos\left(\omega_{r0}t\right) + (1+M) \\
i_{rN} &= i_{r5N}\cos\left(\omega_{r0}t\right) - \left[v_{cr5N}-(1+M)\right]\sin\left(\omega_{r0}t\right)
\end{aligned}
\tag{93}
$$

where $i_{r5N} = \dfrac{i_{r5}Z_0}{V_{in}}$, $v_{cr5N} = \dfrac{v_{cr5}}{V_{in}}$

The above equation can be rewritten as

$$
\begin{aligned}
i_{rN} &= \sqrt{i_{r5N}^{\ 2} + \left[v_{cr5N}-(1+M)\right]^2}\sin\left[\omega_{r0}\left(t-t_5\right)+\theta_3\right] \\
v_{crN} &= -\sqrt{i_{r5N}^{\ 2} + \left[v_{cr5N}-(1+M)\right]^2}\cos\left[\omega_{r0}\left(t-t_5\right)+\theta_3\right] + (1+M)
\end{aligned}
\tag{94}
$$

where

$$\cos\theta_3 = -\frac{\left[v_{cr5N} - (1+M)\right]}{\sqrt{{i_{r5N}}^2 + \left[v_{cr5N} - (1+M)\right]^2}}, \sin\theta_3 = \frac{i_{r5N}}{\sqrt{{i_{r5N}}^2 + \left[v_{cr5N} - (1+M)\right]^2}}$$

$$\theta_3 = \arctan\left(-\frac{i_{r5N}}{v_{cr5N} - (1+M)}\right), r_3 = \sqrt{{i_{r5N}}^2 + \left[v_{cr5N} - (1+M)\right]^2}$$

Let $r_2 = \sqrt{{i_{r3N}}^2 + \left[v_{cr3N} + (1-M)\right]^2}$, then

$$\begin{aligned} v_{crN} &= -r_3 \cos\left(\omega_{r0}(t-t_5) + \theta_3\right) + (1+M) \\ i_{rN} &= r_3 \sin\left(\omega_{r0}(t-t_5) + \theta_3\right) \end{aligned} \tag{95}$$

i_{r5N}, v_{cr5N}, i_{r6N}, and v_{cr6N} can be expressed as

$$\begin{aligned} i_{r5N} &= r_3 \sin(\theta_3) \\ v_{cr5N} &= -r_3 \cos(\theta_3) + (1+M) \\ i_{r6N} &= r_3 \sin(\varphi_3 + \theta_3) \\ v_{cr6N} &= -r_3 \cos(\varphi_3 + \theta_3) + (1+M) \end{aligned} \tag{96}$$

The magnetizing current i_{mN} can still be referred to Eq.(90), and the output current of the rectifier bridge is expressed as

$$i_{rec} = nI_n\left(i_{rN} - i_{mN}\right) = nI_n\left(r_3 \sin\left(\omega_{r0}(t-t_5) + \theta_3\right) - r_2 \sin(\theta_2) + \frac{M}{L_n}\omega_{r0}(t-t_3)\right) \tag{97}$$

From t_3 to t_6, the average value of i_{rec} over half a switching cycle can be expressed as follows.

$$\begin{aligned} \overline{i_{rec2}} &= \frac{2}{T_s}\int_{t_3}^{t_6} i_{rec}dt = \frac{2nI_n}{T_s}\int_{t_3}^{t_6}\left(i_{rN} - i_{mN}\right)dt = \frac{2nI_n}{T_s}\int_{t_3}^{t_6} i_{rN}dt - \int_{t_3}^{t_6} i_{mN}dt \\ &= \frac{2nI_n}{T_s}\int_{t_3}^{t_5} r_2 \sin\left[\omega_{r0}(t-t_3) + \theta_2\right]dt + \frac{2nI_n}{T_s}\int_{t_5}^{t_6} r_3 \sin\left[\omega_{r0}(t-t_5) + \theta_3\right]dt - 0 \\ &= \frac{2nI_n}{T_s\omega_{r0}}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2 + \theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3 + \theta_3)\right) \end{aligned} \tag{98}$$

Section VI. Calculation of steady-state operating point for NP mode

Because of the semi-period symmetry, the i_{r0N} and v_{cr0N} at t_0 are equal to the negative of i_{r3N} and v_{cr3N} respectively. Therefore, (99) can be obtained.

$$\begin{aligned} i_{r3N} &= r_1 \sin(\varphi_1+\theta_1) = -i_{r0N} = -r_0 \sin(\theta_0) \\ v_{cr3N} &= -r_1 \cos(\varphi_1+\theta_1)-(1+M) = -v_{cr0N} = -[-r_0 \cos(\theta_0)+(1-M)] \end{aligned} \tag{99}$$

Mode P transitions to Mode N at t_2, and the resonant current i_{rN} equal to the magnetizing current i_{mN} at t_3, (100) can be obtain.

$$\begin{aligned} i_{r2N} &= r_0 \sin(\varphi_0+\theta_0) = r_1 \sin(\theta_1) \\ v_{cr2N} &= -r_0 \cos(\varphi_0+\theta_0)+(1-M) = -r_1 \cos(\theta_1)-(1+M) \\ i_{rec}(t_3) &= nI_n\left(i_{rN}(t_3)-i_{mN}(t_3)\right) \\ &= nI_n\left(r_1 \sin(\varphi_1+\theta_1)-r_0 \sin(\theta_0)-\frac{M\omega_{r0}}{L_n}\frac{T_s}{2}\right) = nI_n\left(-2r_0 \sin(\theta_0)-\frac{M\omega_{r0}}{L_n}\frac{T_s}{2}\right) = 0 \end{aligned} \tag{100}$$

At steady state, $\overline{i}_{rec1} = \overline{i}_{rec2}$, $M = \overline{i}_{rec1}R$ .According to the definition of M, φ_0 and φ_1, (32) can be obtained.

$$\begin{aligned} &M = \frac{nV_o}{V_{in}} = \frac{n\overline{i}_{rec1}R}{V_{in}} = \frac{2n^2RI_n}{V_{in}T_s\omega_{r0}}\left(r_0 \cos(\theta_0)-r_0 \cos(\varphi_0+\theta_0)+r_1 \cos(\theta_1)-r_1 \cos(\varphi_1+\theta_1)\right) \\ &\varphi_0+\varphi_1-\frac{\omega_{r0}T_s}{2} = 0 \end{aligned} \tag{101}$$

Therefore, the following system of equations can be obtained

$$\begin{cases} r_0 \sin(\theta_0)+\dfrac{M\omega_{r0}T_s}{4L_n} = 0 \\ r_0 \sin(\varphi_0+\theta_0)-r_1 \sin(\theta_1) = 0 \\ -r_0 \cos(\varphi_0+\theta_0)+r_1 \cos(\theta_1)+2 = 0 \\ r_1 \sin(\varphi_1+\theta_1)+r_0 \sin(\theta_0) = 0 \\ -r_1 \cos(\varphi_1+\theta_1)-r_0 \cos(\theta_0)-2*\mathrm{M} = 0 \\ M-\dfrac{2n^2RI_n}{V_{in}T_s\omega_{r0}}\left(r_0 \cos(\theta_0)-r_0 \cos(\varphi_0+\theta_0)+r_1 \cos(\theta_1)-r_1 \cos(\varphi_1+\theta_1)\right) = 0 \\ \varphi_0+\varphi_1-\dfrac{\omega_{r0}T_s}{2} = 0 \end{cases} \tag{102}$$

$[r_0 \quad \theta_0 \quad \varphi_0 \quad r_1 \quad \theta_1 \quad \varphi_1 \quad M]$ is defined as the variables to be solved under the steady state. By using the Newton-Raphson iteration method, the solution of the equations can be calculated, so the steady-state operating point of the system will be obtained, and then steady-state current and voltage values I_{r0N}, I_{r2N}, I_{r3N}, I_{r5N}, I_{r6N}, V_{r0N}, V_{r2N}, V_{r3N}, V_{r5N}, and V_{r6N} at different moments can be obtained.

Section VII. Small-signal model of the LLC converter for NP mode

Set $x=[i_{r0N}, v_{cr0N}, v_o]^T$ as state variables, $u=[v_{in}, t_s]^T$ as input variables, and $y=v_o$ as output variable. The state-space expression for the system can be expressed as (103), where C=[0, 0, 1].

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{103}$$

The large-signal model of the LLC converter over one switching cycle is expressed as follows:

$$\begin{cases} \dot{i}_{r0N} = \dfrac{i_{r6N} - i_{r0N}}{t_s} \\ \dot{v}_{cr0N} = \dfrac{v_{cr6N} - v_{cr0N}}{t_s} \\ \dot{v}_o = \dfrac{1}{C_o}\left(\langle i_{rec}\rangle - \dfrac{v_o}{R}\right) \end{cases} \tag{104}$$

In this derivation for the small-signal model of the LLC converter, ***g***, ***h***, ***k***, ***l***, ***m*** represent the partial derivatives of the $\boldsymbol{\theta}$, $\boldsymbol{r}$, $\boldsymbol{i_{rN}}$, $\boldsymbol{v_{crN}}$, and $\boldsymbol{\varphi}$ to the corresponding variables. Add perturbations to the input and state variables at the quiescent-state operating point as follows:

$$\begin{cases} v_{in} = V_{in} + \hat{v}_{in} \\ v_o = V_o + \hat{v}_o \\ t_s = T_s + \hat{t}_s \\ i_{r0N} = I_{r0N} + \hat{i}_{r0N} \\ v_{cr0N} = V_{cr0N} + \hat{v}_{cr0N} \end{cases} \tag{105}$$

From t_0 to t_3 with half a switch period, time-domain expressions are as follows:

$$\begin{cases} i_{r0N} = r_0 \sin(\theta_0) \\ v_{r0N} = -r_0 \cos(\theta_0) + (1-M) \\ i_{r2N} = r_0 \sin(\varphi_0 + \theta_0) = r_1 \sin(\theta_1) \\ v_{r2N} = -r_0 \cos(\varphi_0 + \theta_0) + (1-M) = -r_1 \cos(\theta_1) - (1+M) \\ i_{r3N} = r_1 \sin(\varphi_1 + \theta_1) \\ v_{r3N} = -r_1 \cos(\varphi_1 + \theta_1) - (1+M) \\ i_{rec}(t_3) = nI_n\left(r_1 \sin(\varphi_1 + \theta_1) - r_0 \sin(\theta_0) - \dfrac{M\omega_{r0}}{L_n}\dfrac{T_s}{2}\right) = 0 \\ \overline{i}_{rec1} = \dfrac{nI_n}{\omega_{r0}T_s}\left(r_0\cos(\theta_0) - r_0\cos(\varphi_0+\theta_0) + r_1\cos(\theta_1) - r_1\cos(\varphi_1+\theta_1)\right) \\ \varphi_0 = \dfrac{\omega_{r0}T_s}{2} - \varphi_1 \end{cases} \tag{106}$$

At time t_0, the converter starts to operate in mode P. θ_0 and r_0 can be calculated by

$$\theta_0 = \arctan\left(-\frac{i_{r0N}}{v_{r0N} - (1-M)}\right) \qquad r_0 = \sqrt{i_{r0N}^{\;2} + \left[v_{r0N} - (1-M)\right]^2} \tag{107}$$

After first-order linearization:

$$
\begin{aligned}
\theta_0+\Delta\theta_0 &= \theta_0+\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o \\
&= \theta_0-\frac{v_{r0N}-(1-M)}{\left[v_{r0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{\left[v_{r0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{v}_{r0N}-\frac{i_{r0N}M/v_{in}}{r_0{}^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0{}^2}\hat{v}_o \\
&= \theta_0-\frac{v_{r0N}-(1-M)}{r_0{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{r_0{}^2}\hat{v}_{r0N}-\frac{i_{r0N}M/v_{in}}{r_0{}^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0{}^2}\hat{v}_o \\
&= \theta_0+g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{r0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o \\
r_0+\Delta r_0 &= r_0+\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o \\
&= r_0+\frac{i_{r0N}}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{i}_{r0N}+\frac{v_{r0N}-(1-M)}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{v}_{r0N} \\
&\quad -\frac{\left[v_{r0N}-(1-M)\right]M/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{v}_{in}+\frac{\left[v_{r0N}-(1-M)\right]n/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{r0N}-(1-M)\right]^2}}\hat{v}_o \\
&= r_0+\frac{i_{r0N}}{r_0}\hat{i}_{r0N}+\frac{v_{r0N}-(1-M)}{r_0}\hat{v}_{r0N}-\frac{\left[v_{r0N}-(1-M)\right]M/v_{in}}{r_0}\hat{v}_{in}+\frac{\left[v_{r0N}-(1-M)\right]n/v_{in}}{r_0}\hat{v}_o \\
&= r_0+h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{r0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o
\end{aligned}
\tag{108}
$$

At time t_2, Δi_{r2N} and Δv_{r2N} can be expressed as follows:

$$
\begin{aligned}
i_{r2N}+\Delta i_{r2N} &= r_0\sin(\varphi_0+\theta_0)+\frac{\partial i_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r2N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial i_{r2N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r2N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r2N}}{\partial \varphi_0}\Delta\varphi_0 \\
&= i_{r2N}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{r0N}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
&\quad +\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}\right)\hat{v}_o+\frac{\partial i_{r2N}}{\partial \varphi_0}\Delta\varphi_0 \\
&= i_{r2N}+\left[\sin(\varphi_0+\theta_0)h_{0i}+r_0\cos(\varphi_0+\theta_0)g_{0i}\right]\hat{i}_{r0N} \\
&\quad +\left[\sin(\varphi_0+\theta_0)h_{0v}+r_0\cos(\varphi_0+\theta_0)g_{0v}\right]\hat{v}_{r0N} \\
&\quad +\left[\sin(\varphi_0+\theta_0)h_{0in}+r_0\cos(\varphi_0+\theta_0)g_{0in}\right]\hat{v}_{in} \\
&\quad +\left[\sin(\varphi_0+\theta_0)h_{0o}+r_0\cos(\varphi_0+\theta_0)g_{0o}\right]\hat{v}_o+r_0\cos(\varphi_0+\theta_0)\Delta\varphi_0 \\
&= i_{r2N}+k_{2i}\hat{i}_{r0N}+k_{2v}\hat{v}_{r0N}+k_{2in}\hat{v}_{in}+k_{2o}\hat{v}_o+k_{2m0}\Delta\varphi_0 \\
v_{r2N}+\Delta v_{r2N} &= -r_0\cos(\varphi_0+\theta_0)+(1-M)+\frac{\partial v_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{r2N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial v_{r2N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r2N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{r2N}}{\partial \varphi_0}\Delta\varphi_0 \\
&= v_{r2N}+\left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{r0N}}+\frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
&\quad +\left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial v_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}\right)\hat{v}_o+\frac{\partial v_{r2N}}{\partial \varphi_0}\Delta\varphi_0 \\
&= v_{r2N}+\left[-\cos(\varphi_0+\theta_0)h_{0i}+r_0\sin(\varphi_0+\theta_0)g_{0i}\right]\hat{i}_{r0N} \\
&\quad +\left[-\cos(\varphi_0+\theta_0)h_{0v}+r_0\sin(\varphi_0+\theta_0)g_{0v}\right]\hat{v}_{r0N} \\
&\quad +\left[-\cos(\varphi_0+\theta_0)h_{0in}+r_0\sin(\varphi_0+\theta_0)g_{0in}+\frac{M}{v_{in}}\right]\hat{v}_{in} \\
&\quad +\left[-\cos(\varphi_0+\theta_0)h_{0o}+r_0\sin(\varphi_0+\theta_0)g_{0o}-\frac{n}{v_{in}}\right]\hat{v}_o+r_0\sin(\varphi_0+\theta_0)\Delta\varphi_0 \\
&= v_{r2N}+l_{2i}\hat{i}_{r0N}+l_{2v}\hat{v}_{r0N}+l_{2in}\hat{v}_{in}+l_{2o}\hat{v}_o+l_{2m0}\Delta\varphi_0
\end{aligned}
\tag{109}
$$

$\Delta\varphi_0$ needs to be determined, $\Delta\theta_1$ and Δr_1 can be calculated by

$$
\begin{aligned}
&\theta_1+\Delta\theta_1=\theta_1+\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o+\frac{\partial\theta_1}{\partial\varphi_0}\Delta\varphi_0\\
&=\theta_1+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{r0N}}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{in}}-\frac{M}{V_{in}}\frac{i_{r2N}}{r_1^2}\right)\hat{v}_{in}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_o}+\frac{n}{V_{in}}\frac{i_{r2N}}{r_1^2}\right)\hat{v}_o\\
&+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial\varphi_0}+\frac{\partial\theta_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial\varphi_0}\right)\Delta\varphi_0\\
&=\theta_1+\left[-\frac{v_{r2N}+1+M}{r_1^2}k_{2i}+\frac{i_{r2N}}{r_1^2}l_{2i}\right]\hat{i}_{r0N}+\left[-\frac{v_{r2N}+1+M}{r_1^2}k_{2v}+\frac{i_{r2N}}{r_1^2}l_{2v}\right]\hat{v}_{r0N}\\
&+\left[-\frac{v_{r2N}+1+M}{r_1^2}k_{2in}+\frac{i_{r2N}}{r_1^2}l_{2in}-\frac{M}{V_{in}}\frac{i_{r2N}}{r_1^2}\right]\hat{v}_{in}+\left[-\frac{v_{r2N}+1+M}{r_1^2}k_{2o}+\frac{i_{r2N}}{r_1^2}l_{2o}+\frac{n}{V_{in}}\frac{i_{r2N}}{r_1^2}\right]\hat{v}_o\\
&+\left(-\frac{v_{r2N}+1+M}{r_1^2}k_{2m0}+\frac{i_{r2N}}{r_1^2}l_{2m0}\right)\Delta\varphi_0\\
&=\theta_1+g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0\\
&r_1+\Delta r_1=r_1+\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o+\frac{\partial r_1}{\partial\varphi_0}\Delta\varphi_0\\
&=r_1+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{r0N}}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_{in}}-\frac{M}{V_{in}}\frac{V_{r2N}+1+M}{r_1}\right)\hat{v}_{in}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial v_o}+\frac{n}{V_{in}}\frac{V_{r2N}+1+M}{r_1}\right)\hat{v}_o\\
&+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial\varphi_0}+\frac{\partial r_1}{\partial v_{r2N}}\frac{\partial v_{r2N}}{\partial\varphi_0}\right)\Delta\varphi_0\\
&=r_1+\left(\frac{i_{r2N}}{r_1}k_{2i}+\frac{V_{r2N}+1+M}{r_1}l_{2i}\right)\hat{i}_{r0N}+\left(\frac{i_{r2N}}{r_1}k_{2v}+\frac{V_{r2N}+1+M}{r_1}l_{2v}\right)\hat{v}_{r0N}\\
&+\left(\frac{i_{r2N}}{r_1}k_{2in}+\frac{V_{r2N}+1+M}{r_1}l_{2in}-\frac{M}{V_{in}}\frac{V_{r2N}+1+M}{r_1}\right)\hat{v}_{in}+\left(\frac{i_{r2N}}{r_1}k_{2o}+\frac{V_{r2N}+1+M}{r_1}l_{2o}+\frac{n}{V_{in}}\frac{V_{r2N}+1+M}{r_1}\right)\hat{v}_o\\
&+\left(\frac{i_{r2N}}{r_1}k_{2m0}+\frac{V_{r2N}+1+M}{r_1}l_{2m0}\right)\Delta\varphi_0\\
&=r_1+h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0
\end{aligned}
\tag{110}
$$

At time t_3, $i_{recN}(t_3)=0$, and $i_{recN}(t_3+\Delta t_3)=0$ after the perturbations are added. (111) can be obtained

$$
\begin{aligned}
&i_{recN}(t_3+\Delta t_3)=n\left((r_1+\Delta r_1)\sin(\varphi_1+\Delta\varphi_1+\theta_1+\Delta\theta_1)-(r_0+\Delta r_0)\sin(\theta_0+\Delta\theta_0)-\frac{n(v_o+\Delta v_o)}{(v_{in}+\Delta v_{in})L_n}\frac{\omega_{r0}T_s}{2}\right)\\
&\approx n\left(r_1\sin(\varphi_1+\theta_1)-r_0\sin(\theta_0)-\frac{M}{L_n}\frac{\omega_{r0}T_s}{2}\right)+\frac{\partial i_{recN}(t_3)}{\partial r_0}\Delta r_0+\frac{\partial i_{recN}(t_3)}{\partial r_1}\Delta r_1+\frac{\partial i_{recN}(t_3)}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial i_{recN}(t_3)}{\partial\theta_0}\Delta\theta_0+\\
&\frac{\partial i_{recN}(t_3)}{\partial\theta_1}\Delta\theta_1+\frac{\partial i_{recN}(t_3)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}(t_3)}{\partial v_o}\hat{v}_o+\frac{\partial i_{recN}(t_3)}{\partial t_s}\hat{t}_s\\
&=i_{recN}(t_3)+\frac{\partial i_{recN}(t_3)}{\partial r_0}\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{recN}(t_3)}{\partial r_1}\left(\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o+\frac{\partial r_1}{\partial\varphi_0}\Delta\varphi_0\right)\\
&+\frac{\partial i_{recN}(t_3)}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial i_{recN}(t_3)}{\partial\theta_0}\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{recN}(t_3)}{\partial\theta_1}\left(\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o+\frac{\partial\theta_1}{\partial\varphi_0}\Delta\varphi_0\right)\\
&+\frac{\partial i_{recN}(t_3)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}(t_3)}{\partial v_o}\hat{v}_o+\frac{\partial i_{recN}(t_3)}{\partial t_s}\hat{t}_s
\end{aligned}
$$

$$
= i_{recN}(t_3) + n \begin{bmatrix} -\sin(\theta_0)\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial r_0}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial r_0}{\partial v_{in}}\hat{v}_{in} + \frac{\partial r_0}{\partial v_o}\hat{v}_o\right) \\ +\sin(\varphi_1+\theta_1)\left(\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial r_1}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial r_1}{\partial v_{in}}\hat{v}_{in} + \frac{\partial r_1}{\partial v_o}\hat{v}_o + \frac{\partial r_1}{\partial \varphi_0}\Delta\varphi_0\right) \\ +r_1\cos(\varphi_1+\theta_1)\Delta\varphi_1 - r_0\cos(\theta_0)\left(\frac{\partial \theta_0}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial \theta_0}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial \theta_0}{\partial v_{in}}\hat{v}_{in} + \frac{\partial \theta_0}{\partial v_o}\hat{v}_o\right) \\ +r_1\cos(\varphi_1+\theta_1)\left(\frac{\partial \theta_1}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial \theta_1}{\partial v_{r0N}}\hat{v}_{r0N} + \frac{\partial \theta_1}{\partial v_{in}}\hat{v}_{in} + \frac{\partial \theta_1}{\partial v_o}\hat{v}_o + \frac{\partial r_1}{\partial \varphi_0}\Delta\varphi_0\right) \\ +\frac{\omega_{r0}T_s M/v_{in}}{2L_n}\hat{v}_{in} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}\hat{v}_o - \frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s \end{bmatrix}
$$

$$
= i_{recN}(t_3) + n \begin{bmatrix} -\sin(\theta_0)\left(h_{0i}\hat{i}_{r0N} + h_{0v}\hat{v}_{r0N} + h_{0in}\hat{v}_{in} + h_{0o}\hat{v}_o\right) \\ +\sin(\varphi_1+\theta_1)\left(h_{1i}\hat{i}_{r0N} + h_{1v}\hat{v}_{r0N} + h_{1in}\hat{v}_{in} + h_{1o}\hat{v}_o + h_{1m0}\Delta\varphi_0\right) \\ +r_1\cos(\varphi_1+\theta_1)\Delta\varphi_1 - r_0\cos(\theta_0)\left(g_{0i}\hat{i}_{r0N} + g_{0v}\hat{v}_{r0N} + g_{0in}\hat{v}_{in} + g_{0o}\hat{v}_o\right) \\ +r_1\cos(\varphi_1+\theta_1)\left(g_{1i}\hat{i}_{r0N} + g_{1v}\hat{v}_{r0N} + g_{1in}\hat{v}_{in} + g_{1o}\hat{v}_o + g_{1m0}\Delta\varphi_0\right) \\ +\frac{\omega_{r0}T_s M/v_{in}}{2L_n}\hat{v}_{in} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}\hat{v}_o - \frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s \end{bmatrix}
$$

$$
= i_{recN}(t_3) + n \begin{bmatrix} \left[-\sin(\theta_0)h_{0i} + \sin(\varphi_1+\theta_1)h_{1i} - r_0\cos(\theta_0)g_{0i} + r_1\cos(\varphi_1+\theta_1)g_{1i}\right]\hat{i}_{r0N} + \\ \left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v}\right]\hat{v}_{r0N} + \\ \left[-\sin(\theta_0)h_{0in} + \sin(\varphi_1+\theta_1)h_{1in} - r_0\cos(\theta_0)g_{0in} + r_1\cos(\varphi_1+\theta_1)g_{1in} + \frac{\omega_{r0}T_s M/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}\right]\hat{v}_o \\ -\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s + r_1\cos(\varphi_1+\theta_1)\Delta\varphi_1 + \left[\sin(\varphi_1+\theta_1)h_{1m0} + r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]\Delta\varphi_0 \end{bmatrix} = 0 \tag{111}
$$

Because $i_{recN}(t_3)=0$, the following equation can be obtained.

$$
\begin{bmatrix} \left[-\sin(\theta_0)h_{0i} + \sin(\varphi_1+\theta_1)h_{1i} - r_0\cos(\theta_0)g_{0i} + r_1\cos(\varphi_1+\theta_1)g_{1i}\right]\hat{i}_{r0N} + \\ \left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v}\right]\hat{v}_{r0N} + \\ \left[-\sin(\theta_0)h_{0in} + \sin(\varphi_1+\theta_1)h_{1in} - r_0\cos(\theta_0)g_{0in} + r_1\cos(\varphi_1+\theta_1)g_{1in} + \frac{\omega_{r0}T_s M/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}\right]\hat{v}_o \\ -\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s + r_1\cos(\varphi_1+\theta_1)\Delta\varphi_1 + \left[\sin(\varphi_1+\theta_1)h_{1m0} + r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]\Delta\varphi_0 \end{bmatrix} = 0 \tag{112}
$$

Bringing $\Delta\varphi_0 = \frac{\omega_{r0}\hat{t}_s}{2} - \Delta\varphi_1$ into the above equation. (113) can be obtained.

$$\begin{bmatrix} \left[-\sin(\theta_0)h_{0i}+\sin(\varphi_1+\theta_1)h_{1i}-r_0\cos(\theta_0)g_{0i}+r_1\cos(\varphi_1+\theta_1)g_{1i}\right]\hat{i}_{r0N}+ \\ \left[-\sin(\theta_0)h_{0v}+\sin(\varphi_1+\theta_1)h_{1v}-r_0\cos(\theta_0)g_{0v}+r_1\cos(\varphi_1+\theta_1)g_{1v}\right]\hat{v}_{r0N}+ \\ \left[-\sin(\theta_0)h_{0in}+\sin(\varphi_1+\theta_1)h_{1in}-r_0\cos(\theta_0)g_{0in}+r_1\cos(\varphi_1+\theta_1)g_{1in}+\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_0)h_{0v}+\sin(\varphi_1+\theta_1)h_{1v}-r_0\cos(\theta_0)g_{0v}+r_1\cos(\varphi_1+\theta_1)g_{1v}-\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\ +\frac{\omega_{r0}}{2}\left[\sin(\varphi_1+\theta_1)h_{1m0}+r_1\cos(\varphi_1+\theta_1)g_{1m0}-\frac{M}{L_n}\right]\hat{t}_s \\ +\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]\Delta\varphi_1 \end{bmatrix}=0 \tag{113}$$

$\Delta\varphi_1$ can be calculated by

$$\begin{aligned}\Delta\varphi_1 &= -\frac{\begin{bmatrix} \left[-\sin(\theta_0)h_{0i}+\sin(\varphi_1+\theta_1)h_{1i}-r_0\cos(\theta_0)g_{0i}+r_1\cos(\varphi_1+\theta_1)g_{1i}\right]\hat{i}_{r0N}+ \\ \left[-\sin(\theta_0)h_{0v}+\sin(\varphi_1+\theta_1)h_{1v}-r_0\cos(\theta_0)g_{0v}+r_1\cos(\varphi_1+\theta_1)g_{1v}\right]\hat{v}_{r0N}+ \\ \left[-\sin(\theta_0)h_{0in}+\sin(\varphi_1+\theta_1)h_{1in}-r_0\cos(\theta_0)g_{0in}+r_1\cos(\varphi_1+\theta_1)g_{1in}+\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_0)h_{0v}+\sin(\varphi_1+\theta_1)h_{1v}-r_0\cos(\theta_0)g_{0v}+r_1\cos(\varphi_1+\theta_1)g_{1v}-\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\ +\frac{\omega_{r0}}{2}\left[\sin(\varphi_1+\theta_1)h_{1m0}+r_1\cos(\varphi_1+\theta_1)g_{1m0}-\frac{M}{L_n}\right]\hat{t}_s \end{bmatrix}}{\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]} \\ &= m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{r0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\end{aligned} \tag{114}$$

Δi_{r3N} and Δv_{r3N} can be calculated by

$$\begin{aligned}
&i_{r3N}+\Delta i_{r3N}=(r_1+\Delta r_1)\sin(\varphi_1+\Delta\varphi_1+\theta_1+\Delta\theta_1)=r_1\sin(\varphi_1+\theta_1)+\frac{\partial i_{r3N}}{\partial r_1}\Delta r_1+\frac{\partial i_{r3N}}{\partial\theta_1}\Delta\theta_1+\frac{\partial i_{r3N}}{\partial\varphi_1}\Delta\varphi_1 \\
&=i_{r3N}+\frac{\partial i_{r3N}}{\partial r_1}\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0\right) \\
&+\frac{\partial i_{r3N}}{\partial\theta_1}\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0\right)+\frac{\partial i_{r3N}}{\partial\varphi_1}\Delta\varphi_1 \\
&=i_{r3N}+\sin(\varphi_1+\theta_1)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-h_{1m0}\Delta\varphi_1\right) \\
&+r_1\cos(\varphi_1+\theta_1)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-g_{1m0}\Delta\varphi_1\right)+r_1\cos(\varphi_1+\theta_1)\Delta\varphi_1 \\
&=i_{r3N}+\sin(\varphi_1+\theta_1)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&+r_1\cos(\varphi_1+\theta_1)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&+\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]\left(m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{r0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\right) \\
&=i_{r3N}+\left[\sin(\varphi_1+\theta_1)h_{1i}+r_1\cos(\varphi_1+\theta_1)g_{1i}+\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]m_{1i}\right]\hat{i}_{r0N} \\
&+\left[\sin(\varphi_1+\theta_1)h_{1v}+r_1\cos(\varphi_1+\theta_1)g_{1v}+\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]m_{1v}\right]\hat{v}_{r0N} \\
&+\left[\sin(\varphi_1+\theta_1)h_{1in}+r_1\cos(\varphi_1+\theta_1)g_{1in}+\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]m_{1in}\right]\hat{v}_{in} \\
&+\left[\sin(\varphi_1+\theta_1)h_{1o}+r_1\cos(\varphi_1+\theta_1)g_{1o}+\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]m_{1o}\right]\hat{v}_o \\
&+\left[\sin(\varphi_1+\theta_1)h_{1m0}\frac{\omega_{r0}}{2}+r_1\cos(\varphi_1+\theta_1)g_{1m0}\frac{\omega_{r0}}{2}+\left[r_1\cos(\varphi_1+\theta_1)-\sin(\varphi_1+\theta_1)h_{1m0}-r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]m_{1t}\right]\hat{t}_s \\
&=i_{r3N}+k_{3i}\hat{i}_{r0N}+k_{3v}\hat{v}_{r0N}+k_{3in}\hat{v}_{in}+k_{3o}\hat{v}_o+k_{3t}\hat{t}_s
\end{aligned}$$

$$
\begin{aligned}
&v_{r3N}+\Delta v_{r3N}=\left(-r_1\cos\left(\varphi_1+\theta_1\right)-(1+M)\right)+\frac{\partial v_{r3N}}{\partial r_1}\Delta r_1+\frac{\partial v_{r3N}}{\partial\theta_1}\Delta\theta_1+\frac{\partial v_{r3N}}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial v_{r3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r3N}}{\partial v_o}\hat{v}_o\\
&=v_{r3N}+\frac{\partial v_{r3N}}{\partial r_1}\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0\right)+\frac{\partial v_{r3N}}{\partial\theta_1}\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0\right)+\frac{\partial v_{r3N}}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial v_{r3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r3N}}{\partial v_o}\hat{v}_o\\
&=v_{r3N}-\cos\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-h_{1m0}\Delta\varphi_1\right)\\
&+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-g_{1m0}\Delta\varphi_1+\Delta\varphi_1\right)+\frac{M}{v_{in}}\hat{v}_{in}-\frac{n}{v_{in}}\hat{v}_o\\
&=v_{r3N}-\cos\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)\\
&+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]\left(m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{r0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\right)+\frac{M}{v_{in}}\hat{v}_{in}-\frac{n}{v_{in}}\hat{v}_o\\
&=v_{r3N}+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1i}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1i}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1i}\right]\hat{i}_{r0N}\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1v}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1v}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1v}\right]\hat{v}_{r0N}\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1in}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1in}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1in}+\frac{M}{v_{in}}\right]\hat{v}_{in}\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1o}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1o}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1o}-\frac{n}{v_{in}}\right]\hat{v}_o\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)\frac{\omega_{r0}}{2}h_{1m0}+r_1\sin\left(\varphi_1+\theta_1\right)\frac{\omega_{r0}}{2}g_{1m0}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1t}\right]\hat{t}_s\\
&=v_{r3N}+l_{3i}\hat{i}_{r0N}+l_{3v}\hat{v}_{r0N}+l_{3in}\hat{v}_{in}+l_{3o}\hat{v}_o+l_{3t}\hat{t}_s
\end{aligned}
\tag{115}
$$

The average output current of the rectifier from t_0 to t_3 can be expressed as

$$
\begin{aligned}
&\left\langle i_{rec1}\right\rangle+\Delta\left\langle i_{rec1}\right\rangle=\frac{nI_n}{\omega_{r0}t_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\\
&+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial r_0}\Delta r_0+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial r_1}\Delta r_1+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial\theta_0}\Delta\theta_0+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial\theta_1}\Delta\theta_1+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial\varphi_0}\Delta\varphi_0+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\left\langle i_{rec1}\right\rangle}{\partial t_s}\hat{t}_s\\
&=\left\langle i_{rec1}\right\rangle+\frac{nI_n}{\omega_{r0}t_s}\left[\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)\right]\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{r0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right)\\
&+\frac{nI_n}{\omega_{r0}t_s}\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-h_{1m0}\Delta\varphi_1\right)\\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)\right]\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{r0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right)\\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-g_{1m0}\Delta\varphi_1\right)\\
&+\frac{nI_n}{\omega_{r0}t_s}\left[r_0\sin\left(\varphi_0+\theta_0\right)\right]\left(\frac{\omega_{r0}}{2}\hat{t}_s-\Delta\varphi_1\right)+\frac{nI_n}{\omega_{r0}t_s}\left[r_1\sin\left(\varphi_1+\theta_1\right)\right]\Delta\varphi_1\\
&+\left[\frac{nC_r}{t_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{v}_{in}\\
&+\left[-\frac{nI_n}{\omega_{r0}t_s^{\,2}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{t}_s\\
&=\left\langle i_{rec1}\right\rangle+\frac{nI_n}{\omega_{r0}t_s}\left[\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)\right]\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{r0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right)+\frac{nI_n}{\omega_{r0}t_s}\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{r0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)\\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)\right]\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{r0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right)+\frac{nI_n}{\omega_{r0}t_s}\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{r0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)\\
&+\left[\frac{nC_r}{t_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{v}_{in}+\left[-\frac{nI_n}{\omega_{r0}t_s^{\,2}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{t}_s\\
&+\frac{nI_n}{\omega_{r0}t_s}\left[r_0\sin\left(\varphi_0+\theta_0\right)\right]\frac{\omega_{r0}}{2}\hat{t}_s+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{matrix}-\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]h_{1m0}-\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]g_{1m0}\\-r_0\sin\left(\varphi_0+\theta_0\right)+\left[r_1\sin\left(\varphi_1+\theta_1\right)\right]\end{matrix}\right]\bullet\left(m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{r0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \langle i_{rec1} \rangle + \frac{nI_n}{\omega_{r0} t_s}\left[\begin{array}{l}
\left[\cos(\theta_0) - \cos(\varphi_0 + \theta_0)\right] h_{0i} + \left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1i} \\
+\left[-r_0 \sin(\theta_0) + r_0 \sin(\varphi_0 + \theta_0)\right] g_{0i} + \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1i} \\
+\left[\begin{array}{l} -\left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1m0} - \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1m0} \\ -r_0 \sin(\varphi_0 + \theta_0) + \left[r_1 \sin(\varphi_1 + \theta_1)\right] \end{array}\right] m_{1i}
\end{array}\right] \hat{i}_{r0N} \\
&+ \frac{nI_n}{\omega_{r0} t_s}\left[\begin{array}{l}
\left[\cos(\theta_0) - \cos(\varphi_0 + \theta_0)\right] h_{0v} + \left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1v} \\
+\left[-r_0 \sin(\theta_0) + r_0 \sin(\varphi_0 + \theta_0)\right] g_{0v} + \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1v} \\
+\left[\begin{array}{l} -\left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1m0} - \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1m0} \\ -r_0 \sin(\varphi_0 + \theta_0) + \left[r_1 \sin(\varphi_1 + \theta_1)\right] \end{array}\right] m_{1v}
\end{array}\right] \hat{v}_{r0N} \\
&+ \frac{nI_n}{\omega_{r0} t_s}\left[\begin{array}{l}
\left[\cos(\theta_0) - \cos(\varphi_0 + \theta_0)\right] h_{0in} + \left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1in} \\
+\left[-r_0 \sin(\theta_0) + r_0 \sin(\varphi_0 + \theta_0)\right] g_{0in} + \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1in} \\
+\left[\begin{array}{l} -\left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1m0} - \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1m0} \\ -r_0 \sin(\varphi_0 + \theta_0) + \left[r_1 \sin(\varphi_1 + \theta_1)\right] \end{array}\right] m_{1in} \\
+\frac{\omega_{r0} t_s}{nI_n} \frac{nC_r}{t_s}\left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0 + \theta_0) + r_1 \cos(\theta_1) - r_1 \cos(\varphi_1 + \theta_1)\right)
\end{array}\right] \hat{v}_{in} \\
&+ \frac{nI_n}{\omega_{r0} t_s}\left[\begin{array}{l}
\left[\cos(\theta_0) - \cos(\varphi_0 + \theta_0)\right] h_{0o} + \left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1o} \\
+\left[-r_0 \sin(\theta_0) + r_0 \sin(\varphi_0 + \theta_0)\right] g_{0o} + \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1o} \\
+\left[\begin{array}{l} -\left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1m0} - \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1m0} \\ -r_0 \sin(\varphi_0 + \theta_0) + \left[r_1 \sin(\varphi_1 + \theta_1)\right] \end{array}\right] m_{1o}
\end{array}\right] \hat{v}_{o} \\
&+ \frac{nI_n}{\omega_{r0} t_s}\left[\begin{array}{l}
\left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1m0} \frac{\omega_{r0}}{2} + \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1m0} \frac{\omega_{r0}}{2} \\
+\left[-\frac{1}{t_s}\left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0 + \theta_0) + r_1 \cos(\theta_1) - r_1 \cos(\varphi_1 + \theta_1)\right)\right] \\
+\left[\begin{array}{l} -\left[\cos(\theta_1) - \cos(\varphi_1 + \theta_1)\right] h_{1m0} - \left[-r_1 \sin(\theta_1) + r_1 \sin(\varphi_1 + \theta_1)\right] g_{1m0} \\ -r_0 \sin(\varphi_0 + \theta_0) + \left[r_1 \sin(\varphi_1 + \theta_1)\right] \end{array}\right] m_{1t} \\
+\left[r_0 \sin(\varphi_0 + \theta_0)\right] \frac{\omega_{r0}}{2}
\end{array}\right] \hat{t}_s \\
&= \langle i_{rec1} \rangle + k_{rec1i} \hat{i}_{r0N} + k_{rec1v} \hat{v}_{r0N} + k_{rec1in} \hat{v}_{in} + k_{rec1o} \hat{v}_o + k_{rec1t} \hat{t}_s
\end{aligned}
\tag{116}
$$

From t_3 to t_6 with half a switch period, time-domain expressions are as follows:

$$
\begin{cases}
i_{r3N} = r_2 \sin(\theta_2) \\
v_{r3N} = -r_2 \cos(\theta_2) - (1 - M) \\
i_{r5N} = r_2 \sin(\varphi_2 + \theta_2) = r_3 \sin(\theta_3) \\
v_{r5N} = -r_2 \cos(\varphi_2 + \theta_2) - (1 - M) = -r_3 \cos(\theta_3) + (1 + M) \\
i_{r6N} = r_3 \sin(\varphi_3 + \theta_3) \\
v_{r6N} = -r_3 \cos(\varphi_3 + \theta_3) + (1 + M) \\
i_{rec}(t_6) = nI_n\left(r_3 \sin(\varphi_3 + \theta_3) - r_2 \sin(\theta_2) + \dfrac{M\omega_{r0}}{L_n} \dfrac{T_s}{2}\right) = 0 \\
\overline{i}_{rec2} = \dfrac{nI_n}{\omega_{r0} T_s}\left(r_2 \cos(\theta_2) - r_2 \cos(\varphi_2 + \theta_2) + r_3 \cos(\theta_3) - r_3 \cos(\varphi_3 + \theta_3)\right) \\
\varphi_3 = \dfrac{\omega_{r0} T_s}{2} - \varphi_2
\end{cases}
\tag{117}
$$

At time t_3, θ_2, r_2 can be expressed as:

$$\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{r3N}+(1-M)}\right)\quad r_2=\sqrt{{i_{r3N}}^2+\left[v_{r3N}+(1-M)\right]^2} \tag{118}$$

After first-order linearization

$$\begin{aligned}
&\theta_2+\Delta\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{r3N}+(1-M)}\right)+\frac{\partial\theta_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_2}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_2}{\partial v_o}\hat{v}_o+\frac{\partial\theta_2}{\partial t_s}\hat{t}_s\\
&=\theta_2+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}}+\frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{r0N}}+\frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}+\\
&\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}}+\frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{in}}-\frac{i_{r3N}M/v_{in}}{{r_2}^2}\right)\hat{v}_{in}+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o}+\frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_o}+\frac{i_{r3N}n/v_{in}}{{r_2}^2}\right)\hat{v}_o+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s}+\frac{\partial\theta_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial t_s}\right)\hat{t}_s\\
&=\theta_2+\left[-\frac{v_{r3N}+(1-M)}{{r_2}^2}k_{3i}+\frac{i_{r3N}}{{r_2}^2}l_{3i}\right]\hat{i}_{r0N}+\left[-\frac{v_{r3N}+(1-M)}{{r_2}^2}k_{3v}+\frac{i_{r3N}}{{r_2}^2}l_{3v}\right]\hat{v}_{r0N}+\left[-\frac{v_{r3N}+(1-M)}{{r_2}^2}k_{3in}+\frac{i_{r3N}}{{r_2}^2}l_{3in}+\frac{i_{r3N}M/v_{in}}{{r_2}^2}\right]\hat{v}_{in}\\
&+\left[-\frac{v_{r3N}+(1-M)}{{r_2}^2}k_{3o}+\frac{i_{r3N}}{{r_2}^2}l_{3o}-\frac{ni_{r3N}/v_{in}}{{r_2}^2}\right]\hat{v}_o+\left[-\frac{v_{r3N}+(1-M)}{{r_2}^2}k_{3t}+\frac{i_{r3N}}{{r_2}^2}l_{3t}\right]\hat{t}_s\\
&=\theta_2+g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{r0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s\\
&r_2+\Delta r_2=\sqrt{{i_{r3N}}^2+\left[v_{r3N}+(1-M)\right]^2}+\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s\\
&=r_2+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}}+\frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{r0N}}+\frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}}+\frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_{in}}+\frac{\left[v_{r3N}+(1-M)\right]M/v_{in}}{r_1}\right)\hat{v}_{in}+\\
&\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o}+\frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial v_o}-\frac{\left[v_{r3N}+(1-M)\right]n/v_{in}}{r_1}\right)\hat{v}_o+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s}+\frac{\partial r_2}{\partial v_{r3N}}\frac{\partial v_{r3N}}{\partial t_s}\right)\hat{t}_s\\
&=r_2+\left(\frac{i_{r3N}}{r_2}k_{3i}+\frac{v_{r3N}+(1-M)}{r_2}l_{3i}\right)\hat{i}_{r0N}+\left(\frac{i_{r3N}}{r_2}k_{3v}+\frac{v_{r3N}+(1-M)}{r_2}l_{3v}\right)\hat{v}_{r0N}+\left(\frac{i_{r3N}}{r_2}k_{3in}+\frac{v_{r3N}+(1-M)}{r_2}l_{3in}+\frac{\left[v_{r3N}+(1-M)\right]M/v_{in}}{r_2}\right)\hat{v}_{in}+\\
&\left(\frac{i_{r3N}}{r_2}k_{3o}+\frac{v_{r3N}+(1-M)}{r_2}l_{3o}-\frac{\left[v_{r3N}+(1-M)\right]n/v_{in}}{r_2}\right)\hat{v}_o+\left(\frac{i_{r3N}}{r_2}k_{3t}+\frac{v_{r3N}+(1-M)}{r_2}l_{3t}\right)\hat{t}_s=r_2+h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{r0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s
\end{aligned} \tag{119}$$

At time t_5, Δi_{r5N} and Δv_{r5N} can be calculated by

$$\begin{aligned}
i_{r5N}+\Delta i_{r5N}&=r_2\sin(\varphi_2+\theta_2)+\frac{\partial i_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r5N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial i_{r5N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r5N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r5N}}{\partial t_s}\hat{t}_s+\frac{\partial i_{r5N}}{\partial\varphi_2}\Delta\varphi_2\\
&=i_{r2N}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial i_{r5N}}{\partial\theta_2}\frac{\partial\theta_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{r0N}}+\frac{\partial i_{r5N}}{\partial\theta_2}\frac{\partial\theta_2}{\partial v_{r0N}}\right)\hat{v}_{r0N}\\
&+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial i_{r5N}}{\partial\theta_2}\frac{\partial\theta_2}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_o}+\frac{\partial i_{r5N}}{\partial\theta_2}\frac{\partial\theta_2}{\partial v_o}\right)\hat{v}_o+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial t_s}+\frac{\partial i_{r5N}}{\partial\theta_2}\frac{\partial\theta_2}{\partial t_s}\right)\hat{t}_s+\frac{\partial i_{r5N}}{\partial\varphi_2}\Delta\varphi_2\\
&=i_{r5N}+\left[\sin(\varphi_2+\theta_2)h_{2i}+r_2\cos(\varphi_2+\theta_2)g_{2i}\right]\hat{i}_{r0N}\\
&+\left[\sin(\varphi_2+\theta_2)h_{2v}+r_2\cos(\varphi_2+\theta_2)g_{2v}\right]\hat{v}_{r0N}\\
&+\left[\sin(\varphi_2+\theta_2)h_{2in}+r_2\cos(\varphi_2+\theta_2)g_{2in}\right]\hat{v}_{in}\\
&+\left[\sin(\varphi_2+\theta_2)h_{2o}+r_2\cos(\varphi_2+\theta_2)g_{2o}\right]\hat{v}_o\\
&+\left[\sin(\varphi_2+\theta_2)h_{2t}+r_2\cos(\varphi_2+\theta_2)g_{2t}\right]\hat{t}_s+r_2\cos(\varphi_2+\theta_2)\Delta\varphi_2\\
&=i_{r5N}+k_{5i}\hat{i}_{r0N}+k_{5v}\hat{v}_{r0N}+k_{5in}\hat{v}_{in}+k_{5o}\hat{v}_o+k_{5t}\hat{t}_s+k_{5m2}\Delta\varphi_2
\end{aligned}$$

$$
\begin{aligned}
& v_{r5N}+\Delta v_{r5N}=-r_2\cos\left(\varphi_2+\theta_2\right)-\left(1-M\right)+\frac{\partial v_{r5N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{r5N}}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial v_{r5N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r5N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{r5N}}{\partial t_s}\hat{t}_s+\frac{\partial v_{r5N}}{\partial \varphi_2}\Delta\varphi_2 \\
& =v_{r5N}+\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{r0N}}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
& +\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}}-\frac{M}{v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_o}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_o}+\frac{n}{v_{in}}\right)\hat{v}_o \\
& +\left(\frac{\partial v_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial t_s}+\frac{\partial v_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_s}\right)\hat{t}_s+\frac{\partial v_{r5N}}{\partial \varphi_2}\Delta\varphi_2 \\
& =v_{r5N}+\left[-\cos\left(\varphi_2+\theta_2\right)h_{2i}+r_2\sin\left(\varphi_2+\theta_2\right)g_{2i}\right]\hat{i}_{r0N} \\
& +\left[-\cos\left(\varphi_2+\theta_2\right)h_{2v}+r_2\sin\left(\varphi_2+\theta_2\right)g_{2v}\right]\hat{v}_{r0N} \\
& +\left[-\cos\left(\varphi_2+\theta_2\right)h_{2in}+r_2\sin\left(\varphi_2+\theta_2\right)g_{2in}-\frac{M}{v_{in}}\right]\hat{v}_{in} \\
& +\left[-\cos\left(\varphi_2+\theta_2\right)h_{2o}+r_2\sin\left(\varphi_2+\theta_2\right)g_{2o}+\frac{n}{v_{in}}\right]\hat{v}_o \\
& +\left[-\cos\left(\varphi_2+\theta_2\right)h_{2t}+r_2\sin\left(\varphi_2+\theta_2\right)g_{2t}\right]\hat{t}_s+r_2\sin\left(\varphi_2+\theta_2\right)\Delta\varphi_2 \\
& =v_{r5N}+l_{5i}\hat{i}_{r0N}+l_{5v}\hat{v}_{r0N}+l_{5in}\hat{v}_{in}+l_{5o}\hat{v}_o+l_{5t}\hat{t}_s+l_{5m2}\Delta\varphi_2
\end{aligned}
\tag{120}
$$

θ_3 and r_3 can be expressed as:

$$
\theta_3=\arctan\left(-\frac{i_{r5N}}{\left[v_{cr5N}-\left(1+M\right)\right]}\right) \quad r_3=\sqrt{{i_{r5N}}^2+(v_{r5N}-\left(1+M\right))^2}
\tag{121}
$$

$\Delta\theta_3$ and Δr_3 can be calculated by

$$
\begin{aligned}
& \theta_3+\Delta\theta_3=\theta_3+\frac{\partial\theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial\theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_3}{\partial v_o}\hat{v}_o+\frac{\partial\theta_3}{\partial t_s}\hat{t}_s+\frac{\partial\theta_3}{\partial \varphi_2}\Delta\varphi_2 \\
& =\theta_3+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{r0N}}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{r0N}}\right)\hat{v}_{r0N} \\
& +\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{in}}+\frac{M}{V_{in}}\frac{i_{r5N}}{{r_3}^2}\right)\hat{v}_{in}+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_o}-\frac{n}{V_{in}}\frac{i_{r5N}}{{r_3}^2}\right)\hat{v}_o \\
& +\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial t_s}\right)\hat{t}_s+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial \varphi_2}+\frac{\partial\theta_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial \varphi_2}\right)\Delta\varphi_2 \\
& =\theta_3+\left[-\frac{v_{r5N}-\left(1+M\right)}{{r_3}^2}k_{5i}+\frac{i_{r5N}}{{r_3}^2}l_{5i}\right]\hat{i}_{r0N}+\left[-\frac{v_{r5N}-\left(1+M\right)}{{r_3}^2}k_{5v}+\frac{i_{r5N}}{{r_3}^2}l_{5v}\right]\hat{v}_{r0N} \\
& +\left[-\frac{v_{r5N}-\left(1+M\right)}{{r_3}^2}k_{5in}+\frac{i_{r5N}}{{r_3}^2}l_{5in}+\frac{M}{V_{in}}\frac{i_{r5N}}{{r_3}^2}\right]\hat{v}_{in}+\left[-\frac{v_{r5N}-\left(1+M\right)}{{r_3}^2}k_{5o}+\frac{i_{r5N}}{{r_3}^2}l_{5o}-\frac{n}{V_{in}}\frac{i_{r5N}}{{r_3}^2}\right]\hat{v}_o \\
& +\left[-\frac{v_{r5N}-\left(1+M\right)}{{r_3}^2}k_{5t}+\frac{i_{r5N}}{{r_3}^2}l_{5t}\right]\hat{t}_s+\left(-\frac{v_{r5N}-\left(1+M\right)}{{r_3}^2}k_{5m2}+\frac{i_{r5N}}{{r_3}^2}l_{5m2}\right)\Delta\varphi_2 \\
& =\theta_3+g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\Delta\varphi_2
\end{aligned}
$$

$$
\begin{aligned}
r_3+\Delta r_3 &= r_3+\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s+\frac{\partial r_3}{\partial \varphi_2}\Delta\varphi_2 \\
&= r_3+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{r0N}}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{r0N}}\right)\hat{v}_{r0N}+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_{in}}+\frac{M}{V_{in}}\frac{V_{r5N}-(1+M)}{r_3}\right)\hat{v}_{in} \\
&+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial v_o}-\frac{n}{V_{in}}\frac{V_{r5N}-(1+M)}{r_3}\right)\hat{v}_o+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial t_s}\right)\hat{t}_s+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial \varphi_2}+\frac{\partial r_3}{\partial v_{r5N}}\frac{\partial v_{r5N}}{\partial \varphi_2}\right)\Delta\varphi_2 \\
&= r_3+\left(\frac{i_{r5N}}{r_3}k_{5i}+\frac{V_{r5N}-(1+M)}{r_3}l_{5i}\right)\hat{i}_{r0N}+\left(\frac{i_{r5N}}{r_3}k_{5v}+\frac{V_{r5N}-(1+M)}{r_3}l_{5v}\right)\hat{v}_{r0N} \\
&+\left(\frac{i_{r5N}}{r_3}k_{5in}+\frac{V_{r5N}-(1+M)}{r_3}l_{5in}+\frac{M}{V_{in}}\frac{V_{r5N}-(1+M)}{r_3}\right)\hat{v}_{in}+\left(\frac{i_{r5N}}{r_3}k_{5o}+\frac{V_{r5N}-(1+M)}{r_3}l_{5o}-\frac{n}{V_{in}}\frac{V_{r5N}-(1+M)}{r_3}\right)\hat{v}_o \\
&+\left(\frac{i_{r5N}}{r_3}k_{5t}+\frac{V_{r5N}-(1+M)}{r_3}l_{5t}\right)\hat{t}_s+\left(\frac{i_{r5N}}{r_3}k_{5m2}+\frac{V_{r5N}-(1+M)}{r_3}l_{5m2}\right)\Delta\varphi_2 \\
&= r_3+h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\Delta\varphi_2
\end{aligned}
\tag{122}
$$

At time t_6, $i_{recN}(t_6)$=0, and $i_{recN}(t_6+\Delta t_6)$=0 after the perturbations are added. (40) can be obtained

$$
\begin{aligned}
& i_{recN}(t_6+\Delta t_6)=n\left((r_3+\Delta r_3)\sin(\varphi_3+\Delta\varphi_3+\theta_3+\Delta\theta_3)-(r_2+\Delta r_2)\sin(\theta_2+\Delta\theta_2)+\frac{n(v_o+\Delta v_o)}{(v_{in}+\Delta v_{in})L_n}\frac{\omega_{r0}T_s}{2}\right) \\
&\approx n\left(r_3\sin(\varphi_3+\theta_3)-r_2\sin(\theta_2)+\frac{M\omega_{r0}}{L_n}\frac{T_s}{2}\right)+\frac{\partial i_{recN}(t_6)}{\partial r_2}\Delta r_2+\frac{\partial i_{recN}(t_6)}{\partial r_3}\Delta r_3+\frac{\partial i_{recN}(t_6)}{\partial \varphi_3}\Delta\varphi_3+\frac{\partial i_{recN}(t_6)}{\partial \theta_2}\Delta\theta_2+ \\
&\frac{\partial i_{recN}(t_6)}{\partial \theta_3}\Delta\theta_3+\frac{\partial i_{recN}(t_6)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}(t_6)}{\partial v_o}\hat{v}_o+\frac{\partial i_{recN}(t_6)}{\partial t_s}\hat{t}_s \\
&= i_{recN}(t_6)+\frac{\partial i_{recN}(t_6)}{\partial r_2}\left(\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s\right)+\frac{\partial i_{recN}(t_6)}{\partial r_3}\left(\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s+\frac{\partial r_3}{\partial \varphi_3}\Delta\varphi_2\right) \\
&+\frac{\partial i_{recN}(t_6)}{\partial \varphi_3}\Delta\varphi_3+\frac{\partial i_{recN}(t_6)}{\partial \theta_2}\left(\frac{\partial \theta_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial \theta_2}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial \theta_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial \theta_2}{\partial v_o}\hat{v}_o+\frac{\partial \theta_2}{\partial t_s}\hat{t}_s\right) \\
&+\frac{\partial i_{recN}(t_6)}{\partial \theta_3}\left(\frac{\partial \theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial \theta_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial \theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial \theta_3}{\partial v_o}\hat{v}_o+\frac{\partial \theta_3}{\partial t_s}\hat{t}_s+\frac{\partial \theta_3}{\partial \varphi_2}\Delta\varphi_2\right)+\frac{\partial i_{recN}(t_6)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}(t_6)}{\partial v_o}\hat{v}_o+\frac{\partial i_{recN}(t_6)}{\partial t_s}\hat{t}_s \\
&= i_{recN}(t_6)+n\begin{bmatrix}
-\sin(\theta_2)\left(\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s\right) \\
+\sin(\varphi_3+\theta_3)\left(\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s+\frac{\partial r_3}{\partial \varphi_3}\Delta\varphi_2\right) \\
+r_3\cos(\varphi_3+\theta_3)\Delta\varphi_3-r_2\cos(\theta_2)\left(\frac{\partial \theta_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial \theta_2}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial \theta_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial \theta_2}{\partial v_o}\hat{v}_o+\frac{\partial \theta_2}{\partial t_s}\hat{t}_s\right) \\
+r_3\cos(\varphi_3+\theta_3)\left(\frac{\partial \theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial \theta_3}{\partial v_{r0N}}\hat{v}_{r0N}+\frac{\partial \theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial \theta_3}{\partial v_o}\hat{v}_o+\frac{\partial \theta_3}{\partial t_s}\hat{t}_s+\frac{\partial \theta_3}{\partial \varphi_2}\Delta\varphi_2\right) \\
-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\hat{v}_{in}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\hat{v}_o+\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s
\end{bmatrix} \\
&= i_{recN}(t_6)+n\begin{bmatrix}
-\sin(\theta_2)\left(h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{r0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s\right) \\
+\sin(\varphi_3+\theta_3)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\Delta\varphi_2\right) \\
+r_3\cos(\varphi_3+\theta_3)\Delta\varphi_3-r_2\cos(\theta_2)\left(g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{r0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s\right) \\
+r_3\cos(\varphi_3+\theta_3)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\Delta\varphi_2\right) \\
-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\hat{v}_{in}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\hat{v}_o+\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s
\end{bmatrix}
\end{aligned}
$$

$$
= i_{recN}(t_6) + n\begin{bmatrix}
\left[-\sin(\theta_2)h_{2i} + \sin(\varphi_3+\theta_3)h_{3i} - r_2\cos(\theta_2)g_{2i} + r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N} + \\
\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{r0N} + \\
\left[-\sin(\theta_2)h_{2in} + \sin(\varphi_3+\theta_3)h_{3in} - r_2\cos(\theta_2)g_{2in} + r_3\cos(\varphi_3+\theta_3)g_{3in} - \dfrac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\
+\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v} + \dfrac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\
+\left[-\sin(\theta_2)h_{2t} + \sin(\varphi_3+\theta_3)h_{3t} - r_2\cos(\theta_2)g_{2t} + r_3\cos(\varphi_3+\theta_3)g_{3t} + \dfrac{M}{L_n}\dfrac{\omega_{r0}}{2}\right]\hat{t}_s \\
+r_3\cos(\varphi_3+\theta_3)\Delta\varphi_3 + \left[\sin(\varphi_3+\theta_3)h_{3m2} + r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\Delta\varphi_2
\end{bmatrix} = 0 \tag{123}
$$

Because $i_{recN}(t_6)=0$, the following equation can be obtained.

$$
\begin{bmatrix}
\left[-\sin(\theta_2)h_{2i} + \sin(\varphi_3+\theta_3)h_{3i} - r_2\cos(\theta_2)g_{2i} + r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N} + \\
\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{r0N} + \\
\left[-\sin(\theta_2)h_{2in} + \sin(\varphi_3+\theta_3)h_{3in} - r_2\cos(\theta_2)g_{2in} + r_3\cos(\varphi_3+\theta_3)g_{3in} - \dfrac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\
+\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v} + \dfrac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\
+\left[-\sin(\theta_2)h_{2t} + \sin(\varphi_3+\theta_3)h_{3t} - r_2\cos(\theta_2)g_{2t} + r_3\cos(\varphi_3+\theta_3)g_{3t} + \dfrac{M}{L_n}\dfrac{\omega_{r0}}{2}\right]\hat{t}_s \\
+r_3\cos(\varphi_3+\theta_3)\Delta\varphi_3 + \left[\sin(\varphi_3+\theta_3)h_{3m2} + r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\Delta\varphi_2
\end{bmatrix} = 0 \tag{124}
$$

Bringing $\Delta\varphi_2 = \dfrac{\omega_{r0}\hat{t}_s}{2} - \Delta\varphi_3$ into the above equation. (125) can be obtained.

$$
\begin{bmatrix}
\left[-\sin(\theta_2)h_{2i} + \sin(\varphi_3+\theta_3)h_{3i} - r_2\cos(\theta_2)g_{2i} + r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N} + \\
\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{r0N} + \\
\left[-\sin(\theta_2)h_{2in} + \sin(\varphi_3+\theta_3)h_{3in} - r_2\cos(\theta_2)g_{2in} + r_3\cos(\varphi_3+\theta_3)g_{3in} - \dfrac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\
+\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v} + \dfrac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\
+\begin{bmatrix} -\sin(\theta_2)h_{2t} + \sin(\varphi_3+\theta_3)h_{3t} - r_2\cos(\theta_2)g_{2t} + r_3\cos(\varphi_3+\theta_3)g_{3t} \\ +\dfrac{M}{L_n}\dfrac{\omega_{r0}}{2} + \left[\sin(\varphi_3+\theta_3)h_{3m2} + r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\dfrac{\omega_{r0}}{2} \end{bmatrix}\hat{t}_s \\
+\left[r_3\cos(\varphi_3+\theta_3) - \sin(\varphi_3+\theta_3)h_{3m2} - r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\Delta\varphi_3
\end{bmatrix} = 0 \tag{125}
$$

$\Delta\varphi_3$ can be calculated by

$$
\Delta\varphi_3 = -\frac{\begin{bmatrix}
\left[-\sin(\theta_2)h_{2i} + \sin(\varphi_3+\theta_3)h_{3i} - r_2\cos(\theta_2)g_{2i} + r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N} + \\
\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{r0N} + \\
\left[-\sin(\theta_2)h_{2in} + \sin(\varphi_3+\theta_3)h_{3in} - r_2\cos(\theta_2)g_{2in} + r_3\cos(\varphi_3+\theta_3)g_{3in} - \dfrac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\
+\left[-\sin(\theta_2)h_{2v} + \sin(\varphi_3+\theta_3)h_{3v} - r_2\cos(\theta_2)g_{2v} + r_3\cos(\varphi_3+\theta_3)g_{3v} + \dfrac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\
+\begin{bmatrix} -\sin(\theta_2)h_{2t} + \sin(\varphi_3+\theta_3)h_{3t} - r_2\cos(\theta_2)g_{2t} + r_3\cos(\varphi_3+\theta_3)g_{3t} \\ +\dfrac{M}{L_n}\dfrac{\omega_{r0}}{2} + \left[\sin(\varphi_3+\theta_3)h_{3m2} + r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\dfrac{\omega_{r0}}{2} \end{bmatrix}\hat{t}_s
\end{bmatrix}}{\left[r_3\cos(\varphi_3+\theta_3) - \sin(\varphi_3+\theta_3)h_{3m2} - r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]} \tag{126}
$$

$$
= m_{3i}\hat{i}_{r0N} + m_{3v}\hat{v}_{r0N} + m_{3in}\hat{v}_{in} + m_{3o}\hat{v}_o + m_{3t}\hat{t}_s
$$

Δi_{r6N} and Δv_{r6N} can be calculated as follows:

$$
\begin{aligned}
& i_{r6N}+\Delta i_{r6N}=\left(r_3+\Delta r_3\right)\sin\left(\varphi_3+\Delta\varphi_3+\theta_3+\Delta\theta_3\right)=r_3\sin\left(\varphi_3+\theta_3\right)+\frac{\partial i_{r6N}}{\partial r_3}\Delta r_3+\frac{\partial i_{r6N}}{\partial\theta_3}\Delta\theta_3+\frac{\partial i_{r6N}}{\partial\varphi_3}\Delta\varphi_3 \\
& =i_{r6N}+\frac{\partial i_{r6N}}{\partial r_3}\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_S+h_{3m2}\Delta\varphi_2\right)+\frac{\partial i_{r6N}}{\partial\theta_3}\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_S+g_{3m2}\Delta\varphi_2\right)+\frac{\partial i_{r6N}}{\partial\varphi_3}\Delta\varphi_3 \\
& =i_{r6N}+\sin\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_S+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S-h_{3m2}\Delta\varphi_3\right) \\
& +r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_S+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S-g_{3m2}\Delta\varphi_3\right)+r_3\cos\left(\varphi_3+\theta_3\right)\Delta\varphi_3 \\
& =i_{r6N}+\sin\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_S+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S\right)+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_S+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S\right) \\
& +\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]\left(m_{3i}\hat{i}_{r0N}+m_{3v}\hat{v}_{r0N}+m_{3in}\hat{v}_{in}+m_{3o}\hat{v}_o+m_{3t}\hat{t}_s\right) \\
& =i_{r6N}+\left[\sin\left(\varphi_3+\theta_3\right)h_{3i}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3i}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3i}\right]\hat{i}_{r0N} \\
& +\left[\sin\left(\varphi_3+\theta_3\right)h_{3v}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3v}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3v}\right]\hat{v}_{r0N} \\
& +\left[\sin\left(\varphi_3+\theta_3\right)h_{3in}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3in}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3in}\right]\hat{v}_{in} \\
& +\left[\sin\left(\varphi_3+\theta_3\right)h_{3o}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3o}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3o}\right]\hat{v}_o \\
& +\left[\begin{array}{l}\sin\left(\varphi_3+\theta_3\right)\left(h_{3t}+h_{3m2}\frac{\omega_{r0}}{2}\right)+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3t}+g_{3m2}\frac{\omega_{r0}}{2}\right) \\ +\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3t}\end{array}\right]\hat{t}_s=i_{r6N}+k_{6i}\hat{i}_{r0N}+k_{6v}\hat{v}_{r0N}+k_{6in}\hat{v}_{in}+k_{6o}\hat{v}_o+k_{6t}\hat{t}_s \\
& v_{r6N}+\Delta v_{r6N}=-r_3\cos\left(\varphi_3+\theta_3\right)+(1+M)+\frac{\partial v_{r6N}}{\partial r_3}\Delta r_3+\frac{\partial v_{r6N}}{\partial\theta_3}\Delta\theta_3+\frac{\partial v_{r6N}}{\partial\varphi_3}\Delta\varphi_3+\frac{\partial v_{r6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r6N}}{\partial v_o}\hat{v}_o \\
& =v_{r6N}+\frac{\partial v_{r6N}}{\partial r_3}\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\Delta\varphi_2\right)+\frac{\partial v_{r6N}}{\partial\theta_3}\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\Delta\varphi_2\right)+\frac{\partial v_{r6N}}{\partial\varphi_3}\Delta\varphi_3+\frac{\partial v_{r6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{r6N}}{\partial v_o}\hat{v}_o \\
& =v_{r6N}-\cos\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s-h_{3m2}\Delta\varphi_3\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s-g_{3m2}\Delta\varphi_3+\Delta\varphi_3\right) \\
& -\frac{M}{v_{in}}\hat{v}_{in}+\frac{n}{v_{in}}\hat{v}_o=v_{r6N}-\cos\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{r0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{r0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
& +\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]\left(m_{3i}\hat{i}_{r0N}+m_{3v}\hat{v}_{r0N}+m_{3in}\hat{v}_{in}+m_{3o}\hat{v}_o+m_{3t}\hat{t}_s\right)-\frac{M}{v_{in}}\hat{v}_{in}+\frac{n}{v_{in}}\hat{v}_o \\
& =v_{r6N}+\left[-\cos\left(\varphi_3+\theta_3\right)h_{3i}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3i}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3i}\right]\hat{i}_{r0N} \\
& +\left[-\cos\left(\varphi_3+\theta_3\right)h_{3v}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3v}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3v}\right]\hat{v}_{r0N} \\
& +\left[-\cos\left(\varphi_3+\theta_3\right)h_{3in}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3in}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3in}-\frac{M}{v_{in}}\right]\hat{v}_{in} \\
& +\left[-\cos\left(\varphi_3+\theta_3\right)h_{3o}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3o}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3o}+\frac{n}{v_{in}}\right]\hat{v}_o \\
& +\left[\begin{array}{l}-\cos\left(\varphi_3+\theta_3\right)\left(h_{3t}+\frac{\omega_{r0}}{2}h_{3m2}\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3t}+\frac{\omega_{r0}}{2}g_{3m2}\right) \\ +\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3t}\end{array}\right]\hat{t}_s=v_{r6N}+l_{6i}\hat{i}_{r0N}+l_{6v}\hat{v}_{r0N}+l_{6in}\hat{v}_{in}+l_{6o}\hat{v}_o+l_{6t}\hat{t}_s
\end{aligned}
\tag{127}
$$

The average output current of the rectifier from t_3 to t_6 can be expressed as

$$
\begin{aligned}
&\langle i_{rec2}\rangle + \Delta\langle i_{rec2}\rangle = \frac{nI_n}{\omega_{r0}t_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right) \\
&+\frac{\partial\langle i_{rec2}\rangle}{\partial r_2}\Delta r_2 + \frac{\partial\langle i_{rec2}\rangle}{\partial r_3}\Delta r_3 + \frac{\partial\langle i_{rec2}\rangle}{\partial \theta_2}\Delta\theta_2 + \frac{\partial\langle i_{rec2}\rangle}{\partial \theta_3}\Delta\theta_3 + \frac{\partial\langle i_{rec2}\rangle}{\partial \varphi_2}\Delta\varphi_2 + \frac{\partial\langle i_{rec2}\rangle}{\partial \varphi_3}\Delta\varphi_3 + \frac{\partial\langle i_{rec2}\rangle}{\partial v_{in}}\hat{v}_{in} + \frac{\partial\langle i_{rec2}\rangle}{\partial t_s}\hat{t}_s \\
&= \langle i_{rec2}\rangle + \frac{nI_n}{\omega_{r0}t_s}\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]\left(h_{2i}\hat{i}_{r0N} + h_{2v}\hat{v}_{r0N} + h_{2in}\hat{v}_{in} + h_{2o}\hat{v}_o + h_{2t}\hat{t}_s\right) + \frac{nI_n}{\omega_{r0}t_s}\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]\left(h_{3i}\hat{i}_{r0N} + h_{3v}\hat{v}_{r0N} + h_{3in}\hat{v}_{in} + h_{3o}\hat{v}_o + h_{3t}\hat{t}_s + h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s - h_{3m2}\Delta\varphi_3\right) \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{r0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right) \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]\left(g_{3i}\hat{i}_{r0N} + g_{3v}\hat{v}_{r0N} + g_{3in}\hat{v}_{in} + g_{3o}\hat{v}_o + g_{3t}\hat{t}_s + g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s - g_{3m2}\Delta\varphi_3\right) \\
&+\frac{nI_n}{\omega_{r0}t_s}r_2\sin(\varphi_2+\theta_2)\left(\frac{\omega_{r0}}{2}\hat{t}_s - \Delta\varphi_3\right) + \frac{nI_n}{\omega_{r0}t_s}r_3\sin(\varphi_3+\theta_3)\Delta\varphi_3 + \left[\frac{nC_r}{t_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right]\hat{v}_{in} \\
&+\left[-\frac{nI_n}{\omega_{r0}t_s^2}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right]\hat{t}_s \\
&= \langle i_{rec2}\rangle + \frac{nI_n}{\omega_{r0}t_s}\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]\left(h_{2i}\hat{i}_{r0N} + h_{2v}\hat{v}_{r0N} + h_{2in}\hat{v}_{in} + h_{2o}\hat{v}_o + h_{2t}\hat{t}_s\right) \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]\left(h_{3i}\hat{i}_{r0N} + h_{3v}\hat{v}_{r0N} + h_{3in}\hat{v}_{in} + h_{3o}\hat{v}_o + h_{3t}\hat{t}_s + h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{r0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right) \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]\left(g_{3i}\hat{i}_{r0N} + g_{3v}\hat{v}_{r0N} + g_{3in}\hat{v}_{in} + g_{3o}\hat{v}_o + g_{3t}\hat{t}_s + g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&+\left[\frac{nC_r}{t_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right]\hat{v}_{in} + \left[-\frac{nI_n}{\omega_{r0}t_s^2}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right]\hat{t}_s \\
&+\frac{nI_n}{\omega_{r0}t_s}r_2\sin(\varphi_2+\theta_2)\frac{\omega_{r0}}{2}\hat{t}_s + \frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]\bullet\left(m_{3i}\hat{i}_{r0N} + m_{3v}\hat{v}_{r0N} + m_{3in}\hat{v}_{in} + m_{3o}\hat{v}_o + m_{3t}\hat{t}_s\right) \\
&= \langle i_{rec2}\rangle + \frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l} \left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2i} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3i} \\ +\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2i} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3i} \\ +\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3i} \end{array}\right]\hat{i}_{r0N} \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l} \left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2v} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3v} \\ +\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2v} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3v} \\ +\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3v} \end{array}\right]\hat{v}_{r0N} + \frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l} \left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2in} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3in} \\ +\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2in} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3in} \\ +\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3in} \\ \frac{\omega_{r0}t_s}{nI_n}\left[\frac{nC_r}{t_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right] \end{array}\right]\hat{v}_{in} \\
&+\frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l} \left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2o} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3o} \\ +\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2o} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3o} \\ +\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3o} \end{array}\right]\hat{v}_o + \frac{nI_n}{\omega_{r0}t_s}\left[\begin{array}{l} \left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2t} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]\left(h_{3t} + h_{3m2}\frac{\omega_{r0}}{2}\right) \\ +\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2t} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]\left(g_{3t} + g_{3m2}\frac{\omega_{r0}}{2}\right) \\ +\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3t} \\ -\frac{1}{t_s}\left[\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right] + r_2\sin(\varphi_2+\theta_2)\frac{\omega_{r0}}{2} \end{array}\right]\hat{t}_s \\
&= \langle i_{rec2}\rangle + k_{rec2i}\hat{i}_{r0N} + k_{rec2v}\hat{v}_{r0N} + k_{rec2in}\hat{v}_{in} + k_{rec2o}\hat{v}_o + k_{rec2t}\hat{t}_s
\end{aligned} \tag{128}
$$

The change in output current of the rectifier bridge during one switching cycle is expressed as

$$
\begin{aligned}
\Delta\overline{i}_{rec} &= \Delta\langle i_{rec1}\rangle - \Delta\langle i_{rec2}\rangle \\
&= \left(k_{rec1i}-k_{rec2i}\right)\hat{i}_{r0N} + \left(k_{rec1v}-k_{rec2v}\right)\hat{v}_{r0N} + \left(k_{rec1o}-k_{rec2o}\right)\hat{v}_o + \left(k_{rec1v}-k_{rec2in}\right)\hat{v}_{in} + \left(k_{rec1t}-k_{rec2t}\right)\hat{t}_s
\end{aligned} \tag{129}
$$

According to the large signal model, the state space expression of the LLC converter can be expressed as

$$
\begin{aligned}
\dot{\hat{i}}_{r0N} &= \frac{i_{r6N}+\hat{i}_{r6N}-i_{r0N}-\hat{i}_{r0N}}{t_s+\hat{t}_s} \approx \frac{\hat{i}_{r6N}-\hat{i}_{r0N}}{T_s} = \frac{1}{T_s}\left[\left(k_{6i}-1\right)\hat{i}_{r0N}+k_{6v}\hat{v}_{r0N}+k_{6in}\hat{v}_{in}+k_{6o}\hat{v}_o+k_{6t}\hat{t}_s\right] \\
\dot{\hat{v}}_{r0N} &= \frac{v_{r6N}+\hat{v}_{r6N}-v_{r0N}-\hat{v}_{r0N}}{t_s+\hat{t}_s} \approx \frac{v_{r6N}-v_{r0N}}{T_s} = \frac{1}{T_s}\left[l_{6i}\hat{i}_{r0N}+\left(l_{6v}-1\right)\hat{v}_{r0N}+l_{6in}\hat{v}_{in}+l_{6o}\hat{v}_o+l_{6t}\hat{t}_s\right] \\
\dot{\hat{v}}_o &= \frac{1}{C_o}\left(\Delta\bar{i}_{rec}-\frac{\hat{v}_o}{R}\right) = \frac{1}{C_o}\left[\begin{array}{l}\left(k_{rec1i}-k_{rec2i}\right)\hat{i}_{r0N}+\left(k_{rec1v}-k_{rec2v}\right)\hat{v}_{r0N}+\left(k_{rec1o}-k_{rec2o}-\frac{1}{R}\right)\hat{v}_o \\ +\left(k_{rec1v}-k_{rec2in}\right)\hat{v}_{in}+\left(k_{rec1o}-k_{rec2t}\right)\hat{t}_s\end{array}\right]
\end{aligned}
\tag{130}
$$

$$
\begin{aligned}
&\dot{\hat{x}} = A\hat{x}+B\hat{u} \\
&\hat{y} = C\hat{x} \\
&A = \begin{bmatrix} \frac{k_{6i}-1}{T_s} & \frac{k_{6v}}{T_s} & \frac{k_{6o}}{T_s} \\ \frac{l_{6i}}{T_s} & \frac{l_{6v}-1}{T_s} & \frac{l_{6o}}{T_s} \\ \frac{k_{rec1i}-k_{rec2i}}{C_o} & \frac{k_{rec1v}-k_{rec2v}}{C_o} & \frac{k_{rec1o}-k_{rec2o}-1/R}{C_o} \end{bmatrix} \\
&B = \begin{bmatrix} \frac{k_{6in}}{T_s} & \frac{k_{6t}}{T_s} \\ \frac{l_{6in}}{T_s} & \frac{l_{6t}}{T_s} \\ \frac{k_{rec1in}-k_{rec2in}}{C_o} & \frac{k_{rec1t}-k_{rec2t}}{C_o} \end{bmatrix} \\
&C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
\end{aligned}
\tag{131}
$$

The transfer function of the LLC converter for PO mode can be expressed as

$$
G(s) = C\left(sI-A\right)^{-1}B = \left[G_{vin}(s) \quad G_t(s)\right] \tag{132}
$$

where

$$
\begin{aligned}
G_{vin}(s) &= \frac{\hat{v}_o}{\hat{v}_{in}} \\
G_t(s) &= \frac{\hat{v}_o}{\hat{t}_s}
\end{aligned}
$$

The disturbance is implemented after the half of the switching period delay. The following equation can be obtained.

$$
\begin{aligned}
G_{ts}(s) &= e^{-\frac{T_s}{2}s}G_t(s) \\
G_{vins}(s) &= e^{-\frac{T_s}{2}s}G_{vin}(s)
\end{aligned}
\tag{133}
$$

Section VIII. Small-signal model for PO mode with TSC

The definitions of t_{Z1}, t_{Z2} and t_{cs} are shown below.

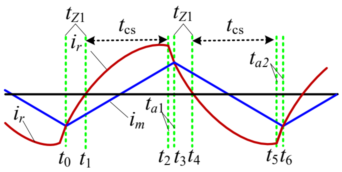

Fig.7 The analysis of control time under TSC for NP mode.

Δt_{Z1}, Δt_{Z2}, Δt_{a2}, and Δt_{a2} can be expressed as follows:

$$
\begin{aligned}
& t_{Z1} = -\frac{\theta_0}{\omega_{r0}}, t_{Z1} = \frac{\pi - \theta_2}{\omega_{r0}}, t_{a1} = \frac{\varphi_1}{\omega_{r0}}, t_{a2} = \frac{\varphi_3}{\omega_{r0}} \\
& \Delta t_{Z1} = -\frac{\Delta\theta_0}{\omega_{r0}}, \Delta t_{Z1} = -\frac{\Delta\theta_2}{\omega_{r0}}, \Delta t_{a1} = \frac{\Delta\varphi_1}{\omega_{r0}}, \Delta t_{a2} = \frac{\Delta\varphi_3}{\omega_{r0}}
\end{aligned}
\tag{134}
$$

The relationship between $\hat{t}_{cs}$ and $\hat{t}_s$ can be shown below.

$$
\begin{aligned}
\hat{t}_s &= \Delta t_{Z1} + 2\hat{t}_{cs} + \Delta t_{Z2} + \Delta t_{a1} + \Delta t_{a2} = \frac{1}{\omega_{r0}}\left(-\Delta\theta_0 - \Delta\theta_1 + \Delta\varphi_1 + \Delta\varphi_3\right) + 2\hat{t}_{cs} \\
&= \frac{1}{\omega_{r0}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right)\hat{i}_{r0N} + \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right)\hat{v}_{r0N} + \\ \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)\hat{v}_{in} + \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right)\hat{v}_o \end{bmatrix} + \frac{\left(-g_{2t} + m_{1t} + m_{3t}\right)}{\omega_{r0}}\hat{t}_s + 2\hat{t}_{cs}
\end{aligned}
\tag{135}
$$

The above equation can be rewritten as

$$
\begin{aligned}
\hat{t}_s &= \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right)\hat{i}_{r0N} + \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right)\hat{v}_{r0N} + \\ \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)\hat{v}_{in} + \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right)\hat{v}_o \end{bmatrix} + \frac{2\omega_{r0}}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\hat{t}_{cs} \\
&= \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right) & \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right) & \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right) \end{bmatrix}\begin{bmatrix} \hat{i}_{r0N} \\ \hat{v}_{r0N} \\ \hat{v}_o \end{bmatrix} \\
&+ \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right) & 2\omega_{r0} \end{bmatrix}\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix} \\
&= A_Z \begin{bmatrix} \hat{i}_{r0N} \\ \hat{v}_{r0N} \\ \hat{v}_o \end{bmatrix} + B_Z \begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix}
\end{aligned}
\tag{136}
$$

where

$$
\begin{aligned}
A_Z &= \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right) \\ \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right) \\ \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right) \end{bmatrix}^T \\
B_Z &= \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right) & 2\omega_{r0} \end{bmatrix}
\end{aligned}
$$

Replace $\hat{t}_s$ in the state space expression with $t_s = A_Z\hat{x} + B_Z\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix}$

$$
\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + B\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = \hat{x} + \begin{bmatrix}B_1 & B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = A\hat{x} + B_1\hat{v}_{in} + B_2\hat{t}_s \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2\left[A_Z\hat{x} + B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}\right] \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2A_Z\hat{x} + B_2B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= \left(A + B_2A_Z\right)\hat{x} + B_1\hat{v}_{in} + \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\left(\left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)\hat{v}_{in} + 2\omega_{r0}\hat{t}_{cs}\right) \\
&= \left(A + B_2A_Z\right)\hat{x} + \left(B_1 + \frac{\left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\right)\hat{v}_{in} + \frac{2\omega_{r0}}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\hat{t}_{cs} \\
&= \left(A + B_2A_Z\right)\hat{x} + \begin{bmatrix}B_1 + \dfrac{\left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2 & \dfrac{2\omega_{r0}}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= A_c\hat{x} + B_c\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}
\end{aligned} \tag{137}
$$

Therefore, the small-signal model of the LLC converter for PO mode with TSC can be expressed as follows with $e^{-\frac{t_s}{2}}$ correction.

$$
G_{cs}\left(s\right) = C\left(sI - A_c\right)^{-1}B_c = \begin{bmatrix}G_{vin\_tc}(s) & G_{tc}(s)\end{bmatrix} \tag{138}
$$

$$
\begin{aligned}
G_{tcs}\left(s\right) &= e^{-\frac{T_s}{2}s}G_{tc}\left(s\right) \\
G_{vin\_tcs}\left(s\right) &= e^{-\frac{T_s}{2}s}G_{vin\_tc}\left(s\right)
\end{aligned} \tag{139}
$$