Detailed derivation of small-signal model for the LLC based on time-domain analysis

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The topology of the full-bridge LLC resonant converter is shown in Fig.1, where v_{in} and v_o represent the input and output voltages. C_o denotes the output capacitor, and R is the load resistance. The primary stage is composed of Q_1-Q_4, and the rectifier stage is composed of D_{r1}-D_{r4}. The resonant tank consists of resonant inductor L_r, resonant capacitor C_r, and magnetizing inductor L_m of the transformer. For the ZVS of the switches, the LLC converter is suggested to work in PO mode for $f_s<f_r$ and NP mode for $f_s>f_r$ [25]. Therefore, the PO mode and NP mode will be analyzed below.

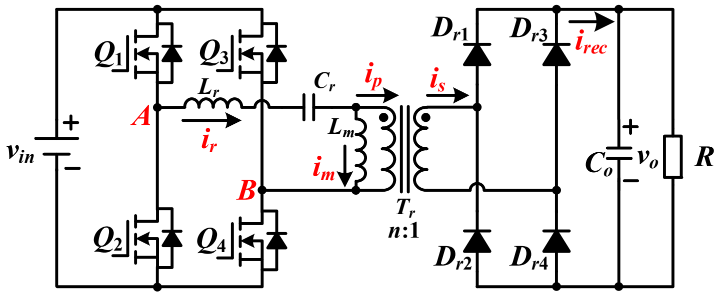

Fig.1 Topology of the LLC resonant converter

To facilitate the subsequent theoretical analysis, the time when the resonant current is equal to the magnetizing current is selected as t_0. The voltage across the resonant tank is v_{AB}. i_r and i_m represent the resonant current and magnetizing current. The transformer secondary current i_s is rectified to i_{rec}.

Variables with the subscript N are normalized in this article, where voltages are normalized with the voltage factor v_{in} and currents are normalized with the current factor I_N=v_{in}/Z_0. Z_0 is the characteristic impedance, expressed as $\sqrt{L_r/C_r}$, and the voltage gain M is defined as M=nv_o/v_{in}.

Section I. Time-domain expressions for PO mode

Typical waveforms and planar trajectory of the LLC converter for PO mode are shown in Fig.2 and Fig.3.

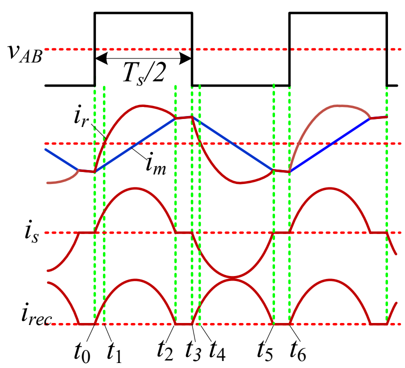

Fig.2 Typical waveforms of the LLC converter for PO mode.

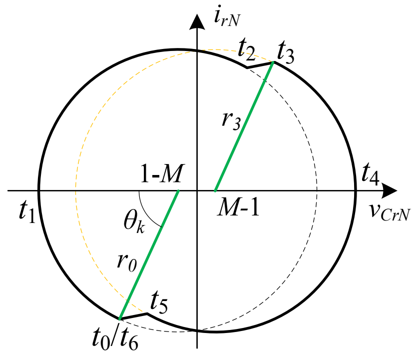

Fig.3 Planar trajectory of the LLC converter for PO mode

[t_0, t_2]

The converter operates in P mode. Setting $t_0 = 0$, only the resonant inductor and resonant capacitor are involved in resonance, and the magnetizing inductor is clamped by the output voltage. ω_{r0} is the resonant angular frequency of both. i_{r0} and v_{cr0} are the values of resonant current and resonant capacitor voltage at t_0, respectively. Similarly, i_{rx} and v_{crx} are the values of resonant current and resonant capacitor voltage at t_x, respectively. They can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r0} Z_0 \sin(\omega_{r0} t) + \left[v_{cr0} - (v_{in} - n v_o)\right]\cos(\omega_{r0} t) + (v_{in} - n v_o) \\ i_r &= i_{r0}\cos(\omega_{r0} t) - \frac{v_{cr0} - (v_{in} - n v_o)}{Z_0}\sin(\omega_{r0} t) \end{aligned} \tag{1}$$

The normalization of the formular is shown in (2).

$$\begin{aligned} v_{crN} &= i_{r0N}\sin(\omega_{r0} t) + \left[v_{cr0N} - (1-M)\right]\cos(\omega_{r0} t) + (1-M) \\ i_{rN} &= i_{r0N}\cos(\omega_{r0} t) - \left[v_{cr0N} - (1-M)\right]\sin(\omega_{r0} t) \end{aligned} \tag{2}$$

where $i_{r0N} = \dfrac{i_{r0} Z_0}{v_{in}}$, $v_{cr0N} = \dfrac{v_{cr0}}{v_{in}}$

In the following analyses, subscript N donates the normalized variable.

Eq.(2) can be rewritten as

$$\begin{aligned} i_{rN} &= \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}\sin(\omega_{r0} t + \theta_0) \\ v_{crN} &= -\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}\cos(\omega_{r0} t + \theta_0) + (1-M) \end{aligned} \tag{3}$$

where

$$\cos\theta_0 = -\frac{\left[v_{cr0N} - (1-M)\right]}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}}, \sin\theta_0 = \frac{i_{r0N}}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}}$$

$$\theta_0 = \arctan\left(\frac{-i_{r0N}}{v_{cr0N} - (1-M)}\right)$$

Set $r_0 = \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}$, then

$$\begin{aligned} i_{rN} &= r_0 \sin(\omega_{r0} t + \theta_0) \\ v_{crN} &= -r_0\cos(\omega_{r0} t + \theta_0) + (1-M) \end{aligned} \tag{4}$$

i_{r0N}, v_{cr0N}, i_{r2N}, and v_{cr2N} can be expressed in (5), where $\varphi_0 = \omega_{r0} t_2$.

$$\begin{aligned} i_{r0N} &= r_0 \sin(\theta_0) \\ v_{cr0N} &= -r_0 \cos(\theta_0) + (1-M) \\ i_{r2N} &= r_0 \sin(\varphi_0 + \theta_0) \\ v_{cr2N} &= -r_0 \cos(\varphi_0 + \theta_0) + (1-M) \end{aligned} \tag{5}$$

The expression of the magnetizing current i_m is shown in (6).

$$i_m = i_{r0} + \frac{n v_o}{L_m} t \tag{6}$$

Equ.(6) is normalized to (7).

$$i_{mN}=\frac{i_{r0}Z_0}{v_{in}}+\frac{nv_oZ_0}{v_{in}L_m}t=i_{r0N}+M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}t=r_0\sin(\theta_0)+\frac{M}{L_n}\omega_{r0}t \tag{7}$$

The current in the secondary winding of the transformer is expressed as

$$i_{s1}=nI_n\left(i_{rN}-i_{mN}\right)=nI_n\left(r_0\sin\left(\omega_{r0}t+\theta_0\right)-r_0\sin\left(\theta_0\right)-\frac{M}{L_n}\omega_{r0}t\right) \tag{8}$$

Since $i_s=0$ from t_2 to t_3, the average value of i_{s1} over half a switching cycle can be expressed as (9).

$$\begin{aligned}\overline{i_{s1}}&=\frac{2}{T_s}\int_0^{t_2}i_{s1}dt=\frac{2nI_n}{T_s}\int_0^{t_2}\left(r_0\sin\left(\omega_{r0}t+\theta_0\right)-r_0\sin\left(\theta_0\right)-\frac{M}{L_n}\omega_{r0}t\right)dt\\&=\frac{2nI_n}{T_s}\left(-\frac{r_0}{\omega_{r0}}\cos\left(\omega_{r0}t+\theta_0\right)-r_0\sin\left(\theta_0\right)t-\frac{M}{2L_n}\omega_{r0}t^2\right)\Bigg|_0^{t_2}\\&=\frac{2nI_n}{T_s}\left(\frac{r_0}{\omega_{r0}}\cos\left(\theta_0\right)-\frac{r_0}{\omega_{r0}}\cos\left(\omega_{r0}t_2+\theta_0\right)-r_0\sin\left(\theta_0\right)t_2-\frac{M}{2L_n}\omega_{r0}t_2^{\ 2}\right)\end{aligned} \tag{9}$$

[t_2, t_3]

The converter operates in O mode. The resonant inductor, the resonant capacitor, and the magnetizing inductor are involved in resonance. ω_{r1} is the resonant angular frequency of them. Z_1 is expressed as $\sqrt{\left(L_r+L_m\right)/C_r}$. v_{cr} and i_r can be expressed as follows.

$$\begin{aligned}v_{cr}&=i_{r2}Z_1\sin\left(\omega_{r1}\left(t-t_2\right)\right)+\left(v_{cr2}-v_{in}\right)\cos\left(\omega_{r1}\left(t-t_2\right)\right)+v_{in}\\i_r&=i_{r2}\cos\left(\omega_{r1}\left(t-t_2\right)\right)-\frac{1}{Z_1}\left(v_{cr2}-v_{in}\right)\sin\left(\omega_{r1}\left(t-t_2\right)\right)\end{aligned} \tag{10}$$

The normalization of (10) is shown in (11).

$$\begin{aligned}v_{crN}&=\frac{i_{r2N}}{Z_0/Z_1}\sin\left(\omega_{r1}\left(t-t_2\right)\right)+\left(v_{cr2N}-1\right)\cos\left(\omega_{r1}\left(t-t_2\right)\right)+1\\i_{rN}&=i_{r2N}\cos\left(\omega_{r1}\left(t-t_2\right)\right)-\frac{Z_0}{Z_1}\left(v_{cr2N}-1\right)\sin\left(\omega_{r1}\left(t-t_2\right)\right)\end{aligned} \tag{11}$$

where $i_{r2N}=\dfrac{i_{r2}Z_0}{v_{in}},v_{cr2N}=\dfrac{v_{cr2}}{v_{in}},L_n=\dfrac{L_m}{L_r},\dfrac{Z_0}{Z_1}=\sqrt{\dfrac{1}{1+L_n}}$

Eq.(11) can be rewritten as

$$\begin{aligned}v_{crN}&=-\sqrt{\left(1+L_n\right)i_{r2N}^{\ 2}+\left(v_{cr2N}-1\right)^2}\cos\left(\omega_{r1}\left(t-t_2\right)+\theta_1\right)+1\\i_{rN}&=\frac{\sqrt{\left(1+L_n\right)i_{r2N}^{\ 2}+\left(v_{cr2N}-1\right)^2}}{\sqrt{1+L_n}}\sin\left(\omega_{r1}\left(t-t_2\right)+\theta_1\right)\end{aligned} \tag{12}$$

where

$$\cos\theta_1=-\frac{\left(v_{cr2N}-1\right)}{\sqrt{\left(1+L_n\right)i_{r2N}^{\ 2}+\left(v_{cr2N}-1\right)^2}},\sin\theta_1=\frac{\sqrt{1+L_n}i_{r2N}}{\sqrt{\left(1+L_n\right)i_{r2N}^{\ 2}+\left(v_{cr2N}-1\right)^2}}$$

$$\theta_1=\arctan\left(-\frac{\sqrt{1+L_n}i_{r2N}}{v_{cr2N}-1}\right)$$

Set $r_1=\sqrt{\left(1+L_n\right)i_{r2N}^{\ 2}+\left(v_{cr2N}-1\right)^2}$, then

$$\begin{aligned} v_{crN} &= -r_1 \cos\left(\omega_{r1}\left(t-t_2\right)+\theta_1\right)+1 \\ i_{rN} = i_{mN} &= \frac{r_1}{\sqrt{1+L_n}} \sin\left(\omega_{r1}\left(t-t_2\right)+\theta_1\right) \end{aligned} \tag{13}$$

i_{r2N}, v_{cr2N}, i_{r3N}, and v_{cr3N} can be expressed in (14), where $\varphi_1=\omega_{r1}(t_3-t_2)$

$$\begin{aligned} v_{cr2N} &= -r_1 \cos\left(\theta_1\right)+1 \\ i_{r2N} = i_{m2N} &= \frac{r_1}{\sqrt{1+L_n}} \sin\left(\theta_1\right) \\ v_{cr3N} &= -r_1 \cos\left(\varphi_1+\theta_1\right)+1 \\ i_{r3N} = i_{m3N} &= \frac{r_1}{\sqrt{1+L_n}} \sin\left(\varphi_1+\theta_1\right) \end{aligned} \tag{14}$$

[t_3, t_5]

Similar to the derivation from t_0 to t_2, v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r3} Z_0 \sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3}+\left(v_{in}-nv_o\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(v_{in}-nv_o\right) \\ i_r &= i_{r3} \cos\left(\omega_{r0}\left(t-t_3\right)\right)-\frac{v_{cr3}+\left(v_{in}-nv_o\right)}{Z_0}\sin\left(\omega_{r0}\left(t-t_3\right)\right) \end{aligned} \tag{15}$$

The normalized equations are expressed in (16).

$$\begin{aligned} v_{crN} &= i_{r3N} \sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3N}+\left(1-M\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(1-M\right) \\ i_{rN} &= i_{r3N} \cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left[v_{cr3N}+\left(1-M\right)\right]\sin\left(\omega_{r0}\left(t-t_3\right)\right) \end{aligned} \tag{16}$$

where $i_{r3N}=\dfrac{i_{r3}Z_0}{v_{in}}, v_{cr3N}=\dfrac{v_{cr3}}{v_{in}}$.

Eq.(16) can be rewritten as

$$\begin{aligned} i_{rN} &= \sqrt{i_{r3N}{}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\sin\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right) \\ v_{crN} &= -\sqrt{i_{r3N}{}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\cos\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right)-\left(1-M\right) \end{aligned} \tag{17}$$

where

$$\cos\theta_2 = -\frac{\left[v_{cr3N}+\left(1-M\right)\right]}{\sqrt{i_{r3N}{}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}}, \sin\theta_2 = \frac{i_{r3N}}{\sqrt{i_{r3N}{}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}}$$

$$\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{cr3N}+\left(1-M\right)}\right)$$

Set $r_2=\sqrt{i_{r3N}{}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}$, then

$$\begin{aligned} i_{rN} &= r_2 \sin\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right) \\ v_{crN} &= -r_2 \cos\left(\omega_{r0}\left(t-t_3\right)+\theta_2\right)-\left(1-M\right) \end{aligned} \tag{18}$$

i_{r3N}, v_{cr3N}, i_{r5N}, and v_{cr5N} can be expressed in (19), where $\varphi_2=\omega_{r0}(t_5-t_3)$

$$
\begin{aligned}
i_{r3N} &= r_2 \sin(\theta_2) \\
v_{cr3N} &= -r_2 \cos(\theta_2) - (1-M) \\
i_{r5N} &= r_2 \sin(\varphi_2 + \theta_2) \\
v_{cr5N} &= -r_2 \cos(\varphi_2 + \theta_2) - (1-M)
\end{aligned} \tag{19}
$$

The expression of the magnetizing current i_m is shown in (20).

$$
i_m = i_{r3} - \frac{nV_o}{L_m}(t - t_3) \tag{20}
$$

The normalized magnetizing current is expressed in (21).

$$
i_{mN} = \frac{i_{r3}Z_0}{v_{in}} - \frac{nv_o Z_0}{v_{in}L_m}(t-t_3) = i_{r3N} - M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}(t-t_3) = r_2 \sin(\theta_2) - \frac{M}{L_n}\omega_{r0}(t-t_3) \tag{21}
$$

The current in the secondary winding of the transformer during t_3 to t_5 is expressed as

$$
i_{s2} = nI_n(i_{rN} - i_{mN}) = nI_n\left(r_2 \sin(\omega_{r0}(t-t_3)+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\omega_{r0}(t-t_3)\right) \tag{22}
$$

Since $i_s = 0$ from t_5 to t_6, the average value of i_{s2} over half a switching cycle can be expressed as (23).

$$
\begin{aligned}
\overline{i_{s2}} &= \frac{2}{T_s}\int_{t_3}^{t_5} i_{s2}dt = \frac{2nI_n}{T_s}\int_{t_3}^{t_5}\left(r_2\sin(\omega_{r0}(t-t_3)+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\omega_{r0}(t-t_3)\right)dt \\
&= \frac{2nI_n}{T_s}\left(-\frac{r_2}{\omega_{r0}}\cos(\omega_{r0}(t-t_3)+\theta_2) - r_2\sin(\theta_2)t + \frac{M}{2L_n}\omega_{r0}(t-t_3)^2\right)\Bigg|_{t_3}^{t_5} \\
&= \frac{2nI_n}{T_s}\left(\frac{r_2}{\omega_{r0}}\cos(\theta_2) - \frac{r_2}{\omega_{r0}}\cos(\omega_{r0}(t_5-t_3)+\theta_2) - r_2\sin(\theta_2)(t_5-t_3) + \frac{M}{2L_n}\omega_{r0}(t_5-t_3)^2\right)
\end{aligned} \tag{23}
$$

The output voltage can be calculated by (24).

$$
v_o = \frac{\overline{i_{s1}} - \overline{i_{s2}}}{2}R \tag{24}
$$

[t_5, t_6]

Similar to the derivation from t_2 to t_3, v_{cr} and i_r can be expressed as follows.

$$
\begin{aligned}
v_{cr} &= i_{r5}Z_1\sin(\omega_{r1}(t-t_5)) + (v_{cr5}+v_{in})\cos(\omega_{r1}(t-t_5)) - v_{in} \\
i_r &= i_{r5}\cos(\omega_{r1}(t-t_5)) - \frac{v_{cr5}+v_{in}}{Z_1}\sin(\omega_{r1}(t-t_5))
\end{aligned} \tag{25}
$$

The normalized equations are expressed in (26).

$$
\begin{aligned}
v_{crN} &= \frac{i_{r5N}}{Z_0/Z_1}\sin(\omega_{r1}(t-t_5)) + (v_{cr5N}+1)\cos(\omega_{r1}(t-t_5)) - 1 \\
i_{rN} &= i_{r5N}\cos(\omega_{r1}(t-t_5)) - \frac{Z_0}{Z_1}(v_{cr5N}+1)\sin(\omega_{r1}(t-t_5))
\end{aligned} \tag{26}
$$

where $i_{r5N} = \frac{i_{r5}Z_0}{v_{in}}, v_{cr5N} = \frac{v_{cr5}}{v_{in}}, L_n = \frac{L_m}{L_r}, \frac{Z_0}{Z_1} = \sqrt{\frac{1}{1+L_n}}$

Eq.(26) can be rewritten as

$$v_{crN}=-\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}\cos\left(\omega_{r1}(t-t_5)+\theta_3\right)-1$$
$$i_{rN}=\frac{\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}}{\sqrt{1+L_n}}\sin\left(\omega_{r1}(t-t_5)+\theta_3\right) \tag{27}$$

where

$$\cos\theta_3=-\frac{(v_{cr5N}+1)}{\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}},\sin\theta_3=\frac{\sqrt{1+L_n}\,i_{r5N}}{\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}}$$
$$\theta_3=\pi+\arctan\left(-\frac{\sqrt{1+L_n}\,i_{r5N}}{v_{cr5N}+1}\right)$$

Set $r_3=\sqrt{(1+L_n)i_{r5N}{}^2+(v_{cr5N}+1)^2}$, then

$$v_{crN}=-r_3\cos\left(\omega_{r1}(t-t_5)+\theta_3\right)-1$$
$$i_{rN}=i_{mN}=\frac{r_3}{\sqrt{1+L_n}}\sin\left(\omega_{r1}(t-t_5)+\theta_3\right) \tag{28}$$

i_{r5N}, v_{cr5N}, i_{r6N}, and v_{cr6N} can be expressed in (29), where $\varphi_3=\omega_{r1}(t_6\text{-}t_5)$

$$v_{cr5N}=-r_3\cos(\theta_3)-1$$
$$i_{r5N}=i_{m5N}=\frac{r_3}{\sqrt{1+L_n}}\sin(\theta_3)$$
$$v_{cr6N}=-r_3\cos(\varphi_3+\theta_3)-1 \tag{29}$$
$$i_{r6N}=i_{m6N}=\frac{r_3}{\sqrt{1+L_n}}\sin(\varphi_3+\theta_3)$$

Section II. Calculation of steady-state operating point for PO mode

Because of the semi-period symmetry, i_{r0N} and v_{cr0N} at t_0 are equal to the negative of i_{r3N} and v_{cr3N} respectively. Therefore, (30) can be obtained.

$$\begin{aligned} i_{r3N} &= \frac{r_1}{\sqrt{1+L_n}}\sin(\varphi_1+\theta_1) = -i_{r0N} = -r_0\sin(\theta_0) \\ v_{cr3N} &= -r_1\cos(\varphi_1+\theta_1)+1 = -v_{cr0N} = r_0\cos(\theta_0)-(1-M) \end{aligned} \tag{30}$$

Mode P transitions to Mode O at t_2, so the resonant current i_{rN} equal to the magnetizing current i_{mN}. (31) can be obtain.

$$\begin{aligned} i_{r2N} &= r_0\sin(\varphi_0+\theta_0) = \frac{r_1}{\sqrt{1+L_n}}\sin(\theta_1) \\ v_{cr2N} &= -r_0\cos(\varphi_0+\theta_0)+(1-M) = -r_1\cos(\theta_1)+1 \\ i_{s1}(t_2) &= nI_n\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)=0 \end{aligned} \tag{31}$$

At steady state, $\overline{i_{s1}} = -\overline{i_{s2}}$, $M = \overline{i_{s1}}R$ .According to the definition of M, φ_0 and φ_1, (32) can be obtained.

$$\begin{aligned} &M = \frac{nv_o}{v_{in}} = \frac{2n^2RI_n}{T_s v_{in}}\left(\frac{r_0}{\omega_{r0}}\cos(\theta_0)-\frac{r_0}{\omega_{r0}}\cos(\omega_{r0}t_2+\theta_0)-r_0\sin(\theta_0)t_2-\frac{M}{2\omega_{r0}L_n}(\omega_{r0}t_2)^2\right) \\ &\frac{\varphi_2}{\omega_{r0}}+\frac{\varphi_2}{\omega_{r1}}=\frac{T_s}{2} \end{aligned} \tag{32}$$

Therefore, the following equations can be obtained

$$\begin{cases} r_0\sin(\varphi_0+\theta_0)-\dfrac{r_1}{\sqrt{1+L_n}}\sin(\theta_1)=0 \\ -r_0\cos(\varphi_0+\theta_0)-M+r_1\cos(\theta_1)=0 \\ \dfrac{r_1}{\sqrt{1+L_n}}\sin(\varphi_1+\theta_1)+r_0\sin(\theta_0)=0 \\ -r_1\cos(\varphi_1+\theta_1)-r_0\cos(\theta_0)+(2-M)=0 \\ r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\dfrac{M}{L_n}\varphi_0=0 \\ M-\dfrac{2n^2RI_n}{T_sV_{in}\omega_{r0}}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\dfrac{M}{2L_n}\varphi_0^{\,2}\right)=0 \\ \dfrac{\varphi_0}{\omega_{r0}}+\dfrac{\varphi_1}{\omega_{r1}}-\dfrac{T_s}{2}=0 \end{cases} \tag{33}$$

$[r_0 \quad \theta_0 \quad \varphi_0 \quad r_1 \quad \theta_1 \quad \varphi_1 \quad M]$ is defined as the variables to be solved under the steady state. By using the Newton-Raphson iteration method, the solution of the equations can be calculated, so the steady-state operating point of the system will be obtained, and then steady-state current and voltage values V_{in}, V_o, T_s, I_{r0N}, I_{r2N}, I_{r3N}, I_{r5N}, I_{r6N}, V_{r0N}, V_{r2N}, V_{r3N}, V_{r5N}, and V_{r6N} at different moments can be obtained.

Section III. Small-signal model of the LLC converter for PO mode with PFM

Set x=[i_{r0N}, v_{cr0N}, v_o]^T as state variables, u=[v_{in}, t_s]^T as input variables, and y=v_o as output variable. The state-space expression for the system can be expressed as (34), where C=[0, 0, 1].

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{34}$$

The large-signal model of the LLC converter over one switching cycle is expressed as follows:

$$\begin{cases} \dot{i}_{r0N} = \dfrac{i_{r6N} - i_{r0N}}{t_s} \\ \dot{v}_{cr0N} = \dfrac{v_{cr6N} - v_{cr0N}}{t_s} \\ \dot{v}_o = \dfrac{1}{C_o}\left(\Delta\bar{i}_{rec} - \dfrac{v_o}{R}\right) \end{cases} \tag{35}$$

In this derivation for the small-signal model of the LLC converter, $\boldsymbol{g}$, $\boldsymbol{h}$, $\boldsymbol{k}$, $\boldsymbol{l}$, and $\boldsymbol{m}$ represent the partial derivatives of the $\boldsymbol{\theta}$, $\boldsymbol{r}$, $\boldsymbol{i_{rN}}$, $\boldsymbol{v_{crN}}$, and $\boldsymbol{\varphi}$ to the corresponding variables. The above variables can be expressed as the quiescent-state operating point plus the disturbances.

$$\begin{cases} v_{in} = V_{in} + \hat{v}_{in} \\ v_o = V_o + \hat{v}_o \\ t_s = T_s + \hat{t}_s \\ i_{r0N} = I_{r0N} + \hat{i}_{r0N} \\ v_{cr0N} = V_{cr0N} + \hat{v}_{cr0N} \end{cases} \tag{36}$$

*In the subsequent derivation of the small-signal modeling, all variables i_{r0N}, v_{cr0N}, M, v_{in}, v_o, r_0, θ_0, φ_0, i_{r2N}, v_{cr2N}, r_2, θ_2, φ_2, etc., represent steady-state values, which can be calculated through iteration of the steady-state operating point equations. ^ and Δ present the small disturbance.

From t_0 to t_3 with half a switch period, time-domain expressions are as follows:

$$\begin{cases} i_{r0N} = r_0 \sin(\theta_0) \\ v_{cr0N} = -r_0 \cos(\theta_0) + (1-M) \\ i_{r2N} = r_0 \sin(\varphi_0 + \theta_0) = \dfrac{r_1}{\sqrt{1+L_n}} \sin(\theta_1) \\ v_{cr2N} = -r_0 \cos(\varphi_0 + \theta_0) + (1-M) = -r_1 \cos(\theta_1) + 1 \\ i_{r3N} = \dfrac{r_1}{\sqrt{1+L_n}} \sin(\varphi_1 + \theta_1) \\ v_{cr3N} = -r_1 \cos(\varphi_1 + \theta_1) + 1 \\ i_{s1}(t_2) = nI_n\left(r_0 \sin(\varphi_0 + \theta_0) - r_0 \sin(\theta_0) - \dfrac{M}{L_n}\varphi_0\right) = 0 \\ \bar{i}_{s1} = \dfrac{nI_n}{\omega_{r0} T_s}\left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0 + \theta_0) - r_0 \sin(\theta_0)\varphi_0 - \dfrac{M}{2L_n}\varphi_0^2\right) \end{cases} \tag{37}$$

At time t_0, the converter starts to operates in the P mode. θ_0 and r_0 can be calculated by

$$\theta_0 = \arctan\left(-\frac{i_{r0N}}{v_{cr0N} - (1-M)}\right) \qquad r_0 = \sqrt{i_{r0N}^2 + \left[v_{cr0N} - (1-M)\right]^2} \tag{38}$$

The first-order linearization of θ_0 and r_0 is expressed in the following.

$$\theta_0+\Delta\theta_0=\theta_0+\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o$$
$$=\theta_0-\frac{v_{cr0N}-(1-M)}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{v}_{cr0N}-\frac{i_{r0N}M/v_{in}}{r_0{}^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0{}^2}\hat{v}_o$$
$$=\theta_0-\frac{v_{cr0N}-(1-M)}{r_0{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{r_0{}^2}\hat{v}_{cr0N}-\frac{i_{r0N}M/v_{in}}{r_0{}^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0{}^2}\hat{v}_o$$
$$=\theta_0+g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{cr0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o$$

where

$$g_{0i}=\frac{\partial\theta_0}{\partial i_{r0N}}=-\frac{v_{cr0N}-(1-M)}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2},\; g_{0v}=\frac{\partial\theta_0}{\partial v_{cr0N}}=\frac{i_{r0N}}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2},$$
$$g_{0in}=\frac{\partial\theta_0}{\partial v_{in}}=-\frac{i_{r0N}M/v_{in}}{\left[v_{cr0N}-(1-M)\right]^2+I_{r0N}{}^2},\; g_{0o}=\frac{\partial\theta_0}{\partial v_o}=\frac{ni_{r0N}/v_{in}}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2}.$$

$$r_0+\Delta r_0=r_0+\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o$$
$$=r_0+\frac{i_{r0N}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{i}_{r0N}+\frac{v_{cr0N}-(1-M)}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{v}_{cr0N}$$
$$-\frac{\left[v_{cr0N}-(1-M)\right]M/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{v}_{in}+\frac{\left[v_{cr0N}-(1-M)\right]n/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{v}_o$$
$$=r_0+\frac{i_{r0N}}{r_0}\hat{i}_{r0N}+\frac{v_{cr0N}-(1-M)}{r_0}\hat{v}_{cr0N}-\frac{\left[v_{cr0N}-(1-M)\right]M/v_{in}}{r_0}\hat{v}_{in}+\frac{\left[v_{cr0N}-(1-M)\right]n/v_{in}}{r_0}\hat{v}_o$$
$$=r_0+h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{cr0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o \tag{39}$$

where

$$h_{0i}=\frac{\partial\theta_0}{\partial i_{r0N}}=\frac{i_{r0N}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}},\quad h_{0v}=\frac{\partial\theta_0}{\partial v_{cr0N}}=\frac{v_{cr0N}-(1-M)}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}},$$
$$h_{0in}=\frac{\partial\theta_0}{\partial v_{in}}=-\frac{\left[v_{cr0N}-(1-M)\right]M/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}},\quad h_{0o}=\frac{\partial\theta_0}{\partial v_o}=\frac{\left[v_{cr0N}-(1-M)\right]n/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}$$

At time t_2, $i_{s1N}(t_2)$=0, and $i_{s1N}(t_2+\Delta t_2)$=0 after the disturbances are added. (75) can be obtained.

$$
\begin{aligned}
& i_{s1N}\left(t_2+\Delta t_2\right)=n\left(\left(r_0+\Delta r_0\right)\sin\left(\varphi_0+\Delta\varphi_0+\theta_0+\Delta\theta_0\right)-\left(r_0+\Delta r_0\right)\sin\left(\theta_0+\Delta\theta_0\right)-\frac{n\left(v_o+\Delta v_o\right)}{\left(v_{in}+\Delta v_{in}\right)L_n}\left(\varphi_0+\Delta\varphi_0\right)\right) \\
& \approx n\left(r_0\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)-\frac{M}{L_n}\varphi_0\right)+\frac{\partial i_{s1N}\left(t_2\right)}{\partial r_0}\Delta r_0+\frac{\partial i_{s1N}\left(t_2\right)}{\partial\varphi_0}\Delta\varphi_0+\frac{\partial i_{s1N}\left(t_2\right)}{\partial\theta_0}\Delta\theta_0+\frac{\partial i_{s1N}\left(t_2\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{s1N}\left(t_2\right)}{\partial v_o}\hat{v}_o \\
& =i_{s1N}\left(t_2\right)+\frac{\partial i_{s1N}\left(t_2\right)}{\partial r_0}\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{s1N}\left(t_2\right)}{\partial\varphi_0}\Delta\varphi_0 \\
& +\frac{\partial i_{s1N}\left(t_2\right)}{\partial\theta_0}\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{s1N}\left(t_2\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{s1N}\left(t_2\right)}{\partial v_o}\hat{v}_o \\
& =i_{s1N}\left(t_2\right)+n\left[\begin{array}{l}\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right)+r_0\cos\left(\varphi_0+\theta_0\right)\Delta\varphi_0-\frac{M}{L_n}\Delta\varphi_0+ \\ r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right)+\frac{\varphi_0 M/v_{in}}{L_n}\hat{v}_{in}-\frac{\varphi_0 n/v_{in}}{L_n}\hat{v}_o\end{array}\right] \\
& =i_{s1N}\left(t_2\right)+n\left[\begin{array}{l}\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{cr0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right)+r_0\cos\left(\varphi_0+\theta_0\right)\Delta\varphi_0+ \\ r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{cr0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right)-\frac{M}{L_n}\Delta\varphi_0+\frac{\varphi_0 M/v_{in}}{L_n}\hat{v}_{in}-\frac{\varphi_0 n/v_{in}}{L_n}\hat{v}_o\end{array}\right] \\
& =i_{s1N}\left(t_2\right)+n\left[\begin{array}{l}\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0i}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0i}\right]\hat{i}_{r0N}+ \\ \left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0v}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0v}\right]\hat{v}_{cr0N}+ \\ \left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0in}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0in}+\frac{\varphi_0 M/v_{in}}{L_n}\right]\hat{v}_{in} \\ +\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0o}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0o}-\frac{\varphi_0 n/v_{in}}{L_n}\right]\hat{v}_o \\ +\left[r_0\cos\left(\varphi_0+\theta_0\right)-\frac{M}{L_n}\right]\Delta\varphi_0\end{array}\right]=0
\end{aligned}
\tag{40}
$$

Therefore, $\Delta\varphi_0$ can be calculated as follows:

$$
\begin{aligned}
& \Delta\varphi_0=\frac{1}{\frac{M}{L_n}-r_0\cos\left(\varphi_0+\theta_0\right)}\left[\begin{array}{l}\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0i}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0i}\right]\hat{i}_{r0N}+ \\ \left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0v}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0v}\right]\hat{v}_{cr0N}+ \\ \left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0in}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0in}+\frac{\varphi_0 M/v_{in}}{L_n}\right]\hat{v}_{in} \\ +\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0o}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0o}-\frac{\varphi_0 n/v_{in}}{L_n}\right]\hat{v}_o\end{array}\right] \\
& =m_{0i}\hat{i}_{r0N}+m_{0v}\hat{v}_{cr0N}+m_{0in}\hat{v}_{in}+m_{0o}\hat{v}_o \\
& \text{where} \\
& m_{0i}=\frac{\partial\varphi_0}{\partial i_{r0N}}=\frac{1}{\frac{M}{L_n}-r_0\cos\left(\varphi_0+\theta_0\right)}\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0i}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0i}\right] \\
& m_{0v}=\frac{\partial\varphi_0}{\partial v_{cr0N}}=\frac{1}{\frac{M}{L_n}-r_0\cos\left(\varphi_0+\theta_0\right)}\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0v}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0v}\right] \\
& m_{0in}=\frac{\partial\varphi_0}{\partial v_{in}}=\frac{1}{\frac{M}{L_n}-r_0\cos\left(\varphi_0+\theta_0\right)}\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0in}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0in}+\frac{\varphi_0 M/v_{in}}{L_n}\right] \\
& m_{0o}=\frac{\partial\varphi_0}{\partial v_o}=\frac{1}{\frac{M}{L_n}-r_0\cos\left(\varphi_0+\theta_0\right)}\left[\left(\sin\left(\varphi_0+\theta_0\right)-\sin\left(\theta_0\right)\right)h_{0o}+r_0\left(\cos\left(\varphi_0+\theta_0\right)-\cos\left(\theta_0\right)\right)g_{0o}-\frac{\varphi_0 n/v_{in}}{L_n}\right]
\end{aligned}
\tag{41}
$$

After $\Delta\theta_0$, Δr_0, and $\Delta\varphi_0$ are known, Δi_{r2N} and Δv_{cr2N} can be calculated as follows:

$$
\begin{aligned}
& i_{r2N}+\Delta i_{r2N}=r_0\sin\left(\varphi_0+\theta_0\right)+\frac{\partial i_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r2N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial i_{r2N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r2N}}{\partial v_o}\hat{v}_o \\
& =i_{r2N}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}+\frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{cr0N}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{cr0N}}+\frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}+\frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}+\frac{\partial i_{r2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_o}\right)\hat{v}_o \\
& =i_{r2N}+\left[\sin\left(\varphi_0+\theta_0\right)h_{0i}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0i}+m_{0i}\right)\right]\hat{i}_{r0N} \\
& +\left[\sin\left(\varphi_0+\theta_0\right)h_{0v}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0v}+m_{0v}\right)\right]\hat{v}_{cr0N} \\
& +\left[\sin\left(\varphi_0+\theta_0\right)h_{0in}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0in}+m_{0in}\right)\right]\hat{v}_{in} \\
& +\left[\sin\left(\varphi_0+\theta_0\right)h_{0o}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0o}+m_{0o}\right)\right]\hat{v}_o \\
& =i_{r2N}+k_{2i}\hat{i}_{r0N}+k_{2v}\hat{v}_{cr0N}+k_{2in}\hat{v}_{in}+k_{2o}\hat{v}_o
\end{aligned}
$$

where

$$
\begin{aligned}
& k_{2i}=\frac{\partial i_{r2N}}{\partial i_{r0N}}=\sin\left(\varphi_0+\theta_0\right)h_{0i}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0i}+m_{0i}\right) \\
& k_{2v}=\frac{\partial i_{r2N}}{\partial v_{cr0N}}=\sin\left(\varphi_0+\theta_0\right)h_{0v}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0v}+m_{0v}\right) \\
& k_{2in}=\frac{\partial i_{r2N}}{\partial v_{in}}=\sin\left(\varphi_0+\theta_0\right)h_{0in}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0in}+m_{0in}\right) \\
& k_{2o}=\frac{\partial i_{r2N}}{\partial v_o}=\sin\left(\varphi_0+\theta_0\right)h_{0o}+r_0\cos\left(\varphi_0+\theta_0\right)\left(g_{0o}+m_{0o}\right)
\end{aligned}
$$

$$
\begin{aligned}
& v_{cr2N}+\Delta v_{cr2N}=-r_0\cos\left(\varphi_0+\theta_0\right)+\left(1-M\right)+\frac{\partial v_{cr2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{cr2N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial v_{cr2N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr2N}}{\partial v_o}\hat{v}_o \\
& =v_{cr2N}+\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}+\frac{\partial v_{cr2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{cr0N}}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{cr0N}}+\frac{\partial v_{cr2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}+\frac{\partial v_{cr2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}+\frac{\partial v_{cr2N}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_o}\right)\hat{v}_o \\
& =v_{cr2N}+\left[-\cos\left(\varphi_0+\theta_0\right)h_{i0}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{i0}+m_{0i}\right)\right]\hat{i}_{r0N} \\
& +\left[-\cos\left(\varphi_0+\theta_0\right)h_{0v}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{0v}+m_{0v}\right)\right]\hat{v}_{cr0N} \\
& +\left[-\cos\left(\varphi_0+\theta_0\right)h_{0in}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{0in}+m_{0in}\right)+\frac{M}{v_{in}}\right]\hat{v}_{in} \\
& +\left[-\cos\left(\varphi_0+\theta_0\right)h_{0o}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{0o}+m_{0o}\right)-\frac{n}{v_{in}}\right]\hat{v}_o \\
& =v_{r2N}+l_{2i}\hat{i}_{r0N}+l_{2v}\hat{v}_{cr0N}+l_{2in}\hat{v}_{in}+l_{2o}\hat{v}_o
\end{aligned}
$$

where

$$
\begin{aligned}
& l_{2i}=\frac{\partial v_{cr2N}}{\partial i_{r0N}}=-\cos\left(\varphi_0+\theta_0\right)h_{i0}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{i0}+m_{0i}\right) \\
& l_{2v}=\frac{\partial v_{cr2N}}{\partial v_{cr0N}}=-\cos\left(\varphi_0+\theta_0\right)h_{0v}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{0v}+m_{0v}\right) \\
& l_{2in}=\frac{\partial v_{cr2N}}{\partial v_{in}}=-\cos\left(\varphi_0+\theta_0\right)h_{0in}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{0in}+m_{0in}\right)+\frac{M}{v_{in}} \\
& l_{2o}=\frac{\partial v_{cr2N}}{\partial v_o}=-\cos\left(\varphi_0+\theta_0\right)h_{0o}+r_0\sin\left(\varphi_0+\theta_0\right)\left(g_{0o}+m_{0o}\right)-\frac{n}{v_{in}}
\end{aligned}
\tag{42}
$$

At time t_2, the converter starts to work in O mode, θ_1 and r_1 can be expressed as follows:

$$
\theta_1=\arctan\left(-\frac{\sqrt{1+L_n}\,i_{r2N}}{v_{cr2N}-1}\right) \qquad r_1=\sqrt{\left(1+L_n\right)i_{r2N}{}^2+\left(v_{cr2N}-1\right)^2} \tag{43}
$$

Therefore, $\Delta\theta_1$ and Δr_1 can be calculated by:

$$
\begin{aligned}
&\theta_1+\Delta\theta_1=\theta_1+\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o \\
&=\theta_1+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{cr0N}}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
&+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_o}\right)\hat{v}_o \\
&=\theta_1+\left[-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2i}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2i}\right]\hat{i}_{r0N}+\left[-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2v}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2v}\right]\hat{v}_{cr0N} \\
&+\left[-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2in}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2in}\right]\hat{v}_{in}+\left[-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2o}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2o}\right]\hat{v}_o \\
&=\theta_1+g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o \\
&\text{where} \\
&g_{1i}=\frac{\partial\theta_1}{\partial i_{r0N}}=-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2i}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2i} \\
&g_{1v}=\frac{\partial\theta_1}{\partial v_{cr0N}}=-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2v}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2v} \\
&g_{1in}=\frac{\partial\theta_1}{\partial v_{in}}=-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2in}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2in} \\
&g_{1o}=\frac{\partial\theta_1}{\partial v_o}=-\frac{\sqrt{1+L_n}\left(v_{cr2N}-1\right)}{r_1^2}k_{2o}+\frac{\sqrt{1+L_n}i_{r2N}}{r_1^2}l_{2o} \\
\\
&r_1+\Delta r_1=r_1+\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o \\
&=r_1+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{cr0N}}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
&+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_o}\right)\hat{v}_o \\
&=r_1+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2i}+\frac{v_{cr2N}-1}{r_1}l_{2i}\right)\hat{i}_{r0N}+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2v}+\frac{v_{cr2N}-1}{r_1}l_{2v}\right)\hat{v}_{cr0N} \\
&+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2in}+\frac{v_{cr2N}-1}{r_1}l_{2in}\right)\hat{v}_{in}+\left(\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2o}+\frac{v_{cr2N}-1}{r_1}l_{2o}\right)\hat{v}_o \\
&=r_1+h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o \\
&\text{where} \\
&h_{1i}=\frac{\partial r_1}{\partial i_{r0N}}=\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2i}+\frac{v_{cr2N}-1}{r_1}l_{2i} \\
&h_{1v}=\frac{\partial r_1}{\partial v_{cr0N}}=\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2v}+\frac{v_{cr2N}-1}{r_1}l_{2v} \\
&h_{1in}=\frac{\partial r_1}{\partial v_{in}}=\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2in}+\frac{v_{cr2N}-1}{r_1}l_{2in} \\
&h_{1o}=\frac{\partial r_1}{\partial v_o}=\frac{\left(1+L_n\right)i_{r2N}}{r_1}k_{2o}+\frac{v_{cr2N}-1}{r_1}l_{2o}
\end{aligned}
\tag{44}
$$

$\Delta\varphi_1$ can be calculated by

$$\varphi_1 = \frac{\omega_{r1} t_s}{2} - \frac{\omega_{r1}}{\omega_{r0}} \varphi_0$$

$$\begin{aligned}\Delta\varphi_1 &= \frac{\omega_{r1}}{2}\hat{t}_s - \frac{\omega_{r1}}{\omega_{r0}}\Delta\varphi_0 = \frac{\omega_{r1}}{2}\hat{t}_s - \frac{\omega_{r1}}{\omega_{r0}} m_{0i}\hat{i}_{r0N} - \frac{\omega_{r1}}{\omega_{r0}} m_{0v}\hat{v}_{cr0N} - \frac{\omega_{r1}}{\omega_{r0}} m_{0in}\hat{v}_{in} - \frac{\omega_{r1}}{\omega_{r0}} m_{0o}\hat{v}_o \\ &= m_{1t}\hat{t}_s + m_{1i}\hat{i}_{r0N} + m_{1v}\hat{v}_{cr0N} + m_{1in}\hat{v}_{in} + m_{1o}\hat{v}_o \end{aligned} \tag{45}$$

where

$$m_{1t} = \frac{\partial\varphi_1}{\partial t_s} = \frac{\omega_{r1}}{2},\ m_{1i} = \frac{\partial\varphi_1}{\partial i_{r0N}} = -\frac{\omega_{r1}}{\omega_{r0}} m_{0i},\ m_{1v} = \frac{\partial\varphi_1}{\partial v_{cr0N}} = -\frac{\omega_{r1}}{\omega_{r0}} m_{0v},$$

$$m_{1in} = \frac{\partial\varphi_1}{\partial v_{in}} = -\frac{\omega_{r1}}{\omega_{r0}} m_{0in},\ m_{1o} = \frac{\partial\varphi_1}{\partial v_o} = -\frac{\omega_{r1}}{\omega_{r0}} m_{0o}$$

After $\Delta\theta_1$, Δr_1, and $\Delta\varphi_1$ are known, Δi_{r3N} and Δv_{cr3N} can be calculated as follows:

$$\begin{aligned}
i_{r3N} + \Delta i_{r3N} &= \frac{r_1}{\sqrt{1+L_n}}\sin(\varphi_1+\theta_1) + \frac{\partial i_{r3N}}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial i_{r3N}}{\partial v_{cr0N}}\hat{v}_{cr0N} + \frac{\partial i_{r3N}}{\partial v_{in}}\hat{v}_{in} + \frac{\partial i_{r3N}}{\partial v_o}\hat{v}_o + \frac{\partial i_{r3N}}{\partial t_s}\hat{t}_s \\
&= i_{r3N} + \left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial i_{r0N}} + \frac{\partial i_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial i_{r0N}} + \frac{\partial i_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial i_{r0N}}\right)\hat{i}_{r0N} + \left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{cr0N}} + \frac{\partial i_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{cr0N}} + \frac{\partial i_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
&+ \left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{in}} + \frac{\partial i_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{in}} + \frac{\partial i_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{in}}\right)\hat{v}_{in} + \left(\frac{\partial i_{r3N}}{\partial r_1}\frac{\partial r_1}{\partial v_o} + \frac{\partial i_{r3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_o} + \frac{\partial i_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_o}\right)\hat{v}_o + \frac{\partial i_{r3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial t_s}\hat{t}_s \\
&= i_{r3N} + \left(\frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1i} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1i}+m_{1i})\right)\hat{i}_{r0N} + \left(\frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1v} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1v}+m_{1v})\right)\hat{v}_{cr0N} \\
&+ \left(\frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1in} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1in}+m_{1in})\right)\hat{v}_{in} + \left(\frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1o} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1o}+m_{1o})\right)\hat{v}_o \\
&+ \frac{r_1}{\sqrt{1+L_n}}\cos(\varphi_1+\theta_1)m_{1t}\hat{t}_s \\
&= i_{r3N} + k_{3i}\hat{i}_{r0N} + k_{3v}\hat{v}_{cr0N} + k_{3in}\hat{v}_{in} + k_{3o}\hat{v}_o + k_{3t}\hat{t}_s
\end{aligned}$$

where

$$k_{3i} = \frac{\partial i_{r3N}}{\partial i_{r0N}} = \frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1i} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1i}+m_{1i})$$

$$k_{3v} = \frac{\partial i_{r3N}}{\partial v_{cr0N}} = \frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1v} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1v}+m_{1v})$$

$$k_{3in} = \frac{\partial i_{r3N}}{\partial v_{in}} = \frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1in} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1in}+m_{1in})$$

$$k_{3o} = \frac{\partial i_{r3N}}{\partial v_o} = \frac{\sin(\varphi_1+\theta_1)}{\sqrt{1+L_n}}h_{1o} + \frac{r_1\cos(\varphi_1+\theta_1)}{\sqrt{1+L_n}}(g_{1o}+m_{1o})$$

$$k_{3t} = \frac{\partial i_{r3N}}{\partial t_s} = \frac{r_1}{\sqrt{1+L_n}}\cos(\varphi_1+\theta_1)m_{1t}$$

$$
\begin{aligned}
&v_{cr3N}+\Delta v_{cr3N}=\left(-r_1\cos\left(\varphi_1+\theta_1\right)+1\right)+\frac{\partial v_{r3N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{cr3N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial v_{cr3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr3N}}{\partial v_{o}}\hat{v}_{o}+\frac{\partial v_{cr3N}}{\partial t_{s}}\hat{t}_{s}\\
&=v_{cr3N}+\left(\frac{\partial v_{cr3N}}{\partial r_1}\frac{\partial r_1}{\partial i_{r0N}}+\frac{\partial v_{cr3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial i_{r0N}}+\frac{\partial v_{cr3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{cr3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{cr0N}}+\frac{\partial v_{cr3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{cr0N}}+\frac{\partial v_{cr3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}\\
&+\left(\frac{\partial v_{cr3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{in}}+\frac{\partial v_{cr3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{in}}+\frac{\partial v_{cr3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{cr3N}}{\partial r_1}\frac{\partial r_1}{\partial v_{o}}+\frac{\partial v_{cr3N}}{\partial \theta_1}\frac{\partial \theta_1}{\partial v_{o}}+\frac{\partial v_{cr3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial v_{o}}\right)\hat{v}_{o}+\frac{\partial v_{cr3N}}{\partial \varphi_1}\frac{\partial \varphi_1}{\partial t_s}\hat{t}_s\\
&=v_{cr3N}+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1i}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}+m_{1i}\right)\right)\hat{i}_{r0N}+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1v}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1v}+m_{1v}\right)\right)\hat{v}_{cr0N}\\
&+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1in}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1in}+m_{1in}\right)\right)\hat{v}_{in}\\
&+\left(-\cos\left(\varphi_1+\theta_1\right)h_{1o}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1o}+m_{1o}\right)\right)\hat{v}_{o}+r_1\sin\left(\varphi_1+\theta_1\right)m_{1t}\hat{t}_s\\
&=v_{r3N}+l_{3i}\hat{i}_{r0N}+l_{3v}\hat{v}_{cr0N}+l_{3in}\hat{v}_{in}+l_{3o}\hat{v}_{o}+l_{3t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
&l_{3i}=\frac{\partial v_{cr3N}}{\partial i_{r0N}}=-\cos\left(\varphi_1+\theta_1\right)h_{1i}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}+m_{1i}\right)\\
&l_{3v}=\frac{\partial v_{cr3N}}{\partial v_{cr0N}}=-\cos\left(\varphi_1+\theta_1\right)h_{1v}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1v}+m_{1v}\right)\\
&l_{3in}=\frac{\partial v_{cr3N}}{\partial v_{in}}=-\cos\left(\varphi_1+\theta_1\right)h_{1in}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1in}+m_{1in}\right)\\
&l_{3o}=\frac{\partial v_{cr3N}}{\partial v_{o}}=-\cos\left(\varphi_1+\theta_1\right)h_{1o}+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1o}+m_{1o}\right)\\
&l_{3t}=\frac{\partial v_{cr3N}}{\partial t_{s}}=r_1\sin\left(\varphi_1+\theta_1\right)
\end{aligned}
\tag{46}
$$

The average output current of the rectifier from t_0 to t_3 can be expressed as

$$
\begin{aligned}
&\bar{i}_{s1}+\Delta\bar{i}_{s1}=\frac{nI_n}{\omega_{r0}t_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\frac{M}{2L_n}\varphi_0^{\ 2}\right)+\frac{\partial\bar{i}_{s1}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\bar{i}_{s1}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\bar{i}_{s1}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\bar{i}_{s1}}{\partial v_{o}}\hat{v}_{o}+\frac{\partial\bar{i}_{s1}}{\partial t_{s}}\hat{t}_{s}\\
&=\bar{i}_{s1}+\left(\frac{\partial\bar{i}_{s1}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial\bar{i}_{s1}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}+\frac{\partial\bar{i}_{s1}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\bar{i}_{s1}}{\partial r_0}\frac{\partial r_0}{\partial v_{cr0N}}+\frac{\partial\bar{i}_{s1}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{cr0N}}+\frac{\partial\bar{i}_{s1}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}+\\
&\left(\frac{\partial\bar{i}_{s1}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial\bar{i}_{s1}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}+\frac{\partial\bar{i}_{s1}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{in}}+\frac{nI_n}{\omega_{r0}t_s}\frac{M}{2v_{in}L_n}\varphi_0^{\ 2}\right)\hat{v}_{in}+\frac{nC_r}{t_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\frac{M}{2L_n}\varphi_0^{\ 2}\right)\hat{v}_{in}\\
&+\left(\frac{\partial\bar{i}_{s1}}{\partial r_0}\frac{\partial r_0}{\partial v_{o}}+\frac{\partial\bar{i}_{s1}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{o}}+\frac{\partial\bar{i}_{s1}}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial v_{o}}-\frac{nI_n}{\omega_{r0}t_s}\frac{n}{2v_{in}L_n}\varphi_0^{\ 2}\right)\hat{v}_{o}+\frac{\partial\bar{i}_{s1}}{\partial t_s}\hat{t}_s\\
&=\bar{i}_{s1}+\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0i}+\\ \left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0i}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0i}\end{array}\right]\hat{i}_{r0N}+\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0v}+\\ \left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0v}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0v}\end{array}\right]\hat{v}_{cr0N}\\
&+\left\{\begin{array}{l}\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0in}+\\ \left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0in}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0in}+\frac{M}{2v_{in}L_n}\varphi_0^{\ 2}\end{array}\right]\\ +\frac{nC_r}{T_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\frac{M}{2L_n}\varphi_0^{\ 2}\right)\end{array}\right\}\hat{v}_{in}+\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0o}+\\ \left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0o}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0o}-\frac{n}{2v_{in}L_n}\varphi_0^{\ 2}\end{array}\right]\hat{v}_{o}\\
&-\frac{nI_n}{\omega_{r0}T_s^{\ 2}}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\frac{M}{2L_n}\varphi_0^{\ 2}\right)\hat{t}_s\\
&=\bar{i}_{s1}+k_{s1i}\hat{i}_{r0N}+k_{s1v}\hat{v}_{cr0N}+k_{s1in}\hat{v}_{in}+k_{s1o}\hat{v}_{o}+k_{s1t}\hat{t}_{s}
\end{aligned}
$$

where

$$
\begin{aligned}
&k_{s1i}=\frac{\partial\bar{i}_{s1}}{\partial i_{r0N}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0i}+\left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0i}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0i}\end{array}\right]\\
&k_{s1v}=\frac{\partial\bar{i}_{s1}}{\partial v_{cr0N}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0v}+\left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0v}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0v}\end{array}\right]\\
&k_{s1in}=\frac{\partial\bar{i}_{s1}}{\partial v_{in}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0in}+\\ \left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0in}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0in}+\frac{M}{2v_{in}L_n}\varphi_0^{\ 2}\end{array}\right]+\frac{nC_r}{T_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\frac{M}{2L_n}\varphi_0^{\ 2}\right)\\
&k_{s1o}=\frac{\partial\bar{i}_{s1}}{\partial v_{o}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}\left(\cos(\theta_0)-\cos(\varphi_0+\theta_0)-\sin(\theta_0)\varphi_0\right)h_{0o}+\left(-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)-r_0\cos(\theta_0)\varphi_0\right)g_{0o}\\ +\left(r_0\sin(\varphi_0+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\varphi_0\right)m_{0o}-\frac{n}{2v_{in}L_n}\varphi_0^{\ 2}\end{array}\right]\\
&k_{s1t}=-\frac{nI_n}{\omega_{r0}T_s^{\ 2}}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)-r_0\sin(\theta_0)\varphi_0-\frac{M}{2L_n}\varphi_0^{\ 2}\right)
\end{aligned}
\tag{47}
$$

From t_3 to t_6

From t_3 to t_6 with half a switch period, time-domain expressions are as follows:

$$\begin{cases} i_{r3N} = r_2 \sin\left(\theta_2\right) \\ v_{cr3N} = -r_2 \cos\left(\theta_2\right) - \left(1-M\right) \\ i_{r5N} = r_2 \sin\left(\varphi_2 + \theta_2\right) = \dfrac{r_3}{\sqrt{1+L_n}} \sin\left(\theta_3\right) \\ v_{cr5N} = -r_2 \cos\left(\varphi_2 + \theta_2\right) - \left(1-M\right) = -r_3 \cos\left(\theta_3\right) - 1 \\ i_{r6N} = \dfrac{r_3}{\sqrt{1+L_n}} \sin\left(\varphi_3 + \theta_3\right) \\ v_{cr6N} = -r_3 \cos\left(\varphi_3 + \theta_3\right) - 1 \\ i_{s2}\left(t_5\right) = nI_n \left(r_2 \sin\left(\varphi_2 + \theta_2\right) - r_2 \sin\left(\theta_2\right) + \dfrac{M}{L_n}\varphi_2 \right) = 0 \\ \overline{i}_{s2} = \dfrac{nI_n}{\omega_{r0} T_s} \left(r_2 \cos\left(\theta_2\right) - r_2 \cos\left(\varphi_2 + \theta_2\right) - \sin\left(\theta_2\right)\varphi_2 + \dfrac{M}{2L_n}{\varphi_2}^2 \right) \end{cases} \tag{48}$$

At time t_3, the converter starts to operates in the P mode. θ_2 and r_2 can be expressed as follows:

$$\theta_2 = \pi + \arctan\left(-\frac{i_{r3N}}{v_{cr3N} + \left(1-M\right)} \right), \quad r_2 = \sqrt{{i_{r3N}}^2 + \left[v_{cr3N} + \left(1-M\right) \right]^2} \tag{49}$$

The first-order linearization of θ_2 and r_2 is shown in the following.

$$\begin{aligned}
&\theta_2 + \Delta\theta_2 = \pi + \arctan\left(-\frac{i_{r3N}}{v_{cr3N} + \left(1-M\right)} \right) + \frac{\partial\theta_2}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial\theta_2}{\partial v_{cr0N}}\hat{v}_{cr0N} + \frac{\partial\theta_2}{\partial v_{in}}\hat{v}_{in} + \frac{\partial\theta_2}{\partial v_o}\hat{v}_o + \frac{\partial\theta_2}{\partial t_s}\hat{t}_s \\
&= \theta_2 + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}} + \frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial i_{r0N}} \right)\hat{i}_{r0N} + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{cr0N}} + \frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{cr0N}} \right)\hat{v}_{cr0N} + \\
&\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}} + \frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{in}} - \frac{i_{r3N}M/v_{in}}{{r_2}^2} \right)\hat{v}_{in} + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o} + \frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_o} + \frac{i_{r3N}n/v_{in}}{{r_2}^2} \right)\hat{v}_o + \left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s} + \frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial t_s} \right)\hat{t}_s \\
&= \theta_2 + \left[-\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3i} + \frac{i_{r3N}}{{r_2}^2}l_{3i} \right]\hat{i}_{r0N} + \left[-\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3v} + \frac{i_{r3N}}{{r_2}^2}l_{3v} \right]\hat{v}_{cr0N} \\
&+ \left[-\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3in} + \frac{i_{r3N}}{{r_2}^2}l_{3in} + \frac{i_{r3N}M/v_{in}}{{r_2}^2} \right]\hat{v}_{in} + \left[-\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3o} + \frac{i_{r3N}}{{r_2}^2}l_{3o} - \frac{ni_{r3N}/v_{in}}{{r_2}^2} \right]\hat{v}_o \\
&+ \left[-\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3t} + \frac{i_{r3N}}{{r_2}^2}l_{3t} \right]\hat{t}_s \\
&= \theta_2 + g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{cr0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s
\end{aligned}$$

where

$$g_{2i} = \frac{\partial\theta_2}{\partial i_{r0N}} = -\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3i} + \frac{i_{r3N}}{{r_2}^2}l_{3i}$$

$$g_{2v} = \frac{\partial\theta_2}{\partial v_{cr0N}} = -\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3v} + \frac{i_{r3N}}{{r_2}^2}l_{3v}$$

$$g_{2in} = \frac{\partial\theta_2}{\partial v_{in}} = -\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3in} + \frac{i_{r3N}}{{r_2}^2}l_{3in} + \frac{i_{r3N}M/v_{in}}{{r_2}^2}$$

$$g_{2o} = \frac{\partial\theta_2}{\partial v_o} = -\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3o} + \frac{i_{r3N}}{{r_2}^2}l_{3o} - \frac{ni_{r3N}/v_{in}}{{r_2}^2}$$

$$g_{2t} = \frac{\partial\theta_2}{\partial t_s} = -\frac{v_{cr3N} + \left(1-M\right)}{{r_2}^2}k_{3t} + \frac{i_{r3N}}{{r_2}^2}l_{3t}$$

$$
\begin{aligned}
r_2+\Delta r_2 &= \sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}+\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s \\
&= r_2+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{cr0N}}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{in}}+\frac{\left[v_{cr3N}+\left(1-M\right)\right]M/v_{in}}{r_1}\right)\hat{v}_{in} \\
&+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_o}-\frac{\left[v_{cr3N}+\left(1-M\right)\right]n/v_{in}}{r_1}\right)\hat{v}_o+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial t_s}\right)\hat{t}_s \\
&= r_2+\left(\frac{i_{r3N}}{r_2}k_{3i}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3i}\right)\hat{i}_{r0N}+\left(\frac{i_{r3N}}{r_2}k_{3v}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3v}\right)\hat{v}_{cr0N} \\
&+\left(\frac{i_{r3N}}{r_2}k_{3in}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3in}+\frac{\left[v_{cr3N}+\left(1-M\right)\right]M/v_{in}}{r_2}\right)\hat{v}_{in}+\left(\frac{i_{r3N}}{r_2}k_{3o}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3o}-\frac{\left[v_{cr3N}+\left(1-M\right)\right]n/v_{in}}{r_2}\right)\hat{v}_o \\
&+\left(\frac{i_{r3N}}{r_2}k_{3t}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3t}\right)\hat{t}_s \\
&= r_2+h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{cr0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
h_{2i} &= \frac{\partial r_2}{\partial i_{r0N}}=\frac{i_{r3N}}{r_2}k_{3i}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3i} \\
h_{2v} &= \frac{\partial r_2}{\partial v_{cr0N}}=\frac{i_{r3N}}{r_2}k_{3v}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3v} \\
h_{2in} &= \frac{\partial r_2}{\partial v_{in}}=\frac{i_{r3N}}{r_2}k_{3in}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3in}+\frac{\left[v_{cr3N}+\left(1-M\right)\right]M/v_{in}}{r_2} \\
h_{2o} &= \frac{\partial r_2}{\partial v_o}=\frac{i_{r3N}}{r_2}k_{3o}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3o}-\frac{\left[v_{cr3N}+\left(1-M\right)\right]n/v_{in}}{r_2} \\
h_{2t} &= \frac{\partial r_2}{\partial t_s}=\frac{i_{r3N}}{r_2}k_{3t}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3t}
\end{aligned}
\tag{50}
$$

At time t_5, $i_{s2N}(t_5)$=0, and $i_{s2N}(t_5+\Delta t_5)$=0 after the disturbances are added. The following equation can be obtained.

$$
\begin{aligned}
& i_{s2N}\left(t_5+\Delta t_5\right)=n\left(\left(r_2+\Delta r_2\right)\sin\left(\varphi_2+\Delta\varphi_2+\theta_2+\Delta\theta_2\right)-\left(r_2+\Delta r_2\right)\sin\left(\theta_2+\Delta\theta_2\right)+\frac{n\left(v_o+\Delta v_o\right)}{\left(v_{in}+\Delta v_{in}\right)L_n}\left(\varphi_2+\Delta\varphi_2\right)\right) \\
& \approx n\left(r_2\sin\left(\varphi_2+\theta_2\right)-r_2\sin\left(\theta_2\right)+\frac{M}{L_n}\varphi_2\right)+\frac{\partial i_{s2N}\left(t_5\right)}{\partial r_2}\Delta r_2+\frac{\partial i_{s2N}\left(t_2\right)}{\partial\varphi_2}\Delta\varphi_2+\frac{\partial i_{s2N}\left(t_2\right)}{\partial\theta_2}\Delta\theta_2+\frac{\partial i_{s2N}\left(t_2\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{s2N}\left(t_2\right)}{\partial v_o}\hat{v}_o \\
& =i_{s2N}\left(t_5\right)+n\left[\begin{array}{l}\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)\Delta r_2+\left(r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}\right)\Delta\varphi_2 \\ +\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)\hat{\theta}_2-\frac{M}{v_{in}L_n}\varphi_2\hat{v}_{in}+\frac{n/v_{in}}{L_n}\varphi_2\hat{v}_o\end{array}\right] \\
& =i_{s2N}\left(t_5\right)+n\left[\begin{array}{l}\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)\left(h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{cr0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s\right)+\left(r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}\right)\Delta\varphi_2 \\ +\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)\left(g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{cr0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s\right)-\frac{M}{v_{in}L_n}\varphi_2\hat{v}_{in}+\frac{n/v_{in}}{L_n}\varphi_2\hat{v}_o\end{array}\right] \\
& =i_{s2N}\left(t_5\right)+n\left[\begin{array}{l}\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2i}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2i}\right]\hat{i}_{r0N}+ \\ \left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2v}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2v}\right]\hat{v}_{cr0N}+ \\ \left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2in}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2in}-\frac{M}{v_{in}L_n}\varphi_2\right]\hat{v}_{in} \\ +\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2o}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2o}+\frac{n/v_{in}}{L_n}\varphi_2\right]\hat{v}_o \\ +\left[\left(\sin\left(\varphi_2+\theta_2\right)-\sin\left(\theta_2\right)\right)h_{2t}+\left(r_2\cos\left(\varphi_2+\theta_2\right)-r_2\cos\left(\theta_2\right)\right)g_{2t}\right]\hat{t}_s \\ +\left(r_2\cos\left(\varphi_2+\theta_2\right)+\frac{M}{L_n}\right)\Delta\varphi_2\end{array}\right]=0
\end{aligned}
\tag{51}
$$

Therefore, $\Delta\varphi_2$ can be calculated as follows:

$$\Delta\varphi_2=\frac{-1}{r_2\cos(\varphi_2+\theta_2)+\frac{M}{L_n}}\begin{bmatrix}\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2i}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2i}\right]\hat{i}_{r0N}+\\ \left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2v}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2v}\right]\hat{v}_{cr0N}+\\ \left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2in}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2in}-\frac{M}{v_{in}L_n}\varphi_2\right]\hat{v}_{in}\\ +\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2o}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2o}+\frac{n/v_{in}}{L_n}\varphi_2\right]\hat{v}_o\\ +\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2t}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2t}\right]\hat{t}_s\end{bmatrix}$$

$$=m_{2i}\hat{i}_{r0N}+m_{2v}\hat{v}_{cr0N}+m_{2in}\hat{v}_{in}+m_{2o}\hat{v}_o+m_{2t}\hat{t}_s$$

where

$$\begin{aligned}
m_{2i}&=\frac{\partial\varphi_2}{\partial i_{r0N}}=\frac{-1}{r_2\cos(\varphi_2+\theta_2)+\frac{M}{L_n}}\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2i}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2i}\right]\\
m_{2v}&=\frac{\partial\varphi_2}{\partial v_{cr0N}}=\frac{-1}{r_2\cos(\varphi_2+\theta_2)+\frac{M}{L_n}}\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2v}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2v}\right]\\
m_{2in}&=\frac{\partial\varphi_2}{\partial v_{in}}=\frac{-1}{r_2\cos(\varphi_2+\theta_2)+\frac{M}{L_n}}\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2in}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2in}-\frac{M}{v_{in}L_n}\varphi_2\right]\\
m_{2o}&=\frac{\partial\varphi_2}{\partial v_o}=\frac{-1}{r_2\cos(\varphi_2+\theta_2)+\frac{M}{L_n}}\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2o}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2o}+\frac{n/v_{in}}{L_n}\varphi_2\right]\\
m_{2t}&=\frac{\partial\varphi_2}{\partial t_s}=\frac{-1}{r_2\cos(\varphi_2+\theta_2)+\frac{M}{L_n}}\left[\left(\sin(\varphi_2+\theta_2)-\sin(\theta_2)\right)h_{2t}+\left(r_2\cos(\varphi_2+\theta_2)-r_2\cos(\theta_2)\right)g_{2t}\right]
\end{aligned}\tag{52}$$

After $\Delta\theta_2$, Δr_2, and $\Delta\varphi_2$ are known, Δi_{r5N} and Δv_{cr5N} can be calculated as follows:

$$
\begin{aligned}
i_{r5N} + \Delta i_{r5N} &= r_2 \sin\left(\varphi_2 + \theta_2\right) + \frac{\partial i_{r5N}}{\partial i_{r0N}} \hat{i}_{r0N} + \frac{\partial i_{r5N}}{\partial v_{cr0N}} \hat{v}_{cr0N} + \frac{\partial i_{r5N}}{\partial v_{in}} \hat{v}_{in} + \frac{\partial i_{r5N}}{\partial v_o} \hat{v}_o + \frac{\partial i_{r5N}}{\partial t_s} \hat{t}_s \\
&= i_{r5N} + \left(\frac{\partial i_{r5N}}{\partial r_2} \frac{\partial r_2}{\partial i_{r0N}} + \frac{\partial i_{r5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial i_{r0N}} + \frac{\partial i_{r5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial i_{r0N}} \right) \hat{i}_{r0N} + \left(\frac{\partial i_{r5N}}{\partial r_2} \frac{\partial r_2}{\partial v_{cr0N}} + \frac{\partial i_{r5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial v_{cr0N}} + \frac{\partial i_{r5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial v_{cr0N}} \right) \hat{v}_{cr0N} \\
&+ \left(\frac{\partial i_{r5N}}{\partial r_2} \frac{\partial r_2}{\partial v_{in}} + \frac{\partial i_{r5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial v_{in}} + \frac{\partial i_{r5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial v_{in}} \right) \hat{v}_{in} + \left(\frac{\partial i_{r5N}}{\partial r_2} \frac{\partial r_2}{\partial v_o} + \frac{\partial i_{r5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial v_o} + \frac{\partial i_{r5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial v_o} \right) \hat{v}_o + \left(\frac{\partial i_{r5N}}{\partial r_2} \frac{\partial r_2}{\partial t_s} + \frac{\partial i_{r5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial t_s} + \frac{\partial i_{r5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial t_s} \right) \hat{t}_s \\
&= i_{r5N} + \left(\sin\left(\varphi_2 + \theta_2\right) h_{2i} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2i} + m_{2i}\right) \right) \hat{i}_{r0N} + \left(\sin\left(\varphi_2 + \theta_2\right) h_{2v} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2v} + m_{2v}\right) \right) \hat{v}_{cr0N} \\
&+ \left(\sin\left(\varphi_2 + \theta_2\right) h_{2in} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2in} + m_{2in}\right) \right) \hat{v}_{in} + \left(\sin\left(\varphi_2 + \theta_2\right) h_{2o} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2o} + m_{2o}\right) \right) \hat{v}_o \\
&+ \left(\sin\left(\varphi_2 + \theta_2\right) h_{2t} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2t} + m_{2t}\right) \right) \hat{t}_s \\
&= i_{r5N} + k_{5i} \hat{i}_{r0N} + k_{5v} \hat{v}_{cr0N} + k_{5in} \hat{v}_{in} + k_{5o} \hat{v}_o + k_{5t} \hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
k_{5i} &= \frac{\partial i_{r5N}}{\partial i_{r0N}} = \sin\left(\varphi_2 + \theta_2\right) h_{2i} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2i} + m_{2i}\right) \\
k_{5v} &= \frac{\partial i_{r5N}}{\partial v_{cr0N}} = \sin\left(\varphi_2 + \theta_2\right) h_{2v} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2v} + m_{2v}\right) \\
k_{5in} &= \frac{\partial i_{r5N}}{\partial v_{in}} = \sin\left(\varphi_2 + \theta_2\right) h_{2in} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2in} + m_{2in}\right) \\
k_{5o} &= \frac{\partial i_{r5N}}{\partial v_o} = \sin\left(\varphi_2 + \theta_2\right) h_{2o} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2o} + m_{2o}\right) \\
k_{5t} &= \frac{\partial i_{r5N}}{\partial t_s} = \sin\left(\varphi_2 + \theta_2\right) h_{2t} + r_2 \cos\left(\varphi_2 + \theta_2\right)\left(g_{2t} + m_{2t}\right)
\end{aligned}
$$

$$
\begin{aligned}
v_{cr5N} + \Delta v_{cr5N} &= -r_2 \cos\left(\varphi_2 + \theta_2\right) - \left(1 - M\right) + \frac{\partial v_{cr5N}}{\partial i_{r0N}} \hat{i}_{r0N} + \frac{\partial v_{cr5N}}{\partial v_{cr0N}} \hat{v}_{cr0N} + \frac{\partial v_{cr5N}}{\partial v_{in}} \hat{v}_{in} + \frac{\partial v_{cr5N}}{\partial v_o} \hat{v}_o + \frac{\partial v_{cr5N}}{\partial t_s} \hat{t}_s \\
&= v_{cr5N} + \left(\frac{\partial v_{cr5N}}{\partial r_2} \frac{\partial r_2}{\partial i_{r0N}} + \frac{\partial v_{cr5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial i_{r0N}} + \frac{\partial v_{cr5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial i_{r0N}} \right) \hat{i}_{r0N} + \left(\frac{\partial v_{r5N}}{\partial r_2} \frac{\partial r_2}{\partial v_{cr0N}} + \frac{\partial v_{cr5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial v_{cr0N}} + \frac{\partial v_{cr5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial v_{cr0N}} \right) \hat{v}_{cr0N} \\
&+ \left(\frac{\partial v_{cr5N}}{\partial r_2} \frac{\partial r_2}{\partial v_{in}} + \frac{\partial v_{cr5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial v_{in}} + \frac{\partial v_{cr5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial v_{in}} - \frac{M}{v_{in}} \right) \hat{v}_{in} + \left(\frac{\partial v_{cr5N}}{\partial r_2} \frac{\partial r_2}{\partial v_o} + \frac{\partial v_{cr5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial v_o} + \frac{\partial v_{cr5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial v_o} + \frac{n}{v_{in}} \right) \hat{v}_o \\
&+ \left(\frac{\partial v_{cr5N}}{\partial r_2} \frac{\partial r_2}{\partial t_s} + \frac{\partial v_{cr5N}}{\partial \theta_2} \frac{\partial \theta_2}{\partial t_s} + \frac{\partial v_{cr5N}}{\partial \varphi_2} \frac{\partial \varphi_2}{\partial t_s} \right) \hat{t}_s \\
&= v_{cr5N} + \left(-\cos\left(\varphi_2 + \theta_2\right) h_{2i} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2i} + m_{2i}\right) \right) \hat{i}_{r0N} + \left(-\cos\left(\varphi_2 + \theta_2\right) h_{2v} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2v} + m_{2v}\right) \right) \hat{v}_{cr0N} \\
&+ \left(-\cos\left(\varphi_2 + \theta_2\right) h_{2in} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2in} + m_{2in}\right) - \frac{M}{v_{in}} \right) \hat{v}_{in} + \left(-\cos\left(\varphi_2 + \theta_2\right) h_{2o} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2o} + m_{2o}\right) + \frac{n}{v_{in}} \right) \hat{v}_o \\
&+ \left(-\cos\left(\varphi_2 + \theta_2\right) h_{2t} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2t} + m_{2t}\right) \right) \hat{t}_s \\
&= v_{r5N} + l_{5i} \hat{i}_{r0N} + l_{5v} \hat{v}_{cr0N} + l_{5in} \hat{v}_{in} + l_{5o} \hat{v}_o + l_{5t} \hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
l_{5i} &= \frac{\partial v_{cr5N}}{\partial i_{r0N}} = -\cos\left(\varphi_2 + \theta_2\right) h_{2i} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2i} + m_{2i}\right) \\
l_{5v} &= \frac{\partial v_{cr5N}}{\partial v_{cr0N}} = -\cos\left(\varphi_2 + \theta_2\right) h_{2v} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2v} + m_{2v}\right) \\
l_{5in} &= \frac{\partial v_{cr5N}}{\partial v_{in}} = -\cos\left(\varphi_2 + \theta_2\right) h_{2in} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2in} + m_{2in}\right) - \frac{M}{v_{in}} \\
l_{5o} &= \frac{\partial v_{cr5N}}{\partial v_o} = -\cos\left(\varphi_2 + \theta_2\right) h_{2o} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2o} + m_{2o}\right) + \frac{n}{v_{in}} \\
l_{5t} &= \frac{\partial v_{cr5N}}{\partial t_s} = -\cos\left(\varphi_2 + \theta_2\right) h_{2t} + r_2 \sin\left(\varphi_2 + \theta_2\right)\left(g_{2t} + m_{2t}\right)
\end{aligned} \qquad (53)
$$

At time t_5, the converter starts to operates in the O mode. r_3 and θ_3 can be expressed as follows:

$$r_3 = \sqrt{(1+L_n) i_{r5N}{}^2 + (v_{cr5N}+1)^2}\ , \quad \theta_3 = \pi + \arctan\left(-\frac{\sqrt{1+L_n}\, i_{r5N}}{v_{cr5N}+1}\right) \tag{54}$$

The first-order linearization of r_3 and θ_3 is shown in the following.

$$\begin{aligned}
\theta_3 + \Delta\theta_3 &= \pi + \arctan\left(-\frac{\sqrt{1+L_n}\, i_{r5N}}{v_{cr5N}+1}\right) + \frac{\partial\theta_3}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial\theta_3}{\partial v_{cr0N}}\hat{v}_{cr0N} + \frac{\partial\theta_3}{\partial v_{in}}\hat{v}_{in} + \frac{\partial\theta_3}{\partial v_o}\hat{v}_o + \frac{\partial\theta_3}{\partial t_s}\hat{t}_s \\
&= \theta_3 + \left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}} + \frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N} + \left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{cr0N}} + \frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} + \\
&\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}} + \frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_{in}}\right)\hat{v}_{in} + \left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o} + \frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_o}\right)\hat{v}_o + \left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s} + \frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial t_s}\right)\hat{t}_s \\
&= \theta_3 + \left[-\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5i} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5i}\right]\hat{i}_{r0N} + \left[-\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5v} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5v}\right]\hat{v}_{cr0N} \\
&+ \left[-\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5in} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5in}\right]\hat{v}_{in} + \left[-\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5o} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5o}\right]\hat{v}_o \\
&+ \left[-\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5t} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5t}\right]\hat{t}_s \\
&= \theta_2 + g_{3i}\hat{i}_{r0N} + g_{3v}\hat{v}_{cr0N} + g_{3in}\hat{v}_{in} + g_{3o}\hat{v}_o + g_{3t}\hat{t}_s
\end{aligned}$$

where

$$g_{3i} = \frac{\partial\theta_3}{\partial i_{r0N}} = -\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5i} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5i}$$

$$g_{3v} = \frac{\partial\theta_3}{\partial v_{cr0N}} = -\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5v} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5v}$$

$$g_{3in} = \frac{\partial\theta_3}{\partial v_{in}} = -\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5in} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5in}$$

$$g_{3o} = \frac{\partial\theta_3}{\partial v_o} = -\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5o} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5o}$$

$$g_{3t} = \frac{\partial\theta_3}{\partial t_s} = -\frac{\sqrt{1+L_n}(v_{cr5N}+1)}{r_3^2}k_{5t} + \frac{\sqrt{1+L_n}\, i_{r5N}}{r_3^2}l_{5t}$$

$$r_3+\Delta r_3 \approx \sqrt{\left(1+L_n\right) i_{r5N}{}^2+\left(v_{cr5N}+1\right)^2}+\frac{\partial r_3}{\partial i_{r0N}} \hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{cr0N}} \hat{v}_{cr0N}+\frac{\partial r_3}{\partial v_{in}} \hat{v}_{in}+\frac{\partial r_3}{\partial v_o} \hat{v}_o+\frac{\partial r_3}{\partial t_s} \hat{t}_s$$

$$=r_3+\left(\frac{\partial r_3}{\partial i_{r5N}} \frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial r_3}{\partial v_{cr5N}} \frac{\partial v_{cr5N}}{\partial i_{r0N}}\right) \hat{i}_{r0N}+\left(\frac{\partial r_3}{\partial i_{r5N}} \frac{\partial i_{r5N}}{\partial v_{cr0N}}+\frac{\partial r_3}{\partial v_{cr5N}} \frac{\partial v_{cr5N}}{\partial v_{cr0N}}\right) \hat{v}_{cr0N}+$$

$$\left(\frac{\partial r_3}{\partial i_{r5N}} \frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial r_3}{\partial v_{cr5N}} \frac{\partial v_{cr5N}}{\partial v_{in}}\right) \hat{v}_{in}+\left(\frac{\partial r_3}{\partial i_{r5N}} \frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial r_3}{\partial v_{cr5N}} \frac{\partial v_{cr5N}}{\partial v_o}\right) \hat{v}_o+\left(\frac{\partial r_3}{\partial i_{r5N}} \frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial r_3}{\partial v_{cr5N}} \frac{\partial v_{cr5N}}{\partial t_s}\right) \hat{t}_s$$

$$=r_3+\left(\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5i}+\frac{v_{cr5N}+1}{r_3} l_{5i}\right) \hat{i}_{r0N}+\left(\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5v}+\frac{v_{cr5N}+1}{r_3} l_{5v}\right) \hat{v}_{cr0N}+$$

$$\left(\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5in}+\frac{v_{cr5N}+1}{r_3} l_{5in}\right) \hat{v}_{in}+\left(\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5i}+\frac{v_{cr5N}+1}{r_3} l_{5i}\right) \hat{v}_o+\left(\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5i}+\frac{v_{cr5N}+1}{r_3} l_{5i}\right) \hat{t}_s$$

$$=r_3+h_{3i} \hat{i}_{r0N}+h_{3v} \hat{v}_{cr0N}+h_{3in} \hat{v}_{in}+h_{3o} \hat{v}_o+h_{3t} \hat{t}_s$$

where

$$h_{3i}=\frac{\partial r_3}{\partial i_{r0N}}=\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5i}+\frac{v_{cr5N}+1}{r_3} l_{5i}$$

$$h_{3v}=\frac{\partial r_3}{\partial v_{cr0N}}=\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5v}+\frac{v_{cr5N}+1}{r_3} l_{5v}$$

$$h_{3in}=\frac{\partial r_3}{\partial v_{in}}=\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5in}+\frac{v_{cr5N}+1}{r_3} l_{5in}$$

$$h_{3o}=\frac{\partial r_3}{\partial v_o}=\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5i}+\frac{v_{cr5N}+1}{r_3} l_{5i} \tag{55}$$

$$h_{3t}=\frac{\partial r_3}{\partial t_s}=\frac{\left(1+L_n\right) i_{r5N}}{r_3} k_{5i}+\frac{v_{cr5N}+1}{r_3} l_{5i}$$

$\Delta\varphi_3$ can be calculated by

$$\varphi_3=\frac{\omega_{r1} t_s}{2}-\frac{\omega_{r1}}{\omega_{r0}} \varphi_2$$

$$\Delta\varphi_3=\frac{\omega_{r1}}{2} \hat{t}_s-\frac{\omega_{r1}}{\omega_{r0}} \hat{\varphi}_2$$

$$=\left(\frac{\omega_{r1}}{2}-\frac{\omega_{r1}}{\omega_{r0}} m_{2t}\right) \hat{t}_s-\frac{\omega_{r1}}{\omega_{r0}} m_{2i} \hat{i}_{r0N}-\frac{\omega_{r1}}{\omega_{r0}} m_{2v} \hat{v}_{cr0N}-\frac{\omega_{r1}}{\omega_{r0}} m_{2in} \hat{v}_{in}-\frac{\omega_{r1}}{\omega_{r0}} m_{2o} \hat{v}_o$$

$$=m_{3t} \hat{t}_s+m_{3i} \hat{i}_{r0N}+m_{3v} \hat{v}_{cr0N}+m_{3in} \hat{v}_{in}+m_{3o} \hat{v}_o \tag{56}$$

where

$$m_{3i}=\frac{\partial \varphi_3}{\partial i_{r0N}}=-\frac{\omega_{r1}}{\omega_{r0}} m_{2i},\ m_{3v}=\frac{\partial \varphi_3}{\partial v_{cr0N}}=-\frac{\omega_{r1}}{\omega_{r0}} m_{2v},\ m_{3in}=\frac{\partial \varphi_3}{\partial v_{in}}=-\frac{\omega_{r1}}{\omega_{r0}} m_{2in},$$

$$m_{3o}=\frac{\partial \varphi_3}{\partial v_o}=-\frac{\omega_{r1}}{\omega_{r0}} m_{2o},\ m_{3t}=\frac{\omega_{r1}}{2}-\frac{\omega_{r1}}{\omega_{r0}} m_{2t}$$

At time t_6, Δi_{r6N} and Δv_{r6N} can be calculated as follows:

$$
\begin{aligned}
& i_{r6N}+\Delta i_{r6N} \\
& =\frac{r_3}{\sqrt{1+L_n}}\sin\left(\varphi_3+\theta_3\right)+\frac{\partial i_{r6N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r6N}}{\partial v_{r0N}}\hat{v}_{cr0N}+\frac{\partial i_{r6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r6N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r6N}}{\partial t_s}\hat{t}_s \\
& =i_{r6N}+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial i_{r0N}}+\frac{\partial i_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial i_{r0N}}+\frac{\partial i_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{cr0N}}+\frac{\partial i_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{cr0N}}+\frac{\partial i_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{in}}+\frac{\partial i_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{in}}+\frac{\partial i_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial v_o}+\frac{\partial i_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_o}+\frac{\partial i_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_o}\right)\hat{v}_o \\
& +\left(\frac{\partial i_{r6N}}{\partial r_3}\frac{\partial r_3}{\partial t_s}+\frac{\partial i_{r6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial t_s}+\frac{\partial i_{r6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial t_s}\right)\hat{t}_s \\
& =i_{r6N}+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3i}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3i}+m_{3i}\right)\right)\hat{i}_{r0N}+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3v}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3i}+m_{3v}\right)\right)\hat{v}_{cr0N} \\
& +\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3in}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3in}+m_{3in}\right)\right)\hat{v}_{in}+\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3o}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3o}+m_{3o}\right)\right)\hat{v}_o \\
& +\left(\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3t}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3t}+m_{3t}\right)\right)\hat{t}_s \\
& =i_{r6N}+k_{6i}\hat{i}_{r0N}+k_{6v}\hat{v}_{cr0N}+k_{6in}\hat{v}_{in}+k_{6o}\hat{v}_o+k_{6t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
& k_{6i}=\frac{\partial i_{r6N}}{\partial i_{r0N}}=\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3i}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3i}+m_{3i}\right),\ k_{6v}=\frac{\partial i_{r6N}}{\partial v_{cr0N}}=\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3v}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3i}+m_{3v}\right) \\
& k_{6in}=\frac{\partial i_{r6N}}{\partial v_{in}}=\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3in}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3in}+m_{3in}\right),\ k_{6o}=\frac{\partial i_{r6N}}{\partial v_o}=\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3o}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3o}+m_{3o}\right) \\
& k_{6t}=\frac{\partial i_{r6N}}{\partial t_s}=\frac{\sin\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}h_{3t}+\frac{r_3\cos\left(\varphi_3+\theta_3\right)}{\sqrt{1+L_n}}\left(g_{3t}+m_{3t}\right)
\end{aligned}
$$

$$
\begin{aligned}
& v_{cr6N}+\Delta v_{cr6N}=-r_3\cos\left(\varphi_3+\theta_3\right)-1+\frac{\partial v_{r6N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{cr6N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial v_{r6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr6N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{cr6N}}{\partial t_s}\hat{t}_s \\
& =v_{cr6N}+\left(\frac{\partial v_{cr6N}}{\partial r_3}\frac{\partial r_3}{\partial i_{r0N}}+\frac{\partial v_{cr6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial i_{r0N}}+\frac{\partial v_{cr6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{cr6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{cr0N}}+\frac{\partial v_{cr6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{cr0N}}+\frac{\partial v_{cr6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial v_{cr6N}}{\partial r_3}\frac{\partial r_3}{\partial v_{in}}+\frac{\partial v_{cr6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_{in}}+\frac{\partial v_{cr6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{cr6N}}{\partial r_3}\frac{\partial r_3}{\partial v_o}+\frac{\partial v_{cr6N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial v_o}+\frac{\partial v_{cr6N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial v_o}\right)\hat{v}_o+\left(\frac{\partial v_{cr5N}}{\partial r_3}\frac{\partial r_3}{\partial t_s}+\frac{\partial v_{cr5N}}{\partial \theta_3}\frac{\partial \theta_3}{\partial t_s}+\frac{\partial v_{cr5N}}{\partial \varphi_3}\frac{\partial \varphi_3}{\partial t_s}\right)\hat{t}_s \\
& =v_{cr6N}+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3i}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}+m_{3i}\right)\right)\hat{i}_{r0N}+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3v}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3v}+m_{3v}\right)\right)\hat{v}_{cr0N} \\
& +\left(-\cos\left(\varphi_3+\theta_3\right)h_{3in}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3in}+m_{3in}\right)\right)\hat{v}_{in}+\left(-\cos\left(\varphi_3+\theta_3\right)h_{3o}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3o}+m_{3o}\right)\right)\hat{v}_o \\
& +\left(-\cos\left(\varphi_3+\theta_3\right)h_{3t}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3t}+m_{3t}\right)\right)\hat{t}_s \\
& =v_{r6N}+l_{6i}\hat{i}_{r0N}+l_{6v}\hat{v}_{cr0N}+l_{6in}\hat{v}_{in}+l_{6o}\hat{v}_o+l_{6t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
& l_{6i}=\frac{\partial v_{cr6N}}{\partial i_{r0N}}=-\cos\left(\varphi_3+\theta_3\right)h_{3i}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}+m_{3i}\right),\quad l_{6v}=\frac{\partial v_{cr6N}}{\partial v_{cr0N}}=-\cos\left(\varphi_3+\theta_3\right)h_{3v}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3v}+m_{3v}\right) \\
& l_{6in}=\frac{\partial v_{cr6N}}{\partial v_{in}}=-\cos\left(\varphi_3+\theta_3\right)h_{3in}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3in}+m_{3in}\right),\ l_{6o}=\frac{\partial v_{cr6N}}{\partial v_o}=-\cos\left(\varphi_3+\theta_3\right)h_{3o}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3o}+m_{3o}\right) \\
& l_{6t}=\frac{\partial v_{cr6N}}{\partial t_s}=-\cos\left(\varphi_3+\theta_3\right)h_{3t}+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3t}+m_{3t}\right)
\end{aligned}
\tag{57}
$$

The average output current of the rectifier from t_3 to t_6 can be expressed as

$$
\begin{aligned}
&\bar{i}_{s2} + \Delta\bar{i}_{s2} = \frac{nI_n}{\omega_{r0}t_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - r_2\sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right) + \frac{\partial\bar{i}_{s2}}{\partial i_{r0N}}\hat{i}_{r0N} + \frac{\partial\bar{i}_{s2}}{\partial v_{cr0N}}\hat{v}_{cr0N} + \frac{\partial\bar{i}_{s2}}{\partial v_{in}}\hat{v}_{in} + \frac{\partial\bar{i}_{s2}}{\partial v_{o}}\hat{v}_{o} + \frac{\partial\bar{i}_{s2}}{\partial t_{s}}\hat{t}_{s} \\
&= \bar{i}_{s2} + \left(\frac{\partial\bar{i}_{s2}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}} + \frac{\partial\bar{i}_{s2}}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}} + \frac{\partial\bar{i}_{s2}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial i_{r0N}}\right)\hat{i}_{r0N} + \left(\frac{\partial\bar{i}_{s2}}{\partial r_2}\frac{\partial r_2}{\partial v_{cr0N}} + \frac{\partial\bar{i}_{s2}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{cr0N}} + \frac{\partial\bar{i}_{s2}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} + \\
&\left(\frac{\partial\bar{i}_{s2}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}} + \frac{\partial\bar{i}_{s2}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}} + \frac{\partial\bar{i}_{s2}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{in}} - \frac{nI_n}{\omega_{r0}t_s}\frac{M}{2v_{in}L_n}\varphi_2^{\,2}\right)\hat{v}_{in} + \frac{nC_r}{t_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right)\hat{v}_{in} \\
&+\left(\frac{\partial\bar{i}_{s2}}{\partial r_2}\frac{\partial r_2}{\partial v_{o}} + \frac{\partial\bar{i}_{s2}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{o}} + \frac{\partial\bar{i}_{s2}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial v_{o}} + \frac{nI_n}{\omega_{r0}t_s}\frac{n}{2v_{in}L_n}\varphi_2^{\,2}\right)\hat{v}_{o} + \left(\frac{\partial\bar{i}_{s2}}{\partial r_2}\frac{\partial r_2}{\partial t_{s}} + \frac{\partial\bar{i}_{s2}}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_{s}} + \frac{\partial\bar{i}_{s2}}{\partial \varphi_2}\frac{\partial \varphi_2}{\partial t_{s}}\right)\hat{t}_{s} \\
&-\frac{nI_n}{\omega_{r0}t_s^{\,2}}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right)\hat{t}_{s} \\
&= \bar{i}_{s2} + \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2i} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2i} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2i}
\end{array}\right]\hat{i}_{r0N} + \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2v} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2v} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2v}
\end{array}\right]\hat{v}_{cr0N} \\
&+\left\{\begin{array}{l}
\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2in} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2in} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2in} - \frac{M}{2v_{in}L_n}\varphi_2^{\,2}
\end{array}\right] \\
+\frac{nC_r}{T_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - r_2\sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right)
\end{array}\right\}\hat{v}_{in} + \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2o} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2o} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2o} + \frac{n}{2v_{in}L_n}\varphi_2^{\,2}
\end{array}\right]\hat{v}_{o} \\
&+\left\{\begin{array}{l}
\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2t} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2t} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2t}
\end{array}\right] \\
-\frac{nI_n}{\omega_{r0}T_s^{\,2}}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - r_2\sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right)
\end{array}\right\}\hat{t}_{s} \\
&= \bar{i}_{s2} + k_{s2i}\hat{i}_{r0N} + k_{s2v}\hat{v}_{cr0N} + k_{s2in}\hat{v}_{in} + k_{s2o}\hat{v}_{o} + k_{s2t}\hat{t}_{s}
\end{aligned}
$$

where

$$
\begin{aligned}
&k_{s2i} = \frac{\partial\bar{i}_{s2}}{\partial i_{r0N}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2i} + \left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2i} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2i}
\end{array}\right] \\
&k_{s2v} = \frac{\partial\bar{i}_{s2}}{\partial v_{cr0N}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2v} + \left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2v} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2v}
\end{array}\right] \\
&k_{s2in} = \frac{\partial\bar{i}_{s2}}{\partial v_{in}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2in} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2in} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2in} - \frac{M}{2v_{in}L_n}\varphi_2^{\,2}
\end{array}\right] + \frac{nC_r}{T_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - r_2\sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right) \\
&k_{s2o} = \frac{\partial\bar{i}_{s2}}{\partial v_{o}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2o} + \left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2o} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2o} + \frac{n}{2v_{in}L_n}\varphi_2^{\,2}
\end{array}\right] \\
&k_{s2t} = \frac{\partial\bar{i}_{s2}}{\partial t_{s}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left(\cos(\theta_2) - \cos(\varphi_2+\theta_2) - \sin(\theta_2)\varphi_2\right)h_{2t} + \\
\left(-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2) - r_2\cos(\theta_2)\varphi_2\right)g_{2t} \\
+\left(r_2\sin(\varphi_2+\theta_2) - r_2\sin(\theta_2) + \frac{M}{L_n}\varphi_2\right)m_{2t}
\end{array}\right] - \frac{nI_n}{\omega_{r0}T_s^{\,2}}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) - r_2\sin(\theta_2)\varphi_2 + \frac{M}{2L_n}\varphi_2^{\,2}\right)
\end{aligned}
\tag{58}
$$

The variation in output current of the rectifier bridge during one switching cycle is expressed as

$$\begin{aligned}\Delta\overline{i}_{rec} &= \Delta\overline{i}_{s1} - \Delta\overline{i}_{s2} \\ &= \left(k_{s1i} - k_{s2i}\right)\hat{i}_{r0N} + \left(k_{s1v} - k_{s2v}\right)\hat{v}_{cr0N} + \left(k_{s1o} - k_{s2o}\right)\hat{v}_o + \left(k_{s1v} - k_{s2in}\right)\hat{v}_{in} + \left(k_{s1t} - k_{s2t}\right)\hat{t}_s\end{aligned} \tag{59}$$

According to the large signal model, the state space expression of the LLC converter can be expressed as

$$\begin{aligned}\dot{\hat{i}}_{r0N} &= \frac{i_{r6N} + \Delta i_{r6N} - i_{r0N} - \hat{i}_{r0N}}{T_s + \hat{t}_s} \approx \frac{\Delta i_{r6N} - \hat{i}_{r0N}}{T_s} = \frac{1}{T_s}\left[\left(k_{6i} - 1\right)\hat{i}_{r0N} + k_{6v}\hat{v}_{cr0N} + k_{6in}\hat{v}_{in} + k_{6o}\hat{v}_o + k_{6t}\hat{t}_s\right] \\ \dot{\hat{v}}_{cr0N} &= \frac{v_{cr6N} + \Delta v_{cr6N} - v_{cr0N} - \hat{v}_{cr0N}}{T_s + \hat{t}_s} \approx \frac{\Delta v_{cr6N} - \hat{v}_{cr0N}}{T_s} = \frac{1}{T_s}\left[l_{6i}\hat{i}_{r0N} + \left(l_{6v} - 1\right)\hat{v}_{cr0N} + l_{6in}\hat{v}_{in} + l_{6o}\hat{v}_o + l_{6t}\hat{t}_s\right] \\ \dot{\hat{v}}_o &= \frac{1}{C_o}\left(\Delta\overline{i}_{rec} - \frac{\hat{v}_o}{R}\right) = \frac{1}{C_o}\begin{bmatrix}\left(k_{s1i} - k_{s2i}\right)\hat{i}_{r0N} + \left(k_{s1v} - k_{s2v}\right)\hat{v}_{cr0N} + \left(k_{s1o} - k_{s2o} - \dfrac{1}{R}\right)\hat{v}_o \\ + \left(k_{s1in} - k_{s2in}\right)\hat{v}_{in} + \left(k_{s1t} - k_{s2t}\right)\hat{t}_s\end{bmatrix}\end{aligned} \tag{60}$$

The above equation can be rewritten in the form of the state space equation, which is shown in the following.

$$\begin{aligned}&\dot{\hat{x}} = A\hat{x} + B\hat{u} \\ &\hat{y} = C\hat{x} \\ &\text{where} \\ &A = \begin{bmatrix}\dfrac{k_{6i} - 1}{T_s} & \dfrac{k_{6v}}{T_s} & \dfrac{k_{6o}}{T_s} \\ \dfrac{l_{6i}}{T_s} & \dfrac{l_{6v} - 1}{T_s} & \dfrac{l_{6o}}{T_s} \\ \dfrac{k_{s1i} - k_{s2i}}{C_o} & \dfrac{k_{s1v} - k_{s2v}}{C_o} & \dfrac{k_{s1o} - k_{s2o} - 1/R}{C_o}\end{bmatrix} \\ &B = \begin{bmatrix}\dfrac{k_{6in}}{T_s} & \dfrac{k_{6t}}{T_s} \\ \dfrac{l_{6in}}{T_s} & \dfrac{l_{6t}}{T_s} \\ \dfrac{k_{s1in} - k_{s2in}}{C_o} & \dfrac{k_{s1t} - k_{s2t}}{C_o}\end{bmatrix} \\ &C = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\end{aligned} \tag{61}$$

Substituting the steady-state operating value V_{in}, V_o, T_s I_{r0N}, I_{r2N}, I_{r3N}, I_{r5N}, I_{r6N}, V_{r0N}, V_{r2N}, V_{r3N}, V_{r5N}, and V_{r6N} into v_{in}, v_o, t_s, i_{r0N}, i_{r2N}, i_{r3N}, i_{r5N}, i_{r6N}, v_{r0N}, v_{r2N}, v_{r3N}, v_{r5N}, and v_{r6N} in the state space equation, the transfer function of the LLC converter for PO mode can be expressed as

$$G\left(s\right) = C\left(sI - A\right)^{-1}B = \left[G_{vin}(s) \quad G_t(s)\right] \tag{62}$$

where

$$\begin{aligned}G_{vin}(s) &= \frac{\hat{v}_o}{\hat{v}_{in}} \\ G_t(s) &= \frac{\hat{v}_o}{\hat{t}_s}\end{aligned}$$

As shown in Fig.4, during the derivation of the small-signal model, disturbances $\hat{v}_{in}$, $\hat{v}_o$, $\hat{i}_{r0N}$, and $\hat{v}_{cr0N}$ are introduced at t_0, and they have an impact on the state trajectory in the whole switching period. However, switching period perturbation $\hat{t}_s$ comes into effect at t_3, resulting in a phase lag of half a switching period.

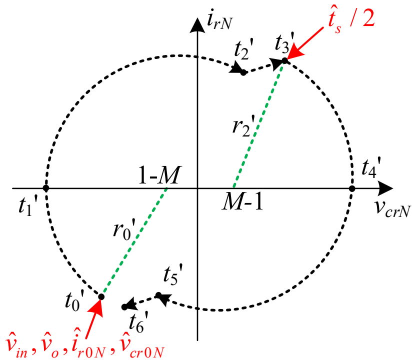

Fig.4. The diagram of periodic perturbation lag $T_s/2$.

Considering the time delay of $T_s/2$, the transfer function from the switching period to the output voltage is revised to (63).

$$G_{ts}(s) = e^{-\frac{T_s}{2}s} G_t(s) \tag{63}$$

Section IV. Small-signal model for PO mode with TSC

The definitions of t_{Z1}, t_{Z2} and t_{cs} are shown below.

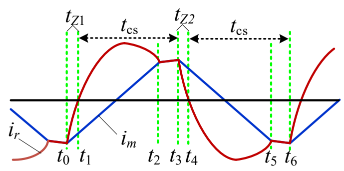

Fig.5 The analysis of control time under TSC for PO mode.

Δt_{Z1} and Δt_{Z2} can be expressed as follows:

$$\begin{aligned}
t_{Z1} &= -\frac{\theta_0}{\omega_{r0}} \\
t_{Z2} &= \frac{\pi-\theta_2}{\omega_{r0}} \\
\Delta t_{Z1} &= -\frac{\Delta\theta_0}{\omega_{r0}} = -\frac{1}{\omega_{r0}}\left(g_{0i}\hat{i}_{r0N} + g_{0v}\hat{v}_{cr0N} + g_{0in}\hat{v}_{in} + g_{0o}\hat{v}_o\right) \\
\Delta t_{Z2} &= -\frac{\Delta\theta_2}{\omega_{r0}} = -\frac{1}{\omega_{r0}}\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{cr0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right)
\end{aligned} \tag{64}$$

The relationship between $\hat{t}_{cs}$ and $\hat{t}_s$ can be shown below.

$$\begin{aligned}
\hat{t}_s &= \Delta t_{Z1} + \Delta t_{Z2} + 2\hat{t}_{cs} \\
&= \frac{1}{\omega_{r0}}\left(-g_{0i}\hat{i}_{r0N} - g_{0v}\hat{v}_{cr0N} - g_{0in}\hat{v}_{in} - g_{0o}\hat{v}_o\right) - \frac{1}{\omega_{r0}}\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{cr0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right) + 2\hat{t}_{cs} \\
&= \frac{1}{\omega_{r0}}\left[\left(-g_{0i}-g_{2i}\right)\hat{i}_{r0N} + \left(-g_{0v}-g_{2v}\right)\hat{v}_{cr0N} + \left(-g_{0in}-g_{2in}\right)\hat{v}_{in} + \left(-g_{0o}-g_{2o}\right)\hat{v}_o\right] - \frac{g_{2t}}{\omega_{r0}}\hat{t}_s + 2\hat{t}_{cs}
\end{aligned} \tag{65}$$

The above equation can be rewritten as

$$\begin{aligned}
\hat{t}_s &= \frac{\omega_{r0}}{\omega_{r0}+g_{2t}} \bullet \frac{1}{\omega_{r0}}\left[\left(-g_{0i}-g_{2i}\right)\hat{i}_{r0N} + \left(-g_{0v}-g_{2v}\right)\hat{v}_{cr0N} + \left(-g_{0in}-g_{2in}\right)\hat{v}_{in} + \left(-g_{0o}-g_{2o}\right)\hat{v}_o\right] + \frac{2\omega_{r0}}{\omega_{r0}+g_{2t}}\hat{t}_{cs} \\
&= \frac{1}{\omega_{r0}+g_{2t}}\left[\left(-g_{0i}-g_{2i}\right)\hat{i}_{r0N} + \left(-g_{0v}-g_{2v}\right)\hat{v}_{cr0N} + \left(-g_{0in}-g_{2in}\right)\hat{v}_{in} + \left(-g_{0o}-g_{2o}\right)\hat{v}_o\right] + \frac{2\omega_{r0}}{\omega_{r0}+g_{2t}}\hat{t}_{cs} \\
&= \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix}-g_{0i}-g_{2i} & -g_{0v}-g_{2v} & -g_{0o}-g_{2o}\end{bmatrix}\begin{bmatrix}\hat{i}_{r0N} \\ \hat{v}_{cr0N} \\ \hat{v}_o\end{bmatrix} + \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix}-g_{0in}-g_{2in} & 2\omega_{r0}\end{bmatrix}\begin{bmatrix}\hat{v}_{in} \\ \hat{t}_{cs}\end{bmatrix} \\
&= A_Z\hat{x} + B_Z\begin{bmatrix}\hat{v}_{in} \\ \hat{t}_{cs}\end{bmatrix}
\end{aligned} \tag{66}$$

where

$$A_Z = \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix}-g_{0i}-g_{2i} & -g_{0v}-g_{2v} & -g_{0o}-g_{2o}\end{bmatrix}$$

$$B_Z = \frac{1}{\omega_{r0}+g_{2t}}\begin{bmatrix}-g_{0in}-g_{2in} & 2\omega_{r0}\end{bmatrix}$$

Replacing $\hat{t}_s$ in the state space expression (61) with $t_s = A_Z\hat{x} + B_Z\begin{bmatrix}\hat{v}_{in} \\ \hat{t}_{cs}\end{bmatrix}$, the state space equation is revised to (67).

$$
\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + B\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = \hat{x} + \begin{bmatrix}B_1 & B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = A\hat{x} + B_1\hat{v}_{in} + B_2\hat{t}_s \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2\left[A_Z\hat{x} + B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}\right] \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2A_Z\hat{x} + B_2B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= \left(A + B_2A_Z\right)\hat{x} + B_1\hat{v}_{in} + \frac{1}{\omega_{r0} + g_{2t}}B_2\left(\left(-g_{0in} - g_{2in}\right)\hat{v}_{in} + 2\omega_{r0}\hat{t}_{cs}\right) \\
&= \left(A + B_2A_Z\right)\hat{x} + \left(B_1 + B_2\frac{-g_{0in} - g_{2in}}{\omega_{r0} + g_{2t}}\right)\hat{v}_{in} + \frac{2\omega_{r0}}{\omega_{r0} + g_{2t}}B_2\hat{t}_{cs} \\
&= \left(A + B_2A_Z\right)\hat{x} + \begin{bmatrix}B_1 + B_2\dfrac{-g_{0in} - g_{2in}}{\omega_{r0} + g_{2t}} & \dfrac{2\omega_{r0}}{\omega_{r0} + g_{2t}}B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= A_c\hat{x} + B_c\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
A_c &= A + B_2A_Z \\
B_c &= \begin{bmatrix}B_1 + B_2\dfrac{-g_{0in} - g_{2in}}{\omega_{r0} + g_{2t}} & \dfrac{2\omega_{r0}}{\omega_{r0} + g_{2t}}B_2\end{bmatrix}
\end{aligned} \tag{67}
$$

Therefore, the small-signal model of the LLC converter for PO mode with TSC can be expressed as follows.

$$
G_{cs}\left(s\right) = C\left(sI - A_c\right)^{-1}B_c = \begin{bmatrix}G_{vin\_tc}(s) & G_{tc}(s)\end{bmatrix} \tag{68}
$$

Considering the time delay of $T_s/2$, the transfer function from the control time to the output voltage is revised as follows:

$$
G_{tcs}\left(s\right) = e^{-\frac{T_s}{2}s}G_{tc}\left(s\right)
$$

Section V. Time-domain expressions for NP mode

Typical waveforms and planar trajectory of the LLC converter for NP mode are shown in Fig.5 and Fig.6.

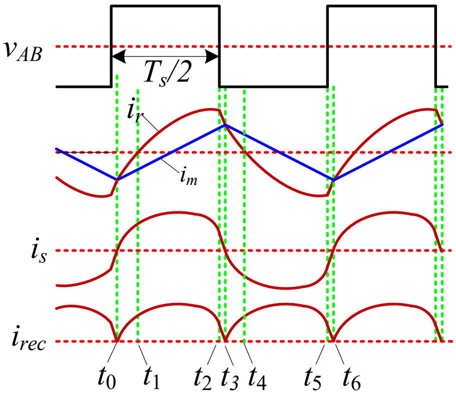

Fig.5 Typical waveforms of the LLC converter for NP mode.

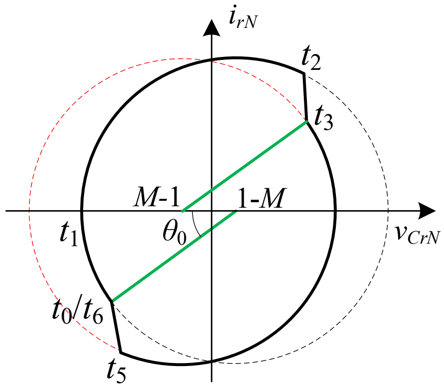

Fig.6 Planar trajectory of the LLC converter for NP mode

[t_0, t_2]

As in the case of PO mode from t_0 to t_2, resonant current and resonant capacitor voltage can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r0}Z_0 \sin(\omega_{r0}t) + \left[v_{cr0} - (v_{in} - nv_o)\right]\cos(\omega_{r0}t) + (v_{in} - nv_o) \\ i_r &= i_{r0}\cos(\omega_{r0}t) - \frac{v_{cr0} - (v_{in} - nv_o)}{Z_0}\sin(\omega_{r0}t) \end{aligned} \tag{69}$$

The normalized equation is shown in the following.

$$\begin{aligned} v_{crN} &= i_{r0N}\sin(\omega_{r0}t) + \left[v_{cr0N} - (1-M)\right]\cos(\omega_{r0}t) + (1-M) \\ i_{rN} &= i_{r0N}\cos(\omega_{r0}t) - \left[v_{cr0N} - (1-M)\right]\sin(\omega_{r0}t) \end{aligned} \tag{70}$$

where $i_{r0N} = \dfrac{i_{r0}Z_0}{v_{in}}, v_{cr0N} = \dfrac{v_{cr0}}{v_{in}}$.

Eq.(70) can be rewritten as

$$\begin{aligned} i_{rN} &= \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}\sin(\omega_{r0}t + \theta_0) \\ v_{crN} &= -\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}\cos(\omega_{r0}t + \theta_0) + (1-M) \end{aligned} \tag{71}$$

where

$$\cos\theta_0 = -\frac{\left[v_{cr0N} - (1-M)\right]}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}}, \sin\theta_0 = \frac{i_{r0N}}{\sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}}$$

$$\theta_0 = \arctan\left(\frac{-i_{r0N}}{v_{cr0N} - (1-M)}\right)$$

Set $r_0 = \sqrt{{i_{r0N}}^2 + \left[v_{cr0N} - (1-M)\right]^2}$, then

$$\begin{aligned} i_{rN} &= r_0 \sin(\omega_{r0}t + \theta_0) \\ v_{crN} &= -r_0\cos(\omega_{r0}t + \theta_0) + (1-M) \end{aligned} \tag{72}$$

i_{r0N}, v_{cr0N}, i_{r2N}, and v_{cr2N} can be expressed in (73), where $\varphi_0=\omega_{r0}t_2$.

$$\begin{aligned} i_{r0N} &= r_0 \sin(\theta_0) \\ v_{cr0N} &= -r_0 \cos(\theta_0)+(1-M) \\ i_{r2N} &= r_0 \sin(\varphi_0+\theta_0) \\ v_{cr2N} &= -r_0 \cos(\varphi_0+\theta_0)+(1-M) \end{aligned} \tag{73}$$

The expression of the magnetizing current i_m is shown in (74).

$$i_m = i_{r0} + \frac{nV_o}{L_m}t \tag{74}$$

Eq.(74) is normalized to (75).

$$i_{mN} = \frac{i_{r0}Z_0}{v_{in}} + \frac{nv_oZ_0}{v_{in}L_m}t = i_{r0N} + M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}t = r_0\sin(\theta_0)+\frac{M}{L_n}\omega_{r0}t \tag{75}$$

The current in the secondary winding of the transformer is expressed as

$$i_{s1} = nI_n(i_{rN}-i_{mN}) = nI_n\left(r_0\sin(\omega_{r0}t+\theta_0)-r_0\sin(\theta_0)-\frac{M}{L_n}\omega_{r0}t\right) \tag{76}$$

[t_2, t_3]

The converter operates in N mode from t_2 to t_3, and the voltage across the resonant tank is changed to $-v_{in}$. v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r2}Z_0\sin(\omega_{r0}t)+\left[v_{cr2}+(v_{in}+nv_o)\right]\cos(\omega_{r0}t)-(v_{in}+nv_o) \\ i_r &= i_{r2}\cos(\omega_{r0}t)-\frac{v_{cr2}+(v_{in}+nv_o)}{Z_0}\sin(\omega_{r0}t) \end{aligned} \tag{77}$$

The normalized equation is shown in the following.

$$\begin{aligned} v_{crN} &= i_{r2N}\sin(\omega_{r0}t)+\left[v_{cr2N}+(1+M)\right]\cos(\omega_{r0}t)-(1+M) \\ i_{rN} &= i_{r2N}\cos(\omega_{r0}t)-\left[v_{cr2N}+(1+M)\right]\sin(\omega_{r0}t) \end{aligned} \tag{78}$$

where $i_{r2N} = \dfrac{i_{r2}Z_0}{v_{in}}, v_{cr2N} = \dfrac{v_{cr2}}{v_{in}}$

Eq.(78) can be rewritten as

$$\begin{aligned} v_{crN} &= \sqrt{i_{r2N}{}^2+\left[v_{cr2N}+(1+M)\right]^2}\cos\left[\omega_{r0}(t-t_2)+\theta_1\right]-(1+M) \\ i_{rN} &= \sqrt{i_{r2N}{}^2+\left[v_{cr2N}+(1+M)\right]^2}\sin\left[\omega_{r0}(t-t_2)+\theta_1\right] \end{aligned} \tag{79}$$

where

$$\cos\theta_1 = -\frac{\left[v_{cr2N}+(1+M)\right]}{\sqrt{i_{r2N}{}^2+\left[v_{cr2N}-(1+M)\right]^2}}, \sin\theta_1 = \frac{i_{r2N}}{\sqrt{i_{r2N}{}^2+\left[v_{cr2N}+(1+M)\right]^2}}$$

$$\theta_1 = \pi + \arctan\left(-\frac{i_{r2N}}{\left[v_{cr2N}+(1+M)\right]}\right)$$

Set $r_1 = \sqrt{i_{r2N}{}^2+\left[v_{cr2N}+(1+M)\right]^2}$, then

$$\begin{aligned} v_{crN} &= r_1 \cos\left[\omega_{r0}\left(t-t_2\right)+\theta_1\right]-\left(1+M\right) \\ i_{rN} &= r_1 \sin\left[\omega_{r0}\left(t-t_2\right)+\theta_1\right] \end{aligned} \tag{80}$$

i_{r2N}, v_{cr2N}, i_{r3N}, and v_{cr3N} can be expressed in (81).

$$\begin{aligned} i_{r2N} &= r_1 \sin\left(\theta_1\right) \\ v_{cr2N} &= -r_1 \cos\left(\theta_1\right)-\left(1+M\right) \\ i_{r3N} &= r_1 \sin\left(\varphi_1+\theta_1\right) \\ v_{cr3N} &= -r_1 \cos\left(\varphi_1+\theta_1\right)-\left(1+M\right) \end{aligned} \tag{81}$$

The magnetizing current i_m can still be referred to Eq.(75), and the output current in the secondary winding of the transformer is expressed as

$$i_{s1} = nI_n\left(i_{rN}-i_{mN}\right) = nI_n\left(r_1 \sin\left(\omega_{r0}\left(t-t_2\right)+\theta_1\right)-r_0 \sin\left(\theta_0\right)-\frac{M}{L_n}\omega_{r0}t\right) \tag{82}$$

From t_0 to t_3, the average value of i_{s1} over half a switching cycle can be expressed as follows.

$$\begin{aligned} \overline{i_{s1}} &= \frac{2}{T_s}\int_0^{t_2} i_{s1}dt = \frac{2nI_n}{T_s}\int_0^{t_3}\left(i_{rN}-i_{mN}\right)dt = \frac{2nI_n}{T_s}\int_0^{t_3} i_{rN}dt-\int_0^{t_3} i_{mN}dt \\ &= \frac{2nI_n}{T_s}\int_0^{t_2} r_0 \sin\left(\omega_{r0}t+\theta_0\right)dt+\frac{2nI_n}{T_s}\int_{t_2}^{t_3} r_1 \sin\left(\omega_{r0}t+\theta_1\right)dt-0 \\ &= \frac{2nI_n}{T_s\omega_{r0}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right) \end{aligned} \tag{83}$$

[t_3, t_5]

Similar to the derivation from t_0 to t_2, v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r3}Z_0 \sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3}+\left(v_{in}-nv_o\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(v_{in}-nv_o\right) \\ i_r &= i_{r3}\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\frac{v_{cr3}+\left(v_{in}-nv_o\right)}{Z_0}\sin\left(\omega_{r0}\left(t-t_3\right)\right) \end{aligned} \tag{84}$$

The normalized equation is shown in the following.

$$\begin{aligned} v_{crN} &= i_{r3N}\sin\left(\omega_{r0}\left(t-t_3\right)\right)+\left[v_{cr3N}+\left(1-M\right)\right]\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left(1-M\right) \\ i_{rN} &= i_{r3N}\cos\left(\omega_{r0}\left(t-t_3\right)\right)-\left[v_{cr3N}+\left(1-M\right)\right]\sin\left(\omega_{r0}\left(t-t_3\right)\right) \end{aligned} \tag{85}$$

where $i_{r3N} = \dfrac{i_{r3}Z_0}{v_{in}}, v_{cr3N} = \dfrac{v_{cr3}}{v_{in}}$

The above equation can be rewritten as

$$\begin{aligned} v_{crN} &= -\sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\cos\left[\omega_{r0}\left(t-t_3\right)+\theta_2\right]-\left(1-M\right) \\ i_{rN} &= \sqrt{{i_{r3N}}^2+\left[v_{cr3N}+\left(1-M\right)\right]^2}\sin\left[\omega_{r0}\left(t-t_3\right)+\theta_2\right] \end{aligned} \tag{86}$$

where

$$\cos\theta_2 = -\frac{\left[v_{cr3N} + (1-M)\right]}{\sqrt{i_{r3N}{}^2 + \left[v_{cr3N} + (1-M)\right]^2}}, \sin\theta_2 = \frac{i_{r3N}}{\sqrt{i_{r3N}{}^2 + \left[v_{cr3N} + (1-M)\right]^2}}$$

$$\theta_2 = \pi + \arctan\left(-\frac{i_{r3N}}{v_{cr3N} + (1-M)}\right)$$

Set $r_2 = \sqrt{i_{r3N}{}^2 + \left[v_{cr3N} + (1-M)\right]^2}$, then

$$\begin{aligned} v_{crN} &= -r_2 \cos\left(\omega_{r0}(t-t_3) + \theta_2\right) - (1-M) \\ i_{rN} &= r_2 \sin\left(\omega_{r0}(t-t_3) + \theta_2\right) \end{aligned} \tag{87}$$

i_{r3N}, v_{cr3N}, i_{r5N}, and v_{cr5N} can be expressed as

$$\begin{aligned} i_{r3N} &= r_2 \sin(\theta_2) \\ v_{cr3N} &= -r_2 \cos(\theta_2) - (1-M) \\ i_{r5N} &= r_2 \sin(\varphi_2 + \theta_2) \\ v_{cr5N} &= -r_2 \cos(\varphi_2 + \theta_2) - (1-M) \end{aligned} \tag{88}$$

The expression of the magnetizing current i_m is shown as follows

$$i_m = i_{r3} - \frac{nV_o}{L_m}(t-t_3) \tag{89}$$

The normalized equation is shown in the following.

$$i_{mN} = i_{r3N} - M\sqrt{\frac{L_r}{C_r}}\frac{1}{L_m}(t-t_3) = r_2\sin(\theta_2) - \frac{M}{L_n}\omega_{r0}(t-t_3) \tag{90}$$

The output current of the rectifier bridge is expressed as

$$i_{s2} = nI_n(i_{rN} - i_{mN}) = nI_n\left(r_2\sin\left(\omega_{r0}(t-t_3)+\theta_2\right) - r_2\sin(\theta_2) + \frac{M}{L_n}\omega_{r0}(t-t_3)\right) \tag{91}$$

[t_5, t_6]

Similar to the derivation from t_0 to t_2, v_{cr} and i_r can be expressed as follows.

$$\begin{aligned} v_{cr} &= i_{r5}Z_0\sin(\omega_{r0}t) + \left[v_{cr5} - (v_{in} + nv_o)\right]\cos(\omega_{r0}t) + (v_{in} + nv_o) \\ i_r &= i_{r5}\cos(\omega_{r0}t) - \frac{v_{cr5} - (v_{in} + nv_o)}{Z_0}\sin(\omega_{r0}t) \end{aligned} \tag{92}$$

The normalized equation is shown in the following.

$$\begin{aligned} v_{crN} &= i_{r5N}\sin(\omega_{r0}t) + \left[v_{cr5N} - (1+M)\right]\cos(\omega_{r0}t) + (1+M) \\ i_{rN} &= i_{r5N}\cos(\omega_{r0}t) - \left[v_{cr5N} - (1+M)\right]\sin(\omega_{r0}t) \end{aligned} \tag{93}$$

where $i_{r5N} = \frac{i_{r5}Z_0}{v_{in}}, v_{cr5N} = \frac{v_{cr5}}{v_{in}}$

The above equation can be rewritten as

$$i_{rN} = \sqrt{{i_{r5N}}^2 + \left[v_{cr5N} - (1+M)\right]^2}\sin\left[\omega_{r0}(t-t_5)+\theta_3\right]$$
$$v_{crN} = -\sqrt{{i_{r5N}}^2 + \left[v_{cr5N} - (1+M)\right]^2}\cos\left[\omega_{r0}(t-t_5)+\theta_3\right]+(1+M) \tag{94}$$

where

$$\cos\theta_3 = -\frac{\left[v_{cr5N}-(1+M)\right]}{\sqrt{{i_{r5N}}^2+\left[v_{cr5N}-(1+M)\right]^2}}, \sin\theta_3 = \frac{i_{r5N}}{\sqrt{{i_{r5N}}^2+\left[v_{cr5N}-(1+M)\right]^2}}$$
$$\theta_3 = \arctan\left(-\frac{i_{r5N}}{v_{cr5N}-(1+M)}\right), r_3 = \sqrt{{i_{r5N}}^2+\left[v_{cr5N}-(1+M)\right]^2}$$

Let $r_3 = \sqrt{{i_{r3N}}^2 + \left[v_{cr3N}+(1-M)\right]^2}$, then

$$\begin{aligned} v_{crN} &= -r_3\cos\left(\omega_{r0}(t-t_5)+\theta_3\right)+(1+M) \\ i_{rN} &= r_3\sin\left(\omega_{r0}(t-t_5)+\theta_3\right) \end{aligned} \tag{95}$$

i_{r5N}, v_{cr5N}, i_{r6N}, and v_{cr6N} can be expressed as

$$\begin{aligned} i_{r5N} &= r_3\sin(\theta_3) \\ v_{cr5N} &= -r_3\cos(\theta_3)+(1+M) \\ i_{r6N} &= r_3\sin(\varphi_3+\theta_3) \\ v_{cr6N} &= -r_3\cos(\varphi_3+\theta_3)+(1+M) \end{aligned} \tag{96}$$

The magnetizing current i_{mN} can still be referred to Eq.(90), and the output current of the rectifier bridge is expressed as

$$i_{s2} = nI_n(i_{rN}-i_{mN}) = nI_n\left(r_3\sin\left(\omega_{r0}(t-t_5)+\theta_3\right)-r_2\sin(\theta_2)+\frac{M}{L_n}\omega_{r0}(t-t_3)\right) \tag{97}$$

From t_3 to t_6, the average value of i_{s2} over half a switching cycle can be expressed as follows.

$$\begin{aligned} \overline{i_{s2}} &= \frac{2}{T_s}\int_{t_3}^{t_6} i_{s2}dt = \frac{2nI_n}{T_s}\int_{t_3}^{t_6}(i_{rN}-i_{mN})dt = \frac{2nI_n}{T_s}\int_{t_3}^{t_6} i_{rN}dt - \int_{t_3}^{t_6} i_{mN}dt \\ &= \frac{2nI_n}{T_s}\int_{t_3}^{t_5} r_2\sin\left[\omega_{r0}(t-t_3)+\theta_2\right]dt + \frac{2nI_n}{T_s}\int_{t_5}^{t_6} r_3\sin\left[\omega_{r0}(t-t_5)+\theta_3\right]dt - 0 \\ &= \frac{2nI_n}{T_s\omega_{r0}}\left(r_2\cos(\theta_2)-r_2\cos(\varphi_2+\theta_2)+r_3\cos(\theta_3)-r_3\cos(\varphi_3+\theta_3)\right) \end{aligned} \tag{98}$$

Section VI. Calculation of steady-state operating point for NP mode

Because of the semi-period symmetry, the i_{r0N} and v_{cr0N} at t_0 are equal to the negative of i_{r3N} and v_{cr3N} respectively. Therefore, (99) can be obtained.

$$\begin{aligned} i_{r3N} &= r_1 \sin(\varphi_1+\theta_1) = -i_{r0N} = -r_0 \sin(\theta_0) \\ v_{cr3N} &= -r_1 \cos(\varphi_1+\theta_1) - (1+M) = -v_{cr0N} = -[-r_0 \cos(\theta_0) + (1-M)] \end{aligned} \tag{99}$$

Mode P transitions to Mode N at t_2, and the resonant current i_{rN} equal to the magnetizing current i_{mN} at t_3, (100) can be obtained.

$$\begin{aligned} & i_{r2N} = r_0 \sin(\varphi_0+\theta_0) = r_1 \sin(\theta_1) \\ & v_{cr2N} = -r_0 \cos(\varphi_0+\theta_0) + (1-M) = -r_1 \cos(\theta_1) - (1+M) \\ & i_{rec}(t_3) = nI_n\left(i_{rN}(t_3) - i_{mN}(t_3)\right) \\ & = nI_n\left(r_1 \sin(\varphi_1+\theta_1) - r_0 \sin(\theta_0) - \frac{M\omega_{r0}}{L_n}\frac{T_s}{2}\right) = nI_n\left(-2r_0 \sin(\theta_0) - \frac{M\omega_{r0}}{L_n}\frac{T_s}{2}\right) = 0 \end{aligned} \tag{100}$$

At steady state, $\overline{i}_{s1} = -\overline{i}_{s2}$, $M = \overline{i}_{s1}R$ .According to the definition of M, φ_0 and φ_1, (101) can be obtained.

$$\begin{aligned} & M = \frac{nV_o}{V_{in}} = \frac{n\overline{i}_{rec1}R}{V_{in}} = \frac{2n^2RI_n}{V_{in}T_s\omega_{r0}}\left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0+\theta_0) + r_1 \cos(\theta_1) - r_1 \cos(\varphi_1+\theta_1)\right) \\ & \varphi_0 + \varphi_1 - \frac{\omega_{r0}T_s}{2} = 0 \end{aligned} \tag{101}$$

Therefore, the following system of equations can be obtained

$$\begin{cases} r_0 \sin(\theta_0) + \dfrac{M\omega_{r0}T_s}{4L_n} = 0 \\ r_0 \sin(\varphi_0+\theta_0) - r_1 \sin(\theta_1) = 0 \\ -r_0 \cos(\varphi_0+\theta_0) + r_1 \cos(\theta_1) + 2 = 0 \\ r_1 \sin(\varphi_1+\theta_1) + r_0 \sin(\theta_0) = 0 \\ -r_1 \cos(\varphi_1+\theta_1) - r_0 \cos(\theta_0) - 2M = 0 \\ M - \dfrac{2n^2RI_n}{V_{in}T_s\omega_{r0}}\left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0+\theta_0) + r_1 \cos(\theta_1) - r_1 \cos(\varphi_1+\theta_1)\right) = 0 \\ \varphi_0 + \varphi_1 - \dfrac{\omega_{r0}T_s}{2} = 0 \end{cases} \tag{102}$$

$\begin{bmatrix} r_0 & \theta_0 & \varphi_0 & r_1 & \theta_1 & \varphi_1 & M \end{bmatrix}$ is defined as the variables to be solved under the steady state. By using the Newton-Raphson iteration method, the solution of the equations can be calculated, so the steady-state operating point of the system will be obtained, and then steady-state current and voltage values I_{r0N}, I_{r2N}, I_{r3N}, I_{r5N}, I_{r6N}, V_{r0N}, V_{r2N}, V_{r3N}, V_{r5N}, and V_{r6N} at different moments can be obtained.

Section VII. Small-signal model of the LLC converter for NP mode with PFM

Set $x=[i_{r0N}, v_{cr0N}, v_o]^T$ as state variables, $u=[v_{in}, t_s]^T$ as input variables, and $y=v_o$ as output variable. The state-space expression for the system can be expressed as (103), where C=[0, 0, 1].

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{103}$$

The large-signal model of the LLC converter over one switching cycle is expressed as follows:

$$\begin{cases} \dot{i}_{r0N} = \dfrac{i_{r6N} - i_{r0N}}{t_s} \\ \dot{v}_{cr0N} = \dfrac{v_{cr6N} - v_{cr0N}}{t_s} \\ \dot{v}_o = \dfrac{1}{C_o}\left(\bar{i}_{rec} - \dfrac{v_o}{R}\right) \end{cases} \tag{104}$$

In this derivation for the small-signal model of the LLC converter, $\boldsymbol{g}$, $\boldsymbol{h}$, $\boldsymbol{k}$, $\boldsymbol{l}$, $\boldsymbol{m}$ represent the partial derivatives of the $\boldsymbol{\theta}$, $\boldsymbol{r}$, $\boldsymbol{i_{rN}}$, $\boldsymbol{v_{crN}}$, and $\boldsymbol{\varphi}$ to the corresponding variables. The above variables can be expressed as the quiescent-state operating point plus the disturbances.

$$\begin{cases} v_{in} = V_{in} + \hat{v}_{in} \\ v_o = V_o + \hat{v}_o \\ t_s = T_s + \hat{t}_s \\ i_{r0N} = I_{r0N} + \hat{i}_{r0N} \\ v_{cr0N} = V_{cr0N} + \hat{v}_{cr0N} \end{cases} \tag{105}$$

*In the subsequent derivation of the small-signal modeling, all variables i_{r0N}, v_{cr0N}, M, v_{in}, v_o, r_0, θ_0, φ_0, i_{r2N}, v_{cr2N}, r_2, θ_2, φ_2, etc., represent steady-state values, which can be calculated through iteration of the steady-state operating point equations. ^ and Δ present the small disturbance.

From t_0 to t_3 with half a switch period, time-domain expressions are as follows:

$$\begin{cases} i_{r0N} = r_0 \sin(\theta_0) \\ v_{cr0N} = -r_0 \cos(\theta_0) + (1-M) \\ i_{r2N} = r_0 \sin(\varphi_0 + \theta_0) = r_1 \sin(\theta_1) \\ v_{cr2N} = -r_0 \cos(\varphi_0 + \theta_0) + (1-M) = -r_1 \cos(\theta_1) - (1+M) \\ i_{r3N} = r_1 \sin(\varphi_1 + \theta_1) \\ v_{cr3N} = -r_1 \cos(\varphi_1 + \theta_1) - (1+M) \\ i_{s1}(t_3) = nI_n \left(r_1 \sin(\varphi_1 + \theta_1) - r_0 \sin(\theta_0) - \dfrac{M \omega_{r0}}{L_n} \dfrac{T_s}{2} \right) = 0 \\ \bar{i}_{s1} = \dfrac{nI_n}{\omega_{r0} T_s} \left(r_0 \cos(\theta_0) - r_0 \cos(\varphi_0 + \theta_0) + r_1 \cos(\theta_1) - r_1 \cos(\varphi_1 + \theta_1) \right) \\ \varphi_0 = \dfrac{\omega_{r0} T_s}{2} - \varphi_1 \end{cases} \tag{106}$$

At time t_0, the converter starts to operate in mode P. θ_0 and r_0 can be calculated by

$$\theta_0 = \arctan\left(-\frac{i_{r0N}}{v_{cr0N} - (1-M)} \right) \qquad r_0 = \sqrt{i_{r0N}^{\ 2} + \left[v_{cr0N} - (1-M) \right]^2} \tag{107}$$

The first-order linearization of θ_0 and r_0 is shown in the following.

$$
\begin{aligned}
&\theta_0+\Delta\theta_0=\theta_0+\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\\
&=\theta_0-\frac{v_{cr0N}-(1-M)}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{i}_{r0N}+\frac{i_{r0N}}{\left[v_{cr0N}-(1-M)\right]^2+i_{r0N}{}^2}\hat{v}_{cr0N}-\frac{i_{r0N}M/v_{in}}{r_0^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0^2}\hat{v}_o\\
&=\theta_0-\frac{v_{cr0N}-(1-M)}{r_0^2}\hat{i}_{r0N}+\frac{i_{r0N}}{r_0^2}\hat{v}_{cr0N}-\frac{i_{r0N}M/v_{in}}{r_0^2}\hat{v}_{in}+\frac{ni_{r0N}/v_{in}}{r_0^2}\hat{v}_o\\
&=\theta_0+g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{cr0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o
\end{aligned}
$$

where

$$
g_{0i}=\frac{\partial\theta_0}{\partial i_{r0N}}=-\frac{v_{r0N}-(1-M)}{r_0^2},\quad g_{0v}=\frac{\partial\theta_0}{\partial v_{cr0N}}=\frac{i_{r0N}}{r_0^2}
$$

$$
g_{0in}=\frac{\partial\theta_0}{\partial v_{in}}=-\frac{i_{r0N}M/v_{in}}{r_0^2},\quad g_{0o}=\frac{\partial\theta_0}{\partial v_o}=\frac{ni_{r0N}/v_{in}}{r_0^2}
$$

$$
\begin{aligned}
&r_0+\Delta r_0=r_0+\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\\
&=r_0+\frac{i_{r0N}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{i}_{r0N}+\frac{v_{cr0N}-(1-M)}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{v}_{cr0N}\\
&-\frac{\left[v_{cr0N}-(1-M)\right]M/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{v}_{in}+\frac{\left[v_{cr0N}-(1-M)\right]n/v_{in}}{\sqrt{i_{r0N}{}^2+\left[v_{cr0N}-(1-M)\right]^2}}\hat{v}_o\\
&=r_0+\frac{i_{r0N}}{r_0}\hat{i}_{r0N}+\frac{v_{cr0N}-(1-M)}{r_0}\hat{v}_{cr0N}-\frac{\left[v_{cr0N}-(1-M)\right]M/v_{in}}{r_0}\hat{v}_{in}+\frac{\left[v_{cr0N}-(1-M)\right]n/v_{in}}{r_0}\hat{v}_o\\
&=r_0+h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{cr0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o
\end{aligned}
\tag{108}
$$

where

$$
h_{0i}=\frac{\partial r_0}{\partial i_{r0N}}=\frac{i_{r0N}}{r_0},\qquad h_{0v}=\frac{\partial r_0}{\partial v_{cr0N}}=\frac{v_{cr0N}-(1-M)}{r_0},
$$

$$
h_{0in}=\frac{\partial r_0}{\partial v_{in}}=-\frac{\left[v_{cr0N}-(1-M)\right]M/v_{in}}{r_0},\qquad h_{0o}=\frac{\left[v_{cr0N}-(1-M)\right]n/v_{in}}{r_0}
$$

At time t_2, Δi_{r2N} and Δv_{r2N} can be expressed as follows:

$$
\begin{aligned}
& i_{r2N}+\Delta i_{r2N}=r_0\sin(\varphi_0+\theta_0)+\frac{\partial i_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r2N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial i_{r2N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r2N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r2N}}{\partial \varphi_0}\Delta\varphi_0 \\
& =i_{r2N}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{cr0N}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial i_{r2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}\right)\hat{v}_o+\frac{\partial i_{r2N}}{\partial \varphi_0}\Delta\varphi_0 \\
& =i_{r2N}+\left[\sin(\varphi_0+\theta_0)h_{0i}+r_0\cos(\varphi_0+\theta_0)g_{0i}\right]\hat{i}_{r0N} \\
& +\left[\sin(\varphi_0+\theta_0)h_{0v}+r_0\cos(\varphi_0+\theta_0)g_{0v}\right]\hat{v}_{cr0N} \\
& +\left[\sin(\varphi_0+\theta_0)h_{0in}+r_0\cos(\varphi_0+\theta_0)g_{0in}\right]\hat{v}_{in} \\
& +\left[\sin(\varphi_0+\theta_0)h_{0o}+r_0\cos(\varphi_0+\theta_0)g_{0o}\right]\hat{v}_o+r_0\cos(\varphi_0+\theta_0)\Delta\varphi_0 \\
& =i_{r2N}+k_{2i}\hat{i}_{r0N}+k_{2v}\hat{v}_{cr0N}+k_{2in}\hat{v}_{in}+k_{2o}\hat{v}_o+k_{2m0}\Delta\varphi_0 \\
& \text{where} \\
& k_{2i}=\frac{\partial i_{r2N}}{\partial i_{r0N}}=\sin(\varphi_0+\theta_0)h_{0i}+r_0\cos(\varphi_0+\theta_0)g_{0i} \\
& k_{2v}=\frac{\partial i_{r2N}}{\partial v_{cr0N}}=\sin(\varphi_0+\theta_0)h_{0v}+r_0\cos(\varphi_0+\theta_0)g_{0v} \\
& k_{2in}=\frac{\partial i_{r2N}}{\partial v_{in}}=\sin(\varphi_0+\theta_0)h_{0in}+r_0\cos(\varphi_0+\theta_0)g_{0in} \\
& k_{2o}=\frac{\partial i_{r2N}}{\partial v_o}=\sin(\varphi_0+\theta_0)h_{0o}+r_0\cos(\varphi_0+\theta_0)g_{0o} \\
& k_{2m0}=\frac{\partial i_{r2N}}{\partial \varphi_0}=r_0\cos(\varphi_0+\theta_0) \\
& \\
& v_{cr2N}+\Delta v_{cr2N}=-r_0\cos(\varphi_0+\theta_0)+(1-M)+\frac{\partial v_{cr2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{cr2N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial v_{cr2N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr2N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{cr2N}}{\partial \varphi_0}\Delta\varphi_0 \\
& =v_{cr2N}+\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial i_{r0N}}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{cr0N}}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial v_{in}}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{cr2N}}{\partial r_0}\frac{\partial r_0}{\partial v_o}+\frac{\partial v_{cr2N}}{\partial \theta_0}\frac{\partial \theta_0}{\partial v_o}\right)\hat{v}_o+\frac{\partial v_{cr2N}}{\partial \varphi_0}\Delta\varphi_0 \\
& =v_{cr2N}+\left[-\cos(\varphi_0+\theta_0)h_{0i}+r_0\sin(\varphi_0+\theta_0)g_{0i}\right]\hat{i}_{r0N} \\
& +\left[-\cos(\varphi_0+\theta_0)h_{0v}+r_0\sin(\varphi_0+\theta_0)g_{0v}\right]\hat{v}_{cr0N} \\
& +\left[-\cos(\varphi_0+\theta_0)h_{0in}+r_0\sin(\varphi_0+\theta_0)g_{0in}+\frac{M}{v_{in}}\right]\hat{v}_{in} \\
& +\left[-\cos(\varphi_0+\theta_0)h_{0o}+r_0\sin(\varphi_0+\theta_0)g_{0o}-\frac{n}{v_{in}}\right]\hat{v}_o+r_0\sin(\varphi_0+\theta_0)\Delta\varphi_0 \\
& =v_{r2N}+l_{2i}\hat{i}_{r0N}+l_{2v}\hat{v}_{cr0N}+l_{2in}\hat{v}_{in}+l_{2o}\hat{v}_o+l_{2m0}\Delta\varphi_0 \\
& \text{where} \\
& l_{2i}=\frac{\partial v_{cr2N}}{\partial i_{r0N}}=-\cos(\varphi_0+\theta_0)h_{0i}+r_0\sin(\varphi_0+\theta_0)g_{0i} \\
& l_{2v}=\frac{\partial v_{cr2N}}{\partial v_{cr0N}}=-\cos(\varphi_0+\theta_0)h_{0v}+r_0\sin(\varphi_0+\theta_0)g_{0v} \\
& l_{2in}=\frac{\partial v_{cr2N}}{\partial v_{in}}=-\cos(\varphi_0+\theta_0)h_{0in}+r_0\sin(\varphi_0+\theta_0)g_{0in}+\frac{M}{v_{in}} \\
& l_{2o}=\frac{\partial v_{cr2N}}{\partial v_o}=-\cos(\varphi_0+\theta_0)h_{0o}+r_0\sin(\varphi_0+\theta_0)g_{0o}-\frac{n}{v_{in}} \\
& l_{2m0}=\frac{\partial v_{cr2N}}{\partial \varphi_0}=r_0\sin(\varphi_0+\theta_0)
\end{aligned}
\tag{109}
$$

$\Delta\varphi_0$ is not the state variable and input variable, but it will be canceled in the analysis of $i_{s1N}(t_3)$=0.

$\Delta\theta_1$ and Δr_1 can be calculated by

$$\begin{aligned}
&\theta_1+\Delta\theta_1=\theta_1+\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{r0N}}\hat{v}_{cr0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o+\frac{\partial\theta_1}{\partial\varphi_0}\Delta\varphi_0\\
&=\theta_1+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{cr0N}}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}\\
&+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{in}}-\frac{M}{v_{in}}\frac{i_{r2N}}{r_1^2}\right)\hat{v}_{in}+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_o}+\frac{n}{v_{in}}\frac{i_{r2N}}{r_1^2}\right)\hat{v}_o\\
&+\left(\frac{\partial\theta_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial\varphi_0}+\frac{\partial\theta_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial\varphi_0}\right)\Delta\varphi_0\\
&=\theta_1+\left[-\frac{v_{cr2N}+1+M}{r_1^2}k_{2i}+\frac{i_{r2N}}{r_1^2}l_{2i}\right]\hat{i}_{r0N}+\left[-\frac{v_{cr2N}+1+M}{r_1^2}k_{2v}+\frac{i_{r2N}}{r_1^2}l_{2v}\right]\hat{v}_{cr0N}\\
&+\left[-\frac{v_{cr2N}+1+M}{r_1^2}k_{2in}+\frac{i_{r2N}}{r_1^2}l_{2in}-\frac{M}{v_{in}}\frac{i_{r2N}}{r_1^2}\right]\hat{v}_{in}+\left[-\frac{v_{cr2N}+1+M}{r_1^2}k_{2o}+\frac{i_{r2N}}{r_1^2}l_{2o}+\frac{n}{v_{in}}\frac{i_{r2N}}{r_1^2}\right]\hat{v}_o\\
&+\left(-\frac{v_{cr2N}+1+M}{r_1^2}k_{2m0}+\frac{i_{r2N}}{r_1^2}l_{2m0}\right)\Delta\varphi_0\\
&=\theta_1+g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0
\end{aligned}\tag{110}$$

where

$$g_{1i}=\frac{\partial\theta_1}{\partial i_{r0N}}=-\frac{v_{cr2N}+1+M}{r_1^2}k_{2i}+\frac{i_{r2N}}{r_1^2}l_{2i},\quad g_{1v}=\frac{\partial\theta_1}{\partial v_{cr0N}}=-\frac{v_{cr2N}+1+M}{r_1^2}k_{2v}+\frac{i_{r2N}}{r_1^2}l_{2v},$$

$$g_{1in}=\frac{\partial\theta_1}{\partial v_{in}}=-\frac{v_{cr2N}+1+M}{r_1^2}k_{2in}+\frac{i_{r2N}}{r_1^2}l_{2in}-\frac{M}{v_{in}}\frac{i_{r2N}}{r_1^2},\quad g_{1o}=\frac{\partial\theta_1}{\partial v_o}=-\frac{v_{cr2N}+1+M}{r_1^2}k_{2o}+\frac{i_{r2N}}{r_1^2}l_{2o}+\frac{n}{v_{in}}\frac{i_{r2N}}{r_1^2},$$

$$g_{1m0}=\frac{\partial\theta_1}{\partial\varphi_0}=-\frac{v_{cr2N}+1+M}{r_1^2}k_{2m0}+\frac{i_{r2N}}{r_1^2}l_{2m0}$$

$$\begin{aligned}
&r_1+\Delta r_1=r_1+\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o+\frac{\partial r_1}{\partial\varphi_0}\Delta\varphi_0\\
&=r_1+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial i_{r0N}}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{cr0N}}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}\\
&+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_{in}}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_{in}}-\frac{M}{v_{in}}\frac{v_{cr2N}+1+M}{r_1}\right)\hat{v}_{in}+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial v_o}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial v_o}+\frac{n}{v_{in}}\frac{v_{cr2N}+1+M}{r_1}\right)\hat{v}_o\\
&+\left(\frac{\partial r_1}{\partial i_{r2N}}\frac{\partial i_{r2N}}{\partial\varphi_0}+\frac{\partial r_1}{\partial v_{cr2N}}\frac{\partial v_{cr2N}}{\partial\varphi_0}\right)\Delta\varphi_0\\
&=r_1+\left(\frac{i_{r2N}}{r_1}k_{2i}+\frac{v_{cr2N}+1+M}{r_1}l_{2i}\right)\hat{i}_{r0N}+\left(\frac{i_{r2N}}{r_1}k_{2v}+\frac{v_{cr2N}+1+M}{r_1}l_{2v}\right)\hat{v}_{cr0N}\\
&+\left(\frac{i_{r2N}}{r_1}k_{2in}+\frac{v_{cr2N}+1+M}{r_1}l_{2in}-\frac{M}{v_{in}}\frac{v_{cr2N}+1+M}{r_1}\right)\hat{v}_{in}+\left(\frac{i_{r2N}}{r_1}k_{2o}+\frac{v_{cr2N}+1+M}{r_1}l_{2o}+\frac{n}{v_{in}}\frac{v_{cr2N}+1+M}{r_1}\right)\hat{v}_o\\
&+\left(\frac{i_{r2N}}{r_1}k_{2m0}+\frac{v_{cr2N}+1+M}{r_1}l_{2m0}\right)\Delta\varphi_0\\
&=r_1+h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0
\end{aligned}$$

where

$$h_{1i}=\frac{\partial r_1}{\partial i_{r0N}}=\frac{i_{r2N}}{r_1}k_{2i}+\frac{v_{cr2N}+1+M}{r_1}l_{2i},\quad h_{1v}=\frac{\partial r_1}{\partial v_{cr0N}}=\frac{i_{r2N}}{r_1}k_{2v}+\frac{v_{cr2N}+1+M}{r_1}l_{2v},$$

$$h_{1in}=\frac{\partial r_1}{\partial v_{in}}=\frac{i_{r2N}}{r_1}k_{2in}+\frac{v_{cr2N}+1+M}{r_1}l_{2in}-\frac{M}{v_{in}}\frac{v_{cr2N}+1+M}{r_1},\quad h_{1o}=\frac{i_{r2N}}{r_1}k_{2o}+\frac{v_{cr2N}+1+M}{r_1}l_{2o}+\frac{n}{v_{in}}\frac{v_{cr2N}+1+M}{r_1},$$

$$h_{1m0}=\frac{i_{r2N}}{r_1}k_{2m0}+\frac{v_{cr2N}+1+M}{r_1}l_{2m0}$$

At time t_3, $i_{s1N}(t_3)=0$, and $i_{s1N}(t_3+\Delta t_3)=0$ after the disturbances are added. Therefore, (111) can be obtained

$$
\begin{aligned}
& i_{s1N}\left(t_3+\Delta t_3\right)=n\left(\left(r_1+\Delta r_1\right)\sin\left(\varphi_1+\Delta\varphi_1+\theta_1+\Delta\theta_1\right)-\left(r_0+\Delta r_0\right)\sin\left(\theta_0+\Delta\theta_0\right)-\frac{n\left(v_o+\Delta v_o\right)}{\left(v_{in}+\Delta v_{in}\right)L_n}\frac{\omega_{r0}T_s}{2}\right) \\
& \approx n\left(r_1\sin\left(\varphi_1+\theta_1\right)-r_0\sin\left(\theta_0\right)-\frac{M}{L_n}\frac{\omega_{r0}T_s}{2}\right)+\frac{\partial i_{s1N}\left(t_3\right)}{\partial r_0}\Delta r_0+\frac{\partial i_{s1N}\left(t_3\right)}{\partial r_1}\Delta r_1+\frac{\partial i_{s1N}\left(t_3\right)}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial i_{s1N}\left(t_3\right)}{\partial\theta_0}\Delta\theta_0+ \\
& \frac{\partial i_{s1N}\left(t_3\right)}{\partial\theta_1}\Delta\theta_1+\frac{\partial i_{s1N}\left(t_3\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{s1N}\left(t_3\right)}{\partial v_o}\hat{v}_o+\frac{\partial i_{s1N}\left(t_3\right)}{\partial t_s}\hat{t}_s \\
& =i_{s1N}\left(t_3\right)+\frac{\partial i_{s1N}\left(t_3\right)}{\partial r_0}\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{s1N}\left(t_3\right)}{\partial r_1}\left(\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o+\frac{\partial r_1}{\partial\varphi_0}\Delta\varphi_0\right) \\
& +\frac{\partial i_{s1N}\left(t_3\right)}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial i_{s1N}\left(t_3\right)}{\partial\theta_0}\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right)+\frac{\partial i_{s1N}\left(t_3\right)}{\partial\theta_1}\left(\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o+\frac{\partial\theta_1}{\partial\varphi_0}\Delta\varphi_0\right) \\
& +\frac{\partial i_{recN}\left(t_3\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{recN}\left(t_3\right)}{\partial v_o}\hat{v}_o+\frac{\partial i_{recN}\left(t_3\right)}{\partial t_s}\hat{t}_s \\
& =i_{s1N}\left(t_3\right)+n\left[\begin{array}{l}
-\sin\left(\theta_0\right)\left(\frac{\partial r_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_0}{\partial v_o}\hat{v}_o\right) \\
+\sin\left(\varphi_1+\theta_1\right)\left(\frac{\partial r_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_1}{\partial v_o}\hat{v}_o+\frac{\partial r_1}{\partial\varphi_0}\Delta\varphi_0\right) \\
+r_1\cos\left(\varphi_1+\theta_1\right)\Delta\varphi_1-r_0\cos\left(\theta_0\right)\left(\frac{\partial\theta_0}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_0}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_0}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_0}{\partial v_o}\hat{v}_o\right) \\
+r_1\cos\left(\varphi_1+\theta_1\right)\left(\frac{\partial\theta_1}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_1}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_1}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_1}{\partial v_o}\hat{v}_o+\frac{\partial r_1}{\partial\varphi_0}\Delta\varphi_0\right) \\
+\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\hat{v}_{in}-\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\hat{v}_o-\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s
\end{array}\right] \\
& =i_{s1N}\left(t_3\right)+n\left[\begin{array}{l}
-\sin\left(\theta_0\right)\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{cr0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right) \\
+\sin\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0\right) \\
+r_1\cos\left(\varphi_1+\theta_1\right)\Delta\varphi_1-r_0\cos\left(\theta_0\right)\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{cr0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right) \\
+r_1\cos\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0\right) \\
+\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\hat{v}_{in}-\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\hat{v}_o-\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s
\end{array}\right] \\
& =i_{s1N}\left(t_3\right)+n\left[\begin{array}{l}
\left[-\sin\left(\theta_0\right)h_{0i}+\sin\left(\varphi_1+\theta_1\right)h_{1i}-r_0\cos\left(\theta_0\right)g_{0i}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1i}\right]\hat{i}_{r0N}+ \\
\left[-\sin\left(\theta_0\right)h_{0v}+\sin\left(\varphi_1+\theta_1\right)h_{1v}-r_0\cos\left(\theta_0\right)g_{0v}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1v}\right]\hat{v}_{cr0N}+ \\
\left[-\sin\left(\theta_0\right)h_{0in}+\sin\left(\varphi_1+\theta_1\right)h_{1in}-r_0\cos\left(\theta_0\right)g_{0in}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1in}+\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\
+\left[-\sin\left(\theta_0\right)h_{0v}+\sin\left(\varphi_1+\theta_1\right)h_{1v}-r_0\cos\left(\theta_0\right)g_{0v}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1v}-\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\
-\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s+r_1\cos\left(\varphi_1+\theta_1\right)\Delta\varphi_1+\left[\sin\left(\varphi_1+\theta_1\right)h_{1m0}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]\Delta\varphi_0
\end{array}\right]=0
\end{aligned}
\tag{111}
$$

Because $i_{s1N}(t_3)=0$, the following equation can be obtained.

$$
\left[\begin{array}{l}
\left[-\sin\left(\theta_0\right)h_{0i}+\sin\left(\varphi_1+\theta_1\right)h_{1i}-r_0\cos\left(\theta_0\right)g_{0i}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1i}\right]\hat{i}_{r0N}+ \\
\left[-\sin\left(\theta_0\right)h_{0v}+\sin\left(\varphi_1+\theta_1\right)h_{1v}-r_0\cos\left(\theta_0\right)g_{0v}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1v}\right]\hat{v}_{cr0N}+ \\
\left[-\sin\left(\theta_0\right)h_{0in}+\sin\left(\varphi_1+\theta_1\right)h_{1in}-r_0\cos\left(\theta_0\right)g_{0in}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1in}+\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\
+\left[-\sin\left(\theta_0\right)h_{0v}+\sin\left(\varphi_1+\theta_1\right)h_{1v}-r_0\cos\left(\theta_0\right)g_{0v}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1v}-\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\
-\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s+r_1\cos\left(\varphi_1+\theta_1\right)\Delta\varphi_1+\left[\sin\left(\varphi_1+\theta_1\right)h_{1m0}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]\Delta\varphi_0
\end{array}\right]=0
\tag{112}
$$

Substituting $\Delta\varphi_0 = \frac{\omega_{r0}\hat{t}_s}{2} - \Delta\varphi_1$ into Eq.(112), Eq.(113) can be obtained.

$$\begin{bmatrix} \left[-\sin(\theta_0)h_{0i} + \sin(\varphi_1+\theta_1)h_{1i} - r_0\cos(\theta_0)g_{0i} + r_1\cos(\varphi_1+\theta_1)g_{1i}\right]\hat{i}_{r0N} + \\ \left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v}\right]\hat{v}_{cr0N} + \\ \left[-\sin(\theta_0)h_{0in} + \sin(\varphi_1+\theta_1)h_{1in} - r_0\cos(\theta_0)g_{0in} + r_1\cos(\varphi_1+\theta_1)g_{1in} + \frac{\omega_{r0}T_s M/v_{in}}{2L_n}\right]\hat{v}_{in} \\ + \left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}\right]\hat{v}_o \\ + \frac{\omega_{r0}}{2}\left[\sin(\varphi_1+\theta_1)h_{1m0} + r_1\cos(\varphi_1+\theta_1)g_{1m0} - \frac{M}{L_n}\right]\hat{t}_s \\ + \left[r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]\Delta\varphi_1 \end{bmatrix} = 0 \tag{113}$$

$\Delta\varphi_1$ can be calculated by

$$\Delta\varphi_1 = -\frac{\begin{bmatrix} \left[-\sin(\theta_0)h_{0i} + \sin(\varphi_1+\theta_1)h_{1i} - r_0\cos(\theta_0)g_{0i} + r_1\cos(\varphi_1+\theta_1)g_{1i}\right]\hat{i}_{r0N} + \\ \left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v}\right]\hat{v}_{cr0N} + \\ \left[-\sin(\theta_0)h_{0in} + \sin(\varphi_1+\theta_1)h_{1in} - r_0\cos(\theta_0)g_{0in} + r_1\cos(\varphi_1+\theta_1)g_{1in} + \frac{\omega_{r0}T_s M/v_{in}}{2L_n}\right]\hat{v}_{in} \\ + \left[-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}\right]\hat{v}_o \\ + \frac{\omega_{r0}}{2}\left[\sin(\varphi_1+\theta_1)h_{1m0} + r_1\cos(\varphi_1+\theta_1)g_{1m0} - \frac{M}{L_n}\right]\hat{t}_s \end{bmatrix}}{\left[r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}\right]}$$

$$= m_{1i}\hat{i}_{r0N} + m_{1v}\hat{v}_{cr0N} + m_{1in}\hat{v}_{in} + m_{1o}\hat{v}_o + m_{1t}\hat{t}_s$$

where

$$\begin{aligned} m_{1i} &= \frac{\partial\varphi_1}{\partial i_{r0N}} = -\frac{-\sin(\theta_0)h_{0i} + \sin(\varphi_1+\theta_1)h_{1i} - r_0\cos(\theta_0)g_{0i} + r_1\cos(\varphi_1+\theta_1)g_{1i}}{r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}} \\ m_{1v} &= \frac{\partial\varphi_1}{\partial v_{cr0N}} = -\frac{-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v}}{r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}} \\ m_{1in} &= \frac{\partial\varphi_1}{\partial v_{in}} = -\frac{-\sin(\theta_0)h_{0in} + \sin(\varphi_1+\theta_1)h_{1in} - r_0\cos(\theta_0)g_{0in} + r_1\cos(\varphi_1+\theta_1)g_{1in} + \frac{\omega_{r0}T_s M/v_{in}}{2L_n}}{r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}} \\ m_{1o} &= \frac{\partial\varphi_1}{\partial v_o} = -\frac{-\sin(\theta_0)h_{0v} + \sin(\varphi_1+\theta_1)h_{1v} - r_0\cos(\theta_0)g_{0v} + r_1\cos(\varphi_1+\theta_1)g_{1v} - \frac{\omega_{r0}T_s n/v_{in}}{2L_n}}{r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}} \\ m_{1t} &= \frac{\partial\varphi_1}{\partial t_s} = -\frac{\frac{\omega_{r0}}{2}\left[\sin(\varphi_1+\theta_1)h_{1m0} + r_1\cos(\varphi_1+\theta_1)g_{1m0} - \frac{M}{L_n}\right]}{r_1\cos(\varphi_1+\theta_1) - \sin(\varphi_1+\theta_1)h_{1m0} - r_1\cos(\varphi_1+\theta_1)g_{1m0}} \end{aligned} \tag{114}$$

Δi_{r3N} and Δv_{r3N} can be calculated by

$$
\begin{aligned}
&i_{r3N}+\Delta i_{r3N}=\left(r_1+\Delta r_1\right)\sin\left(\varphi_1+\Delta\varphi_1+\theta_1+\Delta\theta_1\right)=r_1\sin\left(\varphi_1+\theta_1\right)+\frac{\partial i_{r3N}}{\partial r_1}\Delta r_1+\frac{\partial i_{r3N}}{\partial \theta_1}\Delta\theta_1+\frac{\partial i_{r3N}}{\partial \varphi_1}\Delta\varphi_1\\
&=i_{r3N}+\frac{\partial i_{r3N}}{\partial r_1}\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0\right)\\
&+\frac{\partial i_{r3N}}{\partial \theta_1}\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0\right)+\frac{\partial i_{r3N}}{\partial \varphi_1}\Delta\varphi_1\\
&=i_{r3N}+\sin\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_S-h_{1m0}\Delta\varphi_1\right)\\
&+r_1\cos\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_S-g_{1m0}\Delta\varphi_1\right)+r_1\cos\left(\varphi_1+\theta_1\right)\Delta\varphi_1\\
&=i_{r3N}+\sin\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_S\right)\\
&+r_1\cos\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_S\right)\\
&+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]\left(m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{cr0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\right)\\
&=i_{r3N}+\left[\sin\left(\varphi_1+\theta_1\right)h_{1i}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1i}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1i}\right]\hat{i}_{r0N}\\
&+\left[\sin\left(\varphi_1+\theta_1\right)h_{1v}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1v}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1v}\right]\hat{v}_{cr0N}\\
&+\left[\sin\left(\varphi_1+\theta_1\right)h_{1in}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1in}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1in}\right]\hat{v}_{in}\\
&+\left[\sin\left(\varphi_1+\theta_1\right)h_{1o}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1o}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1o}\right]\hat{v}_o\\
&+\left[\sin\left(\varphi_1+\theta_1\right)h_{1m0}\frac{\omega_{r0}}{2}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\frac{\omega_{r0}}{2}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1t}\right]\hat{t}_s\\
&=i_{r3N}+k_{3i}\hat{i}_{r0N}+k_{3v}\hat{v}_{cr0N}+k_{3in}\hat{v}_{in}+k_{3o}\hat{v}_o+k_{3t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
&k_{3i}=\frac{\partial i_{r3N}}{\partial i_{r0N}}=\sin\left(\varphi_1+\theta_1\right)h_{1i}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1i}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1i}\\
&k_{3v}=\frac{\partial i_{r3N}}{\partial v_{cr0N}}=\sin\left(\varphi_1+\theta_1\right)h_{1v}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1v}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1v}\\
&k_{3in}=\frac{\partial i_{r3N}}{\partial v_{in}}=\sin\left(\varphi_1+\theta_1\right)h_{1in}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1in}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1in}\\
&k_{3o}=\frac{\partial i_{r3N}}{\partial v_o}=\sin\left(\varphi_1+\theta_1\right)h_{1o}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1o}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1o}\\
&k_{3t}=\frac{\partial i_{r3N}}{\partial t_s}=\sin\left(\varphi_1+\theta_1\right)h_{1m0}\frac{\omega_{r0}}{2}+r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\frac{\omega_{r0}}{2}+\left[r_1\cos\left(\varphi_1+\theta_1\right)-\sin\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\cos\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1t}
\end{aligned}
$$

$$
\begin{aligned}
&v_{cr3N}+\Delta v_{cr3N}=\left(-r_1\cos\left(\varphi_1+\theta_1\right)-(1+M)\right)+\frac{\partial v_{cr3N}}{\partial r_1}\Delta r_1+\frac{\partial v_{cr3N}}{\partial \theta_1}\Delta\theta_1+\frac{\partial v_{cr3N}}{\partial \varphi_1}\Delta\varphi_1+\frac{\partial v_{cr3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr3N}}{\partial v_o}\hat{v}_o\\
&=v_{cr3N}+\frac{\partial v_{cr3N}}{\partial r_1}\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\Delta\varphi_0\right)+\frac{\partial v_{cr3N}}{\partial \theta_1}\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\Delta\varphi_0\right)+\frac{\partial v_{cr3N}}{\partial \varphi_1}\Delta\varphi_1+\frac{\partial v_{cr3N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr3N}}{\partial v_o}\hat{v}_o\\
&=v_{cr3N}-\cos\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-h_{1m0}\Delta\varphi_1\right)\\
&+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-g_{1m0}\Delta\varphi_1+\Delta\varphi_1\right)+\frac{M}{v_{in}}\hat{v}_{in}-\frac{n}{v_{in}}\hat{v}_o\\
&=v_{cr3N}-\cos\left(\varphi_1+\theta_1\right)\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)+r_1\sin\left(\varphi_1+\theta_1\right)\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)\\
&+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]\left(m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{cr0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\right)+\frac{M}{v_{in}}\hat{v}_{in}-\frac{n}{v_{in}}\hat{v}_o\\
&=v_{cr3N}+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1i}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1i}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1i}\right]\hat{i}_{r0N}\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1v}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1v}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1v}\right]\hat{v}_{cr0N}\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1in}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1in}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1in}+\frac{M}{v_{in}}\right]\hat{v}_{in}\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)h_{1o}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1o}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1o}-\frac{n}{v_{in}}\right]\hat{v}_o\\
&+\left[-\cos\left(\varphi_1+\theta_1\right)\frac{\omega_{r0}}{2}h_{1m0}+r_1\sin\left(\varphi_1+\theta_1\right)\frac{\omega_{r0}}{2}g_{1m0}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1t}\right]\hat{t}_s\\
&=v_{r3N}+l_{3i}\hat{i}_{r0N}+l_{3v}\hat{v}_{cr0N}+l_{3in}\hat{v}_{in}+l_{3o}\hat{v}_o+l_{3t}\hat{t}_s\\
&where\\
&l_{3i}=\frac{\partial v_{cr3N}}{\partial i_{r0N}}=-\cos\left(\varphi_1+\theta_1\right)h_{1i}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1i}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1i}\\
&l_{3v}=\frac{\partial v_{cr3N}}{\partial v_{cr0N}}=-\cos\left(\varphi_1+\theta_1\right)h_{1v}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1v}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1v}\\
&l_{3in}=\frac{\partial v_{cr3N}}{\partial v_{in}}=-\cos\left(\varphi_1+\theta_1\right)h_{1in}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1in}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1in}+\frac{M}{v_{in}}\\
&l_{3o}=\frac{\partial v_{cr3N}}{\partial v_o}=-\cos\left(\varphi_1+\theta_1\right)h_{1o}+r_1\sin\left(\varphi_1+\theta_1\right)g_{1o}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1o}-\frac{n}{v_{in}}\\
&l_{3t}=\frac{\partial v_{cr3N}}{\partial t_s}=-\cos\left(\varphi_1+\theta_1\right)\frac{\omega_{r0}}{2}h_{1m0}+r_1\sin\left(\varphi_1+\theta_1\right)\frac{\omega_{r0}}{2}g_{1m0}+\left[r_1\sin\left(\varphi_1+\theta_1\right)+\cos\left(\varphi_1+\theta_1\right)h_{1m0}-r_1\sin\left(\varphi_1+\theta_1\right)g_{1m0}\right]m_{1t}
\end{aligned}
\tag{115}
$$

The average output current of the rectifier from t_0 to t_3 can be expressed as

$$\overline{i}_{s1}+\Delta\overline{i}_{s1}=\frac{nI_n}{\omega_{r0}T_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)$$

$$+\frac{\partial\overline{i}_{s1}}{\partial r_0}\Delta r_0+\frac{\partial\overline{i}_{s1}}{\partial r_1}\Delta r_1+\frac{\partial\overline{i}_{s1}}{\partial\theta_0}\Delta\theta_0+\frac{\partial\overline{i}_{s1}}{\partial\theta_1}\Delta\theta_1+\frac{\partial\overline{i}_{s1}}{\partial\varphi_0}\Delta\varphi_0+\frac{\partial\overline{i}_{s1}}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial\overline{i}_{s1}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\overline{i}_{s1}}{\partial t_s}\hat{t}_s$$

$$=\overline{i}_{s1}+\frac{nI_n}{\omega_{r0}T_s}\left[\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)\right]\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{cr0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-h_{1m0}\Delta\varphi_1\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)\right]\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{cr0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s-g_{1m0}\Delta\varphi_1\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[r_0\sin\left(\varphi_0+\theta_0\right)\right]\left(\frac{\omega_{r0}}{2}\hat{t}_s-\Delta\varphi_1\right)+\frac{nI_n}{\omega_{r0}T_s}\left[r_1\sin\left(\varphi_1+\theta_1\right)\right]\Delta\varphi_1$$

$$+\left[\frac{nC_r}{T_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{v}_{in}$$

$$+\left[-\frac{nI_n}{\omega_{r0}T_s^{\,2}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{t}_s$$

$$=\overline{i}_{s1}+\frac{nI_n}{\omega_{r0}T_s}\left[\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)\right]\left(h_{0i}\hat{i}_{r0N}+h_{0v}\hat{v}_{cr0N}+h_{0in}\hat{v}_{in}+h_{0o}\hat{v}_o\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]\left(h_{1i}\hat{i}_{r0N}+h_{1v}\hat{v}_{cr0N}+h_{1in}\hat{v}_{in}+h_{1o}\hat{v}_o+h_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)\right]\left(g_{0i}\hat{i}_{r0N}+g_{0v}\hat{v}_{cr0N}+g_{0in}\hat{v}_{in}+g_{0o}\hat{v}_o\right)$$

$$+\frac{nI_n}{\omega_{r0}T_s}\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]\left(g_{1i}\hat{i}_{r0N}+g_{1v}\hat{v}_{cr0N}+g_{1in}\hat{v}_{in}+g_{1o}\hat{v}_o+g_{1m0}\frac{\omega_{r0}}{2}\hat{t}_s\right)$$

$$+\left[\frac{nC_r}{T_s}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{v}_{in}$$

$$+\left[-\frac{nI_n}{\omega_{r0}T_s^{\,2}}\left(r_0\cos\left(\theta_0\right)-r_0\cos\left(\varphi_0+\theta_0\right)+r_1\cos\left(\theta_1\right)-r_1\cos\left(\varphi_1+\theta_1\right)\right)\right]\hat{t}_s+\frac{nI_n}{\omega_{r0}T_s}\left[r_0\sin\left(\varphi_0+\theta_0\right)\right]\frac{\omega_{r0}}{2}\hat{t}_s$$

$$+\frac{nI_n}{\omega_{r0}T_s}\begin{bmatrix}-\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]h_{1m0}-\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]g_{1m0}\\-r_0\sin\left(\varphi_0+\theta_0\right)+\left[r_1\sin\left(\varphi_1+\theta_1\right)\right]\end{bmatrix}\bullet\left(m_{1i}\hat{i}_{r0N}+m_{1v}\hat{v}_{cr0N}+m_{1in}\hat{v}_{in}+m_{1o}\hat{v}_o+m_{1t}\hat{t}_s\right)$$

$$==\overline{i}_{s1}+\frac{nI_n}{\omega_{r0}T_s}\begin{bmatrix}\left[\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)\right]h_{0i}+\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]h_{1i}\\+\left[-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)\right]g_{0i}+\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]g_{1i}\\+\begin{bmatrix}-\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]h_{1m0}-\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]g_{1m0}\\-r_0\sin\left(\varphi_0+\theta_0\right)+\left[r_1\sin\left(\varphi_1+\theta_1\right)\right]\end{bmatrix}m_{1i}\end{bmatrix}\hat{i}_{r0N}$$

$$+\frac{nI_n}{\omega_{r0}T_s}\begin{bmatrix}\left[\cos\left(\theta_0\right)-\cos\left(\varphi_0+\theta_0\right)\right]h_{0v}+\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]h_{1v}\\+\left[-r_0\sin\left(\theta_0\right)+r_0\sin\left(\varphi_0+\theta_0\right)\right]g_{0v}+\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]g_{1v}\\+\begin{bmatrix}-\left[\cos\left(\theta_1\right)-\cos\left(\varphi_1+\theta_1\right)\right]h_{1m0}-\left[-r_1\sin\left(\theta_1\right)+r_1\sin\left(\varphi_1+\theta_1\right)\right]g_{1m0}\\-r_0\sin\left(\varphi_0+\theta_0\right)+\left[r_1\sin\left(\varphi_1+\theta_1\right)\right]\end{bmatrix}m_{1v}\end{bmatrix}\hat{v}_{cr0N}$$

$$
\begin{aligned}
&+\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_0)-\cos(\varphi_0+\theta_0)\right]h_{0in}+\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1in} \\
+\left[-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)\right]g_{0in}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1in} \\
+\left[\begin{array}{l}
-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0} \\
-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]
\end{array}\right]m_{1in} \\
+\frac{\omega_{r0}T_s}{nI_n}\frac{nC_r}{t_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)+r_1\cos(\theta_1)-r_1\cos(\varphi_1+\theta_1)\right)
\end{array}\right]\hat{v}_{in} \\
&+\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_0)-\cos(\varphi_0+\theta_0)\right]h_{0o}+\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1o} \\
+\left[-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)\right]g_{0o}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1o} \\
+\left[\begin{array}{l}
-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0} \\
-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]
\end{array}\right]m_{1o}
\end{array}\right]\hat{v}_o \\
&+\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}\frac{\omega_{r0}}{2}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}\frac{\omega_{r0}}{2} \\
+\left[-\frac{1}{T_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)+r_1\cos(\theta_1)-r_1\cos(\varphi_1+\theta_1)\right)\right] \\
+\left[\begin{array}{l}
-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0} \\
-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]
\end{array}\right]m_{1t} \\
+\left[r_0\sin(\varphi_0+\theta_0)\right]\frac{\omega_{r0}}{2}
\end{array}\right]\hat{t}_s \\
&=\overline{i}_{s1}+k_{s1i}\hat{i}_{r0N}+k_{s1v}\hat{v}_{cr0N}+k_{s1in}\hat{v}_{in}+k_{s1o}\hat{v}_o+k_{s1t}\hat{t}_s
\end{aligned}
\tag{116}
$$

where

$$
k_{s1i}=\frac{\partial i_{s1}}{\partial i_{r0N}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_0)-\cos(\varphi_0+\theta_0)\right]h_{0i}+\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1i}+\left[-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)\right]g_{0i}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1i} \\
+\left[-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]\right]m_{1i}
\end{array}\right]
$$

$$
k_{s1v}=\frac{\partial i_{s1}}{\partial v_{cr0N}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_0)-\cos(\varphi_0+\theta_0)\right]h_{0v}+\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1v}+\left[-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)\right]g_{0v}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1v} \\
+\left[-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]\right]m_{1v}
\end{array}\right]
$$

$$
k_{s1in}=\frac{\partial i_{s1}}{\partial v_{in}}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_0)-\cos(\varphi_0+\theta_0)\right]h_{0in}+\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1in}+\left[-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)\right]g_{0in}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1in} \\
+\left[-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]\right]m_{1in} \\
+\frac{\omega_{r0}T_s}{nI_n}\frac{nC_r}{T_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)+r_1\cos(\theta_1)-r_1\cos(\varphi_1+\theta_1)\right)
\end{array}\right]
$$

$$
k_{s1o}=\frac{\partial i_{s1}}{\partial v_o}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_0)-\cos(\varphi_0+\theta_0)\right]h_{0o}+\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1o}+\left[-r_0\sin(\theta_0)+r_0\sin(\varphi_0+\theta_0)\right]g_{0o}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1o} \\
+\left[-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]\right]m_{1o}
\end{array}\right]
$$

$$
k_{s1t}=\frac{\partial i_{s1}}{\partial t_s}=\frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}\frac{\omega_{r0}}{2}+\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}\frac{\omega_{r0}}{2}+\left[-\frac{1}{T_s}\left(r_0\cos(\theta_0)-r_0\cos(\varphi_0+\theta_0)+r_1\cos(\theta_1)-r_1\cos(\varphi_1+\theta_1)\right)\right] \\
+\left[-\left[\cos(\theta_1)-\cos(\varphi_1+\theta_1)\right]h_{1m0}-\left[-r_1\sin(\theta_1)+r_1\sin(\varphi_1+\theta_1)\right]g_{1m0}-r_0\sin(\varphi_0+\theta_0)+\left[r_1\sin(\varphi_1+\theta_1)\right]\right]m_{1t}+\left[r_0\sin(\varphi_0+\theta_0)\right]\frac{\omega_{r0}}{2}
\end{array}\right]
$$

From t_3 to t_6 with half a switch period, time-domain expressions are as follows:

$$
\begin{cases}
i_{r3N}=r_2\sin(\theta_2)\\
v_{cr3N}=-r_2\cos(\theta_2)-(1-M)\\
i_{r5N}=r_2\sin(\varphi_2+\theta_2)=r_3\sin(\theta_3)\\
v_{cr5N}=-r_2\cos(\varphi_2+\theta_2)-(1-M)=-r_3\cos(\theta_3)+(1+M)\\
i_{r6N}=r_3\sin(\varphi_3+\theta_3)\\
v_{cr6N}=-r_3\cos(\varphi_3+\theta_3)+(1+M)\\
i_{rec}(t_6)=nI_n\left(r_3\sin(\varphi_3+\theta_3)-r_2\sin(\theta_2)+\dfrac{M\omega_{r0}}{L_n}\dfrac{T_s}{2}\right)=0\\
\overline{i}_{s2}=\dfrac{nI_n}{\omega_{r0}T_s}\left(r_2\cos(\theta_2)-r_2\cos(\varphi_2+\theta_2)+r_3\cos(\theta_3)-r_3\cos(\varphi_3+\theta_3)\right)\\
\varphi_3=\dfrac{\omega_{r0}T_s}{2}-\varphi_2
\end{cases}
\tag{117}
$$

At time t_3, θ_2 and r_2 can be expressed as:

$$
\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{cr3N}+(1-M)}\right)\quad r_2=\sqrt{i_{r3N}{}^2+\left[v_{cr3N}+(1-M)\right]^2}
\tag{118}
$$

The first-order linearization of θ_2 and r_2 is expressed as

$$
\begin{aligned}
&\theta_2+\Delta\theta_2=\pi+\arctan\left(-\frac{i_{r3N}}{v_{r3N}+(1-M)}\right)+\frac{\partial\theta_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_2}{\partial v_o}\hat{v}_o+\frac{\partial\theta_2}{\partial t_s}\hat{t}_s\\
&=\theta_2+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}}+\frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{cr0N}}+\frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}+\\
&\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}}+\frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{in}}-\frac{i_{r3N}M/v_{in}}{r_2^2}\right)\hat{v}_{in}+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o}+\frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_o}+\frac{i_{r3N}n/v_{in}}{r_2^2}\right)\hat{v}_o+\left(\frac{\partial\theta_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s}+\frac{\partial\theta_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial t_s}\right)\hat{t}_s\\
&=\theta_2+\left[-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3i}+\frac{i_{r3N}}{r_2^2}l_{3i}\right]\hat{i}_{r0N}+\left[-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3v}+\frac{i_{r3N}}{r_2^2}l_{3v}\right]\hat{v}_{cr0N}+\left[-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3in}+\frac{i_{r3N}}{r_2^2}l_{3in}+\frac{i_{r3N}M/v_{in}}{r_2^2}\right]\hat{v}_{in}\\
&+\left[-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3o}+\frac{i_{r3N}}{r_2^2}l_{3o}-\frac{ni_{r3N}/v_{in}}{r_2^2}\right]\hat{v}_o+\left[-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3t}+\frac{i_{r3N}}{r_2^2}l_{3t}\right]\hat{t}_s\\
&=\theta_2+g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{cr0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
g_{2i}&=\frac{\partial\theta_2}{\partial i_{r0N}}=-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3i}+\frac{i_{r3N}}{r_2^2}l_{3i}\\
g_{2v}&=\frac{\partial\theta_2}{\partial v_{cr0N}}=-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3v}+\frac{i_{r3N}}{r_2^2}l_{3v}\\
g_{2in}&=\frac{\partial\theta_2}{\partial v_{in}}=-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3in}+\frac{i_{r3N}}{r_2^2}l_{3in}+\frac{i_{r3N}M/v_{in}}{r_2^2}\\
g_{2o}&=\frac{\partial\theta_2}{\partial v_o}=-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3o}+\frac{i_{r3N}}{r_2^2}l_{3o}-\frac{ni_{r3N}/v_{in}}{r_2^2}\\
g_{2t}&=\frac{\partial\theta_2}{\partial t_s}=-\frac{v_{cr3N}+(1-M)}{r_2^2}k_{3t}+\frac{i_{r3N}}{r_2^2}l_{3t}
\end{aligned}
$$

$$
\begin{aligned}
&r_2+\Delta r_2=\sqrt{i_{r3N}{}^2+\left[v_{r3N}+\left(1-M\right)\right]^2}+\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s\\
&=r_2+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial i_{r0N}}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{cr0N}}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_{in}}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_{in}}+\frac{\left[v_{cr3N}+\left(1-M\right)\right]M/v_{in}}{r_1}\right)\hat{v}_{in}+\\
&\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial v_o}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial v_o}-\frac{\left[v_{cr3N}+\left(1-M\right)\right]n/v_{in}}{r_1}\right)\hat{v}_o+\left(\frac{\partial r_2}{\partial i_{r3N}}\frac{\partial i_{r3N}}{\partial t_s}+\frac{\partial r_2}{\partial v_{cr3N}}\frac{\partial v_{cr3N}}{\partial t_s}\right)\hat{t}_s\\
&=r_2+\left(\frac{i_{r3N}}{r_2}k_{3i}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3i}\right)\hat{i}_{r0N}+\left(\frac{i_{r3N}}{r_2}k_{3v}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3v}\right)\hat{v}_{cr0N}+\left(\frac{i_{r3N}}{r_2}k_{3in}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3in}+\frac{\left[v_{cr3N}+\left(1-M\right)\right]M/v_{in}}{r_2}\right)\hat{v}_{in}+\\
&\left(\frac{i_{r3N}}{r_2}k_{3o}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3o}-\frac{\left[v_{cr3N}+\left(1-M\right)\right]n/v_{in}}{r_2}\right)\hat{v}_o+\left(\frac{i_{r3N}}{r_2}k_{3t}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3t}\right)\hat{t}_s\\
&=r_2+h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{cr0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s\\
&\text{where}\\
&h_{2i}=\frac{\partial r_2}{\partial i_{r0N}}=\frac{i_{r3N}}{r_2}k_{3i}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3i}\\
&h_{2v}=\frac{\partial r_2}{\partial v_{cr0N}}=\frac{i_{r3N}}{r_2}k_{3v}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3v}\\
&h_{2in}=\frac{\partial r_2}{\partial v_{in}}=\frac{i_{r3N}}{r_2}k_{3in}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3in}+\frac{\left[v_{cr3N}+\left(1-M\right)\right]M/v_{in}}{r_2}\\
&h_{2o}=\frac{\partial r_2}{\partial v_o}=\frac{i_{r3N}}{r_2}k_{3o}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3o}-\frac{\left[v_{cr3N}+\left(1-M\right)\right]n/v_{in}}{r_2}\\
&h_{2t}=\frac{\partial r_2}{\partial t_s}=\frac{i_{r3N}}{r_2}k_{3t}+\frac{v_{cr3N}+\left(1-M\right)}{r_2}l_{3t}
\end{aligned}
\tag{119}
$$

At time t_5, Δi_{r5N} and Δv_{r5N} can be calculated by

$$
\begin{aligned}
&i_{r5N}+\Delta i_{r5N}=r_2\sin\left(\varphi_2+\theta_2\right)+\frac{\partial i_{r2N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial i_{r5N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial i_{r5N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{r5N}}{\partial v_o}\hat{v}_o+\frac{\partial i_{r5N}}{\partial t_s}\hat{t}_s+\frac{\partial i_{r5N}}{\partial \varphi_2}\Delta\varphi_2\\
&=i_{r2N}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{cr0N}}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}\\
&+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial v_o}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_o}\right)\hat{v}_o+\left(\frac{\partial i_{r5N}}{\partial r_2}\frac{\partial r_2}{\partial t_s}+\frac{\partial i_{r5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_s}\right)\hat{t}_s+\frac{\partial i_{r5N}}{\partial \varphi_2}\Delta\varphi_2\\
&=i_{r5N}+\left[\sin\left(\varphi_2+\theta_2\right)h_{2i}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2i}\right]\hat{i}_{r0N}\\
&+\left[\sin\left(\varphi_2+\theta_2\right)h_{2v}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2v}\right]\hat{v}_{cr0N}\\
&+\left[\sin\left(\varphi_2+\theta_2\right)h_{2in}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2in}\right]\hat{v}_{in}\\
&+\left[\sin\left(\varphi_2+\theta_2\right)h_{2o}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2o}\right]\hat{v}_o\\
&+\left[\sin\left(\varphi_2+\theta_2\right)h_{2t}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2t}\right]\hat{t}_s+r_2\cos\left(\varphi_2+\theta_2\right)\Delta\varphi_2\\
&=i_{r5N}+k_{5i}\hat{i}_{r0N}+k_{5v}\hat{v}_{cr0N}+k_{5in}\hat{v}_{in}+k_{5o}\hat{v}_o+k_{5t}\hat{t}_s+k_{5m2}\Delta\varphi_2\\
&\text{where}\\
&k_{5i}=\frac{\partial i_{r5N}}{\partial i_{r0N}}=\sin\left(\varphi_2+\theta_2\right)h_{2i}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2i}\\
&k_{5v}=\frac{\partial i_{r5N}}{\partial v_{cr0N}}=\sin\left(\varphi_2+\theta_2\right)h_{2v}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2v}\\
&k_{5in}=\frac{\partial i_{r5N}}{\partial v_{in}}=\sin\left(\varphi_2+\theta_2\right)h_{2in}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2in}\\
&k_{5o}=\frac{\partial i_{r5N}}{\partial v_o}=\sin\left(\varphi_2+\theta_2\right)h_{2o}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2o}\\
&k_{5t}=\frac{\partial i_{r5N}}{\partial t_s}=\sin\left(\varphi_2+\theta_2\right)h_{2t}+r_2\cos\left(\varphi_2+\theta_2\right)g_{2t}\\
&k_{5m2}=\frac{\partial i_{r5N}}{\partial \varphi_2}=r_2\cos\left(\varphi_2+\theta_2\right)
\end{aligned}
$$

$$
\begin{aligned}
& v_{cr5N}+\Delta v_{cr5N}=-r_2\cos(\varphi_2+\theta_2)-(1-M)+\frac{\partial v_{cr5N}}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial v_{cr5N}}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial v_{cr5N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr5N}}{\partial v_o}\hat{v}_o+\frac{\partial v_{cr5N}}{\partial t_s}\hat{t}_s+\frac{\partial v_{cr5N}}{\partial \varphi_2}\Delta\varphi_2 \\
& =v_{cr5N}+\left(\frac{\partial v_{cr5N}}{\partial r_2}\frac{\partial r_2}{\partial i_{r0N}}+\frac{\partial v_{cr5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial v_{cr5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{cr0N}}+\frac{\partial v_{cr5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{cr0N}}\right)\hat{v}_{cr0N} \\
& +\left(\frac{\partial v_{cr5N}}{\partial r_2}\frac{\partial r_2}{\partial v_{in}}+\frac{\partial v_{cr5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_{in}}-\frac{M}{v_{in}}\right)\hat{v}_{in}+\left(\frac{\partial v_{cr5N}}{\partial r_2}\frac{\partial r_2}{\partial v_o}+\frac{\partial v_{cr5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial v_o}+\frac{n}{v_{in}}\right)\hat{v}_o \\
& +\left(\frac{\partial v_{cr5N}}{\partial r_2}\frac{\partial r_2}{\partial t_s}+\frac{\partial v_{cr5N}}{\partial \theta_2}\frac{\partial \theta_2}{\partial t_s}\right)\hat{t}_s+\frac{\partial v_{cr5N}}{\partial \varphi_2}\Delta\varphi_2 \\
& =v_{cr5N}+\left[-\cos(\varphi_2+\theta_2)h_{2i}+r_2\sin(\varphi_2+\theta_2)g_{2i}\right]\hat{i}_{r0N} \\
& +\left[-\cos(\varphi_2+\theta_2)h_{2v}+r_2\sin(\varphi_2+\theta_2)g_{2v}\right]\hat{v}_{cr0N} \\
& +\left[-\cos(\varphi_2+\theta_2)h_{2in}+r_2\sin(\varphi_2+\theta_2)g_{2in}-\frac{M}{v_{in}}\right]\hat{v}_{in} \\
& +\left[-\cos(\varphi_2+\theta_2)h_{2o}+r_2\sin(\varphi_2+\theta_2)g_{2o}+\frac{n}{v_{in}}\right]\hat{v}_o \\
& +\left[-\cos(\varphi_2+\theta_2)h_{2t}+r_2\sin(\varphi_2+\theta_2)g_{2t}\right]\hat{t}_s+r_2\sin(\varphi_2+\theta_2)\Delta\varphi_2 \\
& =v_{cr5N}+l_{5i}\hat{i}_{r0N}+l_{5v}\hat{v}_{cr0N}+l_{5in}\hat{v}_{in}+l_{5o}\hat{v}_o+l_{5t}\hat{t}_s+l_{5m2}\Delta\varphi_2
\end{aligned}
\tag{120}
$$

where

$$
\begin{aligned}
& l_{5i}=\frac{\partial v_{cr5N}}{\partial i_{r0N}}=-\cos(\varphi_2+\theta_2)h_{2i}+r_2\sin(\varphi_2+\theta_2)g_{2i} \\
& l_{5v}=\frac{\partial v_{cr5N}}{\partial v_{cr0N}}=-\cos(\varphi_2+\theta_2)h_{2v}+r_2\sin(\varphi_2+\theta_2)g_{2v} \\
& l_{5in}=\frac{\partial v_{cr5N}}{\partial v_{in}}=-\cos(\varphi_2+\theta_2)h_{2in}+r_2\sin(\varphi_2+\theta_2)g_{2in}-\frac{M}{v_{in}} \\
& l_{5o}=\frac{\partial v_{cr5N}}{\partial v_o}=-\cos(\varphi_2+\theta_2)h_{2o}+r_2\sin(\varphi_2+\theta_2)g_{2o}+\frac{n}{v_{in}} \\
& l_{5t}=\frac{\partial v_{cr5N}}{\partial t_s}=-\cos(\varphi_2+\theta_2)h_{2t}+r_2\sin(\varphi_2+\theta_2)g_{2t} \\
& l_{5m2}=\frac{\partial v_{cr5N}}{\partial \varphi_2}=r_2\sin(\varphi_2+\theta_2)
\end{aligned}
$$

θ_3 and r_3 can be expressed as:

$$
\theta_3=\arctan\left(-\frac{i_{r5N}}{\left[v_{cr5N}-(1+M)\right]}\right) \quad r_3=\sqrt{{i_{r5N}}^2+(v_{cr5N}-(1+M))^2} \tag{121}
$$

$\Delta\theta_3$ and Δr_3 can be calculated by

$$
\begin{aligned}
&\theta_3+\Delta\theta_3=\theta_3+\frac{\partial\theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_3}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_3}{\partial v_o}\hat{v}_o+\frac{\partial\theta_3}{\partial t_s}\hat{t}_s+\frac{\partial\theta_3}{\partial\varphi_2}\Delta\varphi_2\\
&=\theta_3+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{cr0N}}+\frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}\\
&+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_{in}}+\frac{M}{v_{in}}\frac{i_{r5N}}{r_3^{\,2}}\right)\hat{v}_{in}+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_o}-\frac{n}{v_{in}}\frac{i_{r5N}}{r_3^{\,2}}\right)\hat{v}_o\\
&+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial t_s}\right)\hat{t}_s+\left(\frac{\partial\theta_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial\varphi_2}+\frac{\partial\theta_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial\varphi_2}\right)\Delta\varphi_2\\
&=\theta_3+\left[-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5i}+\frac{i_{r5N}}{r_3^{\,2}}l_{5i}\right]\hat{i}_{r0N}+\left[-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5v}+\frac{i_{r5N}}{r_3^{\,2}}l_{5v}\right]\hat{v}_{cr0N}\\
&+\left[-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5in}+\frac{i_{r5N}}{r_3^{\,2}}l_{5in}+\frac{M}{v_{in}}\frac{i_{r5N}}{r_3^{\,2}}\right]\hat{v}_{in}+\left[-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5o}+\frac{i_{r5N}}{r_3^{\,2}}l_{5o}-\frac{n}{v_{in}}\frac{i_{r5N}}{r_3^{\,2}}\right]\hat{v}_o\\
&+\left[-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5t}+\frac{i_{r5N}}{r_3^{\,2}}l_{5t}\right]\hat{t}_s+\left(-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5m2}+\frac{i_{r5N}}{r_3^{\,2}}l_{5m2}\right)\Delta\varphi_2\\
&=\theta_3+g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\Delta\varphi_2
\end{aligned}
$$

where

$$g_{3i}=\frac{\partial\theta_3}{\partial i_{r0N}}=-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5i}+\frac{i_{r5N}}{r_3^{\,2}}l_{5i}$$

$$g_{3v}=\frac{\partial\theta_3}{\partial v_{cr0N}}=-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5v}+\frac{i_{r5N}}{r_3^{\,2}}l_{5v}$$

$$g_{3in}=\frac{\partial\theta_3}{\partial v_{in}}=-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5in}+\frac{i_{r5N}}{r_3^{\,2}}l_{5in}+\frac{M}{v_{in}}\frac{i_{r5N}}{r_3^{\,2}}$$

$$g_{3o}=\frac{\partial\theta_3}{\partial v_o}=-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5o}+\frac{i_{r5N}}{r_3^{\,2}}l_{5o}-\frac{n}{v_{in}}\frac{i_{r5N}}{r_3^{\,2}}$$

$$g_{3t}=\frac{\partial\theta_3}{\partial t_s}=-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5t}+\frac{i_{r5N}}{r_3^{\,2}}l_{5t}$$

$$g_{3m2}=\frac{\partial\theta_3}{\partial\varphi_2}=-\frac{v_{cr5N}-(1+M)}{r_3^{\,2}}k_{5m2}+\frac{i_{r5N}}{r_3^{\,2}}l_{5m2}$$

$$
\begin{aligned}
r_3+\Delta r_3 &= r_3+\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s+\frac{\partial r_3}{\partial \varphi_2}\Delta\varphi_2 \\
&= r_3+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial i_{r0N}}+\frac{\partial r_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial i_{r0N}}\right)\hat{i}_{r0N}+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{cr0N}}+\frac{\partial r_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_{cr0N}}\right)\hat{v}_{cr0N}+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_{in}}+\frac{\partial r_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_{in}}+\frac{M}{V_{in}}\frac{v_{cr5N}-(1+M)}{r_3}\right)\hat{v}_{in} \\
&+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial v_o}+\frac{\partial r_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial v_o}-\frac{n}{V_{in}}\frac{v_{cr5N}-(1+M)}{r_3}\right)\hat{v}_o+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial t_s}+\frac{\partial r_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial t_s}\right)\hat{t}_s+\left(\frac{\partial r_3}{\partial i_{r5N}}\frac{\partial i_{r5N}}{\partial \varphi_2}+\frac{\partial r_3}{\partial v_{cr5N}}\frac{\partial v_{cr5N}}{\partial \varphi_2}\right)\Delta\varphi_2 \\
&= r_3+\left(\frac{i_{r5N}}{r_3}k_{5i}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5i}\right)\hat{i}_{r0N}+\left(\frac{i_{r5N}}{r_3}k_{5v}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5v}\right)\hat{v}_{cr0N} \\
&+\left(\frac{i_{r5N}}{r_3}k_{5in}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5in}+\frac{M}{v_{in}}\frac{v_{cr5N}-(1+M)}{r_3}\right)\hat{v}_{in}+\left(\frac{i_{r5N}}{r_3}k_{5o}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5o}-\frac{n}{v_{in}}\frac{v_{cr5N}-(1+M)}{r_3}\right)\hat{v}_o \\
&+\left(\frac{i_{r5N}}{r_3}k_{5t}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5t}\right)\hat{t}_s+\left(\frac{i_{r5N}}{r_3}k_{5m2}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5m2}\right)\Delta\varphi_2 \\
&= r_3+h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\Delta\varphi_2
\end{aligned}
$$

where

$$
\begin{aligned}
h_{3i} &= \frac{\partial r_3}{\partial i_{r0N}}=\frac{i_{r5N}}{r_3}k_{5i}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5i} \\
h_{3v} &= \frac{\partial r_3}{\partial v_{cr0N}}=\frac{i_{r5N}}{r_3}k_{5v}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5v} \\
h_{3in} &= \frac{\partial r_3}{\partial v_{in}}=\frac{i_{r5N}}{r_3}k_{5in}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5in}+\frac{M}{v_{in}}\frac{v_{cr5N}-(1+M)}{r_3} \\
h_{3o} &= \frac{\partial r_3}{\partial v_o}=\frac{i_{r5N}}{r_3}k_{5o}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5o}-\frac{n}{v_{in}}\frac{v_{cr5N}-(1+M)}{r_3} \\
h_{3t} &= \frac{\partial r_3}{\partial t_s}=\frac{i_{r5N}}{r_3}k_{5t}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5t} \\
h_{3m2} &= \frac{\partial r_3}{\partial \varphi_2}=\frac{i_{r5N}}{r_3}k_{5m2}+\frac{v_{cr5N}-(1+M)}{r_3}l_{5m2}
\end{aligned}
\tag{122}
$$

At time t_6, $i_{s2N}(t_6)=0$, and $i_{s2N}(t_6+\Delta t_6)=0$ after the disturbances are added.(123) can be obtained

$$
\begin{aligned}
&i_{s2N}\left(t_6+\Delta t_6\right)=n\left(\left(r_3+\Delta r_3\right)\sin\left(\varphi_3+\Delta\varphi_3+\theta_3+\Delta\theta_3\right)-\left(r_2+\Delta r_2\right)\sin\left(\theta_2+\Delta\theta_2\right)+\frac{n\left(v_o+\Delta v_o\right)}{\left(v_{in}+\Delta v_{in}\right)L_n}\frac{\omega_{r0}T_s}{2}\right)\\
&\approx n\left(r_3\sin\left(\varphi_3+\theta_3\right)-r_2\sin\left(\theta_2\right)+\frac{M\omega_{r0}}{L_n}\frac{T_s}{2}\right)+\frac{\partial i_{s2N}\left(t_6\right)}{\partial r_2}\Delta r_2+\frac{\partial i_{s2N}\left(t_6\right)}{\partial r_3}\Delta r_3+\frac{\partial i_{s2N}\left(t_6\right)}{\partial\varphi_3}\Delta\varphi_3+\frac{\partial i_{s2N}\left(t_6\right)}{\partial\theta_2}\Delta\theta_2+\\
&\frac{\partial i_{s2N}\left(t_6\right)}{\partial\theta_3}\Delta\theta_3+\frac{\partial i_{s2N}\left(t_6\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{s2N}\left(t_6\right)}{\partial v_o}\hat{v}_o+\frac{\partial i_{s2N}\left(t_6\right)}{\partial t_s}\hat{t}_s\\
&=i_{s2N}\left(t_6\right)+\frac{\partial i_{s2N}\left(t_6\right)}{\partial r_2}\left(\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s\right)\\
&+\frac{\partial i_{s2N}\left(t_6\right)}{\partial r_3}\left(\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s+\frac{\partial r_3}{\partial\varphi_3}\Delta\varphi_2\right)\\
&+\frac{\partial i_{s2N}\left(t_6\right)}{\partial\varphi_3}\Delta\varphi_3+\frac{\partial i_{s2N}\left(t_6\right)}{\partial\theta_2}\left(\frac{\partial\theta_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_2}{\partial v_o}\hat{v}_o+\frac{\partial\theta_2}{\partial t_s}\hat{t}_s\right)\\
&+\frac{\partial i_{s2N}\left(t_6\right)}{\partial\theta_3}\left(\frac{\partial\theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_3}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_3}{\partial v_o}\hat{v}_o+\frac{\partial\theta_3}{\partial t_s}\hat{t}_s+\frac{\partial\theta_3}{\partial\varphi_2}\Delta\varphi_2\right)+\frac{\partial i_{s2N}\left(t_6\right)}{\partial v_{in}}\hat{v}_{in}+\frac{\partial i_{s2N}\left(t_6\right)}{\partial v_o}\hat{v}_o+\frac{\partial i_{s2N}\left(t_6\right)}{\partial t_s}\hat{t}_s\\
&=i_{s2N}\left(t_6\right)+n\left[\begin{array}{l}
-\sin\left(\theta_2\right)\left(\frac{\partial r_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_2}{\partial v_o}\hat{v}_o+\frac{\partial r_2}{\partial t_s}\hat{t}_s\right)\\
+\sin\left(\varphi_3+\theta_3\right)\left(\frac{\partial r_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial r_3}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial r_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial r_3}{\partial v_o}\hat{v}_o+\frac{\partial r_3}{\partial t_s}\hat{t}_s+\frac{\partial r_3}{\partial\varphi_3}\Delta\varphi_2\right)\\
+r_3\cos\left(\varphi_3+\theta_3\right)\Delta\varphi_3-r_2\cos\left(\theta_2\right)\left(\frac{\partial\theta_2}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_2}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_2}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_2}{\partial v_o}\hat{v}_o+\frac{\partial\theta_2}{\partial t_s}\hat{t}_s\right)\\
+r_3\cos\left(\varphi_3+\theta_3\right)\left(\frac{\partial\theta_3}{\partial i_{r0N}}\hat{i}_{r0N}+\frac{\partial\theta_3}{\partial v_{cr0N}}\hat{v}_{cr0N}+\frac{\partial\theta_3}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\theta_3}{\partial v_o}\hat{v}_o+\frac{\partial\theta_3}{\partial t_s}\hat{t}_s+\frac{\partial\theta_3}{\partial\varphi_2}\Delta\varphi_2\right)\\
-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\hat{v}_{in}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\hat{v}_o+\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s
\end{array}\right]\\
&=i_{s2N}\left(t_6\right)+n\left[\begin{array}{l}
-\sin\left(\theta_2\right)\left(h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{cr0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s\right)\\
+\sin\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\Delta\varphi_2\right)\\
+r_3\cos\left(\varphi_3+\theta_3\right)\Delta\varphi_3-r_2\cos\left(\theta_2\right)\left(g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{cr0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s\right)\\
+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\Delta\varphi_2\right)\\
-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\hat{v}_{in}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\hat{v}_o+\frac{M}{L_n}\frac{\omega_{r0}}{2}\hat{t}_s
\end{array}\right]\\
&=i_{s2N}\left(t_6\right)+n\left[\begin{array}{l}
\left[-\sin\left(\theta_2\right)h_{2i}+\sin\left(\varphi_3+\theta_3\right)h_{3i}-r_2\cos\left(\theta_2\right)g_{2i}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3i}\right]\hat{i}_{r0N}+\\
\left[-\sin\left(\theta_2\right)h_{2v}+\sin\left(\varphi_3+\theta_3\right)h_{3v}-r_2\cos\left(\theta_2\right)g_{2v}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3v}\right]\hat{v}_{cr0N}+\\
\left[-\sin\left(\theta_2\right)h_{2in}+\sin\left(\varphi_3+\theta_3\right)h_{3in}-r_2\cos\left(\theta_2\right)g_{2in}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3in}-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in}\\
+\left[-\sin\left(\theta_2\right)h_{2v}+\sin\left(\varphi_3+\theta_3\right)h_{3v}-r_2\cos\left(\theta_2\right)g_{2v}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3v}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o\\
+\left[-\sin\left(\theta_2\right)h_{2t}+\sin\left(\varphi_3+\theta_3\right)h_{3t}-r_2\cos\left(\theta_2\right)g_{2t}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3t}+\frac{M}{L_n}\frac{\omega_{r0}}{2}\right]\hat{t}_s\\
+r_3\cos\left(\varphi_3+\theta_3\right)\Delta\varphi_3+\left[\sin\left(\varphi_3+\theta_3\right)h_{3m2}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]\Delta\varphi_2
\end{array}\right]=0
\end{aligned}
\tag{123}
$$

Because $i_{recN}(t_6)$=0, the following equation can be obtained.

$$\begin{bmatrix} \left[-\sin(\theta_2)h_{2i}+\sin(\varphi_3+\theta_3)h_{3i}-r_2\cos(\theta_2)g_{2i}+r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N}+ \\ \left[-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{cr0N}+ \\ \left[-\sin(\theta_2)h_{2in}+\sin(\varphi_3+\theta_3)h_{3in}-r_2\cos(\theta_2)g_{2in}+r_3\cos(\varphi_3+\theta_3)g_{3in}-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\ +\left[-\sin(\theta_2)h_{2t}+\sin(\varphi_3+\theta_3)h_{3t}-r_2\cos(\theta_2)g_{2t}+r_3\cos(\varphi_3+\theta_3)g_{3t}+\frac{M}{L_n}\frac{\omega_{r0}}{2}\right]\hat{t}_s \\ +r_3\cos(\varphi_3+\theta_3)\Delta\varphi_3+\left[\sin(\varphi_3+\theta_3)h_{3m2}+r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\Delta\varphi_2 \end{bmatrix}=0 \tag{124}$$

Substituting $\Delta\varphi_2=\frac{\omega_{r0}\hat{t}_s}{2}-\Delta\varphi_3$ into the above equation, Eq.(125) can be obtained.

$$\begin{bmatrix} \left[-\sin(\theta_2)h_{2i}+\sin(\varphi_3+\theta_3)h_{3i}-r_2\cos(\theta_2)g_{2i}+r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N}+ \\ \left[-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{cr0N}+ \\ \left[-\sin(\theta_2)h_{2in}+\sin(\varphi_3+\theta_3)h_{3in}-r_2\cos(\theta_2)g_{2in}+r_3\cos(\varphi_3+\theta_3)g_{3in}-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\ +\begin{bmatrix} -\sin(\theta_2)h_{2t}+\sin(\varphi_3+\theta_3)h_{3t}-r_2\cos(\theta_2)g_{2t}+r_3\cos(\varphi_3+\theta_3)g_{3t} \\ +\frac{M}{L_n}\frac{\omega_{r0}}{2}+\left[\sin(\varphi_3+\theta_3)h_{3m2}+r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\frac{\omega_{r0}}{2} \end{bmatrix}\hat{t}_s \\ +\left[r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\Delta\varphi_3 \end{bmatrix}=0 \tag{125}$$

$\Delta\varphi_3$ can be calculated by

$$\Delta\varphi_3=-\frac{\begin{bmatrix} \left[-\sin(\theta_2)h_{2i}+\sin(\varphi_3+\theta_3)h_{3i}-r_2\cos(\theta_2)g_{2i}+r_3\cos(\varphi_3+\theta_3)g_{3i}\right]\hat{i}_{r0N}+ \\ \left[-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}\right]\hat{v}_{cr0N}+ \\ \left[-\sin(\theta_2)h_{2in}+\sin(\varphi_3+\theta_3)h_{3in}-r_2\cos(\theta_2)g_{2in}+r_3\cos(\varphi_3+\theta_3)g_{3in}-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}\right]\hat{v}_{in} \\ +\left[-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}\right]\hat{v}_o \\ +\begin{bmatrix} -\sin(\theta_2)h_{2t}+\sin(\varphi_3+\theta_3)h_{3t}-r_2\cos(\theta_2)g_{2t}+r_3\cos(\varphi_3+\theta_3)g_{3t} \\ +\frac{M}{L_n}\frac{\omega_{r0}}{2}+\left[\sin(\varphi_3+\theta_3)h_{3m2}+r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\frac{\omega_{r0}}{2} \end{bmatrix}\hat{t}_s \end{bmatrix}}{\left[r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]}$$

$$=m_{3i}\hat{i}_{r0N}+m_{3v}\hat{v}_{cr0N}+m_{3in}\hat{v}_{in}+m_{3o}\hat{v}_o+m_{3t}\hat{t}_s$$

where

$$m_{3i}=\frac{\partial\varphi_3}{\partial i_{r0N}}=-\frac{-\sin(\theta_2)h_{2i}+\sin(\varphi_3+\theta_3)h_{3i}-r_2\cos(\theta_2)g_{2i}+r_3\cos(\varphi_3+\theta_3)g_{3i}}{r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}}$$

$$m_{3v}=\frac{\partial\varphi_3}{\partial v_{cr0N}}=-\frac{-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}}{r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}}$$

$$m_{3in}=\frac{\partial\varphi_3}{\partial v_{in}}=-\frac{-\sin(\theta_2)h_{2in}+\sin(\varphi_3+\theta_3)h_{3in}-r_2\cos(\theta_2)g_{2in}+r_3\cos(\varphi_3+\theta_3)g_{3in}-\frac{\omega_{r0}T_sM/v_{in}}{2L_n}}{r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}}$$

$$m_{3o}=\frac{\partial\varphi_3}{\partial v_o}=-\frac{-\sin(\theta_2)h_{2v}+\sin(\varphi_3+\theta_3)h_{3v}-r_2\cos(\theta_2)g_{2v}+r_3\cos(\varphi_3+\theta_3)g_{3v}+\frac{\omega_{r0}T_sn/v_{in}}{2L_n}}{r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}}$$

$$m_{3t}=\frac{\partial\varphi_3}{\partial t_s}=-\frac{-\sin(\theta_2)h_{2t}+\sin(\varphi_3+\theta_3)h_{3t}-r_2\cos(\theta_2)g_{2t}+r_3\cos(\varphi_3+\theta_3)g_{3t}+\frac{M}{L_n}\frac{\omega_{r0}}{2}+\left[\sin(\varphi_3+\theta_3)h_{3m2}+r_3\cos(\varphi_3+\theta_3)g_{3m2}\right]\frac{\omega_{r0}}{2}}{r_3\cos(\varphi_3+\theta_3)-\sin(\varphi_3+\theta_3)h_{3m2}-r_3\cos(\varphi_3+\theta_3)g_{3m2}}$$

(126)

Δi_{r6N} and Δv_{r6N} can be calculated as follows:

$$i_{r6N}+\Delta i_{r6N}=\left(r_3+\Delta r_3\right)\sin\left(\varphi_3+\Delta\varphi_3+\theta_3+\Delta\theta_3\right)=r_3\sin\left(\varphi_3+\theta_3\right)+\frac{\partial i_{r6N}}{\partial r_3}\Delta r_3+\frac{\partial i_{r6N}}{\partial\theta_3}\Delta\theta_3+\frac{\partial i_{r6N}}{\partial\varphi_3}\Delta\varphi_3$$
$$=i_{r6N}+\frac{\partial i_{r6N}}{\partial r_3}\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_S+h_{3m2}\Delta\varphi_2\right)+\frac{\partial i_{r6N}}{\partial\theta_3}\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_S+g_{3m2}\Delta\varphi_2\right)+\frac{\partial i_{r6N}}{\partial\varphi_3}\Delta\varphi_3$$
$$=i_{r6N}+\sin\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_S+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S-h_{3m2}\Delta\varphi_3\right)$$
$$+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_S+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S-g_{3m2}\Delta\varphi_3\right)+r_3\cos\left(\varphi_3+\theta_3\right)\Delta\varphi_3$$
$$=i_{r6N}+\sin\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_S+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S\right)+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_S+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_S\right)$$
$$+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]\left(m_{3i}\hat{i}_{r0N}+m_{3v}\hat{v}_{cr0N}+m_{3in}\hat{v}_{in}+m_{3o}\hat{v}_o+m_{3t}\hat{t}_s\right)$$
$$=i_{r6N}+\left[\sin\left(\varphi_3+\theta_3\right)h_{3i}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3i}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3i}\right]\hat{i}_{r0N}$$
$$+\left[\sin\left(\varphi_3+\theta_3\right)h_{3v}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3v}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3v}\right]\hat{v}_{cr0N}$$
$$+\left[\sin\left(\varphi_3+\theta_3\right)h_{3in}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3in}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3in}\right]\hat{v}_{in}$$
$$+\left[\sin\left(\varphi_3+\theta_3\right)h_{3o}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3o}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3o}\right]\hat{v}_o$$
$$+\begin{bmatrix}\sin\left(\varphi_3+\theta_3\right)\left(h_{3t}+h_{3m2}\frac{\omega_{r0}}{2}\right)+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3t}+g_{3m2}\frac{\omega_{r0}}{2}\right)\\+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3t}\end{bmatrix}\hat{t}_s$$
$$=i_{r6N}+k_{6i}\hat{i}_{r0N}+k_{6v}\hat{v}_{cr0N}+k_{6in}\hat{v}_{in}+k_{6o}\hat{v}_o+k_{6t}\hat{t}_s$$

where

$$k_{6i}=\frac{\partial i_{r6N}}{\partial i_{r0N}}=\sin\left(\varphi_3+\theta_3\right)h_{3i}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3i}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3i}$$
$$k_{6v}=\frac{\partial i_{r6N}}{\partial v_{cr0N}}=\sin\left(\varphi_3+\theta_3\right)h_{3v}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3v}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3v}$$
$$k_{6in}=\frac{\partial i_{r6N}}{\partial v_{in}}=\sin\left(\varphi_3+\theta_3\right)h_{3in}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3in}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3in}$$
$$k_{6o}=\frac{\partial i_{r6N}}{\partial v_o}=\sin\left(\varphi_3+\theta_3\right)h_{3o}+r_3\cos\left(\varphi_3+\theta_3\right)g_{3o}+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3o}$$
$$k_{6t}=\frac{\partial i_{r6N}}{\partial t_s}=\sin\left(\varphi_3+\theta_3\right)\left(h_{3t}+h_{3m2}\frac{\omega_{r0}}{2}\right)+r_3\cos\left(\varphi_3+\theta_3\right)\left(g_{3t}+g_{3m2}\frac{\omega_{r0}}{2}\right)+\left[r_3\cos\left(\varphi_3+\theta_3\right)-\sin\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\cos\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3t}$$

$$
\begin{aligned}
v_{cr6N}+\Delta v_{cr6N} &= -r_3\cos\left(\varphi_3+\theta_3\right)+(1+M)+\frac{\partial v_{cr6N}}{\partial r_3}\Delta r_3+\frac{\partial v_{cr6N}}{\partial \theta_3}\Delta\theta_3+\frac{\partial v_{cr6N}}{\partial \varphi_3}\Delta\varphi_3+\frac{\partial v_{cr6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr6N}}{\partial v_o}\hat{v}_o \\
&= v_{cr6N}+\frac{\partial v_{cr6N}}{\partial r_3}\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\Delta\varphi_2\right)+\frac{\partial v_{cr6N}}{\partial \theta_3}\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\Delta\varphi_2\right)+\frac{\partial v_{cr6N}}{\partial \varphi_3}\Delta\varphi_3+\frac{\partial v_{cr6N}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial v_{cr6N}}{\partial v_o}\hat{v}_o \\
&= v_{cr6N}-\cos\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s-h_{3m2}\Delta\varphi_3\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s-g_{3m2}\Delta\varphi_3+\Delta\varphi_3\right) \\
&\quad -\frac{M}{v_{in}}\hat{v}_{in}+\frac{n}{v_{in}}\hat{v}_o \\
&= v_{cr6N}-\cos\left(\varphi_3+\theta_3\right)\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&\quad +\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]\left(m_{3i}\hat{i}_{r0N}+m_{3v}\hat{v}_{cr0N}+m_{3in}\hat{v}_{in}+m_{3o}\hat{v}_o+m_{3t}\hat{t}_s\right)-\frac{M}{v_{in}}\hat{v}_{in}+\frac{n}{v_{in}}\hat{v}_o \\
&= v_{cr6N}+\left[-\cos\left(\varphi_3+\theta_3\right)h_{3i}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3i}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3i}\right]\hat{i}_{r0N} \\
&\quad +\left[-\cos\left(\varphi_3+\theta_3\right)h_{3v}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3v}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3v}\right]\hat{v}_{cr0N} \\
&\quad +\left[-\cos\left(\varphi_3+\theta_3\right)h_{3in}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3in}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3in}-\frac{M}{v_{in}}\right]\hat{v}_{in} \\
&\quad +\left[-\cos\left(\varphi_3+\theta_3\right)h_{3o}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3o}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3o}+\frac{n}{v_{in}}\right]\hat{v}_o \\
&\quad +\begin{bmatrix} -\cos\left(\varphi_3+\theta_3\right)\left(h_{3t}+\frac{\omega_{r0}}{2}h_{3m2}\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3t}+\frac{\omega_{r0}}{2}g_{3m2}\right) \\ +\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3t} \end{bmatrix}\hat{t}_s \\
&= v_{cr6N}+l_{6i}\hat{i}_{r0N}+l_{6v}\hat{v}_{cr0N}+l_{6in}\hat{v}_{in}+l_{6o}\hat{v}_o+l_{6t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
l_{6i} &= \frac{\partial v_{cr6N}}{\partial i_{r0N}} = -\cos\left(\varphi_3+\theta_3\right)h_{3i}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3i}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3i} \\
l_{6v} &= \frac{\partial v_{cr6N}}{\partial v_{cr0N}} = -\cos\left(\varphi_3+\theta_3\right)h_{3v}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3v}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3v} \\
l_{6in} &= \frac{\partial v_{cr6N}}{\partial v_{in}} = -\cos\left(\varphi_3+\theta_3\right)h_{3in}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3in}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3in}-\frac{M}{v_{in}} \\
l_{6o} &= \frac{\partial v_{cr6N}}{\partial v_o} = -\cos\left(\varphi_3+\theta_3\right)h_{3o}+r_3\sin\left(\varphi_3+\theta_3\right)g_{3o}+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3o}+\frac{n}{v_{in}} \\
l_{6t} &= \frac{\partial v_{cr6N}}{\partial t_s} = -\cos\left(\varphi_3+\theta_3\right)\left(h_{3t}+\frac{\omega_{r0}}{2}h_{3m2}\right)+r_3\sin\left(\varphi_3+\theta_3\right)\left(g_{3t}+\frac{\omega_{r0}}{2}g_{3m2}\right)+\left[r_3\sin\left(\varphi_3+\theta_3\right)+\cos\left(\varphi_3+\theta_3\right)h_{3m2}-r_3\sin\left(\varphi_3+\theta_3\right)g_{3m2}\right]m_{3t}
\end{aligned}
\tag{127}
$$

The average output current of the rectifier from t_3 to t_6 can be expressed as

$$
\begin{aligned}
\overline{i}_{s2}+\Delta\overline{i}_{s2} &= \frac{nI_n}{\omega_{r0}t_s}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)+r_3\cos\left(\theta_3\right)-r_3\cos\left(\varphi_3+\theta_3\right)\right) \\
&\quad +\frac{\partial\overline{i}_{s2}}{\partial r_2}\Delta r_2+\frac{\partial\overline{i}_{s2}}{\partial r_3}\Delta r_3+\frac{\partial\overline{i}_{s2}}{\partial \theta_2}\Delta\theta_2+\frac{\partial\overline{i}_{s2}}{\partial \theta_3}\Delta\theta_3+\frac{\partial\overline{i}_{s2}}{\partial \varphi_2}\Delta\varphi_2+\frac{\partial\overline{i}_{s2}}{\partial \varphi_3}\Delta\varphi_3+\frac{\partial\overline{i}_{s2}}{\partial v_{in}}\hat{v}_{in}+\frac{\partial\overline{i}_{s2}}{\partial t_s}\hat{t}_s \\
&= \overline{i}_{s2}+\frac{nI_n}{\omega_{r0}T_s}\left[\cos\left(\theta_2\right)-\cos\left(\varphi_2+\theta_2\right)\right]\left(h_{2i}\hat{i}_{r0N}+h_{2v}\hat{v}_{cr0N}+h_{2in}\hat{v}_{in}+h_{2o}\hat{v}_o+h_{2t}\hat{t}_s\right) \\
&\quad +\frac{nI_n}{\omega_{r0}T_s}\left[\cos\left(\theta_3\right)-\cos\left(\varphi_3+\theta_3\right)\right]\left(h_{3i}\hat{i}_{r0N}+h_{3v}\hat{v}_{cr0N}+h_{3in}\hat{v}_{in}+h_{3o}\hat{v}_o+h_{3t}\hat{t}_s+h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s-h_{3m2}\Delta\varphi_3\right) \\
&\quad +\frac{nI_n}{\omega_{r0}T_s}\left[-r_2\sin\left(\theta_2\right)+r_2\sin\left(\varphi_2+\theta_2\right)\right]\left(g_{2i}\hat{i}_{r0N}+g_{2v}\hat{v}_{cr0N}+g_{2in}\hat{v}_{in}+g_{2o}\hat{v}_o+g_{2t}\hat{t}_s\right) \\
&\quad +\frac{nI_n}{\omega_{r0}T_s}\left[-r_3\sin\left(\theta_3\right)+r_3\sin\left(\varphi_3+\theta_3\right)\right]\left(g_{3i}\hat{i}_{r0N}+g_{3v}\hat{v}_{cr0N}+g_{3in}\hat{v}_{in}+g_{3o}\hat{v}_o+g_{3t}\hat{t}_s+g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s-g_{3m2}\Delta\varphi_3\right) \\
&\quad +\frac{nI_n}{\omega_{r0}T_s}r_2\sin\left(\varphi_2+\theta_2\right)\left(\frac{\omega_{r0}}{2}\hat{t}_s-\Delta\varphi_3\right)+\frac{nI_n}{\omega_{r0}t_s}r_3\sin\left(\varphi_3+\theta_3\right)\Delta\varphi_3+\left[\frac{nC_r}{t_s}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)+r_3\cos\left(\theta_3\right)-r_3\cos\left(\varphi_3+\theta_3\right)\right)\right]\hat{v}_{in} \\
&\quad +\left[-\frac{nI_n}{\omega_{r0}T_s^2}\left(r_2\cos\left(\theta_2\right)-r_2\cos\left(\varphi_2+\theta_2\right)+r_3\cos\left(\theta_3\right)-r_3\cos\left(\varphi_3+\theta_3\right)\right)\right]\hat{t}_s
\end{aligned}
$$

$$
\begin{aligned}
&= \bar{i}_{s2} + \frac{nI_n}{\omega_{r0}T_s}\left[\cos(\theta_2) - \cos(\varphi_2+\theta_2)\right]\left(h_{2i}\hat{i}_{r0N} + h_{2v}\hat{v}_{cr0N} + h_{2in}\hat{v}_{in} + h_{2o}\hat{v}_o + h_{2t}\hat{t}_s\right) \\
&+ \frac{nI_n}{\omega_{r0}T_s}\left[\cos(\theta_3) - \cos(\varphi_3+\theta_3)\right]\left(h_{3i}\hat{i}_{r0N} + h_{3v}\hat{v}_{cr0N} + h_{3in}\hat{v}_{in} + h_{3o}\hat{v}_o + h_{3t}\hat{t}_s + h_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&+ \frac{nI_n}{\omega_{r0}T_s}\left[-r_2\sin(\theta_2) + r_2\sin(\varphi_2+\theta_2)\right]\left(g_{2i}\hat{i}_{r0N} + g_{2v}\hat{v}_{cr0N} + g_{2in}\hat{v}_{in} + g_{2o}\hat{v}_o + g_{2t}\hat{t}_s\right) \\
&+ \frac{nI_n}{\omega_{r0}T_s}\left[-r_3\sin(\theta_3) + r_3\sin(\varphi_3+\theta_3)\right]\left(g_{3i}\hat{i}_{r0N} + g_{3v}\hat{v}_{cr0N} + g_{3in}\hat{v}_{in} + g_{3o}\hat{v}_o + g_{3t}\hat{t}_s + g_{3m2}\frac{\omega_{r0}}{2}\hat{t}_s\right) \\
&+ \left[\frac{nC_r}{T_s}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right]\hat{v}_{in} + \left[-\frac{nI_n}{\omega_{r0}{t_s}^2}\left(r_2\cos(\theta_2) - r_2\cos(\varphi_2+\theta_2) + r_3\cos(\theta_3) - r_3\cos(\varphi_3+\theta_3)\right)\right]\hat{t}_s \\
&+ \frac{nI_n}{\omega_{r0}T_s}r_2\sin(\varphi_2+\theta_2)\frac{\omega_{r0}}{2}\hat{t}_s + \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2) + r_3\sin(\varphi_3+\theta_3) \end{array}\right]\bullet\left(m_{3i}\hat{i}_{r0N} + m_{3v}\hat{v}_{cr0N} + m_{3in}\hat{v}_{in} + m_{3o}\hat{v}_o + m_{3t}\hat{t}_s\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \bar{i}_{s2} + \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2i} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3i} \\
+\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2i} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3i} \\
+\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3i}
\end{array}\right]\hat{i}_{r0N} \\
&+ \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2v} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3v} \\
+\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2v} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3v} \\
+\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3v}
\end{array}\right]\hat{v}_{cr0N} \\
&+ \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2in} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3in} \\
+\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2in} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3in} \\
+\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3in} \\
+\frac{\omega_{r0}T_s}{nI_n}\left[\frac{nC_r}{T_s}\left(r_2\cos(\theta_2)-r_2\cos(\varphi_2+\theta_2)+r_3\cos(\theta_3)-r_3\cos(\varphi_3+\theta_3)\right)\right]
\end{array}\right]\hat{v}_{in}
+ \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2o} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3o} \\
+\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2o} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3o} \\
+\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3o}
\end{array}\right]\hat{v}_o \\
&+ \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2t} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]\left(h_{3t}+h_{3m2}\frac{\omega_{r0}}{2}\right) \\
+\left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2t} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]\left(g_{3t}+g_{3m2}\frac{\omega_{r0}}{2}\right) \\
+\left[\begin{array}{l} -\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} \\ -r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3) \end{array}\right]m_{3t} \\
-\frac{1}{T_s}\left[\left(r_2\cos(\theta_2)-r_2\cos(\varphi_2+\theta_2)+r_3\cos(\theta_3)-r_3\cos(\varphi_3+\theta_3)\right)\right]+r_2\sin(\varphi_2+\theta_2)\frac{\omega_{r0}}{2}
\end{array}\right]\hat{t}_s \\
&= \bar{i}_{s2} + k_{s2i}\hat{i}_{r0N} + k_{s2v}\hat{v}_{cr0N} + k_{s2in}\hat{v}_{in} + k_{s2o}\hat{v}_o + k_{s2t}\hat{t}_s
\end{aligned}
$$

where

$$
\begin{aligned}
k_{s2i} &= \frac{\partial\bar{i}_{s2}}{\partial i_{r0N}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2i} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3i} + \left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2i} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3i} \\
+\left[-\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} - r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3)\right]m_{3i}
\end{array}\right] \\
k_{s2v} &= \frac{\partial\bar{i}_{s2}}{\partial v_{cr0N}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2v} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3v} + \left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2v} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3v} \\
+\left[-\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} - r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3)\right]m_{3v}
\end{array}\right] \\
k_{s2in} &= \frac{\partial\bar{i}_{s2}}{\partial v_{in}} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2in} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3in} + \left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2in} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3in} \\
+\left[-\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} - r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3)\right]m_{3in} \\
+\frac{\omega_{r0}T_s}{nI_n}\left[\frac{nC_r}{T_s}\left(r_2\cos(\theta_2)-r_2\cos(\varphi_2+\theta_2)+r_3\cos(\theta_3)-r_3\cos(\varphi_3+\theta_3)\right)\right]
\end{array}\right] \\
k_{s2o} &= \frac{\partial\bar{i}_{s2}}{\partial v_o} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2o} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3o} + \left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2o} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3o} \\
+\left[-\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} - r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3)\right]m_{3o}
\end{array}\right] \\
k_{s2t} &= \frac{\partial\bar{i}_{s2}}{\partial t_s} = \frac{nI_n}{\omega_{r0}T_s}\left[\begin{array}{l}
\left[\cos(\theta_2)-\cos(\varphi_2+\theta_2)\right]h_{2t} + \left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]\left(h_{3t}+h_{3m2}\frac{\omega_{r0}}{2}\right) + \left[-r_2\sin(\theta_2)+r_2\sin(\varphi_2+\theta_2)\right]g_{2t} + \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]\left(g_{3t}+g_{3m2}\frac{\omega_{r0}}{2}\right) \\
+\left[-\left[\cos(\theta_3)-\cos(\varphi_3+\theta_3)\right]h_{3m2} - \left[-r_3\sin(\theta_3)+r_3\sin(\varphi_3+\theta_3)\right]g_{3m2} - r_2\sin(\varphi_2+\theta_2)+r_3\sin(\varphi_3+\theta_3)\right]m_{3t} \\
-\frac{1}{T_s}\left[\left(r_2\cos(\theta_2)-r_2\cos(\varphi_2+\theta_2)+r_3\cos(\theta_3)-r_3\cos(\varphi_3+\theta_3)\right)\right]+r_2\sin(\varphi_2+\theta_2)\frac{\omega_{r0}}{2}
\end{array}\right]
\end{aligned}
\tag{128}
$$

The disturbance in output current of the rectifier bridge during one switching cycle is expressed as

$$\begin{aligned}\Delta\bar{i}_{rec} &= \Delta\bar{i}_{s1} - \Delta\bar{i}_{s2} \\ &= \left(k_{s1i}-k_{s2i}\right)\hat{i}_{r0N} + \left(k_{s1v}-k_{s2v}\right)\hat{v}_{cr0N} + \left(k_{s1o}-k_{s2o}\right)\hat{v}_o + \left(k_{s1in}-k_{s2in}\right)\hat{v}_{in} + \left(k_{s1t}-k_{s2t}\right)\hat{t}_s\end{aligned} \tag{129}$$

According to the large signal model, the state space expression of the LLC converter can be expressed as

$$\begin{aligned}\dot{\hat{i}}_{cr0N} &= \frac{i_{r6N}+\Delta i_{r6N}-i_{r0N}-\hat{i}_{r0N}}{t_s+\hat{t}_s} \approx \frac{\Delta i_{r6N}-\hat{i}_{r0N}}{T_s} = \frac{1}{T_s}\left[\left(k_{6i}-1\right)\hat{i}_{r0N} + k_{6v}\hat{v}_{cr0N} + k_{6in}\hat{v}_{in} + k_{6o}\hat{v}_o + k_{6t}\hat{t}_s\right] \\ \dot{\hat{v}}_{r0N} &= \frac{v_{cr6N}+\hat{v}_{cr6N}-v_{cr0N}-\hat{v}_{cr0N}}{t_s+\hat{t}_s} \approx \frac{\Delta v_{cr6N}-\hat{v}_{cr0N}}{T_s} = \frac{1}{T_s}\left[l_{6i}\hat{i}_{r0N} + \left(l_{6v}-1\right)\hat{v}_{cr0N} + l_{6in}\hat{v}_{in} + l_{6o}\hat{v}_o + l_{6t}\hat{t}_s\right] \\ \dot{\hat{v}}_o &= \frac{1}{C_o}\left(\Delta\bar{i}_{rec}-\frac{\hat{v}_o}{R}\right) = \frac{1}{C_o}\left[\begin{matrix}\left(k_{s1i}-k_{s2i}\right)\hat{i}_{r0N} + \left(k_{s1v}-k_{s2v}\right)\hat{v}_{cr0N} + \left(k_{s1o}-k_{s2o}-\dfrac{1}{R}\right)\hat{v}_o \\ +\left(k_{s1in}-k_{s2in}\right)\hat{v}_{in} + \left(k_{s1t}-k_{s2t}\right)\hat{t}_s\end{matrix}\right]\end{aligned} \tag{130}$$

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + B\hat{u} \\ \hat{y} &= C\hat{x} \\ A &= \begin{bmatrix}\dfrac{k_{6i}-1}{T_s} & \dfrac{k_{6v}}{T_s} & \dfrac{k_{6o}}{T_s} \\ \dfrac{l_{6i}}{T_s} & \dfrac{l_{6v}-1}{T_s} & \dfrac{l_{6o}}{T_s} \\ \dfrac{k_{s1i}-k_{s2i}}{C_o} & \dfrac{k_{s1v}-k_{s2v}}{C_o} & \dfrac{k_{s1o}-k_{s2o}-1/R}{C_o}\end{bmatrix} \\ B &= \begin{bmatrix}\dfrac{k_{6in}}{T_s} & \dfrac{k_{6t}}{T_s} \\ \dfrac{l_{6in}}{T_s} & \dfrac{l_{6t}}{T_s} \\ \dfrac{k_{s1in}-k_{s2in}}{C_o} & \dfrac{k_{s1t}-k_{s2t}}{C_o}\end{bmatrix} \\ C &= \begin{bmatrix}0 & 0 & 1\end{bmatrix}\end{aligned} \tag{131}$$

Substituting the steady-state operating value V_{in}, V_o, T_s I_{r0N}, I_{r2N}, I_{r3N}, I_{r5N}, I_{r6N}, V_{r0N}, V_{r2N}, V_{r3N}, V_{r5N}, and V_{r6N} into v_{in}, v_o, t_s, i_{r0N}, i_{r2N}, i_{r3N}, i_{r5N}, i_{r6N}, v_{r0N}, v_{r2N}, v_{r3N}, v_{r5N}, and v_{r6N} in the state space equation, the transfer function of the LLC converter for NP mode can be expressed as

$$G(s) = C\left(sI-A\right)^{-1}B = \begin{bmatrix}G_{vin}(s) & G_t(s)\end{bmatrix} \tag{132}$$

where

$$G_{vin}(s) = \frac{\hat{v}_o}{\hat{v}_{in}}, \quad G_t(s) = \frac{\hat{v}_o}{\hat{t}_s}$$

The disturbance of the switching period is implemented after the half of the switching period delay. Considering the time delay of $T_s/2$, the transfer function from the switching period to the output voltage is revised to (133).

$$G_{ts}(s) = e^{-\frac{T_s}{2}s}G_t(s) \tag{133}$$

Section VIII. Small-signal model for NP mode with TSC

The definitions of t_{Z1}, t_{Z2} and t_{cs} are shown below.

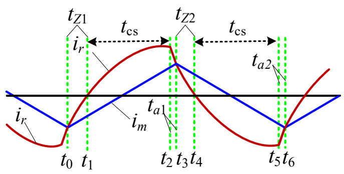

Fig.7 The analysis of control time under TSC for NP mode.

Δt_{Z1}, Δt_{Z2}, Δt_{a2}, and Δt_{a2} can be expressed as follows:

$$\begin{aligned} & t_{Z1} = -\frac{\theta_0}{\omega_{r0}}, t_{Z1} = \frac{\pi - \theta_2}{\omega_{r0}}, t_{a1} = \frac{\varphi_1}{\omega_{r0}}, t_{a2} = \frac{\varphi_3}{\omega_{r0}} \\ & \Delta t_{Z1} = -\frac{\Delta\theta_0}{\omega_{r0}}, \Delta t_{Z1} = -\frac{\Delta\theta_2}{\omega_{r0}}, \Delta t_{a1} = \frac{\Delta\varphi_1}{\omega_{r0}}, \Delta t_{a2} = \frac{\Delta\varphi_3}{\omega_{r0}} \end{aligned} \tag{134}$$

The relationship between $\hat{t}_{cs}$ and $\hat{t}_s$ can be shown below.

$$\begin{aligned} & \hat{t}_s = \Delta t_{Z1} + 2\hat{t}_{cs} + \Delta t_{Z2} + \Delta t_{a1} + \Delta t_{a2} = \frac{1}{\omega_{r0}}\left(-\Delta\theta_0 - \Delta\theta_1 + \Delta\varphi_1 + \Delta\varphi_3\right) + 2\hat{t}_{cs} \\ & = \frac{1}{\omega_{r0}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right)\hat{i}_{r0N} + \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right)\hat{v}_{cr0N} + \\ \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)\hat{v}_{in} + \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right)\hat{v}_o \end{bmatrix} + \frac{\left(-g_{2t} + m_{1t} + m_{3t}\right)}{\omega_{r0}}\hat{t}_s + 2\hat{t}_{cs} \end{aligned} \tag{135}$$

The above equation can be rewritten as

$$\begin{aligned} \hat{t}_s &= \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right)\hat{i}_{r0N} + \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right)\hat{v}_{cr0N} + \\ \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)\hat{v}_{in} + \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right)\hat{v}_o \end{bmatrix} + \frac{2\omega_{r0}}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\hat{t}_{cs} \\ &= \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right) & \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right) & \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right) \end{bmatrix}\begin{bmatrix} \hat{i}_{r0N} \\ \hat{v}_{r0N} \\ \hat{v}_o \end{bmatrix} \\ &+ \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right) & 2\omega_{r0} \end{bmatrix}\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix} \\ &= A_Z\begin{bmatrix} \hat{i}_{r0N} \\ \hat{v}_{r0N} \\ \hat{v}_o \end{bmatrix} + B_Z\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix} \end{aligned} \tag{136}$$

where

$$A_Z = \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0i} - g_{2i} + m_{1i} + m_{3i}\right) \\ \left(-g_{0v} - g_{2v} + m_{1v} + m_{3v}\right) \\ \left(-g_{0o} - g_{2o} + m_{1o} + m_{3o}\right) \end{bmatrix}^T$$

$$B_Z = \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}\begin{bmatrix} \left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right) & 2\omega_{r0} \end{bmatrix}$$

Replace $\hat{t}_s$ in the state space expression with $t_s = A_Z\hat{x} + B_Z\begin{bmatrix} \hat{v}_{in} \\ \hat{t}_{cs} \end{bmatrix}$

$$
\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + B\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = \hat{x} + \begin{bmatrix}B_1 & B_2\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_s\end{bmatrix} = A\hat{x} + B_1\hat{v}_{in} + B_2\hat{t}_s \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2\left[A_Z\hat{x} + B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}\right] \\
&= A\hat{x} + B_1\hat{v}_{in} + B_2A_Z\hat{x} + B_2B_Z\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= \left(A + B_2A_Z\right)\hat{x} + B_1\hat{v}_{in} + \frac{1}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\left(\left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)\hat{v}_{in} + 2\omega_{r0}\hat{t}_{cs}\right) \\
&= \left(A + B_2A_Z\right)\hat{x} + \left(B_1 + \frac{\left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\right)\hat{v}_{in} + \frac{2\omega_{r0}}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\hat{t}_{cs} \\
&= \left(A + B_2A_Z\right)\hat{x} + \left[B_1 + \frac{\left(-g_{0in} - g_{2in} + m_{1in} + m_{3in}\right)}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2 \quad \frac{2\omega_{r0}}{\omega_{r0} + g_{2t} - m_{1t} - m_{3t}}B_2\right]\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix} \\
&= A_c\hat{x} + B_c\begin{bmatrix}\hat{v}_{in}\\ \hat{t}_{cs}\end{bmatrix}
\end{aligned}
\tag{137}
$$

Therefore, the small-signal model of the LLC converter for NP mode with TSC can be expressed as follows.

$$
G_{cs}\left(s\right) = C\left(sI - A_c\right)^{-1}B_c = \left[G_{vin\_tc}(s) \quad G_{tc}(s)\right] \tag{138}
$$

Considering the time delay of $T_s/2$, the transfer function from the control time to the output voltage is revised to (139).

$$
G_{tcs}\left(s\right) = e^{-\frac{T_s}{2}s}G_{tc}\left(s\right) \tag{139}
$$