FIN 3200 Module 4 Guided Notes

Time Value of Money

Basic Definitions

- Present Value:
- Future Value:
- Interest Rate:

Time Value of Money Keys, calculator

- N:
- I/Y:
  - expressed as a percentage, not a decimal
- PMT:
  - Not used with _____, but is used with _____.
- PV:
- FV:

Calculator Notes:

- Be sure you're in end mode
- Set P/Y and C/Y to 1
- remember to clear the registers (CLR TVM) before each problem

Time value of money functions, excel

- NPER:
- RATE:
  - expressed as a decimal, not a percentage.
- PMT:
- PV:

- FV:

Future value of a single sum

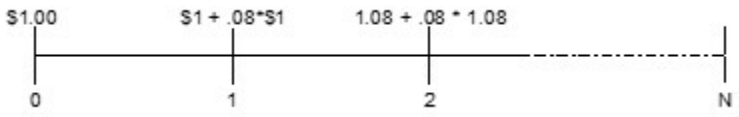

Figure 1: Timeline, FV of $1

$$FV_n = PV * (1 + i)^n$$

- FV:
- PV:
- i:
- n:

Example 1: FV of a single sum

Suppose you have $100, and you invest it at 8% for 3 years. What is the future value?

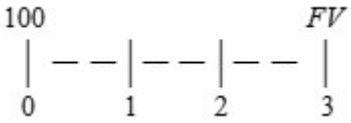

Figure 2: Timeline, FV of $100

$$FV_3 = 100 * (1 + 0.08)^3 = 125.9712$$

Calculator

- N=3
- I/Y=8%
- PV=-100
- FV=

Excel

=FV(0.08,3,,-100)

Effects of compounding:

If interest is compounded:

$$FV = PV * (1 + \frac{i}{m})^{nm}$$

Where m represents:

- semi-annual compounding
- quarterly compounding
- monthly compounding

continuous compounding:

$$FV = PV * e^{in}$$

where e represents:

Example 2: FV of a single sum, monthly compounding

Suppose you have $100, and you invest it at 8% for 3 years. Assuming interest is compounded monthly, What is the future value?

$$FV = 100 * (1 + \frac{0.08}{12})^{3*12} = 127.0237$$

Calculator

- N=3*12
- I/Y=8/12
- PV=-100
- FV=

Excel

=FV(0.0066,36,,-100)

Example 3: FV of a single sum, continuous compounding

Suppose you have $100, and you invest it at 8% for 3 years. Assuming interest is continuously compounded, What is the future value?

$$FV = 100 * e^{0.08*3} = 127.1249$$

Calculator

- 0.08*3
- Press e^x key
- *100
- FV=

Excel

=EXP(0.08x3)*100

Example 4: FV as a general growth formula

Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets per year, how many widgets do you expect to sell in 5 years?

$$FV_5 = 3,000,000 * (1 + 0.15)^5 = 6,034,072$$

Calculator

- N=5
- I/Y=15
- PV=-3,000,000
- FV=

Excel

=FV(0.15,5,,-3000000)

Example 5

Suppose your great aunt gifted you 1,000 when you graduated from high school. Rather than spend this money, you decided to invest it and not touch the balance for 40 years, when you retire. Assuming a rate of 10% per year, what is the future value of your investment? (adapted from Eakins & McNally, 2021).

Present value of a single sum

$$PV = FV_n * \frac{1}{(1+i)^n}$$

Discounting:

Present Value Relationships:

- For a given interest rate, the _____ the time period the _____ the present value.
  - 5 years: $PV = \frac{500}{(1.1)^5} = 310.46$
  - 10 years: $PV = \frac{500}{(1.1)^{10}} = 192.77$

- For a given time period, the _____ the interest rate the _____ the present value.
  - Rate of 10%: $PV = \frac{500}{(1.1)^5} = 310.46$
  - Rate of 15%: $PV = \frac{500}{(1.15)^5} = 248.59$

Example 6: PV of a single sum

Suppose you need 100 in 3 years and can earn an interest rate of 8%. How much would you need to invest today to have $100 in 3 years?

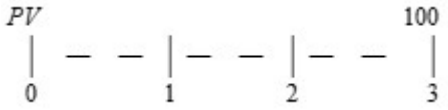

Figure 3: Timeline, PV of $100

$$PV = 100 * \frac{1}{(1+0.08)^3} = 79.3832$$

Calculator

- N=3
- I/Y=8%
- FV=100
- PV=

Excel

=PV(.08,3,,100)

With compounding:

If interest is compounded:

$$PV = FV * \frac{1}{(1+\frac{i}{m})^{nm}}$$

continuous compounding:

$$PV = FV * e^{-in}$$

Example 7: PV of a single sum, monthly compounding

Suppose you need 100 dollars in 3 years. You can earn an interest rate of 8%, compounded monthly. How much would you need to invest today to have $100 in 3 years?

$$PV = 100 * \frac{1}{(1+\frac{0.08}{12})^{3*12}} = 78.7255$$

Calculator

- N=3*12
- I/Y=8/12
- FV=100
- PV=

Excel

=PV(0.00667,36,,100)

Example 8: PV of a single sum, continuous compounding

Suppose you need 100 dollars in 3 years. You can earn an interest rate of 8%, continuously compounded. How much would you need to invest today to have $100 in 3 years?

$$PV = 100 * e^{-0.08*3} = 78.6628$$

Calculator

- -0.08*3
- Press e^x key
- *100
- PV=

Excel

=EXP(-0.08 x 3) x 100

Example 9

You are considering investing in a savings bond that will pay 15,000 in 10 years. If the competitive market interest rate is 6 percent per year, what is the bond worth today? (adapted from Berk & DeMarzo, 2011).

A Note About Interest Rates:

- Nominal:

- Effective annual rate (EAR):

Example 10: Interest rate comparison

Given the 3 following investment options, which should you choose?

- Bank A: 16% compounded annually
- Bank B: 15.5% compounded quarterly:
- Bank C: 15% compounded daily:

Go with _____.

Formula for EAR:

$$EAR = (1 + \frac{nominal}{m})^{nm} - 1$$

where:

- m = number of compounding periods
- n depends on period you're converting to:
  - $n = 1$ for
  - $n = \frac{1}{2}$ for
  - $n = \frac{1}{4}$ for

Example 11: Effective annual rate, n=1

Suppose you have an investment that earns a nominal rate of 18%, compounded monthly. What is the effective annual rate?

$$EAR = (1 + \frac{0.18}{12})^{12} - 1 = 0.1956$$

Calculator

- Push 2nd #2 (INCONV)
- Nom = 18%
- C/Y = 12
- Eff, CPT =

Excel

- =effect(0.18, 12)

NOTE: These only work when n=1. Otherwise, use formula.

Example 12: Effective annual rate, n not equal to 1

Suppose you have a nominal rate of 24%, compounded monthly. What is the effective semi-annual rate?

$$EAR = (1 + \frac{0.24}{12})^{0.5*12} - 1 = 0.1262$$

Example 13

First National Bank charges 12.4 percent compounded monthly on its business loans. First United Bank charges 12.7 percent compounded semi-annually. As a potential borrower, which bank would you go to for a new loan?

Annuity:

- If the first payment occurs at the end of the period, it is called an _____.

- If the first payment occurs at the beginning of the period, it is called an _____.

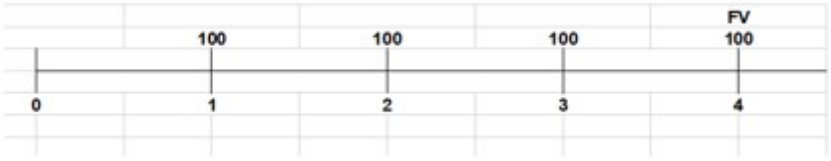

Figure 4: Timeline, Ordinary Annuity

- If final payment and FV occur at the same period its an _____.

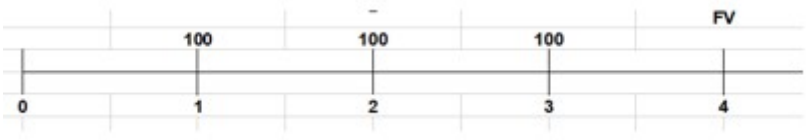

Figure 5: Timeline, Annuity Due

- If final payment is offset from FV by one period its an _____.

Future value of an ordinary annuity

$$FV = pmt * \frac{(1+i)^n - 1}{i}$$

Example 14: FV of an ordinary annuity

Suppose you will receive payments of $100 per year for 4 years. What is the future value of these payments, assuming an 8% interest rate?

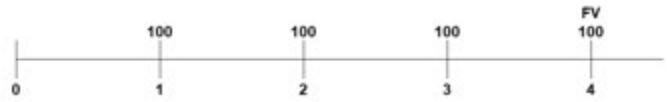

Figure 6: Timeline, FV Ordinary Annuity

$$FV = 100 * \frac{(1+0.08)^4 - 1}{0.08} = 450.6112$$

Calculator (Now we're ready to use the payment key)

- N=4
- I/Y=8
- pmt=-100
- FV=

Excel =FV(.08,4,-100)

Future value of a deferred annuity

Deferred annuity is an annuity with 2 phases:

- Savings Phase
- Income Pahse

Example 15: FV of a deferred annuity

Suppose you have an annuity where you pay $100 for 3 years, then do nothing for the next 4. If the interest rate is 8%,what is the FV at time 7?

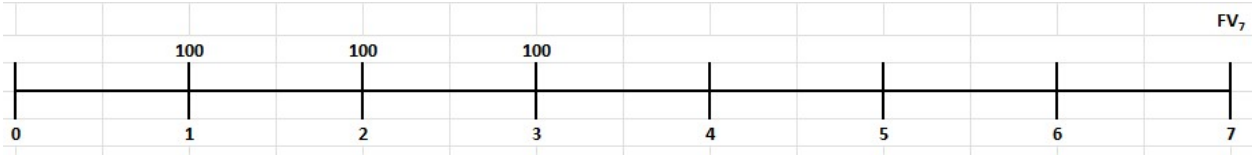

Figure 7: Timeline, FV Deferred Annuity

Two steps to solve:

- Step 1:

Figure 8: Step 1, FV Deferred Annuity

$$FV = 100 * \frac{(1+0.08)^3 - 1}{0.08} = 324.64$$

Calculator

- N=3
- I/Y=8
- PMT=-100
- FV=

Excel =FV(.08,3,-100)

- Step 2:

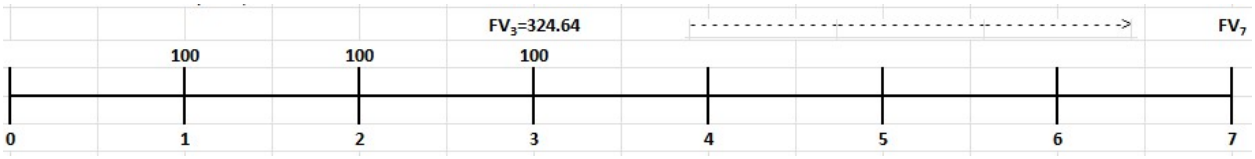

Figure 9: Step 2, FV Deferred Annuity

$$FV = 324.64 * (1+0.08)^4 = 441.669$$

Calculator

- N=4
- I/Y=8
- PV=-324.64
- FV=

Excel =FV(.08,4,,-324.64)

Example 16

Ellen is 35 years old, and has decided it is time to plan seriously for her retirement. Each year until she is 65, she will save 10,000 dollars in a retirement account. If the account earns 10 percent per year, how will will Ellen have saved at age 65?

Present value of an annuity

$$PV = pmt * \frac{1 - \frac{1}{(1+i)^n}}{i}$$
$$PV = pmt * \frac{1 - (1+i)^{-n}}{i}$$

Example 17: PV of an annuity

Suppose you know you will need $100 a year for 4 years, and that you can earn an interest rate of 8%. In order to accomplish this, how much do you need to invest today?

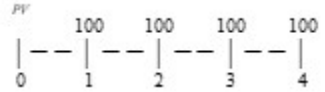

Figure 10: Timeline, PV of an Annuity

$$PV = 100 * \frac{1 - (1 + 0.08)^4}{0.08} = 331.2127$$

Calculator

- N=4
- I/Y=8
- PMT=100
- PV=

Excel =PV(.08,4,-100)

Example 18: PV of a deferred annuity

Suppose your child is going to enter school in 3 years, and will attend for 3 years. You'd like to have the money saved to pay $100 per year for their extra curricular activities. How much do you need to deposit today, assuming an 8% interest rate?

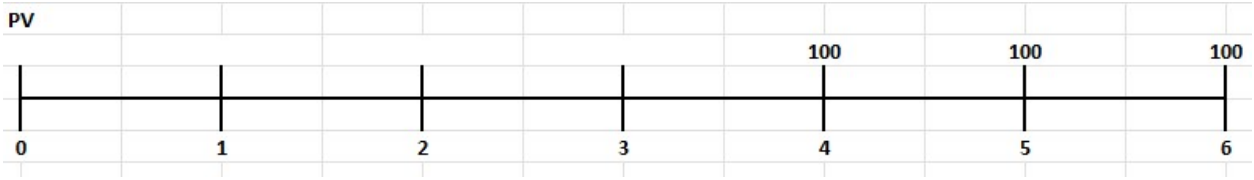

Figure 11: Timeline, PV Deferred Annuity

Two steps to solve:

- Step 1:

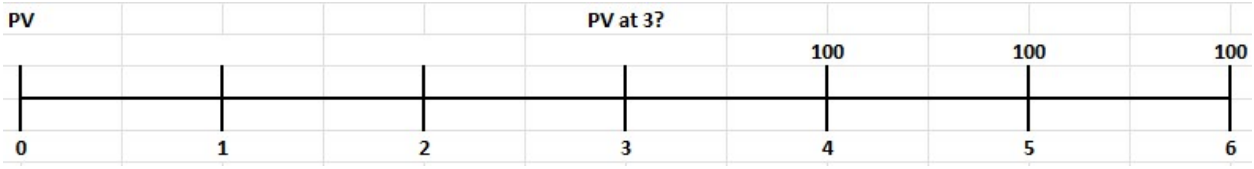

Figure 12: Step 1, PV Deferred Annuity

$$PV = 100 * \frac{1-(1+0.08)^3}{0.08} = 257.7097$$

Calculator

- N=3
- I/Y=8
- PMT=100
- PV=

Excel =PV(.08,3,100)

- Step 2:

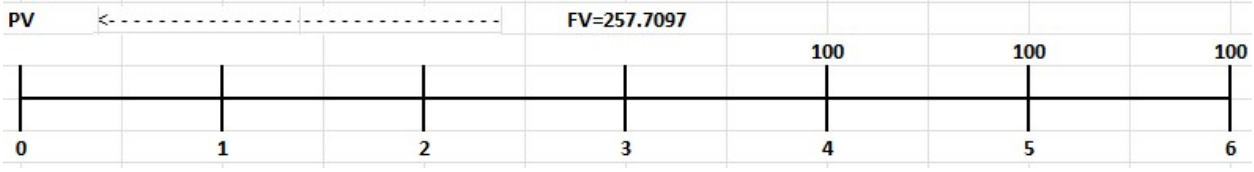

Figure 13: Step 2, PV Deferred Annuity

$$PV = \frac{257.7097}{(1+0.08)^3} = 204.5782$$

Calculator

- N=3
- I/Y=8
- FV=257.7097
- PV=

Excel =PV(.08,3,,257.71)

Example 19

Your grandmother is gifting you $125 a month for four years while you attend college to earn your bachelor's degree. At a 6.5 percent discount rate, what are these payments worth to you on the day you enter college?

Perpetuity

$$PV = \frac{pmt}{i}$$

Example 20: Perpetuity

Suppose you have a perpetuity that pays $100 a period forever, with an interest rate of 8%. What is the PV?

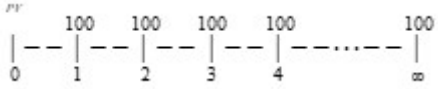

Figure 14: Timeline, Perpetuity

$$PV = \frac{100}{0.08} = 1250$$

Growing Perpetuity

$$PV = \frac{C}{r - g}$$

where:

- C:

- r:
- g:

Example 21: Growing Perpetuity

Suppose you have a perpetuity that pays $100 next year. Thereafter the payments grow at a rate of 3%. Assuming an interest rate of 8%, what is the PV?

$$PV = \frac{100}{0.08 - 0.03} = 2000$$

Example 22

Your buddy in mechanical engineering has invented a money machine. The main drawback of the machine is that it is slow. It takes one year to manufacture 100 dollars. However, once built, the machine will last forever and will require no maintenance. The machine can be built immediately, but it will cost 1,000 to build. Your buddy wants to know if he should invest the money to construct it. If the interest rate is 9.5 percent per year, what should your buddy do? (Adapted from Berk & DeMarzo, 2011)

Mutliple Cash flows

Example 23: multiple cash flows

Suppose you have an investment that will pay us the following cash flows:

- Year 1: 200
- Year 2: 400
- Year 3: 600
- Year 4: 800

Assuming you can earn a 12% interest rate, what is the present value?

$$PV = \frac{200}{(1+0.12)} + \frac{400}{(1+0.12)^2} + \frac{600}{(1+0.12)^3} + \frac{800}{(1+0.12)^4} = 1,432.93$$

Calculator

- Hit CF
  - CF0=0
  - C01=200
  - C02=400
  - C03=600
  - C04=800
- Note: F0 buttons represent frequency.
- Hit NPV
  - I=12
- Hit CPT
- PV=

Excel =NPV(0.12, 200,400,600,800)

Example 24

You just won the TVM lottery. You will receive 1 million dollars today, plus 1.45 million in two years, 1.9 million in four years, and 5 million in six years. If the appropriate discount rate is 7 percent, what is the present value of your winnings?

Example 25: Finding the payment

Suppose you want to borrow $20,000 to buy a new car. You can borrow at 8% per year, compounded monthly. If you take out a 4 year loan, what is your monthly payment?

Calculator

- N=4*12
- I/Y=8/12
- PV=-20,000
- PMT=

Excel =PMT(.00667, 48, -20,000)

Example 26

Holiday Tours (HT) has an employment contract with its newly hired CEO. The contract requires a lump sum payment of $10.4 million be paid to the CEO upon the successful completion of her first three years of service. HT wants to set aside an equal amount of money at the end of each year to cover this anticipated cash outflow and will earn 5.65 percent on the funds. How much must HT set aside each year for this purpose?

Example 27: Finding the number of periods

You ran a little short on your spring break vacation, so you put 1,000 dollars on your credit card. You can afford only the minimum payment of 20 dollars per month. If the credit card has an annual interest rate of 18%, how many years will you need to pay off the $1,000?

Calculator

- I=18/12
- PV=-1,000
- PMT=20
- N=

Excel =(nper(.015, 20, -1000))/12

Example 28

You want to retire on the day you have 750,000 in your savings account. You expect to earn 7 percent, compounded monthly, on your money during your retirement. Your plan is to withdraw $6,000 a month as retirement income from this account. How many months can you be retired until you run out of money?

Example 29: Finding the rate

Suppose you borrow 10,000 dollars from your parents to buy a car. If you agree to pay $207.85 per month for 5 years, what is the interest rate?

Calculator

- N=5*12
- PV=-10,000
- PMT=207.85
- I/Y=

Excel =(rate(60,207.58,-10000))*12

Example 30

Your insurance agent is trying to sell you an annuity that costs 230,000 today. By buying this annuity, your agent promises that you will receive payments of $1,225 a month for the next 30 years. What is the rate of return on this investment?

Example 31

Your brother has offered to give you either 5,000 today or 10,000 in 10 years. If the interest rate is 7 percent per year, which option is preferable?

Calculator

- N=
- I/Y=
- FV=
- PV=

Excel =PV

Example 32: Deferred Annuity

You are planning to save for retirement over the next 30 years. To do this, you will invest 850 per month in a stock account and 350 per month in a bond account. The return on the stock account is expected to be 10 percent, and the bond account will pay 6 percent. When you retire, you will combine your money into an account with a 5 percent return. How much can you withdraw each month during retirement assuming a 25 year withdrawal period?

Step 1:Stock Account

Calculator

- N=30*12
- PMT=-850
- I/Y=10/12
- FV=

Excel =FV(0.1/12, 360, -850)

Step 2: Bond Account

Calculator

- N=30*12
- PMT=-350
- I/Y=6/12
- FV=

Excel =FV(0.06/12, 360, -350)

Step 3: Withdrawal period

Calculator

- N=25*12
- PV=1,921,415+351,580
- I/Y=5/12
- PMT=

Excel =PMT(0.05/12, 300, -2272995)