Exercise 1

We first load a dataset and examine its dimensions.

In [1]:
```
# If you are running this on Google Colab, uncomment and run the following lines; ot
# from google.colab import drive
# drive.mount('/content/drive')
```

In [2]:
```
import math
import numpy as np

xy_data = np.load('Ex1_polyreg_data.npy')
# If running on Google Colab change path to '/content/drive/MyDrive/IB-Data-Science/

np.shape(xy_data)
```

Out[2]:
```
(70, 2)
```

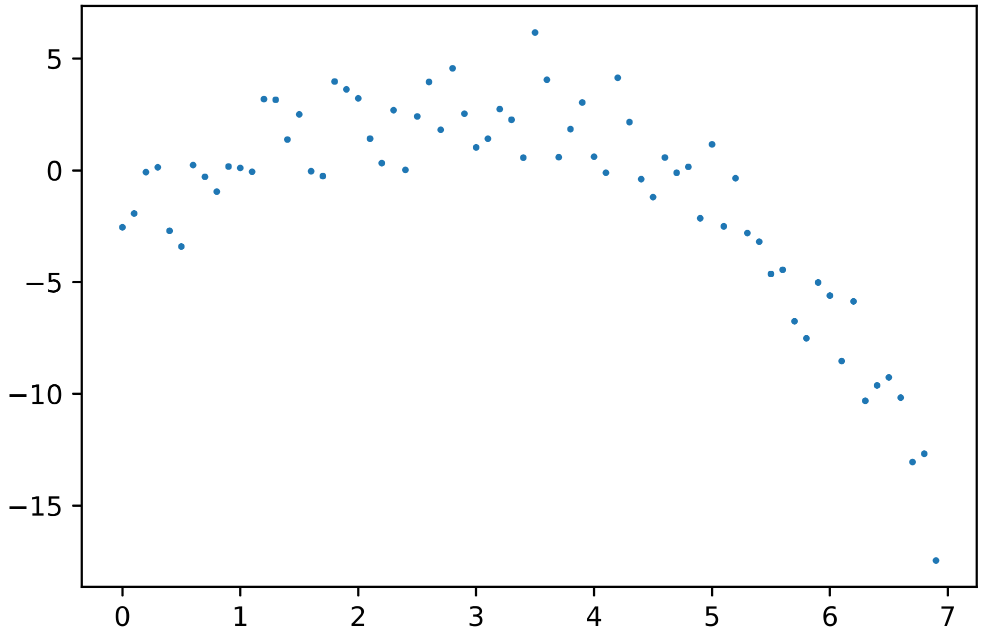
The matrix `xy_data` contains 70 rows, each a data point of the form (x_i, y_i) for $i = 1, \ldots, 70$.

1a) Plot the data in a scatterplot.

In [3]:
```
import matplotlib.pyplot as plt

x = xy_data[:,0]
y = xy_data[:,1]

plt.scatter(x,y,s=2)
plt.show()
```



1b) Write a function `polyreg` to fit a polynomial of a given order to a dataset.

The inputs to the function are a data matrix of dimension $N \times 2$, and $k \geq 0$, the order of the polynomial. The function should compute the coefficients of the polynomial

$\beta_0 + \beta_1 x + \ldots + \beta_k x^k$ via least squares regression, and should return the coefficient vector, the fit, and the vector of residuals.

If specified the degree k is greater than or equal to N, then the function must fit an order $(N-1)$ polynomial and set the remaining coefficients to zero.

NOTE: You are *not* allowed to use the built-in function `np.polyfit` .

In [20]:
```
def polyreg(data_matrix, k):
    """Polynomial fit using Least Squares
    data_matrix: N x 2 array
    k: degree of polynomial
    NOTE: beta-vector is calculated as: (X.T X )-1 X.T y
    """
    assert data_matrix.shape[1] == 2
    N = data_matrix.shape[0]
    degree = N-1 if k > N else k    # Ensure degree of polynomial is less than numbe
    # print(f"Degree of polynomial: {degree}")

    x = data_matrix[:,0]    # Get x values
    y = data_matrix[:,1]     # y
    X = np.column_stack([x**i for i in range(k+1)]) # Create X column matrix

    beta = np.linalg.inv(X.T.dot(X)).dot(X.T).dot(y)    # Least squares
    fit = X.dot(beta)                                   # Fit
    beta = beta if k <= N else np.append(beta, [0 for _ in range(k - N + 1)])   # Ad

    return beta, fit, y-fit
    # The function should return the the coefficient vector beta, the fit, and the v
```

Use the tests below to check the outputs of the function you have written:

In [21]:
```
# Some tests to make sure your  function is working correctly

xcol = np.arange(-1, 1.05, 0.1)
ycol = 2 - 7*xcol + 3*(xcol**2)  # We are generating data accoridng to y = 2 - 7x +
test_matrix = np.transpose(np.vstack((xcol,ycol)))
test_matrix.shape

beta_test = polyreg(test_matrix, k=2)[0]
assert((np.round(beta_test[0], 3) == 2) and (np.round(beta_test[1], 3) == -7) and (n
# We want to check that using the function with k=2 recovers the coefficients exactl

# Now check the zeroth order fit, i.e., the function gives the correct output with k
beta_test = polyreg(test_matrix, k=0)[0]
res_test = polyreg(test_matrix, k=0)[2] #the last output of the function gives the v

assert(np.round(beta_test, 3) == 3.1)
assert(np.round(np.linalg.norm(res_test), 3) == 19.937)
```

In [22]:
```
# Check for k > 3
n = len(test_matrix)
res = polyreg(test_matrix, k=19)
```

1c) Use `polyreg` to fit polynomial models for the data in `xy_data` for $k = 2, 3, 4$:

- Plot the fits for the three cases on the same plot together with the scatterplot of the data. The plots should be labelled and a legend included.

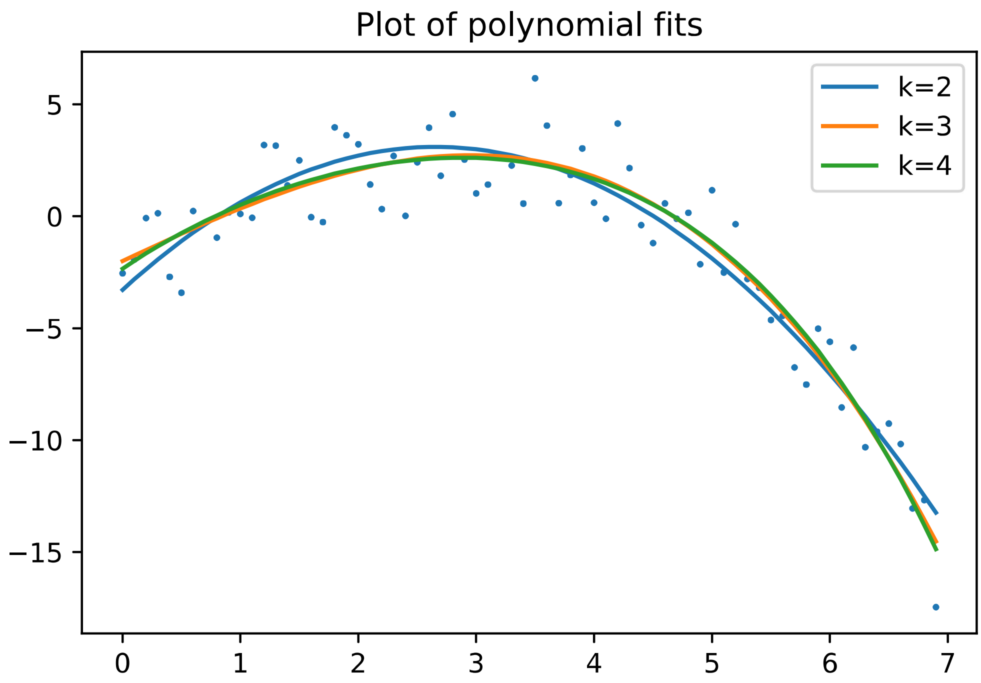
- Compute and print the SSE and R^2 coefficient for each of the three cases.
- Which of the three models you would choose? Briefly justify your choice.

In [23]:
```
# Plot points
plt.scatter(x,y,s=2)

SSE_0 = len(y) * np.var(y) # N * Variance

# Plot fits
for k in range(2,5):
    beta, fit, resid = polyreg(xy_data, k)
    plt.plot(x, fit, label=f"k={k}")
    SSE_k = sum(resid**2)
    print(f"k: {k}\tSSE: {SSE_k:.3g}\tR2: {1 - SSE_k/SSE_0:.3g}")
plt.legend()
plt.title("Plot of polynomial fits")
plt.show()
```

```
k: 2    SSE: 172        R2: 0.888
k: 3    SSE: 152        R2: 0.901
k: 4    SSE: 151        R2: 0.901
```



State which model you choose and briefly justify your choice.

Without knowing what the data is about, it is difficult tell what level of precision is required. A cubic seems to give the best middle groudn in terms of computation vs accuracy as it has R2 > 0.9. Interestingly, k=3 gives a better guassian distribution of residuals

1d) For the model you have chosen in the previous part (either $k = 2/3/4$):

- Plot the residuals in a scatter plot.
- Plot a histogram of the residuals along with a Gaussian pdf with zero mean and the same standard deviation as the residuals.

In [27]:
```
from scipy.stats import norm
```

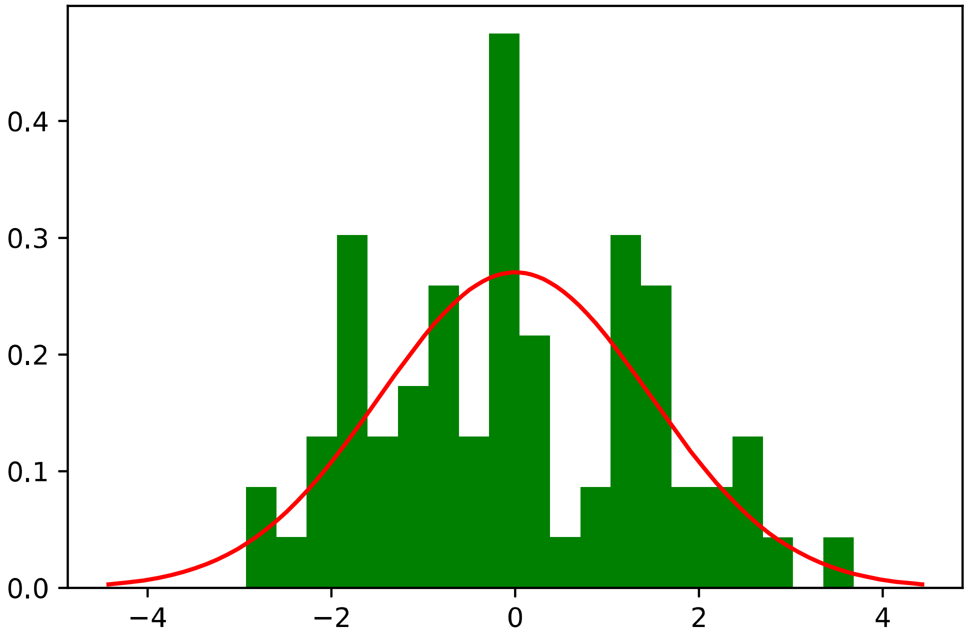
In [32]:
```
# Get fit with k = 3
beta, fit, resid = polyreg(xy_data, k=3)
```

```
print(f'Residuals: Mean={np.mean(resid):.3f}, Var={np.var(resid):.3f}')

# Plot normed histogram of the residuals
n, bins, patches = plt.hist(resid, bins=20, density=True, facecolor='green')

# Plot Gaussian pdf with same mean and variance as the residuals
res_stdev = np.std(resid)  #standard deviation of residuals
xvals = np.linspace(-3*res_stdev,3*res_stdev,1000)
plt.plot(xvals, norm.pdf(xvals, loc=0, scale=res_stdev), 'r')
plt.show()
```

```
Residuals: Mean=0.000, Var=2.177
```



In [ ]: