Physics I - Class Notes. - 2016.

Dear Friends,

I am circulating my class notes to you. I would like to mention some important points.

1) These notes are made only for internal circulation. Please don't post these notes in the internet.

2) While I was making these notes I took help from the following books. Sometimes I have taken examples directly from these books. Some of these books are

(a) Introduction to Mechanics — Kleppner & Kolenkow.

(b) Resnick Halliday Krane.

(c) Young & Freedman.

(d) D. C. Pandey

(e) H. C. Verma

(f) MIT Video Lectures by Prof. Walter Lewin.

(g) Wikipedia.

(h) Tipler & Mosca (i) Dynamics - Meriam.

(j) Internet Resources & other books.

I acknowledge all the above authors & sites.

3) In many places I have added my own views and Thoughts. I never claim that my notes are totally free of errors. There may be some typos, some mistake. You must check & verify yourself. If you have any doubt or confusion or if you feel that something wrong is written somewhere you must contact me immediately. We will ~~get it~~ correct it & rectify the mistake. In science there is no absolute authority unlike religion. It is because religious truths cannot be verified ~~to~~ by mind / brain itself where as scientific truths can be verified by mind / brain only & not beyond That. I will be thankful if you find out any mistake or give your suggestion, add something more interesting so that this note become richer, better & free of any mistake or error.

All the best for Physics-1.

Aneogi

(AMIT NEOGI)

14 Aug, 2016.

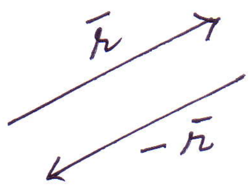
Vectors

- **Vectors** - Magnitude & Direction.
- Why Vector formulation is invented?
  - To Simplify the Writing equation.

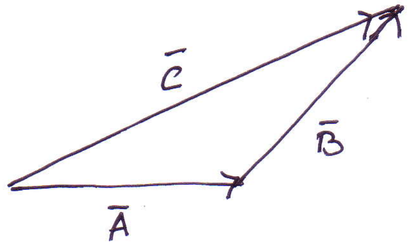
→ $F_x = m a_x$

→ $F_y = m a_y$ ⇒ Can be Written as $\bar{F} = m\bar{a}$

→ $F_z = m a_z$



- **Properties of Vectors**
  - $\bar{A} = \bar{B}$ if direction & Magnitude are Same.
  - Unit Vector: $\hat{A} = \dfrac{\bar{A}}{|\bar{A}|}$ OR $\bar{A} = |\bar{A}|\,\hat{A}$
  - $\bar{B} = x\bar{A}$ (-1) will flip this direction.
  - Addition of Vectors.



$\bar{A} + \bar{B} = \bar{B} + \bar{A}$ (commutative)

$\bar{A} + (\bar{B} + \bar{C}) = (\bar{A} + \bar{B}) + \bar{C}$ (Associative)

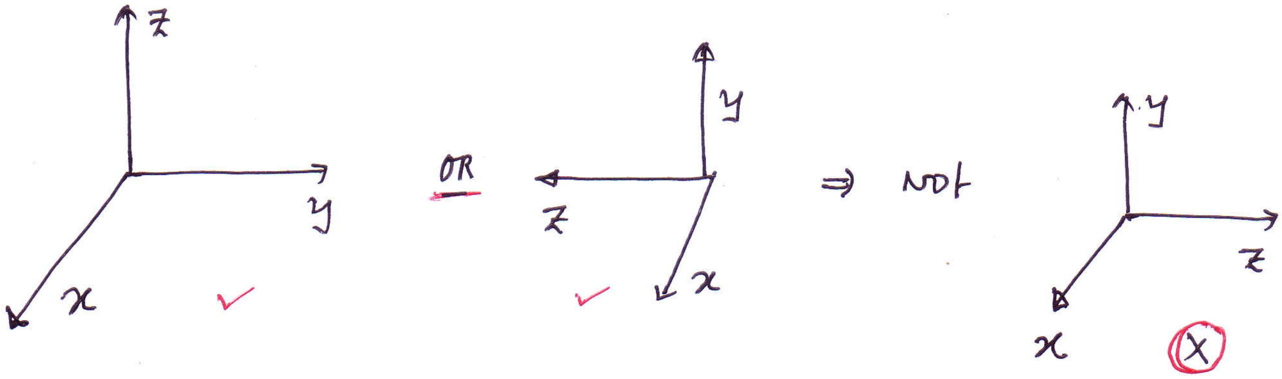
$c(d\bar{A}) = (cd)\bar{A}$ (")

$(c+d)\bar{A} = c\bar{A} + d\bar{A}$ (distributive)

$c(\bar{A} + \bar{B}) = c\bar{A} + c\bar{B}$ (")

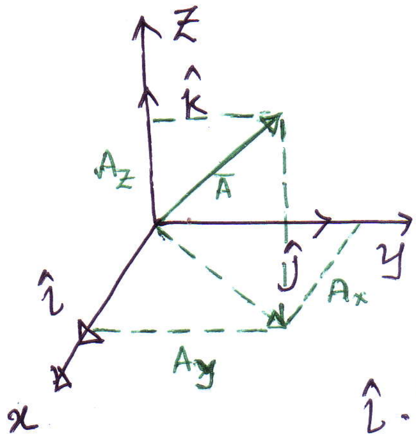
Cartesian Coordinates.

We follow right handed Coordinate system.



- ## Base Vectors.

Base Vectors are a set of orthogonal (perpendicular) Unit vectors one for each dimension



$\hat{i}$ = Unit Vector along x axis.

$\hat{j}$ = unit Vector along y axis.

$\hat{k}$ = Unit vector along z axis.

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1.$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

$$\hat{i}\times\hat{j} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$

$$\boxed{\begin{aligned}\bar{A} &= A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\\ |\bar{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2}\end{aligned}}$$

Multiplication of Vectors.

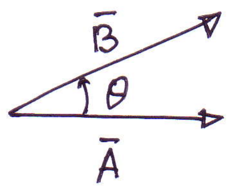
(a) Multiplication of a Vector by a Scalar

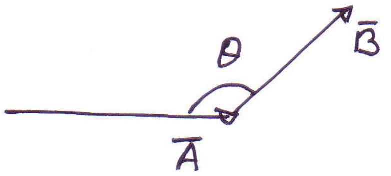
$\bar{C} = b\bar{A} \Rightarrow |C| = b|A|.$

(Multiplication by $-1 \Rightarrow$ new Vector opp in direction to the original vector.

(b) Dot Product.

$\boxed{\bar{A}\cdot\bar{B} = |A|\,|B|\cos\theta}$ (Projection of $\bar{B}$ on $\bar{A}$)





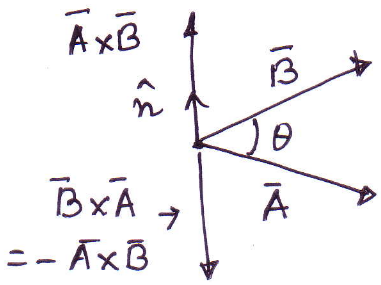
X (wrong).

$\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$ $\qquad A \neq 0,\ B \neq 0.$

$\bar{A}\cdot\bar{B} = 0 \Rightarrow |A| = 0$, OR $|B| = 0$ OR $\theta = 90^\circ$.

(c) Cross Product | Vector Product.

$\boxed{\bar{A}\times\bar{B} = |A|\,|B|\sin\theta\,\hat{n}}$



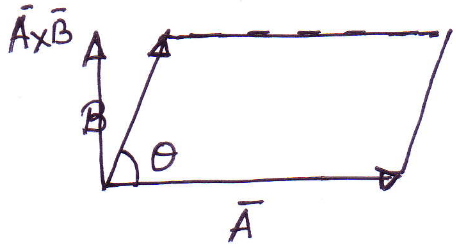
Here θ is the angle between $\bar{A}$ & $\bar{B}$
The direction $\hat{n}$ is defined by right hand rule. $\hat{n}$ is a Unit Vector Perpend. to the Plane Containing $\bar{A}$ & $\bar{B}$.

$\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$

$\bar{A}\times\bar{A} = 0$

Geometrical Meaning.

The magnitude of the ~~parallelogram~~ Cross Product can be interpreted as the positive area of parallelogram having $\bar{A}$ & $\bar{B}$ as sides.



Area = base × height = $AB\sin\theta$

$= \bar{A}\times\bar{B}$

If we think of Area as a Vector we have $\bar{Area} = \bar{A}\times\bar{B}$

Area is a vector.

Dot Product & Vector Product in terms of Components.

A vector can be written in terms of components.

$$A = (A_x, A_y, A_z)$$

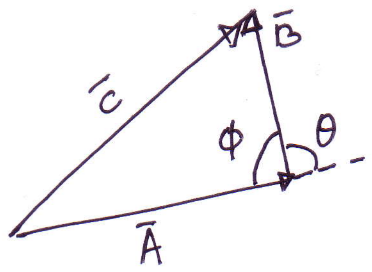
OR $\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

- $\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z$ v. Imp.

- $$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$ v. Imp.

$$= (A_yB_z - B_yA_z)\,\hat{i} - (A_xB_z - B_xA_z)\,\hat{j} + (A_xB_y - B_xA_y)\,\hat{k}.$$

Law of Cosines



$$\bar{C} = \bar{A} + \bar{B}$$

$$\bar{C}\cdot\bar{C} = (\bar{A}+\bar{B})\cdot(\bar{A}+\bar{B})$$

$$|\bar{C}^2| = |\bar{A}^2| + |\bar{B}|^2 + 2|\bar{A}|\,|\bar{B}|\cos\theta$$

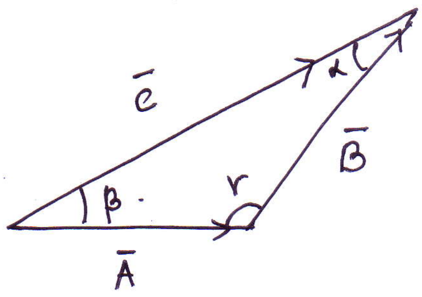
$$C^2 = A^2 + B^2 + 2AB\cos(\pi-\phi).$$

$$\boxed{C^2 = A^2 + B^2 - 2AB\cos\phi.}$$

'$-$' ve sign 4 angle ϕ is imp.

Using this formula one can find out the resultant magnitude ~~of Whic~~ of a Vector which is the Vector sum of two Vectors.

Law of Sines.



$$\bar{C} = \bar{A} + \bar{B}$$

$$\bar{A}\times\bar{C} = \bar{A}\times\bar{A} + \bar{A}\times\bar{B}$$

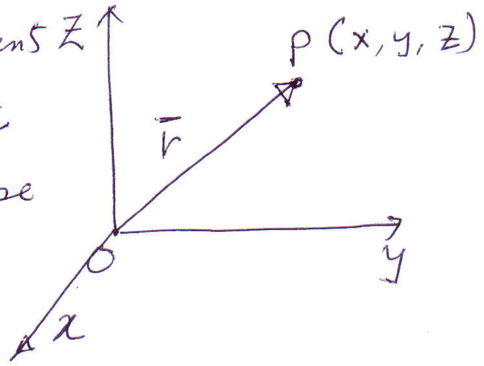
$$AC\sin\beta = AB\sin(\pi-\gamma)$$

$$C\sin\beta = B\sin\gamma.$$

$$\boxed{\frac{C}{\sin\gamma} = \frac{B}{\sin\beta} = \frac{A}{\sin\alpha}}$$

Position Vector

Although Vectors define displacements rather than position of a point, it is in fact possible to describe the position of a point wrt to Origin of a given coordinate system by a Special Vector known as position Vector.

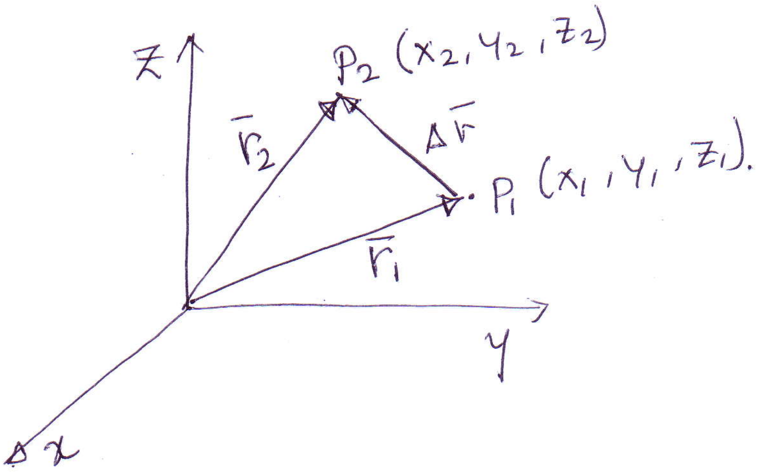


$$\bar{r}(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}.$$

Unlike Ordinary Vector r depends upon coordinate syst.

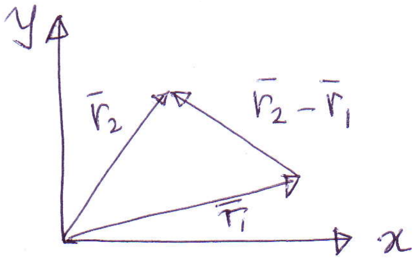
Displacement Vector.

It is independent of coordinate system — a true Vector.



$$\Delta\bar{r} = \bar{r}_2 - \bar{r}_1$$

Velocity and Acceleration



$$\underline{\text{Average Vel}} = \frac{\bar{r}_2 - \bar{r}_1}{\Delta t}$$

Instantaneous Vel

$$\bar{V} = \lim_{\Delta t \to 0} \frac{\bar{r}(t+\Delta t) - \bar{r}(t)}{\Delta t}$$

$$\boxed{\vec{v} = \frac{d\bar{r}}{dt}}$$

Acceleration.

$$\bar{a}_{avg} = \frac{\bar{v}_2 - \bar{v}_1}{\Delta t}$$

$$\bar{a}_{inst} = \lim_{\Delta t \to 0} \frac{\bar{v}(t+\Delta t) - \bar{v}(t)}{\Delta t} = \frac{d\bar{v}}{dt}$$

In term of Components in Cartesian Cordinats.

$$\bar{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{j}.$$

$$\frac{d\bar{r}(t)}{dt} = \frac{dx(t)}{dt}\hat{i} + x(t)\cancel{\frac{d\hat{i}}{dt}} \to 0$$

$$+ \frac{dy(t)}{dt}\hat{j} + y(t)\cancel{\frac{d\hat{j}}{dt}} \to 0$$

$$+ \frac{dz(t)}{dt}\hat{k} + z(t)\cancel{\frac{d\hat{k}}{dt}} \to 0$$

$$\dot{\vec{r}} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k} \qquad |\dot{r}| = \frac{dr}{dt} \quad |\ddot{r}| = \frac{d^2r}{dt^2}.$$

In Cart Coord Unit Vectors are Const Vect $\frac{d\hat{i}}{dt} = \frac{d\hat{j}}{dt} = \frac{d\hat{k}}{dt} = 0$.