# The Relativistic Momentum & KE. (Problems)

The Unit eV. (Electron volt).

In atomic Physics the usual Unit of energy is electron volt (ev) where 1 eV is the energy gained by an electron accelerated through a ~~po~~potential diff of 1 volt. , $W = QV$.

$$1\,eV = (1.602 \times 10^{-19} C)(1.000 V) = 1.602 \times 10^{-19} J.$$

$1\,MeV = 10^{6}\,eV$. $\quad 1\,GeV = 10^{9}\,eV$.

The rest energies of elementary particles are often expressed ~~as~~ in MeV and GeV and the Corresponding rest masses in $MeV/c^2$ and $GeV/c^2$

The advantage of the latter Units is that the rest energy equivalent to a rest mass of say 0.938 $GeV/c^2$ (the rest mass of the proton) is just $E_0 = mc^2$ $= 0.938$ GeV.

\# An electron ($m = 0.511\ MeV/c^2$) and photon ($m=0$) both have momenta of $2.000\ MeV/c$. Find the total energy of each.

(a) $$E = \sqrt{m^2c^4 + p^2c^2} = \sqrt{(0.511\ MeV/c^2)^2 c^4 + \left(2.000\ \frac{MeV}{c}\right)^2 c^2}$$

$$= \sqrt{(0.511\ MeV)^2 + (2.000\ MeV)^2} = 2.064\ MeV.$$

(b) Photon Total Energy $= pc = \left(2.000\ \frac{MeV}{c}\right)c = 2.000\ MeV.$

6. Calculate the momentum of 1 MeV electron.

There is one equation relating momentum & KE.

$$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

Here I am following rest mass invariant approach. Wherever I am writing m, it is rest mass & ~~rest~~ not relativistic mass.

$$\underline{\underline{E}} = \text{Total Energy} = \underline{KE + mc^2}$$

↑ rest energy = $E_0$

$$E = (K + E_0).$$

$$\underline{(K+E_0)^2 = (pc)^2 + (E_0)^2}$$

Rest Energy of Electron

$$\underline{\underline{E_0}} = mc^2 = \frac{9.1\times10^{-31} \times (3\times10^8)^2}{1.6\times10^{-19} \times 10^6} \text{ MeV.} = 0.511 \text{ MeV.}$$

$10^6$ ↳ For Converting in MeV.

$1.6\times10^{-19}$ ↓ For Converting into eV

<u>So</u>

$$(1\text{ MeV} + 0.511\text{ MeV})^2 = (pc)^2 + (0.511\text{ MeV})^2$$

$$p = 1.42 \frac{\text{MeV}}{c}$$

Momentum is often expressed as $\frac{\text{MeV}}{c}$.

7. Calculate the velocity of an electron whose $KE = 2$ Mev.

$$K = \gamma mc^2 - mc^2 \qquad (m = \text{rest mass})$$

$$2\,\text{MeV} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

$$2\,\text{MeV} = \frac{0.511\,\text{MeV}}{\sqrt{1 - v^2/c^2}} - 0.511\,\text{MeV}.$$

[ $E_0 = mc^2 =$ Rest Energy of an electron]
$= 0.511$ MeV.

$$v = 0.98c.$$

8. As is written in your nots that some authors accept the concept of relativistic mass that is mass is changing with velocity where as many authors dont. Here it is asked about mass & rest mass so we have to do it that way - That is Relativistic Mass.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0$$

Now from time dilation

$$\Delta t = \gamma \Delta t_0 \quad \Rightarrow \quad \gamma = \frac{\Delta t}{\Delta t_0} = \frac{7}{2}$$

So $m = \frac{7}{2}(m_{oe}\,207) = 724.5\,m_{oe}.$
$= 6.59 \times 10^{-28}$ kg.

[ $m_{oe}$ = Rest mass of Electron
$= 9.1 \times 10^{-31}$ kg]

(9) For observer O we have.

$$u_x = (0.8)\, c \cos 30^\circ = 0.693c$$

$$u_y = (0.8)\, c \sin 30^\circ = 0.400\, c.$$

Using Lorentz transformation, we have for observer O'

$$u_x' = \frac{u_x - v}{1 - \left(\frac{v}{c^2}\right) u_x} = \frac{0.693\,c - (-0.6c)}{1 - \frac{(-0.6)c\,(0.693c)}{c^2}} = 0.913\,c.$$

$$u_y' = \frac{u_y \sqrt{1 - v^2/c^2}}{1 - \left(\frac{v}{c^2}\right) u_x} = \frac{(0.4c)\sqrt{1-(0.6)^2}}{1 - \left(\frac{0.6c}{c^2}\right) 0.693c} = 0.226c$$

The Speed measured by observer O' is

$$u' = \sqrt{u_x'^2 + u_y'^2} = \sqrt{(0.913\,c)^2 + (0.226c)^2} = 0.941c$$

The angle $\phi'$ the velocity makes with - x axis is

$$\tan\phi' = \frac{u_y'}{u_x'} = \frac{0.226c}{0.913\,c} = 0.248$$

$$\phi' = 13.9c$$

(Set your Calculator to degree)