General Points (3 Imp points)

(1) Torque in a stationary Object & Rotating object:
When a torque is applied to a stationary object it moves in the direction of torque.
When a torque is applied to a rotating / spinning object, & if the value of Torque is much less compared to the angular mom. — then the ang mom. vector rotates. (Just like Centripetal force gives rise to circular motion)

(2) When a Vector rotates that means angular mom rotates then

$$\frac{d\bar{L}}{dt} = \bar{\omega} \times \bar{L} = \bar{\tau}_{ext.} \quad \Big| \quad \frac{d\bar{r}}{dt} = \bar{\omega} \times \bar{r}$$

• precessional ang vel. written as Ω or ω_p.

• Very very useful Formula.

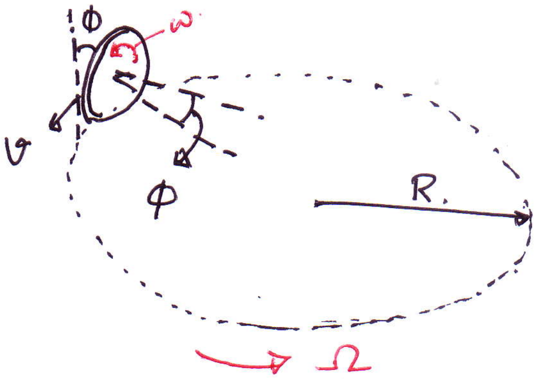
(3) While using the formula no 2 — τ_{ext} or L can be calculated about a particular point
- 1) fixed pt on earth
- 2) about CM

(4) Diff between ω_p & ω_s.

ω_p: ω of precession
- also written as Ω
- You may also call this as orbital motion like that of earth

ω_s: Spin angular velocity
$\omega_s >> \omega_p$.
It gives rise to large angular momentum L_s

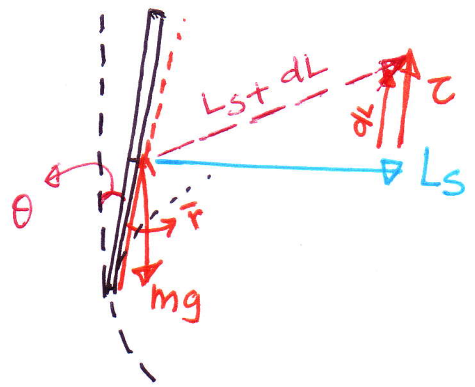
\# If you start a coin rolling on a table with care, you can make it roll in a circle. As you can see from the drawing, the coin "leans" inward with its axis tilted. The radius of the coin is b, the radius of the circle it follows on the table is R, and its velocity is v. Assume that there is no slipping. Find the angle ϕ that the axis makes with the horizontal. (Kleppner – 7.6)



⇒ It is again a very interesting problem from where we can learn lot of things. All these problems are based on gyroscopic principle.

If a body is stationary & you apply a torque, we know very well its motion. It will rotate in the direction of torque.

But if the body is in circular motion / rotating about an axis & its angular momentum vector because of its rotation L_s is quite large compared to torque then the angular momentum vector L_s rotates. This kind of situations are observed in different cases like Gyroscope, bicycle etc. The above example is exactly like that of a bicycle – bicycle does not fall in motion but front wheel either moves towards the left or right due to torque. – dL.



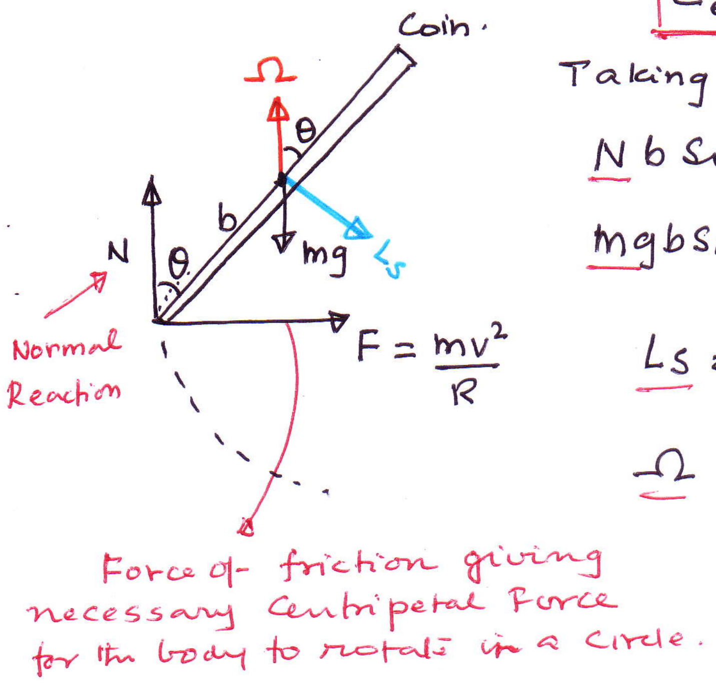
Since the resultant L vector rotates the coin will rotate (changes direction) in a circle.

So because of torque the angular ~~Vecto~~ momentum vector rotates. So we need to apply.

(1) $$\frac{d\bar{L}}{dt} = \bar{\omega} \times \bar{L} = \tau_{ext}.$$

The second important point about these problems. See all the forces acting on the body. We are tempted to take the torque about a point where the coin touchs the ground. But that is an accelerating point — a changing point. We need to take the torque about a fixed pt or about the center of mass. Since we don't find any fixed point, so we need to calculate torque about center of mass. That is very important.

$$\boxed{\bar{\tau}_{ext} = \bar{\omega} \times \bar{L}}$$



Taking Torque about CM.

$$N b \sin\theta - F b \cos\theta = \Omega L_s \sin(90+\theta)$$

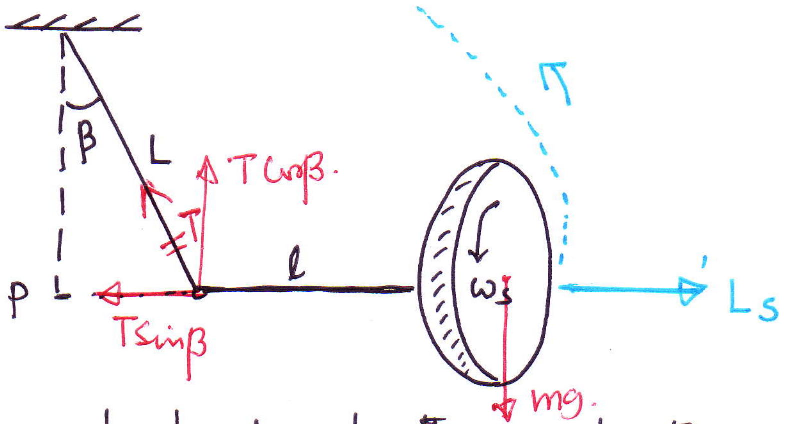
$$mgb\sin\theta - \frac{mv^2}{R} b \cos\theta = \Omega L_s \cos\theta.$$

$$L_s = I\omega_s = \frac{mb^2}{2}\omega_s = \frac{mb^2}{2}\frac{V}{b} = \frac{mbV}{2}$$

$$\Omega = \frac{V}{R}$$

$$\Rightarrow \boxed{\tan\theta = 3v^2/2gR}$$

7.3



We have to find out the angle β. β is very small such that $\sin\beta \approx \beta$.

- Here we can apply the same equation.

$$\frac{d\bar{L}}{dt} = \bar{\omega} \times \bar{L} = \bar{\tau}_{ext}$$

Force $\quad T\cos\beta = Mg \quad -(1)$

$$T\sin\beta = \frac{MV^2}{l + L\sin\beta} = M\Omega^2 (l + L\sin\beta) \quad -(2)$$

?

Now we can calculate the Torque about some fixed pt P or about the center of Mass. It will be simpler if we calculate about COM. In the notes given otherwise.

Torque $\quad T\cos\beta\, l = \Omega L_s = \Omega I_0 \omega_s$

$\Downarrow$

Mg so $Mgl = \Omega I_0 \omega_s \quad -(3)$

Above all the three eqns are given eqns.

$\sin\beta \approx \beta \qquad \cos\beta \simeq 1.$

$T\sin\beta = M\Omega^2 (l + L\sin\beta)$

$T\beta = M\Omega^2 (l + L\beta)$

$\cancel{M}g\,\beta = \cancel{M}\Omega^2 (l + L\beta) \Rightarrow$

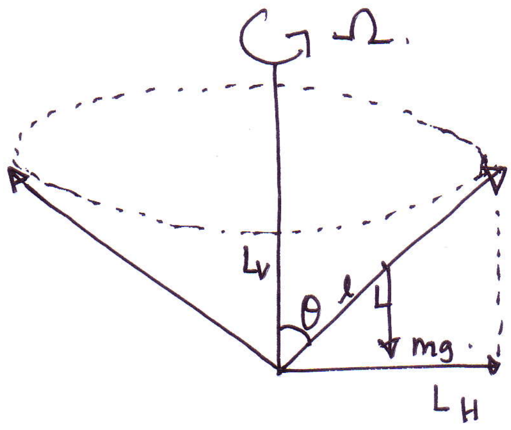
$$\left(\frac{l + L\beta}{\beta}\right) = \frac{g}{\Omega^2} = \frac{g\,(I_0\omega_s)^2}{(Mgl)^2}$$

$\frac{l}{\beta} + L = (\quad)$

$\frac{l}{\beta} = (\quad) - L$

$\beta = l/(\quad) - L$

Method-2 (Simpler) Top precessing with Ω



$$\left(\frac{dL}{dt}\right) = \Omega L \sin\theta$$

$$L = I\omega_s$$

$$\Omega I\omega_s \sin\theta = mgl\sin\theta$$

$$\Omega = \frac{mgl}{I\omega_s}$$

- Earlier in the notes I have solved the problem of Top preceseing with angular velocity Ω & calculated its value using cylindrical polar coordinats

- The above solution is much simpler using the concept of rotating vector that is L_s vector rotates

So $$\frac{dL_s}{dt} = \Omega \times L_s = \Omega L_s \sin\theta$$

Using this concept the problem can be solved in one step only.