

Difference between addition and union of two sets:

Generally, $A+B$ is called the “Minkowski Sum”. It denotes the set of sums of an element of A and an element of B . On the other hand, $A \cup B$ refers to the union with the common elements removed.

Example 1:

$$\{1,2\} + \{1,2,4\} = \{1+1,1+2,1+4,2+1,2+2,2+4\} = \{2,3,4,5,6\}, \text{ whereas}$$

$$\{1,2\} \cup \{1,4,5\} = \{1,2,4,5\}$$

Elaboration on “Minkowski Sum” (If you are interested):

In geometry, the Minkowski sum (also known as dilation) of two sets of position vectors A and B in Euclidean space is formed by adding each vector in A to each vector in B , i.e., the set

$$A + B = \{a + b \mid a \in A, b \in B\}$$

Analogously, the Minkowski difference (or geometric difference) is defined using the complement operation as

$$A - B = (A^c + B^c)$$

In general, $A - B \neq A + (-B)$. For instance, in a one-dimensional case $A = [-2, 2]$ and $B = [-1, 1]$, the Minkowski difference $A - B = [-1, 1]$, whereas $A + (-B) = A + B = [-3, 3]$.

In a two-dimensional case, Minkowski difference is closely related to erosion (morphology) in image processing.

The concept is named for **Hermann Minkowski**.

For example, if we have two sets A and B , each consisting of three position vectors (informally, three points), representing the vertices of two triangles in \mathbb{R}^2 with coordinates

$$A = \{(1, 0), (0, 1), (0, -1)\}$$

And

$$B = \{(0, 0), (1, 1), (1, -1)\},$$

then their Minkowski sum is

$$A+B=\{(1, 0), (2, 1), (2, -1), (0, 1), (1, 2), (1, 0), (0, -1), (1, 0), (1, -2)\},$$

which comprises the vertices of a hexagon.

For Minkowski addition, the zero set, $\{0\}$, containing only the zero vector, 0 , is an identity element: for every subset S of a vector space,

$$S + \{0\} = S.$$

The empty set is important in Minkowski addition, because the empty set annihilates every other subset: for every subset S of a vector space, its sum with the empty set is empty:

$$S + \emptyset = \emptyset.$$