Solution to Q12.13.3.52

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Question: A bag contain (2n+1) coins. It is known that n of these coins have a head on both sides where as the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.

Solution:

parameter	value	description
Random Variable X	0	Fair coin is selected
	1	Unfair coin is selected
Random Variable Y	0	Head comes in the toss
	1	Tail comes in the toss

TABLE 0
Tosses of coins

$$p_X(0) = \frac{n+1}{2n+1} \tag{1}$$

$$p_X(1) = \frac{n}{2n+1} \tag{2}$$

$$\Pr(Y=0|X=0) = \frac{1}{2} \tag{3}$$

$$\Pr(Y=0|X=1) = 1 \tag{4}$$

Hence, the probability of head is

$$p_X(0)\Pr(Y=0|X=0) + p_X(1)\Pr(Y=0|X=1) = \frac{n+1}{2n+1}\times\frac{1}{2} + \frac{n}{2n+1}\times 1 \tag{5}$$

$$= \frac{3n+1}{2(2n+1)} \tag{6}$$

Now,

$$\frac{3n+1}{2(2n+1)} = \frac{31}{42} \tag{7}$$

$$\implies \frac{3n+1}{2n+1} = \frac{31}{21} \tag{8}$$

$$\implies 63n+21 = 62n+31 \tag{9}$$

$$\implies n = 10 \tag{10}$$