

Solution to Q 9.3.25

Mayank Gupta

Question: A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

- (a) none of the bulb is defective
- (b) exactly two bulbs are defective
- (c) more than 8 bulbs are working properly

Solution:

parameter	value	description
n	10	Number of bulbs in the bag
p	$\frac{1}{50}$	Bulb chosen is defective
q	$\frac{49}{50}$	Bulb chosen is proper
$\mu = np$	$\frac{1}{5}$	Mean of the distribution
$\sigma^2 = npq$	$\frac{49}{250}$	Variance of the distribution

TABLE 0
GAUSSIAN INFO TABLE

(i) Gaussian Distribution

Let Y is the Gaussian obtained by approximating binomial with parameters n, p then By Central limit theroem,

$$Y \sim \mathcal{N}(np, npq) \quad (1)$$

CDF of Y is:

$$F_Y(x) = \Pr(Y \leq x) \quad (2)$$

$$= \Pr(Y - \mu \leq x - \mu) \quad (3)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}\right) \quad (4)$$

Since,

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (5)$$

Q function is defined

$$Q(x) = \Pr(Y > x) \quad \forall x \in Y \sim \mathcal{N}(0, 1) \quad (6)$$

From (4) and (6),

$$F_Y(x) = 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (7)$$

$$= \begin{cases} 1 - Q\left(\frac{x - \mu}{\sigma}\right), & x > \mu \\ Q\left(\frac{\mu - x}{\sigma}\right), & x < \mu \end{cases} \quad (8)$$

(a) If we consider no bulb is defective, we need to find

$$\Pr(Y = 0) = \Pr(Y \leq 1) - \Pr(Y \leq 0) \quad (9)$$

$$= F_Y(1) - F_Y(0) \quad (10)$$

From (8) and Table 0,

$$F_Y(0) = Q\left(\frac{0.2 - 0}{0.196}\right) \quad (11)$$

$$= Q(1) \quad (12)$$

$$= 0.1587 \quad (13)$$

$$F_Y(1) = 1 - Q\left(\frac{1 - 0.2}{0.44}\right) \quad (14)$$

$$= 1 - Q(1.8) \quad (15)$$

$$= 0.964 \quad (16)$$

$$\Pr(Y = 0) = F_Y(1) - F_Y(0) \quad (17)$$

$$= 0.964 - 0.1587 \quad (18)$$

$$= 0.8053 \quad (19)$$

(b) If we consider exactly 2 bulbs to be defective, the we need to find

$$\Pr(Y = 2) = \Pr(Y \leq 2) - \Pr(Y \leq 1) \quad (20)$$

$$= F_Y(2) - F_Y(1) \quad (21)$$

From (8) and Table 0,

$$F_Y(2) = 1 - Q\left(\frac{2 - 0.2}{0.44}\right) \quad (22)$$

$$= 1 - Q(4) \quad (23)$$

$$= 0.999 \quad (24)$$

$$F_Y(1) = 1 - Q\left(\frac{1 - 0.2}{0.44}\right) \quad (25)$$

$$= 1 - Q(1.8) \quad (26)$$

$$= 0.964 \quad (27)$$

$$\Pr(Y = 2) = F_Y(2) - F_Y(1) \quad (28)$$

$$= 0.999 - 0.964 \quad (29)$$

$$= 0.036 \quad (30)$$

(c) If more than 8 bulbs are working properly then either 1 bulb is defective or no bulb is defective we need to find

$$\Pr(Y \leq 1) = F_Y(1) \quad (31)$$

From (8) and Table 0,

$$F_Y(1) = 1 - Q\left(\frac{1 - 0.2}{0.44}\right) \quad (32)$$

$$= 1 - Q(1.8) \quad (33)$$

$$= 0.964 \quad (34)$$

(ii) Binomial Distribution

Lets define a random variable X which represents the number of defective bulbs.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \quad (35)$$

The pmf is given by

$$P_X(r) = {}^nC_r p^r (1-p)^{n-r} \quad (36)$$

(a) If we consider there is no defective bulb,

$$P_X(0) = 0.817 \quad (37)$$

(b) If we consider there are 2 defective bulbs,

$$P_X(2) = 0.0153 \quad (38)$$

(c) If we consider there are more than 8 proper bulbs,

$$P_X(0) + P_X(1) = 0.817 + 0.1667 \quad (39)$$

$$= 0.9837 \quad (40)$$

Number of defective bulbs	Binomial distribution	Gaussian approximation
0	0.817	0.8053
2	0.0153	0.036
≤ 1	0.9837	0.964

TABLE (ii)

COMPARING BINOMIAL DISTRIBUTION AND GAUSSIAN APPROXIMATION

(iii) Binomial vs Gaussian Graph

