Solution to Q1.3.4

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Question

Find the intersection H of altitudes BE_1 and CF_1

Solution

From question 1.3.3 we know the equation of altitudes BE_1 and CF_1 now to find the point of intersection H of altitudes we need to solve these two equations simultaneously

Equation of BE_1

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \end{pmatrix} \tag{1}$$

Equation of CF_1

$$\begin{pmatrix} 5 & -7 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 20 \end{pmatrix} \tag{2}$$

Therefore, we need to solve the following equation:

$$\begin{pmatrix} 1 & 1 \\ 5 & -7 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 20 \end{pmatrix} \tag{3}$$

Solving the above equation by Gauss-Jordan method

$$\begin{pmatrix} 1 & 1 & 2 \\ 5 & -7 & 20 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_2 - 5R_1} \begin{pmatrix} 1 & 1 & 2 \\ 0 & -12 & 10 \end{pmatrix} \tag{4}$$

$$\xleftarrow{R_2 \leftarrow \frac{R_2}{-12}} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{-5}{6} \end{pmatrix} \tag{5}$$

$$\xleftarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{17}{6} \\ 0 & 1 & \frac{-5}{6} \end{pmatrix} \tag{6}$$

Therefore point of intersection H is

$$\mathbf{x} = \begin{pmatrix} \frac{17}{6} \\ \frac{-5}{6} \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} 2.833 \\ -0.833 \end{pmatrix} \tag{8}$$