Solution to Q11.16.3.17

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Question: Determine the probability p, for each of the following events.
(a) An odd number appears in a single toss of a fair die.
(b) At least one head appears in two tosses of a fair coin.
(c) A king, 9 of hearts, or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.
(d) The sum of 6 appears in a single toss of a pair of fair dice.

Solution:

parameter	**value**	**description**
Random Variable X	0	Odd number appears in the throw (1,3,5)
	1	Even number appears in the throw (2,4,6)

TABLE (a)
SINGLE TOSS OF DIE

(a)

$$p = p_X(0) = \frac{1}{2} \tag{1}$$

(b) Let the random variable X denote one single coin toss where obtaining a head is considered as sucess.Then,

$$X \sim \text{Ber}(p) \tag{2}$$

Suppose $X_i, 1 \leq i \leq n$ represent each of the n draws. Define Y as

$$Y = \sum_{i=1}^{n} X_i \tag{3}$$

Then, since the X_i are iid, the pmf of Y is given by

$$Y \sim \text{Bin}(n, p) \tag{4}$$

The cdf of Y is given by

$$F_Y(k) = \Pr(Y \leq k) \tag{5}$$

$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{6}$$

In this case,

$$p = \frac{1}{2}, \ n = 2 \tag{7}$$

We require $\Pr(Y \geq 1)$. Since $n = 2$,

$$\Pr(Y \geq 1) = 1 - \Pr(Y < 1) \tag{8}$$
$$= F_Y(2) - F_Y(0) \tag{9}$$
$$= \sum_{k=1}^{2} p_Y(k) \tag{10}$$
$$= \sum_{k=1}^{2} \binom{n}{k} p^k (1-p)^{n-k} \tag{11}$$
$$= 0.75 \tag{12}$$

parameter	**value**	**description**
Random Variable X	$1 \leq X \leq 13$	Different cards in deck
Random Variable Y	1	Hearts of cards
	2	Spades of cards
	3	Clubs of cards
	4	Diamonds of cards

TABLE (c)
DECK OF CARDS

(c)

$$p_X(13) = \frac{1}{13} \tag{13}$$
$$p_{XY}(9,1) = \frac{1}{52} \tag{14}$$
$$p_{XY}(3,2) = \frac{1}{52} \tag{15}$$
$$p = p_X(13) + p_{XY}(9,1) + p_{XY}(3,2) \tag{16}$$
$$p = \frac{3}{26} \tag{17}$$

random variables	**description**
X	number appearing on first dice
Y	number appearing on second dice
Z	Sum of numbers appearing on both dice

TABLE (d)
SUM OF TWO DICES

(d) We know that

$$p_Z(n) = \begin{cases} 0 & n \leq 1 \\ \frac{n-1}{36} & 2 \leq n \leq 6 \\ \frac{13-n}{36} & 7 \leq n \leq 12 \\ 0 & n \geq 13 \end{cases} \tag{18}$$

Hence, the probability ,

$$p = p_Z(6) = \frac{5}{36} \tag{19}$$

The experiment of rolling the dice was simulated using Python for 10000 samples.

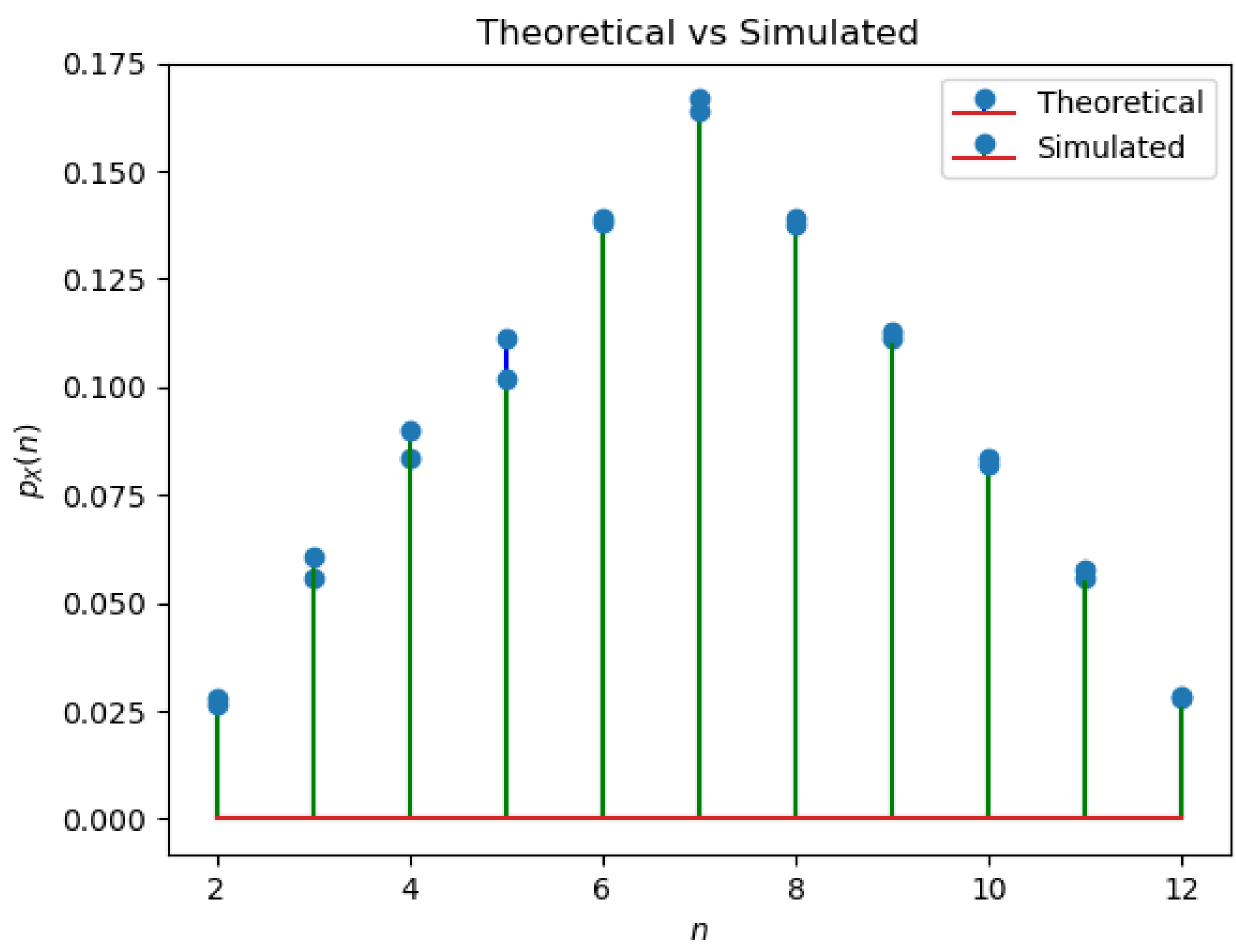

Fig. (d). Plot of $p_Z(n)$.Simulations are close to the analysis.