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Solution to Q 9.3.25

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Question: A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

- (a) none of the bulb is defective
- (b) exactly two bulbs are defective
- (c) more than 8 bulbs are working properly

Solution:

parameter	value	description	
n	10	Number of bulbs in the bag	
p	1 50	Bulb chosen is defective	
q	49 50	Bulb chosen is proper	
$\mu = np$	1/5	Mean of the distribution	
$\sigma^2 = npq$	49 250	Variance of the distribution	

TABLE 0

GAUSSIAN INFO TABLE

(i) Gaussian Distribution

Let Y is the Gaussian obtained by approximating binomial with parameters n,p then By Central limit theroem,

$$Y \sim \mathcal{N}(np, npq) \tag{1}$$

CDF of Y is:

$$F_Y(x) = \Pr\left(Y \le x\right) \tag{2}$$

$$= \Pr\left(Y - \mu \le x - \mu\right) \tag{3}$$

$$=\Pr\left(\frac{Y-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right) \tag{4}$$

Since,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{5}$$

Q function is defined

$$Q(x) = \Pr(Y > x) \ \forall x \in Y \sim \mathcal{N}(0, 1) \tag{6}$$

From (4) and (6),

$$F_Y(x) = 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right)$$
 (7)

$$= \begin{cases} 1 - Q\left(\frac{x - \mu}{\sigma}\right), & x > \mu \\ Q\left(\frac{\mu - x}{\sigma}\right), & x < \mu \end{cases}$$
 (8)

(a) If we consider no bulb is defective, we need to find

$$Pr(Y = 0) = Pr(Y \le 1) - Pr(Y \le 0)$$
 (9)

$$= F_Y(1) - F_Y(0) \tag{10}$$

From (8) and Table 0,

$$F_Y(0) = Q\left(\frac{0.2 - 0}{0.196}\right) \tag{11}$$

$$=Q(1) \tag{12}$$

$$= 0.1587$$
 (13)

$$F_Y(1) = 1 - Q\left(\frac{1 - 0.2}{0.44}\right) \tag{14}$$

$$= 1 - Q(1.8) \tag{15}$$

$$= 0.964$$
 (16)

$$Pr(Y = 0) = F_Y(1) - F_Y(0)$$
(17)

$$= 0.964 - 0.1587 \tag{18}$$

$$= 0.8053$$
 (19)

(b) If we consider exactly 2 bulbs to be defective, the we need to find

$$Pr(Y = 2) = Pr(Y \le 2) - Pr(Y \le 1)$$
 (20)

$$= F_Y(2) - F_Y(1) \tag{21}$$

From (8) and Table 0,

$$F_Y(2) = 1 - Q\left(\frac{2 - 0.2}{0.44}\right) \tag{22}$$

$$= 1 - Q(4) \tag{23}$$

$$= 0.999$$
 (24)

$$F_Y(1) = 1 - Q\left(\frac{1 - 0.2}{0.44}\right) \tag{25}$$

$$= 1 - Q(1.8) \tag{26}$$

$$= 0.964$$
 (27)

$$Pr(Y = 2) = F_Y(2) - F_Y(1)$$
(28)

$$= 0.999 - 0.964 \tag{29}$$

$$= 0.036$$
 (30)

(c) If more than 8 bulbs are working properly then either 1 bulb is defective or no bulb is defetive we need to find

$$\Pr\left(Y \le 1\right) = F_Y(1) \tag{31}$$

From (8) and Table 0,

$$F_Y(1) = 1 - Q\left(\frac{1 - 0.2}{0.44}\right) \tag{32}$$

$$= 1 - Q(1.8) \tag{33}$$

$$= 0.964$$
 (34)

(ii) Binomial Distribution

Lets define a random variable X which represents the number of defective bulbs.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{35}$$

The pmf is given by

$$P_X(r) = {}^{n}C_r p^r (1-p)^{n-r}$$
(36)

(a) If we consider there is no defective bulb,

$$P_X(0) = 0.817 (37)$$

(b) If we consider there are 2 defective bulbs,

$$P_X(2) = 0.0153 \tag{38}$$

(c) If we consider there are more than 8 proper bulbs,

$$P_X(0) + P_X(1) = 0.817 + 0.1667 (39)$$

$$= 0.9837$$
 (40)

Number of defective bulbs	Binomial distribution	Gaussian approximation
0	0.817	0.8053
2	0.0153	0.036
≤ 1	0.9837	0.964

TABLE (ii)

Binomial PMF vs. Normal PDF

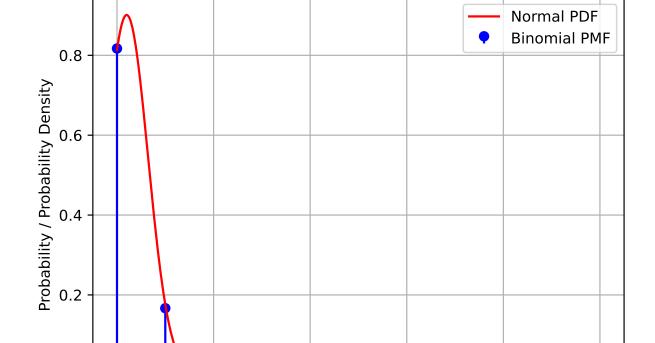
COMPARING BINOMIAL DISTRIBUTION AND GAUSSIAN APPROXIMATION

(iii) Binomial vs Gaussian Graph

0.0

0

2



6

Number of Successes

8

10