Solution to Q 9.3.25

Mayank Gupta

Question: A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probabilty that
(a) none of the bulb is defective
(b) exactly two bulbs are defective
(c) more than 8 bulbs are working properly
Solution:

parameter	**value**	**description**
n	10	Number of bulbs in the bag
p	$\frac{1}{50}$	Bulb chosen is defective
q	$\frac{49}{50}$	Bulb chosen is proper
$\mu = np$	$\frac{1}{5}$	Mean of the distribution
$\sigma^2 = npq$	$\frac{49}{250}$	Variance of the distribution

TABLE 0
GAUSSIAN INFO TABLE

(i) Gaussian Distribution
Let Y is the Gaussian obtained by approximating binomial with parameters n,p then By Central limit theroem,

$$Y \sim \mathcal{N}(np, npq) \tag{1}$$

CDF of Y is:

$$F_Y(x) = \Pr(Y \leq x) \tag{2}$$
$$= \Pr(Y - \mu \leq x - \mu) \tag{3}$$
$$= \Pr\left(\frac{Y-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) \tag{4}$$

Since,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{5}$$

Q function is defined

$$Q(x) = \Pr(Y > x) \; \forall x \in Y \sim \mathcal{N}(0,1) \tag{6}$$

From (4) and (6),

$$F_Y(x) = 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) \tag{7}$$
$$= \begin{cases} 1 - Q\left(\frac{x-\mu}{\sigma}\right), & x > \mu \\ Q\left(\frac{\mu-x}{\sigma}\right), & x < \mu \end{cases} \tag{8}$$

(a) If we consider no bulb is defective, we need to find

$$\Pr(Y = 0) = \Pr(Y \leq 1) - \Pr(Y \leq 0) \tag{9}$$
$$= F_Y(1) - F_Y(0) \tag{10}$$

From (8) and Table 0,

$$F_Y(0) = Q\left(\frac{0.2-0}{0.196}\right) \tag{11}$$
$$= Q(1) \tag{12}$$
$$= 0.1587 \tag{13}$$
$$F_Y(1) = 1 - Q\left(\frac{1-0.2}{0.44}\right) \tag{14}$$
$$= 1 - Q(1.8) \tag{15}$$
$$= 0.964 \tag{16}$$
$$\Pr(Y=0) = F_Y(1) - F_Y(0) \tag{17}$$
$$= 0.964 - 0.1587 \tag{18}$$
$$= 0.8053 \tag{19}$$

(b) If we consider exactly 2 bulbs to be defective, the we need to find

$$\Pr(Y=2) = \Pr(Y \le 2) - \Pr(Y \le 1) \tag{20}$$
$$= F_Y(2) - F_Y(1) \tag{21}$$

From (8) and Table 0,

$$F_Y(2) = 1 - Q\left(\frac{2-0.2}{0.44}\right) \tag{22}$$
$$= 1 - Q(4) \tag{23}$$
$$= 0.999 \tag{24}$$
$$F_Y(1) = 1 - Q\left(\frac{1-0.2}{0.44}\right) \tag{25}$$
$$= 1 - Q(1.8) \tag{26}$$
$$= 0.964 \tag{27}$$
$$\Pr(Y=2) = F_Y(2) - F_Y(1) \tag{28}$$
$$= 0.999 - 0.964 \tag{29}$$
$$= 0.036 \tag{30}$$

(c) If more than 8 bulbs are working properly then either 1 bulb is defective or no bulb is defetive we need to find

$$\Pr(Y \le 1) = F_Y(1) \tag{31}$$

From (8) and Table 0,

$$F_Y(1) = 1 - Q\left(\frac{1-0.2}{0.44}\right) \tag{32}$$
$$= 1 - Q(1.8) \tag{33}$$
$$= 0.964 \tag{34}$$

(ii) Binomial Distribution
Lets define a random variable X which represents the number of defective bulbs.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{35}$$

The pmf is given by

$$P_X(r) = {}^{n}C_r p^r (1-p)^{n-r} \tag{36}$$

(a) If we consider there is no defective bulb,

$$P_X(0) = 0.817 \tag{37}$$

(b) If we consider there are 2 defective bulbs,

$$P_X(2) = 0.0153 \tag{38}$$

(c) If we consider there are more than 8 proper bulbs,

$$P_X(0) + P_X(1) = 0.817 + 0.1667 \tag{39}$$
$$= 0.9837 \tag{40}$$

(iii) Binomial vs Gaussian Graph