Solution to Q 9.3.25

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Question: A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probabilty that
(a) none of the bulb is defective
(b) exactly two bulbs are defective
(c) more than 8 bulbs are working properly
Solution:

parameter	**value**	**description**
n	10	Number of bulbs in the bag
p	$\frac{1}{50}$	Bulb chosen is defective
q	$\frac{49}{50}$	Bulb chosen is proper
μ	$\frac{1}{5}$	Mean of the distribution
σ^2	$\frac{49}{250}$	Variance of the distribution

TABLE 0
Gaussian Info Table

$$\mu = np \tag{1}$$
$$= 10\left(\frac{1}{50}\right) \tag{2}$$
$$= \frac{1}{5} \tag{3}$$
$$\sigma^2 = npq \tag{4}$$
$$= 10\left(\frac{1}{50}\right)\left(\frac{49}{50}\right) \tag{5}$$
$$= \frac{49}{250} \tag{6}$$

(i) Gaussian Distribution
Lets define a random variable Y which represents the number of defective bulbs drawn.

$$Y = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{7}$$

The gaussian distribution function is defined as:

$$P_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{8}$$

The central limit theorm states that we can take a random variable Z such that,

$$Z \approx \frac{Y-\mu}{\sigma} \tag{9}$$

Now, Z is a random variable with $\mathcal{N}(0, 1)$. Hence, the gaussian distribution function changes to:

$$P_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{10}$$

(a) If we consider no bulb is defective,

$$Y = 0 \tag{11}$$
$$Z \approx -\frac{\sqrt{10}}{7} \tag{12}$$

Substituting values in (10),

$$P_Z\left(-\frac{\sqrt{10}}{7}\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{5}{49}} \tag{13}$$
$$= 0.3602 \tag{14}$$

(b) If we consider exactly 2 bulbs to be defective,

$$Y = 2 \tag{15}$$
$$Z \approx \frac{2 - \frac{1}{5}}{\frac{7}{5\sqrt{10}}} \tag{16}$$
$$\approx \frac{9\sqrt{10}}{7} \tag{17}$$

Substituting values in (10),

$$P_Z\left(\frac{9\sqrt{10}}{7}\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{405}{49}} \tag{18}$$
$$= 1.026 \times 10^{-4} \tag{19}$$

(c) If more than 8 bulbs are working properly then either 1 bulb is defective or no bulb is defetive

$$Y = 1 \tag{20}$$
$$Z \approx \frac{1 - \frac{1}{5}}{\frac{7}{5\sqrt{10}}} \tag{21}$$
$$\approx \frac{4\sqrt{10}}{7} \tag{22}$$

Substituting values in (10),

$$P_Z\left(\frac{4\sqrt{10}}{7}\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{80}{49}} \tag{23}$$
$$= 0.078 \tag{24}$$

Required Probabilty

$$P_Z\left(-\frac{\sqrt{10}}{7}\right) + P_Z\left(\frac{4\sqrt{10}}{7}\right) = 0.3602 + 0.078 \tag{25}$$
$$= 0.482 \tag{26}$$

(ii) Binomial Distribution
Lets define a random variable X which represents the number of defective bulbs.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{27}$$

The pmf is given by

$$P_X(r) = {}^nC_r p^r (1-p)^{n-r} \tag{28}$$

(a) If we consider there is no defective bulb,

$$P_X(0) = 0.817 \tag{29}$$

(b) If we consider there are 2 defective bulbs,

$$P_X(2) = 0.0153 \tag{30}$$

(c) If we consider there are more than 8 proper bulbs,

$$P_X(0) + P_X(1) = 0.817 + 0.1667 \tag{31}$$
$$= 0.9837 \tag{32}$$

(iii) Binomial vs Gaussian Graph