Indian Institute of Information Technology, Kota,

Department of Mathematics

Tutorial Sheet (MAT-101: Mathematics I)

Topics: Numerical Analysis- Finite differences, interpolations and numerical differentiations - Forward, Backward, Central differences and relations between them, Newton's forward, backward interpolation formulas and Stirling's central difference interpolation formulas. Lagrange's interpolation formula, Numerical differentiations using Newton's forward, backward, Stirling's central difference interpolation formulas. Numerical integrations - Trapezoidal rule, Simpson's one-third rule, Simpson's 3/8 rule.

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$ using the following data

x	1	1.2	1.4	1.6	1.8	2.0	2.2
$f(x)$	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

Ans: $3.3205, 3.318$

2. Evaluate the following integrals using the trapezoidal rule with $n = 2, 4$ and compare with the exact solution.

   (i) $\int_0^1 \frac{dx}{3+2x}$, **Ans:** $0.25833, 0.25615$

   (ii) $\int_0^2 \frac{dx}{x^2+2x+10}$. **Ans:** $0.15470, 15458$

3. (i) Evaluate $\int_0^2 \frac{dx}{x^2+2x+10}$ using Simpson's 1/3 rule with two and four subintervals. **Ans:** $0.15422, 15454$

   (ii) Evaluate $\int_0^3 \sqrt{1+x^2}dx$ using Simpson's 3/8 rule with 3 and 6 subintervals. **Ans:** $0.186607, 0.186544$

4. Using divided differences show that the following data represents a second-degree polynomial

x	-3	-2	-1	1	2	3	5
$f(x)$	18	12	8	6	8	12	26

   Determine this polynomial and obtain approximate value of $f(0)$.

5. Construct the backward difference table for the data

(i)

x	-3	-2	-1	0	1	2	3
$f(x)$	13	7	3	1	1	3	7

(ii)

x	-4	-2	0	2	4	6
$f(x)$	261	19	1	15	253	1291

6. Show that

   (a) $\nabla f_{i+1} = \Delta f_i$,
   (b) $\nabla^2 f_i = \Delta^2 f_{i+2}$,
   (c) $\nabla^3 f_i = \Delta^3 f_{i+3}$,
   (d) $\nabla\left(\frac{1}{f_i}\right) = -\frac{\nabla f_i}{f_i f_{i+1}}$,
   (e) $\Delta\left(\frac{f_i}{g_i}\right) = \frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_{i+1}}$.

7. Prove that

   (a) $\nabla - \Delta = -\nabla\Delta$
   (b) $\sum_{k=0}^{n} \Delta^2 f_k = \Delta f_{n+1} - \Delta f_0$.

8. Construct the forward difference table for the data

(i)

x	-4	-2	0	2	4	6
$f(x)$	-67	-9	1	11	69	223

(ii)

x	-3	-2	-1	0	1	2
$f(x)$	-2	-4	-4	-2	2	8

9. The population of a town in the decennial census was as given below. Estimate the population for the year 1895. **Ans:** 54.85

x(year)	1891	1901	1911	1921	1931
$f(x)$(population in thousands)	46	66	81	93	101

10. Table below gives the values of $\tan x$ for $0.1 \le x \le 0.3$

x	0.1	0.15	0.2	0.25	0.3
$f(x)(\tan x)$	0.1003	0.1511	0.2027	0.2553	0.3093

Find $\tan 0.12$, $\tan 0.26$, $\tan 0.4$, $\tan 0.5$. **Ans:** $0.1205, 0.2662, 0.4241, 0.5543$