Indian Institute of Information Technology, Kota,

Department of Mathematics

Tutorial Sheet (MAT-101: Mathematics I)

Topics: Integral Calculus - Improper integrals, Area and length of curves, Surface area and volume of solid of revolution. Multiple integrals, Change of order of integration

1. (i) Evaluate $\int (x^2+y^2)dydx$ where area bounded by positive quadrant and $x+y \leq 1$.
   Ans: $\frac{1}{6}$

   (ii) Evaluate the integral by changing the order of integration $\int_0^\infty \int_0^x xe^{\frac{-x^2}{y}} dydx$.
   Ans: $\frac{1}{2}$

2. (i) Evaluate:-

   $$\int\int ydxdy$$

   where **A** is the region of integration by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.
   Ans: $\frac{48a^3}{5}$

   (ii) Evaluate

   $$\int_0^1 \int_0^1 \frac{dydx}{\sqrt{(1-x^2)(1-y^2)}}.$$

   Ans: $\frac{\pi^2}{4}$

3. Change the order of the following double integration:-

   $$\int_0^{a\cos\alpha} \int_{x\tan\alpha}^{\sqrt{a^2-x^2}} f(x,y)dydx.$$

   Ans:

   $$\int_0^{a\sin\alpha} \int_0^{y\cot\alpha} f(x,y)dxdy + \int_{a\sin\alpha}^{a} \int_0^{\sqrt{a^2-y^2}} f(x,y)dxdy.$$

4. Prove that length of the arc of the curve $y = \log(\sec x)$ from $x = 0$ to $x = \frac{\pi}{3}$ is $\ln(2+\sqrt{3})$. **Ans:** $\log(2+\sqrt{3})$

5. Find the volume of the solid generated by revolving the region bounded by the lines $x = 0$, $y = 1$ and the curve $y = \sqrt{x}$ about the line $y = 1$. **Ans:** $\frac{\pi}{6}$

6. Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis or $y = 0$. **Ans:** $\frac{4\pi b^2 a}{3}$

7. The part of the parabola $y^2 = 4ax$ cut off by the latus rectum revolves about the tangent at the vertex. Find the volume of the reel thus generated. **Ans:** $\frac{4\pi a^3}{5}$

8. Find the surface of the solid formed by the revolution of the part of the curve $ay^2 = x^3$ from $x = 0$ to $x = 4a$, which is above the x-axis, about the axis of y. **Ans:** $\left(\frac{128}{1215}\pi a^2[125\sqrt{10} + 1]\right)$

9. Evaluate the integral $\int_0^\infty \sqrt{x}e^{(-x^3)}dx$ using gamma function. **Ans:** $\frac{\sqrt{\pi}}{3}$

10. Evaluate the integral $\int_0^2 x\sqrt{8 - x^3}dx$ using beta function. **Ans:** $48\sqrt{2\pi}\frac{\Gamma\frac{2}{3}}{\Gamma\frac{1}{6}}$