Indian Institute of Information Technology, Kota
Department of Mathematics
B.Tech. Ist Semester | MAT101: Mathematics - I
Tutorial Sheet-2

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1. Find asymptotes of following curves

   (i) $x^3 + y^3 - 3axy = 0$

   (ii) $(x-y)^2(x+2y-1) = 3x + y - 7$

   (iii) $x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0$

   Ans. (i) $x + y + a = 0$
   (ii) $x + 2y - 1 = 0$ and $y = x \pm \frac{2}{\sqrt{3}}$
   (iii) $x + 2y - 1 = 0$, $x - y + 1 = 0$, and $x + y = 0$

2. Show that the asymptotes of the cubic $x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$ cut the curve again in three points which lie on the straight line $x - y + 1 = 0$.

3. Find the equation of the cubic which has the same asymptotes as the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$ and which passes through the points (0,0),(1,0) and (0,1).
   Ans. $x^3 - 6x^2y + 11xy^2 - 6y^3 - x + 6y = 0$

4. Trace the following curves:

   (i) $a^2y^2 = x^3\,(2a - x)$

   (ii) $y^2\,(a^2 + x^2) = x^2\,(a^2 - x^2)$

   (iii) $x^3 + y^3 = 3axy$

   (iv) $r = a\,(1 + \cos\theta)$

   (v) $r = a \sin 3\theta$

5. Verify that $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$ for following functions:

   (i) $f\,(x, y) = ax^2 + 2hxy + by^2$

   (ii) $f\,(x, y) = \log\left(\dfrac{x^2 + y^2}{xy}\right)$

(iii) $f(x,y) = \frac{1}{\sqrt{y}} e^{-\frac{(x-a)^2}{4y}}$

6. $f(x,y) = x^3y - xy^3$, find $\left[\frac{1}{\frac{\partial f}{\partial x}} + \frac{1}{\frac{\partial f}{\partial y}}\right]\Bigg|_{x=1,y=2}$.

   Ans. $-\frac{13}{22}$

7. If by the substitution $u = x^2 - y^2$, $v = 2xy$, $f(x,y) = \theta(u,v)$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4(x^2+y^2)\left(\frac{\partial^2 \theta}{\partial u^2} + \frac{\partial^2 \theta}{\partial v^2}\right)$.

8. If $u = \log \frac{x^4+y^4}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

9. Verify Euler's theorem in the following cases:

   (i) $f(x,y) = 3x^2yz + 5xy^2z + 4z^4$

   (ii) $f(x,y) = \frac{x(x^3-y^3)}{x^3+y^3}$

   (iii) $f(x,y) = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$

10. If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, then evaluate

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2}.$$