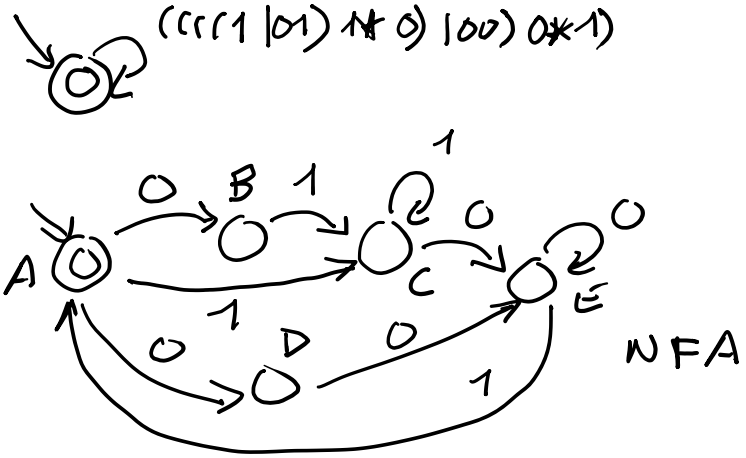
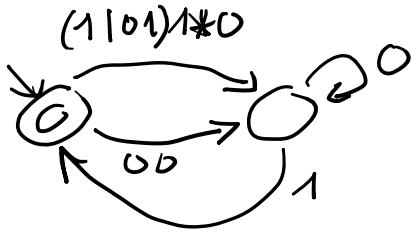
Formal Languages and Compilers Theory test September 22, 2023

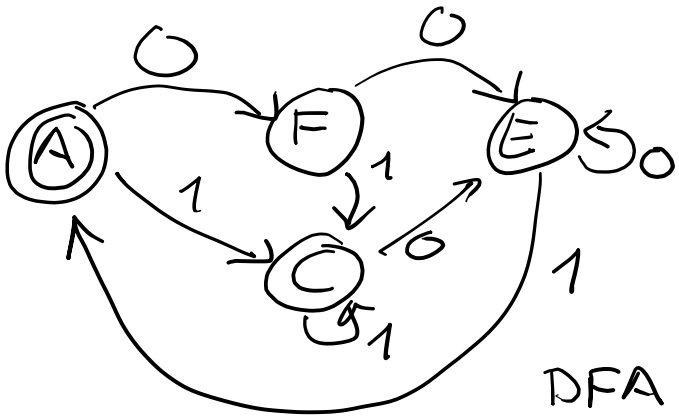
1. (9 points) Find the minimum-state DFA that accepts the language represented by the regular expression (((1|01)1*0)|00)0*1)*.





DFA construction

	0	1
→ * {A}	{B,D}	{C}
{B,D}	{E}	{C}
F {C}	{E}	{C}
C {E}	{E}	{A}
E {E}	{E}	{A}



DFA minimization

Π_0 : {A}, {F, C, E}

Π_1 : {A}, {F, C}, {E}

Π_2 : {A}, {F, C}, {E}

equivalent states

minimum-state DFA

2. (8 points) Given the CFG G=({ S , U , V , R , T } , { a , b , c, d , e } , P , S}, with P={

$S \rightarrow U\ V \mid R\ S$

$V \rightarrow U\ V \mid \varepsilon$

$U \rightarrow a\ S\ c \mid b\ S\ d \mid e$

$R \rightarrow a\ R\ U \mid b\ R\ U \mid a\ R\ T$

$T \rightarrow a\ T \mid \varepsilon$

},

find a CFG G' equivalent to G that does not contain useless symbols and that does not contain epsilon productions.

Symbols that generate a non-empty language: {a,b,c,d,e} U {U,V,T} U {S}
Symbols that generate the empty language: {R}
=> R can be eliminated. Grammar after its elimination:
S -> U V
V -> U V | ε
U -> a S c | b S d | e
T -> a T | ε
Reachable symbols: {S} U {U,V}
Unreachable symbols: {T}
=> T can be eliminated. Grammar after its elimination:
S -> U V
V -> U V | ε
U -> a S c | b S d | e
Symbols that generate ε : {V}
Grammar without ε-transitions:
S -> U V | U
V -> U V | U
U -> a S c | b S d | e
S and V are equivalent, so they can be collapsed. The final grammar is:
S -> U V | U
U -> a S c | b S d | e

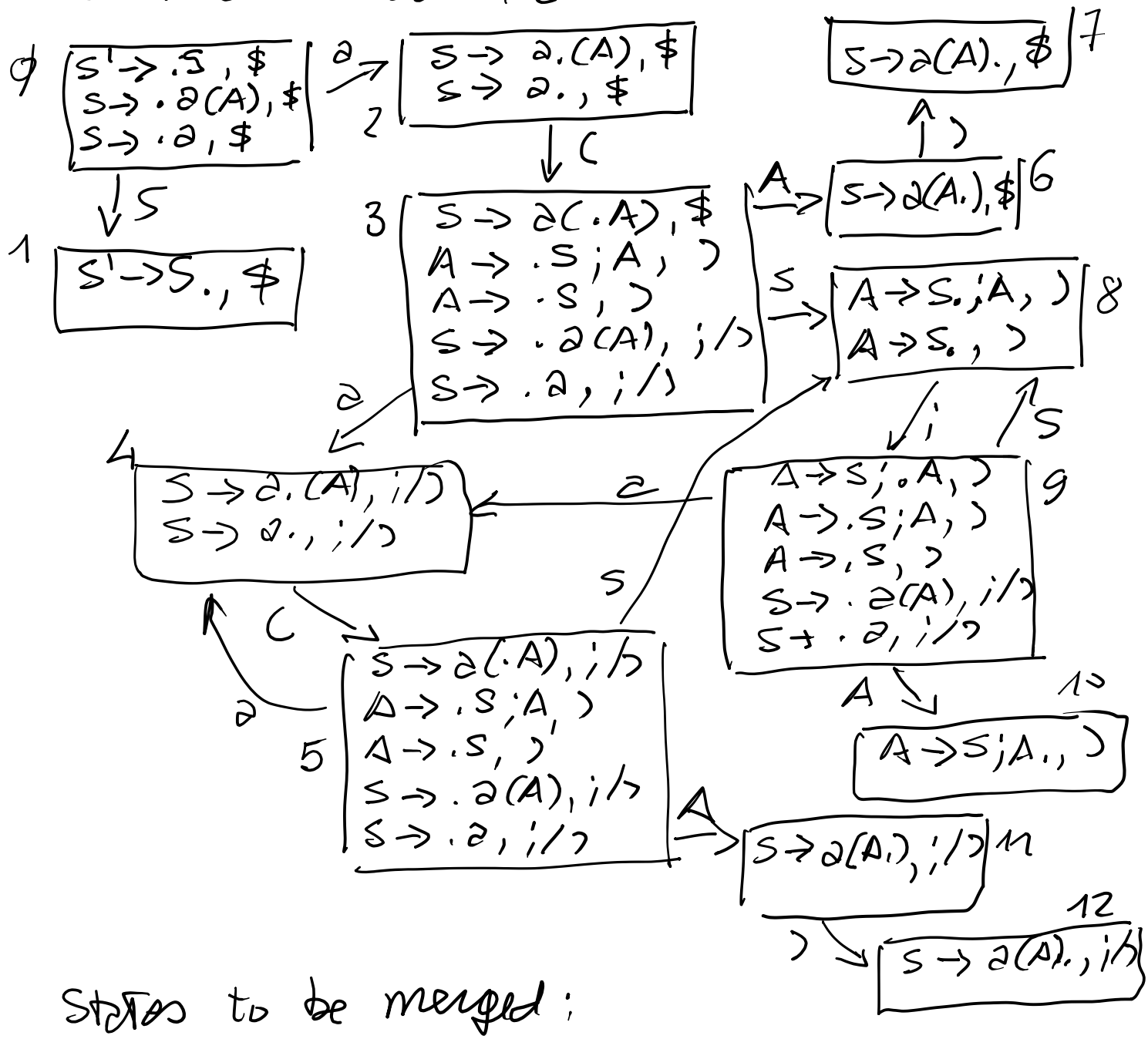
3. (9 points) Build the LALR(1) parsing table for the following grammar whose set of terminal symbols is { a , ; , (,) } and whose start symbol is S

S → a (A) | a 1,2
A → S ; A | S 3,4

Tell if this grammar is LALR(1) or not and motivate your answer.

We introduce a new start symbol S' and new rule S' → S (0)

DFA construction (LR(1))



States to be merged:

2, 4 ⇒ 2-4 6, 11 ⇒ 6-11
3, 5 ⇒ 3-5 7, 12 ⇒ 7-12

LALR(1) Parsing Table:

	a	;	(	)	$	A	S
0	s24						1
1					acc		
24		r2	s35	r2	r2		
35	s24					G11	8
G11			s7_12				
7_12		r1		r1	r1		
8		s9		r4			
9	s24					10	8
10				r3			

As the table does not have conflicts, the given grammar is LALR(1).

4. (6 points) Is the intersection of two regular languages a regular language? Explain how it is possible to prove it.

Yes, it is.
This fact can be proved by using the De-Morgan theorem: if L_1 and L_2 are the two regular languages, their intersection can be computed as $L_1 \cap L_2 = \neg(\neg L_1 \cup \neg L_2)$
As the complement of a regular language is regular and the union of two regular languages is regular, we can conclude that the intersection is regular too.