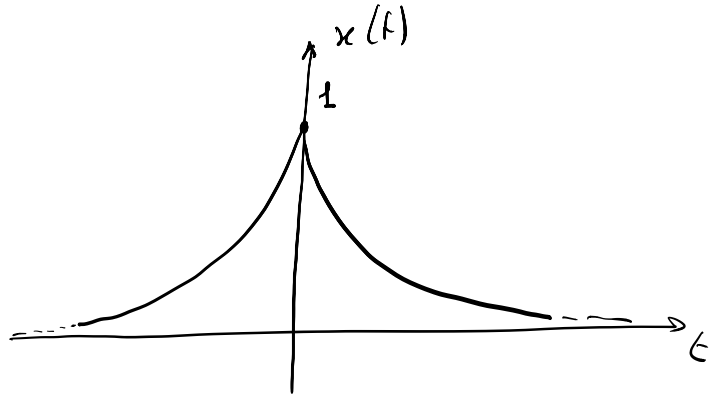
ESERCIZIO - TCF DI UNA ESPONENZIALE BILATERA

$$x(t) = e^{-|t|}$$

$$X(f) = ?$$



$x(t)$ reale e pari $\Rightarrow$ reale e pari $X(f)$

Svolgimento

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} e^{-|t|}\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{0} e^{t}\, e^{-j2\pi f t}\, dt + \int_{0}^{+\infty} e^{-t}\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{0} e^{(1-j2\pi f)t}\, dt + \int_{0}^{+\infty} e^{-(1+j2\pi f)t}\, dt$$

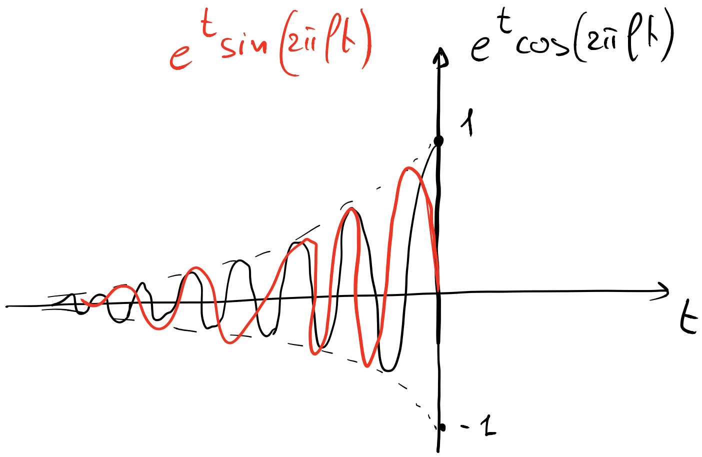
$$= \frac{1}{1-j2\pi f}\, e^{(1-j2\pi f)t}\Big|_{-\infty}^{0} + \frac{1}{-(1+j2\pi f)}\, e^{-(1+j2\pi f)t}\Big|_{0}^{+\infty}$$

$$= \frac{1}{1-j2\pi f}(1-0) - \frac{1}{1+j2\pi f}(0-1) = \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f}$$

$$\lim_{t\to-\infty} e^{(1-j2\pi f)t} = \lim_{t\to-\infty} e^{t}\, e^{-j2\pi f t}$$

$$= \lim_{t\to-\infty} e^{t}\left(\cos(2\pi f t) - j\sin(2\pi f t)\right) = 0$$

$$= \lim_{t\to-\infty} \underbrace{e^{t}\cos(2\pi f t)} - j \lim_{t\to-\infty} e^{t}\sin(2\pi f t) = 0$$

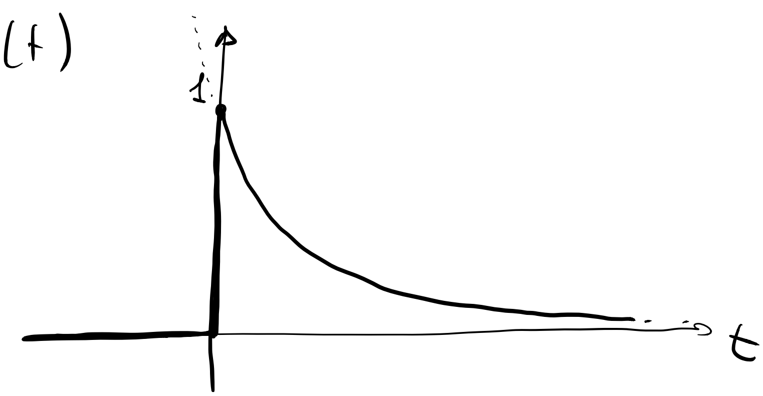


$$X(f) = \underbrace{\frac{1}{1-j2\pi f}}_{z} + \underbrace{\frac{1}{1+j2\pi f}}_{z^*} = \frac{1+\cancel{j2\pi f} + 1-\cancel{j2\pi f}}{\underbrace{1+4\pi^2 f^2}_{|z|^2}} = \boxed{\frac{2}{1+4\pi^2 f^2}}$$

ESERCIZIO – TCF DI UN'ESP. MONOLATERA

$$x(t) = e^{-t}\, u(t)$$

$$X(f) = ?$$



Svolgimento

$$X(f) = \int_{-\infty}^{+\infty} e^{-t} u(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{+\infty} e^{-(1+j2\pi f)t}\, dt = -\frac{1}{1+j2\pi f} e^{-(1+j2\pi f)t}\Big|_{0}^{+\infty}$$

$$= \boxed{\frac{1}{1+j2\pi f}}$$

Hermitiana

$$X(-f) = X^*(f)$$

$$X(-f) = \frac{1}{1+j2\pi(-f)} = \frac{1}{1-j2\pi f}$$

$$X^*(f) = \left(\frac{1}{1+j2\pi f}\right)^* = \frac{1^*}{(1+j2\pi f)^*} = \frac{1}{1-j2\pi f}$$

TEOREMI SULLA TCF

- Linearità

$$x(t) = a\, x_1(t) + b\, x_2(t)$$

$$\Downarrow$$

$$X(f) = a\, X_1(f) + b\, X_2(f)$$

$$X_1(f) = TCF\left[x_1(t)\right]$$

$$X_2(f) = TCF\left[x_2(t)\right]$$

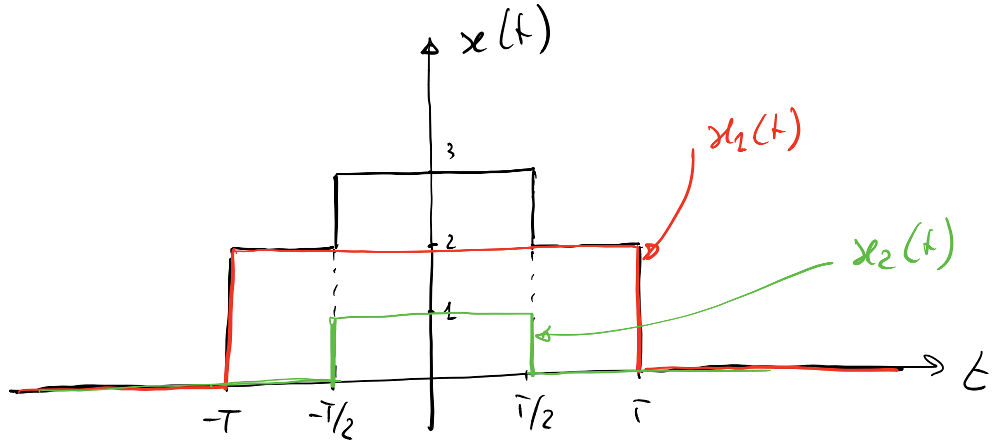
Dimostrazione

$$\int_{-\infty}^{+\infty} \underbrace{[a\,x_1(t) + b\,x_2(t)]}_{x(t)} e^{-j2\pi f t}\,dt$$

$$= a \underbrace{\int_{-\infty}^{+\infty} x_1(t)\, e^{-j2\pi f t}\,dt}_{X_1(f)} + b \underbrace{\int_{-\infty}^{+\infty} x_2(t)\, e^{-j2\pi f t}\,dt}_{X_2(f)}$$

Esempio

Calcolare la TCF del segnale in figura



$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\,dt$$

$$= \int_{-T}^{-T/2} 2\, e^{-j2\pi f t}\,dt + \int_{-T/2}^{T/2} 3\, e^{-j2\pi f t}\,dt + \int_{T/2}^{T} 2\, e^{-j2\pi f t}\,dt$$

$$x(t) = x_1(t) + x_2(t)$$

$$X(f) = X_1(f) + X_2(f)$$

$$X_1(f) = TCF\left[x_1(t)\right] = TCF\left[2\,\mathrm{rect}\left(\frac{t}{2T}\right)\right]$$

$$X_2(f) = TCF\left[x_2(t)\right] = TCF\left[\mathrm{rect}\left(\frac{t}{T}\right)\right]$$

$$A\,\mathrm{rect}\left(\frac{t}{T'}\right) \overset{TCF}{\Longleftrightarrow} AT'\,\mathrm{sinc}(fT')$$

$$X_2(f) = T\,\mathrm{sinc}(fT) \qquad (\text{posto } A = 1)$$

$$X_1(f) = 2 \cdot 2T\,\mathrm{sinc}(f \cdot 2T)$$

$$= 4T\,\mathrm{sinc}(2fT)$$

$$\boxed{X(f) = 4T\,\mathrm{sinc}(2fT) + T\,\mathrm{sinc}(fT)}$$

$$x_1(t) = 2\,\mathrm{rect}\left(\frac{t}{2T}\right) = A\,\mathrm{rect}\left(\frac{t}{T'}\right) \quad \text{con} \quad \begin{array}{l} A = 2 \\ T' = 2T \end{array}$$

$$X_1(f) = AT'\,\mathrm{sinc}(fT') \quad \text{con} \quad \begin{array}{l} A = 2 \\ T' = 2T \end{array}$$

$$= 2 \cdot 2T\,\mathrm{sinc}(f \cdot 2T) = 4T\,\mathrm{sinc}(2fT)$$

·) DUALITÀ

Ip: $x(t) \overset{TCF}{\Longleftrightarrow} X(f)$

Th: $X(t) \overset{TCF}{\Longleftrightarrow} x(-f)$

Dimostrazione

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$$

scambio "t" ed "f"

$$X(t) = \int_{-\infty}^{+\infty} x(f)\, e^{-j2\pi t f}\, df$$

$f' = -f$

$$X(t) = \int_{-\infty}^{+\infty} x(-f')\, e^{-j2\pi (-f') t}\, df'$$

$$= \underbrace{\int_{-\infty}^{+\infty} x(-f')\, e^{j2\pi f' t}\, df'}_{ATCF[x(-f)]}$$

$$X(t) = ATCF[x(-f)]$$

$$x(-f) = TCF[X(t)]$$

$$\Downarrow$$

$$X(t) \overset{TCF}{\Longleftrightarrow} x(-f)$$

Esempio

$$x(t) = \mathrm{rect}\left(\frac{t}{T}\right) \overset{TCF}{\Longleftrightarrow} X(f) = T\,\mathrm{sinc}(fT)$$

Calcoliamo la TCF di

$$\boxed{X(t) = T\,\mathrm{sinc}(Tt)}$$

$$\Downarrow$$

$$x(-f) = \mathrm{rect}\left(\frac{-f}{T}\right) = \boxed{\mathrm{rect}\left(\frac{f}{T}\right)}$$

Calcolare la TCF di

$$x(t) = A\,\mathrm{sinc}(Ft)$$

$$X(f) = A\int_{-\infty}^{+\infty} \mathrm{sinc}(Ft)\, e^{-j2\pi ft}\, dt$$

$$= A\int_{-\infty}^{+\infty} \frac{\sin(\pi F t)}{\pi F t}\, e^{-j2\pi ft}\, dt$$

$$x(t) = A\,\frac{F}{F}\,\mathrm{sinc}(Ft) = \frac{A}{F}\cdot F\,\mathrm{sinc}(Ft)$$

$$\boxed{X(f) = \frac{A}{F}\,\mathrm{rect}\left(\frac{f}{F}\right)}$$

$$x(t) \Leftrightarrow X(f)$$

$$a\,x(t) \Leftrightarrow a\,X(f)$$

·) TEOREMA DEL RITARDO

Ip $x(t) \overset{TCF}{\Longleftrightarrow} X(f)$

$y(t) = x(t - t_0)$

Th: $Y(f) = X(f)\, e^{-j2\pi f t_0}$

Dimostrazione

$$Y(f) = \int_{-\infty}^{+\infty} y(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} x(t-t_0)\, e^{-j2\pi f t}\, dt$$

$t' = t - t_0$

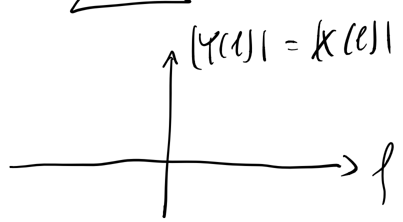
$$= \int_{-\infty}^{+\infty} x(t')\, e^{-j2\pi f (t' + t_0)}\, dt'$$

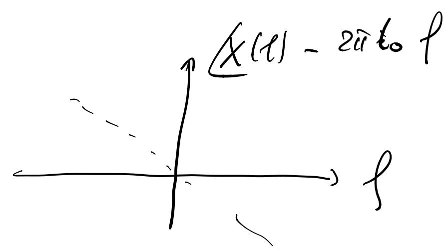
$$= \underbrace{\int_{-\infty}^{+\infty} x(t')\, e^{-j2\pi f t'}\, dt'}_{X(f)}\ e^{-j2\pi f t_0} = X(f)\, e^{-j2\pi f t_0}$$

Disegno dello spettro $Y(f)$

$|Y(f)| = |X(f)|$

$\angle Y(f) = \angle X(f) - 2\pi f t_0$





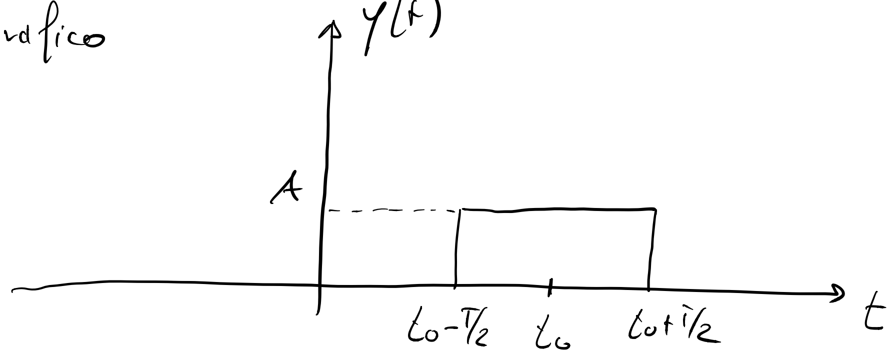
Esempio:

$$y(t) = \mathrm{rect}\left(\frac{t-t_0}{T}\right)$$

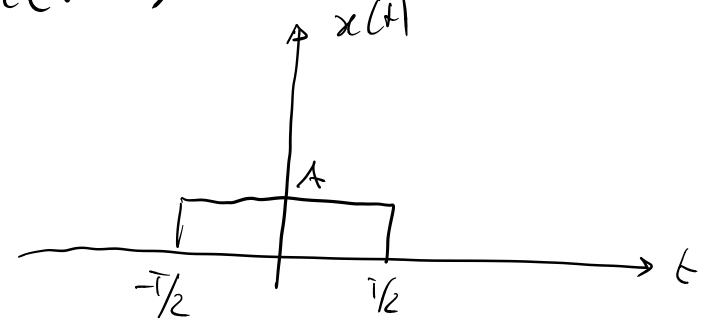
$$Y(f) = TCF[y(t)] = TCF[x(t)]\, e^{-j2\pi f t_0} = \boxed{T\,\mathrm{sinc}(fT)\, e^{-j2\pi f t_0}}$$

$$\begin{cases} y(t) = x(t-t_0) \\ x(t) = \mathrm{rect}\left(\frac{t}{T}\right) \end{cases}$$

esempio grafico



$$y(t) = x(t-t_0)$$



·) TEOREMA DEL CAMBIAMENTO DI SCALA (LINEARE)

Ip: $\begin{cases} x(t) \overset{TCF}{\Leftrightarrow} X(f) \\ y(t) = x(\alpha t) \end{cases}$ $\alpha \neq 0$

Th: $Y(f) = \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right)$

Dimostrazione

due casi $\begin{cases} \alpha > 0 \\ \alpha < 0 \end{cases}$

$\alpha > 0$

$$Y(f) = \int_{-\infty}^{+\infty} x(\alpha t)\, e^{-j2\pi f t}\, dt \qquad t' = \alpha t$$

$$= \int_{-\infty}^{+\infty} x(t')\, e^{-j2\pi f \left(\frac{t'}{\alpha}\right)} \frac{dt'}{\alpha}$$

$$= \frac{1}{\alpha} \int_{-\infty}^{+\infty} x(t')\, e^{-j2\pi \frac{f}{\alpha} t'}\, dt' = \frac{1}{\alpha} X\left(\frac{f}{\alpha}\right)$$

$\alpha < 0 \Rightarrow \alpha = -|\alpha|$

$$Y(f) = \int_{-\infty}^{+\infty} x(\alpha t)\, e^{-j2\pi f t}\, dt \qquad t' = \alpha t = -|\alpha| t$$

$$= \int_{+\infty}^{-\infty} x(t')\, e^{-j2\pi f \frac{t'}{\alpha}} \frac{dt'}{-|\alpha|}$$

$$= \frac{1}{|\alpha|} \int_{-\infty}^{+\infty} x(t')\, e^{-j2\pi \frac{f}{\alpha} t'}\, dt' = \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right)$$

$$Y(f) = \begin{cases} \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right) & \alpha > 0 \\ \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right) & \alpha < 0 \end{cases}$$

$$Y(f) = \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right)$$

Esempio

$$y(t) = \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$x(t) = \mathrm{rect}\left(\frac{t}{T}\right) \overset{TCF}{\Longleftrightarrow} X(f) = T\,\mathrm{sinc}(fT)$$

$$y(t) = x(\alpha t) \qquad \alpha = \frac{1}{2}$$

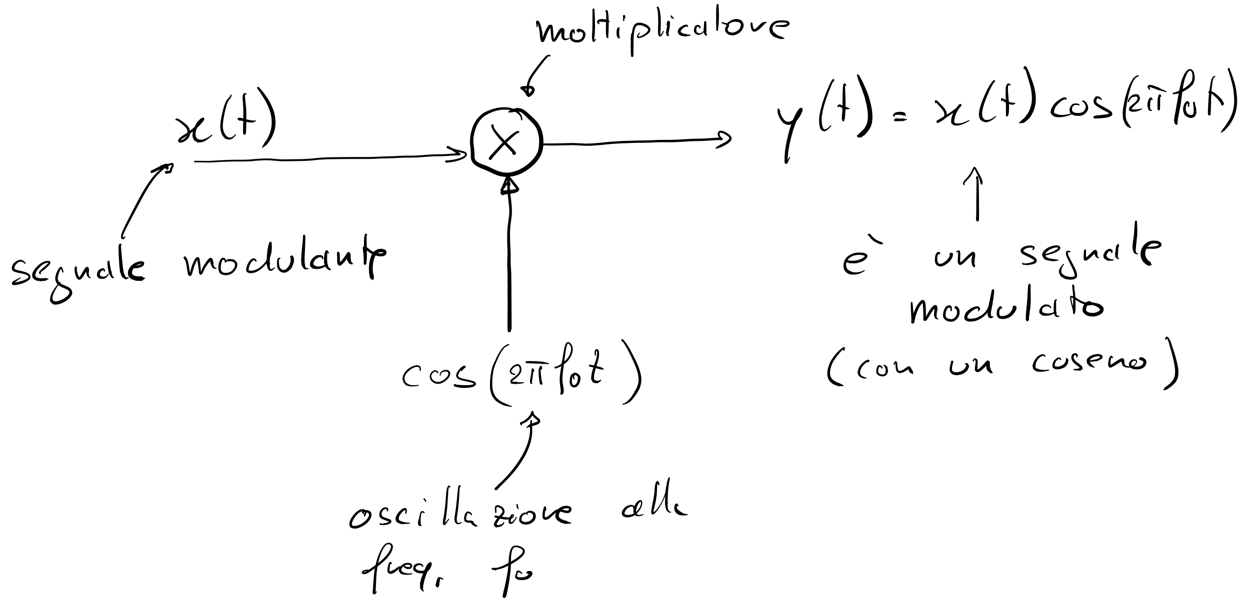
$$y(t) = \mathrm{rect}\left(\frac{\alpha t}{T}\right) \qquad \alpha = \frac{1}{2}$$

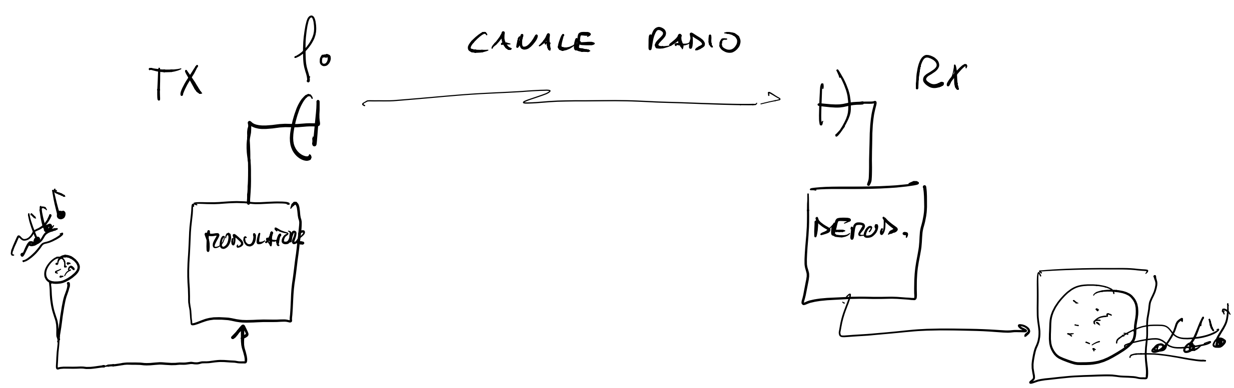
$$Y(f) = \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right) = \frac{1}{|\alpha|} T\,\mathrm{sinc}\left(\frac{f}{\alpha} T\right)$$

$$= \frac{1}{\left|\frac{1}{2}\right|} T\,\mathrm{sinc}\left(\frac{f}{1/2} T\right) = \boxed{2T\,\mathrm{sinc}(2Tf)}$$

-) TEOREMA DELLA MODULAZIONE

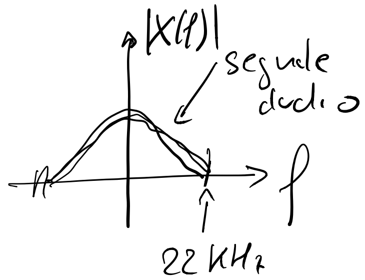
Che cos'è una modulazione?



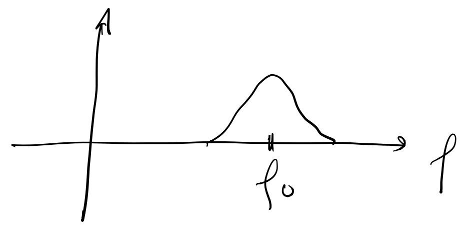


$$F_p \begin{cases} y(t) = x(t)\cos(2\pi f_0 t) \\ x(t) \overset{TCF}{\Leftrightarrow} X(f) \end{cases}$$

$$\text{Th.:} \quad \boxed{Y(f) \overset{!!}{=} \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)}$$



$\xrightarrow{\text{MODUL. con cos a } f_0}$



Dimostrazione

$$Y(f) = \int_{-\infty}^{+\infty} x(t)\cos(2\pi f_0 t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} x(t)\, \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\, e^{-j2\pi f t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi (f-f_0) t}\, dt \; +$$

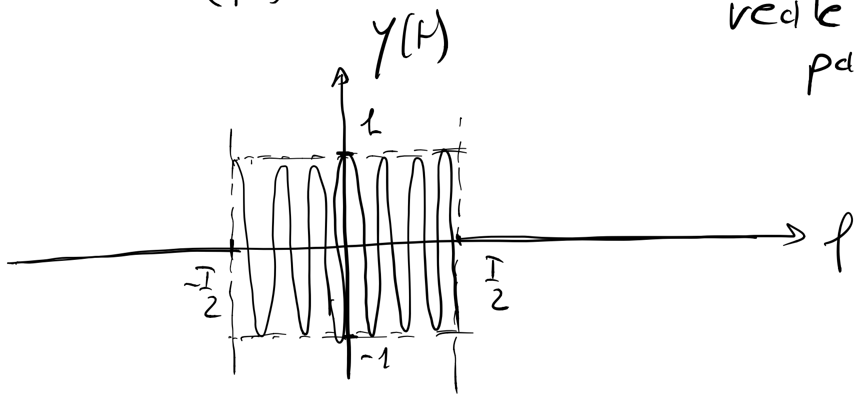
$$+ \; \frac{1}{2}\int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi (f+f_0) t}\, dt$$

$$= \frac{1}{2} X(f-f_0) + \frac{1}{2} X(f+f_0)$$

Esempio

$$y(t) = \mathrm{rect}\left(\frac{t}{T}\right)\cos(2\pi f_0 t)$$

reale e pari



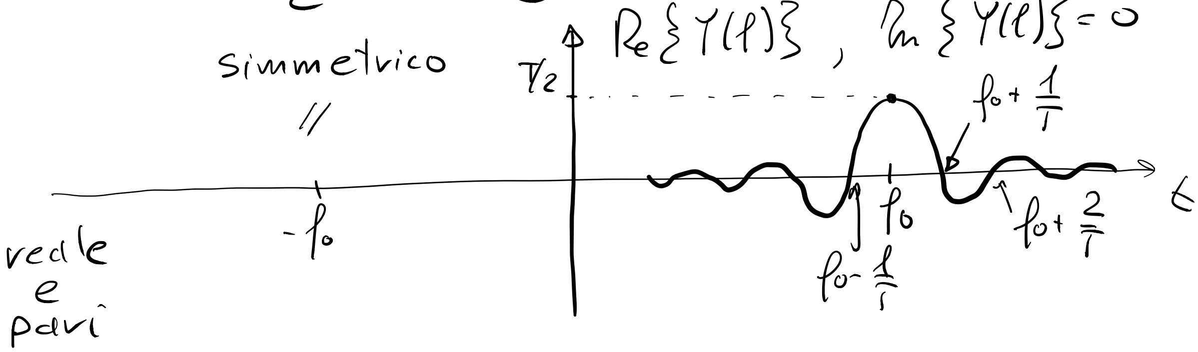
$$Y(f) = \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$$

$$X(f) = \mathrm{TCF}[x(t)] \quad , \quad x(t) = \mathrm{rect}\left(\frac{t}{T}\right)$$

$$y(t) = x(t)\cos(2\pi f_0 t)$$

$$X(f) = T\,\mathrm{sinc}(fT)$$

$$Y(f) = \frac{T}{2}\mathrm{sinc}\left[(f-f_0)T\right] + \frac{T}{2}\mathrm{sinc}\left[(f+f_0)T\right]$$



·) MODULAZIONE CON SENO

Ip: $\begin{cases} x(t) \overset{TCF}{\Longleftrightarrow} X(f) \\ y(t) = x(t) \sin(2\pi f_0 t) \end{cases}$

Th: $Y(f) = \frac{1}{2j} X(f - f_0) - \frac{1}{2j} X(f + f_0)$

Dimostrazione

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$Y(f) = \int_{-\infty}^{+\infty} x(t) \sin(2\pi f_0 t)\, e^{-j2\pi f_0 t}\, dt$$

$$= \frac{1}{2j} \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi(f - f_0)t}\, dt \;+$$

$$- \frac{1}{2j} \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi(f + f_0)t}\, dt$$

$$= \frac{1}{2j} X(f - f_0) - \frac{1}{2j} X(f + f_0)$$

·) MODULAZIONE CON SINUSOIDE (FASE GENERICA)

Ip: $\begin{cases} x(t) \overset{TCF}{\Longleftrightarrow} X(f) \\ y(t) = x(t)\cos(2\pi f_0 t + \varphi) \end{cases}$

Th: $Y(f) = \frac{e^{j\varphi}}{2} X(f-f_0) + \frac{e^{-j\varphi}}{2} X(f+f_0)$

Dim.

$$Y(f) = \int_{-\infty}^{+\infty} x(t)\cos(2\pi f_0 t + \varphi) e^{-j2\pi f t} dt$$

$$= \frac{e^{j\varphi}}{2}\int_{-\infty}^{+\infty} x(t) e^{-j2\pi(f-f_0)t} dt +$$

$$\frac{e^{-j\varphi}}{2}\int_{-\infty}^{+\infty} x(t) e^{-j2\pi(f+f_0)t} dt$$

$$= \frac{e^{j\varphi}}{2} X(f-f_0) + \frac{e^{-j\varphi}}{2} X(f+f_0)$$

·) MODULAZIONE CON ESP. COMPL.

Ip: $\begin{cases} x(t) \overset{TCF}{\Longleftrightarrow} X(f) \\ y(t) = x(t) e^{j2\pi f_0 t} \end{cases}$

Th: $Y(f) = X(f - f_0)$

Dim

$$Y(f) = \int_{-\infty}^{+\infty} x(t)\, e^{j2\pi f_0 t}\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi (f - f_0) t}\, dt = X(f - f_0)$$

Dualità

$$x(t - t_0) \Longleftrightarrow X(f)\, e^{-j2\pi f t_0}$$

$$x(t)\, e^{+j2\pi f_0 t} \Longleftrightarrow X(f - f_0)$$

·) TEOREMA DELLA DERIVAZIONE

Ip: $\begin{cases} x(t) \overset{TCF}{\Longleftrightarrow} X(f) \\ y(t) = \dfrac{d}{dt} x(t) \end{cases}$

Th: $Y(f) = j2\pi f\, X(f)$

Dim.

$$y(t) = \frac{d}{dt} x(t) = \frac{d}{dt}\left[\int_{-\infty}^{+\infty} X(f) e^{j2\pi f t} df\right]$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{+\infty} X(f) \frac{d}{dt}\left[e^{j2\pi f t}\right] df$$

$$= \int_{-\infty}^{+\infty} X(f)\, j2\pi f\, e^{j2\pi f t} df$$

$$= \int_{-\infty}^{+\infty} \underbrace{j2\pi f\, X(f)}_{G(f)} e^{j2\pi f t} df = y(t)$$

$$\Rightarrow \text{ATCF}\left[G(f)\right] = y(t)$$

$$\text{TCF}\left[\text{ATCF}\left[G(f)\right]\right] = \text{TCF}\left[y(t)\right]$$

$$j2\pi f\, X(f) = G(f) = Y(f)$$

·) TEOREMA DELL' INTEGRAZIONE

$$
\text{Ip:} \begin{cases} x(t) \overset{TCF}{\Longleftrightarrow} X(f) \\ y(t) = \int_{-\infty}^{t} x(\alpha)\, d\alpha \\ \int_{-\infty}^{+\infty} x(t)\, dt = 0 \end{cases}
$$

$$
\text{Th:} \quad Y(f) = \frac{X(f)}{j2\pi f}
$$

Dim

$$
y(t) = \int_{-\infty}^{t} x(\alpha)\, d\alpha \Rightarrow x(t) = \frac{d}{dt} y(t)
$$

$$
\Downarrow
$$

$$
Y(f) = \frac{X(f)}{j2\pi f} \Leftarrow X(f) = j2\pi f\, Y(f)
$$

potrebbe divergere e quindi non avere soluzione in $f=0$

$$\Rightarrow \int_{-\infty}^{+\infty} x(t)\,dt = 0 \quad \Rightarrow \quad X(f)\Big|_{f=0} = 0$$

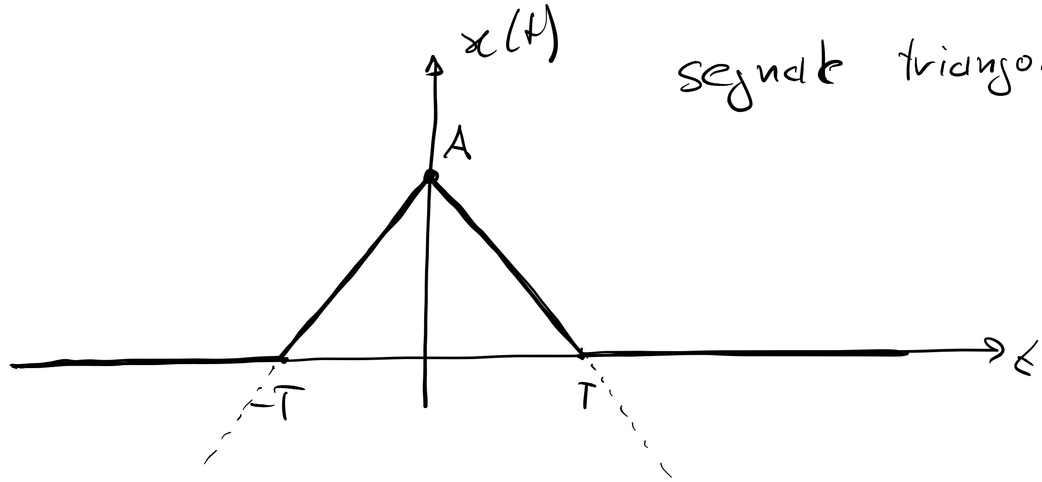
$$X(0) = 0$$

$$\int_{-\infty}^{+\infty} x(t)\,dt = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\,dt\Big|_{f=0}$$

$$= X(f)\Big|_{f=0} = X(0) = 0$$

·) Esempio di applicazione del teorema dell' integrazione

$$x(t) = A\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

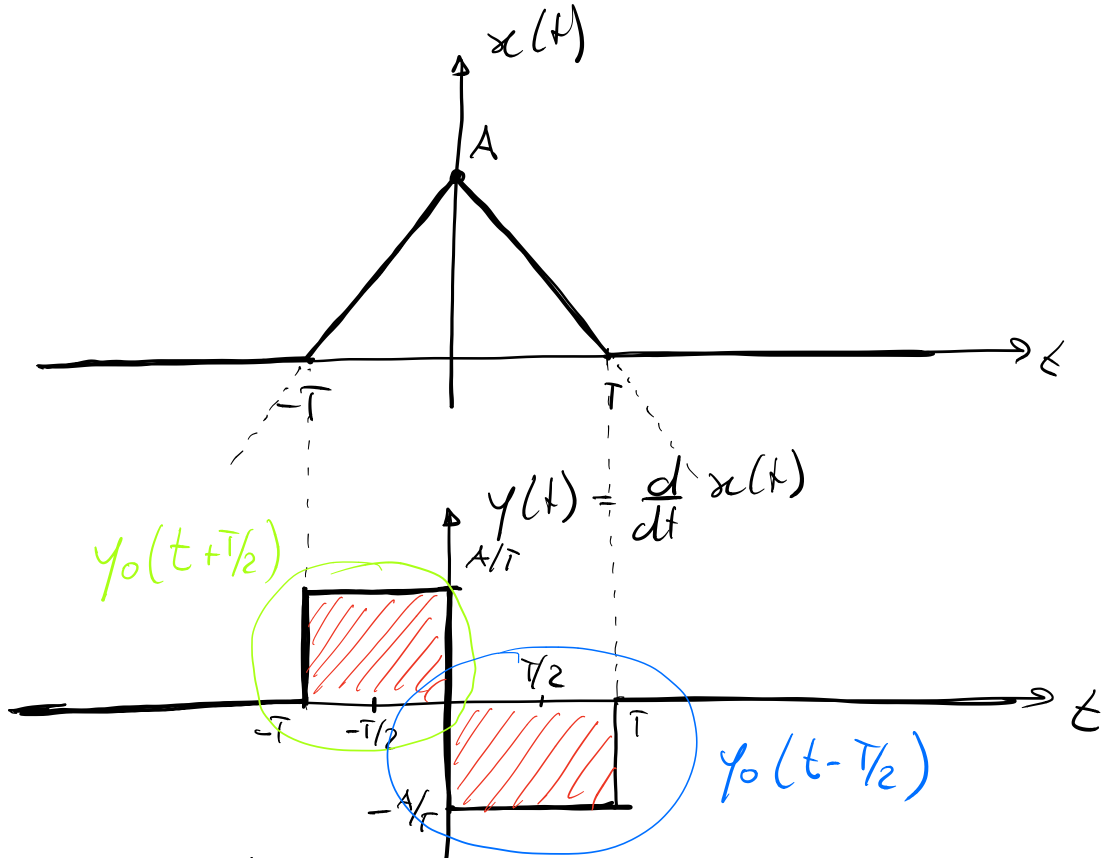


segnale triangolare

$$X(f) = \int_{-\infty}^{+\infty} A\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right) e^{-j2\pi f t}\, dt$$

$$= A\int_{-\infty}^{0}\left(1 + \frac{t}{T}\right) e^{-j2\pi f t}\, dt + A\int_{0}^{+\infty}\left(1 - \frac{t}{T}\right) e^{-j2\pi f t}\, dt$$

altra strada



$$x(t) = \int_{-\infty}^{t} y(\alpha)\, d\alpha \Rightarrow X(f) = \frac{Y(f)}{j2\pi f}$$

$X(f)$ la posso calcolare se

- $Y(f)$ esiste ✓
- $\int_{-\infty}^{+\infty} y(t)\, dt = 0$ ✓

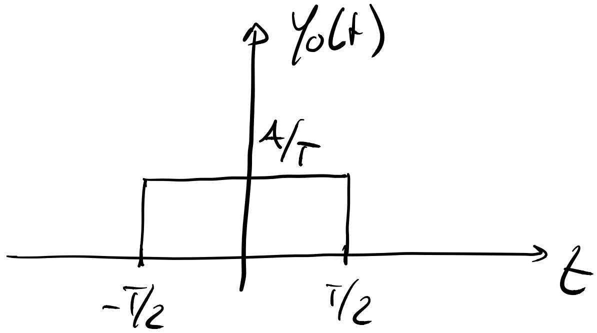
$\Downarrow$

posso applicare il teo

$$X(f) = \frac{Y(f)}{j2\pi f}$$

$\Rightarrow Y(f) = ?$

$$y(t) = y_0\left(t - \left(-T/2\right)\right) - y_0\left(t - T/2\right)$$



$$y_0(t) = \frac{A}{T}\, \mathrm{rect}\left(\frac{t}{T}\right)$$

$$Y(f) = Y_0(f)\, e^{-j2\pi f(-T/2)} - Y_0(f)\, e^{-j2\pi f T/2}$$

$$= Y_0(f)\left[e^{j2\pi f T/2} - e^{-j2\pi f T/2}\right]$$

$$Y_0(f) = \frac{A}{\cancel{T}}\cancel{T}\,\mathrm{sinc}(Tf) = A\,\mathrm{sinc}(Tf)$$

$$Y(f) = A\,\mathrm{sinc}(Tf)\left[e^{j\pi f T} - e^{-j\pi f T}\right]$$

$$X(f) = \frac{Y(f)}{j2\pi f} = \frac{A}{\pi f}\,\mathrm{sinc}(Tf)\,\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j}$$

$$= \frac{AT\,\mathrm{sinc}(Tf)\,\sin(\pi f T)}{\pi f T}$$

$$= AT\,\mathrm{sinc}(Tf)\,\frac{\sin(\pi f T)}{\pi f T} \qquad = \mathrm{sinc}(fT)$$

$$= \boxed{AT\,\mathrm{sinc}^2(Tf)}$$

$$\boxed{A\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \overset{TCF}{\Longleftrightarrow} AT\,\mathrm{sinc}^2(Tf)}$$

Problema: calcolo della TCF di un segnale di cui non conoscete la trasf.

$\Downarrow$

si conosce la trasf della derivata di questo segnale

$\Downarrow$

si calcola la derivata del segnale

$\Downarrow$

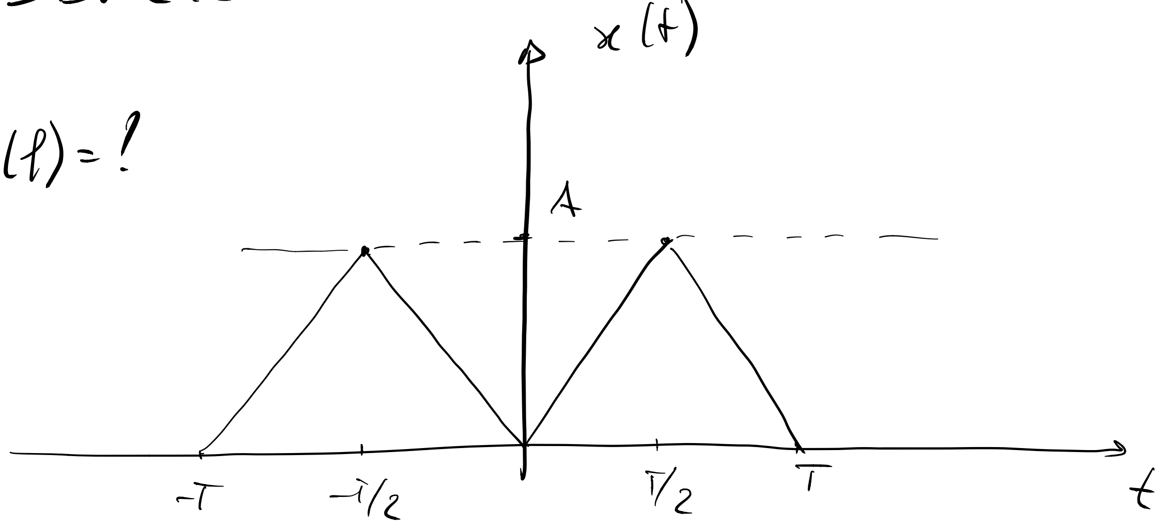
si calcola la TCF della derivata

$\Downarrow$

si applica il Teo dell'int.

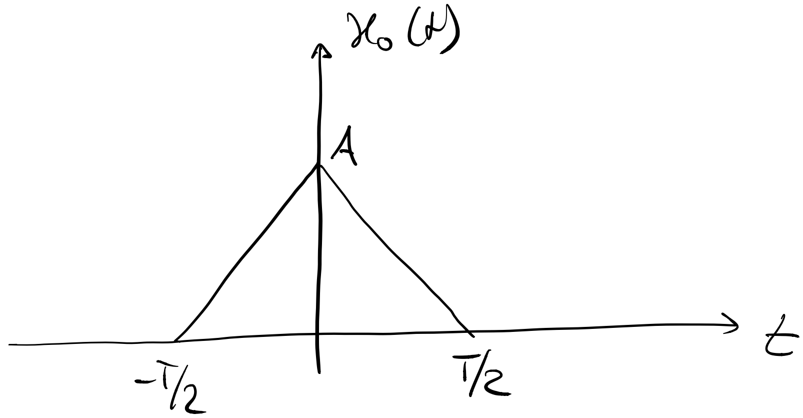
ESERCIZIO

$X(f) = ?$



$$x(t) = \underbrace{x_0(t+T/2)}_{\text{a sx}} + \underbrace{x_0(t-T/2)}_{\text{a dx}}$$

$$x_0(t) = A\left(1 - \frac{|t|}{T/2}\right)\mathrm{rect}\left(\frac{t}{T}\right)$$



$$X(f) = X_1(f) + X_2(f)$$

$$X(f) = \underbrace{X_0(f)\, e^{j2\pi f T/2}}_{X_1(f)} + \underbrace{X_0(f) e^{-j2\pi f T/2}}_{X_2(f)}$$

$$= 2X_0(f)\left(\frac{e^{j\pi f T} + e^{-j\pi f T}}{2}\right)$$

$$= 2\,X_0(f)\cos(\pi f T)$$

$$X_0(f) = \mathrm{TCF}\left[x_0(t)\right]$$

$$\boxed{A\left(1-\frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \overset{TCF}{\Longleftrightarrow} AT\,\mathrm{sinc}^2(Tf)}$$

$$x_0(t) = A\left(1-\frac{|t|}{T/2}\right)\mathrm{rect}\left(\frac{t}{T}\right)$$

$$T' = T/2$$

$$x_0(t) = A\left(1-\frac{|t|}{T'}\right)\mathrm{rect}\left(\frac{t}{2T'}\right)$$

$$X_0(f) = AT'\,\mathrm{sinc}^2(T'f)$$

$$= \frac{AT}{2}\,\mathrm{sinc}^2\left(\frac{Tf}{2}\right)$$

$$\boxed{\begin{aligned} X(f) &= 2X_0(f)\cos(\pi f T) \\ &= AT\,\mathrm{sinc}^2\left(\frac{Tf}{2}\right)\cos(\pi f T) \end{aligned}}$$

altra strada per calcolare $X_0(f)$ $\Rightarrow$ camb. di scala

$$x_0(t) = A\left(1 - \frac{|t|}{T/2}\right) \text{rect}\left(\frac{t}{T}\right)$$

$$x_0(t) = z(\alpha t)$$

$$z(t) = A\left(1 - \frac{|t|}{T}\right) \text{rect}\left(\frac{t}{2T}\right)$$

$$\Downarrow$$

$$Z(f) = AT \,\text{sinc}^2(Tf)$$

$$x_0(t) = z(\alpha t) \Rightarrow \alpha = 2$$

$$z(\alpha t) = A\left(1 - \frac{|\alpha t|}{T}\right) \text{rect}\left(\frac{\alpha t}{2T}\right)$$

$$= A\left(1 - \frac{|2t|}{T}\right) \text{rect}\left(\frac{2t}{2T}\right)$$

$$= A\left(1 - \frac{|t|}{T/2}\right) \text{rect}\left(\frac{t}{T}\right) = x_0(t)$$

$$\Rightarrow X_0(f) = \frac{1}{|\alpha|} Z\left(\frac{f}{\alpha}\right) = \frac{AT}{|\alpha|} \text{sinc}^2\left(T\frac{f}{\alpha}\right)$$

$$= \frac{AT}{2} \text{sinc}^2\left(\frac{Tf}{2}\right)$$