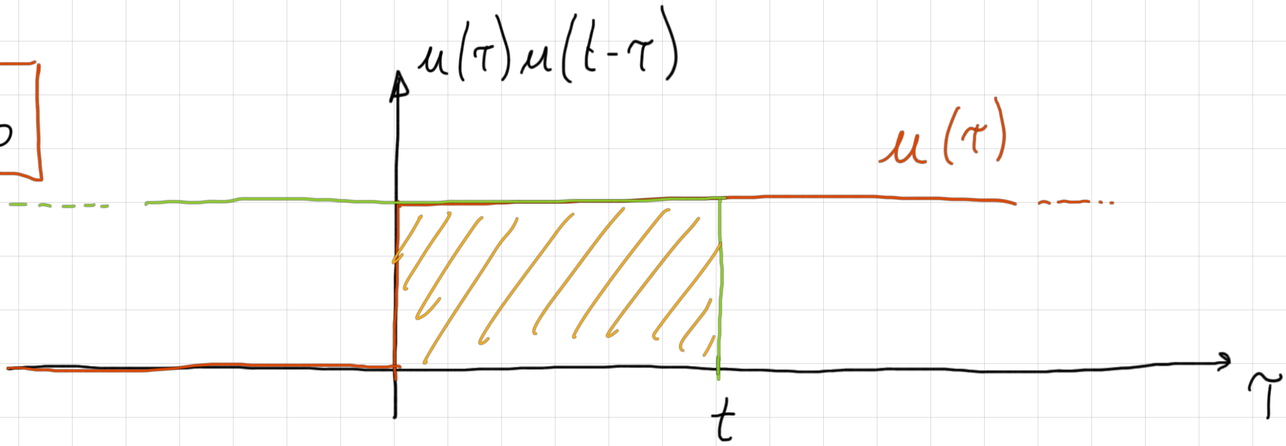
Soluzione Compito Com. Num. 17/01/2019

Es (1)

1) $$h(t) = h_1(t) \otimes h_2(t) = \int_{-\infty}^{+\infty} h_1(\tau)\, h_2(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} e^{-\alpha\tau} u(\tau)\, e^{-\beta(t-\tau)} u(t-\tau)\, d\tau$$

$\boxed{t>0}$



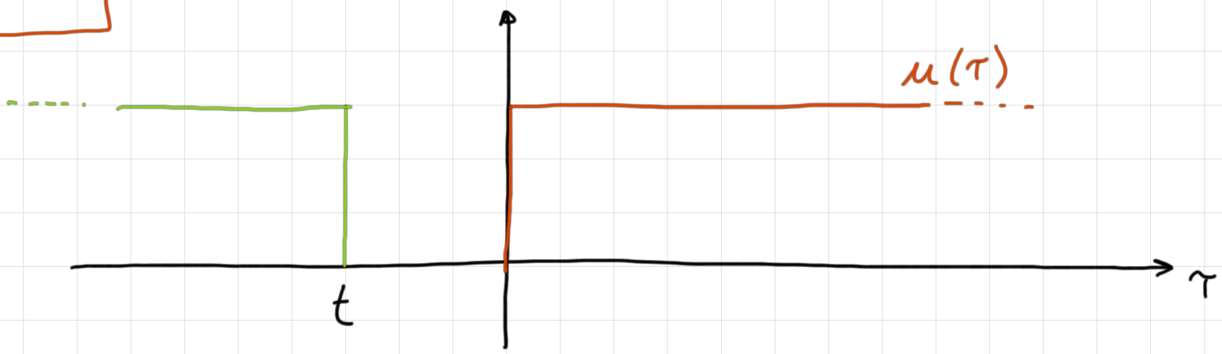
$$= \int_0^t e^{-\alpha\tau} e^{-\beta t} e^{\beta\tau}\, d\tau = e^{-\beta t}\int_0^t e^{(\beta-\alpha)\tau}\, d\tau$$

$$= \frac{e^{-\beta t}}{\beta-\alpha} \cdot e^{(\beta-\alpha)\tau}\Big|_0^t = \frac{e^{-\beta t}}{\beta-\alpha}\left[e^{(\beta-\alpha)t} - 1\right]$$

$$= \frac{e^{-\alpha t} - e^{-\beta t}}{\beta-\alpha} = h(t) \qquad (t>0)$$

$\boxed{t<0}$

$$u(\tau)\,u(t-\tau)=0$$



$$h(t)=0 \qquad (t<0)$$

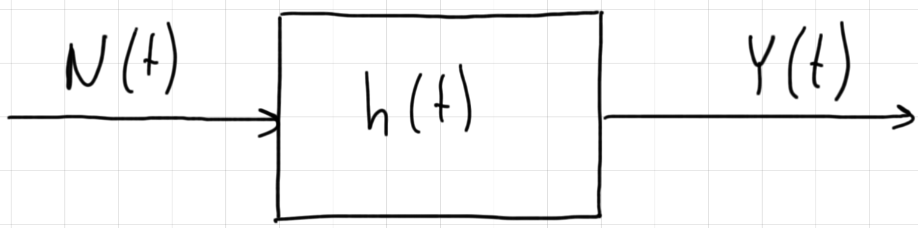
$$h(t)=\frac{e^{-\alpha t}-e^{-\beta t}}{\beta-\alpha}\,u(t)$$

$$H(f)=H_1(f)\,H_2(f) \quad , \quad H_1(f)=TCF\left[h_1(t)\right], \quad H_2(f)=TCF\left[h_2(t)\right]$$

$$H_1(f)=\frac{1}{\alpha+j2\pi f} \quad , \quad H_2(f)=\frac{1}{\beta+j2\pi f}$$

$$\boxed{H(f)=\frac{1}{(\alpha+j2\pi f)(\beta+j2\pi f)}}$$

2)



$N(t)$ è un processo di rumore SSL bianco

con $S_N(f)=\frac{N_0}{2}$, $E[N(t)]=0$

Essendo il sistema lineare e stazionario, allora anche $Y(t)$ è SSL, quindi

$$R_Y(\tau) = R_N(\tau) \otimes h(\tau) \otimes h(-\tau) = R_N(\tau) \otimes R_h(\tau)$$

con $R_h(\tau) = h(\tau) \otimes h(-\tau)$

$$P_Y = R_Y(0) = R_N(\tau) \otimes R_h(\tau)\Big|_{\tau=0} = \frac{N_0}{2} R_h(0)$$

calcoliamo $R_h(0) = \int_{-\infty}^{+\infty} |h(t)|^2 \, dt = \int_{-\infty}^{+\infty} h^2(t) \, dt$ $\quad h(t) \in \mathbb{R}$

$$R_h(0) = \int_{-\infty}^{+\infty} \left[\frac{e^{-\alpha t} - e^{-\beta t}}{\beta - \alpha}\right]^2 u^2(t) \, dt =$$

$$= \frac{1}{(\beta-\alpha)^2} \int_0^{+\infty} e^{-2\alpha t} + e^{-2\beta t} - 2 e^{-(\alpha+\beta)t} \, dt$$

$$= \frac{1}{(\beta-\alpha)^2} \left(\frac{1}{2\alpha} + \frac{1}{2\beta} - \frac{2}{\alpha+\beta} \right)$$

$$\boxed{P_Y = \frac{N_0}{2} \frac{1}{(\beta-\alpha)^2} \left(\frac{1}{2\alpha} + \frac{1}{2\beta} - \frac{2}{\alpha+\beta} \right)}$$

Es ②

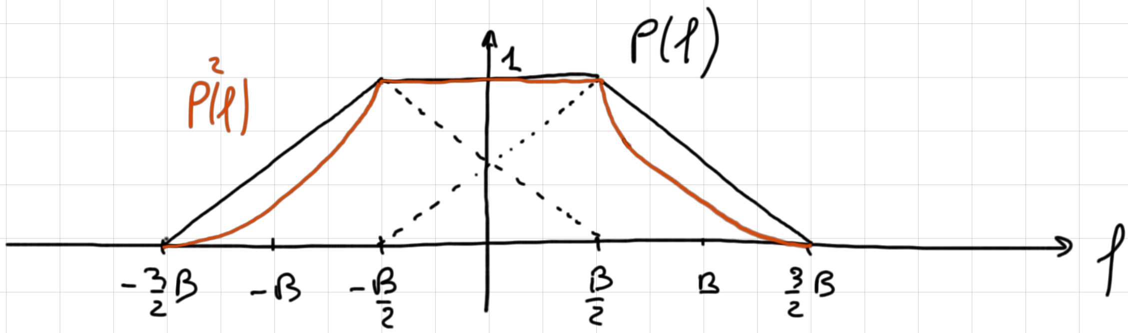
1) $E_s = \frac{1}{2} E[x^2] E_p$

$$E[x^2] = \frac{2}{3}(-1)^2 + \frac{1}{3} 3^2 = \frac{2}{3} + \frac{9}{3} = \frac{11}{3}$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 \, df$$

$$P(f) = \left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right) \otimes \left[\delta\left(f - \frac{B}{2}\right) + \delta\left(f + \frac{B}{2}\right)\right]$$

$$= \left(1 - \frac{|f - \frac{B}{2}|}{B}\right) \text{rect}\left(\frac{f - B/2}{2B}\right) + \left(1 - \frac{|f + \frac{B}{2}|}{B}\right) \text{rect}\left(\frac{f + B/2}{2B}\right)$$



$$E_p = B + \frac{2}{3} B = \frac{5}{3} B$$

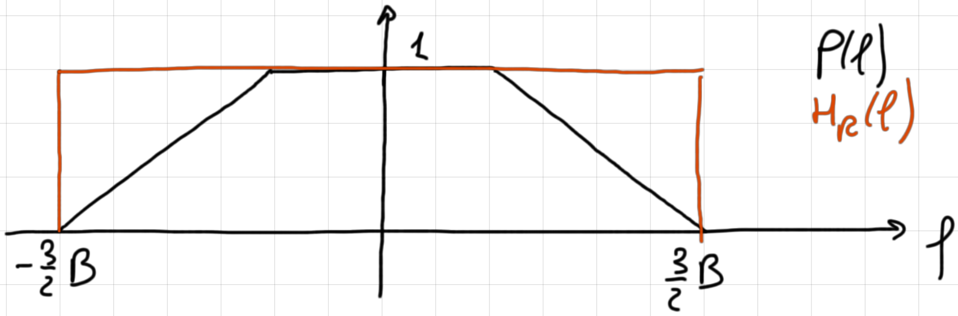
$$\boxed{E_s = \frac{1}{2} \cdot \frac{11}{3} \cdot \frac{5}{3} B = \frac{55}{18} B}$$

2) Non esiste cross-talk per un sistema PAM. Comunque la fase $(\theta - \varphi)$ inficia le prestazioni e bisogna tenerne conto per il calcolo della P_E

3) Condizione di Nyquist

$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P(f)$$



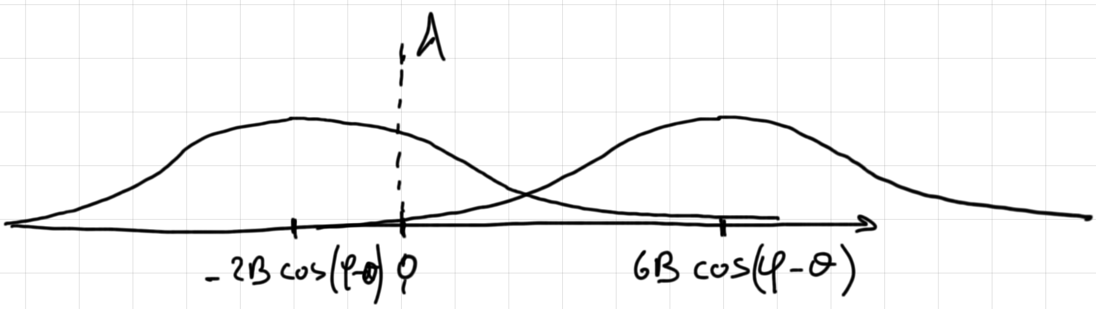
$$h(t) = p(t)$$

$$h[n] = \begin{cases} 2B & n = 0 \\ 0 & n \neq 0 \end{cases}$$

cond. di Nyquist soddisfatta

4) $\boxed{P_{N_u} = N_0 E_{h_R} = 3 N_0 B}$

5) $y[n] = 2B x_k \cos(\varphi - \theta) + n_{un}$



$$\boxed{P_E(b) = \frac{2}{3} Q\left(\frac{2B\cos(\varphi-\theta)}{\sqrt{3N_0B}}\right) + \frac{1}{3} Q\left(\frac{6B\cos(\varphi-\theta)}{\sqrt{3N_0B}}\right)}$$