Esame 30/06/08

$X(f) \rightleftharpoons x(t) = AB\,\mathrm{sinc}^2(Bt) + 2AB\,\mathrm{sinc}^2(Bt)\cos(2\pi f_0 t)$

$X(f) = A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{|f|}{2B}\right) + \left\{2A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left[\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)\right] =\right.$

$= A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{|f|}{2B}\right) + A\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + A\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)$ (1)

$w(t) = x(t) + x(t)\cos(2\pi f_0 t) =$

$W(f) = X(f) + \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0) =$

$= A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + A\left(1-\frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + A\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right) +$

$+ \frac{A}{2}\left(1-\frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \frac{A}{2}\left(1-\frac{|f-2f_0|}{B}\right)\mathrm{rect}\left(\frac{f-2f_0}{B}\right) + \frac{A}{2}\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) +$

$+ \frac{A}{2}\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{B}\right) + \frac{A}{2}\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{B}\right) + \frac{A}{2}\left(1-\frac{|f+2f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right) =$

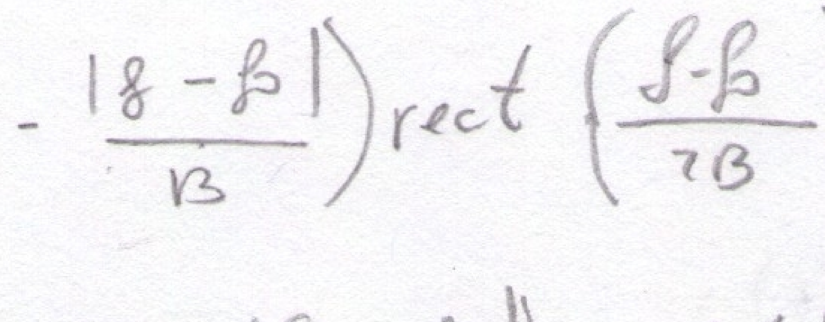
$= 2A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \frac{3}{2}A\left(1-\frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \frac{3}{2}A\left(1-\frac{|f+f|}{B}\right)\mathrm{rect}\left(\frac{f+f}{2B}\right) +$

$+ \frac{A}{2}\left(1-\frac{|f-2f|}{B}\right)\mathrm{rect}\left(\frac{f-2f_0}{2B}\right) + \frac{A}{2}\left(1-\frac{|f+2f_0|}{B}\right)\mathrm{rect}\left(\frac{f+2f_0}{2B}\right)$

$y(t) = \{w(t)\otimes h(t)\}\cdot -j2\pi t$

$Y(f) = \frac{d}{df}\{W(f)H(f)\} = \frac{d}{df}\left(2A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)\right) = \frac{2A}{B}\mathrm{rect}\left(\frac{f+\frac{B}{2}}{B}\right) - \frac{2A}{B}\mathrm{rect}\left(\frac{f-\frac{B}{2}}{B}\right)$

$E_y = \int_{-\infty}^{\infty}|y(t)|^2dt = \int_{-\infty}^{\infty}|Y(f)|^2 f =$

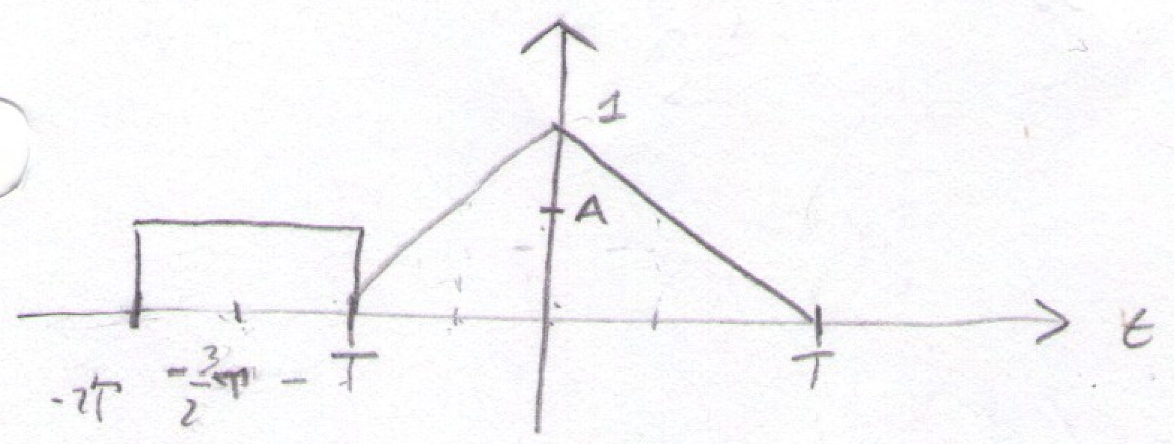


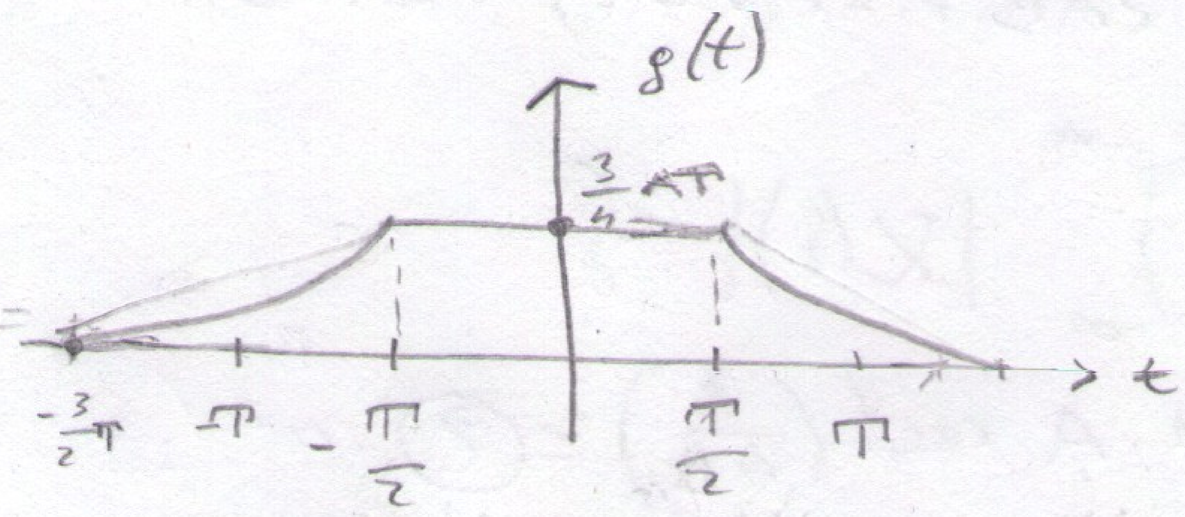
$= \frac{4A^2}{B^2}\cdot 2B = \frac{8A^2}{B}$

$P_y = \emptyset$

• Esercizio 2

$g(t) \triangleq g_{TC}(t) \otimes g_R(t)$





$g(\emptyset) = 1$ solo se $A = \frac{4}{3T}$ (2)

$n(t) \triangleq \left[w(t) \cdot 2\cos(2\pi f_0 t)\right] \otimes g_R(t)$

$\tilde{n}(t) = \tilde{n}_c(t) = \tilde{w}_c(t) \otimes g_R(t)$

$S_{\tilde{n}_c}(f) = S_{\tilde{w}_c}(f) |G_R(f)|^2$; $S_{\tilde{w}}(f) \triangleq \begin{cases} 4 S_w(f+f_0) & f > -f_0 \\ \emptyset & \text{altrove} \end{cases} = 2N_0 \text{ rect}\left(\frac{f}{B}\right)$

$S_{\tilde{w}_c}(f) \triangleq \frac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4} = N_0 \text{ rect}\left(\frac{f}{B}\right)$

$P_{\tilde{n}_c} = \int_{-\infty}^{\infty} N_0 \text{rect}\left(\frac{f}{B}\right) |G_R(f)|^2 df = N_0 \int_{-B}^{B} |G_R(f)|^2 df = N_0 \int_{-B}^{B} |g_R(t)|^2 dt =$

$= N_0 A^2 \int_{-B}^{B} \text{rect}\left(\frac{t}{T}\right) dt = N_0 A^2 T$ (3)

$E_z = P_z T = T \int_{-\infty}^{\infty} S_z(f) df = T \int_{-\infty}^{\infty} \frac{E\{a_i^2\}}{T} |G(f)|^2 df = \int_{-\infty}^{\infty} |G(f)|^2 df = \int_{-\infty}^{\infty} |g(t)|^2 dt =$

$= 2 \cdot \left[\left(\frac{3}{2}T - \frac{T}{2}\right) \cdot \frac{3}{4} AT \cdot \frac{1}{2}\right] + 2 \frac{T}{2} \cdot \frac{3}{4} AT = \frac{3}{4} AT + \frac{3}{4} AT^2 = \frac{3}{2} AT^2$ (1)

$g(mT) = \begin{cases} 1 & m = \emptyset \\ \frac{1}{6} & m = \pm 1 \\ \emptyset & \text{altrove} \end{cases}$, $D_g = \frac{\sum_{m \neq 0} |g(m)|}{g(\emptyset)} \overset{?}{<} 1 = \frac{+\frac{1}{6} + \frac{1}{6}}{1} = \frac{1}{3}$ ok

$q(k) = \sum_{\ell=-N}^{N} p_\ell \, g(k-\ell)$

$\begin{cases} q(\emptyset) = p_{-1} g(1) + p_0 g(\emptyset) + p_1 g(-1) = 1 \\ q(1) = p_{-1} g(2) + p_0 g(1) + p_1 g(\emptyset) = \emptyset \\ q(-1) = p_{-1} g(\emptyset) + p_0 g(-1) + p_1 g(-2) = \emptyset \end{cases} = \begin{cases} \frac{1}{6} p_{-1} + p_0 + \frac{1}{6} p_1 = 1 \\ \frac{1}{6} p_0 + p_1 = \emptyset \\ \frac{1}{6} p_0 + p_{-1} = \emptyset \end{cases}$

$= \begin{cases} p_1 = p_{-1} = -\frac{1}{6} p_0 \\ \frac{1}{6} \cdot -\frac{1}{6} p_0 + p_0 + \frac{1}{6} \cdot -\frac{1}{6} p_0 = 1 \end{cases} = \begin{cases} p_1 = p_{-1} = -\frac{3}{17} \\ p_0 = \frac{18}{17} \end{cases}$ (4)