ESERCIZIO - 20/09/2018 Es. #2

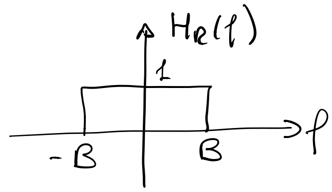
$$r(t) = \sum_i x[i]\, p(t - iT) + w(t)$$

$x[i]$ ind. ed equiprob $\in A_s = [-2, 3]$

$w(t)$ Gaussiano a media nulla con $S_w(f) = \frac{N_0}{2}$

$$p(t) = 2B \operatorname{sinc}(2Bt) + B \operatorname{sinc}^2\left(\frac{B}{2} t\right) \cos(\pi B t)$$

$$H_R(f) = \operatorname{rect}\left(\frac{f}{2B}\right)$$



$$B = \frac{2}{T}$$

$$\hat{x} = \begin{cases} -2 & y < \lambda \\ 3 & y > \lambda \end{cases} \qquad \lambda = 0$$

Calcolare:

1) E_s
2) $S_s(f)$
3) P_{n_u}
4) Assenza di ISI
5) P_E

Soluzione

1) $E_s = E_p \, E[x^2]$

$$E_p = \int_{-\infty}^{+\infty} p^2(t)\, dt = \int_{-\infty}^{+\infty} |P(f)|^2 \, df$$

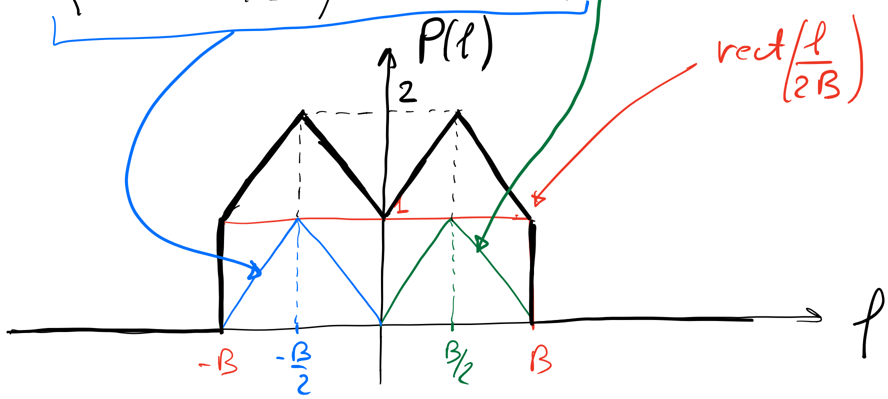
$$P(f) = TCF\left[p(t)\right] = \text{rect}\left(\frac{f}{2B}\right) + \cancel{2}\left(1 - \frac{|f|}{B/2}\right)\text{rect}\left(\frac{f}{B}\right) \otimes$$

$$\otimes\left[\frac{\cancel{1}}{\cancel{2}}\,\delta(f-f_0) + \frac{\cancel{1}}{\cancel{2}}\,\delta(f+f_0)\right]$$

$$f_0 = \frac{B}{2}$$

$$= \text{rect}\left(\frac{f}{2B}\right) + \underbrace{\left(1 - \frac{|f-B/2|}{B/2}\right)\text{rect}\left(\frac{f-B/2}{B}\right)} +$$

$$+ \underbrace{\left(1 - \frac{|f+B/2|}{B/2}\right)\text{rect}\left(\frac{f+B/2}{B}\right)}$$

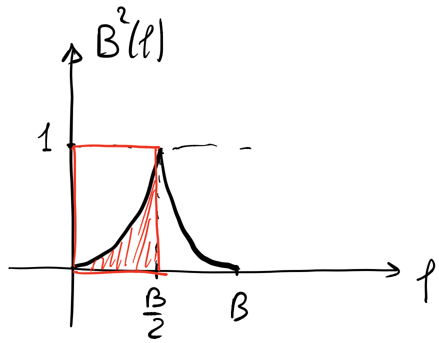


$$E_p = 2\int_0^B \left[A(f) + B(f)\right]^2 df$$

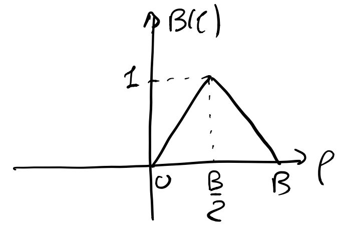
$$A(f) = \text{rect}\left(\frac{f-B/2}{B}\right), \quad B(f) = \left(1 - \frac{|f-B/2|}{B/2}\right)\text{rect}\left(\frac{f-B/2}{B}\right)$$

$$E_p = 2\left[\int_0^B A^2(f)\,df + \int_0^B B^2(f)\,df + 2\int_0^B A(f)\,B(f)\,df\right]$$

$$= 2\left[B + \cancel{2}\cdot\frac{1}{3}\cdot 1\cdot\frac{B}{\cancel{2}} + \cancel{2}\cdot\frac{B}{\cancel{2}}\right] = 2\left(2B + \frac{B}{3}\right) = \frac{14}{3}B$$



$A(f) \cdot B(f) = B(f)$



$$E\left[x^2\right] = \frac{1}{2}(-2)^2 + \frac{1}{2}(+3)^2 = \frac{4}{2} + \frac{9}{2} = \frac{13}{2}$$

$$E_s = \frac{\overset{7}{\cancel{14}}}{3} B \cdot \frac{13}{\cancel{2}} = \boxed{\frac{91}{3} B}$$

2) $S_s(f) = \frac{1}{T} \bar{S}_x(f) \, |P(f)|^2 = \frac{\bar{S}_x(f)}{T} P^2(f)$

$$\bar{S}_x(f) = TFS\left[R_x[m]\right]$$

$$R_x[m] = C_x[m] + \eta_x^2$$

$$\eta_x = E[x] = \frac{1}{2}(-2) + \frac{1}{2}(+3) = -\frac{2}{2} + \frac{3}{2} = \frac{1}{2}$$

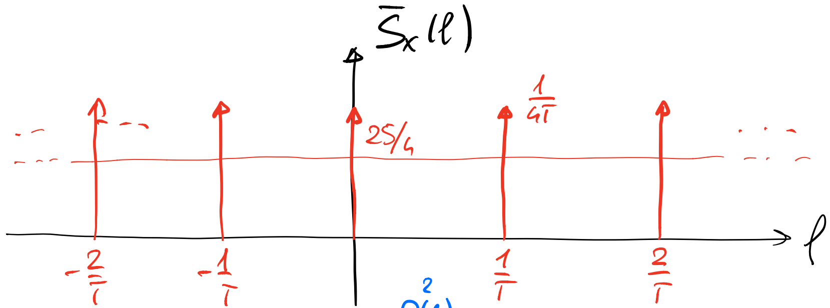
$$\eta_x^2 = \frac{1}{4}$$

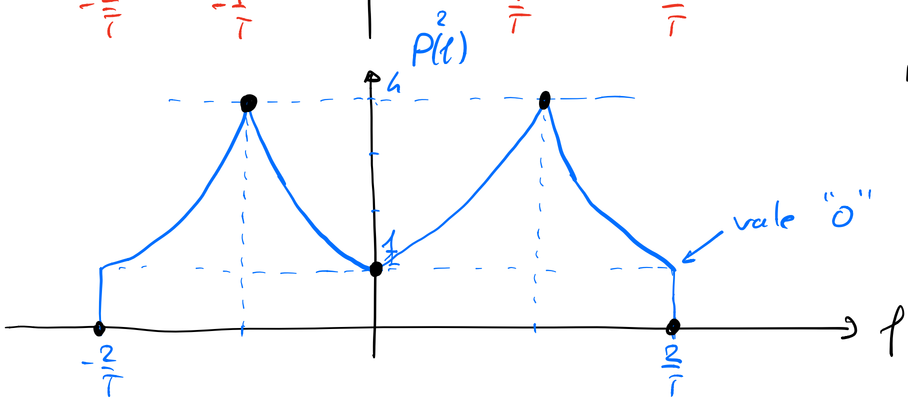
$C_x[m] = \sigma_x^2 \, \delta[m]$ (simboli incorrelati poiché indipendenti)

$$\sigma_x^2 = E\left[x^2\right] - \eta_x^2 = \frac{13}{2} - \frac{1}{4} = \frac{25}{4}$$

$$R_x[m] = \frac{25}{4} \delta[m] + \frac{1}{4}$$

$$\bar{S}_x(f) = \frac{25}{4} + \frac{1}{4} \frac{1}{T} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T}\right)$$





$B = \frac{2}{T}$

$$\boxed{S_s(f) = \frac{25}{4T} P^2(f) + \frac{1}{4T^2} \sum_{n\in[-1,0,1]} P^2\left(\frac{n}{T}\right) \delta\left(f - \frac{n}{T}\right)}$$

3) $$P_{n_u} = \frac{N_0}{2} E_{H_R} = \int_{-\infty}^{+\infty} S_{n_u}(f)\, df$$

$$S_{n_u}(f) = S_n(f) \left|H_R(f)\right|^2 = \frac{N_0}{2} \left|H_R(f)\right|^2$$

$$H_R(f) = rect\left(\frac{f}{2B}\right) \Rightarrow \left|H_R(f)\right|^2 = rect\left(\frac{f}{2B}\right)$$

$$\boxed{P_{n_u} = \frac{N_0}{2} \cdot 2B = N_0 B}$$

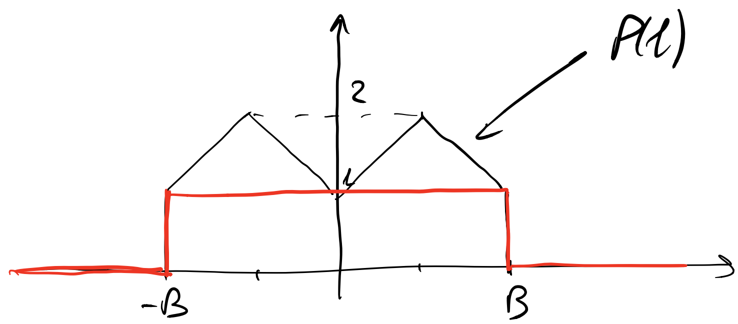
4) $h[n] = K\,\delta[n] = h(0)\,\delta[n]$

$$\bar{H}(f) = K = \frac{1}{T}\sum_{n=-\infty}^{+\infty} H\left(f - \frac{n}{T}\right)$$

$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$c(t) = \delta(t)$$

$$H(f) = P(f)\,H_R(f) = P(f) \Rightarrow h(t) = p(t)$$



$$h(t)\Big|_{t=nT} = 2B\,\mathrm{sinc}(2BnT) + B\,\mathrm{sinc}^2\left(\frac{B}{2}nT\right)\cos(\pi BnT)$$

$T = \frac{2}{B}$

$$= 2B\,\mathrm{sinc}\left(2Bn\frac{2}{B}\right) + B\,\mathrm{sinc}^2\left(\frac{B}{2}n\frac{2}{B}\right)\cos\left(\pi Bn\frac{2}{B}\right)$$

$$= 2B\,\mathrm{sinc}[4n] + B\,\mathrm{sinc}^2[n]\cos[2\pi n]$$

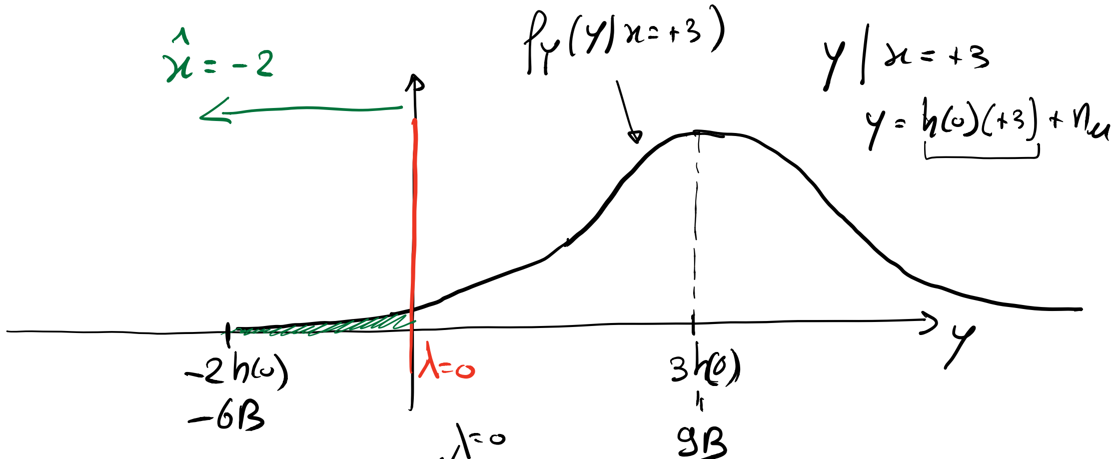
$$= 2B\,\delta[n] + B\,\delta[n] = 3B\,\delta[n]$$

$h(0) = 3B$

$\boxed{\text{soddisfa il criterio di Nyquist nel tempo} \Rightarrow \text{ASSENZA DI ISI}}$

5) $$P_E = P(\hat{x} = -2 \mid x = +3)\, P(x = +3) + P(\hat{x} = +3 \mid x = -2)\, P(x = -2)$$

$$P(x = +3) = P(x = -2) = \frac{1}{2}$$



$$P(\hat{x} = -2 \mid x = +3) = \int_{-\infty}^{\lambda = 0} f_Y(y \mid x = +3)\, dy = F_{Y|x=+3}(\lambda)$$

$$F_{Y|x=+3}(\lambda) = \Phi\left(\frac{\lambda - 3h(0)}{\sqrt{P_{n_u}}}\right) = 1 - Q\left(\frac{\lambda - 3h(0)}{\sqrt{P_{n_u}}}\right)$$

$$= Q\left(\frac{3h(0) - \lambda}{\sqrt{P_{n_u}}}\right)$$

$$\boxed{P_E(b) = \frac{1}{2} Q\left(\frac{9B}{\sqrt{N_0 B}}\right) + \frac{1}{2} Q\left(\frac{6B}{\sqrt{N_0 B}}\right)}$$

ESERCIZIO #2 DEL 16/07/2019

$s(t) = \sum_k x[k]\, p(t - kT)$

$x[k] \in A_s = \{-3, 1\}$ ind. ed equiprob.

$p(t) = 2B\, \mathrm{sinc}^2(2Bt)$

$c(t) = 4B\, \mathrm{sinc}(4Bt) - 2B\, \mathrm{sinc}(2Bt)$

$S_n(f) = \frac{N_0}{2}$

$H_R(f) = \mathrm{rect}\left(\frac{f}{4B}\right)$ banda $2B$

$\lambda = 0$, $T = \frac{1}{B}$

Calcolare:

1) E_s

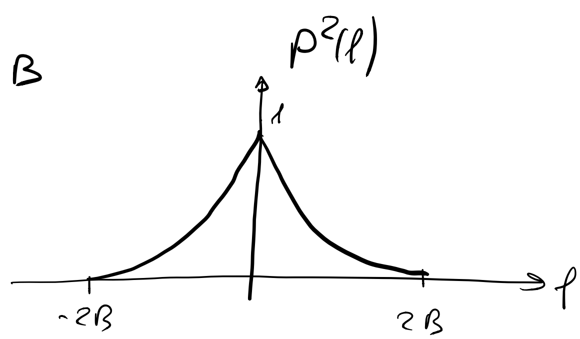
2) P_{n_u}

3) P_E, dopo verifica di assenza di ISI

Soluzione:

1) $E_s = E_p\, E[x^2]$

$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df = \frac{2}{3} 2B = \frac{4}{3} B$

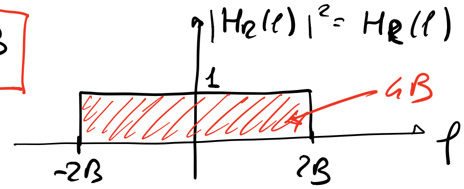


$P(f) = \left(1 - \frac{|f|}{2B}\right) \mathrm{rect}\left(\frac{f}{4B}\right)$

$E[x^2] = \frac{1}{2}(-3)^2 + \frac{1}{2}(+1)^2 = \frac{9}{2} + \frac{1}{2} = 5$

$\boxed{E_s = \frac{20}{3} B}$

2) $P_{n_u} = \frac{N_0}{2} E_{H_R} = \frac{N_0}{2} 4B = \boxed{2N_0B}$

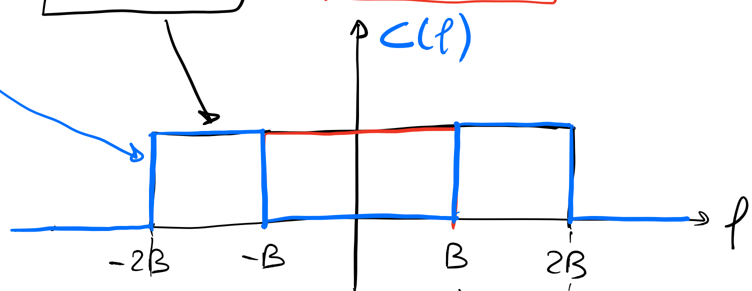


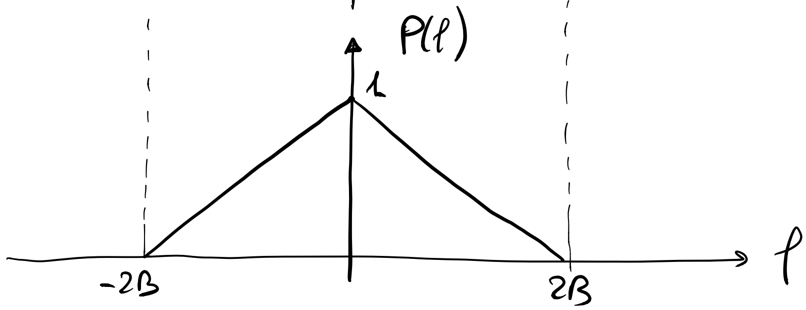
3) Verifica assenza di ISI

$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

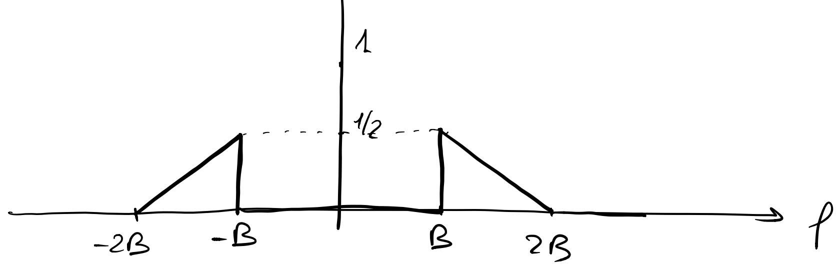
$$H(f) = P(f)\, C(f)\, H_R(f)$$

$$C(f) = \mathrm{rect}\left(\frac{f}{4B}\right) - \mathrm{rect}\left(\frac{f}{2B}\right)$$

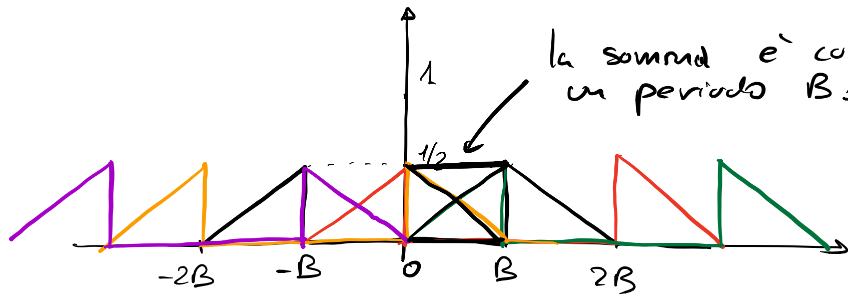




$P(f)C(f) = P(f)C(f)H_R(f) = H(f)$



$$h(0) = h(t)\Big|_{t=0} = \int_{-\infty}^{+\infty} H(f)\, df$$



la somma è costante su un periodo $B = \frac{1}{T} \Rightarrow \bar{H}(f)$ è costante

$\Downarrow$

è soddisfatto Nyquist

$\Downarrow$

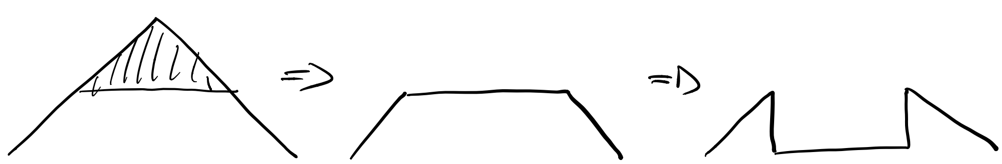
ASSENZA DI ISI

$$h(0) = \frac{B}{2}$$

altra strada

$$h(t) = \text{ATCF}\left[H(f)\right]$$

$$H(f) = \left(1 - \frac{|f|}{2B}\right)\text{rect}\left(\frac{f}{4B}\right) - \frac{1}{2}\left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right) - \frac{1}{2}\,\text{rect}\left(\frac{f}{2B}\right)$$



$$h(t) = 2B\,\text{sinc}^2(2Bt) - \frac{B}{2}\,\text{sinc}^2(Bt) - B\,\text{sinc}(2Bt)$$

$$h(nT) = 2B\,\text{sinc}^2(2BnT) - \frac{B}{2}\,\text{sinc}^2(BnT) - B\,\text{sinc}(2BnT)$$

$$= 2B\,\text{sinc}^2[2n] - \frac{B}{2}\,\text{sinc}^2[n] - B\,\text{sinc}[2n]$$

$$= 2B\,\delta[n] - \frac{B}{2}\,\delta[n] - B\,\delta[n] = \frac{B}{2}\,\delta[n]$$

$\Downarrow$ verificata assenza di ISI

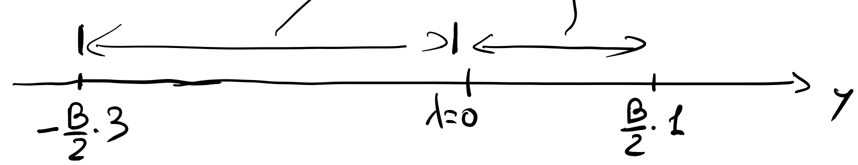
$$h(0) = \frac{B}{2}$$

$\Rightarrow$ verificata la assenza di ISI

$\Rightarrow$ nota $h(0)$

$\Rightarrow$ nota P_{n_u}

$$\boxed{P_E(b) = \frac{1}{2}\, Q\left(\frac{\frac{B}{2}}{\sqrt{2N_0B}}\right) + \frac{1}{2}\, Q\left(\frac{\frac{3}{2}B}{\sqrt{2N_0B}}\right)}$$



ESERCIZIO #2 DEL 26/06/2018

$x \in A_s = \{-1, 2\}$ ind con $P\{-1\} = \frac{1}{3}$, $P\{+2\} = \frac{2}{3}$

$w(t)$ è Gaussiano b. $S_w(f) = \frac{N_0}{2}$

$p(t) = \mathrm{sinc}\left(\frac{2t}{T}\right)$

$h_R(t) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{2t}{T}\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{t}{T}\right)$

$\lambda = 0$

1) E_s

2) $S_s(f)$

3) P_{n_u}

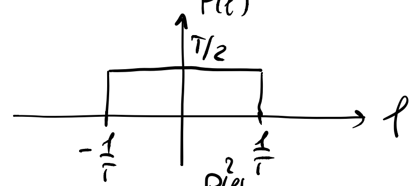
4) Assenza ISI

5) $P_E(b)$

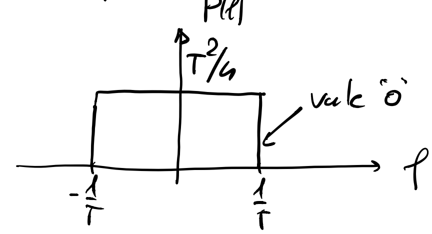
1) $E_s = E[x^2]\, E_p$

$$E[x^2] = \frac{1}{3}(-1)^2 + \frac{2}{3}(+2)^2 = \frac{1}{3} + \frac{8}{3} = 3$$

$$P(f) = \frac{T}{2}\,\mathrm{rect}\left(\frac{f}{2/T}\right)$$



$$E_p = \frac{T^{\cancel{2}}}{{}_{2}\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{T}} = \frac{T}{2}$$



$$\boxed{E_s = 3 \cdot \frac{T}{2} = \frac{3}{2}T}$$

2) $S_s(f) = \frac{1}{T}\,\bar{S}_x(f)\,|P(f)|^2$

$$\bar{S}_x(f) = \mathrm{TFS}\left[R_x[m]\right]$$

$$R_x[m] = \sigma_x^2\,\delta[m] + \eta_x^2$$

$$\eta_x = \frac{1}{3}(-1) + \frac{2}{3}(+2) = -\frac{1}{3} + \frac{4}{3} = 1$$

$$\sigma_x^2 = E[x^2] - \eta_x^2 = 3 - 1 = 2$$

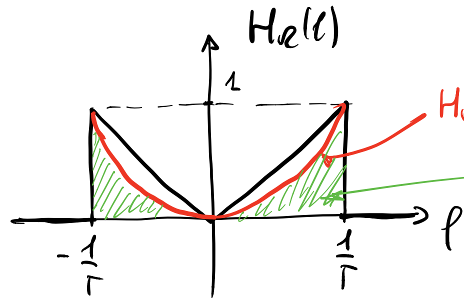
$$\bar{S}_x(f) = 2 + \frac{1}{T}\sum_n \delta\left(f - \frac{n}{T}\right)$$

$$S_s(f) = \frac{1}{T}\left[2\,|P(f)|^2 + \frac{1}{T}\,\delta(f)\,P^2(0)\right]$$

$$= \frac{2}{T}\,\frac{T^2}{4}\,\mathrm{rect}\left(\frac{f}{2/T}\right) + \frac{1}{T^2}\,\frac{T^2}{4}\,\delta(f) = \boxed{\frac{T}{2}\,\mathrm{rect}\left(\frac{f}{2/T}\right) + \frac{1}{4}\,\delta(f)}$$

3) $P_{n_u} = \frac{N_0}{2} E_{H_R} = \frac{N_0}{2} \cdot \frac{2}{3T} = \boxed{\frac{N_0}{3T}}$

$$H_R(f) = \mathrm{rect}\left(\frac{f}{2/T}\right) - \left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right)$$



$$E_{H_R} = \frac{2}{3} \cdot \frac{1}{T} = \frac{2}{3T}$$

4) $H(f) = P(f)\, H_R(f) = \frac{T}{2} H_R(f)$

$$h(t) = \frac{T}{2} h_R(t) = \mathrm{sinc}\left(\frac{2t}{T}\right) - \frac{1}{2} \mathrm{sinc}^2\left(\frac{t}{T}\right)$$

$$h(nT) = \mathrm{sinc}\left(\frac{2}{T} nT\right) - \frac{1}{2} \mathrm{sinc}^2\left(\frac{nT}{T}\right)$$

$$= \mathrm{sinc}[2n] - \frac{1}{2} \mathrm{sinc}^2[n] = \delta[n] - \frac{1}{2} \delta[n]$$

$= \boxed{\frac{1}{2} \delta[n] \implies \text{Nyquist verificato nel tempo}}$

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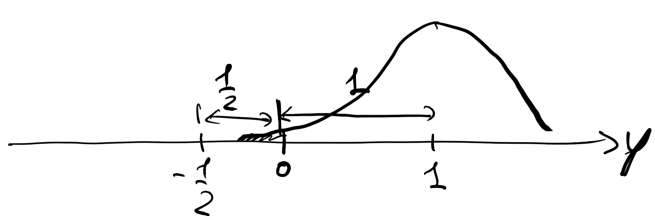
ASSENZA DI ISI

$\boxed{h(0) = \frac{1}{2}}$



$$P_E(b) = P(\hat{x} = -1 \mid x = 2)\, P(x=2) +$$

$$P(\hat{x} = 2 \mid x = -1)\, P(x=-1)$$

$$= \boxed{\frac{2}{3}\, Q\left(\frac{1}{\sqrt{\frac{N_0}{3T}}}\right) + \frac{1}{3}\, Q\left(\frac{\frac{1}{2}}{\sqrt{\frac{N_0}{3T}}}\right)}$$