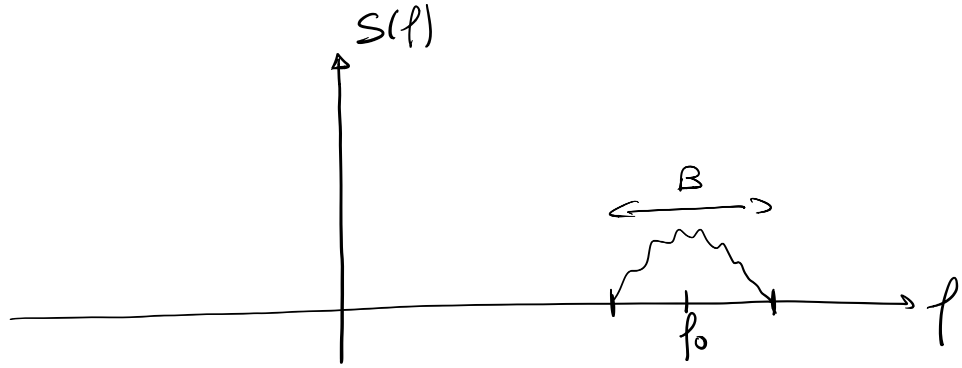
MODULAZIONI NUMERICHE IN BANDA PASSANTE

SEGNALE PASSA-BANDA

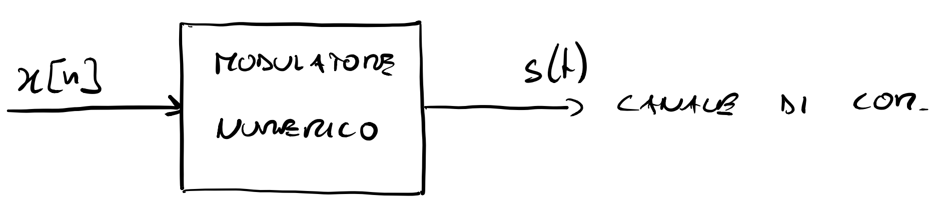


$$s(t) = \underbrace{a(t)}_{\text{inviluppo reale di } s(t)} \cos\Big[2\pi \underbrace{f_0}_{\text{frequenza centrale}} t + \underbrace{\theta(t)}_{\text{fase di } s(t)}\Big]$$

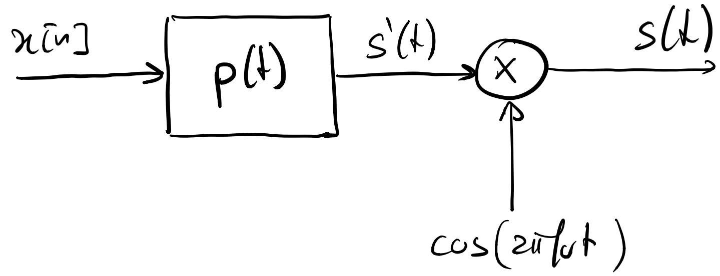
$$s(t) = \mathrm{Re}\left\{ a(t)\, e^{j[2\pi f_0 t + \theta(t)]} \right\}$$

$$\tilde{s}(t) \triangleq a(t)\, e^{j\theta(t)}$$ $\Leftarrow$ INVILUPPO COMPLESSO DI $s(t)$

$$\Rightarrow s(t) = \mathrm{Re}\left\{ \tilde{s}(t)\, e^{j2\pi f_0 t} \right\}$$



MODULATORE NUMERICO PER SISTEMI PASSA-BANDA



esempio di mod numerico in banda passante

·) interpolatore (trasf. da num. ad anal.)

·) traslatore in frequenza (trasf. da banda base a banda passante)

questo elemento non era presente nelle modulazioni in banda base

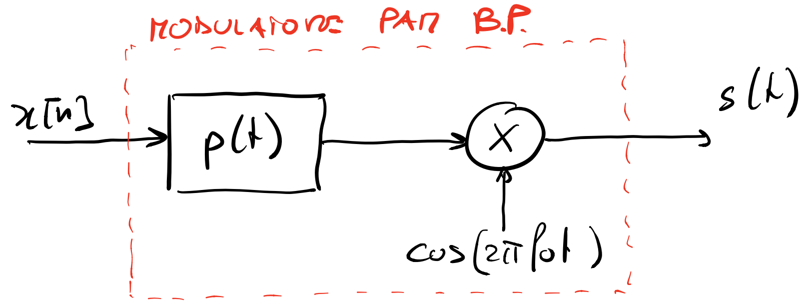
⇒ PAM IN BANDA PASSANTE

$$\Rightarrow \quad s(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p(t - nT_s) \underline{\cos(2\pi f_0 t)}$$ banda passante

per confronto

$$s(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p(t - nT_s)$$ banda base

MODULATORE PAM B.P.



$$x[n] \in A_S = \{\alpha_1, \alpha_2, \dots, \alpha_M\}$$

$\alpha_i = 2i - 1 - M$ per la PAM standard

$s_i(t) = \alpha_i \, p(t) \cos(2\pi f_0 t)$ è il segnale tx relativo al simbolo α_i

$$s_i(t) = \mathrm{Re}\left\{ \tilde{s}_i(t) \, e^{j 2\pi f_0 t} \right\}$$

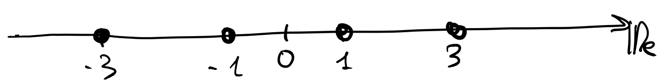
$\tilde{s}_i(t) = \alpha_i \, p(t)$ è reale

$p(t)$ è sempre reale

$\Downarrow$

α_i = sono reali

es $M=4$



spazio dei simboli (COSTELLAZIONE DEI SIMBOLI)

ENERGIA MEDIA PER SIMBOLO TX

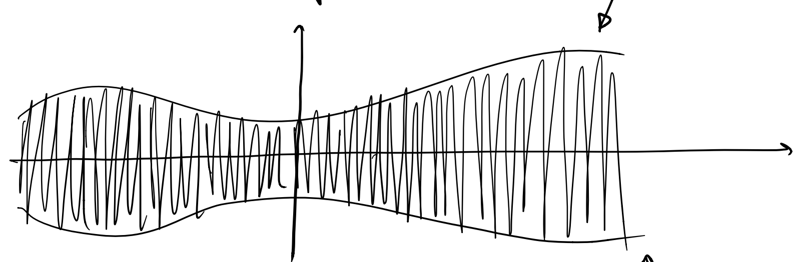
$$E_S = E\left[\int_{-\infty}^{+\infty} \left[x[n] \, p(t - nT_S) \cos(2\pi f_0 t) \right]^2 dt \right]$$

$$= E\left[x^2[n] \right] \int_{-\infty}^{+\infty} p^2(t - nT_S) \cos^2(2\pi f_0 t) \, dt$$

$$= E\left[x^2[n]\right]\int_{-\infty}^{+\infty} p^2(t-nT_s)\left[\frac{1}{2}+\frac{1}{2}\cos(4\pi f_0 t)\right]dt$$

$$= \frac{1}{2}E\left[x^2\right]\overbrace{\int_{-\infty}^{+\infty} p^2(t-nT_s)\,dt}^{E_p} +$$

$$+ \frac{1}{2}E\left[x^2\right]\underbrace{\int_{-\infty}^{+\infty} p^2(t-nT_s)\cos(4\pi f_0 t)\,dt}_{\simeq 0}$$



$$\boxed{E_s = \frac{1}{2}E[x^2]\,E_p}$$

ATTENZIONE AL FATTORE $\frac{1}{2}$ RISPETTO ALLA PAM IN BB.

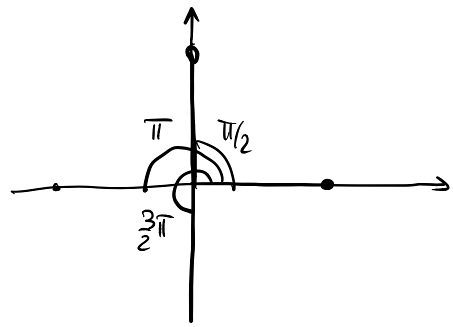
MODULAZIONE DI FASE (Phase Shift Keying PSK)

$$s(t) = \sum_{n=-\infty}^{+\infty} p(t-nT_s)\cos\left(2\pi f_0 t + \theta[n]\right)$$

$$\theta[n] \in A_s = \{\theta_1, \theta_2, .., \theta_M\}$$

$$\theta_i = \frac{2\pi}{M}(i-1)$$

es. $M = 4 \Rightarrow \theta_1 = 0,\ \theta_2 = \frac{\pi}{2},\ \theta_3 = \pi,\ \theta_4 = \frac{3}{2}\pi$



$$s_i(t) = \mathrm{Re}\left\{ \tilde{s}_i(t)\, e^{j2\pi f_0 t} \right\}$$

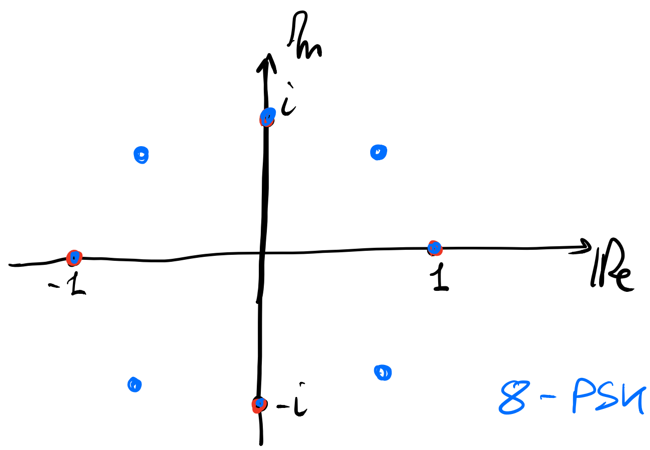
$$\tilde{s}_i(t) = p(t)\, \underbrace{e^{j\theta_i}} = p(t)\,\tilde{x} \quad , \tilde{x} = e^{j\theta_i}$$

⇓

spazio dei simboli (o costellazione dei simboli)

$M = 4$

4-PSK



MODULAZIONE QAM

$$\boxed{\begin{aligned} s(t) = \sum_{n=-\infty}^{+\infty} & x_c[n]\, p(t-nT_s)\cos(2\pi f_0 t) \\ & - x_s[n]\, p(t-nT_s)\sin(2\pi f_0 t) \end{aligned}}$$

SEGNALE MODULATO QAM

$$\tilde{x}[n] = x_c[n] + j\, x_s[n]$$

$$s(t) = \mathrm{Re}\left\{ \tilde{s}(t)\, e^{j 2\pi f_0 t} \right\}$$

$$\tilde{s}(t) = \left[x_c[n] + j\, x_s[n] \right] p(t)$$

$$\tilde{s}_i(t) = \left[\alpha_i^c + j\, \alpha_i^s \right] p(t)$$

$$\alpha_i^c \in A_S^c = \left\{ \alpha_1^c, \alpha_2^c, \dots, \alpha_{M_c}^c \right\}$$

$$\alpha_i^s \in A_S^s = \left\{ \alpha_1^s, \alpha_2^s, \dots, \alpha_{M_s}^s \right\}$$

$$M = M_c \cdot M_s$$

M - QAM $\Rightarrow$ vanno definiti M_c ed M_s

$$\mathrm{Re}\left\{ \tilde{s}(t)\, e^{j 2\pi f_0 t} \right\} = \mathrm{Re}\left\{ \left(x_c[n] + j\, x_s[n]\right) p(t)\, e^{j 2\pi f_0 t} \right\}$$

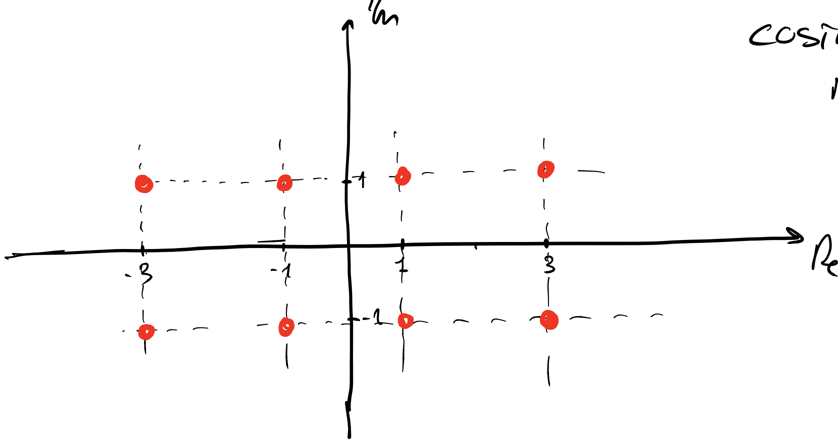
$$= \mathrm{Re}\left\{ x_c[n]\, p(t) \cos 2\pi f_0 t + x_c[n]\, p(t)\, j \sin(2\pi f_0 t) + \right.$$
$$\left. + j\, x_s[n]\, p(t) \cos 2\pi f_0 t - x_s[n]\, p(t) \sin(2\pi f_0 t) \right\}$$

$$= x_c[n]\, p(t) \cos(2\pi f_0 t) - x_s[n]\, p(t) \sin(2\pi f_0 t)$$

$$\alpha_i^c \in A_s^c = \{\alpha_1^c, \alpha_2^c, \dots, \alpha_{M_c}^c\} \quad , \quad \alpha_i^c = 2i - 1 - M_c$$

$$\alpha_i^s \in A_s^s = \{\alpha_1^s, \alpha_2^s, \dots, \alpha_{M_s}^s\} \quad , \quad \alpha_i^s = 2i - 1 - M_s$$

es 8 - QAM $\quad M_c = 4 \quad , \quad M_s = 2$



COSTELLAZIONE DEI SIMBOLI

$$\tilde{x} = x_c + j x_s$$

$$E_s = E\left[\int_{-\infty}^{+\infty} \left[x_c[n]\, p(t-nT_s) \cos(2\pi f_0 t) - x_s[n]\, p(t-nT_s) \sin(2\pi f_0 t) \right]^2 dt \right]$$

$$= E\left[\int_{-\infty}^{+\infty} x_c^2[n]\, p^2(t-nT_s) \cos^2(2\pi f_0 t) + x_s^2[n]\, p^2(t-nT_s) \sin^2(2\pi f_0 t) \right.$$

$$\left. - 2\, x_c[n]\, x_s[n]\, p^2(t-nT_s) \cos(2\pi f_0 t) \sin(2\pi f_0 t)\, dt \right]$$

$\Rightarrow$ tre componenti

$$I^a \quad \int_{-\infty}^{+\infty} p^2(t-nT_s) \cos^2(2\pi f_0 t)\, dt = \frac{1}{2} E_p$$

$$\text{II}^a \quad \int_{-\infty}^{+\infty} p^2(t-nT_s)\,\sin^2(2\pi f_0 t)\,dt = \frac{1}{2}E_p$$

poiché anche in questo caso il $\cos(4\pi f_0 t)$ annulla l'int.

$$\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha$$

$$\text{III}^a \quad \int_{-\infty}^{+\infty} p^2(t-nT_s)\,\sin(4\pi f_0 t)\,dt = 0$$

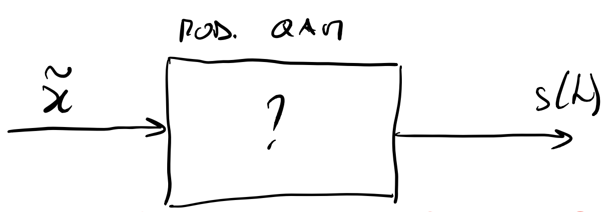
$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

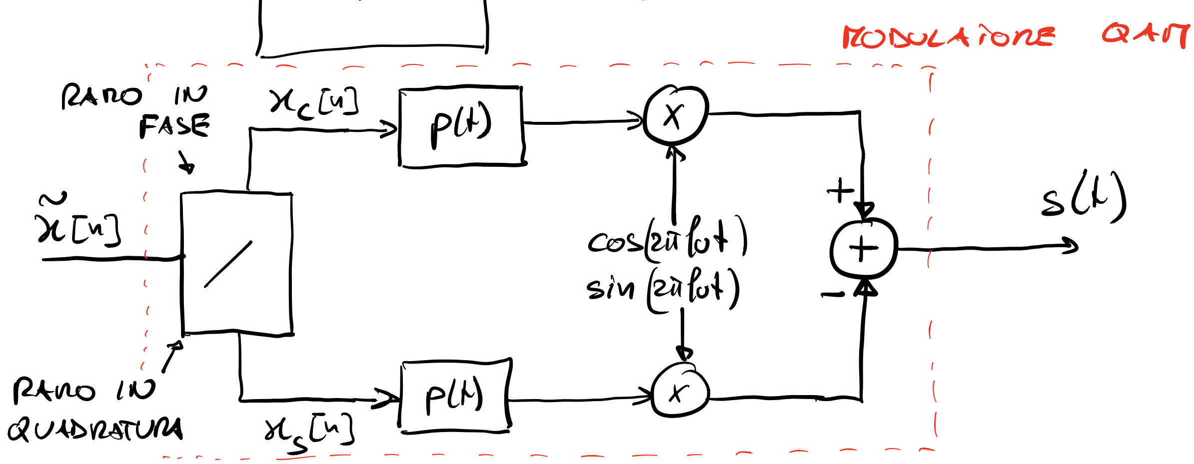
$$E_s = E[x_c^2]\,\frac{1}{2}E_p + E[x_s^2]\,\frac{1}{2}E_p + 0$$

$$= \boxed{\frac{1}{2}\Big[E[x_c^2] + E[x_s^2]\Big]E_p}$$

Parte reale (x_c) $\Rightarrow$ parte in fase

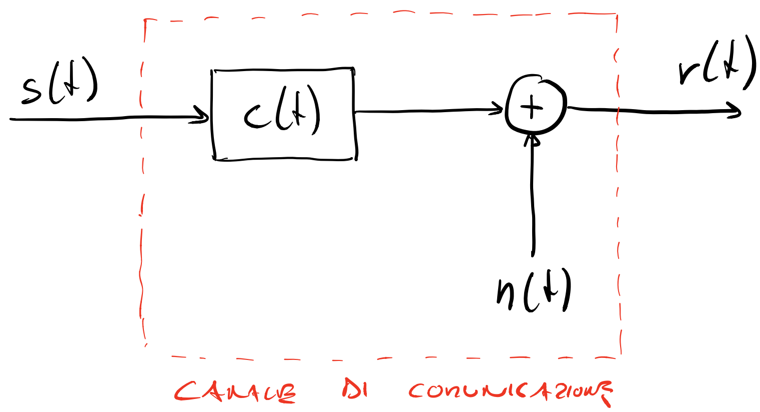
Parte imm. (x_s) $\Rightarrow$ parte in quadratura





$$s(t) = \sum_{n=-\infty}^{+\infty} x_c[n]\, p(t-nT_s) \cos(2\pi f_0 t) - x_s[n]\, p(t-nT_s) \sin(2\pi f_0 t)$$

CANALE DI COMUNICAZIONE PASSA-BANDA



$c(t)$ è la risposta impulsiva di un canale passa-banda

$$C(f) = TCF[c(t)] = \begin{cases} C(f) \neq 0 & f_0 - \frac{B}{2} \leq f \leq f_0 + \frac{B}{2} \\ 0 & \text{altrove} \end{cases}$$

canale passa-banda ideale

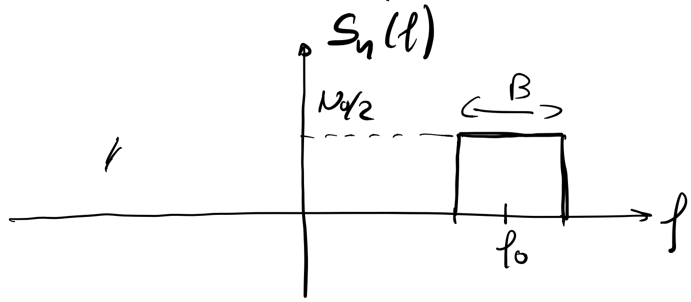
$$\begin{cases} C(f) = 1 & f_0 - \frac{B}{2} \leq f \leq f_0 + \frac{B}{2} \\ 0 & \text{altrove} \end{cases}$$

faremo sempre l'ipotesi che

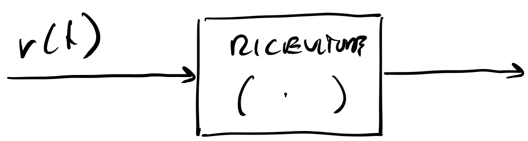
$B = B_T$, B_T = banda del segnale tx

⇒ caratteristiche del rumore passa-banda

- Rumore Gaussiano
- Bianco in banda
  - valor medio nullo
  - DSP piatta in banda

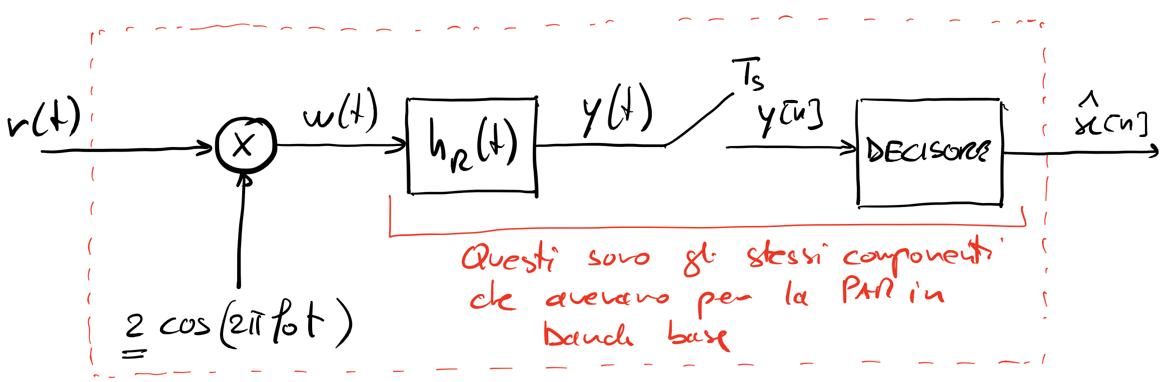


⇒



⇒ PAM IN BANDA PASSANTE

Demodulatore numerico



⇒ CASO DI SOLO SEGNALE UTILE

$$r(t) = s(t) \otimes c(t)$$ (annulliamo il rumore)

$$c(t) = 2\,\tilde{c}(t)\cos(2\pi f_0 t)$$

$\tilde{c}(t)$ è passa-basso

⇒ al ricevitore

$$w(t) = r(t) \cdot 2\cos(2\pi f_0 t)$$

$$= 2\left[s(t) \otimes c(t)\right]\cos(2\pi f_0 t)$$

$$= 2\int_{-\infty}^{+\infty} s(\tau)\, c(t-\tau)\, d\tau$$

$$= 2\int_{-\infty}^{+\infty} s(\tau)\; 2\,\tilde{c}(t-\tau)\cos(2\pi f_0 (t-\tau))\, d\tau \,\cdot$$

$$\cdot \cos(2\pi f_0 t)$$

$$\Rightarrow 2\,s(\tau)\,\tilde{c}(t-\tau)\cos\left[2\pi f_0 (t-\tau)\right] =$$

$$= 2\sum_{n=-\infty}^{+\infty} x[n]\, p(\tau - nT_s)\cos(2\pi f_0 \tau)\,\tilde{c}(t-\tau)\,\cdot$$

$$\cdot \cos\left[2\pi f_0 (t-\tau)\right] d\tau$$

$$= \sum_{n=-\infty}^{+\infty} x[n]\, p(\tau - nT_s)\,\tilde{c}(t-\tau)\left[\cos(2\pi f_0 t) + \cos(4\pi f_0 \tau - 2\pi f_0 t)\right]$$

$$w(t) = 2\left[\int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x[n]\, p(\tau - nT_s)\, \tilde{c}(t-\tau)\, d\tau \cos(2\pi f_0 t) + \right.$$

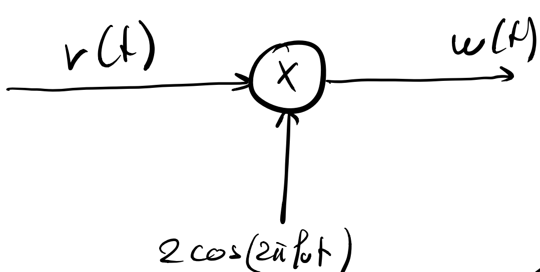
$$\left. + \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x[n]\, p(\tau - nT_s)\, \tilde{c}(t-\tau) \cos(4\pi f_0 \tau - 2\pi f_0 t)\, d\tau \right]$$

$$\cdot \cos(2\pi f_0 t)$$

$$= \sum_{n=-\infty}^{+\infty} x[n] \int_{-\infty}^{+\infty} p(\tau - nT_s)\, \tilde{c}(t-\tau)\, d\tau \,\big(1 + \cos(4\pi f_0 t)\big)$$

$$+ 2 \sum_{n=-\infty}^{+\infty} x[n] \underbrace{\int_{-\infty}^{+\infty} p(\tau - nT_s)\, \tilde{c}(t-\tau) \cos(4\pi f_0 \tau + K)\, d\tau}_{=0} \cos(2\pi f_0 t)$$

$$= \boxed{\sum_{n=-\infty}^{+\infty} x[n]\, p'(t - nT_s) \left[1 + \cos(4\pi f_0 t)\right]} \quad \Big| \quad p'(t) = p(t) \otimes \tilde{c}(t)$$



l'effetto del canale distorce la $p(t)$, trasformandola in una $p'(t)$

$$r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p'(t - nT_s) \cos(2\pi f_0 t)$$

$$w(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p'(t - nT_s) \left[1 + \cos(4\pi f_0 t) \right]$$

con $p'(t) = p(t) \otimes \tilde{c}(t)$

$$c(t) = 2\,\tilde{c}(t)\cos(2\pi f_0 t)$$

per il caso PAM in b.b.

$$s(t) \longrightarrow \boxed{c(t)} \longrightarrow r(t)$$

b.b. $\quad r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p'(t-nT_s)$, $\quad p'(t) = p(t)\otimes c(t)$

b.p. $\quad r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p'(t-nT_s)\cos(2\pi f_0 t)$

$$p'(t) = p(t)\otimes \tilde{c}(t)$$

$$w(t) \longrightarrow \boxed{h_R(t)} \longrightarrow y(t)$$

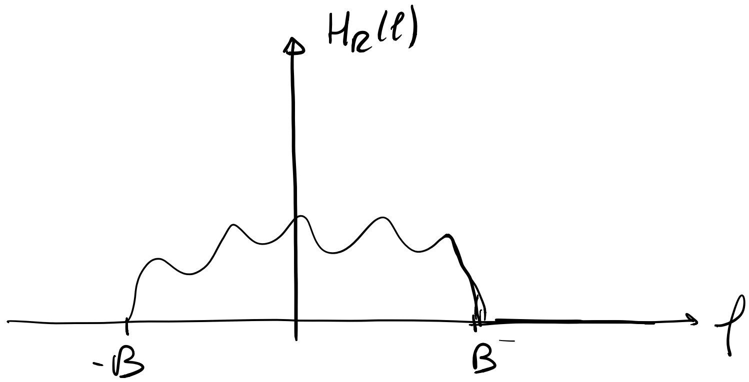
$$y(t) = w(t)\otimes h_R(t)$$

$w(t)$

- b.b. $\Rightarrow \sum_{n=-\infty}^{+\infty} x[n]\, p'(t-nT_s)$
- $2f_0$ $\Rightarrow \sum_{n=-\infty}^{+\infty} x[n]\, p'(t-nT_s)\cos(2\pi f_0 t)$

$h_R(t)$ è un filtro passa-basso di banda B

B è la stessa banda di $p(t)$



tutte le componenti freq. superiori a B vengono tagliate

$$y(t) = \left[\sum_{n=-\infty}^{+\infty} x[n]\, p'(t-nT_s)\right] \otimes h_R(t)$$

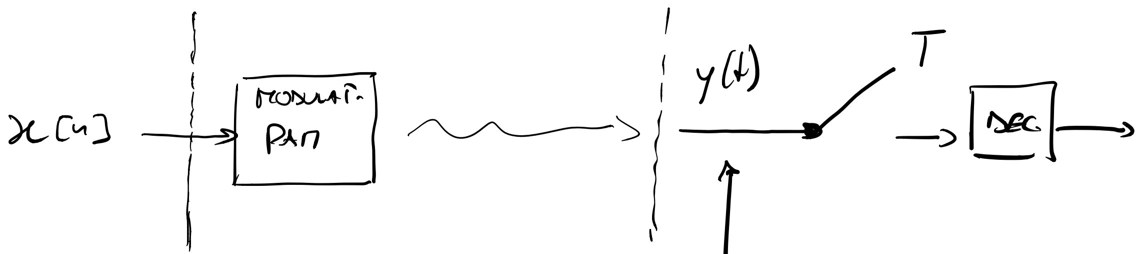
$$= \int_{-\infty}^{+\infty} \sum_n x[n]\, p'(\tau-nT_s)\, h_R(t-\tau)\, d\tau$$

$$= \sum_n x[n] \int_{-\infty}^{+\infty} p'(\tau-nT_s)\, h_R(t-\tau)\, d\tau$$

$$= \sum_n x[n]\, h(t-nT_s) \quad , \quad h(t) = p'(t) \otimes h_R(t)$$

$$\boxed{y(t) = \sum_n x[n]\, h(t-nT_s)}$$ $\Leftarrow$ IDENTICA A QUELLA DELLA PAM IN B.B.

$$\boxed{h(t) = p(t) \otimes \underline{\tilde{c}(t)} \otimes h_R(t)}$$



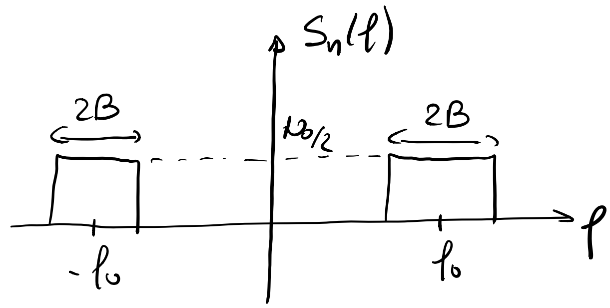
già qui il segnale è indistinguibile da quello di un PAM in bb.

⇒ RUMORE

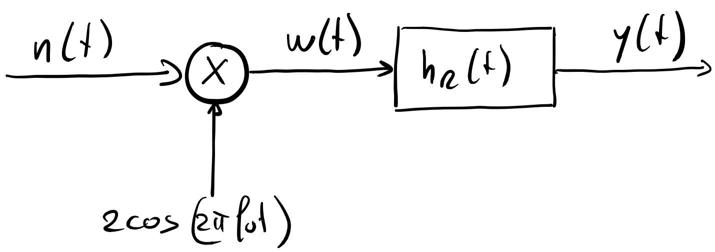
$$r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p'(t-nT_s) + n(t)$$

$n(t)$ Gaussiano bianco in banda

- $S_n(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \mathrm{rect}\left(\frac{f+f_0}{2B}\right)\right]$
- $E[n(t)] = 0$



Al ricevitore



$$R_n(\tau) = \frac{N_0}{2}\, 2B\, \text{sinc}(2B\tau)\left[e^{-j2\pi f_0 \tau} + e^{j2\pi f_0 \tau}\right]$$

$$= 2N_0 B\, \text{sinc}(2B\tau)\cos(2\pi f_0 \tau)$$

$w(t) = n(t)\, 2\cos(2\pi f_0 t)$ è ancora Gaussiano

$$E[w(t)] = \underset{\overset{\shortparallel}{0}}{E[n(t)]}\, 2\cos(2\pi f_0 t) = 0$$

$$R_w(t_1, t_2) = E[w(t_1)\, w(t_2)]$$

$$= E[2n(t_1)\cos(2\pi f_0 t_1)\, 2n(t_2)\cos(2\pi f_0 t_2)]$$

$$= 4\, E[n(t_1)\, n(t_2)]\cos(2\pi f_0 t_1)\cos(2\pi f_0 t_2)$$

$$= 4\, R_n(t_1 - t_2)\cos(2\pi f_0 t_1)\cos(2\pi f_0 t_2)$$

$$= 4N_0 B\, \text{sinc}[2B(t_1-t_2)]\cos[2\pi f_0(t_1-t_2)] \cdot$$

$$\cdot \left[\cos[2\pi f_0(t_1+t_2)] + \cos[2\pi f_0(t_1-t_2)]\right]$$

$$= 2N_0 B\, \text{sinc}[2B(t_1-t_2)]\Big[\cos(4\pi f_0 t_1) + \cos(4\pi f_0 t_2)$$

$$+ \cos[4\pi f_0(t_1-t_2)] + \underline{\underline{1}}\Big]$$

$w(t) =$
- 1 componente staz. in b.b. $2N_0 B\, \text{sinc}[2B(t_1-t_2)]$
- 3 componenti a freq $2f_0$

$$w(t) \longrightarrow \boxed{h_R(t)} \longrightarrow n_u(t)$$

$$n_u(t) = w(t) \otimes h_R(t)$$

$$R_{n_u}(t_1,t_2) = \underbrace{R_w(t_1,t_2) \otimes h_R(t_1)}_{R_w'(t_1,t_2)} \otimes h_R(t_2)$$

$$R_w(t_1,t_2) \otimes h_R(t_1) =$$

$$\Big[2N_0B \,\mathrm{sinc}\left[2B(t_1-t_2)\right] \big(1+\cos(4\pi f_0 t_2)\big)\Big] \otimes h_R(t_1)$$

$$= 2N_0B\left[1+\cos(4\pi f_0 t_2)\right] \mathrm{sinc}\left[2B(t_1-t_2)\right] \otimes h_R(t_1)$$

$$\boxed{R_{\tilde{n}}(t_1-t_2) \triangleq 2N_0B\,\mathrm{sinc}\left[2B(t_1-t_2)\right]}$$

$$= R_{\tilde{n}}(t_1-t_2)\left[1+\cos(4\pi f_0 t_2)\right] \otimes h_R(t_1)$$

$$= \left[1+\cos(4\pi f_0 t_2)\right] \cdot R_{\tilde{n}}(t_1-t_2) \otimes h_R(t_1)$$

$$R_{n_u}(t_1,t_2) = R_w'(t_1,t_2) \otimes h_R(t_2)$$

$$= \left[1+\cos(4\pi f_0 t_2)\right] R_{\tilde{n}}(t_1-t_2) \otimes h_R(t_1) \otimes h_R(t_2)$$

$$= \underbrace{\left[1+\cos(4\pi f_0 t_2)\right] R_{\tilde{n}}(t_1-t_2) \otimes h_R(t_2)} \otimes h_R(t_1)$$

nel filtraggio su t_2 si annulla il $\cos(4\pi f_0 t_2)$

$$= R_{\tilde{n}}(t_1 - t_2) \otimes h_R(t_1) \otimes h_R(t_2)$$

$$\Rightarrow R_{\tilde{n}}(\tau) \otimes h_R(\tau) \otimes h_R(-\tau)$$

si comporta come un processo SSL

$w(t)$ → 1 componente SSL in b.b.

$w(t)$ → 3 ~~componenti non~~ SSL a $2f_0$

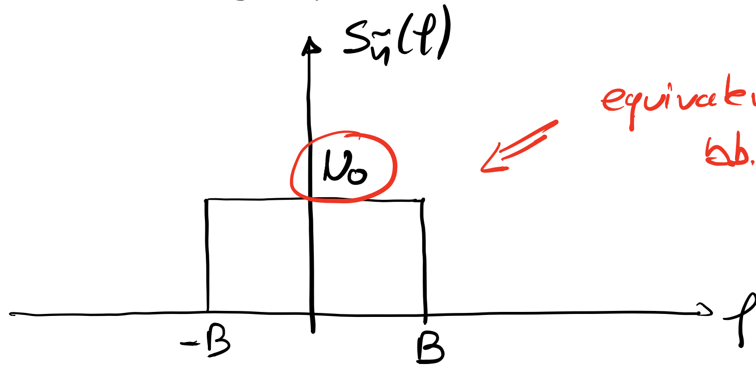
cancellate dal filtro

$$R_{n_u}(\tau) = R_{\tilde{n}}(\tau) \otimes h_R(\tau) \otimes h_R(-\tau)$$

$$S_{n_u}(f) = S_{\tilde{n}}(f)\, |H_R(f)|^2$$

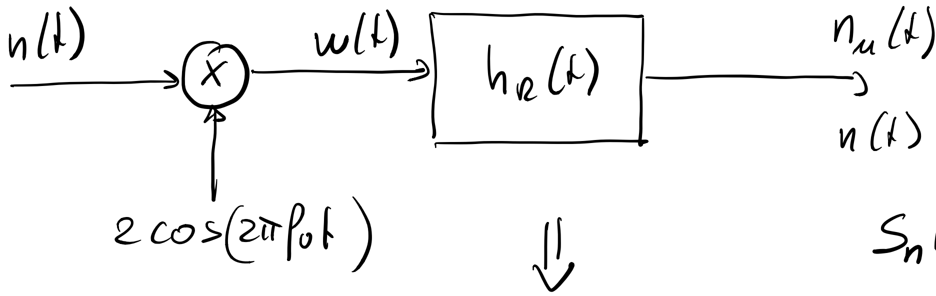
$$R_{\tilde{n}}(\tau) = 2N_0 B \,\mathrm{sinc}(2B\tau)$$

$$S_{\tilde{n}}(f) = N_0 \,\mathrm{rect}\left(\frac{f}{2B}\right)$$



equivalente in b.b.

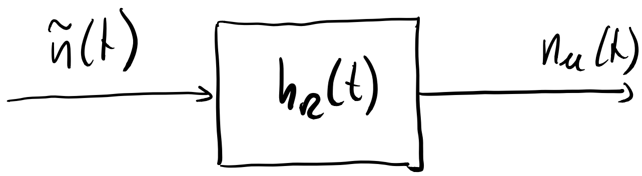
Schema equivalente



$n(t)$ Bianco in banda pass.

$S_n(f) = \frac{N_0}{2}$ in banda

$\Downarrow$



Equivalente in banda base

$\tilde{n}(t)$ bianco in banda base

$$S_{\tilde{n}}(f) = N_0$$

$n_u(t) \Rightarrow$ Gaussiano, con $S_{n_u}(f) = N_0 \, |H_R(f)|^2$

e $E[n_u(t)] = 0$

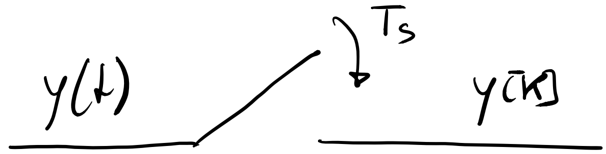
$$E[w(t)] = 0$$

$$E[n_u(t)] = \underset{0}{\underset{\parallel}{\eta_w}}(t) \otimes h_R(t) = 0$$

$$y(t) = \sum_{n=-\infty}^{+\infty} x[n]\, h(t - nT_s) + n_u(t)$$

$h(t) = p(t) \otimes c(t) \otimes h_R(t)$

è caratterizzato



$$\boxed{y[k] = \sum_{n=-\infty}^{+\infty} x[n]\, h[(k-n)T_s] + n_u[k]}$$

STESSA SITUAZIONE CHE ABBIAMO INCONTRATO CON LA PAM IN B.B.

$\Rightarrow$ componente utile

$$\sum_{n=-\infty}^{+\infty} x[n]\, h[(k-n)T_s] =$$

$$= x[k]\, h(0) + \underbrace{\sum_{\substack{n=-\infty \\ n\neq k}}^{+\infty} x[n]\, h[(k-n)T_s]}_{ISI}$$

$\Rightarrow$ Procedura

- Verificare l'assenza di ISI
- calcolare $h(0)$
- calcolare $P_{n_u} = \int_{-\infty}^{+\infty} S_{n_u}(f)\, df = \underline{N_0}\, E_{H_R}$ $\rightarrow$ DIFF. CON B.B.

$$S_{n_u}(f) = \underline{N_0}\, |H_R(f)|^2$$

- calcolare la P_E

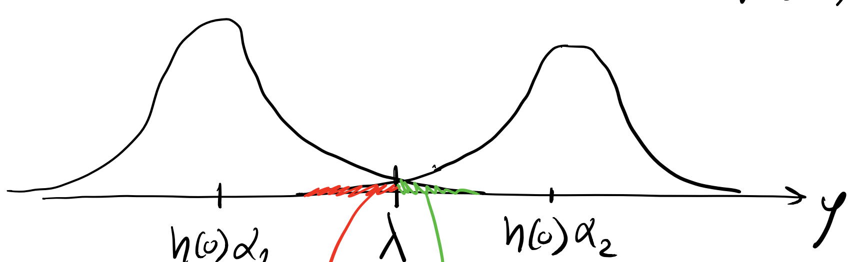
in assenza di ISI

$$y[k] = h(0)\,x[k] + n_u[k]$$

V.A. Gaussiana

$$\in \mathcal{N}(0, P_{n_u})$$

M=2



$$P_E(b) = P\{\hat{x}=\alpha_1 \mid x=\alpha_2\}\, P\{x=\alpha_2\} + P\{\hat{x}=\alpha_2 \mid x=\alpha_1\}\, P\{x=\alpha_1\}$$

È identico a quanto visto per la PAM in bb

Riassumendo le differenze

1) $E_s = \frac{1}{2} E_p\, E[x^2]$

2) $P_{n_u} = N_0\, E_{H_R}$