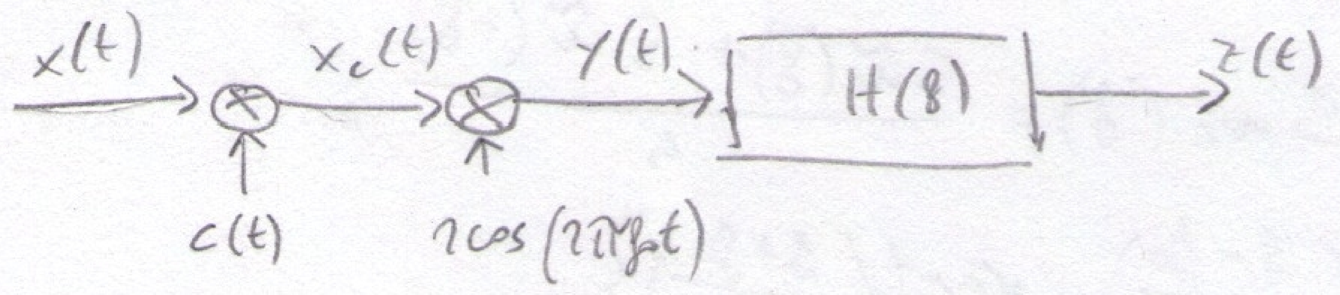
Esame del 13-01-10

• Esercizio 1



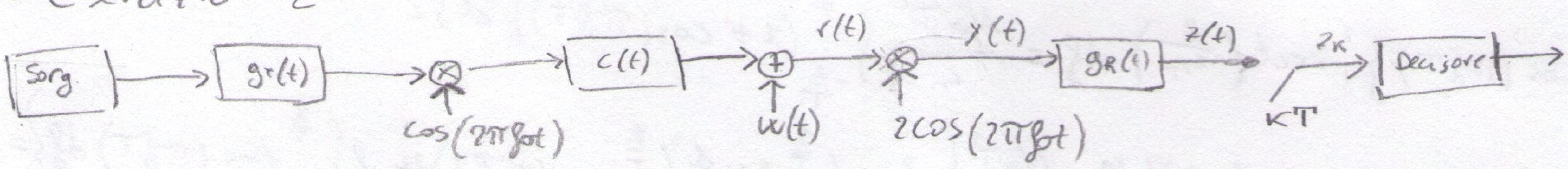
$$x(t) = B\,\mathrm{sinc}^2(Bt) \quad ; \quad c(t) = \sum_n e^{-2\pi f_0 (t-nT)} u(t-nT)$$

$$x_c(t) = x(t)c(t) = B\,\mathrm{sinc}^2(Bt) \sum_n e^{-2\pi f_0 (t-nT)} u(t-nT)$$

$$y(t) = x_c(t)\, 2\cos(2\pi f_0 t) \rightleftharpoons Y(f) = X_c(f-f_0) + X_c(f+f_0) \quad ; \quad Z(f) = Y(f)H(f) = [X_c(f-f_0) + X_c(f+f_0)]H(f)$$

$$z(t) = \emptyset \quad ; \quad E_z = \emptyset$$

• Esercizio 2



1) $E_T = P_{s_T}\, T \quad ; \quad P_{s_T} = \int_{-\infty}^{\infty} S_{s_T}(f)\, df$

$$S_{s_T}(f) = \frac{E\{a_i^2\}}{T} |G_T(f)|^2 = \frac{|G_T(f)|^2}{T} = \frac{1}{T} \left| \frac{T}{2}[1+\cos(\pi f T)]\, \mathrm{rect}\left(\frac{f}{\frac{2}{T}}\right) \right|^2 = \frac{T}{4}[1+\cos(\pi f T)]^2 \mathrm{rect}^2\left(\frac{f}{\frac{2}{T}}\right)$$

$$P_{s_T} = \int_{-\infty}^{\infty} S_{s_T}(f)\, df = \int_{-\infty}^{\infty} \frac{T}{4}[1+\cos(\pi f T)]^2 \mathrm{rect}^2\left(\frac{f}{\frac{2}{T}}\right) df = \frac{T}{4} \int_{-\frac{1}{T}}^{\frac{1}{T}} [1 + \cos^2(\pi f T) + 2\cos(\pi f T)]\, df =$$

$$= \frac{T}{4} \cdot \frac{2}{T} + \frac{T}{4} \int_{-\frac{1}{T}}^{\frac{1}{T}} \left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f T)\right] df + \frac{T}{4} 2 \int_{-\frac{1}{T}}^{\frac{1}{T}} \cos(\pi f T)\, df =$$

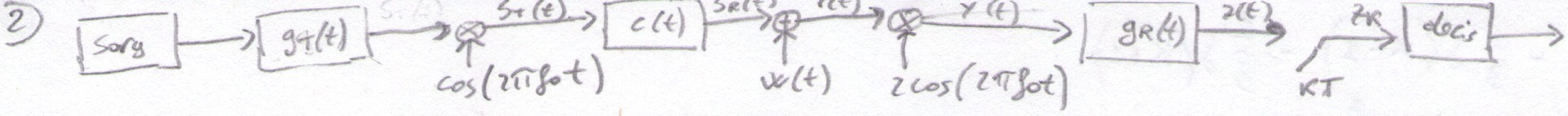
$$= \frac{1}{2} + \frac{1}{4} + \frac{T}{4} \cdot \frac{1}{4} \int_{-\frac{1}{T}}^{\frac{1}{T}} \left(e^{j4\pi f T} + e^{-j4\pi f T}\right) df + \frac{1}{4} \int_{-\frac{1}{T}}^{\frac{1}{T}} \left(e^{j\pi f T} + e^{-j\pi f T}\right) df =$$

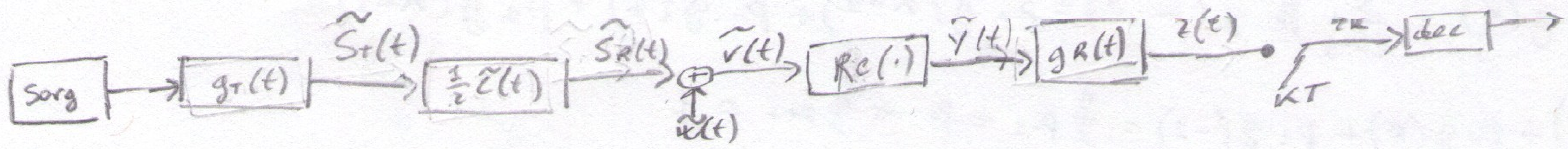
$$= \frac{3}{4} + \frac{T}{8} \cdot \frac{1}{2\pi T} \left[\frac{e^{j4\pi} - e^{-j4\pi}}{2j}\right] + \frac{T}{8} \cdot \frac{1}{2\pi T} \left[\frac{-e^{-j4\pi} + e^{j4\pi}}{2j}\right] + \frac{1}{2} \cdot \frac{1}{\pi T} \left[\frac{e^{j\pi} - e^{-j\pi}}{2j}\right] + \frac{1}{2\pi T} \left[\frac{-e^{-j\pi} + e^{j\pi}}{2j}\right]$$

$$= \frac{3}{4} + \frac{1}{16\pi} \sin(4\pi) + \frac{1}{16\pi} \sin(4\pi) + \frac{1}{2\pi T} \sin(\pi) + \frac{1}{2\pi T} \sin(\pi) = \frac{3}{4}$$

$$E_T = \frac{3}{4} T$$

2)





$$S_T(t) = \underbrace{\sum_i a_i\, g_T(t - iT)}_{S_{Tc}(t)} \cos(2\pi f_0 t) \triangleq Re\left[\tilde{S}_T(t) e^{j2\pi f_0 t}\right] = S_{Tc}(t)\, Re\left[e^{j2\pi f_0 t}\right] = Re\left[S_{Tc}(t) e^{j\pi f_0 t}\right]$$

$$\tilde{S}_T(t) = S_{Tc}(t) = \sum_i a_i\, g_T(t - iT) \quad ; \quad g_{Tc}(t) \triangleq g_T(t) \otimes \tfrac{1}{2}\tilde{c}(t)$$

$$\tilde{S}_R(t) = \sum_i a_i\, g_{Tc}(t - iT) \quad ; \quad \tilde{r}(t) = \sum_i a_i\, g_{Tc}(t - iT) + \tilde{w}(t)$$

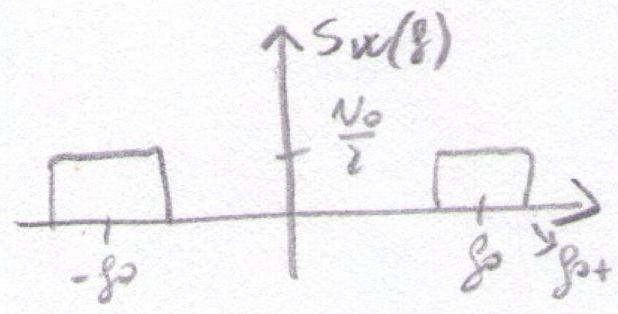
$$\tilde{y}(t) = Re\{\tilde{r}(t)\} = Re\left\{\sum_i a_i\, g_{Tc}(t - iT)\right\} + Re\{\tilde{w}(t)\} = \sum_i a_i\, g_{Tc}(t - iT) + w_c(t)$$

$$z(t) = \tilde{y}(t) \otimes g_R(t) \; ; \; g(t) \triangleq g_{Tc}(t) \otimes g_R(t) \quad ; \quad n_c(t) = w_c(t) \otimes g_R(t)$$

$$z(t) = \sum_i a_i\, g(t - iT) + n_c(t)$$

3) $P_n = \int_{-\infty}^{\infty} S_{n_c}(f)\, df \quad ; \quad S_{n_c}(f) = S_{w_c}(f)\, |G_R(f)|^2 \quad ; \quad S_{w_c}(f) = \dfrac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4}$;

$$S_{\tilde{w}}(f) = \begin{cases} 4 S_w(f + f_0) & f \geq -f_0 \\ \emptyset & \text{altrove} \end{cases} \quad ; \quad S_w(f) = \frac{N_0}{2}\left\{rect\left(\frac{f - f_0}{2/T}\right) + rect\left(\frac{f + f_0}{2/T}\right)\right\}$$



$$S_{\tilde{w}}(f) = 2N_0\, rect\left(\frac{f}{2/T}\right) \quad ; \quad S_{w_c}(f) = \frac{2N_0\, rect\left(\frac{f}{2/T}\right) + 2N_0\, rect\left(\frac{f}{2/T}\right)}{4} = N_0\, rect\left(\frac{f}{2/T}\right)$$

$$S_{n_c}(f) = N_0\, rect\left(\frac{f}{2/T}\right)\left[\frac{T}{2}\left(1 + \cos(\pi f T)\right) rect\left(f\frac{T}{2}\right)\right]^2 = S_{w_c}(f)\left[\frac{T^2}{4}\left[1 + \cos(\pi f T)\right]^2 rect\left(\frac{f}{2/T}\right)\right] =$$

$$= \frac{N_0 T^2}{4}\left[1 + \cos(\pi f T)\right]^2 rect\left(\frac{f}{2/T}\right)$$

$$P_n = \int_{-\infty}^{\infty} \frac{N_0 T^2}{4}\left[1 + \cos(\pi f T)\right]^2 rect\left(\frac{f}{2/T}\right) df = \frac{N_0 T^2}{4}\int_{-\frac{1}{T}}^{\frac{1}{T}} \left(1 + \cos(\pi f T)\right)^2 df =$$

$$= \frac{N_0 T^2}{4}\int_{-\frac{1}{T}}^{\frac{1}{T}} \left[1 + \cos^2(\pi f T) + 2\cos(\pi f T)\right] df = \frac{N_0 T^2}{4}\left\{\frac{2}{T} + \int_{-\frac{1}{T}}^{\frac{1}{T}} \frac{1}{2} df + \frac{1}{2}\int_{-\frac{1}{T}}^{\frac{1}{T}} \cos(2\pi f T)\, df + 2\int_{-\frac{1}{T}}^{\frac{1}{T}} \cos(\pi f T)\, df\right\} =$$

$$= \frac{N_0 T^2}{4}\left\{\frac{2}{T} + \frac{1}{T} + \int_{-\frac{1}{T}}^{\frac{1}{T}} \left(e^{j2\pi f T} + e^{-j2\pi f T}\right) df + 4\int_{-\frac{1}{T}}^{\frac{1}{T}} \left(e^{j\pi f T} + e^{-j\pi f T}\right) df\right\} =$$

$$= \frac{N_0 T^2}{4}\left\{\frac{3}{T} + \frac{1}{\pi T}\left(\frac{e^{j2\pi} - e^{-j2\pi}}{2j}\right) + \frac{1}{j2\pi T}\left(\frac{e^{-j2\pi} + e^{j2\pi}}{2j}\right) + \frac{8}{\pi T}\left(\frac{e^{j\pi} - e^{-j\pi}}{2j}\right) + \frac{8}{j\pi T}\left(\frac{e^{-j\pi} + e^{j\pi}}{2j}\right)\right\} =$$

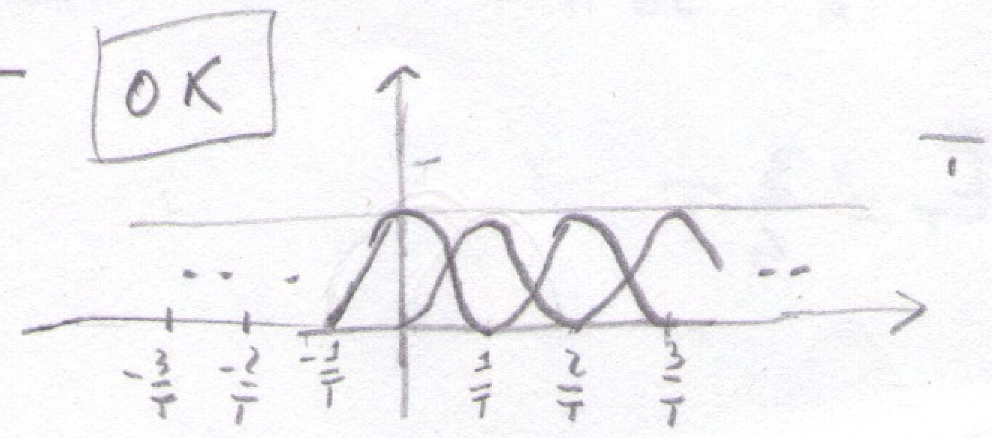
$$= \frac{N_0 T^2}{4}\left\{\frac{3}{T} + \frac{1}{\pi T}\sin(2\pi) + \cancel{\frac{1}{\pi T}\sin(2\pi)} + \frac{8}{\pi T}\sin(\pi) + \cancel{\frac{8}{\pi T}\sin(\pi)}\right\} = \frac{3 N_0 T}{4}$$

4) D_{-1} D_1; -1, 0, 1

$$P_r(e) = P_r\left[\hat{1} \mid -1\right] P_r(-1) + P_r\left[-\hat{1} \mid 1\right] P_r(1)$$

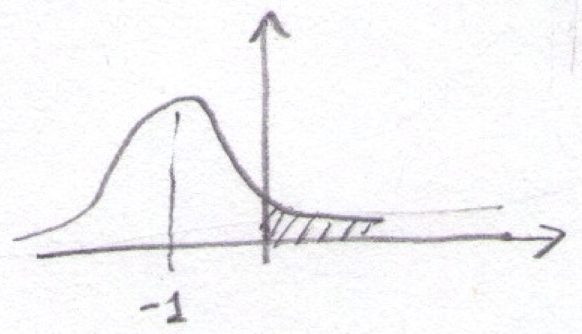
$$G(f) = G_{Tc}(f)\, G_R(f) = \frac{T}{2}\left[1 + \cos(\pi f T)\right] rect\left(\frac{f}{2/T}\right) \cdot rect\left(\frac{f}{2/T}\right) = \frac{T}{2}\left[1 + \cos(\pi f T)\right] rect\left(\frac{f}{2/T}\right)$$

$$G_N(f) = \sum_n G\left(f - \frac{n}{T}\right) = \sum_n \frac{T}{2}\left[1 + \cos\left[\pi T\left(f - \frac{n}{T}\right)\right]\right] rect\left(\frac{f - \frac{n}{T}}{2/T}\right) = T \quad \boxed{OK}$$



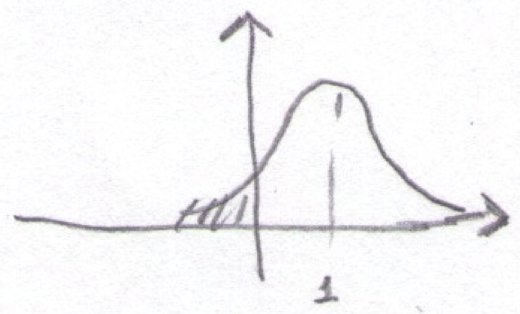
$$z_k = \sum_i a_i\, g([k-i]T) + n_k = a_k\, g(\emptyset) + n_k = a_k + n_k$$

$$z_k\big|_{a_k=-1} = -1 + n_k \qquad n_k \in \mathcal{N}(\emptyset, \sigma_n^2) \Rightarrow z_k\big|_{a_k=-1} \in \mathcal{N}(-1, \sigma_n^2)$$



$$P_r[\hat{1}\,|\,-1] = P_r[\hat{a}_k > \emptyset \,|\, -1] = Q\left(\frac{\emptyset+1}{\sigma_n}\right) = Q\left(\frac{1}{\sigma_n}\right)$$

$$z_k\big|_{a_k=1} = 1 + n_k \in \mathcal{N}(1, \sigma_n^2)$$



$$P_r[-\hat{1}\,|\,1] = P_r[\hat{a}_k < \emptyset \,|\, 1] = 1 - Q\left(\frac{\emptyset-1}{\sigma_n}\right) = Q\left(\frac{1}{\sigma_n}\right)$$

$$\sigma_n^2 = P_n = \frac{3N_0T}{4} \Rightarrow \sigma_n = \sqrt{\frac{3N_0T}{4}}$$

$$P_r(e) = \frac{1}{2}Q\left(\frac{1}{\sigma_n}\right) + \frac{1}{2}Q\left(\frac{1}{\sigma_n}\right) = Q\left(\frac{1}{\sqrt{\frac{3N_0T}{4}}}\right) = Q\left(\frac{2}{\sqrt{3N_0T}}\right)$$