ANALISI ENERGETICA DI SEGNALI APERIODICI

·) Correlazione tra segnali

$$C_{xy}(\tau) \triangleq \int_{-\infty}^{+\infty} x(t)\, y^*\underbrace{(t-\tau)}\, dt$$

diff. tra correlaz. e convoluzione

·) Autocorrelazione

$$C_x(\tau) \triangleq \int_{-\infty}^{+\infty} x(t)\, x^*(t-\tau)\, dt$$

·) Proprietà dell'autocorrelazione

1) $$C_x(0) = \int_{-\infty}^{+\infty} x(t)\, x^*(t)\, dt = \int_{-\infty}^{+\infty} |x(t)|^2\, dt$$

$$= E_x$$

2) $$C_x^*(\tau) = C_x(-\tau)$$ Hermitiana

Dim.

$$C_x(-\tau) = \int_{-\infty}^{+\infty} x(t)\, x^*(t-(-\tau))\, dt$$

$$= \int_{-\infty}^{+\infty} x(t)\, x^*(t+\tau)\, dt \qquad t+\tau = t'$$

$$= \int_{-\infty}^{+\infty} x(t'-\tau)\, x^*(t')\, dt'$$

$$= \int_{-\infty}^{+\infty} x^*(t')\, x(t'-\tau)\, dt' = \left[\int_{-\infty}^{+\infty} x(t')\, x^*(t'-\tau)\, dt' \right]^*$$

$$= C_x^*(\tau)$$

se $x(t)$ è reale $\Rightarrow$ $C_x(-\tau) = C_x(\tau)$

reale e pari

TCF DELLA AUTOCORRELAZIONE

$$C_x(\tau) \overset{TCF}{\Longleftrightarrow} |X(f)|^2 = S_x(f)$$

$\Downarrow$

DENSITÀ SPETTRALE DI ENERGIA

Dim.

$$S_x(f) = \int_{-\infty}^{+\infty} C_x(\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$= \int_{(\tau)\,-\infty}^{+\infty} \underbrace{\int_{(t)\,-\infty}^{+\infty} x(t)\, x^*(t-\tau)\, dt}\; e^{-j2\pi f\tau}\, d\tau$$

$$= \int_{-\infty}^{+\infty} x(t) \int_{-\infty}^{+\infty} x^*(t-\tau)\, e^{-j2\pi f\tau}\, d\tau\, dt \qquad t-\tau = \tau'$$

(t) (τ)

$$= \int_{-\infty}^{+\infty} x(t) \int_{-\infty}^{+\infty} x^*(\tau')\, e^{-j2\pi f(t-\tau')}\, d\tau'\, dt$$

(t) (τ')

$$= \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt \int_{-\infty}^{+\infty} x^*(\tau)\, e^{j2\pi f\tau'}\, d\tau'$$

$$= \underbrace{\int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt}_{X(f)} \underbrace{\left[\int_{-\infty}^{+\infty} x(\tau')\, e^{-j2\pi f\tau'}\, d\tau'\right]^*}_{X^*(f)}$$

$$= X(f)\, X^*(f) = |X(f)|^2$$

$$E_x \stackrel{?}{=} \int_{-\infty}^{+\infty} |X(f)|^2\, df$$

Dim

$$C_x(\tau) = \int_{-\infty}^{+\infty} S_x(f)\, e^{j2\pi f\tau}\, df$$

$$= \int_{-\infty}^{+\infty} |X(f)|^2\, e^{j2\pi f\tau}\, df$$

$$C_x(\tau)\Big|_{\tau=0} = C_x(0) = E_x = \int_{-\infty}^{+\infty} |X(f)|^2\, e^{j2\pi f\tau}\, df\Big|_{\tau=0}$$

$$\boxed{E_x = \int_{-\infty}^{+\infty} |X(f)|^2\, df = \int_{-\infty}^{+\infty} |x(t)|^2\, dt}$$

L'energia di un segnale può essere calcolata in due modi

Esempio

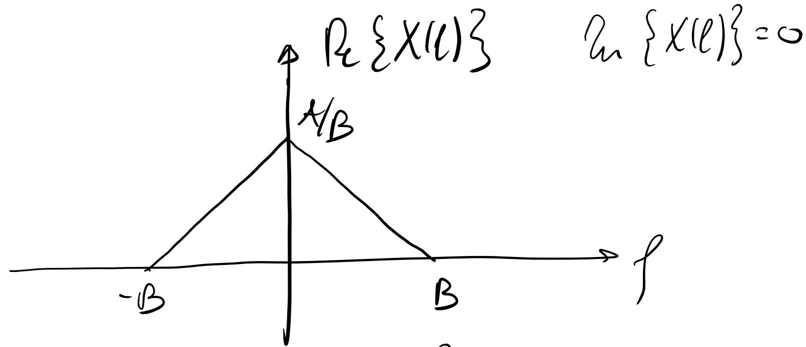
$$x(t) = A\, \mathrm{sinc}^2(Bt)$$

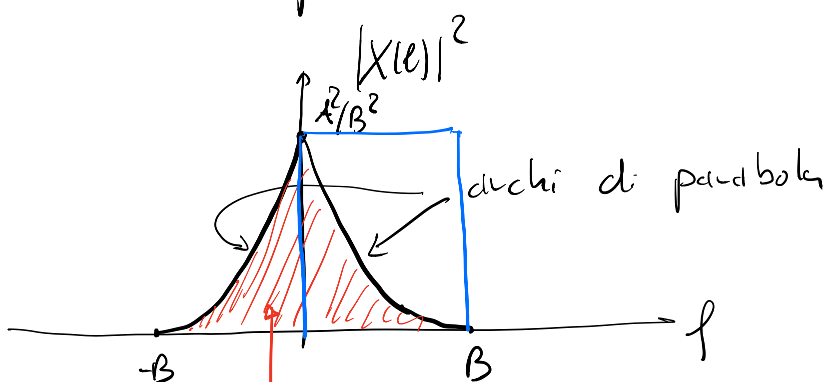
$$E_x = ?$$

$$E_x = A\int_{-\infty}^{+\infty} \left|\mathrm{sinc}^2(Bt)\right|^2 dt$$

$$x(t) \overset{TCF}{\Longleftrightarrow} X(f)$$

$$X(f) = \frac{A}{B}\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$



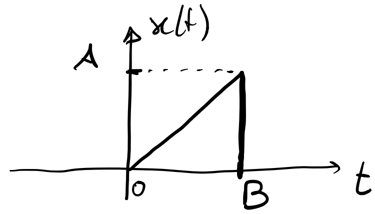


$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2 \, df = \int_{-\infty}^{+\infty} X^2(f) \, df$$

$$= \frac{2}{3} \frac{A^2}{B^2} B = \frac{2}{3} \frac{A^2}{B}$$

In generale:

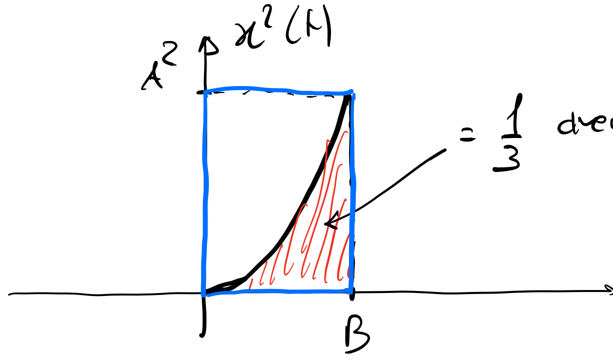
·) quando abbiamo un triangolo



$$x(t) = \frac{A}{B}t$$

$$\int_0^B x^2(t)\,dt = \int_0^B \frac{A^2}{B^2}t^2\,dt = \frac{A^2}{B^2}\left.\frac{t^3}{3}\right|_0^B =$$

$$= \frac{A^2}{3B^2}\left(B^3 - 0\right) = \frac{A^2 B}{3}$$



$= \frac{1}{3}$ area del rettangolo che la contiene

.) TEOREMA DI PARSEVAL

$$\int_{-\infty}^{+\infty} x(t)\,y^*(t)\,dt = \int_{-\infty}^{+\infty} X(f)\,Y^*(f)\,df$$

$\Downarrow$ da cui discende

$$x(t) = y(t)$$

$$\Downarrow$$

$$\int_{-\infty}^{+\infty} |x(t)|^2\,dt = \int_{-\infty}^{+\infty} |X(f)|^2\,df$$

Dim.

$$\int_{-\infty}^{+\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{+\infty} \underbrace{\int_{-\infty}^{+\infty} X(f)\, e^{j2\pi f t}\, df}_{x(t)}\; y^*(t)\, dt$$

(t) (f)

$$= \int_{-\infty}^{+\infty} X(f) \underbrace{\int_{-\infty}^{+\infty} y^*(t)\, e^{j2\pi f t}\, dt}_{Y^*(f)}\, df = \int_{-\infty}^{+\infty} X(f)\, Y^*(f)\, df$$

(f)

·) CORRELAZIONE E CONVOLUZIONE

$$C_{xy}(\tau) = x(\tau) \otimes y^*(-\tau)$$

Dim

$$x(\tau) \otimes y^*(-\tau) = \int_{-\infty}^{+\infty} x(\alpha)\, y^*[-(\tau-\alpha)]\, d\alpha$$

$$= \int_{-\infty}^{+\infty} x(\alpha)\, y^*(\alpha-\tau)\, d\alpha = C_{xy}(\tau)$$

·) PERIODICIZZAZIONE DI SEGNALI APERIODICI

$x(t)$ è aperiodico

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$ è periodico di periodo T_0

Dim

se y è periodico di T_0

$$y(t - kT_0) = y(t)$$

$$y(t-kT_0) = \sum_{n=-\infty}^{+\infty} x(t - kT_0 - nT_0)$$

$$= \sum_{n=-\infty}^{+\infty} x(t - (k+n)T_0) \qquad k+n = n'$$

$$= \sum_{n'=-\infty}^{+\infty} x(t - n'T_0) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

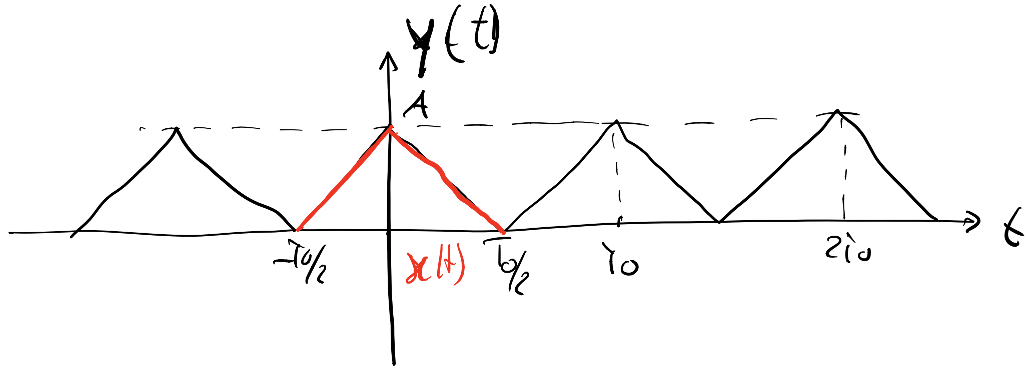
·) RELAZIONE TRA TSF e TCF

$y(t)$ è un segnale periodico ottenuto per periodicizzazione di un segnale $x(t)$ aperiodico

Ip $\begin{cases} y(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0) & T_0 \in \mathbb{R}^+ \\ x(t) \overset{TCF}{\Longleftrightarrow} X(f) \end{cases}$

Th. $\quad Y_n = TSF[y(t)] = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$

$\Rightarrow$ la TSF si ottiene "campionando" la TCF



$y(t)$ periodico $\Rightarrow$ posso sempre trovare almeno un $x(t)$: $y(t) = \sum_{n=-\infty}^{+\infty} x(t-nT_0)$

Calcolo della TSF di $y(t) \Rightarrow Y(f)$

$\Downarrow$

-) trovo $x(t)$: $y(t) = \sum_{n=-\infty}^{+\infty} x(t-nT_0)$
-) calcolo $X(f) = TCF[x(t)]$
-) calcolo $Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$

$y(t) = \sum_{n=-\infty}^{+\infty} x(t-nT_0)$, $\quad x(t) \overset{TCF}{\Longleftrightarrow} X(f)$

$\Rightarrow Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$

Dim.

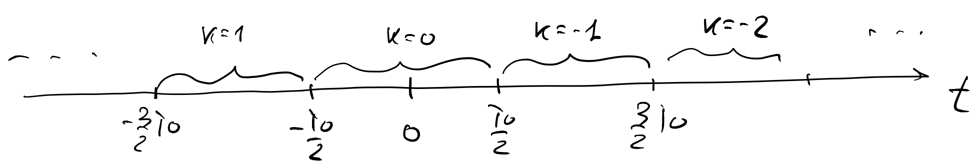
$$Y_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} y(t)\, e^{-j2\pi n f_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_{k=-\infty}^{+\infty} x(t-kT_0)\, e^{-j2\pi n f_0 t}\, dt \qquad t-kT_0 = t'$$

$$= \frac{1}{T_0}\sum_{k=-\infty}^{+\infty}\int_{-\frac{T_0}{2}-kT_0}^{\frac{T_0}{2}-kT_0} x(t')\, e^{-j2\pi n f_0 (t'+kT_0)}\, dt'$$

$$= \frac{1}{T_0}\sum_{k=-\infty}^{+\infty}\int_{-\frac{T_0}{2}-kT_0}^{\frac{T_0}{2}-kT_0} x(t')\, e^{-j2\pi n f_0 t'}\, dt' \; \overbrace{\underbrace{e^{-j2\pi nk}}_{\substack{n,k\in\mathbb{Z}\\ n\cdot k\in\mathbb{Z}}}}^{=1}$$

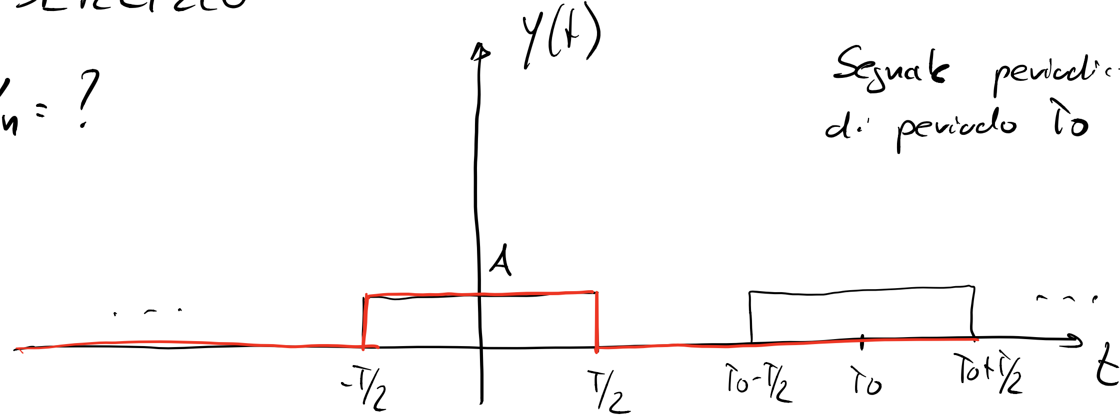
$$= \frac{1}{T_0}\int_{-\infty}^{+\infty} x(t')\, e^{-j2\pi n f_0 t'}\, dt' = \frac{1}{T_0} X(f)\Big|_{f=nf_0}$$



$$= \frac{1}{T_0} X(nf_0) = \frac{1}{T_0} X\left(\frac{n}{T_0}\right) = Y_n$$

ESERCIZIO

$Y_n = ?$



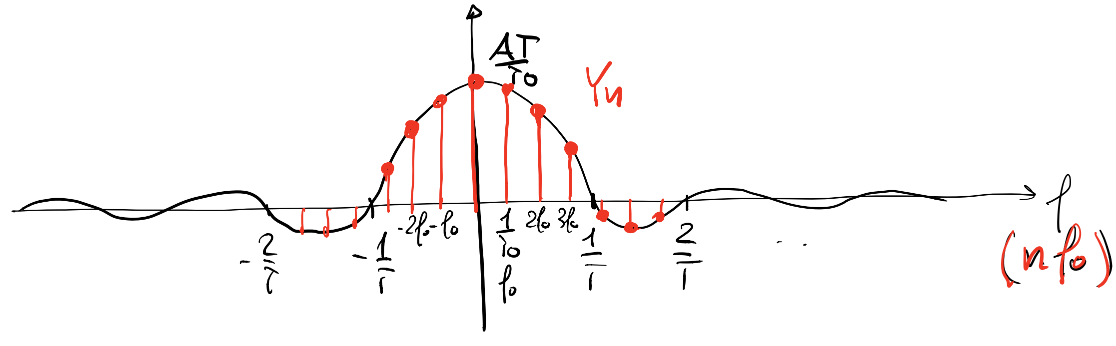
Segnale periodico di periodo To

I) $Y_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} y(t)\, e^{-j2\pi n f_0 t}\, dt$

II) $\begin{cases} y(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0) \\ x(t) = A\, \mathrm{rect}\left(\frac{t}{T}\right) \end{cases}$

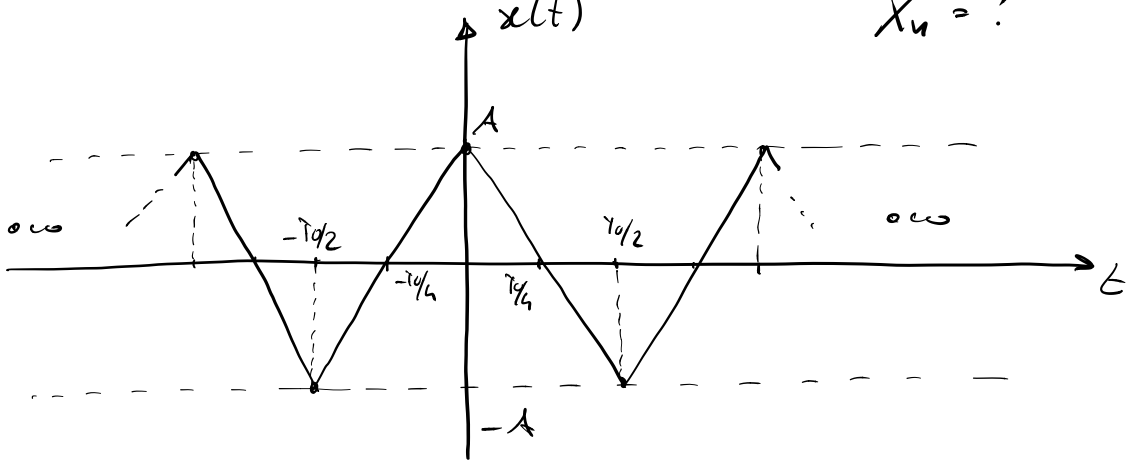
$$Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right) = \boxed{\frac{AT}{T_0} \mathrm{sinc}\left(\frac{T}{T_0} n\right)}$$

$$X(f) = TCF\left[x(t)\right] = AT\, \mathrm{sinc}(Tf)$$

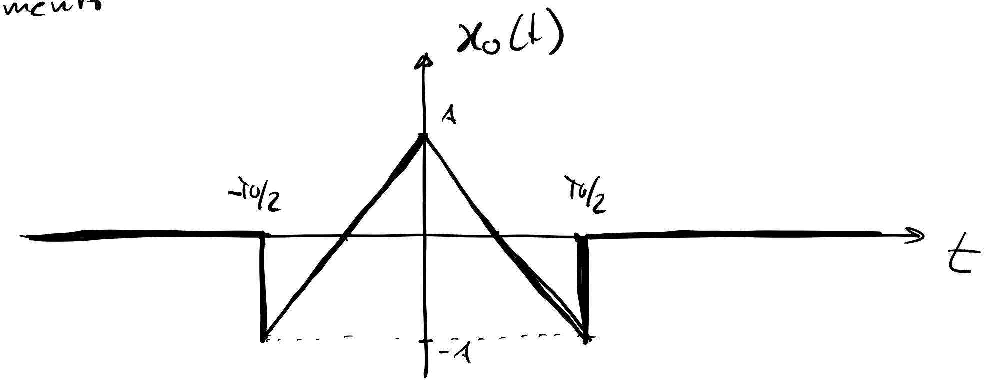


) ESERCIZIO - ONDA TRIANGOLARE

$X_n = ?$



Svolgimento

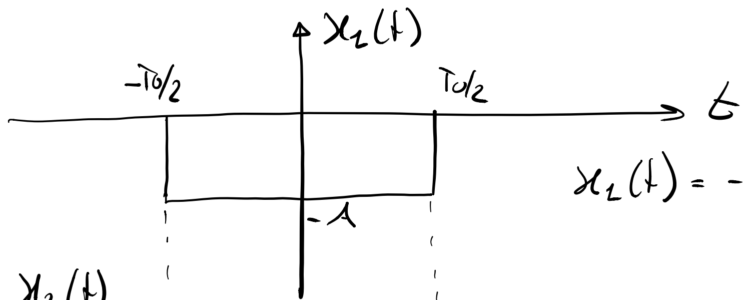


$$x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - nT_0)$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right) \quad , \quad X_0(f) = TCF\left[x_0(t)\right]$$

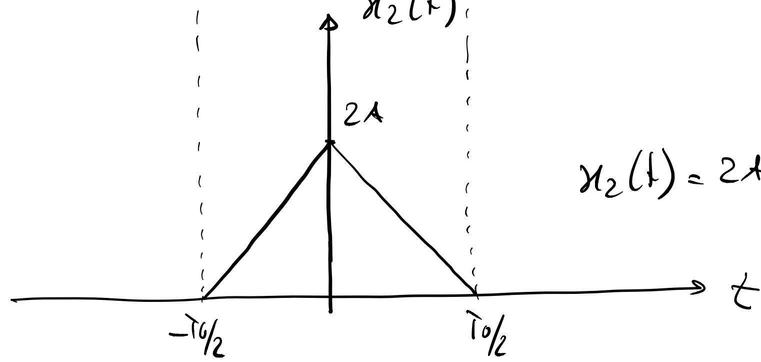
$X_0(f) = ?$ → teo dell' integraz.

→ combinazione di segnali noti



$$x_1(t) = -A\,\mathrm{rect}\left(\frac{t}{T_0}\right)$$

$$x_0(t) = x_1(t) + x_2(t)$$



$$x_2(t) = 2A\left(1 - \frac{|t|}{T_0/2}\right)\mathrm{rect}\left(\frac{t}{T_0}\right)$$

$$X_0(f) = X_1(f) + X_2(f) = A T_0 \left[\mathrm{sinc}^2\left(\frac{T_0}{2} f\right) - \mathrm{sinc}(T_0 f)\right]$$

$$X_1(f) = -A T_0\, \mathrm{sinc}(T_0 f)$$

$$X_2(f) = \cancel{2} A \frac{T_0}{\cancel{2}}\, \mathrm{sinc}^2\left(\frac{T_0}{2} f\right)$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right) = \frac{A \cancel{T_0}}{\cancel{T_0}}\left[\mathrm{sinc}^2\left(\frac{T_0}{2}\frac{n}{T_0}\right) - \mathrm{sinc}\left(T_0 \frac{n}{T_0}\right)\right]$$

$$= A\left[\mathrm{sinc}^2\left(\frac{n}{2}\right) - \mathrm{sinc}(n)\right]$$

·) FORMULE DI POISSON

I) $$\boxed{\sum_{n=-\infty}^{+\infty} x(t - nT_0) = \frac{1}{T_0}\sum_{n=-\infty}^{+\infty} X\left(\frac{n}{T_0}\right) e^{j2\pi n \frac{t}{T_0}}}$$

Dim.

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

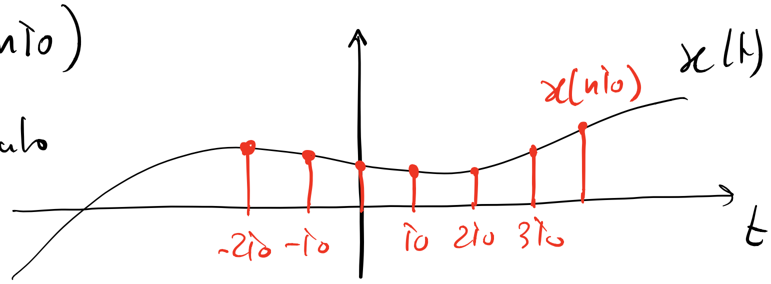
$$\underbrace{Y_n}= \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$$

$$y(t) = \sum_{n=-\infty}^{+\infty} \underbrace{Y_n} e^{j2\pi n f_0 t}$$ (anti-trasf. serie di F.)

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0) = \sum_{n=-\infty}^{+\infty} \frac{1}{T_0} X\left(\frac{n}{T_0}\right) e^{j2\pi \frac{nt}{T_0}}$$

II) $$\boxed{\sum_{n=-\infty}^{+\infty} x(nT_0)\, e^{-j2\pi f n T_0} = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} X\left(f - \frac{n}{T_0}\right)}$$

$x(nT_0)$
segnale campionato



-) APPLICAZIONE DELLE FORMULE DI POISSON ALLA $\delta(t)$

I) $$\sum_{n=-\infty}^{+\infty} x(t - nT_0) = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} X\left(\frac{n}{T_0}\right) e^{j2\pi \frac{nt}{T_0}}$$

$$x(t) = \delta(t)$$

$$X(f) = 1$$

$$\Downarrow$$

$$\boxed{\sum_{n=-\infty}^{+\infty} \delta(t - nT_0) = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} e^{j 2\pi n \frac{t}{T_0}}}$$

II) $$\sum_{n=-\infty}^{+\infty} x(nT_0)\, e^{-j 2\pi f n T_0} = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} X\left(f - \frac{n}{T_0}\right)$$

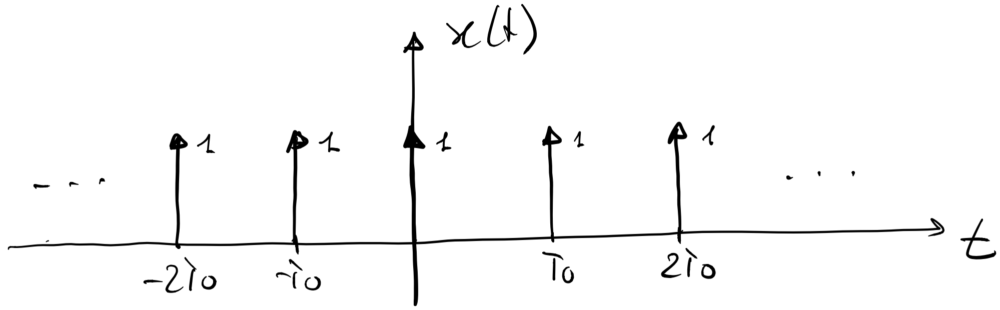
$$X(f) = \delta(f)$$

$$x(t) = 1 \quad \forall t$$

$$\Downarrow$$

$$\boxed{\sum_{n=-\infty}^{+\infty} e^{-j 2\pi f n T_0} = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_0}\right)}$$

·) TRENO DI DELTA DI DIRAC



$$x(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_0)$$

$$X(f) = TCF[x(t)] = TCF\left[\sum_{n=-\infty}^{+\infty} \delta(t-nT_0)\right]$$

$$= \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta(t-nT_0)\, e^{-j2\pi f t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta(t-nT_0)\, \underbrace{e^{-j2\pi f t}}\, dt =$$

$$= \sum_{n=-\infty}^{+\infty} e^{-j2\pi f n T_0} = TCF\left[\sum_{n=-\infty}^{+\infty} \delta(t-nT_0)\right]$$

$$= \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_0}\right)$$ x la II^a form. di Poisson applicata alla $\delta(f)$

$$\sum_{n=-\infty}^{+\infty} \delta(t-nT_0) \overset{TCF}{\Longleftrightarrow} \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_0}\right)$$

-) TCF DI SEGNALI PERIODICIZZATI

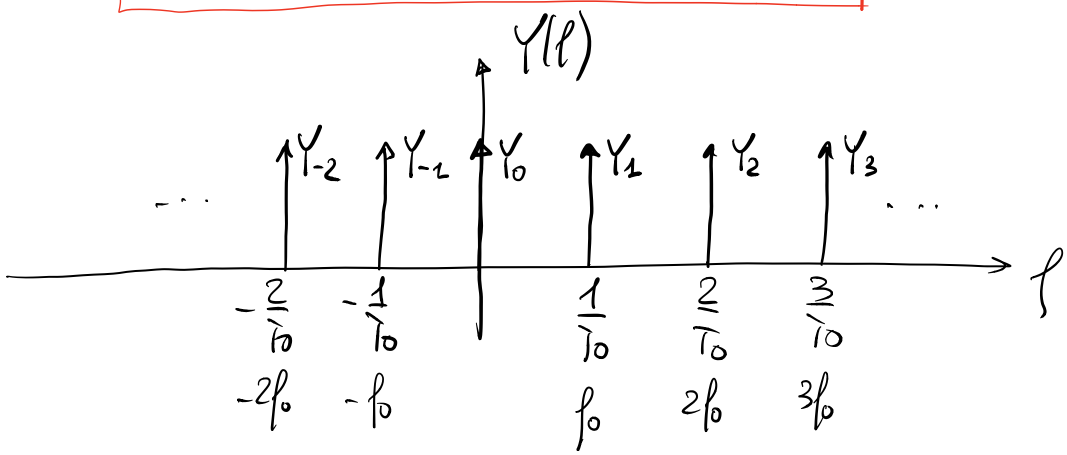
$$\begin{cases} y(t) = \sum_{n=-\infty}^{+\infty} x(t-nT_0), & T_0 \in \mathbb{R}^+ \\ x(t) \overset{TCF}{\Longleftrightarrow} X(f) & \text{aperiodico} \end{cases}$$

$$\boxed{Y(f) = TCF[y(t)] = \frac{1}{T_0}\sum_{n=-\infty}^{+\infty} X\left(\frac{n}{T_0}\right)\delta\left(f-\frac{n}{T_0}\right)}$$

$$= \sum_{n=-\infty}^{+\infty} Y_n \,\delta\left(f-\frac{n}{T_0}\right)$$

$$\boxed{Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)}$$

$$\boxed{Y(f) = \sum_{n=-\infty}^{+\infty} Y_n \,\delta\left(f-\frac{n}{T_0}\right)}$$



esistono solo le freq. armoniche

Dim

$$Y(f) = \int_{-\infty}^{+\infty} y(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(t-nT_0)\, e^{-j2\pi f t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(t-nT_0)\, e^{-j2\pi f t}\, dt \qquad t - nT_0 = t'$$

$$= \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(t') \, e^{-j2\pi f (t' + nT_0)} \, dt'$$

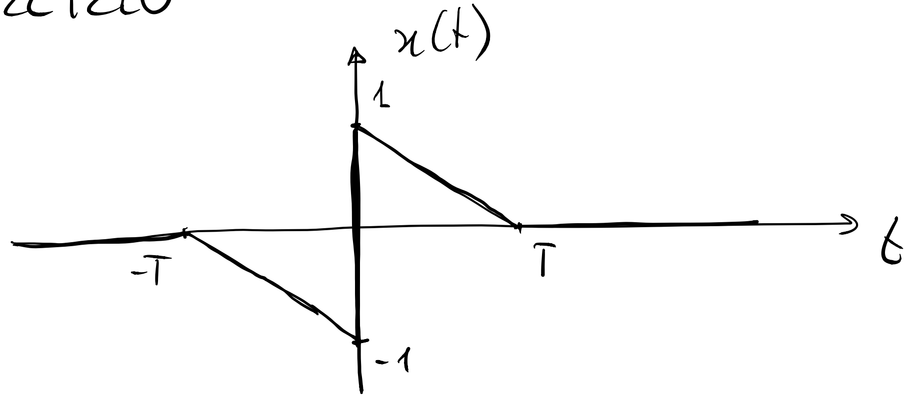
$$= \sum_{n=-\infty}^{+\infty} \underbrace{\int_{-\infty}^{+\infty} x(t') \, e^{-j2\pi f t'} \, dt'}_{X(f)} \; e^{-j2\pi f n T_0}$$

$$= X(f) \sum_{n=-\infty}^{+\infty} e^{-j2\pi f n T_0} = \frac{X(f)}{T_0} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_0}\right)$$

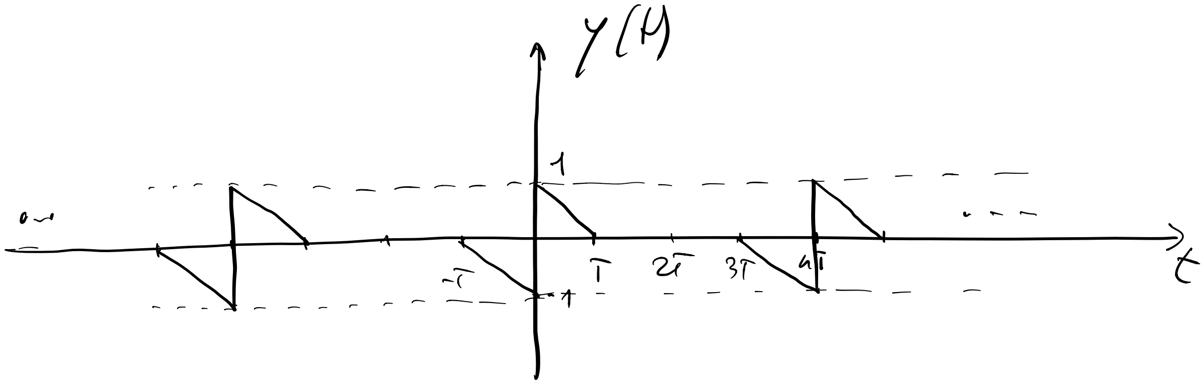
$$= \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} \underbrace{X(f) \, \delta\left(f - \frac{n}{T_0}\right)}_{X\left(\frac{n}{T_0}\right) \delta\left(f - \frac{n}{T_0}\right)}$$

$$= \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} X\left(\frac{n}{T_0}\right) \delta\left(f - \frac{n}{T_0}\right) = \sum_{n=-\infty}^{+\infty} Y_n \, \delta\left(f - \frac{n}{T_0}\right)$$

$\Rightarrow$ ESERCIZIO



$$y(t) = \sum_{n=-\infty}^{+\infty} x(t - 4nT) = \sum_{n=-\infty}^{+\infty} x(t - nT_0) \qquad T_0 \triangleq 4T$$



$$Y(f) = ?$$

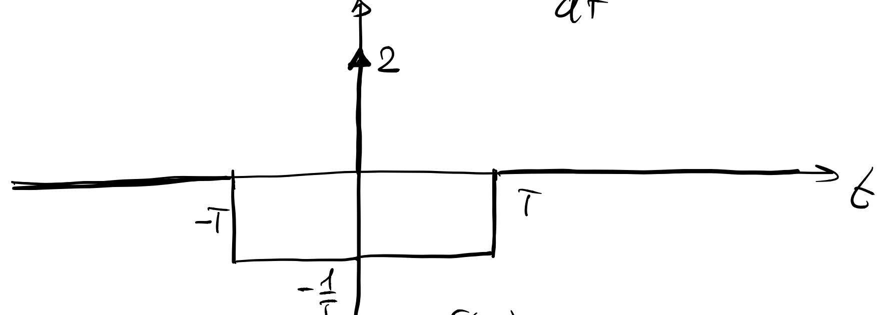
Svolgimento

$$Y(f) = \sum_{n=-\infty}^{+\infty} Y_n \, \delta\left(f - \frac{n}{T_0}\right) \quad , \quad T_0 = 4T$$

$$Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$$

$$X(f) = TCF\left[x(t)\right]$$

$$z(t) = \frac{d}{dt} x(t)$$



$$z(t) = -\frac{1}{T} \operatorname{rect}\left(\frac{t}{2T}\right) + 2\,\delta(t)$$

$$Z(f) = -\frac{1}{T} 2T \operatorname{sinc}(2Tf) + 2$$

$$= 2\left(1 - \operatorname{sinc}(2Tf)\right)$$

$$X(f) = \frac{Z(0)}{2}\delta(f) + \frac{Z(f)}{j2\pi f}$$

teo dell' int completo

$$= 0 + \frac{1}{j\pi f}\left(1 - \operatorname{sinc}(2Tf)\right)$$

$$= \frac{1 - \operatorname{sinc}(2Tf)}{j\pi f}$$

$$Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right) = \frac{1}{j\pi \cancel{T_0} \frac{n}{\cancel{T_0}}}\left(1 - \operatorname{sinc}\left(2T\frac{n}{T_0}\right)\right)$$

$$T_0 = 4T$$

$$= \frac{1}{j\pi n}\left(1 - \operatorname{sinc}\left(\frac{n}{2}\right)\right)$$

$$\boxed{Y(f) = \frac{1}{j\pi}\sum_{n=-\infty}^{+\infty} \frac{1 - \operatorname{sinc}(n/2)}{n}\,\delta\left(f - \frac{n}{4T}\right)}$$

- ESERCIZIO

$$s(t) = \text{sinc}\left(2Bt - \frac{1}{2}\right) + \text{sinc}\left(2Bt + \frac{1}{2}\right)$$

- $x(t) = s(t)\sin(2\pi Bt)$

- $y(t) = \sum_{n=-\infty}^{+\infty} s\left(t - \frac{2}{B}n\right)$

- E_x , P_x , E_y , P_y

Soluzione

$\underline{\underline{x(t)}}$

$$x(t) = s(t)\sin(2\pi Bt)$$

$$E_x = \int_{-\infty}^{+\infty} s^2(t)\sin^2(2\pi Bt)\,dt$$

$\Downarrow$

TEO DI PARSEVAL

$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2\,df$$

$$X(f) = \frac{1}{2j}S(f-B) - \frac{1}{2j}S(f+B)$$ teo. della mod. con seno

$$S(f) = \mathrm{TCF}[s(t)]$$

$$s(t) = \mathrm{sinc}\left[2B\left(t - \frac{1}{4B}\right)\right] + \mathrm{sinc}\left[2B\left(t + \frac{1}{4B}\right)\right]$$

$$s(t) = s_0\left(t - \frac{1}{4B}\right) + s_0\left(t + \frac{1}{4B}\right)$$

$$= s_0(t - t_0) + s_0(t + t_0)$$

$$t_0 = \frac{1}{4B}$$

$$S(f) = S_0(f)e^{-j2\pi f t_0} + S_0(f)e^{j2\pi f t_0}$$

$$= 2S_0(f)\cos(2\pi f t_0)$$

$$= 2S_0(f)\cos\left(\frac{\pi f}{2B}\right)$$

$$s_0(t) = \mathrm{sinc}(2Bt)$$

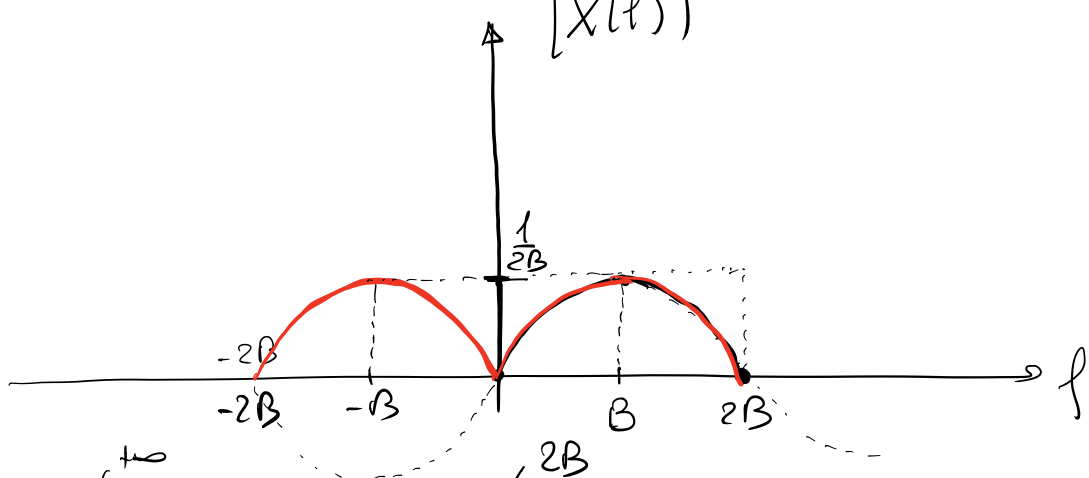
$$S_0(f) = \frac{1}{2B}\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$S(f) = \frac{1}{B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\cos\left(\frac{\pi f}{2B}\right)$$

$$X(f) = \frac{1}{2j}\,S(f-B) - \frac{1}{2j}\,S(f+B)$$

$$= \frac{1}{j2B}\,\mathrm{rect}\left(\frac{f-B}{2B}\right)\cos\left[\frac{\pi(f-B)}{2B}\right] +$$

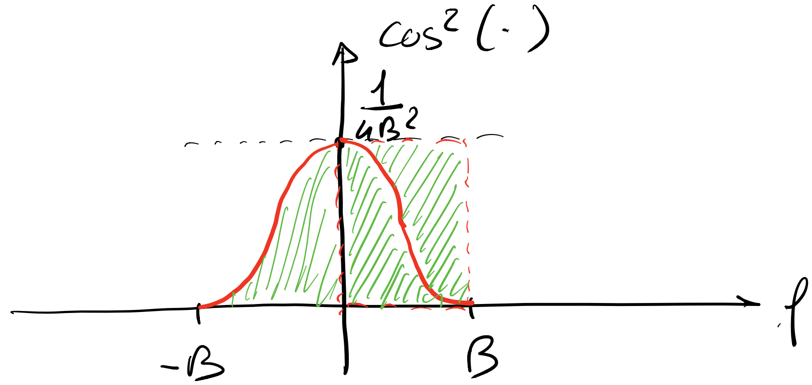
$$- \frac{1}{j2B}\,\mathrm{rect}\left(\frac{f+B}{2B}\right)\cos\left[\frac{\pi(f+B)}{2B}\right]$$



$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2\,df = \frac{2}{4B^2}\int_0^{2B}\cos^2\left(\frac{\pi(f-B)}{2B}\right)df$$

$f-B=f'$

$$= \frac{2}{4B^2}\int_{-B}^{B}\cos^2\left(\frac{\pi f'}{2B}\right)df'$$



$$E_x = \frac{2}{4B^2}\int_{-B}^{B} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi f'}{2B}\right) df'$$

$$= \frac{2B}{4B^2} + \underbrace{\int_{-B}^{B} \cos\left(\frac{2\pi f'}{2B}\right) df'}_{=0} = \frac{2B}{4B^2} = \boxed{\frac{1}{2B}}$$

$$P_x = ?$$

$$E_x = \frac{1}{2B} < \infty \quad \Rightarrow \quad P_x = 0$$