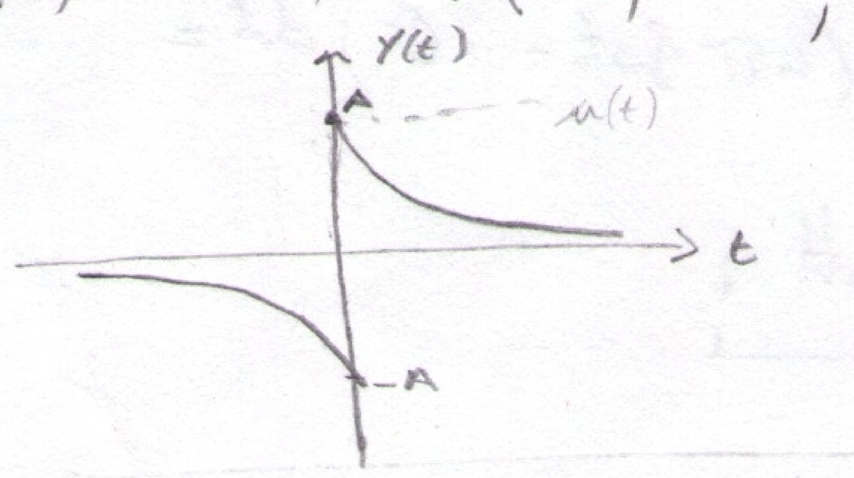
ESERCIZI MARTORELLA

Esercizio 1

$y(t) = x(t) - x(-t)$; $x(t) = Ae^{-t}u(t)$ $A \in \mathbb{R}$



$$p_y(t) = |y(t)|^2 = |Ae^{-t}u(t) - Ae^{t}u(-t)|^2 =$$

$$= A^2e^{-2t}u(t) + A^2e^{2t}u(-t) - 2A^2e^{-t}\cdot e^{t}\underbrace{u(t)u(-t)}_{\emptyset} =$$

$$= A^2e^{-2t}u(t) + A^2e^{2t}u(-t) = A^2\left[e^{-2t}u(t) + e^{2t}u(-t)\right]$$

$$E_y = \int_{-\infty}^{\infty}|y(t)|^2\,dt = \int_{-\infty}^{\infty}A^2\left[e^{-2t}u(t) + e^{2t}u(-t)\right]dt = A^2\int_0^{\infty}e^{-2t}\,dt + A^2\int_{-\infty}^{\emptyset}e^{2t}\,dt =$$

$$= A^2\left[-\frac{1}{2}e^{-2t}\right]_{\emptyset}^{\infty} + A^2\left[\frac{1}{2}e^{2t}\right]_{-\infty}^{\emptyset} = -\frac{A^2}{2}(\emptyset - 1) + \frac{A^2}{2}(1-\emptyset) = A^2$$

$$P_y = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|y(t)|^2\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}A^2\left[e^{-2t}u(t) + e^{2t}u(-t)\right]dt =$$

$$= \lim_{T\to\infty}\frac{A^2}{T}\int_{\emptyset}^{\frac{T}{2}}e^{-2t}\,dt + \lim_{T\to\infty}\frac{A^2}{T}\int_{-\frac{T}{2}}^{\emptyset}e^{2t}\,dt = \lim_{T\to\infty}\frac{A^2}{T}\left[-\frac{1}{2}e^{-2t}\right]_0^{\frac{T}{2}} + \lim_{T\to\infty}\frac{A^2}{T}\left[\frac{1}{2}e^{2t}\right]_{-\frac{T}{2}}^{\emptyset} =$$

$$= \lim_{T\to\infty}-\frac{A^2}{2T}\left(\underbrace{e^{-T}}_{\to\emptyset} - 1\right) + \lim_{T\to\infty}\frac{A^2}{2T}\left(1 - \underbrace{e^{-T}}_{\emptyset}\right) = 2\lim_{T\to\infty}\frac{A^2}{2T} = \emptyset$$

$$y_{eff} = \sqrt{P_y} = \sqrt{\emptyset} = \emptyset$$

$$x_m = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left[Ae^{-t}u(t) - Ae^{t}u(t)\right]dt = \emptyset$$

Esercizio 2_1

$$x(t) = A\cos(2\pi f_0 t + \varphi) + B\sin(4\pi f_0 t) \qquad ; \quad \frac{1}{T} = f_0$$

$$x(t) \rightleftharpoons X_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-j2\pi f_0 n t}\, dt$$

$$X_n = \frac{A}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\pi f_0 t + \varphi)\, e^{-j2\pi f_0 n t}\, dt + \frac{B}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin(2\pi 2 f_0 t)\, e^{-j2\pi f_0 n t}\, dt =$$

$$= \frac{A}{2T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(e^{+j2\pi f_0 t + j\varphi} + e^{-j2\pi f_0 t - j\varphi}\right) e^{-j2\pi f_0 n t}\, dt + \frac{B}{T2j}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(e^{j2\pi 2 f_0 t} - e^{-j2\pi f_0 2 t}\right) e^{-j2\pi f_0 t n}\, dt =$$

$$= \frac{A e^{\pm j\varphi}}{2T} \underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j2\pi f_0 (1-n)t} + e^{-j2\pi f_0 (1+n)t}\, dt}_{} + \frac{B}{2Tj}\underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j2\pi f_0 (2-n)t} - e^{-j2\pi f_0 (2+n)t}\, dt}_{} =$$

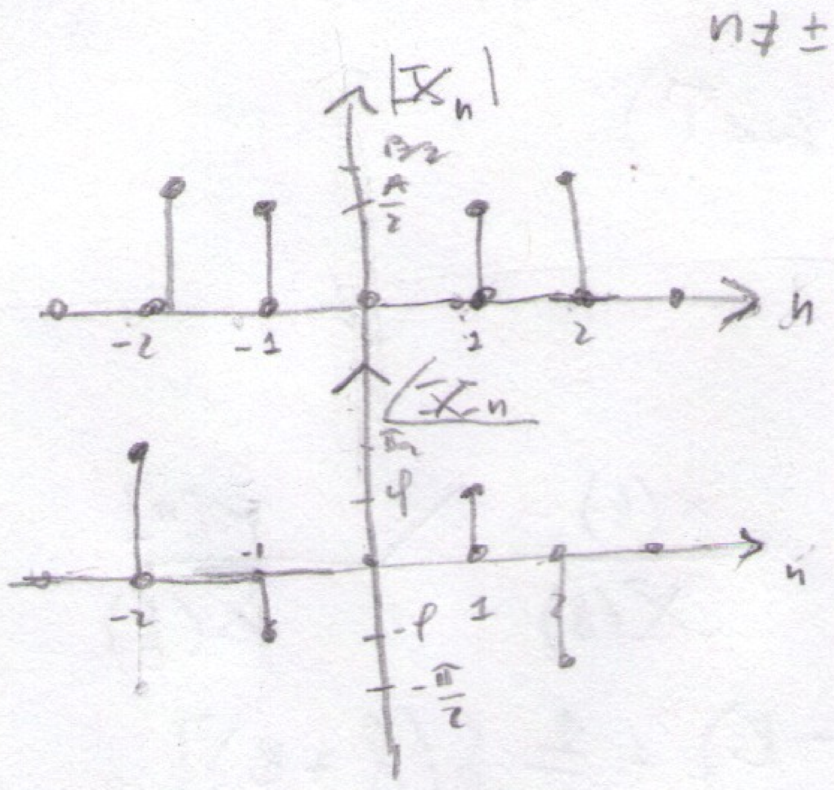
$n = \varnothing \Rightarrow \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\pi f_0 t) \cdot 2 = \varnothing$ $\qquad$ $n = \varnothing \Rightarrow \varnothing$

$n = \pm 1 \Rightarrow T$ $\qquad$ $n = \pm 2 \Rightarrow T$

$n \neq \pm 1 \Rightarrow \varnothing$ $\qquad$ $n \neq \pm 2 \Rightarrow \varnothing$

$$X_{\pm 1} = \frac{A e^{\pm j\varphi}}{2} \quad ; \quad X_{\pm 2} = \mp\frac{B}{2j}$$

$$\frac{1}{j} = -j = e^{-j\frac{\pi}{2}}$$



Esercizio 2_2

$$x(t) = A\cos(2\pi f_0 t + \varphi)\cos(4\pi f_0 t)$$

$$= \frac{A}{2}\left[\cos(6\pi f_0 t + \varphi) + \cos(-2\pi f_0 t - \varphi)\right]$$

$$X_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left\{\frac{A}{2}\left[\cos(6\pi f_0 t + \varphi) + \cos(2\pi f_0 t - \varphi)\right]\right\} e^{-j2\pi f_0 n t}\, dt =$$

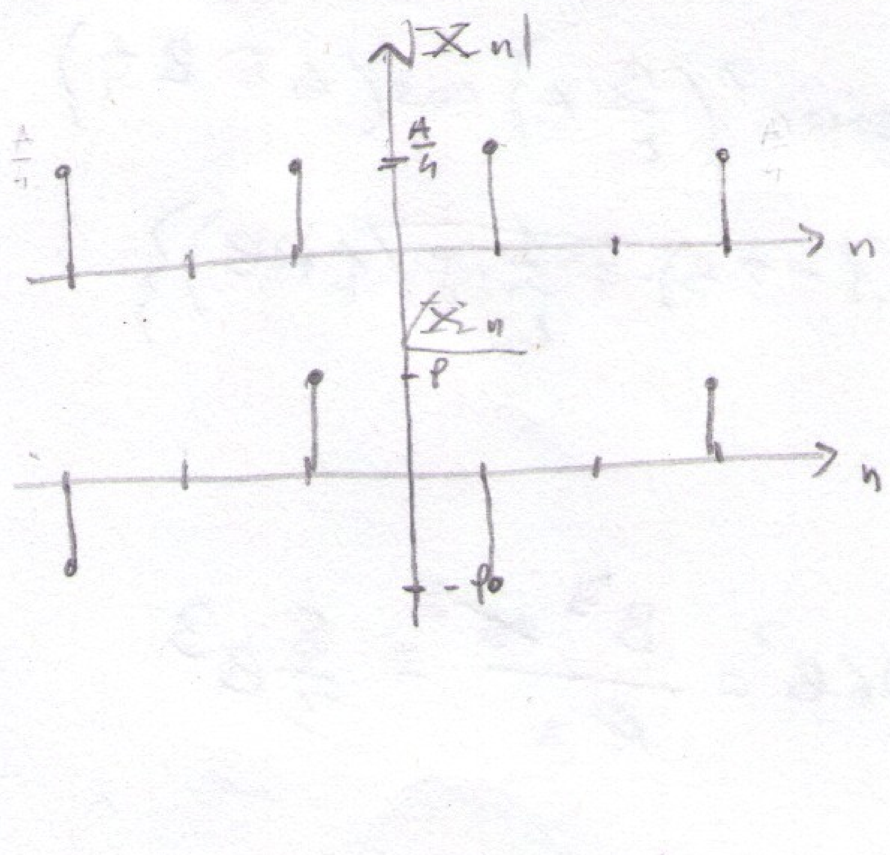
$$= \frac{A}{4T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left[e^{j2\pi 3 f_0 t + j\varphi} + e^{-j2\pi 3 f_0 t - j\varphi}\right] e^{-j2\pi f_0 n t}\, dt + \frac{A}{4T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left[e^{j2\pi f_0 t - j\varphi} + e^{-j2\pi f_0 t + j\varphi}\right] e^{-j2\pi f_0 n t}\, dt =$$

$$= \frac{A}{4T}\underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} \left[e^{j2\pi f_0 (3-n)t + j\varphi} + e^{-j2\pi f_0 (3+n)t - j\varphi}\right] dt}_{} + \frac{A}{4T}\underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} \left[e^{j2\pi f_0 (1-n)t - j\varphi} + e^{-j2\pi f_0 (1+n)t + j\varphi}\right] dt}_{} =$$

$n = 3 \Rightarrow T e^{j\varphi}$ $\qquad$ $n = 1 \Rightarrow T e^{-j\varphi}$

$n = -3 \Rightarrow T e^{-j\varphi}$ $\qquad$ $n = +1 \Rightarrow T e^{j\varphi}$

$n \neq \pm 3 = \varnothing$ $\qquad$ $n \neq \pm 1 = \varnothing$



Esercizio 2-3

$x_0(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$; $x(t) = \sum_n x_0(t - nT_0)$

$$X_k = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-j2\pi f_0 k t}\, dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_n A\,\mathrm{rect}\left(\frac{t-nT_0}{T}\right) e^{-j2\pi f_0 t k}\, dt$$

$$= \frac{1}{T_0}\sum_n A\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \mathrm{rect}\left(\frac{t-nT_0}{T}\right) e^{-j2\pi f_0 k t}\, dt = \frac{1}{T_0} A \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \mathrm{rect}\left(\frac{t}{T}\right) e^{-j2\pi f_0 k t}\, dt =$$

$$= \frac{1}{T_0} A \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j2\pi f_0 k t}\, dt = \frac{1}{T_0}\frac{A}{-j2\pi f_0 k}\left[e^{-j2\pi f_0 k t}\right]_{-\frac{T}{2}}^{\frac{T}{2}} =$$

$$= \frac{1}{T_0} A \frac{2j}{+j2\pi f_0 k}\left(\frac{-e^{-j2\pi f_0 k \frac{T}{2}} + e^{+j2\pi f_0 k \frac{T}{2}}}{2j}\right) = \frac{1}{T_0} A \frac{1}{\cancel{\pi f_0 k}} \frac{\sin\left(2\pi f_0 k \frac{T}{2}\right)}{2\pi f_0 k \frac{T}{2}} \cdot \cancel{2\pi f_0 k}\frac{T}{2}$$

$$= \frac{A}{T_0}\,\mathrm{sinc}(f_0 k T)\cdot T = \frac{AT}{T_0}\,\mathrm{sinc}(f_0 k T)$$

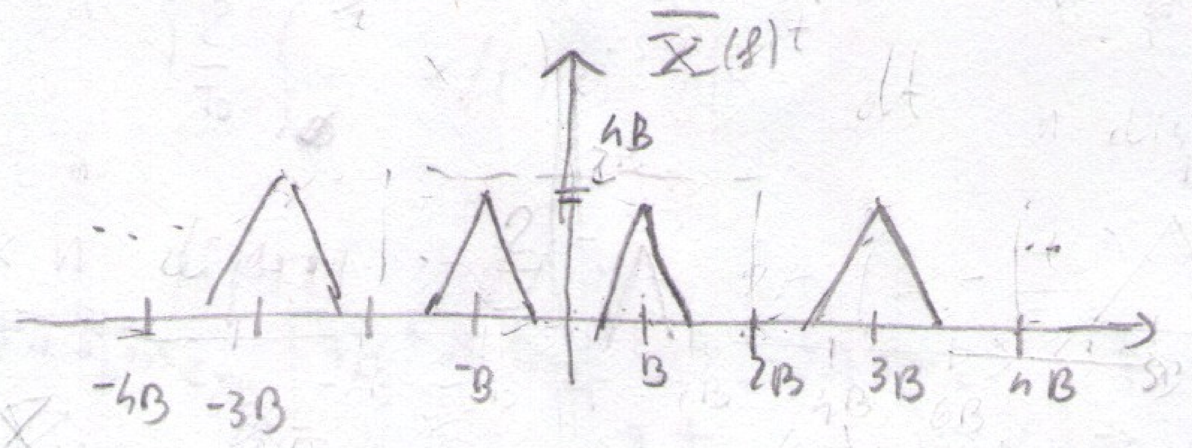
Testo 2

$x(t) = B\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(2\pi B t)$ $\qquad \frac{x(t)}{X(f)} \qquad \frac{x[n]}{\bar{X}(f)}$

$$X(f) = 2\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) \otimes \left[\frac{1}{2}\delta(f-B) + \frac{1}{2}\delta(f+B)\right] =$$

$$= \left(1 - \frac{|f-B|}{B/2}\right)\mathrm{rect}\left(\frac{f-B}{B}\right) + \left(1 - \frac{|f+B|}{B/2}\right)\mathrm{rect}\left(\frac{f+B}{B}\right)$$

$$\bar{X}(f) = \frac{1}{T}\sum_n X\left(f - \frac{n}{T}\right) = 2B\sum_n X(f - 2Bn)$$



$$\bar{X}(f) = 4B\sum_n \left(1 - \frac{|f - 2Bn - B|}{B/2}\right)\mathrm{rect}\left(\frac{f - 2Bn - B}{B}\right)$$

$y(t) = x[n] \otimes p(t) \rightleftharpoons Y(f) = \bar{X}(f)\,P(f)$; $P(f) = \mathrm{rect}\left(\frac{f}{4B}\right)$

$$Y(f) = 4B\left(1 - \frac{|f-B|}{B/2}\right)\mathrm{rect}\left(\frac{f-B}{B}\right) + 4B\left(1 - \frac{|f+B|}{B/2}\right)\mathrm{rect}\left(\frac{f+B}{B}\right) =$$

$$= 8B\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) \otimes \left[\frac{1}{2}\delta(f-B) + \frac{1}{2}\delta(f+B)\right] =$$

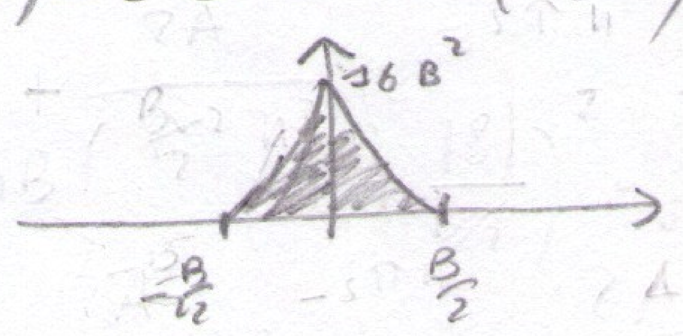
$$y(t) = 4B^2\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(2\pi B t)$$

$$w(t) = 4B^2\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos^2(2\pi B t) = 2B^2\,\mathrm{sinc}^2\left(\frac{B}{2}t\right) + 2B^2\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(4\pi B t)$$

$$W(f) = 4B\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) + 4B\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) \otimes \left\{\frac{1}{2}\delta(f-2B) + \frac{1}{2}\delta(f+2B)\right\}$$

$$Z(f) = 4B\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) \rightleftharpoons z(t) = 2B^2\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)$$

$E_z = \int_{-\infty}^{\infty} |z(t)|^2\, dt = \int_{-\infty}^{\infty} |Z(f)|^2\, df =$



$= \frac{1}{3}\cdot\frac{B}{2}\cdot 16B^2 = \frac{B^3\,\cancel{16}\,8}{\cancel{6}\,3} = \frac{8}{3}B^3$