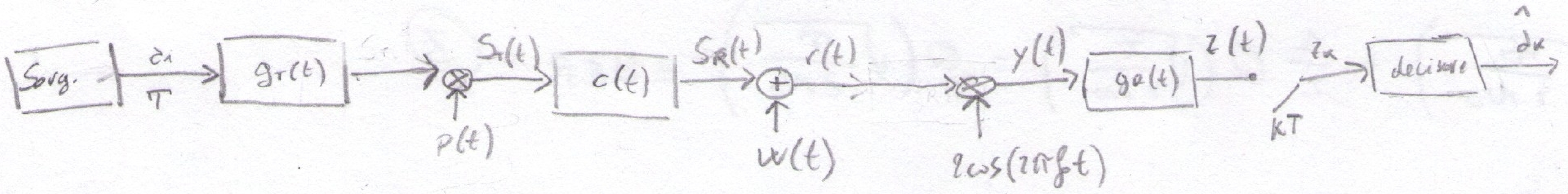
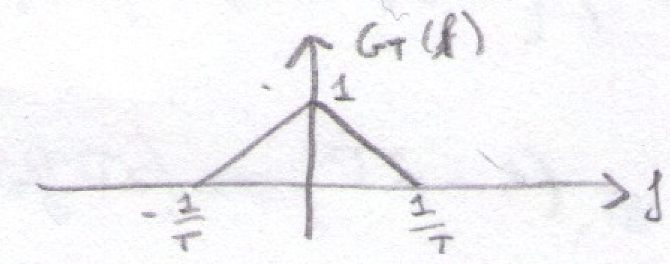
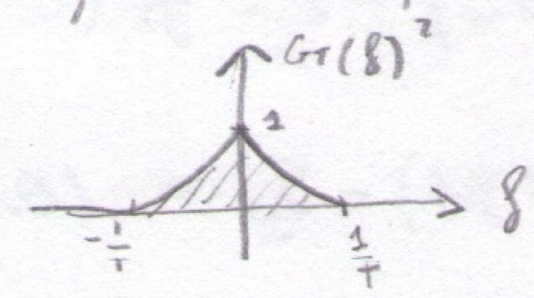
• Esame 16/06/09



$$r(t) = \sum_i a_i\, g_T(t - iT)\, p(t) + w(t) \quad ; \quad p(t) = \sum_n \left[2\, \mathrm{rect}\left(\frac{t - nT_0}{T_0/2}\right) - 1 \right]$$

$$\bar{E}_T = P_{s_T} \cdot T \quad ; \quad P_{s_T} = \int_{-\infty}^{\infty} S_{s_T}(f)\, df \quad ; \quad S_{s_T}(f) = \frac{E\{a_i^2\}}{T} |G_T(f)|^2$$

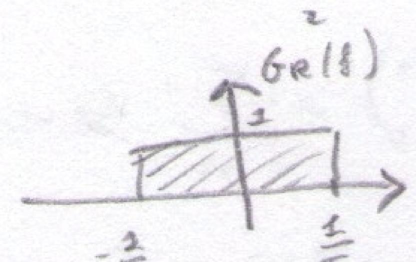
$$E\{a_i^2\} = \sum_{k=0}^{1} m_k^2 p_k = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1 \quad ; \quad S_{s_T}(f) = \frac{1}{T} |G_T(f)|^2$$

$$P_{s_T} = \frac{1}{T} \int_{-\infty}^{\infty} |G_T(f)|^2\, df =$$   $$= \frac{1}{T} \cdot \frac{1}{3} \cdot 2 \cdot \frac{1}{T} = \frac{2}{3T^2}$$

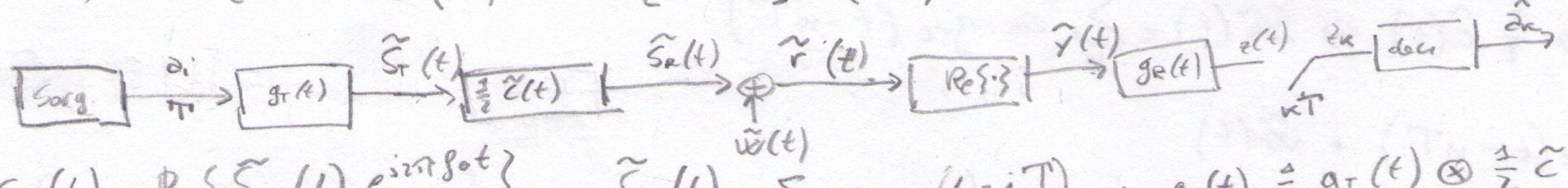
$$\bar{E}_T = \frac{2}{3T^2} \cdot T = \frac{2}{3T} \quad (1)$$

$$S_{\tilde{w}}(f) = \begin{cases} 4 S_w(f + f_0) & f_0 \geq -f \\ \emptyset & \text{altrove} \end{cases} = 4 \frac{N_0}{2} \quad ; \quad S_{\tilde{w}_c}(f) = \frac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4} = N_0$$

$$n_c(t) = \tilde{w}_c(t) \otimes g_R(t)$$

$$P_{n_c} = \int_{-\infty}^{\infty} S_{\tilde{w}_c}(f) |G_R(f)|^2\, df = N_0 \int_{-\infty}^{\infty} |G_R(f)|^2\, df$$  $$= \frac{2 N_0}{T} \quad (2)$$

$$P(e) = P_r[-1 | 1]\, P(1) + P_r[1 | -1]\, P(-1)$$



$$s_T(t) = \mathrm{Re}\{\tilde{s}_T(t)\, e^{j2\pi f_0 t}\} \quad ; \quad \tilde{s}_T(t) = \sum_i a_i\, g_T(t - iT) \quad ; \quad g_{Tc}(t) \triangleq g_T(t) \otimes \frac{1}{2}\tilde{c}(t)$$

$$\tilde{r}(t) = \sum_i a_i\, g_{Tc}(t - iT) + \tilde{w}(t), \quad \tilde{y}(t) = \sum_i a_i\, g_{Tc}(t - iT) + \tilde{w}_c(t)$$

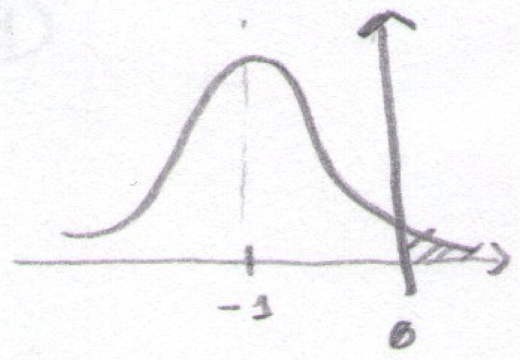
$$z(t) = \tilde{y}(t) \otimes g_R(t) \quad ; \quad g(t) \triangleq g_{Tc}(t) \otimes g_R(t) \quad ; \quad z(t) = \sum_i a_i\, g(t - iT) + n_c(t)$$

$$G(f) = G_{Tc}(f)\, G_R(f) = G_{Tc}(f) = G_T(f)$$

$$T \sum_n G_T\left(f - \frac{n}{T}\right) = T \quad \text{Nyquist OK}$$

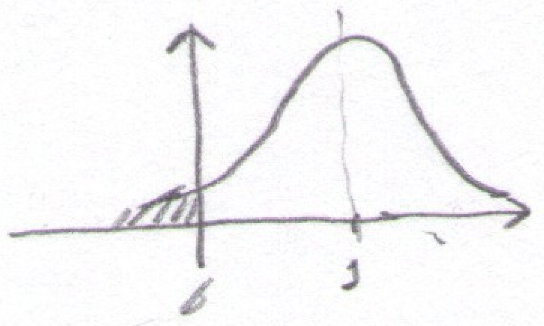
$$z_k = z(kT) = \sum_i a_i\, g[(k - i)T] + n_k = a_k + g(\emptyset) + n_k = a_k + n_k$$

$$z_k \big|_{a_k = -1} = -1 + n_k \in \mathcal{N}(-1, s_n^2)$$



$$P_r\left[\hat{1} \mid -1\right] = Q\left(\frac{0+1}{\delta_{n_c}}\right) = Q\left(\frac{1}{\delta_{n_c}}\right)$$

$$z_k\big|_1 = 1 + n_k \in \mathcal{N}(1, \delta_{n_c}^2)$$



$$P_r\left[-\hat{1} \mid 1\right] = 1 - Q\left(\frac{0-1}{\delta_{n_c}}\right) = 1 - Q\left(-\frac{1}{\delta_{n_c}}\right) = Q\left(\frac{1}{\delta_{n_c}}\right)$$

$$\delta_{n_c}^2 = P_{n_c} = \frac{2N_0}{T} \Rightarrow \delta_{n_c} = \sqrt{\frac{2N_0}{T}}$$

$$P_r(e) = \frac{1}{2} Q\left(\sqrt{\frac{T}{2N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{T}{2N_0}}\right) = Q\left(\sqrt{\frac{T}{2N_0}}\right) = BER \quad 3$$

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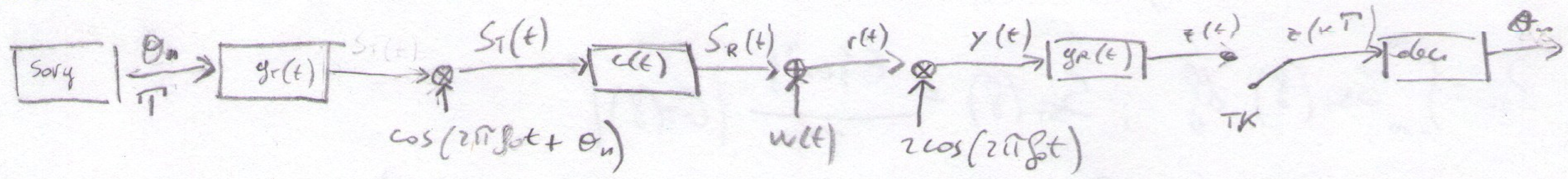
Esercizio 2



$$S_T(t) = \sum_n g_T(t - nT)\cos(2\pi f_0 t + \theta_n) = \sum_n g_T(t - nT)\left[\cos(2\pi f_0 t)\cos(\theta_n) - \operatorname{sen}(2\pi f_0 t)\operatorname{sen}(\theta_n)\right] =$$

$$= \sum_n \cos(\theta_n) g_T(t - nT)\cos(2\pi f_0 t) - \sum_n \operatorname{sen}(\theta_n) g_T(t - nT)\operatorname{sen}(2\pi f_0 t) =$$

$$= \sum_n a_i\, g_T(t - nT)\cos(2\pi f_0 t) - \sum_n b_i\, g_T(t - nT)\operatorname{sen}(2\pi f_0 t) =$$

$$S_T(t) = \operatorname{Re}\left[\tilde{S}_T(t)\, e^{j2\pi f_0 t}\right] =$$

$$= \operatorname{Re}\left\{\left[\sum_n \cos(\theta_n) g_T(t - nT) + j\sum_n \operatorname{sen}(\theta_n) g_T(t - nT)\right]\left[\cos(2\pi f_0 t) + j\operatorname{sen}(2\pi f_0 t)\right]\right\}$$

$$\tilde{S}_T(t) = \sum_n \cos(\theta_n) g_T(t - nT) + j\sum_n \operatorname{sen}(\theta_n) g_T(t - nT) =$$

$$= \sum_n \left[\cos(\theta_n) + j\operatorname{sen}(\theta_n)\right] g_T(t - nT) = \sum_n c_n\, g_T(t - nT)$$

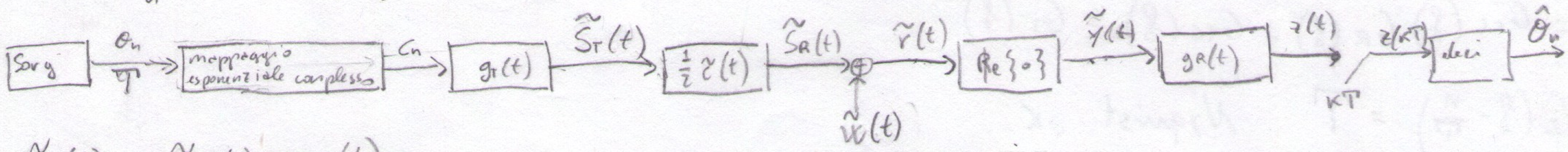
$$g_{Tc}(t) \triangleq g_T(t) \otimes \frac{1}{2}\tilde{c}(t) \Rightarrow \tilde{S}_R(t) = \sum_n c_n\, g_{Tc}(t - nT)$$

$$\tilde{r}(t) = \sum_n c_n\, g_{Tc}(t - nT) + \tilde{w}(t)$$

$$\tilde{y}(t) = \operatorname{Re}\{\tilde{r}(t)\} = \sum_n \cos(\theta_n)\, g_{Tc}(t - nT) + \tilde{w}_c(t)$$

$$g(t) \triangleq g_{Tc}(t) \otimes g_R(t); \qquad \tilde{n}_c(t) = \tilde{w}_c(t) \otimes g_R(t)$$

$$z(t) = \sum_n \cos(\theta_n)\, g(t - nT) + \tilde{n}_c(t)$$



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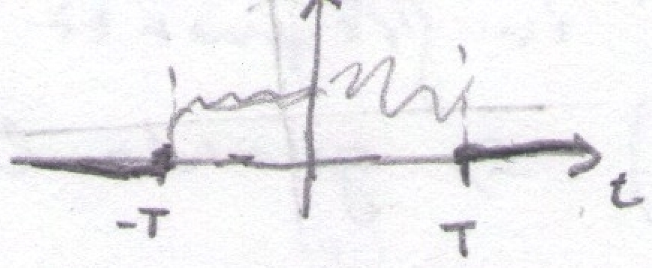
$$\tilde{n}(t) = \tilde{w}(t) \otimes g_R(t)$$

$$S_{\tilde{n}}(f) = S_{\tilde{w}}(f)\,|G_R(f)|^2 \rightleftarrows R_{\tilde{n}}(\tau) = R_{\tilde{w}}(\tau) \otimes g_R(\tau) \otimes g_R(-\tau) \Rightarrow S_{\tilde{n}}(f) = S_{\tilde{w}}(f)\, G_R(f)\, G_R(-f)$$

$$S_{\tilde{w}}(f) = \begin{cases} 4S_w(f+f_0) & f \geq -f_0 \\ 0 & \text{altrove} \end{cases} = 2N_0 \operatorname{rect}\left(\frac{f}{2B}\right); \quad S_{\tilde{w}_c}(f) = N_0 \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$S_{\tilde{n}_c}(f) = S_{\tilde{w}_c}(f)\, G_R(f)\, G_R(-f) = N_0 \operatorname{rect}\left(\frac{f}{2B}\right) A^2 T^2 \operatorname{sinc}^2(fT) = \quad 2$$

$$g(t) \triangleq g_{Tx}(t) \otimes g_R(t) =$$



$$g_N(t) \overset{?}{=} g(mT) = \begin{cases} K & m = \emptyset \\ \emptyset & m \neq \emptyset \end{cases}$$

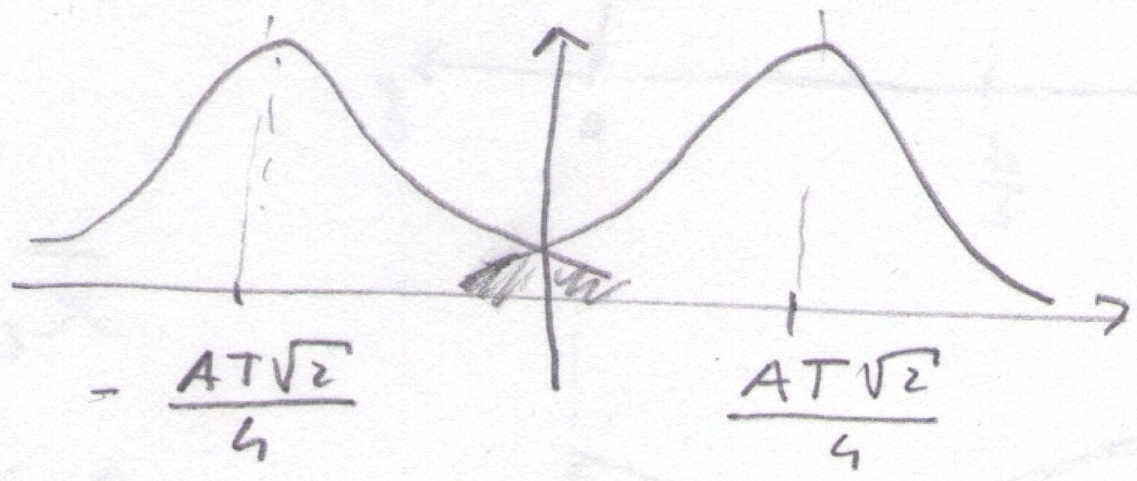
OK 3

$$z_K = z(KT) = \sum_n \cos(\theta_n)\, g([K-n]T) + n_K = \frac{AT}{2}\cos(\theta_K) + n_K$$

$$n_K \triangleq \tilde{n}_c(KT) \quad \in \mathcal{N}(\emptyset, \delta_n^2)$$

$$z_K\Big|_{K=\emptyset} = \frac{AT}{2}\cos(\theta_0) + n_K = \frac{AT}{2}\frac{\sqrt{2}}{2} + n_K$$

$$z_K\Big|_{K=1} = \frac{AT}{2}\cos(\theta_1) + n_K = -\frac{AT}{2}\frac{\sqrt{2}}{2} + n_K$$



$$P_r(\hat{\theta}_0 | \theta_1) = Q\left(\frac{\emptyset + \frac{AT\sqrt{2}}{4}}{\delta_n}\right) \quad ; \quad P_r(\hat{\theta}_1 | \theta_0) = 1 - Q\left(\frac{0 - \frac{AT\sqrt{2}}{4}}{\delta_n}\right)$$

$$\delta_{n_c}^2 = P_{\tilde{n}_c} = \int_{-\infty}^{\infty} S_{n_c}(f)\,df = \int_{-\infty}^{\infty} S_{w_c}(f)\,|G_R(f)|^2\,df = N_0 \int_{-B}^{B} |G_R(f)|^2\,df = N_0 \int_{-B \to -\infty}^{B \to \infty} |g_R(f)|^2\,df =$$

$$= N_0 A^2 \cdot T \Rightarrow \delta_{n_c} = \sqrt{A^2 T N_0}$$

$$P(e) = Q\left(\frac{AT\sqrt{2}}{4} \cdot \frac{1}{\sqrt{A^2 T N_0}}\right) = Q\left(\sqrt{\frac{\cancel{A^2} T^{\cancel{2}}\, 2}{\cancel{16}_{8}\, \cancel{A^2}\, \cancel{T} N_0}}\right) = Q\left(\sqrt{\frac{T}{8 N_0}}\right)$$