(1)

$(\sin\alpha + j\cos\alpha)^n = \sin(n\alpha) + j\cos(n\alpha)$

Dim:

$(\sin\alpha + j\cos\alpha)^n = (\cos(\frac{\pi}{2} - \alpha) + j\sin(\frac{\pi}{2} - \alpha))^n = e^{j(\frac{\pi}{2} - \alpha)^n} = e^{j(\frac{n\pi}{2} - n\alpha)} = \cos(\frac{n\pi}{2} - n\alpha) + j\sin(\frac{n\pi}{2} - n\alpha) = \sin(n\alpha) + j\cos(n\alpha)$

Esercizio

$x(t) = \cos(\omega_0 t)$

$x(t) = \cos(\omega_0 t) \cdot u(t)$

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2. Esercizio di proprietà

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$y(t) = \int_{-\infty}^{t} \cos(\omega_0 \tau)\,d\tau$

$= \int_{-\infty}^{t} \left[\frac{e^{j\omega_0\tau} + e^{-j\omega_0\tau}}{2}\right] d\tau = \int_{-\infty}^{t} \left[\frac{e^{j\omega_0\tau}}{2} + \frac{e^{-j\omega_0\tau}}{2}\right] d\tau$

$= \int_{-\infty}^{t} \left[\frac{e^{j\omega_0\tau}}{2}\right] d\tau + \int_{-\infty}^{t} \left[\frac{e^{-j\omega_0\tau}}{2}\right] d\tau$

$= \frac{1}{2}\int_{-\infty}^{t} e^{j\omega_0\tau} d\tau + \frac{1}{2}\int_{-\infty}^{t} e^{-j\omega_0\tau} d\tau$

$= \frac{1}{2}\left[\frac{e^{j\omega_0\tau}}{j\omega_0}\right]_{-\infty}^{t} + \frac{1}{2}\left[\frac{e^{-j\omega_0\tau}}{-j\omega_0}\right]_{-\infty}^{t}$

$= \frac{1}{2}\left[\frac{e^{j\omega_0 t}}{j\omega_0}\right] + \frac{1}{2}\left[\frac{e^{-j\omega_0 t}}{-j\omega_0}\right]$

$= \frac{1}{2}\left[\frac{e^{j\omega_0 t}}{j\omega_0}\right] - \frac{1}{2}\left[\frac{e^{-j\omega_0 t}}{j\omega_0}\right]$

$= \frac{1}{2}\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j\omega_0}\right]$

$= \frac{1}{2}\left[\frac{2\sin(\omega_0 t)}{j\omega_0}\right]$

$= \frac{\sin(\omega_0 t)}{\omega_0}$

$\boxed{y(t) = \frac{\sin(\omega_0 t)}{\omega_0}}$

Esercizio

$x(t) = \cos(\omega_0 t)$

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$= \int_{-\infty}^{t} \cos(\omega_0 \tau)\,d\tau$

$= \int_{-\infty}^{t} \left[\frac{e^{j\omega_0\tau} +