Elearn

Libro di testo:

| |
|---|
| Teoria dei segnali<br>M. Luise<br>3ª Edizione<br>Mc Graw Hill |

Ricevimento

Giovedì 15:30 -> 17:30
previo appuntamento via email
via Teams stesso canale delle lezioni

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Numeri complessi

$z = a + jb$ (CARTESIANA)

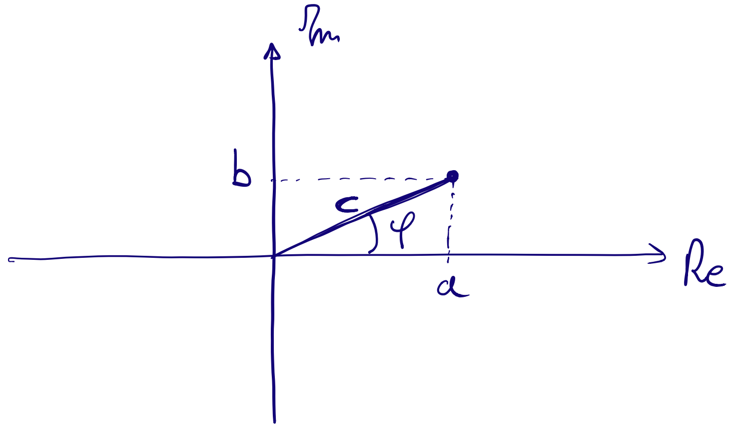
$a = \mathrm{Re}\{z\}$ parte reale

$b = \mathrm{Im}\{z\}$ parte immaginaria

$z = c\, e^{j\varphi}$ (POLARE)

$c$ = modulo

$\varphi$ = fase



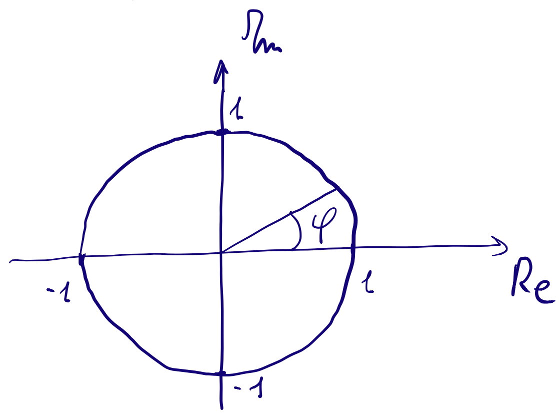
$$c = \sqrt{a^2 + b^2} \qquad a = c \cos\varphi$$

$$\varphi = \operatorname{arctg}\left[\frac{b}{a}\right] \qquad b = c \sin\varphi$$

$$z = a + jb = c\cos\varphi + j\,c\sin\varphi$$

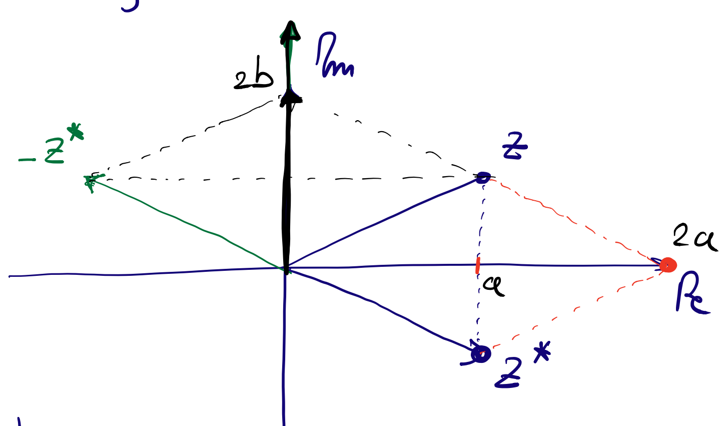
$$= c\left[\cos\varphi + j\sin\varphi\right] = c\,e^{j\varphi}$$

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$




$$a = \mathrm{Re}\{z\} = \frac{1}{2}\left[z + z^*\right]$$

$$b = \mathrm{Im}\{z\} = \frac{1}{2j}\left[z - z^*\right]$$




$$z^* = a - jb$$

$$z + z^* = a + jb + (a - jb) = 2a$$

$$z - z^* = a + jb - (a - jb) = j2b$$

OPERAZIONI ALGEBRICHE

$$z_1 = a_1 + j b_1 = c_1 e^{j\varphi_1}$$

$$z_2 = a_2 + j b_2 = c_2 e^{j\varphi_2}$$

•) $$z = z_1 \pm z_2 = (a_1 + j b_1) \pm (a_2 + j b_2) = a_1 \pm a_2 + j(b_1 \pm b_2)$$

$$z = z_1 + z_2 = c_1 e^{j\varphi_1} + c_2 e^{j\varphi_2}$$

$$\cancel{= (c_1 + c_2)\, e^{j(\varphi_2 + \varphi_1)}}$$

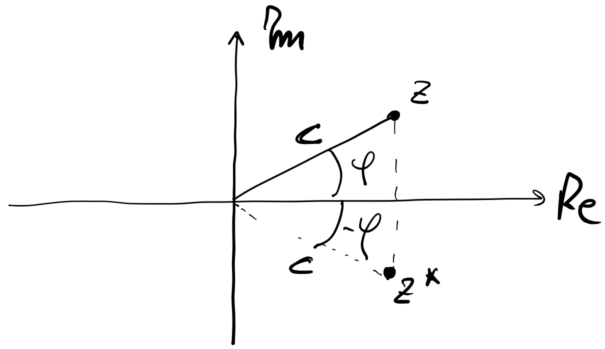
ATTENZIONE

•) $$z = z_1 \cdot z_2 = (a_1 + j b_1)(a_2 + j b_2) = a_1 a_2 + j a_1 b_2 + j a_2 b_1 - b_1 b_2 = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1)$$

$$z = z_1 \cdot z_2 = c_1 e^{j\varphi_1} \cdot c_2 e^{j\varphi_2} = c_1 c_2\, e^{j(\varphi_1 + \varphi_2)}$$

$$|z|^2 = z \cdot z^* = c e^{j\varphi} \cdot c e^{-j\varphi} = c^2 e^{j(\varphi - \varphi)} = c^2$$

$$\Rightarrow |z| = c = \sqrt{z \cdot z^*}$$




$$z^2 \neq |z|^2$$

$$z^2 = z \cdot z = c\, e^{j\varphi} \cdot c\, e^{j\varphi} = c^2 e^{j2\varphi}$$

$$z^2 = |z|^2 \quad \text{quando} \quad z \in Re$$

$$\cdot)\ z = \frac{z_1}{z_2} = \frac{c_1 e^{j\varphi_1}}{c_2 e^{j\varphi_2}} = \frac{c_1}{c_2} e^{j(\varphi_1 - \varphi_2)}$$

$$= \frac{a_1 + j b_1}{a_2 + j b_2} \cdot \frac{a_2 - j b_2}{a_2 - j b_1} = \frac{a_1 a_2 - j a_2 b_2 + j b_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}$$

"1"

Funzioni complesse di variabile reale

$$z(t), \quad z \in \mathbb{C}, \quad t \in \mathbb{R}$$

$$z(t) = a(t) + j\, b(t) = c(t)\, e^{j\varphi(t)}$$

Integrale

$$\int_a^b z(t)\,dt = \int_a^b a(t)\,dt + j\int_a^b b(t)\,dt$$

Derivata

$$\frac{d}{dt}z(t) = \frac{d}{dt}a(t) + j\,\frac{d}{dt}b(t)$$

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## SEGNALI

TIPI
- Deterministico : rappresentabile con delle funzioni analitiche
- Aleatorio : rappresentabili tramite statistiche

DIMENSIONALITÀ

$$\underline{v}(\underline{x}) \quad : \quad \mathbb{R}^n \to \mathbb{R}^m$$

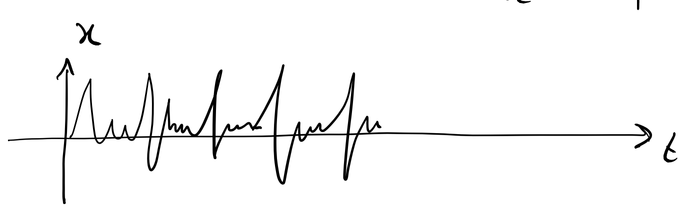
esempi

·) elettrocardiogramma

$$x(t)$$

$t$ = tempo

$x$ = ampiezza



·) audio stereo

$$\begin{bmatrix} l(t) \\ r(t) \end{bmatrix}$$

$t$ = tempo
$l$ = ampiezza canale sx
$r$ = " " dx

·) immagine statica b/w

$$z(x, y)$$

$x$ = coordinata "x"
$y$ = " "y"
$z$ = intensità di grigio

·) immagine statica colori

$$\underline{z}(x, y)$$

$$\underline{z} = \begin{cases} R \\ G \\ B \end{cases}$$

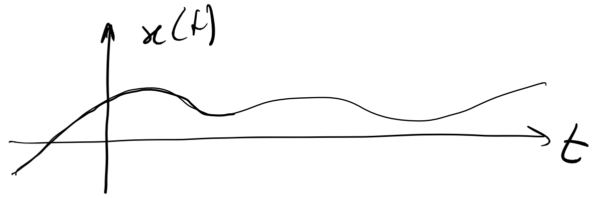
$x$ = coo. "x"
$y$ = coo. "y"

TIPOLOGIE IN BASE ALLA CONTINUITÀ DEI DOMINI

·) Segnale a tempo-continuo

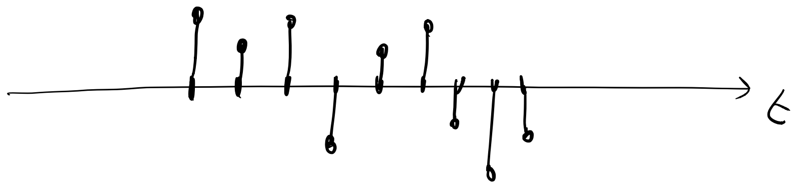
$t$ (dominio) assume con continuità tutti i valori contenuti all'interno di un certo intervallo

$$x(t)$$



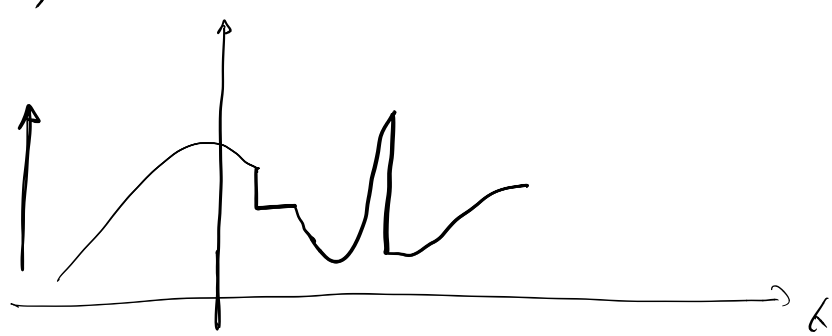
·) Segnale a **tempo - discreto**: la variabile temporale assume solo valori discreti (dominio)

$x[n]$



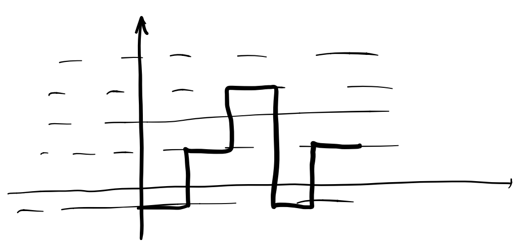


·) Segnale ad ampiezza continua: la grandezza fisica del segnale assume con continuità tutti i valori all'interno di un certo intervallo (codominio)





·) Segnale ad ampiezza discreta: la grandezza fisica può assumere solo valori discreti (appartenenti ad un certo alfabeto numerabile)



dominio

| | tempo-continuo | tempo-discreto |
|---|---|---|
| ampiezza continua | Analogico | Sequenza |
| ampiezza discreta | Quantizzato | Numerico |

codominio

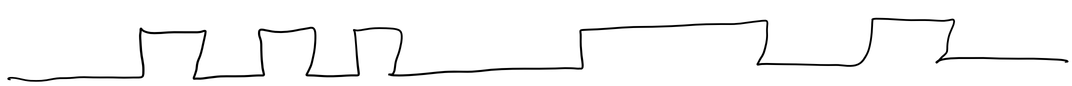
$s(t)$ — segnale analogico

$nT$

$s(nT) = s[n]$

segnale quantizzato

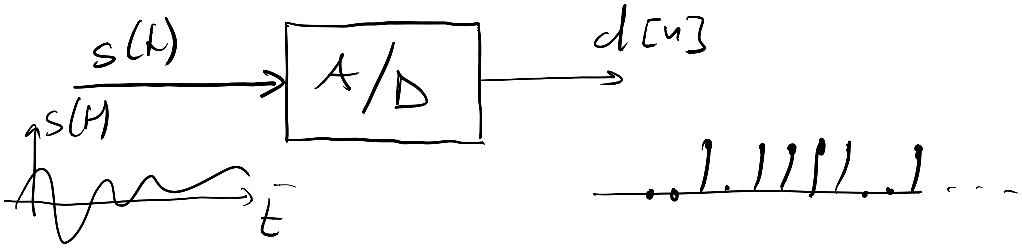
codice "Morse"



sequenza di bit

1 —
0 — ..||.|.||.

I segnali numerici sono gli unici segnali che possono essere completamente rappresentati tramite numeri.




SEGNALI DETERMINISTICI

Proprietà $\Rightarrow$ definizione di quantità

·) Potenza istantanea

$$x(t) \Rightarrow \boxed{p_x(t) \triangleq |x(t)|^2 \qquad \text{POTENZA ISTANTANEA}}$$

·) Energia

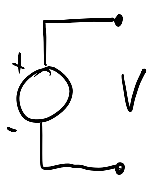
$$\boxed{E_x \triangleq \int_{-\infty}^{+\infty} p_x(t)\,dt = \int_{-\infty}^{+\infty} |x(t)|^2\,dt}$$

Segnali fisicamente realizzabili

$$E_x < \infty$$

Segnali "ideali" possono avere $E_x = \infty$

es. Batteria ideale

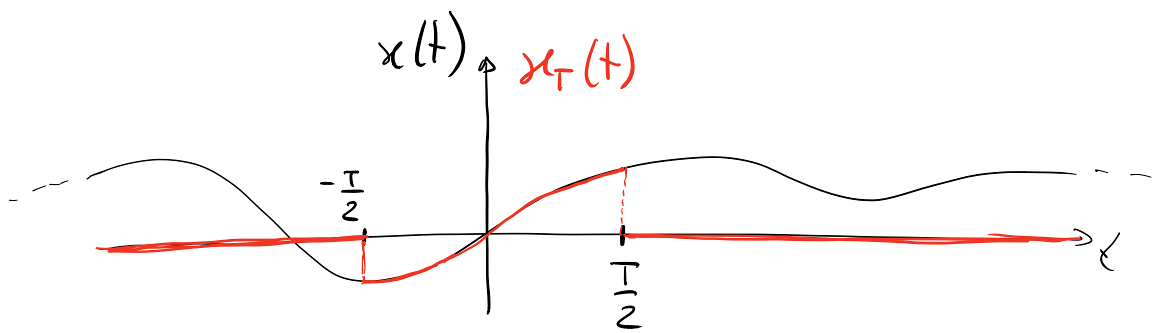



$-\infty < t < +\infty \Rightarrow x(t) = V \quad \forall t$

$$E_x = \int_{-\infty}^{+\infty} V \, dt = \infty$$

$\Rightarrow$ Segnale troncato nel tempo

$$x(t) \Rightarrow x_T(t) \triangleq \begin{cases} x(t) & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0 & \text{altrove} \end{cases}$$




·) $E_{x_T} < \infty$

·) $\lim_{T \to \infty} E_{x_T} = \infty = E_x$

·) POTENZA MEDIA

$P_{x_T} \triangleq \frac{E_{x_T}}{T}$ potenza media del segnale troncato

$$\boxed{P_x = \lim_{T \to \infty} P_{x_T} = \lim_{T \to \infty} \frac{E_{x_T}}{T} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt}$$

POTENZA MEDIA

Si osserva che

$$\boxed{x(t): \quad P_x = K < \infty \Rightarrow E_x = \infty \qquad K \neq 0}$$

$$P_x = \lim_{T\to\infty} \frac{E_{x_T}}{T} = K < \infty$$

$$\Downarrow$$

$$\lim_{T\to\infty} E_{x_T} = \infty = E_x$$

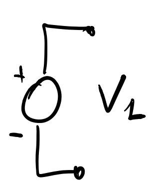
$$\boxed{x(t): \quad E_x = K < \infty \Rightarrow P_x = 0}$$

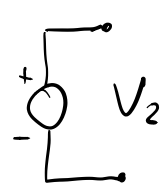
$$E_x = \lim_{T\to\infty} E_{x_T} = \lim_{T\to\infty} P_{x_T} T = K < \infty$$

$$\Downarrow$$

$$\lim_{T\to\infty} P_{x_T} = 0 = P_x$$

Esempio



·) Valore efficace

$$\boxed{x_{eff} \triangleq \sqrt{P_x}}$$

·) Valor medio (temporale)

$$\boxed{x_m \triangleq \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, dt}$$

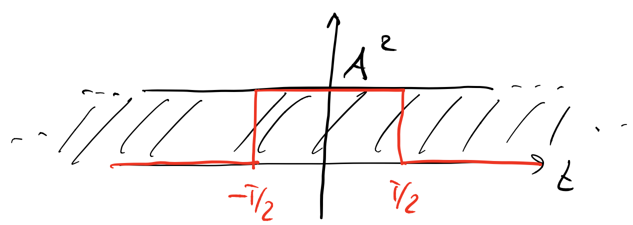
Segnali tipici

·) Costante

$$x(t) = A \qquad \forall t \qquad\qquad A \in \mathbb{R}$$

$$P_x(t) = |x(t)|^2 = A^2$$

$$E_x = \int_{-\infty}^{+\infty} A^2\, dt = \infty$$




$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2\, dt = \cancel{\lim_{T \to \infty}} \frac{1}{\cancel{T}} \cancel{T} A^2 = A^2$$

$$x_{eff} = \sqrt{A^2} = |A|$$

$$x_m = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A\, dt = \lim_{T \to \infty} \frac{1}{T} T A = A$$

- Sinusoidi

$A \in \mathbb{R}$

$$x(t) = \underset{\text{ampiezza}}{A} \cos(2\pi \underset{\text{frequenza}}{f_0} t + \underset{\text{fase (iniziale)}}{\varphi})$$

$$P_x(t) = |x(t)|^2 = x^2(t) = A^2 \cos^2(2\pi f_0 t + \varphi)$$

$$E_x = \int_{-\infty}^{+\infty} P_x(t)\, dt = \int_{-\infty}^{+\infty} A^2 \cos^2(2\pi f_0 t + \varphi)\, dt$$

$$\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$$

$$= \underbrace{\int_{-\infty}^{+\infty} \frac{A^2}{2}\, dt}_{+\infty} + \underbrace{\int_{-\infty}^{+\infty} \frac{A^2}{2} \cos(4\pi f_0 t + 2\varphi)\, dt}$$

$$\frac{A^2}{2} \frac{1}{4\pi f_0} \sin(4\pi f_0 t + 2\varphi) \Big|_{-\infty}^{+\infty}$$

$$+\infty + \left( -\frac{A^2}{8\pi f_0} \leq K \leq \frac{A^2}{8\pi f_0} \right) = +\infty$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2(2\pi f_0 t + \varphi)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2}\, dt + \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} \cos(4\pi f_0 t + 2\varphi)\, dt$$

$$= \cancel{\lim_{T\to\infty}} \frac{1}{\cancel{T}} \frac{A^2}{2} \cancel{T} + \underbrace{\lim_{T\to\infty} \frac{\int_{-T/2}^{T/2} \frac{A^2}{2}\cos(4\pi f_0 t + 2\varphi)\,dt}{T}}_{=0}$$

$$= \frac{A^2}{2}$$

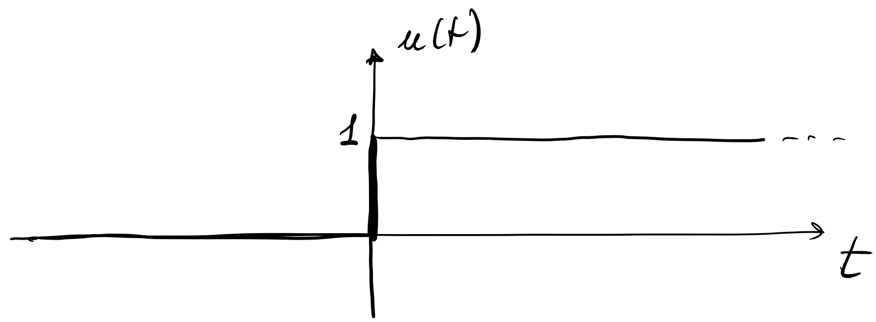
$$\Rightarrow x_{eff} = \sqrt{P_x} = \frac{|A|}{\sqrt{2}}$$

$$\Rightarrow x_m = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A\cos(2\pi f_0 t + \varphi)\,dt$$

$$= \lim_{T\to\infty} \frac{\int_{-T/2}^{T/2} A\cos(2\pi f_0 t + \varphi)\,dt}{T} = 0$$

·) GRADINO

$$u(t) \triangleq \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$




$$P_x(t) = u^2(t) = u(t)$$

$$E_x = \int_{-\infty}^{+\infty} u(t)\,dt = \int_{0}^{+\infty} 1\,dt = \infty$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t)\, dt = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} 1\, dt$$

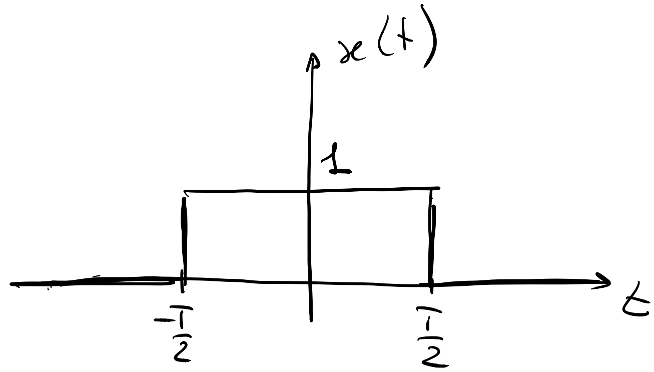
$$= \cancel{\lim_{T \to \infty}} \frac{1}{\cancel{T}} \frac{\cancel{T}}{2} = \frac{1}{2}$$

$$x_{eff} = \frac{1}{\sqrt{2}}$$

$$x_m = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t)\, dt = \frac{1}{2}$$

·) RETTANGOLO

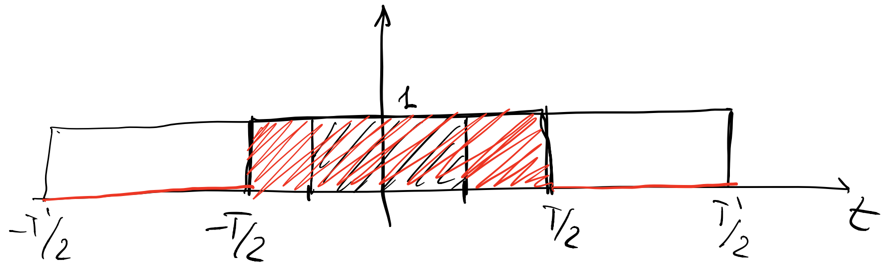
$$x(t) \triangleq rect\left(\frac{t}{T}\right) \triangleq \begin{cases} 1 & -T/2 \leq t \leq T/2 \\ 0 & \text{altrove} \end{cases}$$




$$p_x(t) = |x(t)|^2 = x^2(t) = x(t) = rect\left(\frac{t}{T}\right)$$

$$E_x = \int_{-\infty}^{+\infty} p_x(t)\, dt = \int_{-\infty}^{+\infty} rect\left(\frac{t}{T}\right) dt = \int_{-T/2}^{T/2} 1\, dt = T$$

$$P_x = \lim_{T'\to\infty} \frac{1}{T'} \int_{-\frac{T'}{2}}^{\frac{T'}{2}} \mathrm{rect}\left(\frac{t}{T}\right) dt$$




per $T' > T \to \infty$

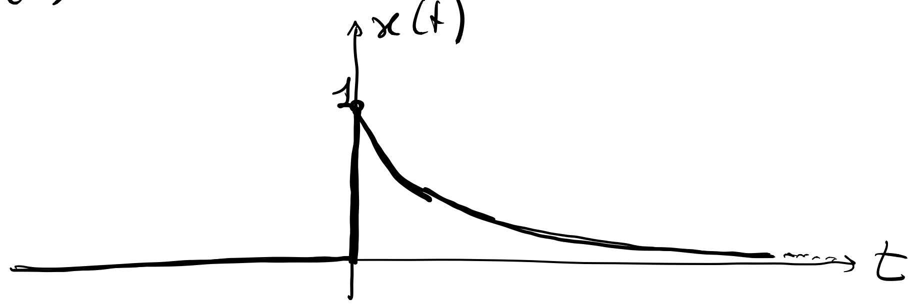
$$\Rightarrow P_x = \lim_{T'\to\infty} \frac{1}{T'} T = 0$$

$\Rightarrow x_{eff} = 0$

$$\Rightarrow x_m = \lim_{T'\to\infty} \frac{1}{T'} \int_{-\frac{T'}{2}}^{\frac{T'}{2}} \mathrm{rect}\left(\frac{t}{T}\right) dt = 0$$

$\Rightarrow$ Esponenziale Unilatera

$$x(t) \triangleq e^{-t} u(t)$$



$$p_x(t) = |x(t)|^2 = e^{-2t}u(t)$$

$$E_x = \int_{-\infty}^{+\infty} e^{-2t}u(t)\,dt = \int_0^{+\infty} e^{-2t}\,dt = -\frac{1}{2}e^{-2t}\Big|_0^{+\infty}$$

$$= 0 - \left(-\frac{1}{2}\right) = \frac{1}{2}$$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} e^{-2t}u(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_0^{T/2} e^{-2t}\,dt = \lim_{T\to\infty} \frac{1}{T}\left(-\frac{1}{2}e^{-2t}\right)_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\frac{1}{2} - \frac{1}{2}e^{-T}\right) = \frac{1}{2}\lim_{T\to\infty} \frac{1-e^{-T}}{T} = 0$$

$$x_{eff} = 0$$

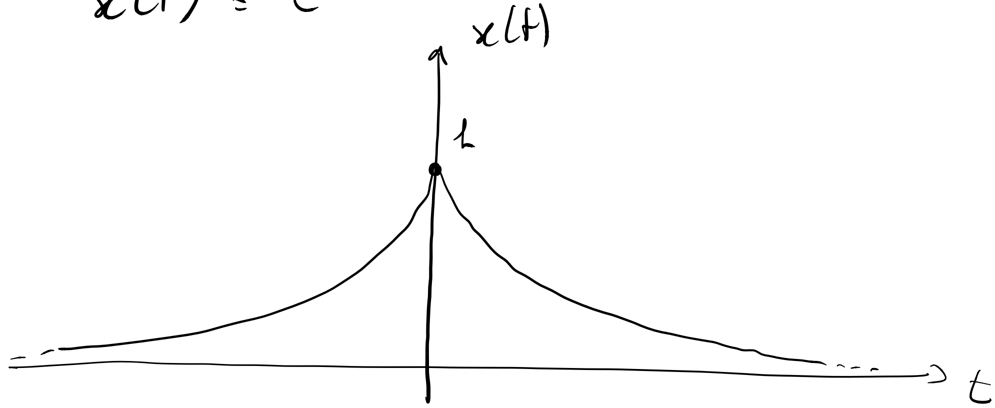
$$x_m = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-t}u(t)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_0^{\frac{T}{2}} e^{-t}\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(-e^{-t}\right)_0^{T/2} = \lim_{T\to\infty} \frac{1}{T}\left(1 - e^{-T/2}\right)$$

$$= \lim_{T\to\infty} \frac{1-e^{-T/2}}{T} = 0$$

·) Esponenziale Bilatera

$$x(t) = e^{-|t|}$$




$$P_x(t) = e^{-2|t|}$$

$$E_x = \int_{-\infty}^{+\infty} e^{-2|t|}\,dt = 2\int_0^{+\infty} e^{-2|t|}\,dt = 2 \cdot \frac{1}{2} = 1$$

$$P_x = 0$$

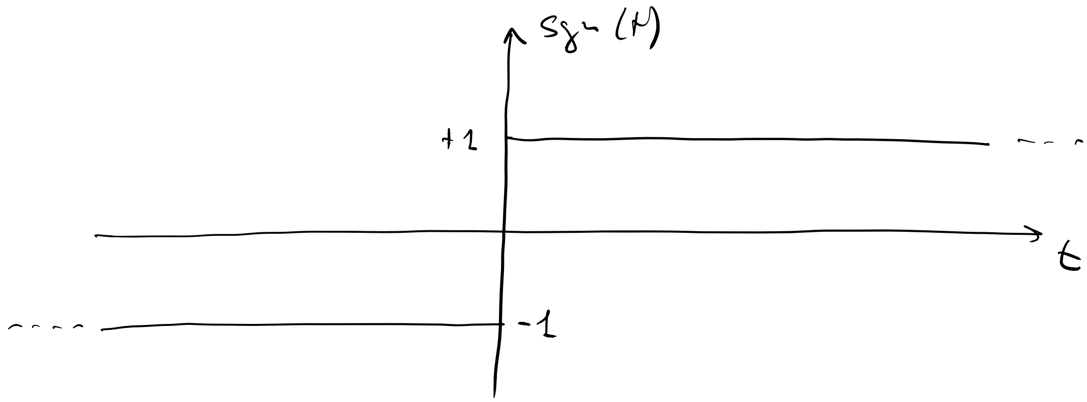
$$x_{eff} = 0$$

$$x_m = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} e^{-|t|}\,dt = \lim_{T\to\infty} \frac{1}{T}\, 2\int_0^{T/2} e^{-t}\,dt$$

$$= \lim_{T\to\infty} \frac{2}{T}\left(1 - e^{-T/2}\right) = 0$$

·) SEGNO

$$x(t) = \mathrm{sgn}(t) = \begin{cases} +1 & t \geq 0 \\ -1 & t < 0 \end{cases}$$




$$P_x(t) = 1 \quad \forall t$$

$$E_x = \int_{-\infty}^{+\infty} 1 \, dt = \infty$$

$$P_x = 1$$

$$x_{eff} = 1$$

$$x_m = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \operatorname{sgn}(t)\, dt = \lim_{T \to \infty} \frac{1}{T} \left[ \int_{-T/2}^{0} -1 \, dt + \int_{0}^{T/2} +1 \, dt \right]$$

$$\underbrace{-T/2 \qquad +T/2}_{=0}$$

$$= 0$$

Osservazione

$$P_x = 0 \Rightarrow \begin{cases} x_m = 0 \\ x_{eff} = 0 \end{cases}$$

$$P_x = 0 \Rightarrow x_m = 0$$

Dim

$x(t) \Rightarrow P_{x} = 0$ I p.

$x(t) = x_m + x'(t)$

$x_m \triangleq$ V. m di $x(t)$

$x'(t)$ = residuo a valor medio nullo

$\hookrightarrow \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x'(t)\,dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [x(t) - x_m]\,dt$

$$= \underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt}_{x_m - x_m = 0} - x_m \underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt}_{\lim_{T \to \infty} \frac{1}{\cancel{T}} \cancel{T} = 1} = 0$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2\,dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x_m + x'(t)|^2\,dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (x_m + x'(t))(x_m + x'(t))^*\,dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left( |x_m|^2 + \underbrace{x_m x'^*(t)}_{z} + \underbrace{x'(t) x_m^*}_{z^*} + |x'(t)|^2 \right) dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x_m|^2 dt + 2\,\mathrm{Re}\left[ \cancel{\underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x_m^* x'(t)\, dt}_{=0}} \right]$$

$$+ \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x'(t)|^2 dt$$

$$P_x = \underbrace{|x_m|^2}_{\geq 0,\ =0} + \underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x'(t)|^2 dt}_{P_{x'} \geq 0,\ =0} = 0$$

$$\Rightarrow |x_m|^2 = 0 \Rightarrow \boxed{x_m = 0}$$