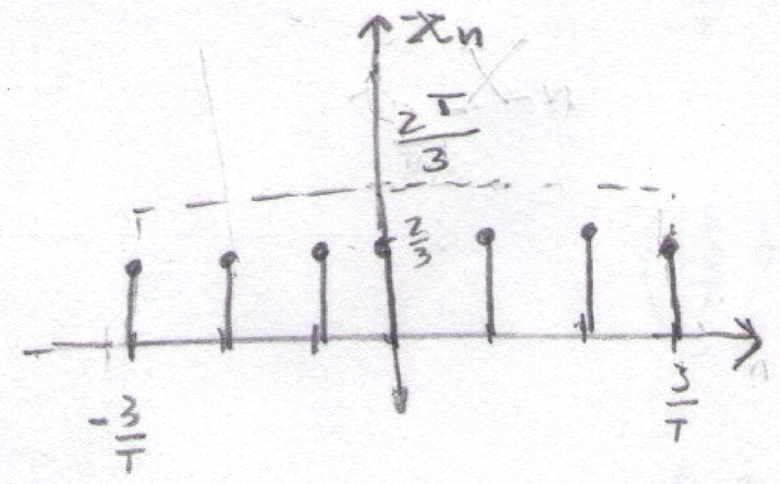
Esame 12/11/09

Esercizio 1

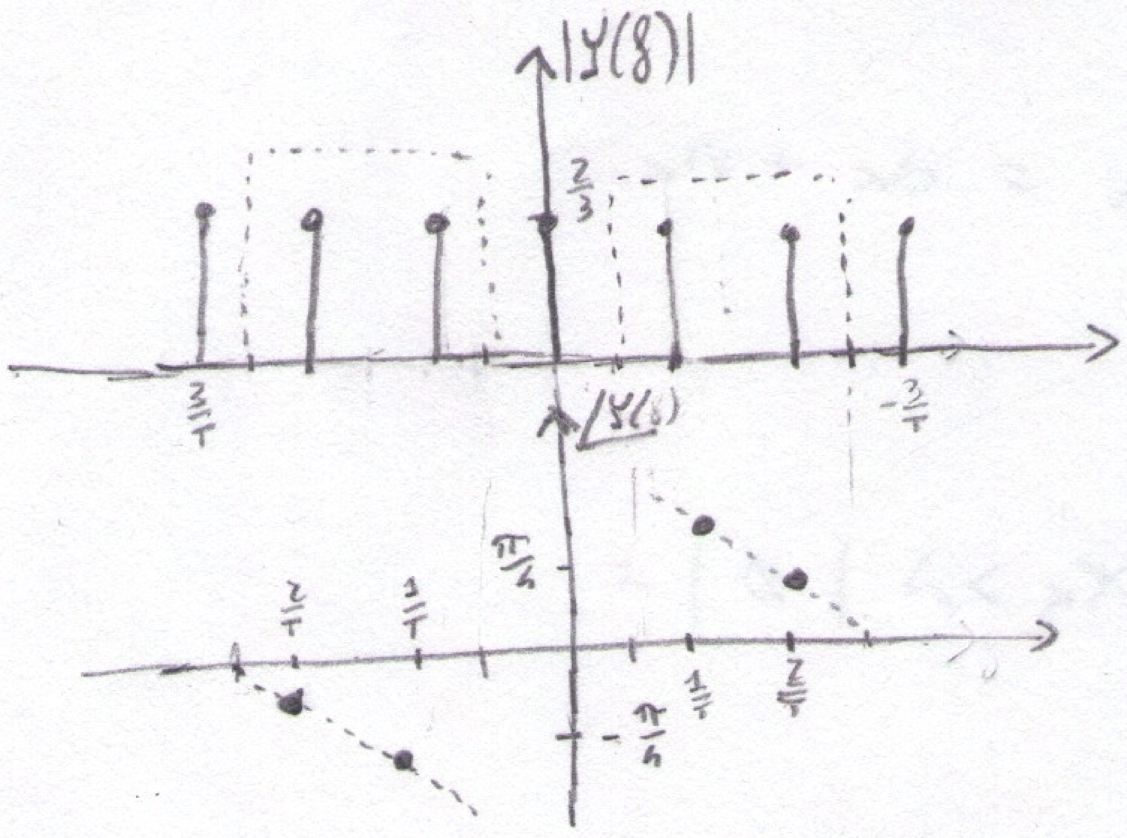
$x(t) = \sum_n 4 \operatorname{sinc}(Bt - 6n)$ = periodico ; $x_0(t) = 4 \operatorname{sinc}(Bt)$

$x(t) \overset{TSF}{\rightleftharpoons} X_n = \frac{1}{T} X_0\left(\frac{n}{T}\right)$; $X_0(f) = \frac{4}{B} \operatorname{rect}\left(\frac{f}{B}\right) = \frac{2}{3} T \operatorname{rect}\left(\frac{f}{6/T}\right)$



$$X(f) = \sum_{n=-3}^{n=3} \frac{2}{3} \delta\left(f - \frac{n}{T}\right)$$

$$Y(f) = X(f) H(f) \begin{cases} |Y(f)| = |X(f)| |H(f)| \\ \angle Y(f) = \angle X(f) + \angle H(f) \end{cases} = Y(f) = |Y(f)| e^{j\angle Y(f)}$$

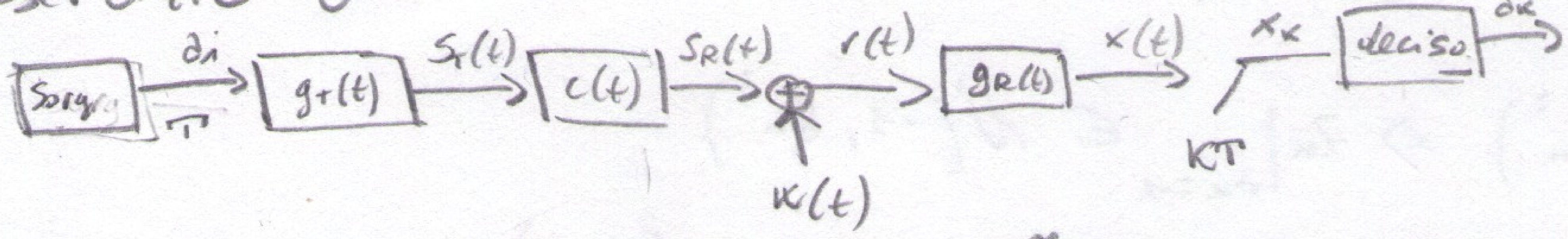


$$Y(f) = \frac{2}{3}\delta\left(f + \frac{2}{T}\right) e^{-j\pi \cancel{T} \frac{2}{\cancel{T}} - j\frac{\pi}{4}} +$$
$$+ \frac{2}{3}\delta\left(f + \frac{1}{T}\right) e^{-j\pi \cancel{T} \cdot \frac{1}{\cancel{T}} - j\frac{\pi}{4}} +$$
$$+ \frac{2}{3}\delta\left(f - \frac{1}{T}\right) e^{-j\pi \cancel{T} \frac{1}{\cancel{T}} + j\frac{\pi}{4}} +$$
$$+ \frac{2}{3}\delta\left(f - \frac{2}{T}\right) e^{-j\pi \cancel{T} \frac{2}{\cancel{T}} + j\frac{\pi}{4}}$$

$$y(t) = \frac{2}{3} e^{j\frac{7}{4}\pi} e^{-j2\pi t \frac{2}{T}} + \frac{2}{3} e^{j\frac{3}{4}\pi} e^{-j2\pi t \frac{1}{T}} + \frac{2}{3} e^{j\pi\frac{5}{4}} e^{j2\pi t \cdot \frac{1}{T}} + \frac{2}{3} e^{j\pi\frac{9}{4}} e^{j2\pi t \cdot \frac{2}{T}}$$

$$P_y = \sum_{n=-\infty}^{\infty} |Y_n|^2 = \left(\frac{2}{3}\right)^2 \cdot 4 = \frac{4}{9} \cdot 4 = \frac{16}{9}$$

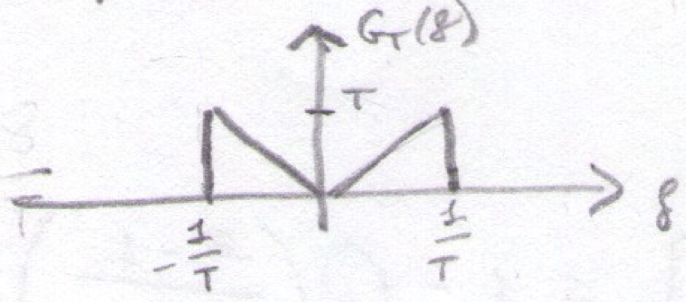
Esercizio 2

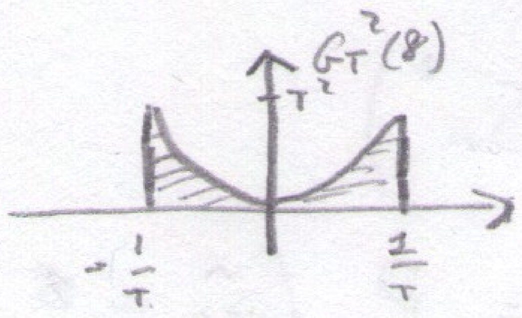


$$E_T = P_{s_T} T = \int_{-\infty}^{\infty} S_{s_T}(f)\, df \cdot T = \int_{-\infty}^{\infty} \frac{E\{a_i^2\}}{T} |G_T(f)|^2 df \cdot T$$

$$E\{a_i^2\} = \sum_{K=0}^{1} p_K m_K^2 = p_0 m_0^2 + p_1 m_1^2 = \emptyset + \frac{1}{2} \cdot 4^2 = 8$$

$$P_{s_T} = \frac{8}{T} \int_{-\infty}^{\infty} |G_T(f)|^2 df =$$



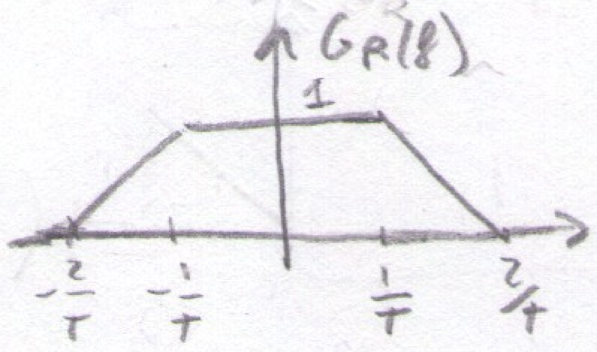


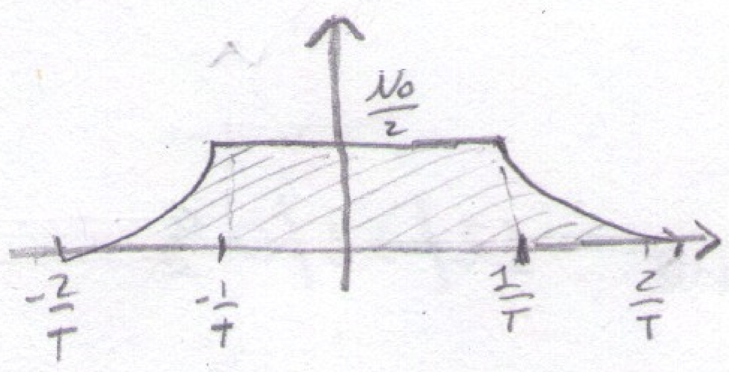
$$= \frac{8}{\cancel{T}} \left\{ 2 \cdot \frac{1}{3} \cdot \cancel{T}^2 \cdot \frac{1}{\cancel{T}} \right\} = \frac{16}{3}$$

$$E_T = \frac{16}{3} T \quad (1)$$

$$n(t) \triangleq w(t) \otimes g_R(t)$$

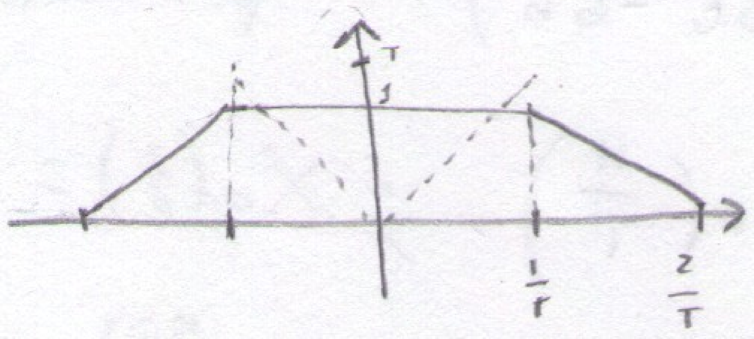
$$P_n = \int_{-\infty}^{\infty} S_n(f)\, df = \int_{-\infty}^{\infty} S_w(f) |G_R(f)|^2 df =$$





$$= \frac{N_0}{\cancel{2}} \cdot \frac{\cancel{2}}{T} + \cancel{2} \cdot \frac{N_0}{\cancel{2}} \cdot \frac{1}{3} \cdot \frac{1}{T} = \frac{N_0}{T} + \frac{N_0}{3T} = \frac{4 N_0}{3T} \quad (2)$$

$$g(t) = g_{TC}(t) \otimes g_R(t) \rightleftharpoons G(f) = G_{TC}(f)\, G_R(f)$$



$$G(f) = T \operatorname{rect}\left(\frac{f}{2/T}\right) - T\left(1 - \frac{|f|}{1/T}\right) \operatorname{rect}\left(\frac{f}{2/T}\right)$$

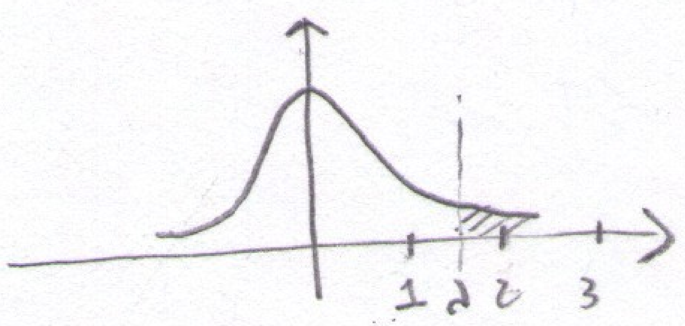
$$g(t) = \cancel{T} \cdot \frac{2}{\cancel{T}} \operatorname{sinc}\left(t \frac{2}{T}\right) - \cancel{T} \cdot \frac{1}{\cancel{T}} \operatorname{sinc}^2\left(t \frac{1}{T}\right) = 2 \operatorname{sinc}\left(t \frac{2}{T}\right) - \operatorname{sinc}^2\left(t \frac{1}{T}\right)$$

$$g_N(mT) = \begin{cases} 1 & m = \emptyset \\ \emptyset & m \neq \emptyset \end{cases} \qquad g(mT) = \begin{cases} 1 & m = \emptyset \\ \emptyset & m \neq \emptyset \end{cases} \quad \text{ok} \quad g(t) = g_N(t)$$

$$P_r(e) = P_r[\hat{\emptyset} | 4]\, P_r[4] + P_r[\hat{4} | \emptyset]\, P_r[\emptyset]$$

$$x_K = x(KT) = \sum_i a_i\, g[(K-i)T] + n(KT) = a_K\, g(\emptyset) + n_K = a_K + n_K$$

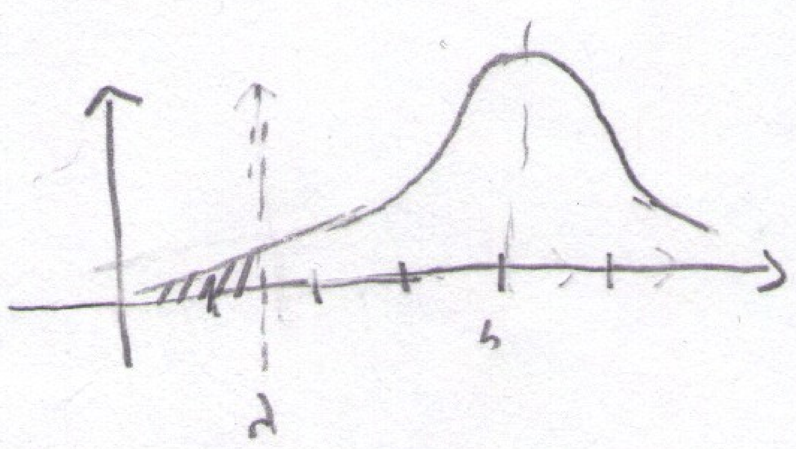
$$x_K|_{a_K=\emptyset} = \emptyset + n_K \in \mathcal{N}(\emptyset, s_n^2)$$



$$P_r[\hat{4} | \emptyset] = P_r(\hat{a}_K = 4 | \emptyset] = P_r[x_K > \lambda | \emptyset] =$$

$$= Q\left(\frac{\frac{3}{2} - \emptyset}{s_n}\right) = Q\left(\frac{3}{2 s_n}\right)$$

$$x_K|_{a_K=4} = 4 + n_K \in \mathcal{N}(4, s_n^2)$$



$$P_r[\hat{\emptyset} | 4] = P_r[\hat{a}_K = \emptyset | 4] = P_r[x_K \le \lambda | 4] =$$

$$= 1 - Q\left(\frac{\frac{3}{2} - 4}{s_n}\right) = 1 - Q\left(-\frac{5}{2 s_n}\right) = Q\left(\frac{5}{2 s_n}\right)$$

$$S_n^2 = P_n = \frac{4N_0}{3T} \Rightarrow S_n = \sqrt{\frac{4N_0}{3T}}$$

$$Q\left(\frac{3}{2S_n}\right) = Q\left(\frac{3}{2\sqrt{\frac{4N_0}{3T}}}\right) = Q\left(\sqrt{\frac{27T}{16N_0}}\right)$$

$$Q\left(\frac{5}{2S_n}\right) = Q\left(\frac{5}{2\sqrt{\frac{4N_0}{3T}}}\right) = Q\left(\sqrt{\frac{75T}{16N_0}}\right)$$

$$P_r(e) = \frac{1}{2}Q\left(\sqrt{\frac{27T}{16N_0}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{75T}{16N_0}}\right)$$