ES#1 - 06/06/2017

$U(t)$ processo Gaussiano stazionario a valor medio nullo

$E[U(t)] = 0$

$R_U(\tau) = \sigma_U^2 \operatorname{sinc}(2B\tau)$

1) $U = U(0) \Rightarrow f_U(u) = ?$

2) $Y(t) = U(t) + 3U(t-T)$

·) $S_Y(f), R_Y(t_1, t_2) = ?$

Svolgimento

1) $U \in \mathcal{N}(\eta_U, \sigma_U^2)$

$$f_U(u) = \frac{1}{\sqrt{2\pi\sigma_U^2}} e^{-\frac{(u-\eta_u)^2}{2\sigma_U^2}}$$

$\eta_u = ?$

$\sigma_U^2 = ?$

$$\eta_u = E\left[U(t)\big|_{t=0}\right] = 0$$

$$\eta_u(t) = 0 \quad \forall t$$

$$\sigma_u^2 = P_u - \eta_u^2 = P_u$$

$$P_u = R_u(\tau)\Big|_{\tau=0} = \int_{-\infty}^{+\infty} S_U(f)\,df = \sigma_U^2 \operatorname{sinc}(2B\tau)\Big|_{\tau=0}$$

$$= \underline{\underline{\sigma_U^2}} \Leftarrow \text{parametro del problema (noto)}$$

$$\boxed{f_U(u) = \frac{1}{\sqrt{2\pi\sigma_U^2}} e^{-\frac{u^2}{2\sigma_U^2}}}$$

2) $Y(t) = U(t) + 3\,U(t-T)$

$$R_Y(t_1, t_2) = E\Big[\big(U(t_1) + 3\,U(t_1 - T)\big)\big(U(t_2) + 3(U(t_2 - T))\big)\Big]$$

$$= E[U(t_1)U(t_2)] + E[U(t_1)U(t_2 - T)] + E[U(t_1 - T)U(t_2)]$$

$$+ E[U(t_1 - T)U(t_2 - T)]$$

$$= R_U(t_2 - t_1) + R_U(t_1 - (t_2 - T)) + R_U(t_1 - T - t_2) + R_U(t_1 - t_2)$$

$$= 2R_U(t_1 - t_2) + R_U(t_1 - t_2 + T) + R_U(t_1 - t_2 - T)$$

$$R_Y(\tau) = 2R_U(\tau) + R_U(\tau + T) + R_U(\tau - T)$$

$$= \boxed{2\sigma_U^2 \operatorname{sinc}(2B\tau) + \sigma_U^2 \operatorname{sinc}(2B(\tau + T)) + \sigma_U^2 \operatorname{sinc}[2B(\tau - T)]}$$

$$S_Y(f) = TCF[R_Y(\tau)] = \frac{\sigma_U^2}{B}\operatorname{rect}\left(\frac{f}{2B}\right) + \frac{\sigma_U^2}{2B}\operatorname{rect}\left(\frac{f}{2B}\right)\left[e^{j2\pi fT} + e^{-j2\pi fT}\right]$$

$$= \boxed{\frac{\sigma_U^2}{B}\operatorname{rect}\left(\frac{f}{2B}\right)\left[1 + \cos(2\pi fT)\right]}$$

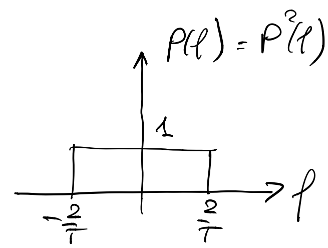
ES #2 06/06/2017

1) $E_s = E[x^2]\,E_p = \boxed{\frac{4}{T}}$

$$E[x^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+1)^2 = 1$$

$$E_p = \frac{4}{T}$$

$$p(t) = \frac{4}{T}\operatorname{sinc}\left(\frac{4}{T}t\right) \Rightarrow P(f) = \operatorname{rect}\left(\frac{f}{4/T}\right)$$

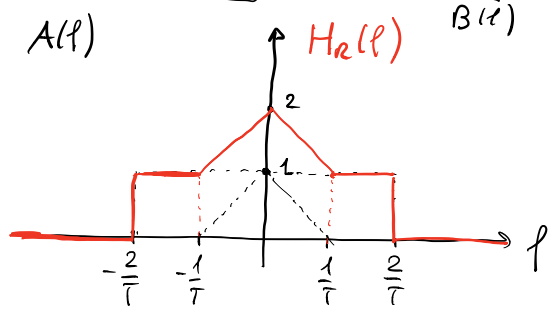


$$S_w(f)=\frac{N_0}{2} \longrightarrow \boxed{h_R(t)} \xrightarrow{S_{n_u}(f)}$$

$$S_{n_u}(f) = \frac{N_0}{2}\left|H_R(f)\right|^2 \Rightarrow P_{n_u} = \frac{N_0}{2} E_{H_R}$$

$$H_R(f) = \left(1-|fT|\right)\mathrm{rect}\left(\frac{fT}{2}\right) + \mathrm{rect}\left(\frac{fT}{4}\right)$$

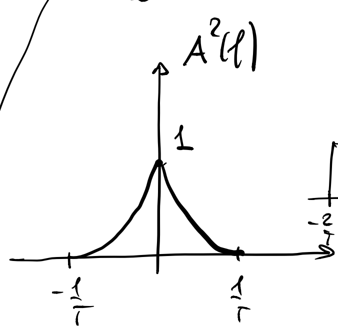
$$= \underbrace{\left(1-\frac{|f|}{1/T}\right)\mathrm{rect}\left(\frac{f}{2/T}\right)}_{A(f)} + \underbrace{\mathrm{rect}\left(\frac{f}{4/T}\right)}_{B(f)}$$

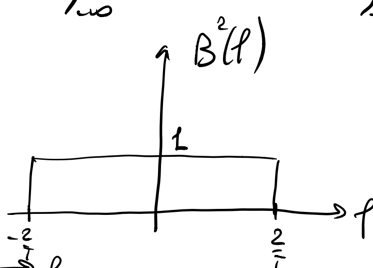


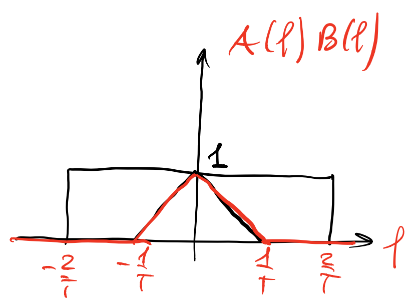
$$H_R(f) = A(f) + B(f)$$

$$H_R^2(f) = A^2(f) + B^2(f) + 2A(f)B(f)$$

$$E_{H_R} = \int_{-\infty}^{+\infty} A^2(f)\,df + \int_{-\infty}^{+\infty} B^2(f)\,df + 2\int_{-\infty}^{+\infty} A(f)B(f)\,df$$







$$= \frac{2}{3}\cdot\frac{1}{T}\cdot 1 + \frac{4}{T} + 2\cdot\frac{1}{T} = \frac{1}{T}\left(\frac{2}{3}+4+2\right) = \frac{20}{3T}$$

$$P_{nu} = \frac{N_0}{\cancel{2}} \cdot \frac{\cancel{20}^{10}}{3T} = \boxed{\frac{10}{3}\frac{N_0}{T}}$$

3) $S_s(f) = \frac{1}{T}\bar{S}_x(f)\,|P(f)|^2$

- simboli antipodali ✓
- " equiprobabili ✓
- " indipendenti ✓

$$S_s(f) = \frac{\sigma_x^2}{T}|P(f)|^2$$

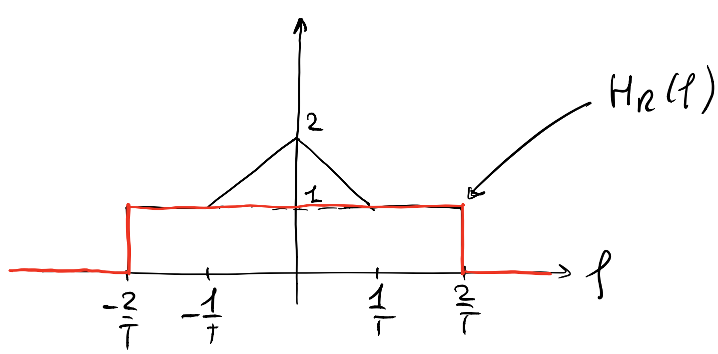
$$\sigma_x^2 = E[x^2] - \eta_x^2 = E[x^2] = 1$$

$$\eta_x = \frac{1}{2}(-1) + \frac{1}{2}(+1) = 0$$

$$S_s(f) = \frac{1}{T}|P(f)|^2 = \boxed{\frac{1}{T}\,\mathrm{rect}\left(\frac{f}{4/T}\right)}$$

4) $h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$$H(f) = P(f)\,H_R(f) = H_R(f)$$



$$h(t) = h_R(t) = TCF^{-1}[H_R(f)]$$

$$h(t) = \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{t}{T}\right) + \frac{4}{T}\,\mathrm{sinc}\left(\frac{4t}{T}\right)$$

$$h(nT) = \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{nT}{T}\right) + \frac{4}{T}\,\mathrm{sinc}\left(\frac{4nT}{T}\right)$$

$$= \frac{1}{T}\,\mathrm{sinc}^2(n) + \frac{4}{T}\,\mathrm{sinc}(4n) = \frac{1}{T}\delta[n] + \frac{4}{T}\delta[n]$$

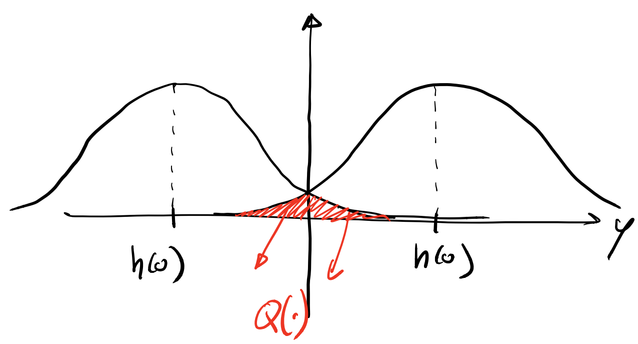
$$= \boxed{\frac{5}{T}\delta[n] \quad \Rightarrow \text{ verifica Nyquist nel tempo}}$$

$$h(0) = \frac{5}{T}$$

prob. a priori dei simboli

5) $$P_E(b) = \frac{1}{2}\,Q\left(\frac{\frac{5}{T}}{\sqrt{\frac{10}{3}\frac{N_0}{T}}}\right) + \frac{1}{2}\,Q\left(\frac{\frac{5}{T}}{\sqrt{\frac{10}{3}\frac{N_0}{T}}}\right)$$

$$= \boxed{Q\left(\frac{\frac{5}{T}}{\sqrt{\frac{10}{3}\frac{N_0}{T}}}\right)}$$



ES #1 07/02/2019

$$x(t) = B\cos(2\pi f_0 t)\,\mathrm{sinc}(Bt) = x_0(t)\cos(2\pi f_0 t), \quad x_0(t) = B\,\mathrm{sinc}(Bt)$$

$$h(t) = B\,\mathrm{sinc}^2(Bt)$$

$f_0 \gg B$

$$w_1(t) = x(t)\cdot\cos(2\pi f_0 t + \varphi)$$

$$y_1(t) = w_1(t) \otimes h(t)$$

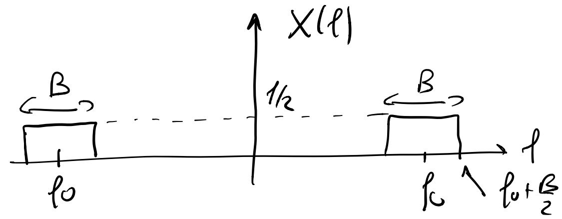
$$Y_1(f) = W_1(f)\, H(f)$$

$$W_1(f) = \frac{e^{j\varphi}}{2} X(f-f_0) + \frac{e^{-j\varphi}}{2} X(f+f_0)$$

$$X(f) = TCF\left[x(t)\right] = \frac{1}{2} X_0(f-f_0) + \frac{1}{2} X_0(f+f_0)$$

$$X_0(f) = \mathrm{rect}\left(\frac{f}{B}\right)$$

$$X(f) = \frac{1}{2}\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f+f_0}{B}\right)$$



$f_0 \gg B$

$$W_1(f) = \frac{e^{j\varphi}}{2}\left[\frac{1}{2}\mathrm{rect}\left(\frac{f-f_0-f_0}{B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f-\cancel{f_0}+\cancel{f_0}}{B}\right)\right] +$$

$$+ \frac{e^{-j\varphi}}{2}\left[\frac{1}{2}\mathrm{rect}\left(\frac{f+\cancel{f_0}-\cancel{f_0}}{B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f+f_0+f_0}{B}\right)\right]$$

$$= \frac{1}{4} e^{j\varphi}\mathrm{rect}\left(\frac{f-2f_0}{B}\right) + \underbrace{\frac{1}{4}\mathrm{rect}\left(\frac{f}{B}\right)\left[e^{j\varphi} + e^{-j\varphi}\right]}_{\text{comp. in b.b.}}$$

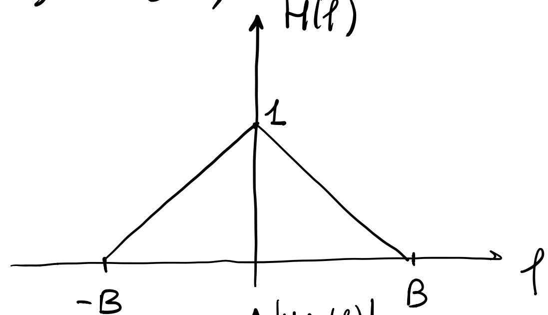
$$+ \frac{1}{4} e^{-j\varphi}\mathrm{rect}\left(\frac{f+2f_0}{B}\right)$$

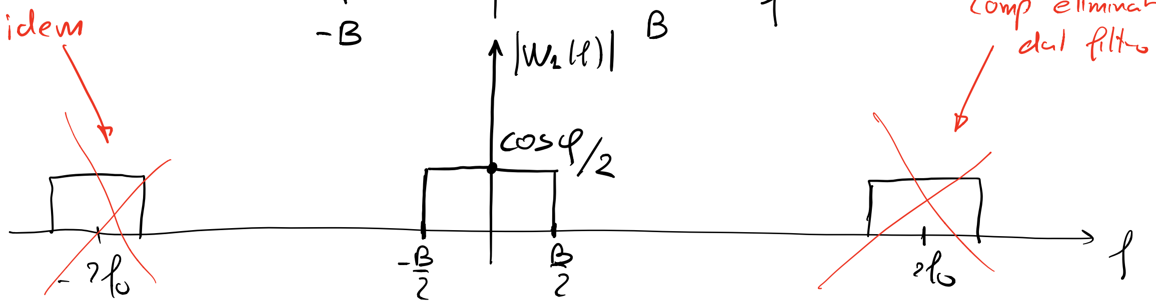
comp. in b.b. $\to \cos\varphi \cdot \mathrm{rect}\left(\frac{f}{B}\right)$

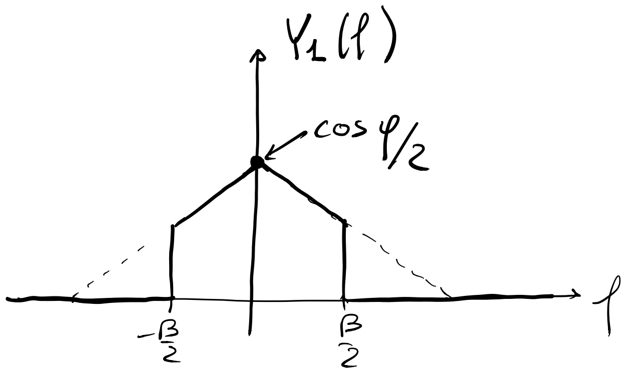
1 comp a $2f_0$

1 comp a $-2f_0$

$$H(f) = \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$







$$W_2(f) = \frac{e^{j\varphi}}{2j} X(f-f_0) - \frac{e^{-j\varphi}}{2j} X(f+f_0)$$

$$X(f) = \frac{1}{2}\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f+f_0}{B}\right)$$

$$= \frac{e^{j\varphi}}{4j}\left[\mathrm{rect}\left(\frac{f-2f_0}{B}\right) + \mathrm{rect}\left(\frac{f}{B}\right)\right] +$$

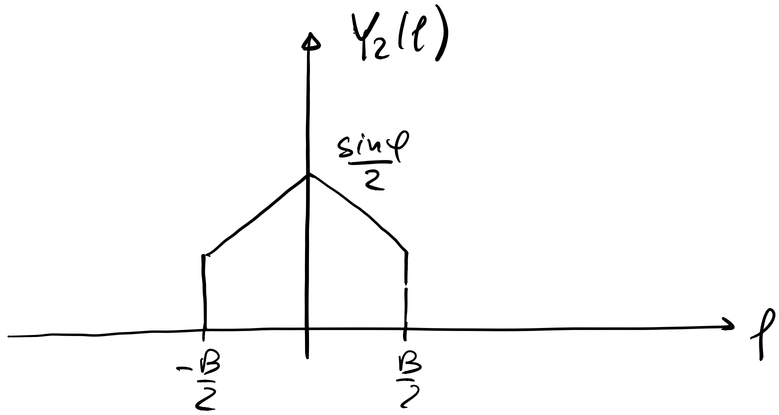
$$- \frac{e^{-j\varphi}}{4j}\left[\mathrm{rect}\left(\frac{f}{B}\right) + \mathrm{rect}\left(\frac{f+2f_0}{B}\right)\right]$$

$$W_{2_{bb}}(f) = \text{rect}\left(\frac{f}{B}\right)\left[\frac{e^{j\varphi}}{4j} - \frac{e^{-j\varphi}}{4j}\right] = \frac{1}{2}\,\text{rect}\left(\frac{f}{B}\right)\sin\varphi$$

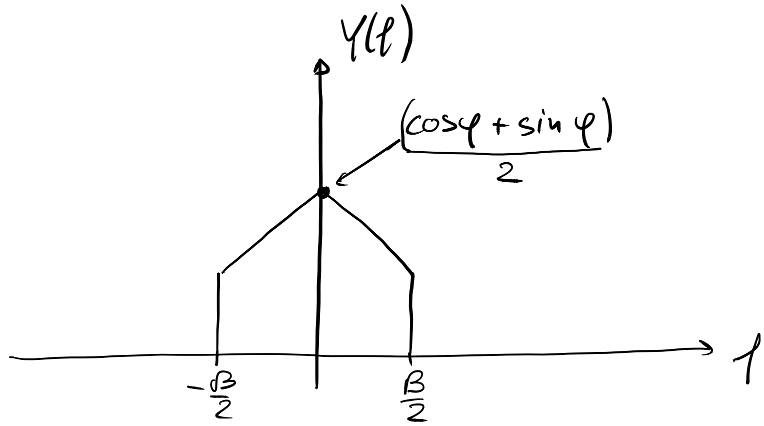
solo comp. in banda base (eliminate le freq. a $\pm 2f_0$)

$$Y_2(f) = W_2(f)\,H(f) = W_{2_{bb}}(f)\,H(f)$$

$=$



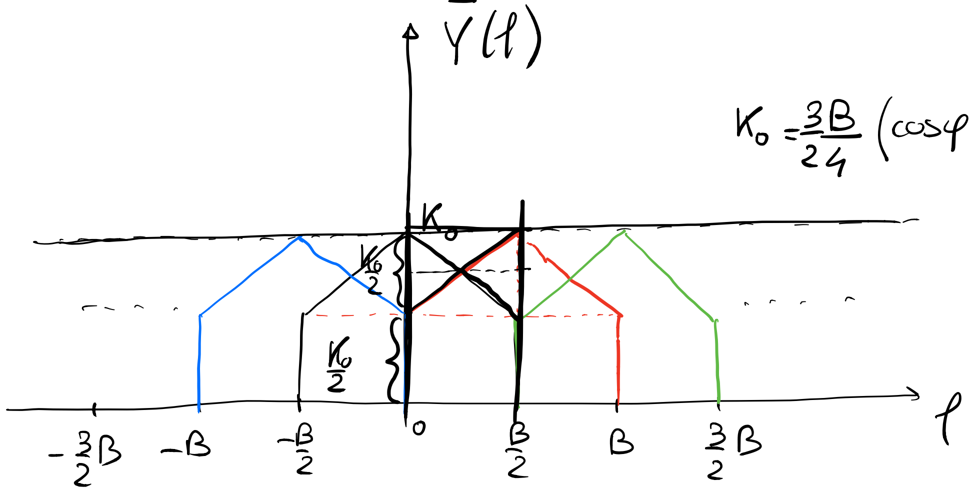
$$Y(f) = Y_1(f) + Y_2(f)$$



$$y(t) \xrightarrow{T=\frac{2}{B}} y[n]$$

$$\bar{Y}(f) = \mathrm{TFS}\left[y[n]\right] = \frac{1}{T}\sum_{n=-\infty}^{+\infty} Y\left(f-\frac{n}{T}\right)$$

$$= \frac{B}{2}\sum_{n=-\infty}^{+\infty} Y\left(f-n\frac{B}{2}\right)$$



$$K_0 = \frac{3B}{24}\left(\cos\varphi + \sin\varphi\right)$$

$$\bar{Y}(f) = K_0$$

$$y[n] = K_0\,\delta[n]$$

$$y[n] \longrightarrow \boxed{p(t)} \longrightarrow z(t)$$

$$z(t) = \sum_{n=-\infty}^{+\infty} y[n]\, p(t-nT)$$

$$= K_0\, p(t) = \boxed{K_0 B \operatorname{sinc}(Bt)}$$

2) $E_2 = \int_{-\infty}^{+\infty} z^2(t)\,dt = \int_{-\infty}^{+\infty} |Z(f)|^2\,df$

$Z(f) = K_0 \,\mathrm{rect}\left(\frac{f}{B}\right)$

$$\boxed{E_2 = K_0^2 B}$$

3) trovare φ che massimizza E_2

$K_0 = \frac{3B}{24}\left(\cos\varphi + \sin\varphi\right)$

$\hat{\varphi} = \arg\max_{\varphi} E_2 = \arg\max_{\varphi} K_0^2 = \arg\max_{\varphi} (\cos\varphi + \sin\varphi)^2$

$K_0^2 = \frac{9B^2}{416}\left(\cos\varphi + \sin\varphi\right)^2$

$(\cos\varphi + \sin\varphi)^2 = \cos^2\varphi + \sin^2\varphi + 2\sin\varphi\cos\varphi$
$= 1 + \sin(2\varphi)$

$\hat{\varphi} = \arg\max_{\varphi} \left(1 + \sin(2\varphi)\right) = \arg\max_{\varphi} \left(\sin(2\varphi)\right)$

$2\varphi = \frac{\pi}{2} \Rightarrow \boxed{\hat{\varphi} = \frac{\pi}{4}}$

ES #1 20/02/2018

$w(t)$ bianco di banda B

$$S_w(f) = \frac{N_0}{2} \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$h(t) = \delta(t) + \delta(t-T)$$

$$w(t) \longrightarrow \underset{\text{SLS}}{\overset{\text{LTI}}{\boxed{h(t)}}} \longrightarrow n_u(t)$$

$$H(f) \begin{cases} \nearrow |H(f)| \\ \searrow \angle H(f) \end{cases}$$

$$H(f) = 1 + e^{-j2\pi fT} = 2e^{-j\pi fT}\left(\frac{e^{j\pi fT} + e^{-j\pi fT}}{2}\right)$$

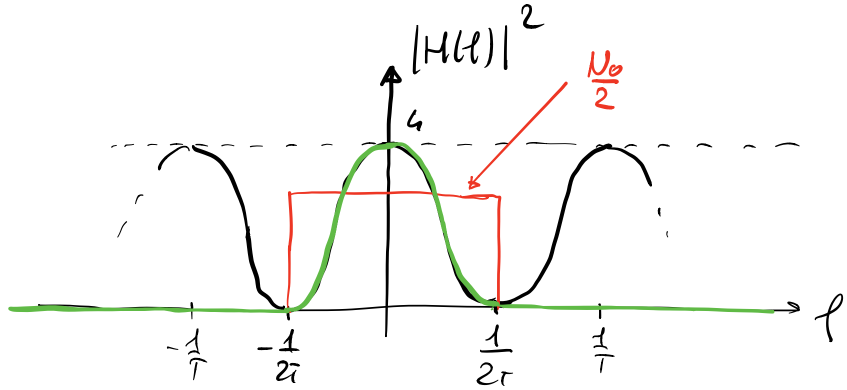
$$= \underbrace{2\cos(\pi fT)}\, e^{-j\underbrace{\pi fT}}$$

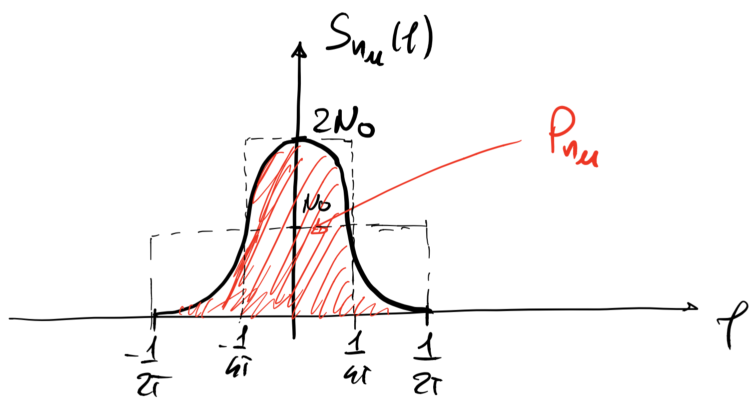
$$\boxed{\begin{aligned} |H(f)| &= 2\cos(\pi fT) \\ \angle H(f) &= -\pi fT \end{aligned}}$$

2) P_{n_u}, $B = \frac{1}{2T}$

$$S_{n_u}(f) = S_w(f)\,|H(f)|^2$$

$$P_{n_u} = \int_{-\infty}^{+\infty} S_{n_u}(f)\,df$$



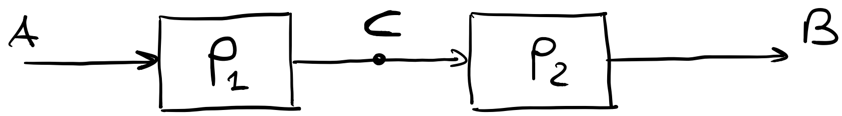


$$\boxed{P_{n_u} = 2N_0 \cdot \frac{1}{2T} = \frac{N_0}{T}}$$

$$P_{n_u} = 2N_0 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \cos^2(\pi f T)\,df = 2N_0 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left[\frac{1}{2} + \frac{1}{2}\cos(2\pi f T)\right] df$$

$$= N_0 \cdot \frac{1}{T} = \frac{N_0}{T}$$

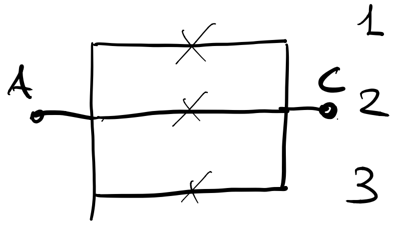
ES #1 20/09/2019



è un parallelo

$$P_2 = \frac{1}{5}$$

$$P_1 = ?$$



$P_{11} = P\{$ramo #1 non conduca$\}$

$P_{12} = P\{$ramo #2 " " $\}$

$P_{13} = P\{$ramo #3 " " $\}$

$$P_1 = P_{11} \cdot P_{12} \cdot P_{13}$$ essendo i rami indipend.

$$P_{11} = 1 - \left[(1 - P_a)(1 - P_b) \right]$$

prob. che il primo relè conduca

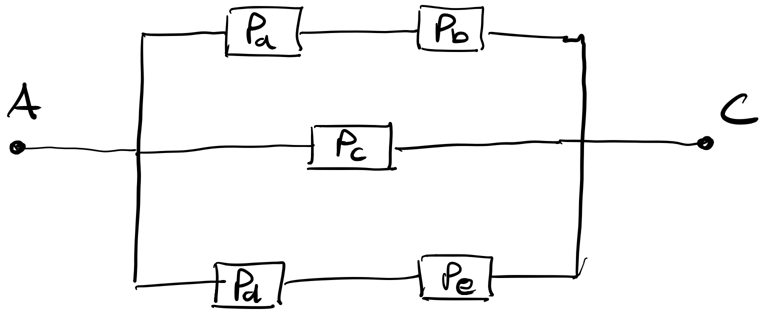
prob. che il secondo relè conduca

$P_{12} = P_c = P\{$il relè nel secondo ramo non conduca$\}$

$$P_{13} = 1 - \left[(1 - P_d)(1 - P_e) \right]$$

prob che il primo relè conduca

prob che il secondo relè conduca



$$\boxed{P_1 = \left[1 - \left[(1-P_a)(1-P_b)\right]\right] \cdot P_c \cdot \left[1 - (1-P_d)(1-P_e)\right]}$$

$$\boxed{P_{AB} = (1-P_1)(1-P_2)}$$

$$P_1 = \left[1 - \left(1-\frac{1}{8}\right)\left(1-\frac{1}{6}\right)\right] \frac{1}{4} \left[1 - \left(1-\frac{1}{10}\right)\left(1-\frac{1}{5}\right)\right]$$

$$= \left[1 - \frac{7}{8}\cdot\frac{5}{6}\right] \frac{1}{4} \left[1 - \frac{9}{\cancel{10}_{5}}\cdot\frac{\cancel{4}^{2}}{5}\right]$$

$$= \left[1 - \frac{35}{48}\right] \frac{1}{4} \left[1 - \frac{18}{25}\right]$$

$$= \frac{13}{48}\cdot\frac{1}{4}\cdot\frac{7}{25} = \frac{91}{4800} \simeq 0.01896$$

$$P_{AB} = \frac{4709}{\cancel{4800}_{1200}}\cdot\frac{\cancel{4}^{1}}{5} = \frac{4709}{6000} \simeq \boxed{0.785}$$