Esame 17/07/08

Esercizio 1

• $X_z(f)$ nel caso a)

$$x_z(t) = x(t) \otimes h_0(t) \rightleftharpoons X_z(f) = X(f)\,H_0(f) = \left\{\left(1-\frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)\right\} \cdot 1 = \left(1-\frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)$$

• $X_2(f)$ nel caso b)

$$X_2(f) = \left(1-\frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right) e^{-j2\pi\frac{T}{2}f}$$

• $z(t)$ nel caso a)

$y_1(t) \triangleq x_1[n] \otimes p_1(t)$

$\bar{X}_1(f) = 1$; $P_1(f) = \text{rect}\left(\frac{f}{2B}\right)e^{-j2\pi\frac{T}{2}f}$

$$Y_1(f) = \text{rect}\left(\frac{f}{2B}\right)e^{-j2\pi\frac{T}{2}f}$$

$y_2(t) \triangleq x_2[n] \otimes p_2(t)$;

$\bar{X}_2(f) = 1$; $P_2(f) = \text{rect}\left(\frac{f}{2B}\right)$

$$Y_2(f) = \text{rect}\left(\frac{f}{2B}\right)$$

$$Z(f) = \text{rect}\left(\frac{f}{2B}\right)e^{-j2\pi\frac{T}{2}f} + \text{rect}\left(\frac{f}{2B}\right) = 2\,\text{rect}($$

$$z(t) = 2B\,\text{sinc}\left[2B\left(t-\frac{T}{2}\right)\right] + 2B\,\text{sinc}(2Bt)$$

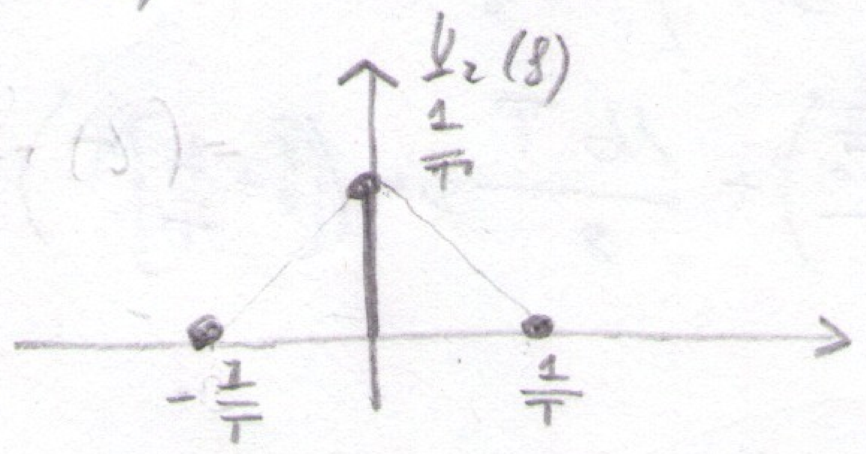
- $z(t)$ nel caso b)

$Y_1(f) = \text{rect}\left(\frac{f}{2B}\right) e^{-j2\pi\frac{T}{2}f} \;\rightleftharpoons\; 2B\,\text{sinc}(2Bt) \otimes \delta\left(t-\frac{T}{2}\right) = y_1(t)$

$x_2(t) = x(t) \otimes h_0(t) = B\,\text{sinc}^2(Bt) \otimes \delta\left(t-\frac{T}{2}\right) = B\,\text{sinc}^2\left(B\left[t-\frac{T}{2}\right]\right)$

$\overline{X}_2(f) = \frac{1}{T}\sum_n X_2\left(\frac{n}{T}\right)\delta\left(f-\frac{n}{T}\right) = \frac{1}{T}\sum_n \left(1-\frac{|\frac{n}{T}|}{B}\right)\text{rect}\left(\frac{\frac{n}{T}}{2B}\right)\delta\left(f-\frac{n}{T}\right)$

$Y_2(f) \triangleq \overline{X}_2(f) H_2(f)$; $H_2(f) = \text{rect}\left(\frac{f}{\frac{2}{T}}\right)$



$Y_2(f) = \frac{1}{T}\delta(f)$

$y_2(t) = \frac{1}{T}$

$z(t) = 2B\,\text{sinc}\left[2B\left(t-\frac{T}{2}\right)\right] + \frac{1}{T}$

- replica fedele? in nessuno dei due casi!
- energia $z(t)$ nel caso a)

$E_z = \int_{-\infty}^{\infty} |z(t)|^2 dt = \int_{-\infty}^{\infty} |Z(f)|^2 df = \int_{-\infty}^{\infty} \left|\text{rect}\left(\frac{f}{2B}\right)\left[\cos(\pi T f) + j\,\text{sen}(\pi T f)\right] + \text{rect}\left(\frac{f}{2B}\right)\right|^2 df =$

$= \int_{-B}^{B} \cos^2(\pi T f)\, df + \int_{-B}^{B} 1 = \emptyset + 2B = 2B$

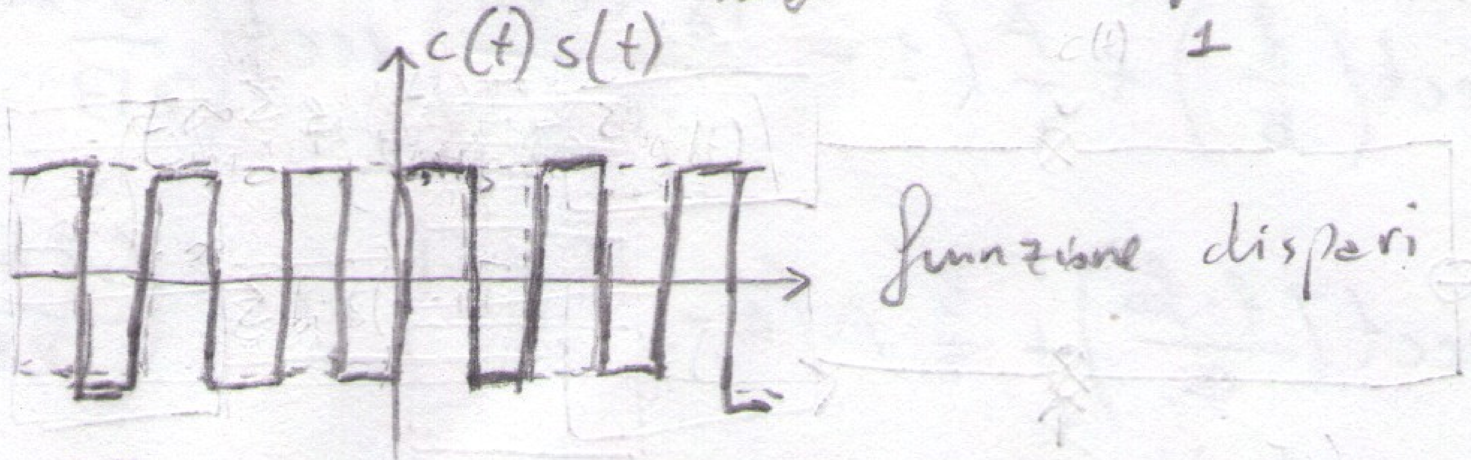
- energia $z(t)$ nel caso b)

$E_z = \int_{-\infty}^{\infty} \left|\text{rect}\left(\frac{f}{2B}\right) e^{-j2\pi\frac{T}{2}f} + \frac{1}{T}\delta(f)\right|^2 df = \int_{-B}^{B}\cos^2(\pi T f) + \int_{-\infty}^{\infty}\frac{1}{T^2}\delta(f)\, df = +\infty$

Esercizio 2

$x(t) = m_c(t)\cdot c(t) - m_s(t)\, s(t) =$

$\overline{E}_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} m_c^2(t)\underbrace{c^2(t)}_{1} dt + \int_{-\infty}^{\infty} m_s^2(t)\underbrace{s^2(t)}_{1} dt - 2\int_{-\infty}^{\infty} m_c(t)\, m_s(t)\, c(t)\, s(t)\, dt =$



funzione dispari

$\int_{-\infty}^{\infty} \sum_i a_i\, g_T(t-iT) \sum_k b_k\, g_T(t-kT)\, s(t)\, c(t)\, dt = \sum_i\sum_k a_i b_k \int_{-\infty}^{\infty} g_T(t-iT)\, g_T(t-kT)\, s(t) c(t)\, dt$

$= \sum_i\sum_k a_i b_k \int_{-\frac{T}{2}}^{\frac{T}{2}} \underbrace{\left[\cos\left(\frac{\pi t}{T}\right)\otimes\delta(t-iT)\right]\left[\cos\left(\frac{\pi t}{T}\right)\otimes\delta(t-kT)\right]}_{\text{sopravvive solo se } k=i \text{ e questa funzione } \boxed{\text{è pari}}} \underbrace{s(t)\, c(t)}_{\text{dispari}}\, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \text{dispari}(t)\, dt = \emptyset$

$\otimes \to$ dispari

$\overline{E}_x = \sum_i a_i \int_{-\infty}^{\infty} g_T^2(t-iT)\, dt + \sum_i b_i \int_{-\infty}^{\infty} g_T^2(t-iT)\, dt = \overline{E}_{m_c(t)} + \overline{E}_{m_s(t)} =$

$\overline{E}_{m_c(t)} = P_{m_c}\cdot T$; $P_{m_c} = \int_{-\infty}^{\infty} S_{m_c}(f)\, df$; $S_{m_c}(f) = \frac{E\{a_i^2\}}{T}|G_T(f)|^2$; $P_{m_c} = \frac{1}{T}\int_{-\infty}^{\infty}|G_T(f)|^2 df =$

$\overline{E}_{m_c(t)} = \int_{-\infty}^{\infty}|g_T(t)|^2 dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos^2\left(\frac{\pi t}{T}\right) dt = \frac{T}{2} + \frac{1}{2}\left|\frac{\text{sen}\left(\frac{\pi t}{T}\right)}{\frac{\pi}{T}}\right|_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{T}{2} + \frac{T}{\pi} = \overline{E}_{m_s(t)}$

$\overline{E}_x = T + 2T\pi$

$$R_n(\tau) \rightleftharpoons S_n(f) = S_w(f)\,|G_R(f)|^2 = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \mathrm{rect}\left(\frac{f+f_0}{B}\right)\right] \cdot \mathrm{TCF}\left[\cos^2\left(\frac{\pi t}{T}\right)\right] \otimes T\,\mathrm{sinc}(fT)$$

$$S_n(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \mathrm{rect}\left(\frac{f+f_0}{B}\right)\right]\left\{\left[\frac{1}{2}\delta(f) + \frac{1}{4}\delta\left(f-\frac{1}{T}\right) + \frac{1}{4}\delta\left(f+\frac{1}{T}\right)\right] \otimes T\,\mathrm{sinc}(fT)\right\} =$$

$$= \left[\frac{N_0}{2}\,\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \frac{N_0}{2}\,\mathrm{rect}\left(\frac{f+f_0}{B}\right)\right] \cdot \left[\frac{T}{2}\mathrm{sinc}(fT) + \frac{T}{4}\mathrm{sinc}\left(\left[f-\frac{1}{T}\right]T\right) + \frac{T}{4}\mathrm{sinc}\left(\left[f+\frac{1}{T}\right]T\right)\right] =$$

$$= \frac{N_0 T}{4}\mathrm{sinc}(fT)\,\mathrm{rect}\left(\frac{f-\frac{1}{T_0}}{B}\right) + \frac{N_0 T}{8}\mathrm{sinc}\left(\left[f-\frac{1}{T}\right]T\right)\mathrm{rect}\left(\frac{f-\frac{1}{T_0}}{B}\right) + \frac{N_0 T}{8}\mathrm{sinc}\left(\left[f+\frac{1}{T}\right]T\right)\mathrm{rect}\left(\frac{f-\frac{1}{T_0}}{B}\right) +$$

$$+ \frac{N_0 T}{4}\mathrm{sinc}(fT)\,\mathrm{rect}\left(\frac{f+\frac{1}{T_0}}{B}\right) + \frac{N_0 T}{8}\mathrm{sinc}\left(\left[f-\frac{1}{T}\right]T\right)\mathrm{rect}\left(\frac{f+\frac{1}{T_0}}{B}\right) + \frac{N_0 T}{8}\mathrm{sinc}\left(\left[f+\frac{1}{T}\right]T\right)\mathrm{rect}\left(\frac{f+\frac{1}{T_0}}{B}\right)$$

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