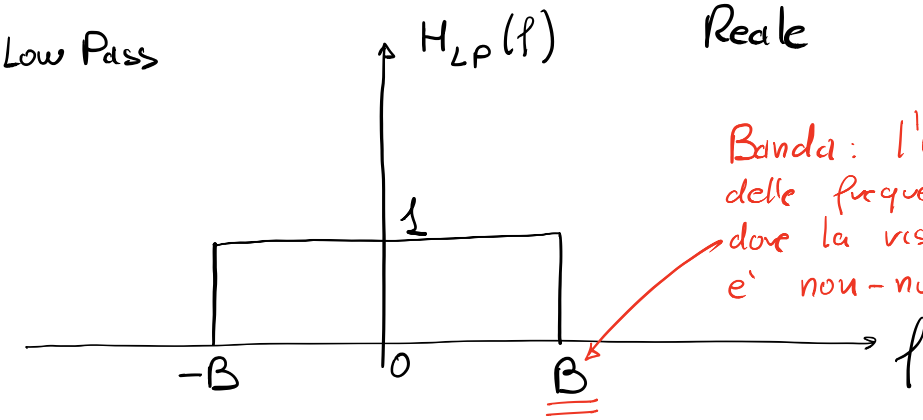
FILTRI IDEALI

·) PASSA BASSO DI BANDA B

LP= Low Pass

Reale



Banda: l'intervallo delle frequenze positive dove la risposta in freq. è non-nulla.

$$H_{LP}(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

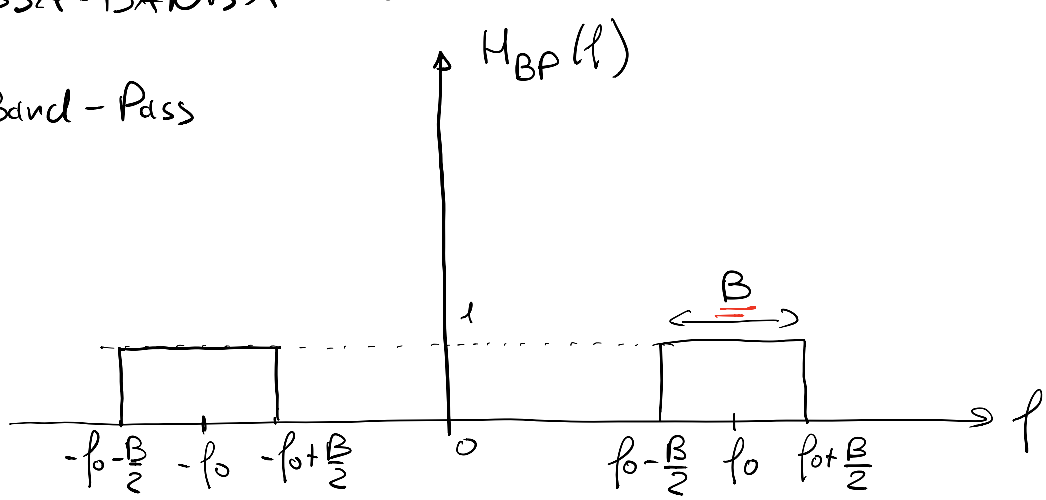
RISPOSTA IN FREQ DI UN LP DI BANDA B

$$h_{LP}(t) = \mathrm{ATCF}\left[H_{LP}(f)\right] = 2B\,\mathrm{sinc}(2Bt)$$

RISPOSTA IMPULSIVA

·) PASSA-BANDA DI BANDA B

BP= Band-Pass



$$H_{BP}(f) = \text{rect}\left(\frac{f-f_0}{B}\right) + \text{rect}\left(\frac{f+f_0}{B}\right)$$

RISPOSTA IN FREQUENZA DI UN BP IDEALE DI BANDA B

$$h_{BP}(t) = B\,\text{sinc}(Bt)\,e^{j2\pi f_0 t} + $$

$$+ B\,\text{sinc}(Bt)\,e^{-j2\pi f_0 t}$$

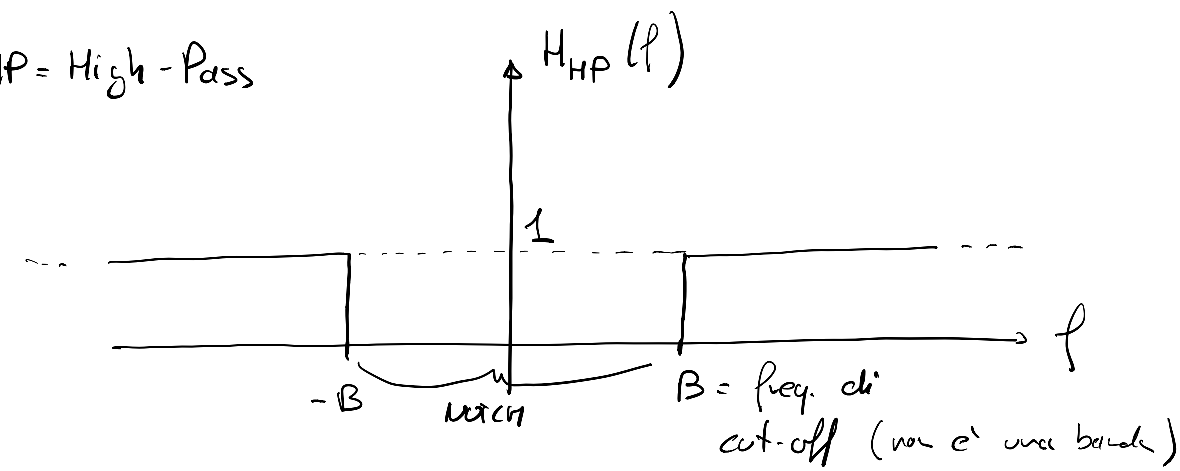
$$= 2B\,\text{sinc}(Bt)\cos(2\pi f_0 t)$$

RISPOSTA IMPULSIVA

fattore $Q = \frac{f_0}{B}$

→ PASSA - ALTO IDEALE

HP = High - Pass



B = banda del 'notch'

$$H_{HP}(f) = 1 - \text{rect}\left(\frac{f}{2B}\right)$$

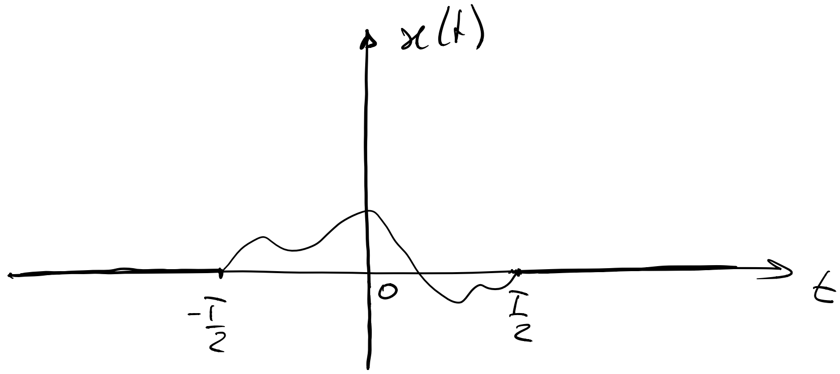
RISPOSTA IN FREQUENZA DI UN FILTRO HP IDEALE

$$h_{HP}(t) = \delta(t) - 2B\,\text{sinc}(2Bt)$$

RISPOSTA IMPULSIVA

DURATA E BANDA DI UN SEGNALE

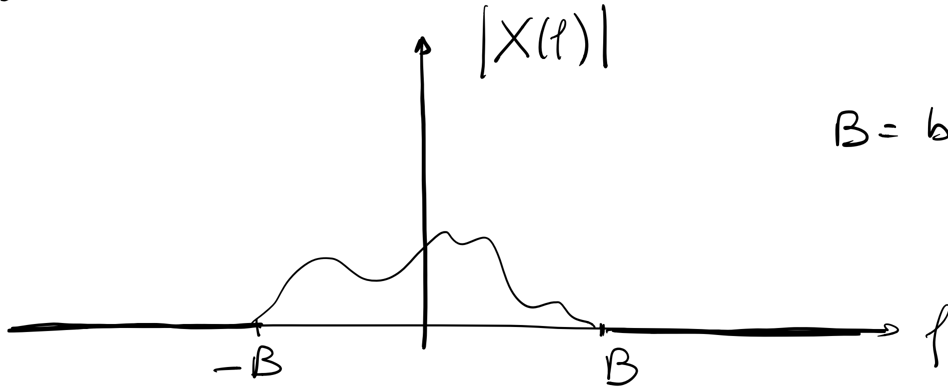
- Segnale a durata "rigorosamente" limitata



T = durata

$$x(t) = 0 \quad \text{per} \quad |t| > \frac{T}{2}$$

- Segnale a banda "rigorosamente" limitata



B = banda

$$|X(f)| = 0 \quad |f| > B$$

PROPRIETÀ

$x(t)$ ha durata rigorosamente limitata

$\Downarrow$

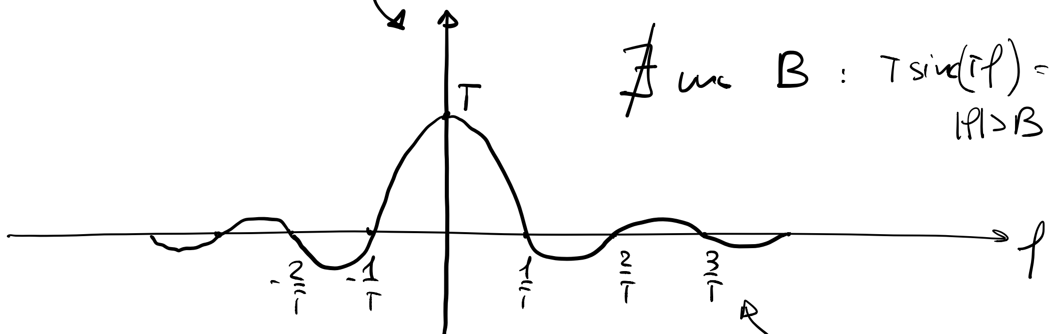
$X(f)$ ha banda infinita

Dimostrazione

se il segnale ha durata T

$$X(f) = TCF[x(t)] = TCF\left[x(t)\, rect\left(\frac{t}{T}\right)\right]$$

$$= \underbrace{X(f)} \otimes \underbrace{T\, sinc(Tf)} = X(f)$$



$\nexists$ un B : $T\, sinc(Tf) = 0 \quad |f| > B$

Banda illimitata

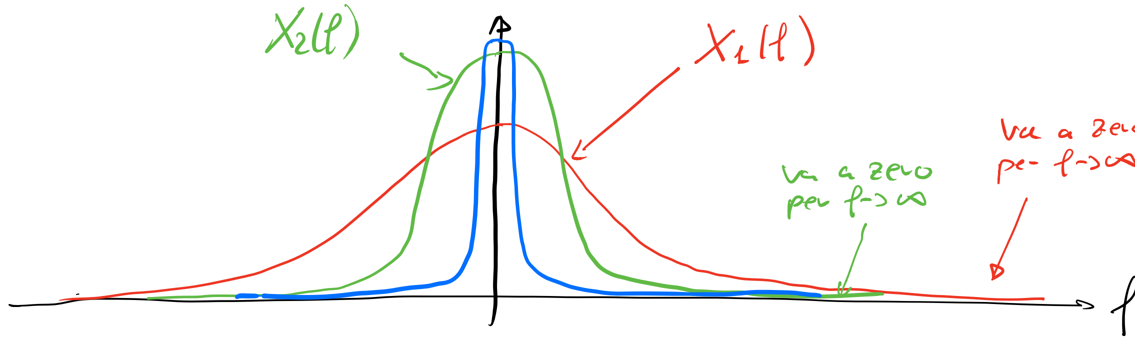
supponiamo "per assurdo" che

$$|X(f)| = 0 \qquad |f| > B$$

$X(f) \otimes T\, sinc(Tf) \Rightarrow$ spettro a banda illimitata

$X(f) \otimes T\, sinc(Tf) = X(f)$ assurdo

$\Rightarrow X(f)$ ha banda illimitata



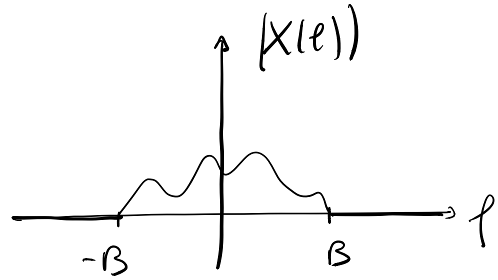
Serve una definizione di banda che possa tener conto della distribuzione frequenziale di un segnale fisico, poiché la definizione di banda rig. limitata non è sufficiente per caratterizzare segnali fisici.

-) PROPRIETÀ DI SEGNALI A BANDA RIG. LIMITATA

$$\boxed{\begin{array}{c} X(f) \text{ ha banda rigorosamente limitata} \\ \Downarrow \\ x(t) \text{ ha durata infinita} \end{array}}$$

Dim.

$$x(t) = \text{ATCF}\left[X(f)\right] = \text{ATCF}\left[X(f)\,\text{rect}\left(\frac{f}{2B}\right)\right]$$



$$= x(t) \otimes \underbrace{2B\,\text{sinc}(2Bt)}_{\text{durata infinita}} = x(t)$$

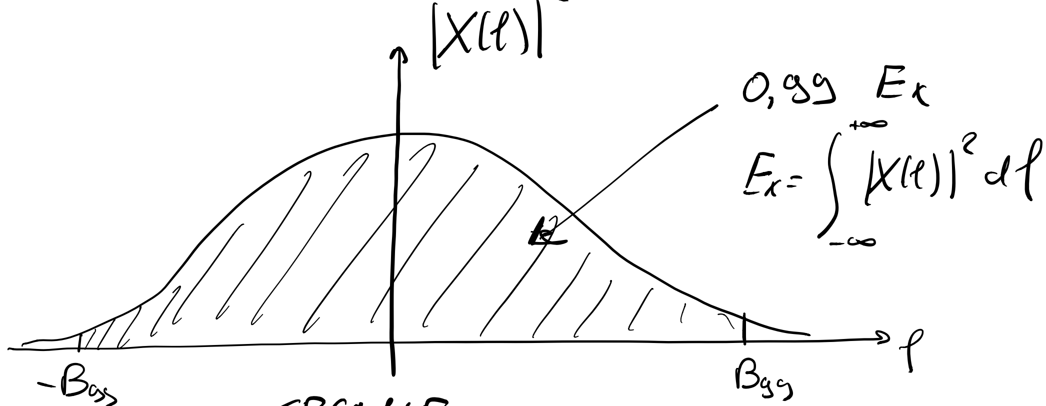
$\Rightarrow x(t)$ deve avere durata infinita

N.B. anche questo è dimostrabile per assurdo

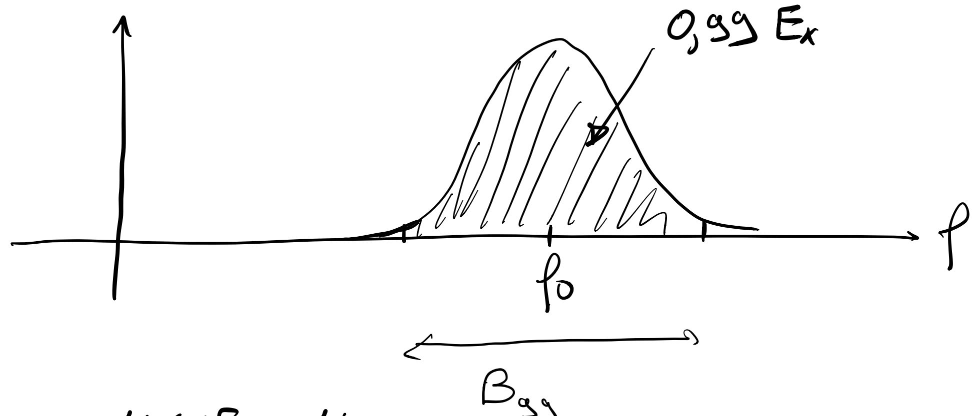
DEFINIZIONI "PRATICHE" DI BANDA

·) Banda al 99% dell'energia

$$B_{99} : \int_{-B_{99}}^{B_{99}} |X(f)|^2 \, df = 0{,}99 \, E_x$$



SEGNALE PASSA-BASSO (IN BANDA BASE)



SI APPLICA ANCHE AI SEGNALI "PASSA-BANDA" (IN BANDA PASSANTE)

·) BANDA A - 3dB

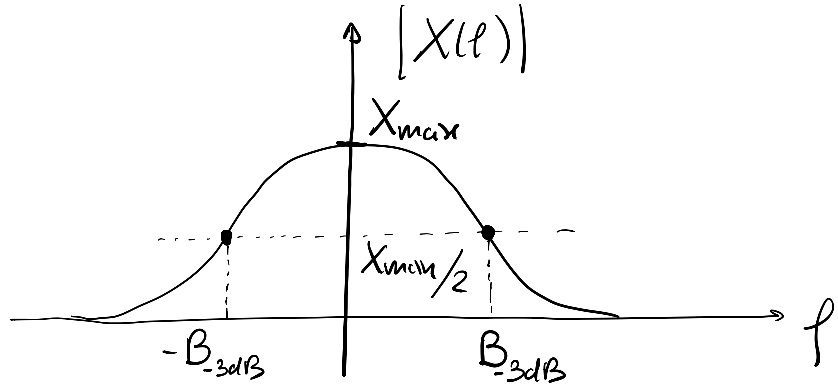
$$dB \triangleq 10 \log_{10} (\cdot)$$

energia	J
potenza	W
tensione	V
distanza	m

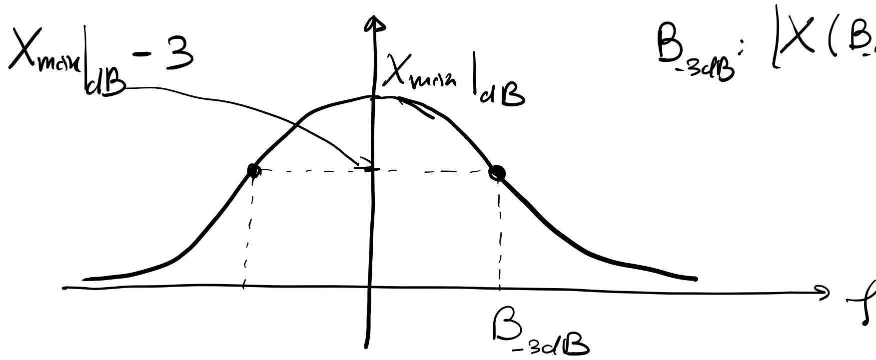
il dB comprime la dinamica di una certa grandezza

$-3dB \Rightarrow \frac{1}{2}$ in lineare

$$10 \log_{10} \frac{1}{2} \simeq -3$$

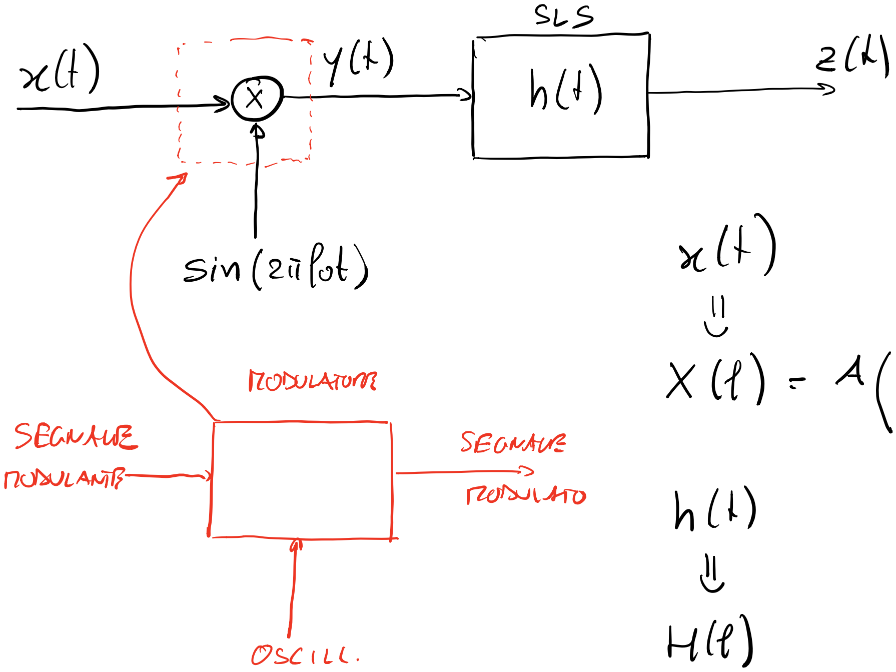


$$B_{-3dB} : \left| X(B_{-3dB}) \right| = \frac{X_{max}}{2}$$



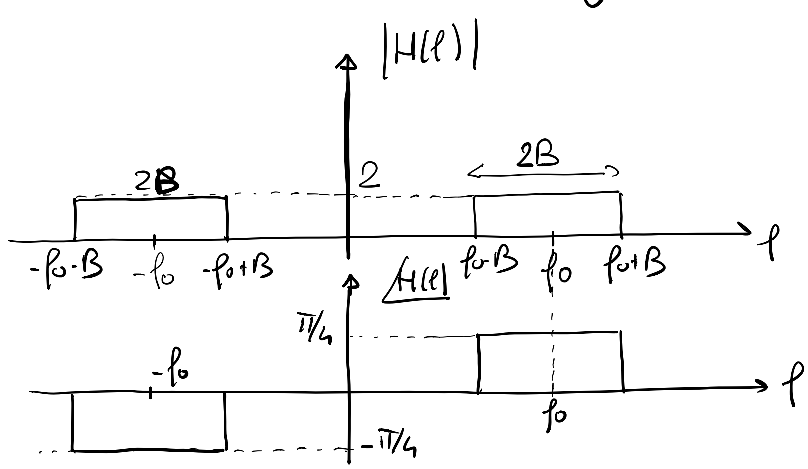
$$B_{-3dB}: |X(B_{-3dB})| = X_{max}|_{dB} - 3$$

) ESERCIZIO - MODULAZIONE E FILTRAGGIO



$$x(t)$$
$$\Downarrow$$
$$X(f) = A\left(1 - \frac{|f|}{B}\right) \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$h(t)$$
$$\Downarrow$$
$$H(f)$$
$$\Downarrow$$



Calcolare

1) L'espressione analitica della $z(t)$

2) L'Energia della $z(t)$

Soluzione

1) $z(t) = y(t) \otimes h(t)$

$$z(t) = \text{ATCF}\left[Z(f) \right] = \text{ATCF}\left[Y(f)\, H(f) \right]$$

$$y(t) = x(t) \sin(2\pi f_0 t)$$

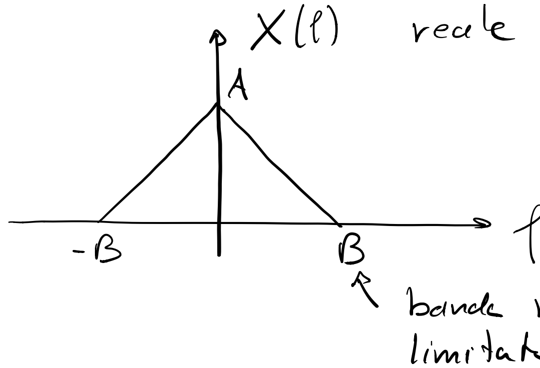
$$\Rightarrow \int_{-\infty}^{+\infty} \underset{\substack{\uparrow \\ \text{sinc} \\ \text{sin}}}{y(\tau)}\; \underset{\substack{\uparrow \\ \text{sinc} \\ \text{sin}}}{h(t-\tau)}\, d\tau$$

questa strada è molto ostica!!

·) in freq.

$$Y(f) = \frac{1}{2j} X(f-f_0) - \frac{1}{2j} X(f+f_0)$$

$$X(f) = A\left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$$



reale

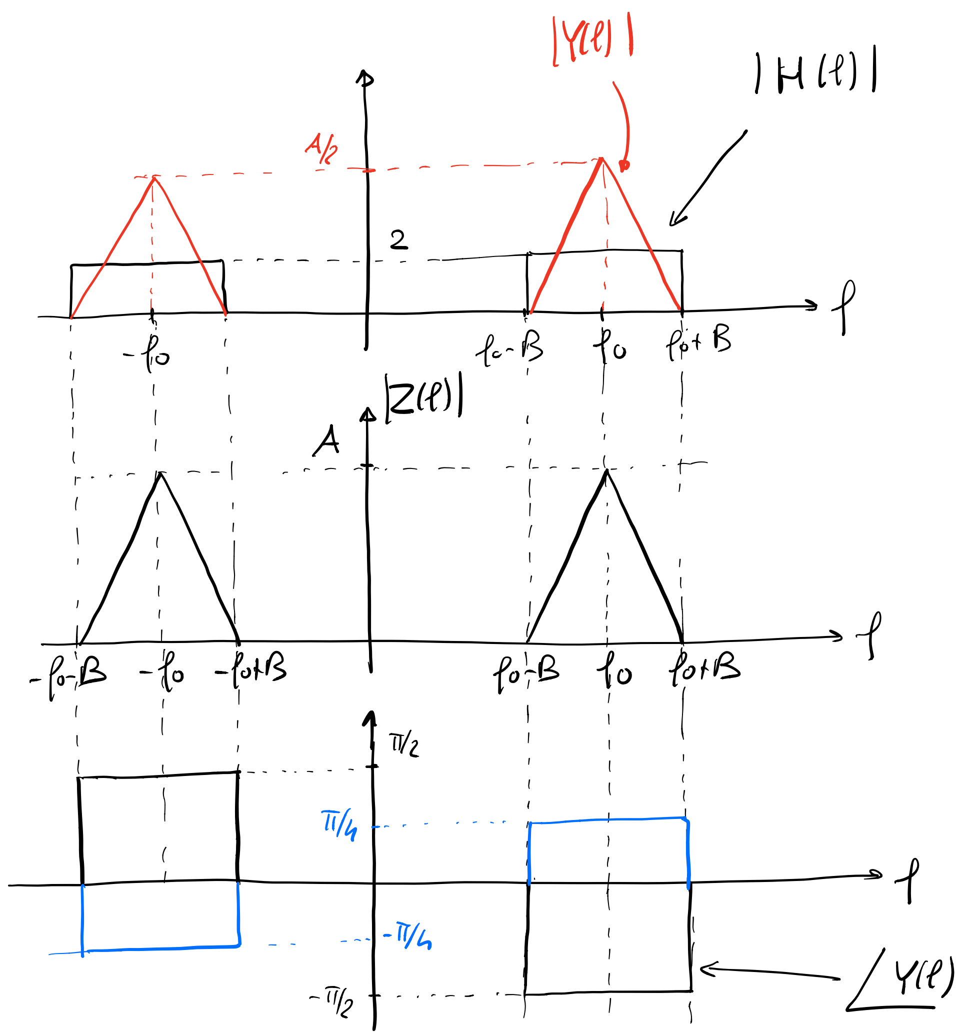
banda rigorosamente limitata "B"

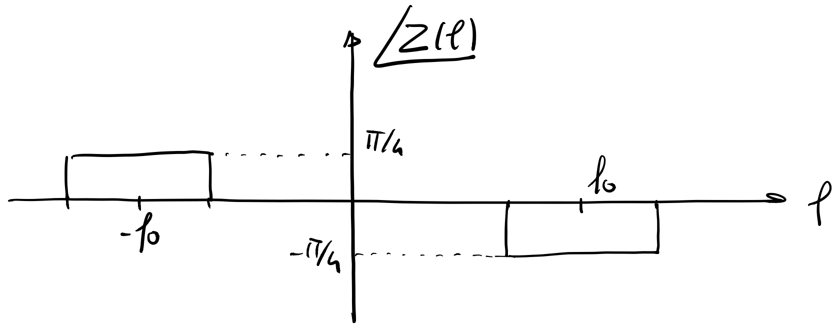
$$Y(f) = \frac{A}{2j}\left(1 - \frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right) - \frac{A}{2j}\left(1 - \frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$

$$Z(f) = Y(f)\, H(f)$$

$$Z(f) = |Z(f)|\, e^{j\angle Z(f)} = |Y(f)|\,|H(f)|\, e^{j(\angle Y(f) + \angle H(f))}$$

$|Z(f)|$





$$Z(f) = |Z(f)|\, e^{j \angle Z(f)} = A\left(1 - \frac{|f+f_0|}{B}\right) \mathrm{rect}\left(\frac{f+f_0}{2B}\right) e^{j\frac{\pi}{4}}$$

$$+ A\left(1 - \frac{|f-f_0|}{B}\right) \mathrm{rect}\left(\frac{f-f_0}{2B}\right) e^{-j\frac{\pi}{4}}$$

$$z(t) = \mathrm{ATCF}\left[Z(f)\right]$$

$$Z_0(f) = A\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Z(f) = Z_0(f + f_0)\, e^{j\frac{\pi}{4}} + Z_0(f - f_0)\, e^{-j\frac{\pi}{4}}$$

$$z(t) = e^{j\frac{\pi}{4}} z_0(t)\, e^{-j2\pi f_0 t} + e^{-j\frac{\pi}{4}} z_0(t)\, e^{j2\pi f_0 t}$$

$$= z_0(t)\left[e^{j\left(2\pi f_0 t - \frac{\pi}{4}\right)} + e^{-j\left(2\pi f_0 t - \frac{\pi}{4}\right)}\right]$$

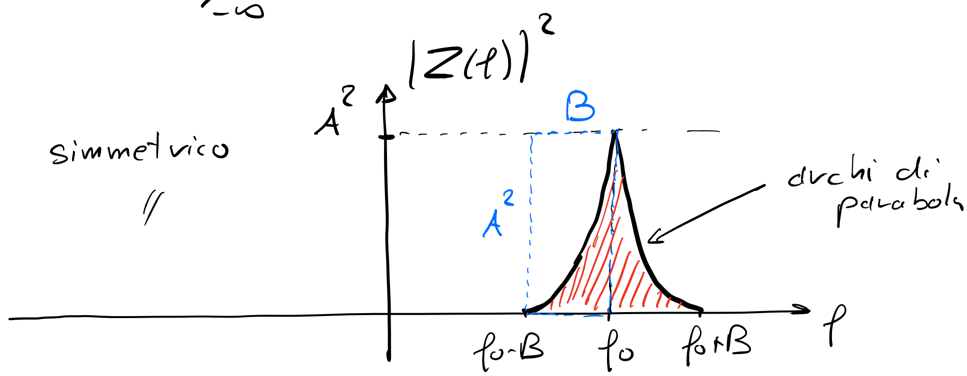
$$= 2 z_0(t) \cos\left(2\pi f_0 t - \frac{\pi}{4}\right)$$

$$z_0(t) = A B\, \mathrm{sinc}^2(Bt)$$

$$\boxed{z(t) = 2AB\, \mathrm{sinc}^2(Bt) \cos\left(2\pi f_0 t - \frac{\pi}{4}\right)}$$

2) $E_z = ?$ → nel dominio del tempo
→ nel dominio della freq.

$$E_z = \int_{-\infty}^{+\infty} |Z(f)|^2 \, df$$



$$E_z = 2\left(2\left(\frac{A^2 B}{3}\right)\right) = \frac{4}{3} A^2 B$$

·) ESERCIZIO – FILTRAGGIO DI SEGNALI PERIODICI

$$x(t) \rightarrow \boxed{h(t)}_{SLS} \rightarrow y(t)$$

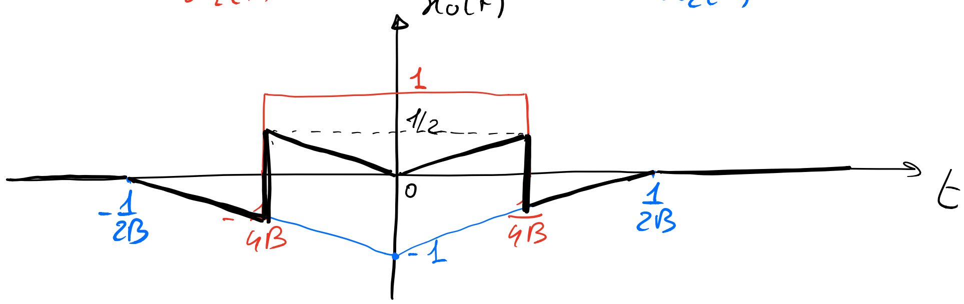
$$x(t) = \sum_{n=-\infty}^{+\infty} \left[\mathrm{rect}\left(\frac{t - \frac{2}{B}n}{\frac{1}{2B}}\right) - \left(1 - \frac{\left|t - \frac{2}{B}n\right|}{\frac{1}{2B}}\right) \mathrm{rect}\left(\frac{t - \frac{2}{B}n}{\frac{1}{B}}\right) \right]$$

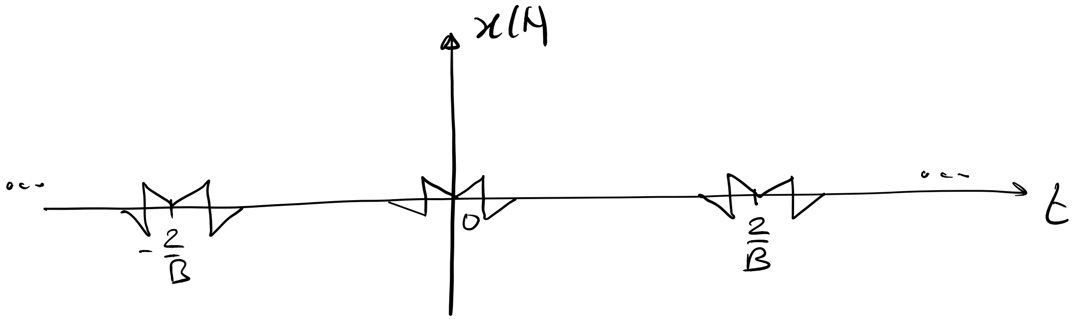
$$h(t) = B \,\mathrm{sinc}^2(Bt)$$

Calcolare
·) $y(t)$
·) P_y, E_y

$$x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - nT_0) \quad , \quad T_0 = \frac{2}{B}$$

$$x_0(t) = \underbrace{\mathrm{rect}\left(\frac{t}{\frac{1}{2B}}\right)}_{x_1(t)} - \underbrace{\left(1 - \frac{|t|}{\frac{1}{2B}}\right)\mathrm{rect}\left(\frac{t}{\frac{1}{B}}\right)}_{x_2(t)}$$



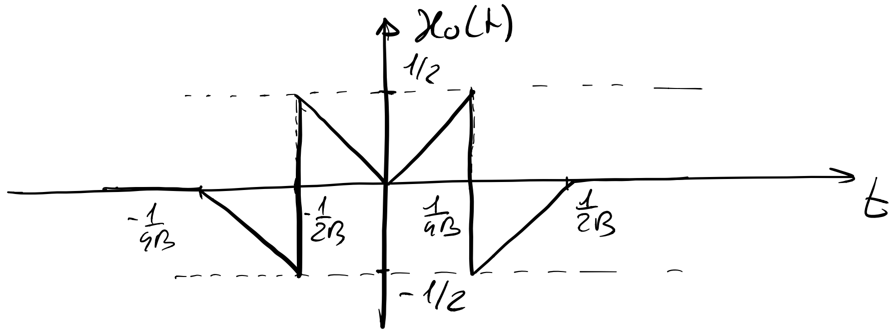


$$y(t) = x(t) \otimes h(t)$$

$$Y(f) = X(f)\, H(f)$$

$$X(f) = \sum_{n=-\infty}^{+\infty} X_n\, \delta\left(f - \frac{n}{T_0}\right)$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right)$$



$$X_0(f) = X_1(f) + X_2(f)$$

$$X_1(f) = \frac{1}{2B} \operatorname{sinc}\left(\frac{f}{2B}\right)$$

$$X_2(f) = -\frac{1}{2B} \operatorname{sinc}^2\left(\frac{f}{2B}\right)$$

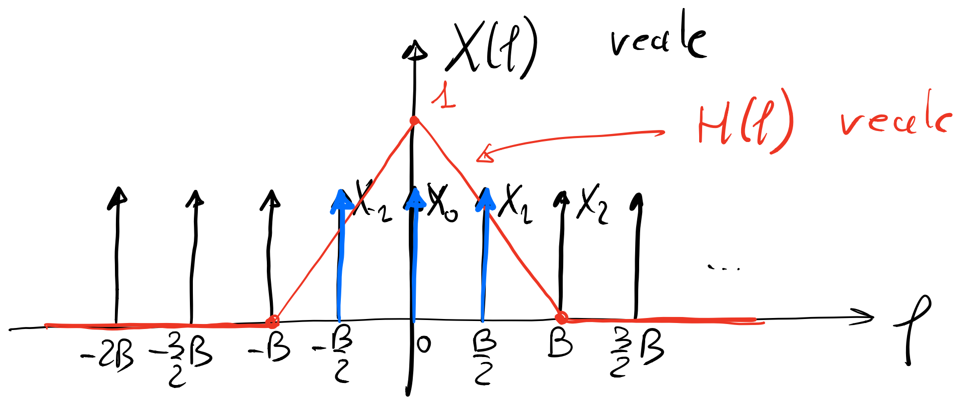
$$X_0(f) = \frac{1}{2B} \left[\operatorname{sinc}\left(\frac{f}{2B}\right) - \operatorname{sinc}^2\left(\frac{f}{2B}\right) \right]$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right) \qquad T_0 = \frac{2}{B}$$

$$= \frac{1}{T_0} X_0\left(\frac{Bn}{2}\right) = \frac{\cancel{B}}{2} \cdot \frac{1}{2\cancel{B}} \left[\operatorname{sinc}\left(\frac{\frac{\cancel{B}n}{2}}{2\cancel{B}}\right) - \operatorname{sinc}^2\left(\frac{\frac{Bn}{2}}{2B}\right) \right]$$

$$= \frac{1}{4} \left[\operatorname{sinc}\left(\frac{n}{4}\right) - \operatorname{sinc}^2\left(\frac{n}{4}\right) \right]$$

$$X(f) = \frac{1}{4} \sum_{n=-\infty}^{+\infty} \left[\operatorname{sinc}\left(\frac{n}{4}\right) - \operatorname{sinc}^2\left(\frac{n}{4}\right) \right] \delta\left(f - \frac{B}{2}n\right)$$



$$H(f) = TCF[h(t)] = \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Y(f) = X_{-1}\,\delta\left(f + \frac{B}{2}\right) + X_0\,\delta(f) + X_1\,\delta\left(f - \frac{B}{2}\right)$$

$$X_{-1} = \frac{1}{4}\left[\mathrm{sinc}\left(\frac{1}{4}\right) - \mathrm{sinc}^2\left(\frac{1}{4}\right)\right] = X_{+1}$$

$$X_0 = \frac{1}{4}\left[1 - 1\right] = 0$$

$$Y(f) = X_1\left[\delta\left(f + \frac{B}{2}\right) + \delta\left(f - \frac{B}{2}\right)\right]$$

$$X_1 = \frac{1}{4}\left[\frac{\sin \pi/4}{\pi/4} - \frac{\sin^2 \pi/4}{\pi^2/16}\right]$$

$$= \frac{1}{4}\left[\frac{\sqrt{2}/2}{\pi/4} - \frac{2/4}{\pi^2/16}\right] = \dots$$

$$y(t) = ATCF\left[Y(f)\right] =$$

$$= 2X_1 \left[\frac{e^{-j2\pi \frac{B}{2} t} + e^{j2\pi \frac{B}{2} t}}{2} \right]$$

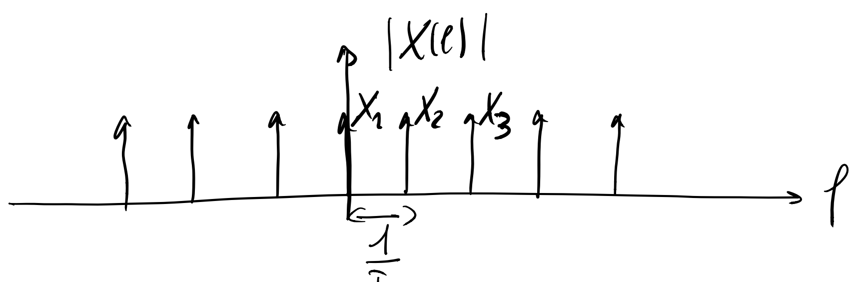
$$= \underbrace{2X_1} \cos(\pi B t)$$

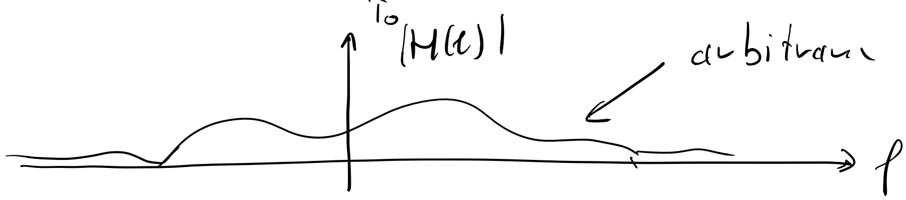
- IN GENERALE SUL FILTRAGGIO DI UN SEGNALE PERIODICO

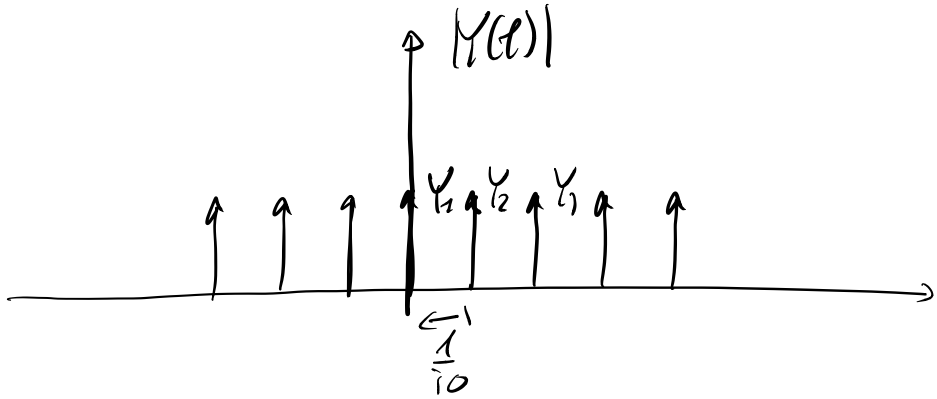
$x(t)$ è periodico di T_0

$\Downarrow$

$y(t)$ è periodico di T_0







$$Y_n = X_n \, H\left(\frac{n}{T_0}\right)$$

$$Y(f) = X(f)\, H(f) =$$

$$= \sum_{n=-\infty}^{+\infty} X_n \, \delta\left(f - \frac{n}{T_0}\right) H(f)$$

$$= \sum_{n=-\infty}^{+\infty} X_n \, \delta\left(f - \frac{n}{T_0}\right) H\left(\frac{n}{T_0}\right)$$

$$= \sum_{n=-\infty}^{+\infty} \underbrace{X_n \, H\left(\frac{n}{T_0}\right)}_{Y_n} \delta\left(f - \frac{n}{T_0}\right)$$

$$= \sum_{n=-\infty}^{+\infty} Y_n \, \delta\left(f - \frac{n}{T_0}\right) \quad , \quad Y_n = X_n \, H\left(\frac{n}{T_0}\right)$$

$y(t) = \text{ATCF}\left[Y(f) \right] \Rightarrow y(t)$ periodico de T_0

$$\boxed{Y(f) = \sum_n Y_n \, \delta\left(f - \frac{n}{T_0}\right) \quad , \quad Y_n = X_n \, H\left(\frac{n}{T_0}\right)}$$