Esercizio 2B

VERS PROVV.

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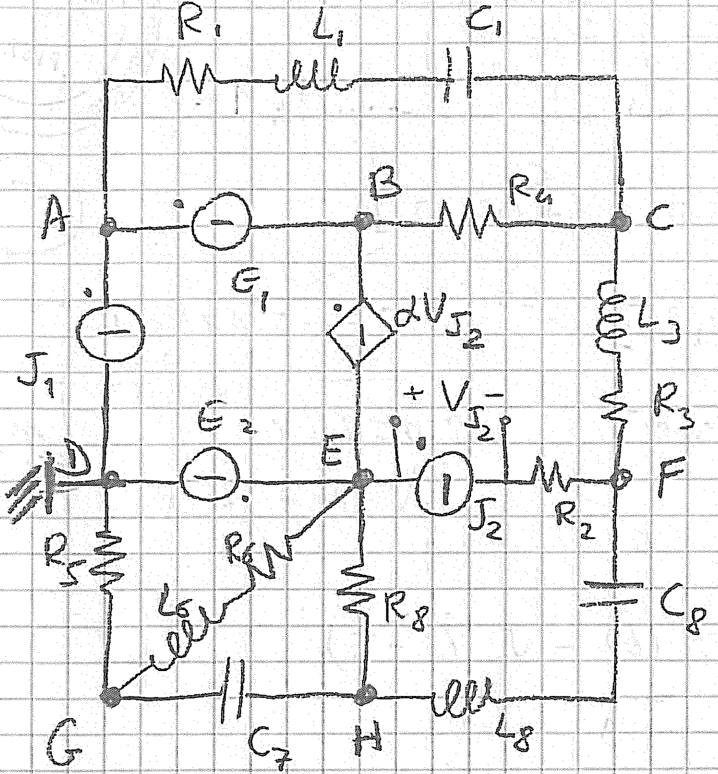

Si sceglie D come nodo di riferimento per le tensioni.

$$\dot V_E = \dot E_2 \;; \quad \dot V_B = \dot E_2 + \alpha \dot V_{J_2} \;; \quad \dot V_A = \dot E_2 + \alpha \dot V_{J_2} + \dot E_1$$

c) $$0 = -\frac{1}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}} \dot V_A - \frac{1}{R_4} \dot V_B + \left(\frac{1}{R_4} + \frac{1}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}} + \frac{1}{R_3 + j\omega L_3}\right) \dot V_C - \frac{1}{R_3 + j\omega L_3} \dot V_F$$

F) $$-\dot J_2 = -\frac{1}{R_3 + j\omega L_3} \dot V_C + \left(\frac{1}{R_3 + j\omega L_3} + \frac{1}{j\omega L_8 + \frac{1}{j\omega C_8}}\right) \dot V_F - \frac{1}{j\omega L_8 + \frac{1}{j\omega C_8}} \dot V_H$$

G) $$0 = -\frac{1}{R_6 + j\omega L_6} \dot V_E + \left(\frac{1}{R_5} + \frac{1}{R_6 + j\omega L_6} + j\omega C_7\right) \dot V_G - j\omega C_7 V_H$$

$$H)\quad 0 = -\frac{1}{R_8}\dot{V}_E - \frac{1}{j\omega L_8 + \frac{1}{j\omega C_8}}\dot{V}_F - j\omega C_7 \dot{V}_G +$$

$$+\left(\frac{1}{R_8} + \frac{1}{j\omega L_8 + \frac{1}{j\omega C_8}} + j\omega C_7\right)\dot{V}_H$$

$$\dot{V}_{J_2} = \dot{V}_E - \dot{V}_F + R_2 \dot{J}_2$$

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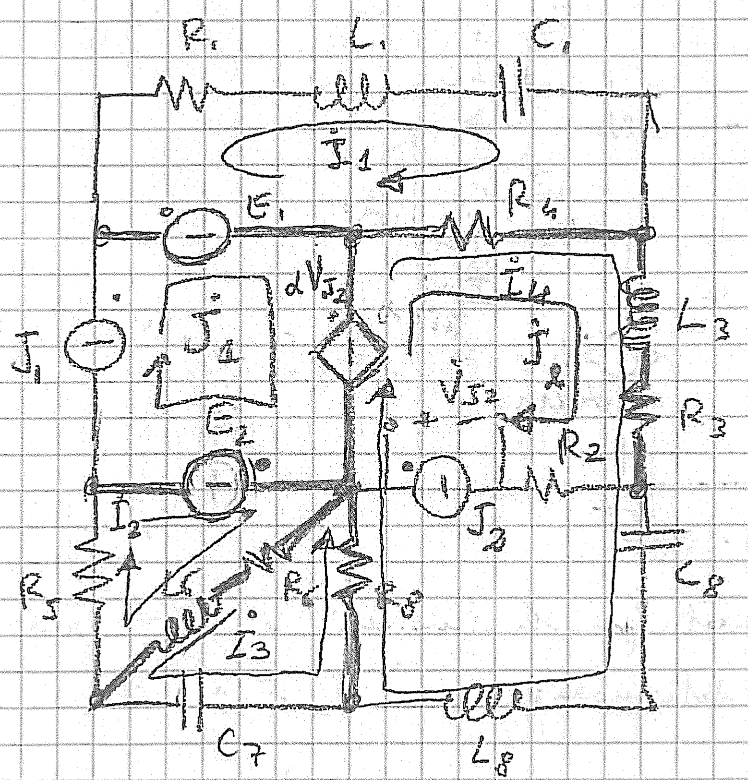
Esercizio 2A



Con riferimento all'albero evidenziato in figura:

$$\dot{E}_1 = \left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}\right)\dot{I}_1 - R_4\dot{I}_4 - R_4\dot{J}_2$$

$$\dot{E}_2 = \left(R_5 + R_6 + j\omega L_6\right)\dot{I}_2 + \left(R_6 + j\omega L_6\right)\dot{I}_3$$

$$0 = \left(R_6 + j\omega L_6\right)\dot{I}_2 + \left(R_6 + j\omega L_6 + R_8 + \frac{1}{j\omega C_7}\right)\dot{I}_3 + R_8\dot{I}_4$$

$$\alpha\dot{V}_{J_2} = -R_4\dot{I}_1 + R_6\dot{I}_3 + \left(R_8 + R_4 + R_3 + j\omega L_3 + \frac{1}{j\omega C_8} + j\omega L_8\right)\dot{I}_4 +$$

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$$+\left(R_4 + R_3 + j\omega L_3\right)\alpha \dot{V}_{J_2}$$

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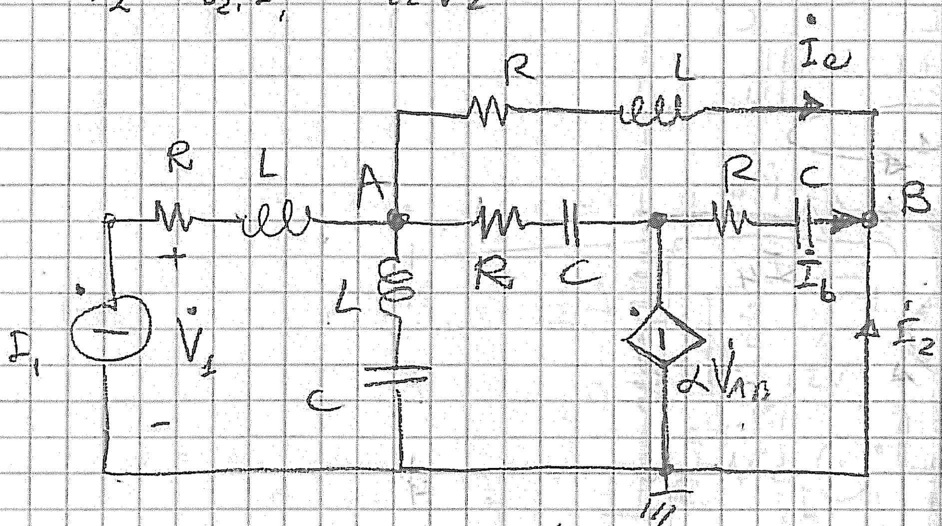
$$\dot{V}_{J_2} = -R_8(\dot{I}_3 + \dot{I}_4) - \left(j\omega L_8 + \frac{1}{j\omega C_8}\right)\dot{I}_4 + R_2 \dot{J}_2$$

Esercizio 3 B

$$\dot{V}_1 = h_{11}\dot{I}_1 + h_{12}\dot{V}_2$$

$$\dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{V}_2$$



Con il riferimento indicato per le tensioni è $V_B = 0$, quindi $\dot{V}_{AB} = \dot{V}_A$ e l'equazione per la determinazione di V_A è:

$$\dot{I}_1 = \dot{V}_A\left(\frac{1}{R + j\omega L} + \frac{1}{R + \frac{1}{j\omega C}} + \frac{1}{j\omega L + \frac{1}{j\omega C}}\right) - \alpha V_A \frac{1}{R + \frac{1}{j\omega C}}$$

$$\dot{V}_A = \frac{1}{\frac{1}{R + j\omega L} + \frac{1}{j\omega L + \frac{1}{j\omega C}} + (1-\alpha)\frac{1}{R + \frac{1}{j\omega C}}}\dot{I}_1 =$$

$$= \bar{K}\dot{I}_1 = (-0.909 + j\,2.727)\,\dot{I}_1$$

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$$\dot{I}_2 = -\dot{I}_a - \dot{I}_b = -\frac{\dot{V}_A}{R+j\omega L} - \alpha\frac{\dot{V}_A}{R+\frac{1}{j\omega C}} =$$

$$= -\left[\frac{\bar{K}}{R+j\omega L} + \frac{\alpha\bar{K}}{R+\frac{1}{j\omega C}}\right]\dot{I}_1$$

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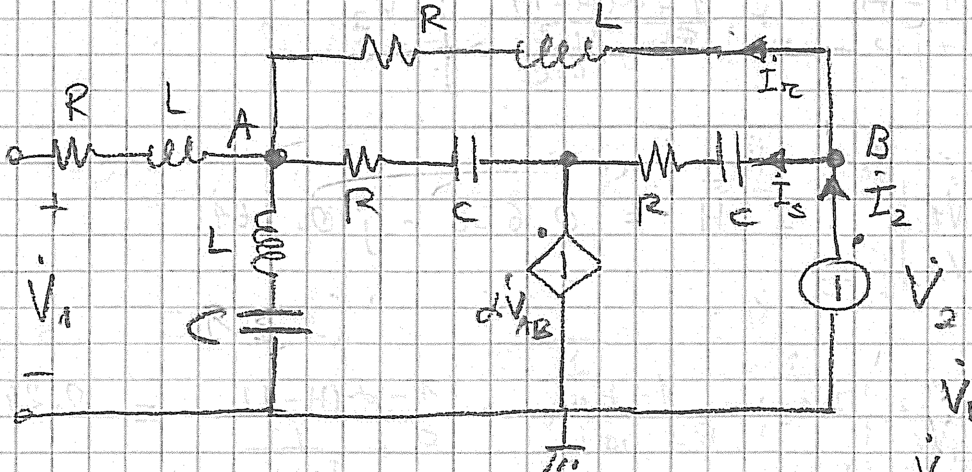
$$\dot{V}_1 = (R+j\omega L)\dot{I}_1 + \dot{V}_A = (R+j\omega L+\bar{K})\dot{I}_1$$

$$h_{11} = \left.\frac{\dot{V}_1}{\dot{I}_1}\right|_{\dot{V}_2=0} = R+j\omega L+\bar{K} = 9.091 + j\,12.727\ \Omega$$

$$h_{21} = \left.\frac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{V}_2=0} = -\left[\frac{\bar{K}}{R+j\omega L} + \frac{\alpha\bar{K}}{R+\frac{1}{j\omega C}}\right] = 0.455 - j\,0.727$$



$$\dot{V}_B = \dot{V}_2$$

$$\dot{V}_{AB} = \dot{V}_A - \dot{V}_2$$

$$0 = \dot{V}_A\left(\frac{1}{R+j\omega L} + \frac{1}{R+\frac{1}{j\omega C}} + \frac{1}{j\omega L+\frac{1}{j\omega C}}\right) - \alpha(\dot{V}_A - \dot{V}_2)\frac{1}{R+\frac{1}{j\omega C}} +$$

$$- \frac{1}{R+j\omega L}\dot{V}_2$$

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$$\dot{V}_A = \frac{\frac{1}{R+j\omega L} - \alpha \frac{1}{R+\frac{1}{j\omega C}}}{\frac{1}{R+j\omega L} + \frac{1}{j\omega L+\frac{1}{j\omega C}} + (1-\alpha)\frac{1}{R+\frac{1}{j\omega C}}} \dot{V}_2 =$$

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$$= \bar{H}\,\dot{V}_2 = 0.636 - j\,0.364\ \dot{V}_2$$

$$\dot{V}_1 = \dot{V}_A = \bar{H}\cdot\dot{V}_2$$

$$\dot{I}_2 = \dot{I}_L + \dot{I}_S = \frac{\dot{V}_2 - \dot{V}_A}{R+j\omega L} + \frac{V_2 - \alpha V_{AB}}{R+\frac{1}{j\omega C}} =$$

$$= \frac{1-\bar{H}}{R+j\omega L}\dot{V}_2 + \frac{1-\alpha(\bar{H}-1)}{R+\frac{1}{j\omega C}}\dot{V}_2$$

$$= \left[\frac{1-H}{R+j\omega L} + \frac{1-\alpha(H-1)}{R+\frac{1}{j\omega C}}\right]\dot{V}_2$$

$$h_{12} = \left.\frac{\dot{V}_1}{\dot{V}_2}\right|_{\dot{I}_1=0} = \bar{H} = 0.636 - j\,0.364$$

$$h_{22} = \left.\frac{\dot{I}_2}{\dot{V}_2}\right|_{\dot{I}_1=0} = \frac{1-H}{R+j\omega L} + \frac{1-\alpha(H-1)}{R+\frac{1}{j\omega C}} = 0.243 + j\,0.11\ \mho$$

Esercizio 3A

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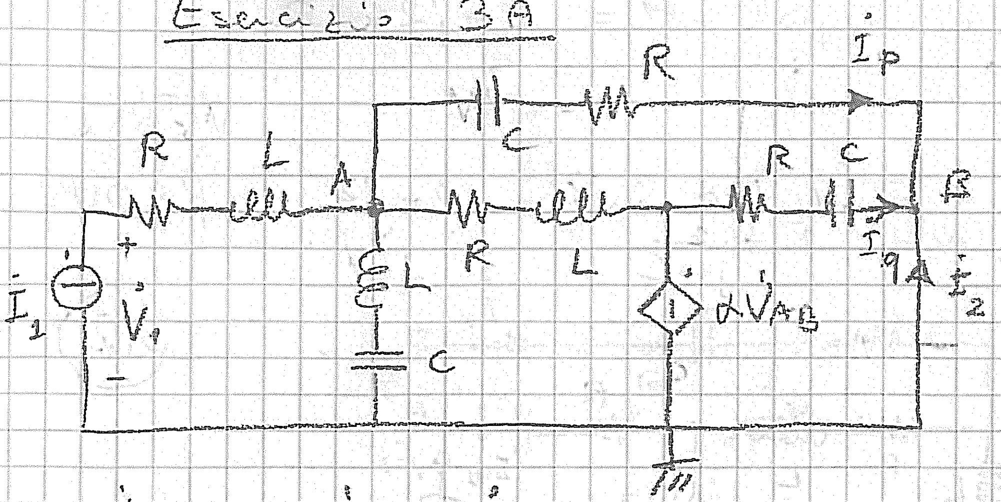

$\dot{V}_B = 0 \qquad \dot{V}_{AB} = \dot{V}_A$

$$\dot{I}_1 = \dot{V}_A \left(\frac{1}{R + j\omega L} + \frac{1}{R + \frac{1}{j\omega C}} + \frac{1}{j\omega L + \frac{1}{j\omega C}} \right) - \alpha V_A \frac{1}{R + j\omega L}$$

$$\dot{V}_A = \frac{1}{\frac{1}{R + \frac{1}{j\omega C}} + \frac{1}{j\omega L + \frac{1}{j\omega C}} + (1-\alpha)\frac{1}{R + j\omega L}} \dot{I}_1 =$$

$$= \overline{M}\, \dot{I}_1 = (-5 + j15)\, \dot{I}_1$$

$$\dot{V}_1 = (R + j\omega L)\dot{I}_1 + \dot{V}_A = (R + j\omega L + \overline{M})\, \dot{I}_1$$

$$\dot{I}_2 = -\dot{I}_p - \dot{I}_q = -\frac{V_A}{R + \frac{1}{j\omega C}} - \frac{\alpha V_A}{R + \frac{1}{j\omega C}} =$$

$$= -\frac{1 + \alpha}{R + \frac{1}{j\omega C}} \overline{M}\, \dot{I}_1$$

$$h_{11} = \left.\frac{\dot V_1}{\dot I_1}\right|_{\dot V_2=0} = R + j\omega L + \bar M = 5 + j25\ \Omega$$

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$$h_{21} = \left.\frac{\dot I_2}{\dot I_1}\right|_{\dot V_2=0} = -\frac{1+\alpha}{R+\frac{1}{j\omega C}}\bar M = 4 - j4$$

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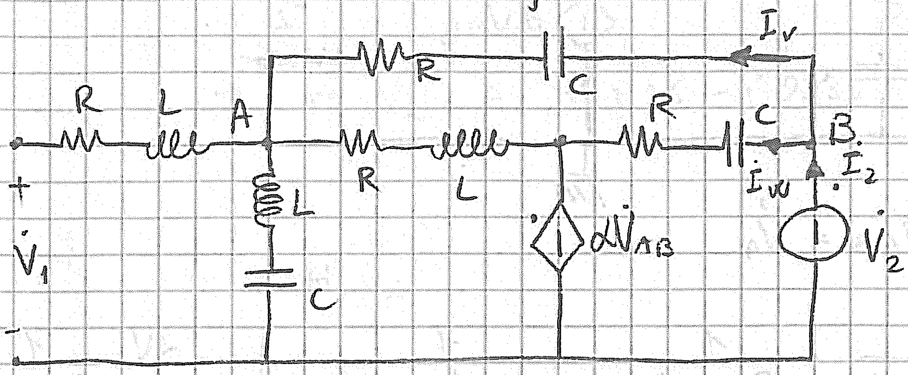


$\dot V_B = \dot V_2$

$\dot V_{AB} = \dot V_A - \dot V_2$

$$0 = \dot V_A\left(\frac{1}{j\omega L + \frac{1}{j\omega C}} + \frac{1}{R + j\omega L} + \frac{1}{R + \frac{1}{j\omega C}}\right) - \frac{1}{R+j\omega L}\alpha(\dot V_A - \dot V_2) + $$
$$- \frac{1}{R+\frac{1}{j\omega C}}\dot V_2$$

$$\dot V_A = \frac{\frac{1}{R+\frac{1}{j\omega C}} - \alpha\frac{1}{R+j\omega L}}{\frac{1}{j\omega L + \frac{1}{j\omega C}} + \frac{1}{R+\frac{1}{j\omega C}} + (1-\alpha)\frac{1}{R+j\omega L}}\dot V_2 =$$

$$= \bar N \dot V_2 = (-2.5 - j2)\dot V_2$$

$$\dot I_2 = \dot I_V + \dot I_W = \frac{\dot V_2 - \dot V_A}{R+\frac{1}{j\omega C}} + \frac{\dot V_2 - \alpha(\dot V_A - \dot V_2)}{R+\frac{1}{j\omega C}} = \frac{1-\bar N}{R+\frac{1}{j\omega C}}\dot V_2 +$$

$$+ \frac{1-\alpha(\bar N - 1)}{R+\frac{1}{j\omega C}}\dot V_2 = (0.88 + j1.24)\dot V_2$$

$$h_{12} = \left.\frac{\dot V_1}{\dot V_2}\right|_{\dot I_1=0} = \left.\frac{\dot V_A}{\dot V_2}\right|_{\dot I_1=0} = \bar N = -2.5 - j2$$

$$h_{22} = \left.\frac{\dot I_2}{\dot V_2}\right|_{\dot I_1=0} = 0.88 + j1.24\ \mho$$

Prova scritta del 09/01/09

Esercizio 4

$$G_m = \frac{P_{10}}{V_{10}^2} = 0.0115\ \mho$$

$$Y_m = \sqrt{3}\,\frac{I_{10}}{V_{10}} = 0.0161\ \mho$$

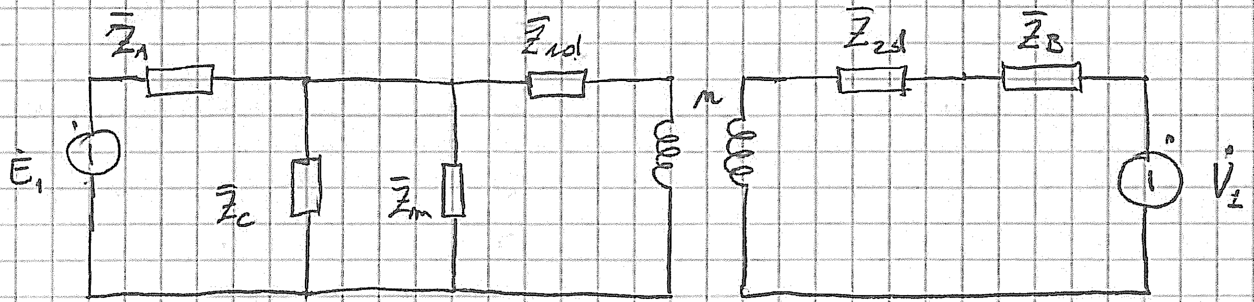
$$B_m = \sqrt{Y_m^2 - G_m^2} = 0.0112\ \mho$$

$$\bar{Z}_m = \frac{1}{\bar{Y}_m} = \frac{1}{G_m - jB_m} = 44.6 + j\,43.42\ \Omega$$

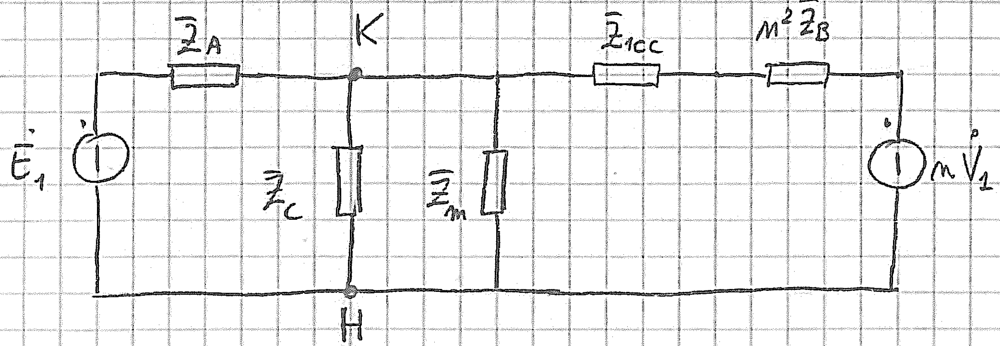
$$\cos\varphi_{cc} = \frac{P_{1cc}}{\sqrt{3}\,V_{1cc}\,I_{1cc}} = 0.585$$

$$\bar{Z}_{1cc} = \frac{V_{1cc}}{\sqrt{3}\,I_{1cc}}\left(\cos\varphi_{cc} + j\sqrt{1-\cos^2\varphi_{cc}}\right) = 1.63 + j\,2.34\ \Omega$$

Il circuito monofase equivalente del sistema è:



Riportando tutto al primario si ha:



Utilizzando il teorema di Millmann si può scrivere:

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$$\dot{V}_{KH} = \frac{\dfrac{\dot{E}_1}{\bar{Z}_A} + \dfrac{m\dot{V}_1}{\bar{Z}_{1cc} + m^2\bar{Z}_B}}{\dfrac{1}{\bar{Z}_A} + \dfrac{1}{\bar{Z}_C} + \dfrac{1}{\bar{Z}_m} + \dfrac{1}{\bar{Z}_{1cc} + m^2\bar{Z}_B}} = 288.5 + j\,217.6 \text{ V}$$

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$$\bar{S}_{\bar{Z}_C} = V_{KH}^2 \left(\frac{1}{\bar{Z}_C}\right)^* = 4.02 + j\,6.03 \text{ kVA}$$

$P_{Z_C} = 4.02$ kW

$Q_{Z_C} = 6.03$ kVAR

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