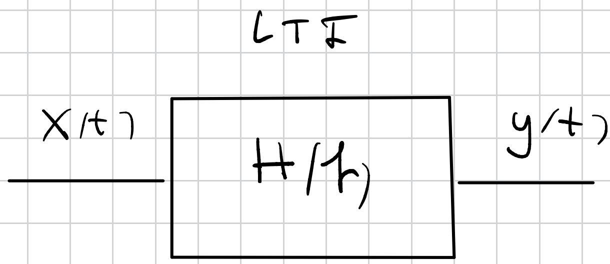
RISPOSTA IN FREQUENZA



METODO 1

$$x(t) = e^{j2\pi f_0 t}$$

$$y(t) = x(t) \otimes h(t)$$

$$= \int_{-\infty}^{\infty} h(\alpha)\, x(t-\alpha)\, d\alpha$$

$$= \int_{-\infty}^{\infty} h(\alpha)\, e^{j2\pi f_0 (t-\alpha)}\, d\alpha$$

$$= \int_{-\infty}^{\infty} e^{j2\pi f_0 t}\, h(\alpha)\, e^{-j2\pi f_0 \alpha}\, d\alpha$$

$$= \underbrace{e^{j2\pi f_0 t}}_{x(t)} \int_{-\infty}^{\infty} h(\alpha)\, e^{-j2\pi f_0 \alpha}\, d\alpha$$

$$= x(t) \underbrace{\int_{-\infty}^{\infty} h(\alpha)\, e^{-j2\pi f_0 \alpha}\, d\alpha}_{H(f_0)}$$

$$= x(t)\, H(f_0)$$

$$H(f_0) = \frac{y(t)}{x(t)} \quad \Bigg| \quad x(t) = e^{j2\pi f_0 t}$$

<u>METODO 2</u>

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi f t}\, dt$$

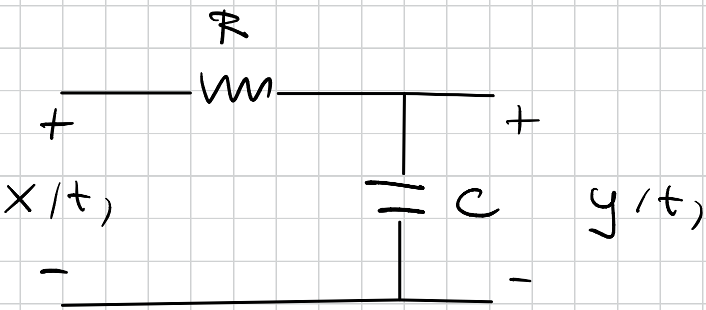
<u>METODO 3</u>

$$Y(f) = X(f)\, H(f)$$

$$H(f) = \frac{Y(f)}{X(f)}$$

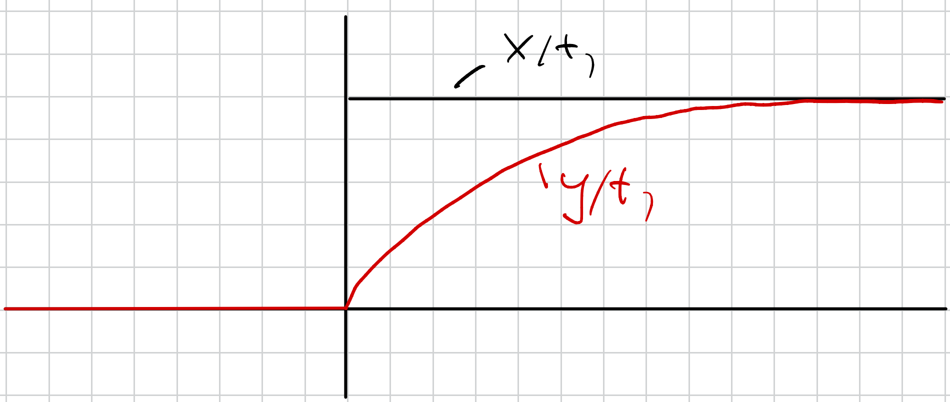
$$H(f) \in \mathbb{C} = |H(f)|\, e^{j\theta(f)}$$

CIRCUITO RC



$$x(t) = u(t)$$

$$y(t) = \left(1 - e^{-\frac{t}{RC}}\right) u(t)$$



$$y(t) = T[u(t)] = \left(1 - e^{-t/RC}\right) u(t)$$

$$h(t) = T[\delta(t)]$$

$$= T\left[\frac{d\,u(t)}{dt}\right]$$

$$= \frac{d}{dt} T[u(t)]$$

$$= \frac{d\,y(t)}{dt}$$

PROPRIETÀ CAMPIONAMENTO

$$= \frac{1}{RC} e^{-t/RC} u(t) +$$

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

$$\underbrace{\left(1 - e^{-t/RC}\right)\delta(t)}_{\delta(t) - e^{-t/RC}\delta(t) = \delta(t) - \delta(t) = \varnothing}$$

$$= \frac{1}{RC} e^{-t/RC} u(t)$$

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j 2\pi f t}\, dt$$

$$\overset{!}{=} \frac{1}{1 + j 2\pi f RC}$$

$$= \frac{1}{1 + j \, f/f_T}$$

$$f_T \triangleq \frac{1}{2\pi RC}$$

$$|H(f)| = \frac{1}{\sqrt{1 + \left(f/f_T\right)^2}}$$

NOTA

USARE SCRIPT IN MATLAB PER GRAFICARE LO SPETTRO.

$$\theta(f) = -\,tg^{-1}\left(f/f_T\right)$$

DECIBEL (dB)

$$|H(f_1)|^2_{dB} = 10 \log_{10} \frac{|H(f_1)|^2}{|H(f_0)|^2}$$

$$= 20 \log_{10} \frac{|H(f_1)|}{|H(f_0)|}$$

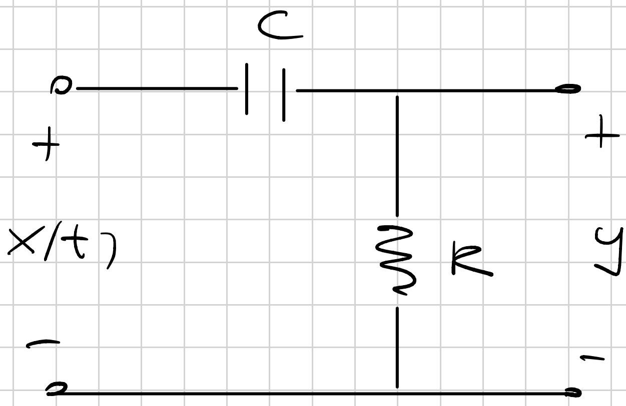
PER IL CIRCUITO RC SCELGO $f_0 = 0\,Hz$ $H(f_0) = 1$

$\lvert H(f_1)\rvert^2$	$(\cdot)\rvert_{dB}$
1	0 dB
2	3 dB
4	6 dB
8	9 dB
10	10 dB
100	20 dB
1000	30 dB

$\lvert\cdot\rvert^2$	$(\cdot)\rvert_{dB}$
50	17 dB
25	14 dB
$1/2$	-3 dB
$1/4$	-6 dB
$\frac{1}{10}$	-10 dB
$\frac{1}{100}$	-20 dB

FILTRO CR

NOTA: USARE SCRIPT IN MATLAB PER IL GRAFICO DELLO SPETTRO



$$H(f) = \frac{j\, f/f_T}{1 + j\, f/f_T}$$

$$= 1 - \frac{1}{1 + j\, f/f_T}$$

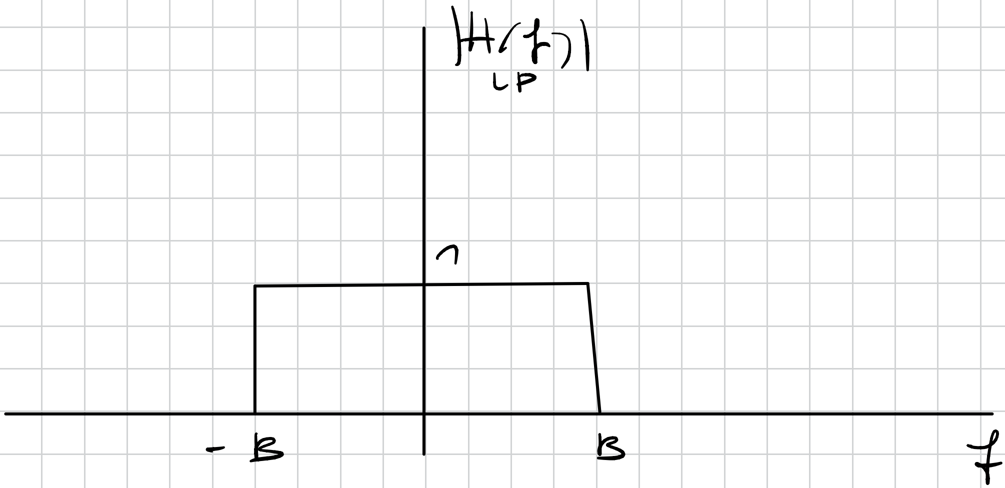
$$= 1 - H_{RC}(f)$$

$$\boxed{|H(f)| = \frac{\sqrt{(f/f_T)^2}}{\sqrt{1 + (f/f_T)^2}}}$$

FILTRI IDEALI

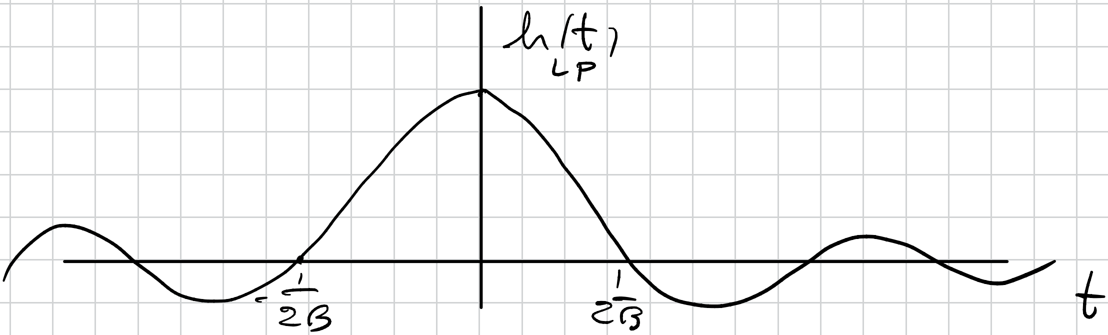
PASSA-BASSO, PASSA-BANDA, ELIMINA BANDA, PASSA-ALTO

FILTRO PASSA-BASSO LOW-PASS (LP)

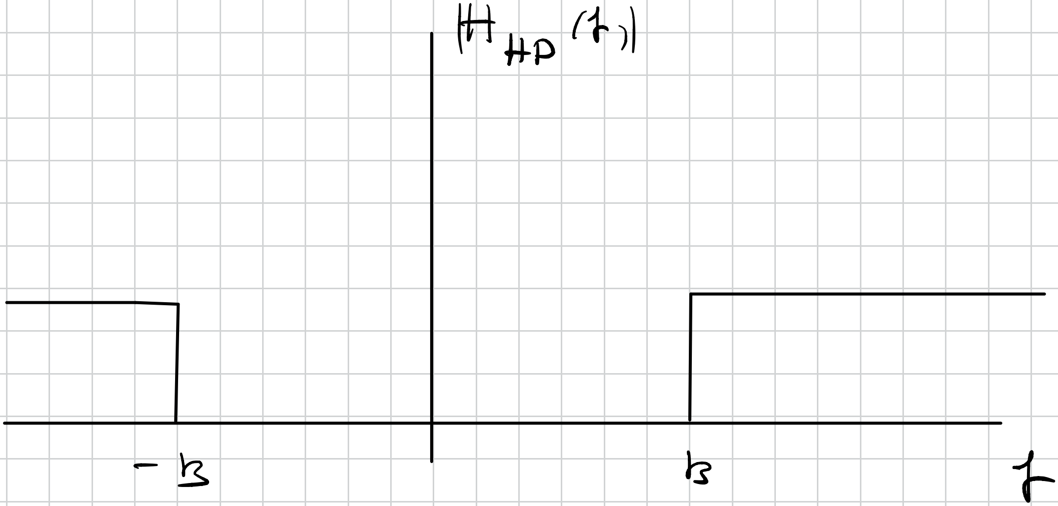


$$H_{LP}(f) = \mathrm{RECT}\left(\frac{f}{2B}\right)$$ RISPOSTA IN FREQUENZA

$$h_{LP}(t) = 2B \,\mathrm{sinc}(2Bt)$$ RISPOSTA IMPULSIVA



FILTRO PASSA ALTO HIGH-PASS (HP)



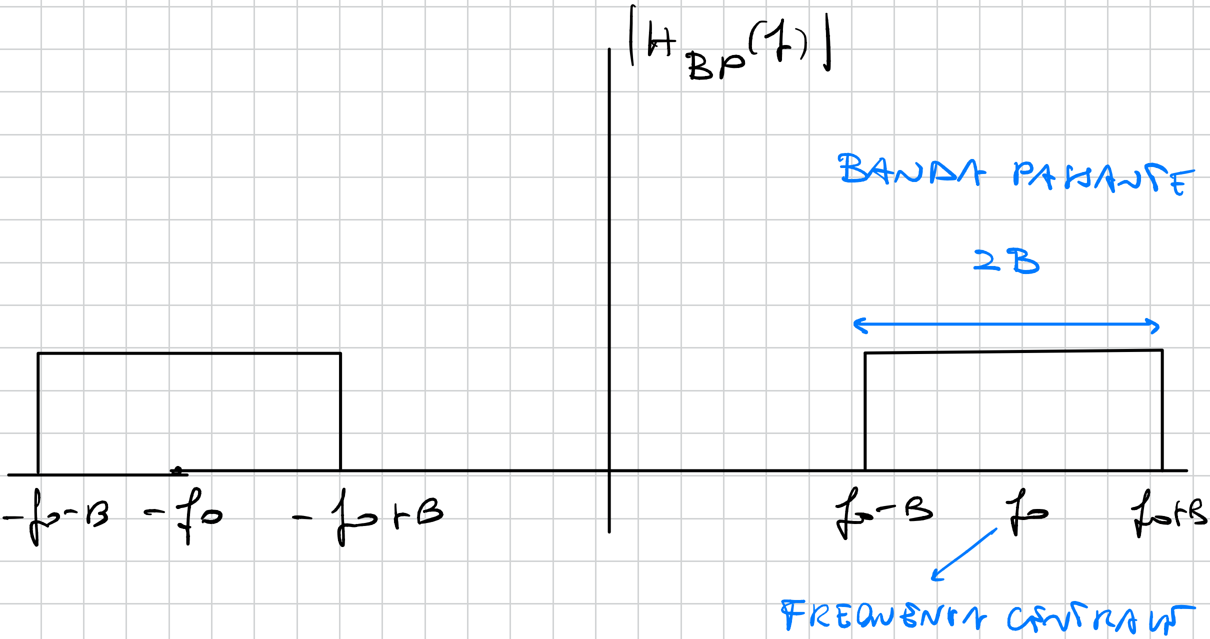
$$H_{HP}(f) = 1 - H_{LP}(f)$$

LA RISPOSTA IMPULSIVA è:

$$h_{HP}(t) = \delta(t) - h_{LP}(t)$$

$$= \delta(t) - 2B\,\mathrm{sinc}(2Bt)$$

FILTRO PASSA BANDA — BAND-PASS (BP)



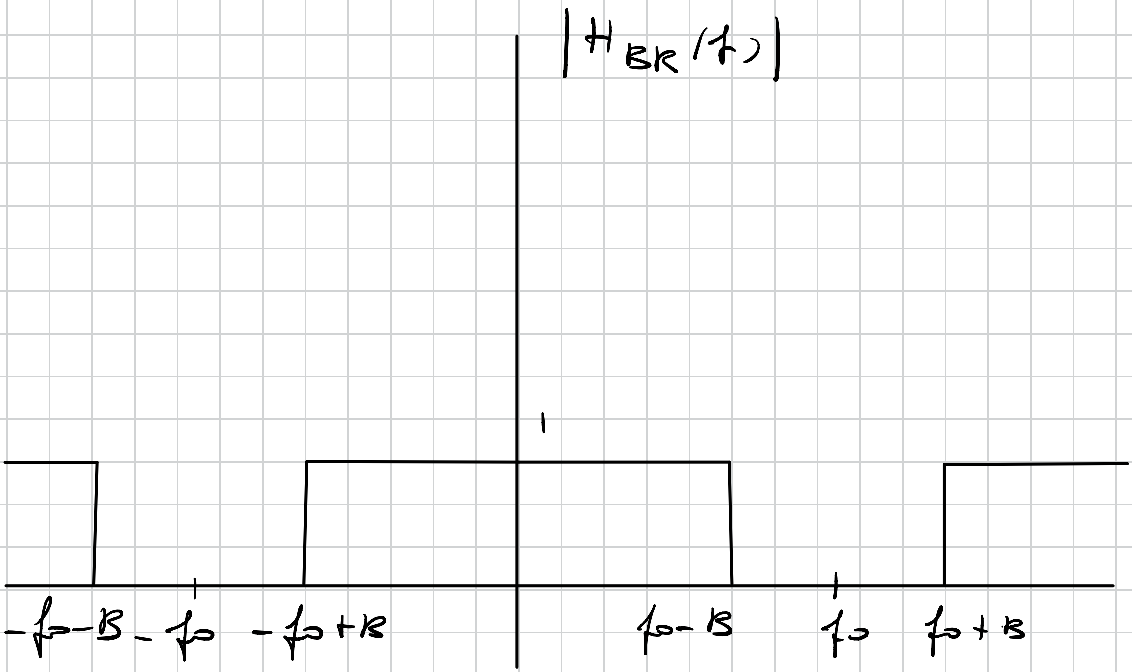
$$H_{BP}(f) = H_{LP}(f-f_0) + H_{LP}(f+f_0)$$

LA RISPOSTA IMPULSIVA È:

$$h_{BP}(t) = h_{BP}(t)\cos(2\pi f_0 t)$$

$$= 2B\,\mathrm{sinc}(2Bt)\cos(2\pi f_0 t)$$

FILTRO ELIMINA - BANDA   BAND-REJECT (BR)



$$H_{BR}(f) = 1 - H_{BP}(f)$$

$$h_{BR}(t) = \delta(t) - h_{BP}(t)$$

$$= \delta(t) - 2B\,\mathrm{sinc}(2Bt)\cos(2\pi f_0 t)$$

NOTA

USARE GLI SCRIPTS IN MATLAB DISPONIBILI NELLA CARTELLA CONDIVISA DEL TEAM. IN PARTICOLARE, QUELLI PER I FILTRI RC e CR, E LO SCRIPT SUI FILTRI IDEALI.