06/06/2018

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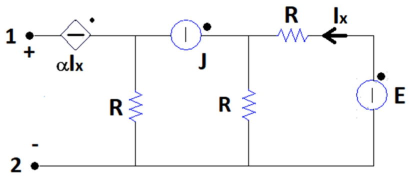
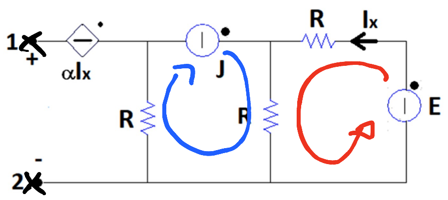
# Es 1

1) Determinare il **circuito equivalente di Thevenin** fra i punti **1** e **2** del circuito in figura.

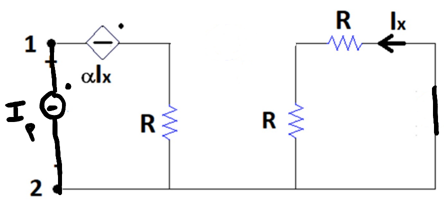



E = 50 V;
J = 2 A;
R = 10 Ω;
α = 15 V/A.




$$V_{TH} = -\alpha I_x - RJ = -22{,}5 - 20 = -42{,}5\ V$$

$$E = 2R I_x + RJ \Rightarrow I_x = \frac{E - RJ}{2R} = 1{,}5\ A$$

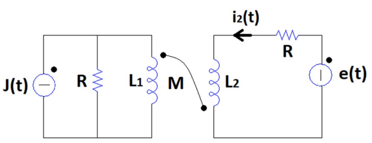



$$I_x = 0 \Rightarrow \alpha I_x = 0$$

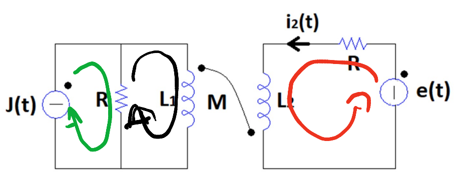
$$R_{TH} = \frac{V_P}{I_P} \qquad V_P = R I_P \qquad R_{TH} = 10\ \Omega$$

# Es 2

2) Determinare **l'andamento temporale della corrente i₂(t)** nel secondo induttore (con il verso mostrato in figura).




$J(t) = \sqrt{2}\sin(1000t)$ A;
$e(t) = 10\sqrt{2}\cos(1000t)\ V$;
R = 10 Ω;
$L_1$ = 10 mH;
$L_2$ = 15 mH.
M = 10 mH;



$$\begin{cases} j\omega L_1 I_1 - RJ + R I_1 - j\omega L_2 I_2 = 0 \\ E = R I_2 + j\omega L_2 I_2 - j\omega M I_1 \end{cases}$$

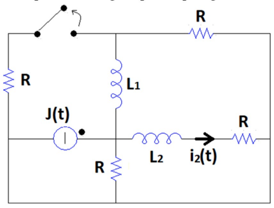
$$I_1 = \frac{RJ}{j\omega L_1 + R} + \frac{j\omega L_2 I_2}{j\omega L_1 + R} = \alpha + \beta I_2 = 0{,}5 - 0{,}5j + (0{,}5 + 0{,}5j)\, I_2$$

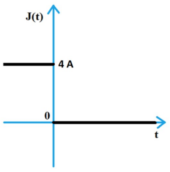
$$E = I_2 (R + j\omega L_2 - \beta j\omega M) - \alpha j\omega M$$

$$\dot{I}_2 = \frac{E + \alpha j\omega M}{(R + j\omega L_2 - \beta j\omega M)} = 0{,}877\, e^{-0{,}2662j}$$

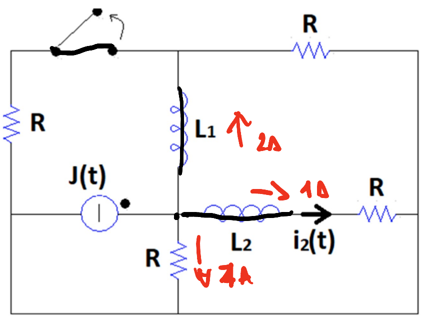
Es 3

3) Determinare l'andamento temporale della corrente **$i_2(t)$** per **$-\infty < t < +\infty$**, considerando l'**andamento a gradino della corrente erogata del generatore di corrente J(t)**, come in figura, e che il **tasto si apre** per t=0. Il circuito è ipotizzato a regime per tempi negativi.




R = 30 Ω;
$L_1$ = 0.1 H;
$L_2$ = 0.2 H;

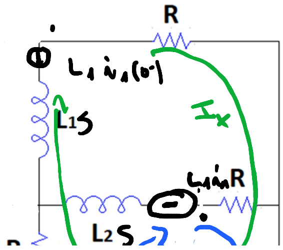


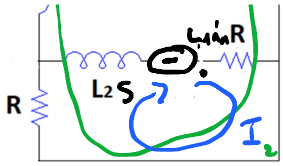

$t < 0$

$i_{L_1}(0^-) = 2A$ $\qquad$ $i_2 = 1A$

$i_{L_2}(0^-) = 1A$

$t \geq 0$






$$\begin{cases} 2R\,I_1(s) + I_1(s)\,L_1 s + I_2 R = 2L_1 \\ I_2\,2R + I_2\,L_2 s + I_1 R = L_2 \end{cases}$$

$$I_1 = \frac{2L_1 - I_2(s)\,R}{2R + L_1 s}$$

$$2R\,I_2 + I_2\,L_2 s + R\left(\frac{2L_1 - I_2(s)\,R}{2R + L_1 s}\right) = L_2$$

$$I_2\left(2R + L_2 s - \frac{R^2}{2R + L_1 s}\right) = L_2 - \frac{2L_1 R}{2R + L_1 s}$$

$$I_2\left(\frac{2R(2R + L_1 s) + (2R + L_1 s)\,L_2 s - R^2}{2R + L_1 s}\right) = \frac{L_2(2R + L_1 s) - 2L_1 R}{2R + L_1 s}$$

$$I_2 = \frac{L_2(2R + L_1 s) - 2L_1 R}{(2R + L_2 s)(2R + L_1 s) - R^2} = \frac{0{,}02\,s + 6}{0{,}02\,s^2 + 18s + 2700}$$

$$I_2 = \frac{s + 300}{s^2 + 900s + 135000}$$

$$s_{1,2} = \frac{-900 \pm \sqrt{900^2 - 4 \cdot 135000}}{2} = \begin{cases} -190{,}19 \\ -709{,}807 \end{cases}$$

$$A_1 = \lim_{s \to s_1} \frac{s + 300}{(s + 709{,}807)} = 0{,}2113$$

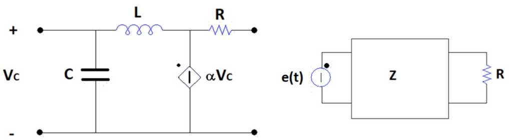
$$A_2 = \lim_{s \to s_2} \frac{s + 300}{s + 190{,}19} = 0{,}7886$$

$$i_2(t) = \begin{cases} \left(0{,}2113\,e^{-190{,}19t} + 0{,}7886\,e^{-709{,}807t}\right)u(t) & t \geq 0 \\ 1 & t < 0 \end{cases}$$

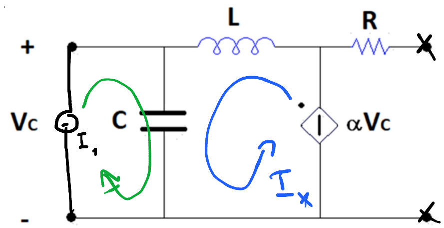
$$i_2(t) = \begin{cases} \left(0{,}2113\, e^{-[\text{illegible}]\,t} + 0{,}7886\, e^{-[\text{illegible}]\,t}\right) u(t) & t \geq 0 \\ 1 & t < 0 \end{cases}$$

# Es 4

4) Determinare la rappresentazione a parametri **Z** della rete a due porte indicata in figura (a sinistra), ipotizzata a regime periodico sinusoidale a pulsazione **ω**. Successivamente, considerando che la rete a due porte **Z** precedentemente determinata è alimentata da un generatore di tensione e chiusa su una resistenza come indicato in figura (a destra), calcolare la potenza attiva erogata dal generatore di tensione **e(t)**.




R = 10 Ω;
L = 10 mH;
C = 300 μF;
α = 2;
ω = 1000 rad/s;
e(t) = $10\sqrt{2}\sin(1000t)V$.




$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$\begin{cases} V_C = \dfrac{(I_1 + I_X)}{j\omega C} \\ \left(j\omega L + \dfrac{1}{j\omega C}\right) I_X + \dfrac{I_1}{j\omega C} = \alpha V_C \end{cases}$$

$$\left(j\omega L + \frac{1}{j\omega C}\right) I_X + \frac{I_1}{j\omega C} = \frac{\alpha (I_1 + I_X)}{j\omega C}$$

$$I_X \left(j\omega L + \frac{1}{j\omega C} - \frac{\alpha}{j\omega C}\right) = I_1 \left(\frac{\alpha - 1}{j\omega C}\right)$$

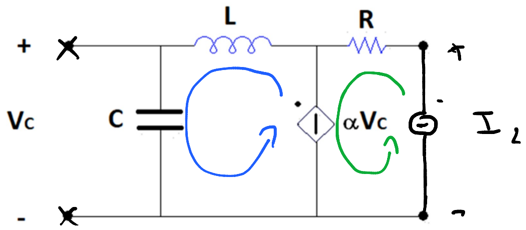
$$I_1(\alpha - 1) = I_X(-\omega^2 LC + 1 - \alpha)$$

$$I_X = \frac{I_1(\alpha - 1)}{-\omega^2 LC + 1 - \alpha} = \gamma I_1 = -0{,}25\, I_1$$

$$V_c = V_1 = \frac{3/4\, I_1}{Jwc} = \frac{3}{4Jwc} = -2{,}5\,\delta$$

$$V_2 = \alpha V_c = \alpha z_{11} = z_{21} = -5\,\delta$$

$I_1 = 0$




$$R I_2 + \alpha V_c = V_2$$

$$\frac{I_x}{Jwc} = V_c$$

$$I_x\left(JwL + \frac{1}{Jwc}\right) = \alpha V_c$$

$$I_x\left(JwL + \frac{1}{Jwc}\right) = \alpha \frac{I_x}{Jwc}$$

$$I_x\left(JwL + \frac{1}{Jwc} - \frac{\alpha}{Jwc}\right) = 0 \qquad I_x = 0$$

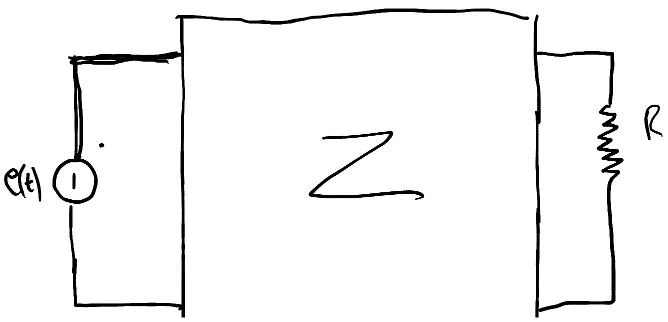
$V_c = 0$

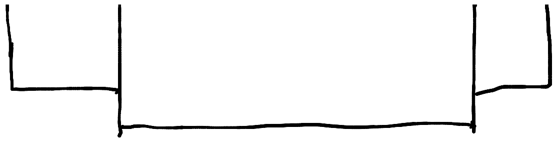
$V_2 = R I_2 \qquad z_{22} = 10$

$z_{12} = 0$

$$V_1 = z_{12} I_2$$

$$Z = \begin{bmatrix} -2{,}5\,\delta & 0 \\ -5\,\delta & 10 \end{bmatrix}$$





$$\begin{cases} V_1 = e(t) \\ V_2 = -R I_2 \\ V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

$$I_1 = \frac{V_1}{z_{11}} = \frac{E}{z_{11}} = 4jA$$

$$P_z = \mathrm{Re}\left\{ V I^* \right\} = 10 \cdot (0 - 4j) = \underset{P_e}{0} \underset{Q_e}{- 40j}$$