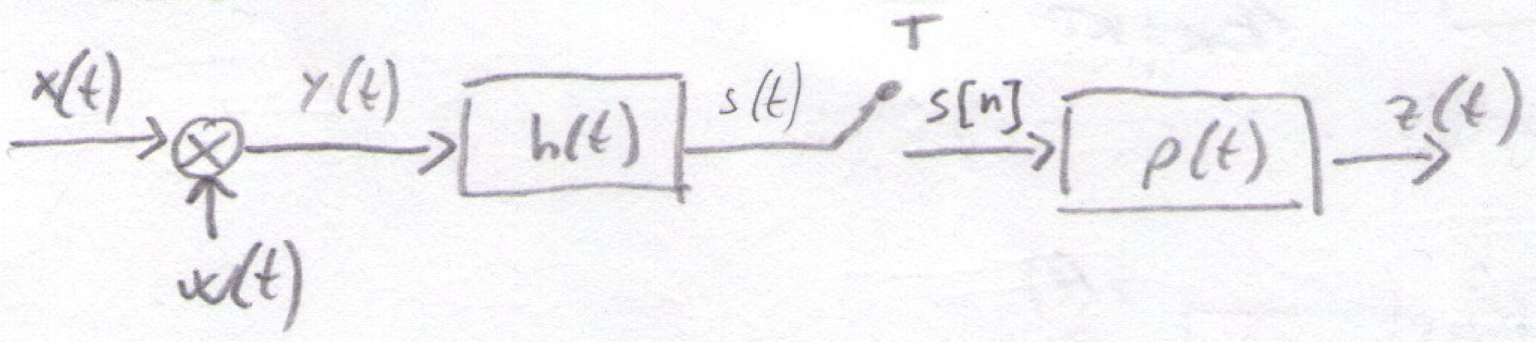
Esame 05-07-2010

• Esercizio 1



$f_0 \gg T$

$$y(t) = x(t)\,w(t) = A\,\mathrm{rect}\left(\frac{t}{2T}\right)\cos(2\pi f_0 t)\cdot \sin(2\pi f_0 t + \phi) =$$

$$= A\,\mathrm{rect}\left(\frac{t}{2T}\right)\left\{\frac{1}{2}\left[\mathrm{sen}(2\pi f_0 t + 2\pi f_0 t + \phi) + \sin(2\pi f_0 t + \phi - 2\pi f_0 t)\right]\right\} =$$

$$= A\,\mathrm{rect}\left(\frac{t}{2T}\right)\left\{\frac{1}{2}\left[\mathrm{sen}(4\pi f_0 t + \phi) + \sin(\phi)\right]\right\}$$

$$= \frac{A}{2}\,\mathrm{rect}\left(\frac{t}{2T}\right)\mathrm{sen}(4\pi f_0 t + \phi) + \frac{A}{2}\sin(\phi)\,\mathrm{rect}\left(\frac{t}{2T}\right)$$

$$s(t) = y(t) \otimes h(t) \rightleftharpoons S(f) = Y(f)\,H(f) = \left\{\frac{A}{2}\cdot 2T\,\mathrm{sinc}(f2T)\left[\frac{e^{j\phi}}{2j}\delta(f-f_0) - \frac{e^{-j\phi}}{2j}\delta(f+f_0)\right] +\right.$$

$$\left. + \frac{A}{2}\sin(\phi)\cdot 2T\cdot \mathrm{sinc}(f\,2T)\right\} T\,\mathrm{sinc}(f\cdot 2T) =$$

$$S(f) = AT^2\,\mathrm{sinc}^2(f2T)\left\{\frac{e^{j\phi}}{2j}\delta(f-f_0) - \frac{e^{-j\phi}}{2j}\delta(f+f_0)\right\} + AT^2\sin(\phi)\,\mathrm{sinc}^2(f2T) =$$

$$s(t) = \frac{AT}{2}\left(1-\frac{|t|}{2T}\right)\mathrm{rect}\left(\frac{t}{4T}\right)\mathrm{sen}(4\pi f_0 t + \phi) + \frac{AT}{2}\sin(\phi)\left(1-\frac{|t|}{2T}\right)\mathrm{rect}\left(\frac{t}{4T}\right)$$

$$s[n] = \frac{AT}{2}\left(1-\frac{|nT|}{2T}\right)\mathrm{rect}\left(\frac{nT}{4T}\right)\mathrm{sen}(4\pi f_0 nT + \phi) + \frac{AT}{2}\sin(\phi)\left(1-\frac{|nT|}{2T}\right)\mathrm{rect}\left(\frac{nT}{4T}\right) =$$

$$= \frac{AT}{2}\left(1-\frac{|nT|}{2T}\right)\mathrm{rect}\left(\frac{n}{4}\right)\mathrm{sen}(4\pi f_0 nT + \phi) + \frac{AT}{2}\sin(\phi)\left(1-\frac{|nT|}{2T}\right)\mathrm{rect}\left(\frac{n}{4}\right)$$

$$z(t) = \sum_{n=-\infty}^{\infty} s[n]\,p(t-nT) =$$

$$= \sum_{n=-\infty}^{\infty}\left\{\frac{AT}{2}\left(1-\frac{|nT|}{2T}\right)\mathrm{rect}\left(\frac{n}{4}\right)\mathrm{sen}(4\pi f_0 nT + \phi) + \frac{AT}{2}\sin(\phi)\left(1-\frac{|nT|}{2T}\right)\mathrm{rect}\left(\frac{n}{4}\right)\right\}\underbrace{\mathrm{rect}\left(\frac{t-nT}{T}\right)}_{\text{costante}} =$$

$$z(t) = s(t)$$

$$E_z = \int_{-\infty}^{\infty}|z(t)|^2 dt = \int_{-\infty}^{\infty}\left[\frac{A^2T^2}{4}\left(1-\frac{|t|}{2T}\right)^2\mathrm{rect}^2\left(\frac{t}{4T}\right)\mathrm{sen}^2(4\pi f_0 t + \phi) + \frac{A^2T^2}{4}\sin^2(\phi)\left(1-\frac{|t|}{2T}\right)^2\mathrm{rect}^2\left(\frac{t}{4T}\right) +\right.$$

$$\left. + 2\frac{AT}{2}\left(1-\frac{|t|}{2T}\right)\mathrm{rect}\left(\frac{t}{4T}\right)\mathrm{sen}(4\pi f_0 t + \phi)\;\frac{AT}{2}\sin(\phi)\left(1-\frac{|t|}{2T}\right)\mathrm{rect}\left(\frac{t}{4T}\right)\right] =$$

$$= \int_{-\infty}^{\infty}\left[\frac{A^2T^2}{4}\left(1-\frac{|t|}{2T}\right)^2\mathrm{rect}^2\left(\frac{t}{4T}\right)\cdot\underbrace{\frac{1}{4}}_{\text{energia segnale modulato}} + \frac{A^2T^2}{4}\sin^2(\phi)\left(1-\frac{|t|}{2T}\right)^2\mathrm{rect}^2\left(\frac{t}{4T}\right) + \emptyset\right]dt$$

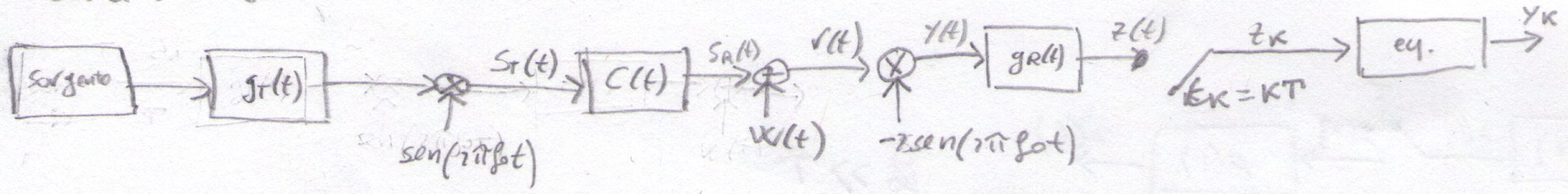
↳ la rect è pari, il seno è dispari
pari · dispari · pari = dispari
$\int_{-\infty}^{\infty}$ dispari $= \emptyset$

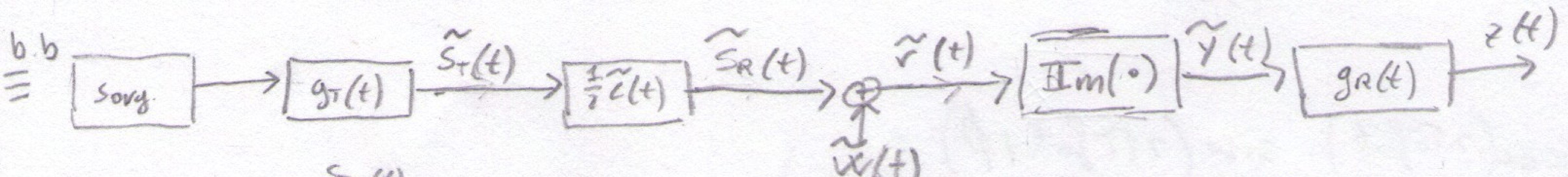
$$= \int_{-\infty}^{\infty}\frac{A^2T^2}{16}\left(1-\frac{|t|}{2T}\right)^2\mathrm{rect}^2\left(\frac{t}{4T}\right)dt + \int_{-\infty}^{\infty}\frac{A^2T^2}{4}\sin^2(\phi)\left(1-\frac{|t|}{2T}\right)^2\mathrm{rect}^2\left(\frac{t}{4T}\right)dt =$$

$$= \frac{2}{3}\,2T\cdot\frac{A^2T^2}{16} + \frac{2}{3}\cdot 2T\cdot\frac{A^2T^2}{4}\sin^2(\phi) = \frac{A^2T^3}{12}\left(1+\sin^2(\phi)\right)$$

$P_z = \emptyset$ per th

• Esercizio 2





$s_T(t) = \overbrace{\sum_i a_i g_T(t-iT)}^{s_{Ts}(t)} \operatorname{sen}(2\pi f_0 t) \triangleq \Re\left[\tilde s_T(t)\, e^{j2\pi f_0 t}\right]$

$s_T(t) = s_{Ts}(t)\, \Re\{-j e^{j\omega_0 t}\} = \Re\{\underbrace{s_{Ts}(t)(-j)}\, e^{j\omega_0 t}\}$

$\tilde s_T(t) = -j\, s_{Ts}(t) = -j \sum_i a_i g_T(t-iT)$

$\tilde s_R(t) = \tilde s_T(t) \otimes \frac12 \tilde c(t) \Rightarrow g_{Tc}(t) \triangleq g_T(t) \otimes \frac12 \tilde c(t) \Rightarrow \tilde s_R(t) = -j \sum_i a_i g_{Tc}(t-iT)$

$\tilde r(t) = \tilde s_R(t) + \tilde w(t) = -j \sum_i a_i g_{Tc}(t-iT) + \tilde w(t)$

$\tilde y(t) = \Im\{\tilde r(t)\} = -\sum_i a_i g_{Tc}(t-iT) + w_n(t)$

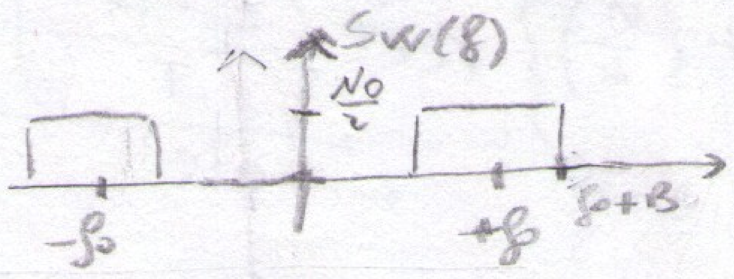
$g(t) \triangleq g_T(t) \otimes \frac12 \tilde c(t) \otimes g_R(t)$; $n_s(t) = w_n(t) \otimes g_R(t)$

$z(t) = \tilde y(t) \otimes g_R(t) = -\sum_i a_i g(t-iT) + n_s(t)$

2) $S_{n_s}(f) = S_{w_s}(f)\, |G_R(f)|^2$

$g_R(t) = \delta(t) + \frac13\left[\delta(t-T) + \delta(t+T)\right] \rightleftharpoons G_R(f) = 1 + \frac13\left[e^{-j2\pi fT} + e^{j2\pi fT}\right] 2 = 1 + \frac23 \cos(2\pi fT)$

$S_{w_s}(f) = \frac{S_{\tilde w}(f) + S_{\tilde w}(-f)}{4}$; $S_{\tilde w}(f) = \begin{cases} 4 S_w(f+f_0) & f \geq -f_0 \\ 0 & \text{altrove} \end{cases}$; $S_w(f) = \frac{N_0}{2}\left\{\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \mathrm{rect}\left(\frac{f+f_0}{2B}\right)\right\}$



$S_{\tilde w}(f) = 2N_0\, \mathrm{rect}\left(\frac{f}{2B}\right)$; $S_{w_s}(f) = \frac{2N_0\, \mathrm{rect}\left(\frac{f}{2B}\right) + 2N_0\, \mathrm{rect}\left(\frac{-f}{2B}\right)}{4} = N_0\, \mathrm{rect}\left(\frac{f}{2B}\right)$

$|G_R(f)|^2 = \left(1 + \frac23 \cos(2\pi fT)\right)^2 = 1 + \frac49 \cos^2(2\pi fT) + \frac43 \cos(2\pi fT)$

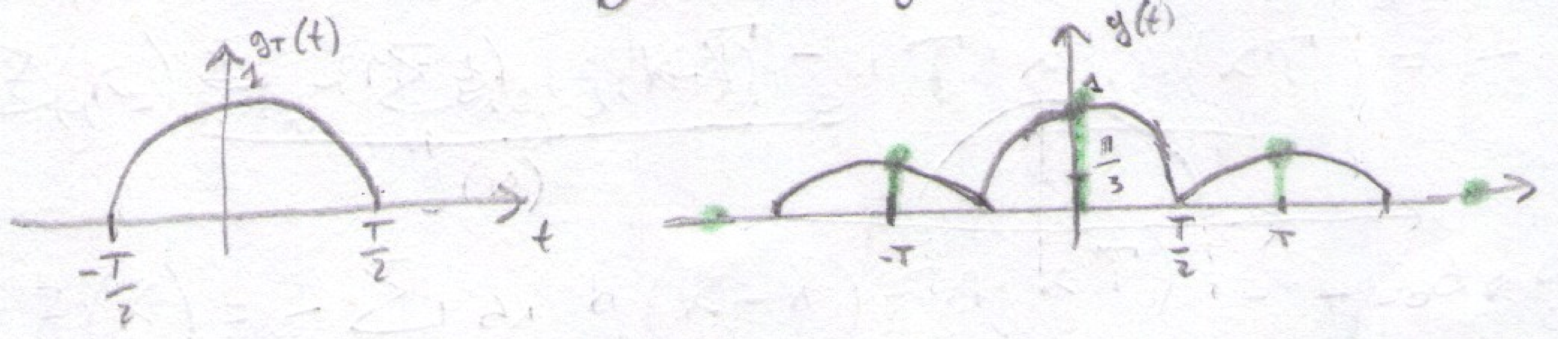
$S_{n_s}(f) = N_0\, \mathrm{rect}\left(\frac{f}{2B}\right)\left[1 + \frac49 \cos^2(2\pi fT) + \frac43 \cos(2\pi fT)\right]$

$P_n = \int_{-\infty}^{\infty} S_{n_s}(f)\, df = \int_{-\infty}^{\infty} \left\{N_0\, \mathrm{rect}\left(\frac{f}{2B}\right)\left[1 + \frac49 \cos^2(2\pi fT) + \frac43 \cos(2\pi fT)\right]\right\} df =$

$= N_0 \int_{-B}^{B} 1\, df + \frac{4}{9} N_0 \frac12 \int_{-B}^{B} 1\, df + \frac49 N_0 \frac12 \int_{-B}^{B} \cos(4\pi fT)\, df + \frac43 N_0 \int_{-B}^{B} \cos(2\pi fT)\, df =$

= 'ste fungie di minchia!!!

3) $g(t) = g_{Tc}(t) \otimes g_R(t) = g_T(t) \otimes \left[\delta(t) + \frac13 \delta(t-T) + \delta(t+T)\right] = g_T(t) + \frac13 g_T(t-T) + \frac13 g(t+T)$



$g(kT) = \begin{cases} 1 & k = 0 \\ \frac13 & k = \pm 1 \\ 0 & \text{altrove} \end{cases}$

$z_k = z(t_k) = -\sum_i a_i\, g[kT - iT] + n(t_k) = -\sum_i a_i\, g[(k-i)T] + n_k =$

$= -a_k\, g(0) - a_{k+1}\, g(-T) - a_{k-1}\, g(T) + n_k = -a_k - \frac13 a_{k+1} - \frac13 a_{k-1} + n_k$

$g(k) = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1 \end{cases}$; $D_g = \frac{\sum_{k-i \neq 0} |g(k-i)|}{|g(0)|} = \frac{g(k+1) + g(k-1)}{g(0)} = \frac{\frac13 + \frac13}{1} = \frac23 < 1$ OK

$$g(K) = -\delta(K) - \frac{1}{3}\delta(K-1) - \frac{1}{3}\delta(K+1)$$

$$q(K) = p(K) \otimes g(K) = \sum_{\ell=-1}^{1} p_\ell \, g(K-\ell) = p_{-1}\, g(K+1) + p_0\, g(K) + p_1\, g(K-1)$$

$$\begin{cases} q(0) = p_{-1}\, g(1) + p_0\, g(0) + p_1\, g(-1) = \frac{1}{3} p_{-1} + p_0 + \frac{1}{3} p_{+1} = 1 \\ q(-1) = p_{-1}\, g(0) + p_0\, g(-1) + p_1\, g(-2) = p_{-1} + \frac{1}{3} p_0 = 0 \\ q(1) = p_{-1}\, g(2) + p_0\, g(1) + p_1\, g(0) = \frac{1}{3} p_0 + p_1 = 0 \end{cases}$$

$$\begin{cases} p_{-1} = -\frac{1}{3} p_0 \Rightarrow p_{-1} = -\frac{1}{\cancel{3}_1} \cdot \frac{\cancel{9}^3}{7} = -\frac{3}{7} \\ p_1 = -\frac{1}{3} p_0 \Rightarrow p_1 = -\frac{3}{7} \\ \left(\frac{1}{3} \cdot -\frac{1}{3} p_0\right) + p_0 + \left(\frac{1}{3} \cdot -\frac{1}{3} p_0\right) = 1 \equiv -\frac{1}{9} p_0 + p_0 - \frac{1}{9} p_0 = 1 \equiv \frac{-1+9-1}{9} p_0 = 1 \Rightarrow p_0 = \frac{9}{7} \end{cases}$$