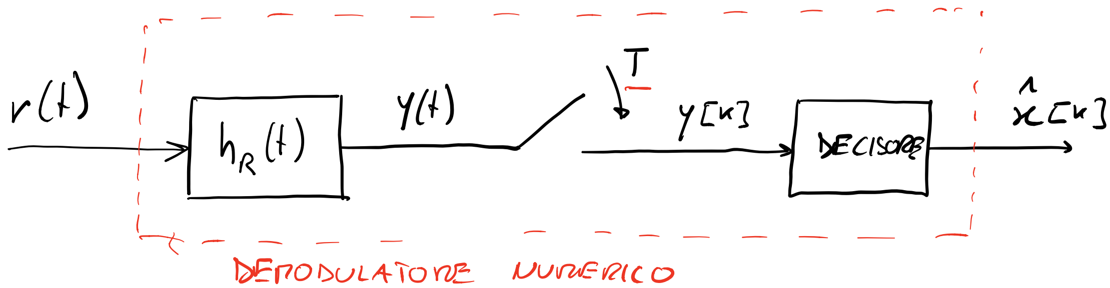
ESERCIZIO - 06/02/2017



$$r(t) = \sum_{i=-\infty}^{+\infty} x[i]\, p(t - iT) + w(t)$$

$x[i]$ sono i simboli trasmessi indipendenti ed equiprob.

$A_s = \{-1 \,;\, 2\}$

$w(t)$ rumore Gaussiano bianco con DSP $S_w(f) = \frac{N_0}{2}$

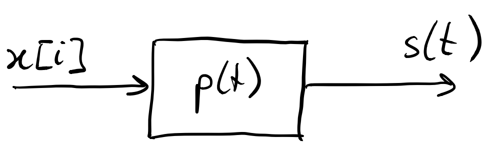
$$p(t) = \frac{2}{T}\,\text{sinc}^2\!\left(\frac{2t}{T}\right)\cos\!\left(\frac{4\pi t}{T}\right)$$

$$h_R(t) = \frac{4}{T}\,\text{sinc}\!\left(\frac{4t}{T}\right)$$

1) Calcolare l'energia media per simbolo trasmesso
2) DSP di $s(t)$
3) Potenza del rumore in uscita al filtro di ric.

Soluzione

$T_s = T$



modulatore numerico

1) $E_S = E\left[x^2[i]\right] E_p = E\left[\int_{-\infty}^{+\infty} x^2[i]\, p^2(t-iT)\, dt\right]$

$$E\left[x^2\right] = P\{\alpha_1\}\,\alpha_1^2 + P\{\alpha_2\}\,\alpha_2^2$$

$$= \frac{1}{2}(-1)^2 + \frac{1}{2}(+2)^2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$$

VALOR QUADRATICO MEDIO DEI SIMBOLI

$$E_p = \int_{-\infty}^{+\infty} p^2(t)\, dt = \int_{-\infty}^{+\infty} |P(f)|^2\, df$$

$$P(f) = TCF\left[p(t)\right] = TCF\left[\underbrace{\frac{2}{T}\,\text{sinc}^2\left(\frac{2t}{T}\right)\cos\left(\frac{4\pi t}{T}\right)}_{x(t)\cos(2\pi f_0 t)}\right]$$

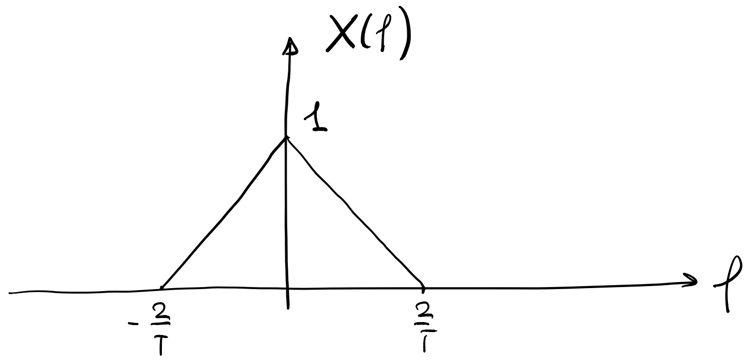
$$x(t)\cos\left(2\pi f_0 t\right), \quad f_0 = \frac{2}{T}$$

$$P(f) = \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$$

dal teo della modulazione

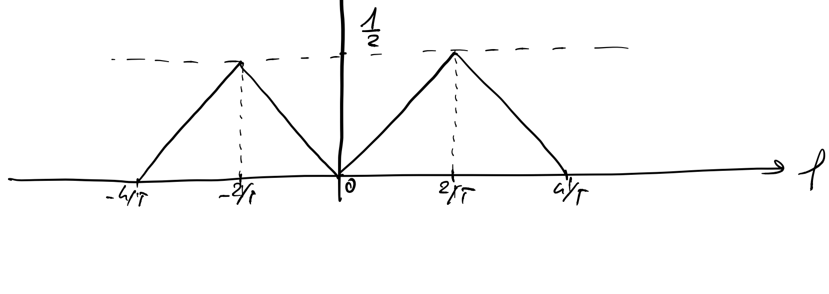
$$X(f) = TCF\left[\frac{2}{T}\,\text{sinc}^2\left(\frac{2t}{T}\right)\right] = TCF\left[\frac{2}{T}\,\text{sinc}^2\left(\frac{t}{T/2}\right)\right]$$

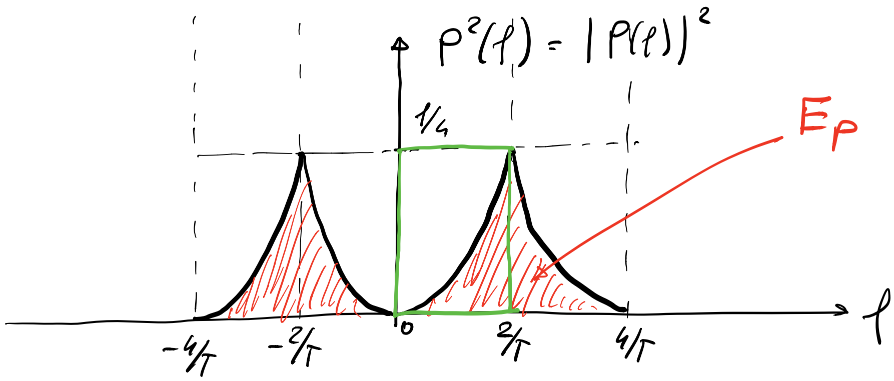
$$= \frac{\cancel{2}}{\cancel{T}}\,\frac{\cancel{T}}{\cancel{2}}\left(1 - \frac{|f|}{2/T}\right)\text{rect}\left(\frac{f}{4/T}\right)$$



$$P(f) = \frac{1}{2}\left(1 - \frac{\left|f-\frac{2}{T}\right|}{\frac{2}{T}}\right)\text{rect}\left(\frac{f-\frac{2}{T}}{\frac{4}{T}}\right)$$

$$+ \frac{1}{2}\left(1 - \frac{\left|f+\frac{2}{T}\right|}{\frac{2}{T}}\right)\text{rect}\left(\frac{f+\frac{2}{T}}{\frac{4}{T}}\right)$$





$$E_p = \cancel{4} \cdot \frac{1}{3}\,\frac{2}{T} \cdot \frac{1}{\cancel{4}} = \frac{2}{3T}$$

$$\boxed{E_s = \frac{5}{\cancel{2}} \cdot \frac{\cancel{2}}{3T} = \frac{5}{3T}}$$

2) DSP di $s(t)$

$$S_s(f) = \frac{1}{T_s}\,\bar{S}_x(f)\,|P(f)|^2$$

$$\bar{S}_x(f) = \text{TFS}\left[R_x[m]\right]$$

$$R_x[m] = C_x[m] + \eta_x^2$$

$$\eta_x = E[x] = \frac{1}{2}(-1) + \frac{1}{2}(+2) = -\frac{1}{2} + \frac{2}{2} = \frac{1}{2}$$

La formula semplificata

$$S_s(f) = \frac{\sigma_x^2}{T_s}\,|P(f)|^2$$

non può essere utilizzata poiché i simboli non sono simmetrici

$\Rightarrow$ NON SI TRATTA DI UNA PAM standard ($\alpha_i = 2i - 1 - M$)

$\eta_X^2 = \frac{1}{4}$

$C_X[m]$ autocovarianza

$$C_X[m] = E\left[\left(x[n] - \eta_X\right)\left(x[n-m] - \eta_X\right)\right]$$
$$= E\left[\left(X - \eta_X\right)\left(Y - \eta_X\right)\right]$$

$\Rightarrow$ i simboli sono indipendenti

$\Downarrow$

incorrelati

$\Downarrow$

a covarianza nulla

due V.A indip. sono anche incorrelate

$\Rightarrow$ per tutti i valori di $m \neq 0$ $\Rightarrow$ $\boxed{C_X[m] = 0}$

$\Rightarrow$ per $m=0$ $\Rightarrow$ $C_X[0] = E\left[\left(X[n] - \eta_X\right)^2\right] = \sigma_X^2$

$$\Rightarrow C_X[m] = \sigma_X^2\,\delta[m]$$

$$\delta[m] = \begin{cases} 1 & m=0 \\ 0 & m \neq 0 \end{cases}$$

$$\Rightarrow R_X[m] = \sigma_X^2\,\delta[m] + \eta_X^2 = \begin{cases} \sigma_X^2 + \eta_X^2 = E[x^2] & m=0 \\ \eta_X^2 & m \neq 0 \end{cases}$$

$$\sigma_X^2 = E\left[x^2\right] - \eta_X^2 = \frac{5}{2} - \frac{1}{4} = \frac{9}{4}$$

$$\Rightarrow R_X[m] = \frac{9}{4}\,\delta[m] + \frac{1}{4}$$

$$\Rightarrow \bar{S}_x(f) = TFS\left[R_x[m] \right] =$$

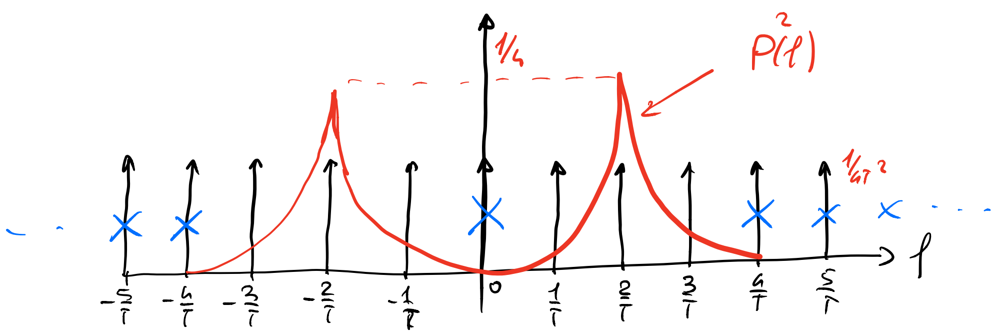
$$= \sum_{m=-\infty}^{+\infty} R_x[m]\, e^{-j2\pi f m T} = \sum_m \frac{9}{4}\delta[m]\, e^{-j2\pi f m T}$$

$$+ \sum_m \frac{1}{4} e^{-j2\pi f m T} = \frac{9}{4} + \frac{1}{4}\sum_m e^{-j2\pi f m T}$$

$$= \frac{9}{4} + \frac{1}{4}\,\frac{1}{T}\sum_{m=-\infty}^{+\infty}\delta\left(f - \frac{m}{T}\right)$$

dalla II^a formula di Poisson

$$S_s(f) = \frac{1}{T}\left[\frac{9}{4} + \frac{1}{4T}\underbrace{\sum_m \delta\left(f - \frac{m}{T}\right)}_{\text{treno di delta}}\right] |P(f)|^2$$



$$\boxed{\begin{aligned} S_s(f) &= \frac{9}{4T}|P(f)|^2 + \frac{1}{4T^2}\sum_m \left|P\left(\frac{m}{T}\right)\right|^2 \delta\left(f - \frac{m}{T}\right) \\ &= \frac{9}{4T}|P(f)|^2 + \frac{1}{4T^2}\sum_{m\in M} P^2\left(\frac{m}{T}\right)\delta\left(f - \frac{m}{T}\right) \end{aligned}}$$

$$M = \{\pm 3, \pm 2, \pm 1\}$$

$$P^2(f) = \frac{1}{4}\left(1 - \frac{|f - \frac{2}{T}|}{\frac{2}{T}}\right)^2 \mathrm{rect}\left(\frac{f - \frac{2}{T}}{\frac{4}{T}}\right) + \frac{1}{4}\left(1 - \frac{|f + \frac{2}{T}|}{\frac{2}{T}}\right)^2 \mathrm{rect}\left(\frac{f + \frac{2}{T}}{\frac{4}{T}}\right)$$

3) Potenza di rumore in uscita a $h_R(t)$

$$w(t) \longrightarrow \boxed{h_R(t)} \longrightarrow w_u(t)$$

$$P_{w_u} = \int_{-\infty}^{+\infty} S_{w_u}(f)\, df = R_{w_u}(\tau)\Big|_{\tau=0} = R_w(0)$$

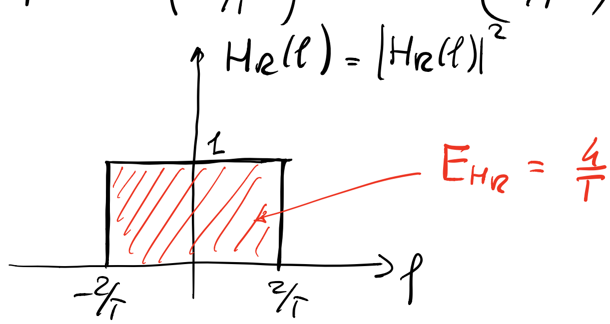
$$S_w(f) = \frac{N_0}{2}$$ rumore bianco

$$P_{w_u} = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H_R(f)|^2\, df = \frac{N_0}{2} E_{h_R} = \frac{N_0}{2} \cdot \frac{4}{T} = \boxed{\frac{2N_0}{T}}$$

$$R_{w_u}(\tau) = \frac{N_0}{2}\delta(\tau) \otimes \underbrace{h(\tau) \otimes h(-\tau)}_{}$$

↳ è complicata

$$H_R(f) = \frac{\cancel{4}}{\cancel{T}} \frac{\cancel{T}}{\cancel{4}} \operatorname{rect}\left(\frac{f}{4/T}\right) = \operatorname{rect}\left(\frac{f}{4/T}\right)$$



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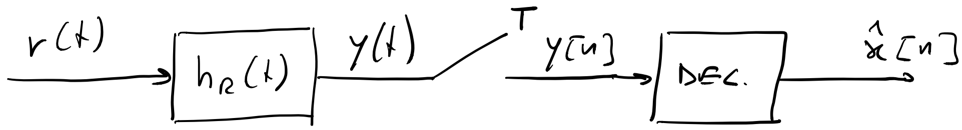


$$r(t) = A_0 \sum_k x[n]\, p(t - kT)$$

$x[k] \in A_s \{-1, 1\}$ equiprob. ed indip.

$w(t)$ è Gaussiano bianco $S_w(f) = \frac{N_0}{2}$

$c(t) = \delta(t)$ canale senza distorsioni

$$p(t) = \frac{4}{T} \operatorname{sinc}\left(\frac{4t}{T}\right)$$

$$H_R(f) = (1 - |fT|)\operatorname{rect}\left(\frac{fT}{2}\right) + \operatorname{rect}\left(\frac{fT}{4}\right)$$

1) E_s
2) $S_s(f)$
3) P_{w_u}

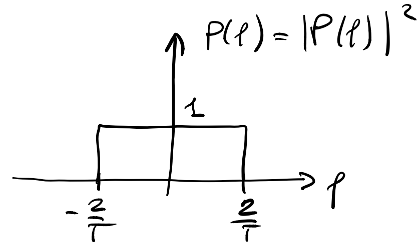
Soluzione

$$E_s = E[x^2]\, E_p = 1 \cdot \frac{4}{T} = \boxed{\frac{4}{T}}$$

$$E[x^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+1)^2 = 1$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\, df = \frac{4}{T}$$

$$P(f) = \operatorname{rect}\left(\frac{f}{4/T}\right)$$



2) $S_s(f) = \dfrac{\sigma_x^2}{T} |P(f)|^2$

- simboli antipodali (± 1)
- " ind.
- " equiprob.

$\sigma_x^2 = E[x^2] - \eta_x^2 = 1$

$\eta_x = \frac{1}{2}(-1) + \frac{1}{2}(+1) = 0$

$$\boxed{S_s(f) = \frac{1}{T} \operatorname{rect}\left(\frac{f}{1/T}\right)}$$

$$S_s(f) = \frac{1}{T} \bar{S}_x(f) |P(f)|^2$$

$$R_x[m] = C_x[m] + \underbrace{\eta_x^2}_{=0} = C_x[m] = \sigma_x^2 \delta[m]$$

$$\bar{S}_x(f) = TFS\left[R_x[m]\right] = \sigma_x^2$$

$$\Downarrow$$

$$S_s(f) = \frac{\sigma_x^2}{T} |P(f)|^2$$

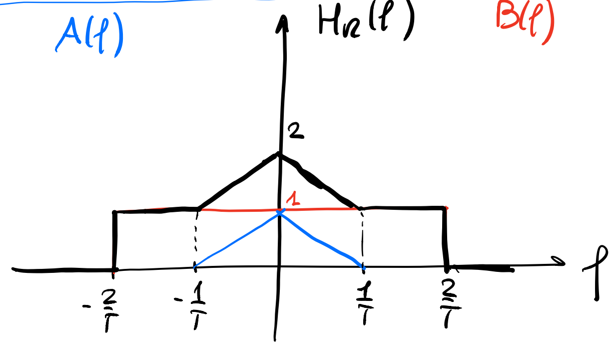
3) P_{w_u}

$$w(t) \longrightarrow \boxed{H_R(f)} \longrightarrow w_u(t)$$

$$S_{w_u}(f) = S_w(f) |H_R(f)|^2 = \frac{N_0}{2} |H_R(f)|^2$$

$$P_{w_u} = \frac{N_0}{2} E_{H_R} = \frac{N_0}{2} \cdot \frac{20}{3T} = \frac{10}{3T} N_0 = \boxed{\frac{10}{3} \frac{N_0}{T}}$$

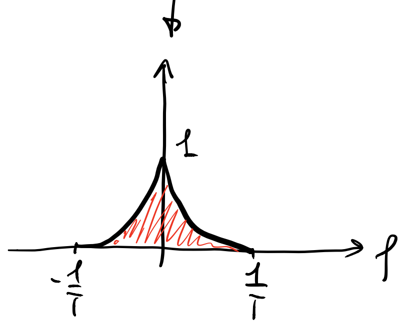
$$H_R(f) = \underbrace{\left(1 - \frac{|f|}{1/T}\right)\mathrm{rect}\left(\frac{f}{2/T}\right)}_{A(f)} + \underbrace{\mathrm{rect}\left(\frac{f}{4/T}\right)}_{B(f)}$$



$$H_R(f) = A(f) + B(f)$$

$$|H_R(f)|^2 = H_R^2(f) = \left[A(f) + B(f)\right]^2 = A^2(f) + B^2(f) + 2A(f)B(f)$$

$$E_{H_R} = \int_{-\infty}^{+\infty} A^2(f)\,df + \int_{-\infty}^{+\infty} B^2(f)\,df + 2\int_{-\infty}^{+\infty} \underbrace{A(f)\,B(f)}_{A(f)\cdot B(f) = A(f)}\,df$$



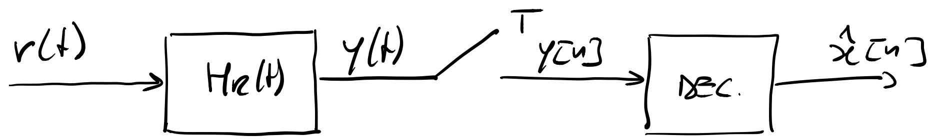
$$E_A = 2 \cdot \frac{1}{3} \cdot 1 \cdot \frac{1}{T} = \frac{2}{3T}$$

$$E_B = \frac{4}{T}$$

$$E_{AB} = 2 \cdot \frac{1}{T} = \frac{2}{T}$$

$$E_{H_R} = E_A + E_B + E_{AB} = \frac{1}{T}\left(\frac{2}{3} + 4 + 2\right) = \frac{20}{3T}$$

ESERCIZIO – 18/07/2017



$x[i]$ ind. equipr. $\in A_s = \{-3, 2\}$

$w(t)$ Gaussiano bianco con DSP $\frac{N_0}{2}$

$p(t) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{2t}{T}\right)$

$H_R(f) = \frac{T}{2}\left(1 + \cos\left(\pi f T\right)\right)\mathrm{rect}\left(\frac{fT}{2}\right)$

1) E_s
2) $S_s(f)$
3) P_{w_u}

Soluzione

1) $E_s = E[x^2]\,E_p$

2) $S_s(f) = ?$

$$S_s(f) = \frac{1}{T}\,\bar{S}_x(f)\,|P(f)|^2$$

$$R_x[m] = C_x[m] + \eta_x^2 \longrightarrow \eta_x = E[x]$$

$C_x[m]$ $\downarrow$ ind. dei simboli $\Rightarrow C_x[m] = \sigma_x^2\,\delta[m]$

$$R_x[m] = \sigma_x^2\,\delta[m] + \eta_x^2$$

$$\boxed{\bar{S}_x(f) = \sigma_x^2 + \eta_x^2\,\frac{1}{T}\sum_m \delta\left(f - \frac{m}{T}\right)}$$

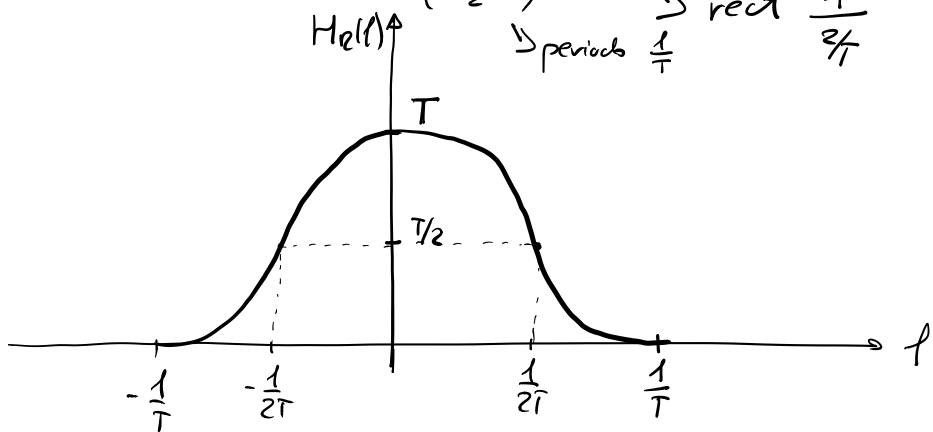
$$P(f) = TCF\left[p(t)\right]$$

3) $P_{w_u} = \frac{N_0}{2} E_{H_R}$

$$H_R(f) = \frac{T}{2}\left(1 + \cos\left(\pi f T\right)\right) \operatorname{rect}\left(\frac{fT}{2}\right)$$

$\cos\left(2\pi \frac{T}{2} f\right) \Rightarrow$ periodo $\frac{1}{T}$

$\Rightarrow \operatorname{rect} \frac{f}{2/T}$



$$E_{H_R} = \int_{-\frac{1}{T}}^{\frac{1}{T}} \frac{T^2}{2^2}\left(1 + \cos\left(\pi f T\right)\right)^2 df = \frac{T^2}{4} \int_{-\frac{1}{T}}^{\frac{1}{T}} 1 + \cos^2\left(\pi f T\right) + \underbrace{2\cos\left(\pi f T\right)}_{=0} df$$

$$\cos^2\left(\pi f T\right) \to \frac{1}{2} + \underbrace{\frac{1}{2}\cos\left(2\pi f T\right)}_{\Rightarrow 0}$$

$$= \frac{T^2}{4}\left[\frac{2}{T} + \frac{1}{T} + 0 + 2 \cdot 0\right] =$$

$$= \frac{T^{\cancel{2}}}{4} \frac{3}{\cancel{T}} = \frac{3T}{4}$$

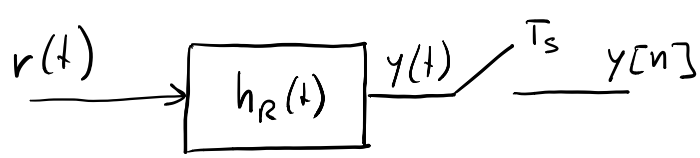
$$P_{w_u} = \frac{N_0}{2} \cdot E_{H_R} = \frac{N_0}{2} \cdot \frac{3}{4} T = \boxed{\frac{3}{8} N_0 T}$$

PRESTAZIONI DEI SISTEMI DI COMUNICAZIONE NUMERICI IN BANDA BASE

⇒ Due fenomeni "peggiorativi"

1) INTERFERENZA INTER-SIMBOLICA (ISI)
2) RUMORE

⇒ ISI in assenza di rumore



ASSENZA DI ISI

$$y[n] = f(x[n])$$

campione prelevato all'istante "n"

simbolo trasmesso all'istante "n"

PRESENZA DI ISI

$$y[n] = f(\underbrace{\ldots, x[n-1], x[n], x[n+1], \ldots}_{\text{vari simboli trasmessi ad istanti diversi da "n"}})$$

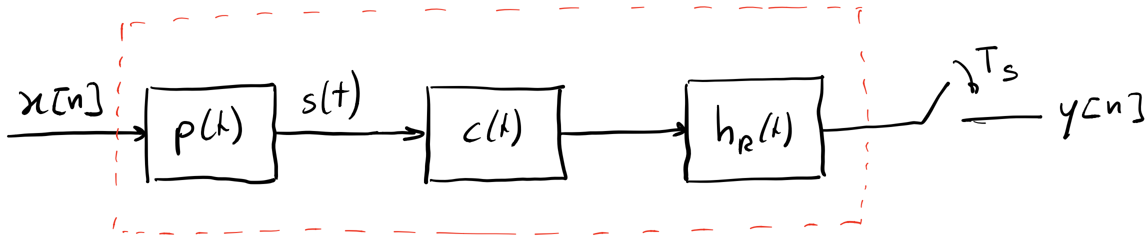
$$r(t) = s(t) \otimes c(t) + \cancel{n(t)}$$

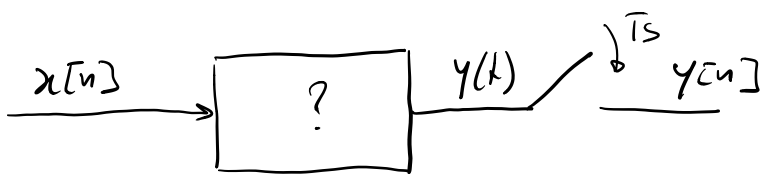
siamo in condizioni di assenza di rumore

$$= \left[\sum_{k=-\infty}^{+\infty} x[k]\, p(t-kT_s)\right] \otimes c(t)$$

$$y(t) = r(t) \otimes h_R(t) = \left[\sum_k x[k]\, p(t-kT_s)\right] \otimes c(t) \otimes h_R(t)$$

⇒ Per poter valutare la presenza o meno di ISI devo considerare $p(t)$, $c(t)$ e $h_R(t)$





$$y(t) = r(t) \otimes h_R(t) = \underbrace{\left[\sum_k x[n]\, p(t-kT_s)\right] \otimes c(t)} \otimes h_R(t)$$

$$s(t) \otimes c(t) = \int_{-\infty}^{+\infty} \sum_k x[k]\, p(\tau - kT_s)\, c(t-\tau)\, d\tau$$

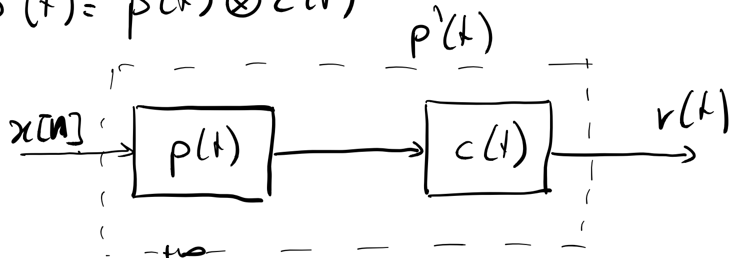
$$= \sum_k x[k] \int_{-\infty}^{+\infty} p(\tau - kT_s)\, c(t-\tau)\, d\tau \qquad \tau - kT_s = \tau'$$

$$= \sum_k x[n] \int_{-\infty}^{+\infty} p(\tau')\, c(t - \tau' - kT_s)\, d\tau'$$

$$= \sum_k x[n] \underbrace{\int_{-\infty}^{+\infty} p(\tau')\, c\left[(t-kT_s) - \tau'\right] d\tau'}_{p'(t)\big|_{t=t-kT_s} = \left. p(t) \otimes c(t) \right|_{t=t-kT_s}}$$

$$s(t) \otimes c(t) = \sum_{k} x[k]\, p'(t-kT_s)$$

$$p'(t) = p(t) \otimes c(t)$$



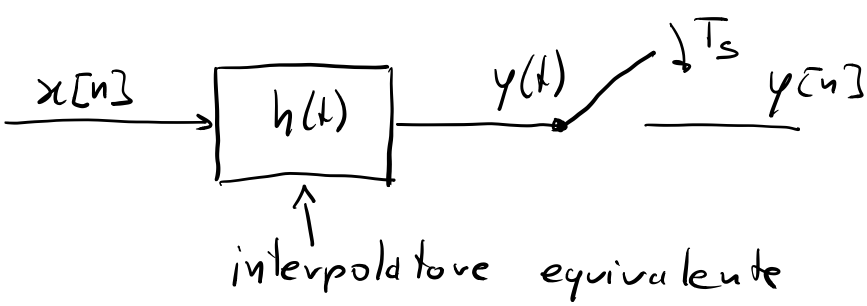
$$r(t) = \sum_{k=-\infty}^{+\infty} x[k]\, p'(t-kT_s)$$

$$y(t) = r(t) \otimes h_R(t) = \left[\sum_{k=-\infty}^{+\infty} x[k]\, p'(t-kT_s)\right] \otimes h_R(t)$$

$$= \cdots\cdots = \underbrace{\sum_{k=-\infty}^{+\infty} x[k]\, h(t-kT_s)}$$

$$h(t) = p'(t) \otimes h_R(t)$$

$$\boxed{h(t) = p'(t) \otimes h_R(t) = p(t) \otimes c(t) \otimes h_R(t)}$$



$$y(t) = \sum_{k=-\infty}^{+\infty} x[k]\, h(t - kT_s)$$

$$y(t)\Big|_{t=nT_s} = y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h(nT_s - kT_s)$$

$$= \sum_{k} x[k]\, h[(n-k)T_s]$$

$$= \underbrace{x[n]\, h(0)} + \underbrace{\sum_{\substack{k=-\infty \\ (k \neq n)}}^{+\infty} x[k]\, h[(n-k)T_s]}$$

componente di $y[n]$
che dipende esclusivamente
da $x[n]$

ISI
componente di $y[n]$
che dipende da tutti
gli altri simboli

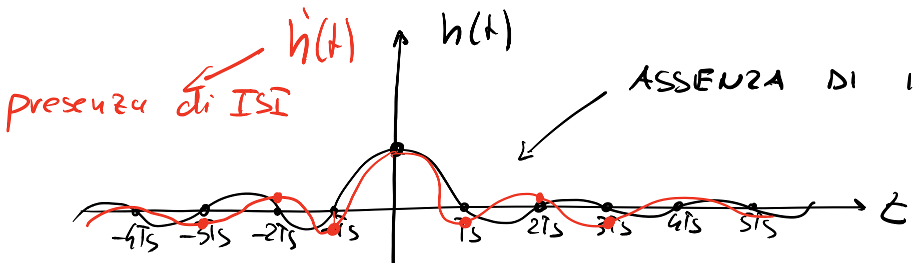
I° CRITERIO DI NYQUIST PER L'ASSENZA DI ISI

$$h[(n-k)T_s] = 0 \qquad n \neq k$$

$$h[n' T_s] = 0 \qquad n' = n-k \neq 0$$

$$\Downarrow$$

$$h(nT_s) = 0 \qquad n \neq 0$$



Dimostriamolo partendo dalle condizione

$$h(t)\Big|_{t=nT_s} = 0 \qquad n \neq 0$$

$$y[n] = h(0)\,x[n] + \sum_{\substack{k=-\infty \\ k\neq n}}^{+\infty} x[k]\, \underbrace{h\big((n-k)T_s\big)}_{=0}$$

$$h(n'T_s) = 0 \qquad n' \neq 0$$

$$\Downarrow$$

$$h\big((n-k)T_s\big) = 0 \qquad n-k \neq 0$$

$$n' = n-k$$

$$h[n'] = 0 \qquad n' \neq 0$$

$$\Downarrow$$

$$h[n'] = 0 \qquad n-k \neq 0$$
$$n \neq k$$

$$\Downarrow$$

$$h[(n-k)T_s] = 0 \qquad n \neq k$$