ESERCIZI

oscilla a FREQUENZA f_0 → oscilla a FREQ. $2f_0$

$$x(t) = A\cos(2\pi f_0 t + \varphi) + B\,\text{sen}(4\pi f_0 t)$$

1) Calcolare SPETTRO di $x(t)$

2) Calcolare SPETTRO DI AMPIEZZA E FASE

Svolgimento:

1) È periodico?

$x(t) = x(t - nT_0)$ → Dobbiamo definire un T_0 del segnale

periodo $T_0 = \frac{1}{f_0}$ ⇒ f_0 = fondamentale

$2f_0$ → determina che il segnale è periodico

DIMOSTRIAMOLO

$$x(t-nT_0) = A\cos(2\pi f_0(t-nT_0)+\varphi) + B\,\text{sen}(4\pi f_0(t-nT_0)) =$$

$$= A\cos(2\pi f_0 t - 2\pi \cancel{f_0} n \cancel{T_0} + \varphi) + B\,\text{sen}(4\pi f_0 t - 4\pi \cancel{f_0} n \cancel{T_0}) =$$

$$= A\cos(2\pi f_0 t + \varphi) + B\,\text{sen}(4\pi f_0 t)$$ PERIODICO di T_0

SPETTRO $$X_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \Big[\underbrace{A\cos(2\pi f_0 t + \varphi)}_{x_1(t)} + \underbrace{B\,\text{sen}(4\pi f_0 t)}_{x_2(t)}\Big] e^{-j2\pi n f_0 t}\, dt =$$

•) APPLICO LA LINEARITÀ

$$X_n = X_{1n} + X_{2n}$$

$$X_{1n} = TSF[x_1(t)] \quad , \quad X_{2n} = TSF[x_2(t)]$$

$$X_{1n} = \frac{A}{T_0}\int_{-T_0/2}^{T_0/2} \frac{e^{j(2\pi f_0 t+1)} + e^{-j(2\pi f_0 t+1)}}{2} \cdot e^{-j2\pi n f_0 t}\, dt$$

$$\frac{A}{2T_0}\, e^{j1}\int_{-T_0/2}^{T_0/2} e^{j2\pi f_0(1-n)t}\, dt + \frac{A}{2T_0}\, e^{-j1}\int_{-T_0/2}^{T_0/2} e^{-j2\pi(1+n)f_0 t}$$

$$X_{1n} = \begin{cases} \frac{A}{2\cancel{T_0}}\, e^{j1}\cdot \cancel{T_0} & n=1 \\ \frac{A}{2\cancel{T_0}}\, e^{-j1}\cdot \cancel{T_0} & n=-1 \\ 0 & n \neq \pm 1 \end{cases}$$

$$\{RE\}\quad \cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\{Im\}\quad \operatorname{sen}\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

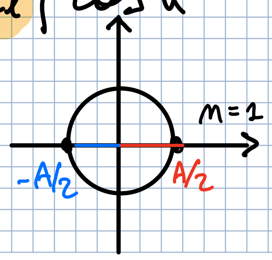
$$X_{2n} = \frac{B}{T_0}\int_{-T_0/2}^{T_0/2} \frac{e^{j2\pi(2f_0)t} - e^{-j2\pi(2f_0)t}}{2j}\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{B}{j2T_0}\int_{-T_0/2}^{T_0/2} e^{j2\pi(2-n)f_0 t}\, dt - \frac{B}{j2T_0}\int_{-T_0/2}^{T_0/2} e^{-j2\pi(2+n)f_0 t}\, dt =$$

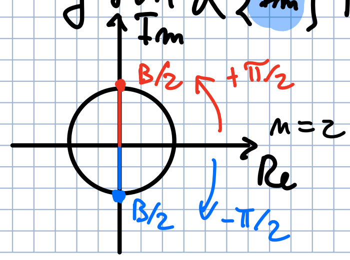
$$X_{2n} = \begin{cases} \frac{B}{j2\cancel{T_0}}\cdot \cancel{T_0} & n=2 \\ -\frac{B}{j2\cancel{T_0}}\cdot \cancel{T_0} & n=-2 \\ 0 & n \neq \pm 2 \end{cases} \Rightarrow X_n = \begin{cases} \frac{A}{2}\, e^{j1} & n=1 & (1) \\ \frac{A}{2}\, e^{-j1} & n=-1 & (2) \\ \frac{B}{2}\, e^{-j\pi/2} & n=2 & (3) \\ \frac{B}{2}\, e^{j\pi/2} & n=-2 & (4) \\ 0 & n \neq \pm 1, \pm 2 \end{cases}$$

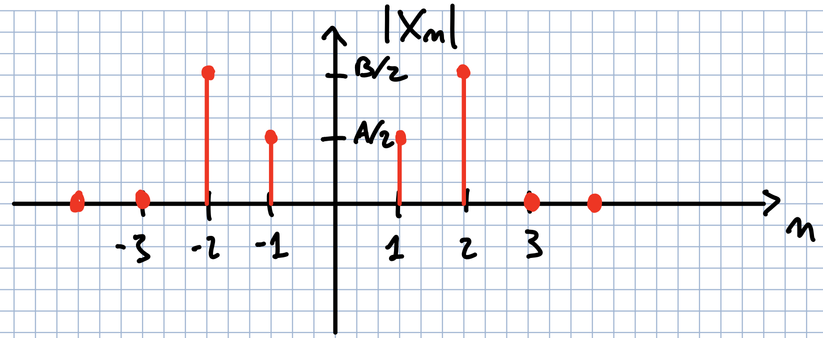
$\{Re\}\ \cos\alpha \quad + \quad j\,\operatorname{sen}\alpha\ \{Im\} +$

(1) e (2)

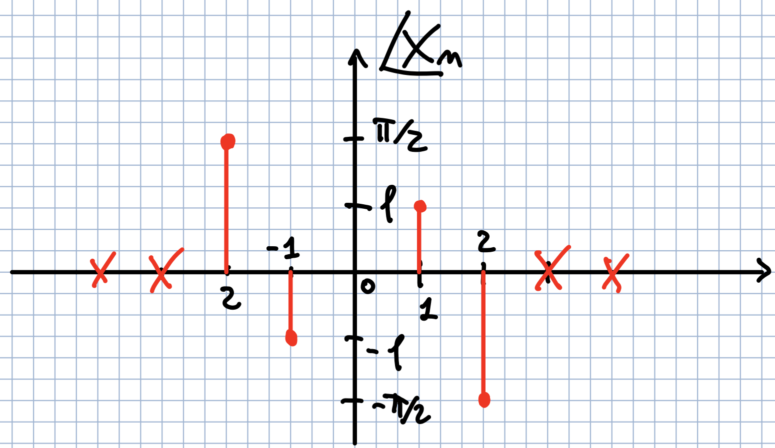


(3) e (4)





SPETTRO DI AMPIEZZA



SPETTRO DI FASE

ES. 2 - PRODOTTO DI SINUSOIDI

$$z(t) = A\cos(2\pi f_0 t + \varphi)\cdot\cos(4\pi f_0 t)$$

•) Calcolare e disegnare lo SPETTRO di $z(t)$

SVOLGIMENTO:

$$Z_n = TSF[z(t)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$$

$$z(t) = \frac{A}{2}\cos(2\pi f_0 t + \varphi + 4\pi f_0 t) + \frac{A}{2}\cos(2\pi f_0 t + \varphi - 4\pi f_0 t) =$$

$$= \frac{A}{2}\cos(\underbrace{6\pi f_0 t + \varphi}_{3f_0}) + \frac{A}{2}\cos(\underbrace{-2\pi f_0 t + \varphi}_{f_0}) = \rightarrow \frac{A}{2}\cos(2\pi f_0 t - \varphi)$$

$$f_0 = \frac{1}{T_0} \quad \rightarrow \text{ È PERIODICO}$$

$$z(t - nT_0) = z(t) \qquad T_0 = \frac{1}{f_0}$$

$$z(t - nT_0) = \frac{A}{2}\cos(6\pi f_0(t - nT_0) + \varphi) + \frac{A}{2}\cos(2\pi f_0(t - nT_0) - \varphi) =$$

$$= \frac{A}{2}\cos(6\pi f_0 t - 6\pi n \cancel{f_0}\cancel{T_0} + \varphi) + \frac{A}{2}\cos(2\pi f_0 t - 2\pi n \cancel{f_0}\cancel{T_0} - \varphi) =$$

$$= \frac{A}{2}\cos(6\pi f_0 t + \varphi) + \frac{A}{2}\cos(2\pi f_0 t - \varphi) = z(t)$$

$$Z_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} z(t)\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{A}{2T_0}\int_{-T_0/2}^{T_0/2} \frac{e^{j(6\pi f_0 t + \varphi)} + e^{-j(6\pi f_0 t + \varphi)}}{2}\, e^{-j2\pi n f_0 t}\, dt \; +$$

$$+ \frac{A}{2T_0}\int_{-T_0/2}^{T_0/2} \frac{e^{j(2\pi f_0 t - \varphi)} + e^{-j(2\pi f_0 t - \varphi)}}{2}\, e^{-j2\pi n f_0 t}\, dt =$$

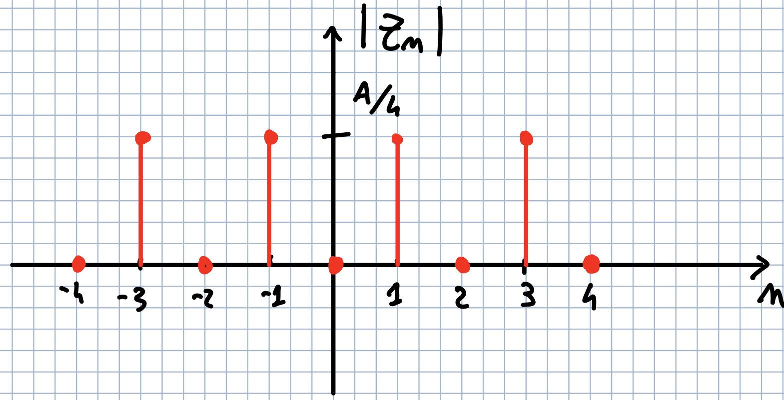
$$= \frac{A}{4T_0} e^{j1} \int_{-T_0/2}^{T_0/2} e^{j2\pi(3-n)f_0 t} \, dt + \frac{A}{4T_0} e^{-j1} \int_{-T_0/2}^{T_0/2} e^{-j2\pi(3+n)f_0 t} \, dt \; +$$

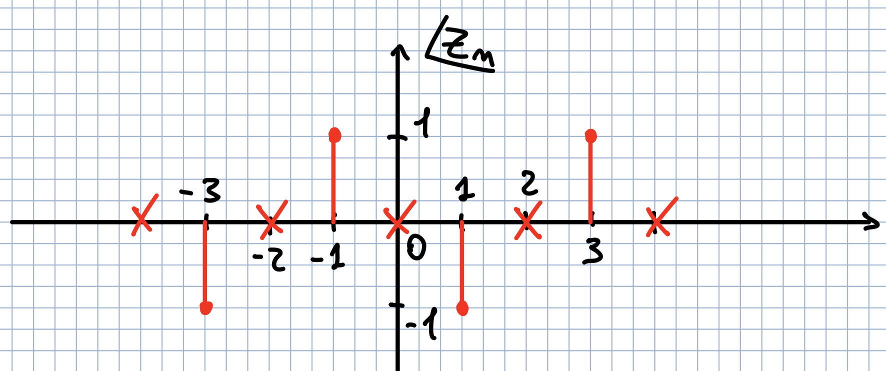
$n=3$; $n=-3$

$$+ \frac{A}{4T_0} e^{-j1} \int_{-T_0/2}^{T_0/2} e^{j2\pi(1-n)f_0 t} \, dt + \frac{A}{4T_0} e^{j1} \int_{-T_0/2}^{T_0/2} e^{-j2\pi(1+n)f_0 t} \, dt$$

$n=1$; $n=-1$

$$Z_n = \begin{cases} \frac{A}{4\cancel{T_0}} e^{j1} \cancel{T_0} & n = 3 \\ \frac{A}{4\cancel{T_0}} e^{-j1} \cancel{T_0} & n = -3 \\ \frac{A}{4\cancel{T_0}} e^{-j1} \cancel{T_0} & n = 1 \\ \frac{A}{4\cancel{T_0}} e^{j1} \cancel{T_0} & n = -1 \\ 0 & n \neq \pm 1, \pm 3 \end{cases}$$





MODULO → S. PARI

FASE → SIMM. DISPARI

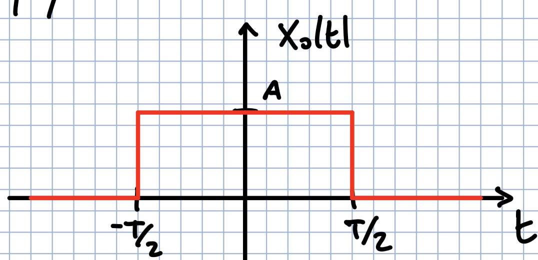
SI NOTA LA SIMMETRIA HERMITIANA

$z(t) \Rightarrow Z_n$

REALE $\quad$ HERMITIANA

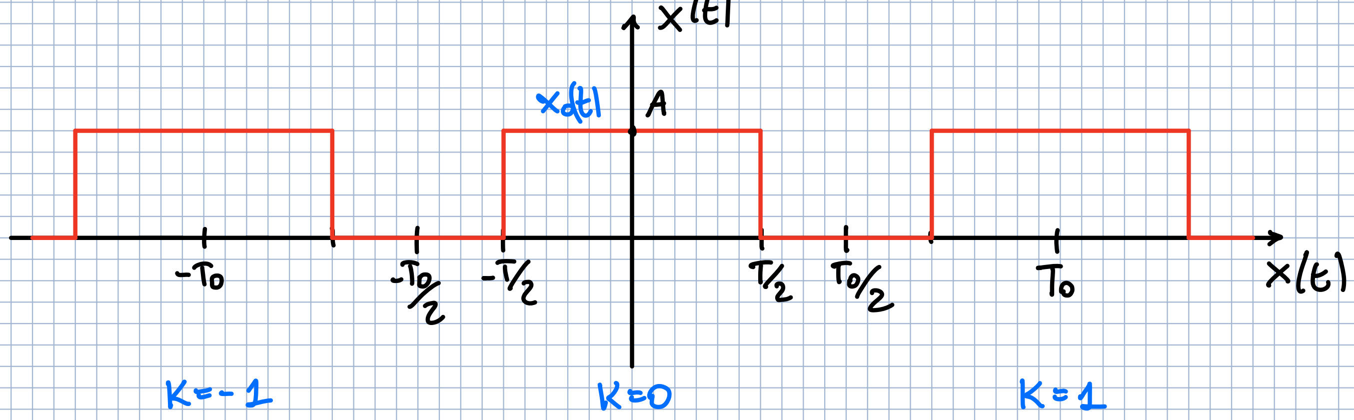
ES. - TRENO DI IMPULSI RETTANGOLARI

$$X_0(t) = A \, \mathrm{rect}\left(\frac{t}{T}\right)$$



$$x(t)=\sum_{k=-\infty}^{+\infty} x_0(t-kT_0) = \qquad T<T_0$$

$$= A\sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-kT_0}{T}\right)$$



x(t) è PERIODICO ?

$$x(t-nT_0)=x(t)$$

$$\sum_{k=-\infty}^{+\infty} x(t-nT_0-kT_0)=\sum_{k=-\infty}^{+\infty} x(t-(n+k)T_0) = \qquad n+k=k'$$

$$=\sum_{k'=-\infty}^{+\infty} x(t-k'T_0)=x(t) \rightarrow \text{SI}$$

$$X_n = ?$$

$$X_n=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,e^{-j2\pi n f_0 t}\,dt=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x_0(t)\,e^{-j2\pi n f_0 t}\,dt=$$

$$=\frac{A}{T_0}\int_{-T_0/2}^{T_0/2} \mathrm{rect}\left(\frac{t}{T}\right)e^{-j2\pi n f_0 t}\,dt=\frac{A}{T_0}\int_{-T/2}^{T/2} e^{-j2\pi n f_0 t}\,dt=$$

$$=\frac{A}{\cancel{T_0}}\left(-\frac{1}{j2\pi n \cancel{f_0}}\right)e^{-j2\pi n f_0 t}\Big|_{-T/2}^{T/2}=-\frac{A}{j2\pi n}\left(e^{-j2\pi n f_0\frac{T}{2}}-e^{+j2\pi n f_0 \frac{T}{2}}\right)=$$

$$=\frac{A}{\pi n}\left(\frac{e^{j\pi n f_0 T}-e^{-j\pi n f_0 T}}{2j}\right)=\frac{A}{\pi n}\cdot \mathrm{sen}(\pi n f_0 T)$$

$= \frac{A \sin(\pi n f_0 T)}{\pi n} =$ PARTO DA QUESTA E MOLTIPLICO per $\frac{f_0 T}{f_0 T}$

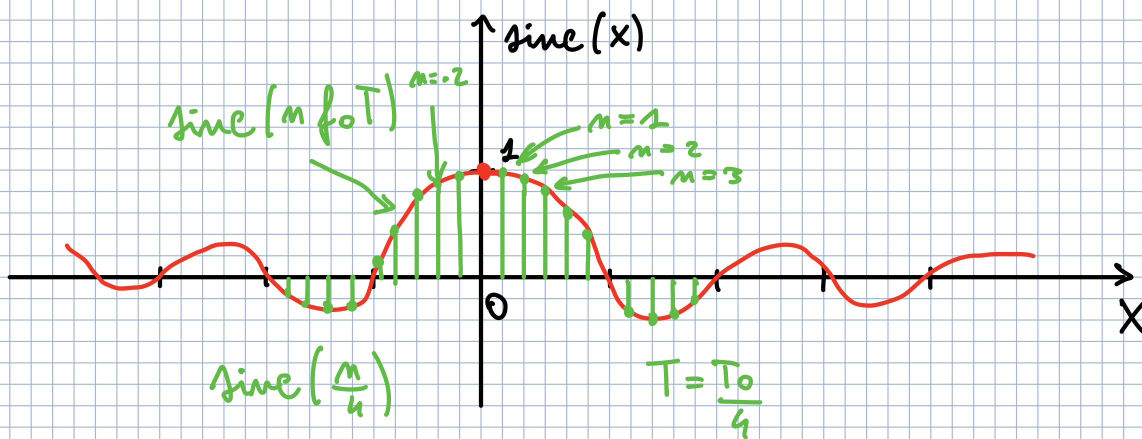
$$\boxed{\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}}$$

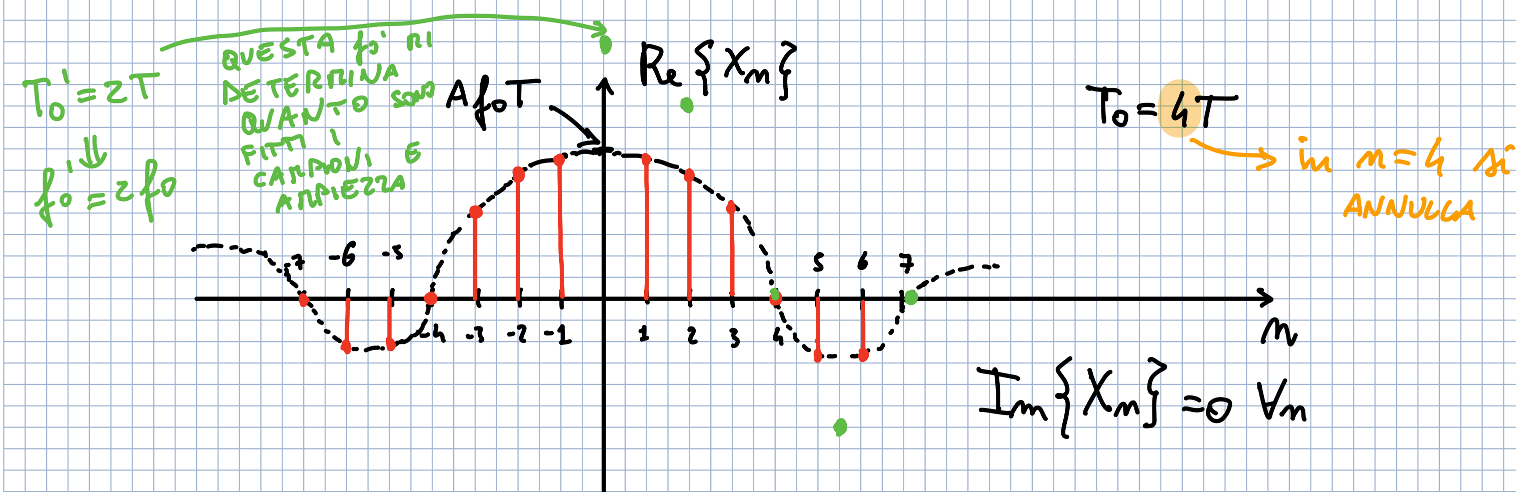
OMETTO π

$$A f_0 T \frac{\sin(\pi n f_0 T)}{\pi n f_0 T} = \boxed{A f_0 T \,\mathrm{sinc}(n f_0 T)}$$

$$\boxed{A \sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t - nT_0}{T}\right) \xleftrightarrow{TSF} A f_0 T \,\mathrm{sinc}(n f_0 T)}$$

$\mathrm{sinc}(x)$ $\quad$ x var. continua $\in \mathbb{R}$





$x(t)$ è REALE E PARI $\Longrightarrow$ X_n è REALE E PARI

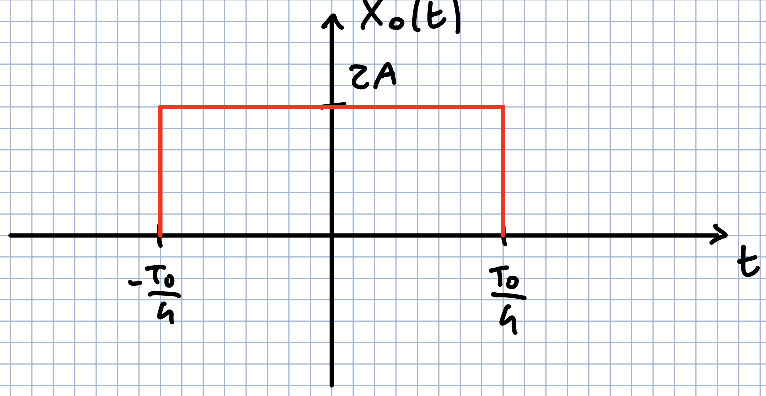
SINC è PARI perché è il RAPPORTO TRA 2 FUNZIONI DISPARI

ESERCIZIO

$$x_0(t) = 2A \, \mathrm{rect}\left(\frac{t}{T_0/2}\right)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} x_0(t-kT_0) - A \qquad \boxed{X_n = ?}$$

SVOLG.



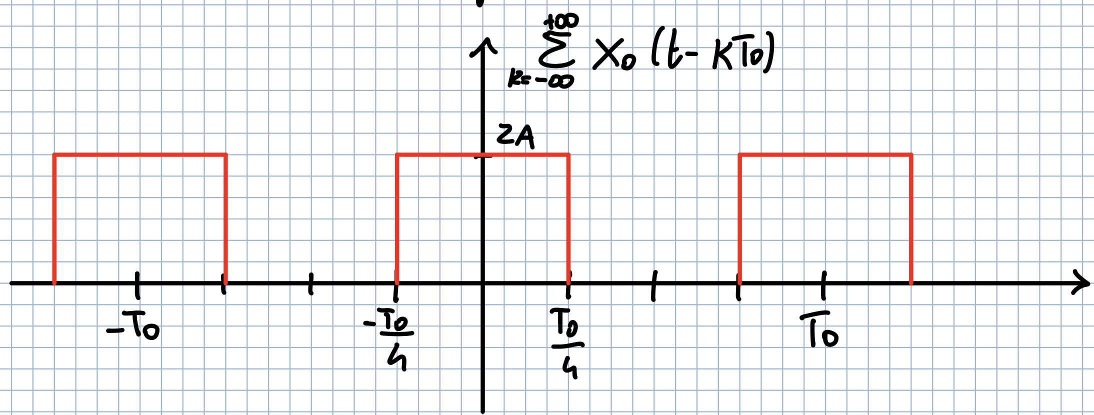
HA AMPIEZZA 2A

HA DURATA DI $\frac{T_0}{2}$

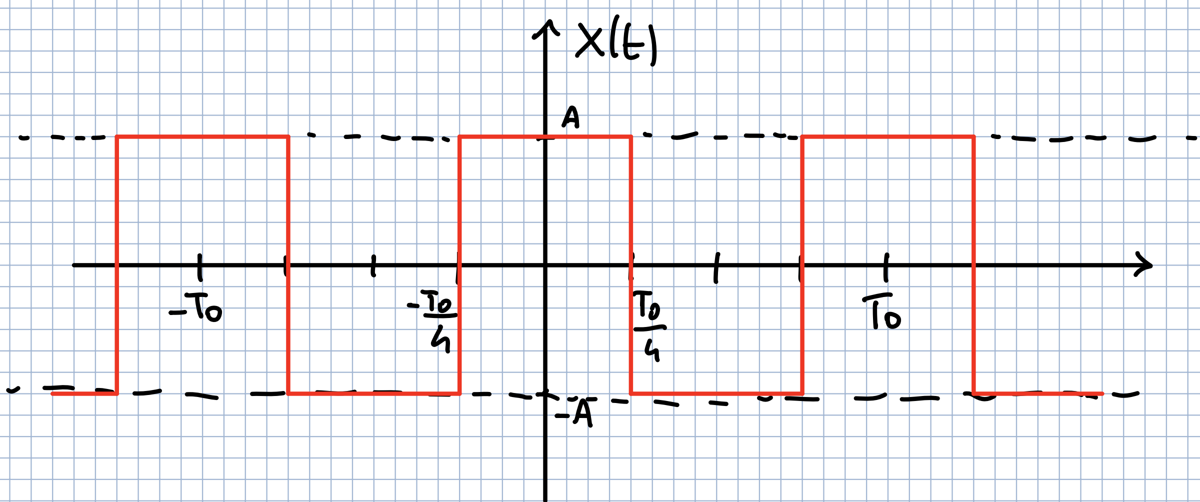
⇓

VA DA $-\frac{T_0}{4}$ a $\frac{T_0}{4}$

per ottenere la $x(t)$ devo periodicizzare la rect con T_0 e sottrarre A



PERIODICIZZO



I^a STRADA

•) è un segnale $x(t)$ REALE E PARI $\Rightarrow$ X_n REALE E PARI

·) è un segnale alternativo $\rightarrow X_n = \begin{cases} 0 & n \text{ PARI} \\ \frac{2}{T_0}\int_0^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt & n \text{ DISP.} \end{cases}$

II^ STRADA

$$x_0(t) = 2A \,\mathrm{rect}\left(\frac{t}{T_0/2}\right)$$

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = \sum_{k=-\infty}^{+\infty} x_0(t-kT_0) = 2A\sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-kT_0}{T_0/2}\right)$$

$$x_2(t) = -A$$

·) LINEARITA' $\Rightarrow X_n = X_{1n} + X_{2n}$

$$X_{1n} = TSF[x_1(t)]$$

$$X_{2n} = TSF[x_2(t)]$$

$$A\sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-kT_0}{T}\right) \Longleftrightarrow A\frac{T}{T_0}\,\mathrm{sinc}\left(n\frac{T}{T_0}\right)$$

f_0 f_0

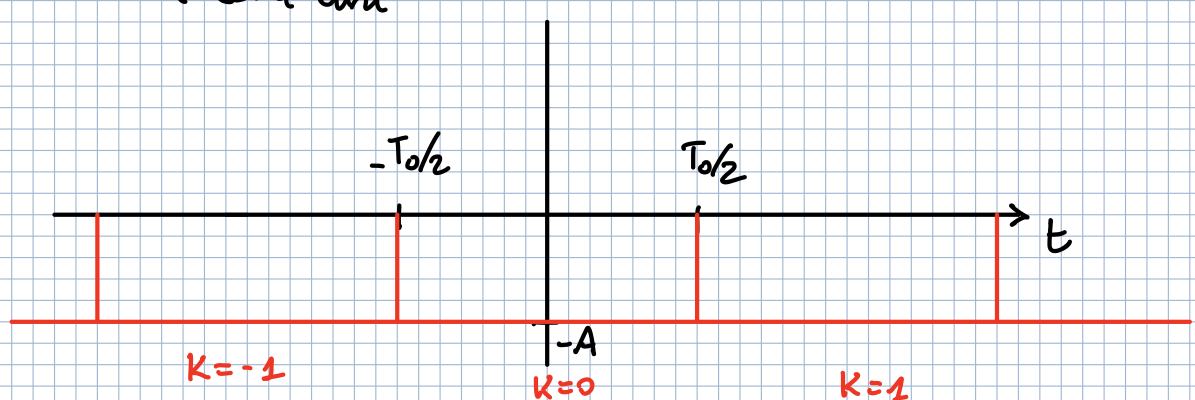
$$2A\sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-kT_0}{T_0/2}\right) \Longleftrightarrow \cancel{2}A\frac{\cancel{T_0}/\cancel{2}}{\cancel{T_0}}\,\mathrm{sinc}\left(n\frac{\cancel{T_0}/2}{\cancel{T_0}}\right) =$$

$$X_{1n} = A\,\mathrm{sinc}\left(\frac{n}{2}\right)$$

$$X_{2n} = -A = -A\sum_{k=-\infty}^{+\infty} \mathrm{rect}\,\frac{t-kT_0}{T_0} \Longleftrightarrow -A\frac{T_0}{T_0}\,\mathrm{sinc}\left(n\frac{T_0}{T_0}\right) = -A\,\mathrm{sinc}(n)$$

$\downarrow$ è come dire

$\| \; X_{2n}$



$$X_n = A\,\mathrm{sinc}\left(\frac{n}{2}\right) - A\,\mathrm{sinc}(n)$$

1^a STRADA

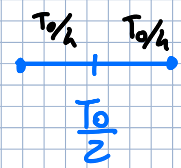
$$X_n = \begin{cases} 0 & n \text{ PARI} \\ \dfrac{2}{T_0}\displaystyle\int_0^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt & n \text{ DISPARI} \end{cases}$$

$$\mathrm{rect}\left(\frac{t}{T}\right) \triangleq \begin{cases} 1 & -T/2 \le t \le T/2 \\ 0 & \text{ALTROVE} \end{cases}$$

X_n DISPARI =

$$= \frac{2}{T_0}\int_0^{T_0/2}\left[2A\,\mathrm{rect}\left(\frac{t}{T_0/2}\right) - A\right] e^{-j2\pi n f_0 t}\, dt =$$

* HA DURATA $\frac{T_0}{2}$



$$= \frac{4A}{T_0}\int_0^{T_0/4 \, *} 1 \cdot e^{-j2\pi n f_0 t}\, dt - \frac{2A}{T_0}\int_0^{T_0/2} e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{\cancel{4}^{2}A}{\cancel{T_0}}\left(-\frac{1}{j\cancel{2}\pi n \cancel{f_0}}\right) e^{-j2\pi n f_0 t}\Big|_0^{T_0/4} - \frac{\cancel{2}A}{\cancel{T_0}}\left(-\frac{1}{j\cancel{2}\pi n \cancel{f_0}}\right) e^{-j2\pi n f_0 t}\Big|_0^{T_0/2} =$$

$$= -\frac{2A}{j\pi n}\left(e^{-j\pi n f_0 \frac{T_0}{2}} - 1\right) + \frac{A}{j\pi n}\left(e^{-j\pi n f_0 T_0} - 1\right) =$$

$$= \frac{2A}{j\pi n}\left(1 - e^{-j\frac{\pi n}{2}}\right) + \frac{A}{j\pi n}\Big(\underbrace{\overbrace{e^{-j\pi n}}^{-1} - 1}_{= -2}\Big) = \boxed{n \text{ DISPARI}}$$

$$= \cancel{\frac{2A}{j\pi n}} - \cancel{\frac{2A}{j\pi n}} - \frac{2A}{j\pi n}\cdot e^{-j\frac{\pi n}{2}} = \frac{2A\, e^{j\frac{\pi n}{2}}}{j\pi n} = \frac{2A\, \cancel{j}(-1)^{\frac{n-1}{2}}}{\cancel{j}\pi n} = \frac{2A(-1)^{\frac{n-1}{2}}}{\pi n}$$

$$X_n = A\,\mathrm{sinc}\left(\frac{n}{2}\right) - A\,\mathrm{sinc}(n)$$

$$\mathrm{sinc}(x) = \frac{\mathrm{sen}(\pi x)}{\pi x}$$

OMETTO π

n PARI $\Rightarrow X_n = 0$

$$\frac{2A\,\mathrm{sen}\,\pi n/2}{\pi n} - \frac{A\,\mathrm{sen}\,\pi n}{\pi n} = \begin{cases} n=0 \to 0 \\ n=2 \to 0 \\ n=4 \to 0 \end{cases}$$

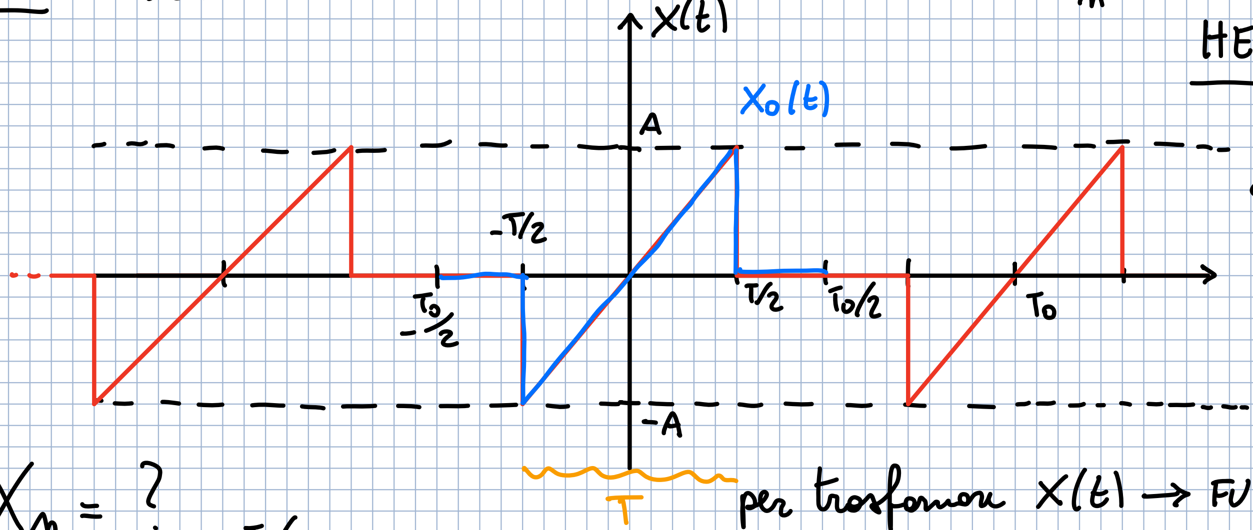
il seno per N DISPARI è val. -1 e 1

n DISPARI $\quad A\,\mathrm{sinc}\left(\frac{n}{2}\right) = \dfrac{A\,\mathrm{sinc}\,\pi n/2}{\pi n/2} = \dfrac{A(-1)^{\frac{n-1}{2}}}{\pi n/2} = \dfrac{2A(-1)^{\frac{n-1}{2}}}{\pi n}$

ES. DISEGNO → TSF

X_n DISPARI E IMM. PURA

HERMITIANA



$X_n = ?$

per trasformare $x(t)$ → FUNZIONE ANALITICA

$$X_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt =$$

in $\frac{T}{2}$ vale A $\quad m = \frac{A}{T/2} = \frac{2A}{T}$

↑ pendenza, è una rampa

$$x_0(t) = \frac{2A}{T} t \,\mathrm{rect}\left(\frac{t}{T}\right)$$

t → DERIVATA

e^{-j} → INTEGRALE

$$X_n = \frac{1}{T_0}\int_{-T/2}^{T/2} \frac{2At}{T} e^{-j2\pi n f_0 t}\, dt = \frac{2A}{T_0 T}\int_{-T/2}^{T/2} t\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{2A}{T_0 T}\cdot\left[t\cdot\left(-\frac{1}{j2\pi n f_0}\right)\cdot e^{-j2\pi n f_0 t}\Big|_{-T/2}^{T/2} - \int_{-T/2}^{T/2} e^{-j2\pi n f_0 t}\, dt\right] =$$

$$= \frac{2A}{T_0 T}\cdot\left[\left(-\frac{1}{j\pi n f_0}\right)\underbrace{\left(\frac{T}{2} e^{-j\pi n f_0 \frac{T}{2}} + \frac{T}{2} e^{+j2\pi n f_0 \frac{T}{2}}\right)}_{} + \right.$$

$\rightarrow \cos\alpha = \dfrac{e^{j\alpha}+e^{-j\alpha}}{2}$

$$\left. - \left(-\frac{1}{j2\pi n f_0}\right)\cdot e^{-j2\pi n f_0 t}\Big|_{-T/2}^{T/2}\right] =$$

$\mathrm{sen}\,\alpha = \dfrac{e^{j\alpha}-e^{-j\alpha}}{2j}$

$$= \frac{2A}{T_0 T}\left[\frac{jT}{2\pi n f_0}\cos(\pi n f_0 T) + \frac{1}{j2\pi n f_0}\left(e^{-j\pi n f_0 T} - e^{j\pi n f_0 T}\right)\right] =$$

$2j$

DA FINIRE !

$$y_0(t) = \frac{A}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$$

$$Y(f) = Y_0(f)\,e^{-j2\pi f(-T/2)} - Y_0(f)\,e^{-j2\pi f T/2} =$$

$$= Y_0(f)\left[e^{j2\pi f T/2} - e^{-j2\pi f T/2}\right]$$

TCF rect

$$Y_0(f) = \frac{A}{\cancel{T}} \cdot \boxed{\cancel{T}\,\mathrm{sinc}(Tf)} = A\,\mathrm{sinc}(Tf)$$

$$Y(f) = A\,\mathrm{sinc}(Tf)\left[e^{j\pi f T} - e^{-j\pi f T}\right]$$

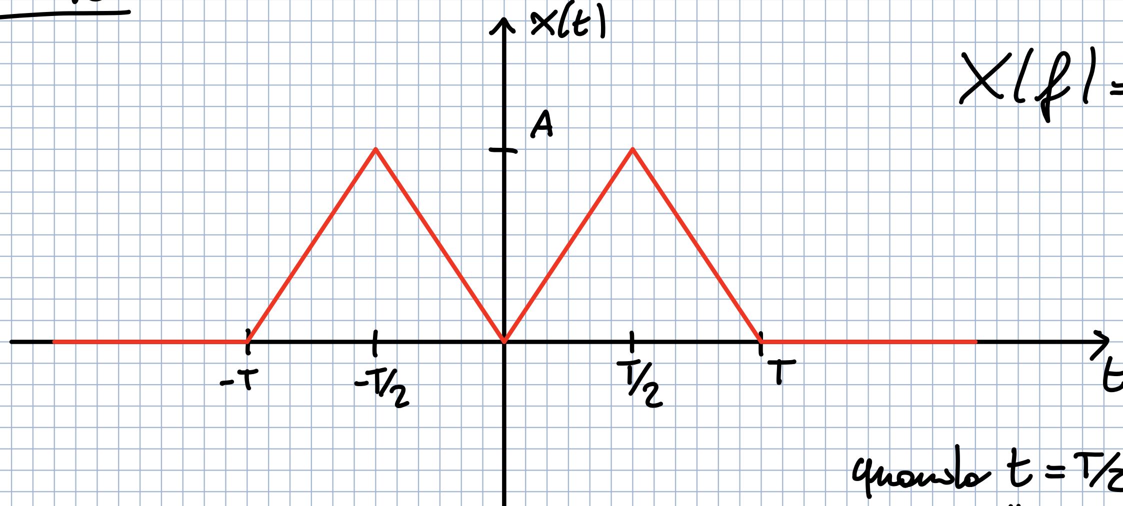
$$X(f) = \frac{Y(f)}{j2\pi f} = \frac{A}{\pi f}\,\mathrm{sinc}(Tf)\,\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j} = \frac{A}{\pi f}\,\mathrm{sinc}(Tf)\,\mathrm{sen}(\pi f T) =$$

$$= AT\,\mathrm{sinc}(Tf)\,\frac{\mathrm{sen}(\pi f T)}{\pi f T} = AT\,\mathrm{sinc}(Tf)\,\mathrm{sinc}(Tf) = \boxed{AT\,\mathrm{sinc}^2(Tf)}$$

MOLTIPLICO per $\frac{T}{T}$

$$\boxed{A\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \overset{TCF}{\Longleftrightarrow} AT\,\mathrm{sinc}^2(Tf)}$$

ESERCIZIO



$X(f) = ?$

$$x(t) = \underbrace{X_0(t + T/2)}_{SX} + \underbrace{X_0(t - T/2)}_{DX}$$

quando $t = T/2$ $\quad X_0(t) = 0$

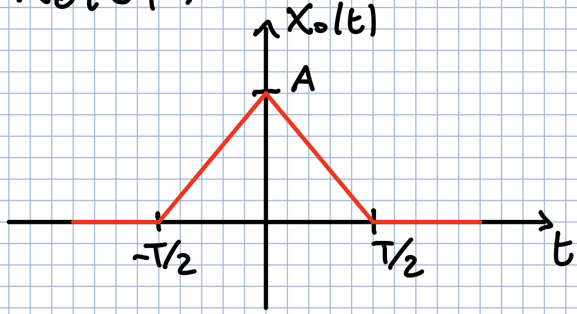
$\Downarrow$

in $\frac{T}{2}$ si ANNULLA

e la rect è grande quanto la base del triangolo cioè T

$$X_0(t) = A\left(1 - \frac{|t|}{T/2}\right)\mathrm{rect}\left(\frac{t}{T}\right)$$

DISEGNAMO $x_0(t)$:



per LINEARITÀ

$$X(f) = X_1(f) + X_2(f)$$

$$X(f) = \underbrace{X_0(f)\, e^{+j2\pi f T/2}}_{X_1(f)} + \underbrace{X_0(f)\, e^{-j2\pi f T/2}}_{X_2(f)} =$$

c'è + perché c'è un ritardo a sx

c'è − perché c'è ritardo a dx

$$= 2X_0(f)\underbrace{\left(e^{j\pi f T} + e^{-j\pi f T}\right)}_{2} = 2X_0(f)\cos(\pi f T)$$

$$X_0(f) = \mathrm{TCF}[x_0(t)]$$

$$\boxed{A\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \overset{\mathrm{TCF}}{\Longleftrightarrow} AT\,\mathrm{sinc}^2(Tf)}$$

$$x_0(t) = A\left(1 - \frac{|t|}{T/2}\right)\mathrm{rect}\left(\frac{t}{T}\right) \qquad T' = \frac{T}{2}$$

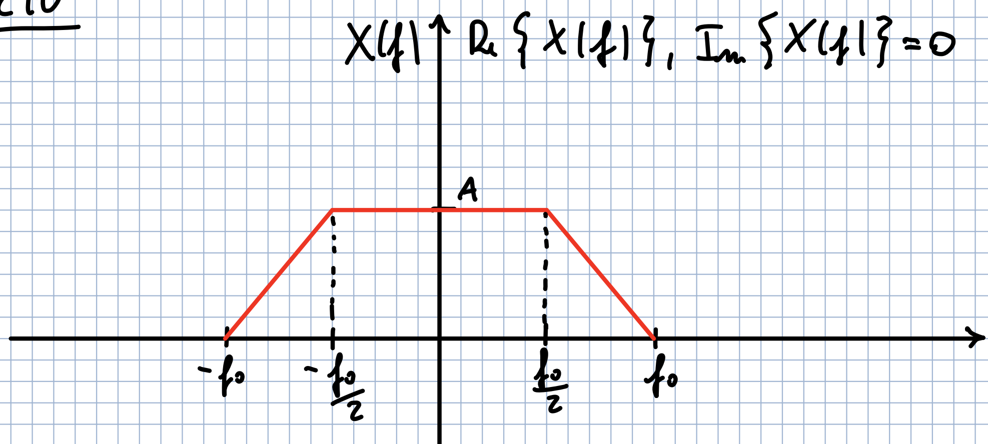
$$x_0(t) = A\left(1 - \frac{|t|}{T'}\right)\mathrm{rect}\left(\frac{t}{2T'}\right)$$

$$\Downarrow$$

$$X_0(f) = AT'\,\mathrm{sinc}^2(T'f) = \frac{AT}{2}\,\mathrm{sinc}^2\left(\frac{Tf}{2}\right)$$

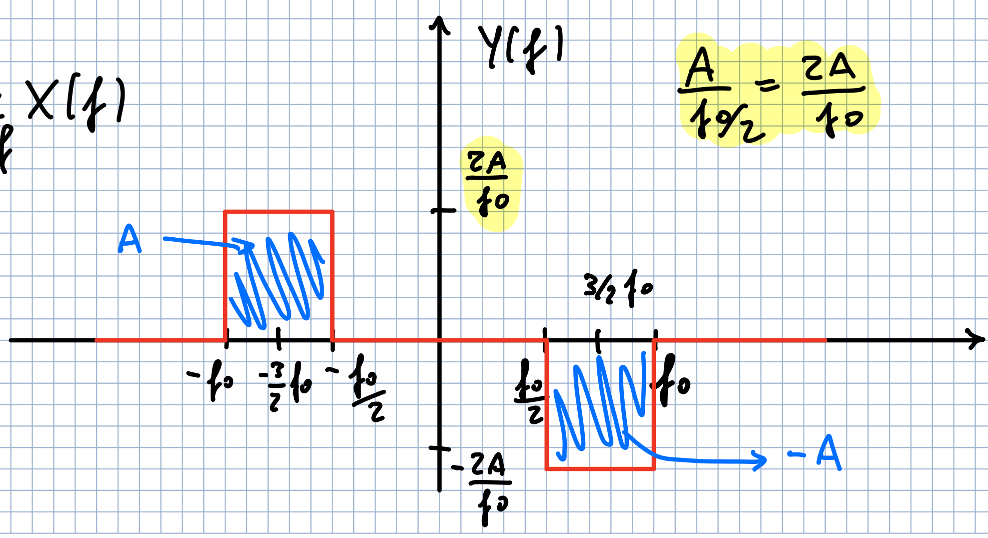
$$X(f) = 2X_0(f)\cos(\pi f T) = \boxed{AT\,\mathrm{sinc}^2\left(\frac{Tf}{2}\right)\cos(\pi f T)}$$

ESERCIZIO



$$x(t) = ATCF[X(f)]$$

$$Y(f) = \frac{d}{df} X(f)$$



$$\frac{A}{f_0/2} = \frac{2A}{f_0}$$

$$A + (-A) = 0$$

*

$$X(f) = \int_{-\infty}^{f} Y(\alpha)\, d\alpha \quad \Rightarrow \quad x(t) = -\frac{y(t)}{-j2\pi t}$$

$$\int_{-\infty}^{+\infty} Y(\alpha)\, d\alpha \stackrel{?}{=} 0 \quad * \text{ SI} \longrightarrow \frac{2A}{f_0} \cdot \frac{f_0}{2} = A + (-A) = 0$$

$h \cdot b =$ AREA RETT.

$$y(t) = \text{ATCF}[Y(f)]$$

per calcolare $Y(f)$ individuiamo il centro dei rettangoli

$$Y(f) = \frac{2A}{f_0}\,\text{rect}\left(\frac{f-\left(-\frac{3}{2}f_0\right)}{\underbrace{f_0/2}_{\text{(BASE) durata rect}}}\right) - \frac{2A}{f_0}\,\text{rect}\left(\frac{f-\left(\frac{3}{2}f_0\right)}{f_0/2}\right)$$

$$y(t) = \frac{2A}{f_0}\frac{f_0}{2}\,\text{sinc}\left(\frac{f_0}{2}t\right)e^{-j2\pi\frac{3}{2}f_0 t} - \frac{2A}{f_0}\frac{f_0}{2}\,\text{sinc}\left(\frac{f_0}{2}t\right)e^{j2\pi\frac{3}{2}f_0 t}$$

$$\boxed{\text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow T\,\text{sinc}(Tf)}$$

$$\left[\begin{array}{l} x(t-t_0) \Leftrightarrow X(f)\,e^{-j2\pi f t_0} \\ X(f-f_0) \Leftrightarrow x(t)\,e^{j2\pi f_0 t} \end{array}\right]$$

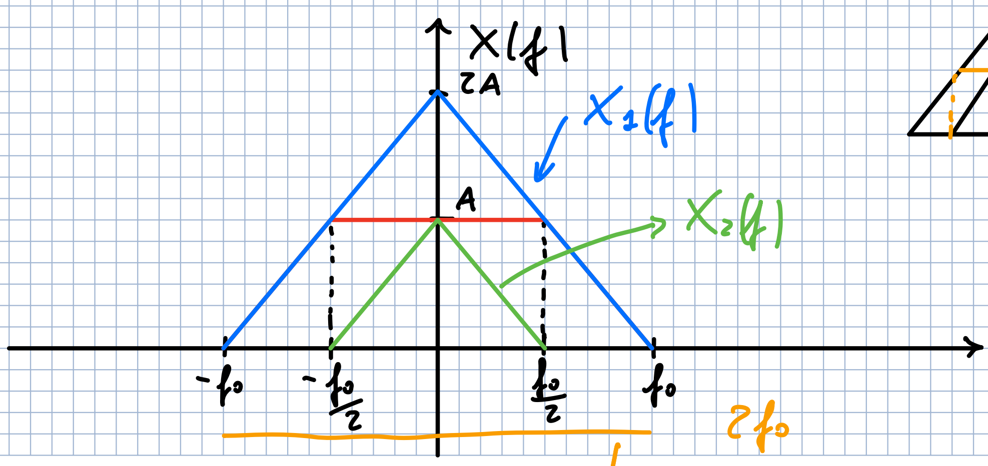
TH. RITARDO

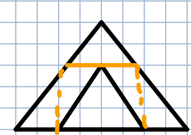
$$y(t) = A\,\text{sinc}\left(\frac{f_0}{2}t\right)\left[e^{-j3\pi f_0 t} - e^{j3\pi f_0 t}\right]$$

$$x(t) = -\frac{y(t)}{j2\pi t} = \frac{A}{\pi t}\,\text{sinc}\left(\frac{f_0}{2}t\right)\left[\frac{e^{j3\pi f_0 t} - e^{-j3\pi f_0 t}}{2j}\right] = \frac{A}{\pi t}\,\text{sinc}\left(\frac{f_0}{2}t\right)\text{sen}(3\pi f_0 t)$$

$$= 3f_0 A\,\text{sinc}\left(\frac{f_0}{2}t\right)\frac{\text{sen}(3\pi f_0 t)}{3f_0\pi t} = 3f_0 A\,\text{sinc}\left(\frac{f_0}{2}t\right)\text{sinc}(3f_0 t)$$

POTEVA ESSERE SVOLTO IN UN'ALTRA MANIERA





$$X(f) = X_1(f) - X_2(f)$$

$$X_1(f) = 2A\left(1-\frac{|f|}{f_0}\right)\mathrm{rect}\left(\frac{f}{2f_0}\right)$$

QUESTO TRIANGOLO ASSUME VALORE MASSIMO in $f=0 \rightarrow 2A$ E DEVE ANDARE A 0 QUANDO $f = f_0 = -f_0$

$$X_2(f) = A\left(1-\frac{|f|}{f_0/2}\right)\mathrm{rect}\left(\frac{f}{f_0}\right)$$

$$2A\left(1-\frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \overset{TCF}{\Longleftrightarrow} 2T\,\mathrm{sinc}^2(Tf)$$

$$x_1(t) = 2A f_0\,\mathrm{sinc}^2(f_0 t)$$

$$2Af_0\,\mathrm{sinc}^2(f_0 t) \Longleftrightarrow 2A\left(1-\frac{|-f|}{f_0}\right)\mathrm{rect}\left(\frac{-f}{2f_0}\right)$$

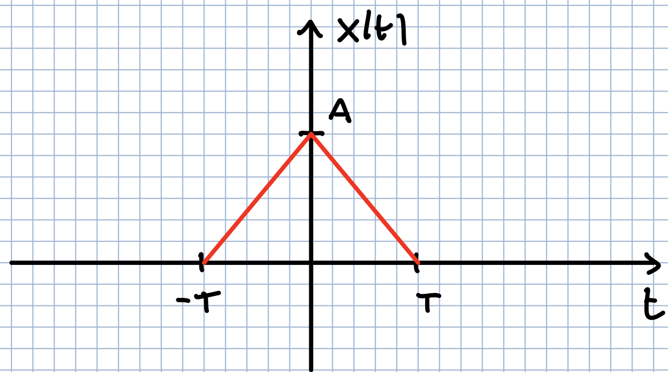
$T = f_0$

$$x_2(t) = A\frac{f_0}{2}\,\mathrm{sinc}^2\left(\frac{f_0}{2}t\right) \rightarrow T = \frac{f_0}{2}$$

$$x(t) = 2Af_0\,\mathrm{sinc}^2(f_0 t) - A\frac{f_0}{2}\,\mathrm{sinc}^2\left(\frac{f_0}{2}t\right) = 3f_0 A\,\mathrm{sinc}\left(\frac{f_0}{2}t\right)\mathrm{sinc}(3f_0 t)$$

È UGUALE AL RIS. DI PRIMA

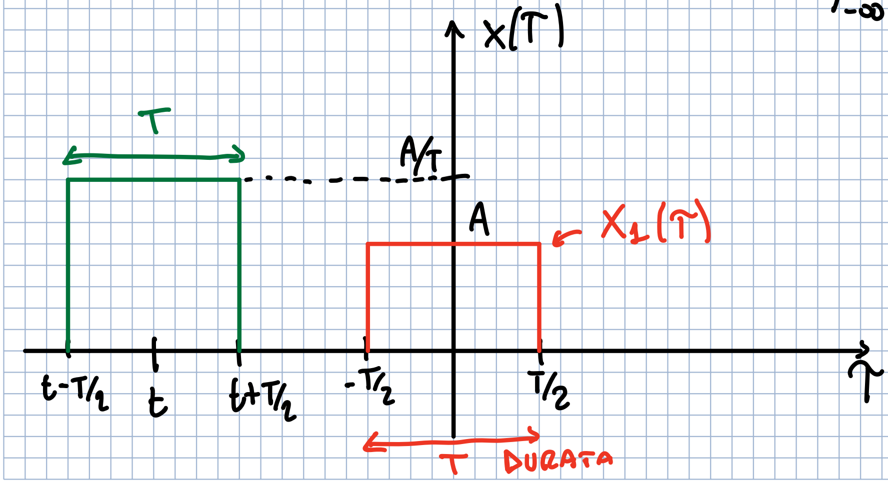
ESERCIZIO



$$X(f) = ?$$

$$x(t) = A\left(1-\frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \Longleftrightarrow AT\,\mathrm{sinc}^2(Tf)$$

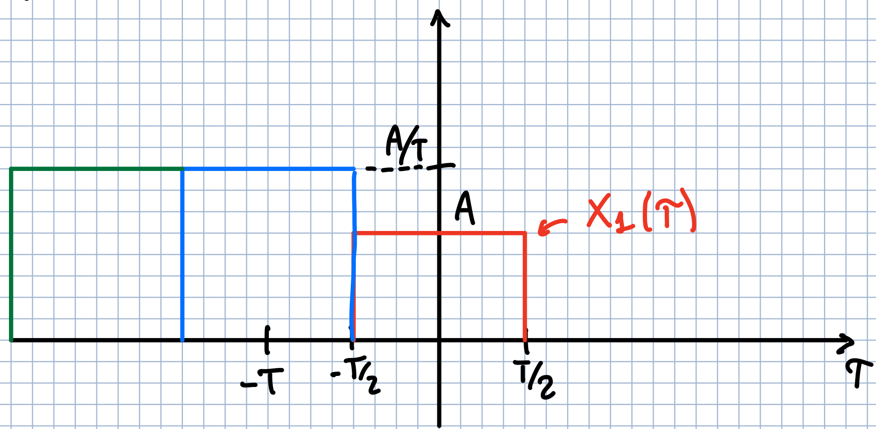
$$x(t) = x_1(t) \otimes x_2(t) = \int_{-\infty}^{+\infty} x_1(\tau)\,x_2(t-\tau)\,d\tau$$



$$x_1(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$$

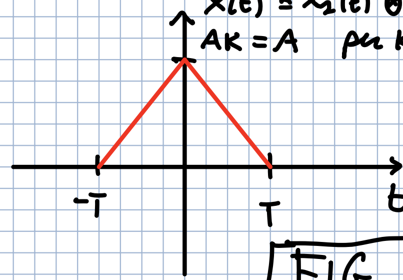
$$x_2(t) = \frac{K}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$$

QUANDO $t = -T$



Quando t è in 0 ho la sovrapposizione perfetta man mano che sovrappongo il prodotto aumenta fino ad essere $K \cdot \frac{A}{\cancel{T}} \cancel{T} = KA$

$x(t) = x_1(t) \otimes x_2(t)$
$AK = A$ per $K = 1$



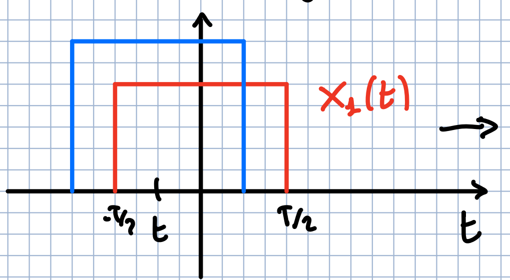
$\boxed{\text{FIG. 1}}$

QUANDO $t < -T$

$x(t) = x_1(\tau)\, x_2(t-\tau) = 0$

$\hookrightarrow x(t) = 0$ perchè i rettangoli non si sovrapp.

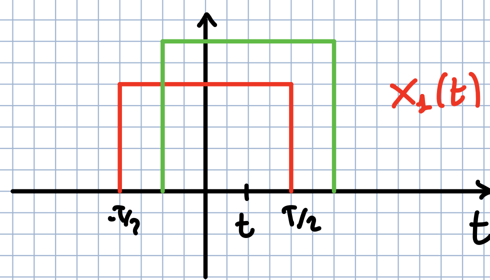
per $-T \leq t \leq 0$



IL PRODOTTO AUMENTA FINO A KA (FIG. 1)

$$A\left(1 + \frac{t}{T}\right)$$

per $0 \leq t \leq T$

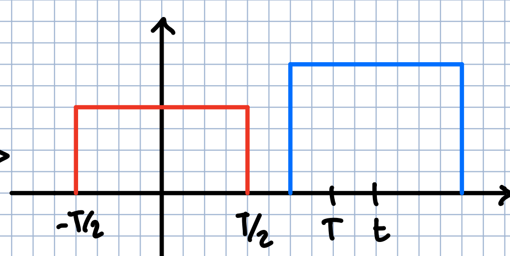


IL PRODOTTO DIMINUISCE FINO A 0 per $t = T$

$$A\left(1 - \frac{t}{T}\right)$$

per $t > T$

IL PRODOTTO è NULLO perchè i rettangoli non si sovrappongono $\Rightarrow$



$$A\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right) = x(t)$$

$x_1(t) = A\, \mathrm{rect}\left(\frac{t}{T}\right) \qquad x_2(t) = \frac{1}{T}\, \mathrm{rect}\left(\frac{t}{T}\right)$

$$X(f) = \underbrace{AT\, \mathrm{sinc}(fT)}_{X_1(f)} \cdot \underbrace{\frac{1}{T} \cdot T\, \mathrm{sinc}(fT)}_{X_2(f)} = AT\, \mathrm{sinc}^2(fT)$$

TRASFORMATE NOTEVOLI

$$A\left(1-\frac{|t|}{T}\right)\text{rect}\left(\frac{t}{2T}\right) \overset{TCF}{\Longleftrightarrow} AT\,\text{sinc}^2(fT)$$

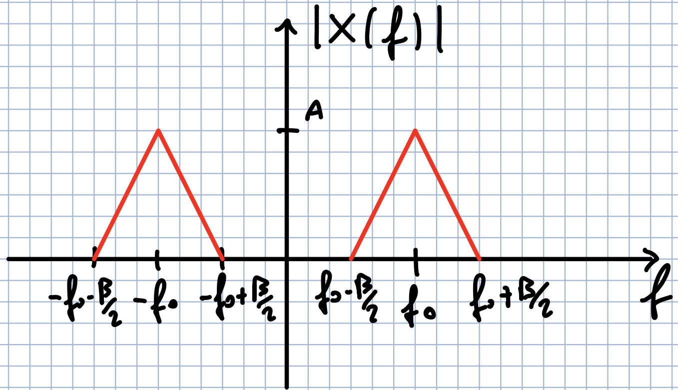
$$A f_0\,\text{sinc}^2(f_0 t) \overset{TCF}{\Longleftrightarrow} A f_0 \cdot \frac{1}{f_0}\left(1-\frac{|f|}{f_0}\right)\text{rect}\left(\frac{f}{2f_0}\right)$$

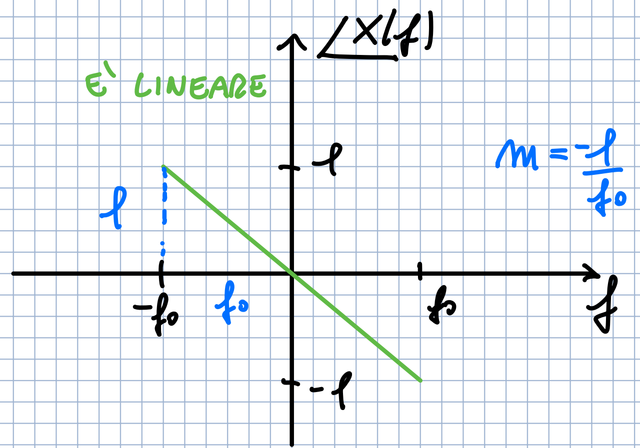
$$A\,\text{rect}\left(\frac{t}{T}\right) \overset{TCF}{\Longleftrightarrow} AT\,\text{sinc}(fT)$$

$$A f_0\,\text{sinc}(f_0 t) \overset{TCF}{\Longleftrightarrow} \frac{A f_0}{f_0}\,\text{rect}\left(\frac{f}{f_0}\right)$$

ESERCIZIO - Calcolo di un' ANTITRASFORMATA

Questa coppia di grafici mi definisce lo spettro





$x(t) = \text{ATCF}[X(f)]$

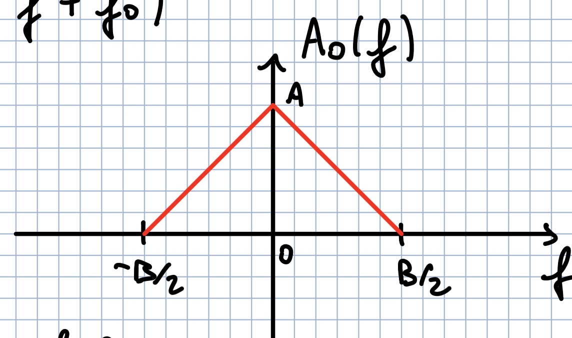
$X(f) = |X(f)|\,e^{j\angle X(f)}$

REPLICA A DX

REPLICA A SX

$|X(f)| = A(f) = A_0(f-f_0) + A_0(f+f_0)$

$A_0(f) = A\left(1-\frac{|f|}{B/2}\right)\text{rect}\left(\frac{f}{B}\right)$



FASE $F(f) = -\frac{1}{f_0} f$

$X(f) = [A_0(f-f_0) + A_0(f+f_0)]\,e^{-j\frac{1}{f_0}f} =$

$t_0 = \frac{1}{2\pi f_0}$

$$= \left[A\left(1-\frac{|f-f_0|}{B/2}\right) rect\left(\frac{f-f_0}{B}\right) + A\left(1-\frac{|f+f_0|}{B/2}\right) rect\left(\frac{f+f_0}{B}\right)\right] \cdot e^{-j2\pi f t_0} =$$

$$X(f) = A(f)\, e^{-j2\pi f t_0} \quad \rightarrow \text{ Per il th. del ritardo}$$

$$x(t) = a(t - t_0) \quad *$$

$$a(t) = ATCF[A(f)]$$

$$A(f) = A\left(1-\frac{|f-f_0|}{B/2}\right) rect\left(\frac{f-f_0}{B}\right) + A\left(1-\frac{|f+f_0|}{B/2}\right) rect\left(\frac{f+f_0}{B}\right)\Big] =$$

$$= A_0(f - f_0) + A_0(f + f_0)$$

* DA TEMPO A FREQUENZA SEGNO CONCORDE

* DA FREQUENZA A TEMPO SEGNO DISCORDE

$$a(t) = a_0(t)\, e^{+j2\pi f_0 t} + a_0(t)\, e^{-j2\pi f_0 t} =$$

$$= a_0 \cos(2\pi f_0 t) + j\, a_0 \cancel{\operatorname{sen}(2\pi f_0 t)} + a_0 \cos(2\pi f_0 t) - j\, a_0 \cancel{\operatorname{sen}(2\pi f_0 t)} =$$

$$= 2a_0 \cos(2\pi f_0 t)$$

$$\boxed{T = \frac{B}{2}}$$

$$x(t) = 2a_0(t-t_0) \cos(2\pi f_0 (t-t_0))$$

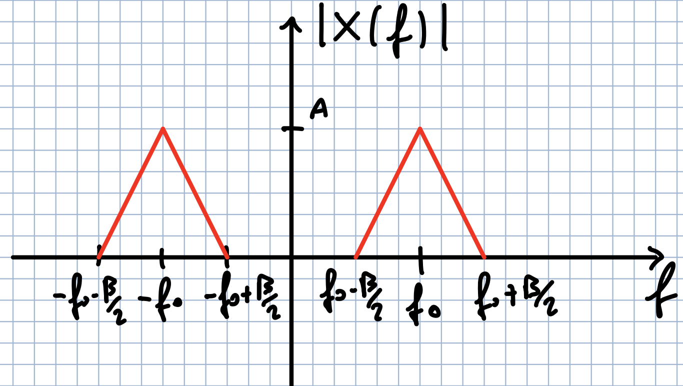
$$a_0(t) = ATCF[A_0(f)] = ATCF\left[A\left(1-\frac{|f|}{B/2}\right) rect\left(\frac{f}{B}\right)\right] =$$

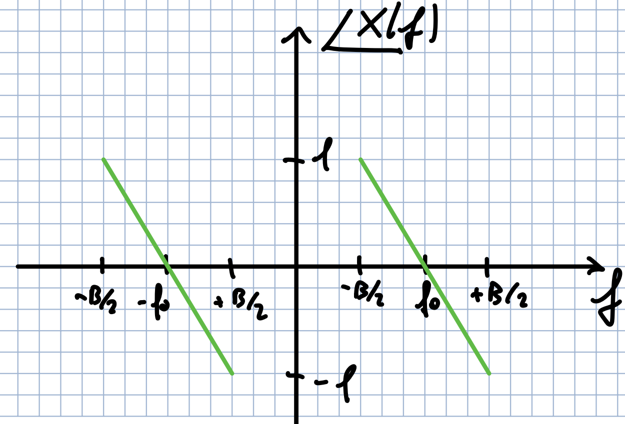
$$= A \cdot \frac{B}{2}\, \operatorname{sinc}^2\left(\frac{B}{2} t\right)$$

$$x(t) = AB\, \operatorname{sinc}^2\left[\frac{B}{2}(t-t_0)\right] \cos\left[2\pi f_0 (t-t_0)\right], \quad t_0 = \frac{1}{2\pi f_0}$$

ESERCIZIO

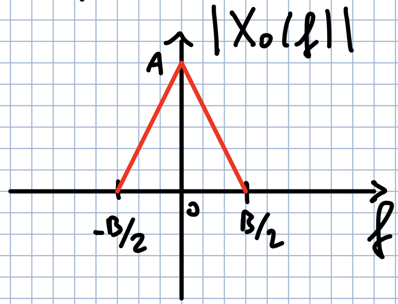
Questa coppia di grafici mi definisce lo spettro

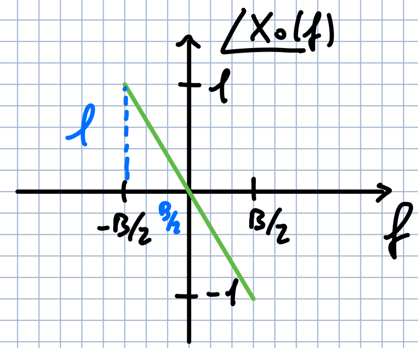




$$x(t) = \text{ATCF}[X(f)]$$

$$X_0(f) = |X_0(f)|\, e^{j\angle X_0(f)}$$





$$m = -\frac{1}{B/2}$$

$$|X_0(f)| = A\left(1 - \frac{|f|}{B/2}\right)\text{rect}\left(\frac{f}{B}\right)$$

$$\angle X_0(f) = -\frac{1}{B/2} f = -2\pi f t_0 \ , \quad t_0 = \frac{1}{2\pi B/2} = \frac{1}{\pi B}$$

$$X(f) = X_0(f-f_0) + X_0(f+f_0)$$

$$x(t) = x_0(t)\, e^{j2\pi f_0 t} + x_0(t)\, e^{-j2\pi f_0 t} = 2x_0(t)\cos(2\pi f_0 t)$$

$$x_0(t) = \text{ATCF}[X_0(f)] = \text{ATCF}\left[A(f)\, e^{-j2\pi f t_0}\right] = a(t-t_0)$$

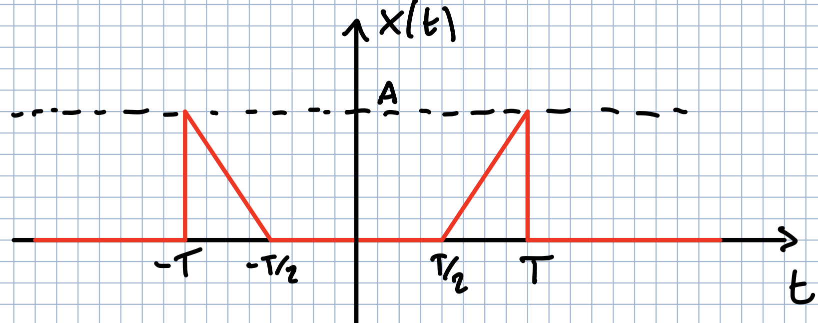
$$a(t) = \text{ATCF}[A(f)] = \text{ATCF}\left[A\left(1 - \frac{|f|}{B/2}\right)\text{rect}\left(\frac{f}{B}\right)\right] = \frac{AB}{2}\text{sinc}^2\left(\frac{B}{2}t\right)$$

$$x_0(t) = \frac{AB}{2}\text{sinc}^2\left[\frac{B}{2}(t-t_0)\right]$$

$$x(t) = 2x_0(t)\cos(2\pi f_0 t) = \underbrace{AB\,\text{sinc}^2\left[\frac{B}{2}(t-t_0)\right]}_{\text{SEGNALE MODULANTE}}\ \underbrace{\cos(2\pi f_0 t)}_{\text{OSCILLAZIONE}}$$

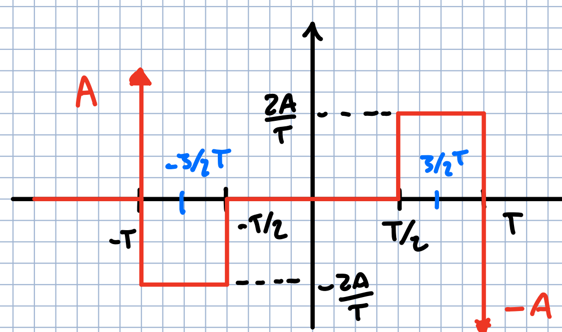
•) ESERCIZIO

$X(f) = ?$



TH. DELL' INT. COMPLETO.

$$y(t) = \frac{d}{dt} x(t) \quad \Rightarrow \quad x(t) = \int_{-\infty}^{t} y(\alpha)\, d\alpha$$



- DA $-\infty$ a t la DER. è $= 0$
- IN $-T$ C'È UNA DISCONTINUITÀ POSITIVA CHE È LA DERIVATA DI UN GRADINO 0 1 +
- DA $-T$ A $-T/2$ HO UNA RAMPA NEGATIVA LA CUI DERIVATA È UN RETTANGOLO CON CENTRO IN $-3/2\,T$
- IN $-T$ C'È UNA DISCONTINUITÀ NEGATIVA CHE È LA DERIVATA DI UN GRADINO 1 0 −

$$X(f) = \frac{Y(0)}{2}\delta(f) + \frac{Y(f)}{j2\pi f}$$

CENTRATA IN $3/2T$

$$y(t) = A\delta(t-\underbrace{(-T)}_{\text{CENTRATA IN } -T}) - \frac{2A}{T}\,\mathrm{rect}\left(\frac{t-(-\frac{3}{2}T)}{T/2}\right) + \frac{2A}{T}\,\mathrm{rect}\left(\frac{t-\frac{3}{2}T}{T/2}\right) - A\delta(t-\underbrace{T}_{\text{CENTRATA IN } T})$$

$$Y(f) = TCF[y(t)]$$

$$Y(f) = A e^{+j2\pi fT} - \frac{2A}{T}\cdot\frac{T}{2}\,\mathrm{sinc}\left(\frac{T}{2}f\right)e^{j2\pi\frac{3}{2}Tf} + \frac{2A}{T}\cdot\frac{T}{2}\,\mathrm{sinc}\left(\frac{T}{2}f\right)e^{-j2\pi\frac{3}{2}Tf} - Ae^{-j2\pi fT} =$$

$$= A\left[e^{j2\pi fT} - e^{-j2\pi fT}\right] - A\,\mathrm{sinc}\left(\frac{T}{2}f\right)\left[e^{j3\pi fT} - e^{-j3\pi fT}\right] =$$

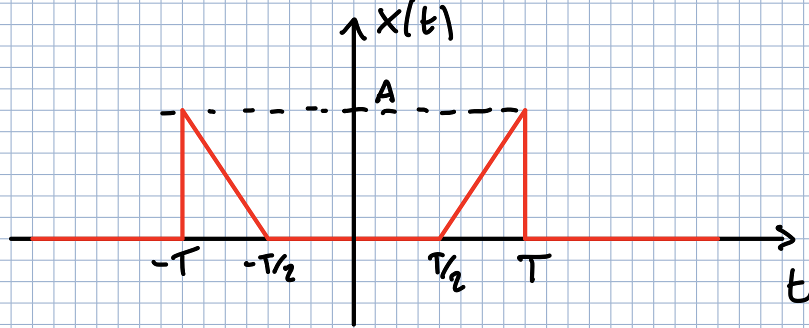
$$= j2A\sin(2\pi fT) - j2A\,\mathrm{sinc}\left(\frac{T}{2}f\right)\sin(3\pi fT) =$$

$$X(f) = \frac{Y(0)}{2}\delta(f) + \frac{Y(f)}{j2\pi f} \qquad Y(0) = 0 + 0 = 0$$

$$X(f) = \frac{A}{\pi f}\sin(2\pi fT) - \frac{A}{\pi f}\,\mathrm{sinc}\left(\frac{T}{2}f\right)\sin(3\pi fT) =$$

$$= \boxed{2AT\,\mathrm{sinc}(2fT) - 3AT\,\mathrm{sinc}\left(\frac{T}{2}f\right)\mathrm{sinc}(3fT)}$$

ALTRA STRADA:



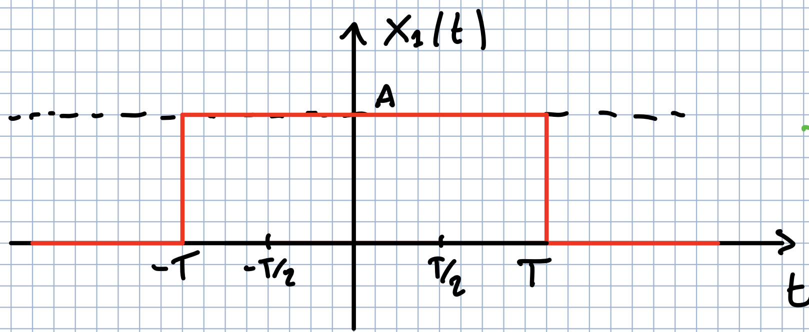
- DA $-\infty$ A $-T$ = 0
- IN $-T$ VALE **A**

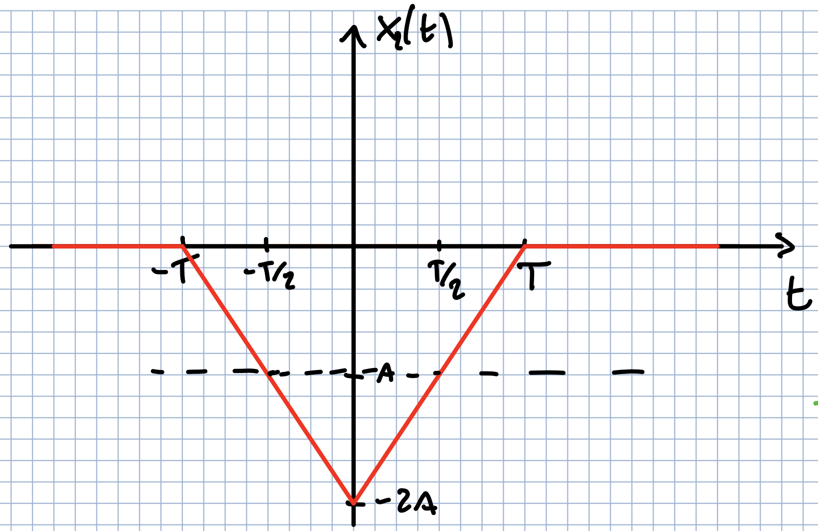
$-T$ $\begin{cases} \text{LA ① IN } -T \text{ VALE } A \\ \text{LA ② IN } -T \text{ VALE } 0 \\ \text{LA ③ IN } -T \text{ VALE } 0 \end{cases}$ $A + 0 + 0 \Rightarrow$ **A**

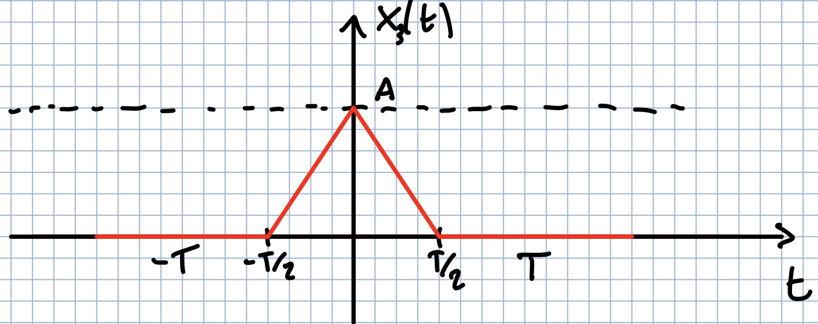
- IN $-T/2$ VALE **0**

$-T/2$ $\begin{cases} \text{LA ① } = A \\ \text{LA ② } = -A \\ \text{LA ③ } = 0 \end{cases} \Rightarrow A + (-A) = 0$

- IN 0







- IN 0 VALE 0

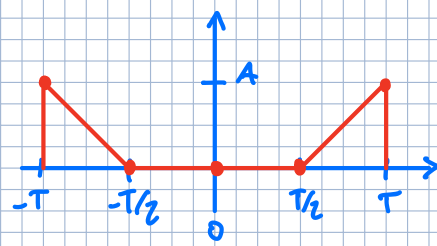
$$0 \begin{cases} LA \ 1 = A \\ LA \ 2 = -2A \Rightarrow A + (-2A) + A = 0 \\ LA \ 3 = A \end{cases}$$

- IN T/2 VALE 0

$$T/2 \begin{cases} LA \ 1 = A \\ LA \ 2 = -A \Rightarrow A + (-A) = 0 \\ LA \ 3 = 0 \end{cases}$$

- IN T VALE A

$$T \begin{cases} LA \ 1 = A \\ LA \ 2 = 0 \Rightarrow A + 0 + 0 = A \\ LA \ 3 = 0 \end{cases}$$



$$x_1(t) = A \, \text{rect}\left(\frac{t}{2T}\right) \Longleftrightarrow A \cdot 2T \, \text{sinc}(2\pi f)$$

$$x_2(t) = -2A\left(1 - \frac{|t|}{T}\right) \text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow -2A \cdot T \, \text{sinc}^2(Tf)$$

$$x_3(t) = A\left(1 - \frac{|t|}{T/2}\right) \text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow A \cdot \frac{T}{2} \, \text{sinc}^2\left[\frac{T}{2} f\right]$$

$$X(f) = 2AT\left[\text{sinc}(2Tf) - \text{sinc}^2(Tf)\right] + \frac{AT}{2} \, \text{sinc}^2\left(\frac{T}{2} f\right)$$

ES. - MODULAZIONE DI AMPIEZZA

$$y(t) = x(t) \cos(2\pi f_0 t)$$

$$x(t) = AB \, \text{sinc}^2(Bt)$$

SVOLG.

I) TH. MODULAZIONE

$$Y(f) = \frac{1}{2} X(f - f_0) + \frac{1}{2} X(f + f_0)$$

$$X(f) = TCF[x(t)]$$

II) $Y(t) = X(t) \cdot z(t)$ $\qquad z(t) = \cos(2\pi f_0 t)$

$$Y(f) = X(f) \otimes Z(f)$$

$$X(f) = A\left(1 - \frac{|f|}{B}\right) \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$Z(f) = TCF[\cos(2\pi f_0 t)] = \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)$$

$$Y(f) = A\left(1 - \frac{|f|}{B}\right) \operatorname{rect}\left(\frac{f}{2B}\right) \otimes \left[\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right] =$$

$$= A\left(1 - \frac{|f|}{B}\right) \operatorname{rect}\left(\frac{f}{2B}\right) \otimes \frac{1}{2}\delta(f-f_0) + A\left(1 - \frac{|f|}{B}\right) \operatorname{rect}\left(\frac{f}{2B}\right) \otimes \frac{1}{2}\delta(f+f_0) =$$

$$= \frac{A}{2}\left(1 - \frac{|f-f_0|}{B}\right) \operatorname{rect}\left(\frac{f-f_0}{2B}\right) + \frac{A}{2}\left(1 - \frac{|f+f_0|}{B}\right) \operatorname{rect}\left(\frac{f+f_0}{2B}\right) =$$

$$= \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$$

ES. – TCF DI SINUSOIDI MULTI COMPONENTE

$$x_1(t) = A\cos(2\pi f_1 t)$$
$$x_2(t) = B\,\text{sen}(2\pi f_2 t)$$
$$x(t) = x_1(t) + x_2(t)$$

$\rightarrow$ calcola: $TSF[x(t)]$, $TCF[x(t)]$

SVOLG.

TSF $\Rightarrow$ devo procurarmi T_0 per $x(t)$

$$X_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt \qquad f_0 = \frac{1}{T_0}$$

$x(t)$ è periodica? ho due componenti f_1 e f_2

Affinché $x(t)$ sia periodica deve esistere una f_0 : f_1 e f_2 siano un multiplo di f_0

$$f_1 = K_1 f_0 \quad , \quad f_2 = K_2 f_0 \qquad K_1, K_2 \text{ interi positivi}$$

$$\frac{f_1}{f_2} = \frac{K_1}{K_2} \Rightarrow \text{numero razionale} \Rightarrow x(t) \text{ è periodica}$$

$\Rightarrow$ LA TCF mi permette di calcolare $X(f)$ indipendentemente dai valori f_1 e f_2, anche se $x(t)$ non è periodico

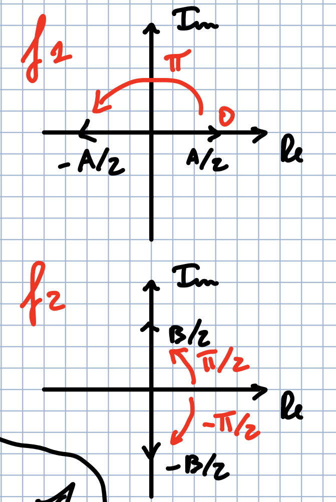
$$TCF[x(t)] = X_1(f) + X_2(f)$$

$$X_1(f) = \frac{A}{2}\delta(f-f_1) + \frac{A}{2}\delta(f+f_1)$$

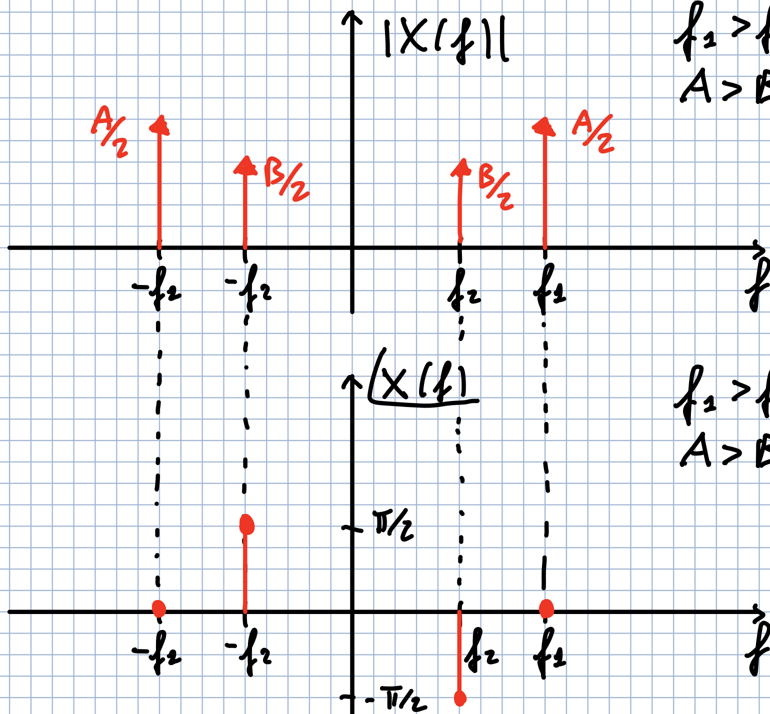
$$X_2(f) = \frac{B}{2j}\delta(f-f_2) - \frac{B}{2j}\delta(f+f_2)$$

$$\frac{A}{2}\cos(2\pi f_1 t) + 0j$$

$$\frac{B}{2}j\,\mathrm{sen}(2\pi f_2 t) + 0$$



DISEGNO :

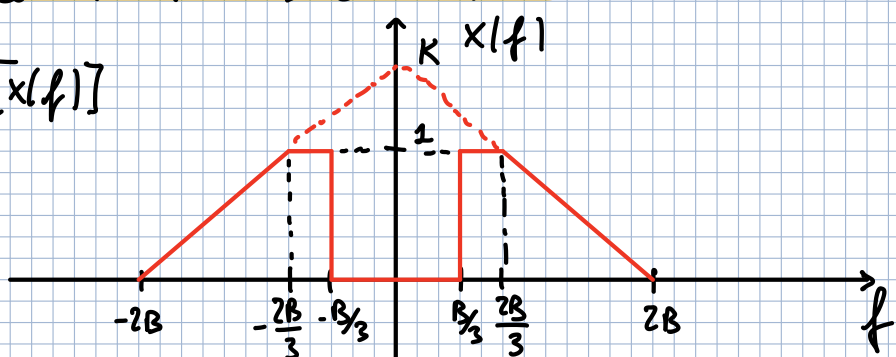


$f_1 > f_2$
$A > B$

$f_1 > f_2$
$A > B$

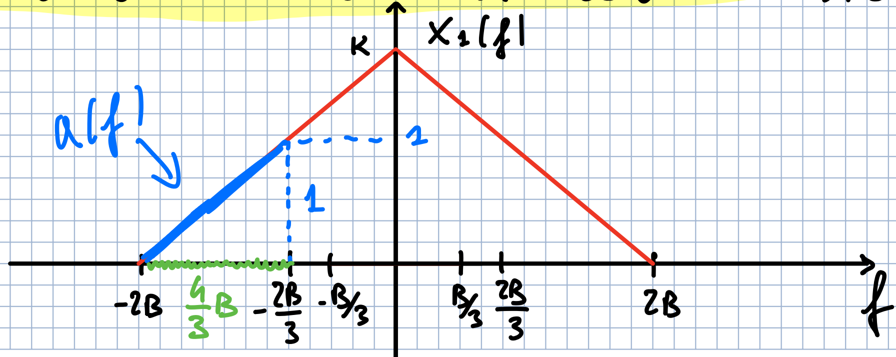
ES. Calcola ANTITRASFORMATA

$$x(t) = ATCF[X(f)]$$

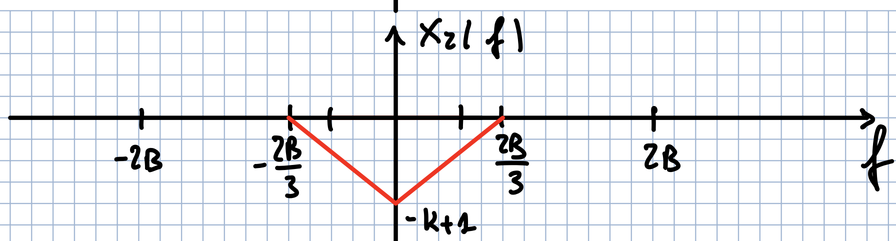


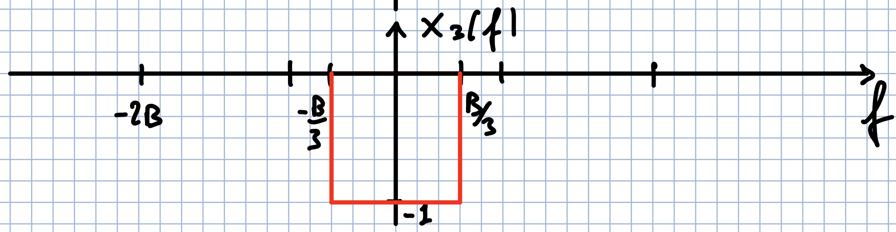
SVOLG.

I) COMPOSIZIONI DI SEGNALI CON TRASFORMATA NOTA



$$2B - \frac{2B}{3} = \frac{4}{3}B$$





per trovare K devo trovare la retta passante per $(-2B)$ e K

$$a(f) = c\,f + K = \frac{3}{4B} f + K$$

c è il coeff. angolare della retta che ha pendenza $\frac{1}{2B - \frac{2B}{3}} = \frac{3}{4B} = c$

SICCOME PASSA NEL PUNTO f di COORDINATA $(-2B)$:

$$\Rightarrow \frac{3}{4B}(-2B) + K = 0 \quad \Rightarrow K = \frac{3}{2}$$

$$X_1(f) = \frac{3}{2}\left(1 - \frac{|f|}{2B}\right) \mathrm{rect}\left(\frac{f}{4B}\right)$$

$$X_2(f) = -\frac{1}{2}\left(1 - \frac{|f|}{\frac{2B}{3}}\right) \mathrm{rect}\left(\frac{f}{\frac{4B}{3}}\right)$$

$\Rightarrow$ DEVO FARE LE ATCF di tutti e 3

$$X_3(f) = -\,\mathrm{rect}\left(\frac{f}{\frac{2B}{3}}\right)$$

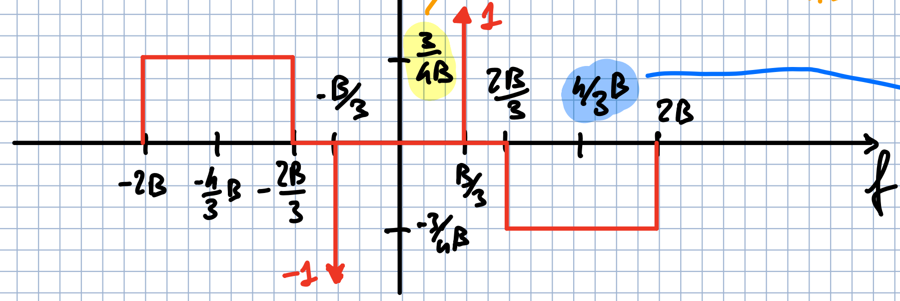
$$x(t) = x_1(t) + x_2(t) + x_3(t) =$$

$$= \frac{3}{2} \cdot 2B\,\mathrm{sinc}^2(2Bt) - \frac{1}{2} \cdot \frac{2B}{3}\,\mathrm{sinc}^2\left(\frac{2B}{3}t\right) - \frac{2B}{3}\,\mathrm{sinc}\left(\frac{2B}{3}t\right) =$$

$$= \boxed{3B\,\mathrm{sinc}^2(2Bt) - \frac{B}{3}\,\mathrm{sinc}^2\left(\frac{2B}{3}t\right) - \frac{2B}{3}\,\mathrm{sinc}\left(\frac{2B}{3}t\right)}$$

II) DERIVAZIONE E TH. INTEGR.

FACCIO LA DERIVATA DI $X(f)$ e faccio il grafico di $Y(f) = \frac{d}{df} X(f)$



$\frac{3}{4B}$ è la pendenza della rampa

è il punto medio del rettangolo

$$\frac{\frac{2B}{3}+2B}{2}=\frac{4}{3}B$$

$$Y(f)=\frac{3}{4B}\,\mathrm{rect}\left(\frac{f+\frac{4}{3}B}{\frac{4}{3}B}\right)-\delta\left(f+\frac{B}{3}\right)+\delta\left(f-\frac{B}{3}\right)-\frac{3}{4B}\,\mathrm{rect}\left(\frac{f-\frac{4}{3}B}{\frac{4}{3}B}\right)$$

$$y(t)=\frac{\cancel{3}}{\cancel{4B}}\cdot\frac{\cancel{4B}}{\cancel{3}}\,\mathrm{sinc}\left(\frac{4}{3}Bt\right)\Big[\underbrace{e^{-j2\pi\frac{4}{3}Bt}-e^{j2\pi\frac{4}{3}Bt}}_{-\sin}\Big]+e^{j2\pi\frac{B}{3}t}-e^{-j2\pi\frac{B}{3}t}=$$

TCF della 2 rect

$$=-j2\,\mathrm{sinc}\left(\frac{4}{3}Bt\right)\sin\left(\frac{8}{3}\pi Bt\right)+j2\,\mathrm{sen}\left(\frac{2}{3}\pi Bt\right)$$

$Y(0)=0 \Rightarrow$ POSSO APPLICARE IL TH. INTEGR. IN FREQUENZA *

$$x(t)=-\frac{y(t)}{j2\pi t}=+\,\mathrm{sinc}\left(\frac{4}{3}Bt\right)\frac{\sin\left(\frac{8}{3}\pi Bt\right)}{\pi t}-\frac{\mathrm{sen}\left(\frac{2}{3}\pi Bt\right)}{\pi t}=$$

$$=\boxed{\frac{8}{3}B\,\mathrm{sinc}\left(\frac{4}{3}Bt\right)\mathrm{sinc}\left(\frac{8}{3}Bt\right)-\frac{2}{3}B\,\mathrm{sinc}\left(\frac{2}{3}Bt\right)}$$ → è uguale all'altra

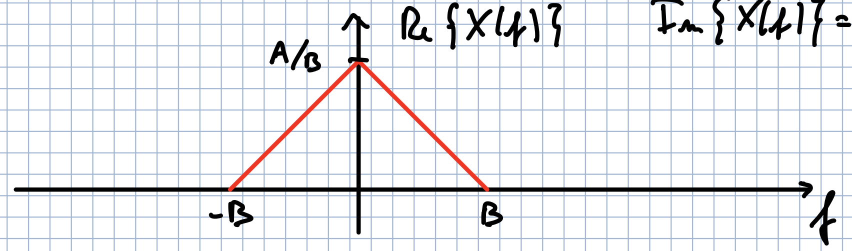
ESEMPIO

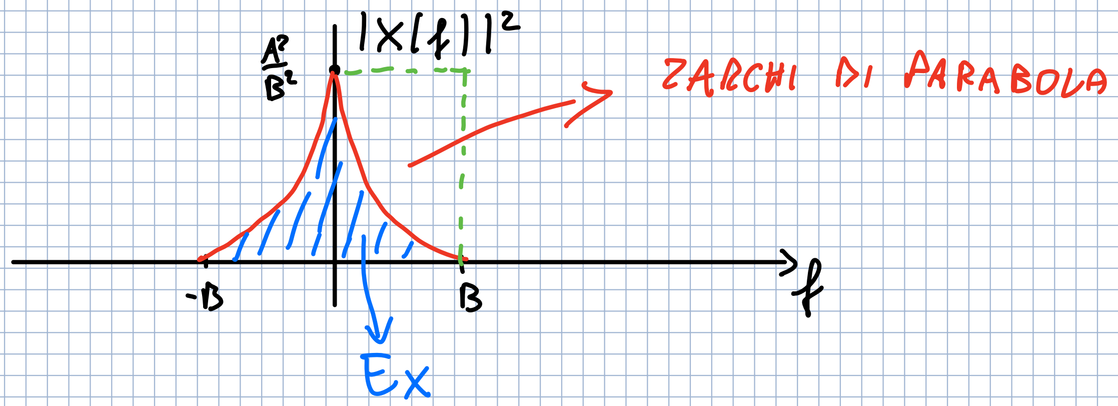
$$x(t)=A\,\mathrm{sinc}^2(Bt)$$

$E_x=?$ $\qquad E_x=A\int_{-\infty}^{+\infty}\left|\mathrm{sinc}^2(Bt)\right|^2dt$ $\qquad$ NO!

E' MEGLIO CALCOLARE LA TCF $x(t) \Leftrightarrow X(f)$

$$X(f) = \frac{A}{B}\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

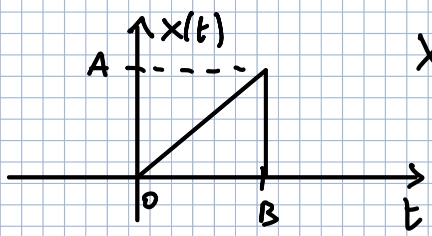




$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2 \, df = \int_{-\infty}^{+\infty} X^2(f) \, df = \frac{2}{3}\frac{A^2}{B^2} B = \frac{2}{3}\frac{A^2}{B}$$

IN GENERALE:

- QUANDO ABBIAMO UN TRIANGOLO

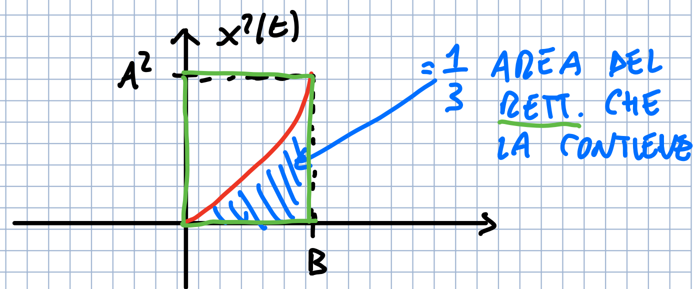


$$x(t) = \frac{A}{B} t$$

SE VOGLIO L'ENERGIA:

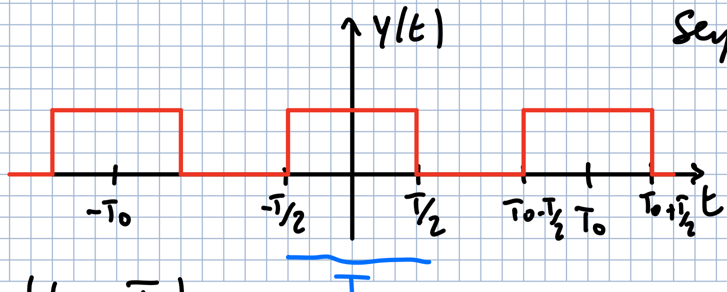
$$\int_0^B x^2(t) \, dt = \int_0^B \frac{A^2}{B^2} t^2 \, dt = \frac{A^2}{B^2} \frac{t^3}{3} \Big|_0^B =$$

$$= \frac{A^2}{3B^2}\left(B^3 - 0\right) = \frac{A^2 B}{3}$$



ESERCIZIO

$Y_m = ?$



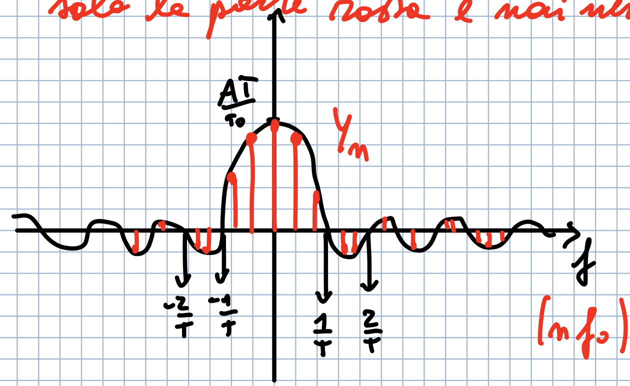
Segnale periodico di periodo T_0

$$\begin{cases} y(t) = \sum_{n=-\infty} x(t-nT_0) \\ x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right) \end{cases}$$

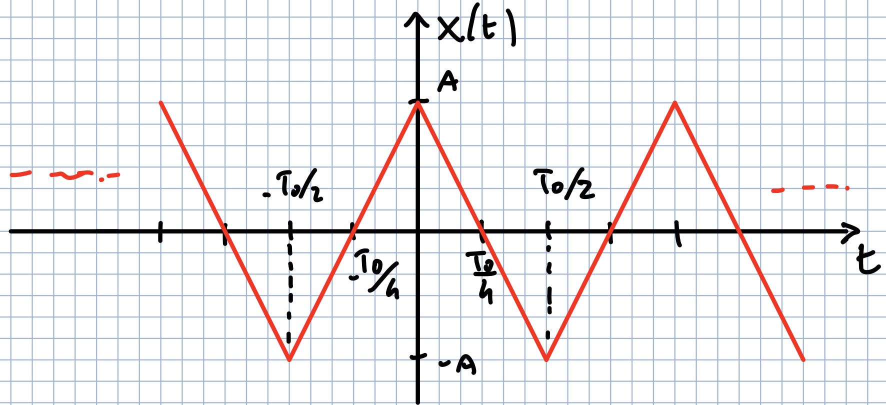
se mi chiede Y_n devo disegnare solo la parte rossa e non nera

$$X(f) = AT\,\mathrm{sinc}(Tf)$$

$$Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right) = \boxed{\frac{AT}{T_0}\,\mathrm{sinc}\left(\frac{T}{T_0}n\right)}$$



ES. – ONDA TRIANGOLARE

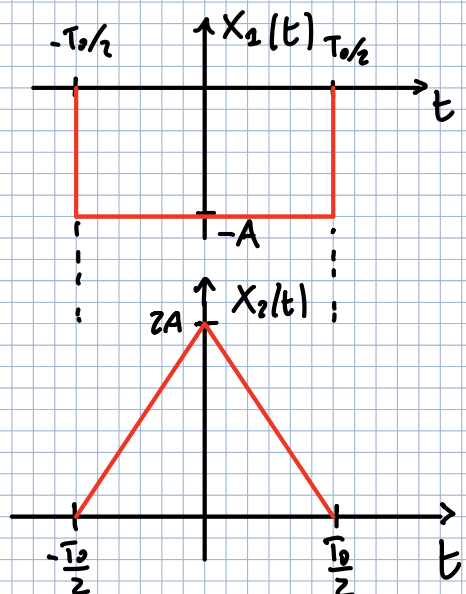


$$X_n = ?$$

$$x(t) = \sum_{n=-\infty}^{+\infty} x_0(t-nT_0) \qquad X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right),\ X_0(f) = TCF[x_0(t)]$$

$$X_0(f) = ?$$

$$x_0(t) = x_1(t) + x_2(t)$$



$$x_1(t) = -A\,\mathrm{rect}\left(\frac{t}{T_0}\right)$$

$$x_2(t) = 2A\left(1 - \frac{|t|}{T_0/2}\right)\mathrm{rect}\left(\frac{t}{T_0}\right)$$

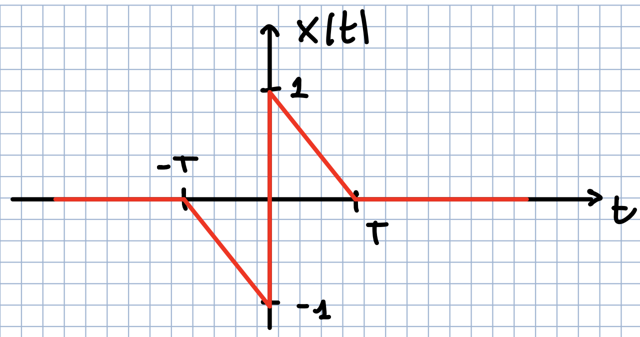
$$X_0(f) = X_1(f) + X_2(f) = AT_0\left[\mathrm{sinc}^2\left(\frac{T_0}{2}f\right) - \mathrm{sinc}(T_0 f)\right]$$

$$X_1(f) = -AT_0\,\mathrm{sinc}(T_0 f)$$

$$X_2(f) = \cancel{2}A\frac{T_0}{\cancel{2}}\,\mathrm{sinc}^2\left(\frac{T_0}{2}f\right)$$

$$X_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right) = A\left[\mathrm{sinc}^2\left(\frac{\cancel{T_0}}{2}\frac{n}{\cancel{T_0}}\right) - \mathrm{sinc}\left(\cancel{T_0}\frac{n}{\cancel{T_0}}\right)\right] = A\left[\mathrm{sinc}^2\left(\frac{n}{2}\right) - \mathrm{sinc}(n)\right]$$

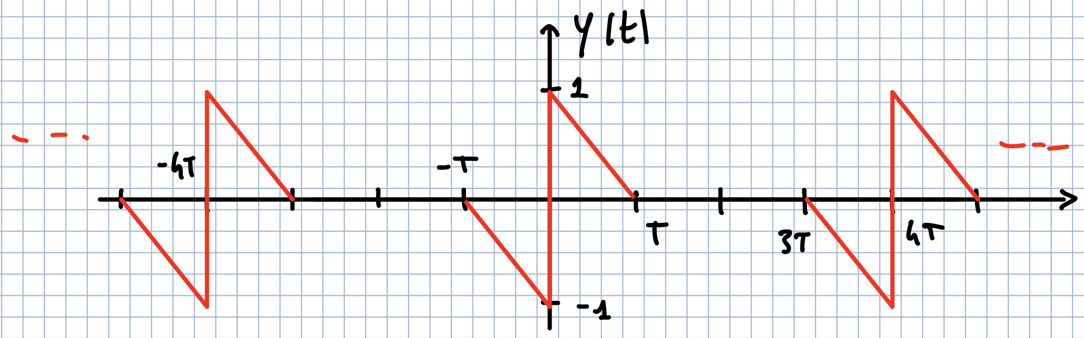
ESERCIZIO



$$y(t) = \sum_{n=-\infty}^{+\infty} x(t-4nT) =$$
$$= \sum_{n=-\infty}^{+\infty} x(t-nT_0)$$

$\boxed{T_0 = 4T}$ PERIODO

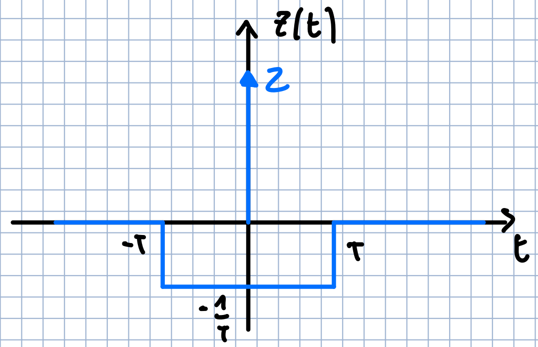
SI RIPETE OGNI 4T



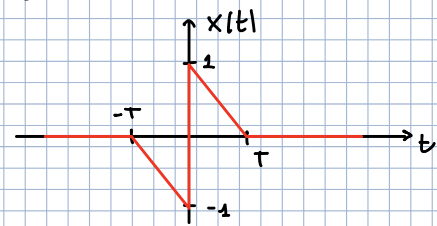
$Y(f) = ?$

$$Y(f) = \sum_{n=-\infty}^{+\infty} Y_n \delta\left(f - \frac{n}{T_0}\right), \quad T_0 = 4T, \quad Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$$

$X(f) = TCF[x(t)]$ poniamo $z(t) = \frac{d}{dt} x(t)$



- IN -T HO UNA → RAMPA NEGATIVA DI PENDENZA $-\frac{1}{T}$



- IN 0 HO UN SALTO CHE VA DA -1 A 1, QUINDI UN GRADINO ALTO 2 → $\delta(t)$ di AREA 2
- IN -T HO UNA RAMPA NEGATIVA DI PENDENZA $-\frac{1}{T}$

$$z(t) = -\frac{1}{T} \operatorname{rect}\left(\frac{t}{2T}\right) + 2\delta(t)$$

$$Z(f) = -\frac{1}{T} \cdot 2T \operatorname{sinc}(2Tf) + 2 = 2\left(1 - \operatorname{sinc}(2Tf)\right)$$

$$X(f) = \frac{Z(0)}{2} + \frac{Z(f)}{j2\pi f}$$ → PER TH. INT. COMPLETO

$Z(0) = 2(1-1) = 0$ LA $\operatorname{SINC}(0) = 1$

$$X(f) = 0 + \frac{1}{j\pi f}\left(1 - \operatorname{sinc}(2Tf)\right) = \frac{1 - \operatorname{sinc}(2Tf)}{j\pi f}$$

$$Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right) = \frac{1}{T_0} \cdot \left(\frac{1 - \operatorname{sinc}\left(2T \cdot \frac{n}{T_0}\right)}{j\pi \frac{n}{T_0}}\right) = \frac{1}{4T}\left(\frac{1 - \operatorname{sinc}\left(2T \cdot \frac{n}{4T}\right)}{j\pi \frac{n}{4T}}\right) =$$

$$= \frac{1}{j\pi}\left(\frac{1 - \operatorname{sinc}\left(\frac{n}{2}\right)}{n}\right)$$

$$Y(f) = \frac{1}{j\pi} \sum_{n=-\infty}^{+\infty} \left(\frac{1 - \operatorname{sinc}\left(\frac{n}{2}\right)}{n}\right) \delta\left(f - \frac{n}{4T}\right)$$

ESERCIZIO

$$s(t) = \text{sinc}\left(2Bt - \frac{1}{2}\right) + \text{sinc}\left(2Bt + \frac{1}{2}\right)$$

- $x(t) = s(t)\sin(2\pi Bt)$ $\rightarrow f_0 = B$
- $y(t) = \sum_{n=-\infty}^{+\infty} s\left(t - \frac{2}{B}n\right)$

Si chiede di calcolare: E_x, P_x, E_y, P_y

SOLUZIONE:

$$x(t) = s(t)\,\text{sinc}(2\pi Bt)$$

$$E_x = \int_{-\infty}^{+\infty} s^2(t)\sin^2(2\pi BT)\,dt$$

CONVIENE USARE TH. PARSEVAL

$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2\,df$$

$$X(f) = \frac{1}{2j}S(f-f_0) - \frac{1}{2j}S(f+f_0) \Longrightarrow$$

- MODULAZIONE CON SENO

Ip. $\begin{cases} x(t) \overset{TCF}{\Longleftrightarrow} X(f) \\ y(t) = x(t)\sin(2\pi f_0 t) \end{cases}$

Th. $Y(f) = \frac{1}{2j}X(f-f_0) - \frac{1}{2j}X(f+f_0)$

poichè $f_0 = B$

$$X(f) = \frac{1}{2j}S(f-B) - \frac{1}{2j}S(f+B)$$

$$S(f) = TCF[s(t)]$$

$$s(t) = \text{sinc}\left[2B\left(t - \frac{1}{4B}\right)\right] + \text{sinc}\left[2B\left(t + \frac{1}{4B}\right)\right] \Leftarrow$$

LA SCRIVIAMO COSÌ PER POTERLA VEDERE NELLA FORMA CHE CONOSCIAMO, E USARE TH. RITARDO

$$s(t) = s_0\left(t - \frac{1}{4B}\right) + s_0\left(t + \frac{1}{4B}\right) = \qquad t_0 = \frac{1}{4B}$$

$$= s_0(t - t_0) + s_0(t + t_0)$$

$$S(f) = S_0(f)e^{-j2\pi f t_0} + S_0(f)e^{j2\pi f t_0} = 2S_0(f)\cos(2\pi f t_0) =$$

$$= 2S_0(f)\cos\left(2\pi f \cdot \frac{1}{4B}\right) = 2S_0(f)\cos\left(\frac{\pi f}{2B}\right)$$

$$s_0(t) = \text{sinc}(2Bt) \rightarrow \text{SINC NON TRASLATA}$$

$$S_0(f) = \frac{1}{2B}\text{rect}\left(\frac{f}{2B}\right)$$

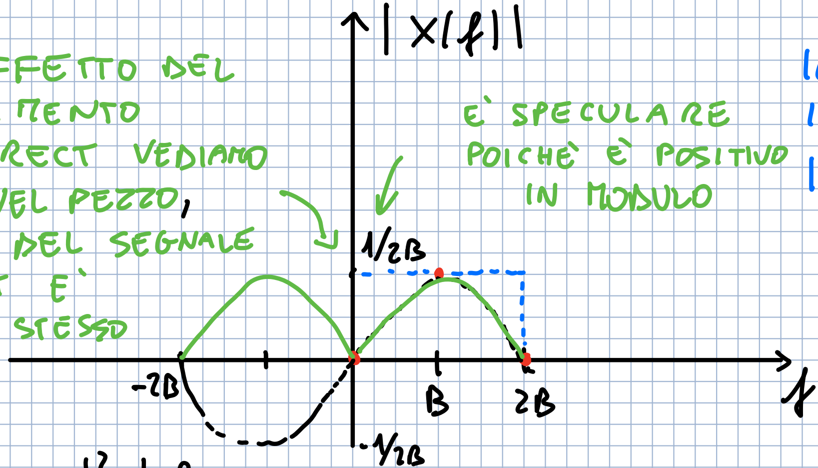
$$S(f) = \frac{1}{B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\cos\left(\frac{\pi f}{2B}\right)$$

DA QUI:

$$X(f) = \frac{1}{j2B}\,\mathrm{rect}\left(\frac{f-B}{2B}\right)\cos\left[\frac{\pi(f-B)}{2B}\right] - \frac{1}{j2B}\,\mathrm{rect}\left(\frac{f+B}{2B}\right)\cos\left[\frac{\pi(f+B)}{2B}\right]$$

PER EFFETTO DEL TRONCAMENTO DELLA RECT VEDIAMO SOLO QUEL PEZZO, IL PRODOTTO DEL SEGNALE CON LA RECT È IL SEGNALE STESSO

È SPECULARE POICHÈ È POSITIVO IN MODULO



IL COSENO È CENTRATO IN B E HA MAX IN B → 1

IN F = 2B VALE

$$\cos\left(\frac{\pi(2B-B)}{2B}\right) = 0$$

IN F=0 → VALE 0

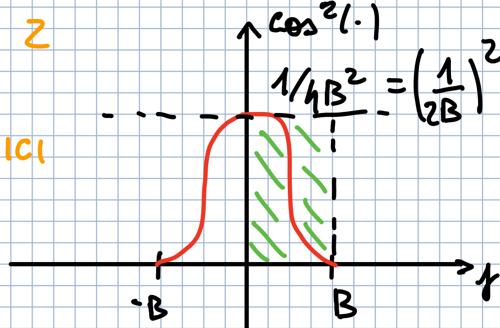
IL PERIODO DEL COSENO È 4B, MA È TRASLATO IN B

$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2\, df =$$

$$f - B = f'$$

$$= \frac{2}{4B^2}\int_0^{2B} \cos^2\left(\frac{\pi - (f-B)}{2B}\right) df = \frac{2}{4B^2}\int_0^{2B} \cos^2\left(\frac{\pi f'}{2B}\right) df' = \frac{2}{4B^2}\cdot\int_{-B}^{B} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi f'}{2B}\right) df' =$$

MOLTIPLICO PER 2 PERCHÈ HO 2 COSENI SIMMETRICI



$$= \frac{2B}{4B^2} + \int_{-B}^{B} \cos\left(\frac{2\pi f'}{2B}\right) df' = \frac{2B}{4B^2}$$

$$= \frac{1}{2B}$$

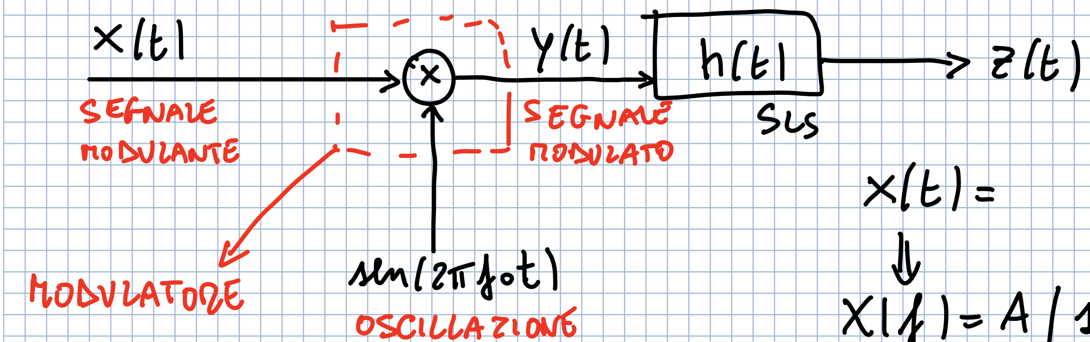
$$B \cdot \frac{1}{4B^2} = \frac{1}{4B}$$ AREA DI UN SOLO COSENO (VEDI FIG.)

$$\frac{1}{4B}\cdot 2 = \frac{1}{2B}$$

$$P_x = ?$$

$$E_x = \frac{1}{2B} < \infty \Rightarrow P_x = 0$$

ESERCIZIO – MODULAZIONE E FILTRAGGIO

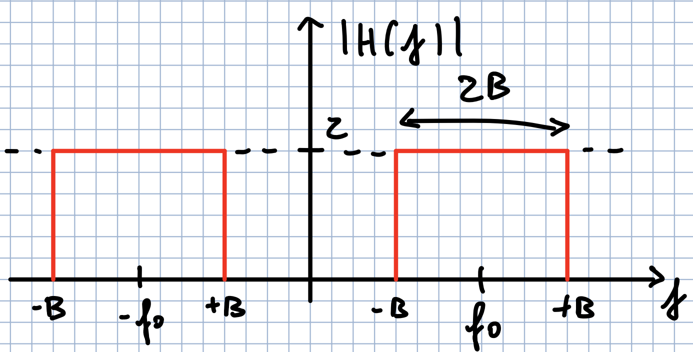


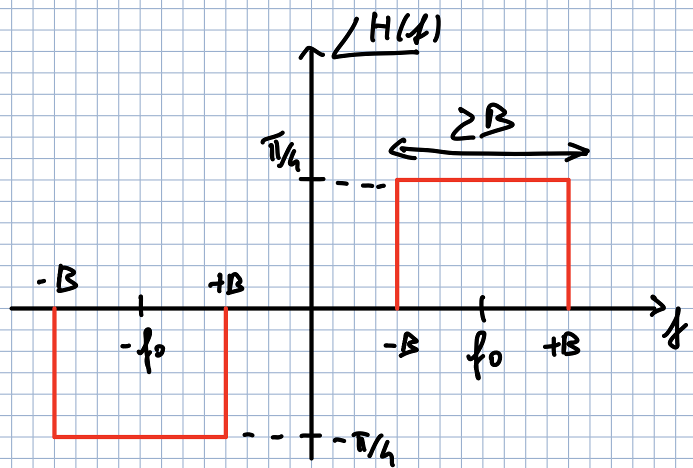
$x(t) =$

$\Downarrow$

$$X(f) = A\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

$h(t) \Rightarrow H(f)$ grafico





Calcolare:

1) ESPRESSIONE ANALITICA $z(t)$

2) ENERGIA $z(t)$

SOLUZIONE

1) $z(t) = y(t) \otimes h(t)$

$z(t) = \text{ATCF}[Z(f)] = \text{ATCF}[Y(f)\,H(f)]$ → DEVO TROVARE $Y(f)$ e $H(f)$

$y(t) = x(t)\,\mathrm{sen}(2\pi f_0 t)$ → NON CONVIENE USARE GLI INTEGRALI

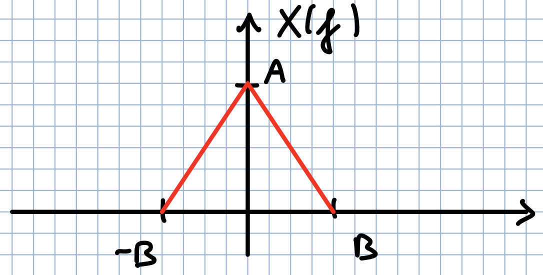
MEGLIO USARE IL DOMINIO IN FREQUENZA

MODULO IL SENO

•) IN FREQUENZA

$$Y(f)=\frac{1}{2j}X(f-f_0)-\frac{1}{2j}X(f+f_0)$$

$$X(f)=A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

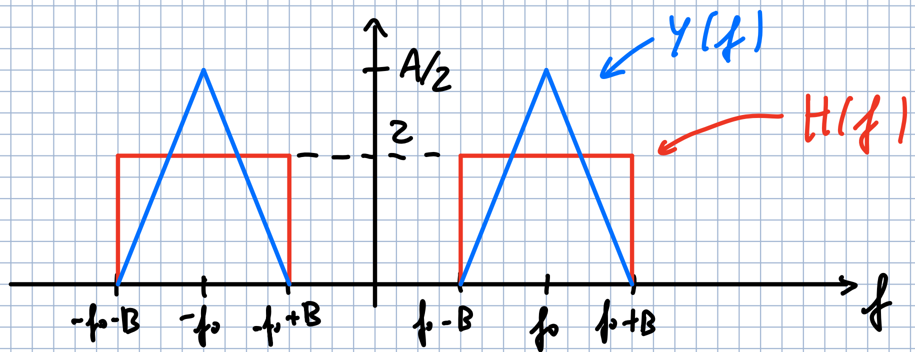


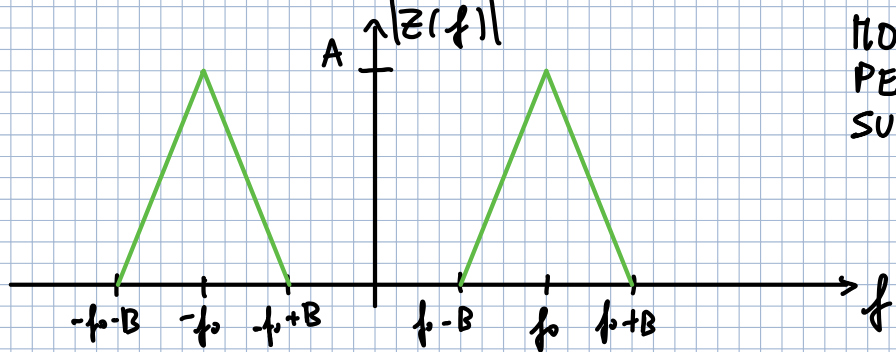
BANDA RIGOROSAMENTE LIMITATA

$$Y(f)=\frac{1}{2j}A\left(1-\frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right)-\frac{1}{2j}A\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$

$$Z(f)=Y(f)H(f)$$

$$Z(f)=|Z(f)|e^{j\angle Z(f)}=|Y(f)||H(f)|e^{j(\angle Y(f)+\angle H(f))}=$$

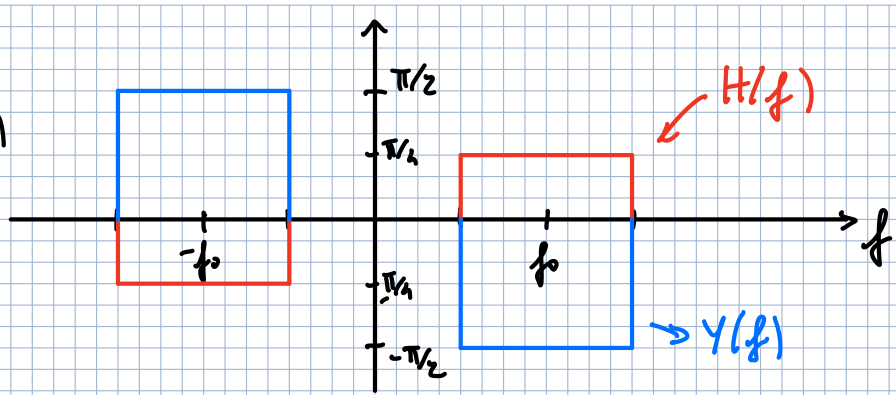




MOLTIPLICO IL TRIANGOLO PER FUNZIONE COSTANTE SU TUTTA LA BANDA 2B

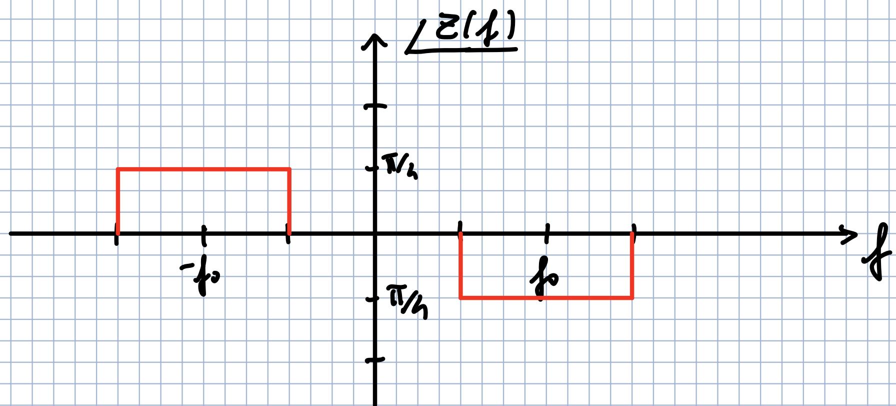
$$\frac{A}{2}\cdot 2=A$$

FASI DI $Y(f)$ e $H(f)$



FACCIO LA SOMMA DELLE FASI

$\angle Z(f) = \angle Y(f) + \angle H(f)$



$$Z(f) = |Z(f)|\, e^{j\angle Z(f)} = A\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)e^{j\pi/4}$$
$$+\,A\left(1-\frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right)e^{-j\pi/4}$$

$$z(t) = \mathrm{ATCF}[Z(f)] =$$

$$Z_0(f) = A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Z(f) = Z_0(f+f_0)\,e^{j\pi/4} + Z_0(f-f_0)\,e^{-j\pi/4}$$

$$z(t) = e^{j\pi/4} z_0(t)\, e^{-j2\pi f_0 t} + e^{-j\pi/4} z_0(t)\, e^{j2\pi f_0 t} =$$

$$= z_0(t)\left[e^{-j(2\pi f_0 t - \pi/4)} + e^{j(2\pi f_0 t - \pi/4)}\right] = 2 z_0(t)\cos(2\pi f_0 t - \pi/4)$$

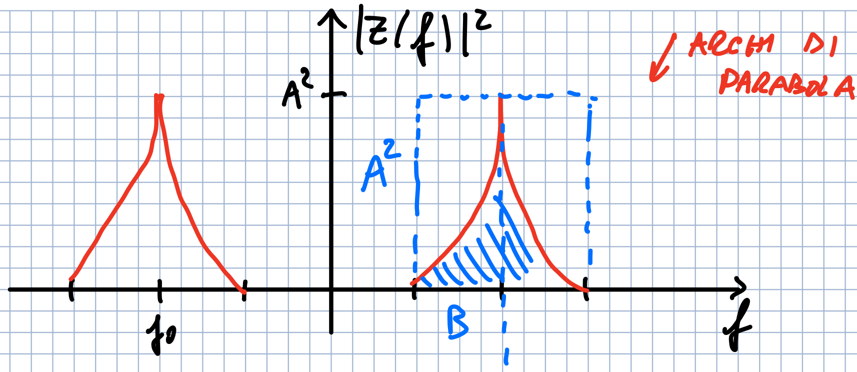
$$z_0(t) = AB\,\mathrm{sinc}^2(Bt)$$

$$z(t) = 2AB\,\mathrm{sinc}^2(Bt)\cos(2\pi f_0 t - \pi/4)$$

2) $E_z =$ → nel tempo (DIFFICILE) $\int$ sinc

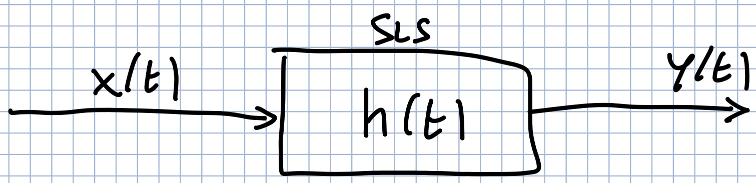
→ in frequenza (FACILE)

$$E_z = \int_{-\infty}^{+\infty} |Z(f)|^2 \, df$$



$$E_z = 2\left(2\left(\frac{A^2 B}{3}\right)\right) = \frac{4}{3} A^2 B$$

ESERCIZIO - FILTRAGGIO DI SEGNALI PERIODICI



$$x(t) = \sum_{n=-\infty}^{+\infty} \left[\operatorname{rect}\left(\frac{t - \frac{2}{B}n}{\frac{1}{2B}}\right) - \left(1 - \frac{\left|t - \frac{2}{B}n\right|}{\frac{1}{2B}}\right) \operatorname{rect}\left(\frac{t - \frac{2}{B}n}{\frac{1}{B}}\right) \right]$$

$$h(t) = B \operatorname{sinc}^2(Bt)$$

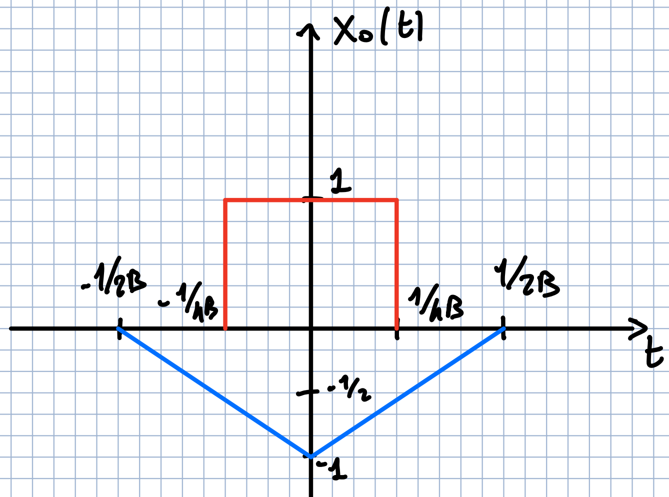
Calcolare:

1) $y(t)$ e 2) P_y, E_y

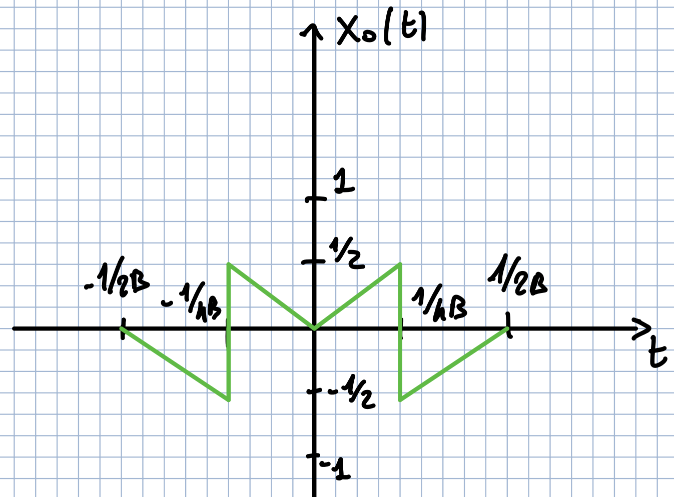
SVOLG.

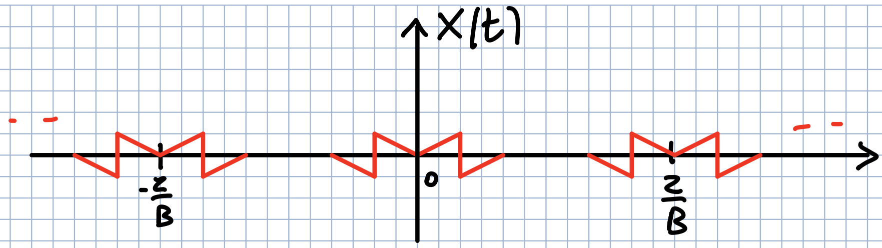
$$x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - nT_0) \quad , \quad T_0 = \frac{2}{B}$$

$$x_0(t) = \operatorname{rect}\left(\frac{t}{\frac{1}{2B}}\right) - \left(1 - \frac{|t|}{\frac{1}{2B}}\right) \operatorname{rect}\left(\frac{t}{\frac{1}{B}}\right)$$



$\Rightarrow$





$$y(t) = x(t) \otimes h(t)$$

$$Y(f) = X(f) \cdot H(f)$$

$$X(f) = \sum_{n=-\infty}^{+\infty} X_n \, \delta\left(f - \frac{n}{T_0}\right)$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right) \qquad x_0(t) = \mathrm{rect}\left(\frac{t}{\frac{1}{2B}}\right) - \left(1 - \frac{|t|}{\frac{1}{2B}}\right)\mathrm{rect}\left(\frac{t}{\frac{1}{B}}\right)$$

$$X_0(f) = X_1(f) + X_2(f)$$

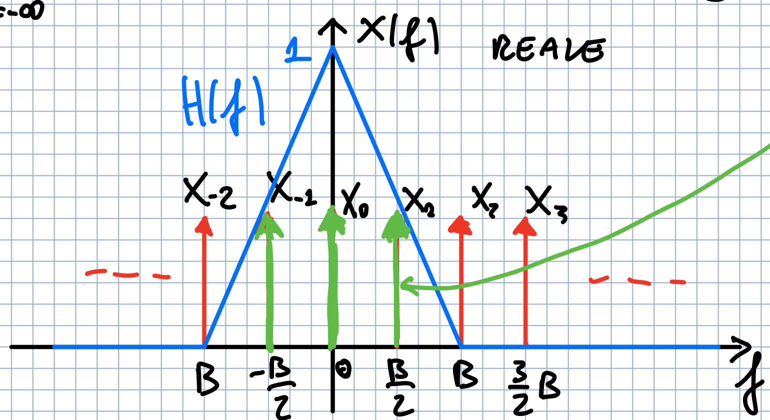
$$X_1(f) = \frac{1}{2B}\,\mathrm{sinc}\left(\frac{1}{2B} f\right) \qquad X_2(f) = -\frac{1}{2B}\,\mathrm{sinc}^2\left(\frac{1}{2B} f\right)$$

$$X_0(f) = \frac{1}{2B}\left[\mathrm{sinc}\left(\frac{1}{2B} f\right) - \mathrm{sinc}^2\left(\frac{1}{2B} f\right)\right]$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right) \qquad T_0 = \frac{2}{B}$$

$$X_n = \frac{\cancel{B}}{2} \cdot \frac{1}{2\cancel{B}}\left[\mathrm{sinc}\left(\frac{1}{2\cancel{B}} \cdot \frac{\cancel{B}n}{2}\right) - \mathrm{sinc}^2\left(\frac{1}{2\cancel{B}} \cdot \frac{\cancel{B}n}{2}\right)\right] = \frac{1}{4}\left[\mathrm{sinc}\left(\frac{n}{4}\right) - \mathrm{sinc}^2\left(\frac{n}{4}\right)\right]$$

$$X(f) = \frac{1}{4}\sum_{n=-\infty}^{+\infty}\left[\mathrm{sinc}\left(\frac{n}{4}\right) - \mathrm{sinc}^2\left(\frac{n}{4}\right)\right]\delta\left(f - \frac{B}{2}n\right)$$



SONO LE UNICHE FREQUENZE CHE SOPRAVVIVONO POICHÉ STANNO ALL'INTERNO DI H(f) E VADO COSÌ A SCRIVERE Y(f)

$$H(f) = \mathrm{TCF}[h(t)] = \mathrm{TCF}[B\,\mathrm{sinc}^2(Bt)] = B\left(1 - \frac{|t|}{B}\right)\mathrm{rect}\left(\frac{t}{2B}\right)$$

$$Y(f) = X_{-1}\,\delta\left(f + \frac{B}{2}\right) + X_0\,\delta(f) + X_1\,\delta\left(f - \frac{B}{2}\right)$$

ESSENDO PARI LA SINC IL - SPARISCE

$$X_{-1} = \frac{1}{4}\left[\mathrm{sinc}\left(-\frac{1}{4}\right) - \mathrm{sinc}^2\left(\frac{1}{4}\right)\right] = X_{+1}$$

$$X_0 = \frac{1}{4}\left[1 - 1\right] = 0$$

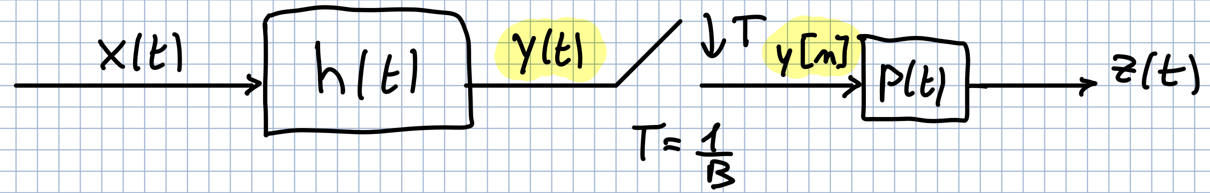
$$Y(f) = X_1\left[\delta\left(f + \frac{B}{2}\right) + \delta\left(f - \frac{B}{2}\right)\right]$$

$$X_1 = \frac{1}{4}\left[\frac{\mathrm{sen}\left(\pi \cdot \frac{1}{4}\right)}{\pi \cdot \frac{1}{4}} - \frac{\mathrm{sen}^2\left(\pi \cdot \frac{1}{4}\right)}{\pi^2 \cdot \frac{1}{16}}\right] = \frac{1}{4}\left[\frac{\frac{\sqrt{2}}{2}}{\pi/4} - \frac{2/4}{\pi^2/16}\right] = K$$

$$y(t) = \mathrm{ATCF}\left[Y(f)\right] =$$

$$= 2X_1\left[\frac{e^{-j2\pi\frac{B}{2}t} + e^{j2\pi\frac{B}{2}t}}{2}\right] = 2X_1\cos(\pi B t)$$

I COMPITINO 2019



$x(t) = 2AB\, \text{sinc}(2Bt) + AB\, \text{sinc}^2(Bt)$

$h(t) = B\, \text{sinc}(Bt)$ $\qquad z(t) = ?$

$p(t) = 2B\, \text{sinc}(2Bt)$ $\qquad E_z = ?$

SVOLG.

VANNO TROVATE $y(t)$ e $y[n]$

$y(t) = x(t) \otimes h(t)$

$Y(f) = X(f) \cdot h(f)$ $\quad$ -> per cui ricaviamo $X(f)$ e $H(f)$

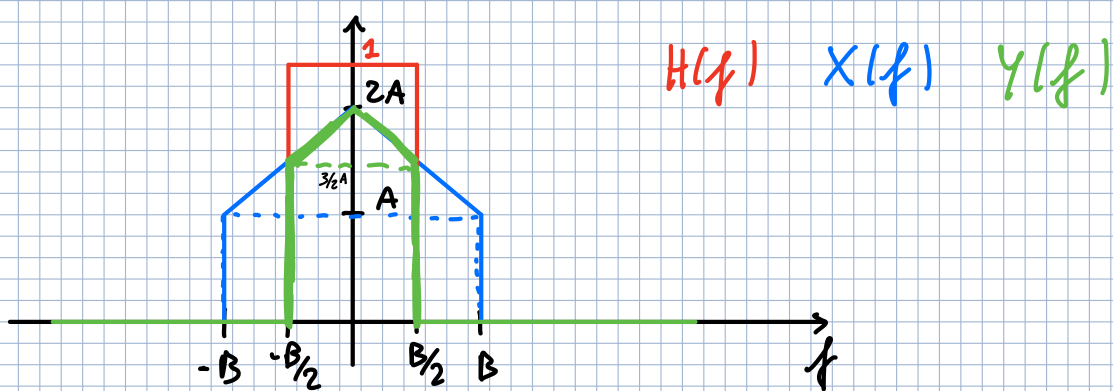
$X(f) = X_1(f) + X_2(f) = A\, \text{rect}\left(\frac{f}{2B}\right) + A\left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)$

$X_1(f) = A\, \text{rect}\left(\frac{f}{2B}\right)$ $\qquad X_2(f) = A\left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)$

$H(f) = \text{rect}\left(\frac{f}{B}\right)$

$Y(f)$ PER VIA GRAFICA

⇓



$Y(f) = \frac{3}{2}A\, \text{rect}\left(\frac{f}{B}\right) + \frac{A}{2}\left(1 - \frac{|f|}{B/2}\right)\text{rect}\left(\frac{f}{B}\right)$ $\quad$ USCITA DAL FILTRO $h(t)$

⇒ CAMPIONAMENTO di $y(t)$

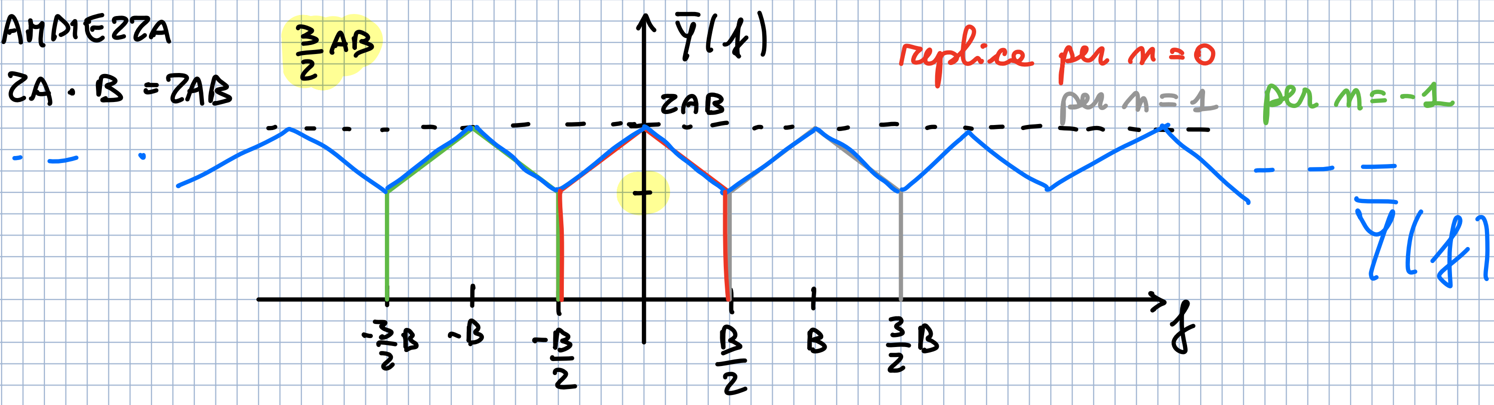
$$y[n] = y(nT)$$

Siccome $T = \frac{1}{B} \Rightarrow B = \frac{1}{T}$

$$\overline{Y}(f) = TFS\left[y[n]\right] = \frac{1}{T}\sum_{n=-\infty}^{+\infty} Y\left(f - \frac{n}{T}\right) = B\sum_{n}^{+\infty} Y(f - nB)$$

AMPIEZZA

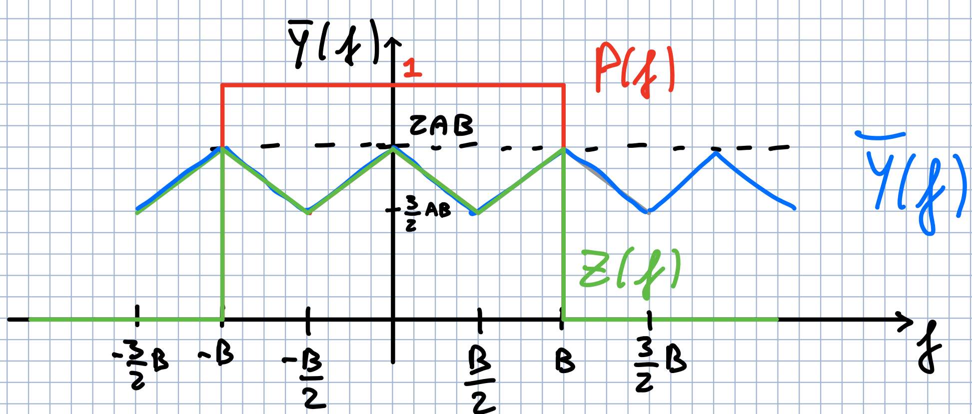
$2A \cdot B = 2AB$

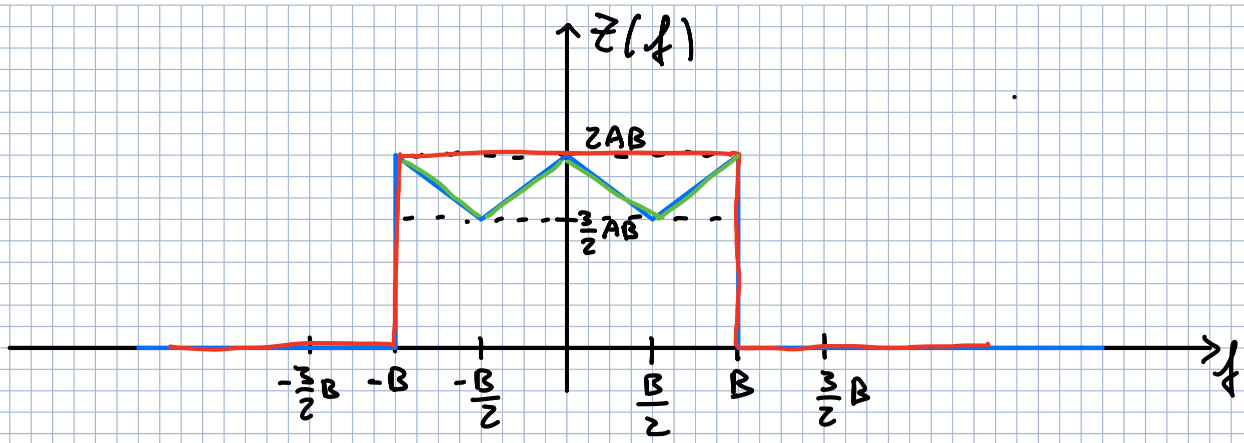


⇒ INTERPOLAZIONE

$$Z(f) = \overline{Y}(f) \cdot P(f)$$

$$P(f) = rect\left(\frac{f}{2B}\right)$$





$$Z(f) = 2AB\,\mathrm{rect}\left(\frac{f}{2B}\right) - \frac{AB}{2}\left[\left(1-\frac{|f-\frac{B}{2}|}{B/2}\right)\mathrm{rect}\left(\frac{f-\frac{B}{2}}{B}\right) + \left(1-\frac{|f+\frac{B}{2}|}{B/2}\right)\mathrm{rect}\left(\frac{f+\frac{B}{2}}{B}\right)\right]$$

$$= 2AB\,\mathrm{rect}\left(\frac{f}{2B}\right) - \frac{AB}{2}\left(1-\frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) \otimes \left[\delta\left(f-\frac{B}{2}\right)+\delta\left(f+\frac{B}{2}\right)\right]$$

$$z(t) = 2AB\cdot 2B\,\mathrm{sinc}(2Bt) - \frac{AB}{2}\cdot\frac{B}{2}\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\left[e^{j2\pi\frac{B}{2}t} + e^{-j2\pi\frac{B}{2}t}\right] =$$

$$= 4AB^2\,\mathrm{sinc}(2Bt) - \frac{AB^2}{2}\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\left[\frac{e^{j\pi Bt} + e^{-j\pi Bt}}{2}\right]$$

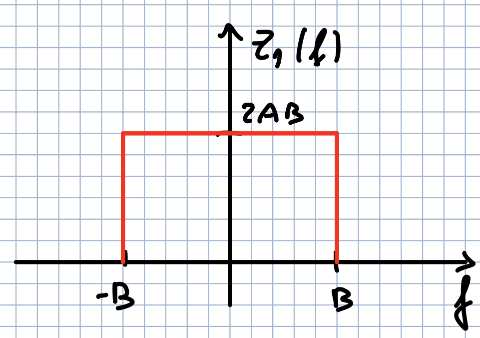
$$= \boxed{4AB^2\,\mathrm{sinc}(2Bt) - \frac{AB^2}{2}\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(\pi Bt)}$$

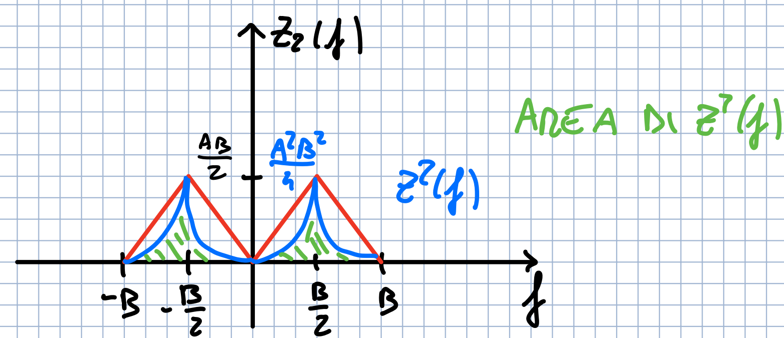
$$E_z = \int_{-\infty}^{+\infty} |z(t)|^2\,dt = \int_{-\infty}^{+\infty} |Z(f)|^2\,df$$

$$Z(f) = Z_1(f) - Z_2(f)$$

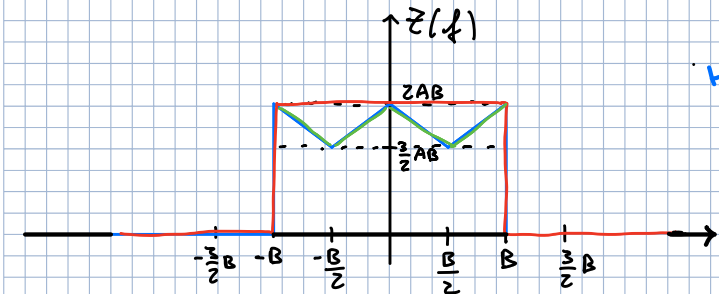
$$Z_1(f) = 2AB\,\mathrm{rect}\left(\frac{f}{2B}\right)$$

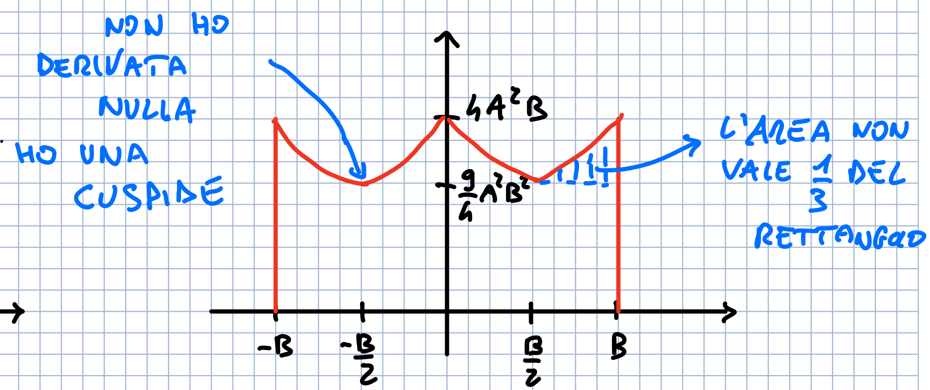
$$Z_2(f) = \frac{AB}{2}\left[\left(1-\frac{|f-\frac{B}{2}|}{B/2}\right)\mathrm{rect}\left(\frac{f-\frac{B}{2}}{B}\right) + \left(1-\frac{|f+\frac{B}{2}|}{B/2}\right)\mathrm{rect}\left(\frac{f+\frac{B}{2}}{B}\right)\right]$$





$$|Z(f)|^2 = Z^2(f)$$



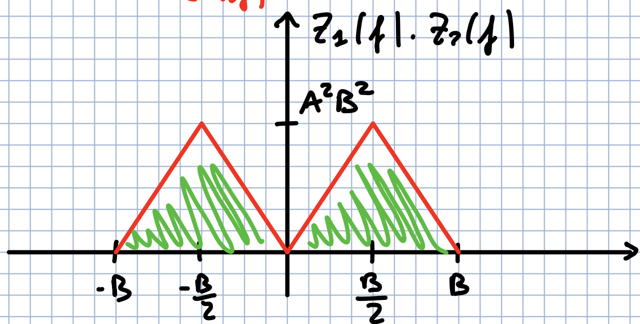


$$Z^2(f) = Z_1^2(f) + Z_2^2(f) - 2Z_1(f)Z_2(f)$$

$$E_Z = 2B \cdot 4A^2B^2 + 4\underbrace{\left(\frac{B}{2}\cdot\frac{A^2B^2}{4}\right)}_{\text{AREA TRIANGOLO } Z^2(f)}\cdot\underbrace{\frac{1}{3}}_{\text{INTEGRALE DI UN ARCO DI PARABOLA}} - 2\cdot\cancel{2}\left(\frac{B\cdot A^2B^2}{\cancel{2}}\right)$$

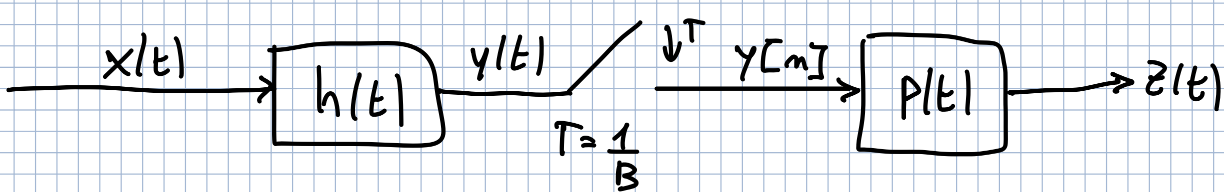
AMPIEZZA

$$2AB\cdot\frac{AB}{2} = A^2B^2$$



$$E_Z = 8A^2B^3 + \frac{A^2B^3}{6} - 2A^2B^3 = \boxed{\frac{37}{6}A^2B^3}$$

ESERCIZIO - COMPITINO 2018



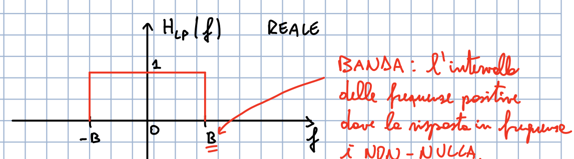
$$x(t) = 3B\,\mathrm{sinc}\left(\frac{3B}{2}t\right) - \frac{B}{2}\,\mathrm{sinc}\left(\frac{B}{2}t\right)$$

$h(t)$ è un passa-basso ideale di banda B

$$p(t) = 2B\,\mathrm{sinc}(2Bt)$$

·) PASSO BASSO DI BANDA B

LP = LOW PASS



RISPOSTA IN FREQUENZA DI UN FILTRO LP DI BANDA B

$$H_{LP}(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$h_{LP}(t) = \mathrm{ATCF}[H_{LP}(f)] = 2B\,\mathrm{sinc}(2Bt)$$

1) $y(t)$

2) Verificare se $y[n]$ è OTTENUTA rispettando Nyquist

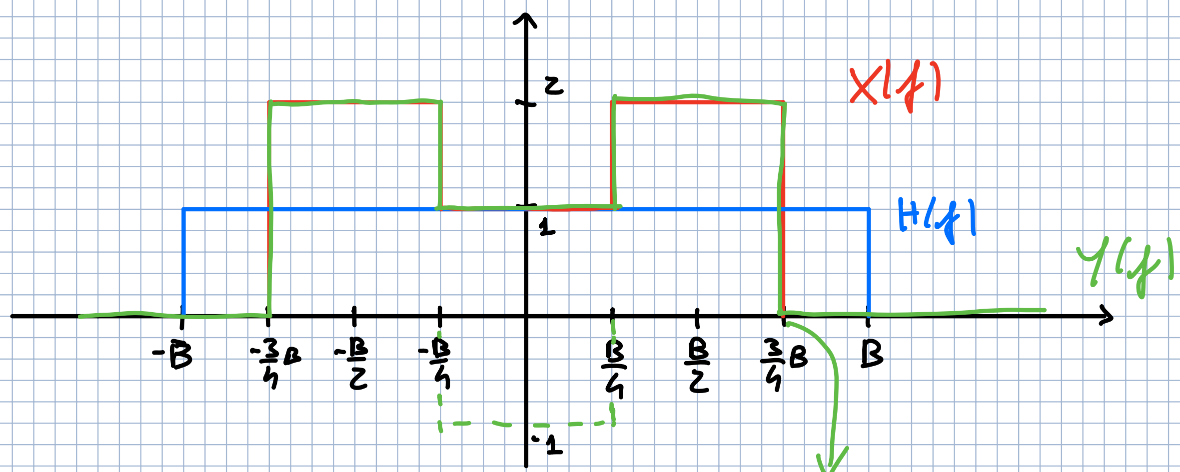
3) $z(t)$, 4) E_z, P_z

SVOLGIMENTO

1) $y(t) = x(t) \otimes h(t)$

$Y(f) = X(f) \cdot H(f)$

$X(f) = 2\,\mathrm{rect}\left(\frac{f}{\frac{3B}{2}}\right) - \mathrm{rect}\left(\frac{f}{B/2}\right) \qquad H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$



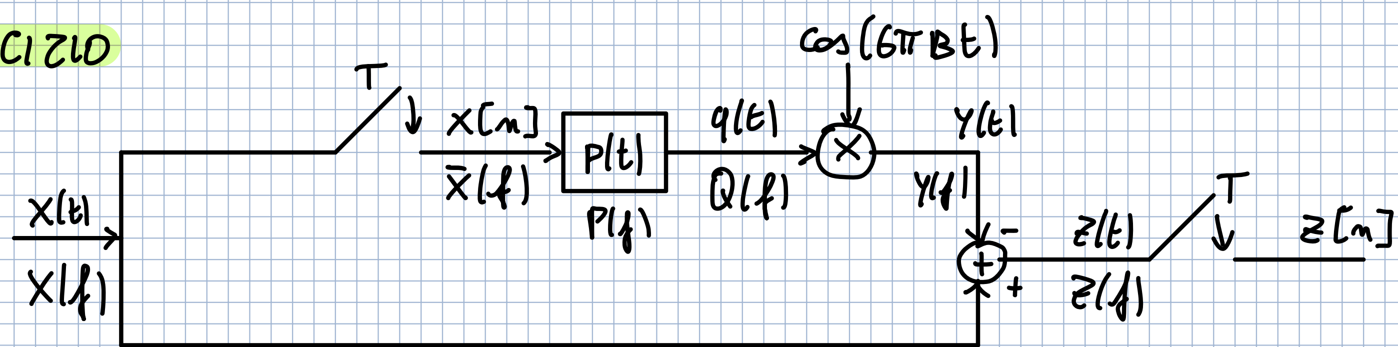
$Y(f) = X(f)$ BANDA DI $x(t) = y(t)$ È RIG. LIM. $= B'$

$y(t) = x(t)$

2) $B' = \frac{3}{4}B \Rightarrow \boxed{T_{Ny} \le \frac{1}{2B'}} = \frac{1}{2 \cdot \frac{3}{4}B} = \frac{2}{3B}$

$\Rightarrow T = \frac{1}{B} \rightarrow \frac{1}{B} > \frac{2}{3B} \Rightarrow$ NON È RISPETTATA LA COND. DI NYQUIST!!

ESERCIZIO



$x(t) = 2AB\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\cos(6\pi Bt)\right] \qquad T = \frac{1}{3B}$

$p(t)$ interpolatore CARDINALE DI BANDA B

$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$

1) Calcolare e disegnare lo spettro di $x(t)$

2) Calcolare $z(t)$

3) Calcolare e disegnare lo spettro di $z(t)$

4) E_z, P_z

5) Calcolare $z[n]$

MODULAZIONE COSENO

$$y(t) = x(t) \cdot \cos(2\pi f_0 t)$$

$$Y(f) = \frac{X(f-f_0) + X(f+f_0)}{2}$$

$\cos(2\pi f_0 t)$, $f_0 = 3B$

SVOLGIMENTO

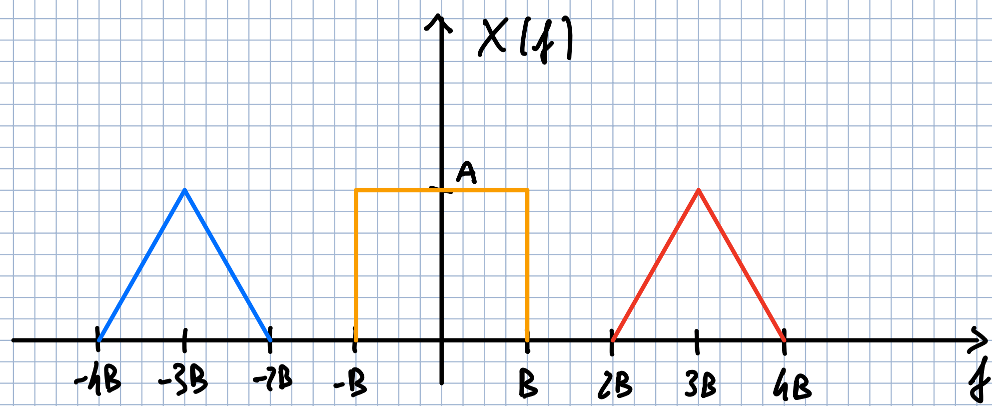
$$x(t) = 2AB\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\cos(6\pi Bt)\right]$$

$$X(f) = 2AB\left[\frac{1}{2B}\mathrm{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\cdot\frac{1}{B}\left(1 - \frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \right.$$

$$\left. + \frac{1}{2}\cdot\frac{1}{B}\left(1 - \frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)\right]$$

$$= \underbrace{A\,\mathrm{rect}\left(\frac{f}{2B}\right)} + \underbrace{A\left(1 - \frac{|f-3B|}{B}\right)\mathrm{rect}\left(\frac{f-3B}{2B}\right)} + \underbrace{A\left(1 - \frac{|f+3B|}{B}\right)\mathrm{rect}\left(\frac{f+3B}{2B}\right)}$$

1



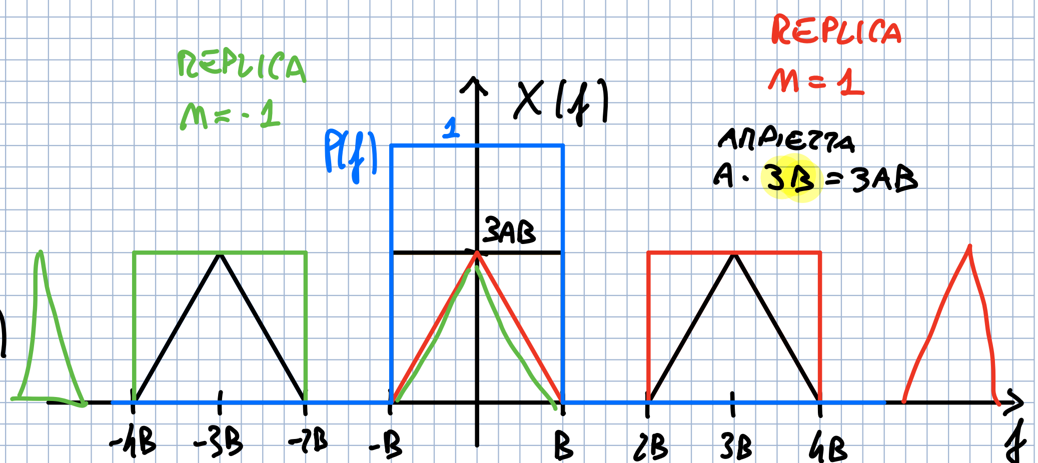
2 $z(t) = ?$

$$z(t) = x(t) - y(t)$$

$$y(t) = q(t)\cos(6\pi Bt)$$

$$q(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p(t - nT)$$

$$x[n] = x(nT)$$

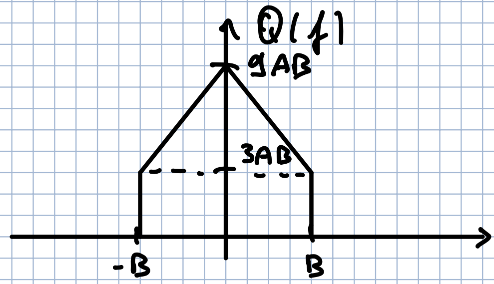


$$\overline{X}(f) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} X\left(f - \frac{m}{T}\right) \qquad T = \frac{1}{3B} \Downarrow \frac{1}{T} = 3B$$

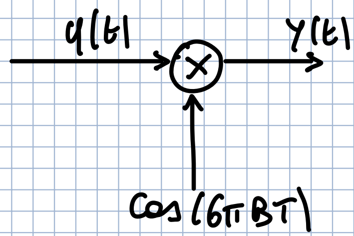
$$= 3B \sum X(f - 3Bm)$$

$Q(f) = \bar{X}(f)\,P(f) =$

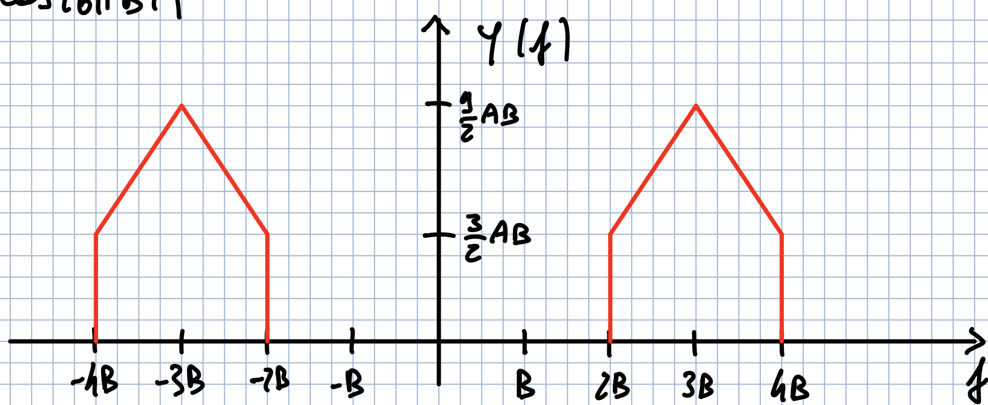
$= 3AB\left[\mathrm{rect}\left(\frac{f}{2B}\right) + 2\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)\right]$

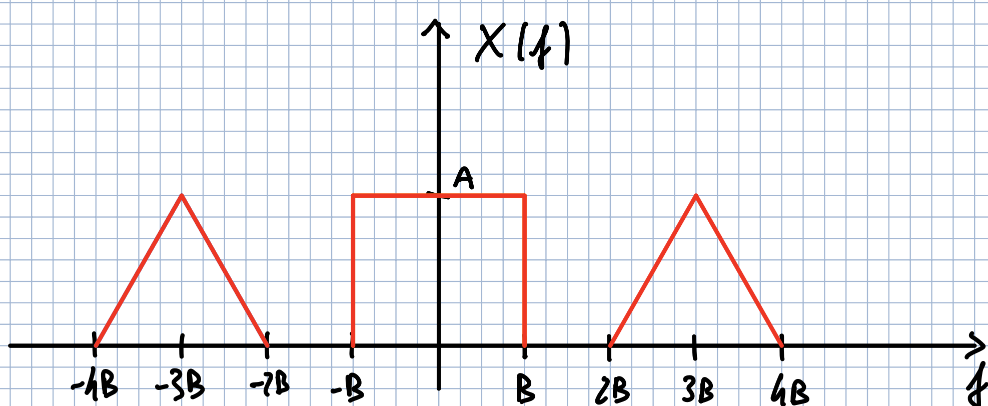


$y(t) = q(t)\cdot\cos(6\pi BT)$

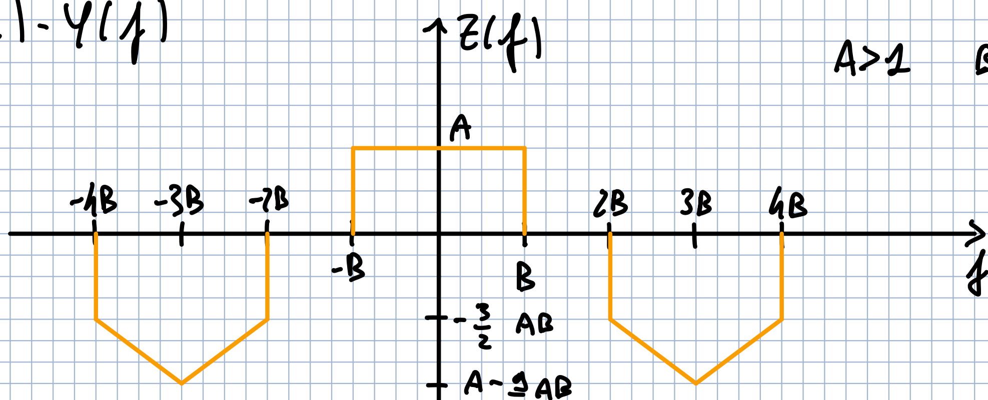


$Y(f) = \frac{1}{2}Q(f-3B) + \frac{1}{2}Q(f+3B)$





$Z(f) = X(f) - Y(f)$



$A > 1 \quad B > 1 \quad AB > 1$

$$Y(f) = \frac{3}{2}AB\left[\mathrm{rect}\left(\frac{f-3B}{2B}\right) + 2\left(1-\frac{|f-3B|}{B}\right)\mathrm{rect}\left(\frac{f-3B}{2B}\right)\right] +$$

$$+\frac{3}{2}AB\left[\mathrm{rect}\left(\frac{f+3B}{2B}\right) + 2\left(1-\frac{|f+3B|}{B}\right)\mathrm{rect}\left(\frac{f+3B}{2B}\right)\right] =$$

$$z(t) = x(t) - y(t)$$

$$y(t) = q(t)\cdot\cos(6\pi BT)$$

$$q(t) = \mathrm{ATCF}[Q(f)] = 3AB\cdot 2B\,\mathrm{sinc}(2Bt) + 6AB\cdot B\,\mathrm{sinc}^2(Bt)$$

$$y(t) = 6AB^2\left(\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\right)\cos(6\pi BT) =$$

$$\underline{z(t)} = 2AB\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\cos(6\pi Bt)\right] -$$

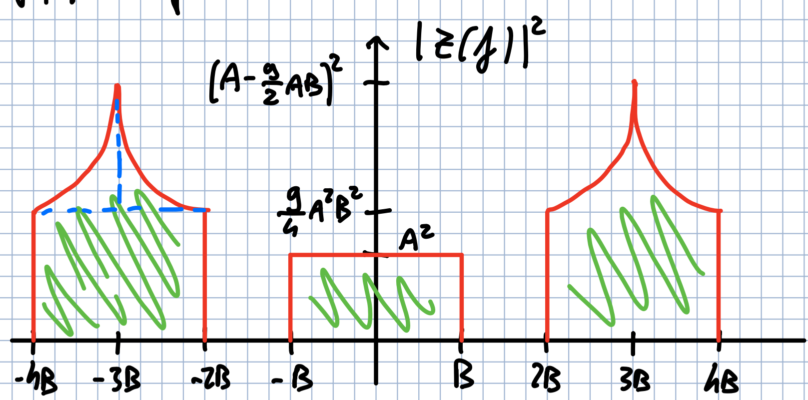
$$-6AB^2\left(\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\right)\cos(6\pi BT) =$$

$$= \boxed{(2AB - 6AB^2)\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\cos(6\pi Bt)\right]} \qquad T = \frac{1}{3B}$$

$$z[n] = z(nT) = (2AB - 6AB^2)\left[\mathrm{sinc}\left(\frac{2Bn}{3B}\right) + \mathrm{sinc}^2\left(\frac{Bn}{3B}\right)\cos\left(6\pi\frac{Bn}{3B}\right)\right] =$$

$$(2AB - 6AB^2)\left[\mathrm{sinc}\left(\frac{2}{3}n\right) + \mathrm{sinc}^2\left(\frac{n}{3}\right)\underbrace{\cos(\cancel{2\pi n})}_{\text{FA SEMPRE } 1\ \forall n}\right]$$

•) $E_z = \int_{-\infty}^{+\infty} |Z(f)|^2\, df$



10

CALCOLIAMO L'AREA SOTTESA

$$E_z = 2BA^2 + 2\left[(2B)\cdot\frac{9}{4}A^2B^2 + 2\cdot\frac{1}{3}\cdot\left(B\cdot\left[\left(A-\frac{9}{2}AB\right)^2 - \frac{9}{4}A^2B^2\right]\right)\right] =$$

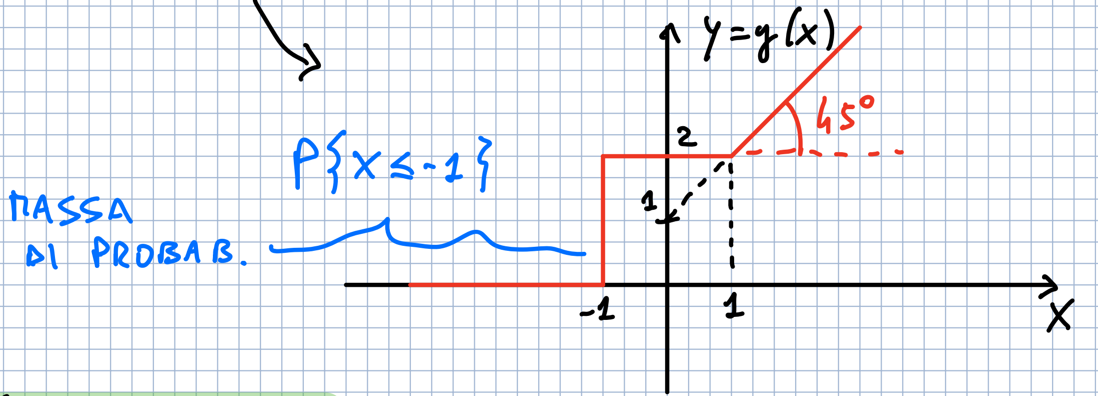
$$2A^2B + 2\left[\frac{9}{2}A^2B^3 + \frac{2B}{3}\cdot\left[A^2 + \frac{81}{4}A^2B^2 - 9A^2B - \frac{9}{4}A^2B^2\right]\right) =$$

$$= 2A^2B + 2\left[\frac{9}{2}A^2B^3 + \frac{2B}{3}\left[A^2 + 18A^2B^2 - 9A^2B\right]\right]$$

ESERCIZIO – TRASFORMAZIONE DI UNA V.A.

$X \in \mathcal{N}(0,4) \qquad \eta_x = 0 \quad , \quad \sigma_x^2 = 4$

$Y = g(X) \Rightarrow f_Y(y) = ?$



SVOLGIMENTO

$$f_Y(y) = \sum_{i=1}^{N} \frac{f_X(y_i)}{|g'(y_i)|} \qquad \{y_i\} \text{ sono le soluzioni } g^{-1}(y) = x$$

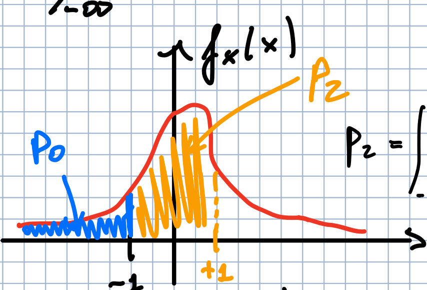
$\boxed{Y<0}$ NON CI SONO SOLUZIONI $f_Y(y) = 0$

$\boxed{Y=0} \Rightarrow f_Y(y) = p_0\,\delta(y) \qquad p_0 = P\{X \le 1\}$

$$p_0 = \int_{-\infty}^{-1} \frac{1}{\sqrt{2\pi 4}} \cdot e^{-\frac{x^2}{2\cdot 4}}\,dx$$

$\boxed{0<Y<2} \Rightarrow f_Y(y) = 0$

$\boxed{Y=2} \Rightarrow f_Y(y) = p_2\,\delta(y-2)$

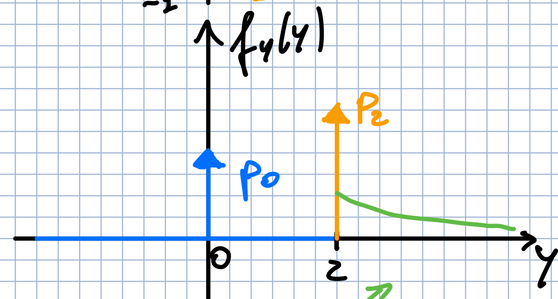


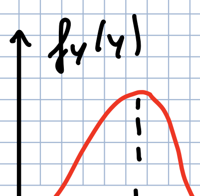
$$P_2 = \int_{-1}^{1} \frac{1}{\sqrt{8\pi}}\,e^{-\frac{x^2}{8}}\,dx$$

$\boxed{Y>2} \quad y = g(x) = \underbrace{x+1}_{\text{RETTA}} \qquad x = y - 1$

$|g'(x)| = |1| = 1$

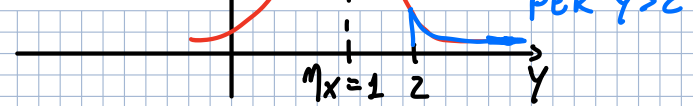
$$f_Y(y) = \frac{\frac{1}{\sqrt{8\pi}}\,e^{-\frac{(y-1)^2}{8}}}{1} = \underbrace{\frac{1}{\sqrt{8\pi}}\,e^{-\frac{(y-1)^2}{8}}}_{\eta_x = 1}$$





STESSA VARIANZA di X

VALE SOLO PER Y>2



ESERCIZIO – 09 APRILE/2018 #1

X ha una ddp esp. negativa con parametro λ, con:

$$F_X(x) = \begin{cases} 1 - e^{-\frac{x}{\lambda}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

1) Prob. che la durata sia > 1000 ore è $= e^{-1} \simeq 0{,}368 \Leftarrow$ OSSERVO

Calcolare λ

X indica la durata di una lampadina in ore

$P\{X > 1000\} = e^{-1}$

$P\{X \leq 1000\} = F_X(1000)$

$P\{X \leq x\} = F_X(x) \nearrow x = 1000$

$X \leq 1000$ è l'evento complementare di $X > 1000$

$P\{X > 1000\} = 1 - P\{X \leq 1000\}$

$P\{X > 1000\} = 1 - F_X(1000) = e^{-1}$

$F_X(1000) = 1 - e^{-1}$

$1 - e^{-\frac{1000}{\lambda}} = 1 - e^{-1}$

$e^{-\frac{1000}{\lambda}} = e^{-1}$ $\quad \boxed{\lambda = 1000}$

2) Usando $\lambda = 1000$ si calcoli il tempo x_0 : probabilità di una durata $< x_0$ sia $=$ a $0{,}09$

$P\{X \leq x_0\} = 0{,}09$

$F_X(x_0) = 0{,}09$

$1 - e^{-\frac{x_0}{1000}} = 0{,}09$

$e^{-\frac{x_0}{1000}} = 0{,}91$

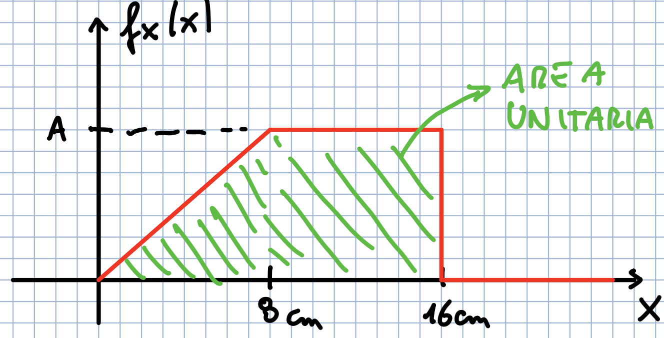
$e^{-\frac{x_0}{1000}} = e^{\ln 0{,}91}$

$-\frac{x_0}{1000} = \ln 0{,}91$

$x_0 = -\ln 0{,}91^{1000} \simeq 94{,}31$ ore

ESERCIZIO - 20/02/2020 #1

X V.A. con ddp $\qquad X$ = distanza dal centro



1) Calcolare A:

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$$ → AREA UNITARIA

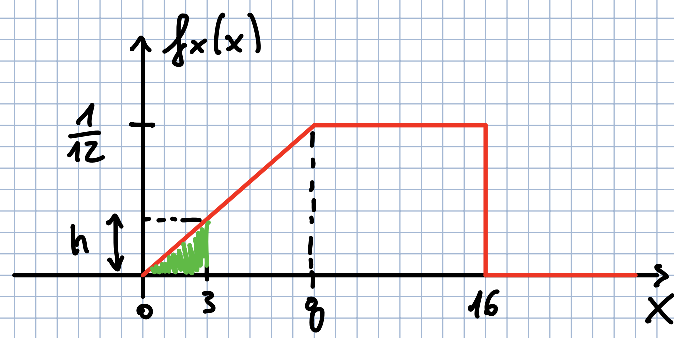
$$\frac{A\cdot \cancel{8}^{4}}{\cancel{2}} + 8A = 1 \qquad A = \frac{1}{12}$$

2) Calcolare la prob. che si colpisca il bersaglio entro i 3 cm di distanza dal centro

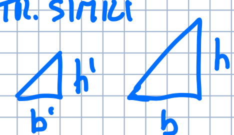
$P\{X \leq 3\,cm\}$

$F_X(x)\Big|_{x=3cm}$ => non l'abbiamo

$f_X(x)$ => ce l'ho => $P\{X\leq 3\,cm\} = \int_{-\infty}^{3cm} f_X(x)\,dx = \int_{0}^{3} f_X(x)\,dx = \frac{1}{32}\cdot 3\cdot\frac{1}{2} = \boxed{\frac{3}{64}}$



$h \Rightarrow \underbrace{\frac{1}{12}\cdot\frac{1}{8}}_{\text{TR. SIMILI}} = \frac{h}{3} \qquad h = \frac{\cancel{3}^{1}}{\cancel{12}_{4}\cdot 8} = \frac{1}{32}$



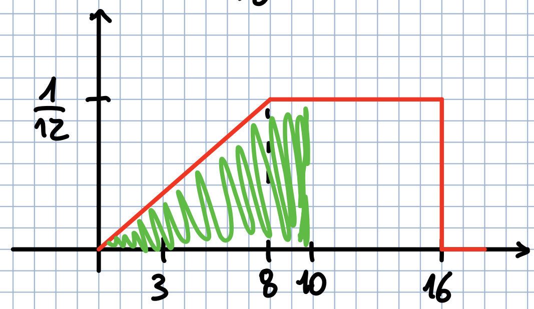
$$\frac{h}{b} = \frac{h'}{b'}$$

3) Calcolare la stessa probabilità al punto 2, sapendo che la freccetta non è andata più in là di 10 cm

$$P\{X \leq 3\,cm \mid \underbrace{X \leq 10\,cm}_{\uparrow\ \text{OSSERVATO}}\}$$

$$P\{A|B\} = \frac{P\{AB\}}{P\{B\}} \rightarrow P\{X\leq 3 \mid X\leq 10\} = \frac{P\{X\leq 3, X\leq 10\}}{P\{X\leq 10\}} = \frac{P\{X\leq 3\}}{P\{X\leq 10\}}$$

$P\{X \le 10\} = \int_0^{10} f_X(x)\,dx = \frac{1}{2}$



AREA $= \underbrace{\left(8 \cdot \frac{1}{12} \cdot \frac{1}{2}\right)}_{\text{TRIANG.}} + \underbrace{2 \cdot \frac{1}{12}}_{\text{RETT.}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

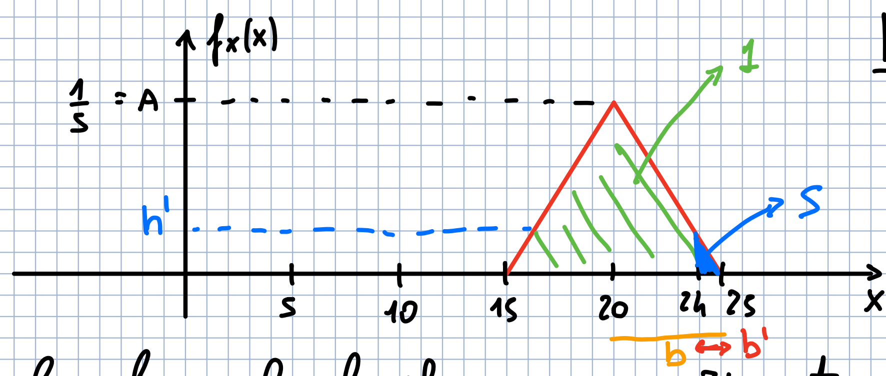
$\frac{P\{X \le 3\}}{P\{X \le 10\}} = \frac{\frac{3}{\cancel{64}_{32}}}{\frac{1}{\cancel{2}}} = \boxed{\frac{3}{32}}$

ESERCIZIO

X rappresenta il risultato del lancio (distanza)

$$f_X(x) = A\left(1 - \frac{|x-20|}{5}\right) rect\left(\frac{x}{10}\right)$$

a) Calcolare A:



$\frac{b \cdot h}{2} = Area = 1$

$h = \frac{2 \cdot 1}{10} = \frac{2}{10} = \boxed{\frac{1}{5} = A}$

b) Calcolare la prob. che il peso superi 24 metri

$$P\{X > 24\,m\} = 1 - P\{X \le 24\,m\} = 1 - F_X(24) = \underbrace{\int_{24}^{+\infty} f_X(x)\,dx}_{\text{AREA BLU}}$$

$$= 1 - \int_{-\infty}^{24} f_X(x)\,dx = S = \frac{b' \cdot h'}{2} = \frac{1 \cdot \frac{1}{25}}{2} = \boxed{\frac{1}{50}}$$

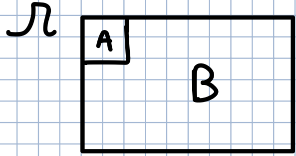
DEVO CALCOLARE h' per risolvere l'integrale

$\frac{h}{b} = \frac{h'}{b'}$ $\qquad \frac{A}{5} = \frac{h'}{1} \Rightarrow h' = \frac{\frac{1}{5}}{5} = \frac{1}{25}$

c) Calcolare la probabilità che il peso superi la soglia record almeno 1 volta su 3 lanci

$$P\{\text{almeno una volta su } N \text{ lanci}\} = 1 - P\{\text{nessuna volta}\}$$

I 2 EVENTI SONO COMPLEMENTARI



B = almeno 1 volta

A = mai

$P = \frac{1}{50}$ prob. su un lancio

$$P\{A\} = (1-p)(1-p)(1-p) = (1-p)^3 = \left(\frac{49}{50}\right)^3 = 0,$$

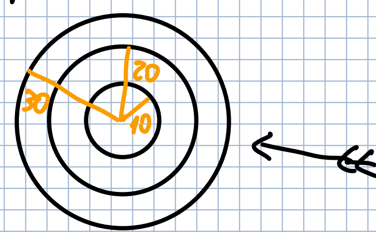
PROB. CHE NON SI VERIFICHI AL 1° LANCIO — AL 2° — AL 3°

$$P\{B\} = \boxed{1 - \left(\frac{49}{50}\right)^3 \simeq 0,059}$$

ESERCIZIO

X = rappresenta l'accuratezza (DISTANZA DAL CENTRO)

$$f_X(x) = x e^{-x} u(x)$$



a) Calcola la distanza media dal centro del punto in cui si conficca la freccia

$$E[X] = \int_{-\infty}^{+\infty} x \cdot f_X(x)\, dx = \int_{-\infty}^{+\infty} x \cdot x e^{-x} u(x)\, dx =$$

$$= \int_0^{+\infty} x^2 e^{-x}\, dx = -e^{-x} \cdot x^2 \Big|_0^{+\infty} - \int_0^{+\infty} -e^{-x} \cdot 2x\, dx = 2\int_0^{+\infty} x e^{-x}\, dx =$$

$$2\left[-e^{-x} \cdot x \Big|_0^{+\infty} - \int_0^{+\infty} -e^{-x}\, dx\right] = 2\left[0 + \int_0^{+\infty} e^{-x}\, dx\right] = 2 \cdot \left(-e^{-x}\right)\Big|_0^{+\infty} = 2(1-0) = 2$$

$$\boxed{\eta_x = E[X] = 2}$$

b) Prob. che venga colpita la sagoma

$$P\{X \leq 30\,cm\} =$$

$$= \int_{-\infty}^{30} f_X(x)\,dx = \int_{-\infty}^{30} x\,e^{-x} u(x)\,dx = \int_{0}^{30} x\,e^{-x}\,dx = -e^{-x}\cdot x \Big|_0^{30} - \int_0^{30} -e^{-x}\cdot 1\,dx =$$

$$\left(-e^{-30}\cdot 30 + e^{0}\cdot 0\right) + \left(-e^{-x}\Big|_0^{30}\right) = -\frac{30}{e^{30}} + \left(-e^{-30} - (-1)\right) = \boxed{1 - \frac{31}{e^{30}} \simeq 1}$$

c) Prob. di colpire la sagoma nella corona circolare più esterna (TRA 20 e 30)

$$P\{20 < X \le 30\} = \int_{20}^{30} f_X(x)\,dx = P\{X \le 30\} - P\{X \le 20\}$$

$$P\{X \le 20\} = -20\,e^{-20} + \left(1 - e^{-20}\right) = 1 - 21\,e^{-20}$$

$$P\{20 < X \le 30\} = 1 - \frac{31}{e^{30}} - \left(1 - 21\,e^{-20}\right) = \boxed{21\,e^{-20} - 31\,e^{-30}}$$

$\Rightarrow$ per i processi

$$x(t) \Rightarrow X(t)$$
$$x_T(t) \Rightarrow X_T(t)$$
$$S_x(w_i;f) = \lim_{T\to\infty} \frac{|X_T(w_i;f)|^2}{T}$$

$\Rightarrow$ TH. DI WIENER

$$S_x(f) = \int_{-\infty}^{+\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau \qquad \boxed{S_x(f) = TCF[R_x(\tau)]}$$

ESERCIZIO

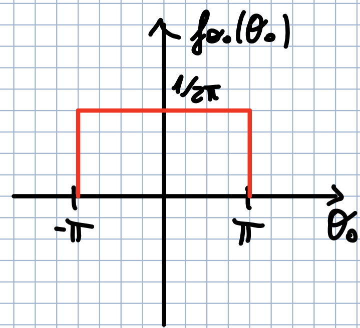
$X(t) = 1 + 9\cos(2\pi f_0 t + 2\theta_0)$ $\qquad \theta_0$ è una V.A. $\in \mathcal{U}[-\pi,\pi]$

Calcolare:

1) V. MEDIO e 2) DENSITÀ SPETTRALE DI POTENZA

SOLUZIONE

1) $\eta_x(t) = E[x(t)] = \int_{-\infty}^{+\infty} x\, f_x(x;t)\, dx$



$X(t) = g(\theta_0;t) \qquad f_{\theta_0}(\theta_0) = \frac{1}{2\pi}\, rect\left(\frac{\theta_0}{2\pi}\right)$

USO TH. V. MEDIO

$$E[g(x)] = \int_{-\infty}^{+\infty} g(x)\, f_x(x)\, dx$$

$$E[x(t)] = E[g(\theta_0;t)] = \int_{-\infty}^{+\infty} [1 + 9\cos(2\pi f_0 t + 2\theta_0)] \cdot f_{\theta_0}(\theta_0)\, d\theta_0$$

INTEGRO SU INTERVALLO DI $f_{\theta_0}(\theta_0) \to -\pi, \pi$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} [1 + 9\cos(2\pi f_0 t + 2\theta_0)]\, d\theta_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta_0 + \frac{9}{2\pi}\underbrace{\int_{-\pi}^{\pi} \cos(2\theta_0 + \varphi_0)\, d\theta_0}_{=0} =$$

$\varphi_0 = 2\pi f_0 t$

$= 1 + 0 = 0 \qquad \boxed{\eta_x(t) = \eta_x = 1}$

$\theta \in \mathcal{U}[0, 2\pi)$

$f_x(x) = \frac{1}{b-a}\, rect\left(\frac{x - \frac{b+a}{2}}{b-a}\right)$

$f_\theta(\theta) = \frac{1}{2\pi}\, rect\left(\frac{\theta - \pi}{2\pi}\right)$

2) DDP

$$S_x(f) = TCF[R_x(\tau)]$$

$$R_X(t_1,t_2) = E[X(t_1)X(t_2)] =$$

$$= E[(1+9\cos(2\pi f_0 t_1 + 2\theta_0))(1+9\cos(2\pi f_0 t_2 + 2\theta_0))] =$$

$$= E[1] + E[9\cos(2\pi f_0 t_2 + 2\theta_0)] + E[9\cos(2\pi f_0 t_1 + 2\theta_0)] +$$

$$+ E[81\cos(2\pi f_0 t_1 + 2\theta_0)\cos(2\pi f_0 t_2 + 2\theta_0)] =$$

TH. V. MEDIO $\Rightarrow$ $$= 1 + \underbrace{\frac{9}{2\pi}\int_{-\pi}^{\pi}\cos(2\pi f_0 t_2 + 2\theta_0)\,d\theta_0}_{\text{FA } 0} + \underbrace{\frac{9}{2\pi}\int_{-\pi}^{\pi}\cos(2\pi f_0 t_1 + 2\theta_0)\,d\theta_0}_{\text{FA } 0}$$

$$+ \underbrace{\frac{81}{2}E[\cos(4\theta_0 + 2\pi f_0(t_1+t_2))]}_{\text{L'INTEGRALE È 0}} + \frac{81}{2}E[\cos(2\pi f_0(t_1-t_2))] =$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(4\theta_0 + \ell)\,d\theta_0 = 0$$

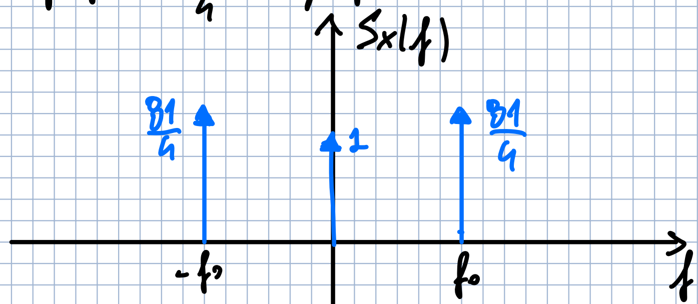
$$= 1 + \frac{81}{2}\cos(2\pi f_0(t_1-t_2)) =$$

$$= 1 + \frac{81}{2}\cos(2\pi f_0 \tau) = R_X(\tau)$$

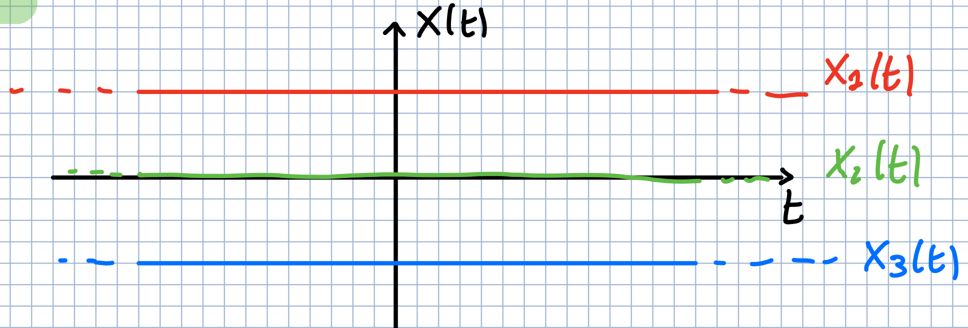
$$S_X(f) = TCF[R_X(\tau)]$$ TH. WIENER

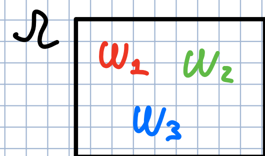
$$S_X(f) = TCF\left[1 + \frac{81}{2}\cos(2\pi f_0 \tau)\right] = \delta(f) + \frac{81}{2}\left[\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right]$$

$$= \delta(f) + \frac{81}{4}\delta(f-f_0) + \frac{81}{4}\delta(f+f_0)$$



ESERCIZIO





Processo aleatorio che ammette 3 realizzazioni equiprobabili:

$$P\{w_1\} = P\{w_2\} = P\{w_3\} = \frac{1}{3}$$

Calcolare:

1) $\eta_X(t)$ e 2) $R_X(t_1,t_2)$ e 3) Dire se il processo è SSL

SOLUZIONE

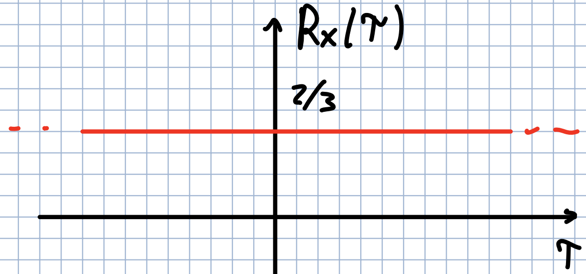
1) $\eta_x(t) = E[x(t)] = \int_{-\infty}^{+\infty} x \, f_x(x;t) \, dx$

$\Rightarrow x(t) = A$ A V. A. discreta processo aleatorio parametrico

$E[x(t)] = E[A] = \sum_{N=1}^{3} p_n x_n = p_1 x_1 + p_2 x_2 + p_3 x_3 =$

$= \frac{1}{3}(1) + \frac{1}{3}(0) + \frac{1}{3} \cdot -1 = 0 \quad \rightarrow \quad \eta_x(t) = 0$

2) $R_x(t_1, t_2) = E[X(t_1) X(t_2)] = E[A^2] = \sum_{1}^{3} p_n x_n^2 = \frac{1}{3} \cdot 1^2 + \frac{1}{3} \cdot 0^2 = \frac{1}{3} \cdot -1^2 = \frac{2}{3}$



3) È SSL poiché V. MEDIO è COSTANTE e R_x DIPENDE SOLO DA τ

ESERCIZIO

$x(t) = A \cos(2\pi f_0 t + \varphi)$ $\quad A \in \mathcal{U}[1, 2)$ $\quad \varphi \in \mathcal{U}[-\pi, \pi]$ $\quad$ A e φ sono INDIPENDENTI

Dire se $x(t)$ è SSL

SOLUZIONE

Devo prima calcolare V. MEDIO e capire se R_x dipende da τ

1) $\eta_x(t) = E[x(t)] = E[A \cos(2\pi f_0 t + \varphi)]$

A e φ sono INDIPENDENTI

$\Downarrow$

A e $\cos(2\pi f_0 t + \varphi)$ sono INDIPENDENTI

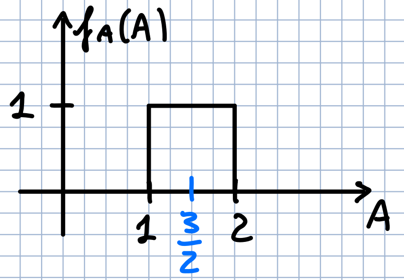
$= E[A] \cdot E[\cos(2\pi f_0 t + \varphi)] =$

$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A \cos(2\pi f_0 t + \varphi) \, f_{A,\varphi}(A, \varphi) \, dA \, d\varphi =$

$\Downarrow f_A(A) f_\varphi(\varphi)$

$$= \int_{-\infty}^{+\infty} A\, f_A(A)\, dA \int_{-\infty}^{+\infty} \cos(2\pi f_0 t + \varphi)\, f_\varphi(\varphi)\, d\varphi =$$

$$f_A(A) = \frac{1}{b-a}\,\mathrm{rect}\left(\frac{A-\frac{b+a}{2}}{b-a}\right) = \frac{1}{2-1}\,\mathrm{rect}\left(\frac{A-\frac{3}{2}}{2-1}\right) = \mathrm{rect}\left(\frac{A-3/2}{1}\right)$$



$$E[A] = \int_{-\infty}^{+\infty} A\, f_A(A)\, dA = \int_{1}^{2} A\,\mathrm{rect}\left(\frac{A-3/2}{1}\right) dA$$

$$= \int_{1}^{2} A\, dA = \frac{A^2}{2}\Bigg|_1^2 = 2 - \frac{1}{2} = \frac{3}{2}$$

$$E[\cos(2\pi f_0 t + \varphi)] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(2\pi f_0 t + \varphi)\, d\varphi = 0$$

$$\boxed{\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha-\beta) + \cos(\alpha+\beta)}$$

$\eta_X(t) = \frac{3}{2} \cdot 0 = 0$ COSTANTE

2) $R_X(t_1, t_2) = E[X(t_1)X(t_2)] =$

$$= E[(A\cos(2\pi f_0 t_1 + \varphi)) \cdot (A\cos(2\pi f_0 t_2 + \varphi))] =$$

$$= E\left[A^2 \frac{1}{2}\cos(2\pi f_0(t_1 - t_2))\right] + E\left[\frac{1}{2}A^2\cos(2\varphi + 2\pi f_0(t_1+t_2))\right] =$$

$$= \frac{1}{2}E[A^2] \cdot E[\cos(2\pi f_0(t_1 - t_2))] + \frac{1}{2}E[A^2] \cdot E[\cos(2\varphi + 2\pi f_0(t_1+t_2))] =$$

$$E[A^2] = \int_{-\infty}^{+\infty} A^2 f_A(A)\, dA = \int_1^2 A^2\,\mathrm{rect}\left(\frac{A-3/2}{1}\right) dA = \int_1^2 A^2\, dA = \frac{A^3}{3}\Bigg|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$$E[\cos(2\varphi + 2\pi f_0(t_1+t_2))] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(2\varphi + \theta_0)\, d\varphi = 0$$

$$E[\cos(2\pi f_0(t_1 - t_2))] = \cos(2\pi f_0(t_1 - t_2)) = \cos(2\pi f_0 \tau)$$

$$R_X(t_1, t_2) = \frac{1}{2} \cdot \frac{7}{3} \cdot \cos(2\pi f_0(t_1 - t_2)) + \frac{1}{2} \cdot \frac{7}{3} \cdot 0$$

$$\boxed{R_X(\tau) = \frac{7}{6}\cos(2\pi f_0 \tau)}$$

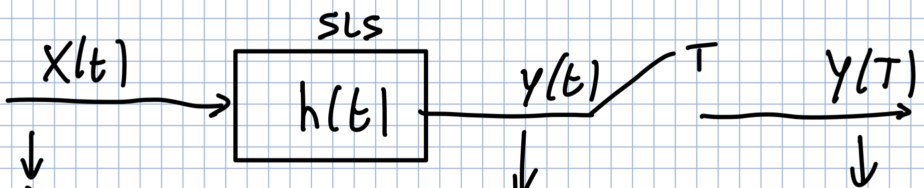
IL PROCESSO È SSL

- Un processo aleatorio Gaussiano SSL è anche SSS

$$SSL \iff SSS$$

- FILTRAGGIO DI P. A. GAUSSIANI

X(t) GAUSSIANO $\quad$ Y(t) è GAUSSIANO



$\eta_x(t) \rightarrow \eta_y(t)$

$R_x(t_1,t_2) \rightarrow R_y(t_1,t_2)$

GAUSSIANO $\rightarrow$ GAUSSIANO

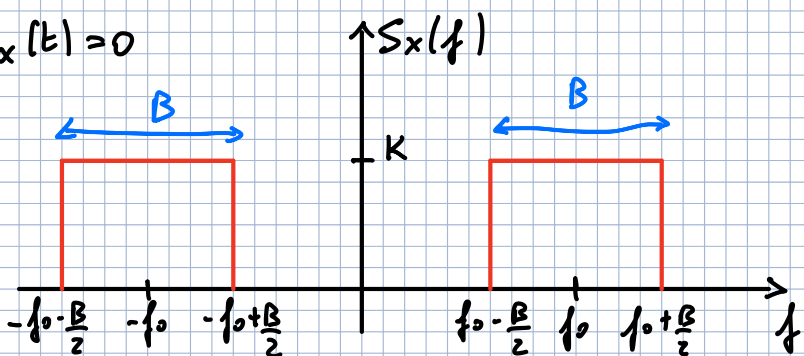
V.A. GAUSSIANA

RUMORE BIANCO

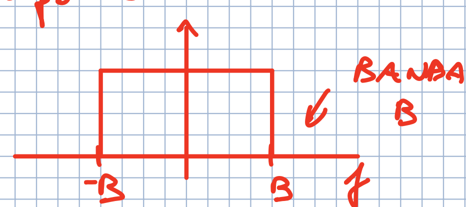
$$\eta_x(t) = 0 \qquad R_x(\tau) = K\,\delta(\tau) \qquad S_x(f) = K \quad \forall f$$

RUMORE BIANCO IN BANDA

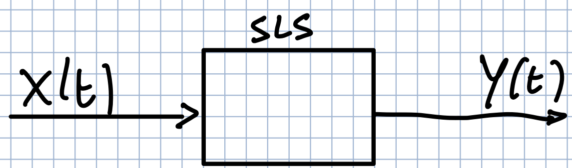
$\eta_x(t) = 0$



per $f_0 = 0$



ESERCIZIO - 05/06/18



X(t) GAUSSIANO BIANCO

$$\frac{d}{dt}y(t) + 2y(t) = x(t)$$

$$R_x(\tau) = \frac{N_0}{2}\,\delta(\tau)$$

$$R_x(\tau) = \underbrace{S_x(f)}_{=K}\,\delta(\tau)$$

1) $R_y(\tau)$, $S_y(f)$ e DISEGNARE GRAFICI

2) P_x, P_y

3) Scrivere la ddp di $Y(t_0)$

4) Calcolare la probabilità che $Y > \sqrt{\frac{N_0}{2}}$

SOLUZIONE

1) $\frac{d}{dt} y(t) + 2y(t) = x(t)$

$$\Downarrow \text{TCF}$$

$$j2\pi f\, Y(f) + 2Y(f) = X(f)$$

$$Y(f)[2 + j2\pi f] = X(f)$$

$$\frac{Y(f)}{X(f)} = H(f) = \frac{1}{2+j2\pi f}$$

$$(2 + j2\pi f)^2 = 4 - 4\pi^2 f^2 + j8\pi f$$

$$S_Y(f) = S_X(f)|H(f)|^2 = \frac{N_0}{2}\left|\frac{1}{2+j2\pi f}\right|^2 = \frac{N_0}{2}\left|\frac{1}{2(1+j\pi f)}\right|^2 = \boxed{\frac{N_0}{2}\,\frac{1}{4}\cdot\frac{1}{1+\pi^2 f^2}}$$

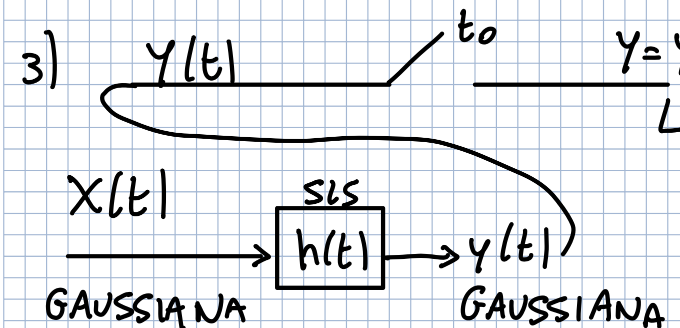
$$R_Y(\tau) = \text{ATCF}[S_Y(f)] = \boxed{\frac{N_0}{8} e^{-2|\tau|}}$$

2) P_X, P_Y

$$P_X = \int_{-\infty}^{+\infty} S_X(f)\, df = \int_{-\infty}^{+\infty} \frac{N_0}{2}\, df = \boxed{\infty}$$

$$P_Y = R_Y(\tau)\Big|_{\tau=0} = R_Y(0) = \boxed{\frac{N_0}{8}} \quad \text{o} \quad \int_{-\infty}^{+\infty} S_Y(f)\, df$$

3)



↳ V.A. GAUSSIANA

ddp $Y = ?$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}} e^{-\frac{(y-\eta_Y)^2}{2\sigma_Y^2}}$$

$\eta_Y = ?$ $\quad \sigma_Y^2 = ?$

$$\eta_Y = \eta_y(t_0) \qquad \eta_y(t)\Big|_{t=t_0} = \eta_Y$$

$$\eta_y(t) = \eta_x(t) \otimes h(t) = 0$$

$\eta_x(t) = 0$ perché il processo $X(t)$ è BIANCO $\rightarrow R_X(t_1, t_2) = R_X(\tau)$ $\nearrow$ $\eta_x(t) = 0$

$$\sigma_Y^2 = \sigma_Y^2(t)\Big|_{t=t_0}$$

$$\sigma_Y^2(t) = P_Y(t) - \eta_Y^2(t) = P_Y(t)$$

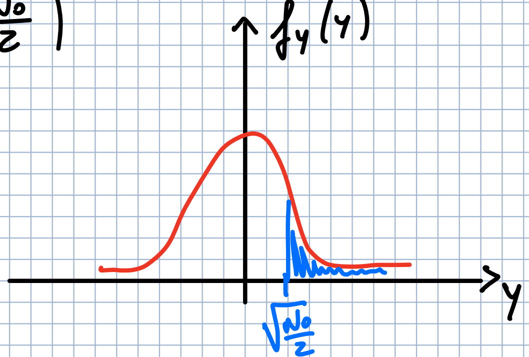
$$\sigma_Y^2(t_0) = P_Y(t_0) = P_Y = R_Y(0) \qquad \text{COSTANTE}$$

$$\sigma_Y^2 = P_Y = \frac{N_0}{8}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi \frac{N_0}{8}}}\, e^{-\frac{y^2}{2\frac{N_0}{8}}} = \boxed{\frac{2}{\sqrt{\pi N_0}}\, e^{-\frac{y^2}{N_0/4}}}$$

$f_Y(y; t_0) \Rightarrow f_Y(y)$ → NON IMPORTA DOVE SI CAMPIONA, IL RISULTATO E' INDIPENDENTE DA t_0

a) $P\left(Y > \sqrt{\frac{N_0}{2}}\right)$



$$P\left(Y > \sqrt{\tfrac{N_0}{2}}\right) = \int_{\sqrt{\frac{N_0}{2}}}^{+\infty} f_Y(y)\, dy$$

$$Q(x) = \int_x^{+\infty} f_N(n)\, dn \qquad N \in \mathcal{N}(0,1)$$

$$Y \in \mathcal{N}\left(0, \tfrac{N_0}{8}\right)$$

$$N = \frac{Y - \eta_Y}{\sigma_Y} \qquad x = \frac{\sqrt{\frac{N_0}{2}} - 0}{\sqrt{\frac{N_0}{8}}} = \sqrt{4} = 2$$

$$P\left(Y > \sqrt{\tfrac{N_0}{2}}\right) = P\{N > 2\} = Q(2)$$

ESERCIZIO - 11/11/19

$$P_w = \int_{-\infty}^{+\infty} S_w(f) = \sigma^2$$

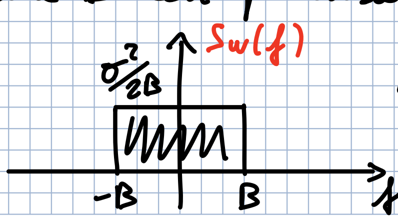
$Y(t) = X(t) + w(t)$

$X(t) = A$, A V.A. con media nulla ($\eta_A = 0$) e varianza σ_A^2

$w(t)$ = rumore bianco in banda B con potenza σ^2 (V. QUADR. MEDIO)

1) Verificare se $Y(t)$ è SSL

2) Calcolare $S_Y(f)$



$w(t)$ è INDIP. da $x(t)$

SOLUZIONE

1) $y(t)$ è SSL ? $\nearrow$ $\eta_y(t) = \eta_y$ ✓ 1
$\searrow$ $R_y(t_1,t_2) = R(t_1-t_2) = R(\tau)$ ✓ 2

$$\eta_y(t) = E[y(t)] = E[x(t)+w(t)] = E[x(t)] + E[w(t)] =$$

$E[x(t)] = \boxed{\eta_x = 0}$ poichè A è una V.A. con media $=0$ $\eta_x = 0$

$E[w(t)] = 0$ poichè è un rumore bianco $\boxed{\eta_w = 0}$, $R_w(\tau) = K\delta(\tau)$

$\eta_y(t) = E[y(t)] = 0+0 = 0$ $\boxed{\text{COSTANTE}}$ 1

$$R_y(t_1,t_2) = E[y(t_1)\cdot y(t_2)] = E\Big[[x(t_1)+w(t_1)][x(t_2)+w(t_2)]\Big] =$$

$$= E[x(t_1)x(t_2)] + E[x(t_1)w(t_2)] + E[w(t_1)x(t_2)] + E[w(t_1)w(t_2)] =$$

$$= R_x(t_1,t_2) + \underbrace{E[x(t_1)]}_{\eta_x=0} \underbrace{E[w(t_2)]}_{\eta_w=0} + \underbrace{E[w(t_1)]}_{\eta_w=0} \cdot \underbrace{E[x(t_2)]}_{\eta_x=0} + R_w(t_1,t_2) =$$

$$= R_x(t_1,t_2) + R_w(t_1,t_2)$$

- $R_x(t_1,t_2) = E[x(t_1)x(t_2)] = E[A\cdot A] = E[A^2] = \sigma_A^2 + \underbrace{\eta_A^2}_{0} = \sigma_A^2$

$R_x(\tau) = \sigma_A^2$ COSTANTE

- $R_w(t_1,t_2) = R_w(\tau) = \dfrac{\sigma^2}{\cancel{2B}} \cdot \cancel{2B}\,\text{sinc}(2B\tau)$

$$\boxed{S_w(f) = \frac{\sigma^2}{2B}\,\text{rect}\left(\frac{f}{2B}\right)}$$

$$\begin{cases} R_y(\tau) = \sigma_A^2 + \sigma^2\,\text{sinc}[2B(\tau)] & \text{dipende da } \tau = t_1 - t_2 \quad (2) \\ \eta_y(t) = 0 & \text{COST.} \quad (1) \end{cases}$$

$\hookrightarrow$ $y(t)$ è SSL

2) $S_y(f) = ?$

$$S_y(f) = TCF[R_y(\tau)] = \sigma_A^2\,\delta(f) + \frac{\sigma^2}{2B}\,\text{rect}\left(\frac{f}{2B}\right)$$

ESERCIZIO - 6/06/17

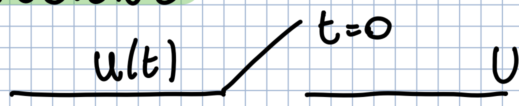
$u(t)$ GAUSSIANO SSL

$\eta_u(t) = 0$ $\quad R_u(\tau) = \sigma_u^2\,\text{sinc}(2B\tau)$

1) Si estragga una V.A. $U = U(0)$. Si descriva la ddp di U

2) Dato $y(t) = u(t) + 3\,U(t-T)$. Calcolare: $S_y(f)$, $R_y(\tau)$

SOLUZIONE

1) 

V.A

$f_u(u) = ?$

$u(t)$ GAUSSIANO $\Rightarrow$ $U \in \mathcal{N}(\eta_u, \sigma_u^2)$

$u(t)$ a v.m. nullo $\Rightarrow$ $\eta_u(t) = 0$ $\Rightarrow$ $\eta_u = 0$

$\sigma_u^2(t) = P_u(t) - \eta_u^2(t) = P_u - 0 = P_u$

$P_u = R_u(0) = \sigma_u^2 \cdot 1 = \sigma_u^2 \quad \rightarrow \quad U \in \mathcal{N}(0, \sigma_u^2)$

$$\boxed{f_u(u) = \frac{1}{\sqrt{2\pi\sigma_u^2}}\, e^{-\frac{u^2}{2\sigma_u^2}}}$$

2) $Y(t) = U(t) + 3\,U(t-T)$

$S_y(f) = TCF[R_y(\tau)]$

$$R_y(t_1, t_2) = E[Y(t_1)Y(t_2)] = E[(U(t_1) + 3U(t_1-T))(U(t_2) + 3U(t_2-T))] =$$

$$= E[U(t_1)U(t_2)] + 3[U(t_1-T)U(t_2)] +$$

$$+ 3E[U(t_1)U(t_2-T)] + \underbrace{9E[U(t_1-T)U(t_2-T)]}_{} = \quad t_1 - \cancel{T} - t_2 + \cancel{T}$$

$$R_U(t_1-t_2) + 3R_U(t_1-T-t_2) + 3R_U(t_1-t_2+T) + 9R_U(t_1-t_2) =$$

$$= 10R_U(t_1-t_2) + 3R_U(t_1-t_2-T) + 3R_U(t_1-t_2+T) =$$

$$= 10R_U(\tau) + 3R_U(\tau-T) + 3R_U(\tau+T)$$

$$S_y(f) = 10S_U(f) + 3S_U(f)e^{-j2\pi fT} + 3S_U(f)e^{j2\pi fT} =$$

$$= 10S_U(f) + 3S_U(f)\left(\frac{e^{-j2\pi fT} + e^{j2\pi fT}}{2}\right)\cdot 2 =$$

$$= 10S_U(f) + 6S_U(f)\cos(2\pi fT) = \boxed{S_U(f)[10 + 6\cos(2\pi fT)]} = R_y(\tau)$$

$$S_U(f) = TCF[R_U(\tau)] = \frac{\sigma_U^2}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$R_u(\tau) = \sigma_u^2\,\mathrm{sinc}(2B\tau) \quad \Rightarrow \boxed{S_y(f) = \frac{\sigma_u^2}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right)[10 + 6\cos(2\pi fT)]}$$

ESERCIZIO - 18/07/2017

$X(t)$

$$S_X(f) = \frac{\sigma_X^2}{2B} \, \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$X(t) \longrightarrow \underset{\text{SLS}}{\boxed{h(t)}} \longrightarrow Y(t)$$

$$h(t) = \frac{1}{2}\delta(t) + \delta(t-T) + \delta(t-2T)$$

1) Calcolare $S_Y(f)$

2) Disegnare il grafico di $S_Y(f)$ per $B = \frac{1}{2T}$

3) Calcolare $R_Y(\tau)$

4) Disegnare il grafico di $R_Y(\tau)$ per $B = \frac{1}{2T}$

SOLUZIONE

Due STRADE

$$S_X(f) \Rightarrow R_X(\tau) \Rightarrow R_Y(\tau) = R_X(\tau) \otimes h(-\tau) \otimes h(\tau)$$

$$\Downarrow$$

$$S_Y(f) = \mathrm{TCF}[R_Y(\tau)]$$

$$h(t) \to H(f) \Rightarrow S_Y(f) = S_X(f)\,|H(f)|^2$$

I STRADA

$$R_X(\tau) = \mathrm{ATCF}[S_X(f)] = \sigma_X^2 \,\mathrm{sinc}(2B\tau)$$

$$h(t) = \frac{1}{2}\delta(t) + \delta(t-T) + \delta(t-2T)$$

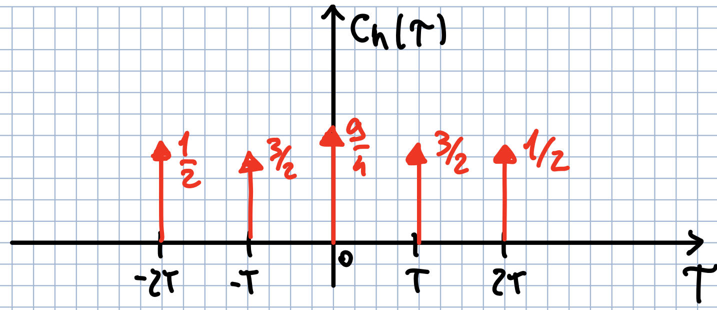
$$h(-t) = \frac{1}{2}\delta(t) + \delta(t+T) + \delta(t+2T)$$

$$\underbrace{h(t) \otimes h(-t)}_{C_h(\tau) = \int_{-\infty}^{+\infty} h(t)\,h(t-\tau)\,dt} = \frac{1}{4}\delta(t) + \frac{1}{2}\delta(t+T) + \frac{1}{2}\delta(t+2T) +$$

$$+ \frac{1}{2}\delta(t-T) + \delta(t) + \delta(t+T) +$$

$$+ \frac{1}{2}\delta(t-2T) + \delta(t-T) + \delta(t) =$$

$$= \frac{9}{4}\delta(t) + \frac{3}{2}\delta(t+T) + \frac{1}{2}\delta(t+2T) + \frac{3}{2}\delta(t-T) + \frac{1}{2}\delta(t-2T)$$



$$R_y(\tau) = R_x(\tau) \otimes C_h(\tau) =$$

DA SX A DX

$$= \frac{1}{2} R_x(\tau+2T) + \frac{3}{2} R_x(\tau+T) + \frac{9}{4} R_x(\tau) + \frac{3}{2} R_x(\tau-T) + \frac{1}{2} R_x(\tau-2T) =$$

$$= \sigma_x^2 \Big[\frac{1}{2}\operatorname{sinc}(2B(\tau+2T)) + \frac{3}{2}\operatorname{sinc}(2B(\tau+T)) + \frac{9}{4}\operatorname{sinc}(2B\tau) +$$
$$+ \frac{3}{2}\operatorname{sinc}(2B(\tau-T)) + \frac{1}{2}\operatorname{sinc}(2B(\tau-2T)) \Big]$$

$$S_y(f) = \frac{\sigma_x^2}{2B}\operatorname{rect}\left(\frac{f}{2B}\right)\left[\frac{1}{2}e^{j2\pi f 2T} + \frac{3}{2}e^{j2\pi fT} + \frac{9}{4} + \frac{3}{2}e^{-j2\pi fT} + \frac{1}{2}e^{-j2\pi f 2T}\right] =$$

$$= \boxed{\frac{\sigma_x^2}{2B}\operatorname{rect}\left(\frac{f}{2B}\right)\left[\frac{9}{4} + \cos(4\pi fT) + 3\cos(2\pi fT)\right]}$$

II STRADA

$$h(t) = \frac{1}{2}\delta(t) + \delta(t-T) + \delta(t-2T)$$

$$H(f) = \frac{1}{2} + e^{-j2\pi fT} + e^{-j2\pi f 2T}$$

$$|H(f)|^2 = H(f)H^*(f) = \left(\frac{1}{2} + e^{-j2\pi fT} + e^{-j4\pi fT}\right)\left(\frac{1}{2} + e^{j2\pi fT} + e^{j4\pi fT}\right) =$$

$$= \frac{1}{4} + \frac{1}{2}e^{j2\pi fT} + \frac{1}{2}e^{j4\pi fT} + \frac{1}{2}e^{-j2\pi fT} + 1 + e^{j2\pi fT} + \frac{1}{2}e^{-j4\pi fT} + e^{-j2\pi fT} + 1 =$$

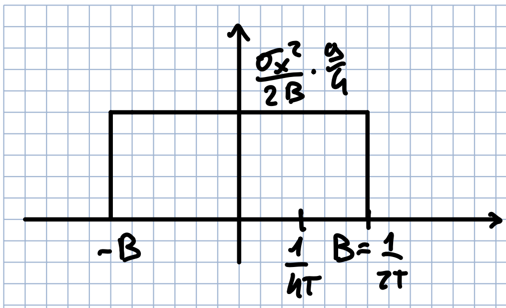
$$\frac{9}{4} + \frac{3}{2}e^{j2\pi fT} + \frac{1}{2}e^{j4\pi fT} + \frac{3}{2}e^{-j2\pi fT} + \frac{1}{2}e^{-j4\pi fT} = \frac{9}{4} + \cos(4\pi fT) + 3\cos(2\pi fT)$$

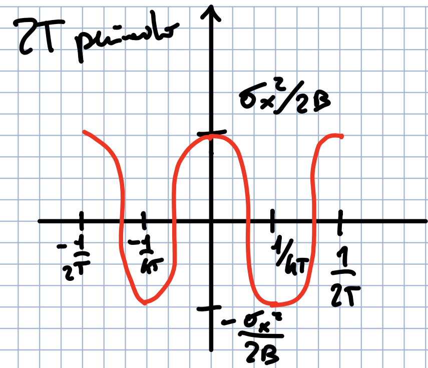
$$S_y(f) = S_x(f)\,|H(f)|^2 = \boxed{\frac{\sigma_x^2}{2B}\operatorname{rect}\left(\frac{f}{2B}\right)\left[\frac{9}{4} + \underset{\substack{\Downarrow \\ 2T}}{\cos(4\pi fT)} + 3\underset{\substack{\Downarrow \\ T}}{\cos(2\pi fT)}\right]}$$

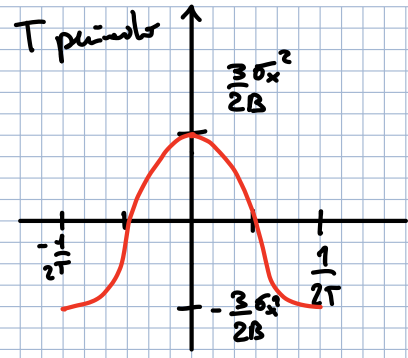
$$R_y(\tau) = \text{ATCF}\left[S_y(f)\right] =$$

MODULAZIONE COSENO

$$= \frac{9}{4}\sigma_x^2 \operatorname{sinc}(2B\tau) + \frac{1}{2}\overbrace{\left[\sigma_x^2 \operatorname{sinc}(2B(\tau-2T)) + \sigma_x^2 \operatorname{sinc}(2B(\tau+2T))\right]} +$$
$$+ \frac{3}{2}\left[\sigma_x^2 \operatorname{sinc}(2B(\tau-T)) + \sigma_x^2 \operatorname{sinc}(2B(\tau+T))\right]$$

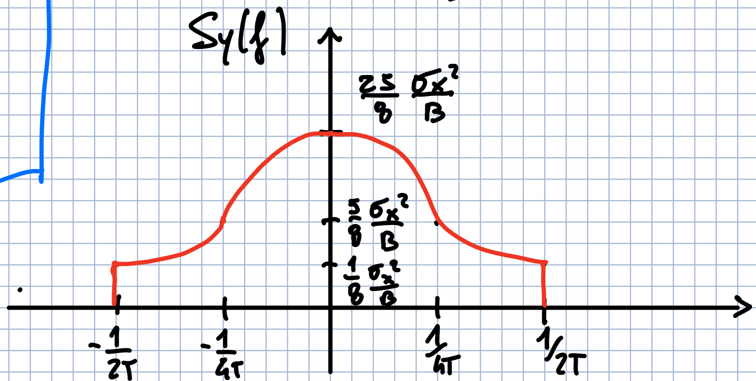






$$\boxed{\begin{aligned} S_Y(f) &= S_Y(-f) \\ S_Y(f) &\geq 0 \end{aligned}}$$

!!



ESERCIZIO - 13/01/20

$C_X(\tau)$ di un processo $X(t)$

$C_X(\tau) = A e^{-\alpha|\tau|} \cos(2\pi f_0 t)$

$f_X(x;t)$ è $\mathcal{U}[0,10]$ $= \frac{1}{b-a}\cdot \text{rect}\left(\frac{x-\frac{b+a}{2}}{b-a}\right) = \frac{1}{10}\,\text{rect}\left(\frac{x-5}{10}\right)$

$\Rightarrow S_X(f) = ?$

$S_X(f) = TCF[R_X(\tau)]$

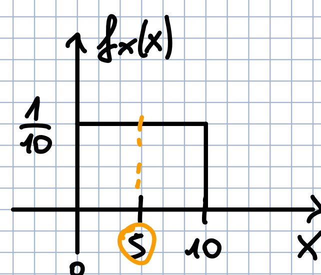
$R_X(\tau) = C_X(\tau) + \eta_X^2(\tau)$

$\eta_X(t) = \int_{-\infty}^{+\infty} x\, f_X(x;t)\, dx$

$f_X(x;t)$ non dipende da "t" poiché è distribuita tra 0 e 10 INDIPENDENT. DAL TEMPO

$\eta_X = \int_{-\infty}^{+\infty} x\, f_X(x)\, dx =$

$= \int_0^{10} x\, \frac{1}{10}\,\text{rect}\left(\frac{x-5}{10}\right) dx = \frac{1}{10}\int_0^{10} x\, dx = \frac{1}{10}\left.\frac{x^2}{2}\right|_0^{10} = 5$



$R_X(\tau) = A e^{-\alpha|\tau|} \cos(2\pi f_0 t) + 25$

$S_X(f) = TCF[A e^{-\alpha|\tau|} \cos(2\pi f_0 t)] + 25\,\delta(f) =$

$S_X'(f) = \frac{A}{2}[S_{X_0}(f-f_0) + S_{X_0}(f+f_0)]$

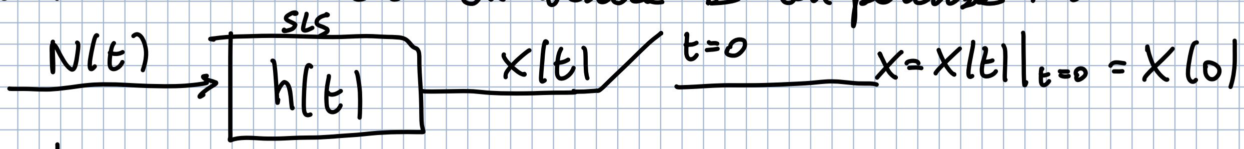
$$S_{x_0}(f) = TCF\left[e^{-\alpha|\tau|}\right] = \frac{1}{\alpha - j2\pi f}\cdot\frac{\alpha + j2\pi f}{\alpha + j2\pi f} + \frac{1}{\alpha + j2\pi f}\cdot\frac{\alpha - j2\pi f}{\alpha - j2\pi f} = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

$$S_x'(f) = \frac{A}{2}\left[\frac{2\alpha}{\alpha^2 + 4\pi^2(f-f_0)^2} + \frac{2\alpha}{\alpha^2 + 4\pi^2(f+f_0)^2}\right] = \frac{\alpha A}{\alpha^2 + 4\pi^2(f-f_0)^2} + \frac{\alpha A}{\alpha^2 + 4\pi^2(f+f_0)^2}$$

$$S_x(f) = \frac{\alpha A}{\alpha^2 + 4\pi^2(f-f_0)^2} + \frac{\alpha A}{\alpha^2 + 4\pi^2(f+f_0)^2} + 25\delta(f)$$

ESERCIZIO - 17/07/18

N(t) GAUSSIANO BIANCO di banda B con potenza $N_0 B$

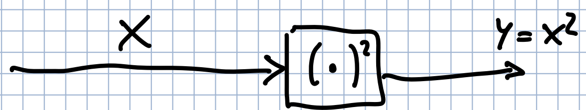


$$h(t) = e^{-2t}u(t)$$

1) Calcolare la DSP di $X(t)$ => $S_x(f)$ e POTENZA

2) Si campioni a $t=0$ e si scrive la ddp di X

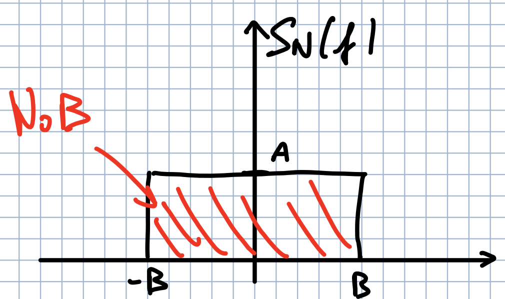
3)



Si calcoli la ddp di Y

SOLUZIONE

1) $S_x(f) = S_N(f)\,|H(f)|^2$



$R_N(\tau) = 2AB\,\mathrm{sinc}(2B\tau)$

$R_x(\tau) = R_N(\tau) \otimes h(\tau) \otimes h(-\tau)$

NON CONVIENE!!

$A = ?$ $\Rightarrow$ $P_N = \underline{N_0 B} = \int_{-\infty}^{+\infty} S_N(f)\,df = 2AB = N_0 B \Rightarrow A = \frac{N_0}{2}$

$$S_N(f) = \frac{N_0}{2}\,\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$H(f) = \frac{1}{2 + j2\pi f} \qquad |H(f)|^2 = H(f)H^*(f) = \frac{1}{2+j2\pi f}\cdot\frac{1}{2-j2\pi f} = \frac{1}{4 + 4\pi^2 f^2}$$

$$\boxed{S_X(f) = \frac{N_0}{2}\frac{1}{4+4\pi^2 f^2}\,\mathrm{rect}\left(\frac{f}{2B}\right)} \quad 1$$

$$P_X = \int_{-\infty}^{+\infty} S_X(f)\,dx = \frac{N_0}{2}\int_{-B}^{B}\frac{1}{4+4\pi^2 f^2}\,df = \frac{N_0}{4\pi}\,\mathrm{arctg}\left(\frac{B}{\pi}\right)$$

2) $x(t) \xrightarrow{t=0} X \to$ V.A. GAUSSIANA

$X(t)$ è GAUSSIANO $\qquad X \in \mathcal{N}(\eta_X, \sigma_X^2)$

$$\Downarrow$$

$\eta_X(t)=0 \Rightarrow \eta_X = 0$ poiché è un processo bianco

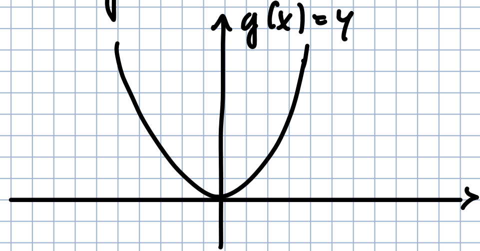
$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}}\, e^{-\frac{(x-\eta_X)^2}{2\sigma_X^2}}$$

$$\sigma_X^2 = P_X - \eta_X^2 = P_X$$

$$\boxed{f_X(x) = \frac{1}{\sqrt{2\pi P_X}}\, e^{-\frac{x^2}{2P_X}}}$$

3) $\xrightarrow{X} \boxed{(\cdot)^2} \to Y = X^2$

$$g(x) = x^2$$



$\boxed{Y<0} \Rightarrow f_Y(y) = 0$

$\boxed{Y=0} \Rightarrow g'(x) = 0 \Rightarrow$ AS. VERT.

$\boxed{Y>0} \Rightarrow$ 2 SOL. $\Rightarrow \pm\sqrt{Y}$

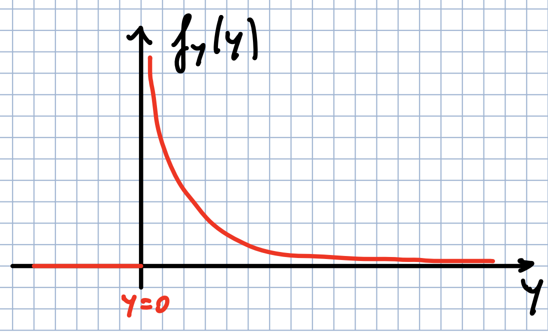
$$f_Y(y) = \frac{f_X(\sqrt{y})}{|g'(\sqrt{y})|} + \frac{f_X(-\sqrt{y})}{|g'(-\sqrt{y})|} =$$

$g'(x) = 2x$ → in $\sqrt{y} = 2\sqrt{y}$; → in $-\sqrt{y} = -2\sqrt{y}$

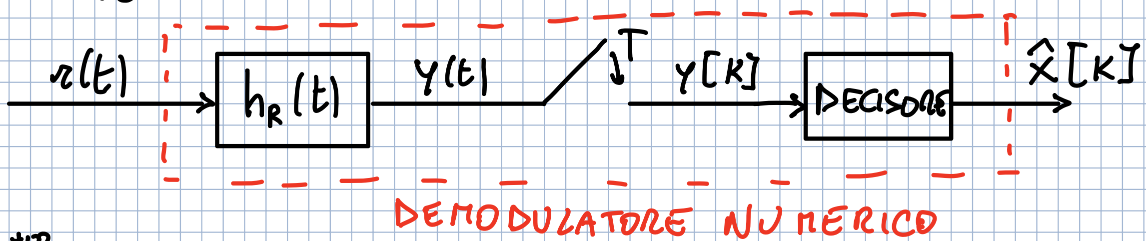
$$= \frac{\frac{1}{\sqrt{2\pi P_X}}e^{-\frac{y}{2P_X}}}{2\sqrt{y}} + \frac{\frac{1}{\sqrt{2\pi P_X}}e^{-\frac{y}{2P_X}}}{2\sqrt{y}} = \frac{\cancel{2}\,a}{\cancel{2}\sqrt{y}} = \boxed{\frac{1}{\sqrt{2\pi P_X\, y}}\, e^{-\frac{y}{2P_X}}} \qquad y>0$$

$$\lim_{y\to 0^+} f_Y(y) = +\infty$$

in $y=0$
HO UN
AS. VERT.



ESERCIZIO - 6/02/17 #2



$$r(t) = \sum_{i=-\infty}^{+\infty} x[i]\, p(t - iT) + w(t)$$

$x[i]$ sono i simboli trasmessi INDIPENDENTI e EQUIPROBABILI

$A_s = \{-1, 2\}$

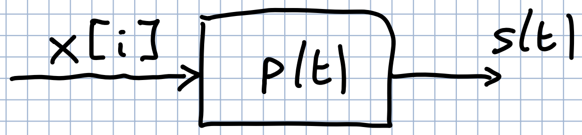
$w(t)$ rumore Gaussiano bianco con DSP $S_w(f) = \frac{N_0}{2}$

$p(t) = \frac{2}{T}\,\text{sinc}^2\left(\frac{2t}{T}\right)\cos\left(\frac{4\pi t}{T}\right)$

$h_R(t) = \frac{4}{T}\,\text{sinc}\left(\frac{4t}{T}\right)$

1) Calcolare l'energia media per simbolo trasmesso
2) DSP di $s(t)$
3) Potenza del rumore in uscita al filtro di ricezione $h_R(t)$

SOLUZIONE



$E[x[n]] = \sum_{i=1}^{M} \alpha_i P\{\alpha_i\}$

$T_s = T$

1) $E_s = E[x^2[i]]\,E_p = E\left[\int_{-\infty}^{+\infty} x^2[i]\,p^2(t-iT)\,dt\right]$

$E[x^2] = P\{\alpha_1\}\cdot\alpha_1^2 + P\{\alpha_2\}\,\alpha_2^2 = \frac{1}{2}(-1)^2 + \frac{1}{2}(+2)^2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$ VAL. QUADR. MEDIO DEI SIMBOLI

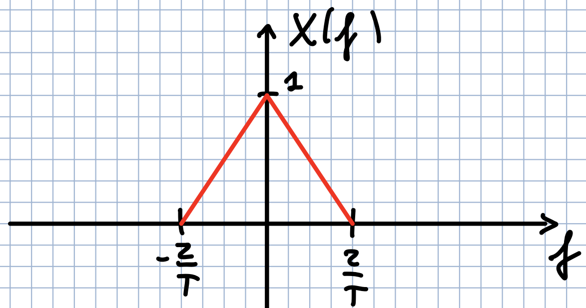
$E_p = \int_{-\infty}^{+\infty} p^2(t)\,dt = \int_{-\infty}^{+\infty} |P(f)|^2\,df$

$p(t) = \underbrace{\frac{2}{T}\,\text{sinc}^2\left(\frac{2t}{T}\right)}_{x(t)}\cos\left(\frac{4\pi t}{T}\right) = x(t)\cos(2\pi f_0 t)$, $\boxed{f_0 = \frac{2}{T}}$

$P(f) = TCF[p(t)] = \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$ TH. MOD. COSENO

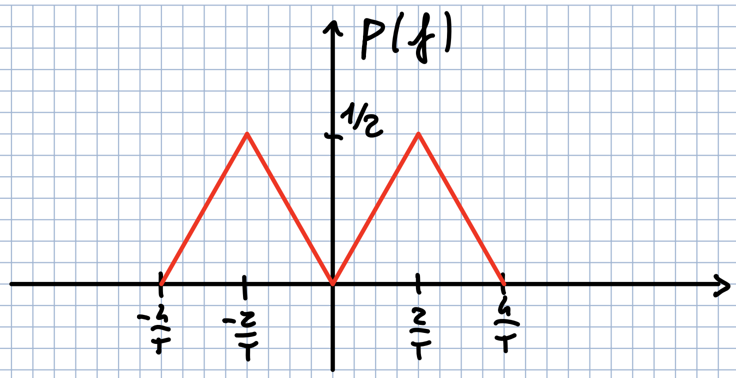
$X(f) = TCF[x(t)] = TCF\left[\frac{2}{T}\,\text{sinc}^2\left(\frac{2t}{T}\right)\right] = TCF\left[\frac{2}{T}\,\text{sinc}^2\left(\frac{t}{T/2}\right)\right] =$

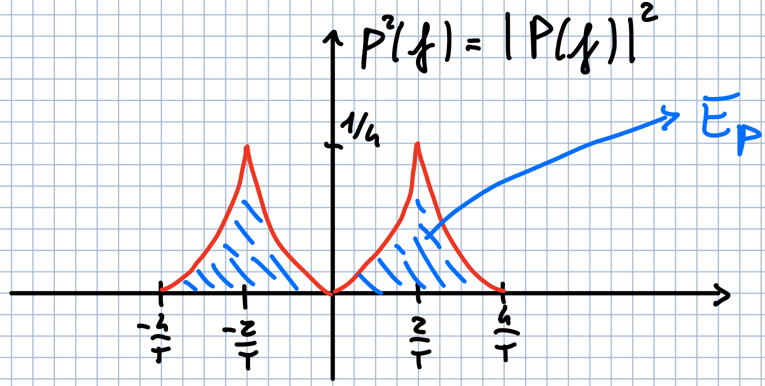
$= \frac{2}{T}\cdot\frac{T}{2}\left(1 - \frac{|f|}{2/T}\right)\text{rect}\left(\frac{f}{\frac{4}{T}}\right) = \left(1 - \frac{|f|}{2/T}\right)\text{rect}\left(\frac{f}{4/T}\right)$



$f_0 = \frac{2}{T}$

$P(f) = \frac{1}{2}\left(1 - \frac{|f-\frac{2}{T}|}{2/T}\right)\text{rect}\left(\frac{f-\frac{2}{T}}{4/T}\right) + \frac{1}{2}\left(1 - \frac{|f+\frac{2}{T}|}{2/T}\right)\text{rect}\left(\frac{f+\frac{2}{T}}{4/T}\right)$





$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 = 4 \cdot \frac{1}{3}\left(\frac{2}{T} \cdot \frac{1}{4}\right) = \frac{2}{3T}$$

$$\boxed{E_s = \frac{5}{2} \cdot \frac{2}{3T} = \frac{5}{3T}} \quad 1$$

2) DSP di $s(t)$

$$S_s(f) = \frac{1}{T_s} \bar{S}_x(f) |P(f)|^2$$

$$S_x(f) = TFS[R_x[m]]$$

$$R_x[m] = C_x[m] + \eta_x^2$$

$$\eta_x = E[x] = \frac{1}{2}(-1) + \frac{1}{2}(2) = -\frac{1}{2} + \frac{2}{2} = \frac{1}{2}$$

la formula semplificata

$$S_s(f) = \frac{\sigma_x^2}{T_s} |P(f)|^2$$

NON PUO' ESSERE USATA POICHE' I SIMBOLI NON SONO SIMMETRICI

$\Rightarrow$ NON SI TRATTA DI PAM STANDARD

$$\alpha_i = (2i - 1 - M)$$

$$\eta_x^2 = \frac{1}{4}$$

$C_x[m]$ AUTOCOVARIANZA

$$C_x[m] = E[(x[n] - \eta_x)(x[n-m] - \eta_x)] = E[(X - \eta_x)(Y - \eta_x)]$$

$\Rightarrow$ I SIMBOLI SONO INDIPENDENTI $\Rightarrow$ INCORRELATI $\Rightarrow C_x = 0$

2 V.A. INDIPENDENTI SONO INCORRELATE

$$\Downarrow$$

$$C_x[m]=0$$

$\Rightarrow$ per tutti i valori di $m \neq 0 \Rightarrow C_x[m]=0$

$\Rightarrow$ per $m=0 \Rightarrow C_x[0]=E\left[(x[n]-\eta_x)^2\right]=\sigma_x^2$

$\Rightarrow C_x[m]=\sigma_x^2\delta[m]$ $\qquad \delta[m]=\begin{cases}1 & m=0\\ 0 & m\neq 0\end{cases}$

$$\Rightarrow R_x[m]=\sigma_x^2\delta[m]+\eta_x^2=\begin{cases}\sigma_x^2+\eta_x^2=E[x^2] & m=0\\ \eta_x^2 & m\neq 0\end{cases}$$

$$\sigma_x^2=E[x^2]-\eta_x^2=\frac{5}{2}-\frac{1}{4}=\frac{9}{4}$$

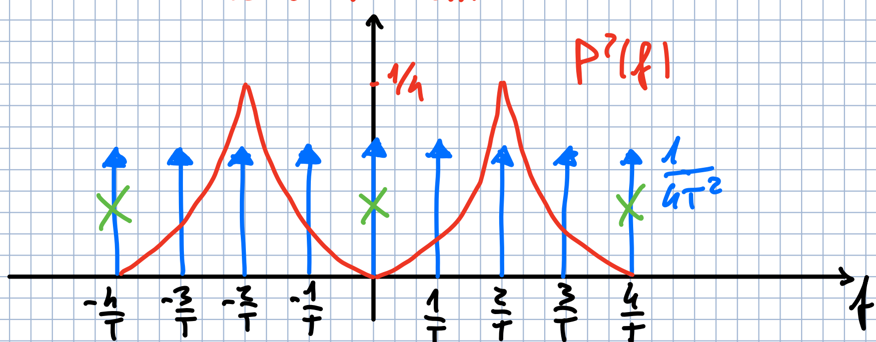
$$\Rightarrow R_x[m]=\frac{9}{4}\delta[m]+\frac{1}{4}$$

$$\Rightarrow \bar{S}_x(f)=TFS\left[R_x[m]\right]=\sum_{m=-\infty}^{+\infty}R_x[m]e^{-j2\pi fmT}=$$

$$=\sum_m \frac{9}{4}\delta[m]e^{-j2\pi fmT}+\sum_m\frac{1}{4}e^{-j2\pi fmT}=\frac{9}{4}+\frac{1}{4}\sum_m e^{-j2\pi fmT}=$$

$$=\frac{9}{4}+\frac{1}{4}\cdot\frac{1}{T}\sum_{m=-\infty}^{+\infty}\delta\left(f-\frac{m}{T}\right)$$

$$S_s(f)=\frac{1}{T}\Bigg[\frac{9}{4}+\frac{1}{4}\cdot\underbrace{\frac{1}{T}\sum_{m=-\infty}^{+\infty}\delta\left(f-\frac{m}{T}\right)}_{\text{TRENO di DELTA}}\Bigg]|P(f)|^2$$



$$\boxed{\begin{aligned}S_s(f)&=\frac{9}{4T}|P(f)|^2+\frac{1}{4T^2}\sum_m\left|P\left(\frac{m}{T}\right)\right|^2\delta\left(f-\frac{m}{T}\right)\\&=\frac{9}{4T}|P(f)|^2+\frac{1}{4T^2}\sum_{m\in M}P^2\left(\frac{m}{T}\right)\delta\left(f-\frac{m}{T}\right)\end{aligned}}$$

2

$$M=\{\pm 3,\pm 2,\pm 1\}$$

$$P^2(f) = \frac{1}{4}\left(1 - \frac{|f - \frac{2}{T}|}{2/T}\right)^2 \mathrm{rect}\left(\frac{f - \frac{2}{T}}{4/T}\right) + \frac{1}{4}\left(1 - \frac{|f + \frac{2}{T}|}{2/T}\right)^2 \mathrm{rect}\left(\frac{f + \frac{2}{T}}{4/T}\right)$$

3) Potenza del rumore in uscita al filtro di ricezione $h_R(t)$

$$P_{w_u} = \int_{-\infty}^{+\infty} S_{w_u}(f)\,df = \int_{-\infty}^{+\infty} S_w(f)|H_R(f)|^2 df = R_{w_u}(\tau)\Big|_{\tau=0} = R_w(0)$$

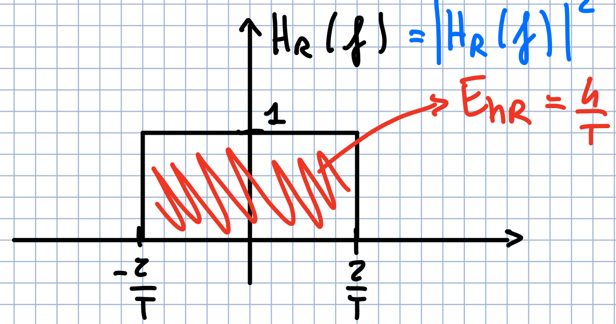
$$\boxed{P_Y = \int_{-\infty}^{+\infty} S_Y(f)\,df = \int_{-\infty}^{+\infty} S_x(f)|H(f)|^2 df}$$

POT. MEDIA PROCESSO AL. USCITA

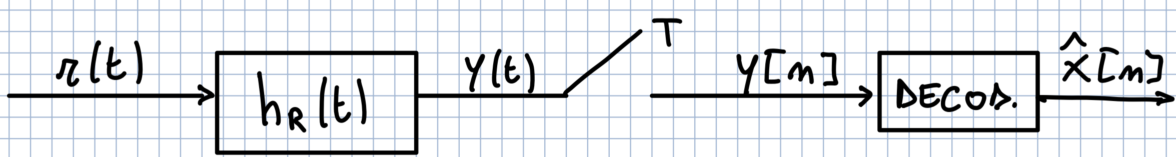
$S_w(f) = \frac{N_0}{2}$ RUMORE BIANCO

$$P_{w_u} = \frac{N_0}{2}\int_{-\infty}^{+\infty}|H_R(f)|^2 df = \frac{N_0}{2} E_{h_R} = \frac{N_0}{2}\cdot\frac{4}{T} = \boxed{\frac{2N_0}{T}}$$ 3

$$H_R(f) = \frac{4}{T}\,\frac{T}{4}\,\mathrm{rect}\left(\frac{f}{4/T}\right) = \mathrm{rect}\left(\frac{f}{4/T}\right)$$



ESERCIZIO - 06/06/17



$r(t) = A_0 \sum_k x[k]\, p(t - kT)$

$x[k] \in A_s\{-1, 1\}$ EQUIPROB. E INDIP.

$w(t)$ è GAUSSIANO BIANCO $S_w(f) = \frac{N_0}{2}$

$c(t) = \delta(t)$ canale senza distorsioni

$p(t) = \frac{4}{T}\,\mathrm{sinc}\left(\frac{4t}{T}\right)$

$H_R(f) = \left(1 - |fT|\right)\mathrm{rect}\left(\frac{fT}{2}\right) + \mathrm{rect}\left(\frac{fT}{4}\right)$

1) E_s 2) $S_s(f)$ 3) P_{w_u}

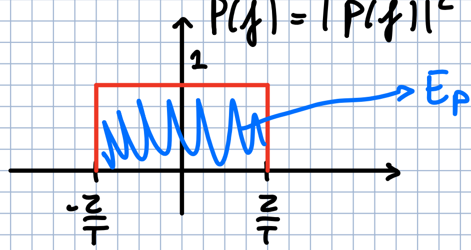
SOLUZIONE

1) $E_s = E[x^2]\, E_p$

$$E[x^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 \, df = \frac{4}{T}$$

$$P(f) = rect\left(\frac{f}{4/T}\right)$$



$$E_s = 1 \cdot \frac{4}{T} = \boxed{\frac{4}{T}} \quad 1$$

2) $S_s(f) = \frac{\sigma_x^2}{T} |P(f)|^2 =$

·) SIMBOLI SIMMETRICI, INDIP. E EQUIPROB. (± 1)

$$\sigma_x^2 = E[x^2] - \eta_x^2 = 1$$

$$\eta_x = \frac{1}{2}(-1) + \frac{1}{2}(1) = 0$$

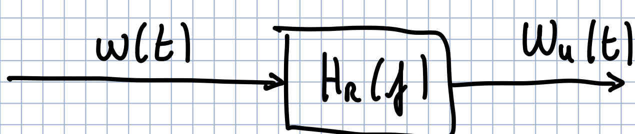
$$\boxed{S_s(f) = \frac{1}{T} rect\left(\frac{f}{4/T}\right)} \quad 2$$

$$S_s(f) = \frac{1}{T} \bar{S}_x(f) |P(f)|^2$$

$$R_x[m] = C_x[m] + \underbrace{\eta_x^2}_{=0} = C_x[m] = \sigma_x^2 \delta[m]$$

$$\bar{S}_x(f) = TFS[R_x[m]] = \sigma_x^2$$

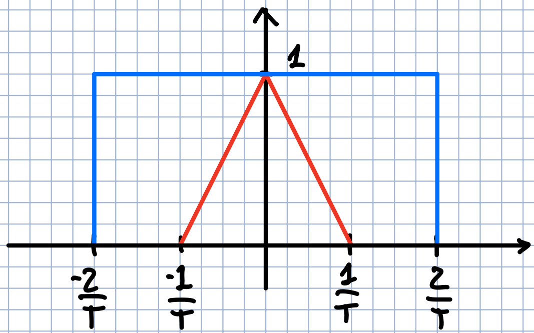
3) P_{w_u}



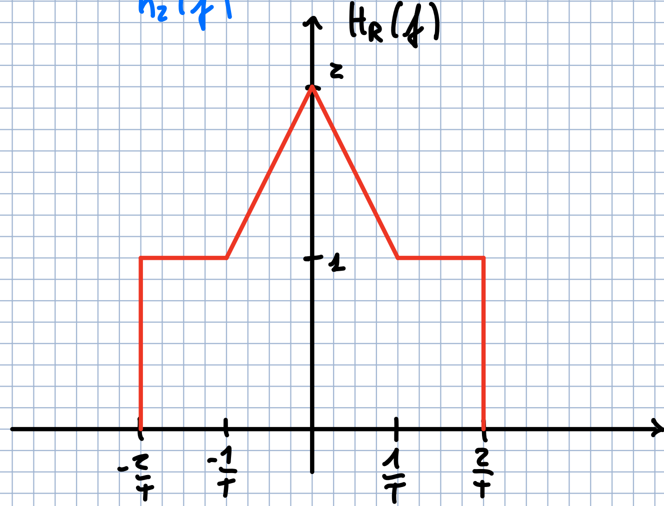
$$S_{w_u}(f) = S_w(f) |H_R(f)|^2 = \frac{N_0}{2} |H_R(f)|^2$$

$$P_{w_u} = \int_{-\infty}^{+\infty} S_w(f) |H_R(f)|^2 df = \frac{N_0}{2} E_{h_R}$$

$$H_R(f) = \underbrace{\left(1 - \frac{|f|}{1/T}\right) \text{rect}\left(\frac{f}{2/T}\right)}_{h_1(f)} + \underbrace{\text{rect}\left(\frac{f}{4/T}\right)}_{h_2(f)}$$

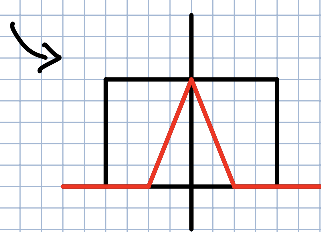


$\Rightarrow$



$$H_R(f) = h_1(f) + h_2(f)$$

$$|H_R(f)|^2 = h_1(f)^2 + h_2(f)^2 + \underbrace{2 h_1(f) h_2(f)}$$



$$h_1(f)^2 = \frac{4}{T}$$

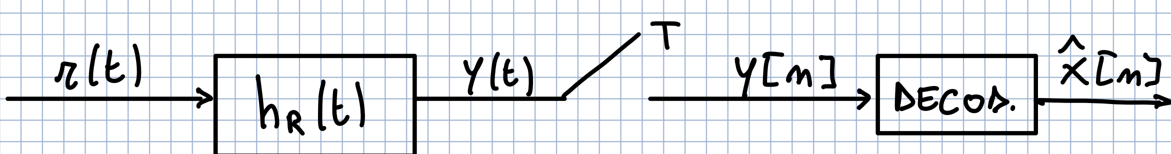
$$h_2(f)^2 = 2\left(\frac{1}{3} \cdot \left(\frac{1}{T}\right)\right) = \frac{2}{3T}$$

$$2 h_1(f) h_2(f) = \cancel{2} \cdot \frac{2}{T} \cdot 1 \cdot \frac{1}{\cancel{2}} = \frac{2}{T}$$

$$|H_R(f)|^2 = \frac{4}{T} + \frac{2}{3T} + \frac{2}{T} = \frac{20}{3T} = E_{h_R}$$

$$P_{w_u} = \frac{N_0}{2} E_{h_R} = \frac{N_0}{2} \cdot \frac{20}{3T} = \boxed{\frac{10 N_0}{3T}}$$ 3

ESERCIZIO – 18/07/17



$x[i]$ indipendenti equiprob. $\in A_s = \{-3, 2\}$

$w(t)$ GAUSSIANO BIANCO con DSP $\frac{N_0}{2}$

$$p(t) = \frac{2}{T} \text{sinc}\left(\frac{2t}{T}\right)$$

$$H_R(f) = \frac{T}{2}\left(1 + \cos(\pi f T)\right) \text{rect}\left(\frac{fT}{2}\right)$$

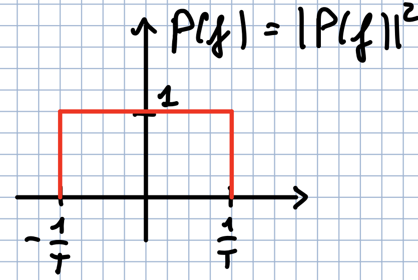
1) E_s
2) $S_s(f)$
3) P_{w_u}

SOLUZIONE

1) $E_s = E[x^2]\, E_p$

$$E[x^2] = P\{\alpha_1\} \cdot \alpha_1^2 + P\{\alpha_2\}\, \alpha_2^2 = \frac{1}{2}(-3)^2 + \frac{1}{2}(+2)^2 = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\, df = \frac{2}{T}$$



$$P(f) = \frac{2}{T} \cdot \frac{T}{2}\, \mathrm{rect}\left(\frac{f}{2/T}\right) = \mathrm{rect}\left(\frac{f}{2/T}\right)$$

$$E_s = \frac{13}{2} \cdot \frac{2}{T} = \boxed{\frac{13}{T}}$$

2) $S_s(f) = \frac{1}{T} \bar{S}_x(f)\, |P(f)|^2$

$$\bar{S}_x(f) = TFS[R_x[m]]$$

$$R_x[m] = C_x[m] + \eta_x^2$$

$$\eta_x = E[x] = \frac{1}{2}(-3) + \frac{1}{2}(2) = -\frac{3}{2} + \frac{2}{2} = -\frac{1}{2} \qquad \eta_x^2 = \frac{1}{4}$$

$$C_x[m] = E[(x[n] - \eta_x)(x[n-m] - \eta_x)]$$

ESSENDO I SIMBOLI INDIP. $\rightarrow$ SONO ANCHE INCORRELATI

$$C_x[m] = 0 \quad \text{per } m \neq 0$$

per $m = 0$ $\quad C_x[0] = E[(x - \eta_x)^2] = \sigma_x^2$

$$\boxed{\sigma_x^2 = E[x^2] - \eta_x^2}$$

$$C_x[m] = \sigma_x^2\, \delta[m]$$

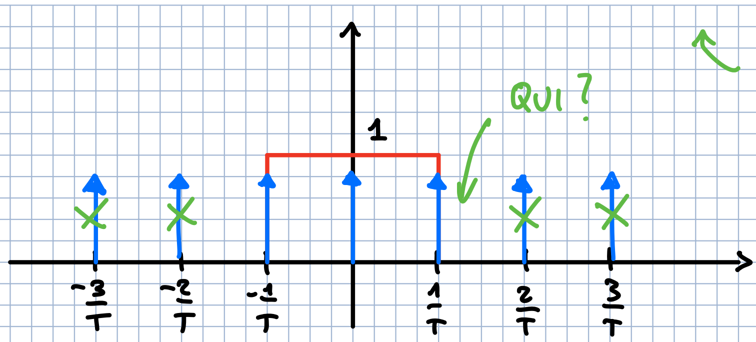
$$R_x[m] = \sigma_x^2\, \delta[m] + \eta_x^2 = \begin{cases} \sigma_x^2 + \eta_x^2 = E[x^2] & m = 0 \\ \eta_x^2 & m \neq 0 \end{cases}$$

$$\sigma_x^2 = E[x^2] - \eta_x^2 = \frac{13}{2} - \frac{1}{4} = \frac{25}{4}$$

$$R_x[m] = \frac{25}{4}\, \delta[m] + \frac{1}{4}$$

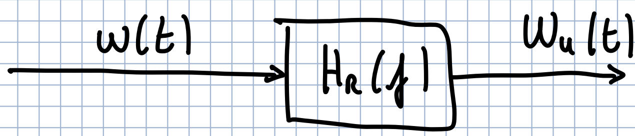
$$\bar{S}_x(f) = TFS[R_x[m]] = \frac{25}{4} + \frac{1}{4} \cdot \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta\left(f - \frac{m}{T}\right)$$

$$S_s(f) = \frac{1}{T}\left[\frac{25}{4} + \frac{1}{4} \cdot \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta\left(f - \frac{m}{T}\right)\right] |P(f)|^2$$



COME LA SCRIVO?
È SBAGLIATO?

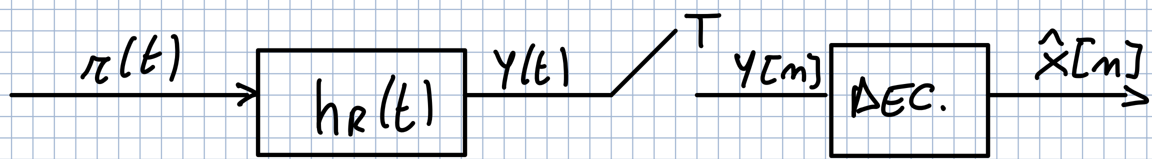
3) P_{w_u}



$$S_{w_u}(f) = S_w(f)\,|H_R(f)|^2 = \frac{N_0}{2}\,|H_R(f)|^2$$

$$P_{w_u} = \int_{-\infty}^{+\infty} S_w(f)\,|H_R(f)|^2\,df = \frac{N_0}{2}\int_{-\infty}^{+\infty} |H_R(f)|^2\,df = \frac{N_0}{2} E_{h_R}$$

ESERCIZIO 6/02/17



$A_s = \{-1, 2\}$ equiprob.

$w(t)$ GAUSSIANO BIANCO con DSP $\frac{N_0}{2}$

$p(t) = \frac{2}{T} \operatorname{sinc}^2\left(\frac{2t}{T}\right) \cos\left(\frac{4\pi t}{T}\right)$

$h_R(t) = \frac{4}{T} \operatorname{sinc}\left(\frac{4t}{T}\right)$

$c(t) = \delta(t)$

1) Verificare l'assenza di ISI

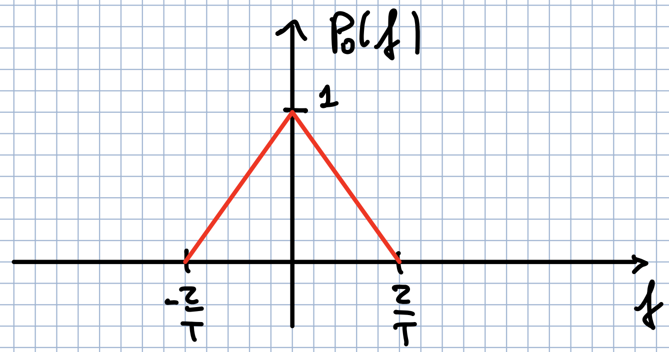
$$h(t) \;\circ\!\!-\!\!\bullet\; H(f)$$

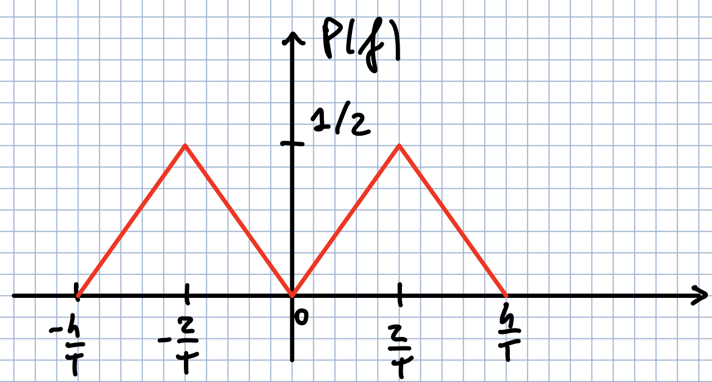
$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f)$$

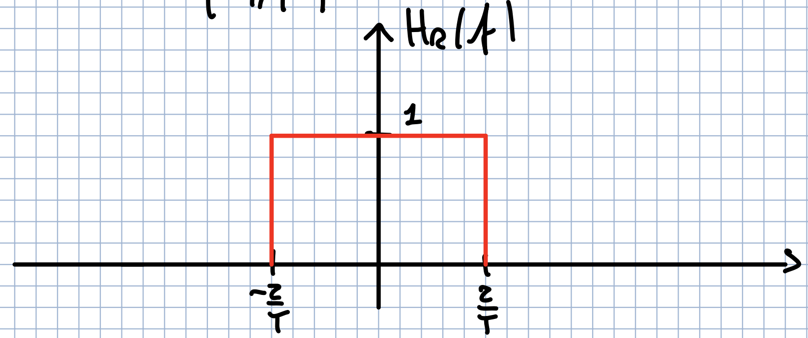
$$P_0(f) = \left(1 - \frac{|f|}{2/T}\right) \mathrm{rect}\left(\frac{f}{4/T}\right) \qquad P(f) = \frac{1}{2} P_0(f - f_0) + \frac{1}{2} P_0(f + f_0)$$

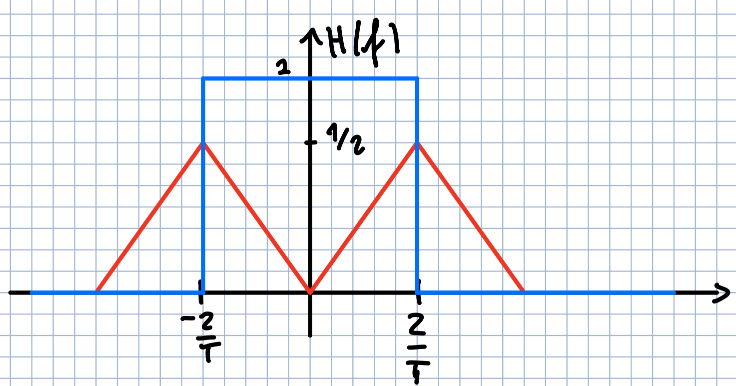
$$f_0 = \frac{2}{T}$$

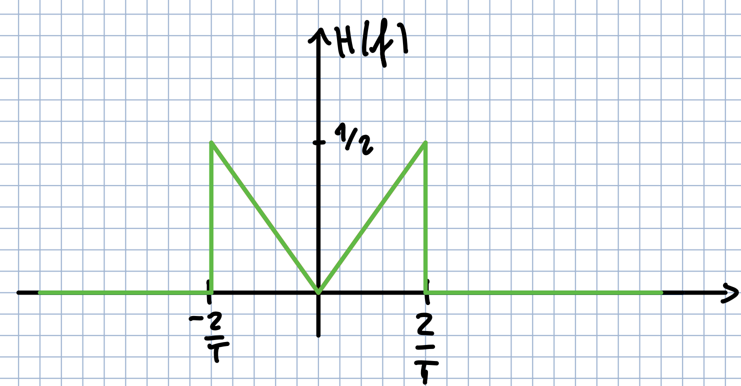




$$H_R(f) = \mathrm{rect}\left(\frac{f}{4/T}\right)$$





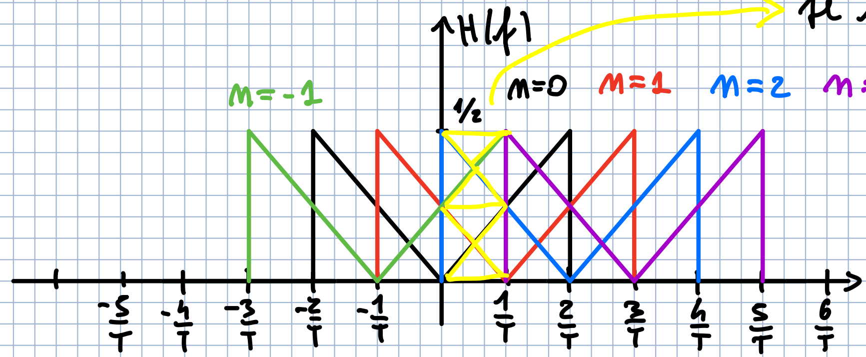


$$H(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{4/T}\right) - \frac{1}{2}\left(1 - \frac{|f|}{2/T}\right) \mathrm{rect}\left(\frac{f}{4/T}\right)$$

$$h(t) = \frac{1}{2}\cdot\frac{4}{T}\,\mathrm{sinc}\left(\frac{4}{T}t\right) - \frac{1}{2}\cdot\frac{2}{T}\,\mathrm{sinc}^2\left(\frac{2}{T}t\right) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{4}{T}t\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{2}{T}t\right)$$

$$h(t)\Big|_{t=nT} = \frac{2}{T}\,\mathrm{sinc}\left(\frac{4}{T}nT\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{2}{T}nT\right) = \frac{2}{T}\,\mathrm{sinc}(4n) - \frac{1}{T}\,\mathrm{sinc}^2(2n)$$

LA SINC SI ANNULLA QUANDO L'ARGOMENTO È INTERO tranne per $n=0$ $\rightarrow$ $= \frac{2}{T}\delta[n] - \frac{1}{T}\delta[n] = \frac{1}{T}\delta[n]$ $\Rightarrow$ è verificata la cond. di Nyquist nel tempo



se è costante su un periodo $\left(\frac{1}{T}\right)$ è costante ovunque

PRESO UN INTERVALLO A PIACERE

Devo dimostrare che $\sum_{m=-\infty}^{+\infty} H\left(f - \frac{m}{T}\right) = \text{COST}$ $\rightarrow$ è periodica di $\frac{1}{T}$

se aggiungo $\frac{1}{T}$ $\rightarrow$ $\frac{1}{T}\sum_{m=-\infty}^{+\infty} H\left(f - \frac{m}{T}\right) = \frac{1}{T}\cdot\frac{1}{2} = \frac{1}{2T} = \text{COST.}$

$\Rightarrow$ ISI È ASSENTE

5) $P_E(b) = P\{\hat{b} = \alpha_1 \mid b = \alpha_2\} + P\{\hat{b} = \alpha_2, b = \alpha_1\} =$

$= P\{\hat{b} = \alpha_1 \mid b = \alpha_2\} P\{\alpha_2\} + P\{\hat{b} = \alpha_2, b = \alpha_1\} P\{\alpha_1\} =$

$$P_E(b) = P\{\alpha_1\}\, Q\left(\frac{\lambda - h(0)\alpha_1}{\sigma_{nu}}\right) + P\{\alpha_2\}\, Q\left(\frac{h(0)\alpha_2 - \lambda}{\sigma_{nu}}\right)$$

$P_{nu} = \frac{N_0}{2} E_{hR} = \frac{N_0}{2}\cdot\frac{4}{T} = \frac{2N_0}{T}$ $\quad \sigma_{nu} = \sqrt{\frac{2N_0}{T}}$ $\quad E_{hR} = \frac{4}{T}$

$$P_E(b) = \frac{1}{2}\cdot Q\left(\frac{0 - \frac{1}{T}\cdot -1}{\sqrt{\frac{2N_0}{T}}}\right) + \frac{1}{2}\, Q\left(\frac{\frac{1}{T}\cdot 2 - 0}{\sqrt{\frac{2N_0}{T}}}\right) = \frac{1}{2}\, Q\left(\frac{1/T}{\sqrt{\frac{2N_0}{T}}}\right) + \frac{1}{2}\, Q\left(\frac{2/T}{\sqrt{\frac{2N_0}{T}}}\right)$$

ESERCIZIO - 20/09/18

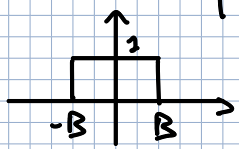
$$r(t) = \sum_i x[i]\, p(t - iT) + w(t)$$

$x[i]$ ind. e equipr. $\in A_S = [-2, 3]$

$w(t)$ Gaussiano a $\eta_x = 0$ e $S_w(f) = \frac{N_0}{2}$ RUMORE BIANCO

$$p(t) = 2B\,\mathrm{sinc}(2Bt) + B\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(\pi B t)$$

$$H_R(f) = \mathrm{rect}\left(\frac{f}{2B}\right) \rightarrow$$



$$\boxed{B = \frac{2}{T}}$$

$$\hat{x} = \begin{cases} -2 & y \leq \lambda \\ 3 & y > \lambda \end{cases} \qquad \lambda = 0$$

Calcolare:

1) E_S
2) $S_S(f)$
3) P_{nu}
4) Assenza di ISI
5) $P_E(b)$

SOLUZIONE

1) $E_S = E_P\, E[x^2]$

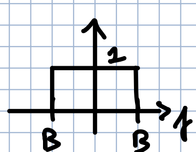
$$E_P \int_{-\infty}^{+\infty} p(t)\, dt = \int_{-\infty}^{+\infty} |P(f)|^2 df$$

$$p(t) = \underbrace{2B\,\mathrm{sinc}(2Bt)}_{p_1(t)} + \underbrace{B\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(\pi B t)}_{p_2(t)}$$

$$2\pi f_0 t = \pi B t$$

$$f_0 = \frac{\cancel{\pi} B \cancel{t}}{2 \cancel{\pi} \cancel{t}} = \frac{B}{2}$$

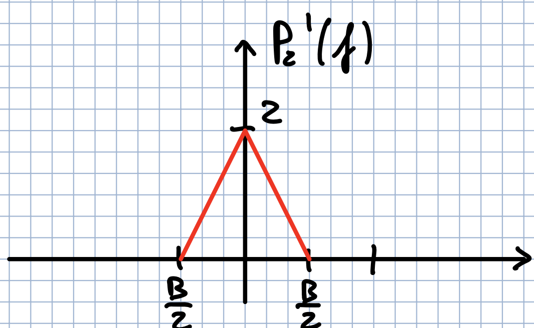
$$\underbrace{P_1(f)} = \mathrm{rect}\left(\frac{f}{2B}\right) \rightarrow$$

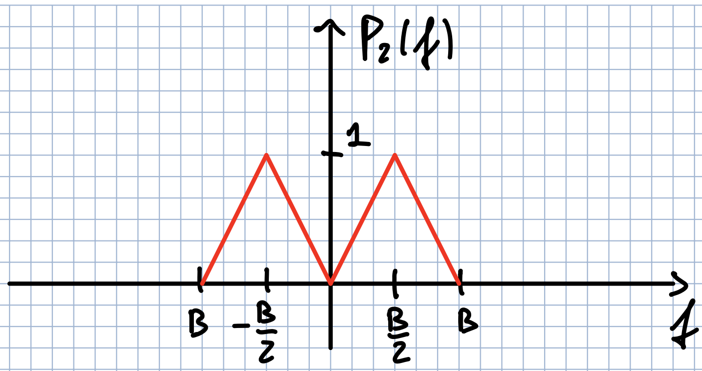


$$P_2'(f) = B \cdot \frac{2}{B}\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) =$$

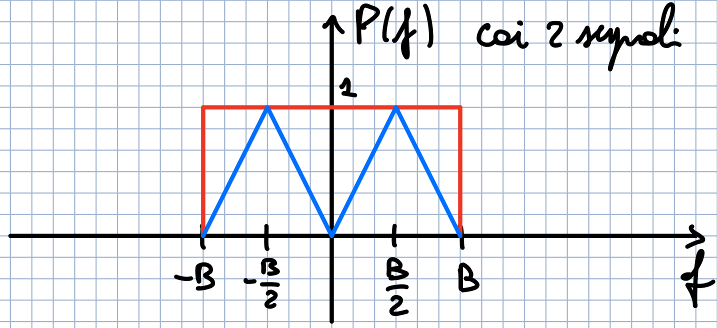
$$= 2\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$

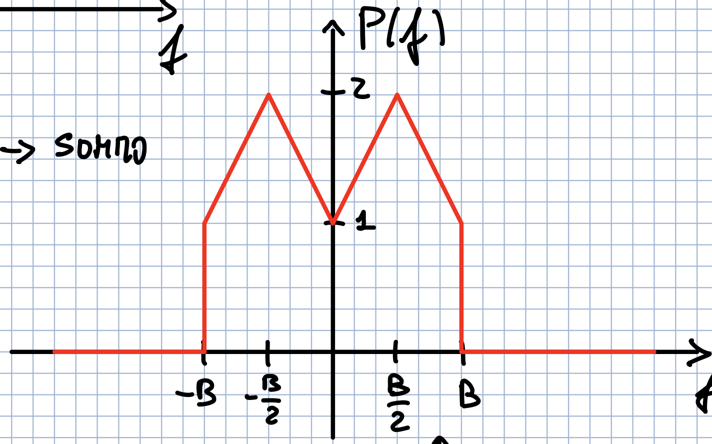
$$\underbrace{P_2(f)} = \frac{1}{2}P_2'(f - f_0) + \frac{1}{2}(f + f_0)$$





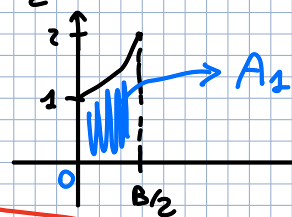
$P(f)$ coi 2 simboli $\rightarrow$ SOMMO





$$P(f) = m\, f + b \qquad m = \frac{1}{\frac{B}{2}} = \frac{2}{B}$$

$$\Downarrow$$



$$x(f) = a f + b \qquad \text{per } 0 \leq t \leq \frac{B}{2}$$

PASSA NEI PUNTI

$$(\overset{f}{0}, 1) \Rightarrow a \cdot 0 + b = 1 \Rightarrow b = 1$$

$$\left(\frac{B}{2}, 2\right) \Rightarrow a \cdot \frac{B}{2} + b = 2 \Rightarrow \frac{B}{2} a = 1 \quad a = \frac{2}{B}$$

$$\boxed{\text{retta} = \frac{2}{B} f + 1}$$

AL QUADRATO

$$= \left(\frac{4}{B^2} f^2 + 1 + \frac{4}{B} f\right)$$

$$A_1 = \int_0^{B/2} \frac{4}{B^2} f^2\, df + \int_0^{B/2} 1\, df + 2\int_0^{B/2} \frac{2}{B} f\, df = \frac{4}{B^2} \frac{f^3}{3}\Bigg|_0^{B/2} + f\Bigg|_0^{B/2} + \frac{4}{B} \frac{f^2}{2}\Bigg|_0^{B/2} =$$

$$A_1 = \frac{4}{B^2} \cdot \left(\frac{\frac{B^3}{8}}{3}\right) + \frac{B}{2} + \frac{4}{B}\left(\frac{\frac{B^2}{4}}{2}\right) = \frac{\cancel{4}^{1}}{\cancel{B^2}} \cdot \frac{B^{\cancel{3}}}{\cancel{24}_6} + \frac{B}{2} + \frac{\cancel{4}}{\cancel{B}} \cdot \frac{B^{\cancel{2}}}{\cancel{8}_2} = \frac{B}{6} + \frac{B}{2} + \frac{B}{2} = \frac{7}{6} B$$

$$4 \cdot A_1 = \frac{7}{\cancel{6}_3} B \cdot \cancel{4}^{2} = \boxed{\frac{14}{3} B}$$

$$E[x^2] = \frac{1}{2}(-2)^2 + \frac{1}{2}(3)^2 = 2 + \frac{9}{2} = \frac{13}{2}$$

$$E_s = \frac{14}{3} B \cdot \frac{13}{2} = \boxed{\frac{91}{3} B} \quad 1$$

2) $S_s(f) = \frac{1}{T} \bar{S}_x(f) |P(f)|^2 = \frac{\bar{S}_x(f)}{T} \cdot P^2(f)$

$\bar{S}_x(f) = TFS[R_x[m]]$

$R_x[m] = C_x[m] + \eta_x^2$

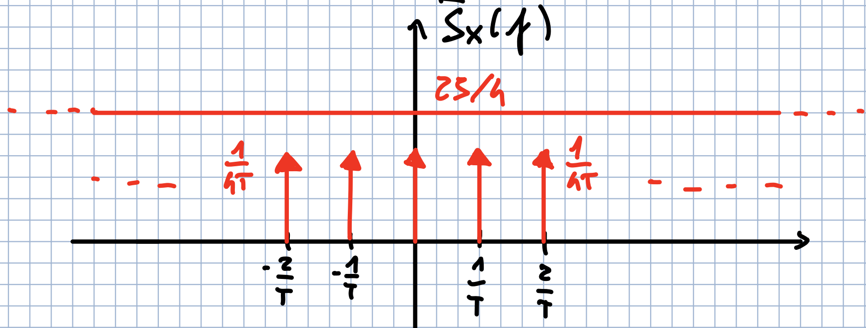
$\eta_x = E[x] = \frac{1}{2} \cdot (-2) + \frac{1}{2} \cdot (3) = -\frac{2}{2} + \frac{3}{2} = \frac{1}{2}$ $\quad \eta_x^2 = \frac{1}{4}$

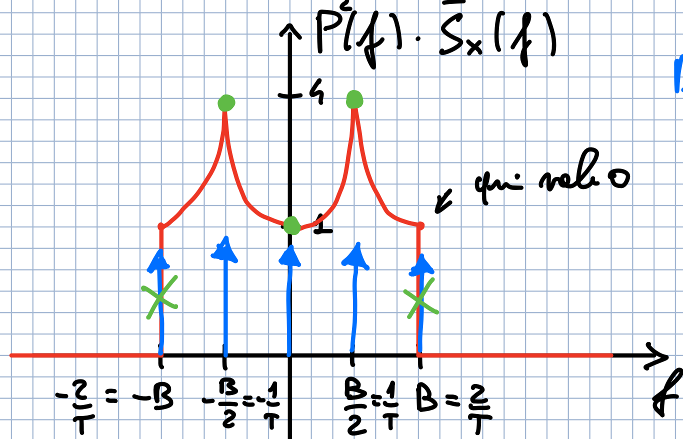
$C_x[m] = \sigma_x^2 \delta[m]$ (i simboli sono indipendenti → INCORRELATI)

$\sigma_x^2 = E[x^2] - \eta_x^2 = \frac{13}{2} - \frac{1}{4} = \frac{25}{4}$

$R_x[m] = \frac{25}{4} \delta[m] + \frac{1}{4}$

$\bar{S}_x(f) = \frac{25}{4} + \frac{1}{4} \cdot \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta\left(f - \frac{m}{T}\right)$





$B = \frac{2}{T}$

2 $\boxed{S_s(f)} = \frac{1}{T} \cdot \frac{25}{4} P^2(f) + \frac{1}{4T^2} \sum_{m \in [-1,0,1]} P\left(\frac{m}{T}\right) \delta\left(f - \frac{m}{T}\right)$

3) $P_{mu} = \frac{N_0}{2} E_{hR} = \int_{-\infty}^{+\infty} S_{mu}(f)\, df = \int_{-\infty}^{+\infty} \frac{N_0}{2} |H_R(f)|^2 df = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H_R(f)| df$

$S_{mu}(f) = S_m(f) \cdot |H_R(f)|^2 = \frac{N_0}{2} \cdot |H_R(f)|^2$

$H_R(f) = \text{rect}\left(\frac{f}{2B}\right)$

$E_{h_R} = 2B \qquad P_{n_u} = \frac{N_0}{2} \cdot 2B = \boxed{N_0 B}$ 3

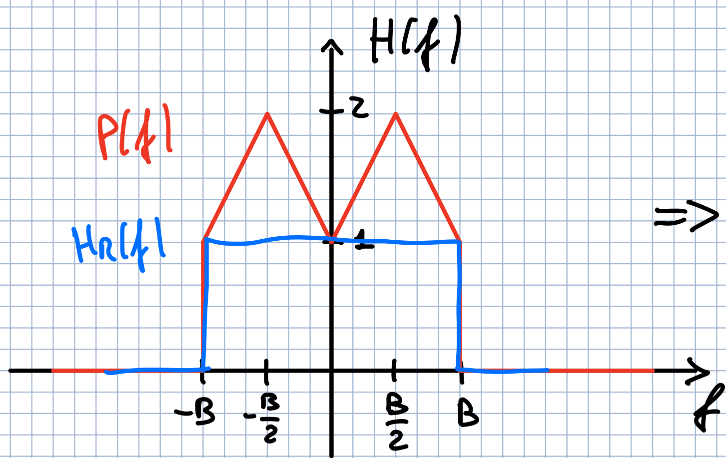
4) Verificare assenza di ISI

$h[n] = K\delta[n] = h(0)\,\delta[n] \qquad \bar{H}(f) = K = \frac{1}{T}\sum_{n=-\infty}^{+\infty} H\left(f - \frac{n}{T}\right)$

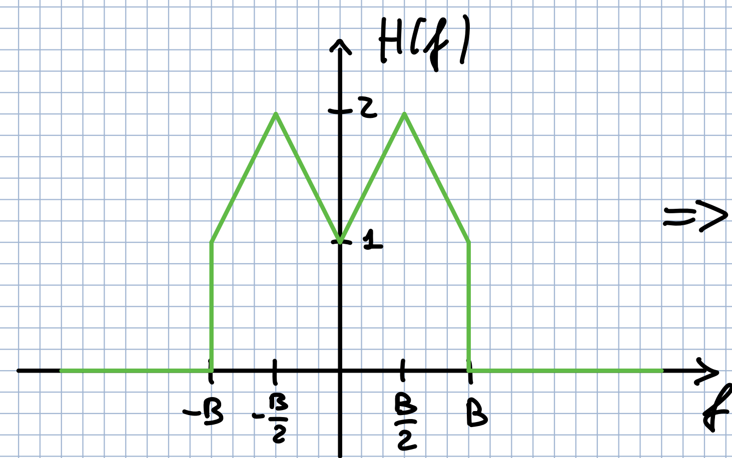
$h(t) = p(t) \otimes c(t) \otimes h_R(t) \qquad c(t) = \delta(t)$

$h(t) = p(t) \otimes h_R(t)$

$H(f) = P(f)\,H_R(f)$



$\Rightarrow$



$\Rightarrow$

$h(t) = p(t) = 2B\,\mathrm{sinc}(2Bt) + B\,\mathrm{sinc}^2\left(\frac{B}{2}t\right)\cos(\pi B t)$

$h\big|_{t=nT} = 2B\,\mathrm{sinc}(2BnT) + B\,\mathrm{sinc}^2\left(\frac{B}{2}nT\right)\cos(\pi B n T) = \qquad B = \frac{2}{T}$

$= 2B\,\mathrm{sinc}\left(2\cancel{B}n \cdot \frac{2}{\cancel{B}}\right) + B\,\mathrm{sinc}^2\left(\frac{\cancel{B}}{\cancel{2}} \cdot n \cdot \frac{\cancel{2}}{\cancel{B}}\right)\cos\left(\pi \cancel{B} n \frac{2}{\cancel{B}}\right) \qquad T = \frac{2}{B}$

$= 2B\,\mathrm{sinc}(4n) + B\,\mathrm{sinc}^2(n)\cos(2\pi n) =$

(TEMPO)

$= 2B\,\delta[n] + B\,\delta[n] = 3B\,\delta[n] \Rightarrow$ SODDISFA NYQUIST

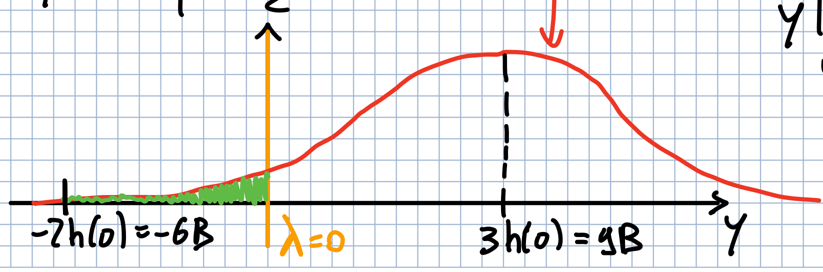
$\downarrow$

ASSENZA DI ISI

$h(0) = 3B$

5) $P_E(b) = P\{\hat{x} = -2 \mid x = 3\}\,P\{x = 3\} + P\{\hat{x} = 3 \mid x = -2\}\,P\{x = -2\}$

$P\{x = -2\} = P\{x = 3\} = \frac{1}{2}$



$y \mid x = 3$

$y = h(0)(+3) + n_u$

$$P\{\hat{x}=-2 \mid x=3\} = \int_{-\infty}^{\lambda=0} f_Y(y \mid x=3)\,dy = F_{Y|x=3}(\lambda) =$$

$$= \Phi\left(\frac{\lambda-3h(0)}{\sqrt{P_{nu}}}\right) = 1 - Q\left(\frac{\lambda-3h(0)}{\sqrt{P_{nu}}}\right) = Q\left(\frac{3h(0)-\lambda}{\sqrt{P_{nu}}}\right)$$

$$P_E(b) = P\{\alpha_1\}\, Q\left(\frac{\lambda-h(0)\alpha_1}{\sigma_{nu}}\right) + P\{\alpha_2\}\, Q\left(\frac{h(0)\alpha_2-\lambda}{\sigma_{nu}}\right) \qquad \alpha_1=-2, \quad \alpha_2=3$$

$$\boxed{P_E(b) = \frac{1}{2}\, Q\left(\frac{9B}{\sqrt{N_0 B}}\right) + \frac{1}{2}\, Q\left(\frac{6B}{\sqrt{N_0 B}}\right)}$$ 5

ESERCIZIO #2 16/07/19

$s(t) = \sum_k x[k]\, p(t-kT)$

$x[k] \in A_s = \{-3, 1\}$ INDIP. E EQUIPROB.

$p(t) = 2B\, \text{sinc}^2(2Bt)$

$c(t) = 4B\, \text{sinc}(4Bt) - 2B\, \text{sinc}(2Bt)$

$S_n(f) = \frac{N_0}{2}$ RUMORE BIANCO

$H_R(f) = \text{rect}\left(\frac{f}{4B}\right)$ BANDA 2B

$\lambda = 0$, $T = \frac{1}{B}$

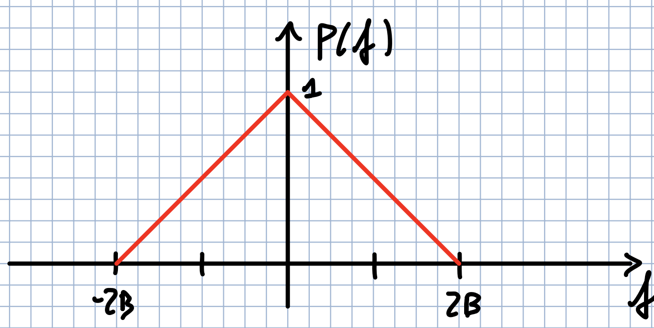
Calcolare:
1) E_s
2) P_{nu}
3) P_E, dopo verifica ASSENZA ISI

SOLUZIONE

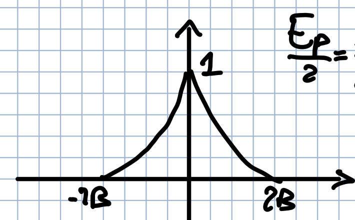
$$E_s = E_p\, E[x^2] = \frac{4B}{3} \cdot 5 = \boxed{\frac{20B}{3}}$$ 1

$$E[x^2] = \frac{1}{2}(-3)^2 + \frac{1}{2}(1)^2 = \frac{9}{2} + \frac{1}{2} = 5$$

$$P(f) = \left(1 - \frac{|f|}{2B}\right) \text{rect}\left(\frac{f}{4B}\right)$$



$$E_p = \frac{4B}{3} = \int_{-2B}^{2B} |P(f)|^2\, df$$

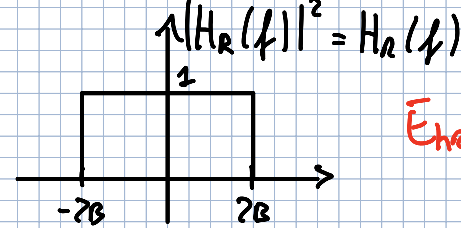


$$\frac{E_p}{2} = \frac{1}{3} \cdot 2B = \frac{2B}{3}$$

$$\frac{E_p}{2} = \int_0^{2B} 1\,df + \frac{1}{4B^2}\int_0^{2B} f^2\,df - \frac{1}{B}\int_0^{2B} f\,df = \frac{1}{4B^2}\frac{f^3}{3}\Big|_0^{2B} - \frac{1}{B}\frac{f^2}{2}\Big|_0^{2B} =$$

$$2B + \frac{1}{\cancel{4B^2}}\,\frac{\cancel{8}^{\,2}\cancel{B^3}}{3} - \frac{1}{\cancel{B}}\,\frac{\cancel{4}^{\,2}B^{\cancel{2}}}{\cancel{2}} = \frac{2B}{3} - 2B + 2B = \frac{2B}{3} \qquad \frac{2B}{3}\cdot 2 = \frac{4}{3}B = E_p$$

2) $P_{nu} = \frac{N_0}{2} E_{hR} = \frac{N_0}{2}\cdot 4B = \boxed{2N_0B}$ (2)



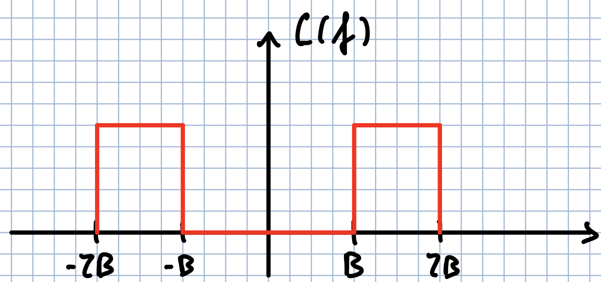
$E_{hR} = 4B$

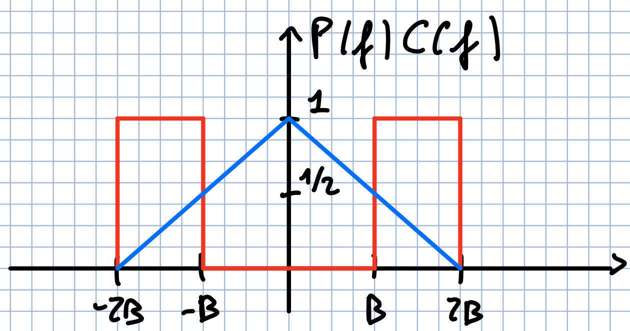
3) Verifica assenza di ISI

$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

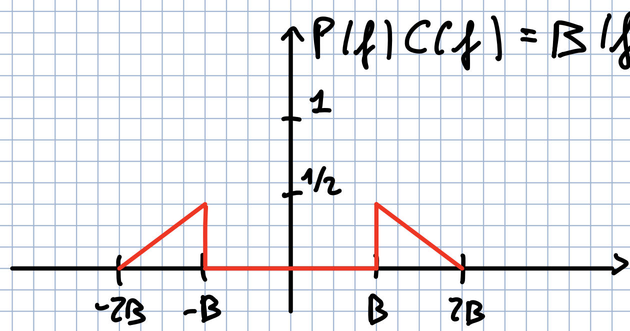
$$H(f) = P(f)\,C(f)\,H_R(f)$$

$$C(f) = \mathrm{rect}\left(\frac{f}{4B}\right) - \mathrm{rect}\left(\frac{f}{2B}\right)$$

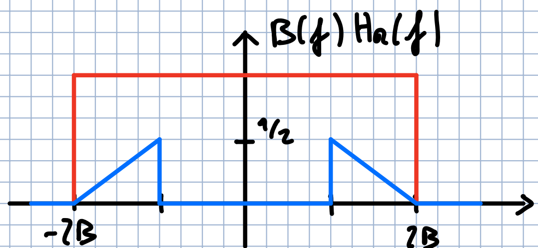


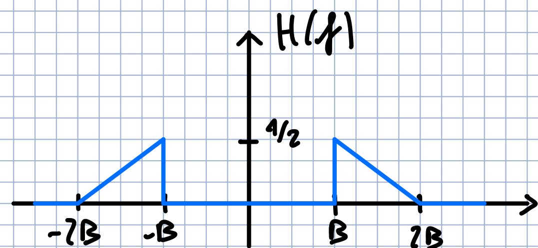


$\Rightarrow$



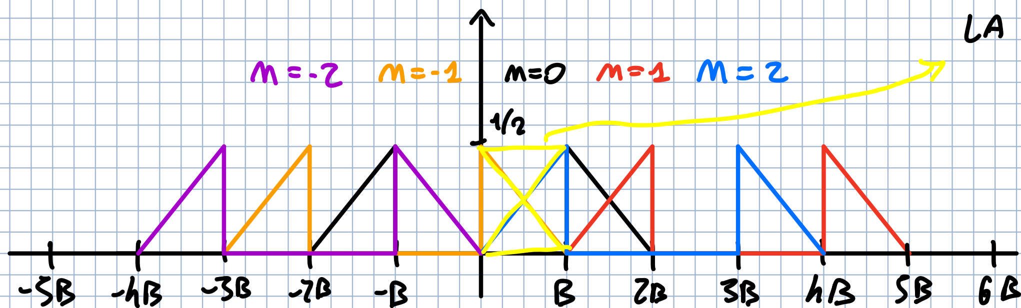
$\Rightarrow$





$$h(0) = h(t)\big|_{t=0} = \int_{-\infty}^{+\infty} H(f)\,df = \frac{B}{2}$$

Devo dimostrare che $\sum_{m=-\infty}^{+\infty} H\left(f-\frac{m}{T}\right) = \text{COST}$



LA SOMMA E' COSTANTE in $B = \frac{1}{T}$

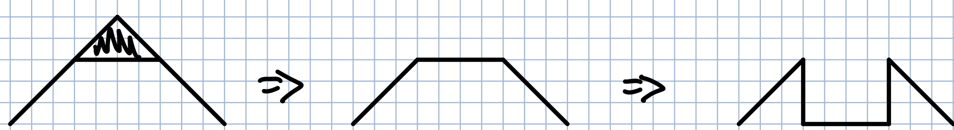
$\Downarrow$

ASSENZA ISI

ALTRO MODO

$h(t) = \text{ATCF}[H(f)]$

$$h(t) = \left(1-\frac{|f|}{2B}\right)\text{rect}\left(\frac{f}{4B}\right) - \frac{1}{2}\left(1-\frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right) - \frac{1}{2}\,\text{rect}\left(\frac{f}{2B}\right)$$



$\Leftarrow T = \frac{1}{B}$

$$h(t) = 2B\,\text{sinc}^2(2Bt) - \frac{B}{2}\,\text{sinc}^2(Bt) - B\,\text{sinc}(2Bt)$$

$$h(mT) = 2B\,\text{sinc}^2(2m) - \frac{B}{2}\,\text{sinc}^2(m) - B\,\text{sinc}(2m) =$$

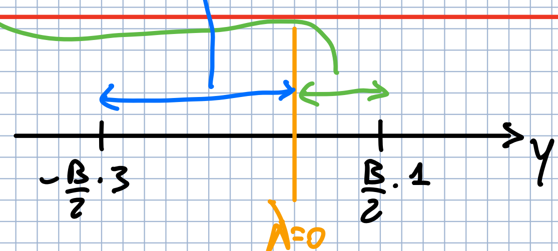
$$= 2B\,\delta[m] - \frac{B}{2}\,\delta[m] - B\,\delta[m] = \frac{B}{2}\,\delta[m] \rightarrow \text{VERIFICA ASSENZA ISI}$$

$\rightarrow h(0)\,\delta[m]$

- Verificata assenza ISI
- nota $h(0)$
- nota P_{mu}

$$\boxed{P_E(b) = \frac{1}{2}\,Q\left(\frac{\frac{B}{2}}{\sqrt{2N_0B}}\right) + \frac{1}{2}\,Q\left(\frac{\frac{3}{2}B}{\sqrt{2N_0B}}\right)}$$

3



ESERCIZIO # 2 26/06/18

$x \in A_s = \{-1, 2\}$ IND. con $P\{-1\} = \frac{1}{3}$ $P\{+2\} = \frac{2}{3}$

$w(t)$ GAUSSIANO BIANCO con $S_w(f) = \frac{N_0}{2}$

$p(t) = \mathrm{sinc}\left(\frac{2t}{T}\right)$

$h_R(t) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{2}{T}t\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{t}{T}\right)\ ,\quad \lambda = 0$

Calcolare

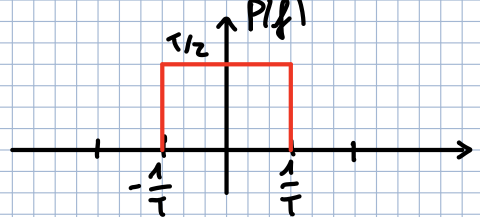
1) E_s 2) $S_s(f)$ 3) P_{nu} 4) Assenza ISI 5) $P_e(b)$

SOLUZIONE

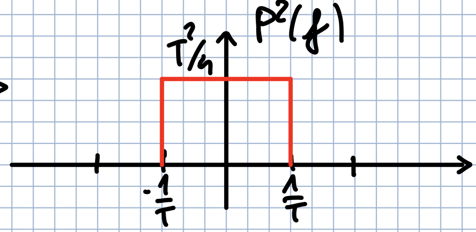
1) $E_s = E_p\, E[x^2] = \frac{T}{2}\cdot 3 = \boxed{\frac{3}{2}T}$

$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\, df$

$P(f) = \frac{T}{2}\,\mathrm{rect}\left(\frac{f}{2/T}\right)$



$\Rightarrow$



$E_p = \frac{2}{T}\cdot\frac{T^2}{4} = \frac{T}{2}$

$E[x^2] = \frac{1}{3}(-1)^2 + \frac{2}{3}(2)^2 = \frac{1}{3} + \frac{8}{3} = 3$

2) $S_s(f) = \frac{1}{T}\,\bar{S}_x(f)\,|P(f)|^2$

$\bar{S}_x(f) = TFS\left[R_x[m]\right]$

$R_x[m] = C_x[m] + \eta_x^2$

$C_x[m] = \sigma_x^2\,\delta[m] + \eta_x^2$ (I SIMBOLI SONO INCORRELATI)

$\eta_x = E[x] = \frac{1}{3}(-1) + \frac{2}{3}\cdot 2 = -\frac{1}{3} + \frac{4}{3} = 1$

$\sigma_x^2 = E[x^2] - \eta_x^2 = 3 - 1 = 2$

$R_x[m] = 2\delta[m] + 1$

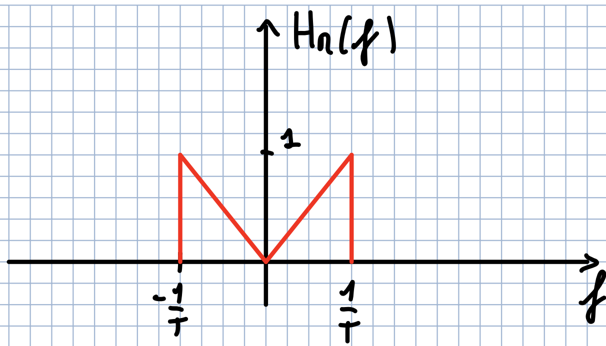
$\bar{S}_x(f) = 2 + \frac{1}{T}\sum_m \delta\left(f - \frac{m}{T}\right)$

$S_s(f) = \frac{2}{T}|P(f)|^2 + \frac{1}{T^2}\delta(f)\,P(0)^2$

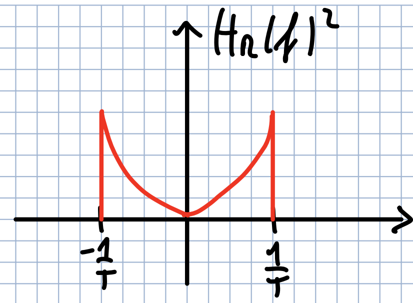
$= \frac{2}{T}\frac{T^2}{4}\,\mathrm{rect}\left(\frac{f}{2/T}\right) + \frac{1}{T^2}\frac{T^2}{4}\delta(f) = \boxed{\frac{T}{2}\,\mathrm{rect}\left(\frac{f}{2/T}\right) + \frac{1}{4}\delta(f)}$ 2

3) $P_{nu} = \frac{N_0}{2}E_{h_R} = \frac{N_0}{2}\cdot\frac{2}{3T} = \boxed{\frac{N_0}{3T}}$ 3

$H_R(f) = \mathrm{rect}\left(\frac{f}{2/T}\right) - 1\left(1 - \frac{|f|}{1/T}\right)\mathrm{rect}\left(\frac{f}{2/T}\right)$



$\Rightarrow$



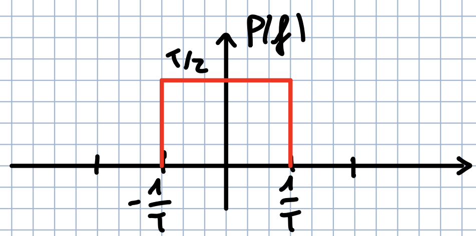
$$E_{h_R} = 2 \cdot \frac{1}{3}\left(\frac{1}{T}\right) = \frac{2}{3T}$$

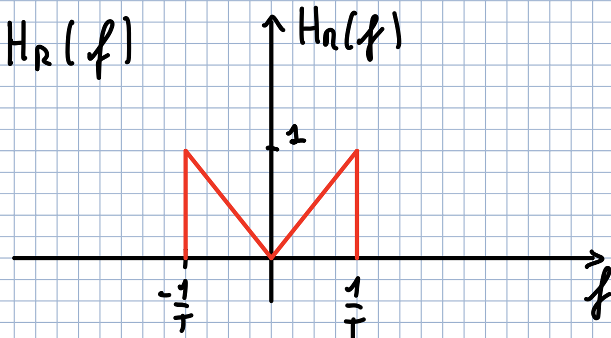
4) Verificare ASSENZA ISI

$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

$$h(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = \frac{T}{2} H_R(f)$$





$$h(t) = \frac{T}{2} h_R(t) = \text{sinc}\left(\frac{2t}{T}\right) - \frac{1}{2}\text{sinc}^2\left(\frac{t}{T}\right)$$

$$h(nT) = \text{sinc}\left(\frac{2nT}{T}\right) - \frac{1}{2}\text{sinc}^2\left(\frac{nT}{T}\right) = \text{sinc}(2n) - \frac{1}{2}\text{sinc}^2(n)$$

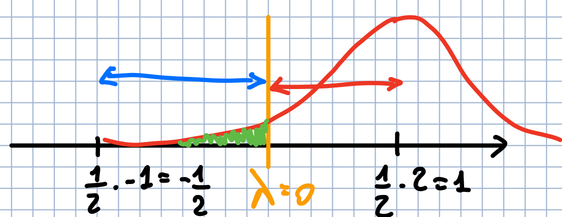
$$= \delta[n] - \frac{1}{2}\delta[n] = \frac{1}{2}\delta[n] \Rightarrow \text{Nyquist verificato nel tempo}$$

$\searrow h(0) = \frac{1}{2}$

$\Downarrow$

ASSENZA ISI

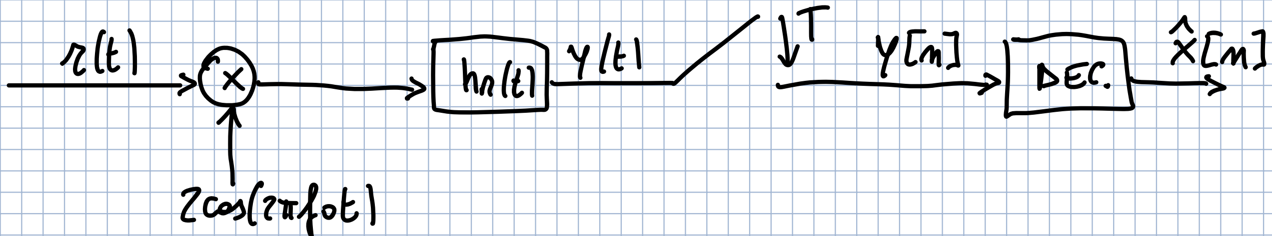
5)



$$P_E(b) = P\{\hat{x} = -1 \mid x = 2\}\, P\{x = 2\} + P\{\hat{x} = 2 \mid x = -1\}\, P\{x = -1\} =$$

$$= \frac{2}{3} Q\left(\frac{1}{\sqrt{\frac{N_0}{3T}}}\right) + \frac{1}{3} Q\left(-\frac{-\frac{1}{2}}{\sqrt{\frac{N_0}{3T}}}\right) = \boxed{\frac{2}{3} Q\left(\frac{1}{\sqrt{\frac{N_0}{3T}}}\right) + \frac{1}{3} Q\left(\frac{\frac{1}{2}}{\sqrt{\frac{N_0}{3T}}}\right)}$$

ES. 29-05-17



$$s(t) = \sum_k x[k]\, p(t-kT) \cos(2\pi f_0 t)$$

$x[k] \in A_s = \{-1, 2\}$ indip. $P\{-1\} = \frac{3}{5}$ $P\{+2\} = \frac{2}{5}$

$$p(t) = 2B \operatorname{sinc}(2Bt) + B \operatorname{sinc}\left(2B\left(t - \frac{1}{2B}\right)\right) + B \operatorname{sinc}\left(2B\left(t + \frac{1}{2B}\right)\right)$$

$f_0 \gg B$, $T = \frac{1}{B}$

$c(t) = \delta[t]$

$n(t)$ bianco in banda $S_n(f) = \frac{N_0}{2}\left[\operatorname{rect}\left(\frac{f-f_0}{2B}\right) + \operatorname{rect}\left(\frac{f+f_0}{2B}\right)\right]$

$h_R(t)$ è un passa basso ideale di banda B

$\lambda = 0$

Calcolare:

1) E_s 2) P_{mu} 3) Dire se $y[n]$ ha il max SNR 4) $P_E(b)$

SOL.

1) $E_s = \frac{1}{2} E[x^2] E_p$

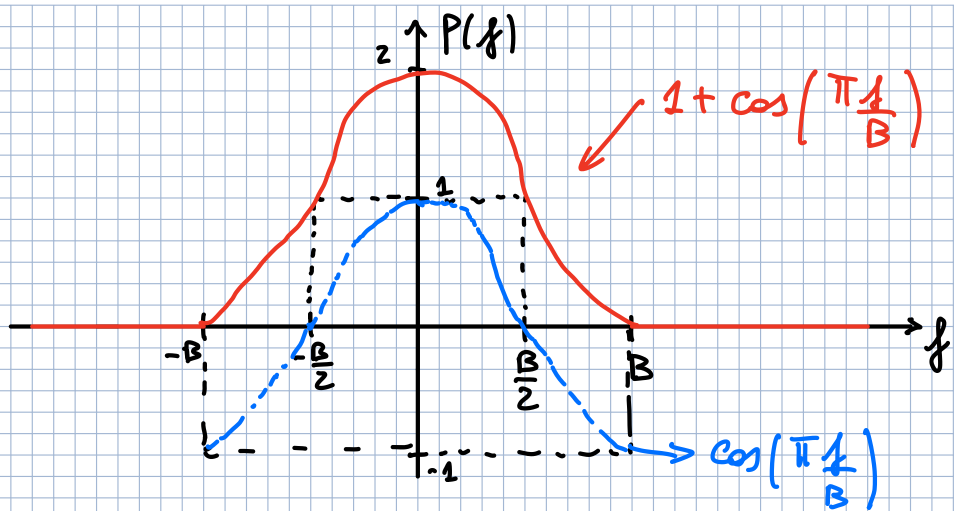
$$E[x^2] = \frac{3}{5}(-1)^2 + \frac{2}{5}(+2)^2 = \frac{3}{5} + \frac{8}{5} = \frac{11}{5}$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 df$$

$$P(f) = \operatorname{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\operatorname{rect}\left(\frac{f}{2B}\right) e^{-j2\pi f \frac{1}{2B}} + \frac{1}{2}\operatorname{rect}\left(\frac{f}{2B}\right) e^{j2\pi f \frac{1}{2B}} =$$

$$= \operatorname{rect}\left(\frac{f}{2B}\right)\left[1 + \frac{1}{2}\left(e^{j\frac{\pi f}{B}} + e^{-j\frac{\pi f}{B}}\right)\right] =$$

$$= \operatorname{rect}\left(\frac{f}{2B}\right)\left[1 + \cos\left(\frac{\pi f}{B}\right)\right] = \operatorname{rect}\left(\frac{f}{2B}\right) + \operatorname{rect}\left(\frac{f}{2B}\right)\cos\left(\frac{\pi f}{B}\right)$$



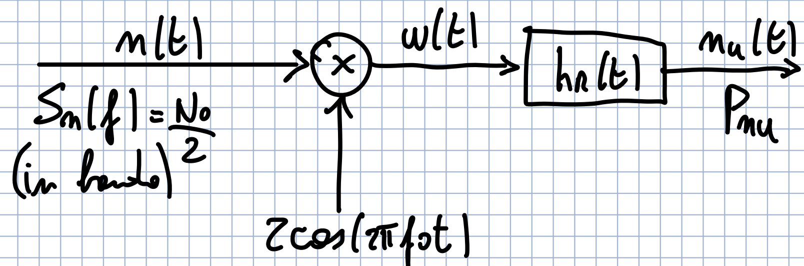
$$E_p = \int_{-B}^{B} \left[1+\cos\left(\frac{\pi f}{B}\right)\right]^2 df = \int_{-B}^{B} 1\, df + \int_{-B}^{B} \cos^2\left(\frac{\pi f}{B}\right) df + 2\underbrace{\int_{-B}^{B} \cos\left(\frac{\pi f}{B}\right) df}_{=0} =$$

perchè è simmetrico

$$= 2B + \frac{1}{2}\int_{-B}^{B}\left[1+\cos\left(\frac{2\pi f}{B}\right)\right] df = 2B + B + 0 = 3B$$

$$E_s = \frac{1}{2}\,\frac{11}{5}\cdot 3B = \boxed{\frac{33}{10}B}$$ 1

2)



$$S_{nu}(f) = N_0\,|H_R(f)|^2$$ quando $S_n(f)$ è BIANCO in banda

$$P_{nu} = N_0\, E_{hR} = \boxed{2N_0 B}$$ 2

$$E_{hR} = \int_{-\infty}^{+\infty} |H_R(f)|^2 df = 2B \qquad H_R(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

3) Dire se $y[n]$ ha il max SNR

$$SNR = \frac{P_s}{P_{nu}} \Rightarrow MAX$$

$\Downarrow$

il filtro di ricezione è "adattato"

$$r(t) = \sum_{n} x[n]\, p'(t-nT) \cos(2\pi f_0 t) + n(t)$$

$$h_R(t) = h_{FA} = K s_i(-t)$$

$$s_i(t) = x[n]\, p'(t-nT)$$

$$p'(t) = p(t) \otimes \tilde{c}(t) \qquad \Rightarrow \qquad \tilde{c}(t) = \delta(t)$$

$$p'(t) = p(t) = 2B\, \text{sinc}(2Bt) + \underbrace{B\, \text{sinc}\left(2B\left(t - \frac{1}{2B}\right)\right) + B\, \text{sinc}\left(2B\left(t + \frac{1}{2B}\right)\right)}$$

$$h_R(t) = 2B\, \text{sinc}(2Bt)$$

$$h_R(t) \neq K s_i(-t) = \underbrace{K\, x[n]}_{=K'}\, 2B\, \text{sinc}(2Bt) + \dots$$

$$2B\, \text{sinc}(2Bt) \neq K'\, 2B\, \text{sinc}(2Bt) + \dots$$

IL FILTRO DI RICEZIONE NON E' ADATTATO

$\Downarrow$

Y[n] non può avere SNR max

$\boxed{K p'(-t) = h_R(t)}$ → CONDIZIONE FILTRO ADATTATO

$\Downarrow$

Y[n] con SNR MAX.

4) $P_E(b)$

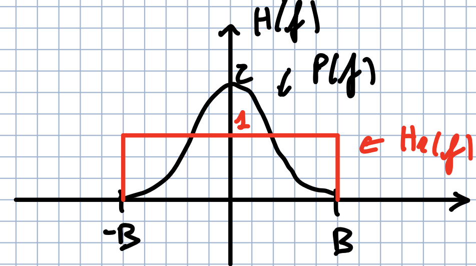
•) Verifica assenza di ISI

$$h(mT) = h[m] = K\delta[m] \qquad \text{o} \qquad \sum_{m} H\left(f - \frac{m}{T}\right) = K$$

$$h(t) = \underbrace{p(t) \otimes \tilde{c}(t)}_{p'(t) = p(t)} \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P(f)$$

$$h(t) = p(t)$$



$$h(mT) = p(mT) = 2B\, \text{sinc}(2BmT) + B\, \text{sinc}\left(2B\left(mT - \frac{1}{2B}\right)\right) + B\, \text{sinc}\left(2B\left(mT + \frac{1}{2B}\right)\right) =$$

$T = \frac{1}{B}$

$$= 2B\, \text{sinc}(2m)\, B\, \text{sinc}\left(2B\left(\frac{2m}{2B} - \frac{1}{2B}\right)\right) + B\, \text{sinc}\left(2B\left(\frac{2m}{2B} + \frac{1}{2B}\right)\right) =$$

$$= 2B\,\delta[n] + \underbrace{B\,\mathrm{sinc}[2n-1]}+\underbrace{B\,\mathrm{sinc}[2n+1]} = 2B\,\delta[n]$$

L'ARGOMENTO NON SI ANNULLA MAI PER NESSUN INTERO n, quindi vale 0 $\forall n$

$\Downarrow$ SODDISFA NYQUIST

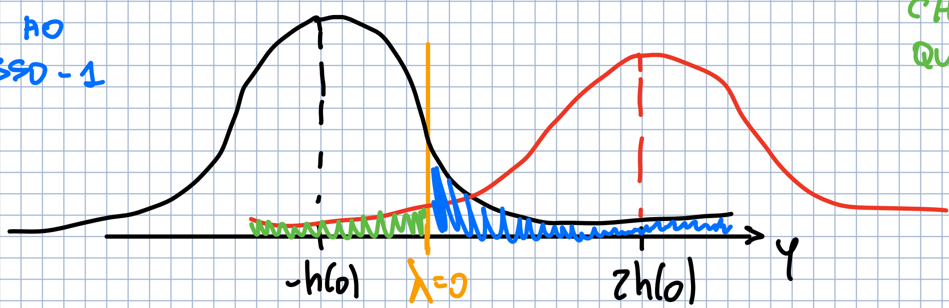
$$\begin{cases} h(0) = 2B \\ h[n] = 0 \quad \forall n \neq 0 \end{cases}$$

$$y[n] = h(0)\,x[n] + n_u[n] \qquad \text{ANCHE IN B. PASSANTE}$$

$$P_E(b) = P\{-1\}\underbrace{P\{\hat{x}=+2 \mid x=-1\}} + P\{2\}\underbrace{P\{\hat{x}=-1 \mid x=2\}}$$

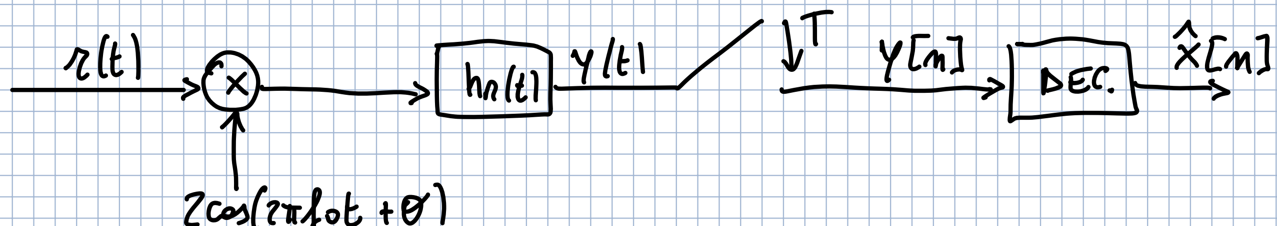
CHE DECIDO +2 QUANDO HO TRASMESSO -1

CHE DECIDO -1 QUANDO HO TRASMESSO 2



$$= \frac{3}{5} Q\left(\frac{h(0)}{\sqrt{P_{nu}}}\right) + \frac{2}{5} Q\left(\frac{2h(0)}{\sqrt{P_{nu}}}\right) = \boxed{\frac{3}{5} Q\left(\frac{2B}{\sqrt{2N_0B}}\right) + \frac{2}{5} Q\left(\frac{4B}{\sqrt{2N_0B}}\right)}$$

ESERCIZIO - #2 20/02/18



$$s(t) = \sum_k x[k]\, p(t-kT_s)\cos(2\pi f_0 t + \varphi)$$

$$x[n] \in A_s = \{-2, +3\} \qquad P\{-2\} = \frac{3}{4} \qquad P\{+3\} = \frac{1}{4}$$

$$P(f) = \begin{cases} \sqrt{1-|fT|} & |fT| \leq 1 \\ 0 & \text{altrove} \end{cases} \qquad f_0 \gg \frac{1}{T}$$

$$S_n(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{2/T}\right) + \mathrm{rect}\left(\frac{f+f_0}{2/T}\right)\right]$$

$$H_R(f) = P(f) \quad , \quad \lambda = 0$$

Calcolare: (ASSENZA DI CROSS TALK)

1) E_s 2) trovare il valore di θ per cui si minimizza la prob. di errore

3) P_{mu} 4) $P_E(b)$ 5) Verificare se $y[n]$ ha max SNR

SVOLGIM.

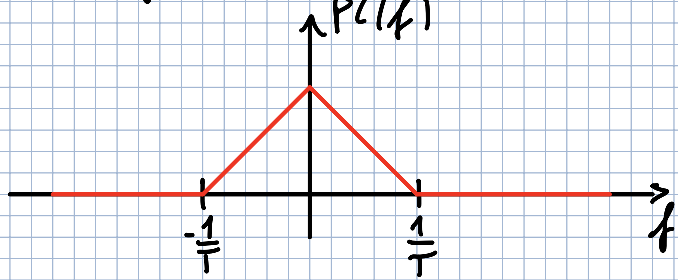
1) $E_s = \frac{1}{2} E[x^2] E_p = \frac{1}{2} \cdot \frac{21}{4} \cdot \frac{1}{T} = \boxed{\frac{21}{8T}}$ 1

$E[x^2] = \frac{3}{4}(-2)^2 + \frac{1}{4}(+3)^2 = 3 + \frac{9}{4} = \frac{21}{4}$

$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df$

$P^2(f) = \begin{cases} 1 - |fT| & |fT| \leq 1 \\ 0 & \text{altrove} \end{cases}$

$= \left(1 - \frac{|f|}{1/T}\right) \text{rect}\left(\frac{f}{2/T}\right)$



$E_p = \frac{1}{T}$

2) $y(t) = \sum_m x[m] h(t - mT) \underbrace{\cos(\theta - \varphi)} + m_u(t)$

$\boxed{\theta = \varphi}$ $\cos(\theta - \varphi) = 1$

$y(t) = \sum_m x[m] h(t - mT) + m_u(t)$ MINIMA $P_E(b)$

3) $P_{mu} = N_0 E_{hR}$

$E_{hR} = \int_{-\infty}^{+\infty} |H_R(f)|^2\, df = \int_{-\infty}^{+\infty} P^2(f)\, df = \frac{1}{T}$

$\boxed{P_{mu} = \frac{N_0}{T}}$

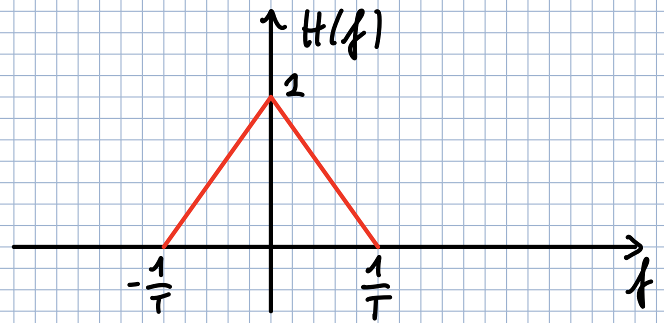
4) $P_E(b)$

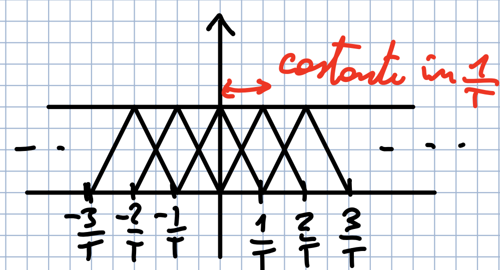
VERIFICO ASSENZA DI ISI

$h(t) = h(nT) = K\delta[n]$ o $\sum_m H\left(f - \frac{m}{T}\right) = \text{COST}$

$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$$H(f) = P(f)\,H_R(f) = P^2(f)$$





$\Downarrow$ ASSENZA ISI IN FREQ.

$$h(t) = \frac{1}{T}\,\text{sinc}^2\left(\frac{t}{T}\right)$$

$$h(nT) = \frac{1}{T}\,\text{sinc}^2\left(\frac{nT}{T}\right) = \frac{1}{T}\,\text{sinc}^2(n) = \frac{1}{T}\,\delta[n]$$

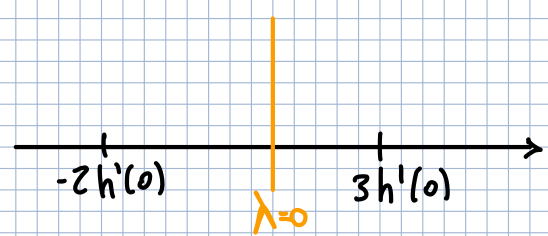
$\Downarrow$

ASSENZA DI ISI NEL TEMPO

$$h(0) = \frac{1}{T}$$

$$y[n] = h'(0)\,x[n] + n_u[n] \qquad \text{con } h'(0) = \frac{\cos(\theta-1)}{T}$$

$$P_E(b) = P\{3\}\,P\{\hat{x} = -2 \mid x = 3\} + P\{-2\}\,P\{\hat{x} = 3 \mid x = -2\}$$



$$\boxed{P_E(b) = \frac{1}{4}\,Q\left(\frac{\frac{3}{T}\cos(\theta-1)}{\sqrt{\frac{N_0}{T}}}\right) + \frac{3}{4}\,Q\left(\frac{\frac{2}{T}\cos(\theta-1)}{\sqrt{\frac{N_0}{T}}}\right)}$$

5) $y[n]$ ha max SNR. $\Rightarrow$ Il FILTRO $h_R(t)$ è "ADATTATO"? SI

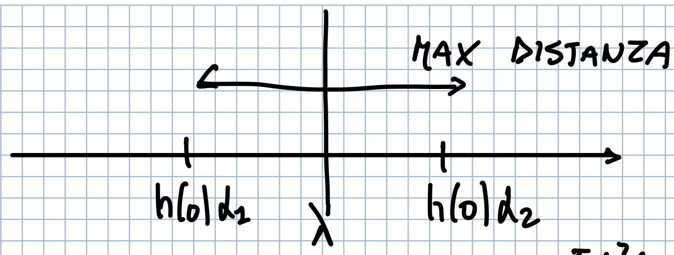
$$h_R(t) = K\,p'(-t)$$

$$p'(t) = p(t)$$

$p(t)$ è REALE E PARI $\Leftarrow$ $P(f)$ REALE E PARI

$$p(-t) = p(t) = h_R(t) \rightarrow h_R(t) = p(-t) = p'(-t)$$

HA IL MAX SNR $\rightarrow$ $\boxed{h_R(t) = K\,p'(t)}$ $\quad K = 1$



Pmedia del segnale in uscita al campionatore

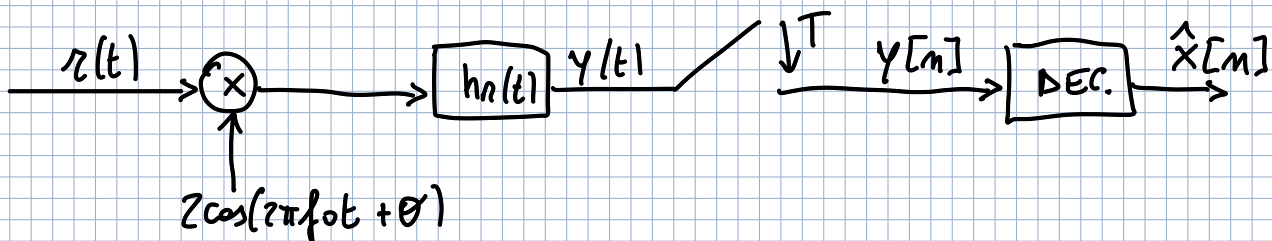
$$\text{MAX SNR} \Rightarrow \frac{E[h^2(0)\,x^2[n]]}{P_{nu}} = \frac{h^2(0)\,E[x^2]}{P_{nu}} =$$

$$= \frac{E[x^2]}{\frac{N_0}{T}}\,h^2(0) \longrightarrow \text{MAX } h^2(0) \Rightarrow \text{MAX } h(0)$$

$\downarrow$

MINIMA $P_E(b)$ $\leftarrow$ MAX DISTANZA TRA I SIMBOLI

ESERCIZIO - 17/01/19 #2



$$s(t) = \sum_k x[k]\,p(t-kT_s)\cos(2\pi f_0 t + \varphi)$$

$x[n]$ INDIPENDENTI $\in A_s:\{-1, 3\}$ $P\{-1\} = \frac{2}{3}$ $P\{3\} = \frac{1}{3}$

$p(t) = 2B\,\text{sinc}^2(Bt)\cos(\pi B t)$

$2\pi f_0 t = \pi B t$

$c(t) = \delta(t)$

$f_0 = \frac{\cancel{\pi} B \cancel{t}}{2\cancel{\pi}\cancel{t}} = \frac{B}{2}$

$w(t)$ GAUSSIANO BIANCO in banda

$$S_w(f) = \frac{N_0}{2}\left[\text{rect}\left(\frac{f-f_0}{2/T}\right) + \text{rect}\left(\frac{f+f_0}{2/T}\right)\right]$$

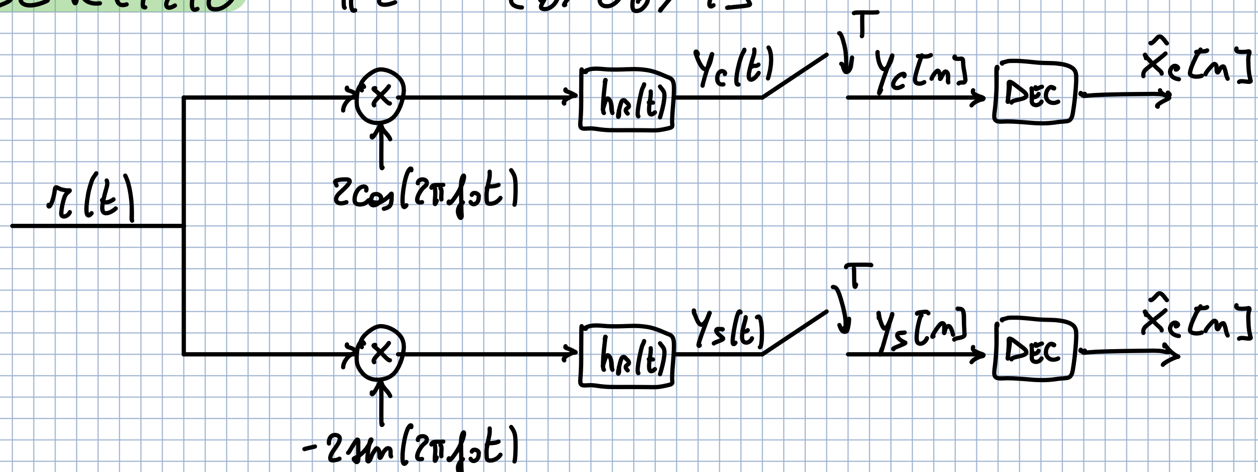
$h_R(t)$ è un passa basso ideale in banda $\frac{3}{2}B$ $\quad T = \frac{1}{B}$

1) E_s 2) valore di θ per min $P_E(b)$

3) Assenza ISI 4) P_{nu} 5) $P_E(b)$

DA FINIRE

ESERCIZIO #2 28/06/19



$$s(t)=\sum_n x_c[n]\,p(t-nT)\cos(2\pi f_0 t)-\sum_n x_s[n]\,p(t-nT)\sin(2\pi f_0 t)$$

$x_c[n]\in A_s^c=\{-1,2\}$

IND. E EQUIPR.

$x_s[n]\in A_s^s=\{-2,1\}$

$p(t)=\text{sinc}\left[B\left(t-\frac{1}{2B}\right)\right]+\text{sinc}\left[B\left(t+\frac{1}{2B}\right)\right]$ $\qquad B=\frac{2}{T}$

$c(t)=\delta(t)$

$n(t)$ bianco in banda $\Rightarrow \frac{N_0}{2}=S_n(f)$

$h_R(t)=p(t)$

$\lambda=0$

1) E_s 2) P_{muc} e P_{mus} 3) $P_E(M)$

SOLUZIONE

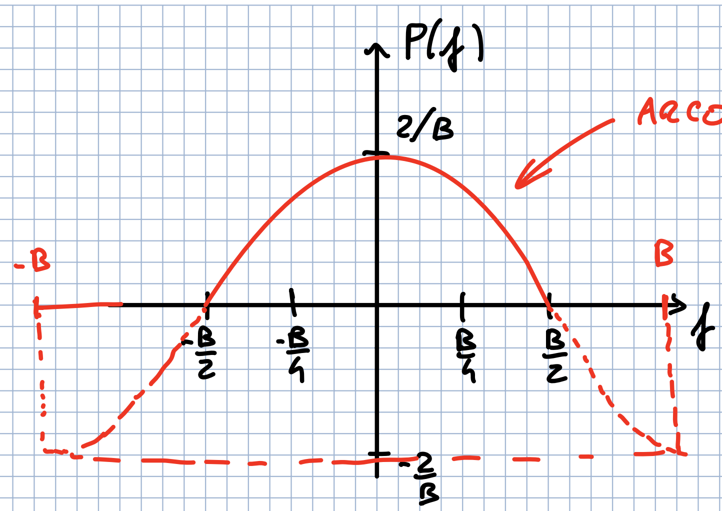
1) $E_s=\frac{1}{2}\left[E[x_c^2]+E[x_s^2]\right]E_p$

$$E[x_c^2]=\frac{1}{2}(-1)^2+\frac{1}{2}(2)^2=\frac{1}{2}+\frac{4}{2}=\frac{5}{2}$$

$$E[x_s^2]=\frac{1}{2}(-2)^2+\frac{1}{2}(1)^2=\frac{5}{2}$$

$$P(f)=\frac{1}{B}\,\text{rect}\left(\frac{f}{B}\right)\left[e^{-j2\pi f\frac{1}{2B}}+e^{j2\pi f\frac{1}{2B}}\right]=$$

$$=\frac{2}{B}\,\text{rect}\left(\frac{f}{B}\right)\cos\left(\frac{\pi f}{B}\right)=$$



$$E_p = \int_{-\frac{B}{2}}^{B/2} \left(\frac{2}{B}\cos\left(\frac{\pi f}{B}\right)\right)^2 = \frac{4}{B^2}\int \cos^2\left(\frac{\pi f}{B}\right) df = \frac{4}{B^2}\int_{-B/2}^{B/2} \frac{1}{2} + \underbrace{\frac{1}{2}\cos\left(\frac{2\pi f}{B}\right)}_{0} df$$

$$\frac{2}{B^2}\cdot f \Big|_{-\frac{B}{2}}^{B/2} = \left(\frac{\cancel{2}}{B^{\cancel{2}}}\cdot\frac{\cancel{B}}{\cancel{2}} + \frac{\cancel{2}}{B^{\cancel{2}}}\,\frac{\cancel{B}}{\cancel{2}}\right) = \frac{1}{B} + \frac{1}{B} = \frac{2}{B}$$

$$E_s = \frac{1}{2}\cdot\left(\frac{5}{2}+\frac{5}{2}\right)\cdot\frac{2}{B} = \boxed{\frac{5}{B}} \quad 1$$

2) $P_{nuc} = P_{nus} = N_0 E_{hR} = N_0 E_p = \boxed{\frac{2N_0}{B}}$ 2

3) $h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t)$

$$H(f) = P(f)\,C(f)\,H_R(f) = P(f)\,H_R(f)$$

$$\frac{2}{B}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos\left(\frac{\pi f}{B}\right)\frac{2}{B}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos\left(\frac{\pi f}{B}\right) =$$

$$= \frac{4}{B^2}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos^2\left(\frac{\pi f}{B}\right) = \frac{4}{B^2}\,\mathrm{rect}\left(\frac{f}{B}\right)\left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi f}{B}\right)\right]$$

$$B = \frac{2}{T} \qquad T = \frac{2}{B}$$

$$h(t) = \frac{4}{B}\,\mathrm{sinc}(Bt)\otimes\left[\frac{1}{2}\delta(t) + \frac{1}{4}\delta\left(t-\frac{1}{B}\right) + \frac{1}{4}\left(t+\frac{1}{B}\right)\right] =$$

$$= \frac{2}{B}\,\mathrm{sinc}(Bt) + \frac{1}{B}\,\mathrm{sinc}\left[B\left(t-\frac{1}{B}\right)\right] + \frac{1}{B}\,\mathrm{sinc}\left[B\left(t+\frac{1}{B}\right)\right] =$$

$$h(nT) = h = \left(n\frac{2}{B}\right) = \frac{2}{B}\,\mathrm{sinc}(2n) + \frac{1}{B}\,\mathrm{sinc}\left[B\left(n\frac{2}{B}-\frac{1}{B}\right)\right] + \frac{1}{B}\,\mathrm{sinc}\left[B\left(\frac{2n}{B}-\frac{1}{B}\right)\right] =$$

$$= \frac{2}{B}\,\mathrm{sinc}(2n) + \underbrace{\frac{1}{B}\,\mathrm{sinc}(2n-1)}_{\text{FA } \forall n \text{ INTERO}} + \underbrace{\frac{1}{B}\,\mathrm{sinc}(2n+1)}_{0} = \boxed{\frac{2}{B}\,\delta[n]}$$

$$\frac{2}{B} = h(0)$$

$\Downarrow$

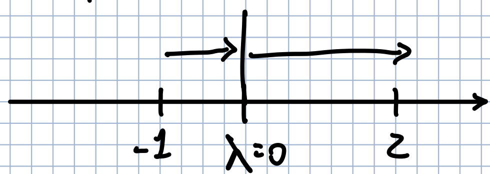
VERIFICATO NYQUIST

$\Downarrow$

ASSENZA ISI

$$P_E(M) = P_E^c(b)\cdot\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b)\,P_E^s(b)$$

$$P_E^I(b) = P(\hat{x} = -1 \mid x = 2)\,P(x=2) + P(\hat{x} = 2 \mid x = -1)\,P(x=2)$$

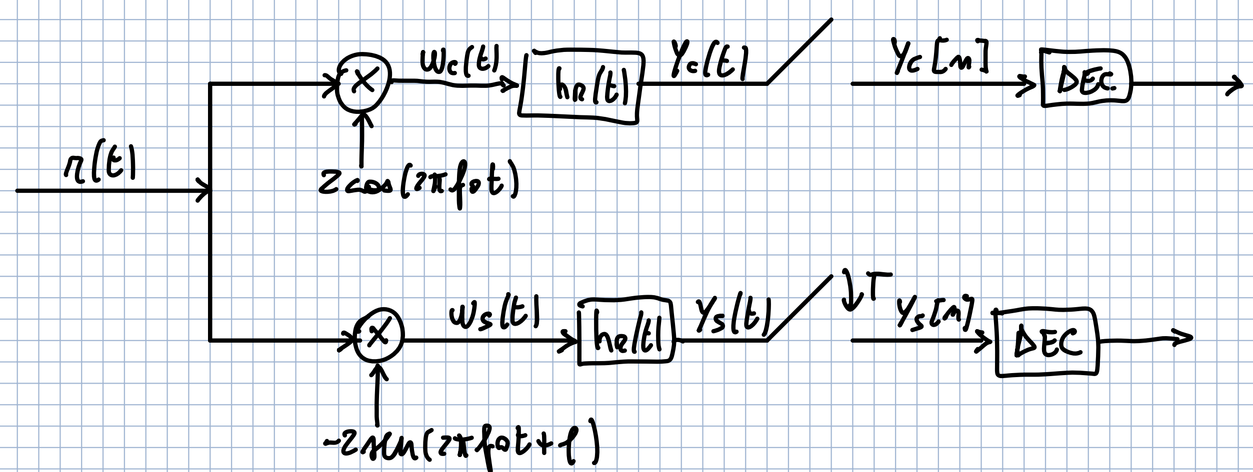


$$P_E^I(b) = \frac{1}{2}\,Q\left(\frac{\frac{4}{B}}{\sqrt{\frac{2N_0}{B}}}\right) + \frac{1}{2}\,Q\left(\frac{\frac{2}{B}}{\sqrt{\frac{2N_0}{B}}}\right)$$

$$P_E^Q(b) = P_E^I(b) = P_E(b)$$

$$\boxed{P_E(M) = 2P_E(b)\left(1 - P_E(b)\right) + P_E^2(b)}$$ 3

ESERCIZIO #2 08/09/17



$$s(t) = \sum_n x_c[n]\,p(t - nT_s)\cos(2\pi f_0 t + \varphi) - \sum_n x_s[n]\,p(t - nT_s)\sin(2\pi f_0 t + \varphi)$$

$x_c[n] \in A_s^c = \{-2, 2\}$ $\quad$ $x_s[n] \in A_s^s = \{-1, 1\}$ $\quad$ IND. E EQUIPR.

$p(t) \Rightarrow P(f) = \sqrt{1-|fT|}\,\mathrm{rect}\left(\frac{fT}{2}\right)$ $\qquad f_0 \gg \frac{1}{T}$

$c(t) = \delta(t)$ $\qquad n(t)$ GAUSS. BIANCO con $S_n(f) = \frac{N_0}{2}$

$h_R(t) = p(t)$ $\qquad \lambda = 0$

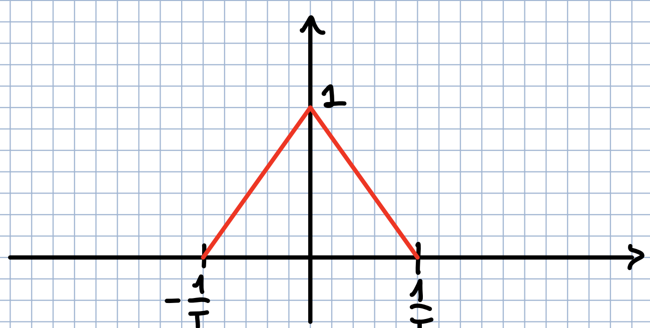
1) E_s 2) $P_{n_{uc}}$ e $P_{n_{us}}$ 3) Dire se è presente cross-talk 4) $P_E^c(b)$ e $P_E^s(b)$

SOLUZIONE

1) $E_s = \frac{1}{2} E[x_c^2 + x_s^2]\, E_p$

$E[x_c^2] = \frac{1}{2}(-2)^2 + \frac{1}{2}(2)^2 = 4$ $\qquad E[x_s^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$

$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\, df$ $\qquad |P(f)|^2 = \left(1 - \frac{|f|}{1/T}\right)\mathrm{rect}\left(\frac{f}{2/T}\right)$



$E_p = \frac{2}{T}\cdot 1 \cdot \frac{1}{2} = \frac{1}{T}$

$E_s = \frac{1}{2}\cdot 5 \cdot \frac{1}{T} = \boxed{\frac{5}{2T}}$ 1

2) $P_{n_{uc}} = P_{n_{us}} = P_{n_u} = N_0 E_{h_R} = N_0 E_p = \boxed{\frac{N_0}{T}}$ 2

3) fase del segnale TX $= \varphi$

IN RX:

fase ramo I $= 0$

fase ramo Q $= \varphi$

$\Delta_1 = \varphi_{TX} - \varphi_{RX} =$
- ramo in fase $= \varphi$
- ramo in quadr. $= 0$

- È presente cross-talk poichè sul ramo in fase la Δ_1 è $\neq 0$
- Non è presente cross-talk sul ramo in quadratura poichè la $\Delta_1 = 0$

4) $P_E^s(b)$ poichè $P_E^I(b)$ non è calcolabile per via del cross-talk

- Verifica ASSENZA ISI ✓
- Calcola $h(0)$ ✓
- Calcola P_{mus} ✓
- Calcola la $P_E^s(b)$

ASSENZA DI ISI

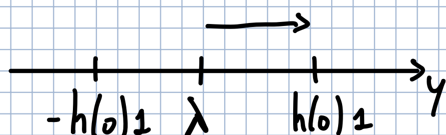
$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t) = p(t) \otimes p(t)$$

$$H(f) = P^2(f)$$

$$h(t) = \text{ATCF}\left(P(f)\right) = \frac{1}{T} \text{sinc}^2\left(\frac{1}{T} t\right)$$

$$h(nT) = \frac{1}{T} \text{sinc}^2\left(\frac{1}{T} \cdot nT\right) = \frac{1}{T} \text{sinc}^2(n) = \underbrace{\frac{1}{T} \delta[n]}_{\text{ASSENZA ISI}} \Rightarrow h(0) = \frac{1}{T}$$

$P_E^s(b)$



$$P_E^s(b) = \frac{1}{2} Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right) + \frac{1}{2} Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right) = \boxed{Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right)}$$ 3

ES #2 13/11/17

$$s(t) = \sum_K x_c[K] p(t - KT) \cos(2\pi f_0 t) - \sum_K x_s[K] p(t-KT) \sin(2\pi f_0 t)$$

$x_c[K] \in A_s^c = \{-1, 2\}$ $\quad x_s[K] \in A_s^s = \{-1, 1\}$ $\quad$ IND. E EQUIP.

$$p(t) = 2B\, \text{sinc}(2Bt) - B\, \text{sinc}\left(2B\left(t - \frac{1}{2B}\right)\right) - B\, \text{sinc}\left(2B\left(t + \frac{1}{2B}\right)\right)$$

$c(t) = \delta(t)$ $\quad S_n(f) = \frac{N_0}{2}\left[\text{rect}\left(\frac{f-f_0}{2B}\right) + \text{rect}\left(\frac{f+f_0}{2B}\right)\right]$ $\quad f_0 \gg B$

$h_R(t)$ passa basso di banda B $\quad T = \frac{1}{B}$

$\lambda = 0$

1) E_s 2) P_{muc} e P_{mus} 3) $P_E(b)$

SOLUZIONE

1) $E_s = \frac{1}{2} E[x_c^2 + x_s^2] E_p$

$E[x_c^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(2)^2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

$E[x_s^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$

$P(f) = \text{rect}\left(\frac{f}{2B}\right) - \frac{1}{2}\text{rect}\left(\frac{f}{2B}\right)\left[e^{-j2\pi f \frac{1}{2B}} + e^{j2\pi f \frac{1}{2B}}\right] =$

$= \text{rect}\left(\frac{f}{2B}\right) - \text{rect}\left(\frac{f}{2B}\right)\cos\left(\frac{\pi f}{B}\right) = \text{rect}\left(\frac{f}{2B}\right)\left[1 - \cos\left(\frac{\pi f}{B}\right)\right]$

$P^2(f) = 1 + \cos^2\left(\frac{\pi f}{B}\right) - 2\cos\left(\frac{\pi f}{B}\right)$

$$\int_{-B}^{B} 1 + \cos^2\left(\frac{\pi f}{B}\right) - 2\cos\left(\frac{\pi f}{B}\right) df = 2B + \frac{1}{2}\cdot 2B + \frac{1}{2}\underbrace{\int_{-B}^{B}\cos\left(\frac{2\pi f}{B}\right)df}_{=0} - 2\underbrace{\int_{-B}^{B}\cos\left(\frac{\pi f}{B}\right)df}_{=0}$$

$= 2B + B = 3B = E_p$

$E_s = \frac{1}{2}\left[\frac{5}{2} + 1\right]\cdot 3B = \boxed{\frac{21}{4}B}$

2) $P_{n_{uc}} = P_{n_{us}} = P_{n_u} = N_0 E_{h_R} =$ $\qquad H_R(f) = \text{rect}\left(\frac{f}{2B}\right)$

$= \boxed{N_0 2B}$ $\qquad E_{h_R} = 2B$

3) $P_E(M) = P_E^c(b)(1 - P_E^s(b)) + P_E^s(1 - P_E^c(b)) + P_E^c(b) P_E^s(b)$

ASSENZA ISI

$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

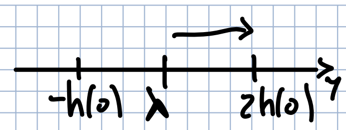
$H(f) = P(f) H_R(f) = P(f) \quad \rightarrow \quad h(t) = p(t)$

$h(nT) = h\left(\frac{n}{B}\right) = 2B\,\text{sinc}\left(\frac{2Bn}{B}\right) - B\,\text{sinc}\left(2B\left(\frac{n}{B} - \frac{1}{2B}\right)\right) - B\,\text{sinc}\left(2B\left(\frac{n}{B} + \frac{1}{2B}\right)\right) =$

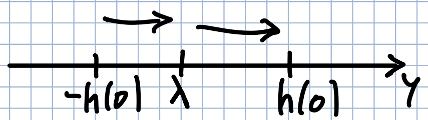
$= 2B\,\delta[n] - B\,\text{sinc}(2n-1) - B\,\text{sinc}(2n+1) = 2B\,\delta[n]$

$\Downarrow$

$h(0) = 2B$ $\qquad$ VERIFICATO NYQUIST



$$\boxed{P_E^c(b)} = \frac{1}{2} Q\left(\frac{4B}{\sqrt{2N_oB}}\right) + \frac{1}{2}\left(\frac{2B}{\sqrt{2N_oB}}\right)$$



$$P_E^s(b) = \frac{1}{2} Q\left(\frac{2B}{\sqrt{2N_oB}}\right) + \frac{1}{2}\left(\frac{2B}{\sqrt{2N_oB}}\right) = \boxed{Q\left(\frac{2B}{\sqrt{2N_oB}}\right)}$$ 3

ES #1 26/06/18

1	2	3
$P(I_1)$ 50%	$P(I_2)$ 30%	$P(I_3)$ 20%
$P_{D1} = 0{,}02$	$P_{D2} = 0{,}05$	$P_{D3} = 0{,}01$

IMPIANTI PRODUZ.

prob. di produrre lavatrici difettose

1) $P_\Delta = P\{\text{lavatrice difettosa}\}$

$\Downarrow$

TH. PR. TOT.

$$P(A) = \sum_{i=1}^{3} P(A \mid I_i) P(I_1) =$$

$$= P_{D1} P(I_1) + P_{D2} P(I_2) + P_{D3} P(I_3) = 0{,}5 \cdot 0{,}02 + 0{,}3 \cdot 0{,}05 + 0{,}2 \cdot 0{,}01 =$$

$$= \boxed{2{,}7\%}$$

$$P(\bar{A}) = 1 - P(A) = \boxed{97{,}3\%}$$

2) $$P(I_2 \mid A) = \frac{P(A \mid I_2) P(I_2)}{P(A)} = \frac{0{,}05 \cdot 0{,}3}{0{,}027} = 0{,}555 \simeq \boxed{55{,}5\%}$$

$P(A \mid I_2) = P_{D2} = 0{,}05$

$P(I_2) = 0{,}3$

$P(A) = 0{,}027$

$$P(I_1|A) = \frac{P(A|I_1)P(I_1)}{P(A)} = \frac{0{,}02 \cdot 0{,}5}{0{,}027} = 0{,}37 \simeq \boxed{37\%}$$

$P(A|I_1) = P_{D1} = 0{,}02$

$P(I_1) = 0{,}5$

$P(A) = 0{,}027$

$$P(I_3|A) = 1 - \left(P(I_1|A) + P(I_2|A)\right) = 0{,}074 \simeq 7{,}4\%$$

ES #1 6/06/17

$U(t)$ stazionario $\eta_u = 0$ e $R_u(\tau) = \sigma_u^2 \operatorname{sinc}(2B\tau)$

$Y(t) = U(t) + 3U(t-T)$

1) $U(0) = ?$ e ddp

2) $S_Y(f)$ e $R_Y(\tau)$

SVOLG.

$U \in \mathcal{N}(\eta_u, \sigma_u^2)$ $\qquad f_u(u) = \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{(u-\eta_u)^2}{2\sigma_u^2}}$

$P_u = \eta_u^2 + \sigma_u^2$

$\Downarrow$

$P_u = \sigma_u^2$ $\qquad P_u = R_u(\tau)\Big|_{\tau=0} = \sigma_u^2 = \int_{-\infty}^{+\infty} S_u(f)\,df$

$$\boxed{f_u(u) = \frac{1}{\sqrt{2\pi\sigma_u^2}} e^{-\frac{u^2}{2\sigma_u^2}}} \quad 1$$

2) $R_Y(t_1,t_2) = E[Y(t_1)Y(t_2)] = E[(U(t_1) + 3U(t_1-T))(U(t_2) + 3U(t_2-T))]$

$= E[U(t_1)U(t_2)] + E[U(t_1)U(t_2-T)] + E[U(t_1-T)U(t_2-T)] + E[U(t_1-T)U(t_2)] =$

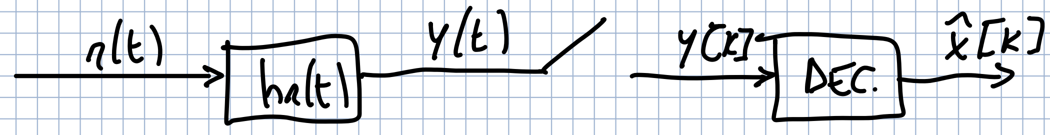
$= R_u(t_1-t_2) + R_u(t_1-(t_2-T)) + R_u(t_1-t_2) + R_u(t_1-T-t_2)$

$R_Y(\tau) = 2R_u(\tau) + R_u(\tau+T) + R_u(\tau-T) =$

$2\sigma_u^2 \operatorname{sinc}(2B\tau) + \sigma_u^2 \operatorname{sinc}(2B(\tau+T)) + \sigma_u^2 \operatorname{sinc}(2B(\tau-T))$

$$S_Y(f) = TCF[R_Y(\tau)] = 2\sigma_a^2 \cdot \frac{1}{2B} \text{rect}\left(\frac{f}{2B}\right) + \frac{\sigma_a^2}{2B} \text{rect}\left(\frac{f}{2B}\right)\left[e^{j2\pi f\tau} + e^{-j2\pi f\tau}\right]$$

#2 6/6/17



$t_k = kT$

$r(t) = s(t) + w(t)$

$s(t) = A_0 \sum_k x[k]\, p(t - kT)$

$x[k] \in A_s = \{-1, 1\}$ INDIP. E EQUIPR.

$w(t)$ GAUSSIANO A MEDIA NULLA $S_w(f) = \frac{N_0}{2}$

$c(t) = \delta(t)$

$p(t) = \frac{4}{T} \text{sinc}\left(\frac{4}{T} t\right)$ $\qquad H(f) = (1 - |fT|)\,\text{rect}\left(\frac{fT}{2}\right) + \text{rect}\left(\frac{fT}{4}\right)$

$\lambda = 0$

SVOLG.

$E_s = E[x^2]\, E_p = \frac{4}{T}$

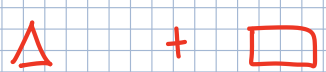
$E[x^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$

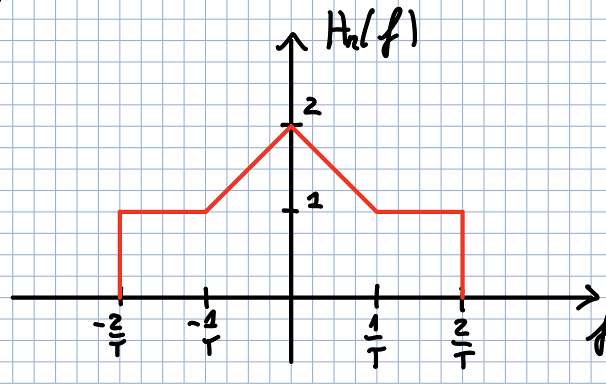
$P(f) = \text{rect}\left(\frac{f}{4/T}\right) \qquad E_p = \frac{4}{T}$

$n(t) = w(t) \otimes h_R(t)$

$S_n(f) = S_w(f)\,|H_R(f)|^2 = \frac{N_0}{2} \cdot \frac{20}{3T} = \frac{10 N_0}{3T}$

$H_R(f) = \left(1 - \frac{|f|}{1/T}\right) \text{rect}\left(\frac{f}{2/T}\right) + \text{rect}\left(\frac{f}{4/T}\right)$





$y(f) = mf + q \qquad m = T \quad \rightarrow \quad y(f) = -Tf + 2$

$\left(\frac{1}{T}, 1\right) \quad \begin{cases} 1 = m \cdot \frac{1}{T} + q \\ 2 = 0 + q \end{cases} \quad m = -T \qquad (-Tf+2)^2 = T^2 f^2 + 4 - 4Tf$

$(0, 2)$

$$2\int_0^{1/T} T^2 f^2 - 4Tf + 4 \, df = 2 \cdot T \frac{f^3}{3}\Big|_0^{1/T} - 2 \cdot 4T \cdot \frac{f^2}{2}\Big|_0^{1/T} + 2 \cdot 4 f\Big|_0^{1/T}$$

$$2T \cdot \left(\frac{\frac{1}{T^3}}{3}\right) - 8T \cdot \left(\frac{\frac{1}{T^2}}{2}\right) + \frac{8}{T} = \frac{2}{3T} - \frac{4}{T} + \frac{8}{T} = \frac{14}{3T}$$

$$\frac{14}{3T} + \frac{2}{T} = \frac{20}{3T} = E_{hR}$$

3) $S_s(f) = \frac{1}{T} \bar{S}_x(f) |P(f)|^2$ ma essendo equiprobabili, ind. e antipodali uso questa

$$S_s(f) = \frac{\sigma_x^2}{T} |P(f)|^2$$

$\sigma_x^2 = E[x^2] - \eta_x^2$

$E[x^2] = 1 \qquad \eta_x = \frac{1}{2} \cdot -1 + \frac{1}{2} \cdot 1 = 0 \qquad \sigma_x^2 = 1$

$$S_s(f) = \boxed{\frac{1}{T} \operatorname{rect}\left(\frac{f}{4/T}\right)}$$

4) ASSENZA ISI

$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$H(f) = p(f) H_R(f) = H_R(f)$

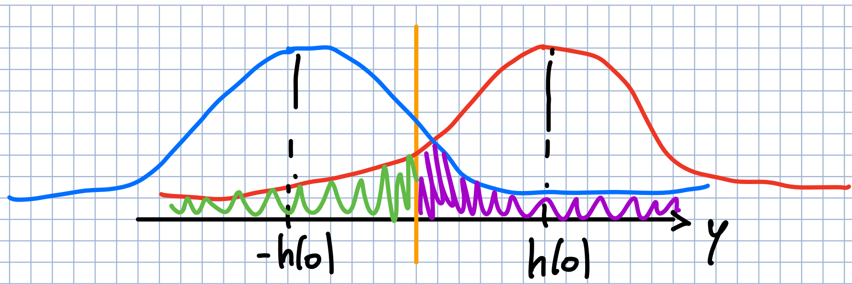
$H_R(f) = \left(1 - \frac{|f|}{1/T}\right) \operatorname{rect}\left(\frac{f}{2/T}\right) + \operatorname{rect}\left(\frac{f}{4/T}\right)$

$h(t) = \text{ATCF}[H_R(f)] = \frac{1}{T} \operatorname{sinc}^2\left(\frac{1}{T} \cdot t\right) + \frac{4}{T} \operatorname{sinc}\left(\frac{4}{T} t\right)$

$h(nT) = \frac{1}{T} \operatorname{sinc}^2(n) + \frac{4}{T} \operatorname{sinc}(4n) = \frac{1}{T} \delta[n] + \frac{4}{T} \delta[n] = \frac{5}{T} \delta[n]$

$h(0) = \frac{5}{T}$

$P_E(b) = P\{\hat{x} = -1 | x = 1\} P\{x = 1\} + P\{\hat{x} = 1 | x = -1\} P\{x = -1\}$



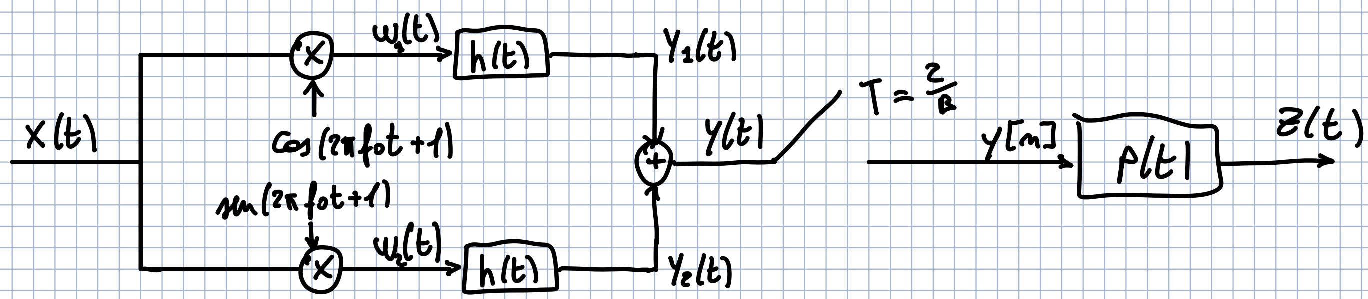
$$P_E(b) = \frac{1}{2} Q\left(\frac{\frac{S}{T}}{\sqrt{\frac{10N_0}{3T}}}\right) + \frac{1}{2} Q\left(\frac{\frac{S}{T}}{\sqrt{\frac{10N_0}{3T}}}\right) = Q\left(\frac{\frac{S}{T}}{\sqrt{\frac{10N_0}{3T}}}\right)$$

#1 7/02/19

$$x(t) = B\cos(2\pi f_0 t)\,\mathrm{sinc}(Bt) = \overbrace{B\,\mathrm{sinc}(Bt)}^{X_0(t)}\cos(2\pi f_0 t)$$

$$h(t) = B\,\mathrm{sinc}^2(Bt)$$

$$p(t) = B\,\mathrm{sinc}(Bt) \qquad \frac{1}{T} = f_0$$



Calcolare: 1) $z(t)$ 2) E_z 3) il valore di φ : E_z sia max

$$w_1(t) = x(t)\cos(2\pi f_0 t + \varphi)$$

$$y_1(t) = w_1(t) \otimes h(t)$$

$$Y_1(f) = W_1(f)\,H(f)$$

$$W_1(f) = \frac{e^{j\varphi}}{2} X(f-f_0) + \frac{e^{-j\varphi}}{2} X(f+f_0)$$

$$X_0(f) = \mathrm{rect}\left(\frac{f}{B}\right)$$

$$X(f) = \frac{1}{2}\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f+f_0}{B}\right)$$

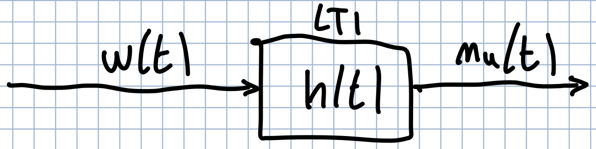
$$W_1(f) = \frac{e^{j\varphi}}{4}\mathrm{rect}\left(\frac{f-2f_0}{B}\right) + \frac{e^{-j\varphi}}{4}\mathrm{rect}\left(\frac{f+2f_0}{B}\right) \qquad \frac{B}{2} = f_0$$

$$H(f) = \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

DA FINIRE

1   20/02/18

$W(t)$ stazionario bianco in banda B con $S_w(f) = \frac{N_0}{2} \operatorname{rect}\left(\frac{f}{2B}\right)$



$$h(t) = \delta(t) + \delta(t-T)$$

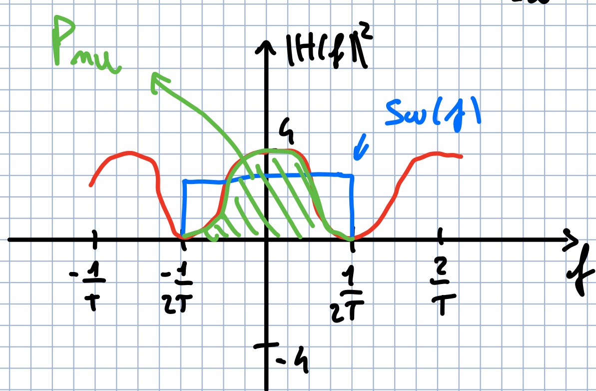
$$H(f) = 1 + e^{-j2\pi fT} = 2e^{-j\pi fT}\left(\frac{e^{j\pi fT} + e^{-j\pi fT}}{2}\right) = 2\cos(\pi fT)e^{-j\pi fT}$$

$$|H(f)| = 2\cos(\pi fT)$$

$$\angle H(f) = -\pi fT$$

2) $P_{mu} = ?$ $B = \frac{1}{2T}$

$$S_{mu} = S_w(f)\,|H(f)|^2 \qquad P_{mu} = \int_{-\infty}^{+\infty} S_{mu}(f)\,df$$



$$|H(f)|^2 = 4\cos^2(\pi fT)$$

$\pi fT = 1 \rightarrow f = 0$

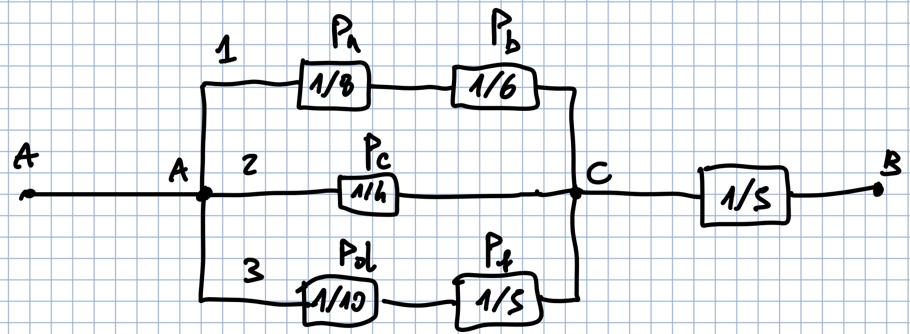
$\pi fT = -1 \rightarrow f = \frac{1}{T}$

$\pi fT = 0 \rightarrow f = -\frac{1}{2T}$

$\pi fT = 0 \rightarrow f = \frac{1}{2T}$

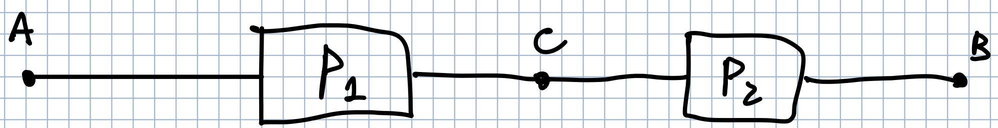
$$2N_0\int_{-\frac{1}{2T}}^{1/2T} \cos^2(\pi fT) = 2N_0\frac{1}{2}\int_{-\frac{1}{2T}}^{1/2T} 1\,df + 0 = 2N_0\cdot\frac{1}{2}\left(\frac{1}{2T}+\frac{1}{2T}\right) = 2N_0\frac{1}{2}\cdot\frac{1}{T} = \frac{N_0}{T}$$

#1 20/09/19



RAMO 1

A {almeno 1 relè guasto} B {1 relè funziona }



$P_1 = P_{11} \cdot P_{12} \cdot P_{13} =$

$P_{11} = 1 - [(1-P_a)(1-P_b)] = 0,27$

$P_{12} = 1 - [(1-P_c)] = 1 - 1 + P_c = P_c = 0,25$

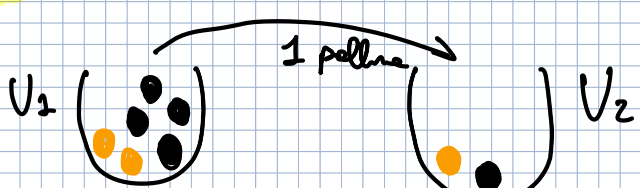
$P_{13} = 1 - [(1-P_d)(1-P_f)] = 0,28$

$P_1 = 0,0189$ → Prob. che non funzioni

$P_2 = \frac{1}{5}$ → prob. che non funzioni

$P_{AB} = (1-P_1)(1-P_2) = 0,785$ → prob. che funzioni

#2 FILA B 29/4/19



Estraiamo 1 pallina dalla seconda.

1) Prob. che la pallina da U_2 sia ARANC.

$A_1 = \{$ pallina estratta da U_1 sia arancione $\}$

$A_2 = \{$ pallina estratta da U_2 sia ARANC. $\} = ?$

1) $P(A_2) = P(A_2 | A_1)\, P(A_1) + P(A_2 | \overline{A_1})\, P(\overline{A_1}) = \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3} = \frac{4}{9}$

$P(A_2 | A_1) = \frac{2}{3}$

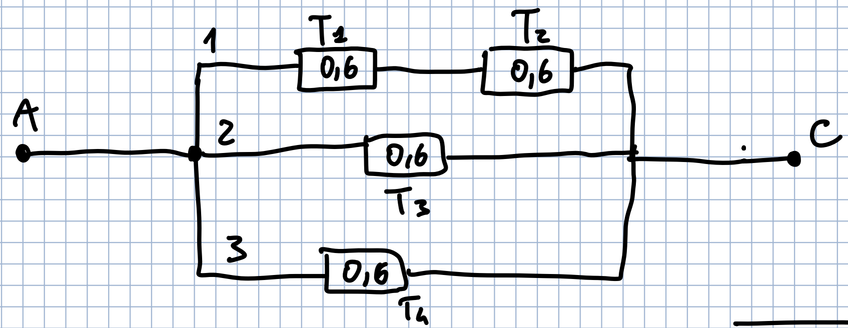
$P(A_1) = \frac{1}{3}$

$P(A_2 | \overline{A_1}) = \frac{1}{3}$

$P(\overline{A_1}) = \frac{2}{3}$

$$P(A_1 | A_2) = \frac{P(A_2 | A_1)\cdot P(A_1)}{P(A_2)} = \frac{\frac{\cancel{2}}{\cancel{3}}\cdot\frac{1}{\cancel{3}}}{\frac{4}{\cancel{9}}} = \frac{1}{2}$$

#1 FILA A 7/4/18



1) $\overline{T_{con.}} = T_{11}\cdot T_{12}\cdot T_{13} = 0{,}3024$ $\boxed{T_{con} = 1 - 0{,}30 = 0{,}7}$

$T_{11} = 1 - (1 - T_1)(1 - T_2) = 0{,}84$

$T_{12} = 1 - (1 - T_2) = 0{,}6$

$T_{13} = 1 - (1 - T_4) = 0{,}6$

2) P_{AB} → sapendo che T_1 è chiuso

$T_{con} = 1 - \overline{T_{CON}} = 1 - 0{,}216 = \boxed{0{,}78}$

$\overline{T_{CON}} = 0{,}6^3 = 0{,}216$

3) $A = \{$ terminali connessi $\}$ ⇒ $P(A) = 0{,}7$

$B = \{ T_3$ chiuso $\}$ ⇒ $P(B) = 0{,}4$

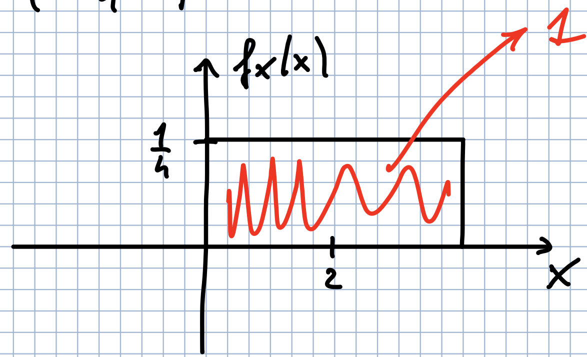
$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)} = \frac{0{,}49\cdot 0{,}4}{0{,}7} = 0{,}28$$

$$P(A|B) = 1 - (\overset{T_{11}}{0{,}84}\cdot\overset{T_{12}}{0{,}6}) = 0{,}49$$

#2

X V.A. UNIFORME $[0,4)$

$$f_X(x) = \frac{1}{b-a}\,\mathrm{rect}\left(\frac{x-\frac{b+a}{2}}{b-a}\right) = \frac{1}{4}\,\mathrm{rect}\left(\frac{x-2}{4}\right)$$



$$E[X^2] = \sigma_X^2 + \eta_X^2$$

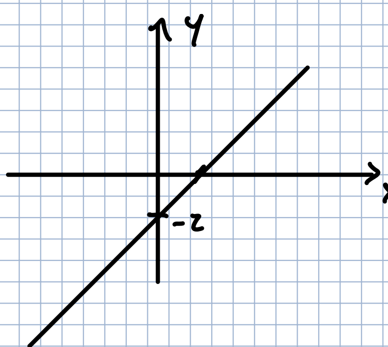
$$\eta_X = \int_0^4 x\, f_X(x)\, dx = 4$$

$$\frac{1}{4}\int_0^4 x\, dx = \frac{1}{4}\,\frac{x^2}{2}\Big|_0^4 = \frac{1}{4}\cdot\frac{16}{2} = 2 = \eta_X$$

$$E[X^2] = \int_0^4 x^2 f_X(x)\, dx = \frac{1}{4}\int_0^4 x^2\, dx = \frac{1}{4}\cdot\frac{x^3}{3}\Big|_0^4 = \frac{1}{4}\cdot\frac{64}{3} = \frac{16}{3}$$

$$\sigma_X^2 = E[X^2] - \eta_X^2 = \frac{16}{3} - 4 = \frac{4}{3}$$

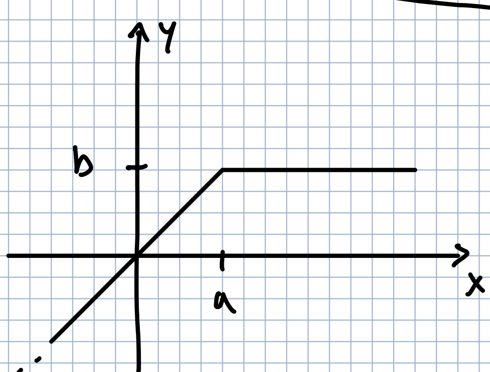
2) Ricavare DDP $f_Y(y)$ di $Y = 3X-2$ $\quad y = g(x) = 3x-2$



$$x = \frac{y+2}{3} = \frac{y}{3} + \frac{2}{3}$$

$$|g'(x)| = 3$$

$$f_Y(y) = \frac{f_X\left(\frac{y}{3}+\frac{2}{3}\right)}{\left|g'\left(\frac{y}{3}+\frac{2}{3}\right)\right|} = \frac{\frac{1}{4}\,\mathrm{rect}\left(\frac{x-2}{4}\right)}{3} = \boxed{\frac{1}{12}\,\mathrm{rect}\left(\frac{x-2}{4}\right)}$$



$$y = g(x) \qquad f_X = \frac{1}{\sqrt{2\pi\sigma_X^2}}\, e^{-\frac{x^2}{2\sigma_X^2}}$$

$$g(x) = \begin{cases} \frac{b}{a}x & x \le a \\ b & x > a \end{cases}$$

$$f_Y(y) = \frac{f_X(x)}{|g'(x)|}$$

$\boxed{y < b}$ $\quad Y = g(x) = \frac{b}{a} x \Rightarrow x = \frac{a}{b} y$

$$|g'(x)| = \frac{b}{a}$$

$\boxed{y = b}$ $\quad A\,\delta(y-b)$

$$A = \int_a^{+\infty} f_X(x)\,dx = Q\left(\frac{a}{\sigma_x}\right)$$

#1 $\quad$ 17/01/19

$\alpha \neq \beta$

$$h_1(t) = e^{-\alpha t} u(t)$$
$$h_2(t) = e^{-\beta t} u(t)$$

$$\xrightarrow{x(t)} \boxed{h_1(t)} \xrightarrow{y(t)} \boxed{h_2(t)} \xrightarrow{z(t)}$$

$$h(t) = h_1(t) \otimes h_2(t)$$
$$H(f) = H_1(f)\, H_2(f)$$

$$H_1(f) = \frac{1}{\alpha + j2\pi f} \qquad H_2(f) = \frac{1}{\beta + j2\pi f}$$

$$H(f) = \frac{1}{\alpha + j2\pi f} \cdot \frac{1}{\beta + j2\pi f}$$

2) $S_n(f) = \frac{N_0}{2}$

$$S_{nu}(f) = \frac{N_0}{2} |H(f)|^2 = \frac{N_0}{2} \frac{1}{\alpha^2 + 4\pi^2 f^2} \cdot \frac{1}{\beta^2 +}$$

#1 $\quad$ 13/01/20

$$C_{xx}(\tau) = A e^{-\alpha|\tau|} \cos(2\pi f_0 \tau) \qquad [0, 10]$$

$$f_X(x;t) = \frac{1}{10} \operatorname{rect}\left(\frac{x-5}{10}\right)$$

$S_X(f) = ?$

$$C_{xx}(\tau) = R_{xx}(\tau) - \eta_x^2(\tau) \Rightarrow R_{xx}(\tau) = C_{xx}(\tau) - \eta_x^2(\tau)$$

$$\eta_x = \int_0^{10} x\, f_x(x;\cancel{t})\, dx = \frac{1}{10}\frac{x^2}{2}\Big|_0^{10} = \frac{100}{20} = 5$$

$$R_{xx}(\tau) = A e^{-\alpha|\tau|} \cos(2\pi f_0 \tau) + 25$$

$$S_x(f) = TCF[R_{xx}(\tau)] \Rightarrow \frac{1}{2}X_0(f-f_0) + \frac{1}{2}X_0(f+f_0) + 25\delta(f)$$

$$X_0(f) = \frac{2\alpha A}{\alpha^2 + 4\pi^2 f^2}$$

$$S_x(f) = \frac{\alpha A}{\alpha^2 + 4\pi^2(f-f_0)^2} + \frac{\alpha A}{\alpha^2 + 4\pi^2(f+f_0)^2} + 25\delta(f)$$

#1 25/02/19

X V.A. ha DDP $\rightarrow f_x(x) = \lambda e^{-\lambda x} u(x)$ $\lambda > 0$

Y V.A. UNIFORME $[0,1]$ $f_Y(y) = rect\left(\frac{y-\frac{1}{2}}{1}\right)$

1) $Z = X + Y$

$$E[Z] = E[X] + E[Y] = \frac{1}{\lambda} + \frac{1}{2}$$

$$E[X] = \int_0^{+\infty} x \lambda e^{-\lambda x} u(x)\, dx = \lambda \int_0^{+\infty} x \cdot e^{-\lambda x}\, dx =$$

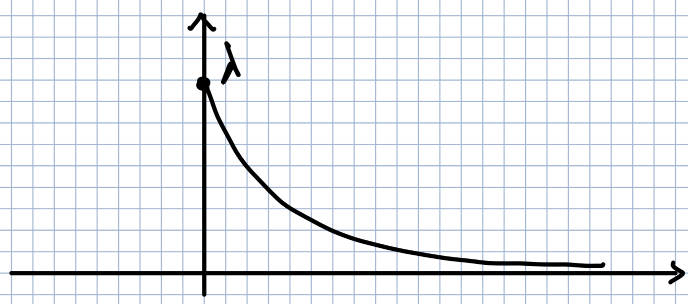
$$= \lambda\left[-\frac{1}{\lambda} e^{-\lambda x} x \Big|_0^{+\infty} + \frac{1}{\lambda}\int_0^{+\infty} e^{-\lambda x}\, dx\right] = \lambda\left[-\frac{1}{\lambda}\cdot\frac{1}{e^{\lambda x}}\cdot x\Big|_0^{+\infty} + \frac{1}{\lambda}\cdot -\frac{1}{\lambda}\cdot\frac{1}{e^{\lambda x}}\Big|_0^{+\infty}\right]$$

$$\lambda\left(0 - \left(-\frac{1}{\lambda^2}\right)\right) = \lambda \cdot \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

$$E[Y] = \int_0^1 y\, f_Y(y)\, dy = \int_0^1 y\, dy = \frac{y^2}{2}\Big|_0^1 = \frac{1}{2}$$

2) $f_Z(z) = f_X(z) \otimes f_Y(z)$

$$f_z(z) = \int_{-\infty}^{+\infty} f_x(t) \cdot f_y(z-t)\, dt$$



DA FINIRE

2 PAM in b.p.

$$s(t) = \sum_K x[K]\, p(t-KT) \cdot \cos\left(2\pi f_0 t + \frac{\pi}{3}\right)$$

$x[K] \in A_S = \{-1, 2\}$ IND. e EQUIPR.

$p(t) = 4B\,\mathrm{sinc}^2(2Bt) - B\,\mathrm{sinc}^2(Bt)$ $\quad f_0 \gg B \quad T = \frac{1}{B}$

$c(t) = \delta(t)$

$w(t)$ è GAUSS. BIANCO in banda

$h_R(t)$ è un filtro passa basso di banda $2B$

$\lambda = 0$

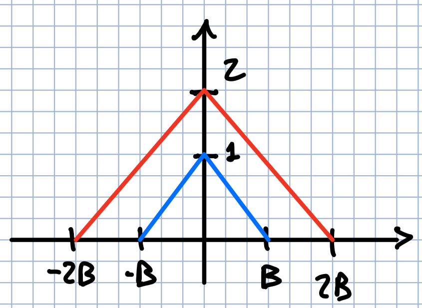
1) E_s 2) P_m 3) $P_E(b)$ in funzione di θ 4) θ minimo con min $P_E(b)$

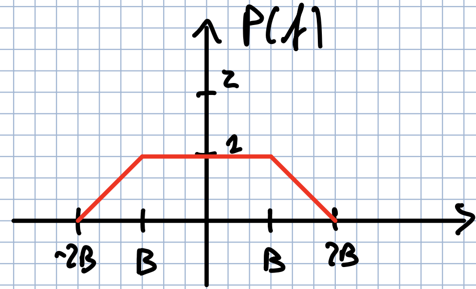
SVOLG.

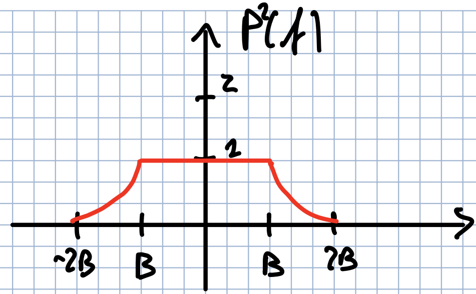
$$E_S = \frac{1}{2} E[x^2]\, E_P$$

$$E[x^2] = \frac{1}{2}\cdot(-1)^2 + \frac{1}{2}(2^2) = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$$

$$E_P = \int_{-\infty}^{+\infty} |P(f)|^2\, df \qquad P(f) = 2\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right) - \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$







$$2\left(\frac{1}{3}\cdot B\right) + 2B = \frac{2B}{3} + 2B$$

$$B\left(\frac{2}{3}+2\right) = \frac{8}{3}B = E_p$$

$$E_s = \frac{1}{\cancel{2}}\cdot\frac{5}{\cancel{2}}\cdot\frac{\cancel{8}}{3}B = \boxed{\frac{10}{3}B}$$

2) $P_{nu} = N_0 E_{hr} = \boxed{N_0 4B}$

$$H_R(f) = rect\left(\frac{f}{2B}\right) \qquad E_{hr} = 4B$$

3) $h'(t) = p(t)\otimes c(t)\otimes h_R(t) = p(t)\otimes h_R(t)$

$$H'(f) = P(f)\,H_R(f) = P(f)$$

$$h'(t) = p(t)$$

$$p(t) = 4B\,\mathrm{sinc}^2(2Bt) - B\,\mathrm{sinc}^2(Bt)$$

$$h'(nT) = h'\left(\frac{n}{B}\right) = 4B\,\mathrm{sinc}^2\left(\frac{2\cancel{B}n}{\cancel{B}}\right) - B\,\mathrm{sinc}^2\left(\frac{Bn}{B}\right) =$$

$$= 4B\,\mathrm{sinc}^2(2n) - B\,\mathrm{sinc}^2(n) = 4B\delta[n] - B\delta[n] =$$

$$= 3B\delta[n] \qquad h'(0) = 3B$$

$$h(t) = h'(0)\cos\left(\theta - \frac{\pi}{3}\right)$$

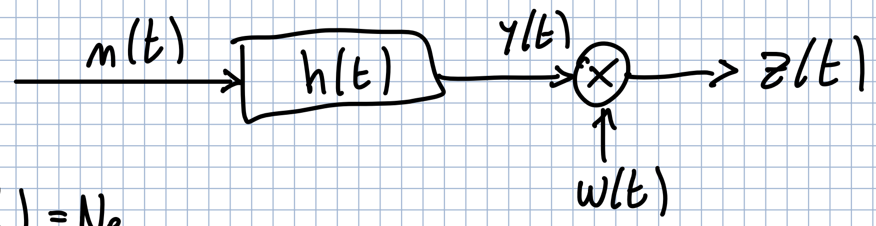
$$h(t) = 3B\cos\left(\theta - \frac{\pi}{3}\right)$$

$$P_E(b) = \frac{1}{2}Q\left(\frac{3B\cos(\theta-\frac{\pi}{3})}{\sqrt{N_0 4B}}\right) + \frac{1}{2}Q\left(\frac{6B\cos(\theta-\frac{\pi}{3})}{\sqrt{4N_0B}}\right)$$

4) si ha $P_E(b)$ minima quando l'argomento di Q è MAX

$$\cos\left(\theta-\frac{\pi}{3}\right) = 1 \Rightarrow \boxed{\theta = \frac{\pi}{3}}$$

1   6/02/17



$$h(t) = e^{-2t}\,u(t)$$

$$S_n(f) = \frac{N_0}{2}$$

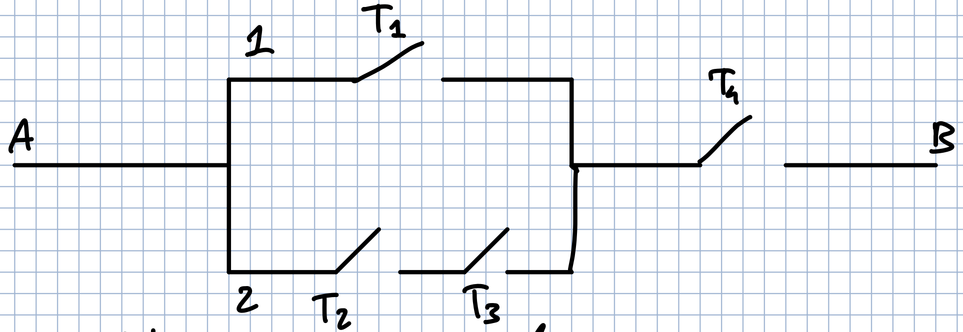
$$w(t) = A\sin(2\pi f_0 t)$$

A V.A. UNIFORME $[1, 2]$ $$f_A(A) = \mathrm{rect}\left(\frac{A-\frac{3}{2}}{1}\right)$$

$$S_y(f) = S_n(f)\,|H(f)|^2$$

$$H(f) = \frac{1}{2+j2\pi f} \qquad S_y(f) = \frac{N_0}{2}\,\frac{1}{4+4\pi^2 f^2}$$

1   12/04/19



$T_i \Rightarrow$ probabilità di essere aperto $= \frac{1}{2}$

1) Calcolare la prob. che A e B siano connessi quando sono tutti INDIP. tra loro

T_i

P_{T_i} = chiuso

P_S = serie chiuso

$\overline{P_{||}}$ = parallelo aperto

$P_{||}$ = parallelo chiuso

$$P_{AB} = P_{||}\cdot P_{T_4} = \frac{5}{8}\cdot\frac{1}{2} = \frac{5}{16}$$

$$P_{||} = 1 - \overline{P_{||}} = 1 - \frac{3}{8} = \frac{5}{8}$$

$$\overline{P_{||}} = \overline{P_{T_1}}\cdot\overline{P_S} = \frac{1}{2}\cdot\left(1-\frac{1}{4}\right) = \frac{3}{8}$$

$$P_S = P_{T_2}\cdot P_{T_3} = \frac{1}{4}$$

2) $P_{AB} = P_{||}\cdot P_{T_4} = \frac{1}{4}$ T_2 e T_3 chiusi

$$P_{11} = P_{T2} \cdot \overline{P_{T2}} + P_{T1} \cdot P_{T2} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

#2 26/06/18 (87)

$r(t) = \sum_i x[i]\,p(t-iT) + w(t)$

$x[i] \in A_s = [-1, 2]$ $\quad P(x=-1) = \frac{1}{3}$ $\quad P\{2\} = \frac{2}{3}$

$w(t)$ è GAUSSIANO con $\eta_w = 0$ e $S_w(f) = \frac{N_0}{2}$

$p(t) = \mathrm{sinc}\left(\frac{2}{T}t\right)$

$h_R(t) = \frac{2}{T}\mathrm{sinc}\left(\frac{2}{T}t\right) - \frac{1}{T}\mathrm{sinc}^2\left(\frac{t}{T}\right)$ $\qquad \lambda = 0$

1) E_s 2) $S_s(f)$ 3) P_{mu} 4) ASSENZA ISI 5) $P_E(b)$

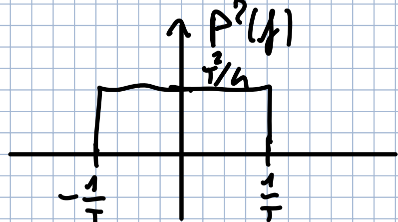
SVOLG.

1) $E_s = E[x^2]\,E_p = 3 \cdot \frac{T}{2} = \boxed{\frac{3T}{2}}$

$E[x^2] = \frac{1}{3}(-1)^2 + \frac{2}{3}(2)^2 = \frac{1}{3} + \frac{8}{3} = 3$

$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\,df$

$P(f) = \frac{T}{2}\,\mathrm{rect}\left(\frac{f}{2/T}\right)$



$E_p = \frac{2}{\cancel{T}} \cdot \frac{T^{\cancel{2}}}{\cancel{4}\,2} = \frac{T}{2}$

2) $S_s(f) = \frac{1}{T}\bar{S}_x(f)\,|P(f)|^2$

$\bar{S}_x(f) = \mathrm{TFS}\left[R_x[m]\right]$

$R_x[m] = C_x[m] + \eta_x^2$

$C_x[m] = \sigma_x^2\,\delta[m] + \eta_x^2$

$\eta_x = E[x] = \frac{1}{3}\cdot -1 + \frac{2}{3}\cdot 2 = -\frac{1}{3} + \frac{4}{3} = 1$

$E[x^2] = \sigma_x^2 + \eta_x^2 \Rightarrow \quad \sigma_x^2 = E[x^2] - \eta_x^2 = 3 - 1 = 2$

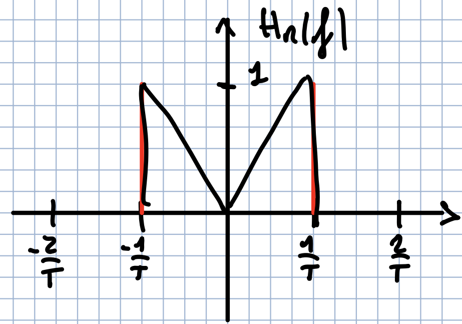
$C_x[m] = 2\,\delta[m] + 1$

$R_x[m] = 2\,\delta[m] + 1$

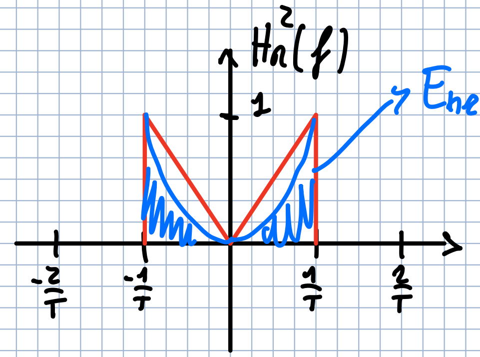
$$\tilde{S}_x(f) = \frac{2}{T}|P(f)|^2 + \frac{1}{T^2}\delta(f)|P(0)|^2$$

3) $P_{nu} = \frac{N_0}{2} E_{hR} = \frac{N_0}{2}\cdot\frac{2}{3T} = \boxed{\frac{N_0}{3T}}$ $\qquad h_R(t) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{2}{T}t\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{t}{T}\right)$

$$H_R(f) = \mathrm{rect}\left(\frac{f}{\frac{2}{T}}\right) - \left(1 - \frac{|f|}{1/T}\right)\mathrm{rect}\left(\frac{f}{2/T}\right)$$



$\Rightarrow$



$$2\cdot\left(\frac{1}{3}\cdot\frac{1}{T}\right) = \frac{2}{3T}$$

4) ASSENZA ISI

$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$H(f) = P(f)\,H_R(f) = \frac{T}{2}H_R(f)$

$h(t) = \frac{T}{2}h_R(t)$

$h(t) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{2}{T}t\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{t}{T}\right)$

$\frac{T}{2}\cdot h(nT) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{2}{T}nT\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{nT}{T}\right) = \frac{2}{T}\,\mathrm{sinc}(2n) - \frac{1}{T}\,\mathrm{sinc}^2(n) =$

$\frac{\cancel{T}}{2}\cdot\frac{1}{\cancel{T}}\,\delta[n] \qquad h(0) = \frac{1}{2}$

$\Downarrow$

ASSENZA ISI

5) $P_E(b) = P_E(\hat{x}=-1 \mid x=2)\,P(x=2) + P(\hat{x}=2 \mid x=-1)\,P(-1)$

$$P_E(b) = \frac{2}{3}Q\left(\frac{1}{\sqrt{\frac{N_0}{3T}}}\right) + \frac{1}{2}Q\left(\frac{1/2}{\sqrt{\frac{N_0}{3T}}}\right)$$

#2 6/02/17

PAM BASE

$r(t) = \sum_i x[i]\, p(t - iT) + w(t)$

$x[i] \in A_S = [-1, 2]$ INDIP. E EQUIPR.

$w(t)$ GAUSSIANO A $m_w = 0$ $S_w(f) = \frac{N_0}{2}$

$p(t) = \frac{2}{T}\,\text{sinc}^2\left(\frac{2}{T}t\right)\cos\left(\frac{4\pi t}{T}\right) \longrightarrow 2\pi f_0 t = \frac{4\pi t}{T}$

$f_0 = \frac{\cancel{4\pi t}}{\cancel{2\pi t}\, T} = \frac{2}{T}$

$h_R(t) = \frac{4}{T}\,\text{sinc}\left(\frac{4}{T}t\right)$ $\lambda = 0$

1) E_s 2) P_{mu} 3) $S_s(f)$ 4) ISI 5) P_E

SVOLG.

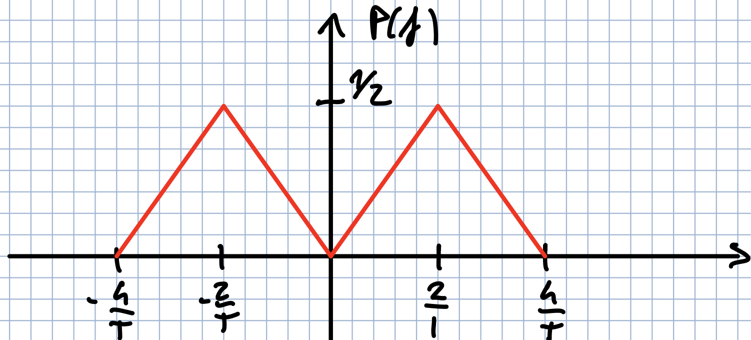
1) $E_s = E[x^2]\, E_p = \frac{5}{\cancel{2}} \cdot \frac{\cancel{2}}{3T} = \boxed{\frac{5}{3T}}$

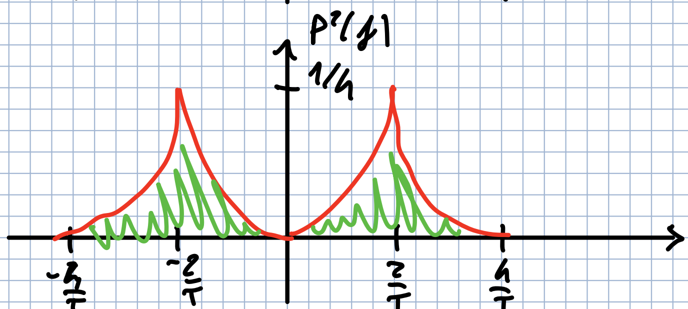
$E[x^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(2^2) = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

$P(f) = \frac{1}{2}P_0(f - f_0) + \frac{1}{2}P_0(f + f_0)$

$P_0(f) = \left(1 - \frac{|f|}{2/T}\right)\text{rect}\left(\frac{f}{4/T}\right)$

$P(f) = \frac{1}{2}\left(1 - \frac{|f - 2/T|}{2/T}\right)\text{rect}\left(\frac{f - 2/T}{4/T}\right) + \frac{1}{2}\left(1 - \frac{|f + 2/T|}{2/T}\right)\text{rect}\left(\frac{f + 2/T}{4/T}\right)$





$E_p = \cancel{4} \cdot \left(\frac{1}{3} \cdot \frac{2}{T} \cdot \frac{1}{\cancel{4}}\right) = \frac{2}{3T}$

2) $P_{mu} = \frac{N_0}{2} E_{h_R} = \boxed{\frac{N_0}{2} \cdot \frac{4}{T}}$ $h_R(t) = \frac{4}{T}\,\text{sinc}\left(\frac{4}{T}t\right)$ $H(f) = \text{rect}\left(\frac{f}{4/T}\right)$

$E_{hR} = \frac{4}{T}$

3) $S_s(f) = \frac{1}{T} \bar{S}_x(f) |P(f)|^2$

$\bar{S}_x(f) = TFS[R_x[m]]$

$R_x[m] = C_x[m] + \eta_x^2$

$\eta_x = E[x] = \frac{1}{2}(-1) + \frac{1}{2}(2) = -\frac{1}{2} + 1 = \frac{1}{2}$

$C_x[m] = E[(x[n] - \eta_x)(x[n-m] - \eta_x)] = E[(X - \eta_x)(Y - \eta_x)]$

I SIMBOLI SONO INDIP. $\Rightarrow$ QUINDI INCORRELATI

$C_x[m] = 0$ per $m \neq 0$ $\quad C[0] = \sigma_x^2$

$C_x[m] = \sigma_x^2 \delta[m]$

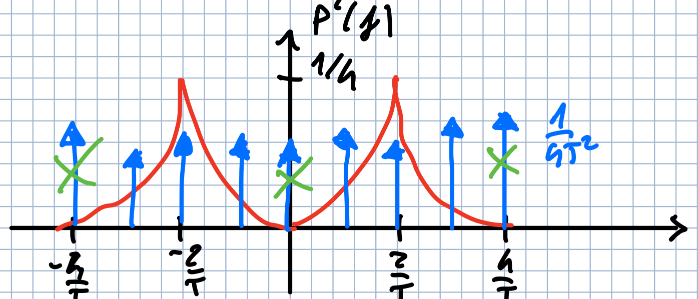
$\sigma_x^2 = E[x^2] - \eta_x^2 = \frac{5}{2} - \frac{1}{4} = \frac{9}{4}$

$R_x[m] = \frac{9}{4}\delta[m] + \frac{1}{4}$

$\bar{S}_x(f) = TFS[R_x[m]] = \sum_m R_x[m] e^{-j2\pi f m T} =$

$= \frac{9}{4} + \frac{1}{4} \cdot \frac{1}{T} \sum_m \delta\left(f - \frac{m}{T}\right)$

$S_s(f) = \left[\frac{9}{4T} + \frac{1}{4T^2} \sum_m \delta\left(f - \frac{m}{T}\right)\right] |P(f)|^2$

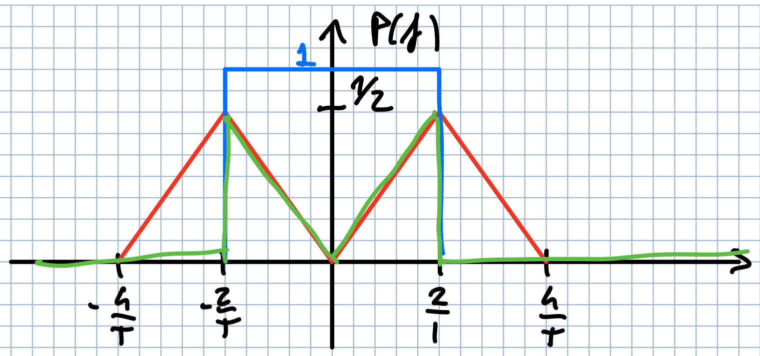


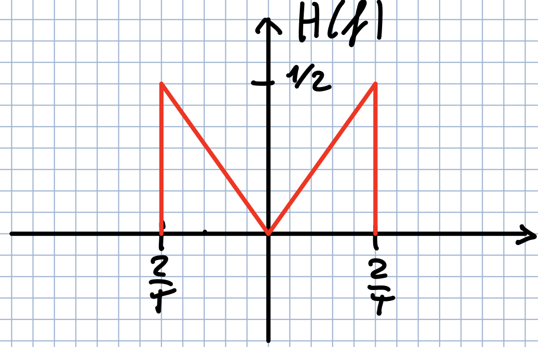
4) ASSENZA ISI $\quad \sum_m H\left(f - \frac{m}{T}\right) = K \quad h(nT) = K\delta[n]$

$K = h(0)$

$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$H(f) = P(f) H_R(f) =$





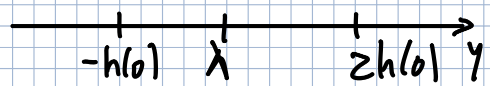
$$H(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{4/T}\right) - \frac{1}{2}\left(1 - \frac{|f|}{2/T}\right)\mathrm{rect}\left(\frac{f}{4/T}\right)$$

$$h(t) = \frac{1}{2}\cdot\frac{4}{T}\,\mathrm{sinc}\left(\frac{4}{T}t\right) - \frac{1}{2}\cdot\frac{2}{T}\,\mathrm{sinc}^2\left(\frac{2}{T}t\right) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{4}{T}t\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{2}{T}t\right)$$

$$h(nT) = \frac{2}{T}\,\mathrm{sinc}\left(\frac{4}{\cancel{T}}n\cancel{T}\right) - \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{2}{T}nT\right) = \frac{2}{T}\delta[n] - \frac{1}{T}\delta[n] = \frac{1}{T}\delta[n]$$

$$h(0) = \frac{1}{T}$$

5) $P_E(b)$



$$P_E(b) = \frac{1}{2}Q\left(\frac{2/T}{\sqrt{\frac{2N_0}{T}}}\right) + \frac{1}{2}Q\left(\frac{1/T}{\sqrt{\frac{2N_0}{T}}}\right)$$

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1 11/11/19

$Y(t) = X(t) + W(t)$

$X(t) = A$ $\quad$ A V.A. con $\eta_A = 0$ e σ_A^2

$W(t)$ rumore bianco in banda B INDIP. da A con $P_W = \sigma_W^2$

1) $Y(t)$ è SSL?

$$E[y(t)] = E[x(t) + w(t)] = E[x(t)] + \underbrace{E[w(t)]}_{0} = \overset{\eta_A}{0} + \overset{\eta_W}{0} \quad \checkmark$$

$$R_y(t_1,t_2) = E\left[\left(x(t_1) + w(t_1)\right)\left(x(t_2) + w(t_2)\right)\right] =$$

$$= E[x_1(t_1)\,x(t_2)] + \underbrace{E[x(t_1)\,w(t_2)]}_{0} + \underbrace{E[w(t_1)\cdot x(t_2)]}_{0} + E[w(t_1)\,w(t_2)]$$

$$= R_x(t_1,t_2) + R_w(t_1,t_2)$$

$R_Y(t_1,t_2) = R_X(t_1,t_2) + R_W(t_1,t_2)$

$R_X(\tau) = E[x(t_1)\,x(t_2)] = E[A \cdot A] = E[A^2] = \sigma_A^2 + \underbrace{m_X^2}_{=0} = \sigma_A^2$

$R_X(\tau) = \sigma_A^2 = \text{COST.}$

$R_W(\tau) = TCF[S_W(f)] =$ $\qquad P_W = \sigma^2 B =$

$= \frac{N_0}{\cancel{2}} \cdot \cancel{2}B \,\text{sinc}(2B\tau) = N_0 B \,\text{sinc}(2B\tau)$ $\qquad S_W(f) = \frac{N_0}{2}\,\text{rect}\left(\frac{f}{2B}\right)$

dipende da τ

$Y(t)$ è SSL

#2 11/11/19

QAM

$x_c \in A_S^c = (-2, 1)$

$x_s \in A_S^s = (-1, 1)$ SONO INDIP. con $P\{x_c = -2\} = \frac{1}{3}$

$p(f) = \sqrt{1 - |fT|}\,\text{rect}\left(\frac{f}{2/T}\right)$ $\quad f_0 \gg \frac{1}{T}$ $\quad P\{x_c = 1\} = \frac{2}{3}$

$c(t) = \delta(t)$ $\quad P\{x_s = -1\} = \frac{1}{2}$

$S_W(f) = \frac{N_0}{2}$ $\quad P\{x_s = 1\} = \frac{1}{2}$

$h_R(t) = p(t)$

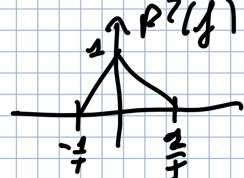
$\lambda = 0$

1) $E_s = \frac{1}{2}\left[E[x_c^2] + E[x_s]^2\right] E_p = \frac{1}{2} \cdot 3 \cdot \frac{1}{T} = \frac{3}{2T}$

$E[x_c^2] = \frac{1}{3} \cdot (-2)^2 + \frac{2}{3}(1)^2 = \frac{4}{3} + \frac{2}{3} = 2$

$E[x_s^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = \frac{1}{2} + \frac{1}{2} = 1$

$p^2(f) = \left(1 - \frac{|f|}{1/T}\right)\text{rect}\left(\frac{f}{2/T}\right)$



$E_p = \frac{2}{T} \cdot 1 \cdot \frac{1}{2} = \frac{1}{T}$

2) $P_{n_{UC}} = P_{n_{US}} = N_0 E_{h_R} = N_0 E_p = \boxed{\frac{N_0}{T}}$

3) ASSENZA ISI TEMPO

$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t)\, h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P^2(f)$$

OK !

$$h(t) = p^2(t) = \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{1}{T}t\right) = \frac{1}{T}\delta[n] \qquad \frac{1}{T} = h(0)$$

4) $P_E(b) = P_E^C(1 - P_E^S) + P_E^S(1 - P_E^C) + P_E^C \cdot P_E^S$

$$P_E^C = \frac{2}{3} Q\left(\frac{1/T}{\sqrt{\frac{N_0}{T}}}\right) + \frac{1}{3} Q\left(\frac{2/T}{\sqrt{\frac{N_0}{T}}}\right)$$

$$P_E^S = \frac{1}{2} Q\left(\frac{1/T}{\sqrt{\frac{N_0}{T}}}\right) + \frac{1}{2} Q\left(\frac{1/T}{\sqrt{\frac{N_0}{T}}}\right) = Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right)$$

#2 6/6/17

PAM b.b.

$r(t) = s(t) + w(t) \qquad s(t) = A_0 \sum_n x[n]\, p(t - nT)$

$x[k] \in A_S = (-1, 1)$ IND. EQUIPR. e ANTIPODI

$w(t)$ è GAUSS. $\eta_w = 0 \quad S_w(f) = \frac{N_0}{2}$

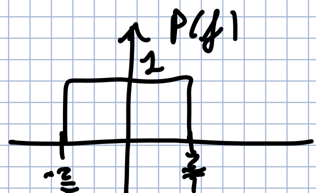
$c(t) = \delta(t)$

$p(t) = \frac{4}{T}\,\mathrm{sinc}\left(\frac{4}{T}t\right) \qquad H_R(f) = \left(1 - \frac{|f|}{1/T}\right)\mathrm{rect}\left(\frac{f}{2/T}\right) + \mathrm{rect}\left(\frac{f}{4/T}\right)$

$\lambda = 0$

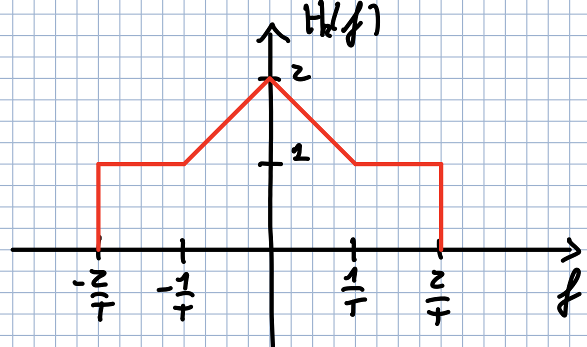
1) $E_s = E[x^2]\, E_p \cdot A_0^2 = \frac{4}{T} \cdot A_0^2$

$P(f) = \mathrm{rect}\left(\frac{f}{4/T}\right)$



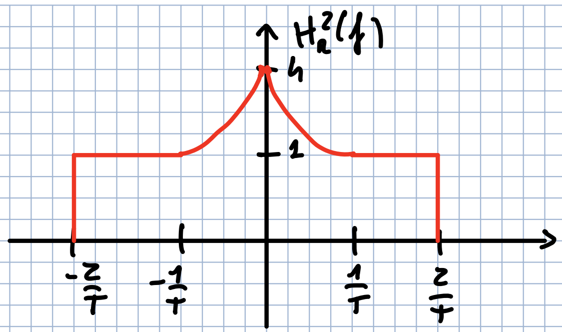
$E_p = \frac{4}{T}$

$E[x^2] = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$



2) $P_{n_u} = S_w(f)\,|H_R(f)|^2 = \frac{N_0}{2} E_{h_R}$

$$= \frac{20}{3T}\frac{N_0}{2} = \boxed{\frac{10 N_0}{3T}}$$



retta da 0 a $\frac{1}{T}$

$$h = mf + q = -Tf + 2$$

PASSA PER $\left(\frac{1}{T}, 1\right)$ $(0, 2)$ $\begin{cases} 1 = \frac{m}{T} + q \\ 2 = q \end{cases}$

$$(-Tf+2)^2 = \frac{1}{T^2}f^2 + 4 - \frac{4}{T}f$$

$$\int_0^{1/T} T^2 f^2 - 4Tf + 4 \, df = T^2 \frac{f^3}{3}\Bigg|_0^{1/T} - 4T\frac{f^2}{2}\Bigg|_0^{1/T} + 4f\Bigg|_0^{1/T} = \frac{\cancel{T^2}}{3}\cdot\frac{1}{T^3} - \frac{\cancel{4T}}{\cancel{2}}\cdot\frac{1}{T^2} + \frac{4}{T} =$$

$$= \frac{1}{3T} - \frac{2}{T} + \frac{4}{T} = \frac{7}{3T}$$

$$2\cdot\frac{7}{3T} + 2\cdot\frac{1}{T} = \frac{14}{3T} + \frac{2}{T} = \frac{20}{3T}$$

3) $S_s(f) = \frac{1}{T}\cdot\sigma_x^2 |P(f)|^2$

$E[x^2] = \sigma_x^2 + \eta_x^2$

$\sigma_x^2 = E[x^2] - \eta_x^2 = 1 - 0 = 1$

$E[s(t)] = 0$

$S_s(f) = \frac{1}{T}\, \mathrm{rect}\left(\frac{f}{4/T}\right)$

oppure se non mi ricordavo di usare questa formula

$S_s(f) = \frac{1}{T}\bar{S}_x(f)\,|P(f)|^2$

$\bar{S}_x(f) = TCF[R_x[m]]$

$R_x[m] = C_x[m] + \eta_x^2$

$\eta_x = \frac{1}{2}\cdot -1 + \frac{1}{2}\cdot 1 = 0$

$R_x[m] = C_x[m]$

$C_x[m] = E[x^2] - \eta_x^2 = 1 - 0 = 1$ per $m = 0$

$R_x[m] = 1\,\delta[m]$

$\bar{S}_x(f) = 1$ $\qquad S_s(f) = \frac{1}{T}\,\mathrm{rect}\left(\frac{f}{4/T}\right)$

4) ASSENZA ISI

$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = H_R(f)$$

$$H_R(f) = \left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right) + \mathrm{rect}\left(\frac{f}{4/T}\right)$$

$$h(t) = \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{1}{T}t\right) + \frac{4}{T}\,\mathrm{sinc}\left(\frac{4}{T}t\right)$$

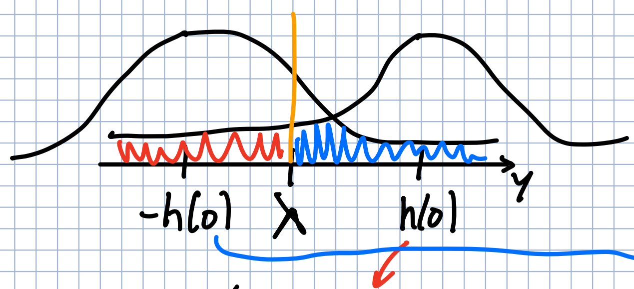
$$h(nT) = \frac{1}{T}\,\mathrm{sinc}^2\left(\frac{1}{T}nT\right) + \frac{4}{T}\,\mathrm{sinc}\left(\frac{4}{T}nT\right) = \frac{1}{T}\,\mathrm{sinc}^2(n) + \frac{4}{T}\,\mathrm{sinc}(4n) =$$

$$= \frac{1}{T}\delta[n] + \frac{4}{T}\delta[n] = \frac{5}{T}\delta[n] \qquad \frac{5}{T} = h(0)$$

$$\Downarrow$$

VERIFICATA ASSENZA ISI

5) $P_E(b)$



$$P_E(b) = \frac{1}{2} Q\left(\frac{5/T}{\sqrt{\frac{10 N_0}{3T}}}\right) + \frac{1}{2} Q\left(\frac{5/T}{\sqrt{\frac{10 N_0}{3T}}}\right) = Q\left(\frac{5/T}{\sqrt{\frac{10 N_0}{3T}}}\right)$$

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$U(t)$ un processo GAUSSIANO $m_u = 0$ e $R_u(\tau) = \sigma_u^2\, \mathrm{sinc}(2B\tau)$

1) Calcolare la V.A. $U = U(0)$ e ddp $f_u(u) = ?$