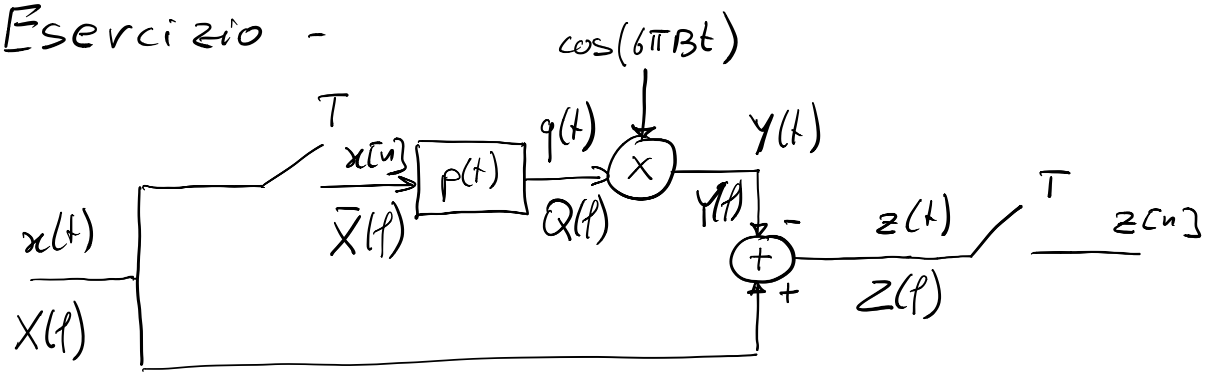
## Esercizio -




$$x(t) = 2AB\left[\text{sinc}(2Bt) + \text{sinc}^2(Bt)\cos(6\pi Bt)\right]$$

$$T = \frac{1}{3B}$$

$p(t)$ interpolatore cardinale di banda $B$

$$P(f) = \text{rect}\left(\frac{f}{2B}\right)$$

1) Calcolare e disegnare lo spettro di $x(t)$
2) Calcolare l'espressione analitica di $z(t)$
3) Calcolare e disegnare lo spettro di $z(t)$
4) Calcolare $E_z$ e $P_z$
5) Calcolare $z[n]$

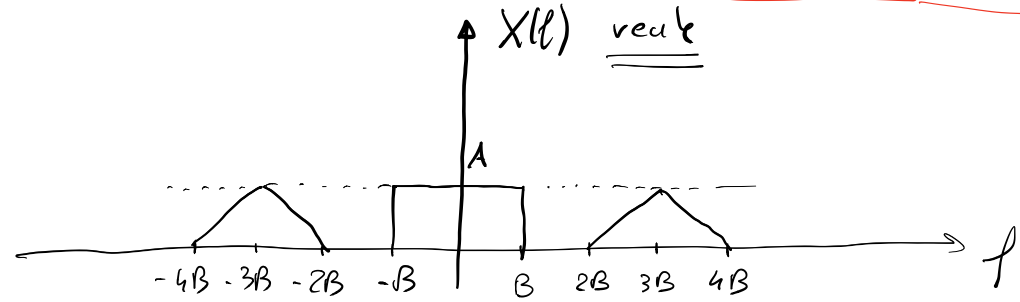
Svolgimento

$$\cos 2\pi \underset{3B}{f_0} t$$

1) $$x(t) = 2AB\left[\text{sinc}(2Bt) + \text{sinc}^2(Bt)\cos(6\pi Bt)\right]$$

$$X(f) = 2AB\left[\frac{1}{2B}\text{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\frac{1}{B}\left(1 - \frac{|f-f_0|}{B}\right)\text{rect}\left(\frac{f-f_0}{2B}\right) + \frac{1}{2}\frac{1}{B}\left(1 - \frac{|f+f_0|}{B}\right)\text{rect}\left(\frac{f+f_0}{2B}\right)\right]$$

$$= A\left[\mathrm{rect}\left(\frac{f}{2B}\right) + \left(1 - \frac{|f-3B|}{B}\right)\mathrm{rect}\left(\frac{f-3B}{2B}\right) + \left(1 - \frac{|f+3B|}{B}\right)\mathrm{rect}\left(\frac{f+3B}{2B}\right)\right]$$




2) $z(t) = ?$

$$z(t) = x(t) - y(t)$$

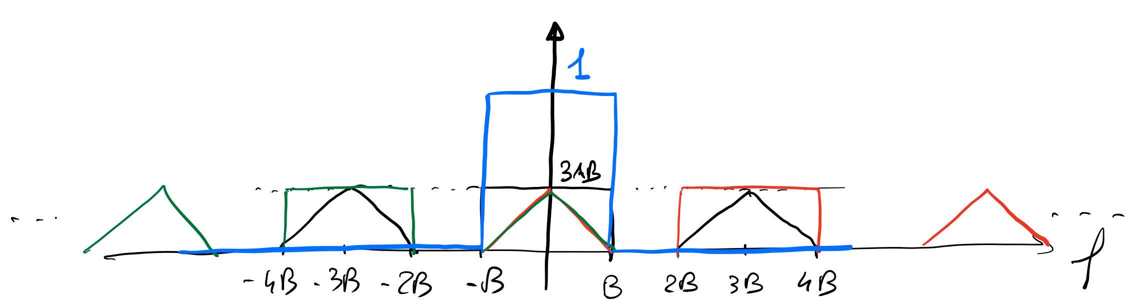
$$y(t) = q(t)\cos(6\pi B t)$$

$$q(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p(t-nT)$$

$$x[n] = x(nT)$$

$$\bar{X}(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X\left(f - \frac{n}{T}\right) \qquad \frac{1}{T} = 3B$$

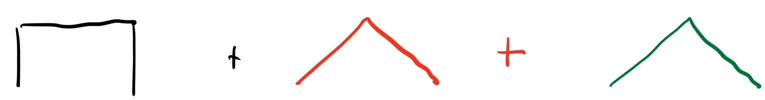
$$= 3B \sum_{n=-\infty}^{+\infty} X(f - 3Bn)$$

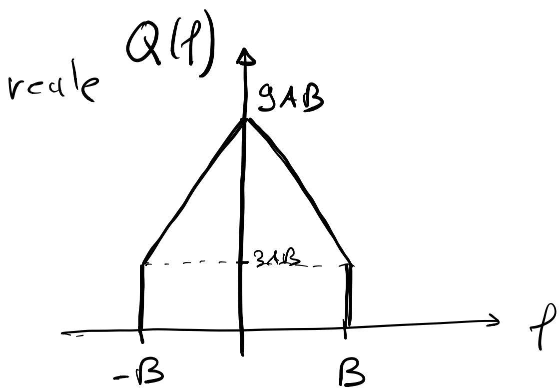


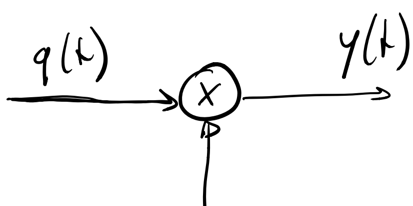
$$Q(f) = \tilde{X}(f)\, P(f)$$

$$= 3AB\left[\operatorname{rect}\left(\frac{f}{2B}\right) + 2\left(1 - \frac{|f|}{B}\right)\operatorname{rect}\left(\frac{f}{2B}\right)\right]$$

$$\shortparallel$$

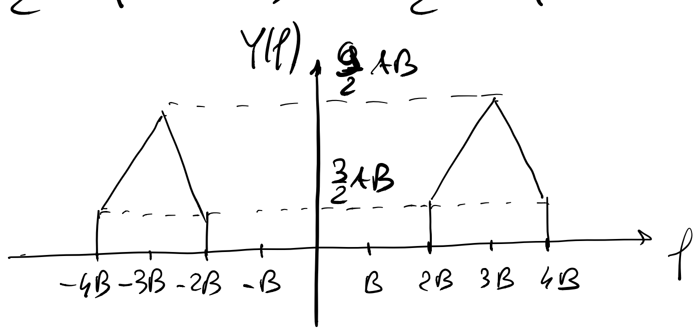


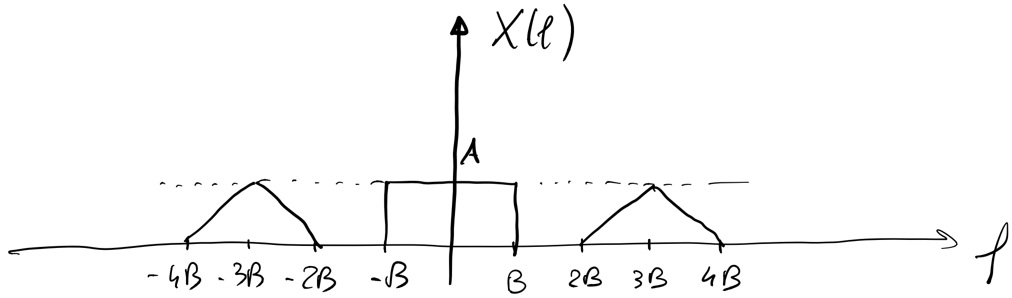



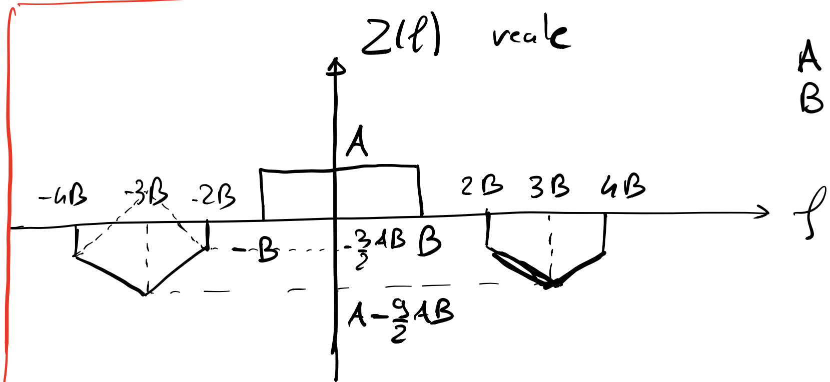




$$\cos(6\pi B t) = \cos(2\pi f_0 t) \quad , \quad f_0 = 3B$$

$$Y(f) = \frac{1}{2} Q(f - 3B) + \frac{1}{2} Q(f + 3B)$$






A > 1
B > 1

$$\boxed{Z(f) = X(f) - Y(f)}$$

$$Y(f) = \frac{3}{2}AB\left[\operatorname{rect}\left(\frac{f-3B}{2B}\right) + 2\left(1-\frac{|f-3B|}{B}\right)\operatorname{rect}\left(\frac{f-3B}{2B}\right)\right] + \frac{3}{2}AB\left[\operatorname{rect}\left(\frac{f+3B}{2B}\right) + 2\left(1-\frac{|f+3B|}{B}\right)\operatorname{rect}\left(\frac{f+3B}{2B}\right)\right]$$

$$z(t) = x(t) - y(t)$$

$$y(t) = q(t)\cos(6\pi Bt)$$

$$q(t) = \text{ATCF}\left[Q(f)\right] = 6AB^2\operatorname{sinc}(2Bt) + 6AB^2\operatorname{sinc}^2(Bt)$$

$$Q(f) = 3AB\operatorname{rect}\left(\frac{f}{2B}\right) + 6AB\left(1-\frac{|f|}{B}\right)\operatorname{rect}\left(\frac{f}{2B}\right)$$

$$y(t) = 6AB^2\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\right]\cos(6\pi Bt)$$

$$z(t) = 2AB\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\cos(6\pi Bt)\right] +$$

$$- 6AB^2\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\right]\cos(6\pi Bt)$$

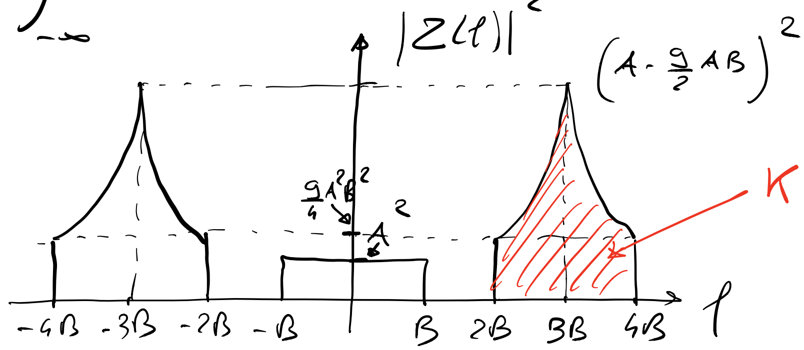
$$= \left[2AB - 6AB^2\cos(6\pi Bt)\right]\mathrm{sinc}(2Bt) + (2AB - 6AB^2)\cdot \mathrm{sinc}^2(Bt)\cos(6\pi Bt)$$

$$z[n] = z(nT) = \left[2AB - 6AB^2\cos\left(\cancel{6}^{2}\pi B\frac{n}{\cancel{3B}}\right)\right]\mathrm{sinc}\left(2B\frac{n}{3B}\right)$$

$$+ \left[2AB - 6AB^2\right]\mathrm{sinc}^2\left(B\frac{n}{3B}\right)\cos\left(\cancel{6}^{2}\pi B\frac{n}{\cancel{3B}}\right)$$

$$= \boxed{\left[2AB - 6AB^2\right]\mathrm{sinc}\left(\frac{2}{3}n\right) + \left[2AB - 6AB^2\right]\mathrm{sinc}^2\left(\frac{n}{3}\right)}$$

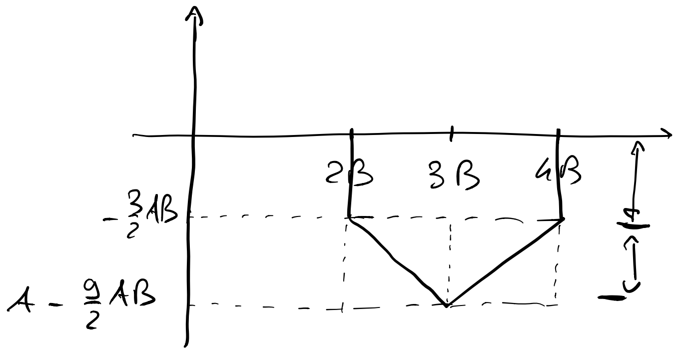
$$\cdot)\ E_Z = \int_{-\infty}^{+\infty} |Z(f)|^2\, df$$




$$E_Z = 2A^2B + 2A_K$$

$$A_K = \int_{2B}^{4B} K^2(f)\, df$$

$$K(f) = \underbrace{-\frac{3}{2}AB\,\mathrm{rect}\left(\frac{f}{2B}\right)}_{K_1(f)} + \underbrace{\left[A - \frac{9}{2}AB + \frac{3}{2}AB\right]\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)}_{K_2(f)}$$




$$K^2(f) = K_1^2(f) + K_2^2(f) + 2K_1(f)K_2(f)$$

$$A_K = \int_{-\infty}^{+\infty} K^2(f)\,df = \int_{-\infty}^{+\infty} K_1^2(f)\,df + \int_{-\infty}^{+\infty} K_2^2(f)\,df + 2\int_{-\infty}^{+\infty} K_1(f)K_2(f)\,df$$

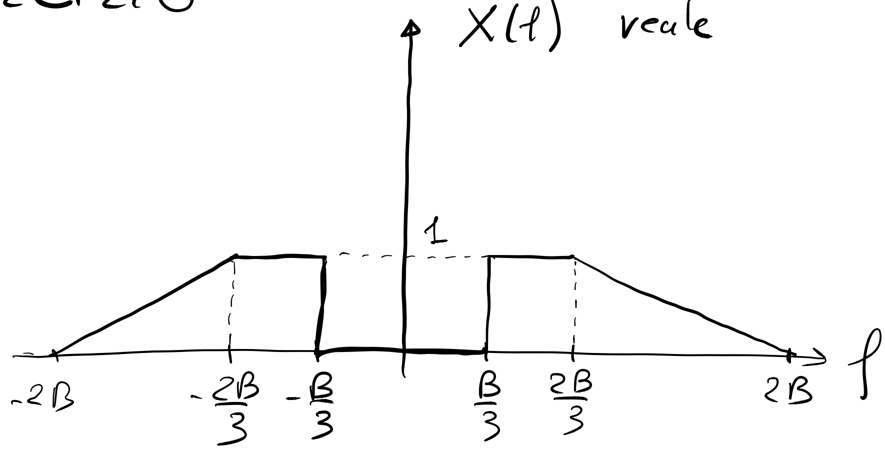
$$= \frac{9}{4}A^2B^2 \cdot 2B + \frac{1}{3}\cdot 2B \cdot \left(A - \frac{9}{2}AB\right)^2 + \frac{\cancel{2}}{\cancel{2}}\left(2B \cdot \left(A - \frac{9}{2}AB\right)\left(-\frac{3}{2}AB\right)\right)$$

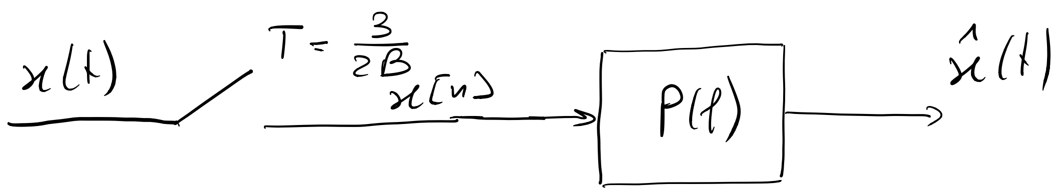
$$= \boxed{\frac{9}{2}A^2B^3 + \frac{2}{3}A^2B\left(1 - \frac{9}{2}B\right)^2 - 3BA\left(A - \frac{9}{2}AB\right)}$$

Energia finita $\Rightarrow$ Potenza media è nulla

$$P_z = 0$$

ESERCIZIO



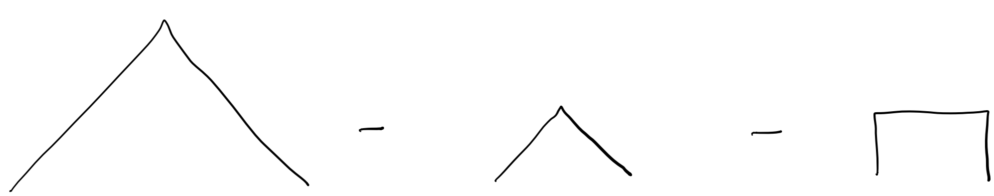

$$P(f) = \mathrm{sinc}^2\left(\frac{3fT}{2}\right)\left(1 + j2\pi f\right)$$

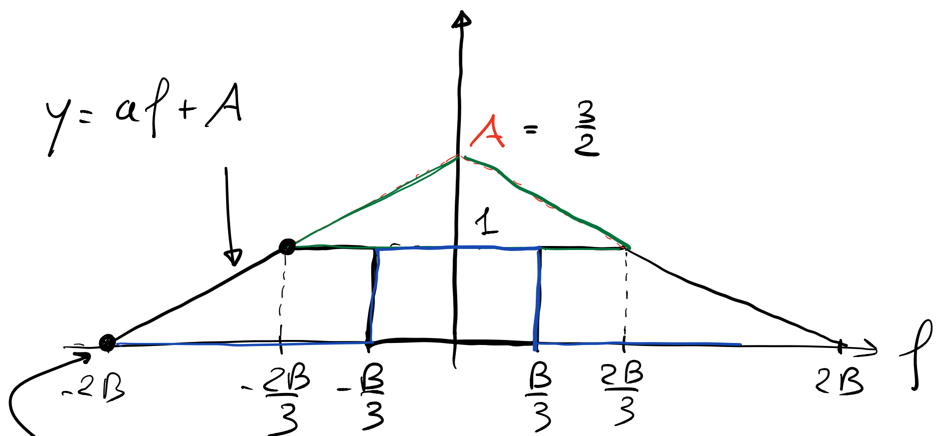
1) Calcolare $x(t)$, $x[n]$, $\hat{x}(t)$

Svolgimento

$$x(t) = \mathrm{ATCF}\left[X(f)\right]$$

I° strada: composizione di funzioni con TCF nota






$A = ?$

$$\begin{cases} a(-2B) + A = 0 \\ a\left(-\frac{2}{3}B\right) + A = 1 \end{cases} \Rightarrow A = 2Ba \Rightarrow a = \frac{A}{2B}$$

$$\frac{A}{\cancel{2B}}\left(-\frac{\cancel{2B}}{3}\right) + A = 1 \Rightarrow A\left(1 - \frac{1}{3}\right) = 1$$

$$A\,\frac{2}{3} = 1 \Rightarrow \boxed{A = \frac{3}{2}}$$

$$X(f) = X_1(f) + X_2(f) - X_3(f)$$

$$X_1(f) = \frac{3}{2}\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right)$$

$$X_2(f) = \frac{1}{2}\left(1 - \frac{|f|}{\frac{2}{3}B}\right)\mathrm{rect}\left(\frac{f}{\frac{4}{3}B}\right)$$

$$X_3(f) = \mathrm{rect}\left(\frac{f}{\frac{2}{3}B}\right)$$

$$x(t) = x_1(t) - x_2(t) - x_3(t)$$

$$= 3B\,\mathrm{sinc}^2(2Bt) - \frac{B}{3}\,\mathrm{sinc}^2\left(\frac{2}{3}Bt\right) - \frac{2}{3}B\,\mathrm{sinc}\left(\frac{2}{3}B\right)$$

$$x(t) \xrightarrow{T = \frac{3}{2B}} x[n] = x(nT) = x\left(n\frac{3}{2B}\right)$$

$$x[n] = 3B\,\mathrm{sinc}^2\left(\cancel{2B}\frac{3}{\cancel{2B}}n\right) - \frac{B}{3}\,\mathrm{sinc}^2\left(\frac{\cancel{2}B}{\cancel{3}}\frac{\cancel{3}}{\cancel{2B}}n\right) - \frac{2}{3}B\,\mathrm{sinc}\left(\frac{\cancel{2B}}{\cancel{3}}\frac{\cancel{3}n}{\cancel{2B}}\right)$$

$$= 3B\,\mathrm{sinc}^2(3n) - \frac{B}{3}\,\mathrm{sinc}^2(n) - \frac{2}{3}B\,\mathrm{sinc}(n)$$

$$= 2B\,\delta[n] = \begin{cases} 2B & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$\delta[n] \longrightarrow \boxed{P(f)} \longrightarrow \hat{x}(t)$$

$$\underline{\hat{x}(t)} = \sum_{n=-\infty}^{+\infty} \delta[n]\, p(t - nT) = \underline{p(t)}$$

$$p(t) = \mathrm{ATCF}\left[P(f)\right]$$

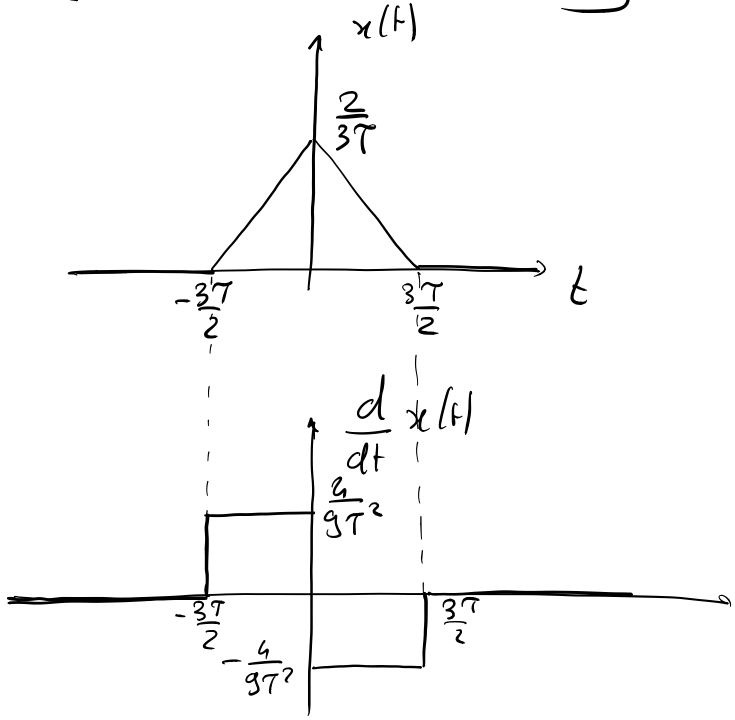
$$P(f) = \mathrm{sinc}^2\left(\frac{3fT}{2}\right)\left(1 + j2\pi f\right)$$

$$= \underbrace{\mathrm{sinc}^2\left(\frac{3fT}{2}\right)}_{X(f)} + \underbrace{j2\pi f}\ \underbrace{\mathrm{sinc}^2\left(\frac{3fT}{2}\right)}$$

$$j2\pi f\, X(f) \overset{TCF}{\longleftrightarrow} \frac{d}{dt}x(t)$$

$$p(t) = \underbrace{\frac{2}{3\tau}\left(1 - \frac{|t|}{\frac{3\tau}{2}}\right)\mathrm{rect}\left(\frac{t}{\frac{6\tau}{2}}\right)}_{x(t)} +$$

$$+ \frac{d}{dt}\left[\frac{2}{3\tau}\left(1 - \frac{|t|}{\frac{3\tau}{2}}\right)\mathrm{rect}\left(\frac{t}{\frac{6\tau}{2}}\right)\right]$$




$$\frac{\frac{2}{3\tau}}{\frac{3\tau}{2}} = \frac{4}{9\tau^2}$$

$$\frac{d}{dt}x(t) = \frac{4}{9\tau^2}\,\mathrm{rect}\left(\frac{t+\frac{3}{4}\tau}{\frac{3}{2}\tau}\right) - \frac{4}{9\tau^2}\,\mathrm{rect}\left(\frac{t-\frac{3}{4}\tau}{\frac{3}{2}\tau}\right)$$

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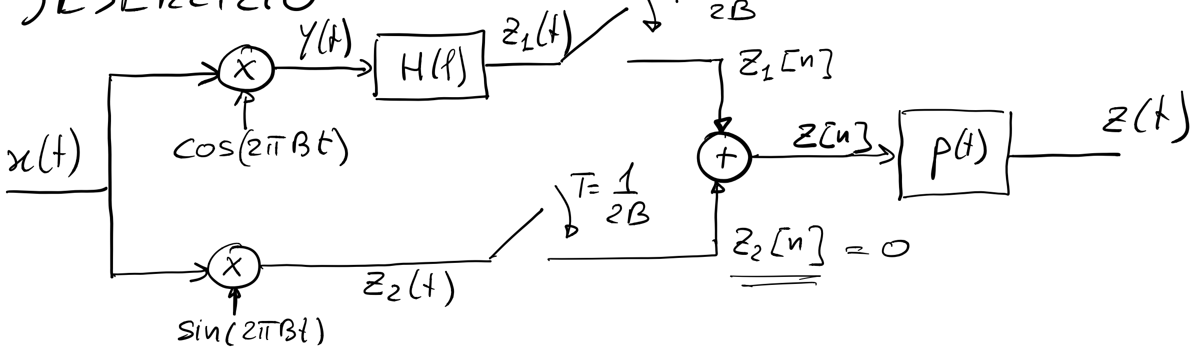


$x(t) = AB\,\text{sinc}^2(Bt)$

$p(t) =$ interpolatore cardinale di banda $B$

$H(f) =$ passa-basso ideale di banda $B$

1) Calcolare l'energia di $x(t)$

2) Disegnare lo spettro di $z_1(t)$

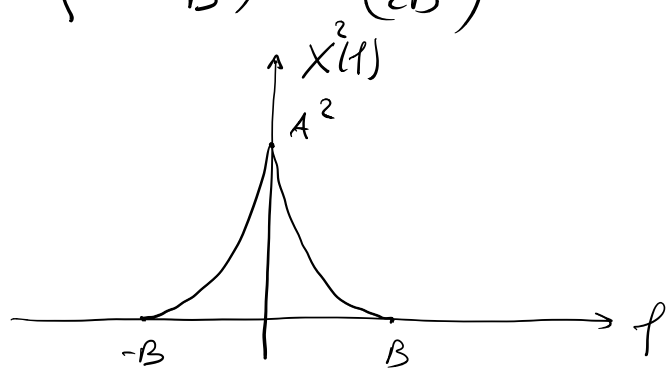
3) Determinare l'espressione analitica di $z(t)$

4) Calcolare l'energia di $z(t)$

Svolgimento

1) $E_x = \int_{-\infty}^{+\infty} |X(f)|^2\, df$

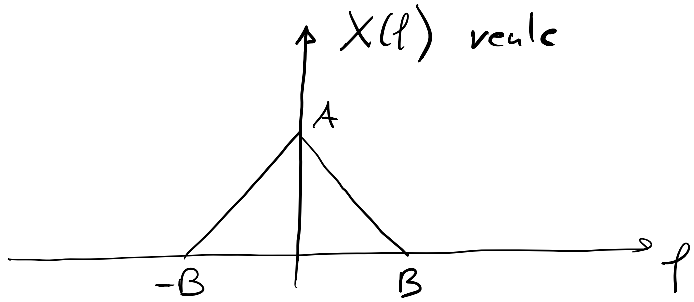
$$X(f) = A\left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$$

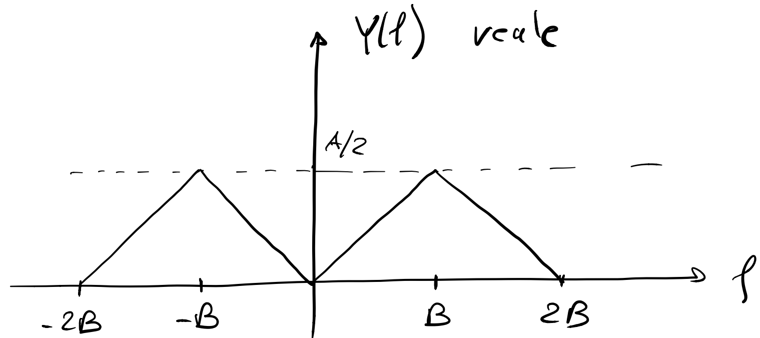


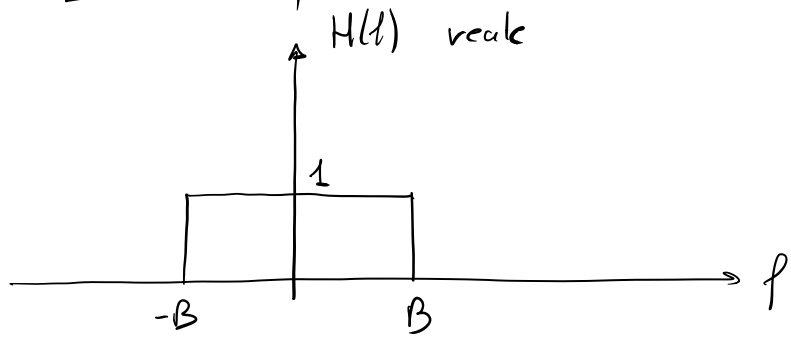

$$E_x = \frac{1}{3}\, 2BA^2 = \frac{2}{3} A^2 B$$

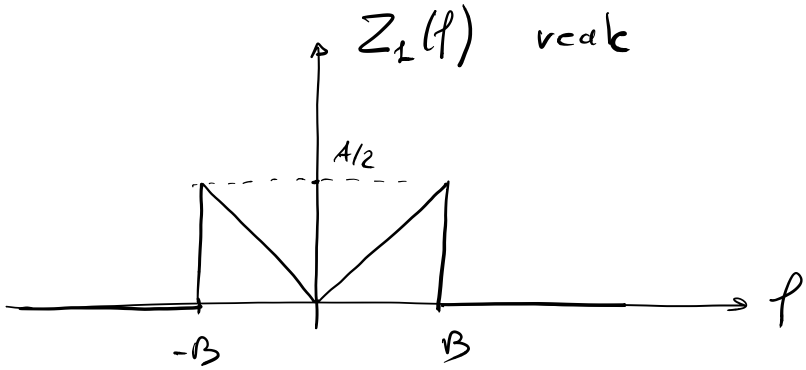
$$Z_1(f) = Y(f)\, H(f)$$

$$Y(f) = \frac{1}{2} X(f - B) + \frac{1}{2} X(f + B)$$



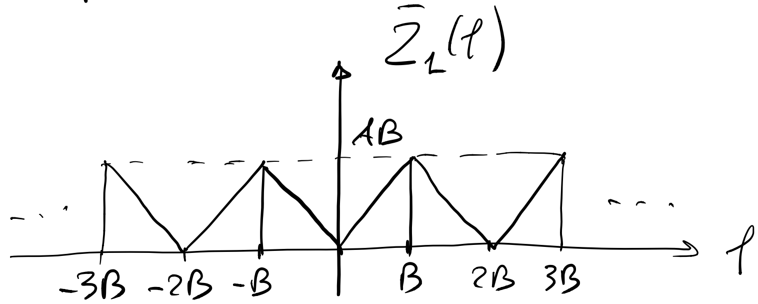

$$Z_1(f) = \frac{A}{2}\,\mathrm{rect}\left(\frac{f}{2B}\right) - \frac{A}{2}\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$z_1(t) = AB\,\mathrm{sinc}(2Bt) - \frac{AB}{2}\,\mathrm{sinc}^2(Bt) \qquad T = \frac{1}{2B}$$

$$z_1[n] = AB\,\mathrm{sinc}\left(2B\frac{n}{2B}\right) - \frac{AB}{2}\,\mathrm{sinc}^2\left(B\frac{n}{2B}\right)$$

$$= AB\,\mathrm{sinc}(n) - \frac{AB}{2}\,\mathrm{sinc}^2\left(\frac{n}{2}\right)$$

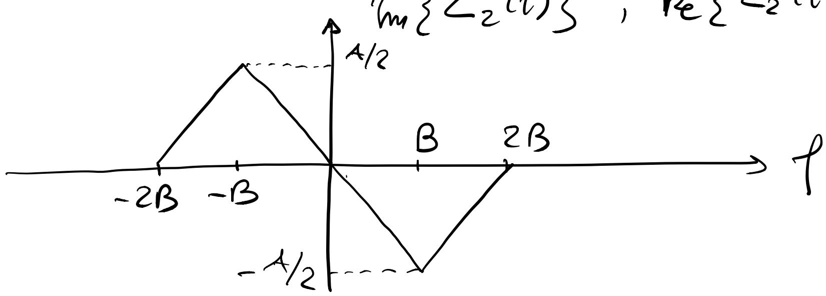
$$\bar{Z}_1(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} Z_1\left(f - \frac{n}{T}\right)$$




$$z_2(t) = x(t)\sin(2\pi B t)$$

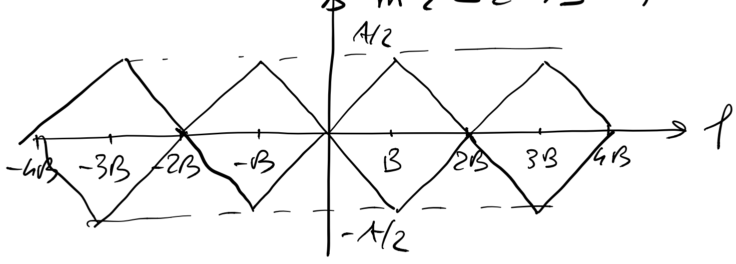
$$Z_2(f) = \frac{1}{2j}X(f-B) - \frac{1}{2j}X(f+B)$$

$\mathrm{Im}\{Z_2(f)\}$ , $\mathrm{Re}\{Z_2(f)\} = 0$




$$\bar{Z}_2(f) = \frac{1}{T}\sum_n Z_2\left(f - \frac{n}{T}\right) = 2B\sum_n Z_2(f - 2Bn) = 0$$

$\mathrm{Im}\{\bar{Z}_2(f)\}$ , $\mathrm{Re}\{Z_2(f)\} = 0$



$$z_2(t) = x(t)\sin(2\pi B t)$$

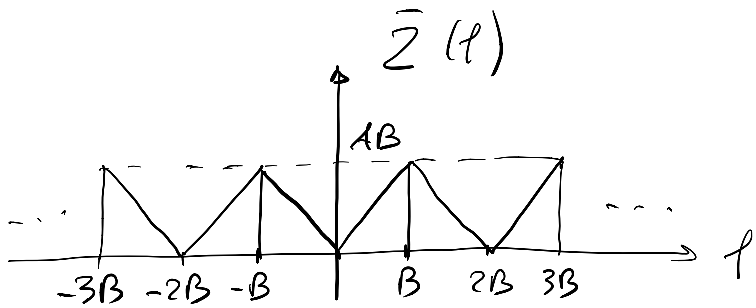
$$= AB\,\mathrm{sinc}^2(Bt)\sin(2\pi B t)$$

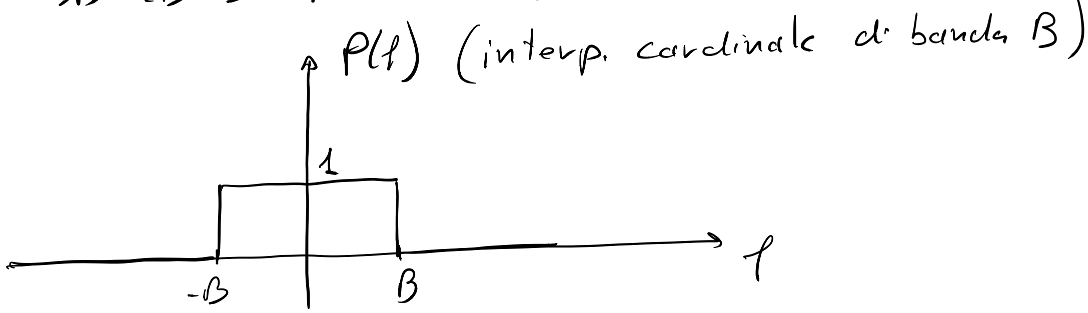
$$z_2[n] = z_2(nT) = z_2\left(\frac{n}{2B}\right) = AB\,\mathrm{sinc}^2\left(B\frac{n}{2B}\right)\sin\left(2\pi \cancel{B}\frac{n}{\cancel{2B}}\right)$$

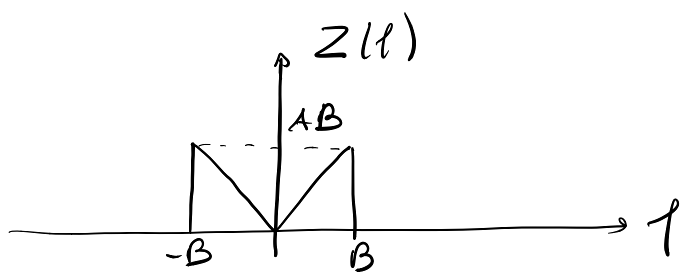
$$= AB\,\mathrm{sinc}^2\left(\frac{n}{2}\right)\underbrace{\sin(n\pi)}_{=0} = 0$$

$$z[n] = z_1[n]$$

$$\bar{Z}(f) = \bar{Z}_1(f)$$

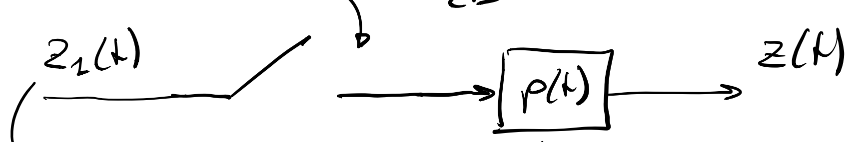


$$z(t) = \underset{\frac{1}{T}}{2B}\, z_1(t) = AB\,\mathrm{sinc}(2Bt) - \frac{AB}{2}\,\mathrm{sinc}^2(Bt)$$

$T = \frac{1}{2B}$ $\Leftarrow$ rispetta Nyquist II




segnale a banda rig. limitata B

I

interp cardinale di banda B

III

$\Downarrow$

applico il teorema del campionamento

$\Downarrow$

$$\boxed{z(t) = \frac{1}{T} z_1(t) = 2B\, z_1(t)}$$