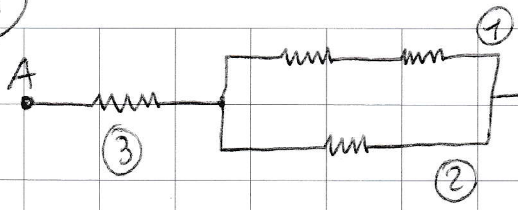
ES. 1



$$P_F^1 = P\{\text{ramo } 1 \text{ funziona}\} = (1-0,4)\cdot(1-0,15) = 0,51$$

$$P_G^1 = P\{\text{ramo } 1 \text{ è guasto}\} = 1 - P_F^1 = 0,49$$

$$P_G^{1//2} = P_G^1 \cdot P_G^2 = 0,49 \cdot 0,2 = 0,098$$

$$P_F^{1//2} = 1 - P_G^{1//2} = 0,902$$

$$P_F^{AB} = P_F^3 \cdot P_F^{1//2} = (1-0,3)\cdot 0,902 = 0,6314$$

ES. 2

$$F_X(x) = P\{X \leq x\} = \left(1 - e^{-\frac{x}{\lambda}}\right) u(x)$$

a) $P\{X > 10 \text{ minuti}\} = e^{-2} \Rightarrow 1 - F_X(10) = e^{-2}$

$$1 - \left(1 - e^{-\frac{10}{\lambda}}\right) = e^{-2}$$

$$e^{-\frac{10}{\lambda}} = e^{-2}$$

$$-\frac{10}{\lambda} = -2 \rightarrow \boxed{\lambda = 5 \text{ min}}$$

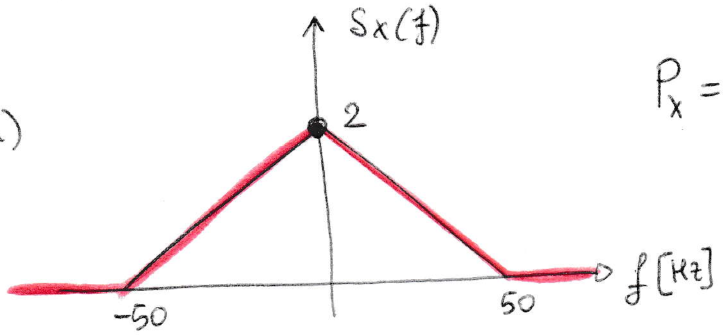
b) $P\{X \leq x_0\} = F_X(x_0) = 0,05 = \frac{1}{20}$

$$1 - e^{-\frac{x_0}{5}} = \frac{1}{20} \Rightarrow e^{-\frac{x_0}{5}} = \frac{19}{20} \Rightarrow -\frac{x_0}{5} = \ln\frac{19}{20}$$

$$x_0 = 5 \cdot \ln\left(\frac{20}{19}\right) \simeq 0,26 \text{ min} \simeq 15 \text{ secondi}$$

ES. 3

a)

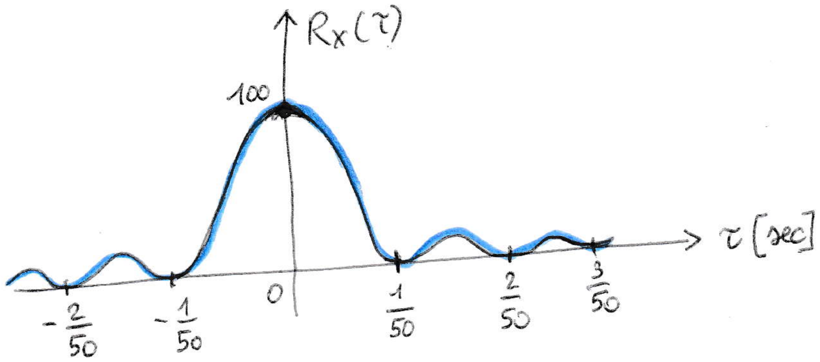


$$P_x = \frac{100 \cdot S_x(0)}{2} = 100$$

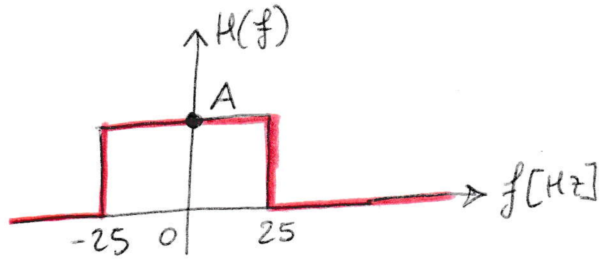
$$\downarrow$$

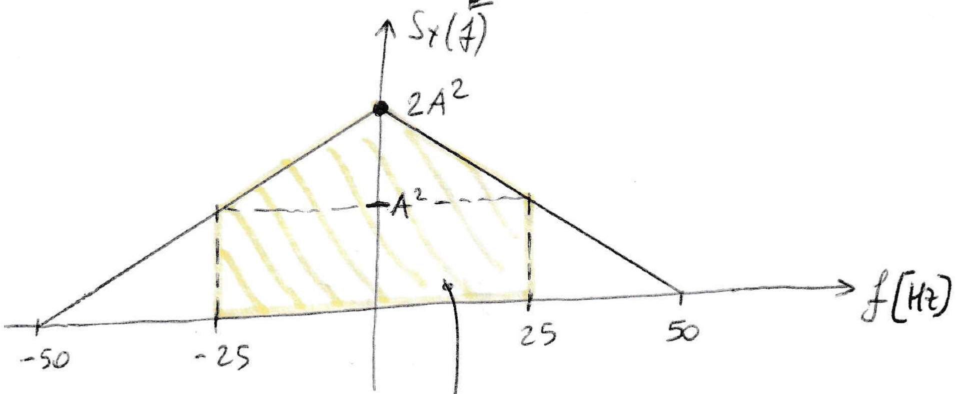
$$\underline{S_x(0) = 2}$$

b) $R_x(\tau) \rightleftharpoons S_x(f) \rightsquigarrow R_x(\tau) = 100 \cdot \mathrm{sinc}^2(50\tau)$



c) $S_Y(f) = S_x(f) \cdot |H(f)|^2$





$$P_Y = 2 \cdot \underbrace{\frac{(2A^2 + A^2)\cdot 25}{2}}_{\text{aree del trapezio}} = 75A^2$$

ESERCIZIO 4

a) $x(t) = e^{-\frac{t}{T}} u(t)$

Calcoliamo la trasformata di Fourier:

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-\frac{t}{T}}\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-\left(\frac{1}{T} + j2\pi f\right)t}\, dt$$

Tenuto conto che:

$$\boxed{\int_{0}^{\infty} e^{-at}\, dt = -\frac{1}{a}\, e^{-at}\Big|_{0}^{\infty} = -\frac{1}{a}(0-1) = \frac{1}{a}}$$

$$= \frac{1}{\frac{1}{T} + j2\pi f} = \frac{T}{1 + j2\pi T f}$$

b) $$X(f) = \frac{T}{1 + j\, f/f_T} \qquad f_T = \frac{1}{2\pi T}$$

$$|X(f)|^2 = \frac{T^2}{1 + \left(f/f_T\right)^2}$$

$$\boxed{\left. |X(f)|^2 \right|_{dB} = 10 \log_{10} \frac{|X(f)|^2}{|X(0)|^2}}$$

$$\left. |X(f)|^2 \right|_{dB} = -3\,dB \longrightarrow \frac{|X(f)|^2}{|X(0)|^2} = \frac{1}{2}$$

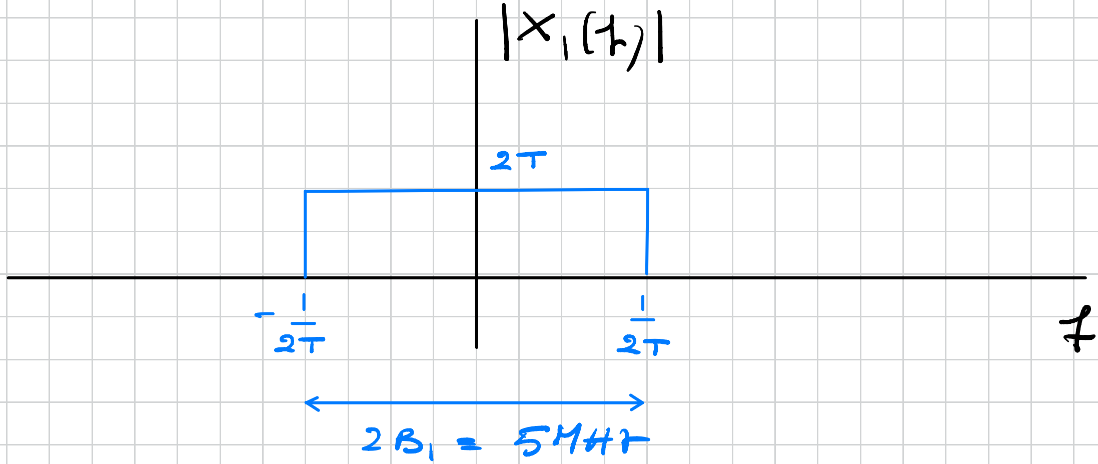
$$\frac{|X(f)|^2}{|X(0)|^2} = \left. \frac{1}{1 + \left(f/f_T\right)^2} \right|_{\boxed{f = f_T}} = \frac{1}{2}$$

Quindi $B_{-3dB} = f_T = \frac{1}{2\pi T} = \frac{1}{2\pi}\, kHz \cong 160\, Hz$

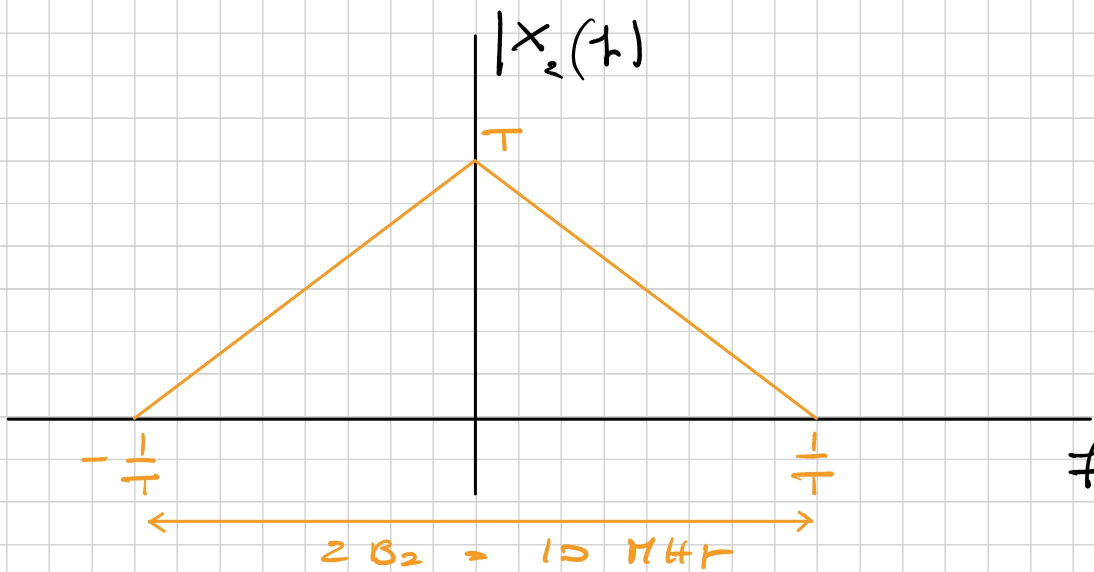
ESERCIZIO 5

a)

$$x_1(t) = \text{sinc}\left(\frac{t}{2T}\right) \rightleftharpoons X_1(f) = 2T\,\text{rect}(2fT)$$



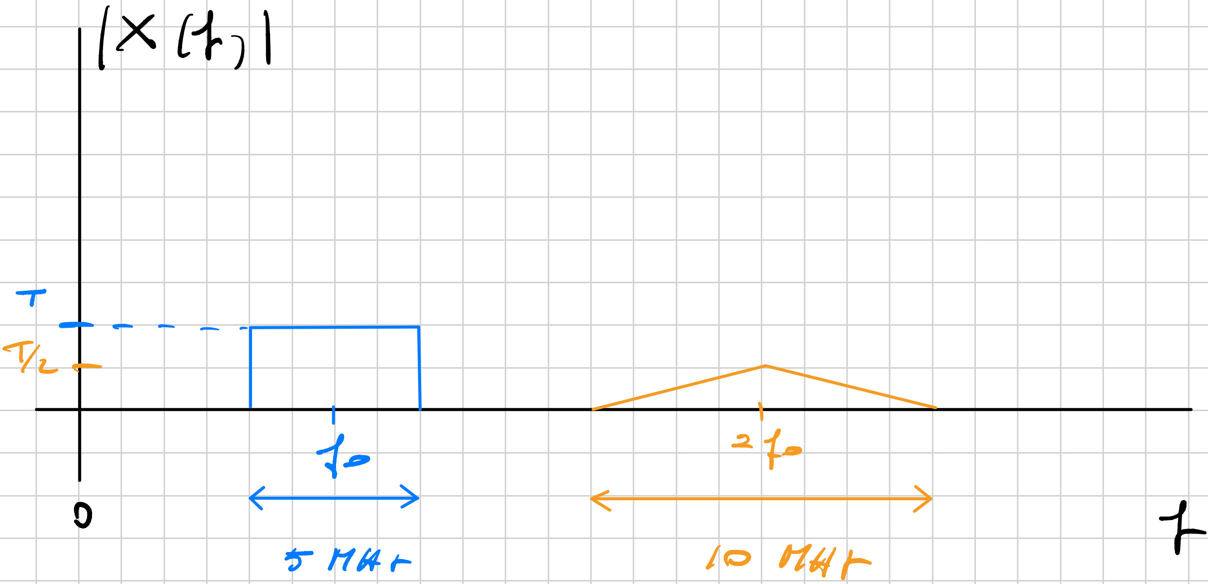
$$x_2(t) = \text{sinc}^2\left(\frac{t}{T}\right) \rightleftharpoons X_2(f) = T^2\,\text{rect}(fT) \otimes \text{rect}(fT$$



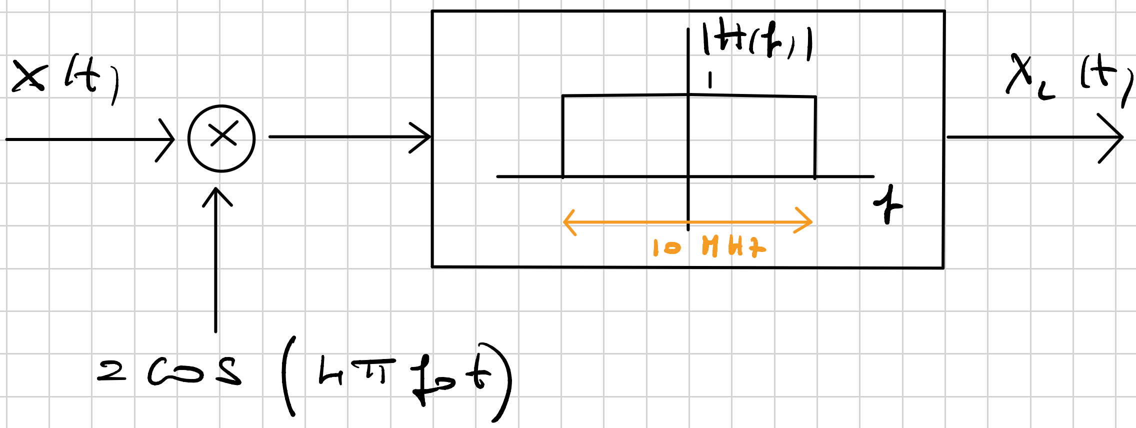
Tenuto di conto che:

$$X(f) = \frac{X_1(f+f_0) + X_1(f-f_0)}{2}$$

$$+ \frac{X_2(f+2f_0) + X_2(f-2f_0)}{2}$$



b) Per recuperare il segnale $x_2(t)$ devo demodularlo con un $\cos(\cdot)$ alla frequenza $2f_0$ e filtrarlo con un filtro passa-basso di banda 5 MHz



ESERCIZIO 6

La matrice controllo di parità è:

$$\underline{\underline{H}} = [\underline{\underline{P}}^T, \underline{\underline{I}}_3] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

La sindrome è pari a:

$$\boxed{\underline{s} = \underline{y}\,\underline{\underline{H}}^T = [0\ 1\ 0]}$$

Siccome $\underline{s} = \underline{y}\,\underline{\underline{H}}^T = \underline{e}\,\underline{\underline{H}}^T$ si trova

$$\boxed{\underline{e} = [0\ 0\ 0\ 0\ 1\ 0]}$$

da cui segue $\underline{y} + \underline{e} = [0\ 1\ 1\ 1\ 1\ 0]$

Esercizio 7

a) La densità spettrale di potenza è

$$S_s(f) = \frac{1}{T_s} S_a(f) |G_r(f)|^2$$

dove

$$S_a(f) = \sum_m R_a(m) e^{-j 2\pi f T_s}$$

$$R_a(m) = \begin{cases} \sigma_a^2 + \eta_a^2 & m = 0 \\ \eta_a^2 & m \neq 0 \end{cases}$$

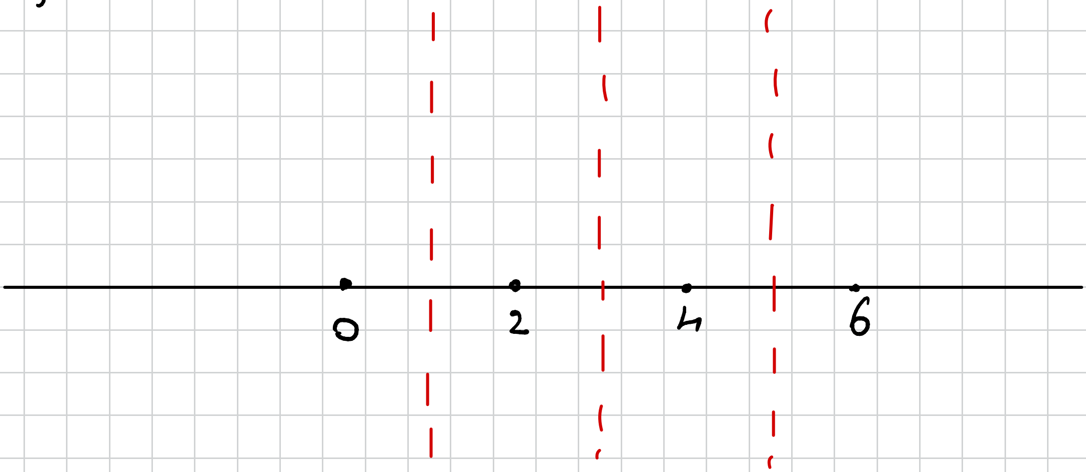
Calcoliamo η_a e σ_a^2:

$$\eta_a = E\{a_i\} = \frac{1}{4}(0 + 2 + 4 + 6) = 3$$

$$\sigma_a^2 = E\{a_i^2\} - \eta_a^2 = \frac{1}{4}(0^2 + 2^2 + 4^2 + 6^2) - 3^2$$

$$= \frac{56}{4} - 9 = 5$$

b)



La probabilità di errore è la stessa di una 4-PAM convenzionale dal momento che dipende da $Q\left(\frac{h-4}{\sigma}\right)$

Quindi:

$$P(e) = \frac{1}{4}\left(2\,P(e\,|\,c_k=2) + 2\,P(e\,|\,c_k=0)\right)$$

$$= \frac{1}{2}\left(2\,Q\left(\frac{1}{\sqrt{4}}\right) + Q\left(\frac{1}{\sqrt{4}}\right)\right) = \frac{3}{2}\,Q\left(\frac{1}{\sqrt{4}}\right)$$

ESERCIZIO 8

a) Tenuto conto che $B = \frac{1+\alpha}{T_S} = \frac{1+\alpha}{r \log_2 M} R_b$

$$\boxed{\eta = \frac{R_b}{B} = \frac{r \log_2 M}{1+\alpha} = \frac{5}{4} \qquad R_b = \eta B = 5 \text{ Mbit/s}}$$

Il tempo necessario per trasmettere il file è

$$\boxed{T_{FILE} = \frac{S_{FILE}}{R_b} = 0.2 \text{ s}}$$

b) La probabilità di errore in ingresso al codificatore è

$$P^{BIT}(e) = Q\left(\frac{1}{\overline{\Gamma_n}}\right) = Q\left(\sqrt{E_s/N_0}\right) = Q\left(\sqrt{\frac{E_b\, r \log_2 M}{N_0}}\right)$$

Se $\frac{E_b}{N_0} = 6 \text{ dB} = 4$ si ottiene $P^{BIT}(e) = Q\left(\sqrt{6}\right) \cong 6 \cdot 10^{-3}$