QAM (Quadrature Amplitude Modulation)

⇒ MODULAZIONE IN BANDA PASSANTE

$$s(t) = \sum_{n=-\infty}^{+\infty} \Big[x_c[n]\, p(t-nT_s) \cos(2\pi f_0 t) + \\ - x_s[n]\, p(t-nT_s) \sin(2\pi f_0 t) \Big]$$

componente in fase

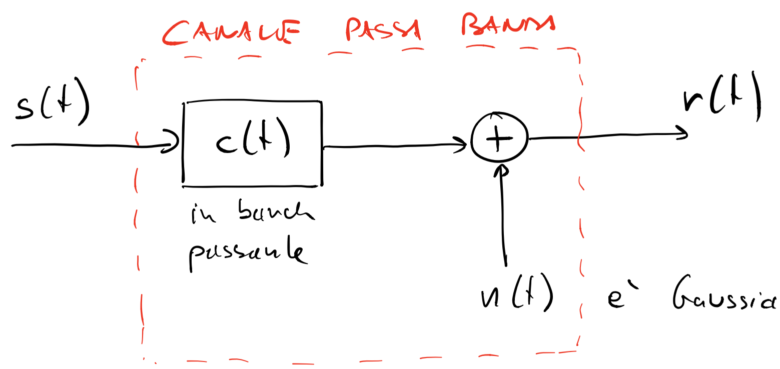
componente in quadratura

$x_c[n]$ simboli in fase

$x_s[n]$ " " quadratura

$$\hat{x}[n] = x_c[n] + j\, x_s[n]$$

$x_c[n]$ e $x_s[n]$ sono indipendenti tra di loro



$n(t)$ è Gaussiano bianco in banda

$$r(t) = s(t) + n(t)$$

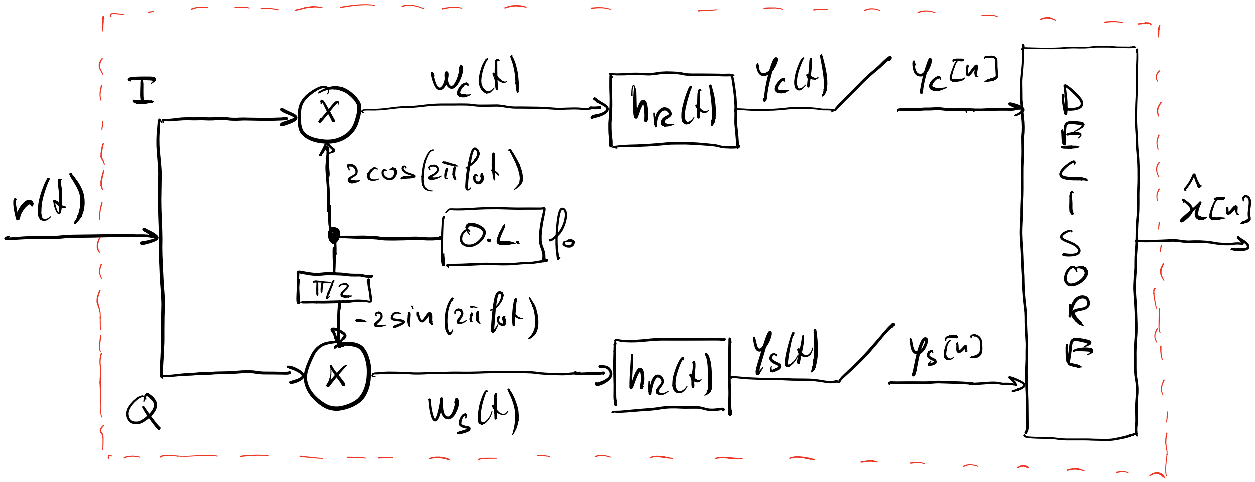
$$= \sum_{n=-\infty}^{+\infty} x_c(t)\, p(t-nT_s) \cos(2\pi f_0 t) + \underbrace{n_c(t) \cos(2\pi f_0 t)}$$

$$- \left[\sum_{n=-\infty}^{+\infty} x_s(t)\, p(t-nT_s) \sin(2\pi f_0 t) + \underbrace{n_s(t) \sin(2\pi f_0 t)} \right]$$

in fase → I

Q ← quadratura

$$n(t) = \underbrace{n_c(t) \cos(2\pi f_0 t)}_{\text{parte in fase}} - \underbrace{n_s(t) \sin(2\pi f_0 t)}_{\text{parte in quadratura}}$$



$$\hat{x}[n] = \hat{x}_c[n] + j\,\hat{x}_s[n]$$

⇒ RAMO IN FASE (I)

- Assenza di rumore

$$w_c(t) = 2\sum_{n=-\infty}^{+\infty} x_c[n]\, p(t-nT_s)\cos(2\pi f_0 t)\cos(2\pi f_0 t) +$$

$$- 2\sum_{n=-\infty}^{+\infty} x_s[n]\, p(t-nT_s)\sin(2\pi f_0 t)\cos(2\pi f_0 t)$$

$$= \sum_n x_c[n]\, p'(t-nT_s)\Big[1 + \underbrace{\cos(4\pi f_0 t)}_{2f_0}\Big] +$$

$$- \sum_n x_s[n]\, p'(t-nT_s)\,\underbrace{\sin(4\pi f_0 t)}_{2f_0}$$

$$= \sum_n x_c[n]\, p'(t-nT_s) + \text{componenti a } 2f_0$$

$$y_c(t) = w_c(t) \otimes h_R(t) = \boxed{\sum_{n=-\infty}^{+\infty} x_c[n]\, h(t-nT_s)}$$

$\Rightarrow y_c(t) = \sum_n x_c[n]\, h(t-nT_s)$ contiene solo i simboli in fase

$\Rightarrow$ è come se la componente in quadratura del segnale trasmesso non esistesse

$\Rightarrow$ RAMO IN FASE (Q)

$$w_s(t) = -2 \sum_n x_c[n]\, p'(t-nT_s) \cos(2\pi f_0 t) \sin(2\pi f_0 t) +$$

$$-(-2) \sum_n x_s[n]\, p'(t-nT_s) \sin(2\pi f_0 t) \sin(2\pi f_0 t)$$

$$= -\sum_n x_c[n]\, p'(t-nT_s) \sin(\underbrace{4\pi f_0 t}_{2f_0}) +$$

$$+ \sum_n x_s[n]\, p'(t-nT_s) \left[1 - \cos(\underbrace{4\pi f_0 t}_{2f_0}) \right]$$

$$= \sum_n x_s[n]\, p'(t-nT_s) + \text{comp. a } 2f_0$$

$$y_s(t) = w_s(t) \otimes h_R(t) = \sum_n x_s[n]\, h(t-nT_s)$$

solo simboli in Q

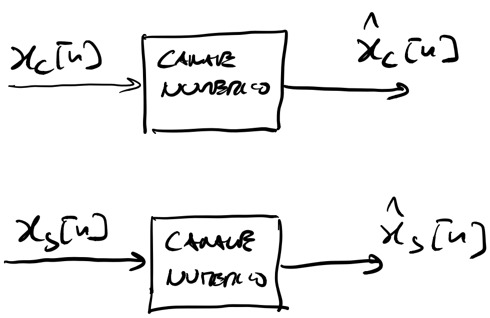
$\Rightarrow$ Le componenti in fase ed in quadratura che erano state sommate al TX sono di nuovo separate al RX.

QAM = 2 PAM → in fase / in quadratura

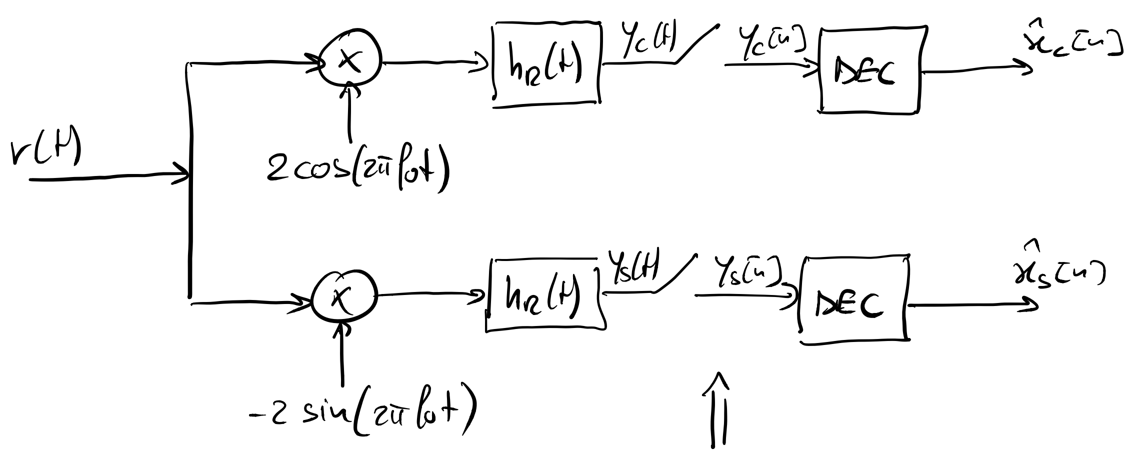
↳ raddoppia il bitrate rispetto alla PAM a parità di banda occupata

⇒ raddoppia η_B (efficienza spettrale)

•) INTERPRETAZIONE DELLA QAM



la decisione sui simboli $x_c[n]$ diventa indipendente da quella sui simboli $x_s[n]$



⇑

questa idea deve essere suffragata dall'indipendenza del rumore

$$r(t) = \underbrace{s(t) \otimes c(t)}_{\text{parte utile}} + n(t)$$

$$s(t) \longrightarrow \boxed{c(t)} \longrightarrow r'(t)$$

$$c(t) = 2\,\tilde{c}(t)\cos(2\pi f_0 t)$$

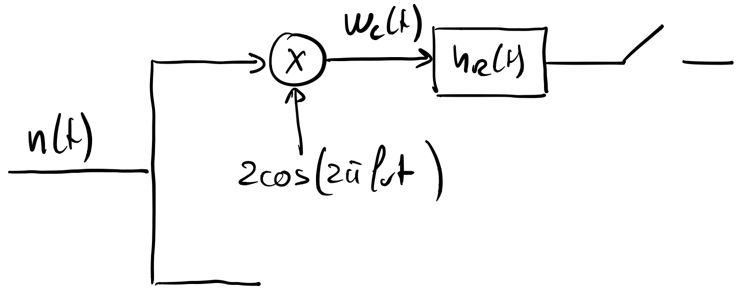
$\Rightarrow$ Non dimostriamo di nuovo come fatto per la PAM in b.passante

$$\boxed{\begin{aligned} \Rightarrow r'(t) &= \sum_{n=-\infty}^{+\infty} x_c[n]\, p'(t-nT_s)\cos(2\pi f_0 t) + \\ &\quad - \sum_{n=-\infty}^{+\infty} x_s[n]\, p'(t-nT_s)\sin(2\pi f_0 t) \\ p'(t) &= p(t) \otimes \tilde{c}(t) \end{aligned}}$$

$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t)$$ $\Leftarrow$ risultato identico al caso PAM in banda passante

$$y_c(t) = \sum_n x_c[n]\, h(t-nT_s) + n_u^{(c)}(t)$$

$$y_s(t) = \sum_n x_s[n]\, h(t-nT_s) + n_u^{(s)}(t)$$



$$n(t) = n_c(t)\cos(2\pi f_0 t) - n_s(t)\sin(2\pi f_0)$$

$$w_c(t) = \underbrace{2\, n_c(t)\cos^2(2\pi f_0 t)} - \underbrace{2 n_s(t)\sin(2\pi f_0 t)\cos(2\pi f_0 t)}$$

$$R^{I}_{w_c}(t_1, t_2) = \underbrace{\underset{\text{in b.b.}}{R_{w_{c0}}(t_1 - t_2)} + \underset{2f_0}{R_{w_{ch}}(t_1, t_2)}}_{\text{relativo alla parte in fase risultato noto dalla PAN in banda passante}}$$

$$R^{Q}_{w_c}(t_1, t_2) = E\left[2\, n_s(t_1)\sin(2\pi f_0 t_1)\cos(2\pi f_0 t_1)\cdot 2 n_s(t_2)\sin(2\pi f_0 t_2)\cos(2\pi f_0 t_2)\right]$$

$\Rightarrow$ non esistono componenti in banda base

$$R_{w_c}(t) = R_{w_{c0}}(t_1 - t_2) + \text{comp a } 2f_0$$

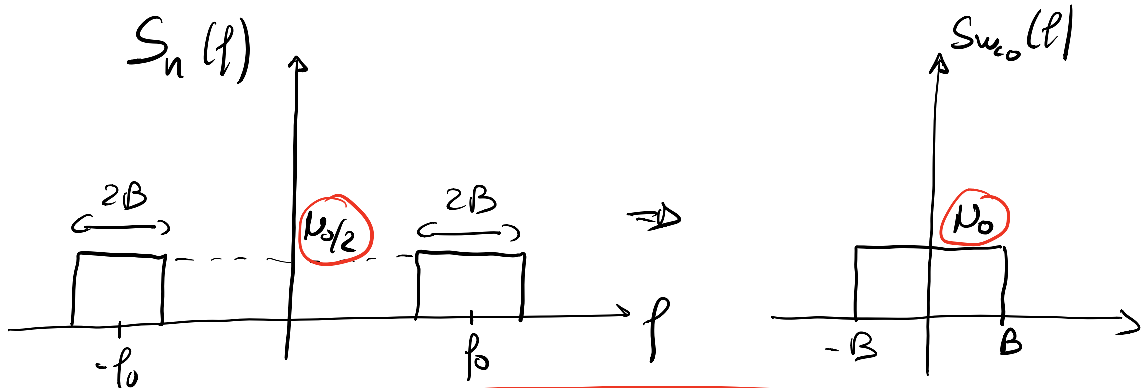
$$R_{n_{uc}}(\tau) = R_{w_{c0}}(\tau) \otimes h_R(\tau) \otimes h_R(-\tau)$$

poiché le componenti a $2f_0$ vengono filtrate

$$S_{n_u}(f) = S_{w_{c_0}}(f)\,|H_R(f)|^2$$

in caso di rumore bianco in banda

$S_{w_{c_0}}(f) = N_0$ 2 volte $\frac{N_0}{2}$



$$\boxed{P_{n_{u_c}} = \int_{-\infty}^{+\infty} S_{n_{u_c}}(f)\,df = N_0 \int_{-\infty}^{+\infty} |H_R(f)|^2\,df = N_0 E_{H_R}}$$

$$E[n_{u_c}(t)] = 0$$

$$E[n(t)] = 0$$

$$E[n_c(t)\cos(2\pi f_0 t) - n_s(t)\sin(2\pi f_0 t)] = 0$$

$$E[n_c(t)] = E[n_s(t)] = 0$$

$$E\left[w_c(t)\right] = E\left[2\underset{=0}{n_c(t)}\cos^2(2\pi f_0 t) - 2\underset{=0}{n_s(t)}\sin(2\pi f_0 t)\cos(2\pi f_0 t)\right]$$

$$= 0$$

$$\underset{\eta_{w_c}=0}{\xrightarrow{w_c(t)}} \boxed{h_R(t)} \underset{\eta_{n_{u_c}}=0}{\xrightarrow{n_{u_c}(t)}}$$

$n_{u_c}(t)$ è Gaussiano con media nulla
e $P_{n_u} = N_0 E_{H_R} = \sigma_{n_u}^2$

$$n_{u_c}(t) \xrightarrow{T} n_{u_c}[n] \leftarrow \text{V.A.} \in \mathcal{N}(0, P_{n_{u_c}})$$

identico al caso PAM

$\Rightarrow$ La stessa cosa si può dimostrare per il ramo in quadratura

$n_{u_s}(t)$ è Gaussiano a media nulla con
$P_{n_{u_s}} = N_0 E_{H_R} = \sigma_{n_{u_s}}^2$

$n_{u_s}[n]$ è una V.A $\in \mathcal{N}(0, P_{n_{u_s}})$

$$\boxed{P_{n_{u_s}} = P_{n_{u_c}} = N_0 E_{H_R}}$$

$$y_c(t) = \sum_n x_c[n]\, h(t - nT_s) + n_{u_c}(t)$$

$$y_s(t) = \sum_n x_s[n]\, h(t - nT_s) + n_{u_s}(t)$$

⇒ Dopo il campionamento

$$y_c[k] = \sum_n x_c[n]\, h((k-n)T_s) + n_{u_c}[k]$$

già caratterizzati

$$= h(0)\, x_c[k] + \underbrace{\sum_{\substack{n=-\infty \\ n \neq k}}^{+\infty} x_c[n]\, h((k-n)T_s)}_{ISI} + n_{u_c}[k]$$

ASSENZA DI ISI VA VERIFICATA TRAMITE IL CRITERIO DI NYQUIST

tempo

frequenza

⇒ Assenza di ISI

$$y_c[n] = h(0)\, x_c[n] + n_{u_c}[n]$$

$$y_s[n] = h(0)\, x_s[n] + n_{u_s}[n]$$

V.A con stesse caratteristiche identicamente dist.

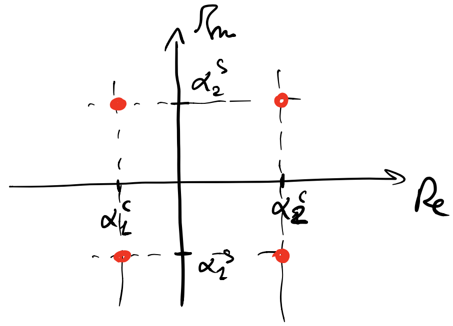
$$h(0) = h(t)\Big|_{t=0} \quad , \quad h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t)$$

$P_E(M) = ?$ $\qquad$ $M=4$ (minimo)

$\Rightarrow$ Caso di 4-QAM

$$x_c[n] \in A_s^{(c)} = \{\alpha_1^c, \alpha_2^c\}$$

$$x_s[n] \in A_s^{(s)} = \{\alpha_1^s, \alpha_2^s\}$$



$$P_E(M) = P_E^c(b)\cdot\left(1 - P_E^s(b)\right) + P_E^s(b)\cdot\left(1 - P_E^c(b)\right) + P_E^c(b)\cdot P_E^s(b)$$

$x = x_c + j\,x_s$ →
- x_c è sbagliato ma x_s è giusto
- x_c è giusto ma x_s è sbagliato
- sia x_c e x_s sono sbagliati

Indipendenza degli errori

$y_c[n]$, $y_s[n]$ indipendenti poichè →
- $x_c[n]$ indip. da $x_s[n]$
- $n_c(t)$ indip. da $n_s(t)$

$\Downarrow$

eventi errore indipendenti

$\Downarrow$

prodotto delle prob di errore

non dimostrato ‖ Condizione di prova sul rumore che deriva dalle caratteristiche di un rumore bianco in banda

⇒ Per calcolare la prob. di errore sul simbolo QAM

- VERIFICARE ASSENZA ISI
- $h(0)$ (uguale per ramo in fase e quadratura)
- $P_{n_{u_c}} = P_{n_{u_s}}$
- $P_E^c(b)$
- $P_E^s(b)$
- Applico la formula ⇒ $P_E(M) = P_E^c(b)\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b)\,P_E^s(b)$

Commento: La PAM in banda passante equivale ad una QAM con il solo ramo in fase

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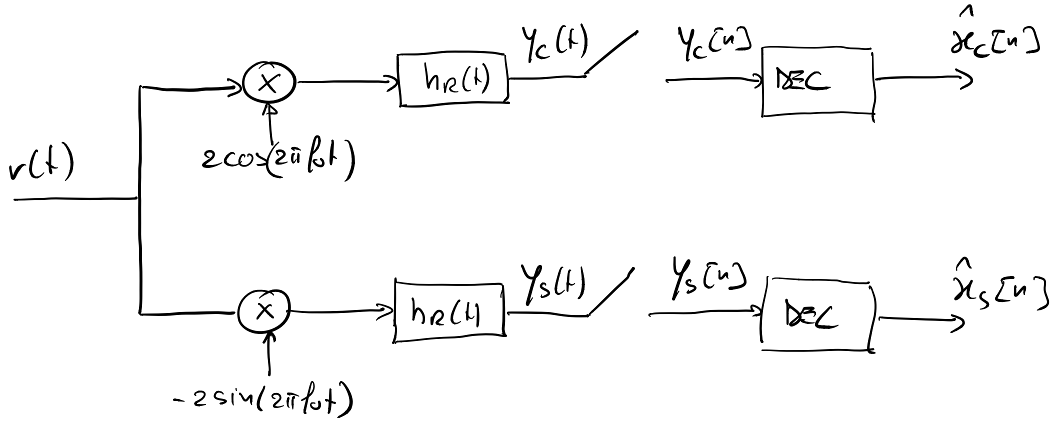
ESERCIZIO #2 28/06/2019



$$s(t) = \sum_n x_c[n]\, p(t-nT)\cos(2\pi f_0 t) - \sum_n x_s[n]\, p(t-nT)\sin(2\pi f_0 t)$$

$x_c[n] \in A_s^c = \{-1, 2\}$

ind. ed equiprob.

$x_s[n] \in A_s^s = \{-2, 1\}$

$$p(t) = \text{sinc}\left[B\left(t - \frac{1}{2B}\right)\right] + \text{sinc}\left[B\left(t + \frac{1}{2B}\right)\right]$$

$B = \frac{2}{T}$

$c(t) = \delta(t)$

$n(t)$ bianco in banda $\Rightarrow \frac{N_0}{2} = S_n(f)$

$h_R(t) = p(t)$

$\lambda = 0$

1) E_s

2) $P_{n_{u_c}}$ e $P_{n_{u_s}}$

3) $P_E(M)$

Soluzione

1) $E_s = \frac{1}{2}\left[E[x_c^2] + E[x_s^2]\right] E_p$

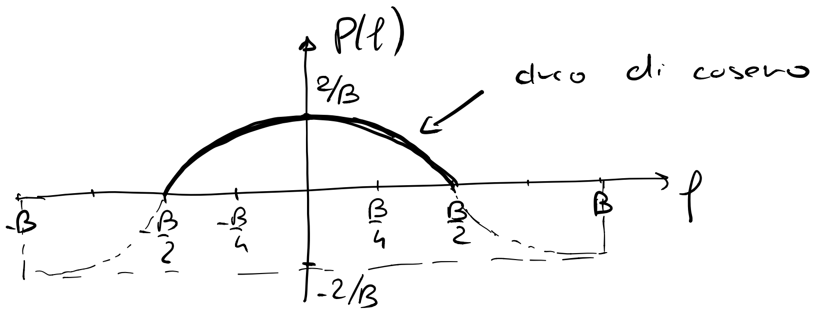
$\Rightarrow E[x_c^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+2)^2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

$$\Rightarrow E[x_s^2] = \frac{1}{2}(-2)^2 + \frac{1}{2}(+1)^2 = \frac{5}{2}$$

$E_p = ?$

$$P(f) = \frac{1}{B}\,\mathrm{rect}\left(\frac{f}{B}\right)\left[e^{-j2\pi f\frac{1}{2B}} + e^{j2\pi f\frac{1}{2B}}\right]$$

$$= \frac{2}{B}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos\left(\frac{\pi f}{B}\right)$$



$$E_p = \frac{4}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}} \cos^2\left(\frac{\pi f}{B}\right) df = \frac{4}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}}\left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi f}{B}\right)\right]df$$

$$= \frac{4}{B^2}\cdot B + \underbrace{\frac{2}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}}\cos\left(\frac{2\pi f}{B}\right)df}_{0} = \frac{4}{B}$$

$$E_s = \frac{1}{2}\left(\frac{5}{2} + \frac{5}{2}\right)\frac{4}{B} = \boxed{\frac{10}{B}}$$

2) $P_{n_{u_c}} = P_{n_{u_s}} = N_0 E_{H_R} = N_0 E_p = \frac{4N_0}{B}$

3) Assenza di ISI

$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P^2(f)$$

$$P^2(f) = \frac{4}{B^2} \operatorname{rect}\left(\frac{f}{B}\right) \cos^2\left(\frac{\pi f}{B}\right) = H(f) = \frac{2}{B^2} \operatorname{rect}\left(\frac{f}{B}\right) \cdot \left[1 + \cos\left(\frac{2\pi f}{B}\right)\right]$$

$$h(t) = \frac{2}{B^2}\, B \operatorname{sinc}(Bt) \otimes \left[\delta(t) + \frac{1}{2}\delta\left(t - \frac{1}{B}\right) + \frac{1}{2}\left(t + \frac{1}{B}\right)\right]$$

$$= \frac{2}{B} \operatorname{sinc}(Bt) + \frac{1}{B} \operatorname{sinc}\left[B\left(t - \frac{1}{B}\right)\right] + \frac{1}{B} \operatorname{sinc}\left[B\left(t + \frac{1}{B}\right)\right]$$

$$\Rightarrow h(nT) = h\left(n\frac{2}{B}\right) = \frac{2}{B} \operatorname{sinc}\left(B\, n\frac{2}{B}\right) + \frac{1}{B} \operatorname{sinc}\left[B\left(n\frac{2}{B} - \frac{1}{B}\right)\right] + \frac{1}{B} \operatorname{sinc}\left[B\left(n\frac{2}{B} + \frac{1}{B}\right)\right]$$

$$= \frac{2}{B} \operatorname{sinc}(2n) + \frac{1}{B} \operatorname{sinc}(2n - 1) + \frac{1}{B} \operatorname{sinc}(2n + 1)$$

$$= \frac{2}{B}\, \delta[n]$$ cond. di Nyquist soddisfatta

$$h(0) = \frac{2}{B}$$

$$P_E(M) = P_E^I(b)\left(1 - P_E^q(b)\right) + P_E^q(b)\left(1 - P_E^I(b)\right) + P_E^I(b)\, P_E^q(b)$$

$$P_E^I(b) = \underbrace{P(\hat{\alpha}_1 \mid \alpha_2)} P(\alpha_2) + \underbrace{P(\hat{\alpha}_2 \mid \alpha_1)} P(\alpha_1)$$

$$y_c[n] = h(0)\, x_c[n] + n_{u_c}[n]$$

$$= \frac{1}{2} Q\left(\frac{h(0)\cdot 2}{\sqrt{P_{n_{u_c}}}}\right) + \frac{1}{2} Q\left(\frac{h(0)\cdot 1}{\sqrt{P_{n_{u_c}}}}\right)$$

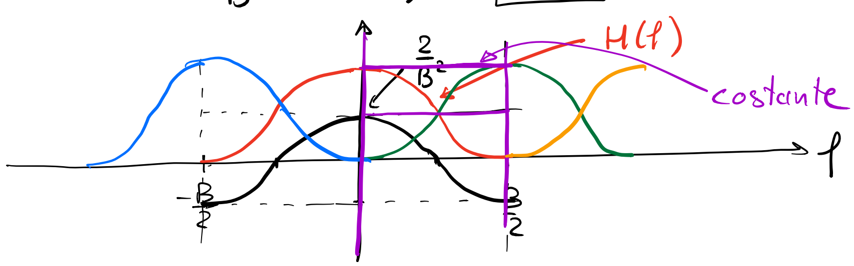
$$= \frac{1}{2} Q\left(\frac{\frac{4}{B}}{\sqrt{\frac{4N_0}{B}}}\right) + \frac{1}{2} Q\left(\frac{\frac{2}{B}}{\sqrt{\frac{4N_0}{B}}}\right)$$

$$\boxed{\begin{aligned} P_E^Q(b) &= \frac{1}{2} Q\left(\frac{\frac{4}{B}}{\sqrt{\frac{4N_0}{B}}}\right) + \frac{1}{2} Q\left(\frac{\frac{2}{B}}{\sqrt{\frac{4N_0}{B}}}\right) \\ P_E^Q(b) &= P_E^I(b) = P_E(b) \end{aligned}}$$

$$\boxed{P_E(M) = 2P_E(b)\left(1 - P_E(b)\right) + P_E^2(b)}$$

-) Verifica dell'assenza di ISI in frequenza

$$H(f) = \frac{2}{B^2} \operatorname{rect}\left(\frac{f}{B}\right) \cdot \left[1 + \underbrace{\cos\left(\frac{2\pi f}{B}\right)}\right] \qquad \frac{1}{T} = \frac{B}{2}$$



$0 \le f \le \frac{B}{2} \Rightarrow$ 2 componenti non nulle

componente rossa + verde

$$\frac{2}{B^2}\left[1+\cos\left(\frac{2\pi f}{B}\right)\right]+\frac{2}{B^2}\left[1+\cos\left(\frac{2\pi\left(f-\frac{B}{2}\right)}{B}\right)\right]$$

$$= \frac{2}{B^2}+\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}\right)+\frac{2}{B^2}+\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}-\frac{2\pi \cancel{B}/2}{\cancel{B}}\right)$$

$$\cos\left(\frac{2\pi f}{B}-\frac{2\pi B/2}{B}\right) = \cos\left(\frac{2\pi f}{B}-\pi\right) = -\cos\left(\frac{2\pi f}{B}\right)$$

$$= \frac{2}{B^2}+\cancel{\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}\right)}+\frac{2}{B^2}-\cancel{\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}\right)} = \frac{4}{B^2} = \text{cost.}$$

$\Downarrow$

ASSENZA DI ISI