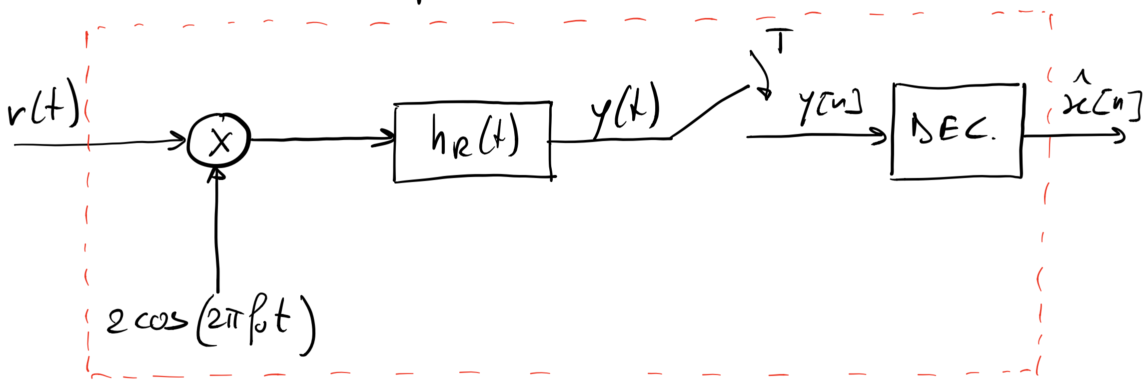
ESERCIZIO - Compitino 29/05/2017



DEMODULATORE NUMERICO PER PAM IN BANDA PASSANTE

$$s(t) = \sum_k x[k]\, p(t-kT)\cos(2\pi f_0 t)$$

$x[k] \in A_s = \{-1, 2\}$ ind. $P\{-1\} = \frac{3}{5}$, $P\{+2\} = \frac{2}{5}$

$$p(t) = 2B\,\mathrm{sinc}(2Bt) + B\,\mathrm{sinc}\left(2B\left(t-\frac{1}{2B}\right)\right) + B\,\mathrm{sinc}\left(2B\left(t+\frac{1}{2B}\right)\right)$$

$f_0 \gg B$, $T = \frac{1}{B}$

$c(t) = \delta(t)$

$n(t)$ bianco in banda $S_n(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \mathrm{rect}\left(\frac{f+f_0}{2B}\right)\right]$

$h_R(t)$ è un passa-basso ideale di banda B

$\lambda = 0$

Calcolare:

1) E_s
2) P_{n_u}
3) Dire se y[n] ha il max SNR
4) Calcolare la $P_E(b)$

Soluzione:

1) $E_s = \frac{1}{2} E[x^2] E_p$

differenza rispetto alla PAM in b.b.

$$E[x^2] = \frac{3}{5}(-1)^2 + \frac{2}{5}(+2)^2 = \frac{3}{5} + \frac{8}{5} = \frac{11}{5}$$

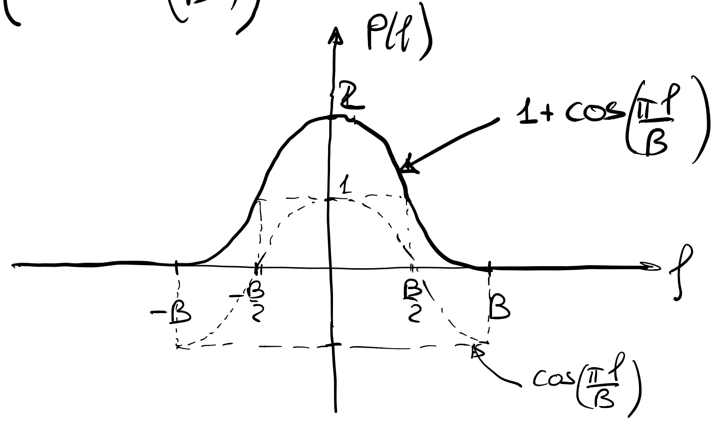
$$E_p = \int_{-\infty}^{+\infty} p^2(t)\,dt = \int_{-\infty}^{+\infty} |P(f)|^2 df = \int_{-\infty}^{+\infty} P^2(f)\,df$$

$P(f)$ dovrà essere reale e pari

$$P(f) = TCF[p(t)] = \text{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\text{rect}\left(\frac{f}{2B}\right) e^{-j2\pi f \frac{1}{2B}} + \frac{1}{2}\text{rect}\left(\frac{f}{2B}\right) e^{j2\pi f \frac{1}{2B}}$$

$$= \text{rect}\left(\frac{f}{2B}\right)\left[1 + \frac{1}{2}\left(e^{j\frac{\pi f}{B}} + e^{-j\frac{\pi f}{B}}\right)\right]$$

$$= \text{rect}\left(\frac{f}{2B}\right)\left(1 + \cos\left(\frac{\pi f}{B}\right)\right)$$

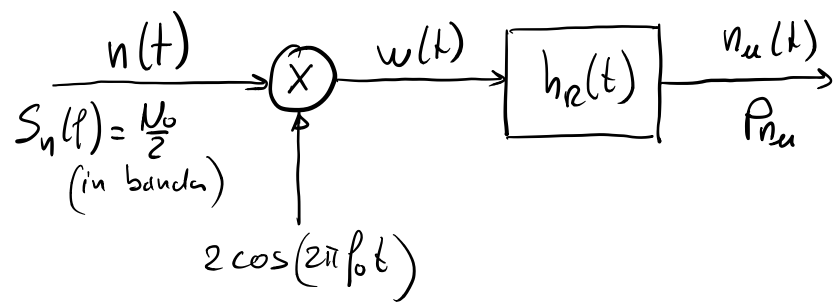


$$E_p = \int_{-B}^{B} \left[1 + \cos\left(\frac{\pi f}{B}\right)\right]^2 df = \int_{-B}^{B} 1\, df + \int_{-B}^{B} \cos^2\left(\frac{\pi f}{B}\right) df + 2\underbrace{\int_{-B}^{B} \cos\left(\frac{\pi f}{B}\right) df}_{0}$$

$$= 2B + \frac{1}{2}\int_{-B}^{B} \left[1 + \cos\left(\frac{2\pi f}{B}\right)\right] df = 2B + B = 3B$$

$$E_s = \frac{1}{2} \cdot \frac{11}{5} \cdot 3B = \boxed{\frac{33}{10} B}$$

2)



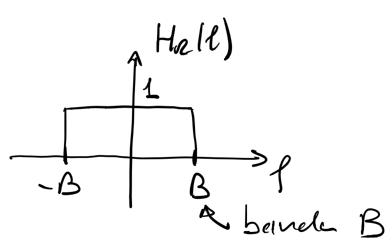
$$S_{n_u}(f) = N_0 \left|H_R(f)\right|^2$$ quando $S_n(f)$ è bianco in banda

$$P_{n_u} = N_0 E_{H_R} = \boxed{2 N_0 B}$$

differenza rispetto alla PAM in b.b.

$$E_{H_R} = \int_{-\infty}^{+\infty} \left|H_R(f)\right|^2 df = 2B$$

$$H_R(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$



3) $y[n]$ ha SNR max

$$SNR = \frac{P_s}{P_{n_u}} \Rightarrow max$$

$$\Downarrow$$

il filtro di ricezione e' "adattato"

$$r(t) = \sum_n x[n]\, p'(t-nT) \cos(2\pi f_0 t) + n(t)$$

$$h_R(t) = h_{FA}(t) = K s_i(-t)$$

$$s_i(t) = x[n]\, p'(t-nT)$$

$$p'(t) = p(t) \otimes \tilde{c}(t) \quad \Rightarrow \quad \tilde{c}(t) = \delta(t)$$

$$\underline{p'(t) = p(t)} = 2B \operatorname{sinc}(2Bt) + B \operatorname{sinc}\left(2B\left(t - \frac{1}{2B}\right)\right) + B \operatorname{sinc}\left(2B\left(t + \frac{1}{2B}\right)\right)$$

$$h_R(t) = 2B \operatorname{sinc}(2Bt)$$

$$h_R(t) \neq K\, s_i(-t) = \underbrace{K\, x[n]}_{K'}\, 2B \operatorname{sinc}(2Bt) +$$

$$2B \operatorname{sinc}(2Bt) \neq \begin{array}{l} B \operatorname{sinc}\left[2B\left(t + \frac{1}{2T}\right)\right] + \\ B \operatorname{sinc}\left[2B\left(t - \frac{1}{2T}\right)\right] \end{array}$$

IL FILTRO DI RICEZIONE NON E' ADATTATO

$\Downarrow$

$\underline{\underline{y[n] \text{ non}}}$ $\underline{\underline{\text{può avere SNR max}}}$

$$\boxed{K p'(-t) = h_R(t)}$$

FILTRO ADATTATO

$\Downarrow$

$y[n]$ con SNR max

4) $P_E(b)$

·) VERIFICA DELL'ASSENZA DI ISI

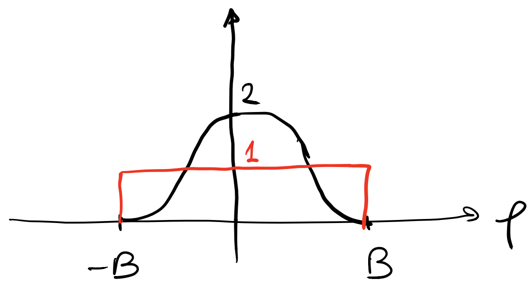
!! VA FATTO SEMPRE, ANCHE QUANDO NON E' ESPLICITAMENTE RICHIESTO

·) $h(nT) = h[n] = K\,\delta[n]$

·) $\sum_n H\left(f - \frac{n}{T}\right) = K$

$$h(t) = \underbrace{p(t) \otimes \tilde{c}(t)}_{\substack{p'(t) \\ \parallel \\ p(t)}} \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P(f)$$



$$h(t) = p(t)$$

$$h(nT) = p(nT) = 2B\,\mathrm{sinc}(2BnT) + B\,\mathrm{sinc}\left(2B\left(nT - \frac{1}{2B}\right)\right)$$

$$+ B\,\mathrm{sinc}\left[2B\left(nT + \frac{1}{2B}\right)\right]$$

$T = \frac{1}{B}$

$$= 2B\,\mathrm{sinc}(2n) + B\,\mathrm{sinc}\left[2B\left(\frac{2n}{2B} - \frac{1}{2B}\right)\right]$$

$$+ B\,\mathrm{sinc}\left[2B\left(\frac{2n}{2B} - \frac{1}{2B}\right)\right]$$

$$= 2B\,\delta[n] + \underbrace{B\,\mathrm{sinc}[2n-1]}_{= 0\ \forall n} + \underbrace{B\,\mathrm{sinc}[2n+1]}_{= 0\ \forall n}$$

$$= 2B\,\delta[n]$$

$\Downarrow$

soddisfa la cond. di Nyquist

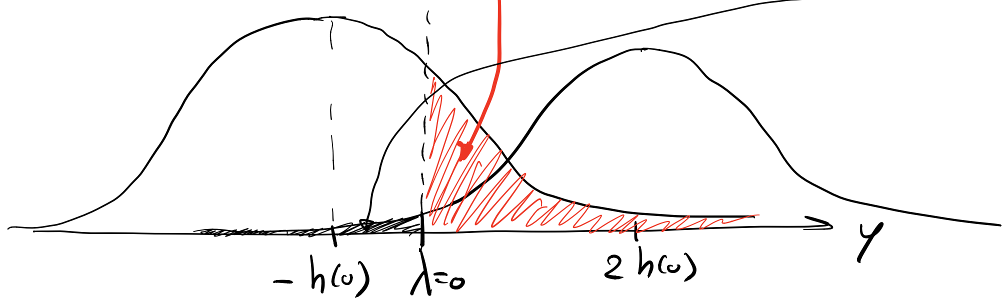
$$\begin{cases} h(0) = 2B \\ h[n] = 0 \quad \forall n \neq 0 \end{cases}$$

V.A $\in \mathcal{N}(0, P_{n_u})$

$$y[n] = h(0)\,x[n] + n_u[n]$$

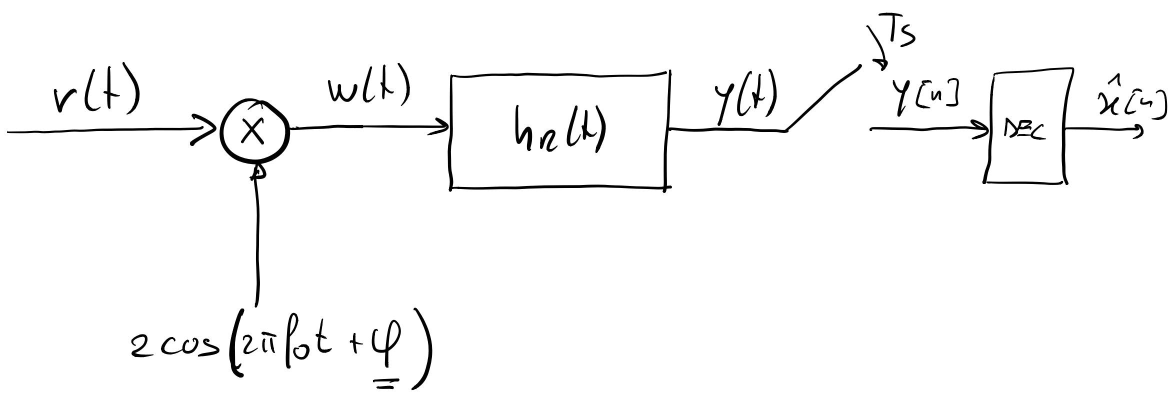
anche nel caso di PAM in b. passante

$$P_E(b) = P\{-1\}\, P\{\hat{x} = +2 \mid x = -1\} + P\{+2\}\, P\{\hat{x} = -1 \mid x = +2\}$$



$$= \frac{3}{5}\, Q\left(\frac{h(0)}{\sqrt{P_{nu}}}\right) + \frac{2}{5}\, Q\left(\frac{2h(0)}{\sqrt{P_{nu}}}\right)$$

$$= \boxed{\frac{3}{5}\, Q\left(\frac{2B}{\sqrt{2N_0B}}\right) + \frac{2}{5}\, Q\left(\frac{4B}{\sqrt{2N_0B}}\right)}$$



tiene conto della differenza di fase tra gli oscillatori in tx ed in rx

$$r(t) = \sum_n x[n]\, p'(t - nT_s) \cos(2\pi f_0 t) + n(t)$$

$\Rightarrow$ parte utile

$$\sum_n x[n]\, p'(t - nT_s) \cos(2\pi f_0 t)$$

$$w(t) = 2 \sum_n x[n]\, p'(t - nT_s) \cos(2\pi f_0 t) \cos(2\pi f_0 t + \varphi)$$

$$= \sum_n x[n]\, p'(t - nT_s) \left[\underbrace{\cos(4\pi f_0 t + \varphi)}_{\text{comp. a } 2f_0} + \cos\varphi \right]$$

$$w(t) \longrightarrow \boxed{h_R(t)} \xrightarrow{\;y(t)\;}$$

P.B.

$$y(t) = \sum_n x[n]\, p'(t - nT_s) \cdot \underbrace{\cos\varphi}$$

termine aggiuntivo

$$y[n] = h(0)\, x[n] \cos\varphi$$ in assenza di rumore ed in assenza di ISI

$$= h'(0)\, x[n] \quad , \quad h'(0) = h(0) \cos\varphi$$

$$|\cos\varphi| \leq 1$$

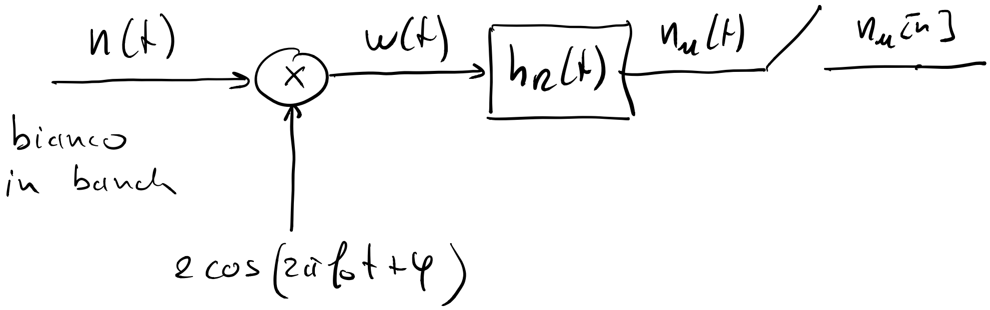
$\Rightarrow$ aggiungo il rumore

$$y[n] = \underbrace{h'(0)\, x[n]}_{\Downarrow} + \underbrace{n_u[n]}$$

??

la parte utile diminuisce di un fattore $\cos\varphi$

RUMORE



$$S_n(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \mathrm{rect}\left(\frac{f+f_0}{2B}\right)\right]$$

$$R_n(\tau) = 2N_0B\,\mathrm{sinc}(2B\tau)\cos(2\pi f_0\tau)$$

$$w(t) = 2n(t)\cos(2\pi f_0 t+\varphi)$$

$$E[w(t)] = \eta_w(t) = E[2n(t)\cos(2\pi f_0 t+\varphi)] =$$

$$= 2\underbrace{E[n(t)]}_{=0}\cos(2\pi f_0 t+\varphi) = 0$$

$$R_w(t_1,t_2) = E[w(t_1)w(t_2)]$$

$$= 4E[n(t_1)n(t_2)]\cos(2\pi f_0 t_1+\varphi)\cos(2\pi f_0 t_2+\varphi)$$

$$= 2\cdot 2N_0B\,\mathrm{sinc}[2B(t_1-t_2)]\cos[2\pi f_0(t_1-t_2)]\cdot$$

$$\cdot\Big[\cos[2\pi f_0(t_1+t_2)+2\varphi] + \cos[2\pi f_0(t_1-t_2)]\Big]$$

$$= 2N_0B\,\mathrm{sinc}[2B(t_1-t_2)]\Big[\cos(4\pi f_0 t_1+2\varphi)+\cos(4\pi f_0 t_2+2\varphi)$$

$$+ 1 + \cos(4\pi f_0(t_1-t_2))\Big]$$

$$= \begin{cases} 2N_0B \,\mathrm{sinc}\left[2B(t_2-t_1)\right] & \text{comp in b.b.} \\ \text{3 componenti a } 2f_0 & \end{cases}$$

$$w(t) \longrightarrow \boxed{h_R(t)} \longrightarrow n_u(t)$$

Filtro
P.B.

$R_{n_u}(\tau)$ è la stessa che abbiamo calcolato quando $\varphi=0$ poiché la componente in b.b. di $w(t) \Rightarrow R_w(\tau)$ non dipende da φ ed è identica quindi al caso $\varphi=0$

$S_{n_u}(f) = N_0 \left|H_R(f)\right|^2$ indipendente da φ

$$n_u[n] \in \mathcal{N}(0, P_{n_u})$$

$P_{n_u} = N_0 E_{H_R}$ (ind. da φ)

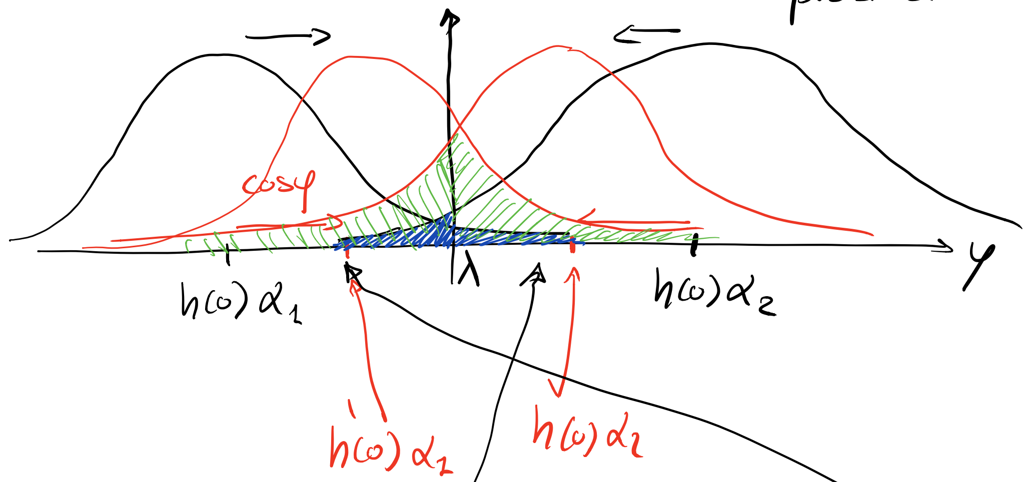
$$y[n] = h'(0)\, x[n] + n_u[n]$$

$h'(0)$: si è ridotto per effetto di φ

$n_u[n]$: è rimasto uguale

⇒ RIDUZIONE DI SNR

Aumenta la prob. di errore



$$P_e(b) = \underbrace{P\{\alpha_1\}}_{\text{restano uguali}} \, \overbrace{P\{\hat{\alpha}_2 \mid \alpha_1\}} + \underbrace{P\{\alpha_2\}} \, \overbrace{P\{\hat{\alpha}_1 \mid \alpha_2\}}$$

aumento per effetto di φ

⇒ caso peggiore $\varphi = \frac{\pi}{2} \Rightarrow \cos\varphi = 0$

$$h'(0) = 0$$

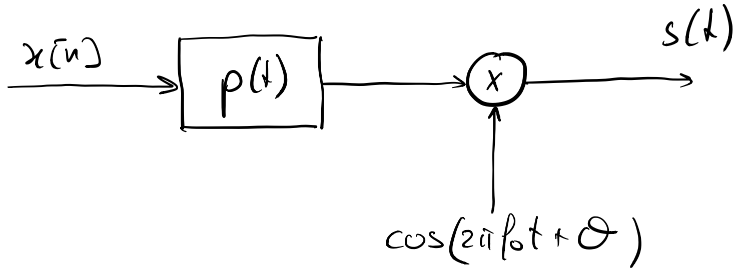
⇒ Cosa cambia quando $\varphi \neq 0$

$$h(0) \Rightarrow h'(0) \Rightarrow Q\left(\quad h'(0) \quad \right)$$

$$\varphi \not\Rightarrow h(t) \Rightarrow h(nT) = 0 \qquad n \neq 0$$

$$\boxed{\cos\varphi} \, h(nT) = 0 \qquad n \neq 0$$

•) IN TX :



$$s(t) = \sum_{n=-\infty}^{+\infty} x[n]\, p(t - nT_s) \cos(2\pi f_0 t + \theta)$$

$$r(t) = s(t) \otimes c(t) + n(t)$$

$$c(t) = 2\, \tilde{c}(t) \cos(2\pi f_0 t)$$

$$s'(t) = s(t) \otimes c(t) =$$

$$= \int_{-\infty}^{+\infty} s(\tau)\, c(t-\tau)\, d\tau$$

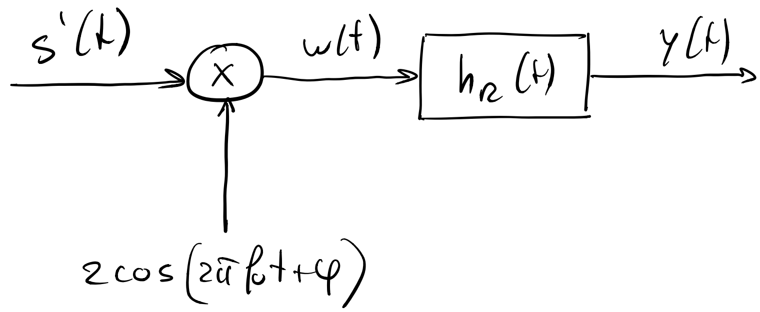
$$= 2 \sum_{n=-\infty}^{+\infty} x[n] \int_{-\infty}^{+\infty} p(\tau - nT_s) \cos(2\pi f_0 \tau + \theta) \cdot \tilde{c}(t-\tau) \cos(2\pi f_0 (t-\tau))$$

$$= \sum_n x[n] \int_{-\infty}^{+\infty} p(\tau - nT_s)\, \tilde{c}(t-\tau) \Big[\cos(2\pi f_0 t + \theta) + \underbrace{\cos(4\pi f_0 \tau + 2\pi f_0 t + \theta)}_{\simeq 0} \Big] d\tau$$

$$= \sum_n x[n]\, p'(t - nT_s) \cos(2\pi f_0 t + \theta)$$

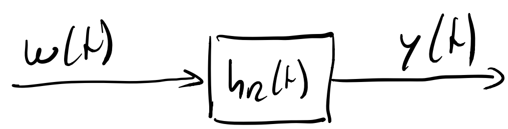
$$s'(t) = \sum_n x[n]\, p'(t - nT_s) \cos(2\pi f_0 t + \theta)$$

$$p'(t) = p(t) \otimes \bar{c}(t)$$



$$w(t) = \sum_n x[n]\, p'(t - nT_s) \left[\cos(\theta - \varphi) + \cos(4\pi f_0 t + \theta + \varphi)\right]$$

$$= \sum_n x[n]\, p'(t - nT_s) \cdot \cos(\theta - \varphi) + \text{comp a } 2f_0$$

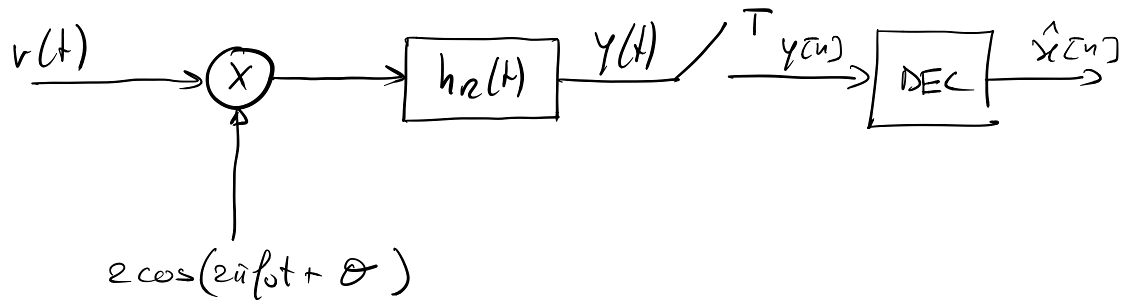


$$y(t) = \sum_n x[n]\, h(t - nT_s) \cdot \underline{\cos(\theta - \varphi)}$$

conta la differenza di fase

$$\Rightarrow \min P_E(b) \Rightarrow \boxed{\theta = \varphi}$$

ESERCIZIO - Es #2 20/02/2018



$$s(t) = \sum_{n} x[n]\, p(t - nT_s) \cos(2\pi f_0 t + \varphi)$$

$x[n] \in A_s = \{-2, +3\}$ $\quad P\{-2\} = \frac{3}{4}$, $P\{+3\} = \frac{1}{4}$

$$P(f) = \begin{cases} \sqrt{1 - |fT|} & fT \le 1 \\ 0 & \text{altrove} \end{cases} \qquad f_0 \gg \frac{1}{T}$$

$$S_N(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{2/T}\right) + \mathrm{rect}\left(\frac{f+f_0}{2/T}\right)\right] \qquad \lambda = 0$$

$$H_R(f) = P(f)$$

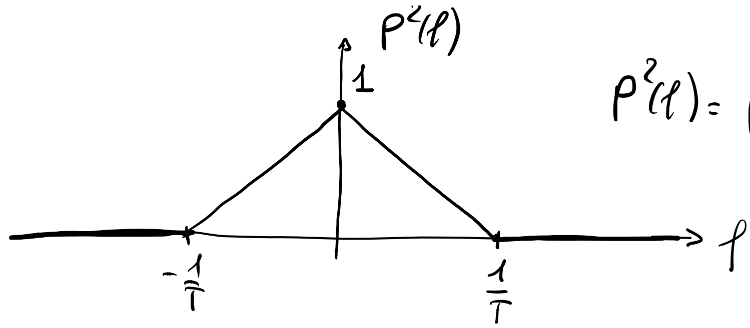
1) E_s
2) trovare il valore di θ per cui si minimizza la prob. di errore
3) P_{n_u}
4) P_{E_b}
5) verificare se $y[n]$ ha max SNR

Soluzione

1) $E_s = \frac{1}{2} E[x^2]\, E_p$

$$E[x^2] = \frac{3}{4}(-2)^2 + \frac{1}{4}(+3)^2 = \frac{12}{4} + \frac{9}{4} = \frac{21}{4}$$

$$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df \qquad P^2(f) = \begin{cases} 1 - |fT| & |fT| < 1 \\ 0 & \text{altrove} \end{cases}$$



$$P^2(f) = \left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right)$$

$$E_p = \frac{1}{T}$$

$$E_s = \frac{1}{2} \cdot \frac{21}{4} \cdot \frac{1}{T} = \boxed{\frac{21}{8T}}$$

2) $$y(t) = \sum_{n=-\infty}^{+\infty} x[n]\, h(t-nT) \cdot \underbrace{\cos(\theta-\varphi)} + n_u(t)$$

$$\boxed{\theta = \varphi} \Rightarrow \cos(\theta - \varphi) = 1$$

$$y(t) = \sum_n x[n]\, h(t-nT) + n_u(t)$$ minima $P_E(b)$

3) $$P_{n_u} = N_0 E_{H_R}$$

$$E_{H_R} = \int_{-\infty}^{+\infty} |H_R(f)|^2 df = \int_{-\infty}^{+\infty} P^2(f)\, df = \frac{1}{T}$$

$$P_{n_u} = \boxed{\frac{N_0}{T}}$$

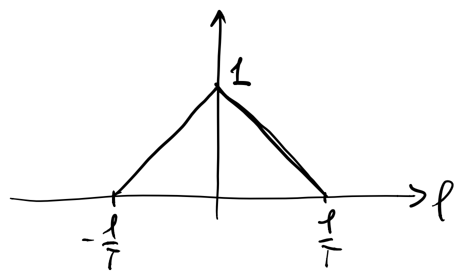
4) $P_E(b)$

•) ASSENZA DI ISI

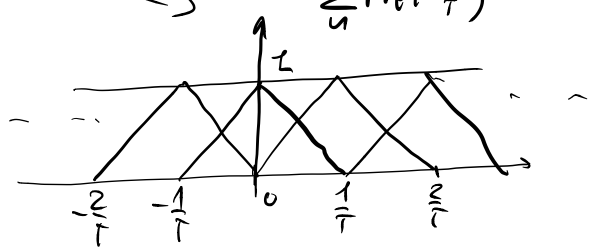
$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t)$$

$$= p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P^2(f)$$



$\Rightarrow$ freq $\sum_n H\left(f - \frac{n}{T}\right)$



Nyquist soddi. in freq.

$$h(t) = \frac{1}{T} \operatorname{sinc}^2\left(\frac{t}{T}\right)$$

$$\Downarrow$$

$$h(nT) = \frac{1}{T} \operatorname{sinc}^2\left(\frac{nT}{T}\right)$$

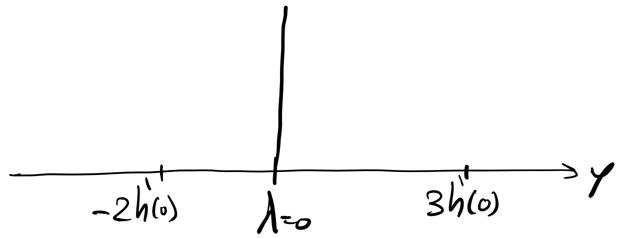
$$= \frac{1}{T} \operatorname{sinc}^2(n) = \frac{1}{T}\, \delta[n] \quad \Rightarrow \quad h(0) = \frac{1}{T}$$

soddisf. Nyquist nel tempo

$$y[n] = h'(0)\, x[n] + n_u[n]$$

$$h'(0) = \frac{\cos(\varphi - \theta)}{T}$$

$$P_E(b) = P\{3\}\, P\{\hat{x} = -2 \mid x = 3\} + P\{-2\}\, P\{\hat{x} = 3 \mid x = -2\}$$



$$\boxed{P_E(b) = \frac{1}{4}\, Q\left(\frac{\frac{3}{T}\cos(\theta-\varphi)}{\sqrt{\frac{N_0}{T}}}\right) + \frac{3}{4}\left(\frac{\frac{2}{T}\cos(\theta-\varphi)}{\sqrt{\frac{N_0}{T}}}\right)}$$

5) $y[n]$ ha max SNR ?

$$\Downarrow$$

il filtro $h_R(t)$ è adattato ?

$$h_R(t) = K\, p^*(-t)$$

$$p^*(t) = p(t)$$

$p(t)$ è reale e pari $\Leftarrow$ $P(f)$ è reale e pari

$$p^*(-t) = p(t) = h_R(t)$$

$$\Downarrow$$

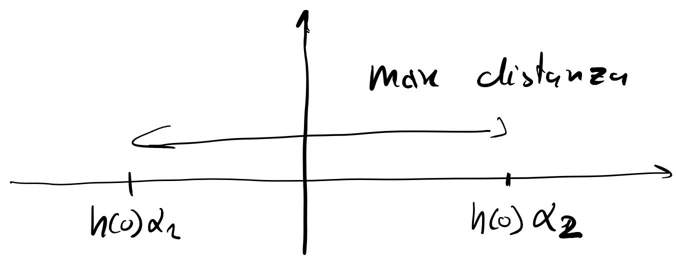
$$h_R(t) = p(-t) = p^*(-t)$$

$$h_R(t) = K\, p^*(-t) \quad , \quad K = 1$$

$$\Downarrow$$

IL FILTRO È ADATTATO

$y[n]$ ha il max SNR



P_{media} del segnale in uscita al camp.

$$\max SNR \Rightarrow \frac{E[h^2(0)\,x^2[n]]}{P_{n_u}} \text{ è max}$$

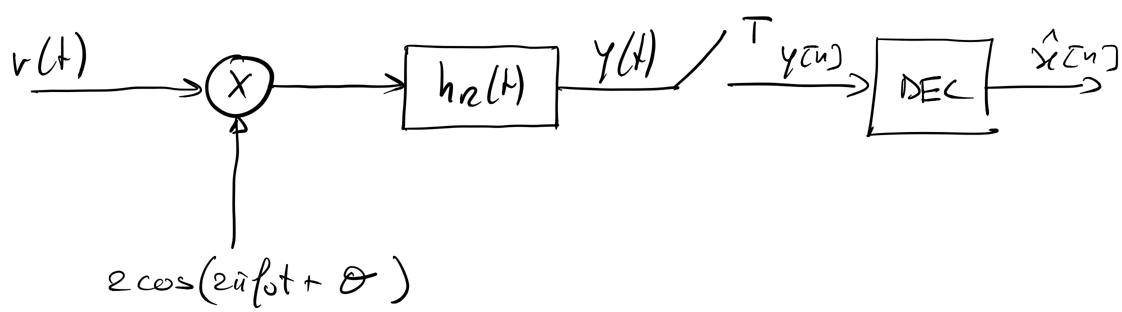
$$= \frac{h^2(0)\,E[x^2]}{P_{n_u}} = \frac{E[x^2]}{\frac{N_0}{T}}\,h^2(0)$$

$\max h^2(0)$

$\Updownarrow$

minima $P_E(b)$ $\Leftarrow$ massima distanza tra i simboli $\Longleftarrow$ $\max h(0)$

ESERCIZIO - 17/01/2019 Es #2



$$s(t) = \sum_k x[n]\,p(t-nT_s)\cos(2\pi f_0 t + \varphi)$$

$x[n]$ ind. $\in A_s = \{-1, +3\}$ $\quad P\{-1\} = \frac{2}{3}$, $P\{+3\} = \frac{1}{3}$

$p(t) = 2B\,\text{sinc}^2(Bt)\cos(\pi B t)$

$c(t) = \delta(t)$

$w(t)$ rumore Gaussiano bianco in banda

$$S_w(f) = \frac{N_0}{2}\left[\text{rect}\left(\frac{f-f_0}{2/T}\right) + \text{rect}\left(\frac{f+f_0}{2/T}\right)\right]$$

$h_R(t)$ è un p.b. ideale di banda $\frac{3}{2}B$, $T = \frac{1}{B}$

$\lambda = 0$

1) E_S

2) valore di θ per min $P_E(b)$

3) Assenza di ISI

4) Calcolo di P_{n_u}

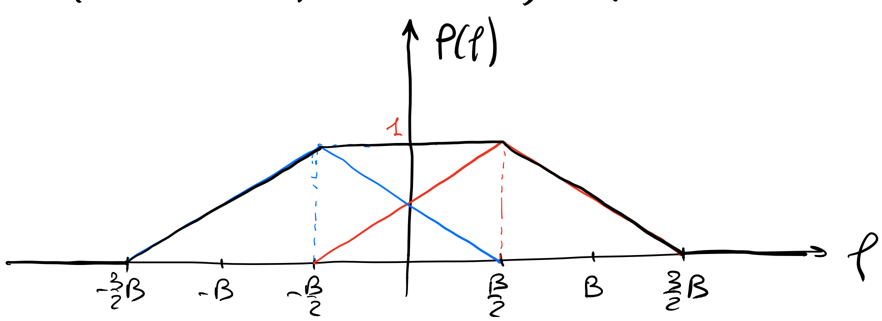
5) $P_E(b)$

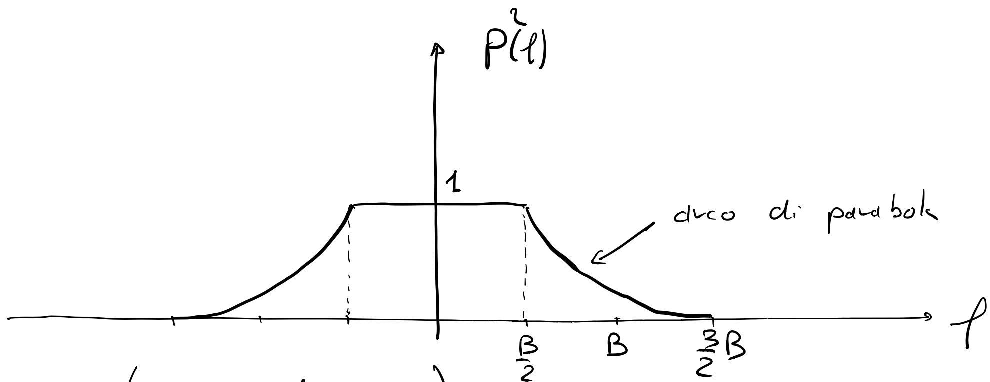
Soluzione

1) $E_S = \frac{1}{2} E[x^2]\, E_P$

$$E[x^2] = \frac{2}{3}(-1)^2 + \frac{1}{3}(+3)^2 = \frac{2}{3} + \frac{9}{3} = \frac{11}{3}$$

$$P(f) = \left(1 - \frac{|f - \frac{B}{2}|}{B}\right)\text{rect}\left(\frac{f-\frac{B}{2}}{2B}\right) + \left(1 - \frac{|f+\frac{B}{2}|}{B}\right)\text{rect}\left(\frac{f+\frac{B}{2}}{2B}\right)$$





$$E_p = 2 \cdot \left(\frac{B}{2} \cdot 1 + \frac{1}{3} \cdot B \cdot 1 \right)$$

$$= 2 \left(\frac{B}{2} + \frac{B}{3} \right) = 2 \cdot \frac{3B + 2B}{6} = \frac{5}{3} B$$

$$E_s = \frac{1}{2} \cdot \frac{11}{3} \cdot \frac{5}{3} = \boxed{\frac{55}{18}}$$

2) $\theta = \varphi$ (dire perché)

3) Nyquist

$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, \tilde{C}(f)\, H_R(f) = P(f)\, H_R(f) = P(f)$$

$\Rightarrow$ $h(t) = p(t)$

$\vdots$

$h(nT) = \ldots$

4) $P_{n_u} = N_0\, E_{H_R}$

5) $P_E(b)$ $\Rightarrow$ applicazione della formula ..