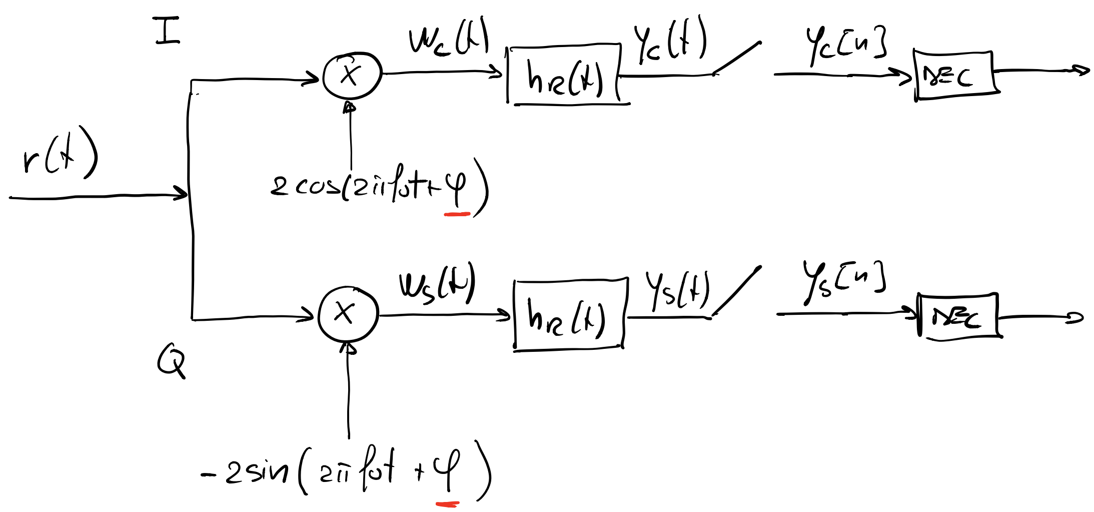


$$r(t) = \sum_{n=-\infty}^{+\infty} x_c[n]\, p'(t-nT_s)\cos(2\pi f_0 t+\vartheta) + $$
$$- \sum_{n=-\infty}^{+\infty} x_s[n]\, p'(t-nT_s)\sin(2\pi f_0 t+\vartheta)$$

in assenza di rumore

$\varphi \neq \vartheta$ in generale

$\Rightarrow$ Ramo in fase (I)

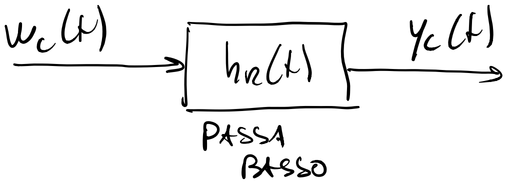
$$w_c(t) = 2\, r(t)\cos(2\pi f_0 t+\varphi)$$

$$= 2\sum_n x_c[n]\, p'(t-nT_s)\cos(2\pi f_0 t+\vartheta)\cos(2\pi f_0 t+\varphi) +$$

$$- 2\sum_n x_c[n]\, p'(t-nT_s)\sin(2\pi f_0 t+\vartheta)\cos(2\pi f_0 t+\varphi)$$

$$= \sum_n x_c[n]\, p'(t-nT_s)\Big[\cos(\underbrace{4\pi f_0}_{2f_0} t+\vartheta+\varphi) + \cos(\underline{\vartheta-\varphi})\Big]$$

$$- \sum_n x_c[n]\, p'(t-nT_s)\Big[\sin(\underbrace{4\pi f_0}_{2f_0} t+\vartheta+\varphi) + \sin(\underline{\vartheta-\varphi})\Big]$$



$$y_c(t) = \cos(\theta-\varphi)\sum_n x_c[n]\, h(t-nT_s) +$$

$$- \sin(\theta-\varphi)\sum_n x_s[n]\, h(t-nT_s)$$

$\Downarrow$

nel ramo in fase sono presenti anche i simboli in quadratura !!

$\Downarrow$

$$\boxed{\text{CROSS - TALK}}$$

Fenomeno negativo che va evitato

$$y_c(t) = \cos(\theta-\varphi)\sum_n x_c[n]\, h(t-nT_s)$$

COMPONENTE UTILE ATTENUATA

+ rumore + interferenza cross-talk (dovuta ai simboli in quadratura)

2 perdite rispetto alla sola attenuazione del caso PAM in banda passante

⇒ La stessa cosa la si può dimostrare per il ramo in quadratura

⇓

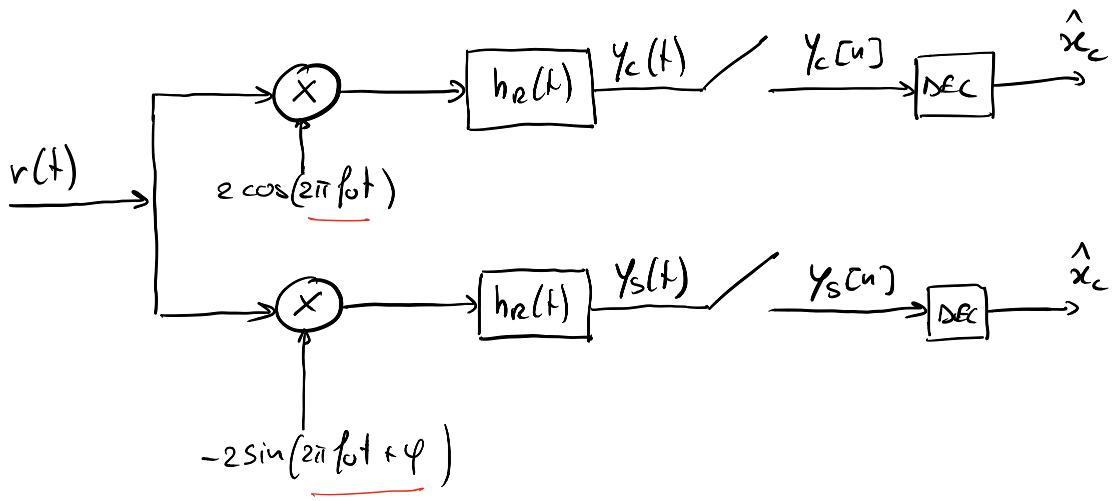
ottengo una attenuazione di $\cos(\theta - \varphi)$

+

interferenza cross-talk (dai simboli in fase)

⇒ Per la QAM, a differenza della PAM in banda passante, quando è presente cross-talk non siamo in grado di calcolare la $P_E(M)$

ESERCIZIO # 2 08/09/2017



$$s(t) = \sum_n x_c[n]\, p(t - nT_s) \cos(2\pi f_0 t + \varphi) +$$

$$- \sum_n x_s[n]\, p(t - nT_s) \sin(2\pi f_0 t + \varphi)$$

$x_c[n] \in A_S^c = \{-2, 2\}$ ind. ed equip.

$x_s[n] \in A_S^s = \{-1, 1\}$

$$p(t) \Rightarrow P(f) = \sqrt{1 - |fT|}\ \mathrm{rect}\left(\frac{fT}{2}\right) \qquad f_0 \gg \frac{1}{T}$$

$c(t) = \delta(t)$

$n(t)$ è Gaussiano bianco con $S_n(f) = \frac{N_0}{2}$

$h_R(t) = p(t)$

$d = 0$

1) E_s

2) $P_{n_{u_c}}$, $P_{n_{u_s}}$

3) Cross-talk

4) $P_E^c(b)$, $P_E^s(b)$ dove possibile

Soluzione

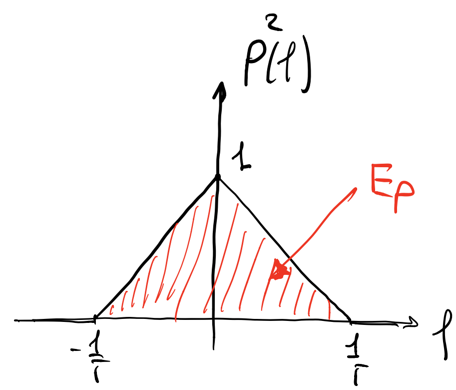
1) $$E_s = \frac{1}{2}\left(E[x_c^2] + E[x_s^2]\right) E_p$$

$$E[x_c^2] = \frac{1}{2}(-2)^2 + \frac{1}{2}(+2)^2 = 4$$

$$E[x_s^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+1)^2 = 1$$

$$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df = \frac{1}{T}$$

$$P^2(f) = \left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right)$$



$$E_s = \frac{1}{2}(4 + 1)\frac{1}{T} = \boxed{\frac{5}{2T}}$$

2) $$P_{n_{u_c}} = P_{n_{u_s}} = P_{n_u} = N_0 E_{h_R} = N_0 E_p = \boxed{\frac{N_0}{T}}$$

3) fase del segnale TX $= \varphi$

fase del ramo I $= 0$

fase del ramo Q $= \varphi$

$$\Delta = \varphi_{TX} - \varphi_{RX} = \begin{cases} \text{ramo in fase} = \varphi \\ \text{ramo in quad.} = 0 \end{cases}$$

- È presente cross-talk sul ramo in fase poiché la differenza di fase $\neq 0$
- Non è presente cross-talk sul ramo in quadratura poiché la differenza di fase $= 0$

4) $P_E^S(b)$

- VERIFICA ASSENZA DI ISI
- CALCOLO $h(0)$
- CALCOLO $P_{n_{us}}$ ✓
- CALCOLO LA $P_E^S(b)$

ASSENZA DI ISI

$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$
$$= p(t) \otimes p(t)$$

$$H(f) = P^2(f) = \left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right)$$

$$h(t) = \frac{1}{T} \mathrm{sinc}^2\left(\frac{t}{T}\right)$$

$$h(nT) = \frac{1}{T} \mathrm{sinc}^2\left(\frac{nT}{T}\right) = \frac{1}{T} \delta[n]$$

verificata l'assenza di ISI

$$h(0) = \frac{1}{T}$$

$$P_E^s(b) = \frac{1}{2} Q\left(\frac{h(0)}{\sqrt{P_{nn}}}\right) + \frac{1}{2} Q\left(\frac{h(0)}{\sqrt{P_{nn}}}\right) = Q\left(\frac{h(0)}{\sqrt{P_{nn}}}\right)$$

prob. a priori dei simboli (-1, 1)

$$= \boxed{Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right)}$$

Testi da svolgere lunedì 24 Maggio

07/02/2019

17/07/2018

26/06/2018 ES #1

20/02/2017

06/06/2017

ES #2 13/11/2017

Soluzione

1) $E_s = \frac{1}{2}\left(E[x_c^2] + E[x_s^2]\right) E_p$

$$E[x_c^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+2)^2 = \frac{5}{2} \qquad x_c \in A_s = \{-1, 2\}$$

$$E[x_s^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+1)^2 = 1 \qquad x_s \in A_s = \{-1, 1\}$$

$$p(t) = 2B\,\mathrm{sinc}(2Bt) - B\,\mathrm{sinc}\left(2B\left(t - \frac{1}{2B}\right)\right) - B\,\mathrm{sinc}\left(2B\left(t + \frac{1}{2B}\right)\right)$$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right) - \left[\frac{1}{2}\,\mathrm{rect}\left(\frac{f}{2B}\right)\left[e^{-j2\pi f\frac{1}{2B}} + e^{j2\pi f\frac{1}{2B}}\right]\right]$$

$$= \mathrm{rect}\left(\frac{f}{2B}\right)\left[1 + \cos\left(\frac{\pi f}{B}\right)\right]$$

$$E_p = \int_{-\infty}^{+\infty} P^2(f)\,df = \int_{-B}^{B}\left[1 + \cos\left(\frac{\pi f}{B}\right)\right]^2 df$$

$$= \int_{-B}^{B} 1\,df + \int_{-B}^{B}\cos^2\left(\frac{\pi f}{B}\right)df + 2\int_{-B}^{B}\cos\left(\frac{\pi f}{B}\right)df$$

$$\cos\left(2\pi f \cdot \frac{1}{2B}\right)$$

periodo

$$= 2B + \frac{1}{2}\int_{-B}^{B} 1\,df + \frac{1}{2}\underbrace{\int_{-B}^{B}\cos\left(\frac{2\pi f}{B}\right)df}_{=0} + 0$$

$$= 2B + \frac{1}{2}\cdot 2B = 3B$$

$$E_s = \frac{1}{2}\left(\frac{5}{2} + 1\right)3B = \boxed{\frac{21}{4}B}$$

2) $P_{n_{uc}} = P_{n_{us}} = N_0 E_{H_R} = \boxed{2N_0B}$ $\quad H_R(f) = rect\left(\frac{f}{2B}\right)$

$\quad\quad\quad\quad\quad\quad\quad\quad \shortparallel$

$\quad\quad\quad\quad\quad\quad\quad\quad P_{n_u}$

passa-basso ideale di banda "B"

3) $P_E(M) = P_E^c(b)\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b)\,P_E^s(b)$

$P_E^c(b)\ ,\ P_E^s(b)$

- ASSENZA DI ISI
- $h(0)$

$\Rightarrow h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$H(f) = P(f)H_R(f) = P(f) \Rightarrow h(t) = p(t)$

$P(f) \neq 0 \quad |f| \leq B$

$$h(t)\Big|_{t=nT} = h(nT) = 2B\,\mathrm{sinc}(2BnT) - B\,\mathrm{sinc}\left[2B\left(nT - \frac{1}{2B}\right)\right] - B\,\mathrm{sinc}\left[2B\left(nT + \frac{1}{2B}\right)\right]$$

$T = \frac{1}{B}$

$$= 2B\,\mathrm{sinc}\left(\cancel{2B}\frac{n}{\cancel{B}}\right) - B\,\mathrm{sinc}\left[2B\left(\frac{n}{B} - \frac{1}{2B}\right)\right] - B\,\mathrm{sinc}\left[2B\left(\frac{n}{B} + \frac{1}{2B}\right)\right]$$

$$h[n] = 2B\,\mathrm{sinc}(2n) - B\,\mathrm{sinc}\underbrace{[2n-1]}_{=\,0\ \forall n} - B\,\mathrm{sinc}\underbrace{[2n+1]}_{=\,0\ \forall n}$$

$$= 2B\,\delta[n]$$

$\Downarrow$ soddisfa il criterio di Nyquist nel tempo

$\Downarrow$

verifica l'assenza di ISI

$$h(0) = 2B$$

$$P_E^c(b) = \frac{1}{2} Q\left(\frac{h(0)}{\sqrt{P_{n_u}}}\right) + \frac{1}{2} Q\left(\frac{2h(0)}{\sqrt{P_{n_u}}}\right)$$

$$P_E^s(b) = \frac{1}{2} Q\left(\frac{h(0)}{\sqrt{P_{n_u}}}\right) + \frac{1}{2}\left(\frac{h(0)}{\sqrt{P_{n_u}}}\right)$$

$$P_E^c(b) = \frac{1}{2} Q\left(\frac{2B}{\sqrt{2N_0B}}\right) + \frac{1}{2} Q\left(\frac{4B}{\sqrt{2N_0B}}\right)$$

$$P_E^s(b) = Q\left(\frac{2B}{\sqrt{2N_0B}}\right)$$

$$P_E(M) = P_E^c(b)\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b)\,P_E^s(b)$$

ES. # 1 26/06/2018

1	2	3	IMPIANTI
$P(I_1) = 50\%$	$P(I_2) = 30\%$	$P(I_3) = 20\%$	produzione
$P_{d_1} = 0.02$	$P_{d_2} = 0.05$	$P_{d_3} = 0.01$	prob. di produrre una lavatrice difett.

1) $P_d = P\{\text{lavatrice difettosa}\} = ?$

$\Downarrow$

È una prob. totale

$\Rightarrow$ Utilizziamo il teorema della prob totale

$$P(A) = \sum_{i=1}^{3} P(A|I_i)\,P(I_i)$$

$$= \boxed{P_{d_1} \cdot P(I_1) + P_{d_2} \cdot P(I_2) + P_{d_3} \cdot P(I_3)}$$

$$P(A) = 0.5 \cdot 0.02 + 0.3 \cdot 0.05 + 0.2 \cdot 0.01$$

$$= 0.01 + 0.015 + 0.002$$

$$= \boxed{0.027} \Rightarrow 2.7\%$$

$$\boxed{P(\bar{A}) = 1 - P(A)} = \boxed{0.973} \Rightarrow 97.3\%$$

prob. che non sia difettosa (totale)

2) $$P(I_2|A) = \boxed{\frac{P(A|I_2)\,P(I_2)}{P(A)}} = \frac{0.05 \cdot 0.3}{0.027} \simeq 0.555$$

$$P(A|I_2) = P_{d_2} = 0.05$$

$$P(I_2) = 0.3$$

$$P(A) = 0.027$$

$$\boxed{P(I_1|A) = \frac{P(A|I_1)\,P(I_1)}{P(A)}} = \frac{0.02 \cdot 0.5}{0.027} \simeq 0.37$$

$$P(I_3|A) = 1 - P(I_1|A) - P(I_2|A) \simeq 0.074$$

ES # 1 19/01/2016

$X(t)$ e $Y(t)$

$$Y(t) = X(t)\cos(2\pi f_0 t + \theta) - X(t)\sin(2\pi f_0 t + \theta)$$

f_0 nota (deterministica)

θ V.A $\in \mathcal{U}[0, 2\pi]$

$X(t)$ e θ sono statisticamente indipendenti

$X(t)$ è stazionario

$$S_X(f) = \frac{1}{B}|f|\,\mathrm{rect}\left(\frac{f}{2B}\right)$$

1) Calcolare $S_Y(f)$ per $f_0 < \begin{matrix} 3B \\ \frac{B}{2} \end{matrix}$

2) Disegnare $S_Y(f)$

Soluzione

$$R_Y(t_1, t_2) = E[Y(t_1)Y(t_2)] =$$

$$= E\{[X(t_1)\cos(2\pi f_0 t_1 + \theta) - X(t_1)\sin(2\pi f_0 t_1 + \theta)] \cdot$$

$$\cdot \left[X(t_2)\cos(2\pi f_0 t_2+\theta) - X(t_2)\sin(2\pi f_0 t_2+\theta)\right] \Big\}$$

$$E\left[X(t_1)X(t_2)\cos(2\pi f_0 t_1+\theta)\cos(2\pi f_0 t_2+\theta)\right] +$$

$$- E\left[X(t_1)X(t_2)\cos(2\pi f_0 t_1+\theta)\sin(2\pi f_0 t_2+\theta)\right] +$$

$$- E\left[X(t_1)X(t_2)\sin(2\pi f_0 t_1+\theta)\cos(2\pi f_0 t_2+\theta)\right] +$$

$$+ E\left[X(t_1)X(t_2)\sin(2\pi f_0 t_1+\theta)\sin(2\pi f_0 t_2+\theta)\right]$$

$$= E\left[X(t_1)X(t_2)\right] \cdot E\left[\cos(2\pi f_0 t_1+\theta)\cos(2\pi f_0 t_2+\theta)\right] +$$

$$\vdots$$

$$\Rightarrow E\left[X(t_1)X(t_2)\right] = R_X(t_1,t_2) = R_X(t_1-t_2)$$

$$\Rightarrow E\left[\cos(2\pi f_0 t_1+\theta)\cos(2\pi f_0 t_2+\theta)\right] =$$

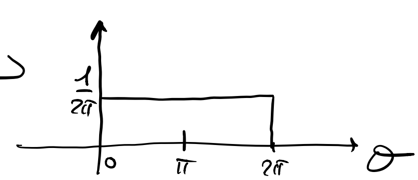
$$= \frac{1}{2} E\left[\cos\left(2\pi f_0 (t_1+t_2) + 2\theta\right)\right] + \frac{1}{2}\cos\left(2\pi f_0 (t_1-t_2)\right)$$

$$\| \qquad\qquad\qquad\qquad \|$$

$$E\left[\cos(2\theta + K_0)\right] \qquad\qquad E[\text{quant. det.}] = \text{quant. det}$$

$$K_0 = 2\pi f_0 (t_1+t_2) \quad \text{quant det.}$$

$$= \int_{-\infty}^{+\infty} \cos(2\theta+K_0)\,\frac{1}{2\pi}\,\mathrm{rect}\left(\frac{\theta-\pi}{2\pi}\right) d\theta$$



$$= \frac{1}{2\pi}\int_0^{2\pi} \cos(2\theta+K_0)\,d\theta = 0$$

$$\Rightarrow E\left[\cos(2\pi f_0 t_1 + \theta)\cos(2\pi f_0 t_2 + \theta)\right] = \frac{1}{2}\cos\left[2\pi f_0 (t_1 - t_2)\right]$$

$$\Rightarrow \cos(2\pi f_0 t_1 + \theta)\sin(2\pi f_0 t_2 + \theta)$$

$$\sin\alpha \cos\beta = \frac{1}{2}\sin(\alpha+\beta) - \frac{1}{2}\sin(\alpha-\beta)$$

$$\sin\left[2\pi f_0(t_2+t_1) + 2\theta\right] \qquad \frac{1}{2}\sin(2\pi f_0 (t_1 - t_2))$$

aleatoria $\qquad$ det

$$E\left[\sin(2\theta + K_0)\right] = 0 \qquad E[\,\cdot\,] =$$

$$\Rightarrow E\left[\cos(2\pi f_0 t_1 + \theta)\sin(2\pi f_0 t_2 + \theta)\right] = -\frac{1}{2}\sin(2\pi f_0 (t_1 - t_2))$$

$$\Rightarrow \sin(\cdot)\cos(\cdot) \Rightarrow \frac{1}{2}\sin(2\pi f_0 (t_1 - t_2))$$

$$\Rightarrow \sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha+\beta) - \frac{1}{2}\cos(\alpha-\beta)$$

$$E[\,\cdot\,] = 0 \qquad E[\,\cdot\,] = -\frac{1}{2}\cos(2\pi f_0 (t_1 - t_2))$$

$$R_Y(t_1, t_2) = R_X(t_1 - t_2)\Big[$$ **da continuare**