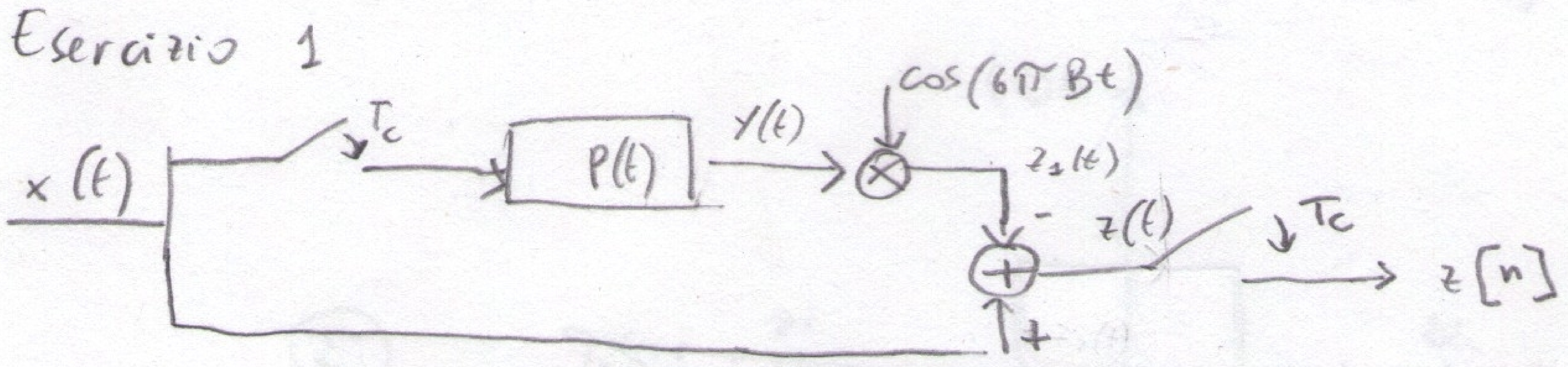
Esame 02/02/09

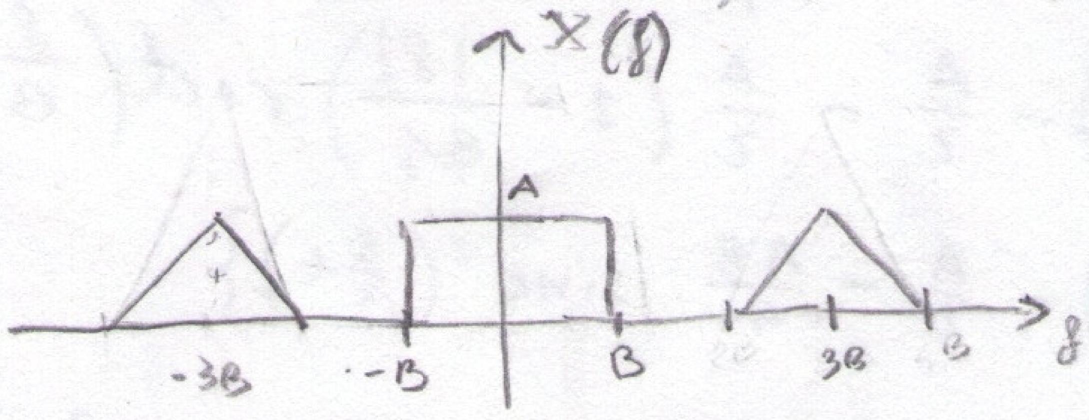
· Esercizio 1

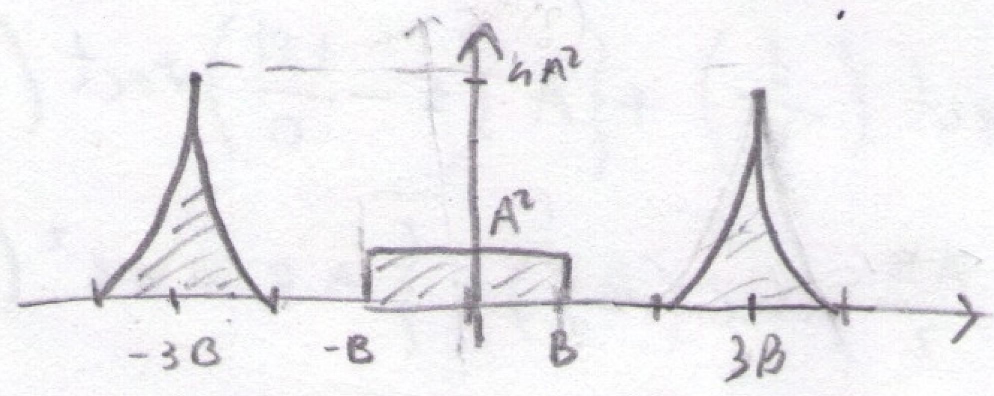



$$x(t) = 2AB\left[\mathrm{sinc}(2Bt) + \mathrm{sinc}^2(Bt)\cos(6\pi Bt)\right] =$$

$$= 2AB\,\mathrm{sinc}(2Bt) + 2AB\,\mathrm{sinc}^2(Bt)\cos(6\pi Bt)$$

$$X(f) = A\,\mathrm{rect}\left(\frac{f}{2B}\right) + 2A\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left\{\frac{1}{2}\delta(f-3B) + \frac{1}{2}\delta(f+3B)\right\}$$



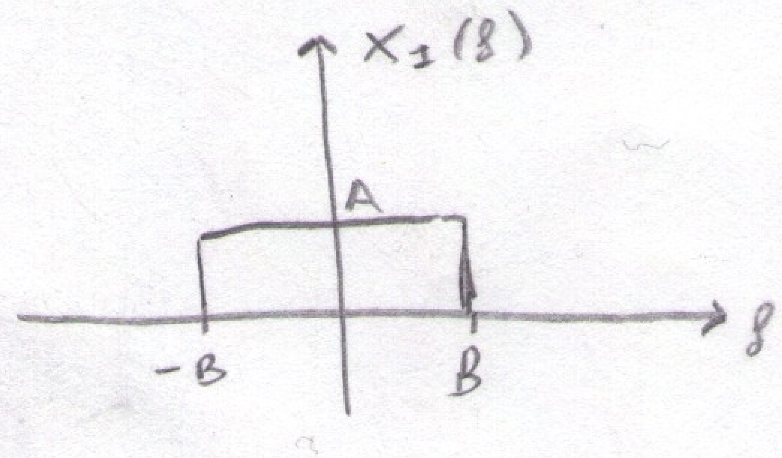



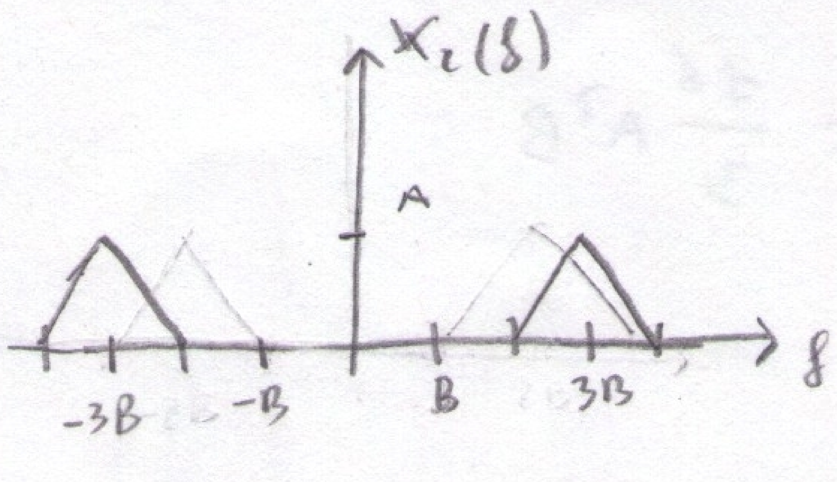

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df = 4\cdot\frac{1}{3}\cdot B\cdot 4A^2 + 2B\cdot A^2 = \frac{16}{3}BA^2 + 2BA^2 = \frac{22}{3}BA^2 \quad (1)$$

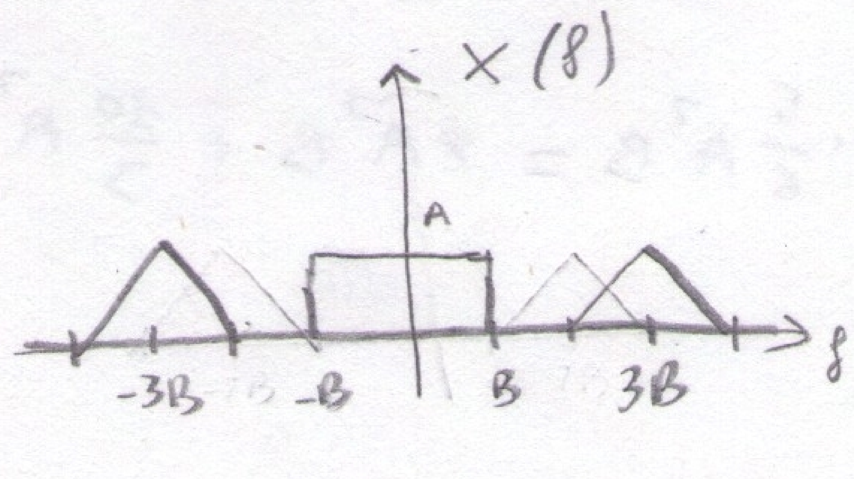
$$P_x = \emptyset$$

$$z(t) = x(t) - \{x[n] \otimes p(t)\}\cos(6\pi Bt)$$

$$x(t) = x_1(t) + x_2(t) \rightleftharpoons X(f) = X_1(f) + X_2(f)$$



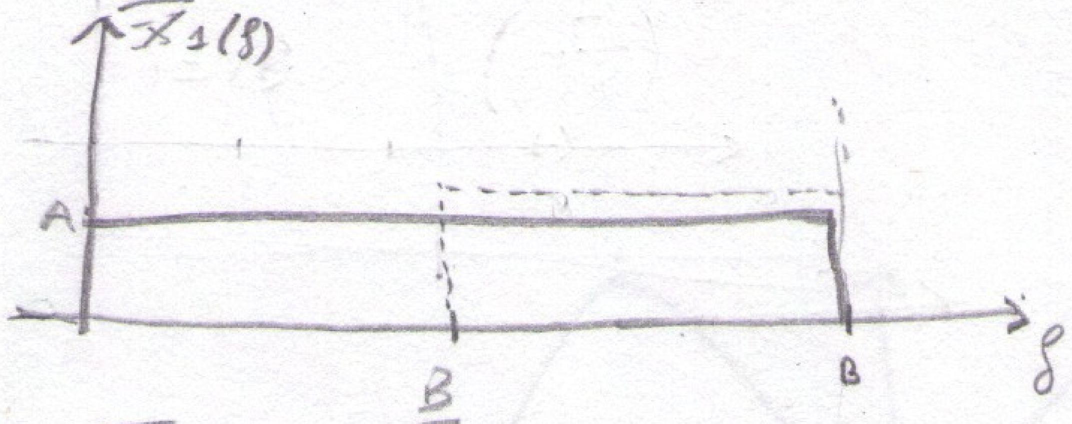



campionare nel tempo = replicare in frequenza

- $p(t) = 2B\,\text{sinc}(2Bt) \rightleftharpoons P(f) = \text{rect}\left(\frac{f}{2B}\right) \Rightarrow$ guardo la trasformata da $-B$ a $B$

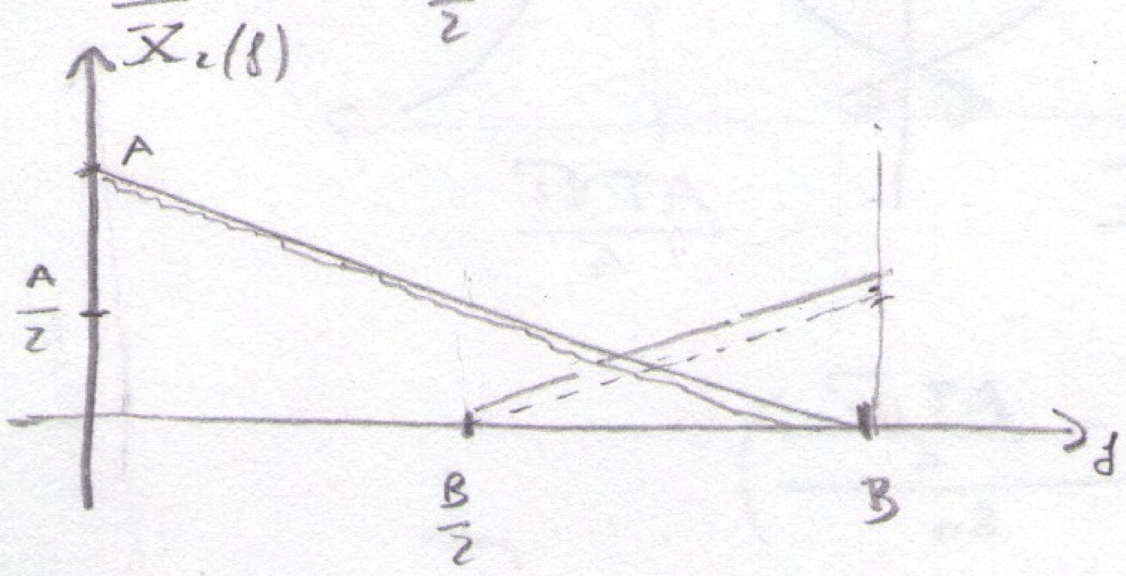
- La trasformata di un segnale reale e pari è reale e pari allora guardo il mio segnale solo da $\emptyset$ a $B$

- $T_c = \frac{2}{3B} \Rightarrow \frac{1}{T_c} = \frac{3B}{2}$



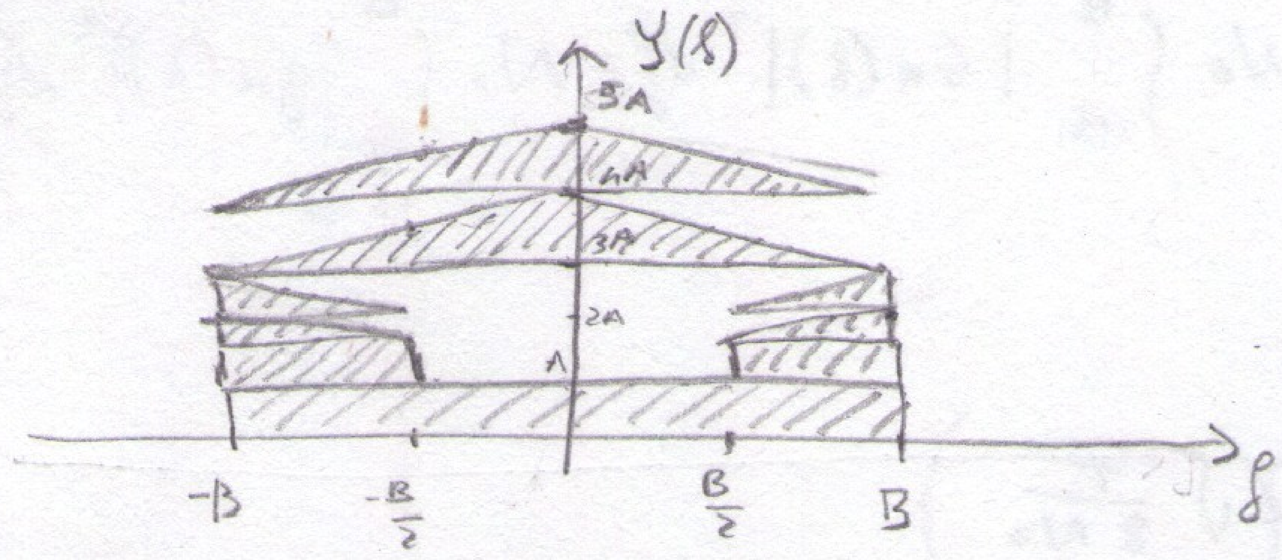

$$\bar{X}_1(f) = \sum_n X_1\left(f - \frac{n}{T_c}\right)$$

$n = \emptyset$ — ; $n = 1$ .... ; $n = -1$

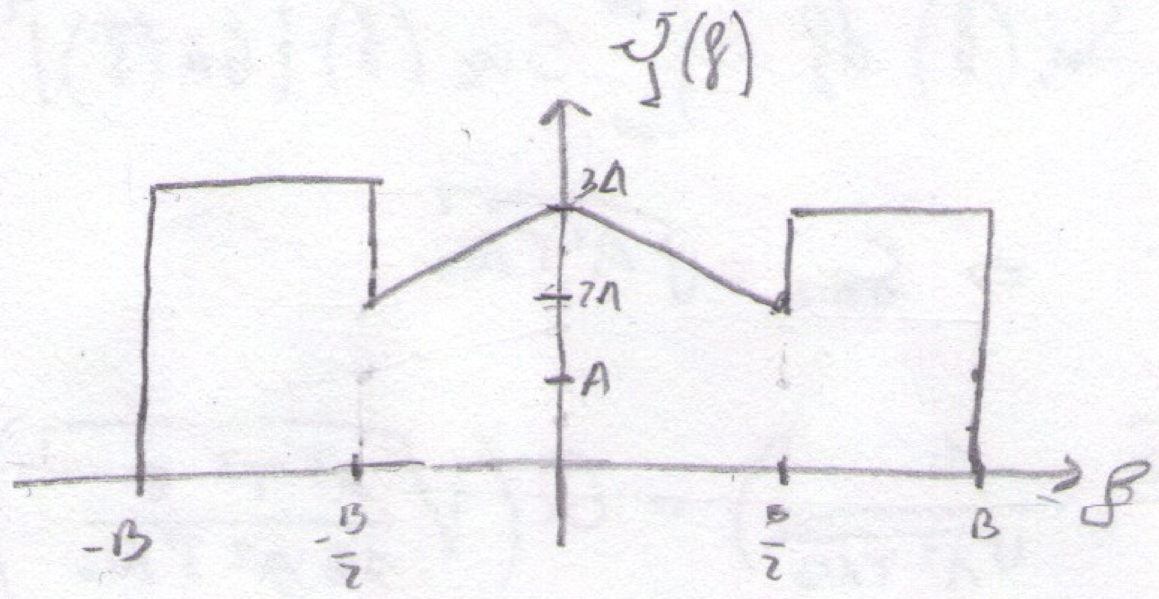



$$\bar{X}_2(f) = \sum_n X_2\left(f - \frac{n}{T_c}\right)$$

$n = 2$ — ; $n = 3$ ... ; $n = -1$ — — — ; $n = -2$ ~~~



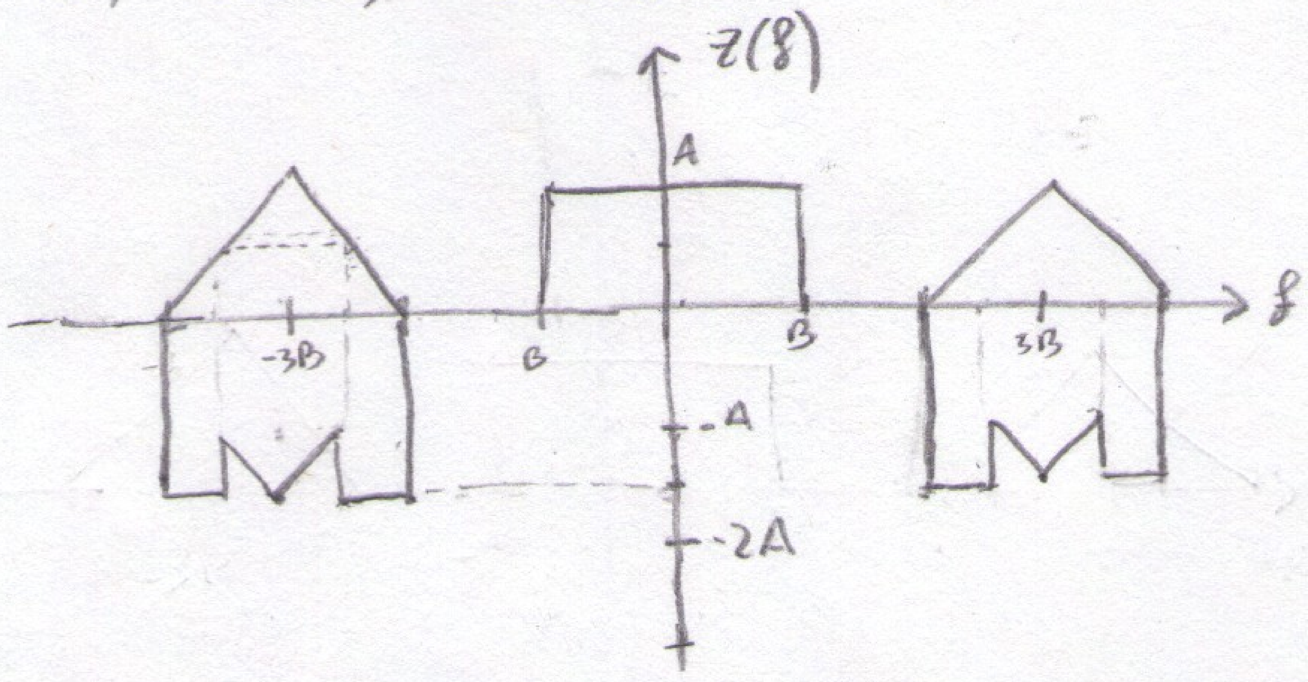

=



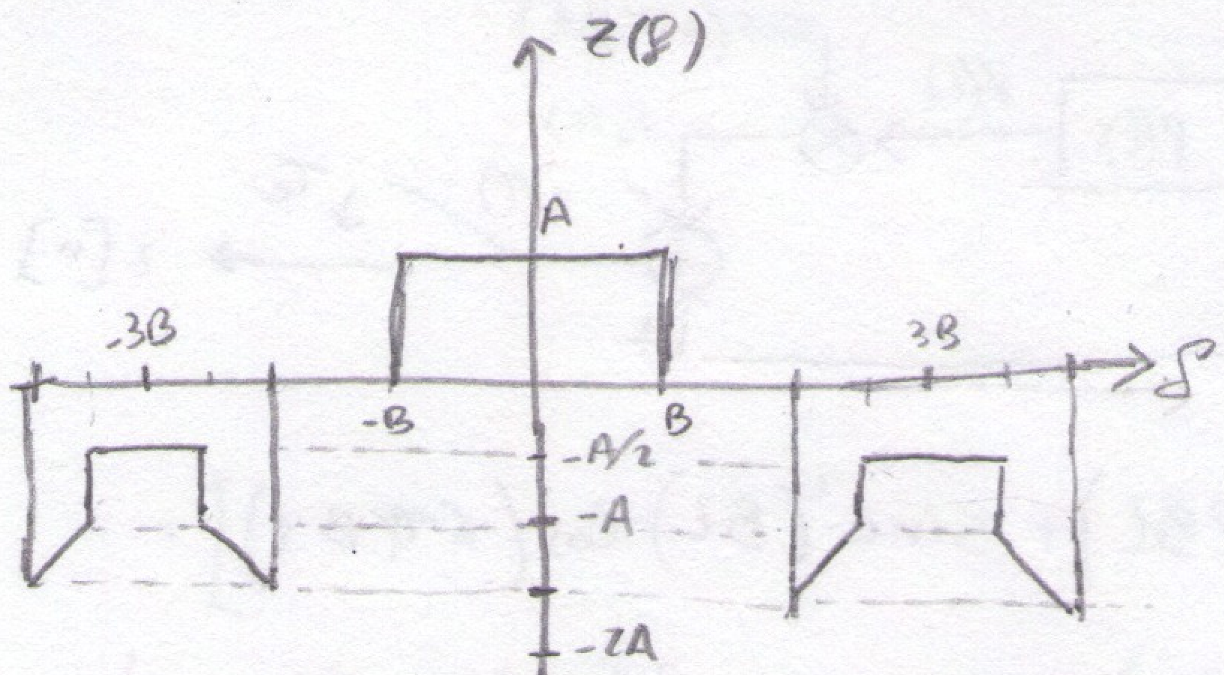

$z(t) =$

$z_1(t) = y(t)\cos(2\pi 3Bt) \rightleftharpoons \frac{1}{2}Y(f-3B) + \frac{1}{2}Y(f+3B) = Z_1(f)$

$Z(f) = X(f) - Z_1(f)$




=




3

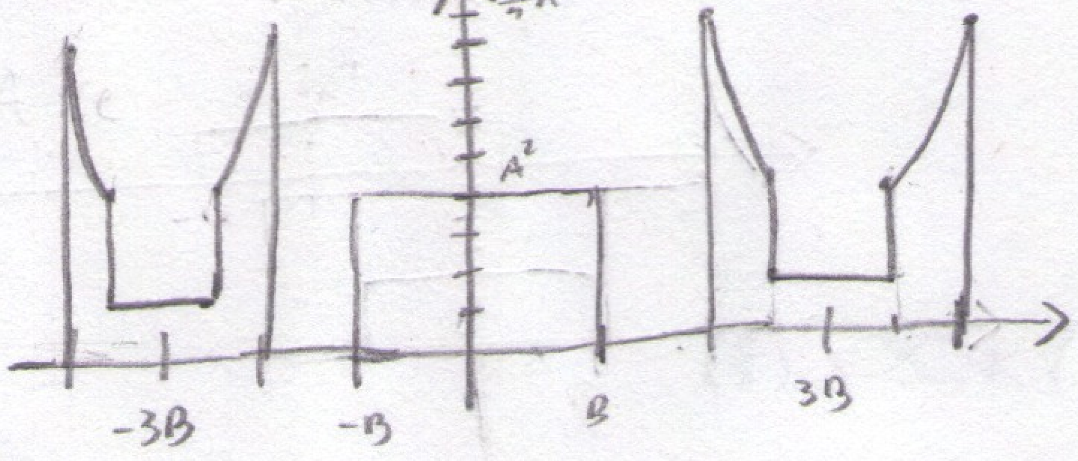
$$Z(f) = A\,\text{rect}\left(\frac{f}{2B}\right) + G(f) \otimes \left\{\frac{1}{2}\delta(f-3B) + \frac{1}{2}\delta\left(f+\frac{3}{B}\right)\right\} \rightleftharpoons z(t) = 2AB\,\text{sinc}(2Bt) + g(t)\cos(6\pi Bt)$$

$$G(f) = -A\,\text{rect}\left(\frac{f}{2B}\right) + \frac{A}{2}\text{rect}\left(\frac{f}{B}\right) + \left\{A\left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right) - \frac{A}{2} - \frac{A}{2}\left(1 - \frac{|f|}{B/2}\right)\text{rect}\left(\frac{f}{B}\right)\right.$$

$$g(t) = -2AB\,\text{sinc}(2Bt) + \frac{AB}{2}\text{sinc}(Bt) + \left\{AB\,\text{sinc}^2(Bt) - \frac{A}{2} - \frac{AB}{4}\text{sinc}^2\left(\frac{B}{2}t\right)\right\}$$ 2

$$z(t) = 2AB\,\text{sinc}(2Bt) + \left\{-2AB\,\text{sinc}(2Bt) + \frac{AB}{2}\text{sinc}(Bt) + \left[AB\,\text{sinc}^2(Bt) - \frac{A}{2} - \frac{AB}{4}\text{sinc}^2\left(\frac{B}{2}t\right)\right]\right\}\cos(6\pi Bt)$$

$$E_z = \int_{-\infty}^{\infty} |Z(f)|^2\,df =$$




$$= A^2 \cdot 2B + 4\left\{\frac{B}{2}\cdot\frac{A^2}{4} + \frac{B}{2}\cdot A^2 + \frac{1}{3}\cdot\frac{B}{2}\cdot\frac{5}{4}A^2\right\} =$$

$$= 2A^2B + 4\left\{\frac{A^2B}{8} + \frac{A^2B}{2} + \frac{5A^2B}{24}\right\} =$$

$$E_z = 2A^2B + 4\cdot\frac{5}{6}A^2B = 2A^2B + \frac{10}{3}A^2B = \frac{16}{3}A^2B$$

$P_z = \emptyset$

Esercizio 2

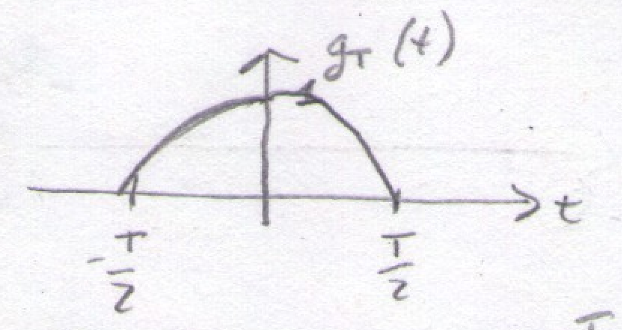
$$S_T(t) = \sum_i a_i \, g_T(t - iT) \cos(2\pi f_0 t) - \sum_i b_i \, g_T(t - iT) \,\mathrm{sen}(2\pi f_0 t)$$

$$S_T(t) = S_{Tc} \cos(2\pi f_0 t) - S_{Ts} \,\mathrm{sen}(2\pi f_0 t)$$

$$E_T = E_{Tc} + E_{Ts} \quad , \quad E_{Tc} = P_{Tc} \cdot \frac{T}{2} = \int_{-\infty}^{\infty} S_{s_{Tc}}(f)\, df \cdot \frac{T}{2}$$

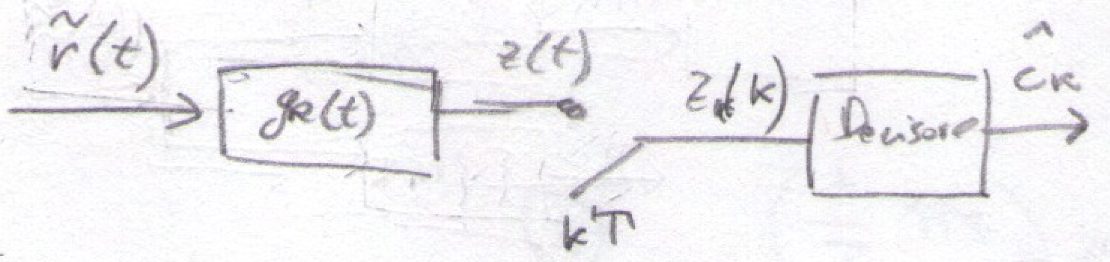
$$S_{s_{Tc}}(f) = \frac{E\{a_i^2\}}{T} |G_T(f)|^2 = \frac{1}{T} |G_T(f)|^2$$

$$P_{Tc} = \int_{-\infty}^{\infty} \frac{1}{T} |G_T(f)|^2 df = \frac{1}{T} \int_{-\infty}^{\infty} |g_T(t)|^2 dt =$$



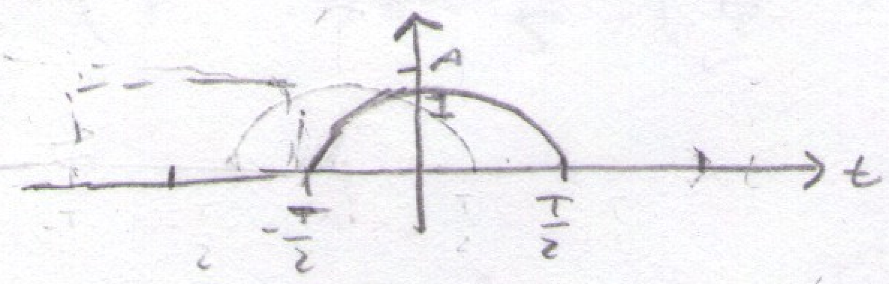
$$= \frac{1}{T} \cdot \frac{1}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} 1\, dt + \frac{1}{T} \frac{1}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos\left(\frac{2\pi}{T} t\right) dt = \frac{1}{2} + \frac{1}{2T} \left[\mathrm{sen}\left(\frac{2\pi}{T} t\right)\right]_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{1}{2} + \frac{1}{2T}[\emptyset - \emptyset] = \frac{1}{2}$$

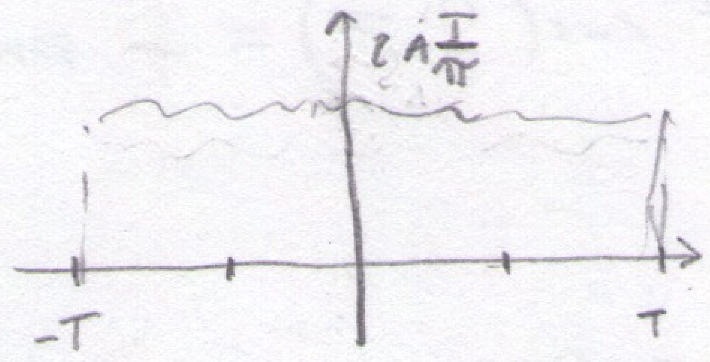
$$E_T = E_{Tc} + E_{Ts} = 2E_{Tc} = \frac{1}{2} \frac{T}{2} \cdot 2 = \frac{T}{2} \quad (1)$$



(2)

$$g(t) \triangleq g_{Tc} \otimes g_R(t)$$





$$g(mT) = \begin{cases} 2A\frac{T}{\pi} & m = \emptyset \\ \emptyset & \text{altrove} \end{cases}$$

per essere di Nyquist $A = \frac{\pi}{2T}$ (3)

$$z_k = c_k + \tilde{n}_k \quad ; \quad \tilde{n}_k \triangleq \tilde{n}(kT) = n_c(kT) + j\, n_s(kT) = n_c + j\, n_s \quad \in \mathcal{N}(\emptyset, \sigma_{\tilde{n}}^2)$$

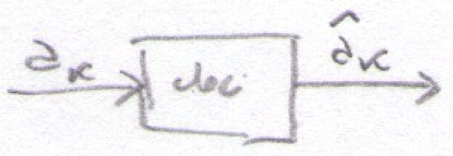
$$z_k = (a_k + n_c) + j\,(b_k + n_s)$$

$$\sigma_{\tilde{n}}^2 = P_{\tilde{n}} = \int_{-\infty}^{\infty} S_{\tilde{n}}(f)\, df$$

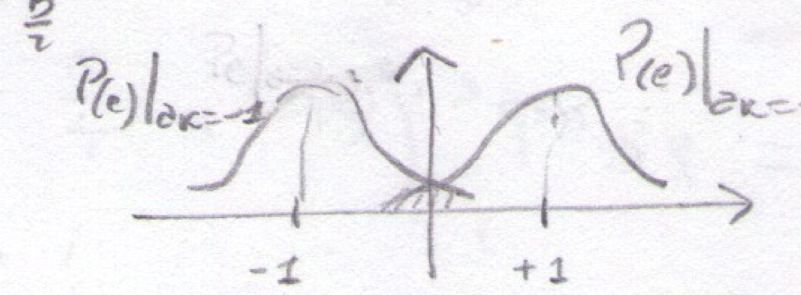
$$S_{\tilde{n}}(f) = S_{\tilde{w}}(f) |G_R(f)|^2 \quad ; \quad S_{\tilde{w}}(f) = \begin{cases} 4 S_w(f + f_0) & f \geq -f_0 \\ \emptyset & \text{altrove} \end{cases} = 2N_0 \,\mathrm{rect}\left(\frac{f}{B}\right)$$

$$S_{\tilde{w}_c}(f) = \frac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4} = N_0 \,\mathrm{rect}\left(\frac{f}{B}\right) \quad ; \quad S_{\tilde{n}_c}(f) = S_{\tilde{w}_c}(f) |G_R(f)|^2$$

$$P_{\tilde{n}_c} = \int_{-\infty}^{\infty} S_{w_c}(f) |G_R(f)|^2 df = N_0 \int_{-\frac{B}{2}}^{\frac{B}{2}} |G_R(f)|^2 df = N_0 \int_{-\frac{B}{2}}^{\frac{B}{2}} |g_R(t)|^2 dt = N_0 A^2 T = N_0 \frac{\pi^2}{4T^2} T = \frac{N_0 \pi^2}{4T}$$



$$P(e) : a_k + n_c$$



$$= 1 - Q\left(\frac{\emptyset - 1}{\sigma_n}\right) + Q\left(\frac{\emptyset + 1}{\sigma_n}\right) =$$

$$P_c(e) = 2Q\left(\frac{1}{\sigma_{\tilde{n}}}\right) = 2Q\left(\sqrt{\frac{4T}{N_0 \pi^2}}\right) = P_s(e)$$

$$P(e) = 4Q\left(\sqrt{\frac{4T}{N_0 \pi^2}}\right)$$