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UNIT - 1

Sample Space

A sample space is defined as the set of possible outcomes of an experiment and is denoted by 'S'.

eg → ① when a coin is tossed $\{H, T\}$

② two coins $\{HH, HT, TH, TT\}$

③ three coins $\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

④ when a dice is thrown, the number of possible outcomes $\{1, 2, 3, 4, 5, 6\}$

⑤ two dice
$$\left\{\begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array}\right\}$$

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Random experiment

An experiment is repeated in which all the possible outcomes are known in advance.

Event and trial

When an experiment is repeated under some conditions, its results in any one of several possible outcomes. Then, the possible outcomes are known as event and cases.

Favourable event

The events which entail (to something necessary) the required happening ~~all~~ are said to be favourable event.

Mutually exclusive event

Two events are called mutually exclusive event when occurance of one of them excludes the occurance of the other.

eg. → while tossing a coin, we get head or tail not both.

Independent event

When the actual happening of one event does not influence in any way. The possibilities of the happening of the other, the two events all called to be an independent events.

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Classical Definition of Probability

Let amount A can happen in 'm' ways and fail in 'n' ways where all ways are equally likely happen to occur. The probability of the happening of an event is

$$P(A) = \frac{\text{No. of favourable case}}{\text{Total no. of mutually exclusive and equally likely cases}}$$

$$= \frac{m}{m+n} = P(\text{say})$$

then of it, failing

$$P(\bar{A}) = \frac{n}{m+n}$$

Or,

$$1 - P(A)$$

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Questions

① What is the P of getting king of heart from the P of cards

⇒ 1/52

② A bag contains 5 red and 10 black balls. What is the probability of getting six black balls.

⇒ The ways of getting 6 black balls out of 10 black balls is

$$^{10}C_6 = \frac{10!}{(10-6)!\,6!} = 210$$

Total ways of choosing 6 balls from all 15 balls is

$$^{15}C_6 = \frac{15!}{6!\,9!} = 5005$$

P (of getting 6 black balls)

$$= \frac{f.o}{T.O}$$

$$= \frac{210}{5005}$$

$$= \frac{6}{143}$$

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③ Getting 2 red and 6 black balls out of 5 red and 10 black balls.

⇒ Way to choose 2 red and 6 black balls out of 15 balls

$$= \frac{^5C_2 \cdot {}^{10}C_6}{^{15}C_8}$$

$$^5C_2 = \frac{5!}{3!\,2!} = 10$$

$$^{10}C_6 = \frac{10!}{4!\,6!} = 210$$

$$^{15}C_8 = \frac{15!}{7!\,8!} = 6435$$

$$\therefore \frac{^5C_2 \cdot {}^{10}C_6}{^{15}C_8}$$

$$= \frac{10 \times 210}{6435}$$

$$= \frac{42}{1287}$$

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Work

① Three coins are tossed. Find the probability of getting

i) 3 Heads — 1/8
ii) 2 Heads — 3/8
iii) At least 2 Heads — 4/8 = 1/2
iv) At least 1 Head — 7/8
v) No tail — 1/8
vi) Atmost 2 Heads — 6/8 = 3/4

② A dice is thrown. Find the probability of getting

i) getting 5 — 1/6
ii) even no. — 3/6 = 1/2
iii) no. ≥ 3 — 4/6 = 2/3
iv) no. ≤ 4 — 4/6 = 2/3

③ In a single throw the two dice. What is the probability obtaining

i) total of 7 — 6/36 = 1/6
ii) total of 13 — 0
iii) total of an even no. — 18/36 = 1/2

When 2 dies are thrown,

S.S. = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1) - - - (2,6),
(3,1) - - - (3,6),
(4,1) - - - (4,6),
(5,1) - - - (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

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① What is the chance that a leap year selected at random will contain 53 Sundays?

→ In a week, there are = 7 days
leap year has = 366 days

Total weeks = 52 weeks and 2 days
(366 ÷ 7)

Two days can be → SM, MT, TW, WTh, ThF, FS, SS

Probability of 53 Sundays = $\frac{2}{7}$.

② What is the chance than a non-leap year selected at random will contain 53 Sundays?

→ In a week = 7 days
leap year has = 365 days

Total weeks = 52 weeks and 1 day
(365 ÷ 7)

1 day can be → S, M, T, W, Th, F, S

Probability of 53 Sundays = $\frac{1}{7}$.

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(3) In a lottery of 50 tickets numbered 1-50, 2 tickets are drawn simultaneously. Find the 'P' when
(i) both are drawn have prime numbers
(ii) none of tickets drawn has prime number

→ S.P. = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 }

(i) Favourable outcomes ⇒ $^{15}C_2$

$$= \frac{15!}{13! \times 2!} = 15 \times 7 = 105$$

Total outcomes = $^{50}C_2$

$$= \frac{50!}{48! \times 2!} = 1225$$

$$\text{Probability} = \frac{^{15}C_2}{^{50}C_2} = \frac{105}{1225}$$

$$= \frac{3}{35}$$

(ii) Probability of not getting prime number in any ticket =

$$1 - \frac{3}{35}$$

$$= \frac{32}{35}$$

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or

Favourable outcomes ⇒ $^{35}C_2$
Total outcomes ⇒ $^{50}C_2$

$$P = \frac{^{35}C_2}{^{50}C_2} = \frac{32}{35}$$

(4) Out of a outstanding students in a college, there are 4 boys and 5 girls. A team of 4 students is to be selected for a quiz program. Find the 'P' that 2 are girls and boys each.

→ Total student = 9
No. of girls = 5
No. of boys = 4

Favourable outcomes = $^4C_2 \cdot {}^5C_2$

$$= \frac{4! \times 5!}{2! \cdot 2! \cdot 3! \cdot 2!}$$

$$= 4 \times 5 \times 3 \times 2$$

$$= 120$$

Total outcomes = $^9C_4 = \frac{9 \times 8 \times 7 \times 6}{24} = 126$

$$\text{Probability} = \frac{120}{126} = \frac{5}{4}$$

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(5) A bag contain 8 white and 4 red balls. 5 balls are drawn at random. P that 2 are red and 3 are white balls

→ Red balls choosen = 2 out of 4
white balls choosen = 3 out of 8

Favourable outcome = $^4C_2 \cdot {}^8C_3$

$$= \frac{4!}{2!\,2!} \times \frac{8!}{5!\times 3!}$$

$$= \frac{4\times 3}{2} \times 8\times 7$$

$$= 16\times 21$$

$$= 336$$

Total outcome = $^{14}C_5$

$$= \frac{14!}{9!\,5!} = \frac{240\cdot 240}{120}$$

$$= 2002$$

Probability = $\frac{336}{2002}$

$$= \frac{168}{1001}$$

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Theorems on Probability

(1) Statement → If A and B are two mutually exclusive and equally likely events of a random experiment, the probability of occurance of the events A and B

$$P(A \text{ or } B) = P(A) + P(B)$$
$$P(A + B) = P(A) + P(B)$$

(2) Statement → The probability of occurance of two independent events is the product of their seperate

$$P(A \cdot B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Questions

(1) Problem in maths is given to 3 students A, B, C. whose chance of solving a problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem will be solved?

→ Probability to solve the problem by student A
$P(A) = 1/2$

$$P(\bar{A}) = 1 - P(A)$$
$$= 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

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Probability to solve the problem by student B

$P(B) = \frac{1}{3}$

$P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

Probability to solve the problem by student C

$P(C) = \frac{1}{4}$

$P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$

Probability not to solve problem by any student is

$P(\bar{A} \cdot \bar{B} \cdot \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

$= \frac{1}{4}$

Probability to solve the problem will be

$P(A.B.C) = 1 - P(\bar{A} \cdot \bar{B} \cdot \bar{C})$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$.

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(2) Five men in a company of 20. Five members are graduates. If three men are selected out of 20 men at random. What is the probability that

(i) they all are graduate. (ii) Atleast one is graduate

total men = 20

graduate = 5

total outcomes to select 3 from 20 = ${}^{20}C_3$

$= \frac{20!}{17! \times 3!}$

$= 20 \times 19 \times 3$

$= 1140$

(i) Favourable outcomes = ${}^{5}C_3$

$= \frac{5!}{2! \cdot 3!} = \frac{5 \times 4}{2} = 10$

Probability $= \frac{\text{Favourable}}{\text{Total}} = \frac{10}{1140} = \frac{1}{114}$.

(ii) Atleast 1 is graduate =>

Favourable outcomes

1 graduate, 2 not + 2 graduate, 1 not + 3 graduate

${}^{5}C_1 \cdot {}^{15}C_2 + {}^{5}C_2 \cdot {}^{15}C_1 + {}^{5}C_3$

$= \frac{5!}{4!} \times \frac{15!}{13! \cdot 2!} + \frac{5!}{3!\,2!} \times \frac{15!}{14!} + \frac{5!}{3!\,2!}$

$= 5 \times 15 \times 7 + 10 \times 15 + 10$

$= 685$

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Probability $= \frac{\text{Favourable}}{\text{Total}}$

$= \frac{685}{1140} = \frac{137}{228}$

(3) 3 cards are drawn at random from an ordinary pack of 52 cards. Find the chance that these are cards of club.

⇒ Cards of club = 13
Cards drawn = 3
Total cards = 52

Total outcomes $= {}^{52}C_3$

$= \frac{52!}{49!\cdot 3!} = \frac{52 \times 51 \times 50}{6}$

$= 52 \times 17 \times 25$

$= 22100$

Favourable outcomes $= {}^{13}C_3$

$= \frac{13!}{10!\cdot 3!} = \frac{13 \times 12 \times 11}{6}$

$= 286$

Probability $= \frac{F.O.}{T.O.} = \frac{286}{22100}$

$= \frac{11}{850}$

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(4) If a die is thrown 2 times. Find the probability that the sum will be 7

⇒ S.S. ⇒ $\{(1,1) \ldots (6,6)\}$ ⇒ Total = 36.

Favourable = 6

Probability that sum is 7 $= \frac{6}{36} = \frac{1}{6}$

(5) We have 5 letters to be post and 5 envelops. Find the probability to post letters in 5 right envelopes.

⇒ Total outcomes $= 5! = 120$

Probability of not posting in right envelope $= 1 - \frac{1}{5!}$

$= \frac{119}{120}$

Probability of posting in right envelope $= 1 - \frac{119}{120}$

$= \frac{1}{120}$

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(1) Two dice are thrown together. Find the probability that the sum is divisible by 2 or 3.

⇒ All cases when two dice are thrown = 36

$$\begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

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Sum divisible by 2 → $\{(1,1),(1,3),(2,2),(3,1),(1,5),(2,4),(3,3),(4,2),(5,1),(2,6),(3,5),(4,4),(5,3),(6,2),(4,6),(5,5),(6,4),(6,6)\} = 18$
(A)

Sum divisible by 3 → $\{(1,2),(2,1),(1,5),(2,4),(3,3),(4,2),(5,1),(6,3),(5,4),(4,5),(3,6),(6,6)\} = 12$
(B)

Sum divisible by 2 and 3 → $\{(1,5),(2,4),(3,3),(4,2),(5,1),(6,6)\} = 6$
$(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{18}{36} + \frac{12}{36} - \frac{6}{36}$$
$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$
$$= \frac{3+2-1}{6} = \frac{2}{3}$$

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Conditional Probability

We denoted conditional probability by $P(E|F)$ where F event is already given and we have to find for event E.

F = already occured

$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)} = \frac{P(E \cap F)}{P(F)}$$

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Definition

If E and F are two events associated with the sample space of the same random experiment the conditional probability $P(E|F)$ of the occurance of E knowing that event F has occured is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

① A die is thrown twice and the sum of numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared atleast once.

Sol: let the events

F = event where sum is 6
E = event where 4 has appeared atleast once

$F = (1,5), (2,4), (3,3), (4,2), (5,1) = 5$
Favourable outcomes = 5
Total outcomes = 36

$$P(F) = \frac{5}{36}$$

$E = (1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (4,1), (4,2), (4,3), (4,5), (4,6) = 11$
Favourable outcomes = 11
Total outcomes = 36

$$P(E) = \frac{11}{36}$$

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$E \cap F = (2,4), (4,2) = 2$

$P(E \cap F) = \frac{2}{36}$

Conditional Probability,

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{2/36}{5/36}$$

$$= \frac{2}{5}$$ Ans

(2) A die is thrown twice and sum of no. appearing is observed to be 7. What is the c.p. that the no. 2 has appeared atleast once.

→ Let the event,

F = event having sum of 7

E = event in which no. 2 has appeared atleast once.

$F = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) = 6$

Favourable outcomes = 6

Total outcomes = 36

$$P(F) = \frac{6}{36} = \frac{1}{6}$$

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$E = (1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (2,1), (2,3), (2,4), (2,5), (2,6) = 11$

$(E \cap F) = \frac{2}{\cancel{36}}$ $\{(2,5), (5,2)\}$ - favourable outcomes

$$P(E \cap F) = \frac{2}{36} = \frac{F.O.}{T.O.}$$

Conditional Probability,

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{2/36}{6/36}$$

$$= \frac{2}{6} = \frac{1}{3}$$ Ans

(3) A die is thrown twice and sum of no. appearing is observed to be 10. What is the conditional probability that the no. 5 has only appeared at first die.

→ Let,

F = event having sum of 10

E = event having no. 5 appeared on first die.

$F = (6,4), (5,5), (4,6)$

Favourable outcomes = 3

Total outcomes = 36

$$P(F) = \frac{3}{36}$$

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$E = (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$

$E \cap F = (5,5)$

Favourable outcome = 1

Total outcomes = 36

$$P(E \cap F) = \frac{1}{36}$$

Conditional Probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{1/36}{3/36}$$

$$= \frac{1}{3}.$$ Ans

(4) Find the Probability of drawing a king when a card is drawn from a well shuffled pack of cards. It is also given that the card is drawn is a face card.

→ let, F = event card is face card

E = event card drawn is king.

Favourable outcomes of $F = 12$

Total outcomes $= 52$

$$P(F) = \frac{12}{52}$$

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$$P(E) = \frac{4}{52}$$

$$P(E \cap F) = \frac{4}{52} = \frac{F.O.}{T.O.}$$

Conditional probability, $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$$= \frac{4/52}{12/52}$$

$$= \frac{1}{3}.$$ Ans

(6) A pair of dice is thrown, if the sum is 7, find the probability that one of the dice shows three.

→ let, F = event having sum is 7

E = event that one die shows 3.

$F = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$

Favourable outcomes = 6

Total outcomes = 36

$$P(F) = \frac{6}{36} = \frac{1}{6}.$$

$E = (1,3), (2,3), (4,3), (5,3), (6,3), (3,1), (3,2), (3,4), (3,5), (3,6)$

$$P(E) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{10}{36}$$

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$E \cap F = (3,4), (4,3)$

$P(E \cap F) = \frac{F.O.}{T.O.} = \frac{2}{36}$

$P(E/F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{2/36}{6/36}$

$= \frac{1}{3}$. Ans.

(6) A problem of statistics is given to 3 students A, B, C. What is the P. that problem will be solved?

→ Total outcomes,

$ABC, A\bar{B}\bar{C}, \bar{A}B\bar{C}, \bar{A}\bar{B}C, AB\bar{C}, A\bar{B}C, \bar{A}BC, \bar{A}\bar{B}\bar{C}$,
$= 8$

Favourable outcomes = 7

Probability $= \frac{F.O.}{T.O.} = \frac{7}{8}$

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Ques (1) A bag contains 100 identical tokens on which no. 1 to 100 are marked. A token is drawn randomly, what is the probability that the no. on token is
(i) an even no. (ii) an odd no. (iii) multiple of 3
(iv) multiple of 5 (v) greater than 70

→ Total outcomes = 100

(i) even no. → $\{2, 4, 6, 8, \ldots 100\} = 50$
favourable outcomes = 50

Probability $= \frac{F.O.}{T.O.} = \frac{50}{100} = \frac{1}{2}$

(ii) odd no. → $\{1, 3, 5, \ldots 99\}$
favourable outcomes = 50

Probability $= \frac{F.O.}{T.O.} = \frac{50}{100} = \frac{1}{2}$

(iii) multiple of 3 → $\{3, 6, 9, 12, \ldots 99\}$
favourable outcomes = 33

Probability $= \frac{F.O.}{T.O.} = \frac{33}{100}$

(iv) multiple of 5 → $\{5, 10, 15, 20, \ldots 100\}$
favourable outcomes = 20

Probability $= \frac{F.O.}{T.O.} = \frac{20}{100} = \frac{1}{5}$

(v) greater than 70 → {71, 72, 73, ... 100}
favourable outcomes = 30

$$\text{Probability} = \frac{F.O.}{T.O.} = \frac{30}{100} = \frac{3}{10}$$

(2) The probability that atleast 1 of A and B occur is 0.6. If A and B occur simultaneously with prob. 0.2. Find $P(\bar{A}) + P(\bar{B})$

→ Given, $P(A \cup B) = 0.6$
$P(A \cap B) = 0.2$

As, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.6 = P(A) + P(B) - 0.2$

$P(\bar{A}) = 1 - P(A)$, $P(\bar{B}) = 1 - P(B)$
$P(A) = 1 - P(\bar{A})$, $P(B) = 1 - P(\bar{B})$

$\Rightarrow$ $0.6 = 1 - P(\bar{A}) + 1 - P(\bar{B}) - 0.2$
$0.8 = 2 - P(\bar{A}) - P(\bar{B})$
$P(\bar{A}) + P(\bar{B}) = 2 - 0.8$
$= 1.2$

Independent Events

Two events which are such that one of them has occured doesn't change the probability of occurance of the event f.

$$P(E \text{ and } F) = P(E).P(F)$$

(1) An unbiased die is thrown. find the probability of getting 4, 5 or 6 in first throw and 1, 2, 3 or 4 in the second throw.

→ let, event E = getting 4, 5 or 6 in first throw
event F = getting 1, 2, 3 or 4 in second throw

Total outcomes = 6
favourable outcomes for E = 3
favourable outcomes for F = 4

$P = \frac{F.O.}{T.O.}$ → $P(E) = \frac{3}{6} = \frac{1}{2}$, $P(F) = \frac{4}{6} = \frac{2}{3}$

$P(E \text{ and } F) = P(E).P(F)$
$= \frac{1}{2} \times \frac{2}{3}$
$= \frac{1}{3}$

(2) A bag contains 5 white, 7 red & 4 black balls. If 4 balls are drawn one by one with replacement. What is the probability that none is white.

→ Given, P(A) = 5 white balls
P(B) = 7 red balls
P(C) = 4 black balls
Total balls = 16
Ball is not white = 11 = favourable

with replacement,
at first drawn = $\frac{11}{16}$, second drawn = $\frac{11}{16}$
third drawn = $\frac{11}{16}$, fourth drawn = $\frac{11}{16}$

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Required Prob. $= \frac{11}{16} \times \frac{11}{16} \times \frac{11}{16} \times \frac{11}{16}$

$=$

without replacement,

required probability $= \frac{11}{16_2} \times \frac{10^5}{15_3} \times \frac{9}{14} \times \frac{8}{13} = \frac{11 \times 3}{14 \times 13}$

$=$

(3) A bag contains 3 white and 5 black balls. 2 balls are drawn without replacement. Determine the probability of getting black balls.

→ Given, 3 white and 5 black balls

Total balls = 8

balls drawn = 2

first drawn $= \frac{5}{8} \times \frac{4}{7}$

$= \frac{5}{14}$

(4) A student has 4 subjects I, II, III, IV with probability of passing as $\frac{4}{5}, \frac{3}{4}, \frac{5}{6}, \frac{2}{3}$ respectively. To qualify, he must pass exam I and any 2 of remaining ~~exam~~ exams. Find the prob. of passing the examination.

→ Given, P in passing I exam $= 4/5 = P(A)$

P in passing II exam $= 3/4 = P(B)$

P in passing III exam $= 5/6 = P(C)$

P in passing IV exam $= 2/3 = P(D)$

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Conditions

$P(\bar{D})\,P(A)\,P(B)\,P(C) + P(A)\,P(B)\,P(D)\,P(\bar{C}) + P(A)\,P(C)\,P(D)\,P(\bar{B}) + P(A)\,P(B)\,P(C)\,P(D)$

Total, $ABCD$, $\bar{A}BCD$, $A\bar{B}CD$, $AB\bar{C}D$, $ABC\bar{D}$, $\bar{A}\bar{B}CD$, $\bar{A}B\bar{C}D$, $\bar{A}BC\bar{D}$, $A\bar{B}\bar{C}D$, $A\bar{B}C\bar{D}$, $AB\bar{C}\bar{D}$, $A\bar{B}\bar{C}\bar{D}$, $\bar{A}\bar{B}\bar{C}D$, $\bar{A}\bar{B}C\bar{D}$, $\bar{A}B\bar{C}\bar{D}$, $\bar{A}\bar{B}\bar{C}\bar{D}$

for Required Probability,

$P(\bar{A}) = 1 - \frac{4}{5} = \frac{1}{5}$, $P(\bar{B}) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(\bar{C}) = 1 - \frac{5}{6} = \frac{1}{6}$, $P(\bar{D}) = 1 - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{1}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{6_3} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{5}{6_3} \times \frac{2}{3}$

$+ \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6_3} \times \frac{2}{3}$

$= \frac{1}{6} + \frac{1}{15} + \frac{1}{9} + \frac{1}{3}$

$= \frac{15 + 6 + 10 + 30}{90}$

$= \frac{61}{90}$

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$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{6+3+2+1+3+2+1}{24} = \frac{18}{24} = \frac{3}{4}.$$

Aug 14, 24

Baye's Theorem

Statement → let S be the sample space and let $E_1, E_2, \ldots E_n$ be 'n' mutually exclusive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}, \quad i = 1, 2, 3, \ldots n$$

① Three urns contain (6 red, 4 black), (4 red, 6 black) and (5 red, 5 black) balls respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from a first urn.

Given, first urn has 6 red and 4 black balls
second urn has 4 red and 6 black balls
third urn has 5 red and 5 black balls

$P(A)$ → prob. to choose a red ball
$P(E)$ → prob. to choose urn

let E_1, E_2, E_3 be 3 urns.

$E_i \Rightarrow E_1 \rightarrow$ choose first urn

$E_2 \rightarrow$ choose second urn

$E_3 \rightarrow$ choose third urn

Probability to choose urn from other urns

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$$

Probability of choosing a red ball from urn E_1, E_2, E_3

$$P(A/E_1) = \frac{6}{10} = \frac{3}{5}$$

$$P(A/E_2) = \frac{4}{10} = \frac{2}{5}$$

$$P(A/E_3) = \frac{5}{10} = \frac{1}{2}$$

As per Baye's theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{1/3 \times 6/10}{\frac{1}{3}\times\frac{3}{5} + \frac{1}{3}\times\frac{2}{5} + \frac{1}{3}\times\frac{1}{2}}$$

$$= \frac{6/30}{6+4+5/30}$$

$$= \frac{6}{15} = \frac{2}{5}.$$

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(2) An urn A contains 2 white, 1 black and 3 red balls urn B contains 3 white, 2 black and 4 red balls and urn C contains 4 white, 3 black and 2 red balls. One urn is choosen at ~~dro~~ random and 2 balls are drawn from the urn. If the choosen ball happened to be red & black. What is the prob. that both balls come from urn B.

Given, urn A has 2 white, 1 black & 3 red
urn B has 3 white, 2 black & 4 red
urn C has 4 white, 3 black & 2 red

$P(E)$ = probability to choose an urn
$P(A)$ = probability to choose red & black balls

let, E_1 = choosing ~~A~~ urn A
E_2 = choosing urn B
E_3 = choosing urn C

Probability to choose urn from other urns

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$$

Probability of choosing red and black balls from urn

$$P(A/E_1) = \frac{{}^3C_1 \cdot {}^1C_1}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}$$

$$P(A/E_2) = \frac{{}^4C_1 \cdot {}^2C_1}{{}^9C_2} = \frac{2}{9}$$

$$P(A/E_3) = \frac{{}^2C_1 \cdot {}^3C_1}{{}^9C} = \frac{1}{6}$$

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As per Baye's theorem,

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{9}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{2}{9} + \frac{1}{3} \times \frac{1}{6}}$$

$$= \frac{2/27}{\frac{1}{15} + \frac{2}{27} + \frac{1}{18}}$$

$$= \frac{2/27}{486+540+405/7290}$$

$$= \frac{2 \times 7290}{\cancel{27} \times \cancel{1431}} \quad \cancel{2430}\ \cancel{270}\ \cancel{90}\ 10 \quad 9 \quad \cancel{477}\ 53$$

$$= \frac{20}{53}.$$

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(8) A factory has 3 machines X, Y, Z producing 1000, 2000, 3000 bolts per day, respectively. The machine X produces 1% defective bolts, Y produces 1.5% & Z produces 2% defective bolts. At the end of the day a bolt is drawn at random and it is found to be defective. What is the ~~prob~~ probability that this defective bolt has been produced by the machine X.

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→ Given, X produces 1000 bolts with 1% defective
Y produces 2000 bolts with 1.5% defective
Z produces 3000 bolts with 2% defective.

Let, E_1, E_2, E_3 are the events to ~~produce~~ drawn ~~defective~~ a bolt from X, Y, Z.

Total bolts = 1000 + 2000 + 3000 = 6000

$$P(E_1) = \frac{1000}{6000} = \frac{1}{6}$$

$$P(E_2) = \frac{2000}{6000} = \frac{1}{3}$$

$$P(E_3) = \frac{3000}{6000} = \frac{1}{2}$$

let A be probability of choosing a defective bolt.

$$P(A/E_1) = \frac{10}{1000} = \frac{1}{100} \quad (1\% \text{ of } 1000 = 10)$$

$$P(A/E_2) = \frac{30}{2000} = \frac{3}{200} \quad (1.5\% \text{ of } 2000 = 30)$$

$$P(A/E_3) = \frac{60}{3000} = \frac{2}{100} \quad (2\% \text{ of } 3000 = 60)$$

As per Bayes's theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) P(A/E_3)}$$

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$$P(E_1/A) = \frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{6}\cdot\frac{1}{100} + \frac{1}{3}\cdot\frac{3}{200} + \frac{1}{2}\cdot\frac{2}{100}}$$

$$= \frac{1/600}{\frac{1}{600} + \frac{1}{200} + \frac{1}{100}}$$

$$= \frac{1/600}{(1+3+6)/600}$$

$$= \frac{1}{10}$$

$$= 0.1$$

(4) A card from a bag of 52 cards is lost. From the remaining cards of the pack 2 cards are drawn and are found to be hearts. Find the prob. that a missing card to be a heart.

→ let, A, E_1, E_2, E_3, E_4 be the events

A = drawing 2 heart cards from the remaining
E_1 = missing card is a heart card
E_2 = missing card is a spade card
E_3 = missing card is a club card
E_4 = missing card is a diamond card

$P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{1}{4}$

$P(E_3) = \frac{1}{4}$, $P(E_4) = \frac{1}{4}$

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$$P(A/E_1) = \frac{^{12}C_2}{^{51}C_2} = \frac{12!}{10! \times 2!} \times \frac{49! \times 2!}{51!}$$

$$= \frac{12 \times 11}{\cancel{2}} \times \frac{\cancel{2}}{51 \times 50}$$

$$= \frac{132}{2550}$$

$$P(A/E_2) = \frac{^{13}C_2}{^{51}C_2} = \frac{13!}{11! \times 2!} \times \frac{49! \times 2!}{51!}$$

$$= \frac{13 \times 12}{2} \times \frac{2}{51 \times 50} = \frac{156}{2550}$$

$$P(A/E_3) = \frac{156}{2550}, \quad P(A/E_4) = \frac{156}{2550}$$

As per Baye's theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) P\left(\frac{A}{E_3}\right) + P(E_4) P\left(\frac{A}{E_4}\right)}$$

$$= \frac{1/4 \cdot 132/2550}{\frac{1}{4}\left(\frac{132}{2550}\right) + \frac{1}{4}\left(\frac{156}{2550}\right) + \frac{1}{4}\left(\frac{156}{2550}\right) + \frac{1}{4}\left(\frac{156}{2550}\right)}$$

$$= \frac{\frac{1}{4} \cdot \frac{132}{2550}}{\frac{1}{4}\left(\frac{132 + 156 + 156 + 156}{2550}\right)}$$

$$= \frac{132}{600}$$

$$= \frac{11}{50}.$$

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(5) A letter is known to have come either from TATANAGAR or CALCUTTA on the envelop just 2 consecutive ~~TA~~ letters TA are visible. What is the prob. that the letter has come from (i) TATANAGAR (ii) CALCUTTA

>) let, E_1 and E_2 are the events to select TATANAGAR and CALCUTTA.

$$P(E_1) = \frac{1}{2} \; ; \quad P(E_2) = \frac{1}{2}.$$

let, A be the event of letters TA

$$P(A/E_1) = \frac{2}{8}$$

$$P(A/E_2) = \frac{1}{7}$$

(i) $$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{8}}{\frac{1}{2} \times \frac{2}{8} + \frac{1}{2} \times \frac{1}{7}}$$

$$= \frac{1/8}{(7+4)/56}$$

$$= \cancel{\frac{7}{8}} \; \frac{7}{11}.$$

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(ii) $P(E_2/A) = \dfrac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)}$

$= \dfrac{\frac{1}{2} \cdot \frac{1}{7}}{\frac{1}{2} \times \frac{2}{8} + \frac{1}{2} \times \frac{1}{7}}$

$= \dfrac{1/14}{\frac{1}{8} + \frac{1}{14}} = \dfrac{1/14}{(7+4)/56} = \dfrac{56}{11 \times 14}$

$= \dfrac{4}{11}$.

(6) There are 4 boxes A, B, C, D containing marbles. A contains 1 red, 6 white & 3 black, B contains 6 red, 2 white & 2 black, C contains 8 red, 1 white & 1 black, & D contains 0 red, 6 white & 4 black marbles. One of the boxes is selected at random and a single marble is drawn from it. If the marble is red, what is the prob. that it was drawn from box A.

Ans let, E_1, E_2, E_3, E_4 be the events of choosing the box A, B, C and D respectively.

and A be the red marble.

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$.

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$P(A/E_1) = 1/10$

$P(A/E_2) = 6/10$

$P(A/E_3) = 8/10$

$P(A/E_4) = 0$

As per Baye's theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3) + P(E_4) \cdot P(A/E_4)}$$

$$= \frac{\frac{1}{4} \cdot \frac{1}{10}}{\frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10} + 0}$$

$$= \frac{\frac{1}{10}}{(1+6+8)/10}$$

$$= \frac{1}{15}.$$

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Probability Distribution

1. Random variable → Random variable may be defined as the real valued function on the sample space taking values on the real time $R(-\infty, \infty)$.

OR

Random variable is a function which takes real values which are determined by the outcomes of the random experiment.

1. Discrete Random Variable → If the random variable X assumes only a finite or countably infinite set of values is known as discrete random variable.

for eg. → • Marks obtained by students
• No. of students in college
• Defective mangoes in a basket

1. Continuous Random Variable → If X can assume infinite & uncountable set of values, it is said to be continuous random variable.

Probability function → Let X be a random variable with possible values $x_1, x_2, \ldots x_n$ and associated probabilities $P(x_1), P(x_2), \ldots P(x_n)$ then the set P whose elements are the ordered pairs $(x_i, P(x_i))$ $\forall$ $i = 1, 2, 3, \ldots n$ is called the probability functions of x.

Ques **Distribution Table**

① for tossing 2 coins
{HH, HT, TH, TT}

no. of heads

x	0	1	2
$P(x)$	1/4	2/4 = 1/2	1/4

② for tossing 2 die
{(1,1), (1,2), ---(6,6)}

Sum

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Probability Density function

If we write $P(X=x) = f(x)$ then $f(x)$ is called probability density function of discrete random variable X.

$$f(x) \geq 0 \quad \text{and} \quad \Sigma f(x) = 1$$

where probability Distribution of random variable X

x	x_1	x_2	---	x_n
$P(x)$	$P(x_1)$	$P(x_2)$	---	$P(x_n)$

Distribution function

Let a random variable X take values $x_1, x_2, \ldots$ w.r.t. probabilities $P_1, P_2, \ldots P_n$ and let

$$x_1 < x_2 < x_3 < \ldots < x_n.$$

The distributive function $f(x)$ is defined as

$$F(x) = P[X \leq x]$$

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Properties

① If F is a distributive function of random variable X and if $a<b$, then
$$P(a<x<b) = F(b) - F(a)$$

② $0 \le F(x) \le 1$

③ $F(x) \le F(y)$ if $x<y$.

④ F has no points of discontinuity at $x_1, x_2, \ldots x_n$. At each of these points F is continuous on the right but is discontinuous on left.

⑤ $F(-\infty) = \lim_{x \to \infty^-} F(x) = 0$

$F(+\infty) = \lim_{x \to \infty^+} F(x) = 1$

Ques Find the Probability Distribution of no. of head in four tosses of a coin.

Sol possibilities $\Rightarrow 2^4 = 16$

HHHH, HTHH, THHH, HHTH, HHHT, HHTT, HTHT, THHT, THTH, HTTH, TTHH, TTTH, TTHT, THTT, HTTT, TTTT.

here X denotes the no. of heads obtains in 4 tosses of a coin. X can take value 0, 1, 2, 3, 4

Let p be the prob. of getting head and q be the prob. of getting tail.

$p = 1/2$, $q = 1/2$

X	0	1	2	3	4
P(x)	1/16	4/16	6/16	4/16	1/16

for X = 0
= TTTT
P(0) = 1/16

for X = 1
= HTTT, TTHT, THTT, TTTH
$P(1) = \frac{4}{16} = \frac{1}{4}$

for X = 2
= HHTT, HTHT, HTTH, THHT, THTH, TTHH
$P(2) = \frac{6}{16} = \frac{3}{8}$

for X = 3
= HHHT, HHTH, HTHH, THHH
$P(3) = \frac{4}{16} = \frac{1}{4}$

for X = 4
= HHHH
$P(4) = \frac{1}{16}$

Hence,

X	0	1	2	3	4
P(x)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

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Ques ① Check whether the following can define probability distribution function and explain your answer

$$f(x) = \frac{5-x^2}{6}, \quad x = 0, 1, 2, 3$$

→ Given, $f(x) = \frac{5-x^2}{6}$ where $x = 0, 1, 2, 3$

$$f(0) = \frac{5-0}{6} = \frac{5}{6}$$

$$f(1) = \frac{5-1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$f(2) = \frac{5-4}{6} = \frac{1}{6}$$

$$f(3) = \frac{5-9}{6} = -\frac{4}{6} = -\frac{2}{3}$$

Distribution function,

$$\sum_{x=0}^{3} f(x) = f(0) + f(1) + f(2) + f(3)$$

$$= \frac{5}{6} + \frac{4}{6} + \frac{1}{6} - \frac{4}{6}$$

$$= \frac{6}{6}$$

$$= 1$$

Hence, $\sum_{x=0}^{3} f(x) = 1$

Yes, it defines the probability distribution function.

(2) If $f = \begin{cases} \frac{2x}{x(x+1)} & , x = 1, 2, 3, \ldots n \\ 0 & , \text{else} \end{cases}$

then find $p(x \leq 3)$

$\Rightarrow$ $P(x \leq 3) = P(1) + P(2) + P(3)$

$$= \frac{2 \times 1}{1(1+1)} + \frac{2 \times 2}{2(2+1)} + \frac{2 \times 3}{3(3+1)}$$

$$= \frac{2}{2} + \frac{4}{6} + \frac{6}{12}$$

$$= \frac{12 + 8 + 6}{12}$$

$$= \frac{26}{12} = \frac{13}{6}$$

(3) Given that $f(x) = k/2^x$ is a prob. function for random variable which take on the values of $x = 0, 1, 2, 3, 4$. find k.

$\Rightarrow$ Since, $f(x)$ is a prob. function

$$\sum_{x=0}^{4} f(x) = 1$$

$$f(0) + f(1) + f(2) + f(3) + f(4) = 1$$

$$\frac{k}{2^0} + \frac{k}{2^1} + \frac{k}{2^2} + \frac{k}{2^3} + \frac{k}{2^4} = 1$$

$$\frac{k}{1} + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} + \frac{k}{16} = 1$$

$$k\left(\frac{16 + 8 + 4 + 2 + 1}{16}\right) = 1$$

$$k = \frac{1 \times 16}{31} = \frac{16}{31}.$$

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(4) Given that $f(x) = \begin{cases} cx^2, & x=1,2,\ldots k \\ 0, & \text{else} \end{cases}$ a probability function for a random variable which take on the value $x=0,1,2,3\ldots k$. find c.

→ Since $f(x)$ is a probability function

$$\sum_{x=0}^{k} f(x) = 1$$

$$f(1) + f(2) + \ldots + f(k) = 1$$

$$c\cdot 1^2 + c\cdot 2^2 + \ldots c\cdot k^2 = 1$$

$$c\,(1^2 + 2^2 + 3^2 + \ldots + k^2) = 1$$

$$c\left[\frac{k(k+1)(2k+1)}{6}\right] = 1$$

$$c = \frac{6}{k(k+1)(2k+1)}$$

(5) $f(x) = \begin{cases} c\left(\frac{2}{3}\right)^x, & x=1,2,3,\ldots \\ 0, & \text{else} \end{cases}$

→ Since $f(x)$ is a probability function

$$\sum_{x=1}^{\infty} f(x) = 1$$

$$f(1) + f(2) + f(3) + \ldots = 1$$

$$c\left(\frac{2}{3}\right)^1 + c\left(\frac{2}{3}\right)^2 + c\left(\frac{2}{3}\right)^3 + \ldots = 1$$

$$c\left[\left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots\right] = 1$$

$$c\left[\frac{2}{3} \Big/ \left(1 - \frac{2}{3}\right)\right] = 1$$

$$\left\{ S_n = \frac{a}{1-r}, r<1 \quad = \frac{a(1-r^n)}{1-r} \right\}$$

$$c = \frac{1}{2}$$

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(6) $f(x) = \begin{cases} c & \text{if } x=0 \\ 2c & \text{if } x=1 \\ 3c & \text{if } x=2 \\ 4c & \text{if } x=3 \\ 1.5c & \text{if } x=4 \\ 0.5c & \text{if } x=5 \end{cases}$

Determine
(i) value of c
(ii) ① $P(x<3)$
② $P(0<x\leq 4)$
③ $P(0<x<2)$
④ Determine the dist. fun

Sol) Since $f(x)$ is a prob. function,

(i) $\sum_{x=0}^{5} f(x) = 1$

$f(0)+f(1)+f(2)+f(3)+f(4)+f(5)=1$

$c+2c+3c+4c+1.5c+0.5c=1$

$12c=1$

$c=1/12$

(ii) ① for $P(x<3) = P(0)+P(1)+P(2)$

$= c+2c+3c$

$= 6c$

$= 6\times\frac{1}{12}$

$= \frac{1}{2}$

(ii) ② for $P(0<x\leq 4) = P(1)+P(2)+P(3)+P(4)$

$= 2c+3c+4c+1.5c$

$= 10.5\times\frac{1}{12}$

$= \frac{7}{8}$

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(ii) (3) for $P(0 < x < 2) = P(1)$

$$= 2c$$

$$= 2 \times \frac{1}{12}$$

$$= \frac{1}{6}$$

(ii) (4) Distributive Table,

at $x = 0$, $P(0) = c = \frac{1}{12}$

at $x = 1$, $P(1) = 2c = \frac{2}{12} = \frac{1}{6}$

at $x = 2$, $P(2) = 3c = \frac{3}{12} = \frac{1}{4}$

at $x = 3$, $P(3) = 4c = \frac{4}{12} = \frac{1}{3}$

at $x = 4$, $P(4) = 1.5c = \frac{1.5}{12} = \frac{1}{8}$

at $x = 5$, $P(5) = 0.5c = \frac{0.5}{12} = \frac{1}{24}$

x	0	1	2	3	4	5
$P(x)$	1/12	1/6	1/4	1/3	1/8	1/24

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(7) If random variable X has the following prob. function.

x	-2	-1	0	1	2	3
$f(x)$	0.1	k	0.2	$2k$	0.3	$3k$

(i) find k
(ii) evaluate $P(x<2)$, $P(x \geq 2)$, $P(-2<x<2)$
(iii) Determine the distribution function.

* Since $f(x)$ is a probability function.

(i)
$$\sum_{x=-2}^{3} f(x) = 1$$

$$f(-2) + f(-1) + f(0) + f(1) + f(2) + f(3) = 1$$
$$0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$$
$$0.6 + 6k = 1$$
$$6k = 1 - 0.6$$
$$k = \frac{0.4}{6}$$
$$k = \frac{1}{15}$$

(ii)
$$P(x<2) = P(-2) + P(-1) + P(0) + P(1)$$
$$= 0.1 + k + 0.2 + 2k$$
$$= 0.3 + 3k$$
$$= \frac{0.3}{10} + \frac{3}{15}$$
$$= \frac{9+6}{30} = \frac{15}{30}$$
$$= \frac{1}{2}$$

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$$P(x \geq 2) = P(2) + P(3)$$
$$= 0.3 + 3k$$
$$= 0.3 + \frac{3}{15}$$
$$= \frac{3}{10} + \frac{3}{15}$$
$$= \frac{9+6}{30} = \frac{15}{30} = \frac{1}{2}$$

$$P(-2 < x < 2) = P(-1) + P(0) + P(1)$$
$$= k + 0.2 + 2k$$
$$= 0.2 + 3 \times \frac{1}{15}$$
$$= 0.2 + 0.2 = 0.4 = \frac{2}{5}$$

(iii) $P(-1) = k = \frac{1}{15}$

$P(1) = 2k = \frac{2}{15}$

$P(3) = 3k = \frac{3}{15} = \frac{1}{5}$

x	-2	-1	0	1	2	3
$f(x)$	0.1	1/15	0.2	2/15	0.3	1/5

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(8)

x	0	1	2	3	4	5	6	7
$f(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) find k
(ii) Determine distributive function
(iii) Evaluate $P(x<6)$, $P(x\geq 6)$, $P(0<x<5)$

(i) Since $f(x)$ is variable function

$$\sum_{x=0}^{7} f(x) = 1$$

$$f(0)+f(1)+f(2)+f(3)+f(4)+f(5)+f(6)+f(7)=1$$
$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$
$$9k+10k^2=1$$

$$10k^2+9k-1=0$$
$$10k^2+10k-k-1=0$$
$$10k(k+1)-1(k+1)=0$$
$$(10k-1)(k+1)=0$$

$$k=-1, \quad k=1/10$$

(ii) for $k=-1$,

x	0	1	2	3	4	5	6	7
$f(x)$	0	-1	-2	-2	-3	1	2	6

for $k=1/10=0.1$,

x	0	1	2	3	4	5	6	7
$f(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

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(iii) for $k = -1$,

$$P(x < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$
$$= 0 + (-1) + (-2) + (-2) + (-3) + 1$$
$$= -8 + 1$$
$$= -7$$

$$P(x \geq 6) = P(6) + P(7)$$
$$= 2 + 6$$
$$= 8$$

$$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$$
$$= -1 - 2 - 2 - 3$$
$$= -8$$

for $k = \frac{1}{10} = 0.1$,

$$P(x < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$
$$= 0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$$
$$= 0.81$$

$$P(x \geq 6) = P(6) + P(7)$$
$$= 0.02 + 0.17$$
$$= 0.19$$

$$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$$
$$= 0.1 + 0.2 + 0.2 + 0.3$$
$$= 0.8$$

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Aug 28, 24

Mean or Expected Value and Variance

$$E(x) = \int_a^b x f(x)\, dx$$

OR

$$E(x) = \sum_{i=1}^{n} P_i x_i = \mu$$

Standard deviation (σ) & Variance

Variance, $\sigma^2 = \int_a^b x^2 f(x) - \mu^2$

OR

$$\sigma^2 = \Sigma P x^2 - \mu^2$$

Questions

Ques(1) A random variable x has the following distribution.

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Determine (i) k (ii) Mean (iii) Variance

Ans Since,

(i) $\Sigma P(x) = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$0.6 + 4k = 1$

$4k = 0.4$

$k = \frac{1}{10} = 0.1$

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X	-2	-1	0	1	2	3
P(X)	0.1	0.1	0.2	0.2	0.3	0.1

(ii) Expected value or Mean,

$$E(x) = \sum_{i=1}^{n} P_i x_i$$

$$= P_1x_1 + P_2x_2 + P_3x_3 + \cdots$$

$$= 0.1(-2) + 0.1(-1) + 0.2(0) + 0.2(1) + 0.3(2) + 0.1(3)$$

$$= -0.2 - 0.1 + 0.2 + 0.6 + 0.3$$

$$= 0.8$$

$$= \mu$$

(iii) Variance,

$$\sigma^2 = \sum_{i=1}^{n} P_i x_i^2 - (\sum P_i x_i)^2$$

$$= 0.1 \times 4 + 0.1 \times 1 + 0.2 \times 0 + 0.2 + 0.3(4) + 0.1(9) - (0.8)^2$$

$$= 0.4 + 0.1 + 0.2 + 1.2 + 0.9 - 0.64$$

$$= 2.8 - 0.64$$

$$= 2.16$$

(2) A coin is tossed twice. Find the distribution table, variance and mean.

→ (i) let x be the no. of heads

X	0	1	2
P(X)	1/4	2/4	1/4

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(ii) Mean, $E(x) = \sum_{i=1}^{n} P_i x_i$

$= P_1 x_1 + P_2 x_2 + P_3 x_3$

$= \frac{1}{4}(0) + \frac{2}{4}(1) + \frac{1}{4}(2)$

$= \frac{2}{4} + \frac{2}{4}$

$= \frac{4}{4} = 1 = \mu$

(iii) Variance,

$\sigma^2 = \sum_{i=1}^{n} P_i x_i^2 - (\mu)^2$

$= \frac{1}{4}(0) + \frac{2}{4}(1) + \frac{1}{4}(4) - (1)^2$

$= \frac{2}{4} + 1 - 1$

$= \frac{2}{4}$

$= \frac{1}{2}$

(3) If random variable X has following distribution. Find the value of k, mean, $P(X) \geq 3$.

X	1	2	3	4	8	9
P(X)	k	3k	5k	7k	9k	11k

(i) Since $\sum P(X) = 1$

$$k + 3k + 5k + 7k + 9k + 11k = 1$$

$$36k = 1$$

$$k = \frac{1}{36}$$

X	1	2	3	4	8	9
P(X)	1/36	3/36	5/36	7/36	9/36	11/36

(ii) Mean,

$$E(x) = \sum_{i=1}^{n} p_i x_i$$

$$= p_1 x_1 + p_2 x_2 + p_3 x_3 + \cdots$$

$$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{72}{36} +$$

$$= \frac{221}{36}$$

Variance, $\sigma^2 = \sum_{i=1}^{n} p_i x_i^2 - \left(\sum p_i x_i\right)^2$

$$= \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} +$$

$$+ \frac{891}{36} - \frac{231}{36}$$

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$$= \frac{1637}{36} - \frac{231}{36}$$

$$= \frac{1406}{36}$$

(iii) $P(x) \geq 3$,

$$= P(3) + P(4) + P(5) + P(6)$$

$$= 5k + 7k + 9k + 11k$$

$$= \frac{32}{36}$$

$$= \frac{16}{18} = \frac{8}{9} = 0.89.$$

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

(i) find k (ii) $P(X<4)$ (iii) Mean (iv) σ^2 (v) $P(3<x\leq$

Ans (i) Since, $\Sigma P(x) = 1$

$$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$

$$49k = 1$$

$$k = \frac{1}{49}$$

(ii) $P(x) < 4$,

$$= P(0) + P(1) + P(2) + P(3)$$

$$= k + 3k + 5k + 7k$$

$$= 16k$$

$$= \frac{16}{49}.$$

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(iii) Mean,

$$E(x) = \sum_{i=1}^{n} P_i x_i$$

$$= (0)k + 3k(1) + 5k(2) + 7k(3) + 9(4) + 11k(5) + 13k(6)$$

$$= 3k + 10k + 21k + 36k + 55k + 78$$

$$= 203k$$

$$= \frac{203}{49} = \frac{29}{7}$$

(iv) Variance,

$$\sigma^2 = \sum_{i=1}^{n} P_i x_i^2 - \mu^2$$

$$= k(0) + 3k(1) + 5k(4) + 7k(9) + 9k(16) + 11k(25) + 13k(36) - \left(\frac{29}{7}\right)^2$$

$$= 3k + 20k + 63k + 144k + 275k + - \frac{841}{49}$$

$$= \frac{2106}{49} - \frac{841}{49}$$

$$= \frac{1265}{49}$$

$$=$$

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Bionomial

$$(x+y)^n = {}^nC_0\, x^0 y^{n-0} + {}^nC_1\, x^1 y^{n-1} + {}^nC_2\, x^2 y^{n-2} + \dots + {}^nC_n\, x^n$$

→ Bionomial distribution

$$P(r) = {}^nC_r\, p^r q^{n-r}$$

$$P \Rightarrow p+q=1$$

Ques ① Ten coins are tossed simultaneously, then find the prob. of getting head at least 7 times.

→ when a coin is tossed, then the probability of getting a head is P and not getting a head is q.

$$P = \frac{1}{2}, \quad q = 1 - \frac{1}{2} = \frac{1}{2}$$

Prob. getting at least 7 heads =
P (7 heads) + P(8 heads) + P(9 heads) + P(10 heads)

$$= {}^{10}C_7\, p^7 q^{10-7} + {}^{10}C_8\, p^8 q^{10-8} + {}^{10}C_9\, p^9 q^{10-9} + {}^{10}C_{10}\, q^{10-10} p^{10}$$

$$= \frac{10!}{7!\cdot 3!} \times \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + \frac{10!}{8!\cdot 2!}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + \frac{10!}{9!\cdot 1!}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1$$

$$+ \frac{10!}{0!\cdot 10!}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$$

$$= \frac{10\times 9\times 8}{6}\left(\frac{1}{2}\right)^{10} + \frac{10\times 9}{2}\left(\frac{1}{2}\right)^{10} + 10\left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left(120 + 45 + 10 + 1\right) = \frac{176}{1024} = \frac{11}{64}$$

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Ques ② In a lot of 200 articles, 10 are defective. Find probability of (i) no defective articles
(ii) 1 defective article
(iii) at least 1 defective article
random sample of 20 artic

⇒ Given, selecting 20 article with 10 defective from 20 artic

let, P is the probability of selecting defective ar

$$p = \frac{10}{200} = \frac{1}{20}$$

and probability of non-defective article

$$q = 1 - \frac{1}{20} = \frac{19}{20}$$

(i) Prob. getting no defective article $= {}^{20}C_0\, p^0\, q^{20-0}$

$$= \frac{20!}{20!\cdot 0!} (1)\left(\frac{19}{20}\right.$$

$$= \left(\frac{19}{20}\right)^{20}$$

(ii) Prob. getting one defective article $= {}^{20}C_1\, p^1\, q^{19}$

$$= \frac{20!}{19!\cdot 1!} \left(\frac{1}{20}\right)\left(\right.$$

$$= \left(\frac{19}{20}\right)^{19}$$

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(iii) Atleast one defective article = 1 − no defective article

$$= 1 - \left(\frac{19}{20}\right)^{20}$$

$$= \frac{(20)^{20} - (19)^{20}}{(20)^{20}}$$

(3) If on an average, 1 ship out of 10 is wrecked. Find the prob. that out of 5 ships are expected to arrive the port atleast 4 will arrive safely.

→ let, p be the probability of safe ships

$$p = \frac{9}{10}$$

and q be the probability of wrecked ship

$$q = 1 - \frac{9}{10} = \frac{1}{10}$$

Prob. that atleast 4 ships out of 5 will arrive safely $= {}^5C_4\, p^4 q^1 + {}^5C_5\, p^5 q^0$

$$= \frac{5!}{4!\cdot 1!}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right) + \frac{5!}{5!\cdot 0!}\left(\frac{9}{10}\right)^5$$

$$= \frac{5(9)^4 + (9)^5}{(10)^5}$$

$$= \frac{14(9)^4}{(10)^5}$$

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(4) The prob. that a man aged 60 will live be 70 is 0.65. What is the probability that out of 10 man aged 60 now, atleast 7 would live to be 70.

⇒ Prob. to live upto 70, $p = 0.65$

also, $q = 1 - 0.65 = 0.$

Prob. that atleast 7 out of 10 man will live upto 70 $= {}^{10}C_7 p^7 q^3 + {}^{10}C_8 p^8 q^2 + {}^{10}C_9 p^9 q^1 + {}^{10}C_{10} p^{10} q^1$

$$= \frac{10!}{7!\cdot 3!}(0.65)^7(0.35)^3 + \frac{10!}{8!\cdot 2!}(0.65)^8(0.35)^2 +$$

$$\frac{10!}{9!\cdot 1!}(0.65)^9(0.35) + \frac{10!}{10!\cdot 0!}(0.65)^{10}$$

$$= 120(0.65)^7(0.35)^3 + 45(0.65)^8(0.35)^2 + 10(0.65)^9(0.35) +$$

$$= (0.65)^7\left[120\times(0.35)^3 + 45\times 0.65\times(0.35)^2 + 10\times(0.65)^2\times + (0.65)^3\right.$$

$$= (0.65)^7(5.145 + 3.583125 + 1.47875 + 0.274625$$

$$= (0.65)^7(10.4815)$$

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(5) 6 dice are thrown together at a time, the process is repeated 729 times. How many times do you expect that atleast 3 dice to have 4 or 6.

⇒ Let, favourable probability (having 4 or 6) $p = \frac{2}{6} = \frac{1}{3}$

and $q = 1 - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

Prob. that atleast 3 dice out of 6 dice having 4 or 6 $= P(3) + P(4) + P(5) + P(6)$

$= {}^6C_3 p^3 q^3 + {}^6C_4 p^4 q^2 + {}^6C_5 p^5 q^1 + {}^6C_6 p^6 q^0$

$= \frac{6!}{3!\cdot 3!}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + \frac{6!}{4!\cdot 2!}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \frac{6!}{5!\cdot 1!}\left(\frac{1}{3}\right)^5\left(\frac{1}{3}\right) + \frac{6!}{6!\cdot 0!}\left(\frac{1}{3}\right)^6$

$= \left(\frac{1}{3}\right)^6 \left[20(2)^3 + 15(2)^2 + 6 + 1\right]$

$= \left(\frac{1}{3}\right)^6 (160 + 60 + 7)$

$= \frac{227}{729}$

3rd Sept, 24

(6) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys, 2 girls
(ii) atleast 1 boy
(iii) no girl
(iv) atmost 2 girl

Ratio of boys and girls are same (2 each).

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» Total families = 800
no. of children = 4

(i) 2 Boys, 2 girls
$= 800\ {}^4C_2\ p^2 q^2$

Prob. to choose a girl $= \frac{2}{4} = \frac{1}{2} = p$

Prob. to choose a boy $= \frac{2}{4} = \frac{1}{2} = q$

$$\Rightarrow 800 \times \frac{4!}{2!\,2!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$$

$$= 800 \times \frac{4\times 3}{2} \times \frac{1}{4} \times \frac{1}{4}$$

$$= 100 \times 3$$

$$= 300$$

(ii) atleast 1 boy $= 800\,(1B\,3G + 2B\,2G + 3B\,1G + 4B)$
$= 800\,[{}^4C_1 p^1 q^3 + {}^4C_2 p^2 q^2 + {}^4C_3 p^3 q^1 + {}^4C_4$

Prob. of girl $= 1/2 = p$
Prob. of boy $= 1/2 = q$

$$\Rightarrow 800 \times \left[\frac{4!}{3!\,1!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + \frac{4!}{2!\,2!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + \frac{4!}{3!\,1!}\left(\frac{1}{2}\right)^3 + \frac{4!}{4!\,0!}\left(\frac{1}{2}\right)^4\right]$$

$$= \frac{800}{16}(4+6+4+1)$$

$$= 50 \times 15 = 750$$

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(iii) No girl $\Rightarrow$ 4 boys $= {}^4C_4\, q^4 p^0 \times 800$

$$= \frac{4!}{4!\cdot 0!}\left(\frac{1}{2}\right)^4 \times 800$$

$$= \frac{800}{16}$$

$$= 50$$

(iv) Atmost 2 girls $= 800\left[4B + 1G\,3B + 2B\,2G\right]$

$$= 800\left[{}^4C_0\, q^4 + {}^4C_1\, p^1 q^3 + {}^4C_2\, p^2 q^2\right]$$

$$= 800\left[\frac{4!}{4!\cdot 0!}\left(\frac{1}{2}\right)^4 + \frac{4!}{3!\cdot 1!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + \frac{4!}{2!\cdot 2!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2\right]$$

$$= \frac{800}{2^4}\left[1 + 4 + 6\right]$$

$$= \frac{800 \times 11}{16} = 550$$

(7) 8 coins are thrown. Find the prob. of getting atleast 5 heads.

$\Rightarrow$ coin thrown = 8 times

prob. of getting head, $p = \frac{1}{2}$

prob. of getting tail, $q = \frac{1}{2}$

atleast 5 head, prob $= (5H3T + 6H2T + 7H1T + 8H)$

$$= \left({}^8C_5\, p^5 q^3 + {}^8C_6\, p^6 q^2 + {}^8C_7\, p^7 q^1 + {}^8C_8\, p^8\right)$$

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$$= \frac{8!}{5!\cdot 3!}\left(\frac{1}{2}\right)^8 + \frac{8!}{6!\,2!}\left(\frac{1}{2}\right)^8 + \frac{8!}{7!\cdot 1!}\left(\frac{1}{2}\right)^8 + \frac{8!}{8!\cdot 0!}\left(\right.$$

$$= \left(\frac{1}{2}\right)^8 \left[\frac{8\times7\times6\times5!}{6\times5!} + \frac{8\times7}{2} + 8 + 1\right]$$

$$= \left(\frac{1}{2}\right)^8 (42 + 28 + 9)$$

$$= \frac{79}{256}$$

(8) The prob. that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the prob. that (i) exactly 2 will be defective
(ii) no defective
(iii) atleast 2 pens will be defective

⇒ total pens = 12

prob. of defective $= \frac{1}{10} = p$

prob. of non-defective pen $= 1 - \frac{1}{10} = \frac{9}{10} = q$

(i) exactly 2 will be defective, prob. $= {}^{12}C_2\, p^2 q^{10}$

$$= \frac{12!}{2!\cdot 10!}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^{10}$$

$$= \frac{6\times11\times(9)^{10}}{(10)^{12}} = \frac{66\times(9)^{10}}{(10)^{12}}$$

$=$

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(ii) No defective $\Rightarrow {}^{12}C_0 p^0 q^{12}$

$$= \frac{12!}{12! \times 0!} \times 1 \times \left(\frac{9}{10}\right)^{12}$$

$$= \left(\frac{9}{10}\right)^{12}$$

(iii) atleast 2 defective $= {}^{12}C_2 p^2 q^{10} + {}^{12}C_3 p^3 q^9 + {}^{12}C_4 p^4 q^8 + {}^{12}C_5 p^5 q^7 + {}^{12}C_6 p^6 q^6 + {}^{12}C_7 p^7 q^5 + {}^{12}C_8 p^8 q^4 + {}^{12}C_9 p^9 q^3 + {}^{12}C_{10} p^{10} q^2 + {}^{12}C_{11} p^{11} q^1 + {}^{12}C_{12} p^{12} q^0$

OR

$= 1 - (0D\ 12ND + 1D\ 11ND)$

$$= 1 - \left({}^{12}C_0 p^0 q^{12} + {}^{12}C_1 p^1 q^{11}\right)$$

$$= 1 - \left[\frac{12!}{12! \cdot 0!} \times 1 \times \left(\frac{9}{10}\right)^{12} + \frac{12!}{11! \cdot 1!} \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{11}\right]$$

$$= 1 - \left[\left(\frac{9}{10}\right)^{12} + \frac{12 \times (9)^{11}}{(10)^{12}}\right]$$

$$= 1 - \left(\frac{9^{12} + 12 \times 9^{11}}{10^{12}}\right)$$

$$= 1 - \frac{9^{11}}{10^{12}} (21)$$

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(9) 8 coins are tossed, the prob. of getting atleast 6 heads.

⇒ Total coins = 8

prob. of getting head $= \frac{1}{2} = p$

prob. of getting head $= \frac{1}{2} = q$

atleast 6 heads = 6H2T + 7H1T + 8H

$= {}^8C_6 p^6 q^2 + {}^8C_7 p^7 q^1 + {}^8C_8 p^8$

$= \frac{8!}{6! \cdot 2!}\left(\frac{1}{2}\right)^8 + \frac{8!}{7! \cdot 1!}\left(\frac{1}{2}\right)^8 + \frac{8!}{8!}\left(\frac{1}{2}\right)^8$

$= \frac{28 + 8 + 1}{256} = \frac{37}{256}$

(10) The prob. that a man will hit a target is 1/5, then its his prob. of atleast 1 hit in 10 shots.

⇒ total shots = 10

prob. of hitting a shot = 1/5 = p

prob. of not - hitting a shot $= 1 - \frac{1}{5} = \frac{4}{5} = q$

atleast 1 hit = no hit = zero hit

$= {}^{10}C_0 p^0 q^{10}$

$= \frac{10!}{10!}\left(\frac{4}{5}\right)^{10} = \left(\frac{4}{5}\right)^{10}$

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Poisson Distribution

It is a discrete probability distribution which has the following characteristics:-

① It is the limiting form of the binomial distribution as n becomes infinitely large i.e. $n \to \infty$ ~~and~~ and p, the constant probability of success for each trial becomes infinitely small, i.e. $p \to 0$ in such a manner that $\boxed{np = m}$ remains a finite.

② It consists of a single parameter m only. The entire distribution can be obtained once 'm' is known.

Conditions:-

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→ Mean of Poisson Distribution,

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

$\begin{cases} x \to \text{event} = 0,1,2,3, \\ m \to \text{trial} \end{cases}$

→ Recurrence formula

$$P(x+1) = \frac{m}{x+1} \cdot P(x)$$

Ques The no. of telephone calls arriving on an interval switch board of an office is 90 per hour. Find the prob. that at the most 1 to 3 calls in one minute. ($e^{-1.5} = 0$

⇒ Given, mean = 90 per hour

$m = \frac{90}{60}$ per minute

$= 1.5$ per minute

by poisson distribution,

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

$$P(1) = \frac{e^{-1.5} \cdot (1.5)^1}{1!}$$

$\cancel{= 0.002 \times 1.5}$

$\cancel{= 0.0030}$

$= 0.223 \times 1.5$

$= 0.3495$

$$P(2) = \frac{e^{-1.5}(1.5)^2}{2!} = \frac{0.223 \times 2.25}{2} \cancel{= 000225}$$

$$= 0.262$$

$$P(3) = \frac{e^{-1.5}(1.5)^3}{3!} = \frac{0.223 \times 1.5 \times 1.5 \times 1.5}{6}$$

$$= 0.125$$

Required probability, $P(1) + P(2) + P(3)$

$$= 0.3495 + 0.262 + 0.125$$

$$= 0.7365$$

Ques Avg. no. of accidents on any day on a national highway is 1.8. Determine the prob. that the no. of accidents are ($e^{-1.8} = 0.16529$)
(i) at least one (ii) at most one

→ Given, mean, $m = 1.8$

(i) at least one accident, $P(x \geq 1)$

$$= 1 - P(x=0)$$

$$= 1 - \frac{e^{-m} m^x}{x!}$$

$$= 1 - \frac{e^{-1.8}(1.8)^0}{0!}$$

$$= 1 - 0.16529$$

$$\simeq 0.83471$$

(approx) $\simeq 0.835$

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at most one accident $(P(0) + P(1))$

by poisson distribution,

$$= P(0) + P(1)$$

$$= \frac{e^{-1.8}(1.8)^0}{0!} + \frac{e^{-1.8}(1.8)^1}{1!}$$

$$= 0.16529\,[1 + (1.8)]$$

$$= 0.16529 \times 2.8$$

$$= 0.462812$$