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Jan 27, 25

Discrete
objects with distinct values.
it includes graphs, logical operations, relations etc.

Sets
Collection of distinct & non-contineous elements.

→ Discrete Mathematics
It's a branch of mathematics that concerned with discrete mathematical structure instead of continuous. It includes object with distinct values like graphs, integers, logical based statements etc.

Sets
A well-defined unordered collection of distinct elements is called a set.
for eg.
$A = \{1,2,3,4\}$ → ordered
$B = \{4,3,2,1\}$ → unordered
$C = \{1,1,2,2,3,4\}$ → $C = \{1,2,3,4\}$
· no duplicate elements, distinct values.

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→ **Null set / Empty set**

Set containing no value is called null set. It's denoted by ϕ or $\{\}$.

$A = \phi$, $B = \{\}$

$C = \{\phi\}$ → set contains ϕ

→ **Subset**

If every element of set A is also element of set B, then A is a subset of B. It's denoted by $\subseteq$.

eg.

if, $A = \{1, 2, 3, 4, 5\}$

and, $B = \{1, 2, 3, 4, 5\}$

then, $A \subseteq B$

→ **Proper Subset**

Any subset of A which is not a trival subset of B, is called proper subset.

eg.

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4, 5\}$

$A \subset B$

→ **Cardinality set**

Total number of elements in a set.

eg.

Cardinality of B ($B = \{1, 2, 3, 4, 5\}$ is 5.

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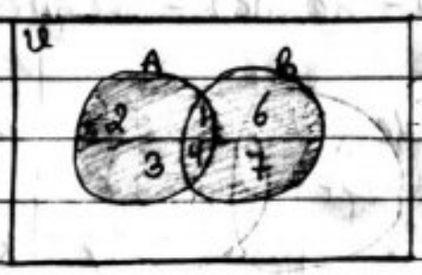
Operations

- Union $(\cup)$
- Intersection $(\cap)$
- Complement $(A^c, \bar{A}, A')$
- Difference $(A - B)$
- Symmetric Difference $(A \Delta B = A-B \cup B-A)$

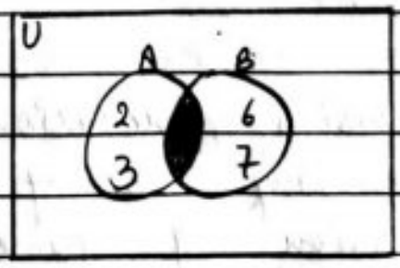
e.g.

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 3, 4\}$
$B = \{1, 4, 6, 7\}$

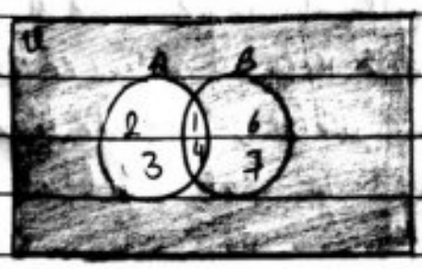
- $A \cup B = \{1, 2, 3, 4\} \cup \{1, 4, 6, 7\} = \{1, 2, 3, 4, 6, 7\}$
- $A \cap B = \{1, 2, 3, 4\} \cap \{1, 4, 6, 7\} = \{1, 4\}$
- $\bar{A} = \{5, 6, 7, 8, 9, 10\}$
  $\bar{B} = \{2, 3, 5, 8, 9, 10\}$
- $A - B = \{1, 2, 3, 4\} - \{1, 4, 6, 7\} = \{2, 3\}$
  $B - A = \{1, 4, 6, 7\} - \{1, 2, 3, 4\} = \{6, 7\}$
- $A \Delta B = (A-B) \cup (B-A) = \{2, 3\} \cup \{6, 7\}$
  $= \{2, 3, 6, 7\}$



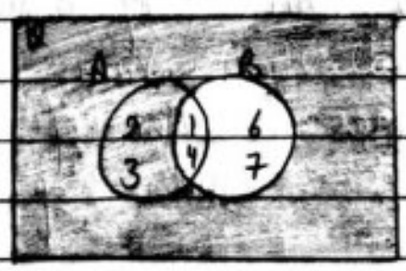
$A \cup B$



$A \cap B$



$\bar{A}$



$\bar{B}$

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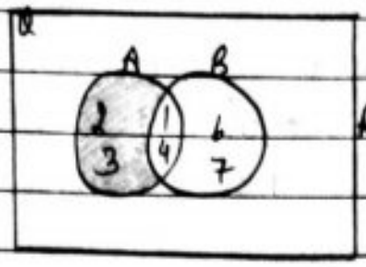
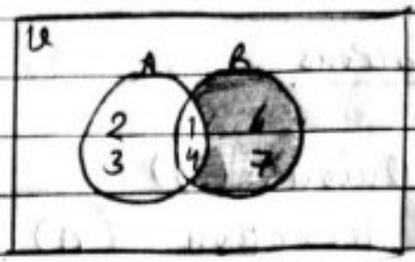
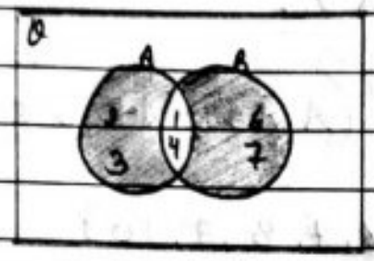


A-B



B-A



$A \Delta B$

28-1-25

Inclusion - Exclusion Principle

It counts the no. of unions in multiple sets.

It's the counting technique that allows us to count no. of elements in the union of multiple sets.

for finite set,

2 → • $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

3 → • $n(A \cup B \cup C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$

Inclusion - Exclusion Principle says that for any no. of finite set A, B, C, D, ... Z. union of these set is given by is equal to sum of size of all single sets minus the sum of all 2 sets intersection, ~~plus~~ sum of all 3 sets intersection, --- sum of all the i set intersection.

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Ques- $n(A) = n(B) = n(C) = 2$, intersections of 2 pairs $= 3$, common intersection $= 4$. find union.

Sol:

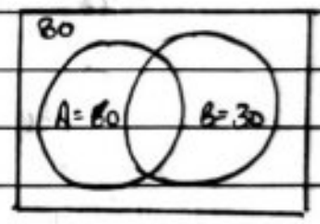
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 2 + 2 + 2 - 3 - 3 - 3 + 4$$

$$= 1$$

Ques- In a survey of group of 80 people, it's found that 60 like eggs and 30 like fish. find the % of people like both egg & fish.

Sol: let, A = eggs, B = fish



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$80 = 60 + 30 - n(A \cap B)$$

$$n(A \cap B) = 10$$

Ques Out of 200 students, 50 takes maths, 140 take economics, 24 took both. How many of these not took any of these course.

Sol: let A be maths and B be economics.

Given, $n(A) = 50$, $n(B) = 140$, $n(A \cap B) = 24$, total $= 200$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 50 + 140 - 24$$

$$= 190 - 24$$

$$= 166$$

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Students who didn't take any subject
$200 - 166$
$= 34$ students

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Multiset

A multiset is an unordered collection of objects or elements in which unlike the classical sets elements are allowed to repeat.

The no. of copies of an element appear in M. set is called multiplicity.

The no. of distinct elements in Multiset & their multiplicity together determine its cardinality, denoted by $C(M)$.

eg.
$A = \{1, 1, 2, 2, 2, 3\} \rightarrow C(A) = \{2.1, 3.2, 1.3\}$
(kitni bar) (kaunsa no.)

Ques Let P & Q are 2 multisets where
$P = \{4.a, 3.b, 1.c\}$ & $Q = \{3.a, 3.b, 2.d\}$
respectively. Find (i) $P \cup Q$ (ii) $P \cap Q$
(iii) $P - Q$ (iv) $Q - P$ (v)

Soln Given, $P = \{4.a, 3.b, 1.c\}$ & $Q = \{3.a, 3.b, 2.d\}$

(i) $P \cup Q = \{4.a, 3.b, 1.c, 2.d\}$ (max)
(ii) $P \cap Q = \{3.a, 3.b\}$
(iii) $P - Q = \{1.a, 1.c\}$
(iv) $Q - P = \{2.d\}$
(v) $P + Q = \{7.a, 6.b, 1.c, 2.d\}$

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Fuzzi Set

let X be the universal set and $x \in X$, a set $\tilde{A}$ is called fuzzy set of X if it can be defined as

$$\tilde{A} = \{x, \mu_{\tilde{A}} : x \in X\}$$

Ex, $\tilde{A} = \left\{ \frac{\mu(x)}{x} \right\}$

Where

$\tilde{A}$ = fuzzy set
x = element
μ = degree of membership

→ **Membership function**

Membership function of fuzzy set $\tilde{A}$ is defined by -

$$\mu_{\tilde{A}} : X$$

$$\tilde{A}(x) = \mu_{\tilde{A}}(x)$$

→ **Representation of fuzzy set**

$$\tilde{A} = \left\{ \frac{u_1}{a}, \frac{u_2}{b}, \frac{u_3}{c}, \ldots \right\}$$

$u_3 \rightarrow$ chances
$a, b, c \rightarrow$ elements

→ **Empty Fuzzy Set**

$$\tilde{A} = \phi \iff \mu_{\tilde{A}}(x) = 0$$

→ **Universal Fuzzy Set**

contains elements as well as chances

$$\tilde{A} = \left\{ \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right\}$$

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→ **Height**

$h(\tilde{A}) = 3$ (Sum of chances)

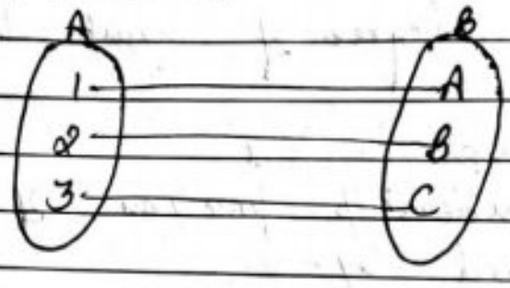
for,

$$\tilde{A} = \left\{ \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right\}$$

Bijection function

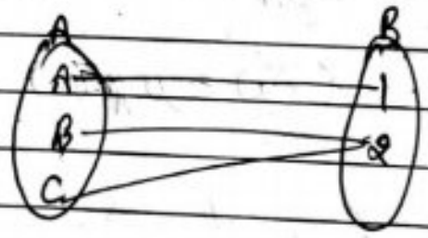
ONE TO ONE FUNCTION

$\boxed{|A| \leq |B|}$

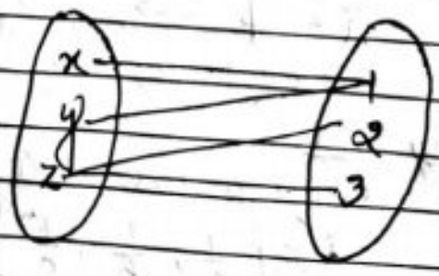


ONTO FUNCTION

$\boxed{|A| \geq |B|}$



A function $f : A \to B$ is bijection if f is both one to one and onto function.



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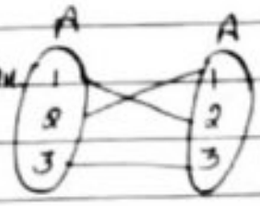
Inverse function

If function $f: X \to Y$ is a bijection then, there always exist preimage of each element of Y, hence the inverse function.

$$x = f(y)$$
$$y = f(x)$$

Let $A = \{1, 2, 3\}$. Define $f: A \to A$ such that $f: \{(1,2), (2,1), (3,3)\}$. Find f^{-1}, f^2 and f^3.

For inverse, f is a bijective function



$f(1) = 2$
$f^{-1}(2) = 1$

$f(2) = 1$
$f^{-1}(1) = 2$

$f(3) = 3$
$f^{-1}(3) = 3$

For square, $f^2 = f \circ f$

$f^2(1) = f \circ f(1) = f.f(1) = f(2) = 1$
$f^2(2) = f \circ f(2) = f.f(2) = f(1) = 2$
$f^2(3) = f \circ f(3) = f.f(3) = f(3) = 3$

For cubic, $f^3 = f \circ f \circ f$

$f^3(1) = f.f.f(1) = f.f(2) = f(1) = 2$
$f^3(2) = f.f.f(2) = f.f(1) = f(2) = 1$
$f^3(3) = f.f.f(3) = f.f(3) = f(3) = 3$

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① Prove the bijective, $f(x) = 5x - 2$

$$f(x) = 5x - 2$$

for one - one,

$$f(x_1) = f(x_2)$$
$$5x_1 - 2 = 5x_2 - 2$$
$$x_1 = x_2$$

✓ Proved

for onto,

$$y = f(x)$$
$$y = 5x - 2$$
$$x = \frac{y+2}{5}$$

$x = 1, y = 1,$

$$1 = \frac{3}{5}$$

$x = 2, y = 2,$

$$2 = \frac{4}{5}$$

$A > B$

✓ Proved

Hence, it is a bijective function.

② Prove bijective, $f(x) = x^3 - 3$

$$f(x) = x^3 - 3$$

for one - one,

$$f(x_1) = f(x_2)$$
$$x_1^3 - 3 = x_2^3 - 3$$
$$x_1^3 = x_2^3$$

✓ Proved

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for note,

$$y = f(x)$$
$$y = x^3 - 3$$
$$x^3 = y + 3$$

$x = 1, y = 1$

$$1 = 1 + 3 = 4$$

$x = 2, y = 2$

$$8 = 5$$

$x = 3, y = 3$

$$27 = 6$$

$A \neq B$

✓ Proved

Hence, it's a bijective function.

$P = \{7.a, 6.b, 5.c\}$
$Q = \{6.a, 5.b, 5.c, 3.d\}$

find,

$P \cup Q = \{7.a, 6.b, 5.c, 3.d\}$
$P \cap Q = \{6.a, 5.b, 5.c\}$
$P - Q = \{1.a, 1.b\}$
$Q - P = \{3.d\}$
$Q + P = \{13.a, 11.b, 10.c, 3.d\}$

fuzzy set, $\tilde{A} = \left\{ \frac{3.2}{a}, \frac{2.7}{b}, \frac{5.6}{c} \right\}$.

find the height of given set.

for height, sum of chances.

$$h(\tilde{A}) = 3.2 + 2.7 + 5.6$$
$$= 11.5$$

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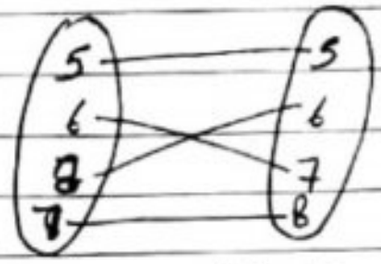
let $A = \{5, 6, 7, 8\}$ define $f: A \to A$ such that $f: \{(5,5), (6,7), (7,8), (8,6)\}$. find f^{-1}, f^2.



$f(5) = 5$ $\quad f(6) = 7$ $\quad f(7) = 8$ $\quad f(8) = 6$

$f^{-1}(5) = 5$ $\quad f^{-1}(7) = 6$ $\quad f^{-1}(8) = 7$ $\quad f^{-1}(6) = 8$

$f^{-1} \Rightarrow f^{-1}(5) = 5, \; f^{-1}(6) = 8, \; f^{-1}(7) = 6, \; f^{-1}(8)$

$f^2 \rightarrow f^2(5) = f \circ f(5) = f(f(5)) = f(5) = 5$

$f^2(6) = f \circ f(6) = f(f(6)) = f(7) = 8$

$f^2(7) = f \circ f(7) = f(f(7)) = f(8) = 6$

$f^2(8) = f \circ f(8) = f(f(8)) = f(6) = 7$

$f^3 \rightarrow f^3(5) = f \circ f \circ f(5) = f \circ f(f(5)) = f \circ f(5) = f(f(5))$
$= f(5) = 5$

$f^3(6) = f \circ f \circ f(6) = f \circ f(f(6)) = f \circ f(7) = f(f(7))$
$= f(8) = 6$

$f^3(7) = f \circ f \circ f(7) = f \circ f(f(7)) = f \circ f(8) = f(f(8))$
$= f(6) = 7$

$f^3(8) = f \circ f \circ f(8) = f \circ f(f(8)) = f \circ f(6) = f(f(6))$
$= f(7) = 8.$

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Composite function

If function $f: A \to B$ and $g: B \to C$ be two functions, then the ~~func~~ function $g \circ f: A \to C$ where $g \circ f(x) = g(f(x))$ for all $x \in A$ is a composite function of f and g.

Example if $f: R \to R$ such that $f(x) = x^2+1$ and $g: R \to R$ such that $g(x) = 2x+1$ are 2 functions then $g \circ f: R \to R$.

$$g \circ f(x) = g[f(x)] = g(x^2+1)$$
$$= 2(x^2+1)+1$$
$$= 2x^2+3$$

$$g \circ f(x) = 2x^2+3.$$

Floor and celing functions

A floor function is the greatest ~~func~~ integer that is less than or equal to x.
It's represented by $\lfloor . \rfloor$.

A ceiling function is the greatest integer that is greater than or equal to x.
It's represented by $\lceil . \rceil$.

Frac function
When we throw away the functional part, this part is called the frac or functional.

$$\text{frac}(x) = x - \text{floor}(x)$$

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~~Growth of function~~

Ques: $f(x) = 2x+1$, find floor & ceiling function
Assume $x = 2.8$. find frac.

$f(x) = 2x+1$
$= 2(2.8)+1 = 5.6+1 = 6.6$

for $f(x)$, floor of $6.6 = \lfloor 6.6 \rfloor$
ceiling of $6.6 = \lceil 7 \rceil$

frac $\rightarrow$ $x - \lfloor (2.8) \rfloor$ and
$2.8 - 2$
$= 0.8$

Ques: Find the value of frac ~~floor~~ $x = 3.6$

$\text{frac}(x) = x - \text{floor of } x$
$= 3.65 - \lfloor 3 \rfloor$
$= 0.65$

Ques: Find the value of floor, ceil & frac. lets,
$f(x) = x+4$ where $x = -3.65$.
$f(x) = -3.65+4 = 0.35$
floor, $\lfloor 0.35 \rfloor = 0$
ceil, $\lceil 0.35 \rceil = 1$

frac $= x - \text{floor}(x)$
$= -3.65 - (-4)$
$= -3.65 + 4$
$= 0.35$

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Composite f^n questions

Suppose $f(x) = 2x+1$, $g(x) = x^2+2$, $h(x) = 3x$.
Find $f(g(h(x)))$.

$$f(g(h(x))) = f(g(3x)) = f((3x)^2+2)$$
$$= f(9x^2+2) = (2(9x^2+2)+1)$$
$$= (18x^2+4+1) = 18x^2+5$$

$f(x) = x-5$, $g(x) = 3x$. Find $g(f(x))$

$$g(f(x)) = g(x-5) = 3(x-5) = 3x-15$$

Out of 300 students, 100 takes Maths, 150 takes English & 24 took both. How many of those will not take any subject.

Let, A = maths, B = English

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 100 + 150 - 24$$
$$= 226$$

Students didn't take any subject $= 300 - 226$
$= 74$

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25-2-25

Growth of functions

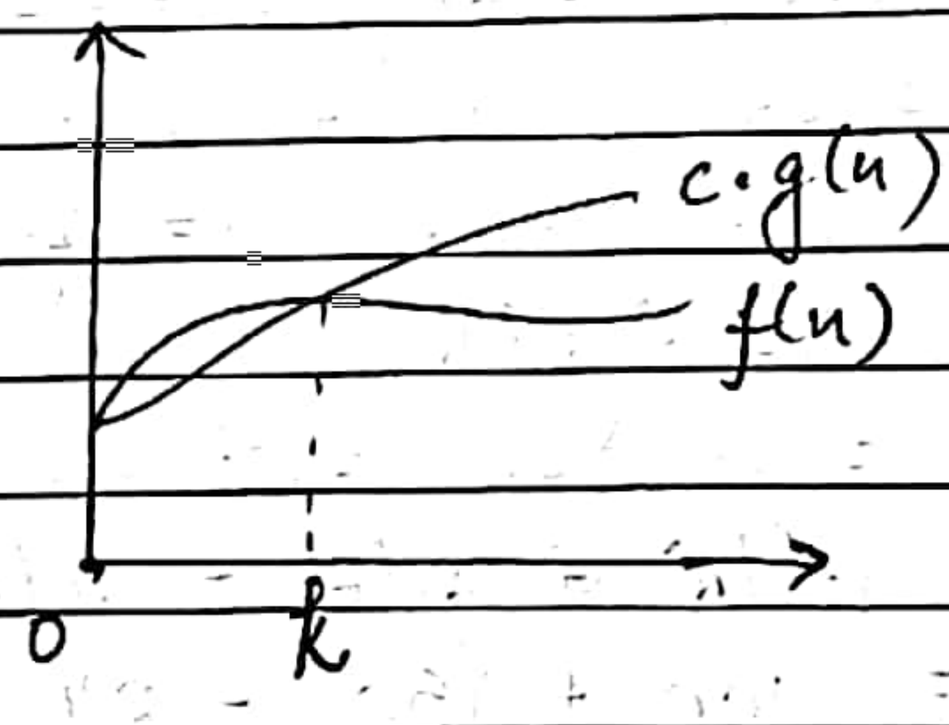
To understand the function's working & function's growth, we use the asymptotic notations.

① Big-O notation:-
at most time, worst case, upper bound location

② Omega - Ω notation:-
at least time, best case, lower bound location

③ Theta - Θ notation:-
exact time, average case, exact time location

• Big-O



$$f(n) \le c.g(n)$$
$$2n^2 + n \le c.g(n)$$
$$2n^2 + n \le c.n^2$$
$$2n^2 + n \le 3n^2$$

hence, $\boxed{c = 3}$

& $3n^2 \ge 2n^2 + n$
$$n^2 \ge n$$
$$\boxed{n \ge 1}$$

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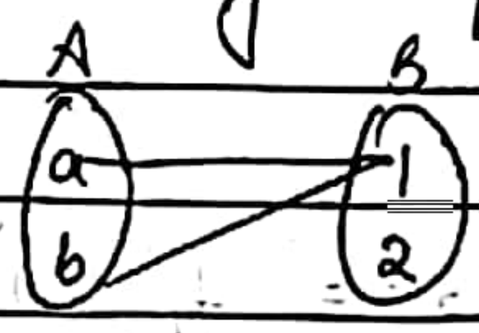
Functions

A relation $f: A \to B$ is called a function

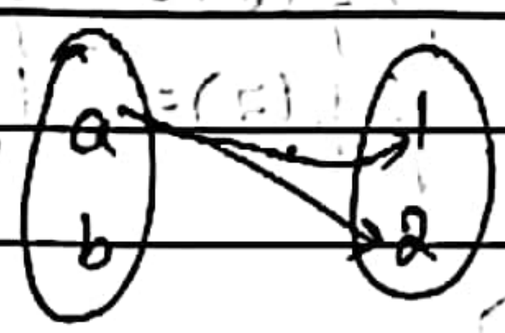
eg. $A = (a, b)$ $B = (3, 4)$

relation, $A \times B = (a,3), (b,3), (a,4), (b,4)$

every element of set A should have (unique relation) an image from set B.

→ A function, 

$(a,1)$, $(b,1)$ (✓)

→ Not a function, 

$(a,1)$ $(a,2)$ (X)

Domain Codomain

Ques Let $A = \{5, 6, 7\}$. Define function $A \to A$ such that $f: (5,6), (6,7), (7,5)$. Find f^2, f^3, f^{-1}.

Ques Find $A \cup B$; $A \cap B$ and $A - B$ where set $A = \{a, b, c, d\}$, $B = \{c, d\}$.

Sol 2

$A = \{a, b, c, d\}$, $B = \{c, d\}$

$A \cup B = \{a, b, c, d\}$
$A \cap B = \{c, d\}$
$A - B = \{a, b\}$

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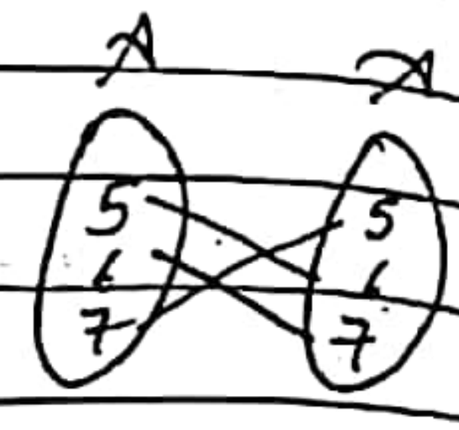
Qol 1 $A = \{5, 6, 7\}$



$f: (5, 6), (6, 7), (7, 5)$

$f(5) = 6$ $\quad$ $f^{-1}(5) = 7$
$f(6) = 7$ $\quad$ $f^{-1}(6) = 5$
$f(7) = 5$ $\quad$ $f^{-1}(7) = 6$

Inverse

Square, $f^2 = f \circ f \Rightarrow$

$f^2(5) = f \circ f(5) = f(f(5)) = f(6) = 7$
$f^2(6) = f \circ f(6) = f(f(6)) = f(7) = 5$
$f^2(7) = f \circ f(7) = f(f(7)) = f(5) = 6$

Cube, $f^3 = f \circ f \circ f \Rightarrow$

$f^3(5) = f \circ f \circ f(5) = f \cdot f(f(5)) = f \cdot f(6) = f(f(6))$
$= f(7) = 5$

$f^3(6) = f \circ f \circ f(6) = f \cdot f(f(6)) = f \cdot f(7) = f(f(7))$
$= f(5) = 6$

$f^3(7) = f \circ f \circ f(7) = f \cdot f(f(7)) = f \cdot f(5) = f(f(5))$
$= f(6) = 7$

Ques $f(x) = x - 5$, $g(x) = x^2 + x$. Find $g(f(x))$.

$g(f(x)) = g(x-5) = (x-5)^2 + (x-5)$
$= x^2 + 25 - 10x + x - 5$
$= x^2 - 9x + 20$

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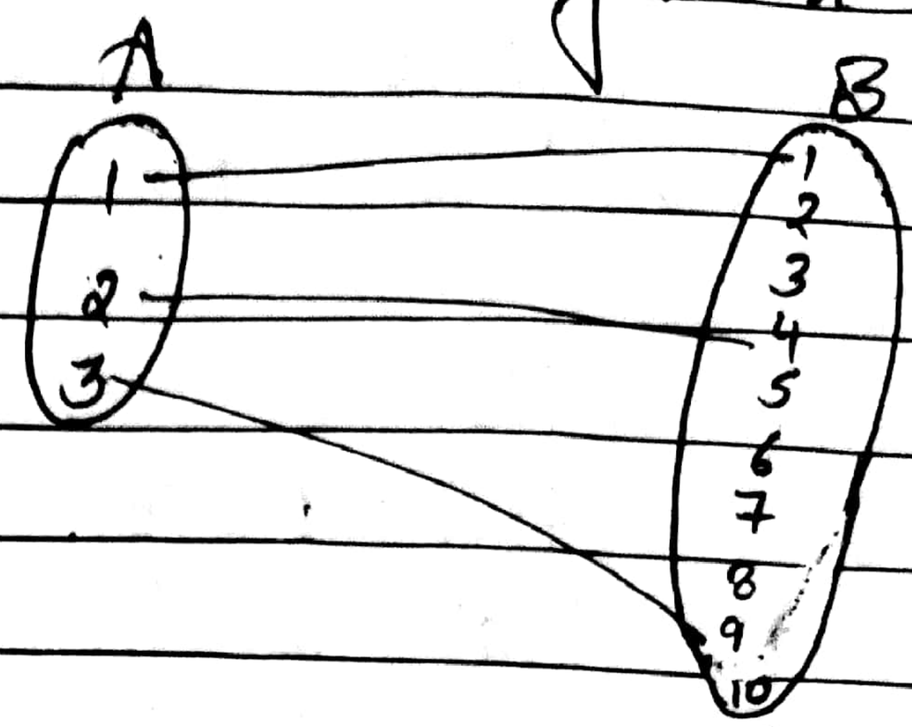
2-3-25

Ques $f = n^2$, find domain, codomain & range. $\forall n = 1, 2, 3$.

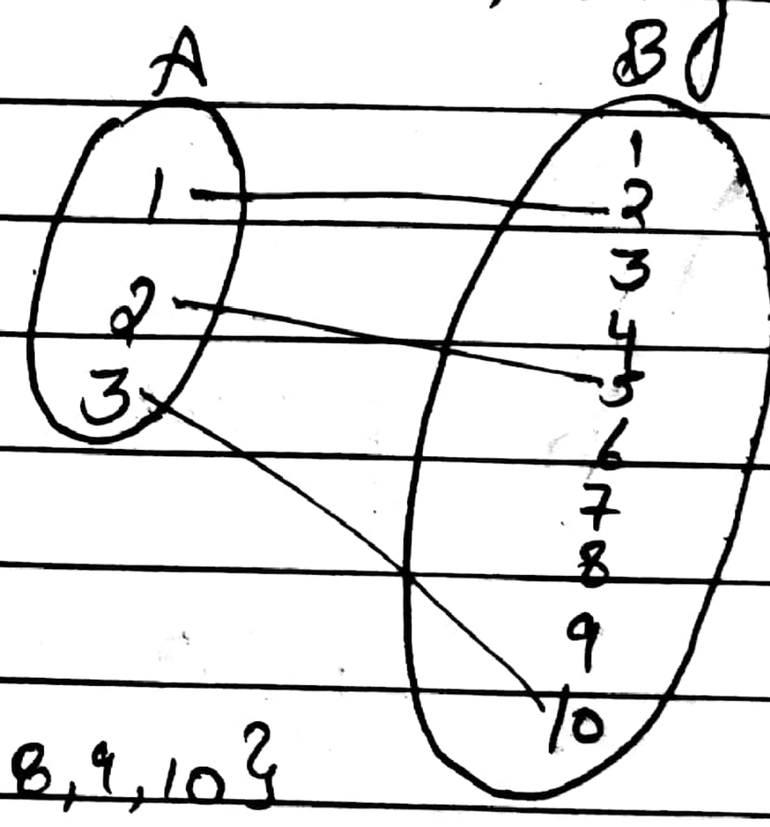
$f(1) = 1$
$f(2) = 4$
$f(3) = 9$



Domain = $\{1, 2, 3\}$
Codomain = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
Range = $\{1, 4, 9\}$

Ques $f(n) = n^2 + 1$. find domain, codomain, range. $n = 1, 2, 3$.

$f(1) = 2$
$f(2) = 5$
$f(3) = 10$



Domain = $\{1, 2, 3\}$
Codomain = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
Range = $\{2, 5, 10\}$

Recursive function

Recursive function is a function which defines in term of itself.

2 types

1. Definition of smallest ~~element~~ argument. such as $f(0)$ & $f(1)$
2. Definition of $f(n)$.

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Ques Calculate the value of following recursively defined function.
$f(0)=1$ & $f(n) = n\, f(n-1)$. Find $f(1), f(2), f(3), f(4)$

Ans

$f(0) = 1$
$f(1) = 1 \cdot f(0) = 1 \cdot 1 = 1$
$f(2) = 2 \cdot f(1) = 2 \cdot 1 = 2$
$f(3) = 3 \cdot f(2) = 3 \cdot 2 = 6$
$f(4) = 4 \cdot f(3) = 4 \cdot 6 = 24$

Ques Consider the following recursive function
if $x < y$, then $f(x,y) = 0$,
if $y \le x$, then $f(x,y) = f(x-y, y) + 1$
find the value of $f(4,7)$ & $f(19,6)$

Ans

for $f(4,7)$, using function $x<y$

$$\boxed{f(4,7) = 0}$$

for $f(19,6)$, using function $y \le x$

$f(19,6) = f(13, 6) + 1$
$= f(7, 6) + 2$
$= f(1, 6) + 3$
$= 0 + 3$

$$\boxed{f(19,6) = 3}$$

Ques Calculate the value of the following function (recursive). $f(0)=3$, $f(n+1) = 2f(n) + 3$
Find $f(1)$, $f(2)$, $f(3)$, $f(4)$.

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$f(0) = 3$

$n=0,\quad f(0+1) = f(1) = 2f(0) + 3 = 2(3) + 3 = 9$

$n=1,\quad f(1+1) = f(2) = 2f(1) + 3 = 2(9) + 3 = 21$

$n=2,\quad f(2+1) = f(3) = 2f(2) + 3 = 2(21) + 3 = 45$

$n=3,\quad f(3+1) = f(4) = 2f(3) + 3 = 2(45) + 3 = 93$

4-3-26

Relations

Reflexive Relation

A relation 'R' on a set 'A' is said to be reflexive if $(x,x) \in R \ \forall \ x \in A$.

$A = \{1, 2, 3\}$

$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$(x,x) \rightarrow$ reflexive.

$A = \{(1,1), (2,2), (3,3)\}$ ✓ reflexive

$A = \{(1,1), (2,2)\}$ ✗ reflexive

Eg: (1) $R1 = \{(x,y);\ x-y$ is an integer$\}$

$R2 = \{(x,y);\ x-y$ is an odd number$\}$

Check which of the above is reflexive on which of all natural number.

→ for reflexive relation, let's take pairs in the form of (x,x)

$N = (1,1)\ (2,2)\ (3,3)\ \text{---}\ (100,100)$

R1 is reflexive → $(x-y=0)$

but R2 is not reflexive → ($x-y=0$ but 0 isn't odd)

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(2) Consider set A in which relation R is defined by (xRy) if & only if $x+3y$ is divisible by 4 for $(x,y) \in A$. Show that R is a reflexive relation on set A.

→ $x+3y \quad \forall (x,y) \in A$

for reflexive relation, $(x,x) \in A$
So, the statement becomes,

$$\Rightarrow x+3x$$
$$= 4x$$

Here, $4x$ is divisible by 4.

Hence, Relation is reflexive.

(3) A relation R is on set A is defined by xRy if & only if $2x+3y$ is divisible by 5 for all $(x,y) \in A$. Check if R is a reflexive relation on A

$$2x+3y \quad \forall (x,y) \in A$$

for reflexive, $(x,x) \in A$
So, eqn becomes,

$$2x+3x$$
$$= 5x$$

It's divisible by 5

Hence, R is reflexive relation

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(4) A relation R on set A if $x-y$ is divisible by 5 for $(x,y) \in A$, Check if R is a reflexive relation on set A.

$$x-y \quad \forall \ (x,y) \in A$$

for reflexive, $(x,x) \in A$
So,

$$x-x$$
$$= 0$$

0 is divisible by 5.
Hence, R is reflexive relation.

(5) A relation R is defined on a set of all real no. 'n' by aRb if & only if $a-b \leq b$ for $(a,b) \in n$. Show that R is not a reflexive relation.

$$a-b \leq b \quad \forall \ (a,b) \in n$$

for reflexive, $(a,a) \in n$
So,

$$a-a \leq a$$
$$0 \leq a$$

a is real number, hence R is not a reflexive relation.

Symmetric Relation

A relation R on set A is said to be symmetric if xRy then yRx for all $x, y \in A$.

if $(x,y) \in R$ & $(y,x) \in R$ $\forall$ $x, y \in A$

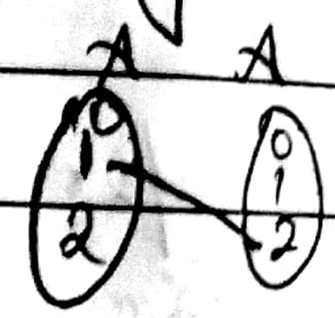
$A = \{1,2\} \rightarrow R = \{(1,2), (2,1)\}$ ✓ Symmetric

$R = \{\ \}$ ✓ Symmetric

Eg. ① let $A = \{0, 1, 2\}$ and R be a relation defined on set A such that $R = \{(0,0), (1,1), (2,2), (1,2)\}$ is a symmetric.

for a relation to be symmetric
$(x,y) \in R$ & $(y,x) \in R$ $\forall$ $x, y \in A$

Here, $(1,2) \in R$ but $(2,1) \notin R$



Hence, R is not a symmetric relation.

② let $A = \{x, y, z\}$ & $R = \{(x,y), (y,z), (x,z), (x,x), (y,x), (z,y), (z,x)\}$. Find if symmetric

for a symmetric relation,
$(x,y) \in R$ & $(y,x) \in R$ $\forall$ $x, y \in A$

Here, all pairs are present.
Hence, R is a symmetric relation.

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Transitive Relation

A relation R on a set A is said to be transitive if xRy & yRz then xRz for all $x, y, z \in A$.

If $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \in R$ $\forall x,y,z \in A$

① Let $A = \{x, y, z\}$ & $R = \{(x,y), (y,z), (x,z), (x,x), (y,x), (z,y), (z,x)\}$.

Here,

all $x, y, z \in A$ has a defined relation in R.

Such that, $(x,y) \in R$, $(y,z) \in R$ then $(x,z) \in R$

for all $x, y, z \in A$

Hence, R is a transitive relation.

② Define a relation R on set $A \in \{a, b, c\}$ as $R = \{(a,b), (b,c), (b,b)\}$. Determine if R is transitive or not.

Here, $(x,y) \in R$, $(y,z) \in R$ but $(x,z) \notin R$

So, the R is not a transitive relation.

(3) Consider a set of natural no. & define a relation are as follows
$R = \{(1,2), (2,3), (1,3)\}$. Check if R is transitive or not.

for transitive relation,
if $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in R$

Here, $(1,2) \in R$, $(2,3) \in R$ and $(1,3) \in R$
Hence, R is a transitive relation.

(4) let us define a ~~set of~~ relation R on set $A = \{1,2,3\}$, check if given relation is symmetric, reflexive & transitive or not
$R = \{(1,1), (2,2), (3,3), (1,2) (2,3), (1,3)\}$

for relation R to be reflexive,
$x, x \in R$

for symmetric, $x, y \in R$ and $y, x \in R$
but all pairs of this are not present.

for transitive, $x, y \in R$ & $y, z \in R$ then $x, z \in R$
Hence,
R is reflexive & transitive

(5) let $A = \{1,2,3,4\}$, $R = \{(1,1), (1,3), (2,2), (3,2) (3,3)$
(6) let $A = \{1,2,3\}$, $R = \{(1,1), (1,2), (1,3) (2,1), (3,1$

(5) Reflexive only
(6) Symmetric & transitive.

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* Partial ordering Relation

A relation R is said to be partially ordering relation if R is antisymmetric, reflexive and transitive.

R ✓ A ✓ T ✓ $\begin{cases} R_1 = \{(1,1), (2,2), (3,3)\} \\ R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\} \end{cases}$

* Antisymmetric Relation

$xRy, yRz \Rightarrow x, y \in A$

It's a type of binary relation on a set where any two distinct element related to each other in one direction can not be related in the opposite relation.

Consider the relation $\leq$ or the set of integer $(A \leq Z)$

* Equivalent Relation

A relation is said to be equivalent if R is reflexive, symmetric and transitive.

Ques: If A is any real no, the $(A, \leq)$ is a partially order relation or equal.

Sol: → $A = \{1, 2, 3\}$, $R_1 = \{(1,2), (1,3), (2,3)\}$, $R_2 = \{(1,2), (2,2), (1,1), (3,3)\}$
$R_3 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$R_1 =$ A, T, R ✓

$R_2 =$ A, T, R ✓

$R_3 =$ T, R Only ✓

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Ques Let $\mathbb{Z} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Define a relation R on a $A = \{\mathbb{Z} \times \mathbb{Z}\}$ by $(a,b)\ R\ (c,d)$ if and only if $10(a+b) \leq 10a + b + 10c + d$.

$$10(a+b) \leq 10a + b + 10c + d$$
$$10(0+1) \leq 10(0) + 1 + 10(2) + 3$$
$$10 \leq 20 + 3 + 1$$
$$10 \leq 24$$

13-3-25 **Irreflexivity Relation**

A relation R on a set A is said to a irreflexive if $x \not R x$ $\forall x$

Ques $A = \{1, 2, 3\}$. $R_1 = \{(1,1), (2,2)\}$ I X R
$R_2 = \{(1,2), (1,3), (2,1)\}$ I ✓ R
$R_3 = \{(1,1), (2,2), (3,3)\}$ I X R
$R_4 = \{\}$ I ✓

Ques $(A, \leq)$ is an irreflexive relation or not. forming 2 relations,
$A = \{1, 2, 3\}$
$R1 = \{(1,1), (2,2), (3,3)\}$ R ✓ I
$R2 = \{(1,2), (2,3), (1,3)\}$ R X I

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Complement & Inverse of a relation

$A = \{a, b\}$
$B = \{1, 2, 3\}$
$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

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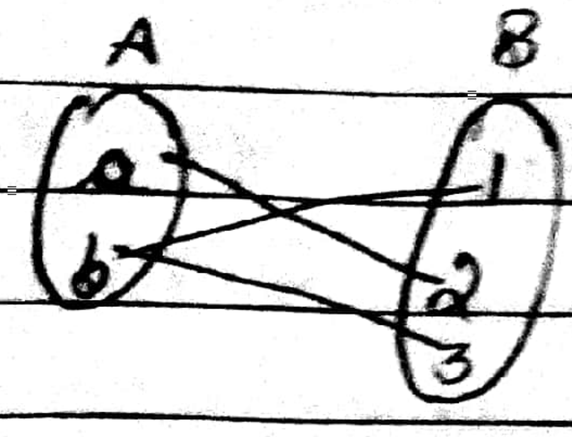
$R = \{(a,2), (b,1), (b,3)\}$



→ Complement of Relation,

$\bar{R}$ or R^c

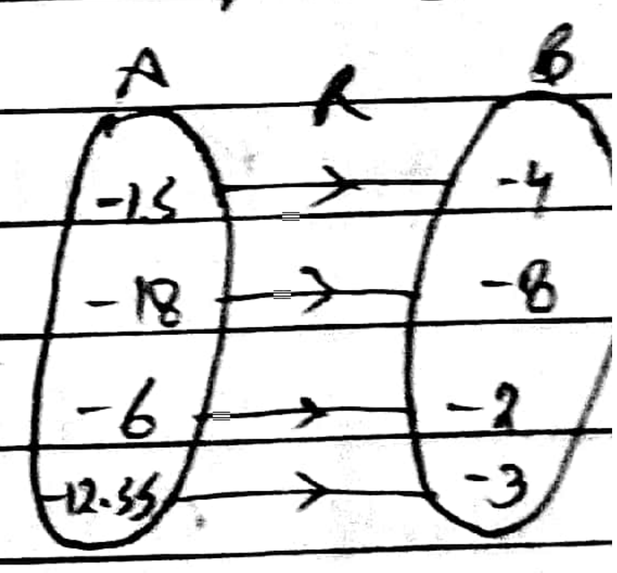
$(a,b) \in A \times B, (a,b) \notin R$

$\bar{R} = \{(a,1), (a,3), (b,2)\}$

→ Inverse of Relation,

$R^{-1} = \{(a,2), (b,1), (b,3)\}$

Ques $R = \{(-15, -4), (-18, -8), (-6, -2), (-12.55, -3)\}$
find $\bar{R}$, R^{-1}



$A = \{-15, -18, -6, -12.55\}$
$B = \{-4, -8, -2, -3\}$

$A \times B = \{(-15,-4), (-15,-8), (-15,-2), (-15,-3), (-18,-4), (-18,-8),$
$(-18,-2), (-18,-3), (-6,-4), (-6,-8), (-6,-2), (-6,-3),$
$(-12.55,-4), (-12.55,-8), (-12.55,-2), (-12.55,-3)\}$

$\bar{R} = \{(-15,-8), (-15,-2), (-15,-3), (-18,-4), (-18,-2), (-18,-3),$
$(-6,-4), (-6,-8), (-6,-3), (-12.55,-4), (-12.55,-8), (-12.55,-2)\}$

$R^{-1} = \{(-15,-4), (-18,-8), (-6,-2), (-12.55,-3)\}$

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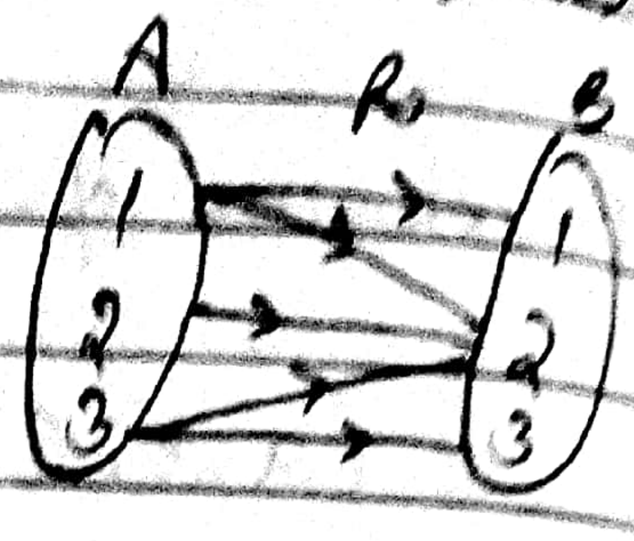
Ques $A = \{1, 2, 3\}$, $R = \{(1,1), (2,2), (3,3), (1,2), (3,$
find B, $\bar{R}$, R^{-1}.



$A = \{1, 2, 3\}$
$B = \{1, 2, 3\}$

$A \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$\bar{R} = \{(1,3), (2,1), (2,3), (3,1)\}$

$R^{-1} = \{(1,1), (2,2), (3,3), (1,2), (3,2)\}$

Partition of Sets

$A = \{a, b, c\}$

Subsets, $P(A) = \{\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}, \{\}\}$

Partition set,

① $P1 = \{\{a\}, \{b\}, \{c\}\}$

Conditions satisfying are -

$1 \cup 2 \cup 3 = A$
$1 \cap 2 \cap 3 = \{\}$

P1, union → $\{a, b, c\}$ ✓
P1, intersection → $\{\}$ ✓

} is a partition

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② $P_2 = \{ \{a\}, \{b, c\} \}$

union ✓
intersection ✓

} is a partition

Ques $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

a) $\{1, 3, 6\}, \{2, 8\}, \{5, 7, 9\}$
✓ b) $\{2, 4, 5, 8\}, \{1, 9\}, \{3, 6, 7\}$
c) $\{1, 5, 7\}, \{2, 4, 8, 9\}, \{3, 5, 6\}$
d) $\{1, 2, 7\}, \{3, 5\}, \{4, 6, 8, 9, 7\}$

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Representation of Relation

Relation can be represent in 2 types
① Relational Matrix
② Di-Graph

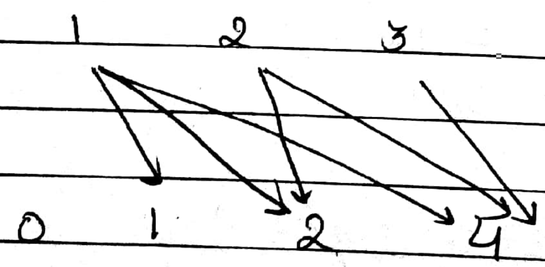
Ques $A = \{1,2,3\}$, $B = \{0,1,2,4\}$, $x, y \in R$ $(x \le y)$
as per the condition $x \le y$,

$R = \{(1,1), (1,2), (1,4), (2,2), (2,4), (3,4)\}$

Relational Matrix :-

$$\begin{array}{c|c} B \rightarrow & \\ A \downarrow & \begin{array}{cccc} 0 & 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Digraph:-



Ques $X = \{1,2,3,4\}$, $R = (x,y)$, $x > y$
as per the condition, $x > y$, $X = Y$.
$R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

Relational matrix,

$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

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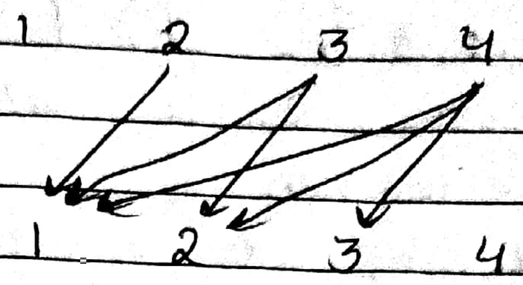
Digraph,



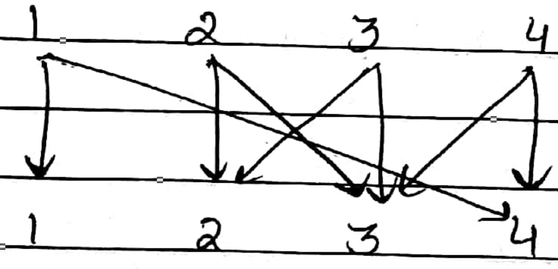
Ques $X = \{1, 2, 3, 4\}$, $R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,3), (4,4)\}$

$X = \{1, 2, 3, 4\}$ and $Y = \{1, 2, 3, 4\}$.

Relational Matrix,

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 3 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 \end{array}$$

Digraph,



Closure of Relation

Let R be a relation in a set A. R may not be satisfy particular property like reflexive, symmetric and transitive. The new relation obtained after adding least no. of pairs so that R satisfy particular property is called closure of relation.

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Ans $A = \{1, 2, 3\}$, $R = \{(1,1), (1,2), (1,3)\}$

the given R is not R, T & S.

Identity relation,

for reflexive, $R^{(r)} = \{(1,1), (2,2), (3,3)\}$

$R1 = R \cup R^{(r)} = \{(1,1), (2,2), (3,3), (1,2), (1,3)\}$

Hence, R1 is a reflexive relation.

Inverse relation,

$R^{-1} = \{(1,1), (2,1), (3,1)\}$

$R2 = R \cup R^{-1} = \{(1,1), (2,1), (1,2), (3,1), (1,3)\}$

R2 is a symmetric relation.

Transitive relation,

$R^{(t)} = R \cup R^2 \cup R^3 \cup \cdots R^n$

$R^2 = R \circ R = R.R = R.\{(1,1), (1,2), (1,3)\}$

can't solve, see further

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Que $A = \{1, 2, 3\}$, $R = \{(1,1), (1,2), (1,3), (2,1), (3,1)\}$

Given relation R is symmetric, but not reflexive and transitive.

→ Identity relation, $R^{(r)} = \{(1,1), (2,2), (3,3)\}$

$R1 = R^{(r)} \cup R = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (3,1)\}$

Hence, $R1$ is reflexive relation.

→ Inverse relation, $R^{-1} = \{\}$.

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Ques $A = \{1,2,3,4\}$, $R = \{(1,2), (2,3), (3,4), (2,1)\}$

for transitive,

$$R^{(t)} = R \cup R^2 \cup R^3 \cup \dots R^n$$

$$R^2 = R \circ R = \{(1,2), (2,3), (3,4), (2,1)\} \circ \{(1,2), (2,3), (3,4), (2,1)\}$$

$$= \{(1,3), (1,1), (2,4), (2,2)\}$$

$$R^3 = R \circ R^2 = \{(1,2), (2,3), (3,4), (2,1)\} \circ \{(1,3), (1,1), (2,4), (2,2)\}$$

$$= \{(1,4), (1,2), (2,1), (2,3)\}$$

$$R^4 = R \circ R^3 = \{(1,2), (2,3), (3,4), (2,1)\} \circ \{(1,4), (1,2), (2,1), (2,3)\}$$

$$= \{(1,3), (1,1), (2,2), (2,4)\}$$

as A has n elements, hence till R^4.

$$R^{(t)} = R \cup R^2 \cup R^3 \cup R^4$$

$$= \{(1,2), (2,3), (3,4), (2,1), (1,3), (1,1), (2,4), (2,2), (1,4), (1,2)\}$$

$\therefore R^{(t)}$ is transitive.

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Ques $A = \{1, 2, 3, 4\}$, $R = \{(1,2), (2,3), (3,4)\}$

Identity relation,

$R^{(r)} = \{(1,1), (2,2), (3,3), (4,4)\}$

$R1 = R \cup R^{(r)} = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,3), (3,4)\}$

$\therefore$ R1 is reflexive relation.

Inverse relation,

$R^{-1} = \{(2,1), (3,2), (4,3)\}$

$R2 = R \cup R^{-1} = \{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3)\}$

$\therefore$ R2 is symmetric relation.

Transitive relation;

$R^{(t)} = R \cup R^2 \cup R^3 \cup R^4$

$R^2 = R \circ R = \{(1,2), (2,3), (3,4)\} \circ \{(1,2), (2,3), (3,4)\} = \{(1,3), (2,4)\}$

$R^3 = R \circ R^2 = \{(1,2), (2,3), (3,4)\} \circ \{(1,3), (2,4)\} = \{(1,4)\}$

$R^4 = R \circ R^3 = \{(1,2), (2,3), (3,4)\} \circ \{(1,4)\} = \{\}$

$R^{(t)} = R \cup R^2 \cup R^3 \cup R^4$

$= \{(1,2), (2,3), (3,4), (1,3), (2,4), (1,4)\}$

$\therefore R^{(t)}$ is transitive relation.

Hence, R is an equivalent relation.

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Warshell's Algorithm

If relation R for a set is not transitive, we need to apply closure to make it a transitive, this closure is known as transitive closure. We use partial method to find the transitive closure.

Ques- $R = \{(a,b), (b,a), (b,c), (c,d)\}$ where $A = \{a,b,c,d\}$

$$A^0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

$$A^1 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 1 & \infty \\ c & 0 & \infty & 0 & 1 \\ d & 0 & \infty & \infty & 0 \end{array}$$

$$A^2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 2 & \infty \\ b & 1 & 0 & 1 & \infty \\ c & \infty & \infty & 0 & \infty \\ d & \infty & \infty & \infty & 0 \end{array}$$

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$$A^3 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 2 & 3 \\ b & 1 & 0 & 1 & 2 \\ c & \infty & \infty & 0 & 1 \\ d & \infty & \infty & \infty & 0 \end{array}$$

$$A^4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 2 & 3 \\ b & 1 & 0 & 1 & 2 \\ c & \infty & \infty & 0 & 1 \\ d & \infty & \infty & \infty & 0 \end{array}$$

by method,

$$A^1 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array} \qquad A^2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 0 \\ b & 1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

$$A^3 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array} \qquad A^4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

hence, $\{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,d)\}$

Lexicographic ordering ($\preccurlyeq$)

eg. $(3,5)$ $(7,8)$

$(3,5) \preccurlyeq (7,8)$

s_1 s_2 s_3 s_4

condition → $s_1 \le s_3$ & $s_2 \le s_4$ (for true)

$3 \le 7$ ✓ & $5 \le 8$ ✓

Hence, it's lexicographic ordering.

It's an order on the cartesian product of 2 or more partial ordered sets. It's also known as lexicographical order, lexicographic order, alphabetical order & dictionary order.

eg. → $(1,1)$ $(1,2)$

$(1,1) \preccurlyeq (1,2)$

$1 \le 1$ ✓ $1 \le 2$ ✓

Hence, it's lexicographical order

→ $(1,3,2)$ $(1,3,5)$

$(1,3,2) \preccurlyeq (1,3,5)$

Yes, it's.

→ $(7,5,4,2) \preccurlyeq (7,6,4,2)$

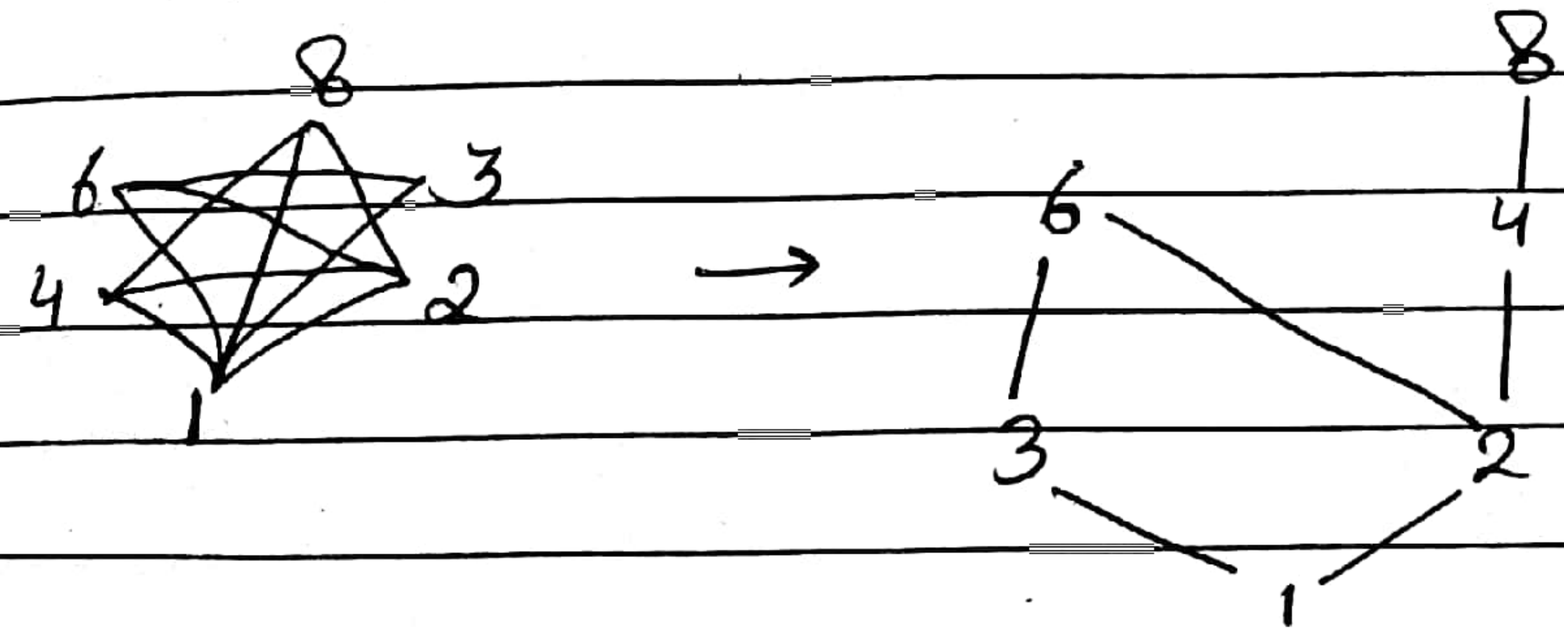
Yes, it's

Hasse Diagrams

It's a graphical representation of partial order relation in which all arrows head are understood to be pointing upward is known as hasse diagram.

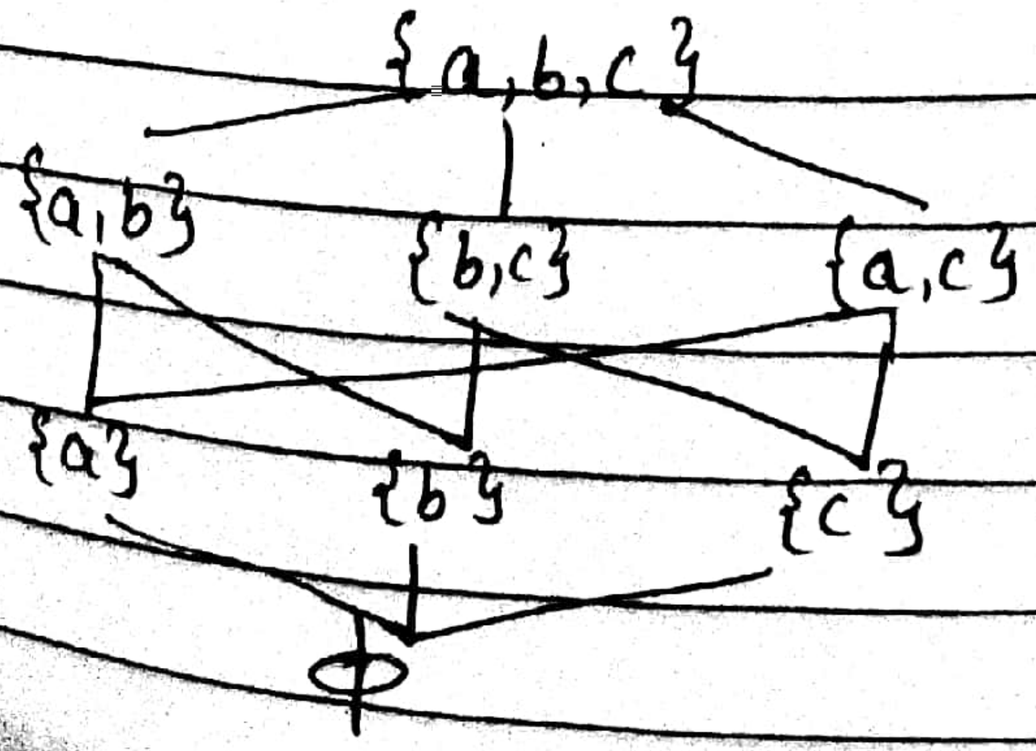
Ques let $A = \{1, 2, 3, 4, 6, 8\}$ be order by relation "a divide b".

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (6,6), (8,8)\}$



→ Hasse diagram is a subset of $S = \{a, b, c\}$ with $P(S) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}, \phi\}$

$R = \{(\phi, \{a\}, \{a,b\}), (\phi, \{a\}, \{c,a\}), (\phi, \{a\}, \{a,b,c\}), (\phi, \{b\}, \{a,b\}), (\phi, \{b\}, \{b,c\}), (\phi, \{b\}, \{a,b,c\}), (\phi, \{c\}, \{b,c\}), (\phi, \{c\}, \{c,a\}), (\phi, \{c\}, \{a,b,c\})\}$



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Lattice

Minimal, Maximal, Upper bound, Lower bound

① **Minimal element**

If no element is related to an element, that element is called minimal element.

② **Maximal element**

In a po-set, if an element is not related to any other element is a maximal element.

③ **Upper bound**

let $(P, \preccurlyeq)$ be a poset and $B \subseteq P$, an element $x \in A$ is a upper ~~pow~~ bound of B. If y related to x for every $y \in B$.

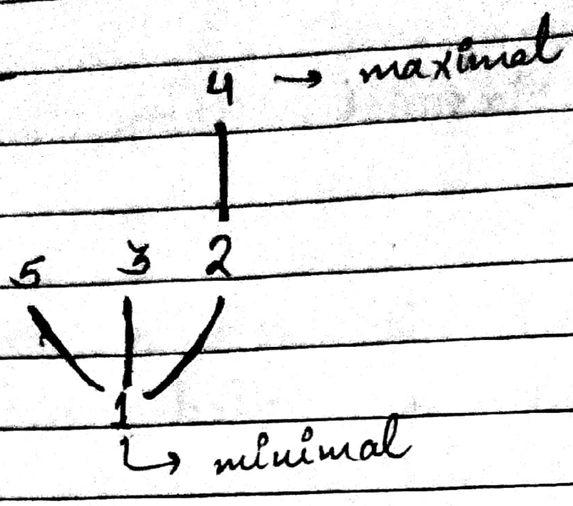
④ **Lower bound**

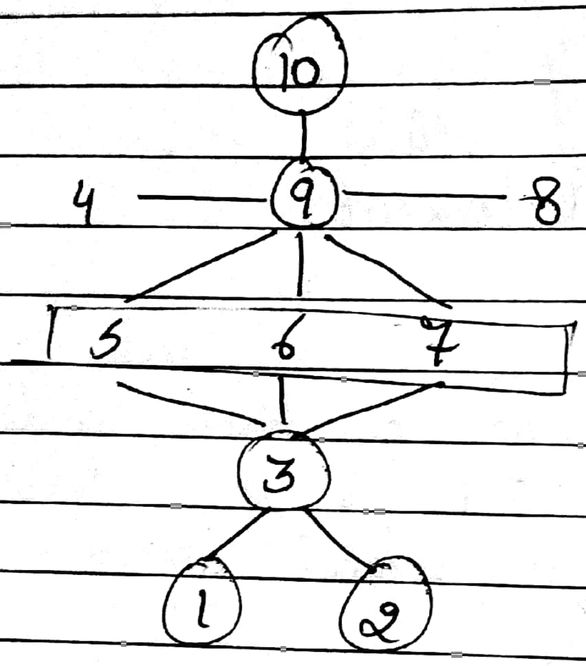
An element $x \in A$ is a lower bound of B if x is related to y for every $y \in B$.

→ **Lattice**

A poset $(P, \preccurlyeq)$ is said to be lattice if every 2 element in the set has uniquely least upper bound & unique greatest lower bound.

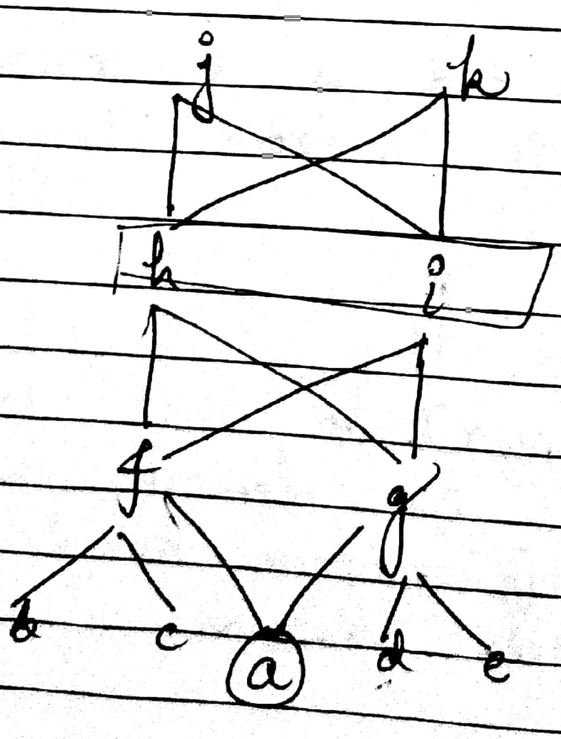
example





least upper bound
10 < 9

greatest lower bound
1 < 2 < 3



least upper bound
j ? k
so no lattice
as only 1 elem is ✓ chosen

& lower → a

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Que: Let S be a non-empty finite set & $P(S)$ be a power set. Show that $(P, \subseteq)$ be a lattice.

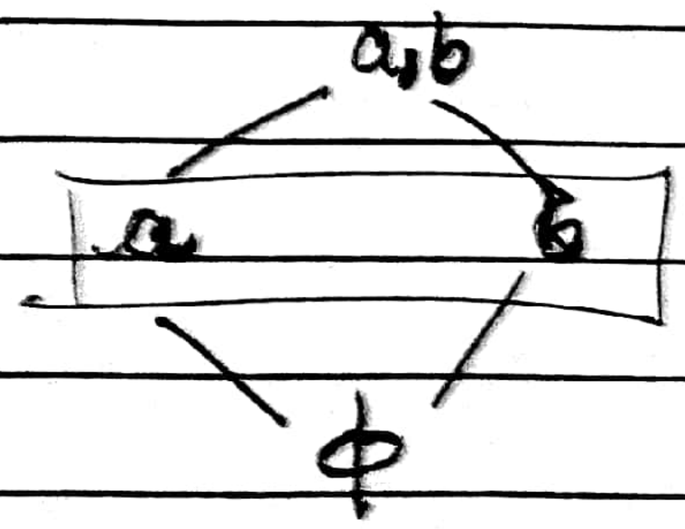
Que: Let L be a set of all factor of 12 & I divisibility relation of L. Show that (L, I) is a lattice.

Que: Let $A = \{2, 3, 4, 6\}$ & let I be a divisibility relation on A. Show that (A, I) is a poset but not a lattice.

① Let S be a non-empty finite set,

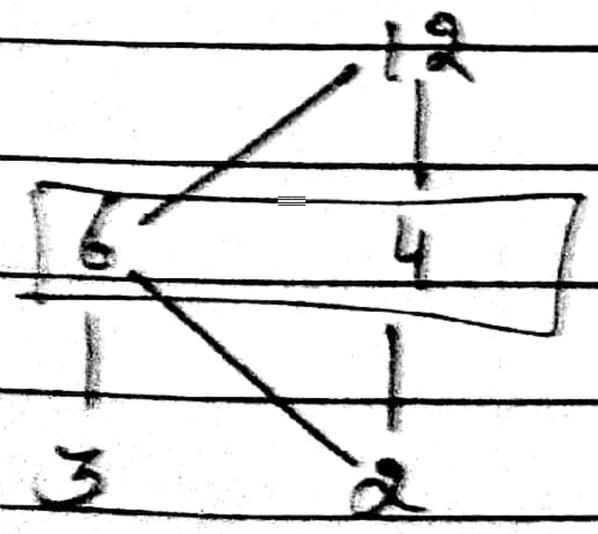
$S = \{a, b\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$



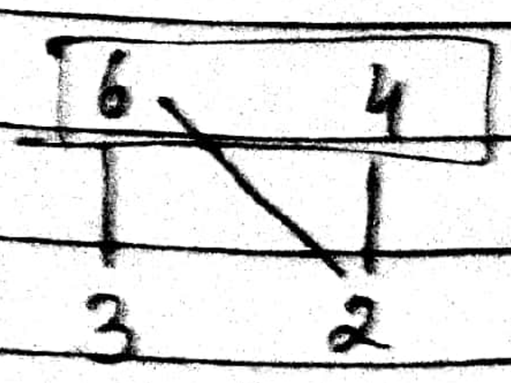
upper → a,b
lower → φ
hence, it's a lattice.

② $L = \{2, 3, 4, 6, 12\}$



upper = 12
lower = 2
hence, it's a lattice.

③ $A = \{2, 3, 4, 6\}$



lower = 2
upper = ?
hence, not a lattice.

Boolean Algebra

$$y = (A+B)(A+C)$$

$A \cdot 0 = 0$	$A+0 = A$
$A \cdot 1 = A$	$A+1 = 1$
$A \cdot A = A$	$A+A = A$
$A \cdot \bar{A} = 0$	$A+\bar{A} = 1$
$\bar{\bar{A}} = A$	

① $y = (A+B)(A+C)$
$= AA + AC + BA + BC$
$= A + AC + BA + BC$
$= A(1+C+B) + BC$
$= A(1) + BC$
$= A + BC$

② $\bar{A}A + \bar{A}\bar{C} + A\bar{B} + \bar{B}\bar{C} + \bar{B}B + \bar{B}C$
$= 0 + \bar{A}\bar{C} + A\bar{B} + \bar{B}\bar{C} + 0 + \bar{B}C$
$= \bar{A}\bar{C} + \bar{B}(A + \bar{C} + C)$
$= \bar{A}\bar{C} + \bar{B}(A+1)$
$= \bar{A}\bar{C} + \bar{B}(1)$
$= \bar{A}\bar{C} + \bar{B}$

③ $(A+B)(A+\bar{B})(\bar{A}+C)$
$= (AA + A\bar{B} + BA + B\bar{B})(\bar{A}+C)$
$= (A + A\bar{B} + BA + B)(\bar{A}+C)$
$= [A(1+\bar{B}+B) + B](\bar{A}+C)$
$= (A+B)(\bar{A}+C) = A\bar{A} + AC + B\bar{A} + BC$
$= 0 + AC + \bar{B}\bar{A} + BC =$

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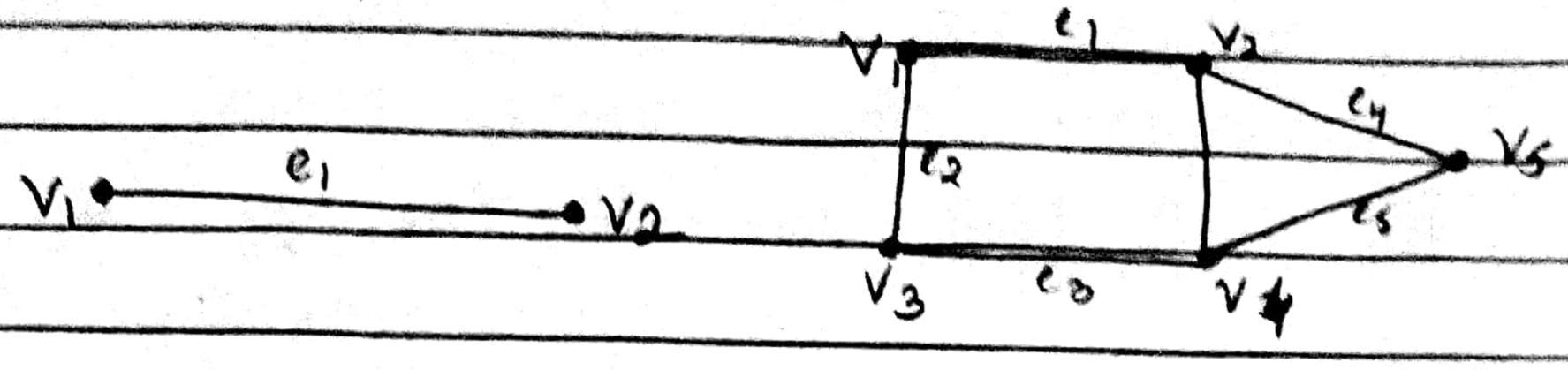
Graph

It's a mathematical structure consisting of 2 sets V & E. V is non-empty set of vertices and E is non-empty set of edges.

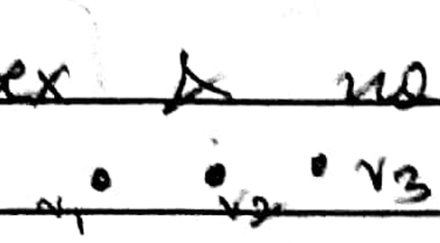
Eg.



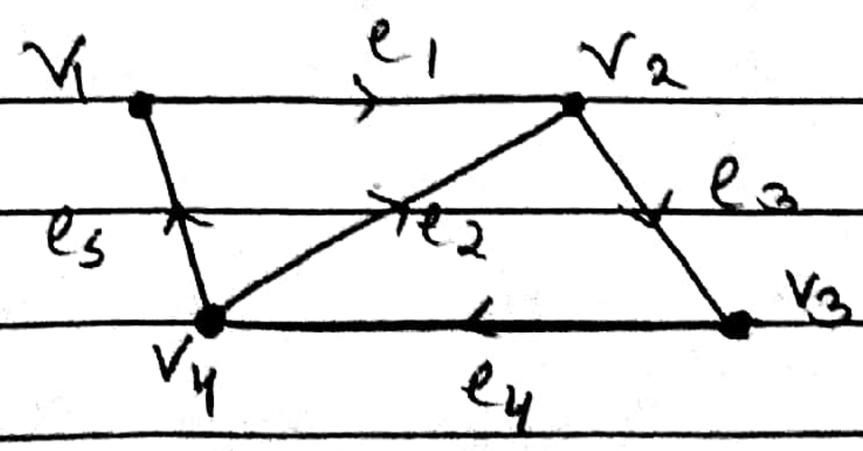
Basic Terminology

① Trivial Graph → consists only 1 vertex & no edge.

② Null Graph → consists n vertex & no edge.



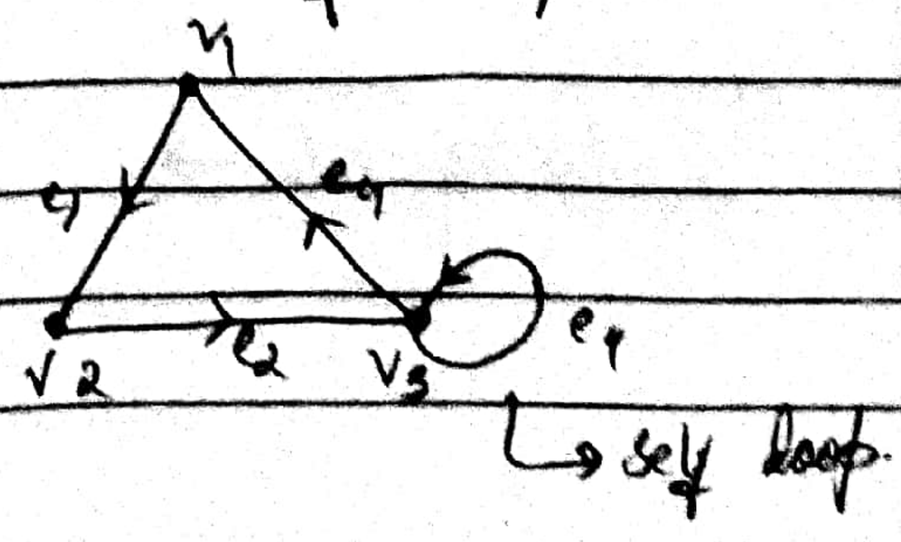
③ Directed Graph → graph consists direction of edges.



④ Undirected Graph → not directed

⑤ Self loop graph → edge has same vertex as both the end vertices is called self loop.

self loop



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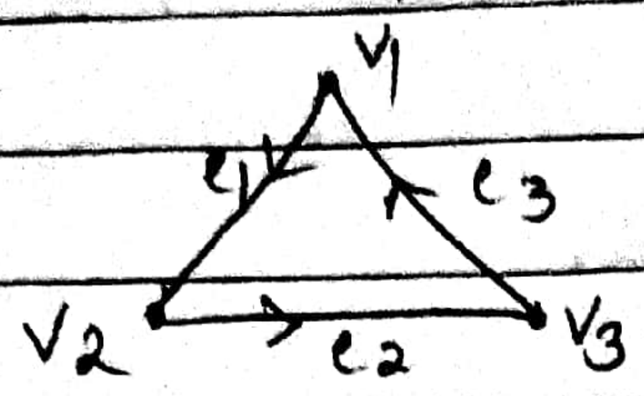
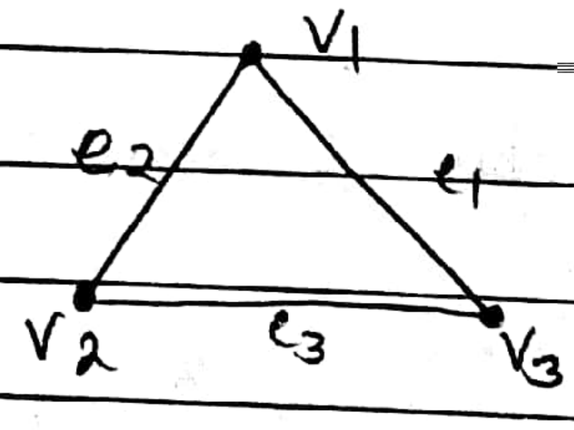
⑥ Proper edge → edge which has no self loop.



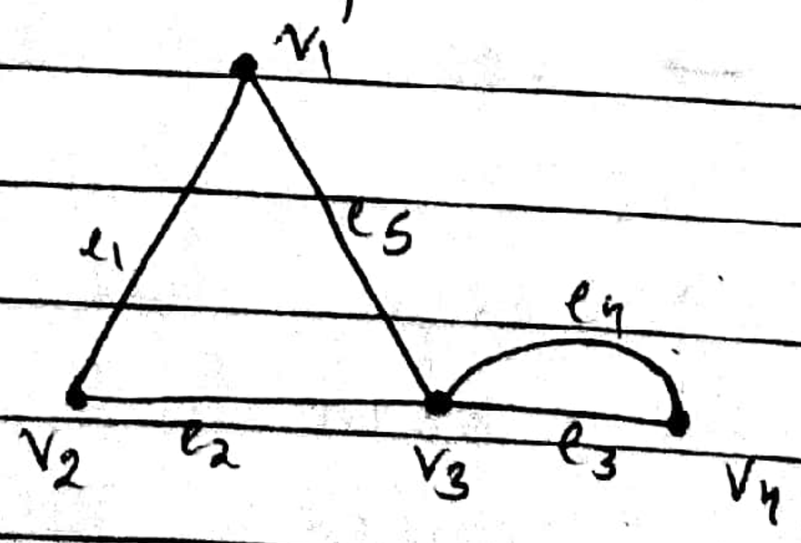
⑦ Multi edge → Collection of 2 or more edges having identically end point.

e_1, e_2, e_3 are multi edges.

⑧ Simple graph → graph have no self loop and multiedge.



⑨ Multigraph → graph does not contain self loop but contain multiedge.



⑩ Pseudo graph → contain both self loop and multi-edge.

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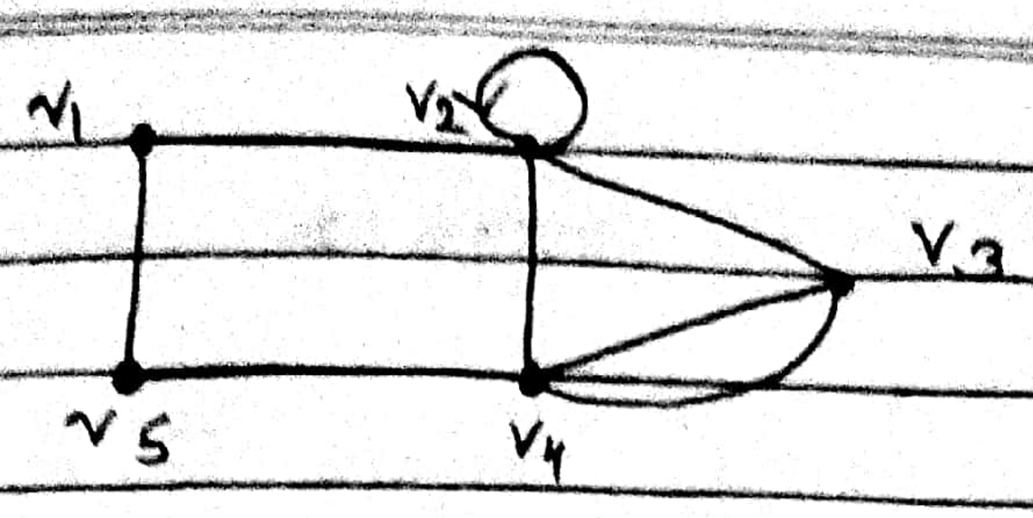
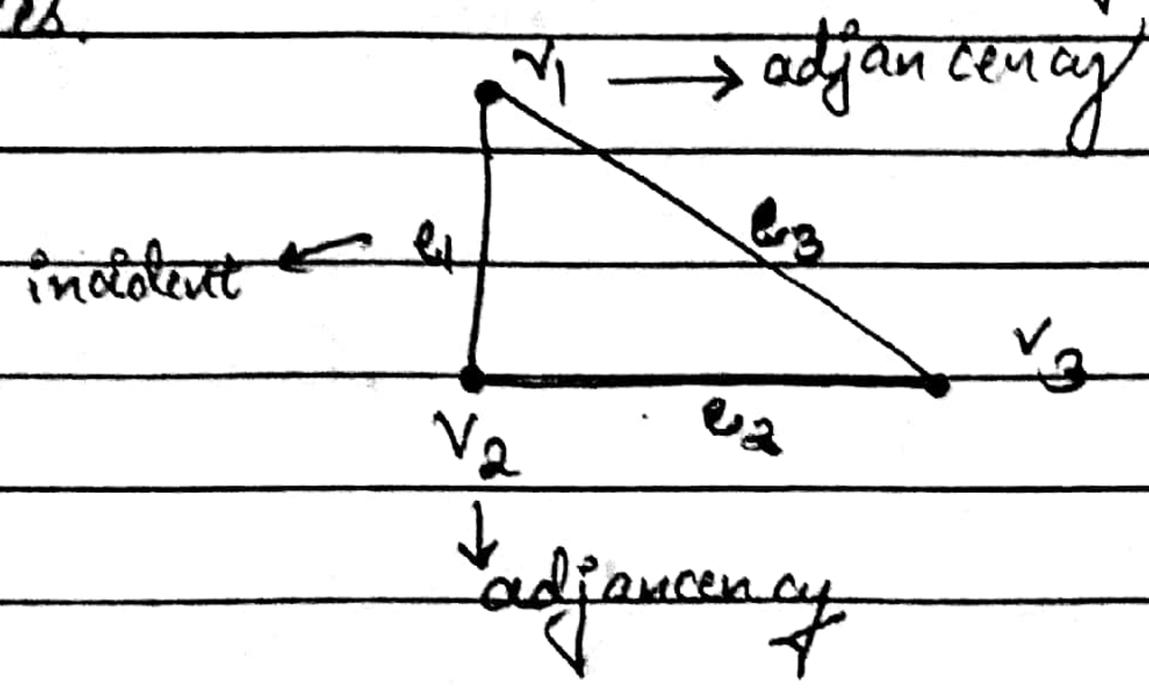


(11) <u>Incident and Adjancency</u> → let e_x be an edge joining two vertex v_i & v_j then e_x is said to be an incident of v_i & v_j.

Two vertex are said to be adjacent if their there exists an edge joining these vertices.

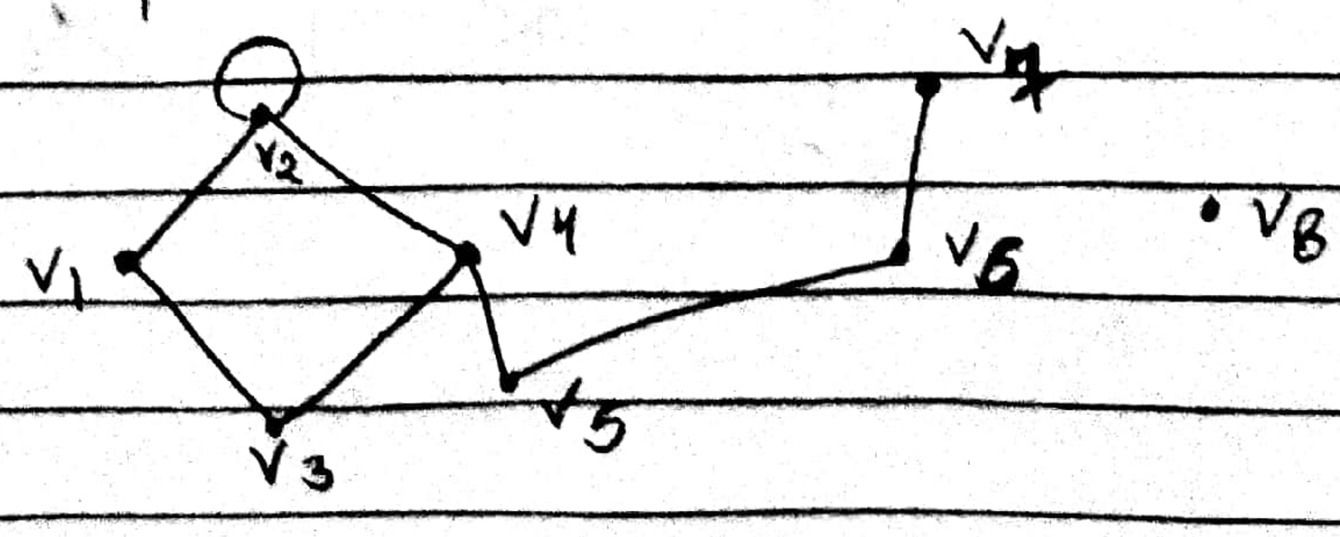


e_1 is incident of v_1 & v_2.
v_1 & v_2 are adjacent of e_1.

(12) # <u>Degree of Vertex</u>

$d(v)$ is equal to number of edges which are incident on v with self loop counted twice.

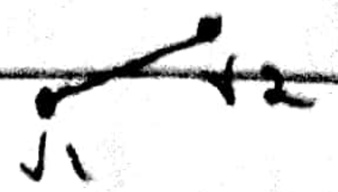
$v_1 = 2$ $v_5 = 2$
$v_2 = 4$ $v_6 = 2$
$v_3 = 2$ $v_7 = 1$
$v_4 = 3$ $v_8 = 0$



(13) Isolated & Pendent Vertex →

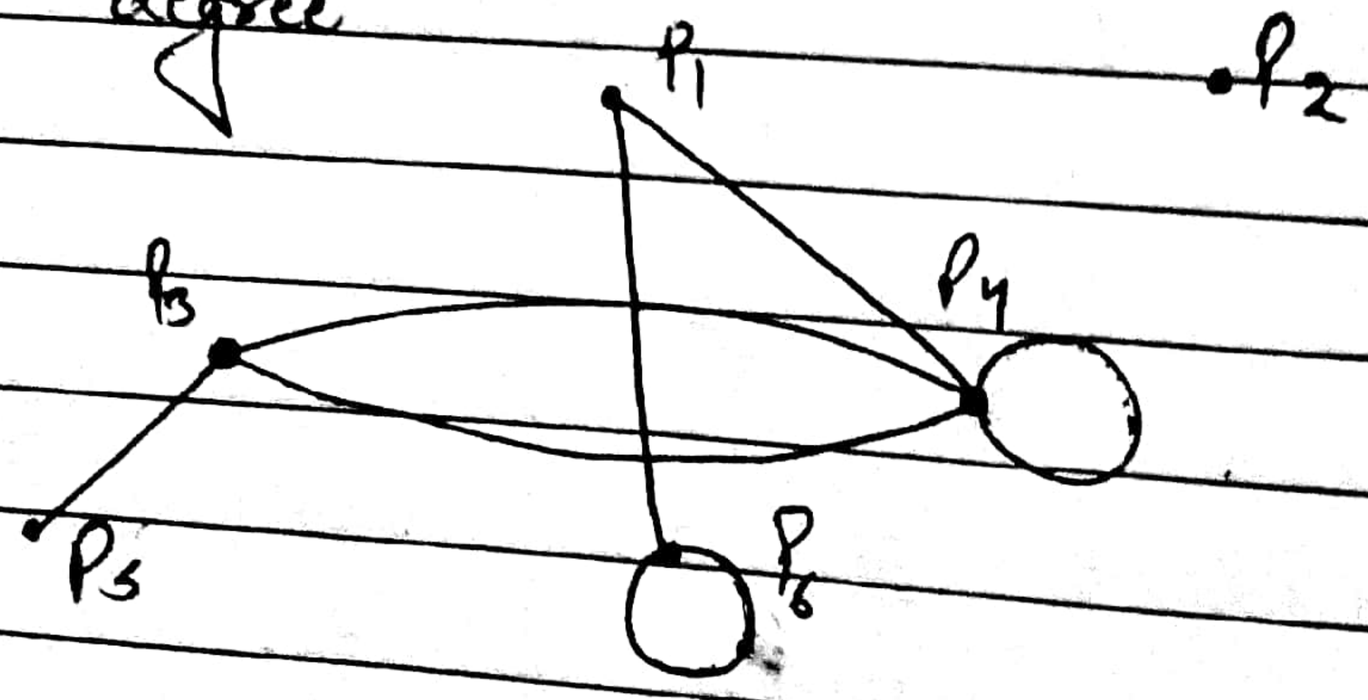
any vertex having degree 0 is called isolated vertex. $\cdot v_1$

any vertex having degree 1 is called pendent vertex.



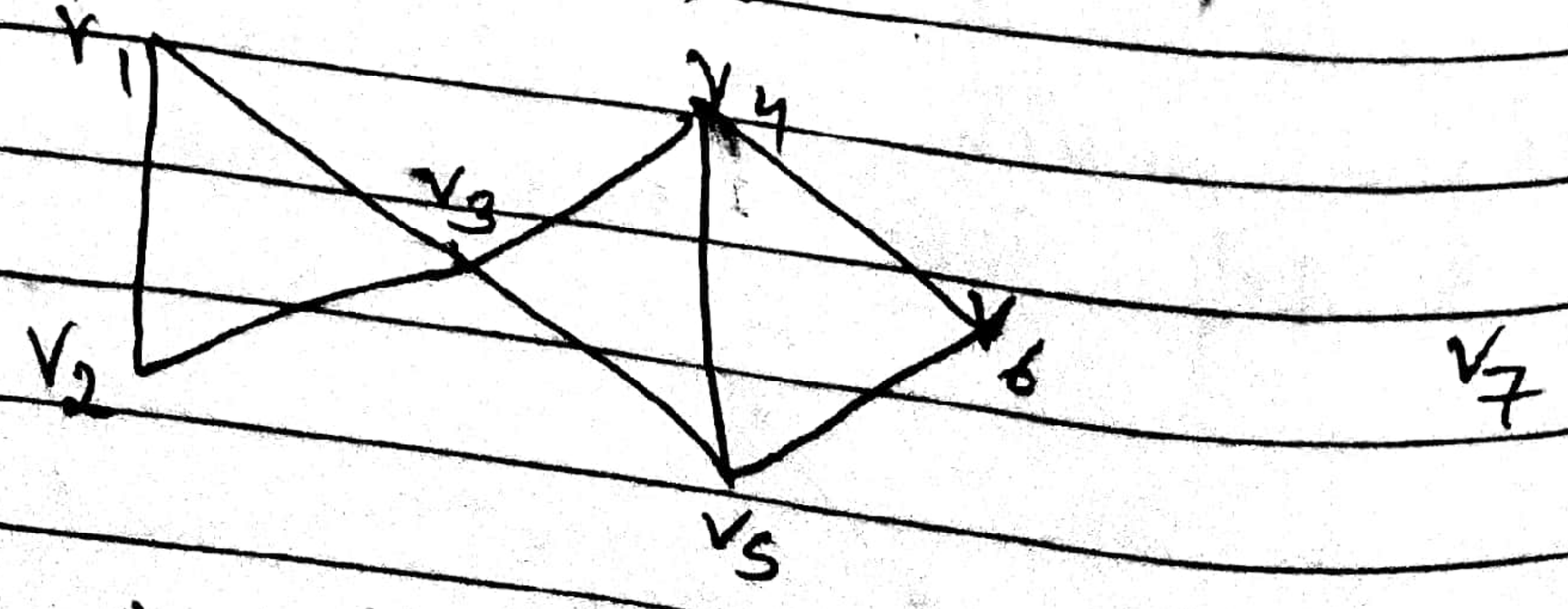
(14) Finite & Infinite graph → Graph with finite no. of vertexs as well as of edge is called finite otherwise infinite.

Ques Find degree



$P_1 = 2$, $P_2 = 0$, $P_3 = 3$, $P_4 = 5$, $P_5 = 1$, P_6

Ques No. of Isolated vertex



only 1 (one), i.e., v_7

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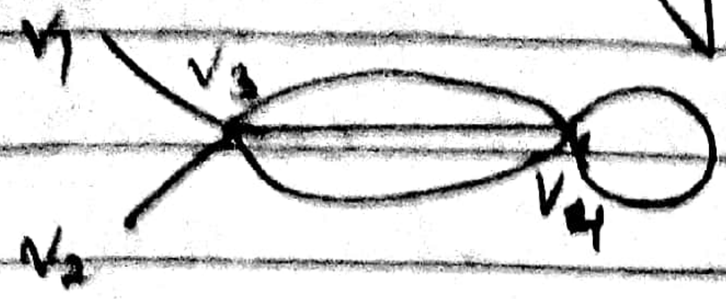
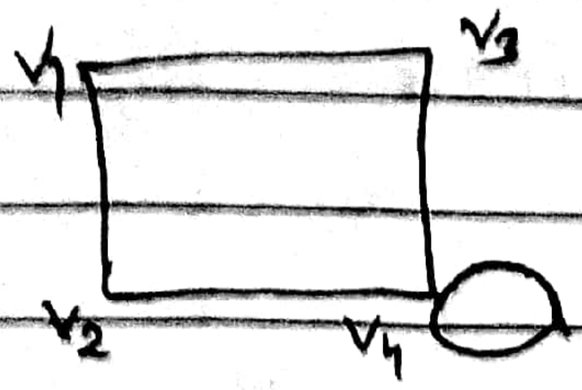
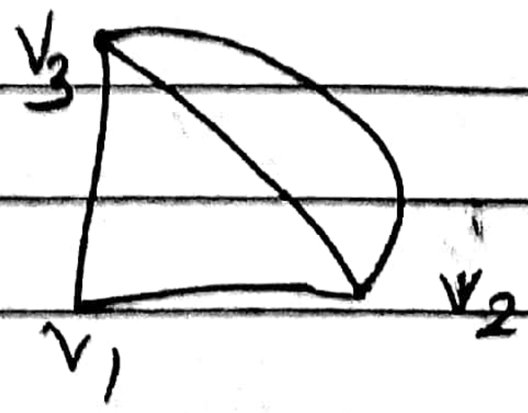
Que Which is pseudograph? (self loop & multiedge)

a) 

✓ both self loop & multiedge

b) 

c) 

d) None

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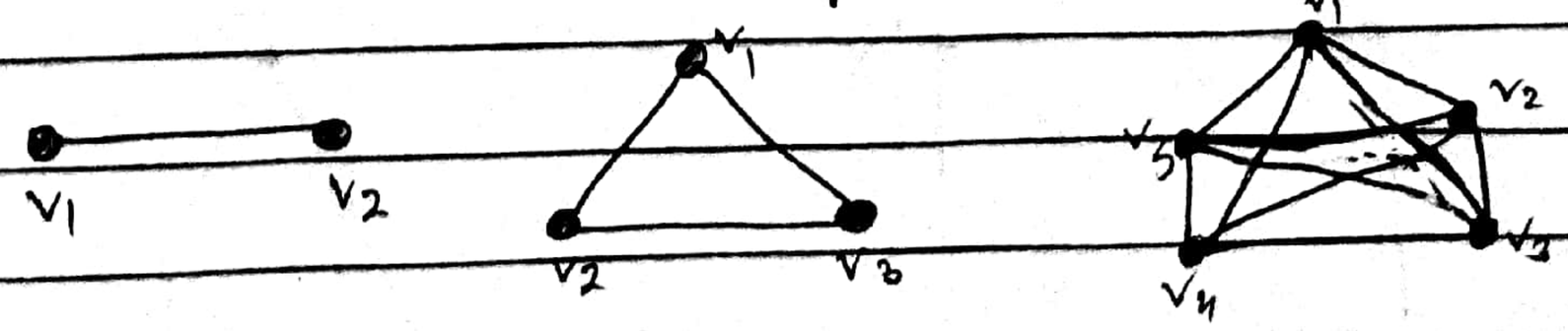
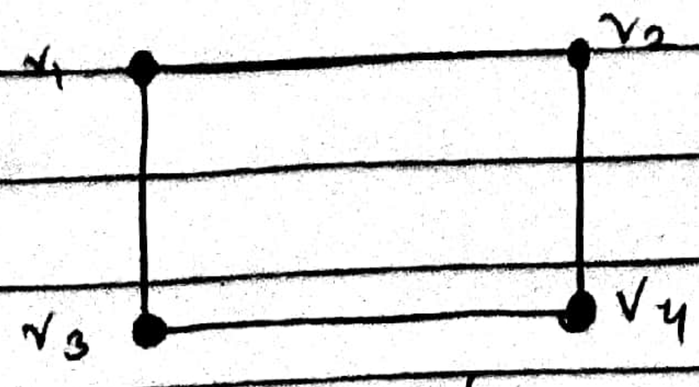
Types of Graph

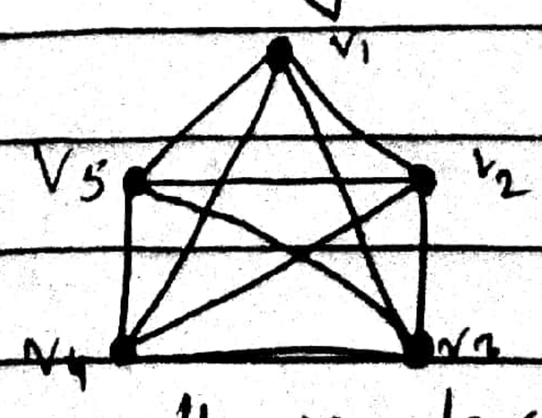
① **Complete Graph** → if each vertex connected to every other vertex.



② **Regular Graph** → Graph G is said to be regular if every vertex has same degree.

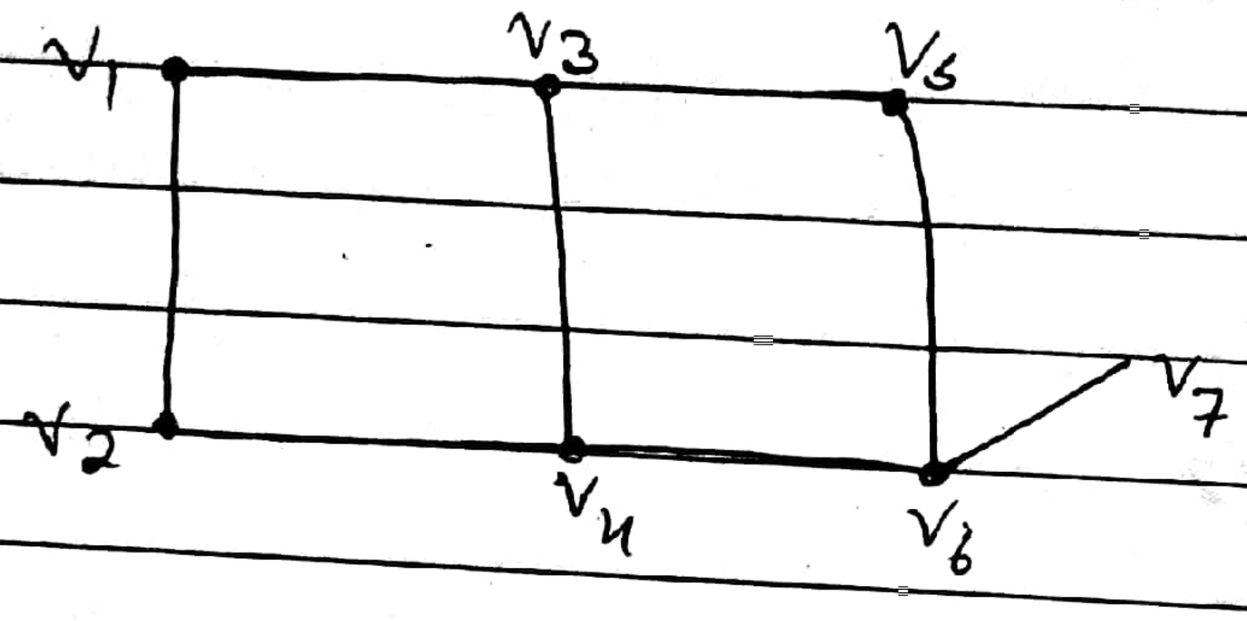


2 - regular graph



4 - regular graph

③ **Bigraph & Bipartide** → Vertex V & graph G can be partitioned of 2 non-empty disjoint subsets X & Y in such a way that ~~in~~ G edge of G has 1 end in X & one end in Y then G is called bigraph & bipartide.

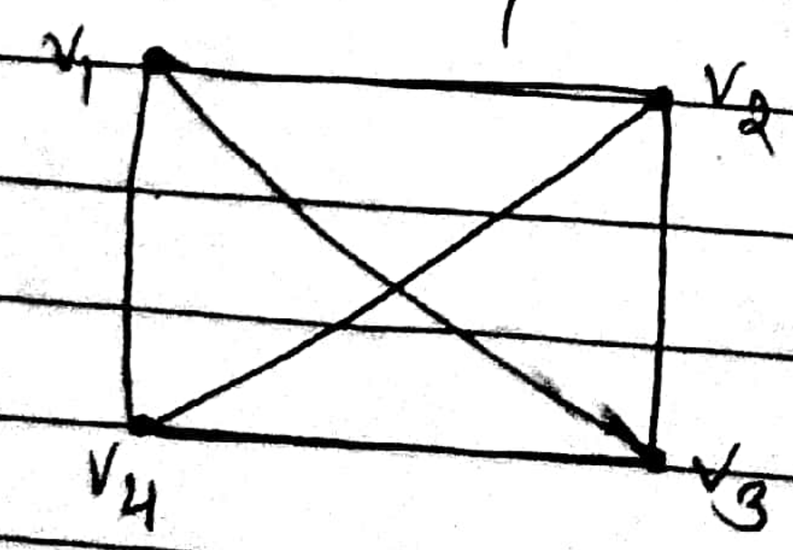


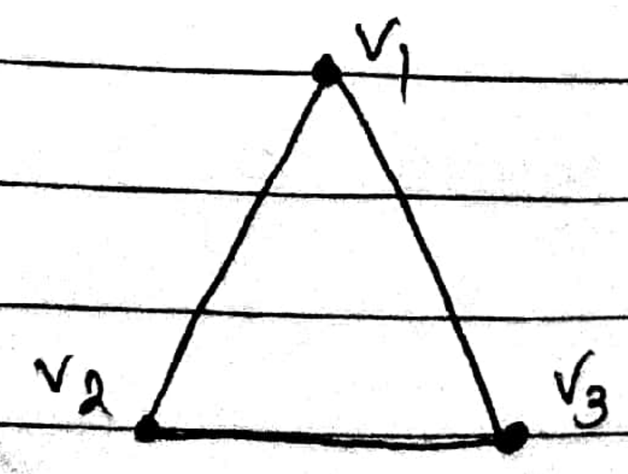
X (v_1, v_4, v_5, v_7)

Y (v_2, v_3, v_6)

Bigraph & Bipartide g

④ **Connected Graph** → An undirected graph said to be conn if there is a path b/w every vertex. If graph is connected then it's not bipartide.





⑤ **Complete Bipartide Graph** → If every vertex in X is disjoint every vertex in Y. It's denoted by K_{mn}.

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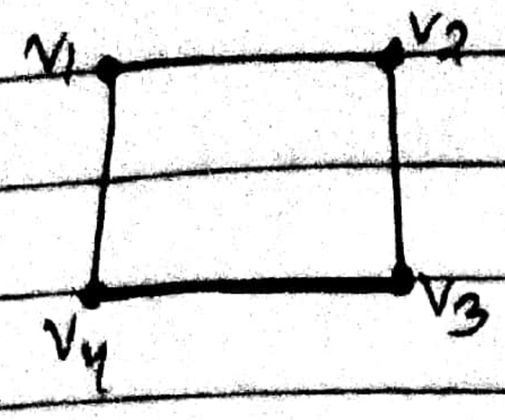
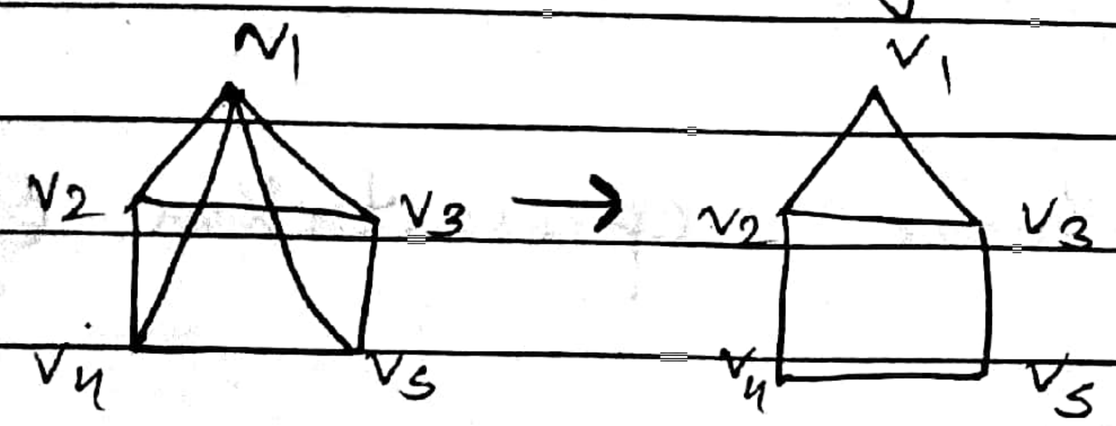
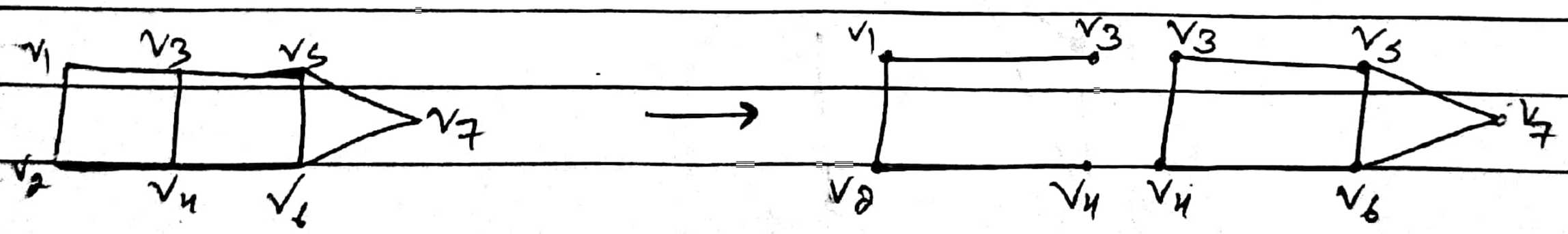


$$X = (\overbrace{v_1, v_3}^{m})$$
$$Y = (\underbrace{v_2, v_4}_{n}) \Rightarrow K_{22}$$

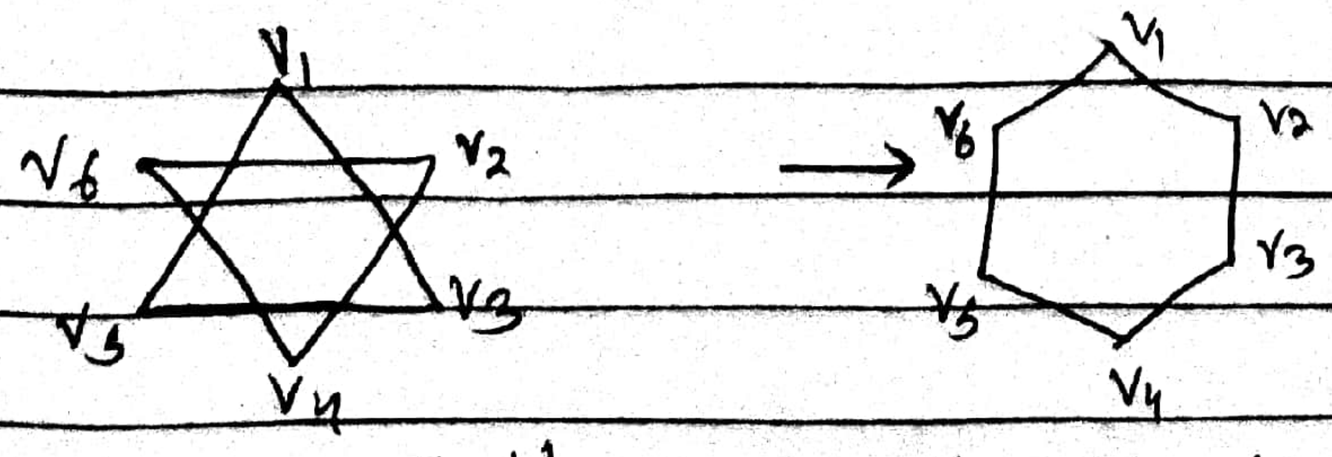
(6) **Subgraph** → let $G(V, E)$ be a graph. let P' be a subset of V and E' be a subset of E, whose end point belongs to V' then $G(V', E')$ is a ~~sugra~~ graph of subgraph.



(7) **Decomposition of Graph** → Graph is ~~said~~ to be decomposed into two subgraphs G_1, G_2. If $G_1 \cup G_2 = G$ and $G_1 \cap G_2 = \phi$.



(8) **Compliment of a Graph** → Compliment of a graph G is defined as



Graph compliment