Baye's Theorm

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

Binomial Distribution

$$P(r) = {}^nC_r \, p^r q^{n-r}$$

n = Total no. of trials

r = No. of successful trials

Poisson Distribution

$$P(r) = \frac{e^{-m} \cdot m^r}{r!}$$

Conditional Probability

The probability of occurance of A under the Condition that B has already occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Counting Principles

$$P(A \text{ or } B) = P(A) + P(B)$$
$$P(AB) = P(A) \cdot P(B)$$

Probability axioms

$$P(A) \geq 0$$
$$P(S) = 1 \qquad S = \text{Sample space}$$
$$P(A \cup B) = P(A) + P(B)$$

Random variable → A random variable is a numerical outcome of a random process or experiment.

Discrete Probability Distribution → Represents probabilities of a discrete random variable, which takes on a Countable number of possible values.

Continuous Probability Distributions → Represents probabilities of a Continuous random variable, which can take any value in a given range or interval.

Expectation (mean) → The expectation of a random variable is average of all its possible values

Variance → It Quantifies how much the values deviate from the expectation

Standard Deviation → It is the square root of variance.

Joint Probability Distribution → It models the likelihood of two or more random variables occuring simultaneously.

Bernoulli Distribution → It is a discrete probability distribution for a random variable that has two possible outcomes : Success (1) with probability p, and failure with probability 1-p.

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

OR

$$\frac{\text{Sample mean} - \text{population mean}}{\text{Population standard deviation} / \sqrt{\text{Sample size}}}$$

Bisection Method

$$x_2 = \frac{x_0 + x_1}{2}$$

Lagrange's Interpolation formula

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 - - -$$

Regula Falsi Method

$$x_2 = x_1 - \frac{(x_1 - x_0) f(x_1)}{f(x_1) - f(x_0)}$$

Newton Raphson's Method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Gauss Elimination Method → Change matrix to upper triangular matrix and Compare values

LU Decomposition Method → Change A into two matrices, upper and lower triangular

Use ⇒ $AX = B$ ⇒ $A = LU$ ⇒ $LUX = B$ ⇒ $UX = C$ ⇒ $LC = B$

Newton's Forward Interpolation Formula

$$y = y_1 + P\Delta y_1 + \frac{P(P-1)}{2!}\Delta^2 y_1 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_1 - - - - - -$$

Newton's Backward Interpolation Formula

$$y = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n - - -$$

where $P = \frac{x - x_0}{h}$ (Forward) OR $\frac{x - x_n}{h}$ (Backward)

Newton's Divided Difference Interpolation

$$y = y_1 + \Delta_d y_1 (x - x_1) + \Delta_d^2 y_1 (x - x_1)(x - x_2) \cdots\cdots$$

Derivative using Newton's Forward Difference

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_1 - \frac{1}{2}\Delta^2 y_1 + \frac{1}{3}\Delta^3 y_1 - \frac{1}{4}\Delta^4 y_1 \cdots\cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_1 - \Delta^3 y_1 + \frac{11}{12}\Delta^4 y_1 - \frac{5}{6}\Delta^5 y_1 \cdots\right]$$

Derivative using Newton's Backward Difference

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n \cdots\right]$$

Taylor's Series

$$y = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{iv} \cdots \quad (\text{where } h = x - x_0)$$

Derivative using Newton's Central Difference

$$\frac{dy}{dx} = \frac{1}{h}\left[\left(\frac{\Delta y_{j-1} + \Delta y_j}{2}\right) - \frac{1}{6}\left(\frac{\Delta^3 y_{j-2} + \Delta^3 y_{j-1}}{2}\right) \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_{j-1} - \frac{1}{12}\Delta^4 y_{j-2}\right]$$

Integration using Trapezoidal rule

$$\int_{x_0}^{x_n} f(x).dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3 --- y_{n-1})\right]$$

Integration using Simpson's 1/3rd rule

$$\int_{x_0}^{x_n} f(x).dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5 -- y_{n-1})+2(y_2+y_4 - y_{n-2})\right]$$

Integration using Simpson's 3/8th rule

$$\int_{x_0}^{x_n} f(x).dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8..) + 2(y_3+y_6+y_9 --)\right]$$

Picard's Iterative method

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx \quad \Big\{ \; y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)\, dx$$

Taylor's Series Method

$$y_n = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{iv}$$

Euler's Method

$$y_1 = y_0 + h f(x_0, y_0) \quad \Big\{ \; y_{n+1} = y_n + h f(x_n, y_n)$$

$\log(1) = 0$ $\qquad$ $\log(0) = $ Undefined

$\log(10) = 1$ $\qquad$ $\log(100) = 2$

$e^1 = 2.718$ $\qquad$ $e^{-1} = 0.367$

$e^0 = 1$

Runge's Method

$y_1 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$; $y_{n+1} = y_n + \frac{1}{6}(k_1 + 4k_2 + k_3)$

where $k_1 = hf(x_0, y_0)$

$k_2 = hf\left(x + \frac{h}{2}, y + \frac{1}{2}k_1\right)$

$k_3 = hf(x+h, y + 2k_2 - k_1)$

Runge Kutta's Method

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$; $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where

$k_1 = hf(x, y)$

$k_2 = hf\left(x + \frac{1}{2}h, y + \frac{1}{2}k_1\right)$

$k_3 = hf\left(x + \frac{1}{2}h, y + \frac{1}{2}k_2\right)$

$k_4 = hf(x+h, y+k_3)$