

Dec-24-0078 (CBCS/NEP)
MAFC-311 (Probability Theory and Statistics)
 [EE, EEE, ECE, CS, CSE AIML, CS, AIDS]
 B.Tech. 3rd

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

- Note : • Attempt five questions in all, one question each from sections A, B, C, D and the section E is compulsory.
 • Use of non-programmable calculator is allowed.

SECTION - A

1. (a) From a lot of 12 items containing 3 defective items, a sample of 4 items are drawn at random without replacement. Let a random variable X denote the number of defective items in the sample. Find the probability distribution of X .
 (b) A continuous random variable has pdf $f(x) = 6x(1-x)$, $0 \leq x \leq 1$. Determine a number b such that $P(X < b) = P(X > b)$.
 (6)
2. (a) Two players A and B play tennis games. Their chances of winning a game are in the ratio 3 : 2 respectively. Find A's chance of winning at least two games out of four games played.
 (b) In a book of 600 pages, there are 60 typographical errors. Assuming Poisson law for the number of errors per page, find the probability that a randomly chosen 4 pages will contain no errors.
 0.61815
 (6)

SECTION - B

3. (a) A random sample of 17 values from a normal population has a mean of 105 cm and the sum of squares of

deviations from this mean is 1225 sq. cm. Is the assumption of a mean of 110 cm for the normal population reasonable ? Test under 1% and 5% levels of significance. Also, obtain the 95% and 99% confidence limits. (value of t for 16 degree of freedom at 1% level of significance is 2.921 and at 5% level of significance is 2.12). (6)

- (b) The theory predicts the proportion of beans in the four groups A, B, C and D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory ? (value of χ^2 for 3 degree of freedom at 5% level of significance is 7.82).
 (6)
4. (a) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05.

x	0	1	2	3	4
f	419	352	154	56	19

(value of χ^2 for 3 degree of freedom at 5% level of significance is 7.82).
 (6)

- (b) The means of two single large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches at 5% level of significance? (given that the critical value z_{α} of Z for two tailed test at 5% level of significance is 1.96).
 (6)

SECTION - C

5. (a) Using Newton-Raphson method, derive formula to find $N^{1/3}$, $N > 0$. Hence find $24^{1/3}$ to four decimal place. Use suitable initial approximation.
 (6)

- (b) Find the Lagrange interpolating polynomial that fits the following data

x	2	1	0	-1
f(x)	3	-3	1	-1

(6)

6. (a) Given the values

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Evaluate f(9), using Newton's divided difference formula. (6)

- (b) A root of the equation $xe^x - 1 = 0$ lies in the interval (0.5, 1). Determine this root correct to three decimal places using secant method. (6)

SECTION - D

7. Obtain the approximate value of $y(1.3)$ for the initial value problem

$$\frac{dy}{dx} = -2xy^2, y(1) = 1$$

Using (i) Euler method, (ii) Taylor series second order method with step size $h = 0.1$. (12)

8. (a) Evaluate

$$\int_1^2 \frac{x^2}{1+x^3} dx$$

using the Simpson's 1/3rd rule with four sub-intervals. (6)

- (b) Solve the initial value problem

$$\frac{dy}{dx} = 2 + \sqrt{xy}, y(2) = 1$$

in the interval [2.0, 2.2] using the Runge-Kutta fourth order method with the step size $h = 0.1$. (6)

SECTION - E (Compulsory)

9. (i) Given

$$f(x) = \begin{cases} 0, & x < 0 \\ ke^{-\frac{x}{4}}, & x \geq 0 \end{cases}$$

Determine the value of k that makes f(x) a pdf.

- (ii) If a random variable X has the density function

$$f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Compute $P(|X| > 1)$.

- (iii) Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means equal to 160 inches² and 91 inches² respectively. Can these be regarded as drawn from the same normal population? (given that $F_{0.05} = 3.73$ at 5% level of significance).

- (iv) Using Lagrange's formula, express the function

$$\frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$$

as a sum of partial functions.

- (v) Evaluate the following integral using trapezoidal rule with $N = 4$.

$$\int_0^1 \frac{dx}{3+2x}$$

- (vi) Using modified Euler's method, find an approximate value of $y(0.1)$ taking $h = 0.1$, given that

$$\frac{dy}{dx} = x + y, y(0) = 1.$$

(6×2=12)