Statistical Machine Learning

Fall 2019, MMF, YSU

Homework No. 01

Due time/date: 9:30 PM, 24 November, 2019

Note: Please use **Python** only in the case the statement of the problem contains (Python) at the beginning. Otherwise, show your calculations on the Jupyter Notebook. Supplementary Problems will not be graded.

Problem 1: Conditional Distribution

a.

Assume X and Y are Discrete r.v.s with the following PMF (= Probability Mass Function):

$Y \setminus X$	-1	0	2
0	0.2	0.1	0
1	0.15	0.1	
2	0.1	0	0.1

- Find the r.v.s $\mathbb{E}(Y|X)$ and $\mathbb{E}(X|Y)$;
- Show that $\mathbb{E}(\mathbb{E}(Y|X)) = \mathbb{E}(Y)$.

b.

Assume X and Y are Continuous r.v.s with $(X,Y) \sim Unif(D)$, where D is the equilateral triangle with the vertices at $(-1,0)$, $(0,1)$ and $(1,0)$.

- For any $x \in [-1,1]$, calculate the Distribution of the r.v. $Y|X = x$;
- Find $\mathbb{E}(Y|X)$, as a r.v.;
- Show that $\mathbb{E}(\mathbb{E}(Y|X)) = \mathbb{E}(Y)$.

c. (Supplementary)

Assume $X \sim Unif[0,1]$ and, for any $x \in [0,1]$, $Y|X = x \sim Unif[0,x]$. Find the Joint PDF of X and Y.

d. (Supplementary)

Give a method to generate Uniformly Distributed points in the Unit Disk.

Problem 2: Bayes Predictors

a. (Supplementary)

Prove that in the K-class Classification problem, the Predictor (Classifier)

$$g^*(x) = \underset{k=1,\ldots,K}{argmax}\, \eta_k(x) = \underset{k=1,\ldots,K}{argmax}\, \mathbb{P}(Y = k|X = x)$$

(see Lecture 02) is indeed a Bayes Predictor. Is it unique?