
EE405A

Local Map Generation &

Motion Planning

School of Electrical Engineering
KAIST

Contents

Notice

Local Map Generation

- Map Generation for Motion Planning
- Occupancy Grid Map

Motion Planning

- Motion Planning Methods
- Motion Primitives-based Planning

Appendix

- A* Algorithm
- RRT* Algorithm

Notice

➤ Final Project Scenario

- Autonomous Racing → **Autonomous Exploration**
- Find **numbers** or **QR codes** as much as possible while exploring corridor environments (Segmentation → **Object detection**-based perception module)
- (Optional) Utilizing GPT for final project mission

➤ Assignment of this week

- The assignment of this week will be provided next week (~ Tuesday).
- Implementing map generation & path planning + Deploying in our gazebo-based simulation.
(Week 3, <https://github.com/Guri-cccc/EE405A-2023-F1-simulation>)

Local Map Generation

Map Generation for Motion Planning
Occupancy Grid Map

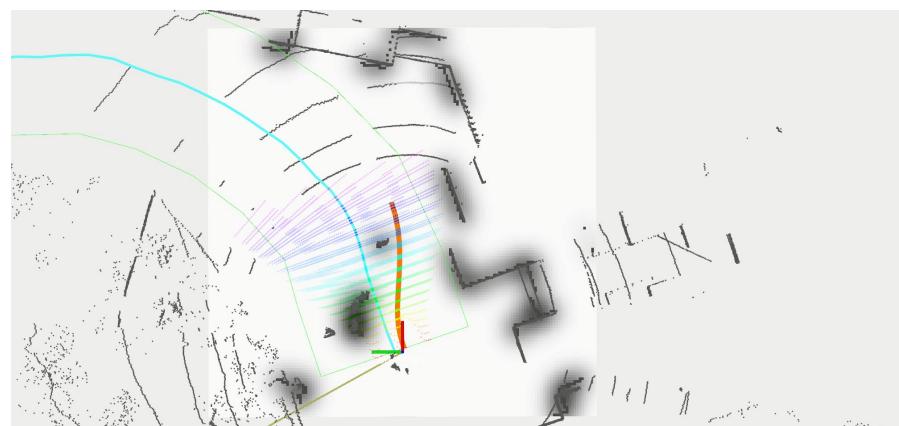
Map Generation For Motion Planning

➤ Map Generation for Motion Planning

- ❑ To navigate in a complex area, a robot should take into account the surrounding environment.
- ❑ As shown in the right side figure, collision-free path can be computed based on the grid map.
- ❑ In order to come up with dynamic obstacles, we need to generate a dynamic occupancy grid map in real-time.
- ❑ An advantage of the occupancy grid map is that it can represent a possibility for the collision.



Generated static map
using Hector SLAM



Example of dynamic occupancy grid map –

We implemented motion planning algorithm for the obstacle avoidance and we generated Gaussian distributed occupancy grid map in real-time.

Occupancy Grid Map

➤ Occupied grid map (Basic)

- We can generate a probabilistic occupancy grid map considering the sensor model(Sonar, Laser, Camera ..)
- Grid map can be generated by accumulating a scan to the corresponding grid cell.

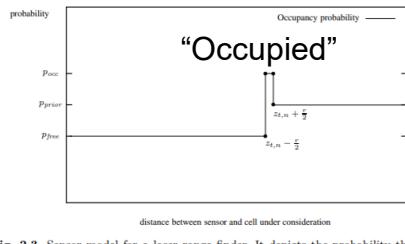


Fig. 2.3. Sensor model for a laser range finder. It depicts the probability that a cell is occupied depending on the distance of that cell from the laser sensor.

Laser sensor model

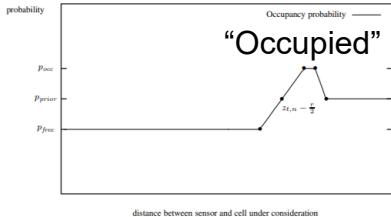
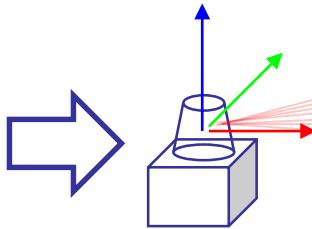


Fig. 2.4. Probability that a cell on the optical axis of the sensor is occupied depending on the distance of that cell from the sensor.

Sonar sensor model



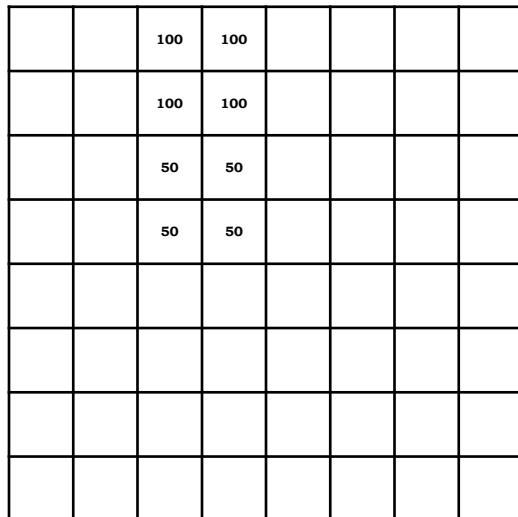
0	5	0	0
0	2	0	0
0	2	0	0
0	0	0	0

Accumulated scan to the grid cell(origin)

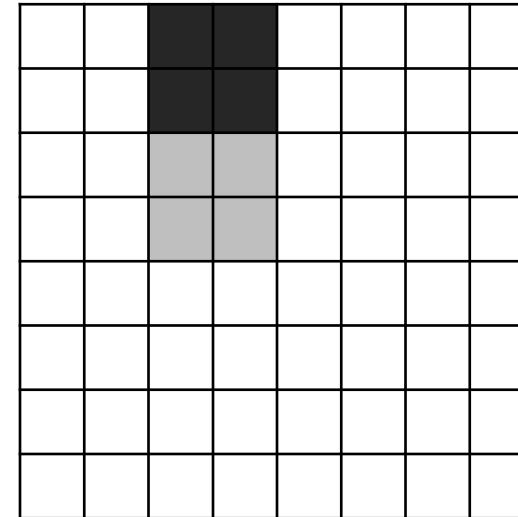
Occupancy Grid Map

➤ Occupied grid map (Basic)

- ❑ You can make a basic occupancy grid map using obstacle height information using 3D depth pointcloud from RealSense D435 camera.
- ❑ Firstly, to detect the position of obstacles, you can use depth point cloud to generate a height map.
- ❑ Using the generated height map, compute the cost map based on the height value.



Heightmap

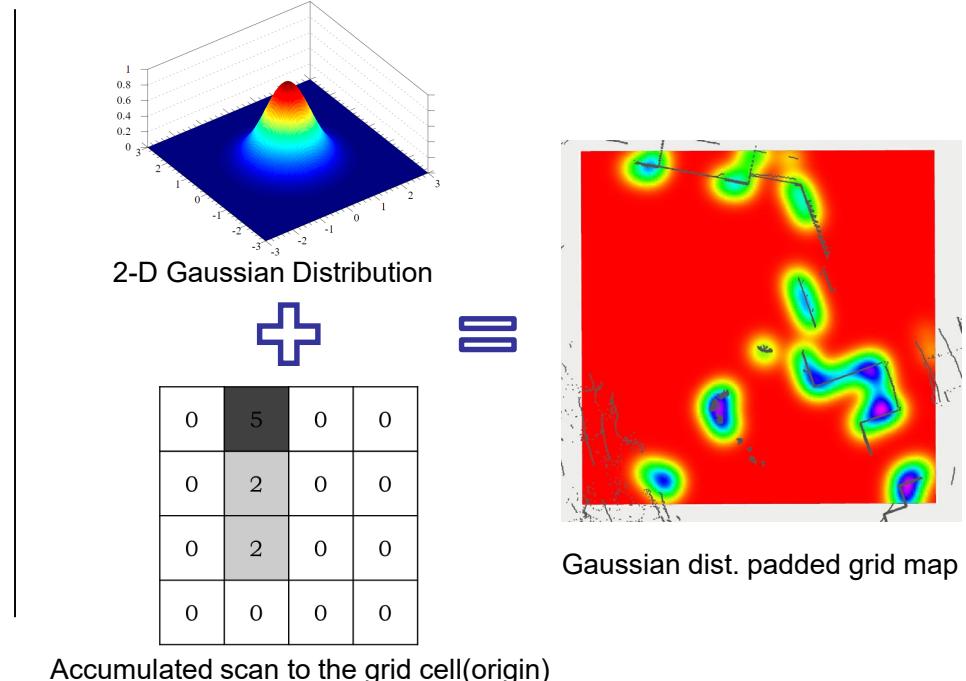
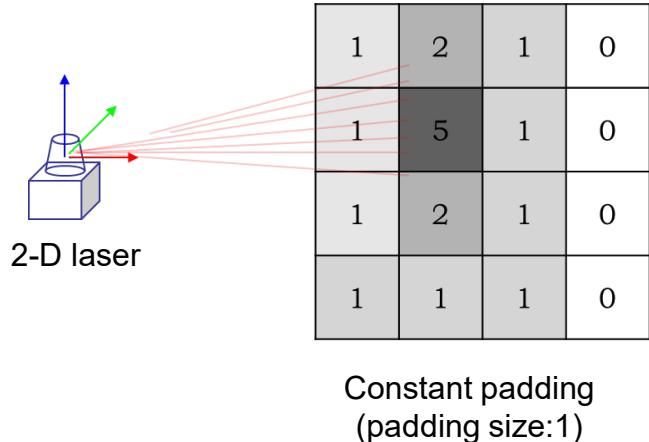


Costmap

Occupancy Grid Map

➤ Occupied grid map (Advanced)

- You can give padding for the collision buffer. Normally we set this padding value considering the size of the vehicle(robot).
- Personally, we gave a Gaussian distribution for the padding to regard it as a collision possibility. You can give a covariance value considering the size of the vehicle.



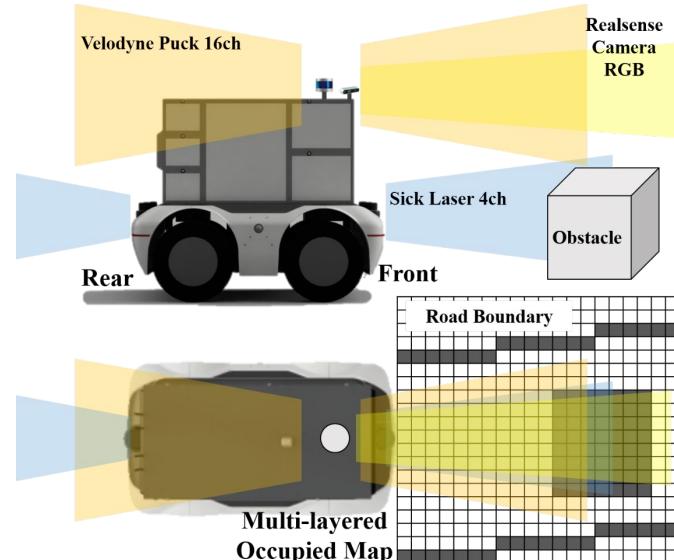
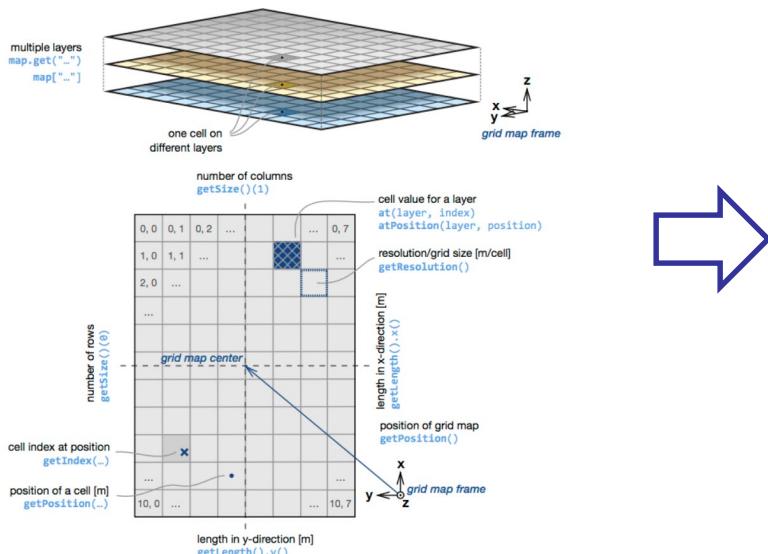
Stachniss, Cyril. *Robotic mapping and exploration*. Vol. 55. Springer, 2009.

Occupancy Grid Map

➤ Occupied grid map (Advanced)

- If you use multiple sensors such as 3-D Laser, camera, 2-D Laser sensors, you can integrate the data from each sensor using a universal grid map library:

https://github.com/anybotics/grid_map



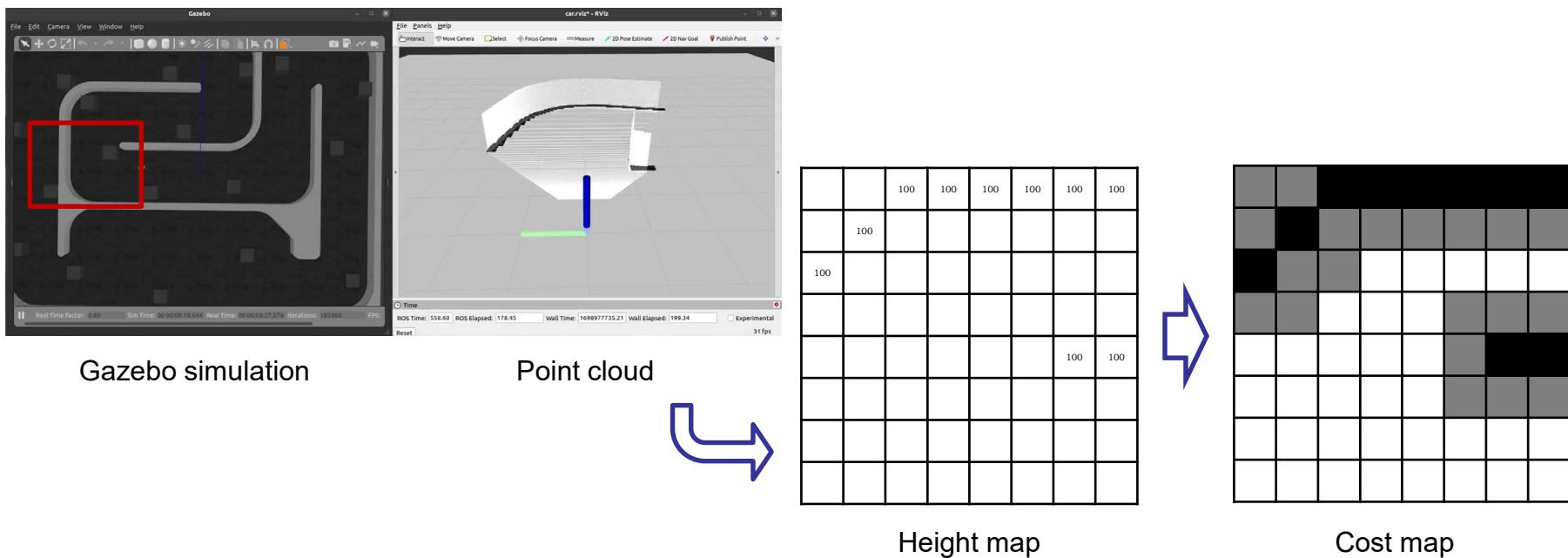
Fankhauser, Péter, and Marco Hutter. "A universal grid map library: Implementation and use case for rough terrain navigation." *Robot Operating System (ROS)*. Springer, Cham, 2016. 99-120.

Example of multi-layered occupied grid map –
From our lab project : cooperative delivery robot for postman

Occupancy Grid Map

➤ Occupied grid map using Point Cloud data from D435i

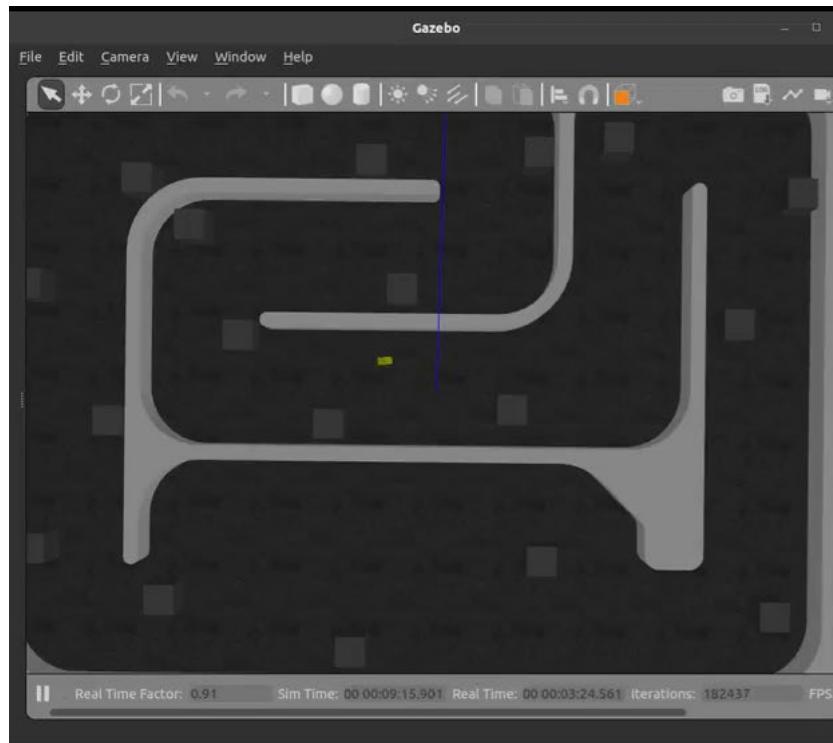
- ❑ Our camera sensor (Realsense D435i) provides point cloud data based on depth image.
- ❑ We can utilize the 3D point cloud to generate real-time local map.
- ❑ Height-based scheme can be used to represent traversable / non-traversable region around the ego vehicle.



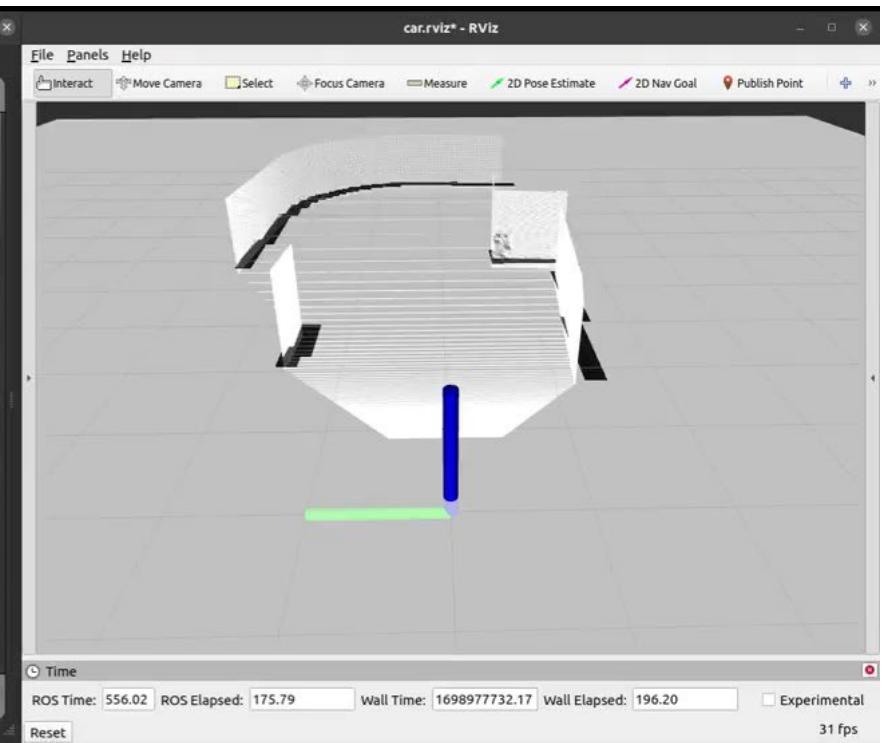
Occupancy Grid Map

➤ Occupied grid map using Point Cloud data from D435i

- ❑ Gazebo simulation: Week 3, <https://github.com/Guri-cccc/EE405A-2023-F1-simulation>.
- ❑ Local cost map generation will be provided as an assignment in next week.



Gazebo simulation



Real-time local cost map generation

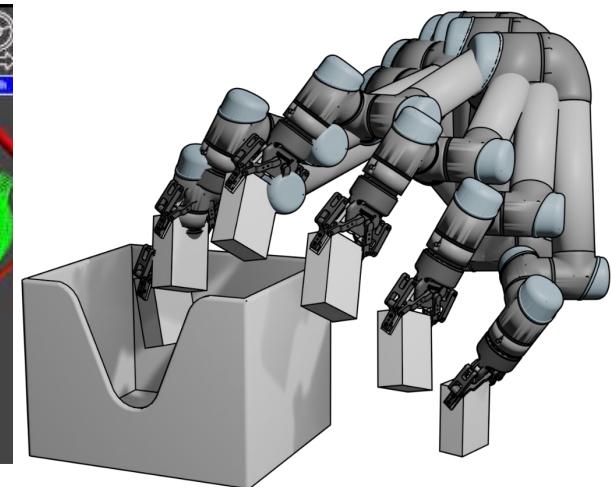
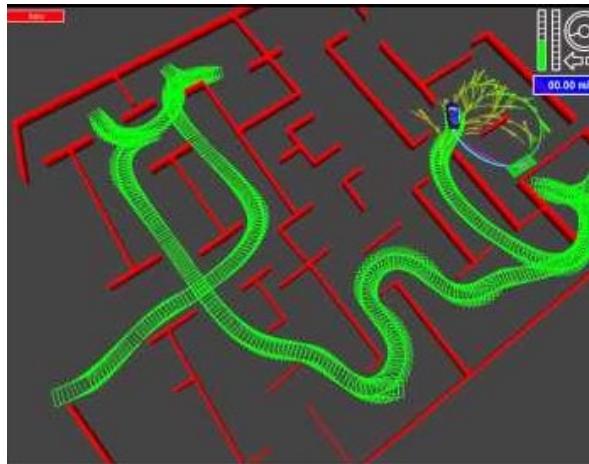
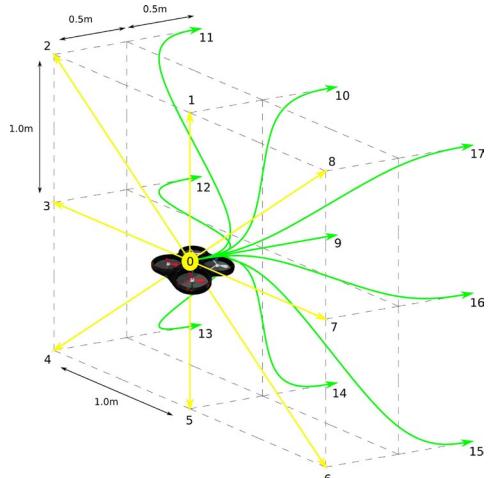
Motion Planning

Motion Planning Methods
Motion Primitive-based Algorithm

Motion Planning Methods

➤ Motion planning

- ❑ Motion planning, also path planning is a computational problem to find a sequence of valid configurations that moves the object from the source to destination.
- ❑ Several major categories for motion planning will be introduced in this lecture:
 - ✓ Graph Search-based motion planning
 - ✓ Random Sampling-based motion planning
 - ✓ Motion Primitives-based planning

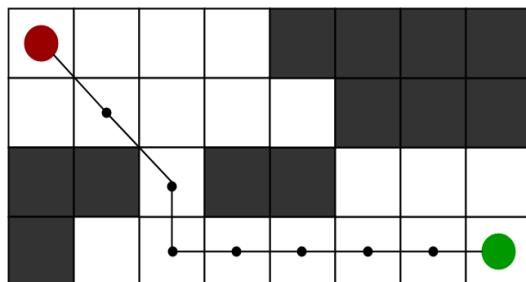


Motion planning in various robotics applications

Motion Planning Methods

➤ Graph search-based motion planning

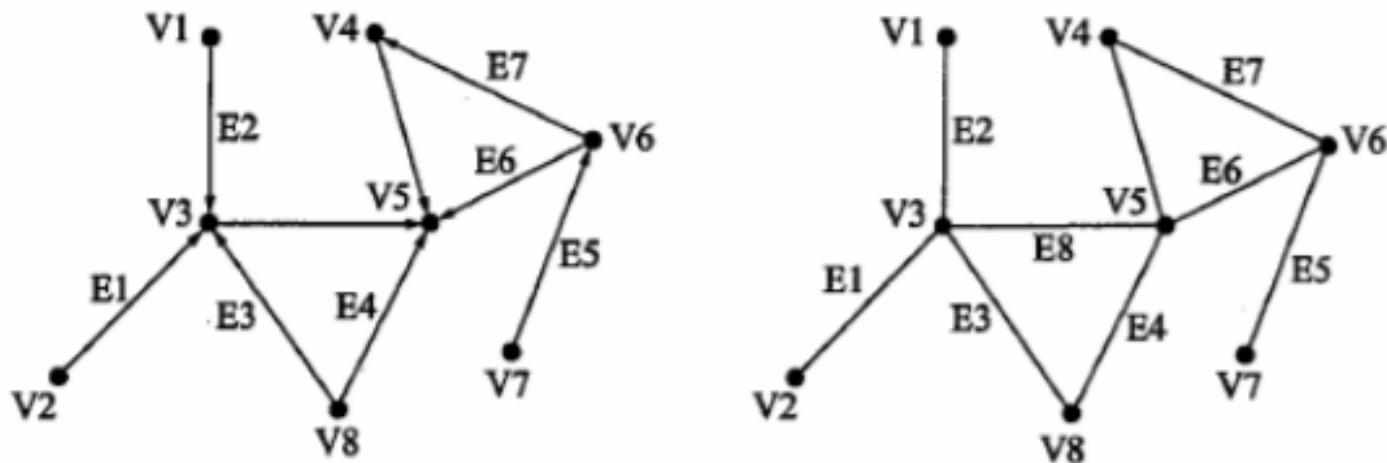
- It represents the environment in a discrete grid (discrete grid can be represented as a graph)
- Grid-based approaches overlay a grid on configuration space, and assume each configuration is identified with a grid point.
- At each grid point, the robot is allowed to move to adjacent grid points as long as the line between them is collision-free.
- This discretizes the set of actions, and search algorithms (like A*) are used to find a path from the start to the goal.



Examples of the grid-based motion planning:
A* and Hybrid A* algorithm

Motion Planning Methods

- Graph search--based motion planning
 - Collection of edges and nodes (vertices)

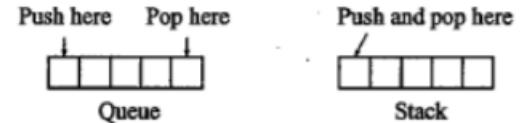


Motion Planning Methods

➤ Graph search-based motion planning

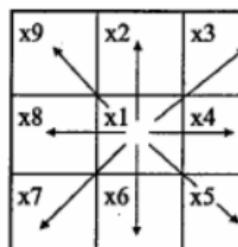
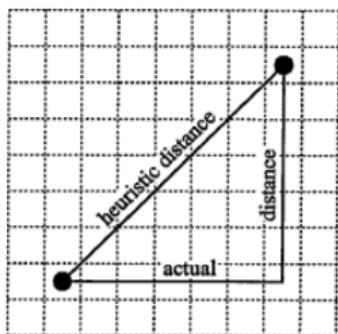
□ Uniform Search

- Use no information obtained from the environment
- Blind search: BFS(*Breadth-First Search*, width), DFS(*Depth-First Search*)

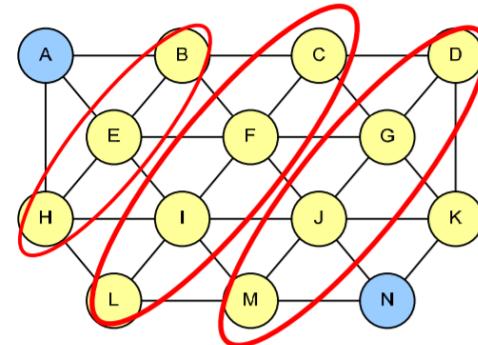


□ Informed Search: A*

- Use evaluation function
- Usually more efficient
- Heuristic search: A*, D*, etc.



$c(x1, x2) = 1$
 $c(x1, x9) = 1.4$
 $c(x1, x8) = 10000, \text{ if } x8 \text{ is in obstacle, } x1 \text{ is a free cell}$
 $c(x1, x9) = 10000.4, \text{ if } x9 \text{ is in obstacle, } x1 \text{ is a free cell}$

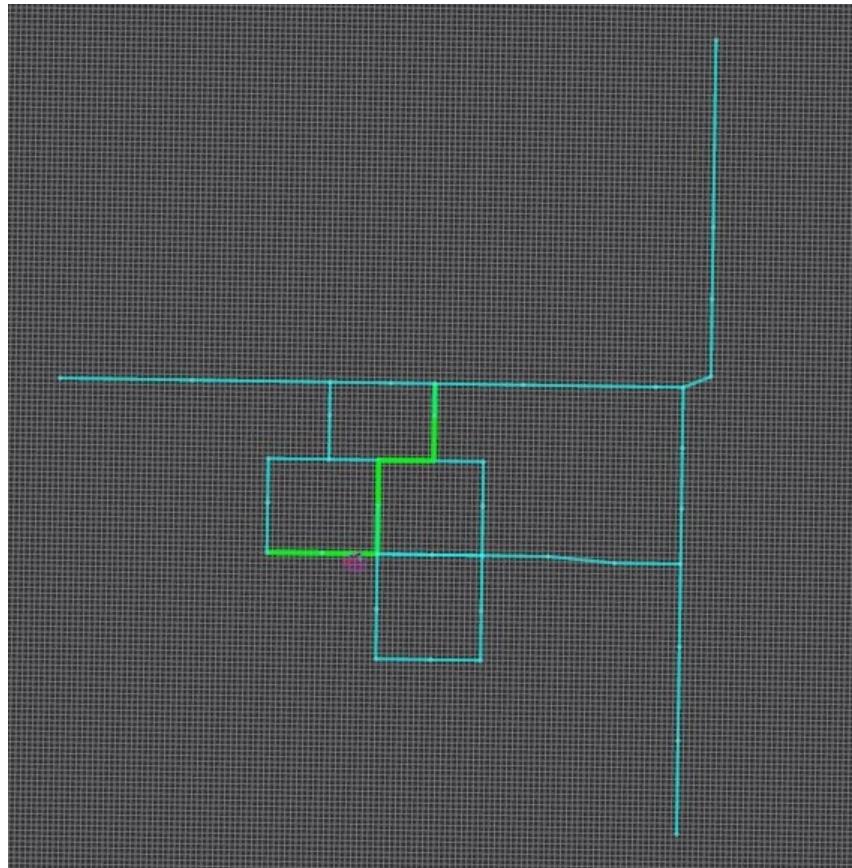


An example of BFS graph search from A to N

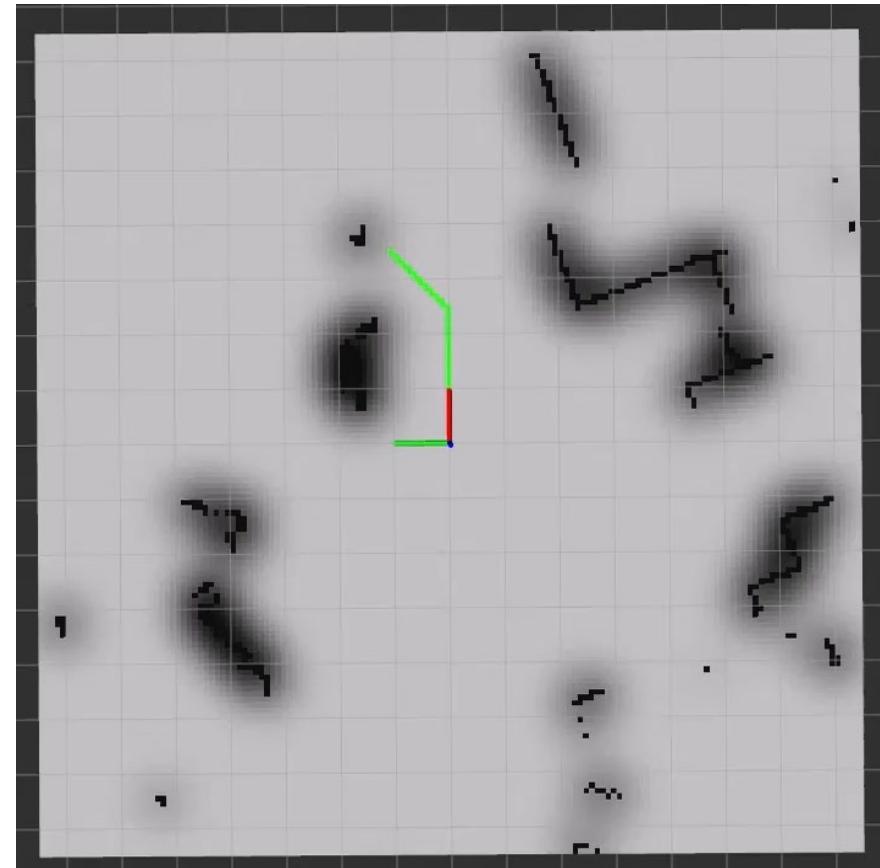
Heuristic-based A* algorithm

A* Algorithm

- Example of implementing A*-based planner



Roadmap-based global path planning

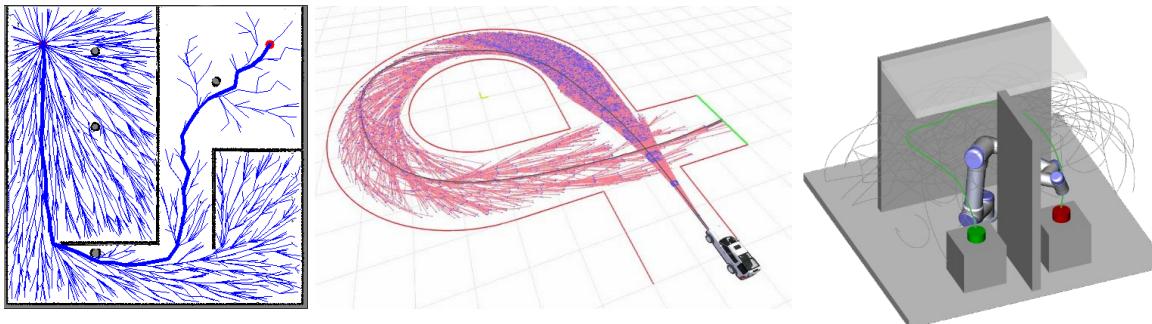


Grid map-based local path planning

Motion Planning Methods

➤ Random sampling-based motion planning

- Random sampling can overcome the disadvantage of the fixed graph discretization that other planner search only over the set of paths.
- Instead of grid search, nodes are sampled in the configuration space C for finding collision-free path in probabilistic manner (e.g., Rapidly exploring Random Tree (RRT)).
- Since their running time is not (explicitly) exponentially dependent on the dimension of the C , they works well for high-dimensional C (i.e., 3 dimensional environment).



Examples of the sampling-based motion planning: RRT and RRT*

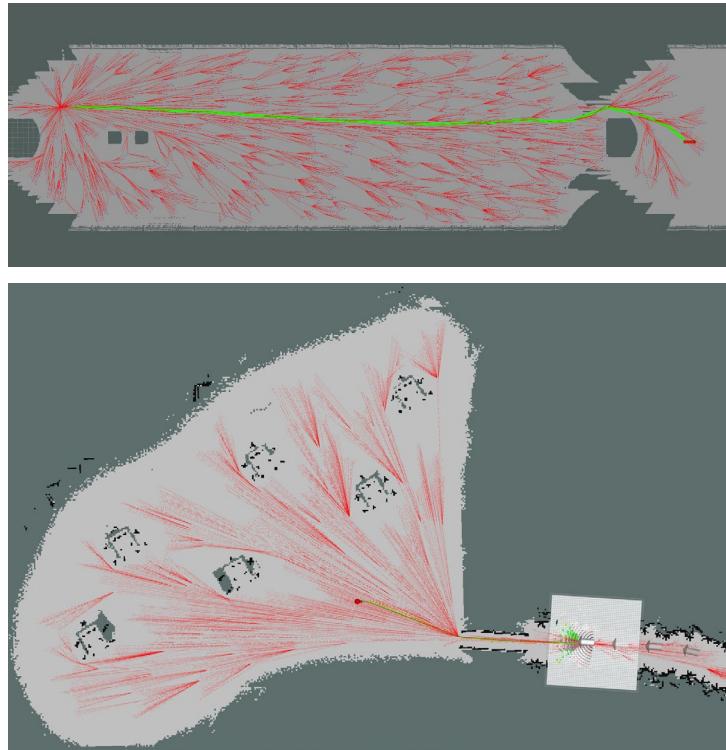
Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow N$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```

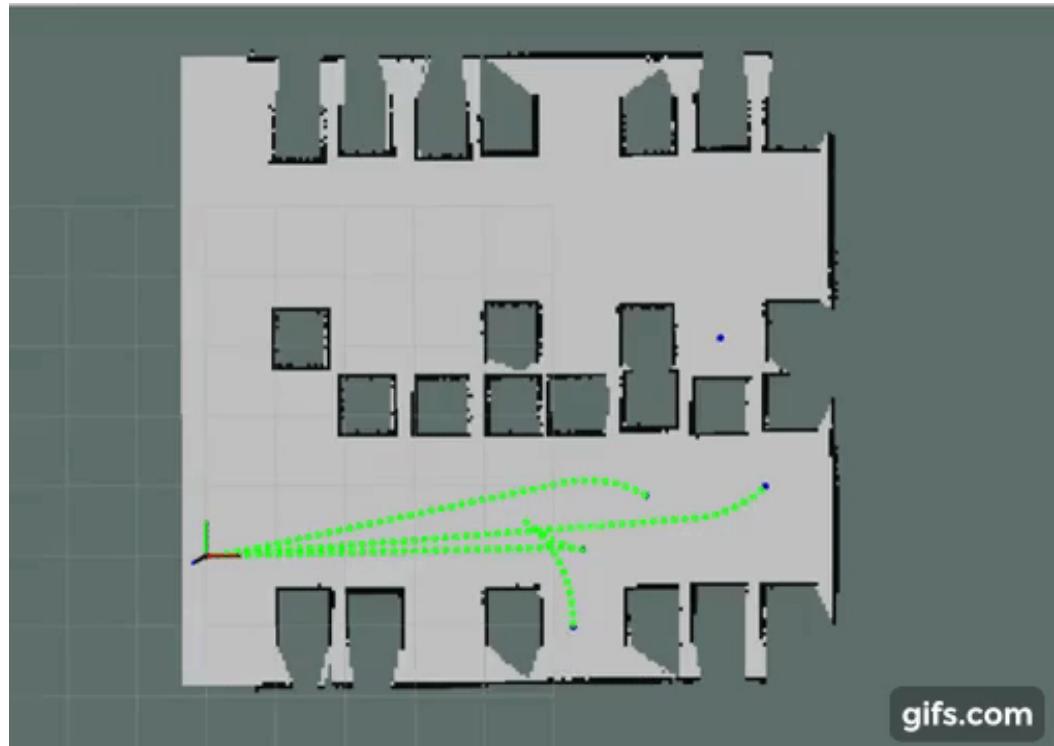
Algorithm of RRT

Motion Planning Methods

- Example of implementing RRT*-based planner



RRT*-based global path planning

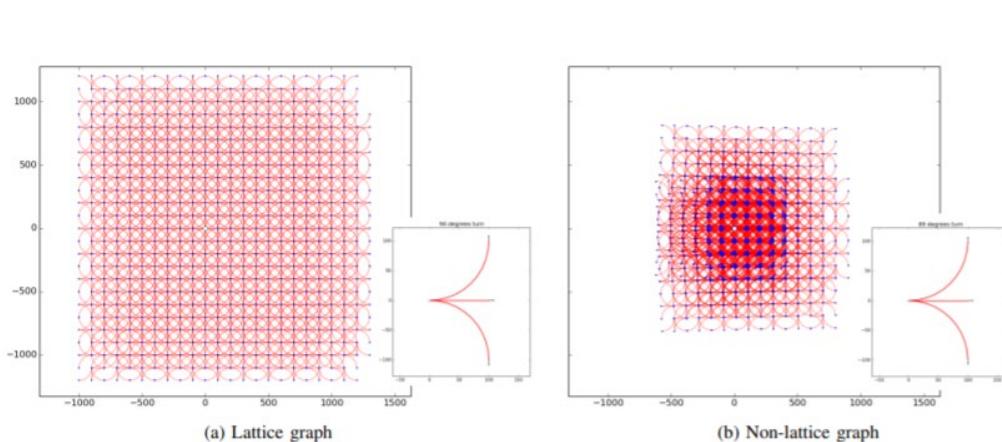


Kino-dynamic RRT*-based path planning

Motion Planning Methods

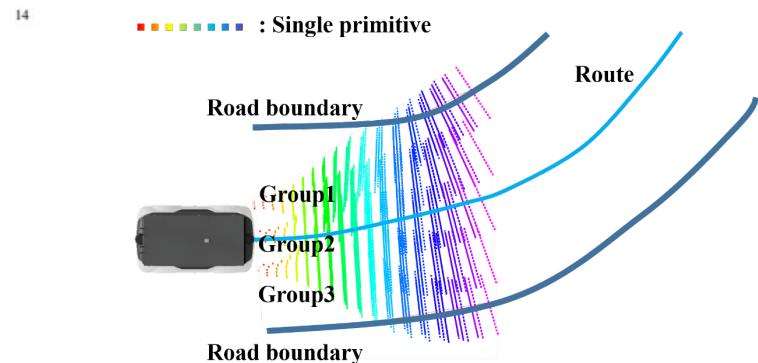
➤ Motion primitives-based planning

- ❑ It generates a sample considering vehicle's constraints to consider a reachability.
- ❑ It builds a cost function to consider the cost minimized(optimal) path in the generated paths.
- ❑ It also discretized the set of actions as like A* algorithm, and find a drivable paths in the candidates.



Examples of lattice graph planner

Paden, Brian, et al. "A survey of motion planning and control techniques for self-driving urban vehicles." *IEEE Transactions on intelligent vehicles* 1.1 (2016): 33-55.

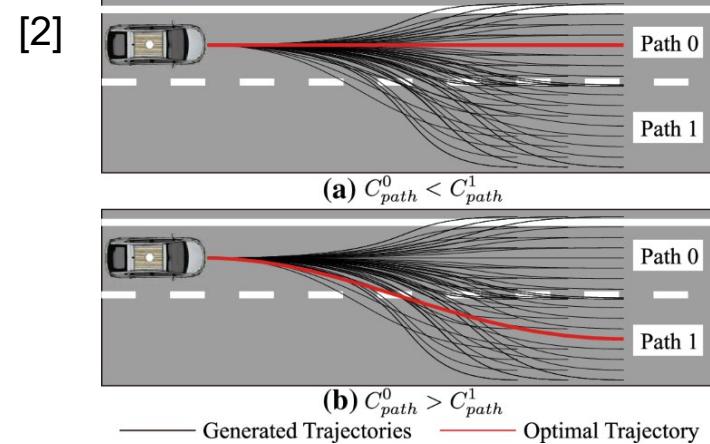
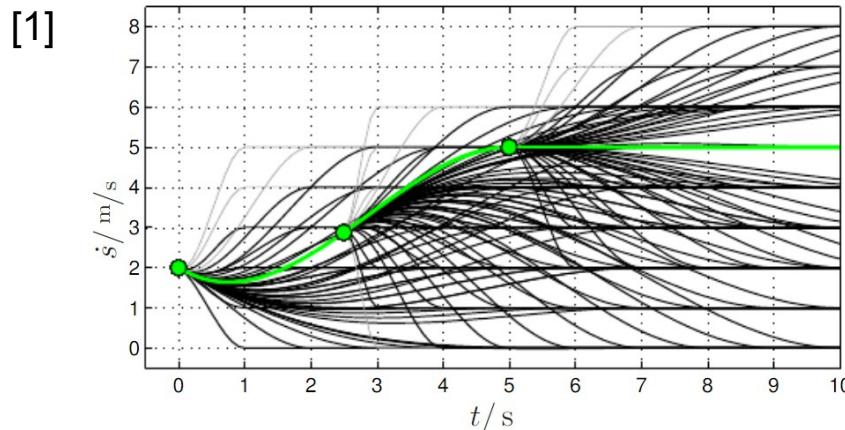


Examples of sampling based planner
- our lab poseman robot project

Motion Primitive-based Algorithm

➤ Motion primitives-based planning algorithm

- ❑ Motion primitives-based path planning relies on a set of precomputed, feasible motions or maneuvers known as “motion primitives.”
- ❑ The core idea behind using motion primitives is to discretize the continuous control space into a finite set of actions that can be efficiently searched.
- ❑ The planner performs 1) Generating motion primitives, 2) Computing cost of primitives, and 3) selecting minimum cost motion primitive as planned path.

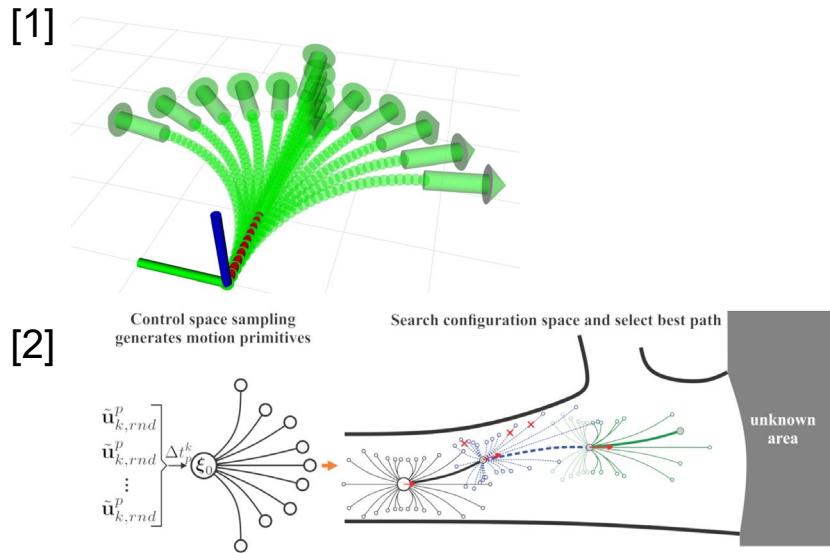


[1]: Werling, M., Ziegler, J., Kammel, S., & Thrun, S. (2010, May). Optimal trajectory generation for dynamic street scenarios in a frenet frame. In 2010 IEEE international conference on robotics and automation (pp. 987-993). IEEE.

[2]: Yoneda, K., Iida, T., Kim, T., Yanase, R., Aldibaja, M., & Suganuma, N. (2018). Trajectory optimization and state selection for urban automated driving. Artificial Life and Robotics, 23(4), 474-480.

Motion Primitive-based Algorithm

➤ Motion primitives-based planning algorithm



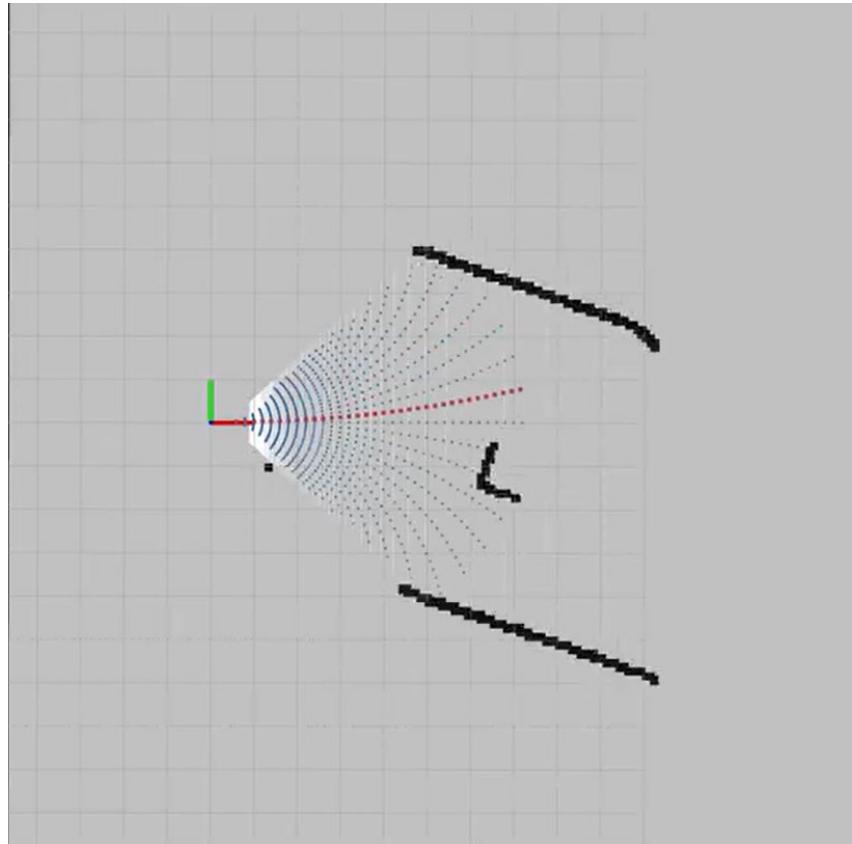
[1]: Scharff Willners, J., Gonzalez-Adell, D., Hernández, J. D., Pairet, È., & Petillot, Y. (2021). Online 3-dimensional path planning with kinematic constraints in unknown environments using hybrid A* with tree pruning. *Sensors*, 21(4), 1152.

[2]: Dharmadhikari, M., Dang, T., Solanka, L., Loje, J., Nguyen, H., Khedekar, N., & Alexis, K. (2020, May). Motion primitives-based path planning for fast and agile exploration using aerial robots. In 2020 IEEE International Conference on Robotics and Automation (ICRA) (pp. 179-185). IEEE.

[3]: Garrote, L., Premebida, C., Silva, M., & Nunes, U. (2014, July). An RRT-based navigation approach for mobile robots and automated vehicles. In 2014 12th IEEE International Conference on Industrial Informatics (INDIN) (pp. 326-331). IEEE.

Motion Primitives-based Algorithm

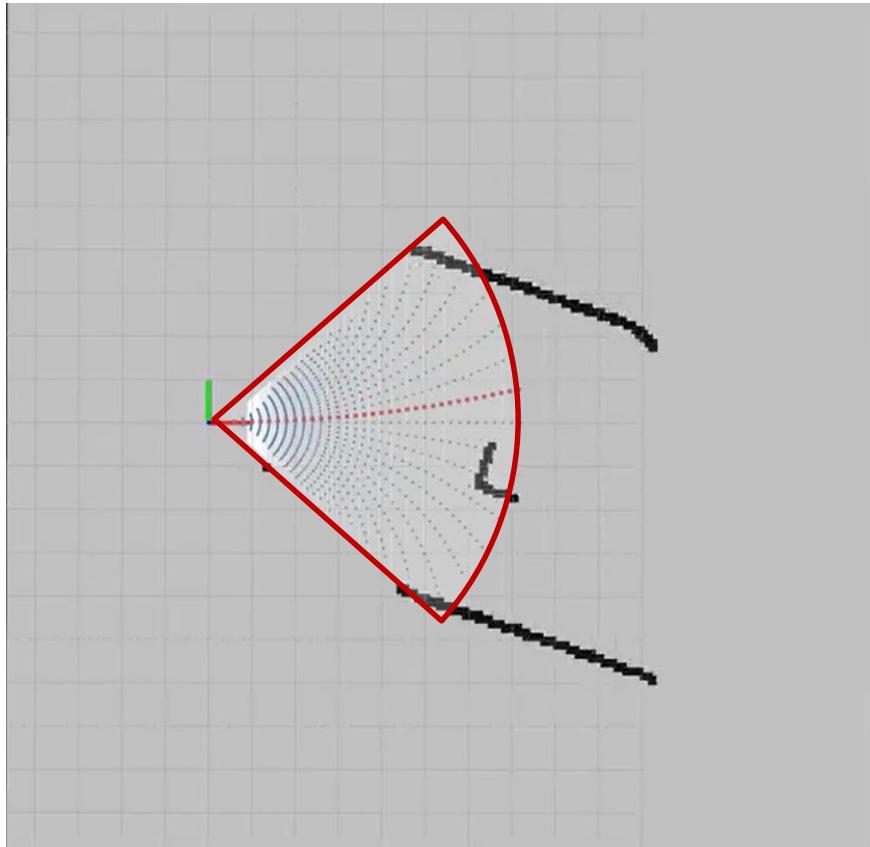
➤ Motion primitives-based planning algorithm



1. Generating motion primitives
2. Computing cost of primitives
3. Selecting minimum cost motion

Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm

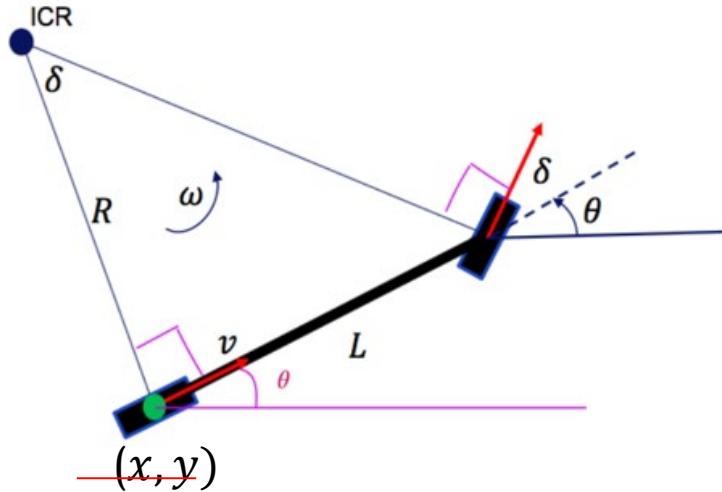


1. Generating motion primitives

- Radial motion primitives
- Kinematic model-based
- Dynamic model-based

Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm



✓ Velocity

$$\begin{aligned}\dot{x} &= v \cos \psi \\ \dot{y} &= v \sin \psi\end{aligned}$$

✓ Acceleration

$$\dot{v} = a$$

✓ Instantaneous Center of Rotation (ICR)

$$\tan \delta = \frac{L}{R} \quad v = R\omega = R\dot{\psi}$$

$$\Rightarrow \dot{\psi} = \frac{v}{L} \tan \delta$$



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \cos \psi \\ v \sin \psi \\ \frac{v}{L} \tan \delta \\ a \end{bmatrix}$$

State : $\{x, y, \psi, v\}$
Control input : $\{\delta, a\}$

x : position x
y : position y
ψ : yaw angle
v : velocity

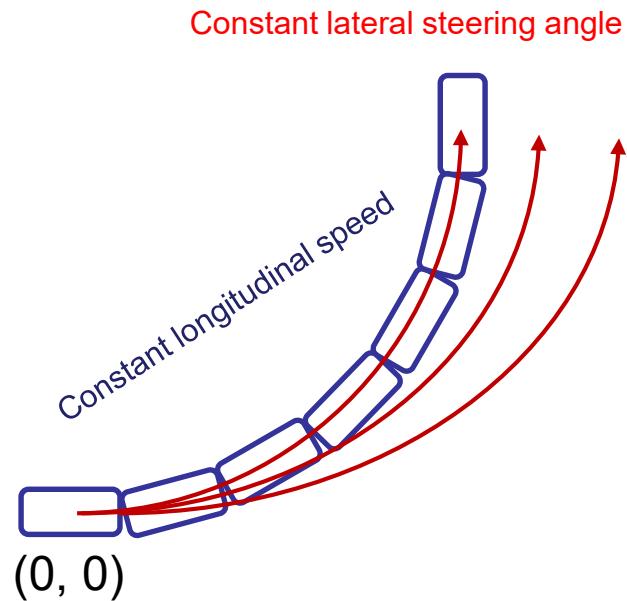
δ : steering angle
a : acceleration

Discrete model in code

```
x_(t+1) = x_t + x_dot * dt
y_(t+1) = y_t + y_dot * dt
ψ_(t+1) = ψ_t + ψ_dot * dt
v_(t+1) = v_t + v_dot * dt
```

Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm



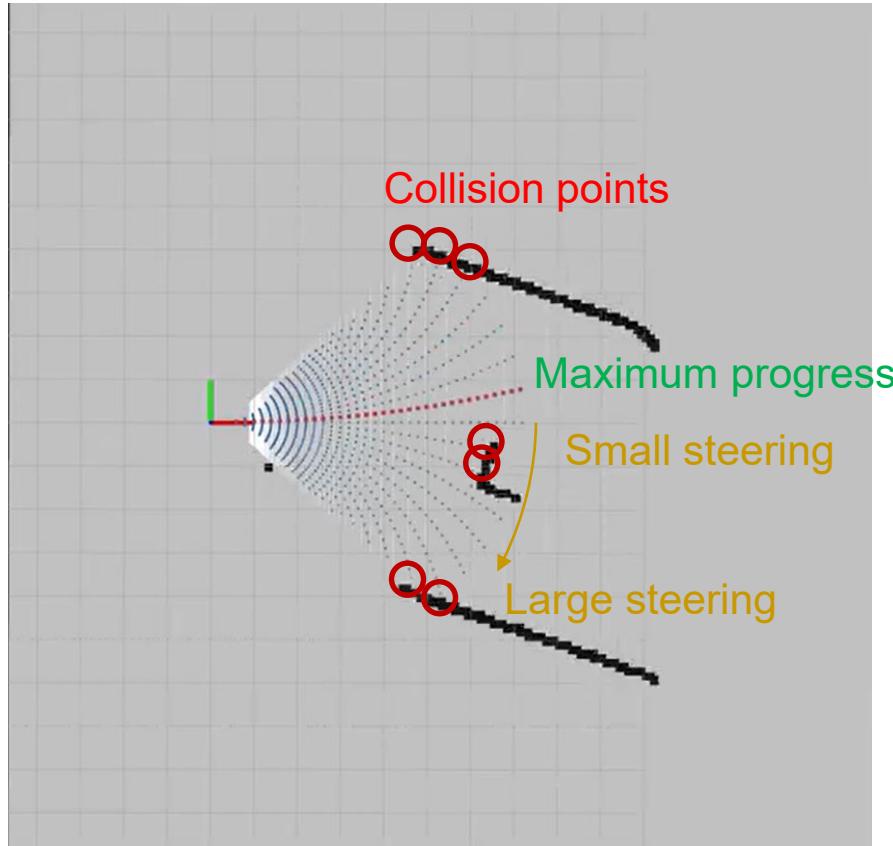
For loop (lateral domain, steering angle)

For loop (longitudinal domain, constant speed)

*Update & Append motions using
the vehicle model equations*

Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm

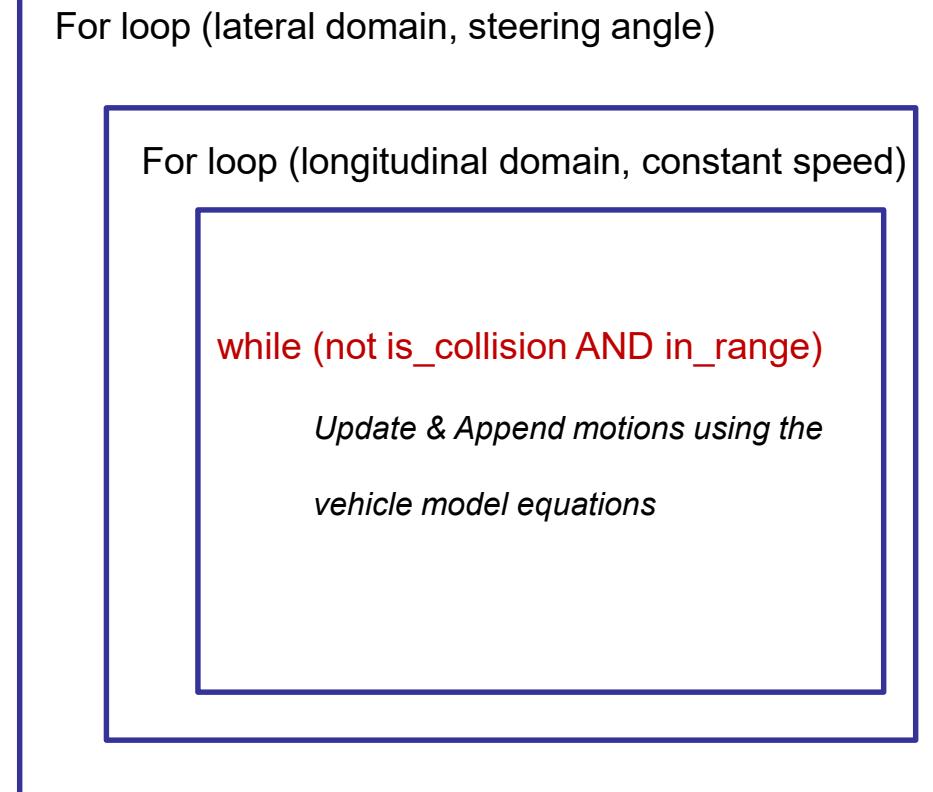
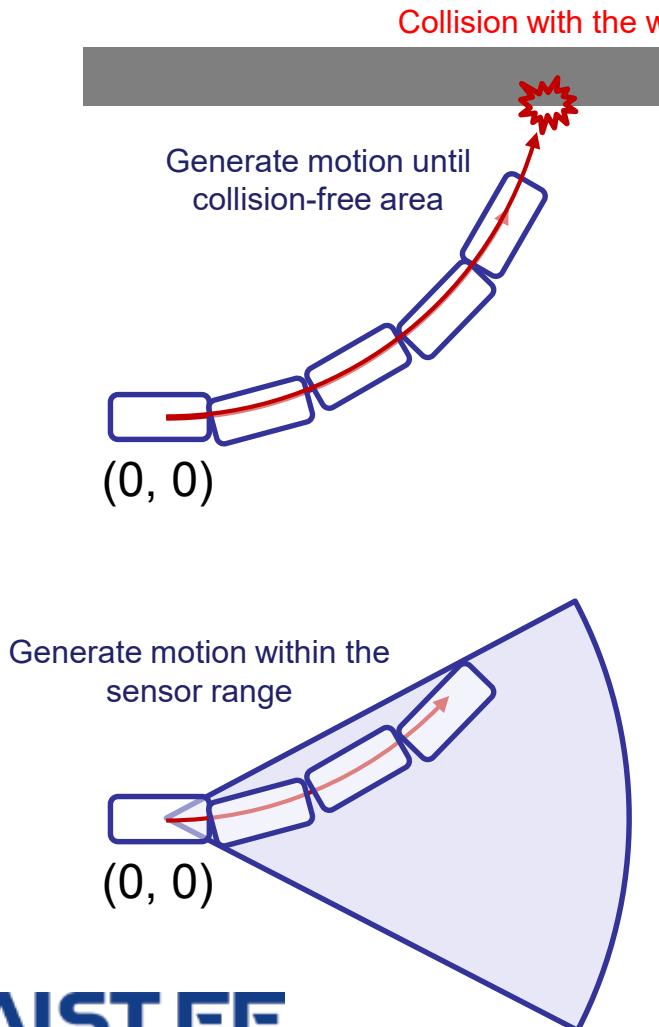


2. Computing cost of primitives

- Collision cost term C_{colli}
- Steering cost term C_{steer}
- Progress term C_{prog}
- ...

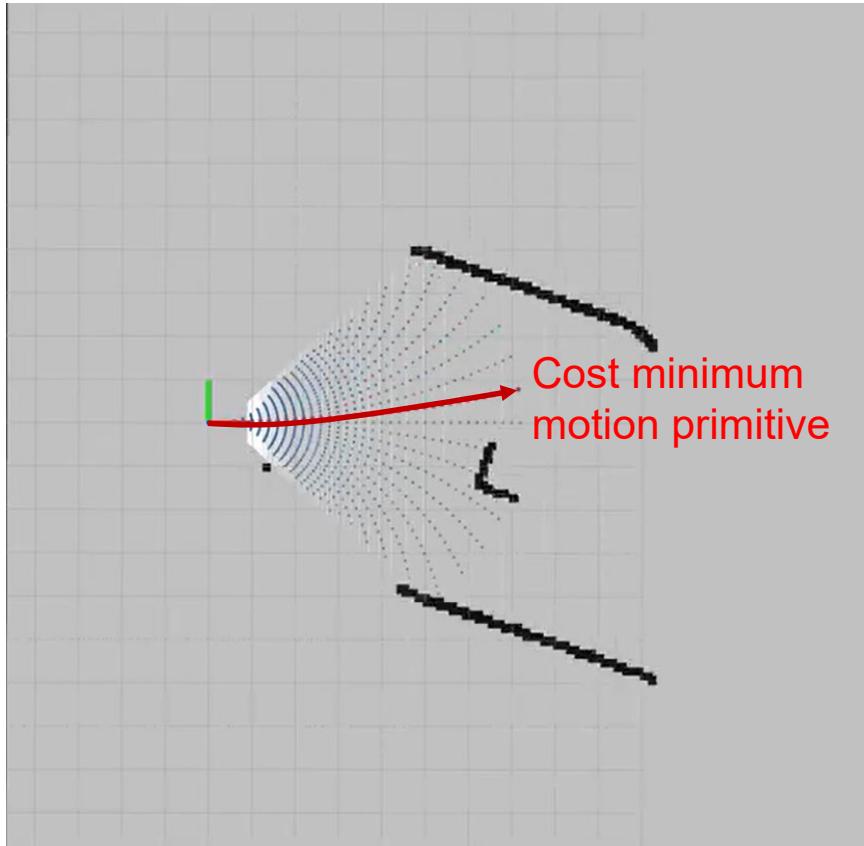
Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm



Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm



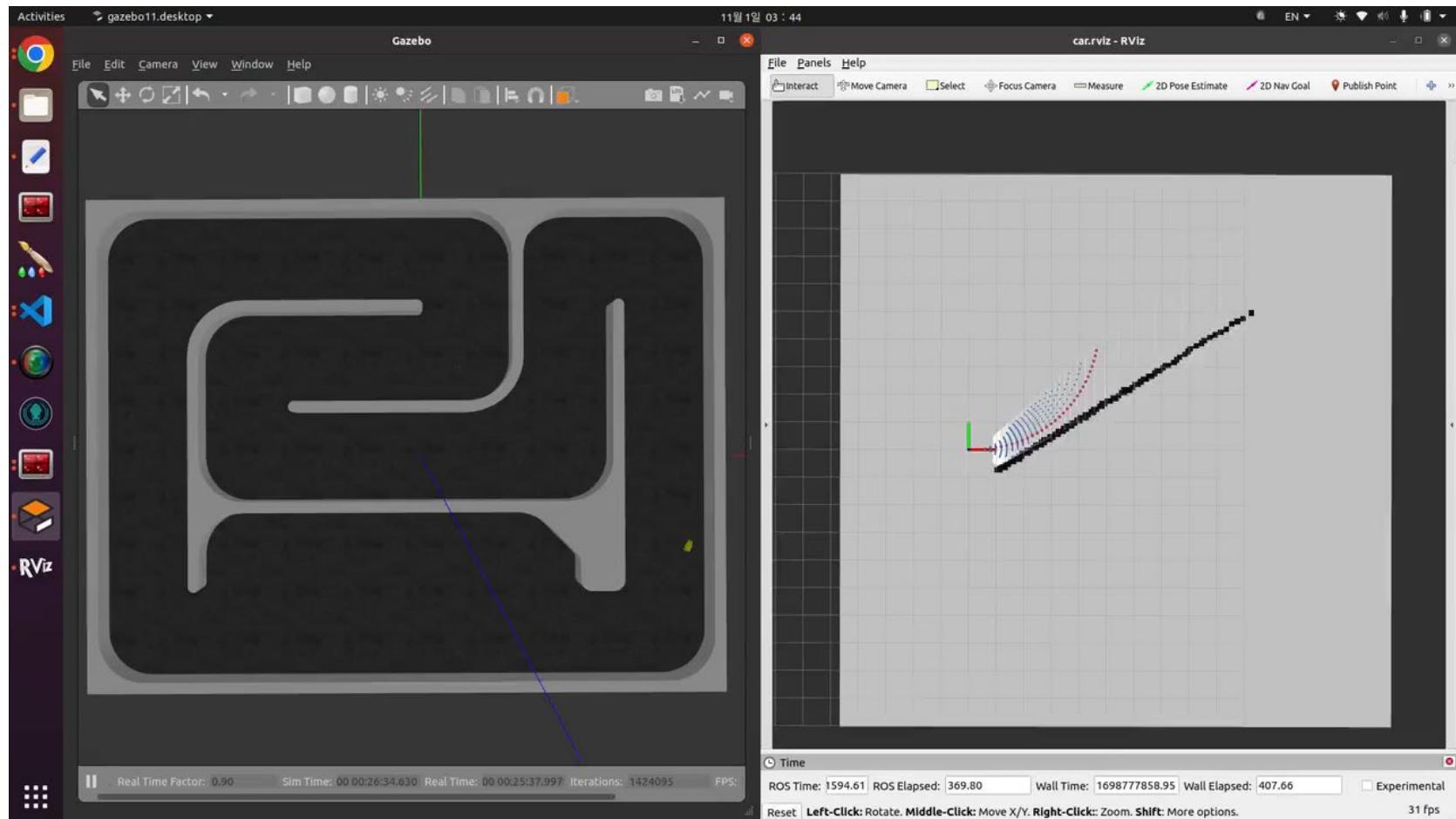
3. Selecting minimum cost motion

- Compute the weighted summation of the cost terms:
$$C_{total} = w_{colli} C_{colli} + w_{steer} C_{steer} + w_{prog} C_{prog} + w_X C_X$$
- Find the minimum cost motion primitive.

Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm

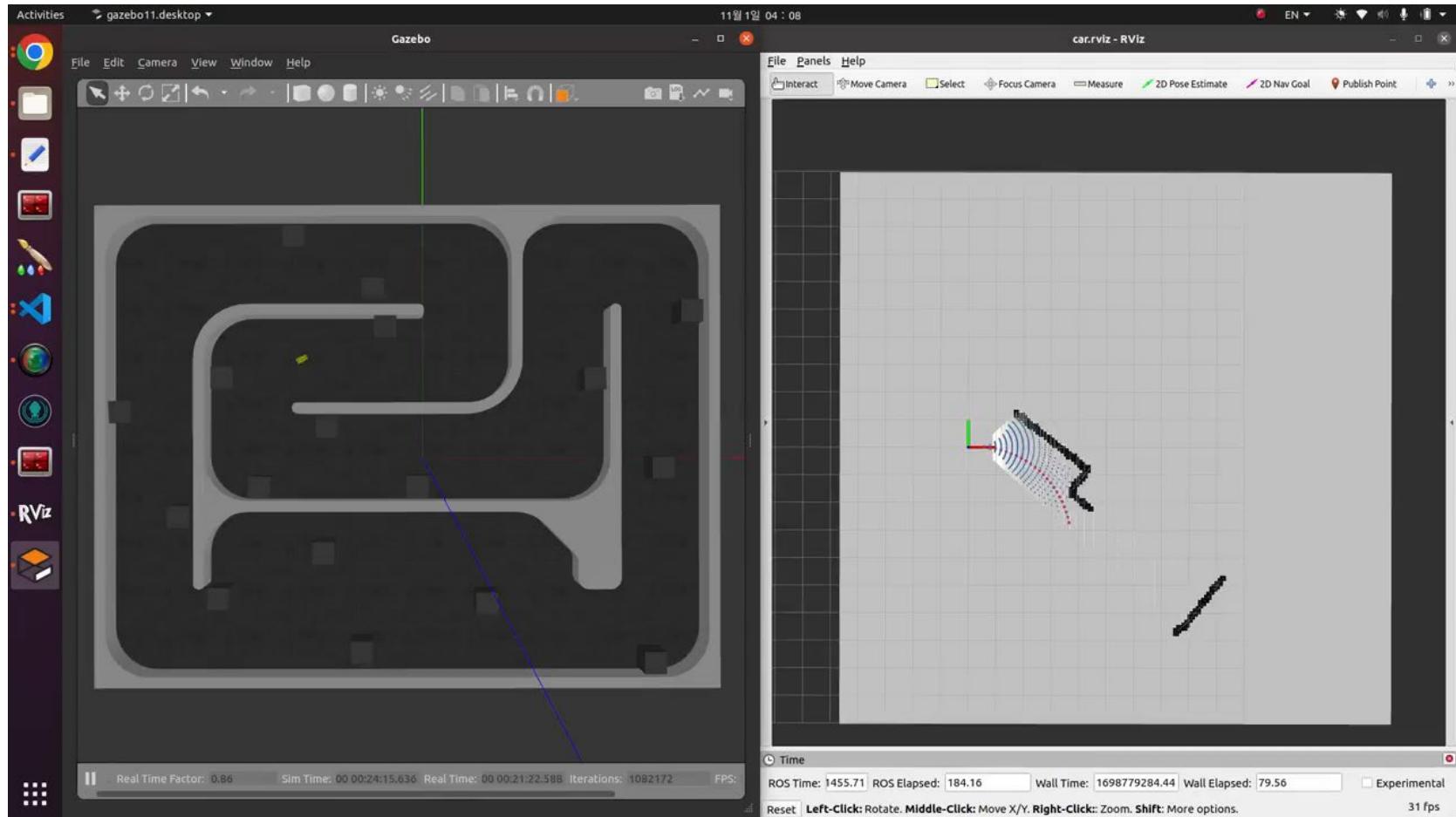
- ❑ Motion primitives-based planner will be provided as an assignment in next week.



Motion Primitives-based Algorithm

➤ Motion primitives-based planning algorithm

- ❑ Motion primitives-based planner will be provided as an assignment in next week.



Appendix

A* Algorithm

- **Evaluation Cost Function** $f(n) = g(n) + h(n)$
 - The value of $f(n)$ determines how “good” a path is.
- **Operating Cost Function** $g(n)$
 - Actual operating cost having been already traversed.
 - ✓ e.g.) Actual ‘distance’ to current location.
- **Heuristic Function** $h(n)$
 - Information used to find the promising node before traversing

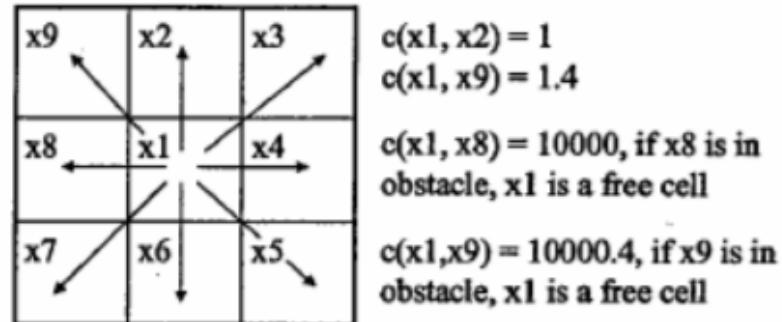
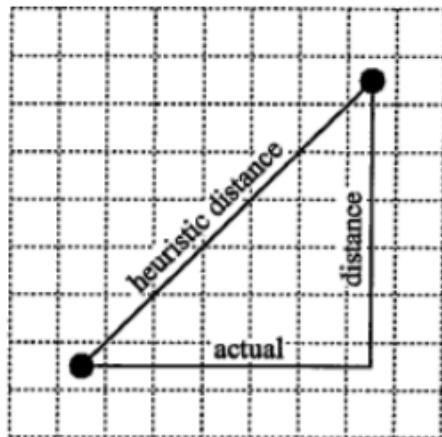


Fig. Cost on a grid

A* Algorithm

➤ 2 lists to store information:

- Open list(O): stores nodes for expansions.
- Closed list(C): stores nodes which we have explored.

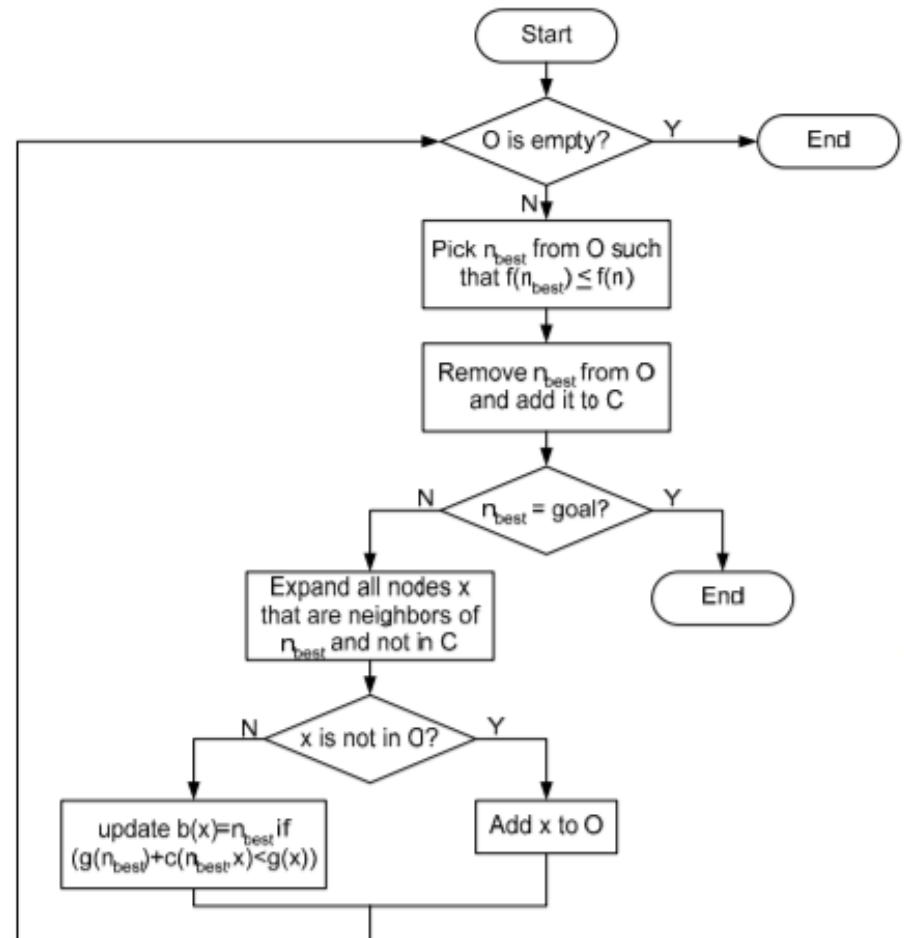
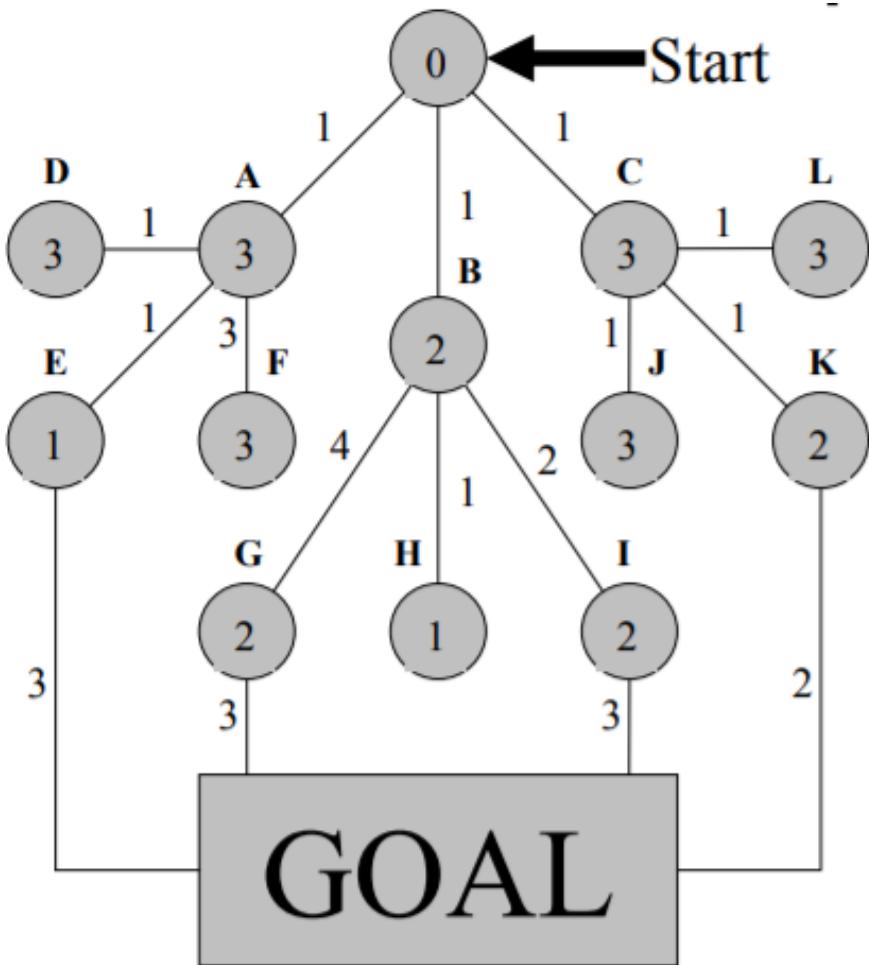


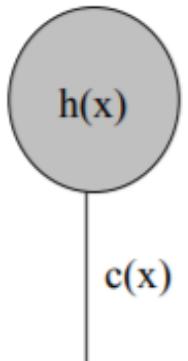
Fig. Flow chart of A* algorithm

A* Algorithm



➤ Legend

- ❑ $h(n)$: heuristic function that estimates the *cost-to-go*.
- ❑ $c(n)$: ‘distance’ from a node to the next node.
- ❑ $g(n)$: sum of all previous arc costs, $c(n)$, from start to n .

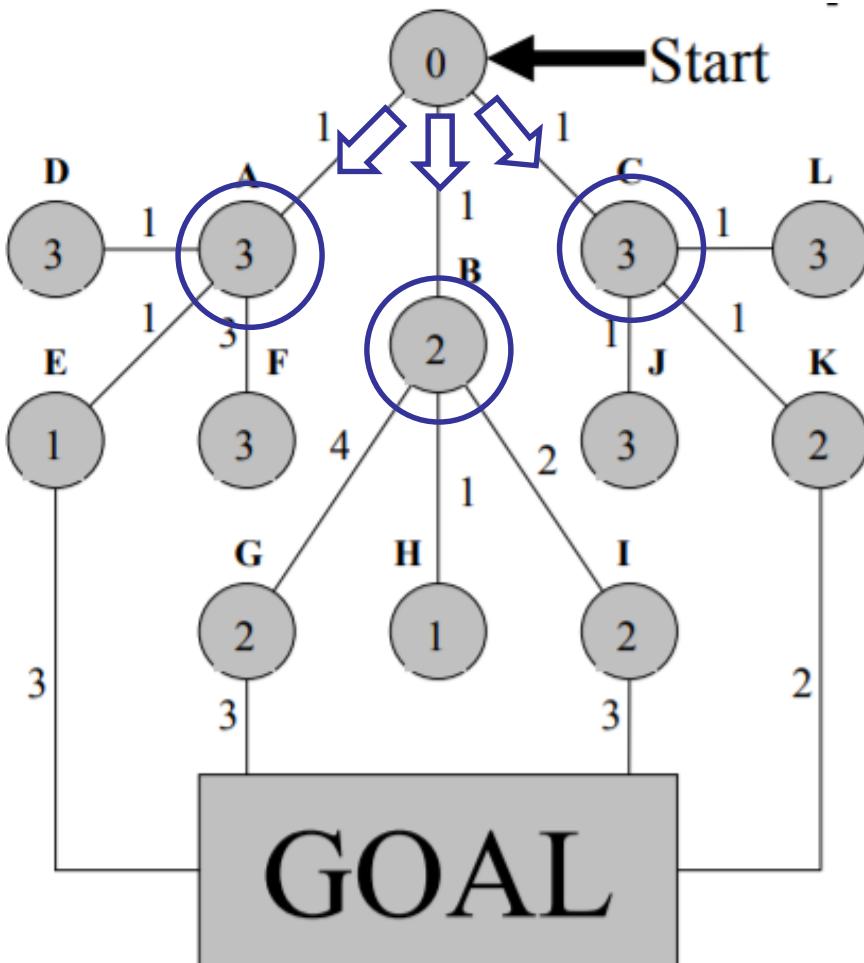


➤ Evaluation Cost Function

$$f(n) = g(n) + h(n)$$

- ❑ e.g.) $g(H) = 1 + 1, h(H) = 1$
 $\therefore f(H) = 3$

A* Algorithm

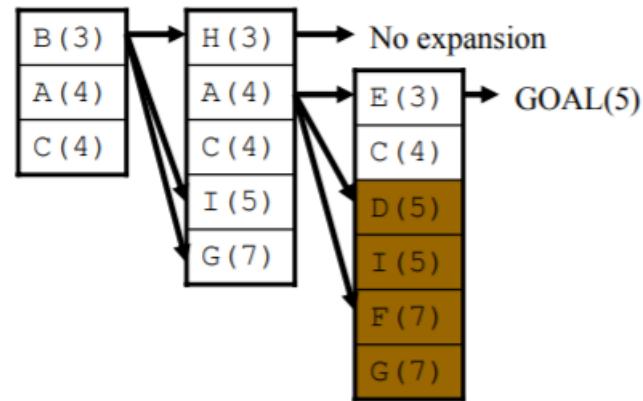
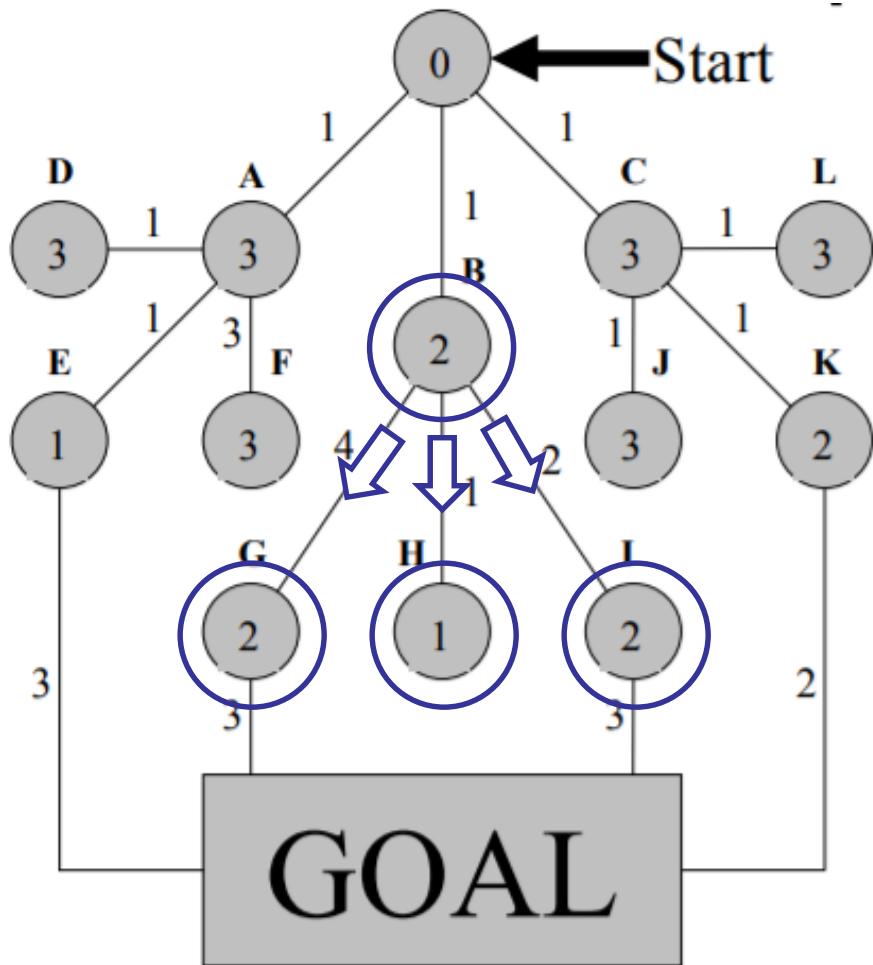


- **Expand the start node**
 - If the goal is not found, expand the first node in the priority queue(in this case, B).
- **Insert nodes to the priority queue**
 - Continue until the goal is found.
 - Or, until the priority queue is empty.
- **Keep a pointer to nodes' respective parents.**
 - e.g.) Nodes A, B and C point to *start*.

B (3)
A (4)
C (4)

H (3)
A (4)
C (4)
I (5)
G (7)

A* Algorithm

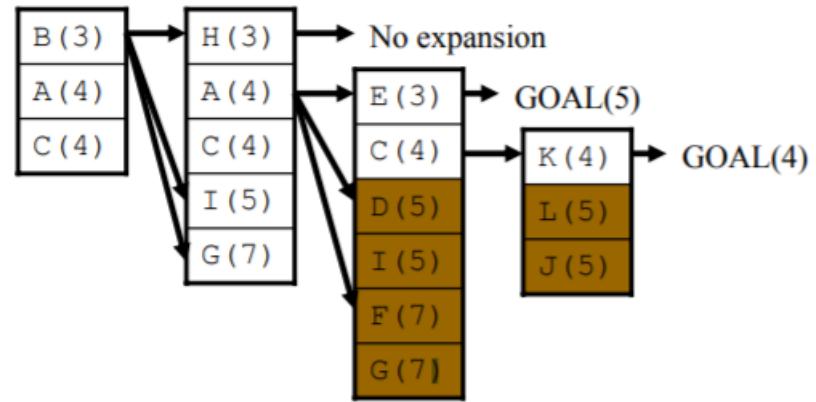
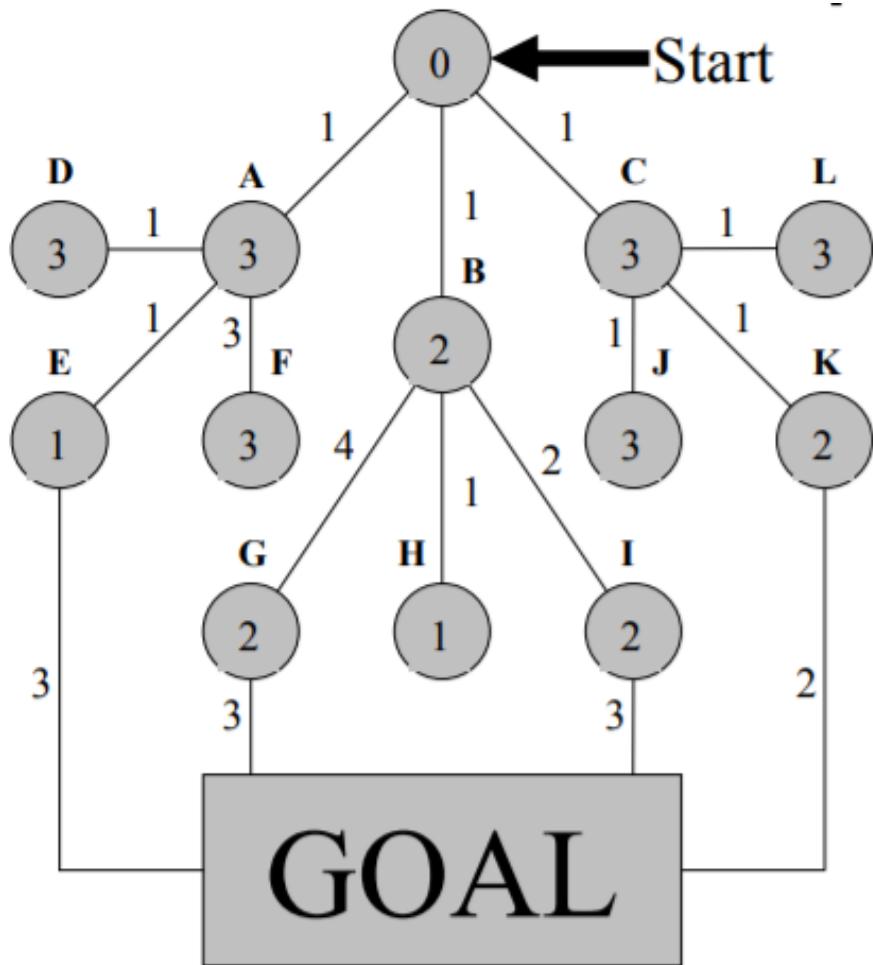


➤ Found a path to the goal

Start → A → E → Goal (cost 5)

- We can throw away nodes with $\text{cost}(\text{priority}) \geq 5$, because arc costs are assumed non-negative.

A* Algorithm



➤ Found a new path to the goal

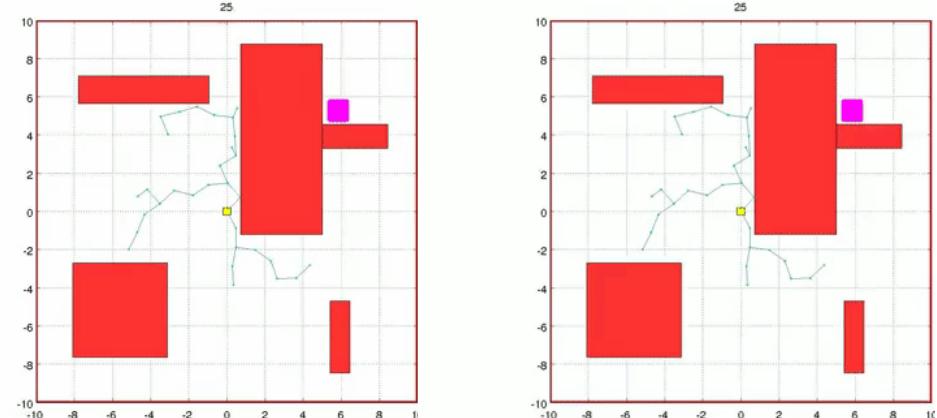
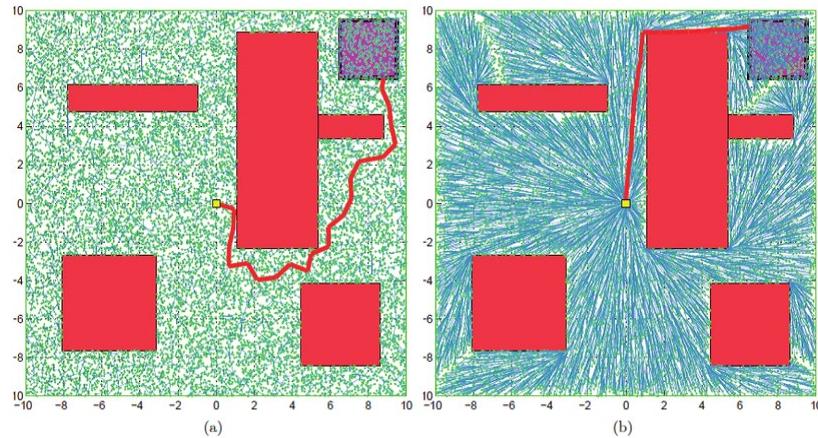
Start→C→K→Goal (cost 4)

- Repeating the process, we could find a shorter path through node K.
- If the priority queue still wasn't empty, we would continue expanding while throwing away nodes with cost higher than 4.

Rapidly exploring Random Tree (RRT)

➤ Rapidly-exploring Random Tree (RRT) [1]

- Rapidly build a space-filling tree toward unexplored collision-free region.
- Incremental tree construction from randomly sampled nodes.
- Inherently biased to grow toward large unsearched area.
- Efficiently search nonconvex, high-dimensional spaces.



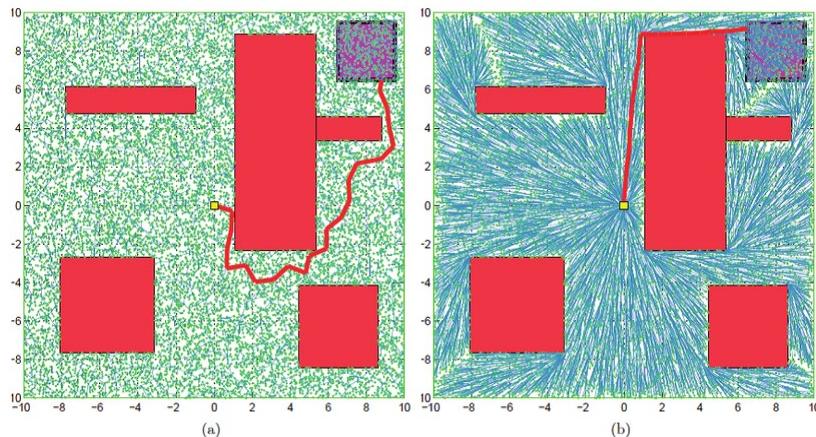
[1]: Rapidly-exploring Random Tree

[2]: Sampling-based Algorithms for Optimal Motion Planning

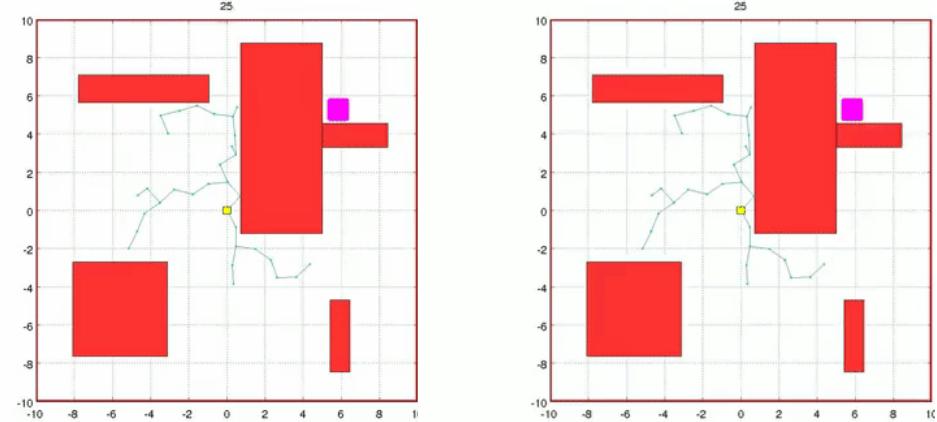
Rapidly exploring Random Tree (RRT)

➤ Sampling-based Algorithms for Optimal Motion Planning (RRT*) [2]

- Solution path from the RRT is not optimal (path is usually curvy, zig-zag, jerky).
- RRT* has the favorable property of an **almost sure convergence** to an optimal solution [2].
- Additional *Rewire*, *Parent* procedures for finding the optimal path are applied.
- RRT*'s optimality is theoretically proven as the number of samples approaches infinity.



The results of the RRT (Left) and RRT* (Right) algorithms



Exploring trees of the RRT (Left) and RRT* (Right) algorithms

[1]: Rapidly-exploring Random Tree

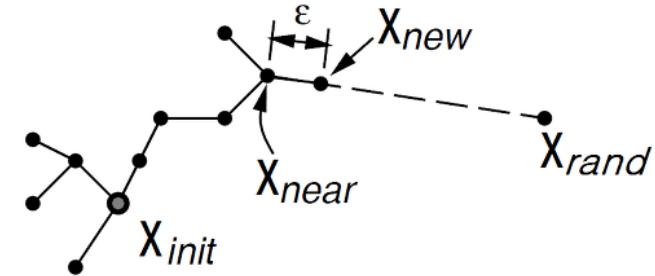
[2]: Sampling-based Algorithms for Optimal Motion Planning

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT [1]

Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```



Set an initial node x_{init} and maximum number of the loop K .

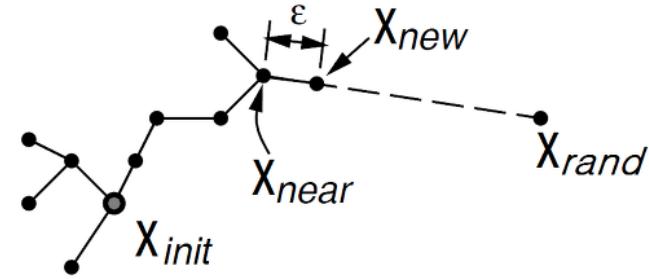
[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT [1]

Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$  ←
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```



Sample a random node x_{rand} in the collision-free space.

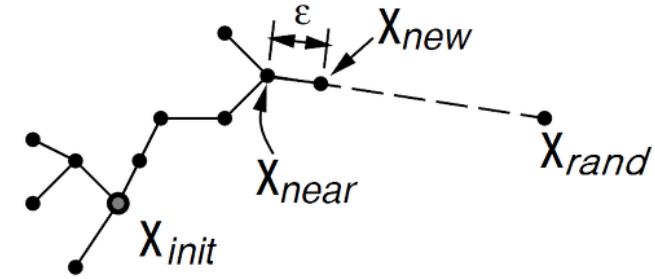
[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT [1]

Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$  ←
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```



Find a nearest node $x_{nearest}$ in the constructed tree with the randomly sampled node x_{rand} .

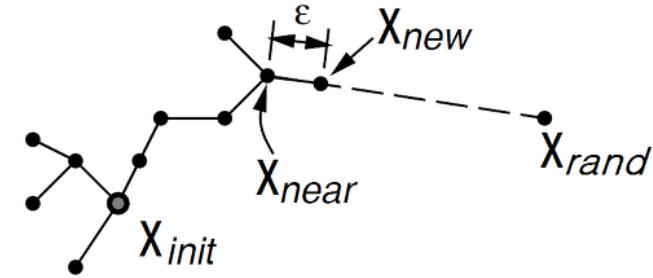
[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT [1]

Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$  ←
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```



Stretch the tree with an ε length from $x_{nearest}$ to x_{rand} and make a new node x_{new} .

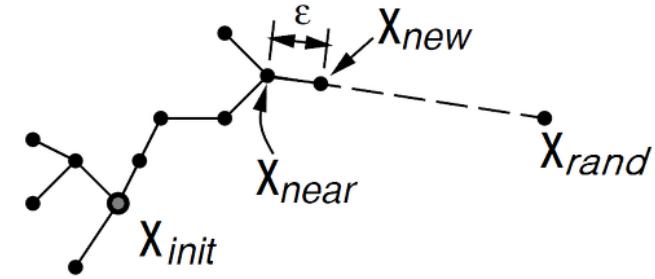
[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT [1]

Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```



If the edge $(x_{nearest}, x_{new})$ is in the obstacle-free region, add the x_{new} in the node set V and add the branch (edge) $(x_{nearest}, x_{new})$ in the edge set E .

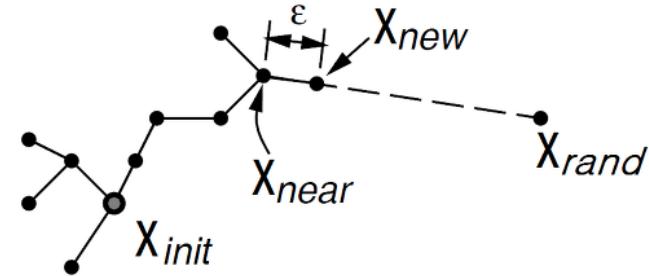
[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT [1]

Algorithm 1 RRT

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
8:      $V \leftarrow V \cup \{x_{new}\}$ 
9:      $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
10:  end if
11: end for
12: return  $(V, E)$ 
```



Repeat this process until finding the solution path.

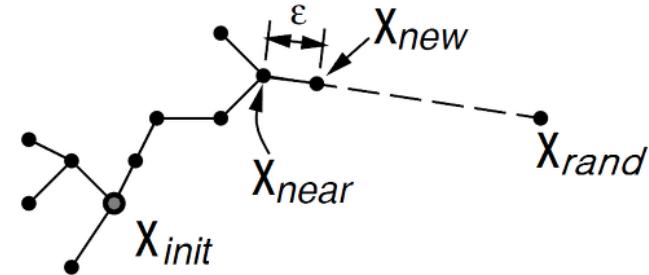
[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]

Algorithm 2 RRT*

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:    $V \leftarrow V \cup \{x_{new}\}$ 
8:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
9:      $X_{near} \leftarrow \text{Near}(V, x_{new})$ 
10:     $x_{nearest} \leftarrow \text{Parent}(X_{near}, x_{nearest}, x_{new})$ 
11:     $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
12:     $E \leftarrow \text{Rewire}(X_{near}, E, x_{new})$ 
13:   end if
14: end for
15: return  $(V, E)$ 
```



← Same processes with RRT.

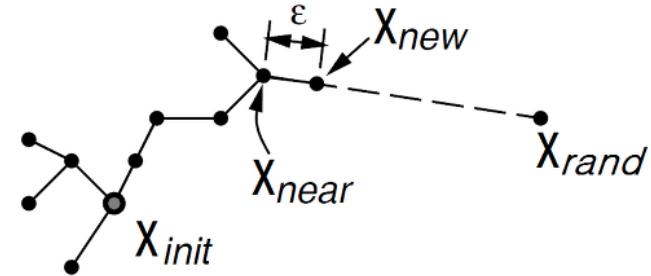
[1]: [An Overview of the Class of Rapidly-Exploring Random Trees](#)

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]

Algorithm 2 RRT*

```
1: Input: initial node  $x_{init}$ , max nodes  $K \leftarrow \mathbb{N}$ 
2:  $V \leftarrow \{x_{init}\}; E \leftarrow \emptyset;$ 
3: for  $i = 0$  to  $K$  do
4:    $x_{rand} \leftarrow \text{Sample}(i)$ 
5:    $x_{nearest} \leftarrow \text{Nearest}(V, x_{rand})$ 
6:    $x_{new} \leftarrow \text{Steer}(x_{nearest}, x_{rand})$ 
7:    $V \leftarrow V \cup \{x_{new}\}$ 
8:   if  $\text{ObstacleFree}(x_{nearest}, x_{new})$  then
9:      $X_{near} \leftarrow \text{Near}(V, x_{new})$ 
10:     $x_{nearest} \leftarrow \text{Parent}(X_{near}, x_{nearest}, x_{new})$ 
11:     $E \leftarrow E \cup \{(x_{nearest}, x_{new})\}$ 
12:     $E \leftarrow \text{Rewire}(X_{near}, E, x_{new})$ 
13:   end if
14: end for
15: return  $(V, E)$ 
```



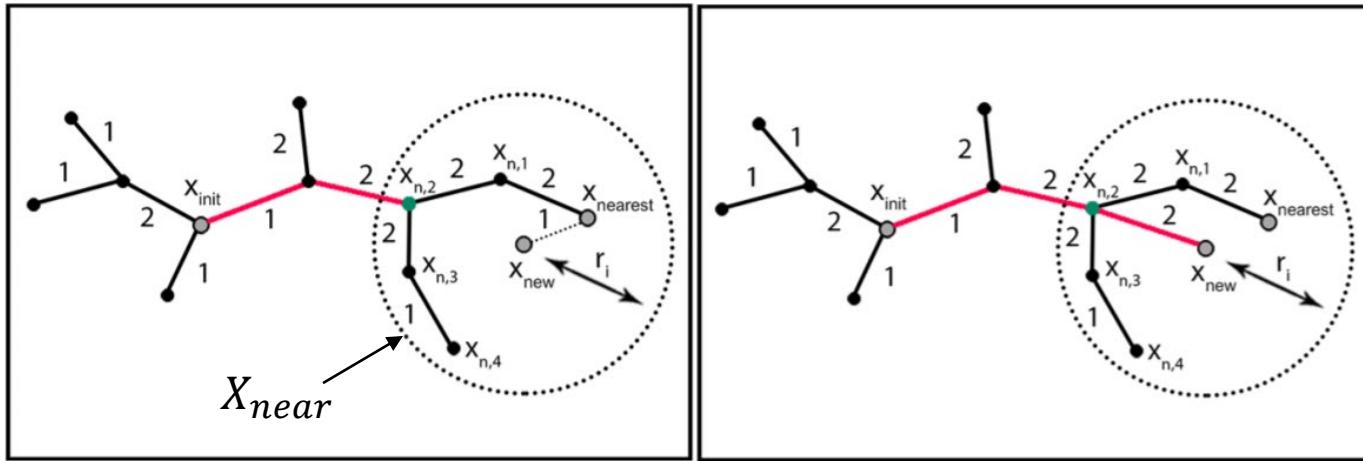
Parent process for finding new parent node of the x_{new} .

Rewire process for rewiring the branches of the tree to find a more optimal path.

[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]



Algorithm 3 Parent

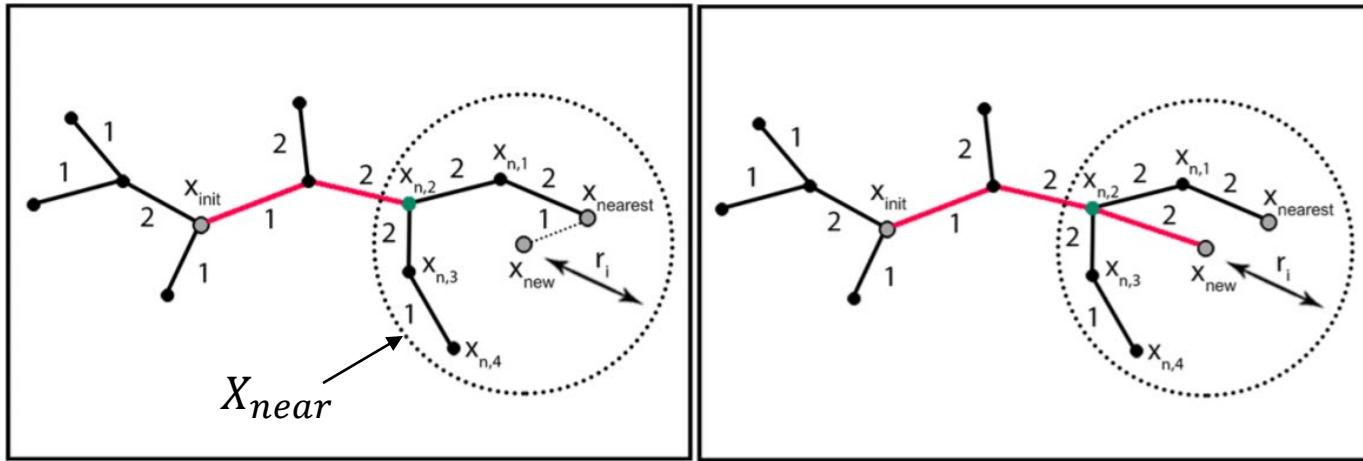
```
1: Input:  $X_{near}$ ,  $x_{nearest}$ ,  $x_{new}$ 
2: for  $x_{near} \in X_{near}$  do
3:   if  $\text{ObstacleFree}(x_{near}, x_{new})$  then
4:      $c' \leftarrow x_{near}.\text{cost} + c(\text{Line}(x_{near}, x_{new}))$ 
5:     if  $c' < x_{new}.\text{cost}$  then
6:        $x_{nearest} \leftarrow x_{near}$ 
7:     end if
8:   end if
9: end for
9: return  $x_{nearest}$ 
```

Parent process gets arguments
 X_{near} , $x_{nearest}$, x_{new} .
 X_{near} is a set of nodes in an area within the range of length r_i .

[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]



Algorithm 3 Parent

```
1: Input:  $X_{near}$ ,  $x_{nearest}$ ,  $x_{new}$ 
2: for  $x_{near} \in X_{near}$  do
3:   if  $\text{ObstacleFree}(x_{near}, x_{new})$  then
4:      $c' \leftarrow x_{near}.\text{cost} + c(\text{Line}(x_{near}, x_{new}))$ 
5:     if  $c' < x_{new}.\text{cost}$  then
6:        $x_{nearest} \leftarrow x_{near}$ 
7:     end if
8:   end if
9: end for
9: return  $x_{nearest}$ 
```



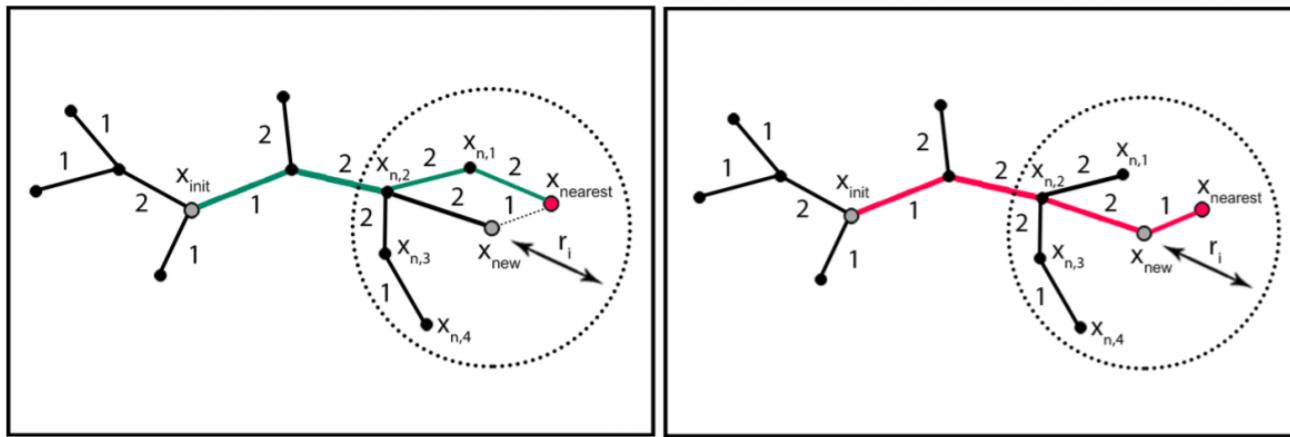
For each node x_{near} in X_{near} ,

- 1) check collision-free of the edge (x_{near}, x_{new}) .
- 2) If no collision, calculate the cost of the path with the edge.
- 3) If the cost is lower (more optimal) than the previous one, update the parent node of the x_{new} as x_{near} .

[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]



Algorithm 4 Rewire

```
1: Input:  $X_{near}$ ,  $E$ ,  $x_{new}$ 
2: for  $x_{near} \in X_{near}$  do
3:   if ObstacleFree( $x_{new}, x_{near}$ ) and  $x_{near}.cost > x_{new}.cost + c(\text{Line}(x_{near}, x_{new}))$  then
4:      $x_{parent} \leftarrow x_{near}.\text{parent}$ 
5:      $E \leftarrow E \setminus \{(x_{parent}, x_{near})\}$ 
6:      $E \leftarrow E \cup \{(x_{new}, x_{near})\}$ 
7:   end if
8: end for
9: return  $E$ 
```

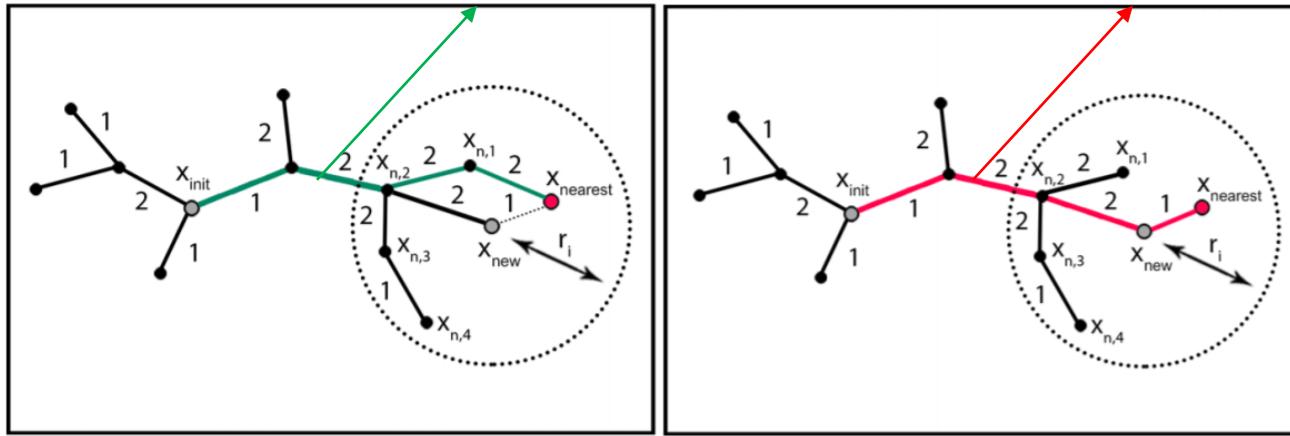
Rewire process get arguments
 X_{near} , E , x_{new} .

[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]

Path cost: $x_{near}.cost > x_{new}.cost + c(\text{Line}(x_{near}, x_{new}))$



Algorithm 4 Rewire

```

1: Input:  $X_{near}$ ,  $E$ ,  $x_{new}$ 
2: for  $x_{near} \in X_{near}$  do
3:   if ObstacleFree( $x_{new}, x_{near}$ ) and  $x_{near}.cost > x_{new}.cost + c(\text{Line}(x_{near}, x_{new}))$  then
4:      $x_{parent} \leftarrow x_{near}.\text{parent}$ 
5:      $E \leftarrow E \setminus \{(x_{parent}, x_{near})\}$ 
6:      $E \leftarrow E \cup \{(x_{new}, x_{near})\}$ 
7:   end if
8: end for
9: return  $E$ 

```

For each node x_{near} in X_{near} ,

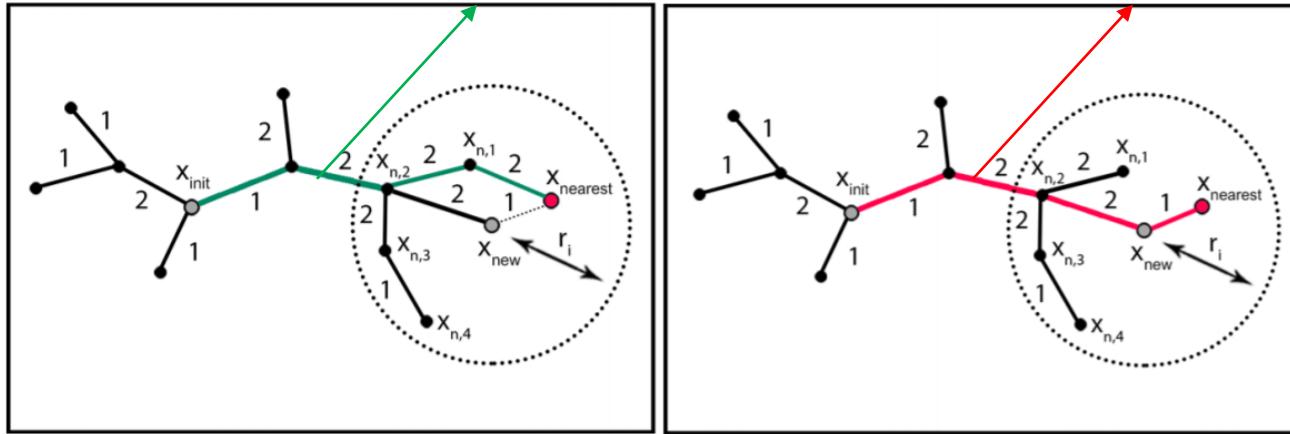
- 1) check collision-free of the edge (x_{near}, x_{new}) .
- 2) calculate the cost of the path with x_{near} and x_{new} .
- 3) if there is no collision and the cost is lower than the previous one, update the parent node of the x_{near} as x_{new} (rewire to the more optimal path).

[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Rapidly exploring Random Tree (RRT)

➤ Pseudo-code for RRT* [1]

Path cost: $x_{near}.cost > x_{new}.cost + c(\text{Line}(x_{near}, x_{new}))$



Algorithm 4 Rewire

```

1: Input:  $X_{near}$ ,  $E$ ,  $x_{new}$ 
2: for  $x_{near} \in X_{near}$  do
3:   if ObstacleFree( $x_{new}, x_{near}$ ) and  $x_{near}.cost > x_{new}.cost + c(\text{Line}(x_{near}, x_{new}))$  then
4:      $x_{parent} \leftarrow x_{near}.\text{parent}$ 
5:      $E \leftarrow E \setminus \{(x_{parent}, x_{near})\}$ 
6:      $E \leftarrow E \cup \{(x_{new}, x_{near})\}$ 
7:   end if
8: end for
9: return  $E$ 

```

← For each node x_{near} in X_{near} ,
 1) check collision-free of the edge (x_{near}, x_{new}) .
 2) calculate the cost of the path with x_{near} and x_{new} .
 3) if there is no collision and the cost is lower than the
 previous one, update the parent node of the x_{near} as x_{new}
 (rewire to the more optimal path).

[1]: An Overview of the Class of Rapidly-Exploring Random Trees

Q & A