Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

Einstein Pudolsky Rosen
Princeton, New Jersey | 1935

Context and Importance

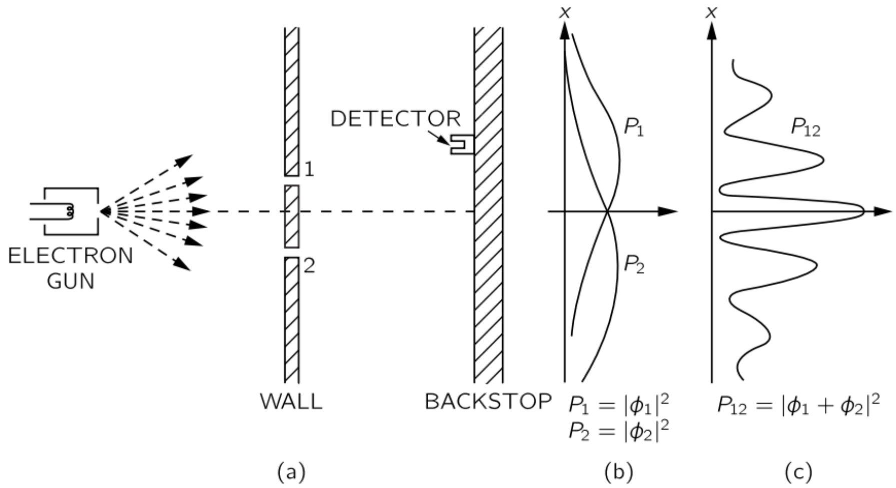
This paper refers to the concept of "non locality" in quantum mechanical description of objective reality. Since the introduction of Newtonian mechanics to describe gravitation, physicists have argued about the causality of the gravitational field between two massive objects (the instantaneous effect of gravity creates a paradox). This cause-effect relation was then resolved by Einstein's general theory of relativity which added the *locality** in the gravitational field. The same was then due for Copenhagen interpretation of quantum mechanics which was presented by Niels Bohr along with other physicists.

**Locality: Object is influenced by its immediate surroundings. Effects of a particular theory propagate through fields/particles with finite speed (less than speed of light). It is meant to avoid immediate "spooky action at a distance."*

Research Question / Core Problem

When the Copenhagen interpretation of quantum mechanics was introduced, it interpreted the wave and particle duality by providing wavefunction $\boldsymbol{\Psi}$ to describe state of a particle in a system. It also says that the moment anyone tried to measure a real value from that wavefunction $\boldsymbol{\Psi}$, it would collapse into a particular observed value.
Imagine a double slit experiment where an electron is released from a laser and that electron propagates through space until it hits a photo-detection plate at the other end of the apparatus. From the moment electron was released to until it hits the photo-detection plate, the description of electron is given by wavefunction $\boldsymbol{\Psi}$, which describes the probability amplitude of finding that electron different point in space at any time t. The moment an electron hits the photo-detection plate, we get a glowing point on the plate that depicts the position of the electron after it's measured, hence its wavefunction collapses at that instant.
Now one can argue that if the wavefunction collapses after measurement at a particular point in space, what happens to the points in space where electrons could have landed as per probability distribution. Theory states that the collapse happens for the entire region of wavefunction, even if it extends to a large distance. This collapse at a different point in space happened instantaneously after the measurement and without any clear causality (cause-effect link), hence this can be considered as a non local event, which Einstein used to call "spooky action at a distance".
The question: **Is the wavefunction of a particle's states as described by quantum mechanics considered to be Correct* and Complete*?**

**Correctness of theory is judged by degree of agreement between conclusions of theory and human experience. Human experience refers to the measurements and experiments conducted.*
**Completeness: Every element of physical reality must have a counterpart in physical theory.*
**An element of physical reality is something whose value can be predicted accurately without disturbing the system.*



Method / Approach

The paper first challenges the completeness of Quantum Mechanics.
Assume a particle with wavefunction $\boldsymbol{\Psi}$, chosen to describe behaviour of a particle. It is a function of a variable (assume p momentum) chosen to describe particle behaviour.
In computational terms, there's a *hermitian operator** available to measure an observable quantity in wavefunction.

$$\psi' \equiv A\psi = a\psi,$$

Here A is the operator, a is the eigenvalue of operator and $\boldsymbol{\Psi}$ is the eigenstate of particle (wavefunction).

**Hermitian Operator: It is a linear transformation applied to an eigenvector (eigenstate) which changes the scale of the vector and does not affect the shape. And gives a real eigenvalue (measurement) after the transformation.*

Example:
Take a wavefunction of a particle with constant momentum p_0 .

$$\psi = e^{(2\pi i/h)\, p_0 x},$$

Apply momentum measurement operator $\hat{P}$ and result is measured out to be p_0 (real value)

$$p = (h/2\pi i)\partial/\partial x, \quad \textit{(momentum operator)}$$

$$\psi' = p\psi = (h/2\pi i)\partial\psi/\partial x = p_0\psi.$$

Try to measure the position (x) of particle with $\hat{x}$ operator. This operator simply involves multiplying the state with x ($\hat{x}\,\psi \;=\; x\,.\,\psi \;\neq p_0\,.\,\psi$). Because $\psi \;=\; e^{i.p_0 x/\hbar}$, so position cannot be measured without disturbing the state. Position x is variable, so any value will distort the wavefunction (change its shape). Alternative is to find the probability density ($|\psi|^2 \;=\; 1$) of measuring the location of the particle between a and b position (probability density calculation is allowed by quantum mechanics).

$$P(a, b) = \int_a^b \bar{\psi}\psi dx = \int_a^b dx = b - a.$$

b-a tells that the probability is equally likely at all points on the graph. In practical terms, if we know the momentum of a particle then its position will be a smudgy line of dots on a photo-detection plate. So the position is still uncertain or can also say position does not have a real value even though it's part of the wavefunction equation.

It confirms the Heisenberg's uncertainty principle:

$$: \Delta x \cdot \Delta p \geq \frac{\hbar}{2},$$

In our wavefunction equation, momentum was constant hence position was uncertain.
Hence $\hat{p}\,.\,\hat{x} \;\neq\; \hat{x}\,.\,\hat{p}$, it's called **non-commuting** operators. Which means:

- Incompatible measurements
- No shared eigenstates
- Order specific ($\hat{p}\,.\,\hat{x} \;\neq\; \hat{x}\,.\,\hat{p}$)
- Non zero ($\hat{p}\,.\,\hat{x} \;-\; \hat{x}\,.\,\hat{p} \;\neq\; 0$)

So now we have come to two possibilities:

1. Either quantum mechanical description of wavefunction is not complete because as we proved above if p is measured then x is not a real value (eigen value). **Quantum mechanics is not complete.**
2. Or when operators (like $\hat{p}\; or\; \hat{x}$) do not commute which means two quantities can't have simultaneous reality. It's a condition for **Quantum mechanics considered to be complete (Copenhagen Interpretation).**

In Quantum Mechanics it is assumed that wavefunction **does** contain the complete description of reality of state of a system. EPR considers this assumption to be reasonable and continues to prove that information obtainable from a wavefunction without disturbing the system will eventually contradict the statement (2).
Think of it like this, EPR already proved statement (1), that the theory is not complete because both values can't be measured to a real value (realism), so that would mean statement (2) is right. But here's the catch, Einstein takes the assumption that quantum mechanics is complete (gives benefit of doubt to

quantum mechanics) and tries to measure both values **without disturbing the system** to prove they were real all along. This would disprove the statement (2) which would prove him right that the quantum mechanical theory is not complete.

Now back to the experiment table,

Imagine two particles (at rest, p=0), let's say electrons, (with known state ψ for each particle) interacting with each other (blasted apart by a split of some bigger particle) for a particular time T. After that time T, the particles stop interacting but their individual states are now unknown but we can get their combined states into a wavefunction using Schrodinger's equation. But quantum mechanics does allow us to find out the state of each particle we need to do measurements without disturbing the system. These measurements would be two different operators we already discussed above $\hat{p}$ *and* $\hat{x}$ (momentum and position).

EPR assumes that there is a system of two particles. System I and System II. They are interacting with each other from time t=0 to t=T. Now with Schrodinger's equation we can find the combined wavefunction of System I+II and write it as ψ for any t>T.

We can't calculate the state of each system individually after the interaction is over. As per quantum mechanics, this can be done by further measurement of state, by the process called **reduction of wave packet**.

Below expression explains how $\psi(x_1, x_2)$ is a wavefunction of particle I and II's combined eigenstates (all possible states of each particle). Here x_i is just a variable used to describe each particle's system. The following infinite series is called a wave **packet**.

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1),$$

Now let's assume we measured the momentum of particle I with $\hat{p}$ (**reduction of wave packet**) the infinite series will be left to a single term

$$\psi_k(x_2) u_k(x_1)$$

The above wave packet was describing a particular physical quantity (say momentum), but if we described the combined state by a different physical quantity (position) then we would have a different wavefunction description.

$$\Psi(x_1, x_2) = \sum_{s=1}^{\infty} \varphi_s(x_2) v_s(x_1),$$

Now measure the position $\hat{x}$ on this wavefunction and we get reduced wave packets as:

$$\varphi_r(x_2)\ v_r(x_1)$$

EPR takes into consideration that if momentum is measured then wavefunction collapses so they consider that if instead of momentum someone decided to measure position, they take separate wavefunction for that. **So don't misinterpret it as both measurements are made together or on the same wavefunction**. It's an "either or" situation where EPR says that an observer could have measured either P or X.

Previous depiction of wavefunction was an abstract but if we talk about particles (electrons) then it can also be written as:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1 - x_2 + x_0)p} dp,$$

x_1 is the position of particle I and x_2 is position of particle II and x_0 is the constant distance between the two particles after the interaction is over. $x_0 = x_2 - x_1$. Particles are assumed to be entangled by the distance between them which is constant.

This is a mathematical form where integral solves out to dirac delta function. For integral to be non zero (to ensure dirac delta is not infinity) the exponential of x must equal to zero, so $x_0 = x_2 - x_1$ is re-written as $x_1 + x_0 - x_2 = 0$.

**When integrating, if a wave is spread out, the integral with dirac delta function helps remove wave wiggles outside the limits of the integral, which helps find a spike in the wave.*

Einstein assumes a possibility where the momentum of two is conserved because they started from p=0. Hence their relative momentum will be equal but opposite so their **relative** position could also be known. He is making the phase angle of two waves to be zero so that they perfectly coincide and then dirac delta function and point to the spike of position of the particle.

Now measure the momentum of particle I, which gives the following eigenstate and its real value comes out to **p**.

$$u_p(x_1) = e^{(2\pi i/h)px_1}$$

The momentum eigenstate for particle II, is

$$\psi_p(x_2) = e^{-(2\pi i/h)(x_2 - x_0)p}. \quad \textit{because of:} \quad P = (h/2\pi i)\partial/\partial x_2,$$

This brings the momentum of particle II to be -p, which confirms the conservation of momentum for two particles.

Now $\hat{x}$ measurement on second wavefunction (position) we discussed above:

$$v_x(x_1) = \delta(x_1 - x)$$

This shows that there's a spike of measurement for particle I as x, which gives a high probability for it to be measured at x. So now we plug this position value back in wavefunction of particle II with respect to conserved momentum

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1 - x_2 + x_0)p} dp$$

Replacing x_1 $with\ x$ and then making dirac delta measurement of this:

$$h\delta(x - x_2 + x_0).$$

$x_2 = x + x_0$ is the final measurement.
This disproves the Heisenberg's uncertainty in terms of measurement:

$$PQ - QP = h/2\pi i,$$

We were able to measure both P (momentum) and Q (position) of particle II without disturbing its system. And both exist in simultaneous reality.

So there were only two possibilities we discussed earlier:

1. Either the wavefunction description given by quantum mechanics is not complete.
2. Or else, if it's complete then the two measurements cannot be simultaneously real.

EPR started with the assumption that quantum mechanical wavefunction description is complete and then disproved statement (2) by showing both real measurements.
Although they did acknowledge that one won't reach the same conclusion if they insisted that both the measurements **must be done simultaneously** and must be a real measurement. This sets a requirement that the realism of the value of p and x is dependent upon the process of measurement done on particle I which does not disturb particle II in any way.
Einstein is insisting that there's some hidden variable in particles that the wavefunction description is missing out on.

Key Result / Main Argument

By this thought experiment, they wanted to prove that the wavefunction description of a system is not complete. They wanted to introduce locality in quantum mechanics by introducing some hidden variables which are currently unknown. The particle II always had position and momentum values, it did not depend on the state of existence of particle I.

Insight / Critique

- Einstein was concerned about two things, the probabilistic nature of quantum mechanics and quantum entanglement. In this paper he addressed the entanglement issue by introducing locality effects to this theory. He was concerned that no distant particle should be impacted without any influence from its immediate surroundings.
- One thing that was unique about this paper was that Einstein subtly introduced the concept of entangled particles in 1935, at this time such a state of elements was an unknown territory because entanglement was not formally introduced in any paper.
- It was assumed that at the time of entangled pair creation the particles would exchange information and would keep that information as it is until measured, these were referred as hidden variables. It was later disproved by Bell's theorem. Quantum mechanics was correct in a way that it would match with the experimental results but it didn't account for causality of "spooky action at a distance"

Reference

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