Midpoint Ellipse Algorithm

1. Input r_x, r_y, and ellipse center (x_c, y_c), and obtain the first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial parameter in region 1 as

$$p1_0 = r_y^2 - r_x^2 r_y + \tfrac{1}{4} r_x^2$$

3. At each x_i position, starting at $i = 0$, if $p1_i < 0$, the next point along the ellipse centered on (0, 0) is $(x_i + 1, y_i)$ and

$$p1_{i+1} = p1_i + 2r_y^2 x_{i+1} + r_y^2$$

otherwise, the next point is $(x_i + 1, y_i - 1)$ and

$$p1_{i+1} = p1_i + 2r_y^2 x_{i+1} - 2r_x^2 y_{i+1} + r_y^2$$

and continue until $2r_y^2 x \geq 2r_x^2 y$

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4. (x_0, y_0) is the last position calculated in region 1. Calculate the initial parameter in region 2 as

$$p2_0 = r_y^2(x_0 + \tfrac{1}{2})^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$

5. At each y_i position, starting at $i = 0$, if $p2_i > 0$, the next point along the ellipse centered on (0, 0) is $(x_i, y_i - 1)$ and

$$p2_{i+1} = p2_i - 2r_x^2 y_{i+1} + r_x^2$$

   otherwise, the next point is $(x_i + 1, y_i - 1)$ and

$$p2_{i+1} = p2_i + 2r_y^2 x_{i+1} - 2r_x^2 y_{i+1} + r_x^2$$

   Use the same incremental calculations as in region 1. Continue until $y = 0$.

6. For both regions determine symmetry points in the other three quadrants.

7. Move each calculated pixel position (x, y) onto the elliptical path centered on (x_c, y_c) and plot the coordinate values

$$x = x + x_c, \qquad y = y + y_c$$

Example

- Given input ellipse parameters r_x =8 and r_y = 6, we illustrate the steps in the midpoint ellipse algorithm by determining raster positions along the ellipse path in the first quadrant.
- Initial values and increments for the decision parameter calculations are

$$2r_y^2 x = 0 \qquad \text{(with increment } 2r_y^2 = 72\text{)}$$
$$2r_x^2 y = 2r_x^2 r_y \qquad \text{(with increment } -2r_x^2 = -128\text{)}$$

Example

- For region 1, the initial point for the ellipse centered on the origin is $(x_0, y_0) = (0, 6)$, and the initial decision parameter value is

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 = -332$$

- Successive midpoint decision parameter values and the pixel positions along the ellipse are listed in the following table.

Example

k	$p1_k$	x_{k+1}, y_{k+1}	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-332	(1,6)	72	768
1	-224	(2,6)	144	768
2	-44	(3,6)	216	768
3	208	(4,5)	288	640
4	-108	(5,5)	360	640
5	288	(6,4)	432	512
6	244	(7,3)	504	384

Example

- We now move out of region 1, since

$$2r_y^2 x > 2r_x^2 y.$$

- For region 2, the initial point is

$$(x_0, y_0) = (7, 3)$$

- and the initial decision parameter is

$$p2_0 = f_{\text{ellipse}}\left(7 + \frac{1}{2}, 2\right) = -23$$

Example

- The remaining positions along the ellipse path in the first quadrant are then calculated as

k	**$p1_k$**	**(x_{k+1}, y_{k+1})**
0	-23	(8,2)
1	361	(8,1)
2	489	(8,0)

Example

- A plot of the calculated positions for the ellipse within the first quadrant is shown bellow: