

We will now build a very simple model for wildfire propagation, so simple indeed that we don't need to concern ourselves with its correctness; it is not correct.

We will model our world as a two dimensional grid made up of discrete cells with their own local state. We will then model the computation as an evolution rule, defined once for all grid cells. This rule may request the state of any cell in the previous generation.

To keep this challenging, we require that the grid be infinitely large, requiring the user to explicitly state the bounds of their computation. At the boundary of said computation the evolution rule will have to stop requesting information of adjacent cells. Without such a requirement, the number of grid cells to compute would exponentially grow as further generations ask about more and more cells. It's impossible to say what this growth factor would be without inspecting the evolution rule itself, as it will be free to request the state of any neighbouring cells from  $generation_{n-1}$ , which will need to be lazily computed and may in turn request information from  $generation_{n-2}$ , and so on.

To make such a computation feasible, we will require that the evolution rule for the system at  $generation_n$  be deterministic with respect to  $generation_{n-1}$ . This allows us to memoise previous generations, and not have to reconstruct the history of the universe at every step.

A computation where we have a rule that may require its own output at a previous generation happens to be a perfect use of the Store Comonad. I will now briefly introduce the Store Comonad.

Recall that the monad  $\text{State } s \rightarrow a$  is a function from initial state  $s$  to final value  $a$  and final state  $s$ .

$$s \rightarrow (a, s)$$

If you flip some arrows around, this State monad becomes a Store Comonad. Observe that  $\text{Store } s \rightarrow a$  is some state  $s$  along with a function that computes an output based on that state.

$$((s \rightarrow a), s)$$

The Store is trivially a Functor, where  $\text{fmap } g (\text{Store } f \ s) = \text{Store } (g \circ f) \ s$ .

The Store is trivially a Comonad, where  $\text{extract } (\text{Store } f \ s) = f \ s$ , and  $\text{duplicate } (\text{Store } f \ s) = \text{Store } (\text{Store } f) \ s$ , and finally  $\text{extend } f = \text{fmap } f \circ \text{duplicate}$ .

With this structure introduced, we can begin.

```
{-# LANGUAGE RecordWildCards #-}
{-# LANGUAGE OverloadedStrings #-}
{-# LANGUAGE ViewPatterns #-}

module Main where
```

```

import Control.Comonad.Store
import Data.MemoCombinators as Memo
import Linear.V2
import Data.Maybe
import Data.Map (Map)
import qualified Data.Map as Map
import qualified SDL
import qualified SDL.Cairo as Cairo
import qualified SDL.Cairo.Canvas as Canvas
import SDL.Cairo.Canvas (Canvas)

```

We can define our evolution rule as a store algebra  $(f\ a\ \rightarrow\ a)$ , where the store configuration is the coordinate that we are computing. This algebra returns a new pixel state and is able to look at any of the previous generation's pixel states with the provided Store. The output of said rule is uniquely determined by the function within the Store that it is parametrised by (generation  $n-1$ 's rule, which *could* be an entirely different rule), and the coordinates represented as a 2D vector. In practice, we will feed the same rule into the simulation at all but the first generation.

Our PixelState will be the instantaneous average temperature and fuel remaining.

```

data PixelState = PixelState { temp :: !Double
                              , fuel :: !Double }
    deriving Show

```

The intuition for the evolution rule is that it takes the entire lazily computed universe at  $generation_{n-1}$ , and computes the state of a single pixel for the  $generation_n$  at a particular coordinate.

```

burn :: Store (V2 Int) PixelState -> PixelState
burn s =

```

To simulate the temperature of the fire, we will use very simple (broken) model of a constant flashpoint, heat output scaled by the logarithm of the fuel, and bound by a oxygen ceiling, heat loss proportional to the amount of heat, and heat transfer itself modelled by a “rolling” average, a sort of cellular blur filter.

We are wherever the this store's coordinates points, at the previous generation.

```

    us { temp = new_temp
        , fuel = new_fuel}
  where
    us = extract s

```

These are our eight neighbouring cells.

```
V2 px py = pos s
them = [ peek (V2 (px + dx) (py + dy)) s
        | dx <- [-1..1], dy <- [-1..1]
        , dx /= 0 || dy /= 0
        ]
```

If we're burning, we burn proportional to the amount of fuel, and limited by oxygen.

```
our_temp = if flashpoint then temp us + burn_speed else temp us
new_fuel = if flashpoint then fuel us - burn_speed else fuel us
burn_speed = max 0 $ min 200 $ fuel us / 2
```

We now need to incorporate this heat with our surrounding cells.

```
ambient_temp = sum (fmap temp them) / fromIntegral (length them)
new_temp = max 0 $
    our_temp + ((ambient_temp - our_temp) * heat_transfer) - heat_loss
```

Some magic numbers.

```
heat_transfer = 0.1
heat_loss = temp us / 100
flashpoint = temp us > 500
```

With our governing rule defined, we need a base case for the computation. This will be our starting generation 0. The initial world is full of emptyPixels, aside from entries in a seed map.

```
emptyPixel :: PixelState
emptyPixel = PixelState { temp = 30, fuel = 4000 }

gen0 :: Store (V2 Int) PixelState
gen0 = store (\v2 -> fromMaybe emptyPixel (Map.lookup v2 seed)) (V2 0 0)
  where
    seed = Map.fromList [ (V2 20 20, emptyPixel { temp = 6000 })
                        , (V2 25 25, emptyPixel { temp = 5000 })
                        , (V2 40 40, emptyPixel { temp = 5000 })
                        , (V2 37 40, emptyPixel { temp = 6000 })
                        ]
```

By passing `gen0` to the burn algebra, we can now compute a single pixel of the next generation. If we were to iterate this twice, then *generation<sub>2</sub>* would request the output of neighbouring pixels at *generation<sub>1</sub>*, which would in turn look up its neighbouring pixels in *generation<sub>0</sub>*. As this process is iterated, we can see how we only compute the pixels relevant to our computation, and on demand.

This could get out of hand very quickly, and there is no point computing the entire universe to learn about 100x100 pixels, so, we bound our computation with a base-case that does not consult its neighbours, the bounds of our system can be thought of as a surrounding box of constant pixels.

```
bound :: (V2 Int, V2 Int) -> a -> (Store (V2 Int) a -> a) -> Store (V2 Int) a -> a
bound (V2 bx by, V2 tx ty) def f s@(pos -> V2 x y)
  | x < bx || y < by || x > tx || y > ty = def
  | otherwise = f s
```

Using data-memocombinators, we can memoise pure functions without dealing with IO. If finer grained control of what we memoise is required, a transformer over might be cleaner.

```
memoise :: Store (V2 Int) a -> Store (V2 Int) a
memoise (runStore -> (f, s)) = store (Memo.wrap to from memoPair f) s
  where
    to (x,y) = V2 x y
    from (V2 x y) = (x,y)
    memoPair = Memo.pair Memo.integral Memo.integral
```

Now we define our simulation as parametrised by a store algebra, and an initial store. We feed result of successive applications of the same algebra over the entire universe back into itself. `simulation alg base !! 10` will retrieve the tenth generation. Only when this tenth generation store is peeked into will the computation actually take place, when all nine previous generations are calculated bottom up. This is why we want memoisation, to avoid rebuilding this layered “cake” each time.

```
simulation :: (Store (V2 Int) a -> a) -> Store (V2 Int) a -> [Store (V2 Int) a]
simulation f base = iterate (extend f . memoise) base
```

As suggested, we’ll need something to peek into a store. Given a bounding box, we evaluate a store at every point within.

```
computeWithin :: (V2 Int, V2 Int) -> Store (V2 Int) a -> [(V2 Int, a)]
computeWithin (V2 x0 y0, V2 xn yn) w = fmap (\s -> (s, peek s w)) coords
```

```

where
  coords = do
    dx <- [x0..xn]
    dy <- [y0..yn]
    return $ V2 dx dy

```

Putting it all together now.

```

main :: IO ()
main = do

```

We create a lazy value, frames, which is a list of “windows” into our universe.

```

let bounds = (V2 10 10, V2 50 50)
let results = simulation (bound bounds emptyPixel burn) gen0
let frames = fmap (computeWithin bounds) results

```

Now for the quick and nasty rendering of our simulation, we will render a Cairo texture within SDL. Get a window and show it.

```

SDL.initialize [SDL.InitVideo]
window <- SDL.createWindow "Burning the world" SDL.defaultWindow
SDL.showWindow window

```

Get a renderer to put our texture on.

```

renderer <- SDL.createRenderer window (-1) SDL.defaultRenderer

```

Associate our Cairo texture with the renderer.

```

texture <- Cairo.createCairoTexture' renderer window
loop renderer texture frames
where

```

```

loop renderer texture (frame:frames) = do

```

A rendering loop like any other rendering loop. Clear the buffer.

```

SDL.clear renderer

```

Draw on the buffer.

```
Canvas.withCanvas texture $
  mapM_ (uncurry drawPixel) frame
```

Swap the buffers.

```
SDL.copy renderer texture Nothing Nothing
SDL.present renderer
loop renderer texture frames
```

We represent each pixel by a circle, bigger and redder circles are hotter.

```
drawPixel :: V2 Int -> PixelState -> Canvas ()
drawPixel coords PixelState{..} = do
  let heat_scaled = max 1 (2000 / temp)
  let fuel_scaled = max 1 (4000 / fuel)
  let redness = round $ 255 / heat_scaled
  Canvas.fill (Canvas.red redness)
  Canvas.circle (fmap ((*10) . fromIntegral) coords) (1 + 9 / fuel_scaled)
```