

TRAIL *OF* **BITS**

Advanced DeFi Invariants

Upcoming workshops

Beginner

- Part 1: The Basics
- Part 2: Breaking ABDKMath (Week of Nov 21, 2022)

Intermediate

- Part 3: Breaking Uniswap I (Week of Nov 28, 2022)
- Part 4: Breaking Uniswap II (Week of Dec 5, 2022)

Advanced

- **Part 5: Advanced DeFi Invariants I (Week of Dec 12, 2022)**
- **Part 6: Advanced DeFi Invariants II (Week of Dec 19, 2022)**

Who am I?

Nat Chin, Security Engineer II

Who You Should Follow

- Troy Sargent ([@0xalpharush](#))
- Josselin Feist ([@montyly](#))
- Anish Naik ([@anishrnaik](#))
- Justin Jacob ([@technovision99](#))

Who are we?

Trail of Bits ([@trailofbits](#))

- We help developers to build safer software
- R&D focused: we use the latest program analysis techniques
- Slither, Echidna, Tealer, Amarna, solc-select, ..

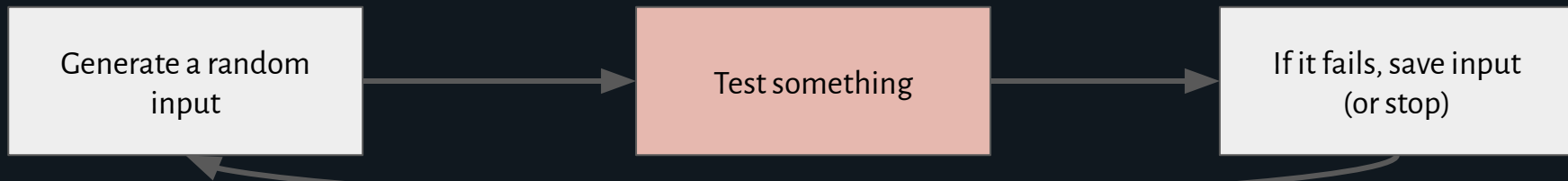
Agenda

- **Recap: Fuzzing**
- **Introduction to Primitive**
- **Architecture**
- **How to find invariants**
- **Code Walkthrough**
- **Invariant Search**
- **Implementing basic invariants**

So...how do I start fuzzing?

- 1. Identify your invariants / system properties in English**
- 2. Convert your properties to code**
- 3. Run Echidna**
- 4. FIND BUGS**

And... what is fuzzing?



Echidna vs Other Fuzzers

- **Echidna is more mature**
- **Allows testing of high gas assumptions**
- **Works with any compilation framework**
- **Supports various API's for testing**
- **Supports hevm/dapptool cheatcodes**

Tips on Identifying Invariants

- **Start with the smallest component first**
- **Analyze all preconditions and postconditions**
- **Determine safe bounds of inputs**
- **Identify inversely-related functions**
- **Focus on the happy and unhappy paths**

Useful Optimizations

- **Tests should have precondition, action, postcondition**
 - Pre-conditions: Scope the input space
 - Action: What we are testing
 - Post-conditions: The “truths” after the action

Coverage **is** your friend! Especially Echidna 2.0.4

```
34 // ----- Margin.sol -----
35 *r function depositIncreasesBalance(uint256 risky, uint256 stable) public {
36 *r     uint256 pre_deposit_bal_risky = margin.balanceRisky;
37 *r     uint256 pre_deposit_bal_stable = margin.balanceStable;
38 *r     Margin.deposit(margin, risky, stable);
39
40 *     assert(margin.balanceRisky - pre_deposit_bal_risky == risky);
41 *     assert(margin.balanceStable - pre_deposit_bal_stable == stable);
42 }
43 mapping (address => Margin.Data) margins;
44 r function withdrawDecreasesBalance(uint256 risky, uint256 stable) public {
45 r     margins[address(this)] = margin;
46 r     uint256 pre_deposit_bal_risky = margins[address(this)].balanceRisky;
47 r     uint256 pre_deposit_bal_stable = margins[address(this)].balanceStable;
48 r     Margin.withdraw(margins, risky, stable);
49
50     assert(pre_deposit_bal_risky - margins[address(this)].balanceRisky == risky);
51     assert(pre_deposit_bal_stable - margins[address(this)].balanceStable == stable);
52 }
53
```

Disclaimer

- This is complicated ✨
- Clone and follow along!



Primitive

Primitive: What is it?

- **Replicating Market Maker**
- **Implements Black-Scholes interest options**

words, words, words

Uniswap vs Primitive

Uniswap

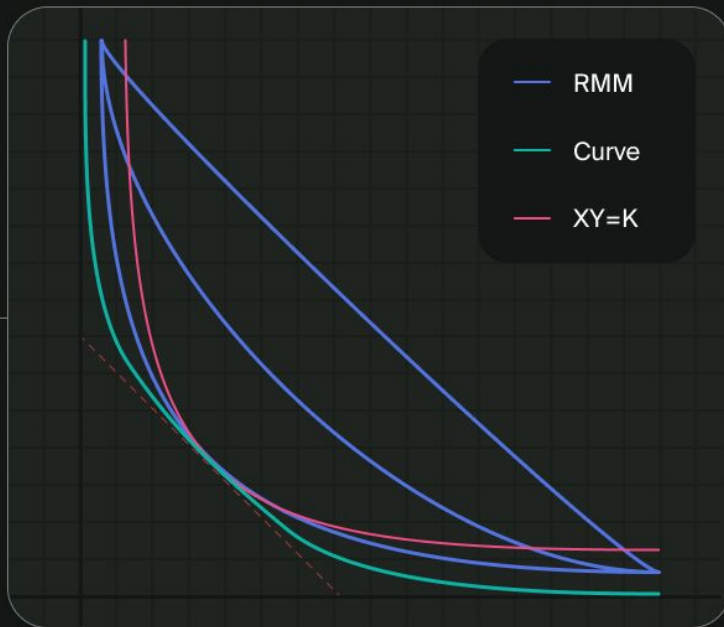
- Price changes on swap
- Pools don't have a concept of time

Primitive

- Price changes on swap and over time
- At expiry, pool consists of an underlying token



Trading Functions



Features

- **Allows creation of pools against 2 tokens**
- **Relies on spot price – no oracles**
- **“Maturity” – specified point in time where curve changes**
- **“Strike Price” – how much the asset will be worth**
- **Price will converge to strike price at maturity**

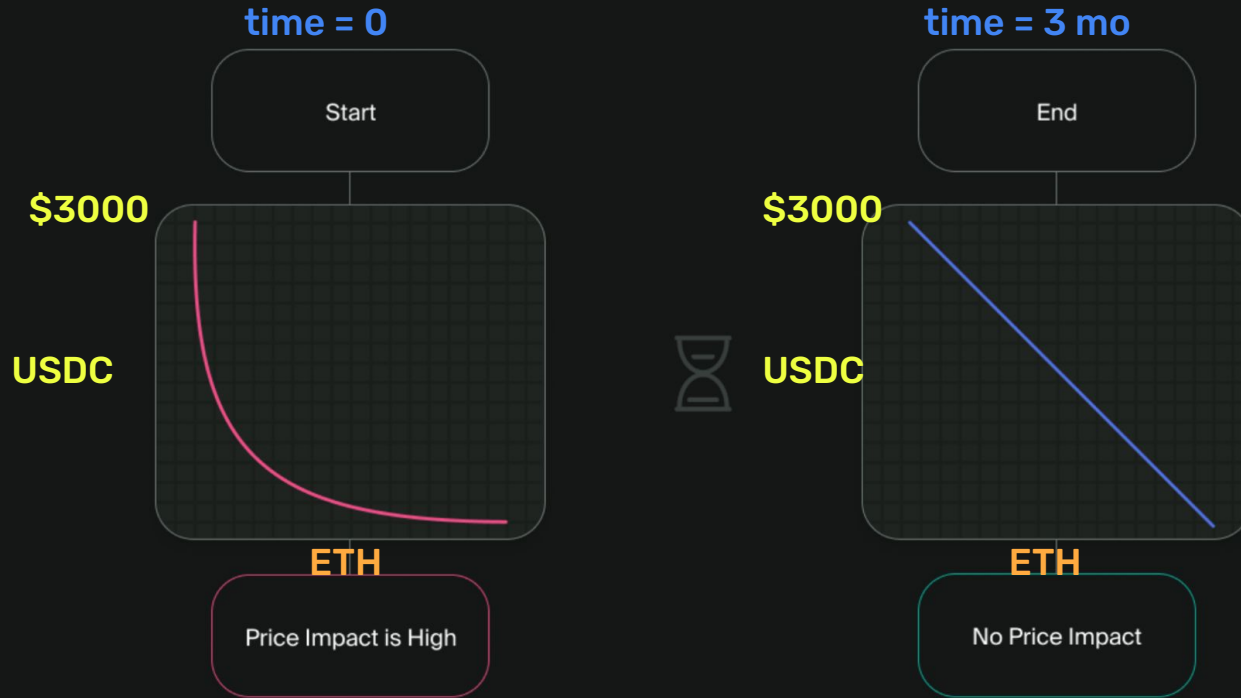
Black-Scholes Options

- **This is how the price changes over time**
- **European Option – right to exercise at expiration**
- **Pricing model depends on:**
 - Strike price (K)
 - Implied volatility (over a period of time)
 - Time to expiry (t)
 - Spot price of underlying (S)

Concrete Example

- Pool consists of **USDC (underlying) - ETH (quote)***
- Strike price = **3000 USDC**
- Maturity of **100 days**
- Implied volatility **150%**

Liquidity Compression - Concentration Over Time



Price Impact on Swaps - Over Time



Sell 1 ETH
@ 2500 USDC



3000 USDC
=
Strike Price

2465 USDC

2425 USDC

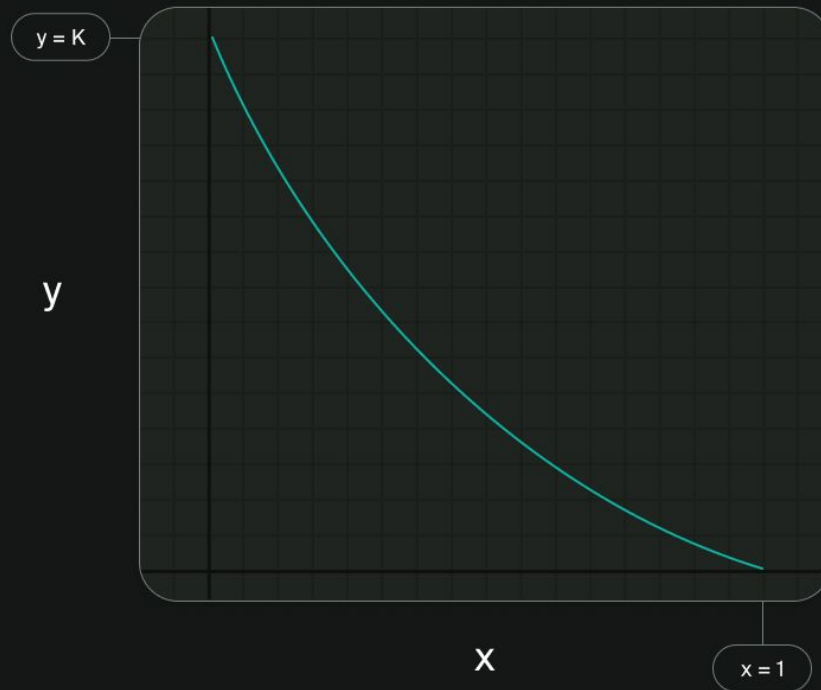
Primitive RMM Curve



| | | | |
|----------|--------------------|-------------|--------------------------|
| K | Strike Price | x | Underlying Asset Reserve |
| σ | Implied Volatility | y | Quote Asset Reserve |
| τ | Time until Expiry | Φ | CDF |
| k | Invariant | Φ^{-1} | Inverse CDF |

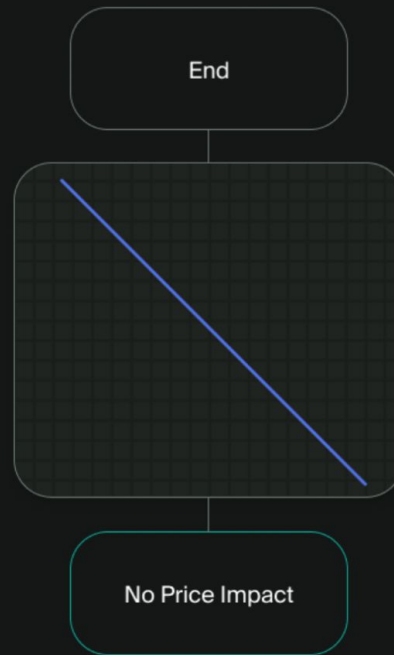
$$y - K\Phi(\Phi^{-1}(1 - x) - \sigma\sqrt{\tau}) = k$$

Trading Function



At maturity

- **Assets cannot be bought/sold**
- **Sells the second asset**
- **Pool prices at the strike price**



System Architecture

Architecture

- **Core system**
 - PrimitiveEngine – controls pools
 - PrimitiveFactory – creates engines
- **Manager: Periphery**

Primitive Engine

- **Create a new pool (curve)**
- **Allocate and remove liquidity from a pool**
- **Swap between tokens**
- **Deposit and withdraw from system**

Creating New Pool

- **Tokens** – underlying and quote token
- **Strike price** – asset's worth at maturity
- **Implied Volatility** – how much price changes
- **Maturity** – expiration
- **Gamma** – trading fee percentage

Reference: [Primitive Whitepaper](#)

What can you do with a curve?

- **Allocate liquidity – supply tokens to curve**
- **Remove liquidity – remove tokens to curve**
- **Create liquidity position – deposit money to Engine**
- **Swap between tokens**

System Architecture

Engine

```
function deposit() external { }  
function withdraw() external { }  
function allocate() external { }  
function remove() external { }  
function swap() external { }
```

Pool

```
function allocate() external { }  
function remove() external { }  
function swap() external { }
```

Version 0.8.0+

Splitting up Deposit and Allocate

- **Allows users to deposit into the system first**

Libraries

- **ABDK Math**
- **Primitive Math***
- **Margin balances**
- **Reserve balances**
- **SafeCast**
- **Transfers**
- **Unit conversions**

Let's look at code!

Lessons Learned

- Use the coverage!
- Start with the libraries
- Don't be afraid to mock
- Understand the system *first*

So on your own time.....

- **Test the Units library**
- **Test the Margin library**
- **Test the other libraries**
- **Look for E2E invariants* 😊**