

# Statistics Notes

B.Tech. CSE

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# 1 Measures of Central Tendency

1. Mean
2. Median
3. Mode

## 1.1 Mean

It is the ratio of sum of all the observations to the total number of observations. let  $x_1, x_2, \dots, x_n$  be all the observations. then:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

### 1.1.1 Properties of Mean

- The sum of deviation of observations from mean is always zero
- the sum of square of deviations of observations is minimum as compared to any other measure.
- suppose there are two sequences:

	Series 1	Series 2
Number of observations	$n_1$	$n_2$
mean of the observations	$\bar{x}_1$	$\bar{x}_2$

then

$$\bar{x} = \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$$

#### Problem 1

If there are 5 and 8 number of observations of 2 series with mean 15 and 18, find the combined mean

*Solution:*

We can get the solution by taking the weighted mean of the two sequences.  
so the required mean is :

$$\begin{aligned} & \frac{5 \times 15 + 8 \times 18}{5 + 8} \\ &= \frac{75 + 144}{13} \\ &= \frac{219}{13} \\ &= 16.846154 \end{aligned}$$

### Problem 2

Class	frequency
0-10	3
10-20	5
20-30	7
30-40	4
40-50	1

*Solution:*

change of origin:

Class	frequency	X	d=X-A	f·d
0-10	3	5	-20	
10-20	5	15	-10	
20-30	7	25	0	
30-40	4	35	10	
40-50	1	45	20	

$$\bar{x} = A + \frac{\sum fd}{n}$$

change of scale

Class	frequency	X	d=X/n	f·d
0-10	3	1	-20	
10-20	5	3	-10	
20-30	7	5	0	
30-40	4	7	10	
40-50	1	9	20	

$$\bar{x} = A + \frac{\sum fd}{n}$$

## 1.2 Median

Steps to find Median in case of Discrete and continuous data:

1. Arrangement of data
2. if  $n$  is odd then the median is the  $\frac{n+1}{2}$ th term
3. if  $n$  is even then the median is the mean of the  $\frac{n}{2}$ th term and  $\frac{n}{2} + 1$ th term

### Problem 3

find the median for the data :

1. 9,9,10,10,12,13,15
2. 9,9,10,10,12,13,14,15

*Solution:*

1. 9,9,10,10,12,13,15 has 7 elements. Therefore our median will be the 4th term in the arranged order  
 $\therefore \text{Median} = 10$
2. 9,9,10,10,12,13,14,15 has 8 elements. Therefore our median will be the mean of the 4th and 5th terms.  
 $\therefore \text{Median} = \frac{10+12}{2} = 11$

#### Problem 4

Finding the median of discrete data.

X	f	cf(cumulative frequency)
1	5	5
2	8	13
3	9	22
<b>4</b>	<b>12</b>	<b>34</b>
5	6	40
6	7	47
7	4	51
Total	51	

find the value of  $x$  which has cumulative frequency just greater than  $\frac{n}{2}$

In case of continuous data:

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right) h}{f}$$

where cf is the cumulative frequency and f is the frequency of the chosen class,  $h$  is the class size

### 1.3 Mode

The observation which occurs the most is called the mode of the data.

In more general terms, the most probable observation in a dataset is the mode of the data.

#### Problem 5

Find mode for the following data: 10,11,15,18,18,18,15,10,18,20

### Problem 6

Find the mean, median and mode for the following data

CI	f
0-10	3
10-20	5
20-30	7
30-40	2
40-50	1
Total	51

#### How to find the mode for continuous data

1. Find the modal class which is having the maximum frequency.
2. based on that input the values into the following formulae:

$$mode = l + h \left( \frac{f_1 - f_2}{2f_1 - f_0 - f_2} \right)$$

#### 1.4 The interconnection between the measures of central tendency

$$Mode = 3Median - 2Mean$$

#### 1.5 Geometric and Harmonic mean

Def<sup>n</sup> :

Geometric mean is defined as the  $n$ th root of the product of  $n$  observations

Mathematically:

$$GM = \sqrt[n]{\prod_{i=0}^n x_i}$$

### Problem 7

Find the Geometric Mean for the values 2,4,8

Def<sup>n</sup> :

Harmonic mean is defined as the reciprocal of arithmetic mean of the reciprocal of all the observations

$$HM = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

#### Theorem:

The following inequality is always true:

$$AM \geq GM \geq HM$$

## 1.6 Histogram

Histogram can also be used to compute the value of mode.

## 1.7 Ogive

Ogives are used to compute the value of median. They are nothing but graphs of cumulative distribution functions.

The point of intersection of two ogives gives the median.

## 1.8 Quartiles

Quartiles are the values which divide the dataset into 4 equal parts. These points are called  $Q_1, Q_2, Q_3$ .

$$Q_1 = l + \frac{\left(\frac{n}{4} - cf\right) h}{f}$$

$$Q_2 = l + \frac{\left(\frac{n}{2} - cf\right) h}{f}$$

$$Q_3 = l + \frac{\left(\frac{3n}{4} - cf\right) h}{f}$$

## 1.9 Deciles

Deciles are the values which divide the dataset into 10 equal parts.

## 1.10 Percentiles

Percentiles are the values which divide the dataset into 100 equal parts.

To find the  $x$ th percentile we can use the following formulae:

$$p_x = l + \frac{\left(\frac{xn}{100} - cf\right) h}{f}$$

## 2 Measures of Spread/Dispersion

Measures of spread are a numerical quantity to signify the variation in the observations.

Dispersion means the scatterment of observations.

There are a number of ways to get a gist of the Dispersion.

1. Range: it is the difference between the maximum and minimum value of the dataset.
2. Quartile Deviation: It is equal to  $\frac{Q_3 - Q_1}{2}$
3. Mean Deviation: It is the arithmetic mean of absolute value of deviation of observations from average.

$$\begin{aligned} MD &= \frac{\sum |x - \bar{x}|}{n} (Discrete) \\ &= \frac{\sum f|x - \bar{x}|}{N} (Continuous) \end{aligned}$$

4. Standard deviation: It is the positive squareroot of arithmetic mean of square of deviation of observations from mean. It is denoted by  $\sigma$ . This is applicable when your mean is an integer.

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} (\text{helpful when mean is integral})$$

$$\sigma = \sqrt{\frac{1}{n} \left( \sum x^2 - \frac{(\sum x)^2}{n} \right)} (\text{helpful when mean is non - integral and } x \text{ is small})$$

This is an alternate derivation of the formulae  $V = E[X^2] - E[X]^2$

$$d = x - a; \sigma = \sqrt{\frac{1}{n} \left( \sum d^2 - \frac{(\sum d)^2}{n} \right)} (\text{helpful when mean is non - integral and } x \text{ is large})$$

5. Variance: It is the square of standard deviation.

$$V = \sigma^2$$

## 2.1 Coefficient of variation

It is another way of measuring the spread. It is analogous to standard deviation with respect to the mean.

$$CV = \frac{\sigma}{\bar{X}} \times 100$$

Suppose there are two sequences with  $n_1$  and  $n_2$  observations.  
 $\bar{x}_1$  and  $\bar{x}_2$  are their means.

their combined variance will be:

$$\sigma^2 = \frac{2}{n_1 + n_2} [n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)]$$

where  $d_i = \bar{x}_i - \bar{x}$ . here  $\bar{x}$  is the combined mean of the series.

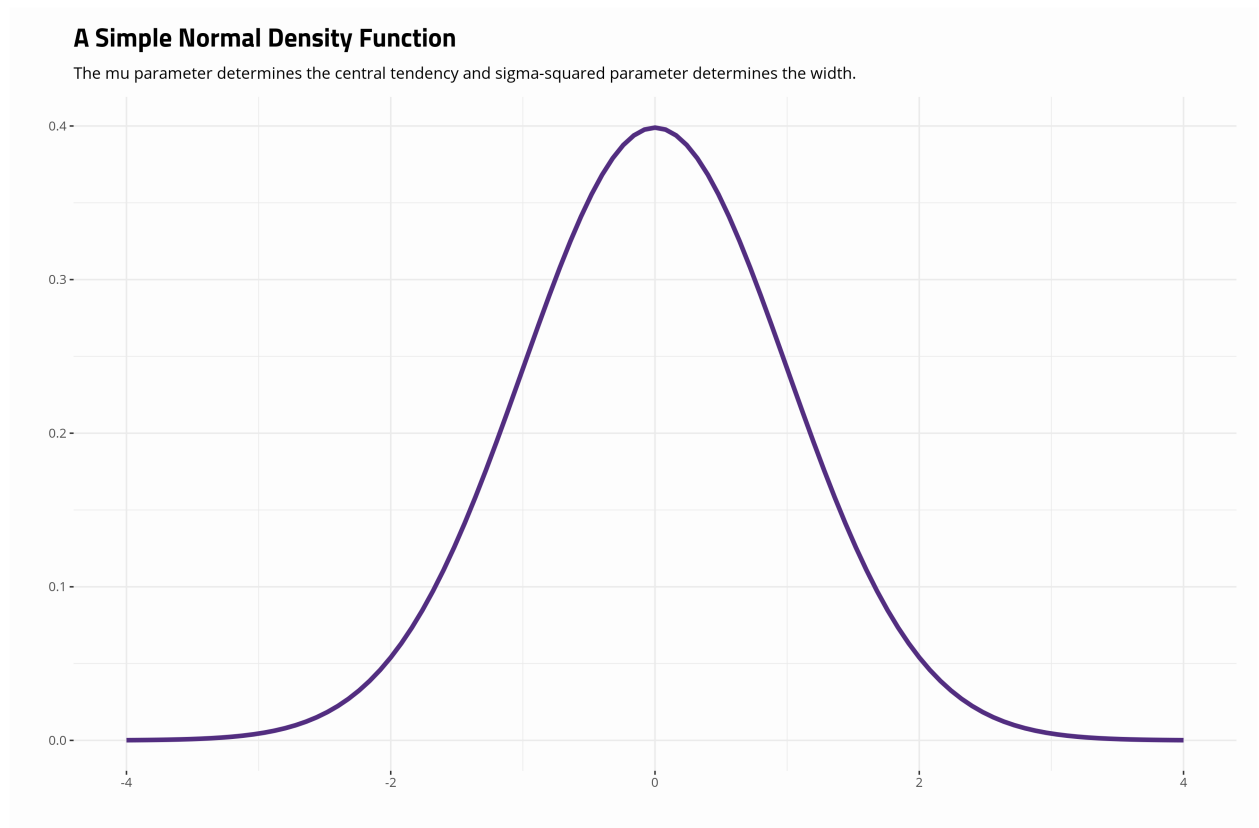
## 2.2 Skewness and Kurtosis

While studying a distribution we can calculate the measures of central tendency and the measures of spread. However even this information is not enough to determine the behavior of the random variable distribution. In order to further narrow down in the analysis of the behavior of the variable we study the skewness of the distribution.

A distribution is called skewed if:

- The Measures of central tendencies do not coincide.
- The curve does not follow gaussian nature.
- The quartiles are not equidistant from the median.

- Some of the positive deviations from median is not equal to the sum of negative deviations from the median.



There can be two types of skewness:

1. Positive skewness (Right skewed)
2. Negative skewness (Left skewed)

$$S_k = \text{Mean} - \text{Mode} \begin{cases} = 0 & \text{if data is symmetrical} \\ > 0 & \text{if data is positively skewed} \\ < 0 & \text{if data is negatively skewed} \end{cases}$$

### 2.2.1 Positive skewness

In this most of the values lie to the right to the peak.

one way to determine this is that  $\text{mode} > \text{median} > \text{mean}$

### 2.2.2 Negative skewness

In this most of the values lie to the left to the peak.

one way to determine this is that  $\text{mode} < \text{median} < \text{mean}$



### 2.2.3 Moments

Arithmetic mean of various powers of deviation of observation from mean.

$$\mu_r = \frac{\sum (x - \bar{x})^r}{n}$$

1.  $\mu_1 = 0$
2.  $\mu_2 = \text{variance}$
3.  $\mu_3 = \text{skewness}$
4.  $\mu_4 = \text{kurtosis}$

### 2.2.4 Coefficient of skewness:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\gamma_1 = \frac{\mu_3}{(\sqrt{\mu_2})^3}$$

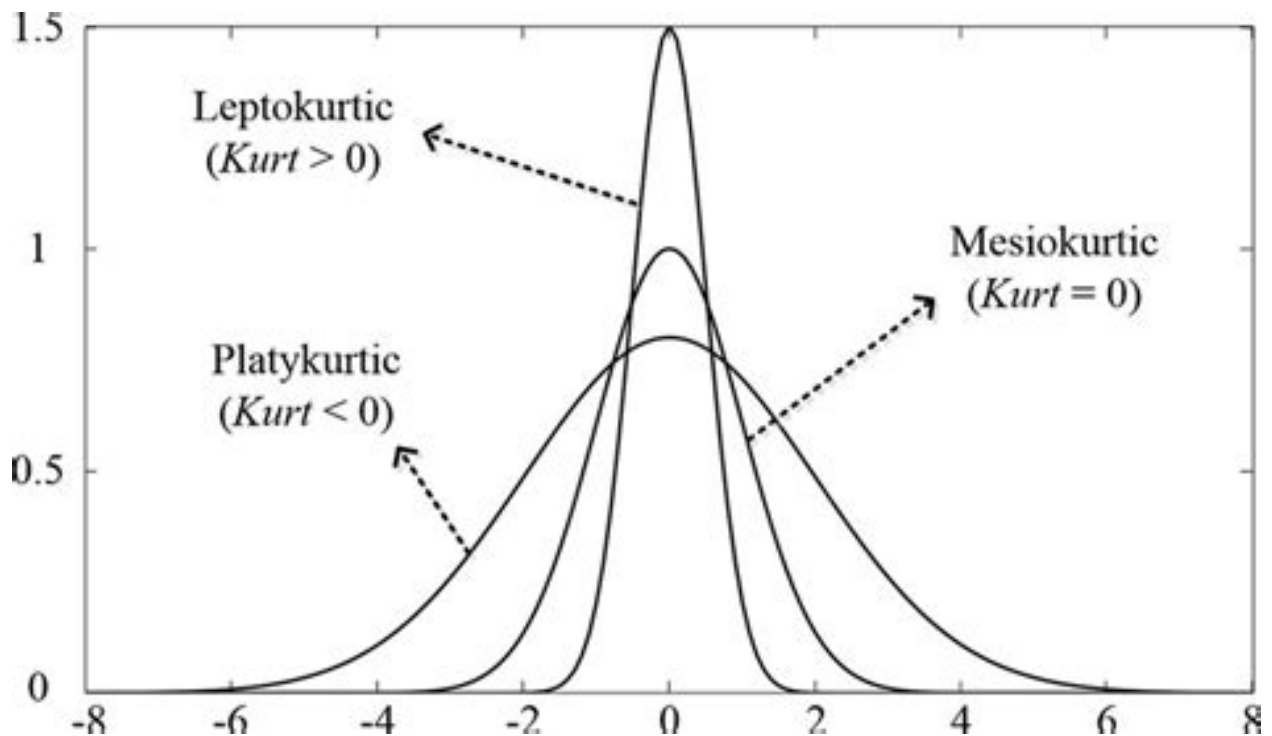
if  $\gamma_1 = 0$  the data is symmetrical.

if  $\gamma_1 > 0$  the data is right skewed.

if  $\gamma_1 < 0$  the data is left skewed.

### 2.2.5 Kurtosis

It measures the flatness or peakness of the distribution.



$$\beta_2 = \frac{\mu_4}{(\mu_2)^2}$$

$$\gamma_2 = \beta_2 - 3$$

$$Distribution = \begin{cases} Mesokurtic, & \text{if } \gamma_2 = 0 \\ Leptokurtic, & \text{if } \gamma_2 > 0 \\ Platykurtic, & \text{if } \gamma_2 < 0 \end{cases}$$

## 3 Probability

Probability is the measure of belief that a certain event will occur.

### 3.1 Prerequisites

Def<sup>n</sup> :

Trial:

Suppose an experiment is repeated under identical and homogeneous conditions does not give unique result but may result in one of several possible outcomes. The experiment is known as a random experiment or a Trial.

For example: the toss of a coin.

The outcome of a Trial is known as an **event**.

Def<sup>n</sup> :

Exhaustive events:

The set of all the simple events that can occur in a trial are called exhaustive events. The cardinality of the sample space can be found out using basic Combinatorics.

#### Problem 8

What would be the sample space of the experiment in which 2 coins are tossed together.

Solution:

{HH,HT,TH,TT}

Def<sup>n</sup> :

Mutually Exclusive Events:

Events are mutually exclusive if no two or more than 2 events occur simultaneously in the same trial

Mathematically, two events  $E$  and  $F$  are mutually exclusive  $\iff E \cap F = \phi$

Def<sup>n</sup> :

Favourable events:

The number of simple events favourable to the happening of an event.

Def<sup>n</sup> :

Equally likely events:

This means that events have equal chances of occurrence.

Def<sup>n</sup> :

Independent events:

Two events are called Independent if the occurrence of one event does not influence the occurrence of the other.

Mathematically, Two events are independent if

$$P(E \cup F) = P(E) \times P(F)$$

### 3.2 Definition of Probability

#### 3.2.1 Mathematical or Empirical Probability

The probability of an event  $E$  is

$$P(E) = \frac{\text{number of simple events favourable to } E}{\text{Total number of simple events}}$$

Suppose there are total of  $n$  number of events and the number of events favourable for a certain event are  $m$ . then the probability will be  $\frac{m}{n}$ .

Then the number of events unfavourable to the same event is  $n - m$  and the probability will be  $\frac{n-m}{n}$

Axiomatically:

$$P(E) = 1 - P(\overline{E})$$

and

$$0 \leq P(E) \leq 1 \quad \forall E \subset \Omega$$

#### 3.2.2 Statistical Probability

The probability of an event  $E$  is

$$P(E) = \frac{\text{number of simple events favourable to } E}{\text{Total number of simple events}}$$

#### 3.2.3 Axiomatic Probability

Consider a sample space  $S$ . The probability is a function that assigns a non-negative value to every event, say  $A$  denoted by  $P(A)$  is called the probability of an event  $A$  if it satisfies the following properties.

1.  $P(A) \in [0, 1] \forall A \subset S$
2.  $P(S) = 1$
3.  $P(\bigcup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i)$  where each  $A_i$  is a mutually exclusive event

### 3.3 Addition law of Probability

If  $A$  and  $B$  are two events and are not mutually exclusive, so :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

### 3.4 Multiplication Law of Probability

$$P(A \cup B) = P(A) \cdot P(B|A)$$

if  $A$  and  $B$  are independent then  $P(B|A) = P(B)$  and

$$P(A \cup B) = P(A) \cdot P(B)$$

### 3.5 Bayes Theorem

If  $E_1, E_2, E_3 \dots E_n$  are  $n$  mutually exclusive events with  $P(E_i) > 0 \forall i$  then for any event  $A$  which is a subset of  $\bigcup_{i=1}^n E_i$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{\sum_{i=1}^n P(E_i)P(A|E_i)}$$

Here  $P(E_i|A)$  is called Posterior Probability.

#### Problem 8

A class consists of 5 girls and 7 boys. If a committee of 3 is to be chosen at random what is the probability that

1. 3 boys are selected (Ans = 0.159)
2. exactly 2 girls are selected (Ans = 0.318)

*Solution:*

- 1.
- 2.