



BINARY NUMBER SYSTEM

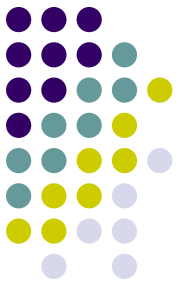
101010101010101010101010101010101010101010101010101010
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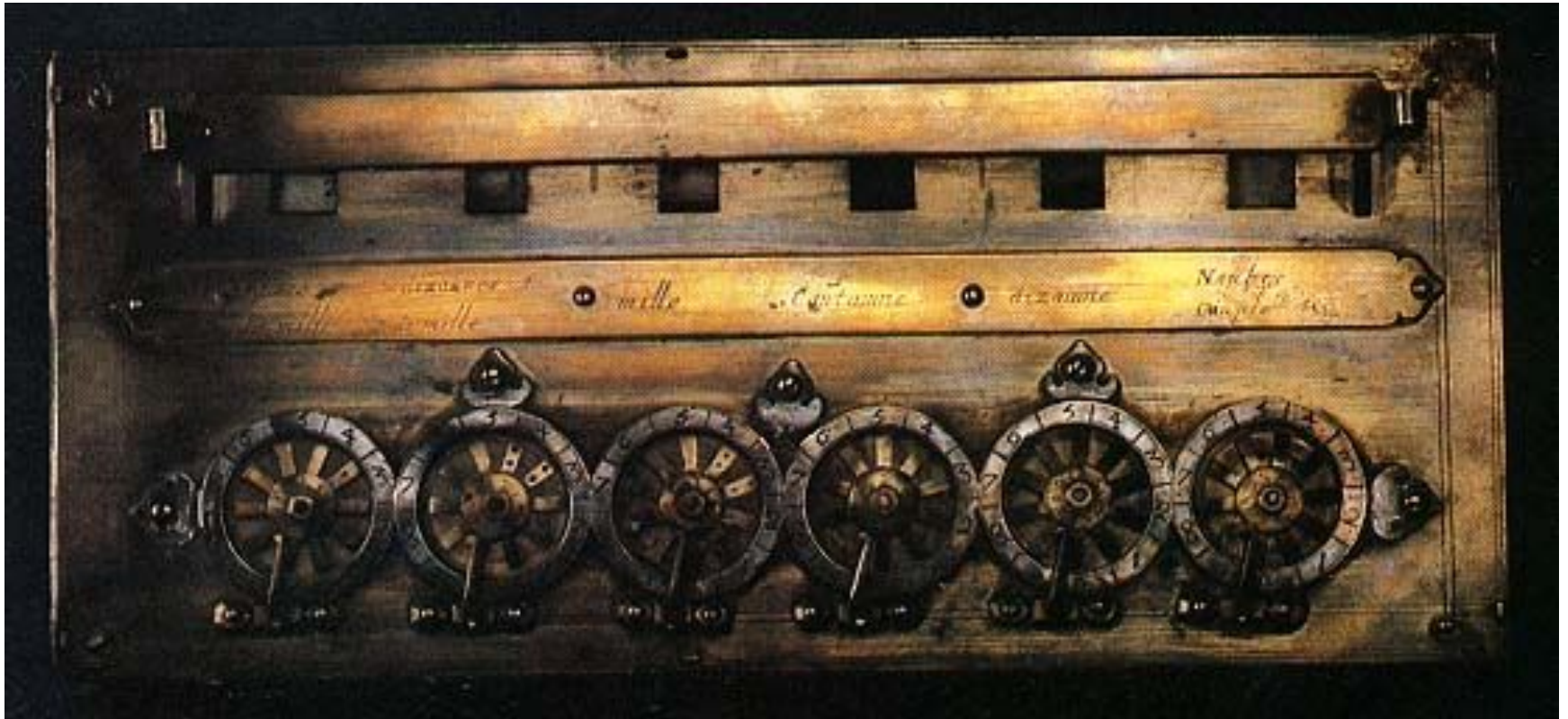
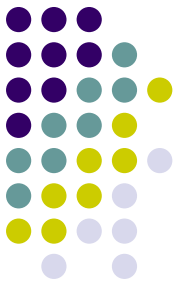
Blaise Pascal

- In 1642 Blaise Pascal, at age 19, invented the ***Pascaline*** as an aid for his father who was a tax collector. Pascal built 50 of this gear-driven one-function calculator (it could only add) but couldn't sell many because of their high cost and because they really weren't that accurate (at that time it was not possible to fabricate gears with the required precision).
- Up until the present age when car dashboards went digital, the odometer portion of a car's speedometer used the very same mechanism as the Pascaline to increment the next wheel after each full revolution of the prior wheel.

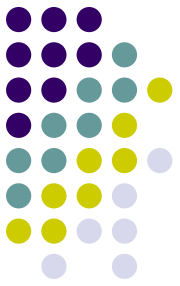
8-digit Pascaline



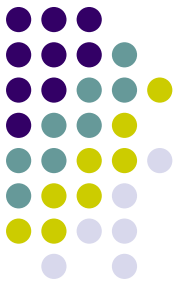
6-digit Pascaline (Cheaper)



Pascaline Insides

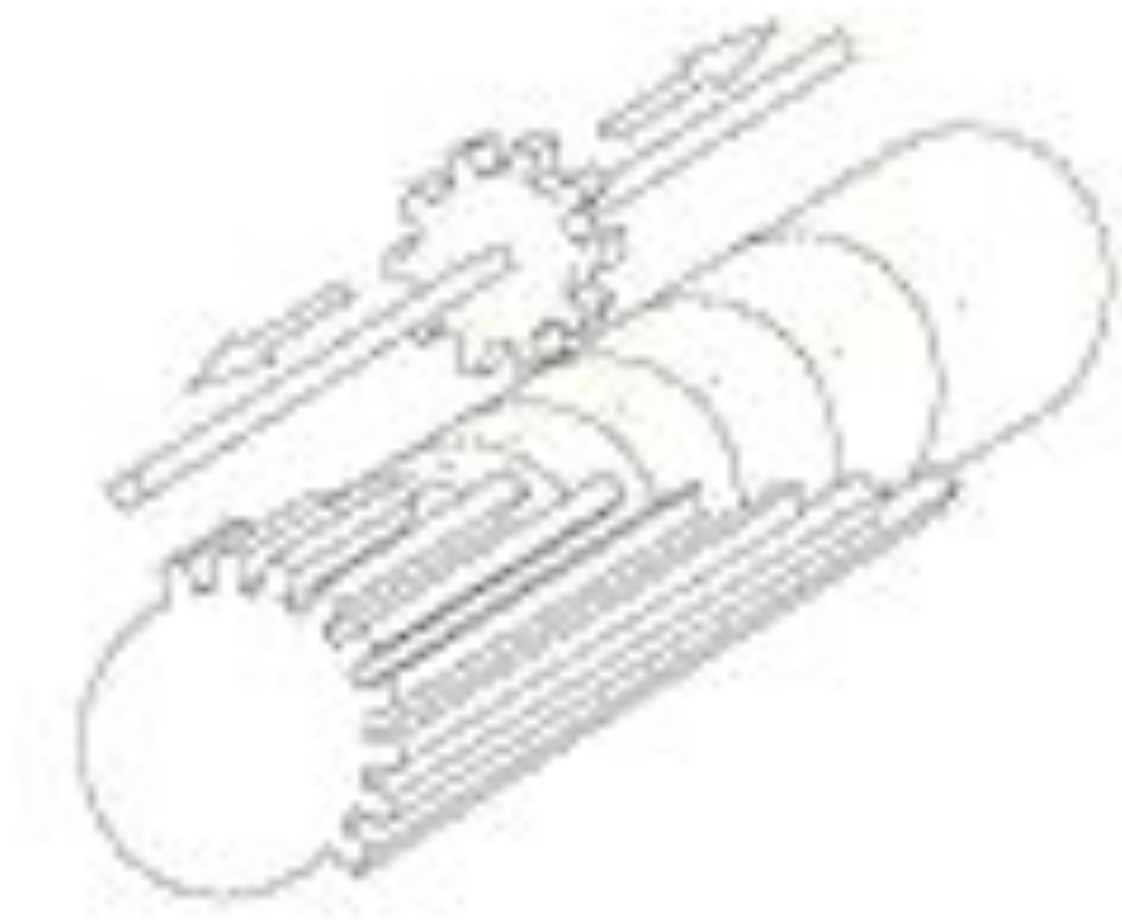
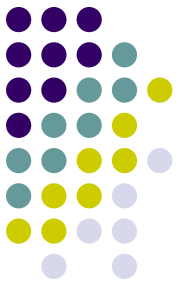


Leibniz



- Just a few years after Pascal, the German Gottfried Wilhelm Leibniz (co-inventor) managed to build a four-function (addition, subtraction, multiplication, and division) calculator that he called the ***stepped reckoner*** because, instead of gears, it employed fluted drums having ten flutes arranged around their circumference in a stair-step fashion. Although the stepped reckoner employed the decimal number system (each drum had 10 flutes), Leibniz was the first to advocate use of the binary number system which is fundamental to the operation of modern computers. Leibniz is considered one of the greatest of the philosophers but he died poor and alone.

Stepped Reckoner



“NORMAL” NUMBER SYSTEM

- Decathlon = 10 events
- The **Decimal** system that is known as our conventional number system uses a BASE of 10.
- There are 10 digits that we can use with this system...

0 1 2 3 4 5 6 7 8 9

NUMBER PLACES (DECIMAL)

<u>100000s</u>	<u>10000s</u>	<u>1000s</u>	<u>100s</u>	<u>TENS</u>	<u>ONES</u>
10^5 =100000	10^4 =10000	10^3 =1000	10^2 =100	10^1 =10	10^0 =1
			2	5	7

Example number: 257

Can be represented as $2 \times 100 + 5 \times 10 + 7 \times 1 = 200 + 50 + 7 = 257$

BINARY NUMBER SYSTEM

- Bi-cycle = 2 wheels
- The Binary number system uses a BASE of 2
- There are two digits that we can use...
- 1 0
- In computers, transistors are little switches that are either ON (1) or OFF (0)

NUMBER PLACES (BINARY)

2^7 = 128	2^6 = 64	2^5 = 32	2^4 = 16	2^3 = 8	2^2 = 4	2^1 = 2	2^0 = 1
				1	0	1	1

Example number 1011.

$$= 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$$

$$= 8 + 0 + 2 + 1$$

$$= 11$$

Therefore, the number 1011 in Binary is equal to 11 (in decimal).

$$1 \times 8 = \underline{8}$$

$$0 \times 4 = \underline{0}$$

$$1 \times 2 = \underline{2}$$





$$1 \times 1 = \underline{1}$$

FOR THE FIRST FEW CONVERSIONS... (BINARY TO DECIMAL)

- Do the following steps:

- Draw the little table
- Write in the binary number (Example: 101101)
- Add them up:
 - $32 + 8 + 4 + 1 = \underline{45}$


Notice they double every time

128	64	32	16	8	4	2	1
		1		1	1		1

CONVERTING FROM DECIMAL TO BINARY

- We start on the **left** this time and try to “take away” the largest number we can
- We use the same table:

128	64	32	16	8	4	2	1
0	0	0	0	1	1	1	1



- Example: **15**
 - 128, 64, 32, and 16 are all too big!
 - So the largest number we can use is 8
 - Now $15 - 8 = 7$ so we now repeat the steps using 7
 - $7 - 4 = 3$ repeat using 3
 - $3 - 2 = 1$
 - THEREFORE, 15 (in decimal) is equal to 1111 in Binary

CONVERTING FROM DECIMAL TO BINARY using repeated division by 2

Convert 14 (decimal) to binary

$14 \div 2 = 7$ with a remainder of 0

$7 \div 2 = 3$ with a remainder of 1

$3 \div 2 = 1$ with a remainder of 1

$1 \div 2 = 0$ with a remainder of 1

Then you read the remainders from bottom to top.

So, 14 (decimal) = 1110 (binary)

ADDING DECIMAL NUMBERS

- When adding decimal numbers (base 10) we carry over when we get to “10 or greater”.

- See:

$$\begin{array}{r} 1 \leftarrow \\ 16 \quad 6+5 = 11 \\ \pm \quad \underline{25} \\ \hline 41 \end{array}$$

NOW ADDING BINARY

$$\begin{array}{r} 101 \\ + \underline{11} \\ \hline \end{array}$$

SUBTRACTING BINARY

$$\begin{array}{r} 101 \\ - \quad \underline{11} \\ \hline \end{array}$$