TEJ4M

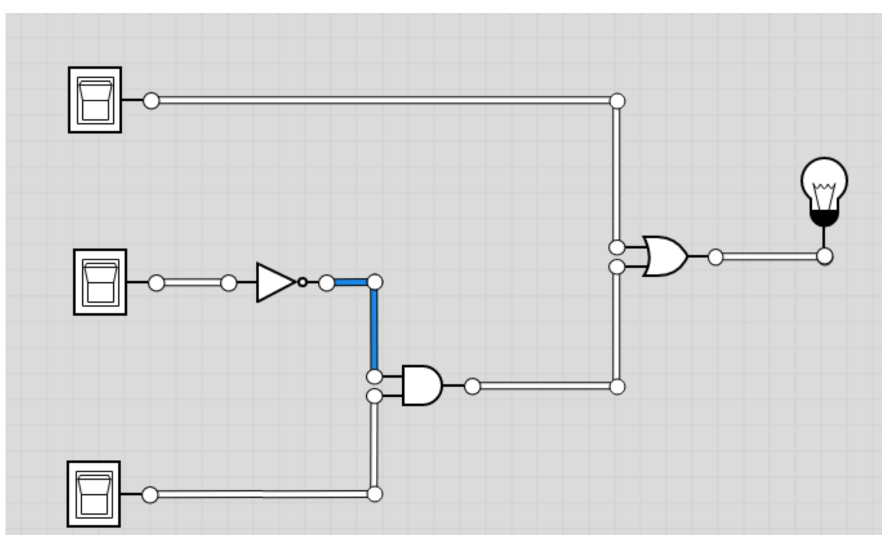
Basic Identities of Boolean Algebra

I. Review of Boolean Algebra:

- Boolean Algebra deals with binary variables and logic operations. The three basic operations are **AND, OR**, and **NOT.**
- A truth table is a combination of inputs and outputs. Let's examine the following example:
  - $Y = A + \ B \bullet C$
  - The Truth table is based on the binary inputs of 0 through $2^n - 1$ (where n = # of inputs)

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

  - Draw the logic circuit diagram below (use http://logic.ly/demo and paste your answer below) :



II. Basic Identities:

1. $X + 0 = X$
2. $X \bullet 1 = X$
3. $X + 1 = 1$
4. $X \bullet 0 = 0$
5. $X + X = X$
6. $X \bullet X = X$
7. $X + \overline{X} = 1$
8. $X \bullet \overline{X} = 0$
9. $\overline{\overline{X}} = X$ *two dashes means NOT, NOT
10. $X + Y = Y + X$
11. $XY = YX$
12. $X + (Y + Z) = (X + Y) + Z$
13. $X(YZ) = (XY)Z$
14. $X(Y + Z) = XY + XZ$
15. $X + YZ = (X + Y)(X + Z)$
16. $\overline{X + Y} = \overline{X} \bullet \overline{Y}$
17. $\overline{X \bullet Y} = \overline{X} + \overline{Y}$

Notes:

- *#1-9 show relationship between a single variable X and its complement (NOT) and binary contstants 1 and 0.*
- *#10-14 are similar to identities from normal algebra.*
- *#15-17 apply only to Boolean algebra and are very useful in manipulation Boolean expressions.*

Additional note:
#16 and 17 use a Theorem established by DeMorgan. Here are the truth tables that verify his theorem:

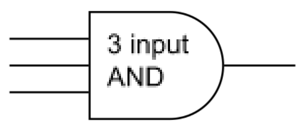
X	Y	$X + Y$	$\overline{X+Y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X} \bullet \overline{Y}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

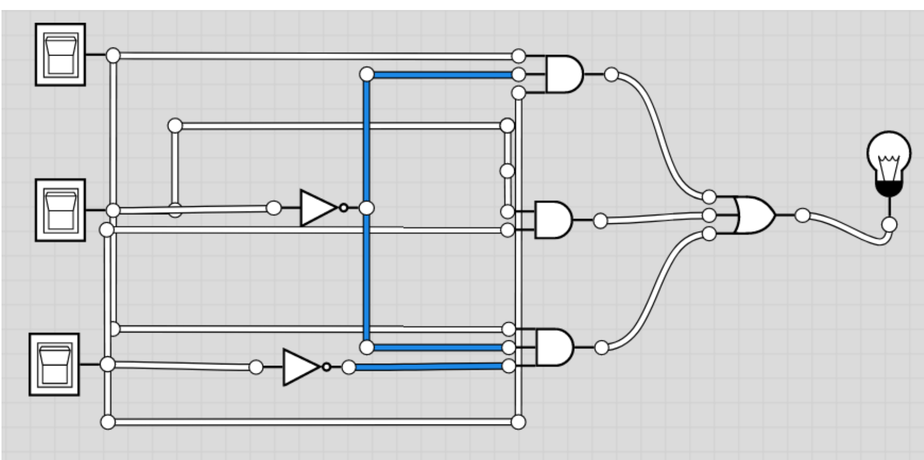
III. **Algebraic Manipulation and Simplification:**

Boolean Algebra is a useful tool for simplifying digital circuits. Consider the following example:

$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$



Draw the circuit diagram for this "complex" expression:



Now lets simplify using our identities: (this is the solution below)

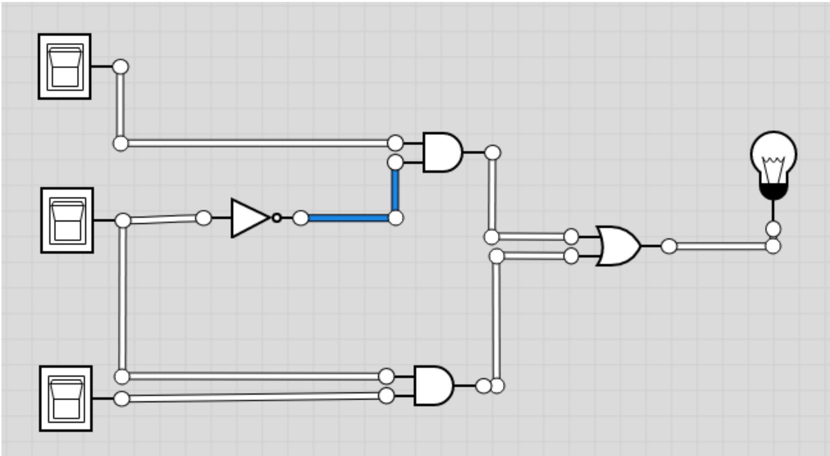
$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$

$F = \overline{X}Y\,(Z + \overline{Z}) + XZ$ (BY IDENTITY #14. $X(Y + Z) = XY + XZ$)

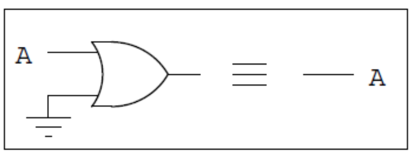
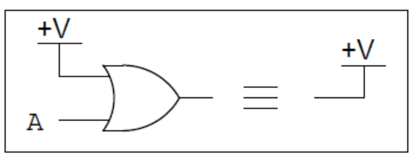
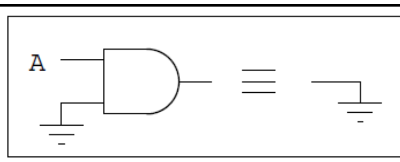
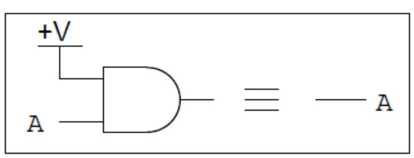
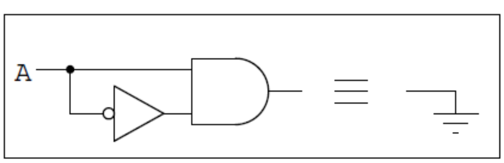
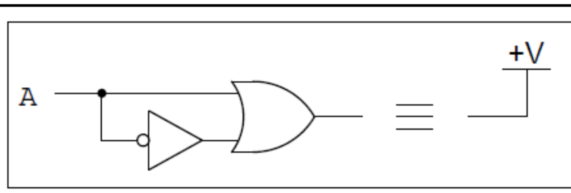
$F = \overline{X}Y \bullet 1 + XZ$ (BY IDENTITY #7. $X + \overline{X} = 1$)

$F = \overline{X}Y + XZ$ (BY IDENTITY #2. $X \bullet 1 = X$)

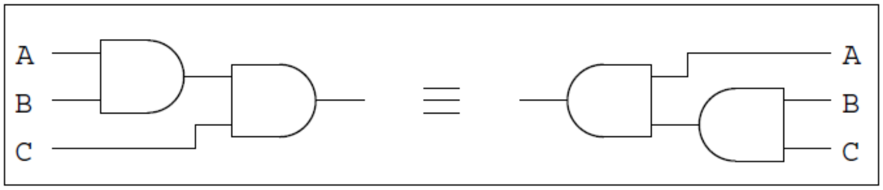
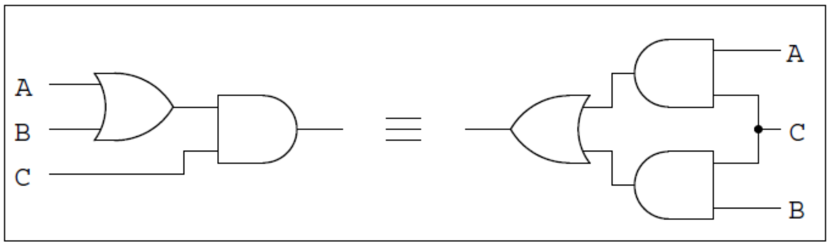
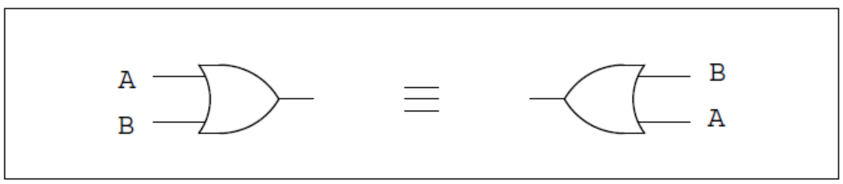
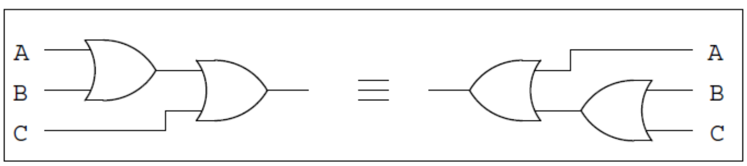
Now, Draw the simplified circuit diagram:



1. Like real-number algebra, Boolean algebra is subject to certain rules which may be applied in the task of simplifying (reducing) expressions. By being able to algebraically reduce Boolean expressions, it allows us to build equivalent logic circuits using fewer components. For each of the equivalent circuit pairs shown, write the corresponding Boolean rule next to it: (NOTE: +V = 1, GND = 0)

Logic Circuit	***Rule/Equation***
	Equation: $0 + A = A$ Rule: Identity #1
	Equation: $1 + A = 1$ Rule: Identity #3
	Equation: $A \bullet 0 = 0$ Rule: Identity #4
	Equation: $1 \bullet A = A$ Rule: Identity #2
	Equation: $A \bullet \overline{A} = 0$ Rule: Identity #8
	Equation: $A + \overline{A} = A$ Rule: Identity: #7

2. Like real-number algebra, Boolean algebra is subject to the laws of commutation, association, and distribution. These laws allow us to build different logic circuits that perform the same logic function. For each of the equivalent circuit pairs shown, write the corresponding Boolean law next to it:

Logic Circuit	***Property Name***
	Equation: (AB) • C = (BC) • A Rule: Identity: #13
	Equation: (A+B) • C = (A • C) + (B • C) Rule: Identify #15
	Equation: A + B = B + A Rule: Identity #10
	Equation: (A + B) + C = (B + C) + A Rule: Identity #12

3. Shown here are six rules of Boolean algebra (these are not the only rules):

$$A + \overline{A} = 1$$
$$A + A = A$$
$$A + 1 = 1$$
$$AA = A$$
$$A + AB = A$$
$$A + \overline{A}B = A + B$$

Determine which rule (or rules) are being used in the following Boolean reductions:

Reduction Example	***Rule(s)/Equation(s)***
$\overline{DF} + \overline{DFC} = \overline{DF}$	Rule #1
$1 + G = 1$	Rule #3
$B + AB = B$	Rule #5
$\overline{FE} + \overline{FE} = \overline{FE}$	Rule #2
$XYZ + \overline{XYZ} = 1$	Rule #1
$GQ + Q = Q$	Rule #5

$\overline{H}\,\overline{H} = \overline{H}$	Rule #4
$\overline{CD} + \overline{CD} = \overline{CD}$	Rule #4
$EF(EF) = EF$	Rule #4
$CD + \overline{C} = \overline{C} + D$	Rule #6

4. A student makes a mistake somewhere in the process of simplifying the following Boolean expression:

$AB + A(B + C)$

$AB + AB + C$

$AB + C$

Determine where the mistake was made, and what the proper sequence of steps should be to simplify the original expression.

The mistake was made at step # 2 as the boolean expression was not expanded properly.

AB + A(B+C)
AB + AB + AC
AB + AC
A (B+C)