

Knowledge (/ 11)

1. [/ 1] Using the single magnitude method of representing negative numbers for a 9-bit signed integer, what is the range of decimal numbers you can represent from the low end to the high end? Answer:

255 is the high end
-256 low end

2. [/ 1] Using the two's complement method of representing negative numbers for a 9-bit signed number, what is the range of decimal (base 10) numbers you can represent from the low end to the high end? Answer:

-128 is the low end
127 is the high end

3. [/ 1] What is one advantage of the computer using two's complement? Hint: Think about how it can be used in arithmetic and how this affects the amount of circuitry used. Answer:

The advantage of using two's complement is that because it is easier to implement in circuitry. Another advantage is that two's complement does not allow a negative zero whereas 1's complement has a "positive" zero and a "negative zero" which does not quite make sense as zero cannot be neither positive or negative.

4. [/ 1] Which specific hardware component of the computer uses two's complement? Answer:

ALU which represents a fundamental building block of the CPU uses two's complement. The ALU stands for Arithmetic Logical Unit and it is used by almost all computers. Microprocessors also use two's complement.

5. [/ 1] Convert the base 10 number -109 to binary using an 8-bit two's complement representation. (show your work)

Answer:

Step 1: Convert 109 to binary

Divided by 2	Remainder
$109/2 = 54$	1
$54/2 = 27$	0
$27/2 = 13$	1

$13/2 = 6$	1
$6/2 = 3$	0
$3/2 = 1$	1
$\frac{1}{2} = 0$	1

$$109_{10} = 01101101$$

Step 2: Use two's complement to convert 109 to negative

$$\begin{array}{r} 10010010 \\ + \quad 1 \\ \hline 10010011 \end{array}$$

$$-109_{10} = 10010011_2$$

6. [/ 1] Convert the base 10 number -109 to binary using an 8-bit single magnitude representation. (show your work)

Answer:

Step 1: Convert 109 to binary

Divided by 2	Remainder
$109/2 = 54$	1
$54/2 = 27$	0
$27/2 = 13$	1
$13/2 = 6$	1
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$$109_{10} = 01101101$$

Step 2: Use Single Magnitude to convert 109 to negative

$$-109_{10} = 11101101_2$$

7. [/ 1] Why is hexadecimal used by humans instead of binary values? [1]

Answer:

Hexadecimal is used by humans instead of binary values simply because they are easier to represent. For instance, hexadecimal uses base 16, meaning that the number of digits used to represent a certain number is usually less than in binary or decimal. In addition, hexadecimal can represent numbers from 0 -15 which is much more than what binary offers which is only from 0-1.

8. [/ 1] Give one specific example of where you would see hexadecimal values while using the computer.

Answer:

Hexadecimal numbers are used quite often by designers and programmers because they provide a human-friendly representation of binary numbers, Therefore hexadecimal is often used when programming, web developing etc. They are also used in the colour codes values.

9. [/ 1] Why does 1 KB (kilobyte) = 1024 Bytes instead of 1000 bytes? Explain clearly with an example.

Answer:

1 KB (kilobyte) = 1024 Bytes instead of 1000 bytes simply because computers are based on the binary system meaning that their hard drives and memory are measured in powers of 2 therefore 1kb is not simply 1000 it is 1024.

Proof:

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

$$2^9 = 512$$

$$2^{10} = 1024$$

10. [/ 2] Highlight or **bold** one of the following: **CD's** RAM Hard Drive(HDD) SSD. Briefly explain how they store 1's and 0's (i.e. what specifically do they use to represent 1's and 0's)

Answer:

CD's store data in the form of binary. If you ever noticed, most CD's contain long spiral tracks of data, along these tracks there are flat reflecting areas and non-reflective arrays. The reflective area represents binary 1, while a non-reflective bump represents the binary 0.

Application [/20]

1. [/ 8] Fill in the missing values. Remember to show your work. [8 marks]

Decimal	Hexadecimal	Binary										
$4 \times 16^3 + 14 \times 16^2 + 12 \times 16^1 + 10 \times 16^0 = 20170_{10}$	4 E C A A = 10 C = 12 E = 14 4 = 4	4										
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		2/2 = 1	0									
		1/2 = 0	1									
		E or 14										
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		7/2 = 3	1									
		3/2 = 1	1									
		1/2 = 0	1									
		C or 12										
		<table><tr><th>Divided by 2</th><th>Remainder</th></tr><tr><td>12/2 = 6</td><td>0</td></tr><tr><td>6/2 = 3</td><td>0</td></tr><tr><td>3/2 = 1</td><td>1</td></tr></table>	Divided by 2	Remainder	12/2 = 6	0	6/2 = 3	0	3/2 = 1	1		
Divided by 2	Remainder											
12/2 = 6	0											
6/2 = 3	0											
3/2 = 1	1											

		<table><tr><td>$\frac{1}{2} = 0$</td><td>1</td></tr></table> <p>A or 10</p> <table><tr><td>Divided by 2</td><td>Remainder</td></tr><tr><td>$10/2 = 5$</td><td>0</td></tr><tr><td>$5/2 = 2$</td><td>1</td></tr><tr><td>$2/2 = 1$</td><td>0</td></tr><tr><td>$1/2$</td><td>1</td></tr></table> <p>$0100\ 1110\ 1100\ 1010_2$</p>	$\frac{1}{2} = 0$	1	Divided by 2	Remainder	$10/2 = 5$	0	$5/2 = 2$	1	$2/2 = 1$	0	$1/2$	1																														
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		41/2	1
		20/2	0
		10/2	0
		5/2	1
		2/2	0
		1/2	1
		10100101010001111101 ₂	

2. [/ 4] Solve using 8-bit two's complement

$\begin{array}{r} 0100\ 0000_{(2)} \\ - 0000\ 0001 \\ \hline 100111111 \end{array}$	$\begin{array}{r} 0000\ 1100_{(2)} \\ - 0010\ 0111 \\ \hline 11100101 \end{array}$
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3. [/ 4] Binary Arithmetic. Solve. [4 marks]

$\begin{array}{r} 1\ 1\ 0\ 1\ 0\ 1\ 1 \\ \times \quad 1\ 0\ 1 \\ \hline 1000010111 \end{array}$	$\begin{array}{r} 11 \overline{) 1\ 0\ 1\ 1\ 0\ 1} \\ \underline{0111} \end{array}$
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4. [/ 2] Hexadecimal Addition. Double-click on the image and show your carry overs clearly using the *scribble* tool. Click on the line symbol to find it. [2 marks]

$$\begin{array}{r} A\ 2\ F\ 1 \\ +\ \ \ 9\ C\ F \\ \hline \end{array}$$

ACC0₁₆

Thinking (6 marks)

1. [/ 2] There is no extra circuitry or hardware for the computer to do multiplication and division directly but it is capable of doing both arithmetic operations. How can it perform multiplication using only the fact that the computer has circuitry for both addition and subtraction? Give an example to support your answer.

The computer can perform multiplications using addition and subtraction simply because multiplication is simply the repeated addition checking to see if the product or quotient has been reached.

For example

$$6 \times 6 = 36$$

$$6+6+6+6+6+6 = 36$$

2. [/ 2] It's a hundred years from now and humans have survived climate change. They have evolved and now have two fingers on each hand. Now, they use a base 4 numbering system. In this new era, how would humans perform this addition question? Double click on the image and clearly show the carryovers.

$$\begin{array}{r} 1 \\ 2\ 3\ 3 \\ +\ 1\ 2\ 3 \\ \hline 1\ 0\ 2\ 2 \end{array}$$

Humans can perform this addition by taking the number and dividing it by 4. Once they get the quotient they can add the remainder along it.

3. [/ 2] Look below to see what happens when you shift all the bits to the right once.

1111001 (initial binary number)

111100 (bits shifted to the right once).

What about if I shift the bits right a second time resulting in the binary number 11110?

Do you see a pattern here? How does it affect the overall value each time? Explain the pattern and specify how the computer can use this technique [2]

The pattern is that the value is being divided by 2 every time we remove a bit from right and it also takes the remainder from the binary. This is because the column weight is decreasing each time you remove the right bit. Computers can use this technique in order to perform division with binary numbers.

Example

$143/2 = 71$ **takes out the remainder**

$$5) \quad \begin{array}{r} 10010010 \\ + 1 \end{array}$$

$$2) \quad \begin{array}{r} 0100 \ 0000 \\ - 0000 \ 0001 \end{array}$$

$$\begin{array}{r} 1111 \ 1110 \\ + 1 \\ \hline 1111 \ 1111 \end{array}$$

$$\begin{array}{r} 0100 \ 0000 \\ + 1111 \ 1111 \\ \hline 10011 \ 1111 \\ \uparrow \\ \text{Overflow} \end{array}$$

$$\begin{array}{r} 0000 \ 1100 \\ - 0010 \ 0111 \end{array}$$

$$\begin{array}{r} 1101 \ 1000 \\ + 1 \\ \hline 1101 \ 1001 \end{array}$$

$$\begin{array}{r} 0000 \ 1100 \\ + 1101 \ 1001 \\ \hline 1110 \ 0101 \end{array}$$

$$\begin{array}{r} 1101011 \\ \times 101 \\ \hline 11101011 \\ 00000000 \\ + 110101100 \\ \hline 1000010111 \end{array}$$

$$\begin{array}{r} 01111 \\ 11 \overline{) 101101} \\ \underline{0} \\ 1011 \\ \underline{11} \\ 100 \\ \underline{11} \\ 0011 \\ \underline{11} \\ 00 \end{array}$$

$$\begin{array}{r}
 \overset{1}{2} \overset{1}{F} 1 \\
 + 9 C F \\
 \hline
 A C C 0
 \end{array}$$

$$\begin{aligned}
 1 + F &= 1 + 15 \\
 &= 16 \\
 &= 16 + 0 \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 F + C &= 15 + 12 + 1 \\
 &= 28 \\
 &= 16 + 12 \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 2 + 9 &= 2 + 9 + \\
 &= 12
 \end{aligned}$$