PH3110 Experimental or Theoretical Project

Python simulations of the resistance of 2D conductors of arbitrary shape

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Abstract: The resistance for 2D conductors of arbitrary shape were calculated by simulating them in Python and solving them numerically using the finite difference method. Two ohmic contacts were attached on two sides, creating a potential difference of 100 V. The potential and the electric field were obtained.

Contents

1 Introduction

With the continuous improvement in technology, electrical devices have become more advanced and diverse. Depending on their function, they may have different components and may also be assembled differently. They may also require specific electrical resistance to function properly. Furthermore, components that require resistors with specific resistances are most commonly circuit boards which are usually very thin. Therefore, the resistor in the circuit board will just be a very thin film, which can be modelled and solved as a two-dimensional (2D) system.

Two dimensional modelling has helped provide important developments in semiconductors, where their behaviour has been confined to two dimensions through limiting the motion of electrons in the third dimension[1]. As a result, the energy levels; which are now quantized in the third dimension, can be ignored. The motion in the remaining two dimensions can be modelled as two dimensional electron gas (2DEG), Therefore, they can essentially be modelled as 2D systems[1].

As stated previously, circuit boards (or any other electrical component) may require resistors that offer a specific amount of resistance. The desired resistance could be achieved by using a conductor made from a specific material or by changing the shape of the conductor. Conducting research to find the appropriate material may be costly and time consuming. So, the alternative method would be to research with different shapes, by modelling them as 2D systems.

Resistance for some shapes could be calculated analytically, by separating and solving the shape as a combination of squares. However, this method only works for simple shapes; as seen later in the report, therefore a more effective method is needed for more complicated shapes.The focus of this report will be the use of Python to simulate the resistance of 2D conductors of arbitrary shape, by solving Laplace's equation using finite difference numerical method. It will also be assumed that the material has uniform conductivity. These are the basic assumptions that will be used when simulating the resistance of 2D shapes.

2 Background

Before trying to compute the resistance of arbitrary 2D shapes, it will help to understand some basic underlying principles and equations.

2.1 Resistivity

Resistivity (ρ) is a characteristic property of any material, it is a measure of the resisting power of a material to the flow of electric current, the inverse of resistivity is conductivity (σ). For isotropic materials, whose properties remain identical in all directions, ρ and σ can be related to the current density ($\overrightarrow{J}$) and the electric field vector ($\overrightarrow{E}$) at any point through Ohm's law [2]

$$\overrightarrow{J} = \frac{\overrightarrow{E}}{\rho} = \sigma \overrightarrow{E}. \tag{2.1}$$

2.2 Resistors with uniform cross-sectional area

For 2D systems, shapes with uniform cross-sectional area refers to squares and rectangles. The resistance of these shapes could be calculated analytically using the following process, using a rectangle of length L as an example, with two ohmic contacts attached, which are low resistance junctions that provide current conductance. Once a potential difference (ΔV) is applied, a uniform $\overrightarrow{E}$ will be produced

$$\overrightarrow{E} = \frac{\Delta V}{L}, \tag{2.2}$$

resulting in a uniform $\overrightarrow{J}$. $\overrightarrow{E}$ and $\overrightarrow{J}$ are parallel to the axis and because of the uniform cross-sectional area, they are also parallel to the sides of the shape. The product of the resultant $\overrightarrow{J}$ and the area of the shape provide the current (I)

$$I = \overrightarrow{J} \times Area = \frac{\overrightarrow{E} \times Area}{\rho}. \tag{2.3}$$

As ΔV is known, resistance can be calculated using Ohm' law, [2],

$$R = \frac{\Delta V}{I}, \tag{2.4}$$

where R is the resistance of the material. Therefore, the equation for R becomes [2],

$$R = \frac{\rho \times L}{A}. \tag{2.5}$$

As the focus of this report is on 2D systems, instead of bulk resistivity ρ, we can use the surface resistivity ρ_{surf}. This is simply ρ divided by the thickness. As ρ has units of $ohms \times length$, the units for ρ_{surf} therefore become ohms. For a rectangle of length L and width W, Eq 2.5 becomes

$$R = \frac{\rho_{surf} \times L}{W}. \tag{2.6}$$

Applying the same principles to a square, the resistance simply becomes

$$R = \rho_{surf}. \tag{2.7}$$

Eq 2.7 is often referred to as ohms per square, which simplifies the process of calculating the resistance in squares and rectangles. By dividing the shape in question into smaller squares, the total resistance in the shape would be the sum of all of the squares. This method however will only provide accurate results for simple shapes with uniform cross-sectional area.

The analytical method works because the $\overrightarrow{E}$ and the $\overrightarrow{J}$ are uniform due to the uniformity of the cross-sectional area. However, shapes with non uniform cross-sectional area will not produce a uniform $\overrightarrow{E}$, hence the analytical method will not work.

2.3 Resistors with varying cross-sectional area

Before any attempt to calculate the resistance, the rules and the behaviour of the fields must be established. In section 2.2, the uniform $\overrightarrow{E}$ was parallel to the axis everywhere in shape due to the uniform cross-sectional area. For 2D shapes with varying cross-sectional area with two ohmic contacts attached, once a potential difference (ΔV) is applied, the resulting $\overrightarrow{E}$ and $\overrightarrow{J}$ will be non uniform. Hence, as seen in Figure 1, the $\overrightarrow{E}$ cannot be parallel to the axis everywhere in the shape.

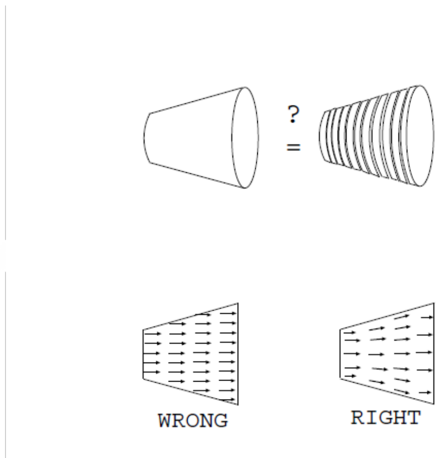

Figure 1: The correct (bottom right) and the incorrect (bottom left) flow of current in a truncated cone, a shape with varying cross-sectional area, that has ohmic contacts attached to the flat surfaces[2].

In Figure 1, the bottom left diagram shows the incorrect flow of current, the current travels parallel to the axis everywhere in the resistor. This implies that the current flows into the cone from the curved sides, which is incorrect. Therefore, current and hence $\overrightarrow{J}$, must flow parallel to the sides. Furthermore, since $\overrightarrow{J}$ and the $\overrightarrow{E}$ travel in the same direction, $\overrightarrow{E}$ must also be parallel to the sides. In the truncated cone, the $\overrightarrow{E}$ and $\overrightarrow{J}$ must also flow perpendicular to the ohmic contacts, as they are located opposite to each other on the flat surfaces. The correct flow can be seen in the bottom right of figure 1.

However, the two conditions that the fields must be parallel to the sides and also be perpendicular to the ohmic contacts contradict each other at the points when the sides meet the ohmic contacts. Therefore, the fields must be 0 at the edge of the resistor. To calculate the resistance of shapes with varying cross sectional area, we must first solve Laplace's equation, which is derived in the following section.

2.4 Laplace's Equation

The equations used in this subsection have been used from and can be found in greater detail in chapter 2 of Introduction to Electrodynamics[3]. To derive Laplace's equation, we begin with the integral of $\overrightarrow{E}$ around a closed loop. For any two points on the loop, a and b, the line integral of $\overrightarrow{E}$ is the same for all possible paths. Due to the lack of dependence on the path, the potential at point r can be defined as

$$V(r) = \int_O^r \overrightarrow{E}.dl, \tag{2.8}$$

where O is the standard reference point. Using this, we can calculate the potential difference between two points, a and b,

$$V(b) - V(a) = \int_O^b \overrightarrow{E}.dl + \int_O^a \overrightarrow{E}.dl = \int_O^b \overrightarrow{E}.dl - \int_a^O \overrightarrow{E}.dl = \int_a^b E.dl. \tag{2.9}$$

By using the Gradient Theorem,

$$\int_a^b (\nabla f).dl = f(b) - f(a), \tag{2.10}$$

and applying it to Eq 2.9 results in

$$\int_a^b (\overrightarrow{\nabla} V).dl = \int_a^b \overrightarrow{E}.dl. \tag{2.11}$$

As this is true for any values of a and b, the integrands therefore must be equal:

$$\overrightarrow{E} = -\overrightarrow{\nabla} V. \tag{2.12}$$

For, the next step we begin with Gauss's law that defines the flux of $\overrightarrow{E}$ through any closed surface S

$$\oint \overrightarrow{E}.da = \frac{1}{\epsilon_0} Q_{enc}, \tag{2.13}$$

where ϵ_0 is the permitivity and Q_{enc} is the total charge enclosed within the surface. Using the divergence Theorem, Gauss's law becomes:

$$\oint_S \overrightarrow{E}.da = \int_V (\overrightarrow{\nabla}.\overrightarrow{E}) d\tau. \tag{2.14}$$

The enclosed charge, Q_{enc} can be defined using the charge density ρ,

$$Q_{enc} = \int_V \rho d\tau. \tag{2.15}$$

Therefore, Gauss's law becomes

$$\int_V (\overrightarrow{\nabla}.\overrightarrow{E})d\tau = \int_V \frac{\rho}{\epsilon_0}d\tau. \tag{2.16}$$

This is true for any volume, therefore

$$\overrightarrow{\nabla}.\overrightarrow{E} = \frac{\rho}{\epsilon_0}. \tag{2.17}$$

The final step involves substituting Eq 2.15 into Eq 2.20, which results in Poisson's equation,

$$\overrightarrow{\nabla}^2 V = -\frac{\rho}{\epsilon_0}[\textbf{?}] \tag{2.18}$$

For areas with no charge, Poisson's equation becomes Laplace's equation,

$$\overrightarrow{\nabla}^2 V = 0. \tag{2.19}$$

2.5 Laplace's Equation in two dimensions

The first step in calculating the resistance with a known value for the potential at the ohmic contacts involves solving Laplace's equation,

$$\overrightarrow{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0, \tag{2.20}$$

in two dimensions[4].

The Python simulation will involve using the finite difference numerical method to solve Laplace's equation. The potential, at equally spaced grid points within the shape, will be calculated using the finite difference method. Therefore, the potential at any grid point, would be the average of the potential at the grid points surrounding it. The first step involves the Taylor series expansion [4] to find the forward and backward difference,

$$u(x+\Delta x) = u(x) + \frac{\partial u}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial^2 u}{\partial x^2}\Delta x^2 + ..., \tag{2.21}$$

as well as,

$$u(x-\Delta x) = u(x) - \frac{\partial u}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial^2 u}{\partial x^2}\Delta x^2 + ... \tag{2.22}$$

By adding Eq 2.21 and Eq 2.22 and rearranging for the second order derivative, we get the equation for the central difference[4],

$$\frac{\partial^2 u}{\partial x^2} = \frac{u(x+\Delta x) - 2u(x) + u(x-\Delta x)}{(\Delta x)^2}. \tag{2.23}$$

Applying Eq 2.23 to equation 2.20, we get

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{V(x+\Delta x) - 2V(x) + V(x-\Delta x)}{(\Delta x)^2} + \frac{V(y+\Delta y) - 2V(y) + V(y-\Delta y)}{(\Delta y)^2} = 0. \quad (2.24)$$

By setting Δx and Δy as 1 and rearranging for the potential, we get the equation for the approximation of the potential at any grid point inside the boundaries of the shape [4],

$$V_{x,y} = \frac{V_{x+1,y} + V_{x-1,y} + V_{x,y+1} + V_{x,y-1}}{4}. \quad (2.25)$$

As this will be solved on python, it will help to rewrite it in terms of unit vectors i and j,

$$V_{i,j} = \frac{V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1}}{4}, \quad (2.26)$$

a visual representation is shown in Figure 2.

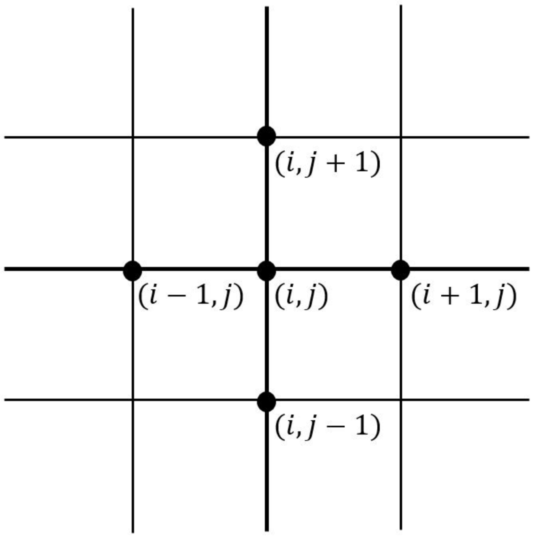

Figure 2: A visualisation of the method of calculating potential for a point on a grid using the neighboring points [5]

2.6 Boundary Conditions

While Eq 2.26 works fine for the interior of the shape, it does not work along the boundaries.

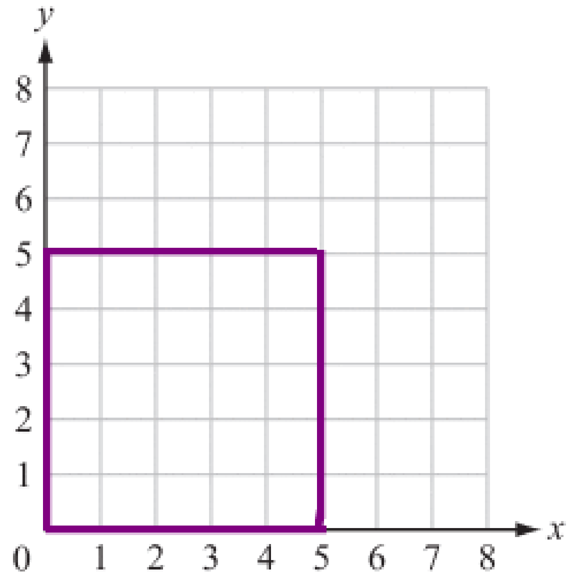

Figure 3: A 5 by 5 square on a Cartesian grid with ohmic contacts along x = 0 and x = 5 [6].

Using Figure 3 as an example, for the boundary along $y = 0$, Eq 2.26 becomes,

$$V_{i,0} = \frac{V_{i+1,0} + V_{i-1,0} + V_{i,1} + V_{i,-1}}{4}. \tag{2.27}$$

The term $V_{i,-1}$ is clearly outside the dimensions of the shape. similar issues would also arise at the boundary along $y = 5$. The boundaries along $x = 0$ and $x = 5$ however will not experience this issue, as they have fixed Dirichlet boundary conditions with values 0 and 100 respectively, as they represent the ohmic contacts. The value at other boundaries must be calculated with Neumann boundary conditions.

This involves using a ghost boundary outside of the shape, which would be along $y = -1$ for the boundary at $y = 0$. The vertical difference at any point along the boundary would then be [7],

$$V_{i,0} \approx \frac{V_{i,-1} - V_{i,1}}{2} = 0. \tag{2.28}$$

Rearranging for $V_{i,-1}$ and substituting back into Eq 2.27 results in [7],

$$V_{i,0} = \frac{V_{i+1,0} + V_{i-1,0} + 2V_{i,1}}{4}. \tag{2.29}$$

Applying the same principle for the bottom right corner results in [7],

$$V_{i,0} = \frac{V_{i-1,0} + V_{i,1}}{2}. \tag{2.30}$$

While Eq 2.29 and Eq 2.30 have been derived for the specified example, they can be easily adapted to boundaries and corners at different coordinates when working with other shapes. In summary, equations 2.26, 2.29 and 2.30 can be used to calculate the potential at all the grid points in a specified 2D box shape.

2.7 Electric Field, Current Density and Resistance

To calculate the resistance using Eq 2.4, both the potential difference and current are necessary. The potential difference is known as it is set as a boundary condition for the problem. The current can be calculated from the potential calculated in section 2.5 with a few more calculations, beginning with the $\vec{E}$.

Once the potential for all grid points has been calculated, the $\vec{E}$ can simply be calculated using Eq 2.12. The final step before calculating current involves $\vec{J}$. For a 2D problem, it is convenient to use the surface current density, $\vec{J}_{surf}$, defined as the current density multiplied by the thickness of the layer which the current passes through.

The total current, flowing from the source ohmic contact to the drain contact, in a 2D resistor, can be thought of as the surface current through any curve across the resistor as shown by the dashed line in Figure 4.

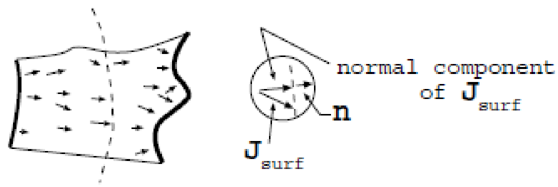

Figure 4: A visualisation of the curve used to calculate current in a 2D shape from the surface current density.

Furthermore, only the normal component of $\vec{J}_{surf}$ to the curve carries current, resulting in the current per unit length of the curve at that point. Therefore the total current would be the sum of all the normal components along the curve and can be calculated using the integral [2],

$$I = \int \vec{J}_{surf}.\vec{n}\,dl. \tag{2.31}$$

Once the current is known, resistance can simply be calculated using Eq 2.4 alongside the known value of potential difference.

2.8 Errors and Uncertainty

The focus of this report is on the finite difference method, which is derived from the Taylor series expansion, therefore the error comes from the accuracy of the approximation. Normally, The error for the Taylor series expansion is the difference between the 'true value' and the approximated value, which becomes more accurate with the number of derivatives taken. Our approach however has no true values for the potential, apart from the ohmic contacts which are fixed. The current approach only uses the second order central difference to calculate the potential, therefore the error is the sum of of all the derivatives ignored in the calculation.

However, calculating potential using derivatives of order greater than 2 becomes increasingly difficult. It would also be greatly limited because of computing power, which would make the process to calculate potential alone much longer. The effect of derivatives of higher orders will be very negligible with an increase in the number of iterations for each shape. Therefore, the uncertainty and error calculations have been omitted.

2.9 Python Simulation

The python simulation used a $n \times m$ matrix to model the shape in question. The matrix essentially creates evenly spaced grid points in the shape, where the potential was calculated with the appropriate equations. Starting with values of 0 at all points, apart from the ohmic contacts, the code ran for at least 10,000 iterations to calculate the potential. While a larger number of iterations would have provided a more accurate result, it would also greatly increase the run time and may require more computing power. A consequence of this was that only box shapes were modeled, mostly combinations between squares and rectangles.

3 Results

The resistance for 8 shapes, made using combinations of squares and rectangles, were calculated, including a square and a rectangle. While the square and the rectangle could be solved analytically, they have also been solved numerically to check the method. Current was calculated using the $\overrightarrow{J}$ at the ohmic contacts, as the current leaving one ohmic contact must be equal to the current entering the other, due to conservation of charge. This was then used to calculate the resistance using ΔV. Table 1 shows the current and resistance calculated for each shape.

Table 1: The values for resistance and current obtained for each shape through the simulation.

$Shape$	$Current(A)$	$Resistance(\Omega)$
1	100.00	1.000
2	25.00	4.000
3	59.68	1.676
4	40.25	2.484
5	46.98	2.128
6	22.00	4.544
7	16.44	6.081
8	18.56	5.387

3.1 Shape 1: Square

First, as seen in Figure 5, a simple square was modelled using the simulation. With a of 100 V, the equipotential lines maintain uniform spacing with each other. The $\overrightarrow{E}$ is also perpendicular to the potential and it flows from the 100 V ohmic contact to the 0 V ohmic contact. The resistance and current were calculated to be 1.00 $\Omega$and 100 A respectively. As explained in section 2.2, the resistance of a square could be used to analytically calculate the resistance of some simple shapes, using the ohms per square method. This is shown in the following section.

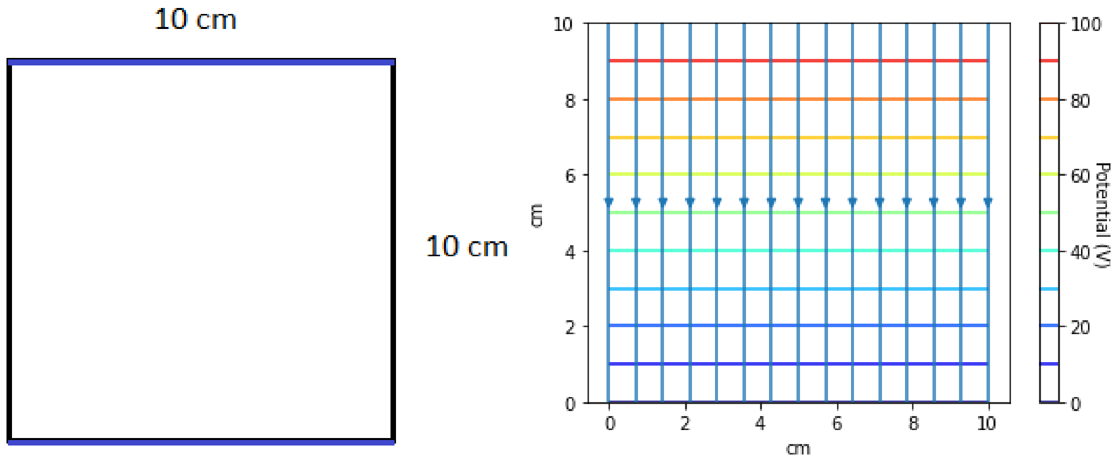

Figure 5: The diagram (left) and the plot (right) for a square. The diagram shows the dimensions of the square, with the ohmic contacts highlighted as blue, creating a Δ V of 100 V from the bottom to the top. The plot, made by using a 10×10 matrix to model the shape, shows the equipotential lines, whose values can be determined using the colour bar. The vertical blue lines show the direction of the $\overrightarrow{E}$ in the square as a result of the applied ΔV.

3.2 Shape 2: Rectangle

The second shape, as seen in Figure 6, is a rectangle with a length to width ratio of 4. Using the ohms per square method, where the resistance of a square is 1.00 Ω as calculated in the previous section, the expected resistance of this rectangle is 4 Ω.

The result from the simulation for the resistance and current was 4 Ω and 25.00 A respectively, as expected from the ohms per square method. The equipotential lines have uniform spacing, while the $\overrightarrow{E}$ flows perpendicular to the potential. This is the same pattern observed for the square, since they both have uniform cross sectional area.

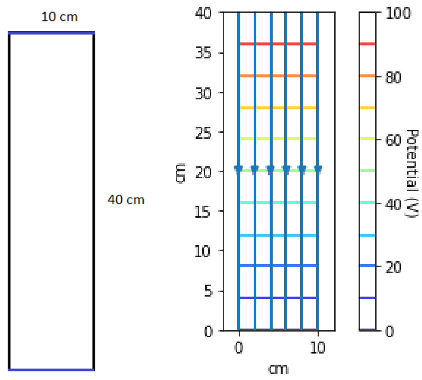

Figure 6: The diagram (left) and the plot (right) for a rectangle. The diagram shows the dimensions of the rectangle, with the ohmic contacts highlighted as blue, creating a Δ V of 100 V from the bottom to the top. The plot, made by using a 40 $\times$ 10 matrix to model the shape, shows the equipotential lines, whose values can be determined using the colour bar. The vertical blue lines show the direction of the $\overrightarrow{E}$ in the rectangle as a result of the applied ΔV.

3.3 Shape 3

Figure 7 shows shape 3, which is a combination of a square and a rectangle.

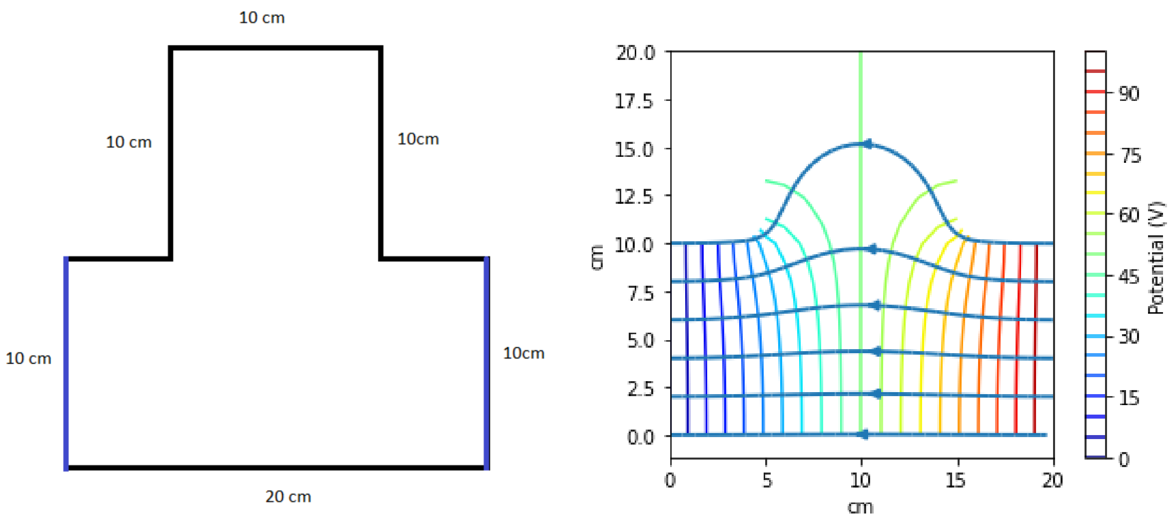

Figure 7: The diagram (left) and the plot (right) for shape 3. The diagram shows the dimensions of shape 3, a combination of a square and a rectangle with the ohmic contacts highlighted as blue, creating a Δ V of 100 V from left to right. The plot, made by using a 20 $\times$ 20 matrix to model the shape, shows the equipotential lines (increments of 5V), whose values can be determined using the colour bar. The horizontal blue lines show the direction of the $\overrightarrow{E}$ in shape 5 as a result of the applied ΔV.

Due to the non-uniform cross sectional area, the equipotential lines and the $\overrightarrow{E}$ are no longer uniform.

Furthermore, as the ohmic contacts are attached to the rectangle, all of the potential in the square arises from the rectangle. As the potential spreads out from the top of the rectangle, the density decreases with distance in the square. This is shown through the equipotential lines, which have the greatest density near the electrodes as well as at the bottom of the rectangle compared to the square.

The electric field lines are parallel to the potential, they also have the greatest density near the bottom of the shape at y = 0. The density decreases as y increases, as seen with the single field line going through the square. However, there are still some patterns to be observed. Due to the line of symmetry present at $x = 10$, the equipotential lines and the $\overrightarrow{E}$ are also symmetrical along $x = 10$, as seen with the equipotential of 50 V.

The calculation for the current is still simple as the total current leaving one electrode is equal to the total entering the second electrode through conservation of charge. The current was calculated to be 59.68 A, resulting in a resistance of 1.676 Ω. Without the square, the expected resistance would be 2 Ω using the principle of ohms per square.

3.4 Shape 4

Figure 8 shows shape 4, which is a combination of two squares and a rectangle.

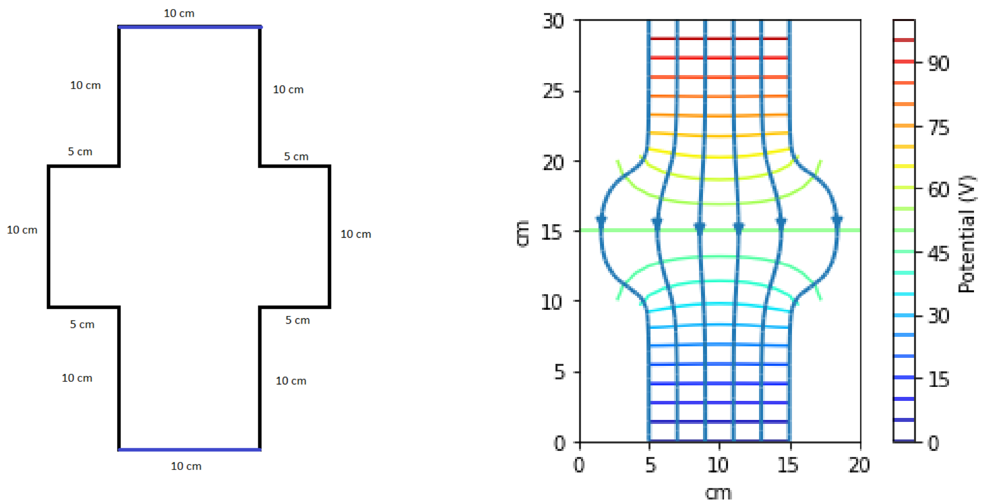

Figure 8: The diagram (left) and the plot (right) for shape 4. The diagram shows the dimensions of shape 4, a combination of two squares and a rectangle with the ohmic contacts highlighted as blue, creating a ΔV of 100 V from top to bottom. The plot, made by using a 30 × 20 matrix to model the shape, shows the equipotential lines (increments of 5V), whose values can be determined using the colour bar. The horizontal blue lines show the direction of the $\overrightarrow{E}$ in shape 5 as a result of the applied ΔV.

Shape 4 has two lines of symmetry, at $x = 10$ and $y = 15$, resulting in a symmetrical $\overrightarrow{E}$ and potential about those points. The equipotential lines have the greatest density at the center of the shape along $x = 10$, with 50

V along $y = 15$, the vertical halfway point in the shape. Since the ohmic contacts are attached at the top and at the bottom of the shape, the lowest density in potential occurs in the rectangle between $x = 0$ and $x = 5$ as well as between $x = 15$ and $x = 20$

The $\overrightarrow{E}$ also follows the same pattern as potential in regards to density throughout the shape. As most of the $\overrightarrow{E}$ is present in the centre of the shape, as seen with the increasing distance between the $\overrightarrow{E}$ lines at $x = 15$.

The resistance and current was calculated to be 2.484 Ω and 40.25 A respectively.

3.5 Shape 5

The dimensions and plot for shape 5 can be seen in Figure 9.

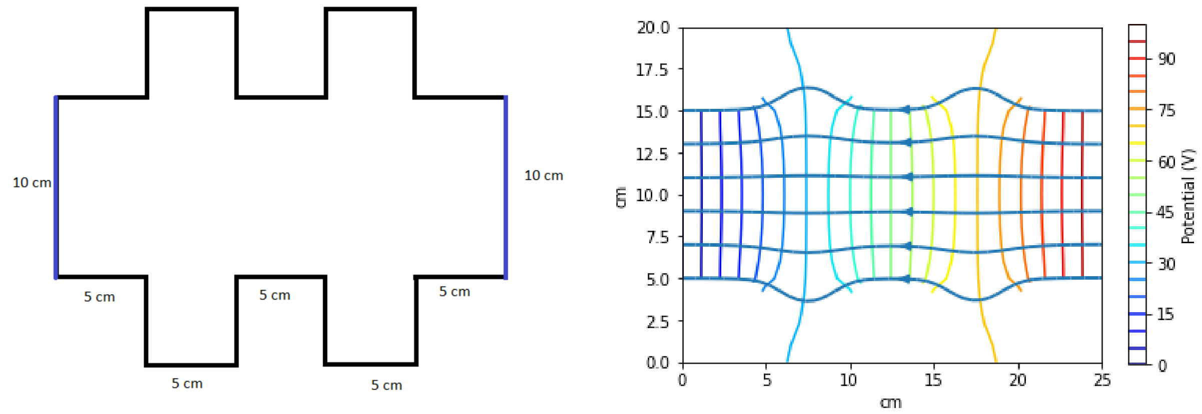

Figure 9: The diagram (left) and the plot (right) for shape 5. The diagram shows the dimensions of shape 5, a combination of 4 squares and a rectangle with the ohmic contacts highlighted as blue, creating a ΔV of 100 V from left to right. The plot, made by using a 20×25 matrix to model the shape,shows the equipotential lines (increments of 5V), whose values can be determined using the colour bar. The horizontal blue lines show the direction of the $\overrightarrow{E}$ in shape 5 as a result of the applied ΔV.

The $\overrightarrow{E}$ as well as the equipotential lines are symmetrical along the lines of symmetry of the shape at $y = 10$ and 12.5, with 50 V at $x = 12.5$. As the ohmic contacts are attached at $x = 0$ and $x = 25$, the density of the potential lines is greatest along $y = 10$, which is the centre of the shape. The potential in the 4 squares enters through the rectangle, whose density decreases with distance from the rectangle, resulting in the lowest density of potential in the shape to be in the squares.

The $\overrightarrow{E}$ follows the same pattern in terms of density in the shape, as seen with the increasing distance between the field lines at $x = 7.5$ and $x = 17.5$, which mark the centre of the squares. The resistance and current were calculated to be 2.128 Ω and 46.98 A respectively. Without the additional 4 squares, the resistance would be 2.5 Ω, using the squares per ohm principle.

3.6 Shape 6

Shape 6 is a reverse 'L' shape, with dimensions shown in Figure 10.

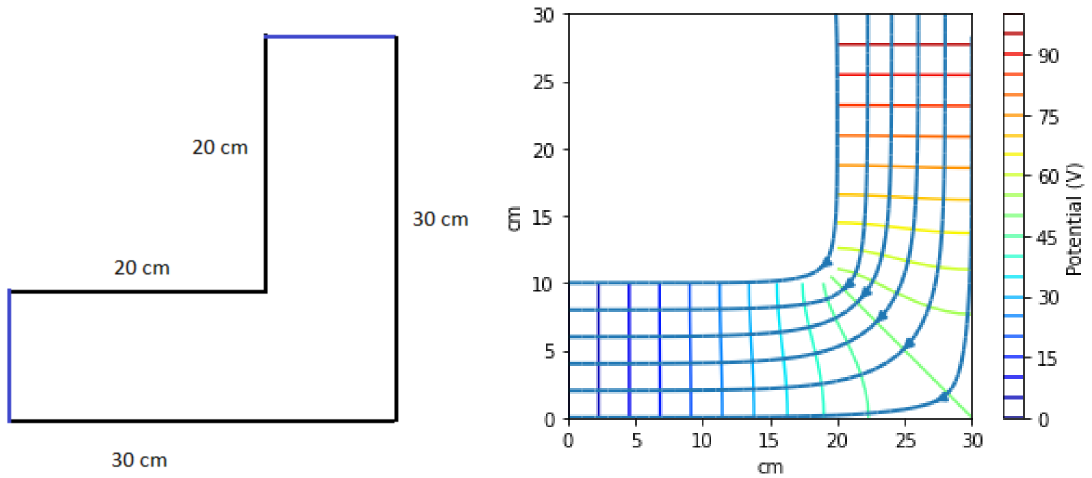

Figure 10: The diagram (left) and the plot (right) for shape 6. The diagram shows the dimensions of shape 6, a reverse 'L' with the ohmic contacts highlighted as blue, creating a ΔV of 100 V from the bottom left to top right. The plot, made by using a 30×30 matrix to model the shape, shows the equipotential lines (increments of 5V), whose values can be determined using the colour bar. The blue lines show the direction of the $\overrightarrow{E}$ in shape 6 as a result of the applied ΔV.

The shape only has one line of symmetry, which goes from the corner at $y = 10$ and $x = 20$ to $y = 0$ and $x = 30$. Both, the $\overrightarrow{E}$ and the potential is symmetrical about this line, as seen by the 50 V equipotential line along the line of symmetry. The density of the potential is greatest along the boundary at $x = 10$ as well as $y = 20$, as this marks the shortest route between the two ohmic contacts. The density in potential is lowest along the boundary at $x = 0$ and $y = 30$, the longest route between the ohmic contacts.

The $\overrightarrow{E}$ shows the same pattern, with the greatest density along the upper boundary of the shape. This can be seen in the increasing distance between the $\overrightarrow{E}$ lines in the bottom right corner of the shape. The resistance and current were calculated to be 4.544 Ω and 22.00 A. Since the shape could be divided into 5 squares, a analytical estimation would yield a resistance of 5 Ω, an increase of about 10%. So even though a rectangle of length 50 cm and width 10 cm could have the exact same area as shape 6, the resistance would still be different.

3.7 Shape 7

Shape 7 is essentially a lowercase 'n' shape, whose dimensions are shown in Figure 11.

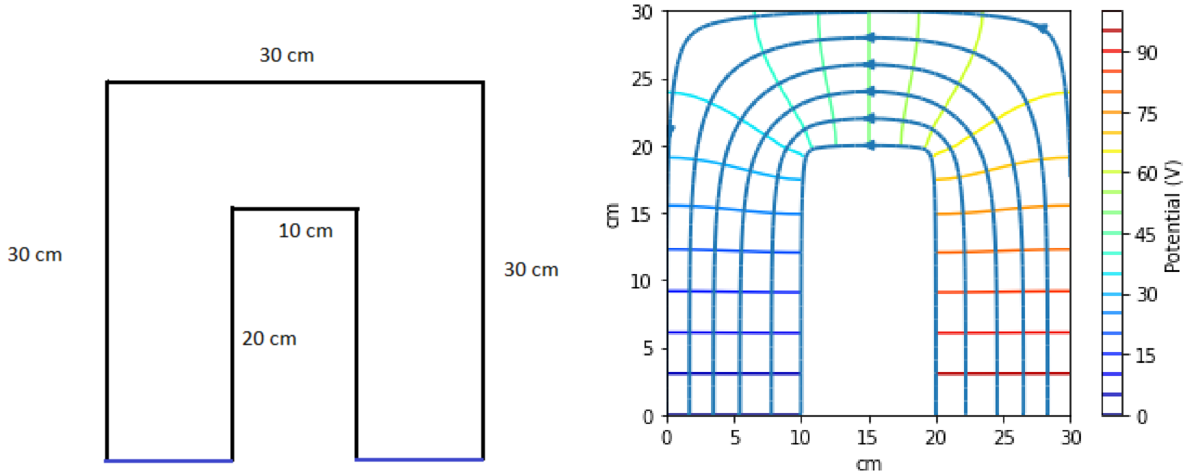

Figure 11: The diagram (left) and the plot (right) for shape 7. The diagram shows the dimensions of shape 7, a lowercase 'n' with the ohmic contacts highlighted as blue, creating a ΔV of 100 V from bottom left to bottom right. The plot, made by using a 30 $\times$ 30 matrix to model the shape, shows the equipotential lines (increments of 5V), whose values can be determined using the colour bar. The horizontal blue lines show the direction of the $\overrightarrow{E}$ in shape 7 as a result of the applied ΔV.

The $\overrightarrow{E}$ and the potential are symmetrical along the sole line of symmetry in the shape at $x = 15$, with a potential of 50 V along the line of symmetry. The greatest density in the potential occurs along the inner boundary of the shape, at $x = 10$, $y = 20$ and $x = 20$. This boundary marks the shortest distance between the two ohmic contacts. This density decreases towards the outer boundary of the shape, at $x = 0$, $y = 30$ and $x = 30$, where the density of potential is the lowest.

The $\overrightarrow{E}$ also has the greatest density along the inner boundary, while the lowest along the outer boundary, as seen with the increasing distance between the $\overrightarrow{E}$ lines in the top left and right corners. The resistance and current were calculated to be 6.081 Ω and 16.44 A. Shape 7 could be divided into 7 equal squares, an estimation for resistance through the ohms per square method would result in 7 Ω, an increase of about 15%.

3.8 Shape 8

Figure 12 shows shape 8, an 'F' shape.

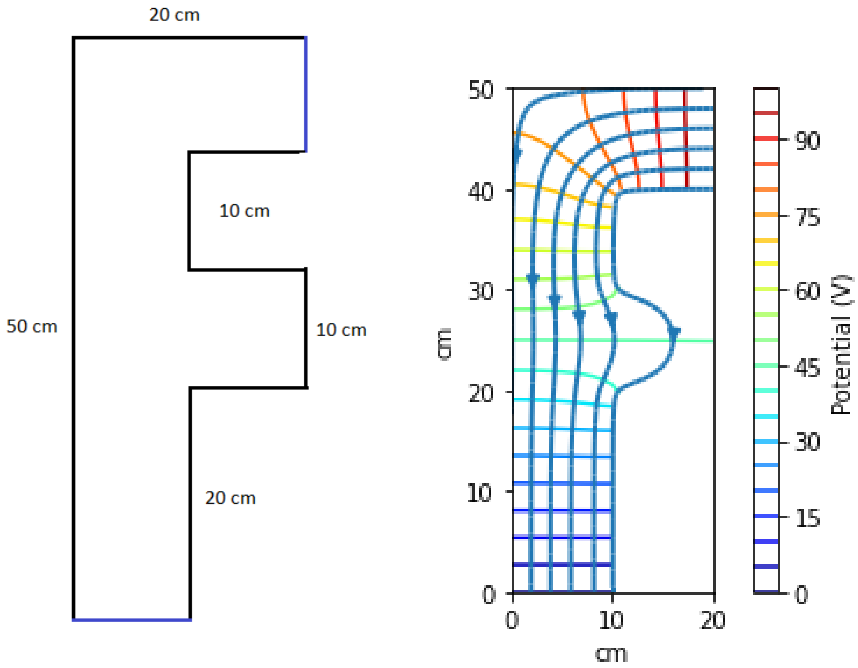

Figure 12: The diagram (left) and the plot (right) for shape 8. The diagram shows the dimensions of shape 8, an 'F' with the ohmic contacts highlighted as blue, creating a ΔV of 100 V from bottom left to top right. The plot, made by using a 50×20 matrix to model the shape, shows the equipotential lines (increments of 5V), whose values can be determined using the colour bar. The horizontal blue lines show the direction of the $\vec{E}$ in shape 7 as a result of the applied ΔV.

The potential has fairly uniform density from the bottom of the shape till around $y = 18$, past which the potential spreads into the square on the right, which has the lowest density of potential in the shape, decreasing the density of the potential. The density of potential increases between $y = 32$ and $y = 38$, past which point the density increases along the inner boundary. A result of this is the low potential density along the outer boundary and in the top left corner of the shape.

The $\vec{E}$ density is fairly uniform at the bottom of the shape but it decreases greatly in the square, as seen by the singular field line on the right. The density of the $\vec{E}$ is also lower in the top left corner of the shape, relative to the top corner in the inner boundary. The resistance and current were calculated to be 5.387 Ω and 18.56 A respectively.

3.9 Analysis of Method

While the aim of the project to simulate the resistance for 2D conductors was achieved, the disadvantages of the approach were also highlighted. The finite difference method only worked for box shapes, which can be separated into squares or rectangles. Another issue is that computing power greatly limited the size of the shapes that could be simulated, as a larger shape would have required a larger matrix.

4 Conclusion

The resistance for the square was 1 Ω whereas it was 4 Ω for the rectangle, with a length to width ratio of 4. The results for the square and the rectangle match the results predicted through an analytical approach using Eq 2.6, which shows a linear relationship between the resistance and length at a constant width. This helps support the reliability of the results obtained from the simulation of the other shapes as well as the accuracy of the numerical method. The values for resistance, from shape 3 to shape 8, were calculated to be 1.676 Ω, 2.484 Ω, 2.128 Ω, 4.544 Ω, 6.981 Ω and 5.387 Ω respectively. The patterns in potential and the $\overrightarrow{E}$ observed in the plots, such as the lines of symmetry present in Shape 5, help support the accuracy the of the finite difference method as well as the calculated resistance. Python can be used to simulate the resistance of 2D systems using the finite difference method, however, the shapes are limited to box type shapes.

For future practice, it would be better to increase the number of iterations for the code as well as to decrease the distance between each grid point, which would provide a more accurate approximation. Higher order derivatives could also be included for better approximation as well as a means to calculate the uncertainty in the final calculations.

5 Bibliography

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