Review of Existing Guidelines

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Innehåll

Review of Existing Guidelines

1 Reviewed Guidelines

There are, today, numerous of national existing guidelines in the design of FRP structures. There are two that are more widely spread: “Structural Design of Polymer Composites – Eurocomp” by The European Structural Polymeric Composites Group, published 2005 and “CUR 96 – Fibre Reinforces Plastics in Civil Engineering Supporting Framework” by Civieltechnisch Centrum Uitvoering Research en Regelgeving, published 2003. None of them are however codes.

In 2015 CEN/TC 250 published “CEN TC 250 WG4 Scientific and technical report. Prospect for new guidance in the design of FRP Structures”, document N1348. The intension of this document is to serve as a basis for further work to achieve a harmonized European view on the design of FRP structures.

In this review I will concentrate on CEN TC 250 WG4 Scientific and technical report, henceforth referred to as “CEN”, since it is a basis for the work of a new part of the EN Eurocodes. I will compare with the Eurocomp and CUR 96 since the authors of CEN have used the other two as a base.

1.1 Assumption

Like other Eurocodes the CEN is based on that a number of assumptions shall be fulfilled if the design and calculations should meet the requirements stated. For instance:

- The design and calculations as well as the manufacturing and production are carried out by qualified personnel and that the work fulfils appropriate standards.

- The structure will be used in accordance with the design assumptions.

Since FRP materials have a temperature-dependent behaviour, there are some assumption on temperature ranges for service conditions:

- Normal temperature, from – 20 ^{o}C to + 40 ^{o}C
- Elevated temperature up to T_g.

2 Basis of Design

2.1 Basic Requirements

FRP-structures should be designed and calculated in accordance with the rules given in EN-1990, EN-1991 and the associated National Annexes.

This follows the setup and spirit of the other parts of the eurocodes. The only odd thing is the emphasis that CEN refer to EN 1997:2011 Geotechnical design – Part 1.

2.2 Durability

The design of the structure should guarantee a constant performance over time in terms of serviceability, strength and stability taking into account both the environmental conditions as well as the maintenance programme. FRP structures should be designed so as to take into account

- The chemical-physical conditions in which the structure is used including UV; temperature; humidity, water and chemicals.
- Accidental loads as fire, lightning strike, impact and explosion

This also follows the intensions of the eurocodes, but since EN 1991-2 doesn´t use EN 1991-1-2 (Part 1-2: General actions – Actions on structures exposed to fire) maybe we could treat that as not a problem for the moment.

2.3 Verification by the Partial Factor Method

2.3.1 Load Effects Calculation

This follows the other parts of the eurocodes.

2.3.2 Design Values of the Properties of Materials, Elements and Products

$$X_d = \eta_c \cdot \frac{X_k}{\gamma_M}, \qquad (2.3)$$

Where X_k is the characteristic value of the property (tension, moment, deflection etc), γ_M is the partial factor covering uncertainty in the resistance model, and geometric deviations if these are not modelled explicitly. η_c is the conversion factor for all the environmental actions and long term effects affecting the behaviour of the material.

The explanation of γ_M differs from the explanation in chapter 2.3.4 that says that γ_M is the "partial material factor" linked to uncertainties in obtaining the correct material properties and production method.

2.3.3 Design Capacity

$$R_d = R\{X_{d,i}, a_{d,i}\}, \qquad (2.4)$$

Where $X_{d,i}$ is mechanical properties and $\alpha_{d,i}$ is geometric properties.

Number (3) says "For elastic moduli mean values can be used." Mean of what? Nominal? Characteristic? Or Design value?

2.3.4 Material Partial Factors

(1) In ULS the factor γ_M for an FRP laminate or structure should be calculated from:

$$\gamma_M = \gamma_{M1} \cdot \gamma_{M2}, \tag{2.6}$$

γ_{M1} is the material partial factor linked to uncertainties in obtaining the correct material properties.

γ_{M1} = 1.0 if production process and quality system are certified by an EOTA-member
1.15 if material properties are derived from tests
1.35 if material properties are derived from theoretical models or values from literature

γ_{M2} is the material partial factor owing to uncertainties due to the nature of the constituent parts depends on the production method. For post-cured laminates values are given I n Table 2.1.

Table 2.1 – Values of γ_{M2}.

Conditions	ULS (strength)	Local stability	Global stability
Production processes and properties of FRP[1] with $V_x \leq 0{,}10$	1.35	1.5	1.35
Production processes and properties of FRP[1] with $0{,}10 < V_x \leq 0{,}17$	1.6	2.0	1.5

[1] The variation coefficient V_x should be determined from tests (EN1990, Annex D).

For non-post cured laminates the values should be multiplied with 1.2.

(2) In SLS the factor γ_M = 1.0.

(3) In the case of sandwich structures with foam core values for γ_{M2} is given in Table 2.2.

(4) For other materials γ_{M2} should be derived by tests according to EN 1990, Annex D.

In case (3) and (4) CEN fail to specify whether it refers to the ULS, SLS or both.

Determining the material partial factor γ_M for joints follow the same approach as for laminates. For bonded joints values for γ_{M1} and γ_{M2} is given in Table 2.3. For bolted joints the partial factors for the joined laminates should be determined as for laminates stated above. The partial factors for the bolts themselves should be determined in accordance to that materials regulations (e.g. steel bolts in accordance to EN 1993-1-8)

This chapter is, except (3) and (4) above, clear to any designing engineer and could very well serve as a client demand.

2.3.5 Approach to Special Problems by Using Conversion Factors

In chapter 2.3.6 values for different conversion factors η_c are given. They are used in order to determine the reduced values of design parameters and follow from either environmental degradation effects or load duration effects.

“(2) Protective coverings already tested as able to mitigate the environmental degradation and to allow service life of the structure to remain unaltered, should be used in aggressive environments. In the presence of an adequate protective system able to counteract a specific environmental effect, the value of the corresponding conversion factor can be assumed to be equal to 1.0.”

Which conversion factor can be assumed to be equal to 1.0? And under which circumstances?

What do they mean by aggressive environment? Exposed to chemicals or an ordinary road environment?

What happens if protective coverings are used in an environment that is not classified as aggressive? Can the conversion factor be assumed to be >1.0?

2.3.6 Relevant Conversion Factors

$$\eta_c = \eta_{ct} \cdot \eta_{cm} \cdot \eta_{cv} \cdot \eta_{cf}, \qquad (2.7)$$

where:

- η_{ct} is the conversion factor for temperature effects;
- η_{cm} is the conversion factor for humidity effects;
- η_{cv} is the conversion factor for creep effects;
- η_{cf} is the conversion factor for fatigue effects.

If appropriate, other conversion factors can be added in the product above, for example in the case of alkaline attack, freezing-thawing cycles, etc.

Maybe we should have conversion factor for freezing-thawing cycles in Sweden since the period November to March is a constant ongoing freezing-thawing cycle in at least the southern parts of the country.

Temperature η_{ct}:

(1) For normal temperature service conditions (See § 1.4(2)), the conversion factor for temperature effects could be as follows:

- for verification of strength: $\eta_{ct} = 0.9$;
- for verification of deformability and stability:
  - at a service temperature of $T_d = T_g - 40$ °C: $\eta_{ct} = 1.0$,
  - at a service temperature of $T_g - 40$ °C $< T_d < T_g - 20$ °C: $\eta_{ct} = 0.9$;
- instead of the momentary T_g, the momentary HDT of the resin can be also used for the calculation.

The statement that $T_d = T_g - 40$ °C: $\eta_{ct} = 1.0$ is a bit strange. What if $T_g = 70$ °C and $T_d = 10$ °C?

Maybe the statement should be: $T_d \leq T_g - 40$ °C: $\eta_{ct} = 1.0$

Humidity η_{cm}:

(1) The values of the conversion factor for humidity effects η_{cm} could be those given in Table 2.5.

Table 2.5 – Values of η_{cm}

Media class	Conversion factor		Influence
	Cured	Non-post cured/ hardened	
I	1.0	1.0	Without influence, e.g. dry goods, indoor climate
II	0.9	0.8	Very small influence, outdoor climate, < 30°C
III	0.8	Not allowed	Small influence, continuously exposed to water, strong UV exposure, 30-40°C

Creep η_{ct}:

Compared with Eurocomp and CUR 96 the CEN has the intension is to be more specific with different duration classes and classifications of loads in to these classes.

Unfortunately, it empties out into a table that is not proofread. You don´t know which column to be read.

Table 2.8 – Conversion factors for creep effects.

Level of proof	Estimated loads	Conversion factors $\eta_{cv}(t_v)$ according to the tabular value $\eta_{cv,20}$ and the duration of exposure t_v					
tabular value $\eta_{cv,20}$ *(20 years)*	---	0.67	0.5	0.4	0.33	0.29	0.25
Permanent 50 years	permanent	0.65	0.48	0.38	0.31	0.27	0.23
Long-term 10 years	permanent, long	0.69	0.51	0.42	0.35	0.30	0.27
Medium-term 6 months	permanent, long, medium	0.74	0.59	0.49	0.43	0.38	0.34
Short-term 1 week	permanent, long, medium, short	0.80	0.67	0.59	0.53	0.49	0.45
Instantaneous 1 minute	permanent, long, medium, short, very short	1.00	1.00	1.00	1.00	1.00	1.00

(2) The corresponding modification conversion factor $\eta_{cv}(t_v)$ could be taken from the Figure 2.1.

(3) As alternative, the modification factor $\eta_{cv}(t_v)$ could be determined by the following relationship:

$$\eta_{cv}(t_v) = (\eta_{cv,20})^T, \quad T = 0.253 + \mathrm{Log}(t_v); \qquad (2.8)$$

Statement (3) gives values that are far from the values stated in Table 2.8.

Fatigue η_{cf}:

Like CUR 96 the CEN states that fatigue has an effect on both stiffness and strength. This results in a conversion factor for SLS. Fatigue as a ULS is dealt with in another chapter.

Adhesive joints

The presence of water, damp atmosphere or aggressive environment can dramatically lower the long term performance of the adhesive joint. Therefor these effects on stiffness and strength shall be determined by test for the specific adhesive type and adherent combination, surface treatment and curing process.

In preliminary design this can be taken care of by using the same conversion factors that are used for the FRP laminate.

3 Materials

Unlike other parts in the Eurocodes, like EN 1992-1-1 (concrete) and EN 1993-1-1 (steel) which refers to material standards or tables for material properties CEN says
(1) The materials used should be suitable for the intended application.

This places greater demands on the designer but also gives her greater freedom.

CEN is applicable for following types of

- fibres: E glass fibres, HS, HT and HM carbon fibre, aramid fibre.
- Thermoset resins: unsaturated polyester, vinyl ester, epoxy, phenolic resins, modified acrylic.

This is a major development since Eurocomp and CUR 96 only are applicable for GFRPs.

Annex B gives indicative values for fibre, resin, ply and laminate properties, to be used in preliminary design. Unfortunately tables with indicative property values for plies and laminates only exist for E glass reinforcement.

4 Durability

Durability in general is treated in the same way as other building materials. There is a focus on thermal effects. Not only fire but also if the structure is operating under raised temperature. If the glass transition temperature (T_g) of the used resin is reached the properties of the resin might change drastically.

5 Basis of Structural Design

5.1 Analysis Criteria

In this chapter, like in other parts of the Eurocode, criteria for analysis is stated. Most of them are obvious but they must be stated in a code like this.

6 Ultimate Limit States

6.1 General

Basic ULS verifications are presented for

- Profiles
- Laminated plates and shells
- Sandwich panels

In Appendix 1 “Comparing Calculation of a Simply Supported H-beam” a calculation of a 15 m long double symmetrical H beam under a uniformly distributed load is performed for ultimate limit state according to respectively CEN, CUR 96 and Eurocomp. The difference and similarities between the different guidelines approaches to various design criteria are compared.

6.2 Profiles

Normal forces, flexure, torsion and shear and combinations of them like “In-Plane Compression-Flexure”

6.2.1 Normal Force

Axial tension and compression are well described.

Tension:

$$N_{t,Sd} \leq N_{t,Rd} \tag{6.1}$$

Compression:

$$N_{c,Sd} \leq N_{c,Rd}\, . \tag{6.5}$$

and

$$N_{c,Rd} = \min\{N_{c,Rd1}, N_{c,Rd2}\}, \tag{6.6}$$

Where $N_{c,Rd1}$ is the design value of the compressive resistance of the profile and $N_{c,Rd2}$ the design compression value due to instability.
$N_{c,Rd2}$ can be determined either through numerical/analytical modelling or through tests.
In the case of double symmetric, pultruded, profiles evaluations of $N_{c,Rd2}$ is provided in Annex C.

6.2.2 Flexure

In-plane flexure is clearly described.

The design value of the bending moment should satisfy the conditions:

$$M_{Sd} \leq M_{Rd1}\, . \tag{6.8}$$

$$M_{Sd} \leq M_{Rd2} \tag{6.10}$$

Where M_{Rd1} is the design value of the flexural resistance of the profile and M_{Rd2} the design value due to instability.
In the case of double symmetric, pultruded, profiles evaluations of M_{Rd2} is provided in Annex D.

A development into making FRP structures more competitive vis-à-vis steel would be if these annexes could be valid for more than double symmetric, pultruded, profiles. For instance box girders with slightly inclined webs.

The combinations of bending moment and axial forces are clearly described.

6.2.3 Shear

The design value of the shear resistance V_{Rd1} is clearly described. But when it comes to the design resistance due to stability V_{Rd2} the authors must do a proper proofreading.

$$V_{Rd2} = V_{loc,Rd} = A_V \cdot f_{V,loc,k} \cdot \qquad (6.20)$$

The expression $f_{V,loc,k}$ is not described and in Annex E some equations are incorrect and some indexes changes through the equations.

Local verification on sections with concentrated loads (e.g. supports)

$$f_{Sd,z} \leq f_{Tc,Rd} , \qquad (6.21)$$

Unfortunately it is not described more than this. There should be some way to determine the participating width, which is a factor both in crushing and buckling. Compare with EN 1993-1-5, Chapter 6 of the steel beams. This is well described in Eurocomp (4.7.4).

6.2.4 Torsion

Torsion is clearly described.

6.3 Plates and shells

To levels of verifications can be considered:

Ply level or laminate level.

Ply level.

The verification can be carried out by using the Tsai-Hill criterion.

$$\left(\frac{\sigma_{1,Sd}}{\sigma_{1,Rd}}\right)^2 + \left(\frac{\sigma_{2,Sd}}{\sigma_{2,Rd}}\right)^2 + \left(\frac{\tau_{12,Sd}}{\tau_{12,Rd}}\right)^2 - \frac{\sigma_{1,Sd} \cdot \sigma_{2,Sd}}{\sigma_{i,Rd}^2} \leq 1 . \qquad (6.25)$$

Other criteria can be used as well.

Laminate level

Stability Verifications

Technical literature provides the values of the critical loads in many situations. In Annex F some of these values for orthotropic symmetrical laminates with a length/width ratio greater than 5 are presented.

Comparing with CUR 96 there are some questions. CEN Appendix F says:

(1) The stability of a plate under uniform compression loads should be verified from:

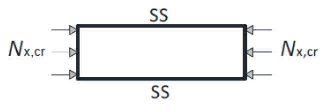
$$\frac{Q_{\mathrm{Ed,c}}}{Q_{\mathrm{Rd,c}}} \leq 1{,}0\ , \tag{15.1}$$

where:

$Q_{Ed,c}$ is the design value for an evenly distributed compression load occurring for each width unit (aligned in the longitudinal direction of the plate);

$Q_{\mathrm{Rd,c}}$ is the design value for buckling resistance and $N_{\mathrm{x,cr}}$ as indicated below.

(2) For a plate length L_y the following applies to the characteristic resistance to buckling:



$$N_{x,cr} = \frac{\pi^2}{L_y^2}\left[2\sqrt{D_{11}D_{22}} + 2(D_{12} + 2D_{66})\right]$$

$N_{x,cr}$ normally stands for Critical buckling force which is the first step towards determining the design value for buckling resistance. The CEN doesn´t tell you how to calculate $Q_{Rd,c}$. Shall the equations in appendix C be used?

In CUR 96 the same equation is used to compare directly with the occurring load.

6.4 Sandwich Panels

In this chapter a number of failure modes are listed. Demands and requirements, especially for preliminary design are well presented.

There are some improvements that are needed though.

A basic figure showing what the x, y, z axes means and what h, b, t_i etc. stand for. A text like the one in (2), shown below would be much easier to understand with a figure.

> (2) In the following the symbols *x* and *y* denote the principal axes of orthotropy of the two faces, supposed to be balanced symmetric laminates with fibres lying along the two orthogonal directions *x* and *y*. The axis orthogonal to the face plane is denoted by *z*. The facings have the same principal axes of orthotropy. The principal axes of orthotropy coincide with the axes of moments. All the relations that are given are expressed in the coordinate system defined by the principal axes of orthotropy.

In all the different failure modes that are described there is a figure to clarify the problem. But they all seem to be in the wrong direction. It is like they all are carrying the load in the weakest direction.

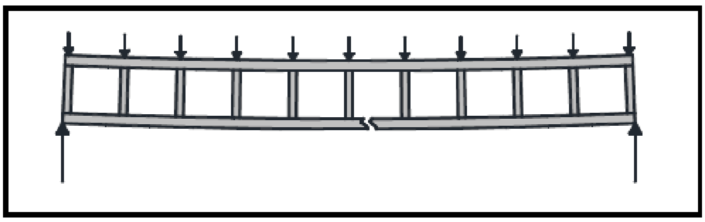


Figure 6.2 – Tensile facing failure.

6.5 Fatigue

Under (1) the description of when fatigue should be taken into account is not in harmony with the rest of the Eurocodes. Reference should be made to, for instance, EN-1991-2

(1) For structures subject to cyclic variations in the size of the load, and where the number of expected load cycles is expected to exceed 5000, while causing the peak stress from cyclic and permanent loads to exceed 15% of the material's design strength, or where the absolute maximum value of the cyclic load is greater than 40 % of the fully factored load, fatigue should be taken into account.

7 Serviceability Limit States

7.1 General

This chapter is clear. Deformation and vibration criteria are treated the same way as in other Eurocodes.

The effect on stiffness due to ageing and creep shall be considered when calculating deformation and vibration, but in Table 2.4 the conversion factor for creep is neglected for "Momentary deformation (SLS)" and "Comfort (vibrations) (SLS)". This feels like a contradiction.

8 Connections

8.1 General

This chapter is clear. Bolted, riveted, bonded or a hybrid combination of these three methods of connections are described.

8.2 Design Criterial

(2) In the case of multi-bolted joints, the shear force on a single bolt cannot be evaluated by statics, as in the case of ductile metals where gross yielding occurs at ULS.

8.3 Bolted Joints

This chapter is clear and easy to follow.

8.4 Adhesive Bonded Joints

8.4.1 General

(1) Bonded connection are not allowed for primary load bearing components, where failure of the connection could lead to progressive collapse or unacceptable risks. In these situations, their use is only allowed in combination with bolted or riveted connections or an alternative backup solution.

Why? Do we lack research in this matter or do adhesive joints have a history of breaking? Or is it psychological?

9 Production, Realization, Management and Maintenance

9.1 General

We haven´t focused on this chapter, but it looks like the major aspects are treated.

Summary

The CEN/TC 250 N1348 document works perfect as a basis for further work. There are some issues and questions regarding the conversion factors and there is a need for a thorough check out of the document regarding equations, indexes and so on.

What we are missing in a larger scale are guidelines for more general sections than pultruded profiles with double symmetric section.

We also need to focus a little bit on stability issues from a more general point of view than what is shown in Annex C to F.

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Appendix 1 Comparing Calculation of a Simply Supported H-beam

Following calculations are meant to compare the results when using different guidelines. It will also see if there is any lack of information that would be of that art that the calculations can´t be done properly. The guidelines are CEN 250/TC 250 N1348 (henceforth referred to as CEN), CUR 96 and Eurocomp.

1. Basic conditions

Length $L = 15$ m

Width/girder $w = 2$ m

Load $q = 5$ kN/m^2

Horizontal load $H = 0{,}6 \times q \times w \times L = 90$ kN

The load-duration is assumed to be short (~10 h).

The production process is assumed to be done under the best circumstances available and the FRP is post-cured.

Outdoor climate $-20^oC < t < 30^oC$, close to seawater.

1.1. Material

Pultruded E-glass FRP with fibre volume $V_f = 50\%$ CEN Table 11.13

$T_g = 80^oC$

$E_{1c} = E_{1t} = (E_L) = 25{,}8$ GPa

$E_{2t} = E_{2c} = (E_T) = 15{,}6$ GPa

$G_{12} = (G_{LT}) = 5{,}6$ GPa

$v_{12} = (v_{LT}) = 0{,}32$

$v_{21} = (v_{TL}) = v_{12} * E_2 / E_1 = 0{,}20$

If we assume a girder with a double symmetrical section somewhat similar to a HEB 900 steel girder we will have:

$b_f = 400$ mm

$t_f = 35$ mm

$h_w = 830$ mm

$t_w = 18{,}5$ mm

$A = 43{,}355 \times 10^{-3}$ m^2

$I_x = 6{,}122 \times 10^{-3}$ m^4

$I_y = 0{,}3738 \times 10^{-3}$ m^4

$I_\omega = 0{,}3215 \times 10^{-3}$ m^6

$$D_{11} = \frac{E_{Lc} \cdot t^3}{12 \cdot (1 - \nu_{LT} \cdot \nu_{TL})},$$

$$D_{12} = \nu_{LT} \cdot D_{22}$$

$$D_{22} = \frac{E_{Tc} \cdot t^3}{12 \cdot (1 - \nu_{LT} \cdot \nu_{TL})}$$

$$D_{66} = \frac{G_{LT} \cdot t^3}{12}$$

Flange:

D_{11} = 0,0985 MNm

D_{12} = 0,0191 MNm

D_{22} = 0,0595 MNm

D_{66} = 0,0200 MNm

Web:

D_{11} = 0,0145 MNm

D_{12} = 0,0028 MNm

D_{22} = 0,0088 MNm

D_{66} = 0,0030 MNm

Failure strain is assumed to be ε =1,8% CEN 11.6.3 (2)

$\sigma_{1tR} = E_1 \times \varepsilon$ =25800 x 0,018 = 464,4 MPa

σ_{1cR} = 464,4 MPa

σ_{2tR} = 280,8 MPa

σ_{2cR} = 280,8 MPa

τ_{12R} = 100,8 MPa

1.2. Safety- and Conversion Factors

1.2.1. CEN:

$R_d = \eta_C \times R_k / \gamma_M$

ULS

Partial Factors CEN 2.3.4

$\gamma_M = \gamma_{M1} \times \gamma_{M2}$.

γ_{M1} = 1.0

γ_{M2} = 1.35

Conversion Factors CEN 2.3.6

$\eta_C = \eta_{Ct} \times \eta_{Cm} \times \eta_{Cv} \times \eta_{Cf}$

η_{Ct} = 0.9

η_{Cm} = 0.8

η_{Cv} = 0.67 This is a guess and probably a low value but the tables and equations are not understandable.

η_{Cf} = 1.0

SLS (deformation)

Partial Factors CEN 2.3.4

γ_{M1} = 1.0

γ_{M2} = 1.0

Conversion Factors CEN 2.3.6

η_{Ct} = 1.0 This is a guess since 2.3.6.1 has its lowest level at T_g - 40°C = 80-40 = 40°C

η_{Cm} = 0.8

η_{Cv} = 1.0 This is not clear. (2.3.6 Table 2.4 versus 7.1 (2))

η_{Cf} = 0.9

1.2.2. CUR 96:

$R_d = R_k / (\gamma_m \times \gamma_C)$

ULS

Partial Factors 6.3

$\gamma_M = \gamma_{M1} \times \gamma_{M2.} \geq 1.5$
$\gamma_{M1} = 1.35$
$\gamma_{M2} = 1.1$

Conversion Factors 6.4

$\gamma_C = \gamma_{Ct} \times \gamma_{Cv} \times \gamma_{Ck} \times \gamma_{Cf}$
$\gamma_{Ct} = 1.1$
$\gamma_{Cv} = 1.1$
$\gamma_{Ck} = 1.0$
$\gamma_{Cf} = 1.0$

SLS (deformation)

Partial Factors

$\gamma_{M1} = 1.0$
$\gamma_{M2} = 1.0$

Conversion Factors

$\gamma_{Ct} = 1.0$
$\gamma_{Cv} = 1.1$
$\gamma_{Ck} = t^n = 10^{0,04} = 1.0965$ (10 h, fabric strip)
$\gamma_{Cf} = 1.1$

1.2.3. Eurocomp:

$R_d = R_k / \gamma_m$

ULS

Partial Factors 2.3.3.2

$\gamma_m = \gamma_{m1} \times \gamma_{m2} \times \gamma_{m3} \geq 1.5$
$\gamma_{m1} = 1.15$
$\gamma_{m2} = 1.1$
$\gamma_{m3} = 1.0$

SLS 2.3.4

Partial Factors

$\gamma_m = \gamma_{m1} \times \gamma_{m2} \times \gamma_{m3} \geq 1.3$
$\gamma_{m1} = 1.15$
$\gamma_{m2} = 1.1$
$\gamma_{m3} = 1.0$

2. Ultimate Limit State

2.1. CEN

Normal force 6.2.1

$N_{t,Sd} \leq N_{t,Rd}$

$N_{t,Rd} = A^* f_{t,d}$,

$f_{t,d} = \eta_C \text{ x } f_{t,k} / \gamma_M =$ 0,9* 0,8* 0,67*1,0*464,4/(1,0* 1,35) = 0,4824/1,35*464,4 = 165,9 MPa

$N_{t,Rd} = 0{,}04335 * 165{,}9*10^3 = 7193$ kN

$N_{c,Sd} \leq N_{c,Rd}$

$N_{c,Rd} = \min (N_{c,Rd1}, N_{c,Rd2})$

$N_{c,Rd1} = A^* f_{c,d}$

$f_{c,d} = \eta_C{}^* f_{c,k} / \gamma_M = f_{t,d} = 165{,}9$ MPa

$N_{c,Rd2}$ is calculated according to Annex C(Chapter 12).

$N_{c,Rd2} = \chi^* N_{loc,Rd}$ 12.1

$N_{loc,Rd} = A^* f^{axial}_{loc,d}$

$$f^{\text{axial}}_{\text{loc,d}} = \frac{\eta_c}{\gamma_M} \cdot \min\{(f^{\text{axial}}_{\text{loc,k}})_f, (f^{\text{axial}}_{\text{loc,k}})_w\}$$

$$\left(f^{\text{axial}}_{\text{loc,k}}\right)_f = \left(f^{\text{axial}}_{\text{loc,k}}\right)^{SS}_f = 4 \cdot G_{LT} \cdot \left(\frac{t_f}{b_f}\right)^2 \quad = 4*5600*(35/400)^2 = 171{,}5 \text{ MPa}$$

$$\left(f^{\text{axial}}_{\text{loc,k}}\right)_w = \left(f^{\text{axial}}_{\text{loc,k}}\right)^{SS}_w = k_c \cdot \frac{\pi^2 \cdot E_{Lc} \cdot t_w^2}{12 \cdot (1 - \nu_{LT} \cdot \nu_{TL}) \cdot b_w^2}, \quad = 31{,}05 \text{ MPa}$$

$$k_c = 2 \cdot \sqrt{\frac{E_{Tc}}{E_{Lc}}} + 4 \cdot \frac{G_{LT}}{E_{Lc}} \cdot \left(1 - \nu_{LT}^2 \cdot \frac{E_{Tc}}{E_{Lc}}\right) + 2 \cdot \nu_{LT} \cdot \frac{E_{Tc}}{E_{Lc}} \quad = 2{,}7566$$

$f^{axial}_{loc,d} = 0{,}4824/1{,}35*31{,}05 = 11{,}1$ MPa

$N_{loc,Rd} = 43{,}355 * 10^{-3} *11{,}1 = 0{,}481$ MN = 481 kN

$$\lambda = \sqrt{\frac{N_{\text{loc,Rd}}}{N_{\text{Eul}}}},$$

$$N_{\text{Eul}} = \frac{\eta_c}{\gamma_M} \cdot \frac{\pi^2 \cdot E_{Lc} \cdot I_{\min}}{L_0^2} \quad = 2475{,}7 \text{ kN}$$

$L_0 = L = 15$ m

$I_{min} = I_X = 6{,}122 *10^{-3}$ m^4 The girder is restrained by the bridge deck in the weaker direction.

$\lambda = 0{,}455$

$\Phi = (1+\lambda^2)/2 = 0{,}604$

$$\chi = \frac{1}{c \cdot \lambda^2} \cdot \left(\Phi - \sqrt{\Phi^2 - c \cdot \lambda^2}\right) \quad = 0{,}923$$

$N_{c,Rd2} = \chi^* N_{loc,Rd} = 480{,}9*0{,}923 =$ **<u>443,9 kN</u>**

$N_{c,Rd1} = A^* f_{c,d} = 0{,}04335*165{,}9*10^3 = 7194$ kN

$N_{c,Sd} = 1{,}5*90 = 135$ kN

Flexure 6.2.2

$M_{Sd} \leq M_{Rd1}$, M_{Rd2}

$M_{Rd1} = \min\{W^* f_{t,d}, W^* f_{c,d}\}$

M_{Rd2} is calculated according to Annex D (Chapter 13).

$M_{Rd2} = \chi_{FT}^* M_{loc,Rd}$ 13.1

$$M_{loc,Rd} = W \cdot f_{loc,d}^{flex}$$

$$f_{loc,d}^{flex} = \frac{\eta_c}{\gamma_M} \cdot \min\{(f_{loc,k}^{axial})_f, (f_{loc,k}^{flex})_w\}$$

$$\left(f_{loc,k}^{flex}\right)_w = \left(f_{loc,k}^{flex}\right)_w^{SS} = k_f \cdot \frac{\pi^2 \cdot E_{Lc} \cdot t_w^2}{12 \cdot (1 - \nu_{LT} \cdot \nu_{TL}) \cdot b_w^2} \quad = 196{,}84 \text{ MPa}$$

$$k_f = 13.9 \cdot \sqrt{\frac{E_{Tc}}{E_{Lc}}} + 22.2 \cdot \frac{G_{LT}}{E_{Lc}} \cdot \left(1 - \nu_{LT}^2 \cdot \frac{E_{Tc}}{E_{Lc}}\right) + 11.1 \cdot \nu_{LT} \cdot \frac{E_{Tc}}{E_{Lc}} \quad = 17{,}477$$

$$\left(f_{loc,k}^{axial}\right)_f = \left(f_{loc,k}^{axial}\right)_f^{SS} = 4 \cdot G_{LT} \cdot \left(\frac{t_f}{b_f}\right)^2 \quad = 171{,}5 \text{ MPa}$$

$M_{loc,Rd} = (04824/1{,}35)*171{,}5*10^3 *(0{,}006122/0{,}45) = 833{,}7$ kNm

$$\chi_{FT} = \frac{1}{c \cdot \lambda_{FT}^2} \cdot \left(\Phi_{FT} - \sqrt{\Phi_{FT}^2 - c \cdot \lambda_{FT}^2}\right) \quad (13.6)$$

The symbols introduced in (13.6) have the following meaning:

c is a coefficient which, in the absence of a more accurate evaluation, can be assumed to be equal to 0.70;

$$\Phi_{FT} = \frac{1 + \lambda_{FT}^2}{2};$$

$$\lambda_{FT} = \sqrt{\frac{M_{loc,Rd}}{M_{FT}}}.$$

$$M_{FT} = \frac{\eta_c}{\gamma_M} \cdot \frac{\pi^2}{L^2} \cdot E_{Lc} \cdot I_{min} \cdot \sqrt{\frac{I_\omega}{I_{min}} \cdot \left(1 + \frac{G_{LT} \cdot I_t}{E_{Lc} \cdot I_\omega} \cdot \frac{L^2}{\pi^2}\right)} \quad = 622{,}2 \text{ kNm}$$

$\lambda_{FT} = 1{,}158$

$\phi_{FT} = 1{,}17$

$\chi_{FT} = 0{,}548$

$M_{Rd2} = 456{,}5$ kNm

$M_{Rd1} = 0{,}006122/0{,}45 * 165{,}9*10^3 = 2257$ kNm

$M_{Sd} = 1{,}5*5*2*15^2/8 = 421{,}8$ kNm

Combination of Bending and Axial Compression Force 6.2.2.3

$$\frac{N_{c,Sd}}{N_{c,Rd1}} + \frac{M_{Sd}}{M_{Rd1}} \leq 1,$$ ⇔ 135/7194+421,8/2257 = 0,246 < 1,0 OK!

$$\frac{N_{c,Sd}}{N_{c,Rd2}} + \frac{M_{Sd}}{M_{Rd2} \cdot \left(1 - \frac{N_{c,Sd}}{N_{Eul}}\right)} \leq 1$$ ⇔ 135/446,6+421,8/(456,5*(1-135/2475,7)) = 1,28

Shear 6.2.3

$V_{Sd} \leq V_{Rd}$

$V_{Rd} = \min(V_{Rd1}, V_{Rd2})$

$V_{Rd1} = A_v * f_{V,Rd}$ (6.19)

$V_{Rd2} = A_v * f_{V,loc,k}$ (6.20)

This is probably incorrect. It should at least be with subscript d instead of k.
When referred to “AnnexE” $f_{V,loc,k}$ change name to $f_{loc,k}^{axial}$.
If we assume that they are the same we can follow the calculations in Annex E (Chapter 14):

$$\left(f_{loc,k}^{axial}\right)_f^{SS} = \frac{12 \cdot (D_{66})_f}{t_f \cdot \left(\frac{b_f}{2}\right)^2}, \qquad (14.6)$$

$$\left(f_{loc,k}^{axial}\right)_w^{SS} = \frac{\pi^2}{t_w \cdot b_w^2} \cdot \left\{2 \cdot \sqrt{(D_{11})_w \cdot (D_{22})_w} + 2 \cdot \left[(D_{12})_w + 2 \cdot (D_{66})_w\right]\right\}. \qquad (14.7)$$

$$\tilde{k} = \frac{(D_{22})_w}{b_w} \cdot \left[1 - \frac{t_f \cdot \left(f_{loc,k}^{axial}\right)_f^{SS} \cdot \frac{1}{(E_{Lc})_f \cdot t_f}}{t_w \cdot \left(f_{loc,k}^{axial}\right)_w^{SS} \cdot \frac{1}{(E_{Lc})_w \cdot t_w}}\right]. \qquad (14.5)$$

The equation 14.5 is probably incorrect to, since the result will be negative if the flange has larger capacity than the web and also because the “t_f´s” and “t_w´s” evens out each other.
In this calculation I have guessed that it should look like this:

$\tilde{k} = (D_{22})_w/b_w*[(1-(f_{loc,k}^{axial})^{SS}_f*(E_{Lc})_w)/((f_{loc,k}^{axial})^{SS}_w*(E_{Lc})_f)] = 18,16$

$$\zeta = \frac{(D_{22})_f}{\tilde{k} \cdot \frac{b_f}{2}} = 0,02$$

$$\rho = \frac{(D_{12})_f}{2 \cdot (D_{66})_f + (D_{12})_f} = 0,323$$

$$\eta = \frac{1}{\sqrt{1 + (7.22 - 3.55 \cdot \rho) \cdot \zeta}} = 0,95$$

$$K = \frac{2 \cdot (D_{66})_f + (D_{12})_f}{\sqrt{(D_{11})_f \cdot (D_{22})_f}}. \quad = 0,771$$

$$\left(f_{\mathrm{loc,k}}^{\mathrm{axial}}\right)_f = \frac{\sqrt{(D_{11})_f \cdot (D_{22})_f}}{t_f \left(\frac{b_f}{2}\right)^2} \left\{ K \cdot \left[15.1 \cdot \eta \cdot \sqrt{1-\rho} + 6 \cdot (1-\rho) \cdot (1-\eta)\right] + \frac{7 \cdot (1-K)}{\sqrt{1+4.12 \cdot \zeta}} \right\}, \text{ for } K \le 1 \qquad (14.3)$$

$$\left(f_{\mathrm{loc,k}}^{\mathrm{axial}}\right)_f = \frac{\sqrt{(D_{11})_f \cdot (D_{22})_f}}{t_f \cdot \left(\frac{b_f}{2}\right)^2} \cdot \left[15.1 \cdot \eta \cdot \sqrt{1-\rho} + 6 \cdot (1-\rho) \cdot (K-\eta)\right], \text{ for } K > 1. \qquad (14.4)$$

$$\left(f_{\mathrm{loc,k}}^{\mathrm{axial}}\right)_f = \cdot \quad (14.3) \qquad = 592,8 \text{ MPa}$$

$$GI_t = 4 \cdot (D_{66})_f \cdot b_f \cdot \left[\frac{1 - \left(f_{\mathrm{loc,k}}^{\mathrm{axial}}\right)_w^{SS} \cdot \left(\frac{1}{E_{Lc} \cdot t_w}\right)}{\left(f_{\mathrm{loc,k}}^{\mathrm{axial}}\right)_f^{SS} \cdot \left(\frac{1}{E_{Lc} \cdot t_f}\right)} \right] \quad = 0,1576$$

$\zeta = \dfrac{(D_{22})_w}{(GI_t) \cdot \frac{b_w}{2}}$; It shall probably be ζ' instead. = 0,1345

$$\xi' = \frac{1}{1 + 10 \cdot \zeta'} \quad = 0,4265$$

$$\left(f_{\mathrm{loc,k}}^{\mathrm{axial}}\right)_w = \frac{\pi^2}{t_w \cdot b_w^2} \cdot \left\{ 2 \cdot \sqrt{1 + 4.139\xi'} \cdot \sqrt{(D_{11})_w \cdot (D_{22})_w} + \left(2 + 0.62 \cdot \xi'^2\right) \cdot \left[(D_{12})_w + 2 \cdot (D_{66})_w\right] \right\} \quad = \underline{43,4 \text{ MPa}}$$

$V_{Rd1} = A_v * f_{V,Rd}$ = (0,4824/1,35)*0,0185*0,83*25800*10^3*1,8/100 = 2548 kN

$V_{Rd2} = A_v * f_{V,loc,d}$ = (0,4824/1,35)*0,0185*0,83*43,4*10^3*= 238 kN

V_{Sd} = 1,5*2*5*15/2 = 112,5 kN

2.2. CUR 96

$$\left(\frac{\sigma_{1,Sd}}{\sigma_{1,Rd}}\right)^2+\left(\frac{\sigma_{2,Sd}}{\sigma_{2,Rd}}\right)^2+\left(\frac{\tau_{12,Sd}}{\tau_{12,Rd}}\right)^2-\frac{\sigma_{1,Sd}\cdot\sigma_{2,Sd}}{\sigma_{i,Rd}^2}\leq 1$$

$\sigma_{1,Sd} = N_{c,Sd}/A + M_{Sd}*z/I_x$ = 34,2 MPa

$\sigma_{2,Sd} =$ = 0 MPa

$\tau_{12,S} = V*S/(I_x*t_w)$ = 7,6 MPa

$\sigma_{1,Rd} = \sigma_{1cR}/(\gamma_M*\gamma_C)$ = 464,4/(1,43*1,5) = 216,5 MPa

$\sigma_{2,Rd} = \sigma_{2cR}/(\gamma_M*\gamma_C)$ = 280,8/(1,43*1,5) = 130,9 MPa

$\tau_{12Rd} = \tau_{12R} /(\gamma_M*\gamma_C)$ = 100,8/(1,43*1,5) = 47,0 MPa

$$\left(\frac{\sigma_{1,Sd}}{\sigma_{1,Rd}}\right)^2+\left(\frac{\sigma_{2,Sd}}{\sigma_{2,Rd}}\right)^2+\left(\frac{\tau_{12,Sd}}{\tau_{12,Rd}}\right)^2-\frac{\sigma_{1,Sd}\cdot\sigma_{2,Sd}}{\sigma_{i,Rd}^2} = (34,2/216,5)^2+0+(6,7/47)^2+0 = 0,051 \leq 1,0$$

Buckling of a right-angled plate.

If we assume that the web is clamped to the flanges:

$N_{x,k} = 4.52*\pi^2/b_w{}^2*[(D_{11}*D_{22})^{1/2}+(D_{12}+2D_{66})/1.84]$ = 1039,4 kN/m

$N_{c,Rd} = N_{x,k}* b_w/(\gamma_c* \gamma_M)$ = 402,2 kN

2.3. Eurocomp

Normal force:

$N_{t,Sd} \leq N_{t,Rd}$

$N_{t,Rd} = A^* \sigma_{x,t,k}/\gamma_M = 0{,}04335*464{,}4*10^3/1{,}5 = 13421$ kN

$N_{c,Sd} \leq N_{c,Rd}$

$N_{c,Rd} = k*\pi^2*E_{1c}*I_x/(L^{2}*\gamma_M) = 6928$ kN (4.7)

k = 1

$I_{min} = I_X = 6{,}122*10^{-3}$ m^4 The girder is restrained by the bridge deck in the weaker direction.

Web buckling, or "an element of the section which can be defined as a long rectangular plate with the two longer edges simply supported".

D_x, D_y etcetera are basically the same as D_{11}, D_{22} mentioned above.

$$\sigma_{c,cr,y} = 2\pi^2\{(D_xD_y)^{1/2} + H_0\}/tb^2 \quad (4.9)$$

where:

$$H_0 = ½\ (\nu_{xy}D_y + \nu_{yx}D_x) + 2(G_{xy}t^3/12)$$

$\sigma_{c,cr,yw} =$ = 31,1 MPa

Flange buckling, "an element of the section which can be defined as a long rectangular plate with one of the longer edges pinned and the other free".

$$\sigma_{c,cr,y} = \pi^2\{(D_x(b/a)^2)+(12D'_{xy}/\pi^2)\}/tb^2$$

$\sigma_{c,cr,yf} =$ = 163,5 MPa

Web buckling is decisive.

$N_{c,Rd} = A^* \sigma_{c,cr,y}/\gamma_M = 0{,}04335*31{,}1*10^3/1{,}5 = 899$ kN

Flexural bending:

$M_{Sd} \leq M_{Rd}$

For ultimate resistance:

$M_{Rd} = W_t* \sigma_{t,k}/\gamma_M = 0{,}006122/0{,}45*464{,}4*10^3/1{,}5 = 4212$ kNm (4.15)

Local buckling resistance:

$M_{Rd} = W_c* \sigma_{c,k}/\gamma_M = 0{,}006122/0{,}45*163{,}5*10^3/1{,}5 = 1483$ kNm (4.16)

$\sigma_{c,k} = \sigma_{c,cr,yf}$

Critical flexural stress in the web

$\sigma_{x,b} \leq \sigma_{x,cr,b}$ (4.24)

$\sigma_{x,b} = \sigma_{1,Sd} = 34{,}2$ MPa

For orthotropic materials:

$$\sigma_{x,cr,b} = \frac{k\pi^2 D_x}{d^2_w t_w} = 225 \text{ MPa} \quad (4.26)$$

Where k = 50 for $D_y/D_x = 1$ and k= 20 if $D_y/D_x = 0{,}5$ *Interpolation/extrapolation?*

Critical shear stress in the web

$\tau_{xy} \leq \tau_{xy,cr,b}$ (4.27)

$\tau_{xy} = V^{ULS}*S/(I_x*t_w) = 112,5*10^{-3}*0,00765/(0,006122*0,0185) = 7,6$ MPa

Where

$V^{ULS} = 1,5*2*5*15/2 = 112,5$ kN

$S = 0,035*0,4*(0,45-0,035/2) +0,0185*0,415^2/2 = 0,00765$ m^3

For orthotropic materials:

$\tau_{cr} = 4k(D_x D^3_y)^{0.25}/d^2_w\ t_w$ = 25 MPa (4.29)

Where k = 8

Combined shear in-plane bending in the web

$(\tau_{xy}/\tau_{xy,cr,b})^2 + (\sigma_{b,x}/\sigma_{x,cr,b})^2 \leq 1$ (4.30)

$(7,6/25)^2 +(34,2/225)^2 = 0,12$

There is nothing said about partial safety factor γ_M in this equation but if that is considered the above would be:

$(7,6/(25/1,5))^2 +(34,2/(225/1,5))^2 = 0,26$

2.4. Summary on ULS

The largest factor for the discrepancies between, in particular, CEN and Eurocomp are that CEN use the critical stresses to calculate design values for N, M and V but in Eurocomp the critical stresses are the design stresses. Also the way that partial safety and conversion factors are treated. Especially since the value of the conversion factor for creep according to CEN had to be guessed and a low value was chosen.

Normal force:

In tensional normal forces the only difference between the guidelines are the safety and conversion factors and since we have to guess the value for the conversion factor for creep in CEN it is no point in getting deeper into that until we have sort that out. CUR 96 does not divide into normal forces, bending moment etc. it only looks at the combined stress criterion, but if you extract the tension due to normal force you will get a similar result.

In compression the variation is larger.
In CEN, Annex C, the calculation is similar to the calculations of buckling resistance of a steel beam (EN 1993-1-1: 2005, 6.3.1) with the exception that here you look at the critical stress of the web and flanges separately. In this case the web is assumed simply supported at the connection with the flange.
As mentioned before CUR 96 only look at the combined stress criterion. But if you, for instance, look at the web as a right-angled plate according to chapter 9.2.2 you get the design buckling load for an evenly distributed load for various boundary conditions.
In the calculations above I have assumed the web to be clamped to the flanges. In that case the $N_{c,Rd}$ is a little bit lower in CUR 96 than it is in CEN.

Eurocomp and CEN are using the same equations to find critical stresses, but when Eurocomp, for instance, states that $N_{c,Rd} = A^* \sigma_{c,cr,y}/ \gamma_{M,}$ CEN also has a reduction factor, "χ" as mentioned above, which takes into consideration the interaction between local and global instability of the element. In this case we assumed that the beam is restrained in the weak direction so the reduction for normal forces is low.

Flexural bending:

For flexural bending it is only interesting to compare CEN and Eurocomp. The two guidelines are using the same equations to find critical stresses. But as for normal forces CEN also consider the interaction between local and global instability. In this case the reduction is more significant. The coefficient "k" or "k_f" in CEN also differs. In Eurocomp k is either 20 or 50, depending on D_{11}/D_{22}. In CEN the value of k_f is calculated and may vary between 11 and 25.
CEN also consider the combination of axial force and bending moment in a way I can´t find in Eurocomp and equation 6.15, where stability is considered is totally decisive.

Shear force:

Since there are so many errors and uncertainties in CEN "Annex E" shear force calculations can hardly be compared. What one can see is that unlike the case for normal force and flexural bending the equations for critical stresses, for shear force, are not the same in CEN and Eurocomp.

3. Summary on Serviceability Limit State

3.1. Deflection

CEN says that deformation should be determined using representative mechanical models. CUR 96 and Eurocomp presents the same, well known, equations to evaluate deflection. Hence we can conclude that the only difference between these guidelines is the partial factors.