Composites: Part B xxx (2012) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Stiffness and strength analysis of four layered laminate bamboo composite at macroscopic scale

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ARTICLE INFO

Article history:
Received 8 February 2012
Received in revised form 1 May 2012
Accepted 17 July 2012
Available online xxxx

Keywords:
A. Lamina/ply
A. Laminates
B. Strength

ABSTRACT

Dry bamboo culms of *Dendrocalamus strictus* were processed into thin laminas and cold pressed using epoxy resin to produce four layered laminated bamboo epoxy composites (LLBCs). Experimental and theoritical values (stiffness and strength) of LLBCs, prepared from middle region laminae, have been evaluated under tensile loading and using rule-of-mixture respectively. Experimental values of LLBCs are closely resembles with theoretical values. Further, a hypothetical four layered unidirectional laminated bamboo composites (HLLBCs) have been proposed where their stresses and strains obtained using constitutive equation of laminate at macroscopic scale are lower than experimental failure limit of LLBCs. This indicates that proposed LLBCs behaves like fibrous composites which are presently in use for a variety of structural applications.

1. Introduction

The tensile, compressive and bending values (stiffness and strength) of bamboo culms for different species are good enough for structural purposes [1,2]. These values are proportional to volume fraction of bamboo fibers. Volume fraction of bamboo fibers is dense in the outer region (60–65%), sparse (15–20%) in the inner region and increases linearly with height by about 20–40% [3]. Bamboo culm or pole can be converted into the engineered bamboo timber for structural purposes. Engineered bamboo timber in the form of LLBCs can be used as wood substitute. Bamboo timber is the suitable arrangement of slivers/laminas with the help of adhesive. While there are several publications on characterization of bamboo composites based on bamboo fibers in polymeric matrix [4–11], very few reports on evaluation of bamboo composites based on laminas exist in the literature [12,13]. The modified epoxy was suggested for preparing laminates [14]. Stiffness and strength analysis have been reported for fibrous composites at macroscopic scale [15,16]. An attempt has been made to study some mechanical properties of bamboo culms along and across the fiber direction. Experimental results indicate that stiffness and strength under tensile loading of bamboo laminae is higher in outer region and lower in inner region. Therefore, LLBCs have been prepared here from middle region laminae. Further, this publication reports the stiffness and strength analysis of LLBCs and HLLBC at macroscopic scale.

2. Materials and methods

Four year old green bamboo (*Dendrocalamus strictus*) culms were obtained from TERI, Gurgaon (Haryana), India. Moisture content of green bamboo collected were 41–45% at the time of felling (Digital moisture meter model MD-4G) which was reduced to 10–12% by sundry. This was done to ensure better adhesion between bamboo laminae and the epoxy resin. A full length bamboo was labeled at nodes and internodes as shown in Fig. 1. Bamboo was cut length-wise into six slats using radial hydraulic splitting machine. Each slat was sliced using sliver cutting machine. Laminae were prepared from slivers as per ASTM standard D3039. Laminae were in the form of rectangular cross-section which were 200 mm overall length, 100 mm gage length and 15 mm wide with a thickness of 1.5 mm (Fig. 2). Tabs were made from bamboo itself which were 50 mm long, 15 mm wide and 1.5 mm thickness with bevel angle of 30–45°. Three specimens were prepared from each location for obtaining average of test results along and across of bamboo culm. Unidirectional LLBCs were fabricated using intermodal laminae as node is weak in tension selected from middle region of bamboo culms. It is noted that width of laminae were generally less due to circular cross section of bamboo culms. Therefore laminae were butt joined using adhesive to make laminates/plies with larger width. To make first layer of laminate, laminas were arranged systematically on die cavity of 250 mm × 100 mm × 15 m (Fig. 3) using adhesive for butt joined.

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http://dx.doi.org/10.1016/j.compositesb.2012.07.048

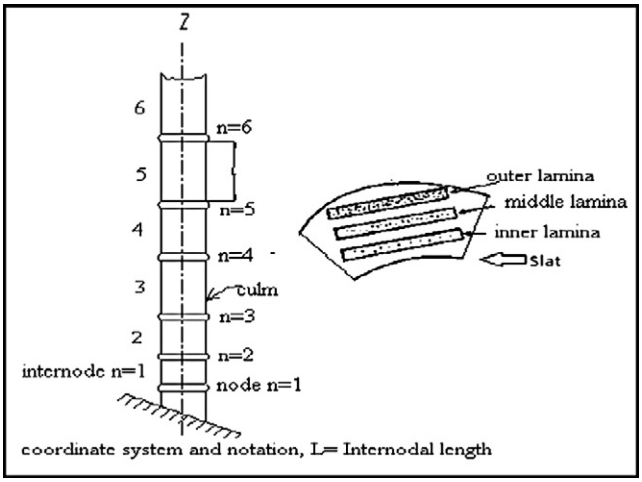

Fig. 1. Bamboo culms and location of laminas on slats.

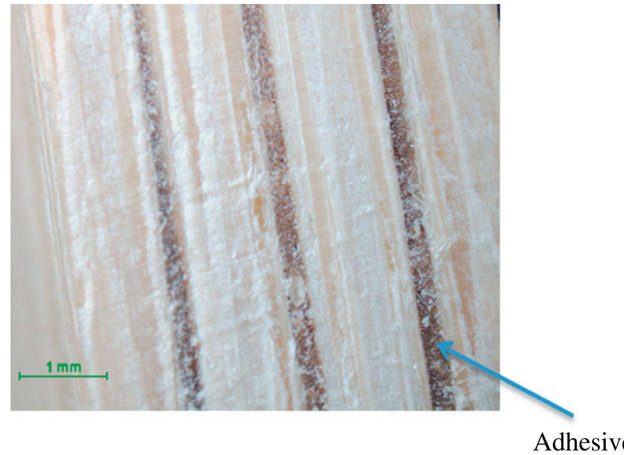

Fig. 4. Cross sectional image of LLBCs cube.

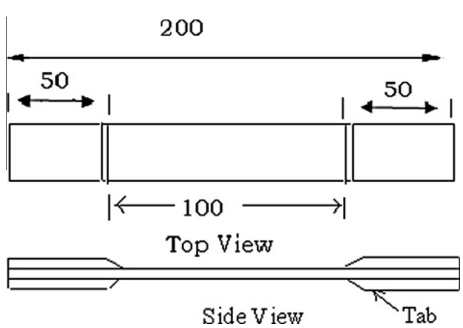

Fig. 2. Sketch of lamina specimens with tab (all dimension in mm).

One piece of lamina (rectangular light thick surface shown in Fig. 4, which is microscopic image) is butted against another and affixed with adhesive (rectangular dark thin surface shown in Fig. 4). To avoid adhesion between epoxy and die, polyesters sheet were used in between. The first layer of laminate was then coated with adhesive for interfacial bonding. Then other laminae were placed over on bottom laminate to make another layer of laminate. In this manner four layers of laminate/ply were stacked together to form one sample of unidirectional LLBCs. This laminate was sandwiched between the plates of die set by applying pressure by tightening of nut and bolt. This ensured straight slivers during solidification of adhesive and squeezed out of excess adhesive. The sample was left for 24 h at room temperature for cross linking of adhesive. The Araldite (LY 556) with curing agent/hardener (HY 951) was used as adhesive for preparation of LLBCs as well as tabs. Tensile strength, Young's modulus, density and curing time at room temperature of adhesive are given in Huntsman parts manufacturing selector guide as 30–35 MPa, 3–10 GPa, 1.3 g/cm^3 and 24 h respectively. The suggested ratio of araldite and hardener are 100:23 by weight. Surfaces of specimens were cleaned with acetone. The sample obtained was subjected to sand grinding from all sides so as to obtain smooth surfaces as shown in Fig. 5. Ten test specimens were prepared from LLBCs samples using cross cutting and grinding machines along fiber direction as per ASTM standards D3039. The specimens were in the form of constant rectangular cross section of 200 mm overall length, 100 mm gauge length and 16 mm wide with a thickness of 4.57–4.69 mm (lamina thickness: 1 mm, adhesive thickness: 0.19–0.23 mm) as shown in Fig. 6. Thickness of adhesive used in LLBCs have been seen (Fig. 4) from Nikon Microscope. Thicknesses of adhesive were measured with the help

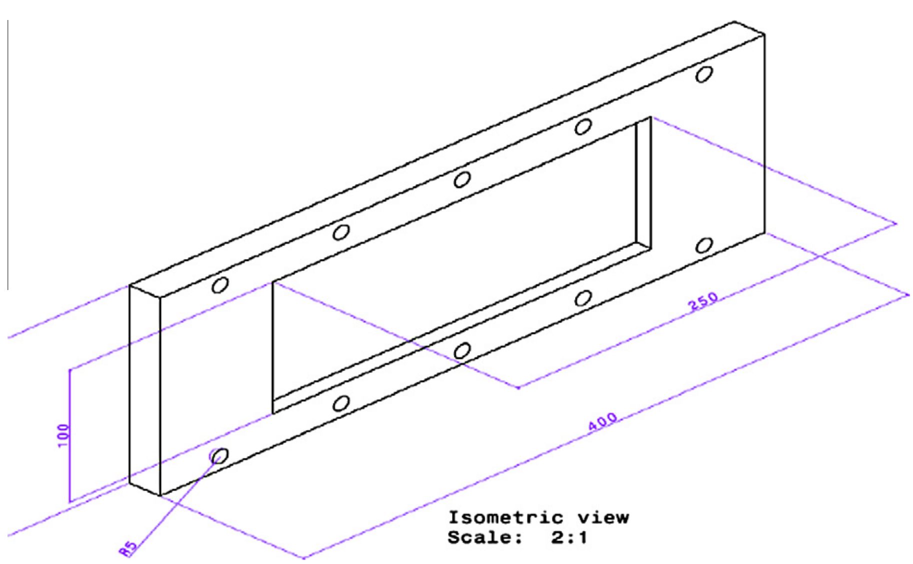

Fig. 3. Die from M.S. Plate.

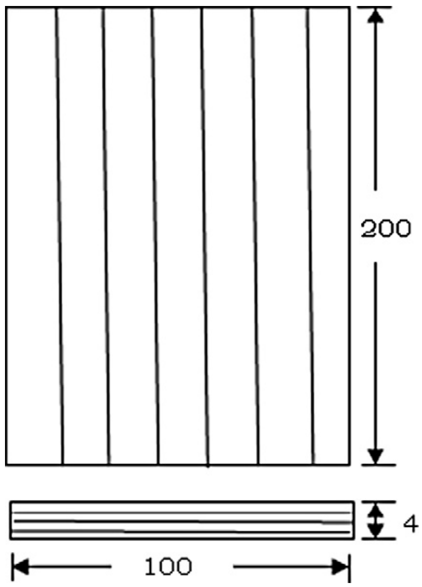

Fig. 5. The bamboo epoxy four layered laminated bamboo composite (LLBCs) sample.

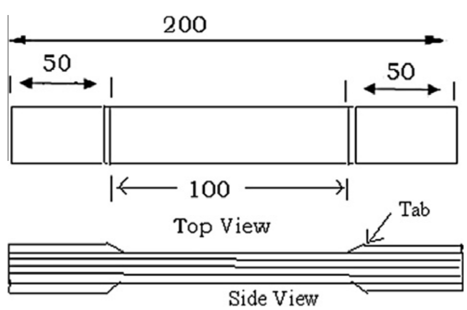

Fig. 6. Sketch of LLBCs specimens.

of image J software. Tabs in LLBCs specimens were 50 mm long, 16 mm wide and 1.5 mm thickness with bevel angle of 30–45° (Fig. 6).

3. Tensile testing

The experiments were performed on universal testing machine (Instron 5T) under tensile loading. The stress–strain curves were obtained for each lamina specimen from the automatic computerized chart recorder with the help of software called testXpert inbuilt in machine. Typical recorded tensile stress–strain curve for internodal lamina is shown in Fig. 7. Tensile failure strength and its Young's modulus i.e. stiffness were recorded from machine for all laminae along the length of bamboo selected from outer, middle and inner region of cross section of culms as shown in Table 1. Strength (i.e. maximum tensile failure stress) and stiffness (i.e. Young modulus) of a lamina from top middle region is 230 MPa and 6.3 GPa respectively (Table 1). Other properties of lamina for same species of bamboo were taken from literature. The summarized elastic constants of lamina obtained from tests are given in Table 2. A hypothetical four layered unidirectional laminated bamboo composite (HLLBC) is proposed for stiffness and strength analysis where elastic constants of all four lamina are similar to top middle lamina as given in Table 2.

Similarly tensile tests were performed on Instron Universal Testing Machine (10T) at a cross head speed of 2 mm/min for ten test specimens that were prepared from LLBCs samples. Typical recorded stress–strain curves for LLBCs specimens under tensile loading is shown in Fig. 8. Average tensile properties recorded by machine for ten specimens are given in Table 3.

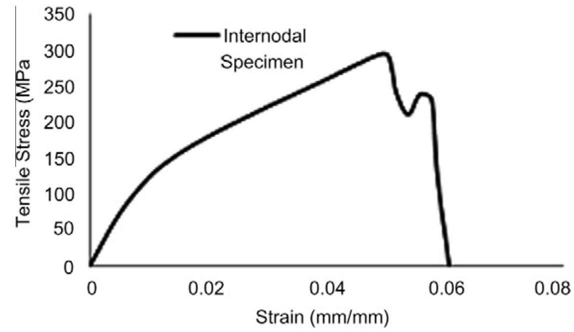

Fig. 7. Typical stress–strain curve in tension for intermodal laminae specimen.

4. Results and discussions

4.1. Experimental Stiffness and strength of LLBCs under tensile loading

Fig. 7 shows that stress–strain curves for laminae specimens are bi-linear up to ultimate stress. The first change of slope location point, in the slope is hypothesized to be due to matrix softening or matrix cracking of laminae. The first change of slope took place at about 42% of ultimate stress. The transition of stress–strain curve from first linear to second linear behavior occurred at a strain level around 0.01–0.012 which corresponds to 109–120 MPa. Strains at upper yield point are less that 3%. As the maximum load is reached, after matrix failure, the fiber failure occurs. Test results given in Table 1 indicate that tensile failure strength and Young modulus both increases from bottom to top of bamboo culms due to increase in volume fraction of bamboo fiber from bottom to top by 20–40% and decrease in cross sectional area of bamboo culms. Similarly, the same tensile properties also increase from inner to outer region across bamboo culms due to increase in volume fraction of bamboo fibers. Maximum tensile strength and stiffness (Young modulus) at the top middle region lamina were 230 MPa and 6.3 GPa respectively. These test results are used at random in HLLBC for stiffness and strength analysis.

Fig. 8 shows typical tensile stress–strain behavior of LLBCs. Tensile stress increased bi-linearly with increasing strain until the point of ultimate load followed by brittle fracture. The bi-linear curves imply that there is only one location where the change of slopes occurs. The first change of slope location point is hypothesized to be due to matrix softening or cracking of layers and the specimens still remained intact until sufficient delamination occurred to cause separation of layers. Even after delamination, the specimen continued to take further load and from cracks until the failure of specimens. Initiation of delamination layers occurred beyond 90% of ultimate stress and before the first lamina failure. Above ultimate point, the stress–strain curves showed sharp, staggered decreases in stress with small increase in strain. Staggered decreases in stress are due to presence of different properties of laminae and adhesive. Strain at upper yield point are up to 20%. It may be due to lamina by lamina and point by point change in strength and stiffness of LLBCs. Tests results of LLBCs on machine for tensile strength and stiffness (Table 3) indicate that average strength and stiffness (210.56 MPa and 4.50 GPa respectively) of LLBCs are comparable with average strength and stiffness (215.9 MPa and 5.57 GPa respectively) of laminae selected from

Table 1
Tensile properties of laminae.

Specimen No.		Internodal number					
		1	4	8	11	14	17
Outer region (Av.)	Stiffness (GPa)	4.6	5.9	6.4	5.23	6.93	8.9
	Strength (MPa)	240	257	250	281	298	302
Middle region (Av.)	Stiffness (GPa)	4.63	6.2	6.3	6.6	6.3	7.56
	Strength (MPa)	175	173	204	226	230	276
Inner Region (Av.)	Stiffness (GPa)	2.1	2.5	2.7	3.63	3.7	4.66
	Strength (MPa)	101	104	169	172	217	212

Decimal are rounded in strength.

Table 2
Elastic constants of top middle bamboo lamina.

Young modulus along fiber direction	$E_1 = 6.3$ GPa
Young modulus across fiber direction	$E_2 = 2$ GPa
Shear modulus	$G_{12} = 0.672$ GPa
Major Poisson ratio	$\upsilon_{12} = 0.32$

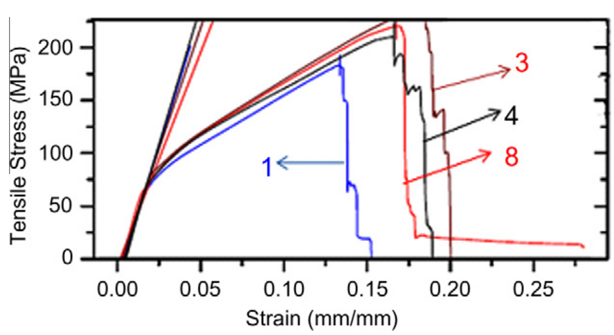

Fig. 8. Typical recorded stress–strain curves in tension for LLBCs.

different locations of bamboo culms. Average tensile strength and modulus is also given in Table 4 for comparative study. Experimental stiffness values of LLBCs can also be found from stress–strain curves. The first modulus was obtained up to the point where first change of slope occurred and the second modulus was obtained in a similar fashion from the point where first change in slope occurred to beyond 90% of ultimate point. The first change took place at about 35% of ultimate stress for all LLBCs specimens. The transition of stress–strain curves from first linear to second linear behavior occurred at a strain level around 0.013–0.025 which corresponds to 75–90 MPa. This might be due to the formation of micro-cracks in interfacial zone and yielding of matrix. From stress–strain curves (Fig. 8), average first and second tangent moduli are 3.5 GPa and 1.3 GPa respectively and chord modulus are 4.4 GPa and 1.02 GPa respectively.

Table 4
Tensile strength and stiffness of LLBCs.

Properties	Stiffness, E (GPa)	Strength, σ (MPa)
Theoretical using rule of mixture	6.26	189.22
Experimental (average)		
Aut Young	4.50	210.56
First tangent	3.5	75
First chord	4.4	0–75
Second tangent	1.3	204
Second chord	1.02	76–204

4.2. Theoretical stiffness and strength of LLBCs using rule of mixture

The rule-of-mixture principle for composites is given by following equations,

$$\sigma_{\mathrm{ROM}} = \sigma_f V_f + \sigma_m(1 - V_f) \quad (1)$$

$$E_{\mathrm{ROM}} = E_f V_f + E_m(1 - V_f) \quad (2)$$

Table 3
Tensile properties of four layered laminated bamboo composites (LLBCs).

Sample no.	1	2	3	4	5	6	7	8	9	10	Average	SD	CV
Displacement at Max. load (mm)	6.78	8.51	9.38	8.44	10.43	8.11	10.79	9.59	8.52	7.90	8.85	1.21	13.70
Max. load (KN)	13.49	16.19	16.79	16.83	15.00	10.36	15.87	16.96	15.92	15.9 6	15.34	2.03	13.26
Max. stress (MPa)	183.35	220.26	240.92	210.5 8	191.07	152.47	251.56	226.54	215.71	213.18	210.5 6	28.8 3	13.69
Strain at Max. load (mm/mm)	0.133	0.168	0.185	0.166	0.205	0.16	0.212	0.189	0.168	0.15 6	0.1742	0.02 37	13.648
Modulus (Aut Young) (MPa)	5018.53	4037.12	4596.46	5227.37	3027.2	2214.48	4356.15	5141.57	5462.83	5937.9	4501.96	1147	25.47

Dimension of test specimens: 200 mm × 16 mm × 4 mm with 50 mm tab length each side.

Please cite this article in press as: Verma CS, Chariar VM. Stiffness and strength analysis of four layered laminate bamboo composite at macroscopic scale. Composites: Part B (2012), http://dx.doi.org/10.1016/j.compositesb.2012.07.048

Tensile strength and Young's modulus of adhesive are 30–35 MPa and 3–10 GPa respectively [14]. Average failure tensile strength (σ_f) and Young's modulus i.e. stiffness (E_f) of middle regions laminae of bamboo culms are 214 MPa and 6.27 GPa respectively. LLBCs were prepared from these laminae. Then, volumetric fraction of bamboo fibers bundle i.e. laminae (V_f) calculated for LLBCs are varies from 0.852 to 0.875 where adhesive thickness are 0.19–0.23 mm. Average adhesive tensile strength (σ_m) and modulus (E_m) of adhesive are 32.5 MPa and 6.2 GPa respectively. Then using rule-of-mixture on laminates where adhesive are assumed as matrix and bamboo fiber bundle as reinforcement, tensile strength and stiffness are 189.22 MPa and 6.26 GPa respectively (Table 4). From stress–strain curves (Fig. 8), average first and second tangent moduli are 3.5 GPa and 1.3 GPa respectively and chord modulus are 4.4 GPa and 1.02 GPa respectively (Table 4). It is found that the first modulus in most of the cases of LLBCs is close to the theoretical prediction using rule-of-mixture, but typically lower than the second modulus. This indicate that the fiber other than the loading direction also contribute to the stiffness. This may be due to fabrication defects such as voids, improper bonding, micro-cracks or fibers breakages existing even before they are tested. Comparative study (Table 4) indicates that theoretical stiffness is more than that of experimental stiffness but strength is slightly less than experimental. These deviations are due to change of properties of laminae from point to point along and across of bamboo culm.

4.3. Theoretical stiffness and strength analysis of HLLBC using constitutive equation of laminate

This section presents stiffness and strength analysis of HLLBC which are assumed to make from lamina selected from top middle region of bamboo culm. Elastic constants of all four laminae of HLLBC are same (Table 2). Equations used here for macroscopic analysis are taken from literatures [15,16].

4.3.1. Stiffness analysis

The process of forming the stress–strain relationship is termed the stiffness analysis where [Stresses σ] = [Stiffness E][Strains ε]. Stiffness of bamboo lamina are discussed here as a specially orthotropic materials. Let combined stress-system in x–y and 1–2 axes is shown in Fig. 9A. Then full stress–strain relationship in terms of reduced stiffness terms denoted by C_{ij} or simply C is given below in matrix form

	ε_1	ε_2	γ_{12}
σ_1	C_{11}	C_{12}	0
σ_2	C_{21}	C_{22}	0
τ_{12}	0	0	C_{33}

(4.1)

where σ and ε indicate direct stress and strain respectively. Similarly τ and γ indicate shear stress and strain respectively. For composite laminae/plies analysis, E_1 = 6.3 GPa, E_2 = 2 GPa, G_{12} = 0.672 GPa and υ_{12} = 0.32 are known. However there is need of minor poisons ratio υ_{21} also. It can be determined as (by assuming homogenous orthotropic)

$$\frac{\upsilon_{21}}{\upsilon_{12}} = \frac{E_2}{E_1}$$

$$\upsilon_{21} = \frac{2}{6.3} \times 0.32 = 0.102$$

And reduced stiffness terms are

$$C_{11} = \frac{E_1}{1 - \upsilon_{12}\upsilon_{31}} = 6.512 \text{ GPa}$$

$$C_{12} = \upsilon_{21} C_{11} = 0.6642 \text{ GPa}$$

$$C_{22} = \frac{E_2}{1 - \upsilon_{12}\upsilon_{21}} = 2.067 \text{ GPa}$$

$$C_{33} = G_{12} = 0.672 \text{ GPa}$$

Therefore,

$$C = \begin{bmatrix} 6.512 & 0.664 & 0 \\ 0.64 & 2.067 & 0 \\ 0 & 0 & 0.672 \end{bmatrix} \text{ GPa} \tag{4.2}$$

Now, HLLBC (Fig. 9B) are assumed to make from specially orthotropic laminae in which the material axes are coincident with the reference axes and all the lamina angles are at 0° to the reference x-axis. The lamina/ply thickness (t_p = 1 mm) and lamina/ply centroidal values of the HLLBC are shown in Fig. 9C.

The first step in calculating the laminate stiffness is to determine reduced stiffness terms of the lamina. Reduced stiffness values of the bamboo lamina are given in Eq. (4.2). Next, the transformed reduced stiffness terms for a lamina angle of 0° is obtained from Eq. (4.3), where $m = \cos\theta = 1$, $n = \sin\theta = 0$.

	C_{11}	C_{22}	C_{12}	C_{33}
	m^4	n^4	$2m^2n^2$	$4m^2n^2$
	n^4	m^4	$2m^2n^2$	$4m^2n^2$
	m^2n^2	m^2n^2	$-2\,m^2n^2$	$(m^2-n^2)^2$
	n^2n^2	m^2n^2	m^4+n^4	$-4m^2n^2$
	m^3n	$-mn^3$	mn^3-m^3n	$2\,(mn^3-m^3n)$
	mn^3	$-\,m^3n$	m^3n-mn^3	$2\,(m^3n-mn^3)$

(4.3)

Then, the transformed reduced stiffness terms $(\overline{C_{ij}})_0$ will be given in the following equation:

$$\begin{bmatrix} 6.512 & 0.664 & 0 \\ 0.664 & 2.067 & 0 \\ 0 & 0 & 0.672 \end{bmatrix} \text{ GPa} \tag{4.4}$$

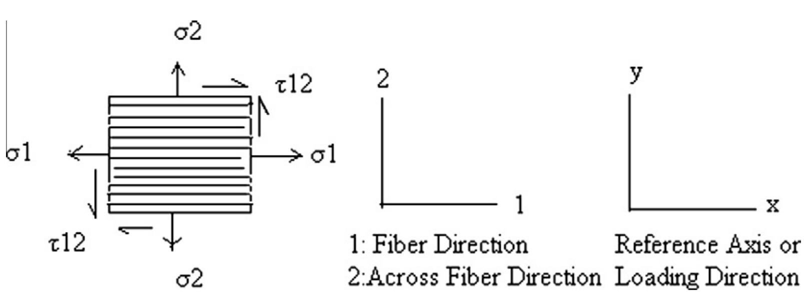

Fig. 9A. Positive combined stress–system in x–y and 1–2 axes.

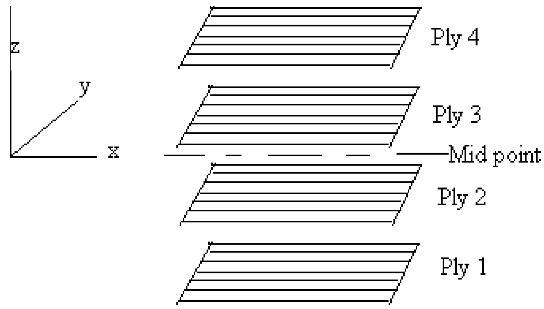

Fig. 9B. Symmetric orthotropic HLLBC (O_4).

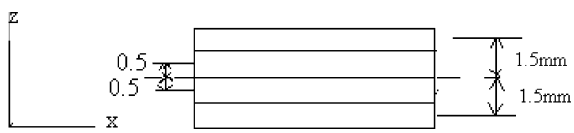

Fig. 9C. Ordinate value (mm) for HLLBC.

Table 5
$\overline{C_{ij}}$ values (GPa) for symmetric HLLBC (O_4).

Lamina/ply	$\theta°$	$\overline{C}_{11}$	$\overline{C}_{22}$	$\overline{C}_{33}$	$\overline{C}_{12}$	$\overline{C}_{12}$	$\overline{C}_{23}$
1	0°	6.512	2.067	0.672	0.664	0	0
2	0°	6.512	2.067	0.672	0.664	0	0
3	0°	6.512	2.067	0.672	0.664	0	0
4	0°	6.512	2.067	0.672	0.664	0	0

From Eqs. (4.2) and (4.4), it is clear that $\overline{C_{ij}} = C_{ij}$ if lamina angle is 0°. Since all the laminae are singly oriented with $\theta = 0°$, the transformed reduced stiffness terms will be same for all the four laminae, and their values are given in Table 5. Now, In order to calculate all the HLLBC stiffness terms, complete constitutive Eq. (4.5) is used [15,16]. For a complex stress system, the loads are assumed to act at the HLLBC mid-plane and the load intensities on the mid-plane are defined below (Fig. 9D), where

- N_x = In-plane direct force intensity (force/length) in the x-direction per unit width (in the y–z plane) of the laminate i.e. HLLBC section.
- N_y = In-plane direct force intensity (force/length) in the y-direction per unit width (in the x–z plane) of the laminate section.
- N_{xy} = In-plane shear force intensity in the x–y direction per unit width (in the y–z or x–z plane) of the laminate section.
- M_x = Moment Intensity (force–length/length) about the y-axis per unit width (in the y–z plane) of the laminate section.
- M_y = Moment Intensity (force–length/length) about the x-axis per unit width (in the x–z plane) of the laminate section.
- M_{xy} = Twisting intensity about the x (y)-axis per unit width (in the y–z or x–z plane) of the laminate section.

	ϵ_x^0	ϵ_y^0	γ_{xy}^0	k_x	k_y	k_{xy}
N_x	A_{11}	A_{12}	A_{13}	B_{11}	B_{12}	B_{13}
N_y	A_{12}	A_{22}	A_{23}	B_{12}	B_{22}	B_{23}
N_{xy}	A_{13}	A_{23}	A_{33}	B_{13}	B_{23}	B_{33}
M_x	B_{11}	B_{12}	B_{13}	D_{11}	D_{12}	D_{13}
M_y	B_{12}	B_{22}	B_{23}	D_{12}	D_{22}	D_{23}
M_{xy}	B_{13}	B_{23}	B_{33}	D_{13}	D_{23}	D_{33}

(4.5)

where

(a) $A_{ij} = \sum_{P=1}^{N} t_P(\overline{C_{ij}})_P$ called extensional stiffness.
(b) $B_{ij} = \sum_{P=1}^{N} (-t_P)\overline{Z_P}(\overline{C_{ij}})_P$ called coupling stiffness
(c) $D_{ij} = \sum_{P=1}^{N} (t_P\overline{Z_P}^2 + \frac{t_P^3}{12})(\overline{C_{ij}})_P$ called bending stiffness
(d) ϵ_x^0, ϵ_y^0, ϵ_{xy}^0 are mid-plane membrane strains and k_x, k_y, k_{xy} are mid-plane membrane curvatures, $\overline{Z_P}$ is the distance of centroid of plane p from the mid-plane of layered laminate HLLBC.
(e) t_P is the thickness of lamina P.
(f) $\overline{C_{ij}}$ the transformed reduced stiffness.

However, for all symmetric laminate, $B_{ij} = 0$. Here laminate is under pure tensile stress only. Therefore there will be no bending stiffness terms, $D_{ij} = 0$. Other ordinate values for each lamina of HLLBCs are given in Table 6. From Table 6, we get the t_p term, ply thickness value for each lamina and from Table 5, we get the transformed reduced stiffness value for each lamina. We use these values for the laminae/plies in the lower half of the laminate. Since the HLLBC is symmetric, therefore, the total extensional stiffness terms A_{ij} of HLLBC will be

$$A_{11} = 2\left\{(t_p \times \overline{C_{11}})_{\text{ply1}} + (t_p \times \overline{C_{11}})_{\text{ply2}}\right\} = 26.048$$

$$A_{22} = 2\left\{(t_p \times \overline{C_{22}})_{\text{ply1}} + (t_p \times \overline{C_{22}})_{\text{ply2}}\right\} = 8.268$$

$$A_{33} = 2\left\{(t_p \times \overline{C_{33}})_{\text{ply1}} + (t_p \times \overline{C_{33}})_{\text{ply2}}\right\} = 2.688$$

$$A_{12} = 2\left\{(t_p \times \overline{C_{12}})_{\text{ply1}} + (t_p \times \overline{C_{12}})_{\text{ply2}}\right\} = 2.656$$

$$A_{13} = A_{23} = 0 \quad \text{as} \quad \overline{C_{13}} = \overline{C_{23}} = 0$$

The extensional stiffness term A_{ij} may be written in a boxed matrix notation form as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 26.048 & 2.656 & 0 \\ 2.656 & 8.268 & 0 \\ 0 & 0 & 2.688 \end{bmatrix} \text{GPa} \times \text{mm}$$

Having got the HLLBC stiffness terms, we are now in position to obtain the HLLBC compliance terms in order to determine the HLLBC equivalent elastic constants.

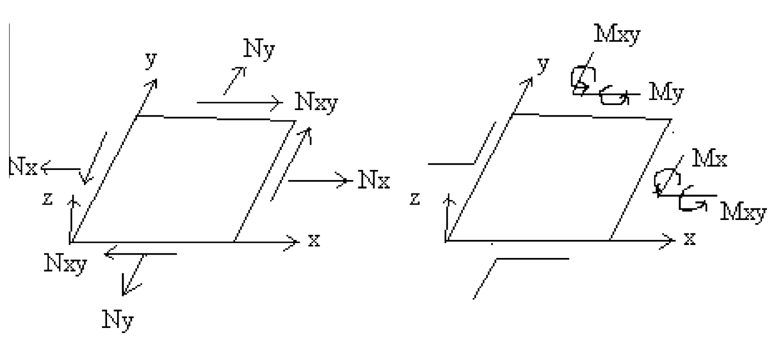

Fig. 9D. Positive system of laminate force and moment intensity at a point.

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Table 6
Ordinate values of HLLBCs.

Lamina/ply	$\theta°$	t_p	$\bar{Z}_P$	$t_p\bar{Z}_p$	$\left(t_n\bar{Z}_p^2 + \frac{t_p^3}{12}\right)$
1	0°	1	−1.5	−1.5	2.333
2	0°	1	−0.5	−0.5	0.333
3	0°	1	0.5	0.5	0.333
4	0°	1	1.5	1.5	2.333

Then extensional compliance matrix a_{ij} will be

$$a_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0.03969 & \frac{-12.75}{1000} & 0 \\ \frac{-12.75}{1000} & 0.1250 & 0 \\ 0 & 0 & 0.3702 \end{bmatrix}$$

where

$$a_{11} = \frac{A_{22}}{AA} = 0.03969, \quad a_{22} = \frac{A_{11}}{AA} = 0.1250, \quad a_{33} = \frac{1}{A_{33}} = 0.3720,$$
$$a_{12} = \frac{-A_{12}}{AA} = a_{21} = -\frac{12.75}{1000}, \quad a_{13} = a_{23} = 0, \quad AA = A_{11}A_{22} - A_{12}^2$$

The HLLBC equivalent elastic constants are obtained from following equation:

$$E_x = \frac{1}{ta_{11}} = \frac{1}{4 \times 0.03960} = 6.3 \text{ GPa}$$

where t is the HLLBC thickness (=4 × t_p mm = 4 mm). Elastic constant calculated for HLLBC is 6.3 GPa which is equal to elastic constant of individual lamina because all lamina are oriented at 0° in HLLBC.

4.3.2. Strength analysis

There are various procedures for strength analysis such as first ply failure, complete ply failure and partial ply failures but complete ply failure generally gives good strength values. Thus we will discuss here about complete ply failure procedure (CPF). For prediction of tensile load N_x that will cause the failure in the symmetric HLLBC where the laminae thickness and width is 1 mm and 16 mm respectively. Let us assume the complete ply failure approach (LPF/CPF) and maximum stress failure theory.

The transformed reduced stiffness terms for all the intact laminae in the HLLBC are given in Table 6. For first lamina/ply failure, we first choose an arbitrary value of applied load, say

$$N_x = -(10 \text{ kN}/16 \text{ mm}) = 625 \text{ N/mm}$$

Using the membrane compliances Eq. (4.6) given below in following matrix

	N_x	N_y	N_{xy}
ϵ_x^0	a_{11}	a_{12}	a_{13}
ϵ_y^0	a_{12}	a_{22}	a_{23}
γ_{xy}^0	a_{31}	a_{32}	a_{33}

x 10^{-3}mm/N (4.6)

Putting N_x = 625, N_y = N_{xy} = 0, (where Area = 4 × 16 mm², unit of N_x is load per unit width), Strains are

$$\varepsilon_x^0 = 0.03969 \times 10^{-3} \times 625 = 24.806 \times 10^{-3}, \quad \varepsilon_y^0 = -7.9687 \times 10^{-3}, \quad \gamma_{xy}^0 = 0$$

We then go back to individual laminae and transform the strains into the material axes (1–2), using Eq. (4.7) which is given below

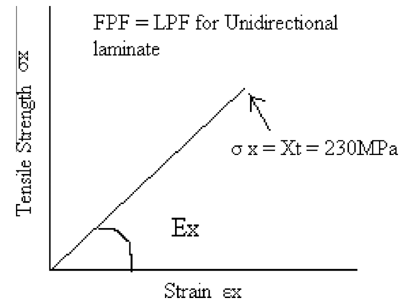

Fig. 10. Tensile strength by complete ply failure method and maximum stress theory for unidirectional four layer laminate.

	ϵ_x	ϵ_y	γ_{xy}
ϵ_1	m^2	n^2	mn
ϵ_2	n^2	m^2	$-mn$
γ_{12}	$-2mn$	$2mn$	m^2-n^2

(4.7)

Laminae/plies 1, 2, 3, 4 at 0°

Lamina/ply 1 at 0°, $m = \cos 0° = 1$, $n = \sin 0° = 0$

Substituting the trigonometric values and the reference axes strain values (x–y) into Eq. (4.7), we get

$$\varepsilon_1 = 24.806 \times 10^{-3}, \varepsilon_2 = -7.9687 \times 10^{-3}, \quad \gamma_{12} = \gamma_{xy}^0 \times 1 = 0$$

Above lamina strains can be transformed into stresses using Eq. (4.8).

Laminae/plies 1, 2, 3 and 4 at 0°

	ε_1	ε_2	γ_{12}
σ_1	6.512	0.664	0
σ_2	0.664	2.067	0
τ_{12}	0	0	0.672

x 10^3 MPa (4.8)

Giving

$$\sigma_1 = 156.24 \text{ MPa}, \quad \sigma_2 = 0, \quad \tau_{12} = 0$$

These results confirm that HLLBC is under pure tensile loading along lamina fiber direction.

Using the maximum stress failure criterion, failure index (FI) will be

$$\text{FI } 1 = \sigma_1/X_t = 156.24/230 = 0.679$$

Above calculation indicate that direct stress of HLLBC is less than maximum tensile failure stress (X_t) of laminae i.e. 230 MPa when modulus E_1 = 6.3 GPa, E_2 = 2 GPa, υ_{12} = 0.32 and G_{12} = 0.672 GPa at N_x = 625 N/mm. Here all laminas/plies are at 0°, therefore maximum failure index will be same for all lamina/ply, that is in case of first lamina/ply which is 0.679 (<1). So, with the applied tensile load N_x = 625 N/mm, no lamina/ply failure has yet occurred. Therefore, the load can increase by a factor of 1.47 (1/0.679) before FPF is predicted by the maximum stress criterion. The load to cause the FPF, by the maximum stress theory is

$$N_x = 625/0.679 = 920.47 \text{ N/mm}$$

Therefore maximum load at failure = 920.47 N/mm × 16 mm = 14.727 kN and maximum stress at failure = 14.727 kN/16 × 4 mm² = 230 MPa.

Note that since the HLLBC is symmetric and only a membrane load is present, then the failure of the first lamina/ply is predicted simultaneously for all the laminas/plies (at 0°). In other words all lamina/plies will fail simultaneously at ultimate stress i.e. 230 MPa because orientation of all lamina/ply is same in HLLBCs. Graph for longitudinal tensile strength by complete ply failure method and maximum stress theory for HLLBC is shown in Fig. 10.

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5. Conclusion

(a) Bi-linear stress–strain curves was observed for all inter-modal laminae of bamboo culm. Tensile strength and Young's modulus of bamboo increases from inner to outer region across any cross section and from bottom to top of bamboo culms.
(b) Bi-linear stress–strain response up to 90% of ultimate stress were also observed for all LLBCs. First matrix failure occurs followed by fibers failure with metallic sound of any one layer and subsequently other layers in LLBCs.
(c) The experimental results indicate that there are good agreements between the estimated and predicted values for stiffness and strength which are satisfactory agreement for initial design purposes. The first chord modulus for almost all laminates is close to the theoretical prediction using rule-of-mixture.
(d) Using elastic constants of top middle region lamina, stresses and strains obtained of HLLBC are lower than experimental failure limit of lamina. This indicates that proposed LLBC behaves like fibrous composites which are presently in use for a variety of structural applications.

Acknowledgements

We are grateful to the Laboratory Incharge of Stress Analysis lab, Numerical Computation Lab and strength of material Lab of IIT Delhi and lab in charge, Material Testing Lab and central workshop, NSIT, New Delhi for assistance rendered in testing.

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