University School of Automation and Robotics
GURU GOBIND SINGH INDRAPRASTHA UNIVERSITY
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Delhi - 110092

Engineering Mechanics

By: Dr. Divya Agarwal

- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon’s theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates
- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.
- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.
- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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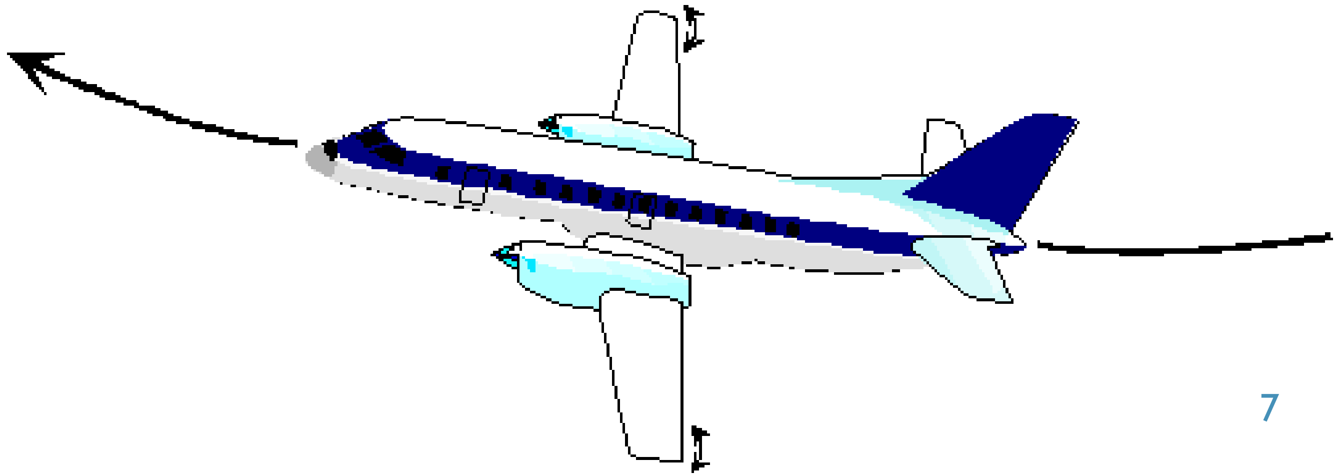
WHAT IS A RIGID BODY?

- A rigid body is an idealization of a body that does not deform or change shape.
- It is defined as a collection of particles with the property that the distance between particles remains unchanged during the course of motions of the body.
- Like the approximation of a rigid body as a particle, this is never strictly true.
- All bodies deform as they move.
- However, the approximation remains acceptable as long as the deformations are negligible relative to the overall motion of the body.
- We will restrict attention to planar motion of rigid bodies. In particular, we will take all rigid bodies to be thin slabs with motion constrained to lie within the plane of slab.
- We will assume vectors of the form {i, j, k}, such that i and j lie in plane, with k as plane normal.

WHAT IS A RIGID BODY?

- As an example, the flutter of an aircraft wing during the course of a flight is clearly negligible relative to the motion of the aircraft as a whole.
- On other hand, if one was interested in stresses induced in wing as a consequence of flutter, these deformations become of primary importance.

Deformations experienced by an aircraft are small relative to its motion.



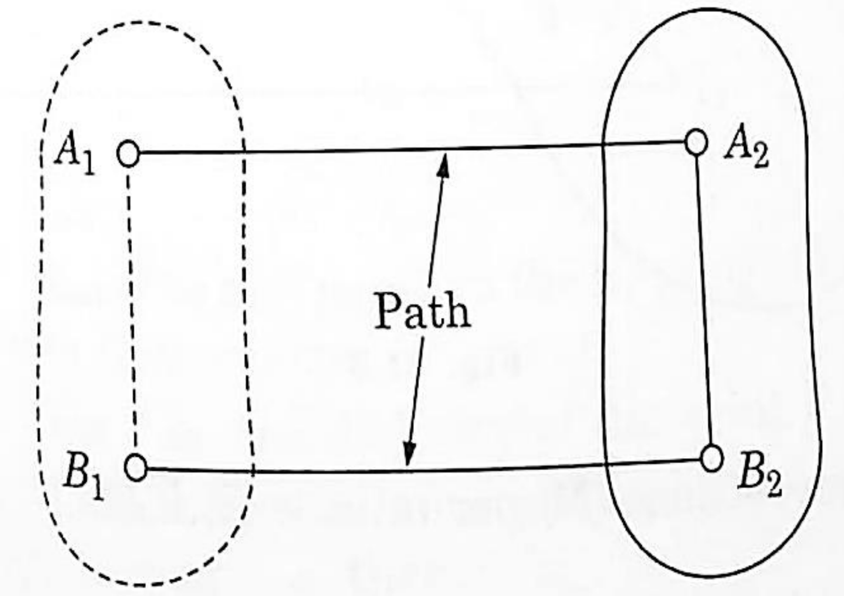
RIGID BODY MOTION

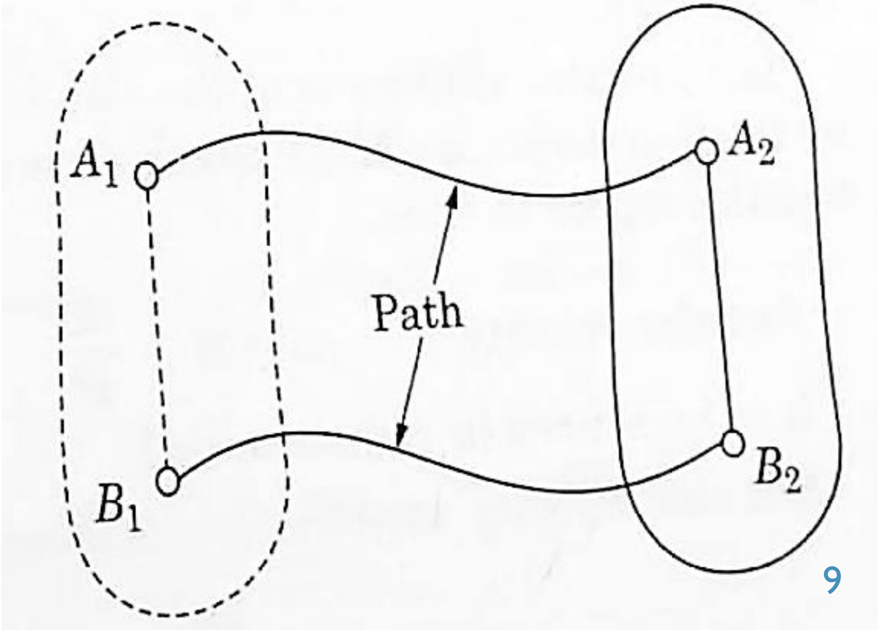
- Rigid body can perform various types of motion from simple translation to quite complex motions.
- In plane motion of a rigid body all particles of the body move in parallel planes.
- Motion of the body, therefore, can be represented by motion in any one of such parallel planes or by a representative plane figure instead of a 3D object.
- Motion of a particle w.r.t. fixed frame of reference is called absolute motion of particle. For example, the motion of a train w.r.t. platform is termed as the absolute motion of the train.
- Motion relative to a set of axes which are moving is called relative motion. For example, motion of a train A w.r.t. another moving train B is relative motion of train A w.r.t. train B
- Various types of plane motion can be grouped as follows:

1. Translation **2. Rotation** **3. General plane motion**

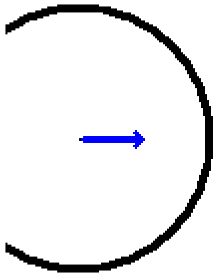
RIGID BODY MOTION - TRANSLATION

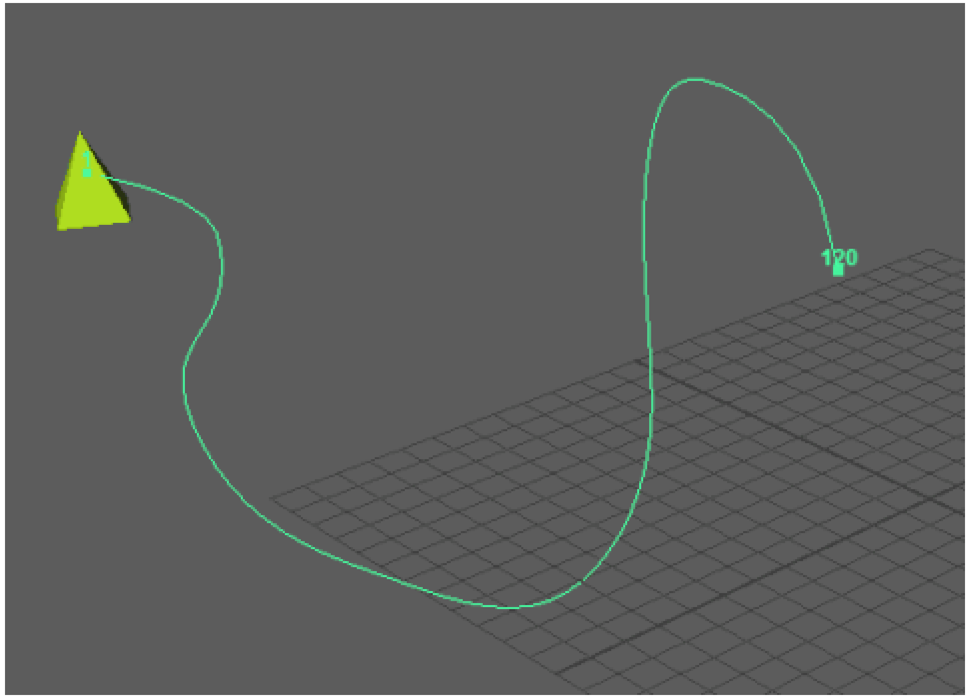
- A rigid body is said to have a translatory motion if an imaginary straight line drawn on the body remains parallel to its original position during the motion.
- It can be observed that all particles of body move along parallel paths in a translatory motion.
- If these paths are straight lines, motion is said to be rectilinear or, if these parts are curved lines, motion is said to be curvilinear translation.
- Some examples are: Moving Bus, plant shaken by a person, stone falling straight at surface of earth





EXAMPLES: TRANSLATION



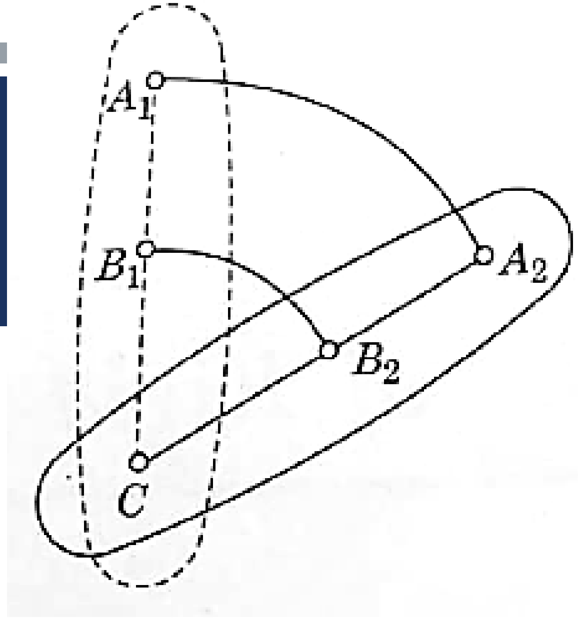


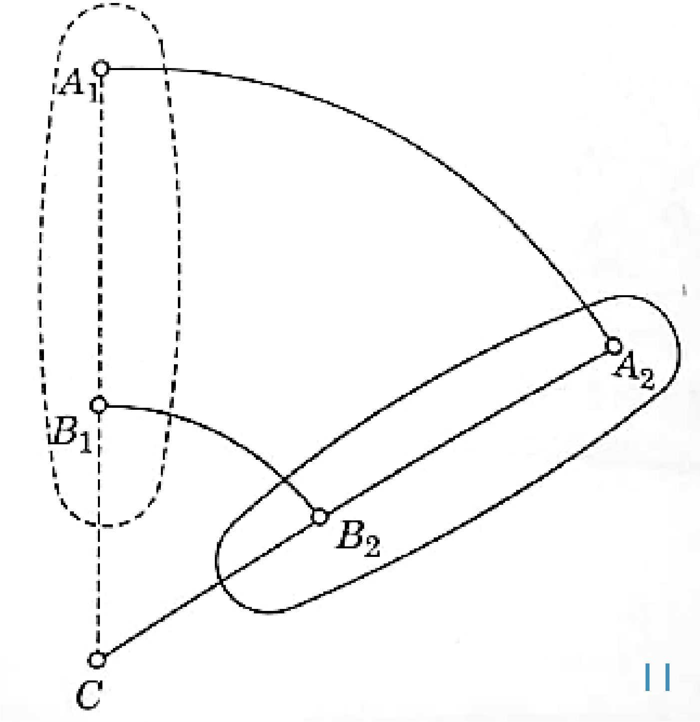
RIGID BODY MOTION - ROTATION

Rotation: It is a type of motion that has a Rotation about a fixed axis. All particles in Rotational motion move in circular paths about the axis of rotation.

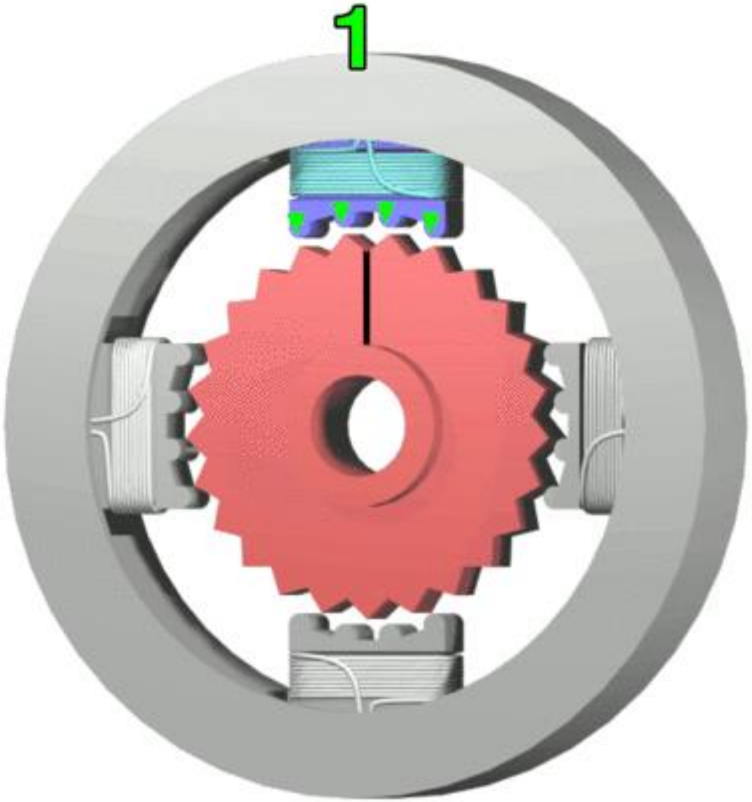
- Common centre of circles may be located in the body or outside the body.
- Rotational Motion of body is completely determined by angular velocity of the rotation.
- Some examples include wheel or rotor of motor.

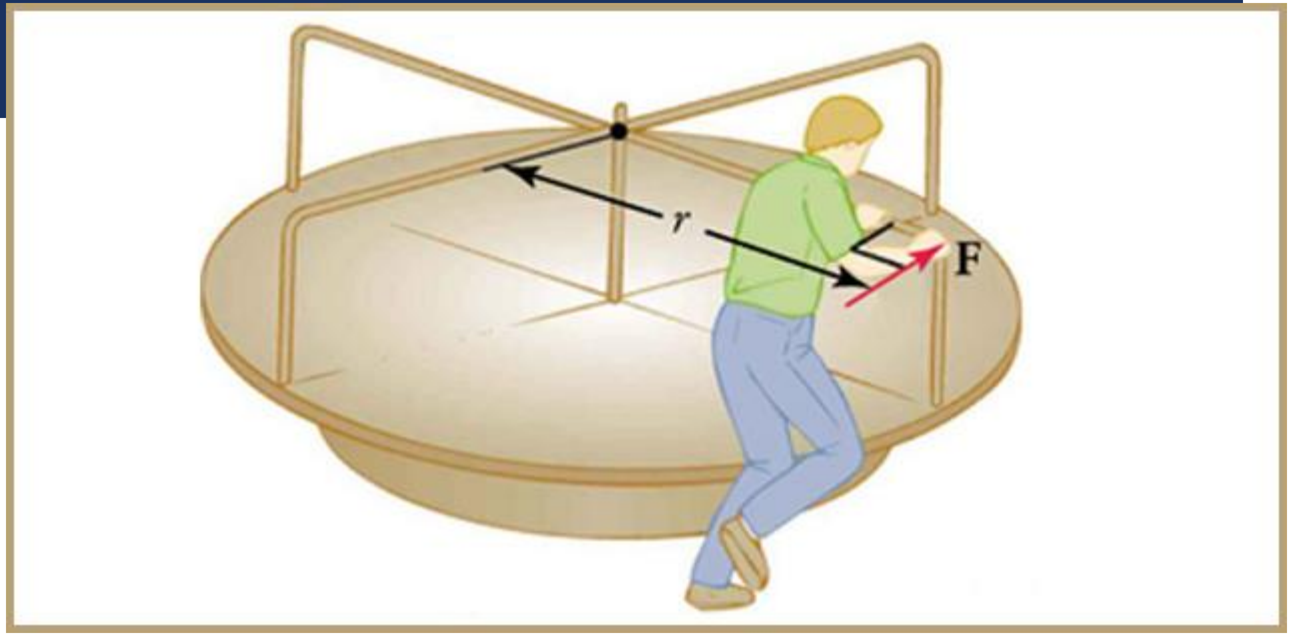
General plane motion. Any plane motion which is neither a translation nor a rotation is known as general plane motion. It can always be reduced to the sum of a translation and a rotation.

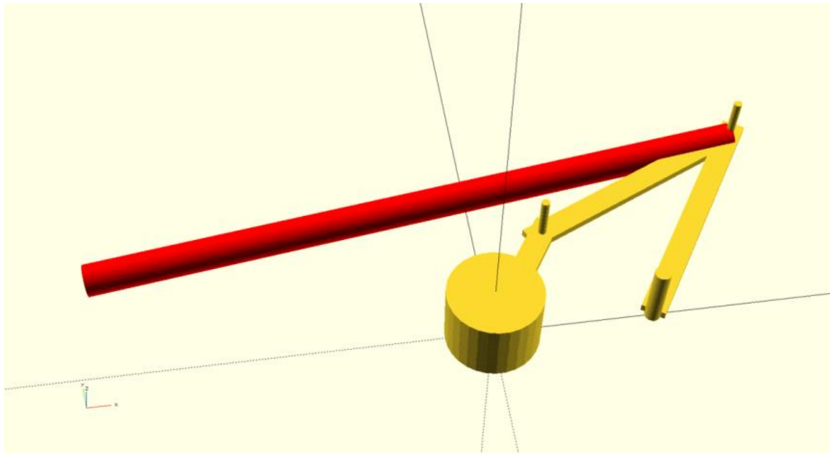




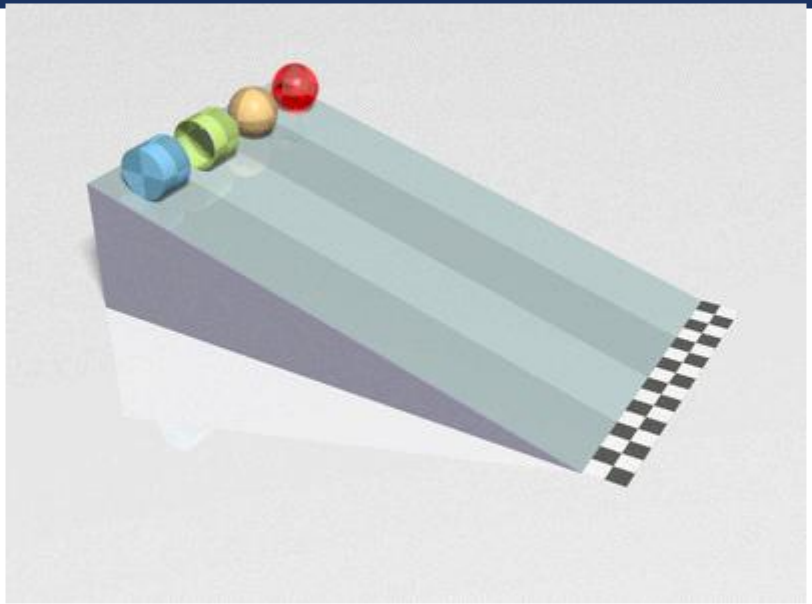
EXAMPLES - ROTATION



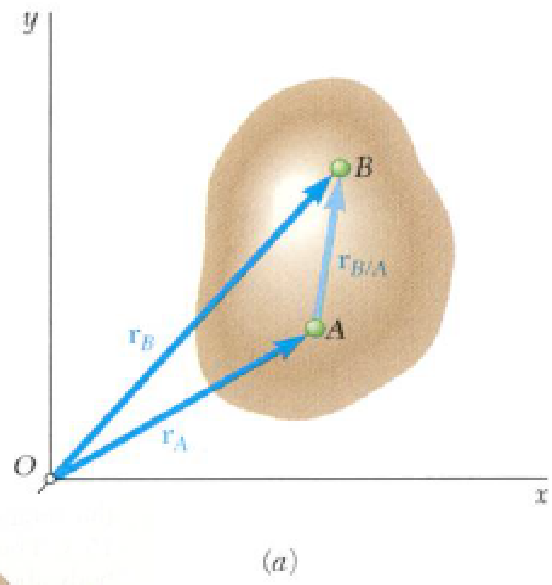


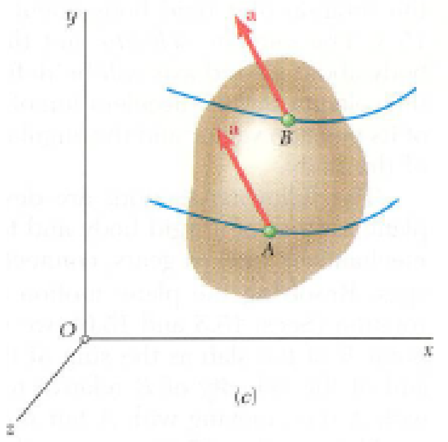


EXAMPLES – GENERAL PLANE MOTION



TRANSLATION





- <u>Consider rigid body in translation</u>:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.
- For any two particles in the body,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

$$\vec{v}_B = \vec{v}_A$$

All particles have the same velocity.

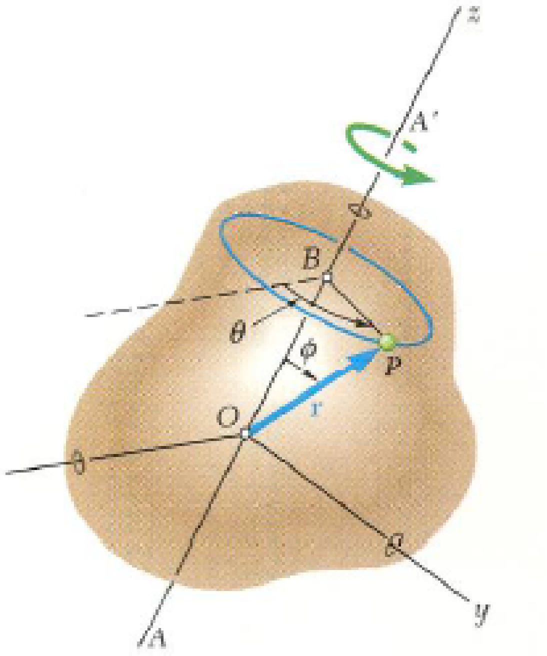
- Differentiating with respect to time again,

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

$$\vec{a}_B = \vec{a}_A$$

All particles have the same acceleration.

ROTATION



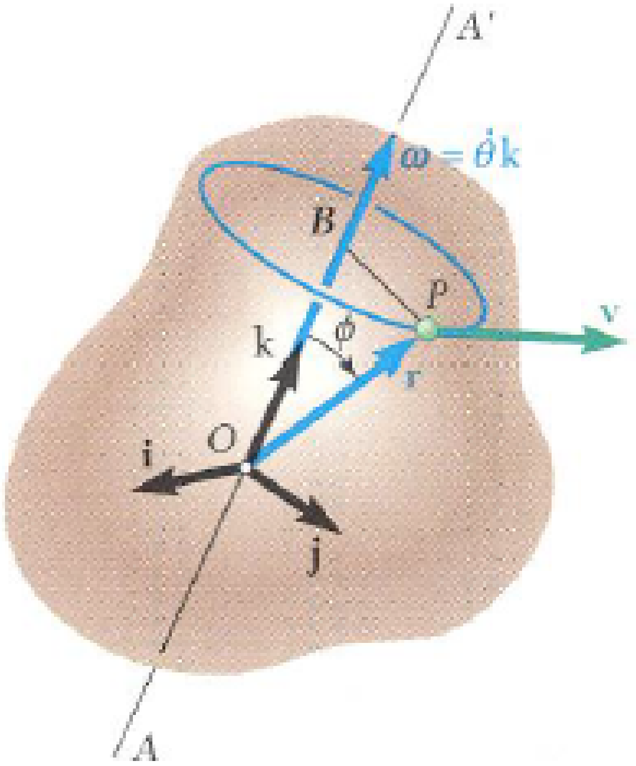
- Consider rotation of rigid body about a fixed axis *AA′*

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

ROTATION

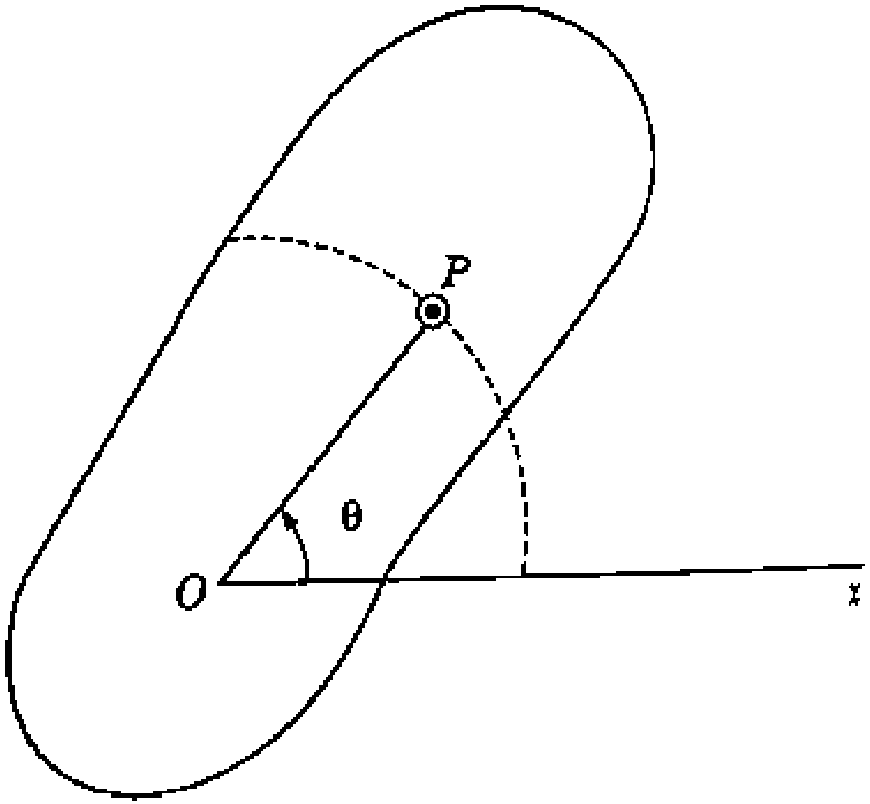


$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \textit{angular velocity}$$

ROTATION – (ANOTHER METHOD)

- Consider a rigid slab which rotates about a fixed axis perpendicular to plane of the slab and intersecting it at point O.
- Let P be a point on the slab.
- Position of slab is completely defined by angle $\boldsymbol{\theta}$ which line OP forms with a fixed direction $O_{x.}$
- Angle $\boldsymbol{\theta}$ is known as angular coordinate of the slab.
- Angular velocity $\boldsymbol{\omega}$ of slab is given by first derivative of angular co-ordinate $\boldsymbol{\theta}$ w.r.t. time.



$$\textit{Angular Velocity}, \omega = \dot{\theta} = \frac{d\theta}{dt}$$

- It is measured in radian per second.
- Angular velocity is sometimes measured in RPM ($\omega = \frac{2\pi N}{60}\ radian/second$)

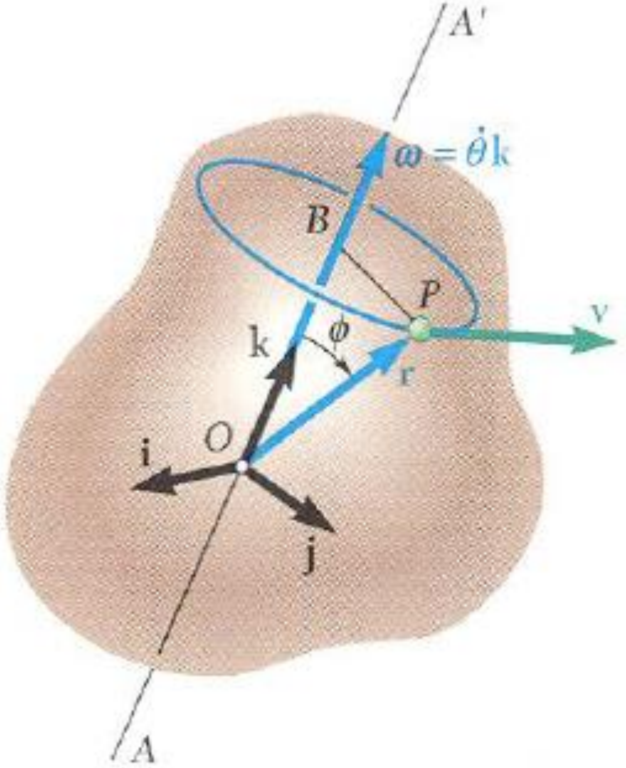
ROTATION - ACCELERATION

- **Differentiating to determine the acceleration,**

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega}\times\vec{r})$$
$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$
$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}$$

and

$$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$$
$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$



- Acceleration of P is combination of <u>two vectors,</u>

$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{\omega}\times\vec{r}$$
$$\vec{\alpha}\times\vec{r} = \text{tangential acceleration component}$$
$$\vec{\omega}\times\vec{\omega}\times\vec{r} = \text{radial acceleration component}$$

ROTATION

- Angular acceleration α of the slab is given by second derivative of angular coordinate $\boldsymbol{\theta}$ w.r.t. time.

$$\boldsymbol{Angular\ acceleration, \alpha = \frac{d\omega}{dt} = \ddot{\theta} = \frac{d^2\theta}{dt^2}}$$

- It is measured in radian/(second)^2.
- **Uniform rotation.** In this case, angular acceleration is zero and slab rotates with constant angular velocity.
- **Uniform accelerated rotation.** In this case angular acceleration α is constant.
- Following formulae relating angular acceleration, angular velocity, angular coordinate and time can be derived in a manner similar to that used in case of uniformly accelerated rectilinear motion.

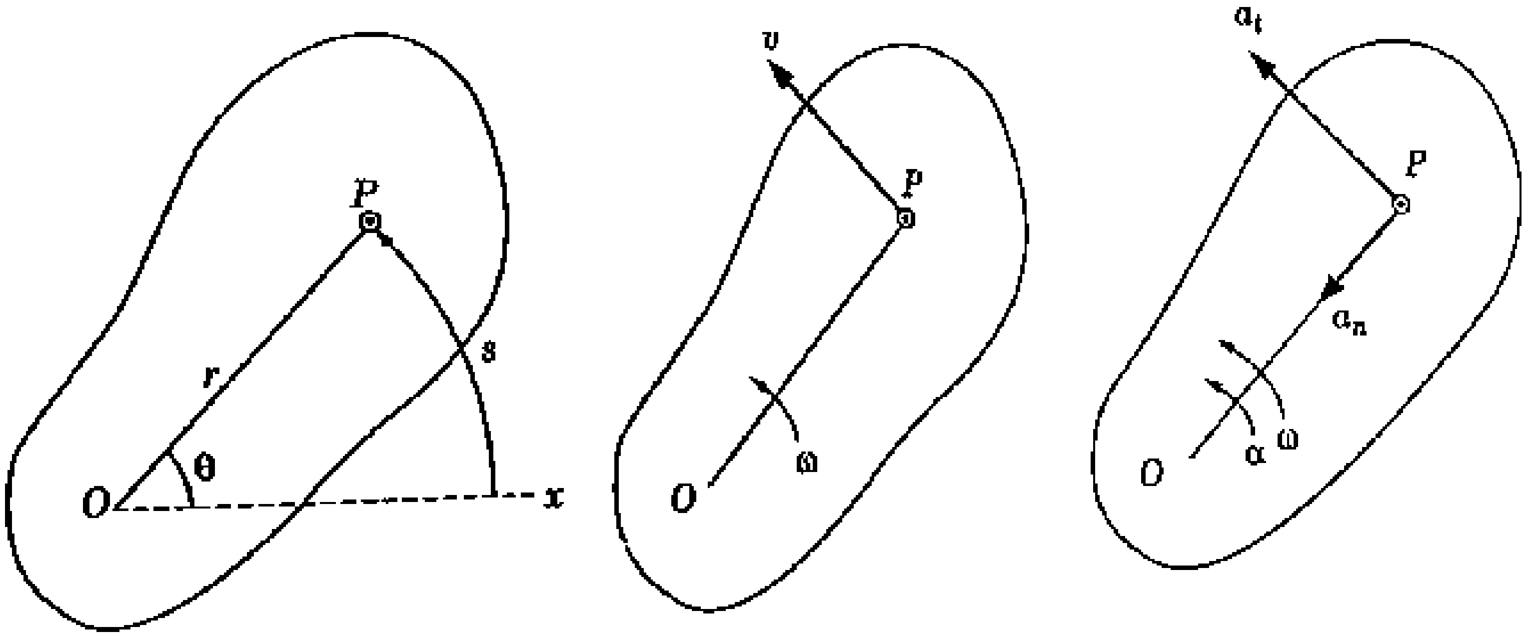
Rectilinear Motion	Angular Motion
$v = u + at$	$\omega = \omega_0 + \boldsymbol{\alpha} t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\boldsymbol{\alpha} t^2$
$v^2 = u^2 + 2as$	$\omega^2 = {\omega_0}^2 + 2\boldsymbol{\alpha}\theta$

LINEAR AND ANGULAR VELOCITY, LINEAR AND ANGULAR ACCELERATION IN ROTATION

- Consider a rigid slab rotating about a fixed axis perpendicular to plane of slab and intersecting it at point O. Let P be any point on rotating slab.
- As slab rotates, point P shall describe a circle with O as centre.
- Let r be radius of point P from O..
- Line OP rotate through an angle θ w.r.t. a reference line O_x describing an arc of length s, then, $s = r\theta$
- Differentiating w.r.t. time,

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \quad \text{or} \quad v = r\omega$$

- i.e., linear velocity is directly proportional to angular velocity
- Where, v is linear velocity of point P and ω angular velocity of slab.



LINEAR AND ANGULAR VELOCITY, LINEAR AND ANGULAR ACCELERATION IN ROTATION

- Angular velocity ω is independent of the choice of point P.
- But magnitude of linear velocity v depends upon distance of point P from point O.
- Tangential and normal components of the linear acceleration of point P by definition are,

$$a_t = \frac{dv}{dt} \quad \textbf{and} \quad a_n = \frac{v^2}{r}$$

- As, $v = r\omega$ **and** $Angular\ acceleration, \alpha = \frac{d\omega}{dt}$
- Therefore, $a_t = \frac{dr\omega}{dt} = \frac{rd\omega}{dt}$

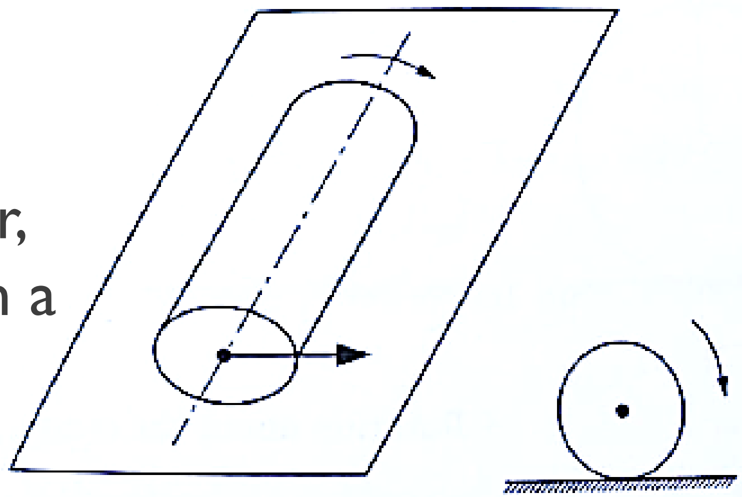
$a_t = r\alpha$ (i.e, tangential component of linear acceleration is directly proportional to angular acceleration)

- And, $a_n = \frac{(r\omega)^2}{r} = r(\omega)^2$ (i.e., normal component of linear acceleration is directly proportional to angular velocity)

GENERAL PLANE MOTION

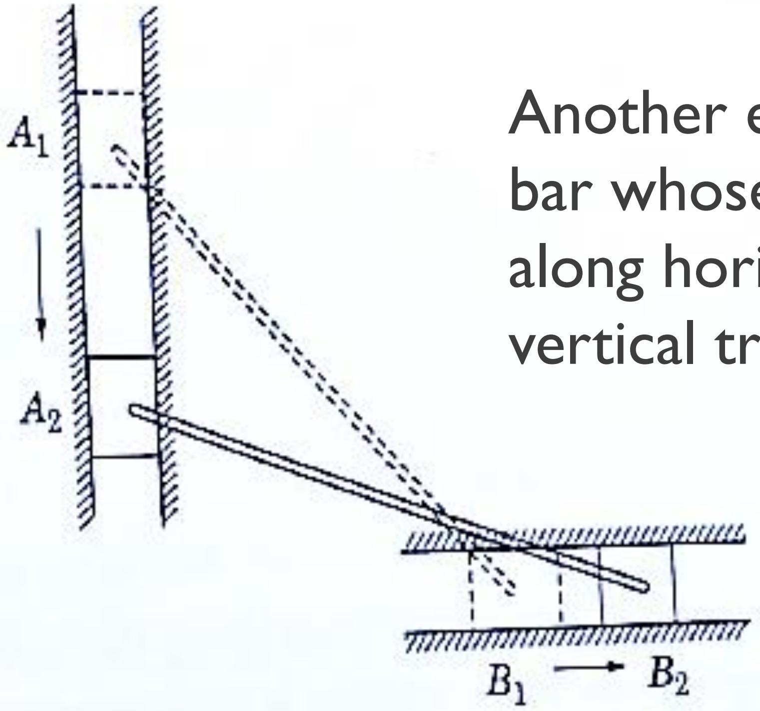
- Any plane motion which is neither a pure translation nor a pure rotation is referred to as a general plane motion.
- A general plane motion has the characteristic of a plane motion, i.e., all particles of body move in parallel planes.

Rolling of a cylinder, without slipping, on a flat or a curved surface



Rolling of a cylinder

Motion of one of the parallel plane



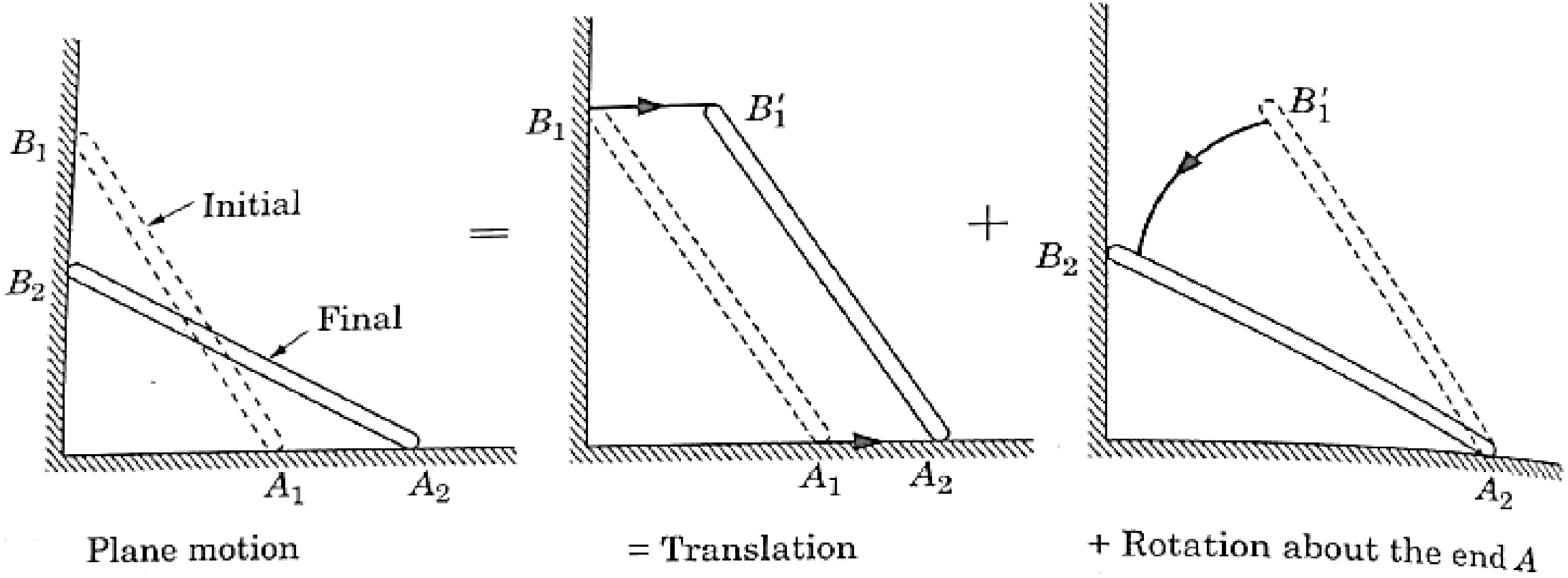
Another example is of a bar whose ends slide along horizontal and vertical tracks.

GENERAL PLANE MOTION

- In a plane motion any chosen particles of rigid body would remain in same fixed plane during the motion and all other particles would move in planes parallel to this plane.
- This concept enables us to study motion of a body by studying motion in one of these parallel planes and points lying in that plane.
- Thus, motion of a cylinder which is rolling without slipping, can be studied by considering the motion of a plane circular figure.
- Also, general plane motion of a rigid body, can always be considered as sum of a plane translation and a rotation about an axis perpendicular to plane motion.

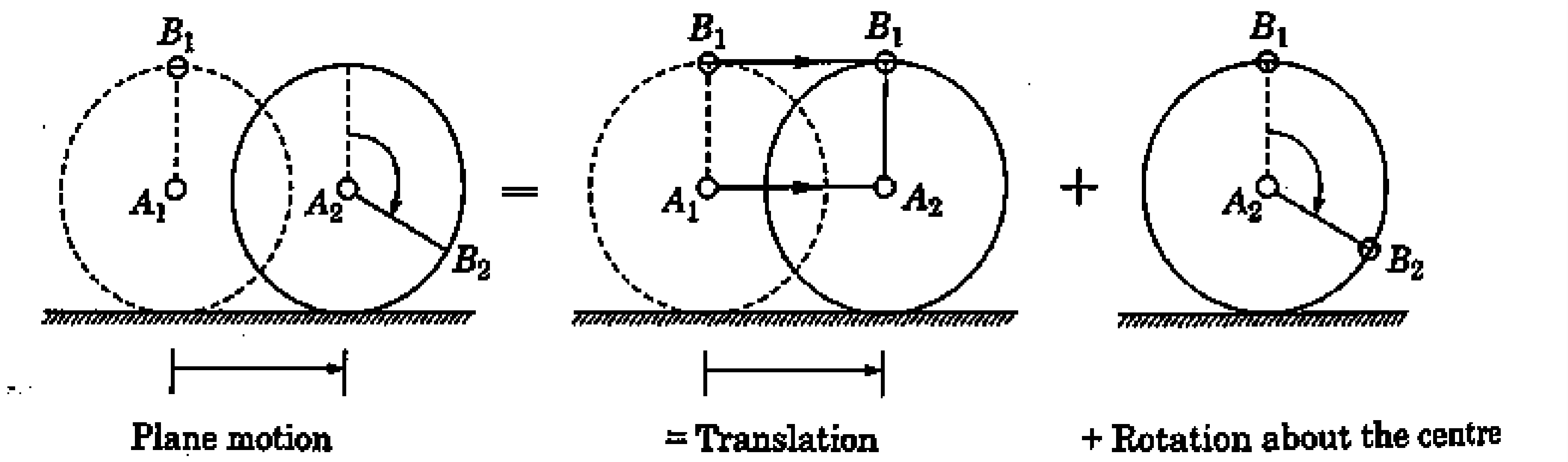
GENERAL PLANE MOTION

- Consider motion of bar AB from initial position A_1B_1 to position A_2B_2.
- This general plane motion can be replaced by a motion of translation of the bar from position A_1B_1 to position A_2B_1' together with a rotation about end A from position A_2B_1' to final position A_2B_2 as shown in figure a.



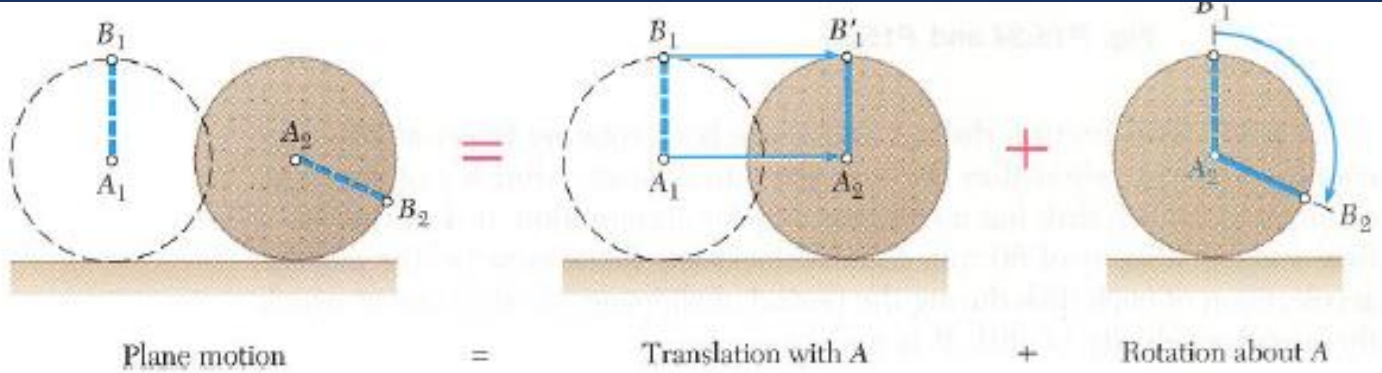
GENERAL PLANE MOTION

- Figure b shows rolling of a cylinder on a horizontal surface. This general plane motion can similarly be replaced by a translation and a rotation

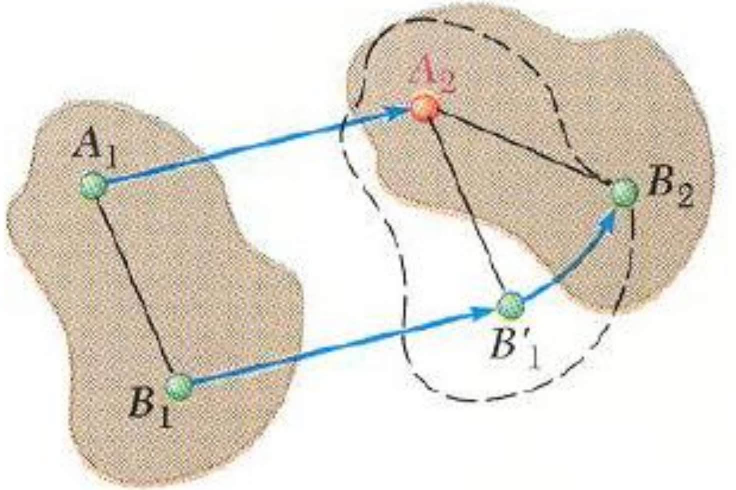


- The above concept of replacing general plane motion of a rigid body by sum of translation and a rotation is an important method to study motion of rigid bodies.

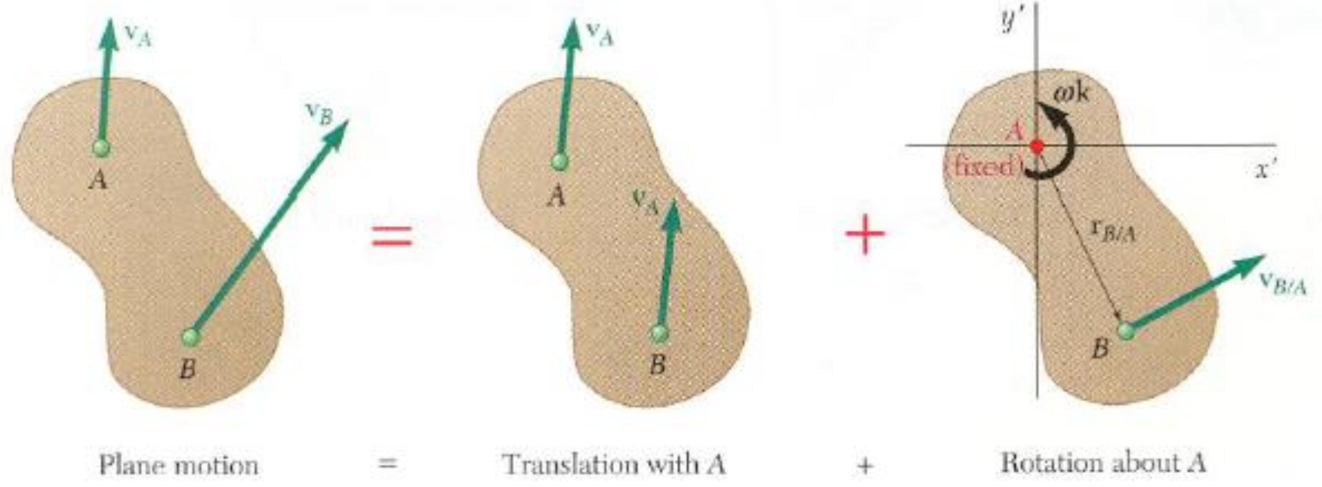
ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION

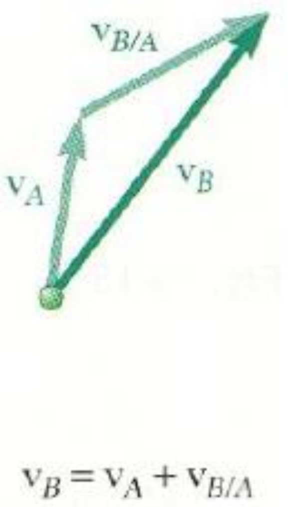


- *General plane motion* is neither a translation nor a rotation.
- General plane motion can be considered as the *sum* of a translation and rotation.
- Displacement of particles *A* and *B* to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2



ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION





- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.

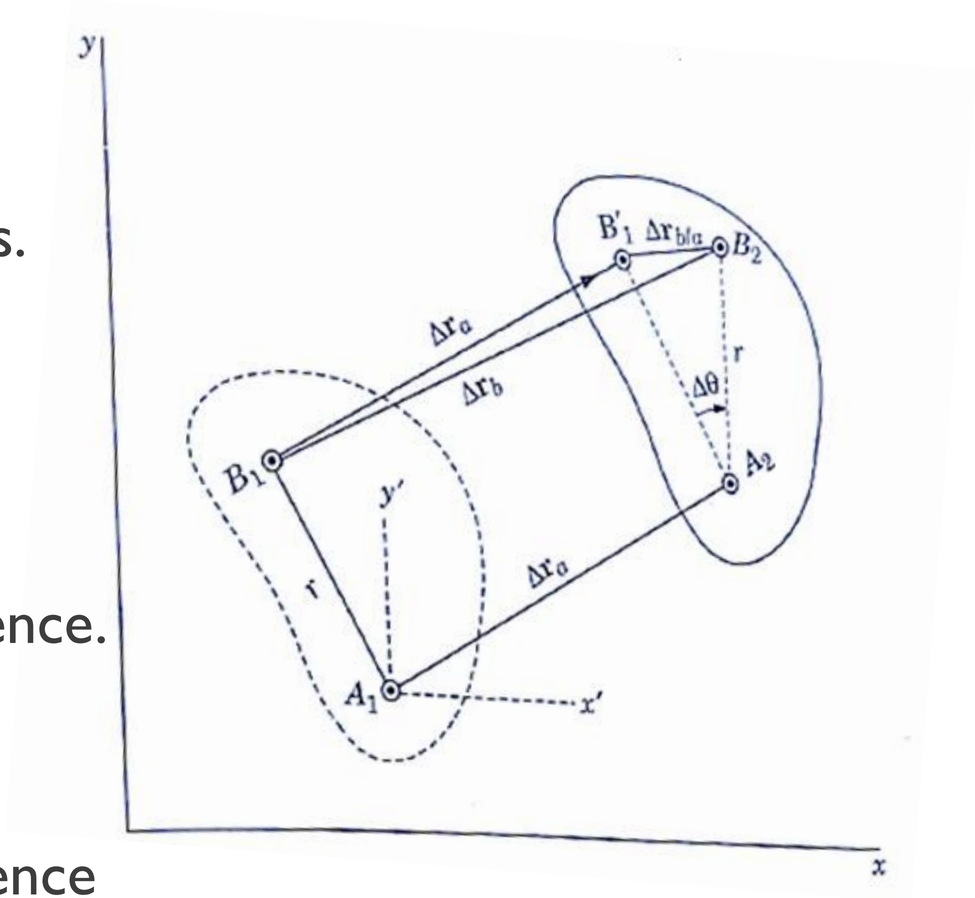
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega \vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega \vec{k} \times \vec{r}_{B/A}$$

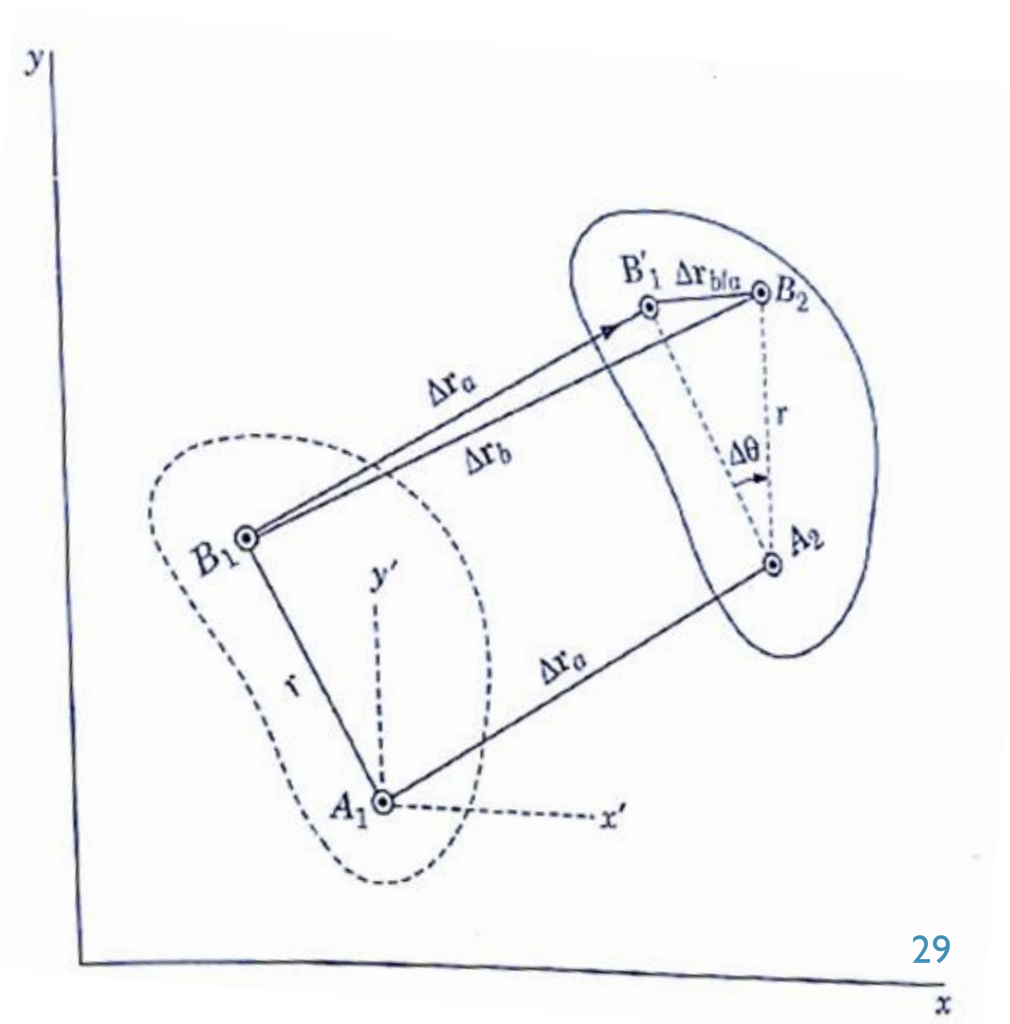
ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION

- Particle motion described using positions, velocities and accelerations vectors referred to fixed reference frames are termed as absolute.
- Not feasible to observe motion considering fixed set of axes.
- Thus, we consider moving reference frame.
- Consider two particles *A* and *B* moving along independent trajectories in the plane, and a fixed reference *O*.
- Let $\mathbf{r}_a$ and $\mathbf{r}_b$ be positions of particles *A* and *B* in fixed reference.
- Instead of observing motion of particle *A* relative to fixed reference, we will attach a non-rotating reference to particle *B* and observe motion of *A* relative to moving reference at *B*.



ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION

- Consider a rigid body in general plane motion w.r.t. fixed axis x-y.
- This motion of rigid body can be completely defined by motion of a plane figure representing motion of one of its parallel planes.
- Let an imaginary line A_1B_1 in body be displaced in time Δt to position A_2B_2.
- This displacement can be considered to be a sum of a translation from A_1B_1 to A_2B_1' and rotation about A_2 from A_2B_1' to A_2B_2.
- Attach a reference axis x'-y' to point A_1 such that as body moves, the axis x'-y' translates, but remains always parallel to the fixed axis x-y.

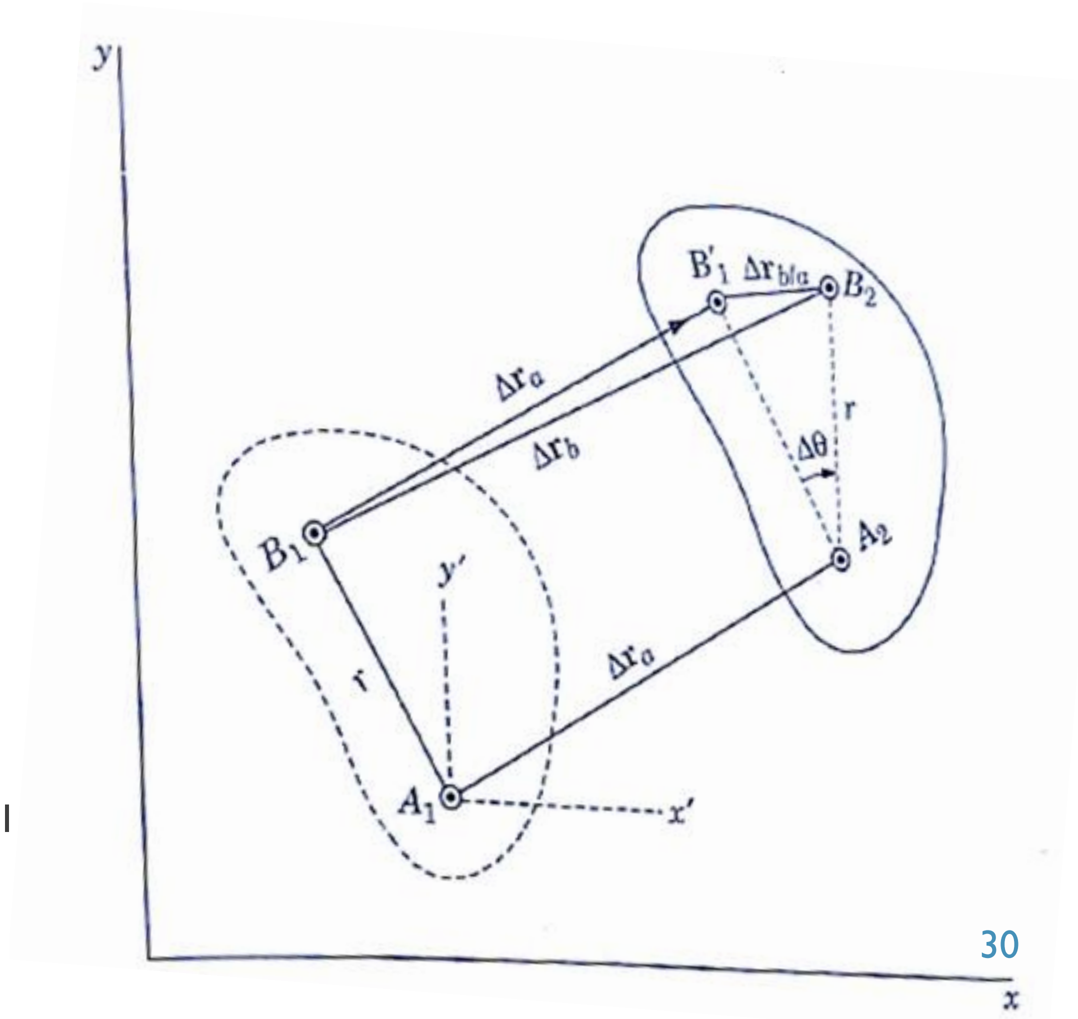


ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION

- Applying triangle law, $\Delta r_b = \Delta r_a + \Delta r_{b/a}$
- Total displacement of A_1B_1 = Translation of A_1B_1 + Displacement due to rotation of end B_1 about A_1.
- Dividing both sides by Δt and $\lim\limits_{\Delta t \to 0}$.

$$v_b = v_a + v_{b/a}$$

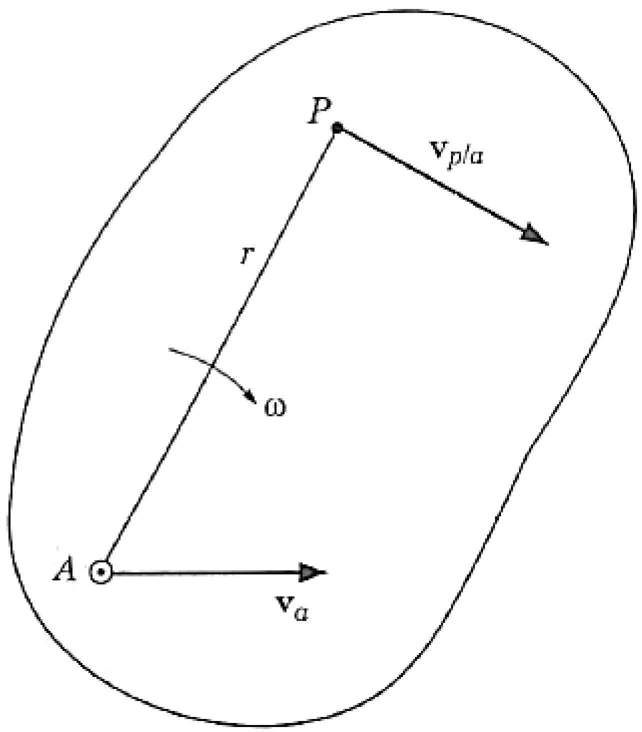
- Where, v_b is velocity of point B_1 w.r.t. fixed axis x-y.
- v_a is absolute velocity of point A_1 and corresponds to translation w.r.t. axis x-y.
- $v_{b/a}$ is relative velocity associated with rotation of point B_1 w.r.t. A_1 and has magnitude $v_{b/a} = r\omega$.
- r- fixed distance of point B_1 from A_1; ω-angular velocity.

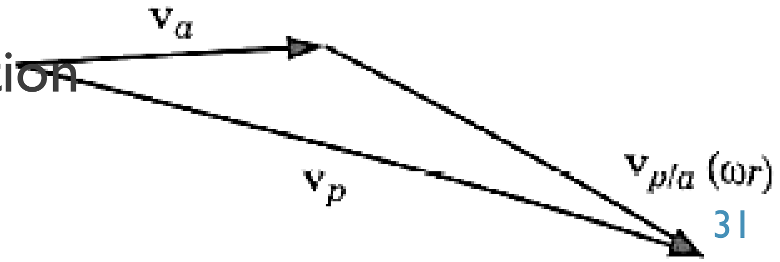


ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION

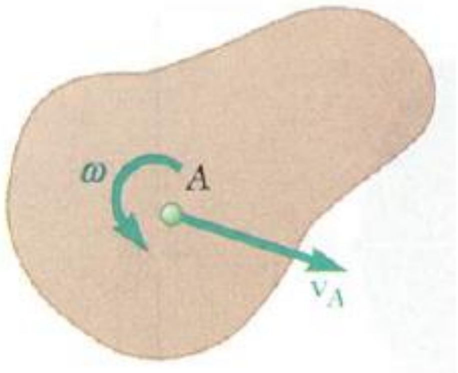
- If in above problem, point A_1 is considered to be a reference point, then such a point is called as a pole and the result can be generalized as,
- Velocity of any point P in rigid body = Vector sum of velocity of pole A and relative velocity of point P w.r.t. pole A. $(\boldsymbol{v_p = v_a + v_{p/a}})$
- Thus, velocity of any point in rigid body in plane motion can be determined if velocity of translation of a reference point called pole is known along with angular velocity of body.
- Similarly, acceleration of a point P is given by,
- Acceleration of any point P in rigid body = Vector sum of acceleration of pole A and relative acceleration of point P w.r.t. pole A.

$$\boldsymbol{\alpha_p = \alpha_a + \alpha_{p/a}}$$



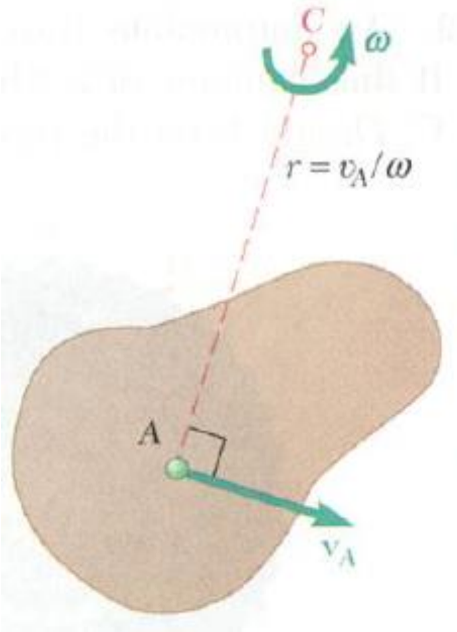


INSTANTANEOUS CENTRE OF ROTATION IN PLANE MOTION



*Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A*.

*The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A*.



*The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.

*As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C.*

INSTANTANEOUS CENTRE OF ROTATION IN PLANE MOTION

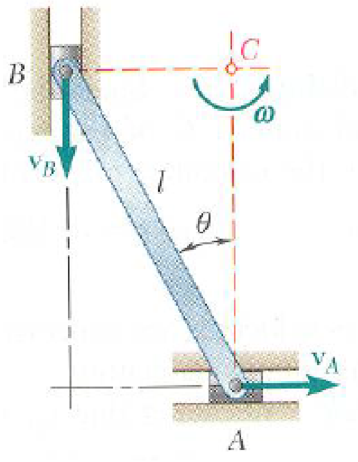


Fig. 15.20

*The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through A and B .with

$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta}$$

Then,

$$v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta}$$

$$= v_A \tan\theta$$

*The velocities of all particles on the rod are as if they were rotated about C.

*The particle at the center of rotation has zero velocity.

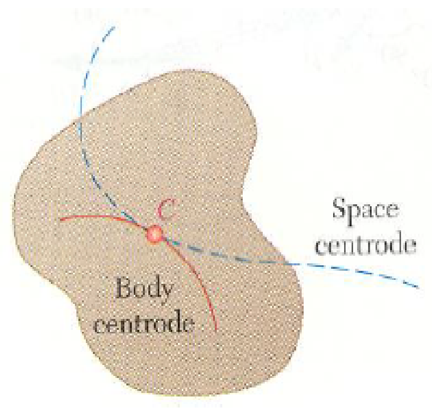
*The particle coinciding with the center of rotation changes with time and the acceleration of the particle at the instantaneous center of

INSTANTANEOUS CENTRE OF ROTATION IN PLANE MOTION

rotation is not zero.

- The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about *C*.

- The trace of the locus of the center of rotation on the body is the body centrode and in space is the space centrode.



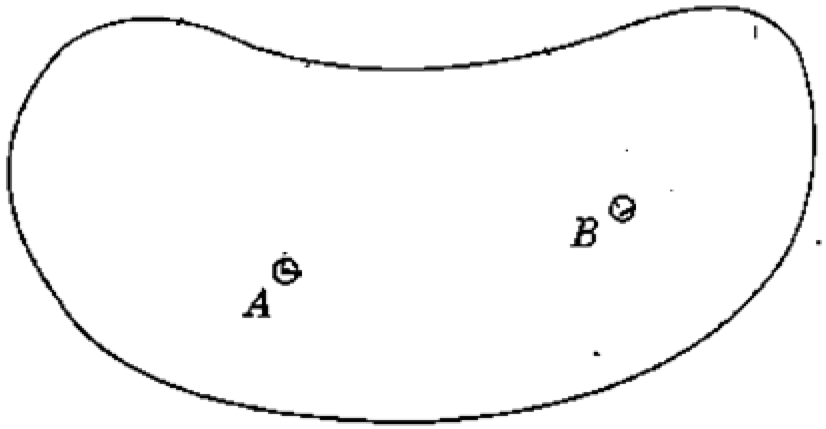
*At the instant shown, what is the approximate direction of the velocity of point G, the center of bar AB?

INSTANTANEOUS CENTRE OF ROTATION IN PLANE MOTION

- In general plane motion, plane motion of a rigid body can be considered to be a combination of translation of a reference point called pole and a rotation about this pole.
- It can also be shown that a rigid body in plane motion, at any given instant of time appears as if rotating about a certain point in the plane of the body.
- This point which is instantaneously at rest and has zero velocity is called the instantaneous centre of rotation.
- i.e., body may seem to be rotating about one point at one instant of time and about another point at the next instant.
- Instantaneous centre therefore, is changing every instant and is not a fixed point.
- Using this concept, velocity of any point in the body can be determined by assuming that point to be rotating, with some angular velocity ω about the instantaneous centre at the instant.

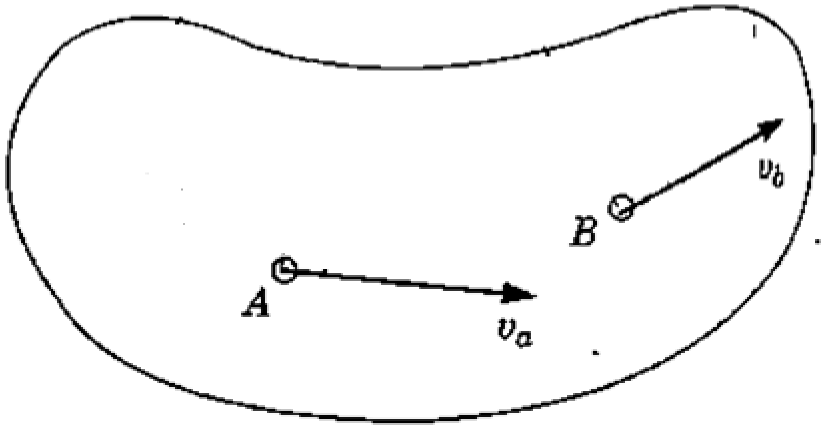
LOCATION OF THE INSTANTANEOUS CENTRE:
1. *WHEN DIRECTIONS OF VELOCITIES OF TWO POINTS A AND B IN THE BODY ARE KNOWN AND ARE UNEQUAL.*

- Consider two points A and B on rigid body in plane motion.



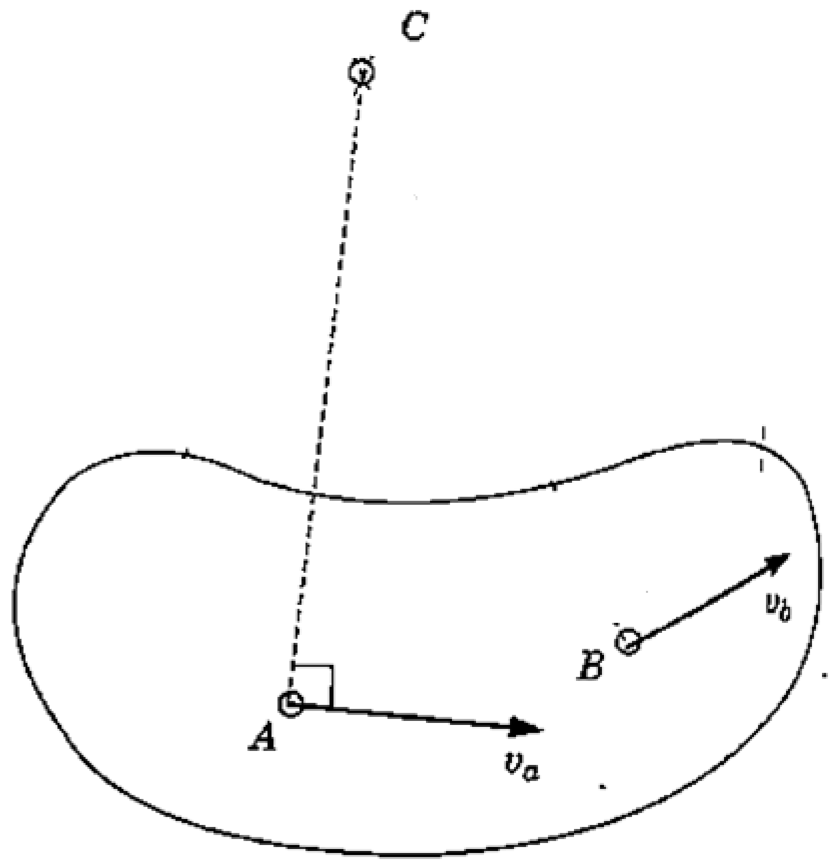
LOCATION OF THE INSTANTANEOUS CENTRE:
I. *WHEN DIRECTIONS OF VELOCITIES OF TWO POINTS A AND B IN THE BODY ARE KNOWN AND ARE UNEQUAL.*

- Consider two points A and B on rigid body in plane motion.
- Let at any instant their velocities be v_a and v_b respectively.



LOCATION OF THE INSTANTANEOUS CENTRE:
I. *WHEN DIRECTIONS OF VELOCITIES OF TWO POINTS A AND B IN THE BODY ARE KNOWN AND ARE UNEQUAL.*

- Consider two points A and B on rigid body in plane motion.
- Let at any instant their velocities be v_a and v_b respectively.
- Draw AC perpendicular to velocity v_a at point A.
- If this velocity v_a is result of rotation about some instantaneous centre then, centre must lie along AC.



LOCATION OF THE INSTANTANEOUS CENTRE:
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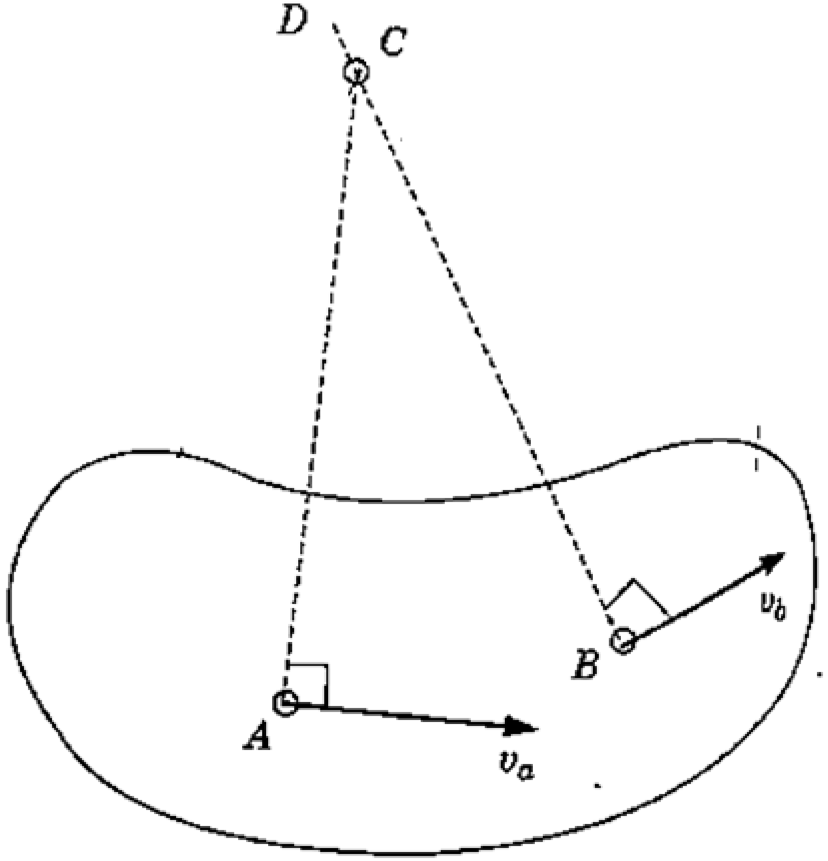
- Consider two points A and B on rigid body in plane motion.
- Let at any instant their velocities be v_a and v_b respectively.
- Draw AC perpendicular to velocity v_a at point A.
- If this velocity v_a is result of rotation about some instantaneous centre then, centre must lie along AC.
- Next draw BD perpendicular to velocity v_b at B.
- If this velocity v_b is result of rotation about some instantaneous centre then, centre must lie along BD.



LOCATION OF THE INSTANTANEOUS CENTRE:
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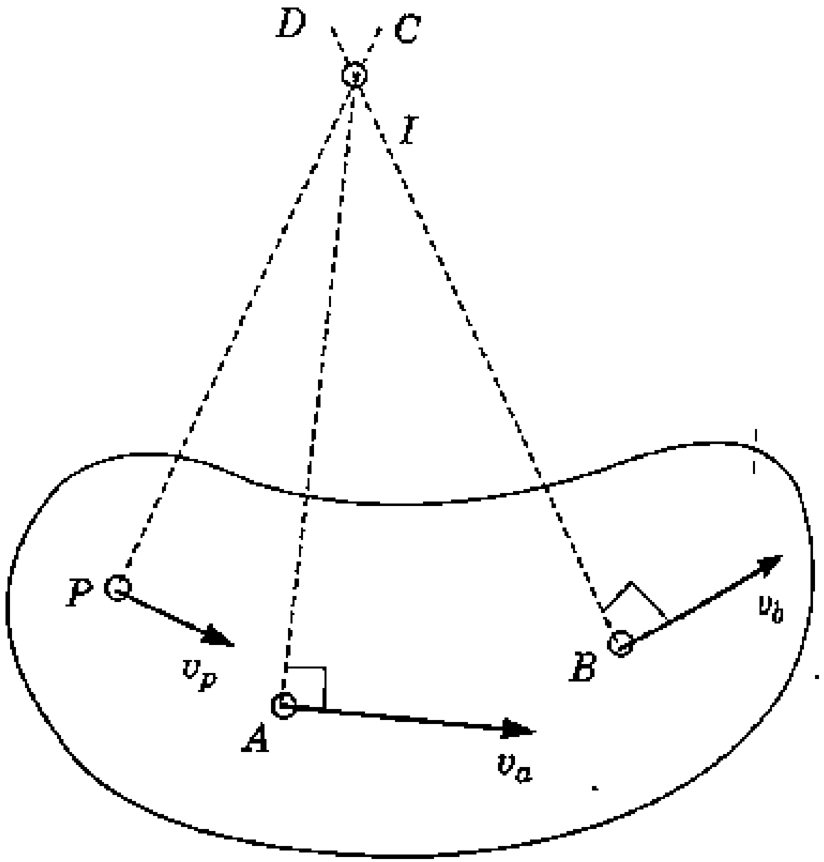
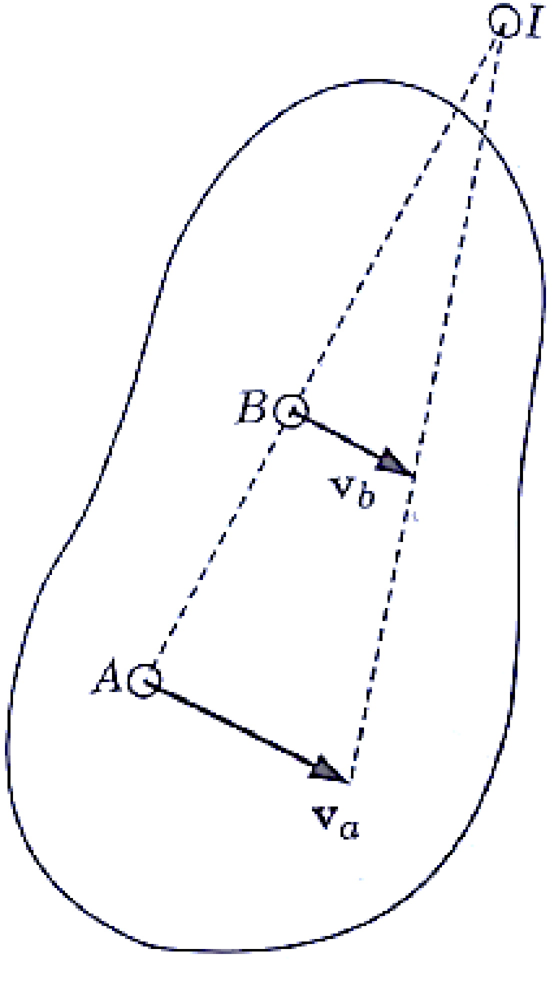
- Consider two points A and B on rigid body in plane motion.
- Let at any instant their velocities be v_a and v_b respectively.
- Draw AC perpendicular to velocity v_a at point A.
- If this velocity v_a is result of rotation about some instantaneous centre then, centre must lie along AC.
- Next draw BD perpendicular to velocity v_b at B.
- If this velocity v_b is result of rotation about some instantaneous centre then, centre must lie along BD.
- Their point of intersection I, therefore, determines instantaneous centre of rotation of body at that instant.
- Angular velocity of body, $\boldsymbol{\omega}$ can be determined as, $\boldsymbol{\omega = \frac{v_a}{IA}}$
- Velocity of any point P is given at that instant by $\boldsymbol{v_p = \omega\, IP}$



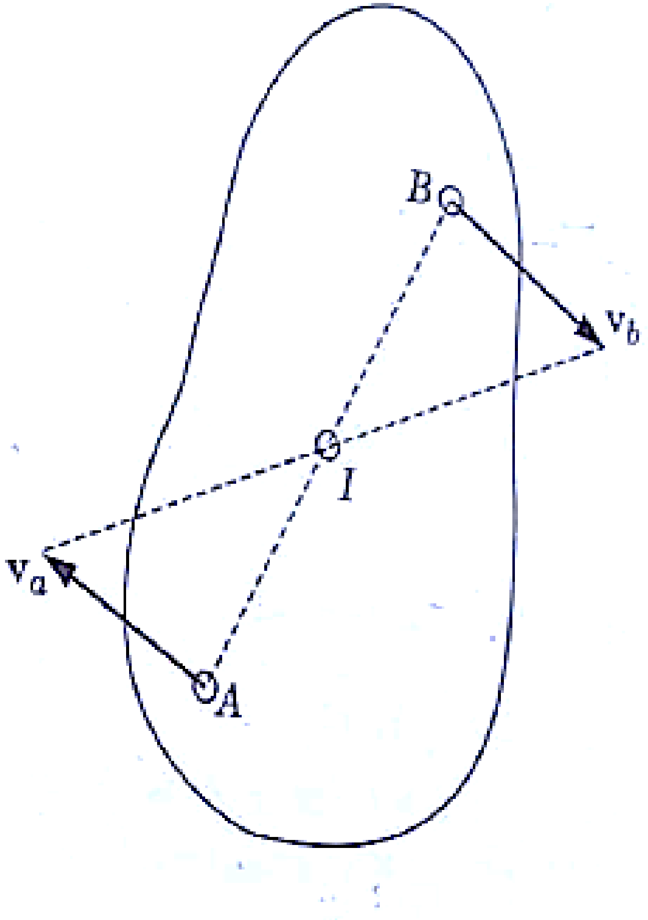
LOCATION OF THE INSTANTANEOUS CENTRE:

2. When velocity v_a and v_b of two points in body are parallel but unequal in magnitude.

- The instantaneous centre I can be found by determining point of intersection of line AB with line joining extremities of the vectors v_a and v_b as shown in figure a and b.



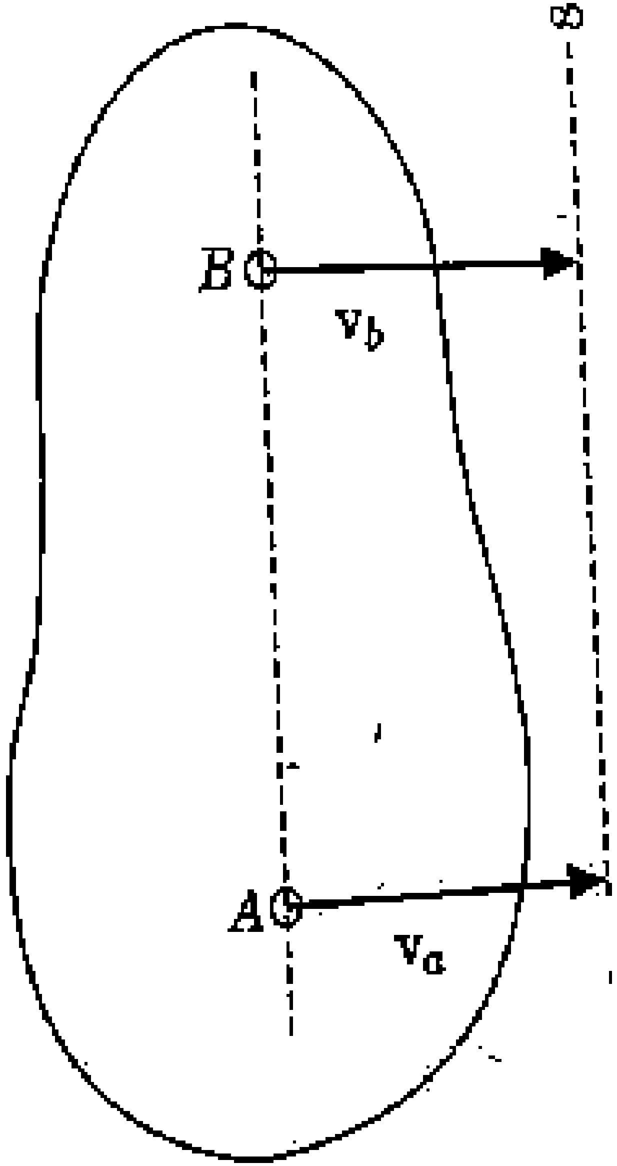
(a)



(b)

LOCATION OF THE INSTANTANEOUS CENTRE:

3. When velocities v_a and v_b of two points are equal and parallel then instantaneous centre is at the infinity and all the points of the body have the same velocity.



SUMMARY

- The instantaneous centre is not a fixed point.
- Its location keeps changing every instant and path traced by it (locus) is called centrode.
- The instantaneous centre may lie on or outside the body.
- The instantaneous centre is a point identified with the body where the velocity is zero.
- Conversely if a point identified with the rigid body is at rest at any instant (within or outside the body) it must be the instantaneous centre of rotation of the body.
- For example, if a circular cylinder rolls without slipping, the point of the contact has zero velocity and is the instantaneous centre of rotation of the cylinder.