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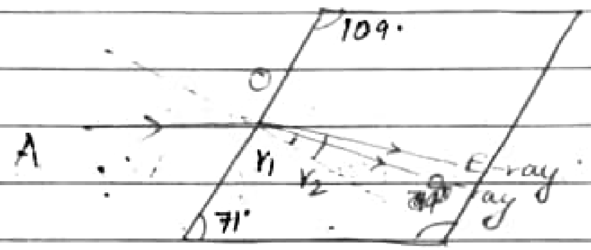
Polarisation

Double refraction or Birefringence

when a ray of light is refracted by a crystal of calcite ~~CaCO2~~ ($CaCO_3$), it gives two refracted rays. this phenomena is called double refraction or birefringence. The structure of calcite crystal is rhombohedral & bounded by 6 parallelograms with opposite angles of 109° & 71°.



In calcite crystal; one refracted ray is called ordinary ray or O-ray which follow the snell's law of refraction. the other refracted ray is called extraordinary ray or E ray which does not follow Snell's law.

* Place your eye vertically above the crystal, it is found that, one image remain stationary & the second image rotates with the rotation of crystal.

→ Stationary image → ordinary ray (O-ray)
→ Rotating image → extraordinary ray (E-ray)

From the figure,

for O ray, $\mu_o = \frac{\sin i}{\sin r_1}$

for E ray, $\mu_e = \frac{\sin i}{\sin r_2}$

from figure, $r_1 < r_2$

$\sin r_2 > \sin r_1$

$\mu_o > \mu_e$

$v_e > v_o$

So the velocity of light for the ordinary ray inside the crystal will be less, so extraordinary ray travel faster as compared to the ordinary rays.

* This happens only when both the rays are plane polarised.

Quarter wave plate

we make a crystal plate of such a thickness that it can introduce a path difference of $\lambda/4$ between the O-ray & E-ray passing through it is known as quarter wave plate.

If the thickness of the calcite or quartz crystal plate is 't' & the refractive index for the ordinary & extraordinary rays are μ_o & μ_e then the path diff. b/w the two rays –

for negative crystal (calcite) : Path diff. $= (\mu_o - \mu_e)t$

for positive crystal (Quartz) : Path diff $= (\mu_e - \mu_o)t$

To produce a path diff. of $\lambda/4$ in calcite;

$$(\mu_o - \mu_e)t = \lambda/4$$

$$\boxed{t = \frac{\lambda}{4(\mu_o - \mu_e)}}$$ Calcite.

For Quartz,

$$(\mu_e - \mu_o)\, t = \lambda/4$$

$$\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$$ Quartz

Half wave plate
path diff $= \lambda/2$ b/w o-ray & e-ray.

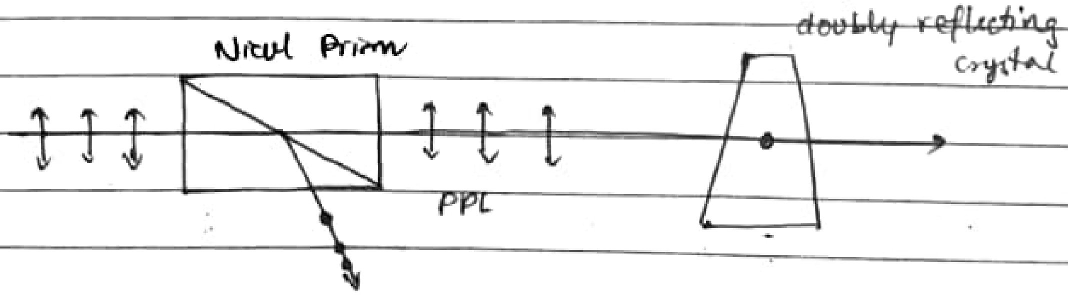
for Calcite, $(\mu_o - \mu_e)t = \lambda/2$

$$\boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}}$$

for Quartz, $(\mu_e - \mu_o)t = \lambda/2$

$$\boxed{t = \frac{\lambda}{2(\mu_e - \mu_o)}}$$

Elliptically & circularly polarised light

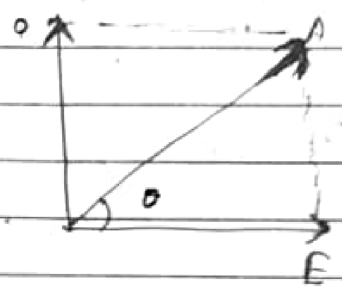


When a monochromatic light passes from the nicole prism, they give the plane polarised light & this ppl incident on doubly reflecting crystal so the incident light split into two components ordinary & extra-ordinary rays. Both the rays travels in the same dirn but with diff velocity. If the thickness of the crystal is 'b' so they create a phase diff. b/w ordinary & ex. ord. rays 'δ'.

Let the amplitude of the ~~and~~ incident wave is 'A' & it makes an angle 'θ' from the optic axis.

amp. of O-ray = $A\sin\theta$

amp of E-ray = $A\cos\theta$



If the phase diff. b/w these two rays is 'δ', so the dist covered by the.

for e rays, $x = A\cos\theta \sin(\omega t+\delta)$

For O rays, $y = A\sin\theta \sin(\omega t)$

Let $A\cos\theta = a$ & $A\sin\theta = b$.

$x = a\sin(\omega t+\delta)$ — ①, $y = b\sin\omega t$

$\Rightarrow \sin\omega t = y/b$, $\cos\omega t = \sqrt{1-\frac{y^2}{b^2}}$

from ①, $\frac{x}{a} = \sin(\omega t+\delta)$

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1-\frac{y^2}{b^2}}\sin\delta$$

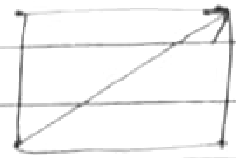
$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1-\frac{y^2}{b^2}}\sin\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - 2\frac{xy}{ab}\cos\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\delta = \sin^2\delta}$$

Case I, $\delta = 0 \Rightarrow \cos\delta = 1, \ \sin\delta = 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

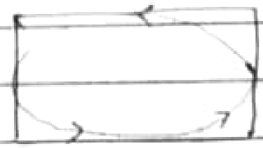


$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\rightarrow \frac{x}{a} - \frac{y}{b} = 0 \Rightarrow \boxed{y = \frac{b}{a}x}$$ straight line eqⁿ

The wave is plane polarised in this condⁿ.

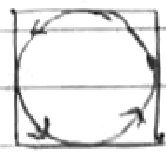
Case II, $\delta = \frac{\pi}{2}$, $\rightarrow \cos\delta = 0, \ \sin\delta = 1$



$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$ (ellipse eqⁿ)

The wave is elliptically polarised.

Case III, $\delta = \frac{\pi}{2}$ but $a = b$

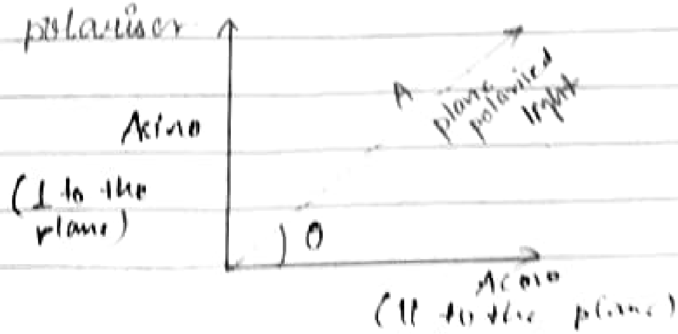


$$\boxed{x^2 + y^2 = a^2}$$ (eqⁿ of circle)

The wave is circularly polarised.

→ Law of Malus -

When a completely plane polarised light incident on analyser, the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle b/w the plane of transmission of any analyser & ~~the~~ polariser



If 'A' be the amplitude of the plane polarised light & 'θ' be the angle b/w the plane of transmission of the analyser & the polariser. Since, only $A\cos\theta$ is transmitted through the analyser, so the intensity of light emerging from the analyser is -

$$I = (A\cos\theta)^2 = A^2\cos^2\theta$$

$$= I_0 \cdot \langle \cos^2\theta \rangle \Rightarrow \boxed{I = \frac{I_0}{2}}$$

So, an ideal polariser is one that transmit 50% of the incident light as plane polarised light.

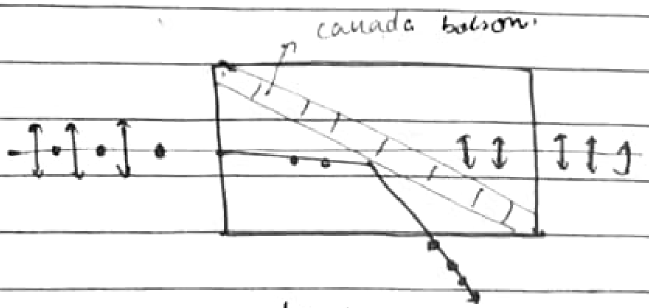
Q Two polarising sheets have their polarising directions parallel so as to transmit maximum intensity of light, so what angle must either sheet be turned so that the intensity becomes one-half of the initial value

$$I = I_0/2 = I_0\cos^2\theta$$

$$\frac{I_0}{2} = I_0\cos^2\theta$$

$$\cos^2\theta = \frac{1}{2} \Rightarrow \theta = 45°$$

• Nicole prism.



$\mu_o = 1.66$

$\mu_e = 1.43$

$\mu_b = 1.55$