Transfer Lines and Similar Automated Manufacturing Systems

L-19

Contents

Automated Production lines

There application is appropriate only under following conditions:

(i) High production demand
(ii) Stable product design
(iii) Long product life
(iv) Multiple operations

When applications satisfy the conditions, they provide the following benefits:

(i) Low direct labor content
(ii) Low product cost because cost of fixed equipment is spread over many units
(iii) High production rates
(iv) Production lead time (the time between beginning of production and completion of finished unit) and work-in-process are minimized.
(v) Factory floor space is minimized.

1) Fundamentals of Automated Production Lines

- An automated production line consists of multiple workstations that are linked together by a work handling system that transfers parts from one station to the next.

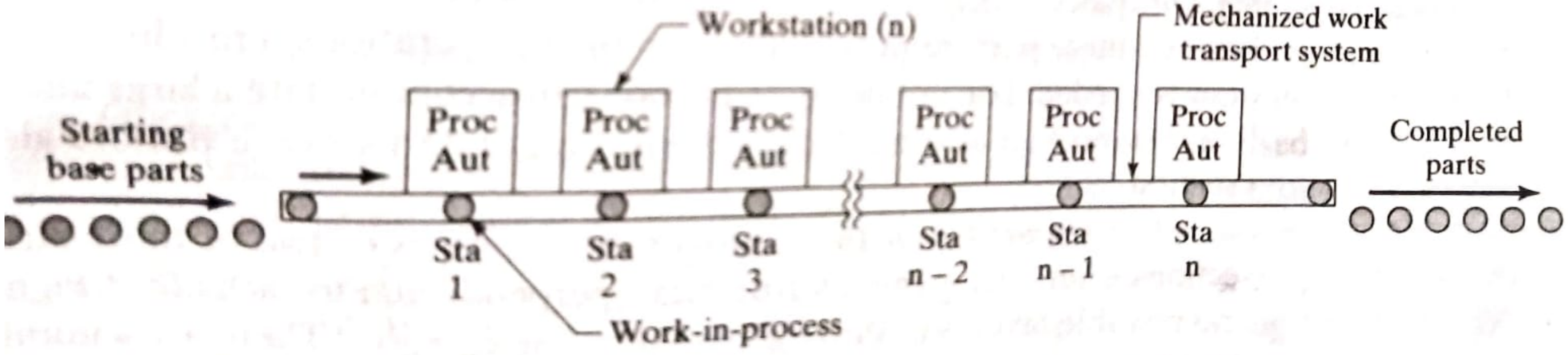


Figure 18.1 General configuration of an automated production line. Key: Proc = processing operation, Aut = automated workstation.

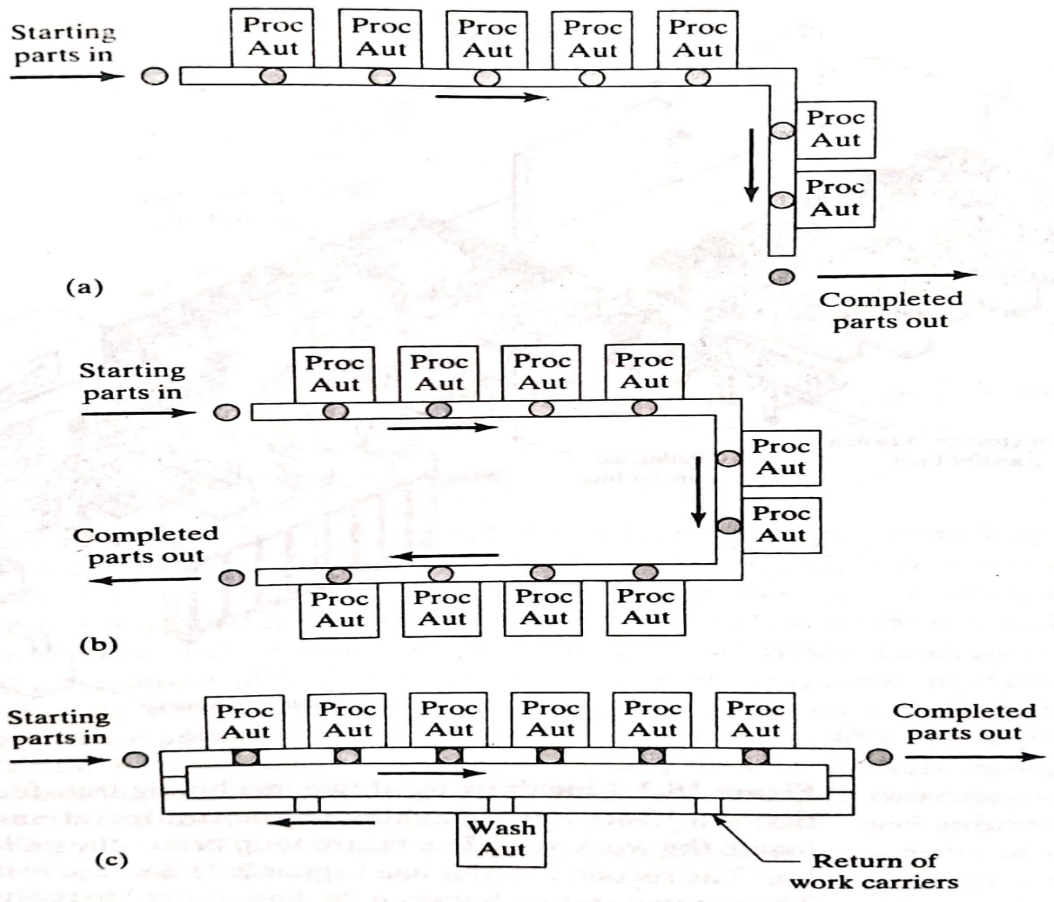
a) System Configurations

It has following types

(i) in-line

(ii) Segmented in-line

(iii) Rotary

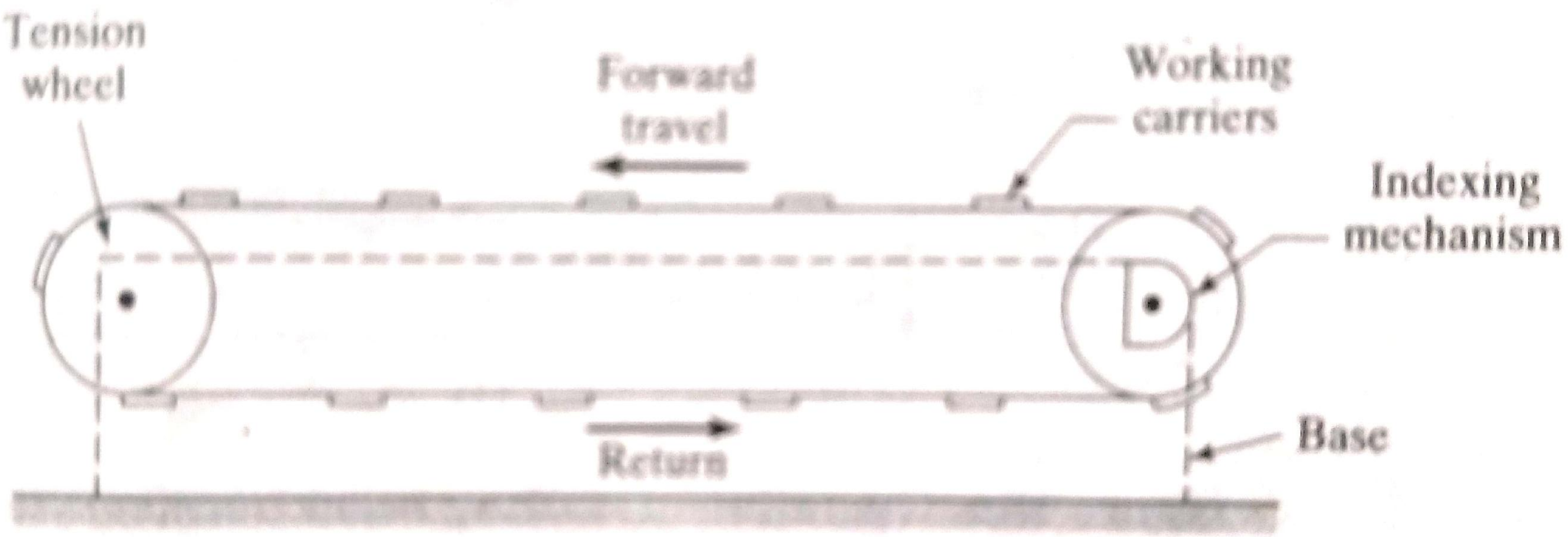


b) Workpart Transfer Mechanisms

The work part transfer mechanism in two parts

(i) Linear transport systems for in-line systems

(ii) Rotary indexing mechanisms for dial indexing machines



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(ii) Rotary Indexing Mechanisms

are explained here: Geneva mechanism and cam drive.

The ***Geneva mechanism*** uses a continuously rotating driver to index the table through a partial rotation, as illustrated in Figure 18.7. If the driven member has six slots for a six-station dial indexing table, each turn of the driver results in 1/6 rotation of the worktable, or 60°. The driver only causes motion of the table through a portion of its own rotation. For a six-slotted Geneva, 120° of driver rotation is used to index the table. The remaining 240° of driver rotation is dwell time for the table, during which the processing operation must be completed on the work unit. In general,

$$\theta = \frac{360}{n_s} \tag{18.1}$$

where θ = angle of rotation of worktable during indexing (degrees of rotation), and n_s = number of slots in the Geneva. The angle of driver rotation during indexing = 2θ, and the angle of driver rotation during which the work table experiences dwell time is ($360-2\theta$). Geneva mechanisms usually have four, five, six, or eight slots, which establishes the maximum number of workstation positions that can be placed around the periphery of the table. Given the rotational speed of the driver, we can determine total cycle time as:

$$T_c = \frac{1}{N} \tag{18.2}$$

where T_c = cycle time (min), and N = rotational speed of driver (rev/min). Of the total cycle time, the dwell time, or available operation time per cycle, is given by:

$$T_s = \frac{(180 + \theta)}{360N} \tag{18.3}$$

where T_s = available service or processing time or dwell time (min), and the other terms are defined above. Similarly, the indexing time is given by:

$$T_r = \frac{(180 - \theta)}{360N} \tag{18.4}$$

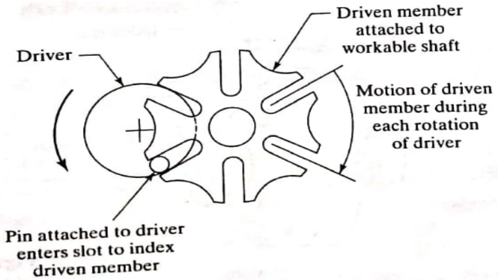

Figure 18.7 Geneva mechanism with six slots.

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EXAMPLE 18.1 Geneva Mechanism for a Rotary Indexing Table

A rotary worktable is driven by a Geneva mechanism with six slots, as in Figure 18.7. The driver rotates at 30 rev/min. Determine the cycle time, available process time, and the lost time each cycle indexing the table.

Solution: With a driver rotational speed of 30 rev/min, the total cycle time is given by Eq. (18.2):

$$T_c = (30)^{-1} = 0.0333 \text{ min} = 2.0 \text{ sec.}$$

The angle of rotation of the worktable during indexing for a six-slotted Geneva is given by Eq. (18.1):

$$\theta = \frac{360}{6} = 60°$$

Eqs. (18.3) and (18.4) give the available service time and indexing time, respectively, as:

$$T_s = \frac{(180 + 60)}{360(30)} = 0.0222 \text{ min} = 1.333 \text{ sec.}$$

$$T_r = \frac{(180 - 60)}{360(30)} = 0.0111 \text{ min} = 0.667 \text{ sec.}$$

Various forms of ***cam drive*** mechanisms, one of which is illustrated in Figure 18.8, are used to provide an accurate and reliable method of indexing a rotary dial table. Although a relatively expensive drive mechanism, its advantage is that the cam can be designed to provide a variety of velocity and dwell characteristics.

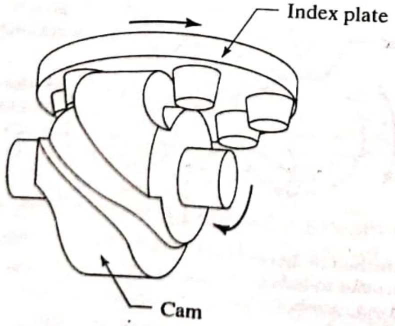

Figure 18.8 Cam mechanism to drive dial indexing table (reprinted from [1]).

C) Storage Buffers

A storage buffer in a production line in a location where parts can be collected and temporarily stored before proceeding to sunsequent (downstream) workstations.

Reasons by storage buffers are used in the automated production lines.

(i) To reduce the effect od station breakdowns

(ii) To provide a bank of parts to supply the line

(iii) To provide a place to put the output of the line

(iv) To allow for curing time or other required delay

(v) To smooth cycle time variations

d) Control of the Production Line

They are of two types:

(i) Control Functions

a) Sequence control

b) Safety monitoring

c) Quality control

(ii) Line Controllable

In this computer, PLC

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In addition to being more reliable, computer control offers the following benefits

a) Opportunity to improve and upgrade the control software, such as adding specific control functions not anticipated in the original system design
b) Recording of data on process performance, equipment reliability and product quality for subsequent analysis. In some cases, product quality records must be maintained for legal reasons
c) Diagnostic routines to expedite maintenance and repair when line breakdowns occur and to reduce the duration of downtime incidents
d) Automatic generation of preventive maintenance schedules indicating when certain preventive maintenance activities should be performed. This helps to reduce the frequency of downtime occurrences
e) Provides a more convenient human-machine interface between the operator and automated line.

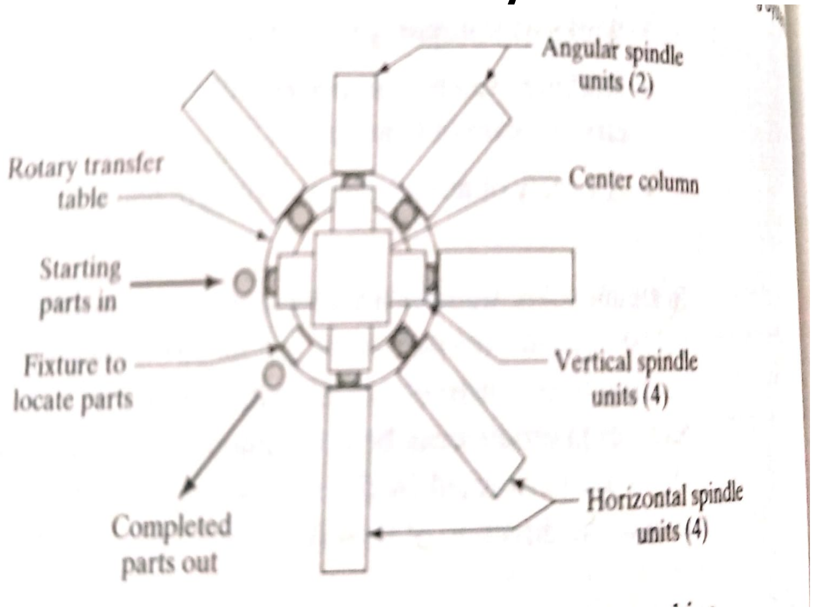
2) Applications of Automated Production Lines

Applications include metal forming and cutting, rolling mill operations, spot welding of automobile car bodies and painting and plating operations

(i) Machining Systems

(a) Transfer lines

(b) Rotary transfer machines and related systems



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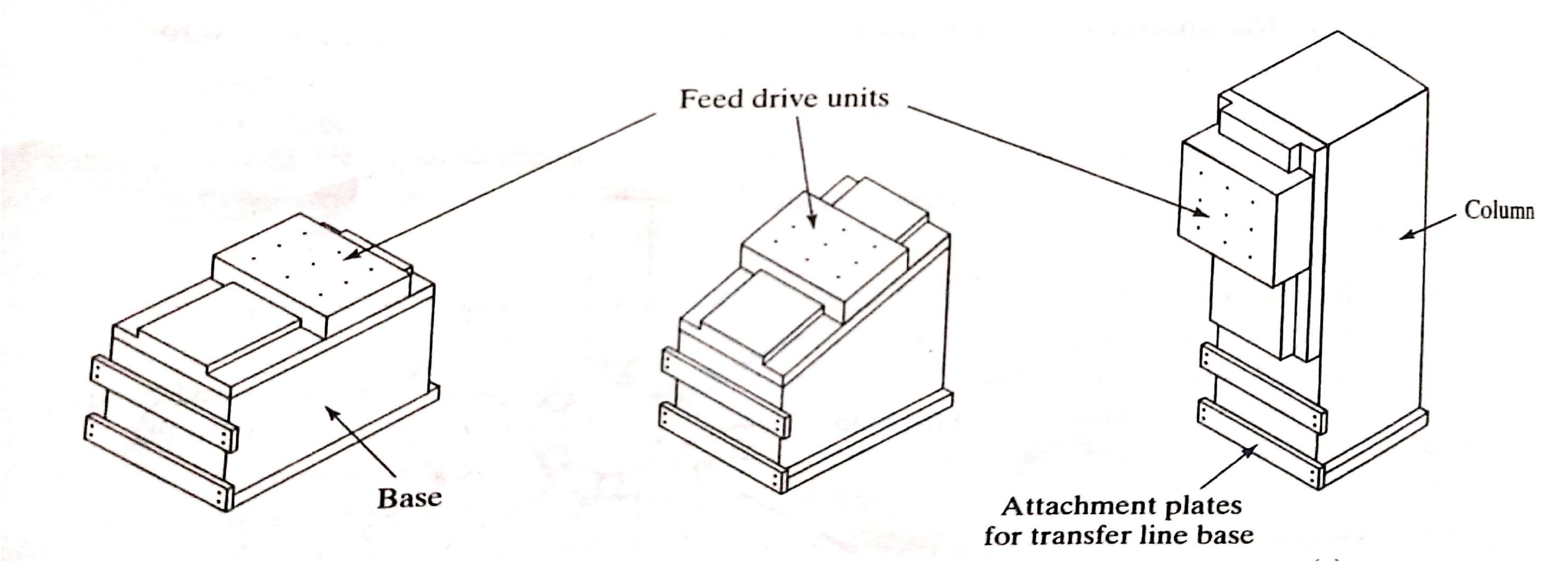
(ii) System Design Considerations



3) Analysis of Transfer Lines with NO internal Storage

In this analysis, two general problem areas must be addressed

(i) Process Technology (ii)systems technology

(a) Basic terminology and performance measures

Following assumptions about the about the operation of the transfer lines and rotary indexing machines.

(i) The workstations perform processing operations, such as machining, not assembly

(ii) Processing times at each statin are constant, through not necessarily equal.

(iii) Synchronous transfer of parts

(iv) No internal storage buffers.

$$T_c = \text{Max}\{T_{si}\} + T_r \tag{18.5}$$

where T_c = ideal cycle time on the line (min); T_{si} = the processing time at station i (min); and T_r = repositioning time, called the transfer time here (min). We use the Max$\{T_{si}\}$ in

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TABLE 18.1 Common Reasons for Downtime on an Automated Production Line

• Tool failures at workstations
• Tool adjustments at workstations
• Scheduled tool changes
• Limit switch or other electrical malfunctions
• Mechanical failure of a workstation
• Mechanical failure of the transfer system
• Stockouts of starting work units
• Insufficient space for completed parts
• Preventive maintenance on the line
• Worker breaks

time for each downtime occurrence. These downtime occurrences cause the actual average production cycle time of the line to be longer than the ideal cycle time given by Eq. (18.5). We can formulate the following expression for the actual average production time T_p:

$$T_p = T_c + FT_d \tag{18.6}$$

where F = downtime frequency, line stops/cycle; and T_d = downtime per line stop, min. The downtime T_d includes the time for the repair crew to swing into action, diagnose the cause of the failure, fix it, and restart the line. Thus, FT_d = downtime averaged on a per cycle basis.

One of the important measures of performance on an automated transfer line is production rate, which can be computed as the reciprocal of T_p:

$$R_p = \frac{1}{T_p} \tag{18.7}$$

where R_p = actual average production rate (pc/min), and T_p is the actual average production time from Eq. (18.6) (min). It is of interest to compare this rate with the ideal production rate given by

$$R_c = \frac{1}{T_c} \tag{18.8}$$

where R_c = ideal production rate (pc/min). It is customary to express production rates on automated production lines as hourly rates (multiply the rates in Eqs. (18.7) and (18.8) by 60).

The machine tool builder uses the ideal production rate R_c in its proposal for the automated transfer line and speaks of it as the production rate at 100% efficiency. Unfortunately, because of downtime, the line will not operate at 100% efficiency. While it may seem deceitful for the machine tool builder to ignore the effect of line downtime on production rate, it should be stated that the amount of downtime experienced on the line is mostly the responsibility of the company using the production line. In practice, most of the reasons for downtime occurrences in Table 18.1 represent factors that must be controlled and managed by the user company.

In the context of automated production systems, line efficiency refers to the proportion of uptime on the line and is really a measure of reliability more than efficiency.

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Nevertheless, this is the terminology of production lines. Line efficiency can be calculated as follows:

$$E = \frac{T_c}{T_p} = \frac{T_c}{T_c + FT_d} \qquad (18.9)$$

where E = the proportion of uptime on the production line, and the other terms have been previously defined.

An alternative measure of performance is the proportion of downtime on the line, which is given by

$$D = \frac{FT_d}{T_p} = \frac{FT_d}{T_c + FT_d} \qquad (18.10)$$

where D = proportion of downtime on the line. It is obvious that

$$E + D = 1.0 \qquad (18.11)$$

An important economic measure of performance of an automated production line is the cost per unit produced. This piece cost includes the cost of the starting blank that is to be processed on the line, the cost of time on the production line, and the cost of any tooling that is consumed (e.g., cutting tools on a machining line). The piece cost can be expressed as the sum of these three factors:

$$C_{pc} = C_m + C_oT_p + C_t \qquad (18.12)$$

where C_{pc} = cost per piece ($/pc); C_m = cost of starting material ($/pc); C_o = cost per minute to operate the line ($/min); T_p = average production time per piece (min/pc); and C_t = cost of tooling per piece ($/pc). C_o includes the allocation of the capital cost of the equipment over its expected service life, labor to operate the line, applicable overheads, maintenance, and other relevant costs, all reduced to a cost per minute (see Section 2.5.3).

Eq. (18.12) does not include factors such as scrap rates, inspection costs, and rework costs associated with fixing defective work units. These factors can usually be incorporated into the unit piece cost in a fairly straightforward way.

EXAMPLE 18.2 Transfer Line Performance

A 20-station transfer line is being proposed to machine a certain component currently produced by conventional methods. The proposal received from the machine tool builder states that the line will operate at a production rate of 50 pc/hr at 100% efficiency. From similar transfer lines, it is estimated that breakdowns of all types will occur with a frequency F = 0.10 breakdown per cycle and that the average downtime per line stop will be 8.0 min. The starting casting that is machined on the line costs $3.00 per part. The line operates at a cost of $75.00/hr. The 20 cutting tools (one tool per station) last for 50 parts each, and the average cost per tool = $2.00 per cutting edge. Based on this data, compute the following: (a) production rate, (b) line efficiency, and (c) cost per unit piece produced on the line.

Solution: (a) At 100% efficiency, the line produces 50 pc/hr. The reciprocal gives the unit time, or ideal cycle time per piece:

$$T_c = \frac{1}{50} = 0.02 \text{ hr/pc} = 1.2 \text{ min.}$$

The average production time per piece is given by Eq. (18.6):

$$T_p = T_c + FT_d = 1.2 + 0.10(8.0) = 1.2 + 0.8 = 2.0 \text{ min/pc.}$$

Production rate is the reciprocal of production time per piece:

$$R_p = \frac{1}{2.0} = 0.500 \text{ pc/min} = \mathbf{30.0\ pc/hr.}$$

Efficiency is the ratio of ideal cycle time to actual average production time, by Eq. (18.9):

$$E = \frac{1.2}{2.0} = 0.60 = \mathbf{60\%}$$

Finally, for the cost per piece produced, we need the tooling cost per piece, which is computed as follows:

$$C_t = (20 \text{ tools})(\$2/\text{tool})/(50 \text{ parts}) = \$0.80/\text{pc}$$

Now the unit cost can be calculated by Eq. (18.12). The hourly rate of $75/hr to operate the line is equivalent to $1.25/min.

$$C_{pc} = 3.00 + 1.25(2.0) + 0.80 = \mathbf{\$6.30/pc.}$$

18.3.2 Workstation Breakdown Analysis

Line downtime is usually associated with failures at individual workstations. Many of the reasons for downtime listed in Table 18.1 represent malfunctions that cause a single station to stop production. Since all workstations on an automated production line without internal storage are interdependent, the failure of one station causes the entire line to stop.

Let us consider what happens when a workstation breaks down. There are two possibilities, in terms of whether a workpart at a station is removed from the line when a breakdown occurs and the resulting effect that this has on the line operation. We refer to the analyses of these two possibilities as the upper-bound approach and the lower-bound approach. In the ***upper-bound approach***, the workstation malfunction has no effect on the part at that station, and therefore the part remains on the line for subsequent processing at the remaining stations. The upper-bound case applies in situations such as minor electrical or mechanical failures at stations, tool changes due to worn cutters, tool adjustments, preventive maintenance at stations, and so on. In these cases, the workpart is unaffected by the station malfunction, and there is no reason to remove the part. In the ***lower-bound approach***, the station malfunction results in damage to the part, and it must therefore be removed from the line and is not available to be processed at subsequent workstations. The lower-bound case arises when a drill or tap breaks in the part during processing, which results in damage to the part. The broken tool must be replaced at the workstation, and the part must be removed from the line and cannot proceed to the next stations for additional processing.

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Upper-Bound Approach. The upper-bound approach provides an upper limit on the frequency of line stops per cycle. In this approach, we assume that the part remains on the line for further processing. It is therefore possible that there will be more than one line stop associated with a given part during its sequence of processing operations. Let p_i = probability or frequency of a failure at station i, where $i = 1, 2, \ldots, n$. Since a part is not removed from the line when a station jam occurs, it is possible (although not probable, thank goodness) that the part will be associated with a station breakdown at every station. The expected number of line stops per part passing through the line is obtained by merely summing the frequencies p_i over the n stations. Since each of the n stations is processing a part each cycle, then the expected frequency of line stops per cycle is equal to the expected frequency of line stops per part; that is,

$$F = \sum_{i=1}^{n} p_i \tag{18.13}$$

where F = expected frequency of line stops per cycle, first encountered in Eq. (18.6); p_i = frequency of station breakdown per cycle, causing a line stop; and n = number of workstations on the line. If all p_i are assumed equal, which is unlikely but useful for approximation and computation purposes, then

$$F = np \tag{18.14}$$

where all p_i are equal, $p_1 = p_2 = \ldots = p_n = p$.

Lower-Bound Approach. The lower-bound approach gives an estimate of the lower limit on the expected frequency of line stops per cycle. In this approach, we assume that a station breakdown results in destruction of the part, resulting in its removal from the line and preventing its subsequent processing at the remaining workstations.

Again let p_i = the probability that a workpart will jam at a particular station i. Then, considering a given part as it proceeds through the line, p_1 = probability that the part will jam at station 1, and $(1 - p_1)$ = probability that the part will not jam at station 1 and will thus be available for processing at subsequent stations. A jam at station 2 is contingent on successfully making it through station 1 and therefore the probability that this same part will jam at station 2 is given by $p_2(1 - p_1)$. Generalizing, the quantity

$$p_i(1 - p_{i-1})(1 - p_{i-2})\ldots(1 - p_2)(1 - p_1) \qquad \text{where } i = 1, 2, \ldots, n$$

is the probability that the given part will jam at any station i. Summing all these probabilities from $i = 1$ through $i = n$ gives the probability or frequency of line stops per cycle. Fortunately there is an easier way to determine this frequency by taking note of the fact that the probability that a given part will pass through all n stations without a line stop is

$$\prod_{i=1}^{n}(1 - p_i)$$

Therefore, the frequency of line stops per cycle is

$$F = 1 - \prod_{i=1}^{n}(1 - p_i) \tag{18.15}$$

If all probabilities p_i are equal, $p_i = p$, then

$$F = 1 - (1 - p)^n \tag{18.16}$$

Because of parts removal in the lower-bound approach, the number of parts coming off the line is less than the number launched onto the front of the line. Therefore, the production rate formula must be amended to reflect this reduction in output. Given that F = frequency of line stops and a part is removed for every line stop, then the proportion of parts removed from the line is F. Accordingly, the proportion of parts produced is $(1 - F)$. This is the yield of the production line. The production rate equation becomes the following:

$$R_{ap} = \frac{1 - F}{T_p} \tag{18.17}$$

where R_{ap} = the average actual production rate of acceptable parts from the line; T_p = the average cycle rate of the transfer machine, given by Eq. (18.6). R_p, which is the reciprocal of T_p, is the average cycle rate of the system.

EXAMPLE 18.3 Upper-Bound vs. Lower-Bound Approaches

A 20-station transfer line has an ideal cycle time T_c = 1.2 min. The probability of station breakdowns per cycle is equal for all stations, and p = 0.005 breakdowns/cycle. For each of the upper-bound and lower-bound approaches, determine (a) frequency of line stops per cycle, (b) average actual production rate, and (c) line efficiency.

Solution: (a) For the upper-bound approach, using Eq. (18.14),

$$F = 20(0.005) = \mathbf{0.10} \text{ line stops per cycle}$$

This is the same value we used in Example 18.2. For the lower-bound approach, using Eq. (18.16),

$$F = 1 - (1 - 0.005)^{20} = 1 - (0.995)^{20} = 1 - 0.0.9046 = \mathbf{0.0954} \text{ line stops per cycle}$$

(b) For the upper-bound approach, the production rate is calculated in Example 18.2 to be

$$R_p = \mathbf{30.0\ pc/hr.}$$

For the lower-bound approach, we must calculate T_p using the new value of F.

$$T_p = 1.2 + 0.0954(0.8) = 1.9631 \text{ min.}$$

Now using Eq. (18.17) to compute production rate, we have

$$R_{ap} = \frac{0.9046}{1.9631} = 0.4608 \text{ pc/min.} = \mathbf{27.65\ pc/hr}.$$

This production rate is about 8% lower than we computed by the upper-bound approach. Note that the cycle rate

$$R_p = (1.9631)^{-1} = 0.5094 \text{ cycles/min.} = 30.56 \text{ cycles/hr}$$

is slightly higher than in the upper-bound case.

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(c) For the upper-bound approach, the line efficiency was computed in Example 18.2 to be

$$E = 0.60 = \mathbf{60\%}.$$

For the lower-bound approach, we have

$$E = \frac{1.2}{1.9631} = 0.6113 = \mathbf{61.13\%}$$

Line efficiency is greater with the lower-bound approach, even though production rate is lower. The reason for this apparent anomaly is that the lower-bound approach, with its assumption of parts removal, leaves fewer parts remaining on the line to jam subsequent workstations. With fewer station jams, line efficiency is higher. However, fewer parts remaining on the line means production rate is lower.

c) What equation tell us

Equations reveals the number of general truths about the operation of automated transfer lines with no internal parts storage. They are:

(i) As the number of workstations on an automated production line increases, line efficiency and production rate are adversely affected

(ii) As the reliability of individual workstations decreases, the line efficiency and production rate are adversely affected

(iii) Comparing the upper-bound and lower-bound cases, the upper-bound calculations lead to higher values of breakdown frequency and production rate but to lower values of line efficiency

4) Analysis of Transfer lines with storage Buffers

In the case of no storage capacity, the production line acts as one stage. When a station breaks down, the entire line stops. This is the case of a production line with no internal storage analyzed in Section 18.3. The efficiency of the line was given by Eq. (18.9). We rewrite it here as the line efficiency of a zero capacity storage buffer:

$$E_0 = \frac{T_c}{T_c + FT_d} \tag{18.18}$$

where the subscript 0 identifies E_0 as the efficiency of a line with zero storage buffer capacity; and the other terms have the same meanings as before.

The opposite extreme is the case where buffer zones of infinite capacity are installed between every pair of stages. If we assume that each buffer zone is half full (in other words, each buffer zone has an infinite supply of parts as well as the capacity to accept an infinite number of additional parts), then each stage is independent of the rest. The presence of infinite storage buffers means that no stage will ever be blocked or starved because of a breakdown at some other stage.

Of course, an infinite capacity storage buffer cannot be realized in practice. If it could, then the overall line efficiency would be limited by the bottleneck stage. That is, production on all other stages would ultimately be restricted by the slowest stage. The downstream stages could only process parts at the output rate of the bottleneck stage. And it would make no sense to run the upstream stages at higher production rates because this would only accumulate inventory in the storage buffer ahead of the bottleneck. As a practical matter, therefore, the upper limit on the efficiency of the entire line is defined by the efficiency of the bottleneck stage. Given that the cycle time T_c is the same for all stages, the efficiency of any stage k is given by:

$$E_k = \frac{T_c}{T_c + F_k T_{dk}} \tag{18.19}$$

where the subscript k is used to identify the stage. According to our argument above, the overall line efficiency would be given by

$$E_\infty = \text{Minimum}\{E_k\} \tag{18.20}$$

where the subscript ∞ identifies E_∞ as the efficiency of a line whose storage buffers all have infinite capacity.

By including one or more storage buffers in an automated production line, we expect to improve the line efficiency above E_0, but we cannot expect to achieve E_∞ simply because buffer zones of infinite capacity are not possible. Hence, the actual value of line efficiency for a given buffer capacity b will fall somewhere between these extremes:

$$E_0 < E_b < E_\infty \tag{18.21}$$

Next, let us consider the problem of evaluating E_b for realistic levels of buffer capacity for a two-stage automated production line.

18.4.2 Analysis of a Two-Stage Transfer Line

Most of the discussion in this section is based on the work of Buzacott [2], who pioneered the analytical research on production lines with buffer stocks. Several of his publications are listed in our references [3]–[7]. Our presentation in this Section will follow Buzacott's analysis in [2].

The two-stage line is divided by a storage buffer of capacity b, expressed in terms of the number of workparts that it can store. The buffer receives the output of stage 1 and forwards it to stage 2, temporarily storing any parts not immediately needed by stage 2 up to its capacity b. The ideal cycle time T_c is the same for both stages. We assume the downtime distributions of each stage to be the same with mean downtime $= T_d$. Let F_1 and $F_2 =$ the breakdown rates of stages 1 and 2, respectively. F_1 and F_2 are not necessarily equal.

Over the long run, both stages must have equal efficiencies. If the efficiency of stage 1 were greater than the stage 2 efficiency, then inventory would build up in the storage buffer until its capacity b is reached. Thereafter, stage 1 would eventually be blocked when it outproduced stage 2. Similarly, if the efficiency of stage 2 were greater than that of stage 1, the inventory in the buffer would become depleted, thus starving stage 2. Accordingly, the efficiencies in the two stages would tend to equalize over time. The overall line efficiency for the two-stage line can be expressed:

$$E_b = E_0 + D_1' h(b) E_2 \tag{18.22}$$

where $E_b =$ overall line efficiency for a two-stage line with buffer capacity b; $E_0 =$ line efficiency for the same line with no internal storage; and the second term on the right-hand side $(D_1' h(b) E_2)$ represents the improvement in efficiency that results from having a storage buffer with $b > 0$. Let us examine the right-hand side terms in Eq. (18.22). The value of E_0 was given by Eq. (18.18), but we write it below to explicitly define the two-stage efficiency when $b = 0$:

$$E_0 = \frac{T_c}{T_c + (F_1 + F_2)T_d} \tag{18.23}$$

The term D_1' can be thought of as the proportion of total time that stage 1 is down, defined as follows:

$$D_1 = \frac{F_1 T_d}{T_c + (F_1 + F_2)T_d} \tag{18.24}$$

The term $h(b)$ is the proportion of the downtime D_1' (when stage 1 is down) that stage 2 could be up and operating within the limits of storage buffer capacity b. Buzacott presents equations for evaluating $h(b)$ using Markov chain analysis. The equations cover several different downtime distributions based on the assumption that both stages are never down at the same time. Four of these equations are presented in Table 18.2.

Finally, E_2 corrects for the assumption in the calculation of $h(b)$ that both stages are never down at the same time. This assumption is unrealistic. What is more realistic is that when stage 1 is down but stage 2 could be producing because of parts stored in the buffer, there will be times when stage 2 itself breaks down. Therefore, E_2 provides an estimate of the proportion of stage 2 uptime when it could otherwise be operating even with stage 1 being down. E_2 is calculated as:

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E_2, the author has found that the equation tends to overestimate line efficiency. With E_2 included, as in our Eq. (18.22), the calculated values are much more realistic. In research subsequent to that reported in [2], Buzacott developed other equations that agree closely with results given by our own Eq. (18.22).

EXAMPLE 18.4 Two-Stage Automated Production Line

A 20-station transfer line is divided into two stages of 10 stations each. The ideal cycle time of each stage is $T_c = 1.2$ min. All of the stations in the line have the same probability of stopping, $p = 0.005$. We assume that the downtime is constant when a breakdown occurs, $T_d = 8.0$ min. Using the upper-bound approach, compute the line efficiency for the following buffer capacities: (a) $b = 0$, (b) $b = \infty$, (c) $b = 10$, and (d) $b = 100$.

Solution: (a) A two-stage line with 20 stations and $b = 0$ turns out to be the same case as in our previous Examples 18.2 and 18.3. To review,

$$F = np = 20(0.005) = 0.10$$

$$E_0 = \frac{1.2}{1.2 + 0.1(8)} = \mathbf{0.60}$$

(b) For a two-stage line with 20 stations (each stage = 10 stations) and $b = \infty$, we first compute F:

$$F_1 = F_2 = 10(0.005) = 0.05$$

$$E_\infty = E_1 = E_2 = \frac{1.2}{1.2 + 0.05(8)} = \mathbf{0.75}$$

(c) For a two-stage line with $b = 10$, we must determine each of the terms in Eq. (18.22). We have E_0 from part (a). $E_0 = 0.60$. And we have E_2 from part (b). $E_2 = 0.75$.

$$D_1' = \frac{0.05(8)}{1.2 + (0.05 + 0.05)(8)} = \frac{0.40}{2.0} = 0.20$$

Evaluation of $h(b)$ is from Eq. (18.27) for a constant repair distribution. In Eq. (18.26), the ratio

$$\frac{T_d}{T_c} = \frac{8.0}{1.2} = 6.667\text{. For } b = 10,\ B = 1 \text{ and } L = 3.333\text{. Thus,}$$

$$h(b) = h(10) = \frac{1}{1+1} + 3.333\left(\frac{1.2}{8.0}\right)\frac{1}{(1+1)(1+2)} = 0.50 + 0.0833 = 0.5833$$

We can now use Eq. (18.22):

$$E_{10} = 0.600 + 0.20(0.5833)(0.75) = 0.600 + 0.0875 = \mathbf{0.6875}$$

(d) For $b = 100$, the only parameter in Eq. (18.22) that is different from part (c) is $h(b)$. For $b = 100$, $B = 15$ and $L = 0$ in Eq. (18.27). Thus, we have

$$h(b) = h(100) = \frac{15}{15+1} = 0.9375$$

Using this value,

$$E_{100} = 0.600 + 0.20(0.9375)(0.75) = 0.600 + 0.1406 = \mathbf{0.7406}$$

The value of $h(b)$ not only serves its role in Eq. (18.22). It also provides information on how much improvement in efficiency we get from any given value of b. Note in Example 18.4 that the difference between E_∞ and $E_0 = 0.75 - 0.60 = 0.15$. For $b = 10$, $h(b) = h(10) = 0.5833$, which means we get 58.33% of the maximum possible improvement in line efficiency using a buffer capacity of 10 ($E_{10} = 0.6875 = 0.60 + 0.5833(0.75-0.60)$). For $b = 100$, $h(b) = h(100) = 0.9375$, which means we get 93.75% of the maximum improvement with $b = 100$ ($E_{100} = 0.7406 = 0.60 + 0.9375(0.75 - 0.60)$).

We are not only interested in the line efficiencies of a two-stage production line. We also want to know the corresponding production rates. These can be evaluated based on knowledge of the ideal cycle time T_c and the definition of line efficiency. According to Eq. (18.9), $E = T_c/T_p$. Since R_p = the reciprocal of T_p, then $E = T_cR_p$. Rearranging, we have

$$R_p = \frac{E}{T_c} \tag{18.32}$$

EXAMPLE 18.5 Production Rates on the Two-Stage Line of Example 18.4

Compute the production rates for the four cases in Example 18.4. The value of $T_c = 1.2$ min is as before.

Solution: (a) For $b = 0$, $E_0 = 0.60$. Applying Eq. (18.32), we have

$$R_p = 0.60/1.2 = 0.5 \text{ pc/min} = 30 \text{ pc/hr.}$$

This is the same value calculated in Example 18.2.

(b) For $b = \infty$, $E_\infty = 0.75$.

$$R_p = 0.75/1.2 = 0.625 \text{ pc/min} = 37.5 \text{ pc/hr.}$$

(c) For $b = 10$, $E_{10} = 0.6875$.

$$R_p = 0.6875/1.2 = 0.5729 \text{ pc/min} = 34.375 \text{ pc/hr.}$$

(d) For $b = 100$, $E_{100} = 0.7406$.

$$R_p = 0.7406/1.2 = 0.6172 \text{ pc/min} = 37.03 \text{ pc/hr.}$$

In Example 18.4, a constant repair distribution was assumed. Every breakdown had the same constant repair time of 8.0 min. It is more realistic to expect that there will be some variation in the repair time distribution. Table 18.2 provides two possible distributions, representing extremes in possible variability. We have already used the constant repair distribution in Example 18.5, which represents the case of no downtime variation. This is covered in Table 18.2 by Eqs. (18.27) and (18.28). Let us consider the opposite extreme, the case of very high variation. This is presented in Table 18.2 as the geometric repair distribution, where h(b) is computed by Eqs. (18.29) and (18.30).

EXAMPLE 18.6 Effect of High Variability in Downtimes

Evaluate the line efficiencies for the two-stage line in Example 18.4, except that the geometric repair distribution is used instead of the constant downtime distribution.

Contd.

Solution: For parts (a) and (b), the values of E_0 and E_∞ will be the same as in previous Example 18.4. $E_0 = 0.600$ and $E_\infty = 0.750$.
(c) For $b = 10$, all of the parameters in Eq. (18.22) remain the same except $h(b)$. Using Eq. (18.29) from Table 18.2, we have

$$h(b) = h(10) = \frac{10(1.2/8.0)}{2 + (10 - 1)(1.2/8.0)} = 0.4478$$

Now using Eq. (18.22), we have

$$E_{10} = 0.600 + 0.20(0.4478)(0.75) = \mathbf{0.6672}$$

(d) For $b = 100$, again the only change is in $h(b)$.

$$h(b) = h(100) = \frac{100(1.2/8.0)}{2 + (100 - 1)(1.2/8.0)} = 0.8902$$

$$E_{100} = 0.600 + 0.20(0.8902)(0.75) = \mathbf{0.7333}$$

Note that when we compare the values of line efficiency for $b = 10$ and $b = 100$ in this example with the corresponding values in Example 18.4, both values are lower here. It must be concluded that increased downtime variability degrades line efficiency.

18.4.3 Transfer Lines with More than Two Stages

If the line efficiency of an automated production line can be increased by dividing it into two stages with a storage buffer between, then one might infer that further improvements in performance can be achieved by adding additional storage buffers. Although we do not present exact formulas for computing line efficiencies for the general case of any capacity b for multiple storage buffers, efficiency improvements can readily be determined for the case of infinite buffer capacity. In Examples 18.5 and 18.6 we have seen the relative improvement in efficiency that result from intermediate buffer sizes between $b = 0$ and $b = \infty$.

EXAMPLE 18.7 Transfer Lines with more than O+ne Storage Buffer

For the same 20-station transfer line we have been considering in previous examples, compare line efficiencies and production rates for the following cases, where in each case the buffer capacity is infinite: (a) no storage buffers, (b) one buffer, (c) three buffers, and (d) 19 buffers. Assume in cases (b) and (c) that the buffers are located in the line to equalize the downtime frequencies; that is, all F_i are equal. As before, the computations are based on the upper-bound approach.

Solution: We have already computed the answer for (a) and (b) in Example 18.4.
(a) For the case of no storage buffer, $E_\infty = \mathbf{0.60}$

$$R_p = 0.60/1.2 = 0.50 \text{ pc/min} = \mathbf{30 \text{ pc/hr.}}$$

(b) For the case of one storage buffer (a two-stage line), $E_\infty = \mathbf{0.75}$

$$R_p = 0.75/1.2 = 0.625 \text{ pc/min.} = \mathbf{37.5 \text{ pc/hr.}}$$

(c) For the case of three storage buffers (a four-stage line), we have

$$F_1 = F_2 = F_3 = F_4 = 5(.005) = 0.025.$$

$$T_p = 1.2 + 0.025(8) = 1.4 \text{ min/pc.}$$

$$E_\infty = 1.2/1.4 = \mathbf{0.8571}$$

$$R_p = 0.8571/1.2 = 0.7143 \text{ pc/min} = \mathbf{42.86 \text{ pc/hr.}}$$

(d) For the case of 19 storage buffers (a 20-stage line, where each stage is one station), we have

$$F_1 = F_2 = \ldots = F_{20} = 1(0.005) = 0.005$$

$$T_p = 1.2 + 0.005(8) = 1.24 \text{ min/pc}$$

$$E_\infty = 1.2/1.24 = \mathbf{0.9677}$$

$$R_p = 0.9677/1.2 = 0.8065 \text{ pc/min} = \mathbf{48.39 \text{ pc/hr.}}$$

This last value is very close to the ideal production rate of $R_c = 50$ pc/hr.

18.4.4 What the Equations Tell Us

The equations and analysis in this section provide some practical guidelines in the design and operation of automated production lines with internal storage buffers. The guidelines can be expressed as follows:

- If E_0 and E_∞ are nearly equal in value, little advantage is gained by adding a storage buffer to the line. If E_∞ is significantly greater than E_0, then storage buffers offer the possibility of significantly improving line performance.
- In considering a multi-stage automated production line, workstations should be divided into stages to make the efficiencies of all stages as equal as possible. In this way, the maximum difference between E_0 and E_∞ is achieved, and no single stage will stand out as a significant bottleneck.
- In the operation of an automated production line with storage buffers, if any of the buffers are nearly always empty or nearly always full, this indicates that the production rates of the stages on either side of the buffer are out of balance, and that the storage buffer is serving little if any useful purpose.
- The maximum possible line efficiency is achieved by (1) setting the number of stages equal to the number of stations, that is, by providing a storage buffer between every pair of stations, and (2) using large capacity buffers.
- The "law of diminishing returns" operates in multi-stage automated lines. It is manifested in two ways: (1) As the number of storage buffers is increased, line efficiency improves at an ever-decreasing rate. (2) As the storage buffer capacity is increased, line efficiency improves at an ever-decreasing rate.

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