# Testing of variance and Std. dev. of normal distⁿ :→

**Step I** $H_0: \sigma^2 = \sigma_0^2$ or $\sigma^2 \leq \sigma_0^2$ or $\sigma^2 \geq \sigma_0^2$

$H_1: \sigma^2 \neq \sigma_0^2$ or $\sigma^2 > \sigma_0^2$ or $\sigma^2 < \sigma_0^2$

**Step II** Test Statistic is

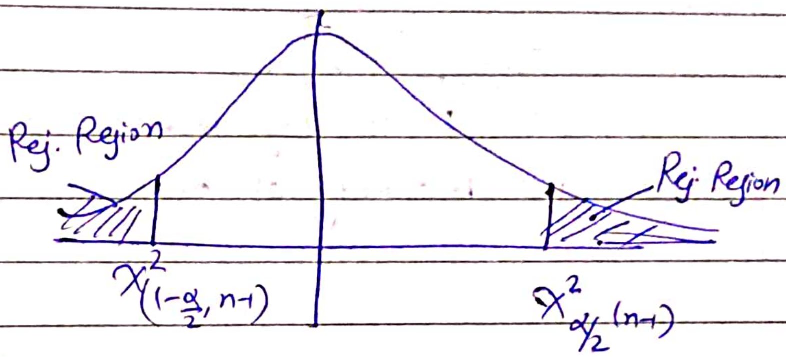
$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi^2_{(n-1)} \text{ — } \boxed{(n-1)\text{ d.f.}}$$

**Step III** $\chi^2_{(n-1)}(\alpha)$ — Table value — $\alpha$ - level of significance.

↳ Reject Ho if

$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or

$\chi_0^2 < \chi^2_{1-\frac{\alpha}{2}, n-1}$




for two-tailed test.

↳ Reject Ho if $\chi_0^2 > \chi^2_{\alpha, n-1}$ for right tail test

↳ Reject Ho if $\chi_0^2 < \chi^2_{1-\alpha, n-1}$, for left tail test.

**Que:-** An automated filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance $s^2 = 0.0153$ of fill volume. If the variance of fill volume exceeds 0.01, an unacceptable prop of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles?

Use $\alpha = 0.05$.

**Solⁿ** $H_0: \sigma^2 \leq 0.01$

$H_1: \sigma^2 > 0.01$

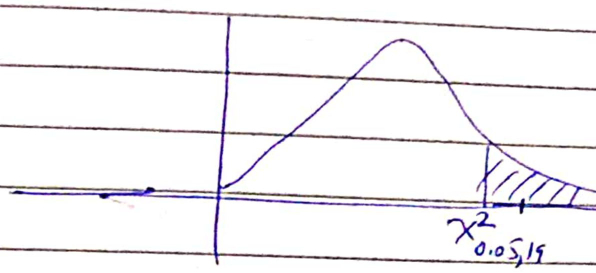
$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19 \times 0.0153}{0.01} = 29.07$$

and $\chi^2_{0.05, 19} = 30.14$ (for right tail test)

$\Rightarrow \chi_0^2 < \chi^2_{0.05,19}$

$\Rightarrow$ Do not reject $H_0$

i.e. variance $\sigma^2 \leq 0.01$.



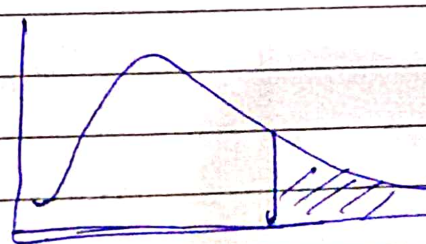

Que An automative part must be machined to close tolerances to be acceptable to Customers. Production specifications call for a max var. in the lengths of the parts of 0.0004. Let the sample var. for 30 units turns out to be $s^2 = 0.0005$. Use $\alpha = 0.05$ to test whether the popⁿ var specifications is being violated.

Solⁿ $H_0: \sigma^2 \leq 0.0004$

$H_1: \sigma^2 > 0.0004$ — (Right tail)

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{29 \times 0.0005}{0.0004} = 36.25$$

$\chi^2_{0.05, 29} = 42.557$



$\because \chi_0^2 < \chi^2_{0.05,29}$

$\Rightarrow$ Do not reject $H_0$.

Que The std. dev. of exam score in the math deptt is 8.6. A Professor believes this value to be less. He samples 20 exam scores and find that the std. dev. is 6.9. Does this sample provide enough evidence to suggest that the true std. dev. < 8.6. ?

**Sol^n**

$\sigma_0^2 = 8.6^2$; $n = 20$; $S^2 = 6.9^2$

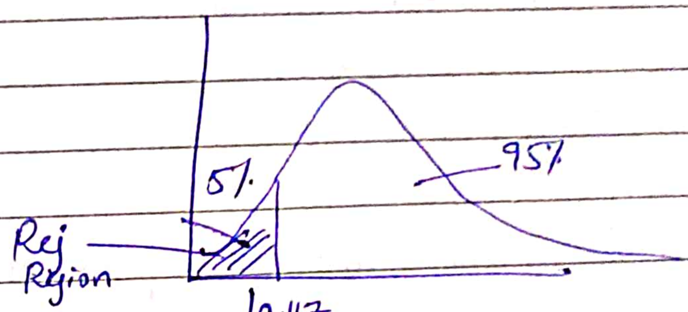
$H_0: \sigma^2 = (8.6)^2$

$H_1: \sigma^2 < (8.6)^2$ — (Left tail test)

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{19 \times (6.9)^2}{(8.6)^2} = 12.23$$

$$\chi^2_{0.05,\,19} = 10.117$$

$\because \chi_0^2 > \chi^2_{0.05,19}$




$\Rightarrow$ Do not reject $H_0$.

**Que**

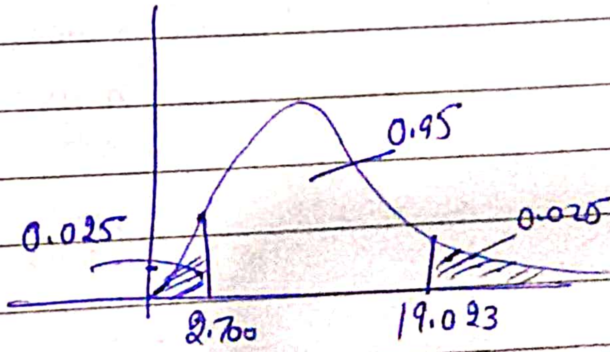
$H_0: \sigma^2 = 8.6$ ; $n = 10$

$H_1: \sigma^2 \neq 8.6$ ; $S^2 = 4.3$ ; $\alpha = 0.05$

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{9 \times 4.3}{8.6} = 4.5$$

$$\chi^2_{9,\,0.025} = 19.023$$

$$\chi^2_{9,\,0.975} = 2.700$$




$$\chi^2_{9,0.975} \leq \chi_0^2 \leq \chi^2_{9,0.025}$$

$\Rightarrow$ Donot reject $H_0$.

# Two Samples Test

→ **Hypothesis testing for difference of popⁿ means:→**

If $X \sim N(\mu, \sigma^2)$

Then $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

For two different sample, if $X_1 \sim N(\mu_1, \sigma_1^2)$

$X_2 \sim N(\mu_2, \sigma_2^2)$

Then $\bar{X}_1 \sim N(\mu_1, \frac{\sigma_1^2}{n_1})$ and $\bar{X}_2 \sim N(\mu_2, \frac{\sigma_2^2}{n_2})$

**Assumptions behind two Independent samples testing:-**

(1) The samples must be randomly selected

(2) The popⁿ from which you are sampling must be normally distributed.

(3) Popⁿs have their Common variances i.e. $\sigma_1^2 = \sigma_2^2 = \sigma^2$ ↳ Pooled variance

Now our target is to find the test statistics for difference b/w sample means

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - E(\bar{X}_1 - \bar{X}_2)}{\sqrt{Var(\bar{X}_1 - \bar{X}_2)}}$$

$$E(\bar{x}_1 - \bar{x}_2) = E(\bar{x}_1) - E(\bar{x}_2) = \mu_1 - \mu_2$$

$$V(\bar{x}_1 - \bar{x}_2) = V(\bar{x}_1) + V(\bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right) \quad (\because \sigma_1^2 = \sigma_2^2 = \sigma^2)$$

$$\therefore Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sigma\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

→ **If popⁿ variance $\sigma^2$ is unknown:-**

then we will use $S^2$ – Sample variance

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$$S^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$$

$\Rightarrow$ Test Statistic becomes

$$t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

For given $S_1^2$ and $S_2^2$

$$S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1+n_2-2}$$

Step I Define hypothesis $H_0: \mu_1-\mu_2 = D$

where D is difference of means

$H_1: \mu_1-\mu_2 \neq D$ (Two-tailed)

or $H_1: \mu_1-\mu_2 > D$ (Right tailed)

or $H_1: \mu_1-\mu_2 < D$ (Left tailed)

Step II Compute Test Statistics

$$t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Step III Conclusion by using table value $t_{n_1+n_2-2}(\alpha)$

Que A random sample of 20 daily workers of State A was found to have avg daily earning of Rs. 44 with sample var 900. Another sample of 20 daily workers of State B was found to earn on an avg Rs. 28 per day with sample var 400. Test whether the workers in State A are earning more 2 than those in State B.

Solⁿ

$H_0: \mu_1-\mu_2 = 2$ or $\mu_1-\mu_2 \le 2$.

$H_1: \mu_1-\mu_2 > 2$

$S_1^2 = 900$; $S_2^2 = 400$; $n_1 = n_2 = 20$, $\mu_1 = 44$, $\mu_2 = 28$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = 648.21$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\cancel{n_1}\, S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = 1.7389$$

$$t_{n_1+n_2-2}(0.05) = t_{38}(0.05) = 1.645$$

$t_{stat} > t_{n_1+n_2-2}(0.05)$

$\Rightarrow$ Reject $H_0$.