Assignment - 03

Ques- 1 The matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ satisfies the matrix Eqn

$A^3 - 6A^2 + 11A - I = 0$, where I is an identity matrix of order 3. find A^{-1}

Soln:-

$$A^3 - 6A^2 + 11A - I = 0 \quad -(1)$$

According to properties of inverse matrix

$$AA^{-1} = I \quad -(2)$$

multiply A^{-1} with Eq (1)

$$\Rightarrow A^3A^{-1} - 6A^2A^{-1} + 11AA^{-1} - A^{-1}I = 0$$

$$\Rightarrow A^2 - 6A + 11I = A^{-1}$$

$$A^2 = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix}$$

$$-6A = \begin{bmatrix} -12 & 0 & 6 \\ -30 & -6 & 0 \\ 0 & -6 & -18 \end{bmatrix}$$

$$11I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix} + \begin{bmatrix} -12 & 0 & 6 \\ -30 & -6 & 0 \\ 0 & -6 & -18 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Ans

Ques-32 ⇒ Examine the following vectors for linear depending and find the relation b/w them, if possible :

$$X_1 = (1,-1,1) \; ; \; X_2 = (2,1,1) \; ; \; X_3 = (3,0,2)$$

Sol^n :-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

using row operations
if $c_1X_1 + c_2X_2 + c_3X_3 = 0$
and $\rho(A) = n$, then X_1, X_2, X_3 are linearly dependent. else they are linearly dependent.

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_2 \to R_2 + 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

further any row cant be multiplies or become zero
so, non zero rows are 2.

∴ Rank of $A = 2$

$\rho(A) = 2 < n$ (No. of variable c_1, c_2, c_3)

so, the vectors are linearly independent.

$c_1 + 2c_2 + 3c_3 = 0 \longrightarrow (1)$

$-c_2 - c_3 = 0$

$c_1 - 2k + 3k = 0$

$\boxed{c_1 = -k} \to (5)$

$c_2 = -c_3 \to (1)$

$c_3 = k \to (3)$

$c_2 = -k \to (4)$

$c_1 + c_2 = -2c_3$

$\boxed{c_1 + c_2 + 2c_3 = 0}$ A

Ques-3 Reduce the matrix A to its normal form where

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

Sol^n:

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 & 1 \\ 4 & 0 & 2 & 6 \\ 0 & 1 & 2 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & -2 & -4 & 4 \\ 0 & 1 & 2 & -2 \end{bmatrix}$$

$R_2 \to \frac{R_2}{-2}$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 1 & 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now, using column operation

$$\Rightarrow \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 3C_2$
$C_4 \to C_4 - C_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 + C_3$
$C_3 \to C_3 + C_4$
$C_4 \to C_1 - 2C_2$

Rank:

$$\boxed{\rho(A) = 2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans

Ques-4 find for what value of A, B the system of linear Equation:

$$x+y+z=6$$
$$x+2y+5z=10$$
$$2x+3y+Bz=A$$

has (i) unique solution
(ii) No solution
(iii) Infinite solution

Sol^n:
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & B \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ A \end{bmatrix}$$

(i) Unique solution:
$\rho(A|B) = \rho(A)$
and $\rho(A|B) = n$
$n =$ no. of variable $= 3$

$$(A|B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 5 & 10 \\ 2 & 3 & B & A \end{array}\right]$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 1 & B-2 & A-12 \end{array}\right]$$

$B-2 \neq 4$
$\boxed{B \neq 6 \;;\; A \in R}$

(iii) for infinite solution → $\rho(A|B) = \rho(A)$
$\rho(A|B) < n = 3$

$\Rightarrow B-2=4 \;;\; A-12=4$
$\boxed{B=6} \;;\; \boxed{A=16}$

(iii) for no solution → $\rho(A|B) \neq \rho(A)$
$\Rightarrow A-12 \neq 4 \;,\; B-2=4$
$\boxed{A \neq 16} \;,\; \boxed{B=6}$

Ques 5 Find the Eigen values and Eigen vector of matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

Sol^n - $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

$AX = \lambda B = A\lambda$

$(A - I\lambda) = 0$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 2 & 1 \\ 0 & 2-\lambda & 1 \\ -1 & 2 & 2-\lambda \end{bmatrix}$$

$\Rightarrow (1-\lambda)\left[(\lambda^2 - 4\lambda + 4)(-2)\right] - 2[+1] + 1[(2-\lambda)]$

characteristic Eq^n

$\Rightarrow (1-\lambda)(\lambda^2 - 4\lambda + 2) - 2 + 2 - \lambda = 0$

$\Rightarrow \lambda^2 - 4\lambda + 2 - \lambda^3 + 4\lambda^2 - 2\lambda - \lambda = 0$

$\Rightarrow -\lambda^3 + 5\lambda^2 - \lambda + 2 = 0$

$\Rightarrow \lambda^3 - 5\lambda^2 + 7\lambda - 2 = 0$

$\Rightarrow 8 - 20 + 14 - 2 = 0$

$\boxed{\lambda = 2}$

$(\lambda - 2)(\lambda^2 - 3\lambda + 1) = 0$

$\lambda = \dfrac{3 \pm \sqrt{9-4}}{2}$

$\boxed{\lambda = \dfrac{3 \pm \sqrt{5}}{2}}$

$$\begin{array}{r} \lambda^2 - 3\lambda + 1 \\ \lambda - 2 \,\big)\, \lambda^3 - 5\lambda^2 + 7\lambda - 2 \\ \lambda^3 - 2\lambda^2 \\ - \quad + \\ \hline -3\lambda^2 + 7\lambda - 2 \\ \cancel{-3\lambda^2} + 6\lambda \\ + \quad - \\ \hline \lambda - 2 \\ \cancel{\lambda} - 2 \\ \hline \times \end{array}$$

for

$$\lambda - 2 \Rightarrow \begin{bmatrix} -1 & 2 & 1 \\ 0 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix}$$

$$\vec{v_1} \Rightarrow \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$-x + 2y + z = 0$

$z = 0$

$-x + 2y = 0$

$\boxed{x = 2y}$, $\boxed{z = 0}$

for;

$$\lambda = \frac{3+\sqrt{5}}{2} \Rightarrow \begin{bmatrix} \frac{-1-\sqrt{5}}{2} & 2 & 1 \\ 0 & \frac{1-\sqrt{5}}{2} & 1 \\ -1 & 2 & \frac{1-\sqrt{5}}{2} \end{bmatrix}$$

$(-1-\sqrt{5})x + 4y + 2z = 0 \quad \text{—} (1)$

$(1-\sqrt{5})y + 2z = 0 \rightarrow (2)$

$-2x + 4y + (1-\sqrt{5})z = 0 \rightarrow (3)$

using (1) and (2)

$\Rightarrow (-1-5)x + (3+\sqrt{5})y = 0$

using (2) and (3)

$\Rightarrow -(1-\sqrt{5})^2 y + 2(1-\sqrt{5})z = 0$

$-4x + 8y + 2(1-\sqrt{5})z = 0$

Ques-6 Diagonalise the following matrices

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

Sol^n:

$(A - I\lambda) = 0$

$$\begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix}$$

$\Rightarrow (3-\lambda)\left[(\lambda^2 - 8\lambda + 15) - 1\right] + 1\left[\lambda - 3 + 1\right] + 1\left[1 + (\lambda - 5)\right]$

$\Rightarrow (3-\lambda)(\lambda^2 - 8\lambda + 14) + (\lambda - 2) + \lambda - 4 = 0$

$\Rightarrow -\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$

$$\boxed{\lambda = 2}$$

$$
\begin{array}{r|l}
 & \lambda^2 - 9\lambda + 18 \\
\hline
\lambda - 2 & \lambda^3 - 11\lambda^2 + 36\lambda - 36 \\
 & \cancel{\lambda^3} - 2\lambda^2 \\
 & - \quad + \\
\hline
 & -9\lambda^2 + 36\lambda - 36 \\
 & -9\lambda^2 + 18\lambda \\
 & + \quad - \\
\hline
 & 18\lambda - 36 \\
 & 18\lambda - 36 \\
\hline
 & \times
\end{array}
$$

$(\lambda - 2)(\lambda^2 - 9\lambda + 18) = 0$

$(\lambda^2 - 6\lambda - 3\lambda + 18) = 0$

$(\lambda - 6)(\lambda - 3)(\lambda - 2) = 0$

$$\boxed{\lambda = 2, 3, 6}$$

for : $\lambda = 2$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$x - y + z = 0$

$-x + 3y - z = 0$

$x - y + z = 0$

$\boxed{2y = 0}$

$\boxed{x = -z}$

$$\vec{v} = \begin{bmatrix} +1 \\ 0 \\ -1 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$-y + z = 0$

$-x + 2y - z = 0$

$x - y = 0$

$\boxed{y=z}$

$\boxed{x=z}$

$\boxed{x=y}$

$$\vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

for $\lambda = 6$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}$$

$-3x - y + z = 0$

$-x + y + z = 0$

$x - y - 3z = 0$

$-4x - 2y = 0$	$4x = -2y$
$x - y - 3z = 0$	$\boxed{y = -2x}$
$+3x - 3z = 0$	$\boxed{x = z}$

$$\vec{v_3} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{adj\,(P)}{|P|}$$

$|P| = 1(1+2) - 1(-2) + 1(1)$

$= 3 + 2 + 1 = 6$

$$adj\,P = \begin{bmatrix} 3 & 0 & -3 \\ 2 & 2 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & -1/3 & 1/6 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{6}\begin{bmatrix} 3 & 0 & -3 \\ 2 & 2 & 2 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$