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Two dimensional random variable:→ Let X and Y be two random variables defined on the sample space S.

i.e. $X: S \to \mathbb{R}$ and $Y: S \to \mathbb{R}$

Then the function $(X,Y): S \to \mathbb{R} \times \mathbb{R}$ is called a two dimensional random variable.

Joint Probability function:→

The function $P_{ij} = P(x = x_i, y = y_j)$ is called the joint prob. fun. of X and Y & is represented as

$X \backslash Y$	y_1	y_2	---	y_m
x_1	P_{11}	P_{12}	----	P_{1m}
x_2	P_{21}	P_{22}	-----	P_{2m}
⋮	⋮	⋮		⋮
x_n	P_{n1}	P_{n2}	---	P_{nm}

→ If (X,Y) is a two-dimensional discrete r.v. then the joint prob. fun of X, Y is called joint prob. mass function of X, Y and is defined as

$$P_{XY}(x_i, y_j) = \begin{cases} P(x = x_i, y = y_j) & ; (x_i, y_j) \in (X,Y) \\ 0 & ; o/w. \end{cases}$$

Note $P(x,y) \geq 0$.

and $\sum_x \sum_y P(x,y) = 1$.

For Continuous ; $P(x,y) \geq 0$.

$$\int_x \int_y P(x,y)\, dy\, dx = 1.$$

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Marginal density functions:→ For two random variables X and Y; the marginal function of X is given as

$$P(x) = \sum_{y} P(X=x, Y=y) \quad \text{— discrete}$$

$$P(x) = \int_{y} P(x,y)\, dy \quad \text{— Continuous.}$$

The marginal function of Y is gives as

$$P(y) = \sum_{x} P(X=x, Y=y) \quad \text{— discrete}$$

$$P(y) = \int_{x} P(x,y)\, dx \quad \text{— Continuous.}$$

Conditional density functions:→

The Conditional density function of X given Y is defined as

$$P(x/y) = \frac{P(X=x, Y=y)}{P(y)} = \frac{P(x,y)}{P(y)} \rightarrow \text{Marginal of } Y.$$

Similarly, the Conditional density function of Y given X is defined as

$$P(Y/x) = \frac{P(X=x, Y=y)}{P(x)} = \frac{P(x,y)}{P(x)} \leftarrow \text{marginal of } X$$

→ Two random variables X and Y are said to be Independent if $P(x,y) = P(x) \cdot P(y) \ \forall \ (x,y) \in (X,Y)$

Ex:→ Consider X & Y – be two r.vs such that

$$P(X=-1, Y=0) = \frac{1}{3}; \quad P(X=0, Y=1) = \frac{1}{3};$$

$$P(X=1, Y=1) = \frac{1}{3}.$$

Find (1) Marginal distⁿ fun of X and Y

(2) Conditional distⁿ fun. of X given Y = 1.

Solⁿ

X | Y	0	1	P(x)
-1	$1/3$	0	$1/3$
0	0	$1/3$	$1/3$
1	0	$1/3$	$1/3$
P(y)	$1/3$	$2/3$	$\boxed{1}$

So Marginal distⁿ fun of X is

X	-1	0	1
P(x)	$1/3$	$1/3$	$1/3$

Marginal distⁿ fun of Y is

Y	0	1
P(Y)	$1/3$	$2/3$

$$P(X=x \mid Y=1) = \frac{P(X=x, Y=1)}{P(Y=1)}$$

X	-1	0	1
$P(X=x\mid Y=1)$	0	$1/2$	$1/2$

Que The joint distⁿ of X and Y is given by

$$P(x,y) = \frac{x+y}{21};\ x=1,2,3;\ y=1,2.$$

Find the marginal distⁿ of X and Y. Find the Conditional distⁿ of Y given $X=3$.

Solⁿ

X/Y	1	2	
1	$2/21$	$3/21$	$5/21$
2	$3/21$	$4/21$	$7/21$
3	$4/21$	$5/21$	$9/21$
	$9/21$	$12/21$	

Marginal distⁿ of X and Y are

X	1	2	3
$P(x)$	$5/21$	$7/21$	$9/21$

Y	1	2
$P(y)$	$9/21$	$12/21$

$$P(Y=y\mid X=3) = \frac{P(Y=y,\ x=3)}{P(x=3)}$$

Y	1	2
$P(Y=y\mid X=3)$	$4/9$	$5/9$

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Que The joint density function of X and Y is given by

$$P(x,y) = K(2x+y); \; x=0,1,2; \; y=0,1,2.$$

(1) Find the value of K
(2) Marginal dist^n of X and Y
(3) Are X and Y Independent variables?
(4) Find the Conditional dist^n of Y for $X=x$

Sol^n

$$\sum_x \sum_y P(x,y) = 1$$

$\Rightarrow 27K = 1$

$\Rightarrow \boxed{K = \frac{1}{27}}$

X\Y	0	1	2
0	0	K	2K
1	2K	3K	4K
2	4K	5K	6K

So

X\Y	0	1	2	Total
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
Total	6/27	9/27	12/27	

So Marginal dist^n of X

X	0	1	2
P(x)	3/27	9/27	15/27

Marginal dist^n of Y

Y	0	1	2
P(y)	6/27	9/27	12/27

Two r.v. x & y are Independent if $P(x,y) = P(x) \cdot P(y)$ $\forall (x,y)$.

$P(0,0) = 0$

$P(0) = 3/27$ and $P(0) = 6/27$

So $P(0,0) \neq P(0) \times P(0)$

$\Rightarrow$ X and Y are not Independent.

– Conditional distⁿ of Y for $X = x$ is

$$P(y|X=x) = \frac{P(x,y)}{P(x)}$$

$\boxed{P(y|x=0)} = \boxed{X=0} = \frac{P(0,y)}{P(0)}$

Y/X	0	1	2
$P(y/x=0)$	0	1/3	2/3

$\boxed{X=1}$

Y/X	0	1	2
$P(Y/x=1)$	2/9	3/9	4/9

$\boxed{X=2}$

Y/X	0	1	2
$P(Y/x=2)$	4/15	5/15	6/15

Que The joint density fun. of X and Y is given by

$$P(x,y) = \frac{2x+y}{31} \ ; \ x = 1,2,3; \ y = 1,2.$$

Find the means of X, Y, XY and X+Y.

Solⁿ $E(X) = \sum x \, p(x)$

where $p(x)$ is Marginal density of X.

$E(Y) = \sum y \, P(y)$ where $P(y)$ – Marginal density of Y.

$E(XY) = \sum\sum xy \cdot P(x,y)$

$E(X+Y) = \sum\sum (x+y) \cdot P(x,y)$

$P(x,y)$ is joint density of X & Y.

P x/y	1	2	Total
1	3/31	4/31	7/31
2	5/31	6/31	11/31
3	7/31	8/31	15/31
Total	15/31	18/31	

Marginal density fun of x

x	1	2	3
P(x)	7/31	11/31	15/31

Marginal density of y

y	1	2
P(y)	15/31	18/31

$$E(x) = 1.\frac{7}{31} + 2.\frac{11}{31} + 3.\frac{15}{31} = \frac{74}{31}$$

$$E(y) = 1.\frac{15}{31} + 2.\frac{18}{31} = \frac{51}{31}$$

$$E(xy) = \sum_x \sum_y xy . P(x,y)$$

$$= 1.1.\left(\frac{3}{31}\right) + 1.2.\left(\frac{4}{31}\right) + \ldots + 3.1.\frac{7}{31} + 3.2.\frac{8}{31}$$

$$E(x+y) = \sum_x \sum_y (x+y) P(x,y)$$

$$= 2.\left(\frac{3}{31}\right) + 3\left(\frac{4}{31}\right) + \ldots + 4.\left(\frac{7}{31}\right) + 5\left(\frac{8}{31}\right)$$

Que

X/Y	1	2	3	4	5	6	Total
0	0	0	1/32	2/32	2/32	3/32	1/4
1	1/16	1/16	1/8	1/8	1/8	1/8	5/8
2	1/32	1/32	1/64	1/64	0	2/64	1/8
Total	3/32	3/32	11/64	13/64	3/16	1/4	

Find $P(X \le 1)$; $P(Y \le 3)$; $P(X+Y \le 4)$.

Sol^n $P(X \le 1) = \underbrace{P(X=0) + P(X=1)}_{\text{Marginal density fun of } X}$

$= \frac{1}{4} + \frac{5}{8} = \frac{7}{8}$

$P(Y \le 3) = \cancel{P(Y=0)} + P(Y=1) + P(Y=2) + P(Y=3)$

$= \frac{3}{32} + \frac{3}{32} + \frac{11}{64}$

$= \frac{23}{64}$

$P(X+Y \le 4) \rightarrow$ Pairs which satisfy $X+Y \le 4$ are

(0,1) (0, 2) (0,3) (0,4)

(1,1) (1,2) (1,3), (2,1) (2,2)

So $P(X+Y \le 4) = \frac{13}{32}$.

HW

Que Three balls are drawn at random from a box Containing 2W, 3R, 4B balls. If X denotes the no. of White balls drawn and Y denotes the no. of red balls drawn. Find the joint Prob. dist^n of (X, Y).

Ans

Y/X	0	1	2
0	1/21	1/7	1/21
1	3/14	2/7	1/28
2	1/7	1/14	0
3	1/84	0	0