# UNIT–2

## A·C Circuit

Made By :- Ms. Neeraj

### Introduction:-

→ As discussed earlier, the flow of current in the circuits was steady and in only one direction that is, direct current (DC)

→ The use of DC is limited to few applications, eg:→ electroplating, charging of batteries, electric traction, electronic circuit etc.

→ But for large scale power Genʳ, transmission, distribution and utilization, an AC system is adopted.

The Most important advantages of AC system over DC system:-

1) An A·C Voltage can be Stepped up & stepped down by the use of transformer

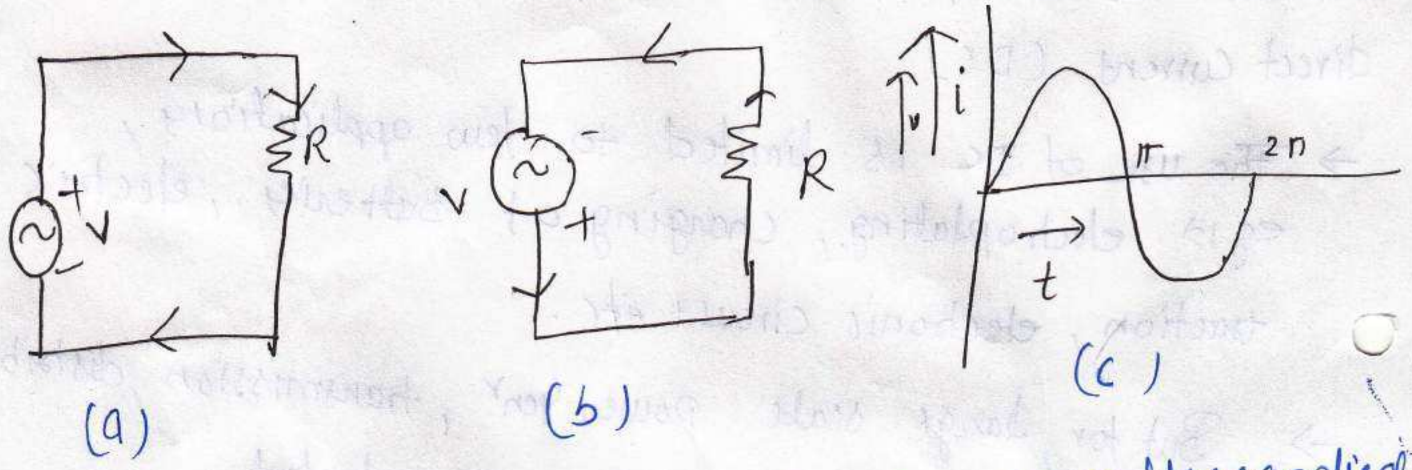
2) The AC Motors are cheaper in cost, simple in construction

3) Switches, circuit breakers for AC system is simpler.

→ AC system is universally adopted for Genʳ, transmission distribution & utilization of electrical energy.

## Alternating voltage & current:-

A Voltage that changes its polarity and Magnitude at Regular intervals of time is called an alternating Voltage.




fig→ Alternating voltage & current (a) AC voltage applied across Resistor, flow of current during Ist half cycle (b) AC voltage applied across Resistor flow of current during next half cycle.

→ Current flow in one direction & then in opposite direction when the polarity is Reversed.

## Wave form:-

→ An alternating voltage or current changes with Respect to time is Known as wave form or wave shape.

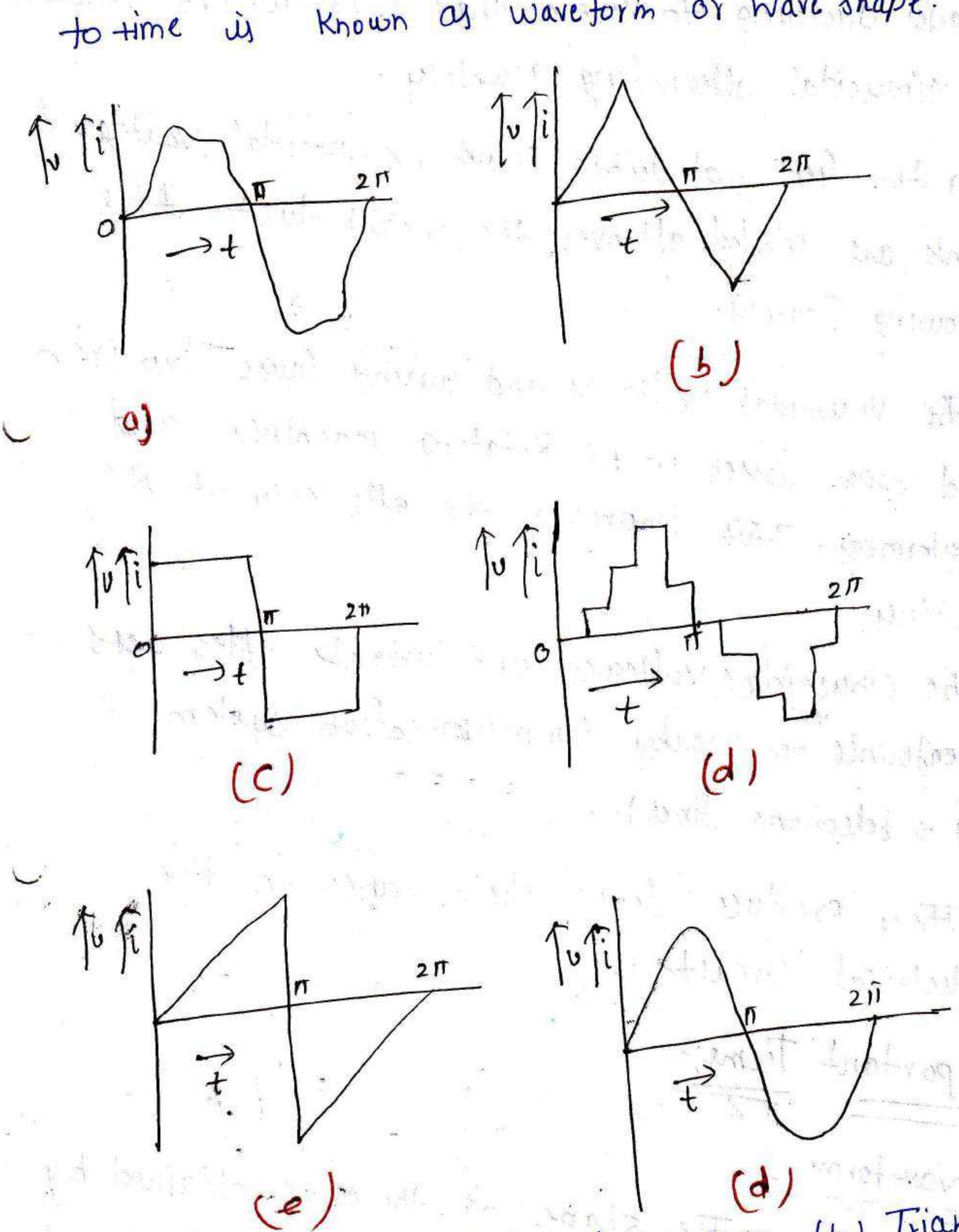



fig → AC wave shapes a) General ac wave (b) Triangular wave (C) Square wave (d) periodic wave (e) Triangular saw tooth wave (f) Sinusoidal wave

## (*) Sinusoidal Alternating Quantity:-

→ An alternating quantity (i.e voltage or current) that varies according to sine of angle θ ($\theta = \omega t$) is known as sinusoidal alternating quantity.

→ For the Genr of electric power, sinusoidal voltages & currents are selected all over the world due to the following Reasons.

1) The sinusoidal voltages and currents cause low iron and copper losses in AC Rotating machines and transformers. This improves the efficiency of AC machines.

2) The sinusoidal voltages and currents offer less interference to nearby communication system (e.g ⇒ telephone lines)

3) They produce least disturbance in the electrical circuits.

## Important Terms:-

1) Waveform:-

The shape of the curve obtained by plotting the instantaneous values of alternating quantity (voltage or current) along Y-axis and time or angle $\theta = \omega t$ along X-axis is called ~~the~~ wave form or wave shape.

2) Instantaneous Value:-

→ The value of an alternating quantity, that is, voltage or current at any instant is called its instantaneous value and is Represented by v or i.

3) Cycle:- When an alternating quantity goes through a complete set of +ve & -ve values or goes through 360 electrical degrees, it is said to have complete one cycle.

4) Alternation:- one half-cycle is called alternation. An alternation spans 180 electrical degrees.

5) Time period:- The time taken in seconds to complete one cycle by an alternating quantity is called time period. It is generally denoted by 'T'.

6) Frequency:- The number of cycles made per second by an alternating quantity is called frequency. It is measured in cycle per second (c/s) or hertz (Hz) and is denoted by f.

7) Amplitude:- The maximum value (+ve or -ve) attained by an alternating quantity in one cycle is called its amplitude or peak value or Maximum value. The maximum value of voltage and current is generally denoted by $E_m$ or $V_m$ & $I_m$.

## Peak Value:-

→ The maximum value attained by an alternating quantity during one cycle is called peak value. This is also called maximum value or crest value or amplitude.

→ Represented by → $E_m$ & $I_m$

## Average Value:-

The arithmatic average of all the instantaneous values considered an alternating quantity (current or voltage) over one cycle is called average value.

→ In case of symmetrical waves (such as sinusoidal current or voltage wave), the +ve half is exactly equal to the -ve half, so that, the average value over a complete cycle is zero. Hence, to determine the average value of A.C, only +ve half cycle is consider.

$$I_{av} = \frac{\text{Area of Alternation}}{\text{Base}} = \frac{i_1 + i_2 + i_3 + \cdots i_n}{n}$$

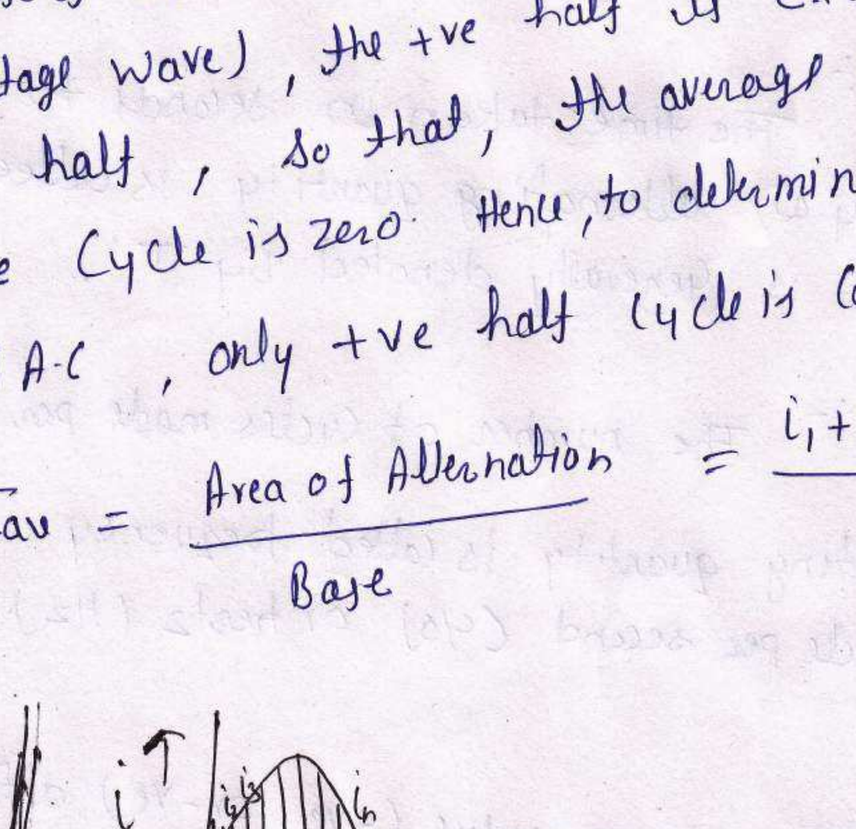



fig → +ve half cycle divided into n equal parts.

## ⊗ Average Value of Sinusoidal Current

$$I_{av} = \frac{I_m}{\pi/2}$$

$$\boxed{I_{av} = 0.637 I_m}$$

⊗ $I_{rms} = I_{effective} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$

$$\boxed{I_{rms} = 0.707 I_m}$$

**Defination:-** The steady current when flow through a Resistor of ~~Resistance R~~ for a given time ~~t seconds~~ which produces the same amount of heat as produced by an alternating current when flows through the same resistor for the same time is called effective or RMS value of an alternating current.

**Form factor:-** The ratio of rms value to average value of an alternating is called form factor.

$$\text{form factor} = \frac{I_{rm}}{I_{av}} = \frac{I_m/\sqrt{2}}{\cancel{2I_m}\ 2I_m/\pi}$$

$$= \frac{\pi I_m}{2\sqrt{2}\, I_m} = 1.11$$

$$\boxed{\text{form factor} = 1.11}$$

## Peak factor:-

→ The ratio of max. value to rms value of an alternating quantity is called peak factor.

$$\text{Peak factor} = \frac{I_m}{I_{rms}} \text{ or } \frac{E_m}{E_{rms}}$$

$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2}$$

$$\boxed{\text{Peak factor} = 1.4142}$$

**Q:-** The equation of an alternating current $i = 42.42 \sin 628t$ find → 1) Max value 2) frequency 3) rms value 4) Average value & 5) form factor.

**Solⁿ** Given that

$$i = 42.42 \sin 628t$$

we know that

$$i = I_m \sin \omega t$$

$$I_m = 42.42A \ \& \ \omega = 628 \text{ rad/sec}$$

1) $I_m = 42.42 \text{ A}$

2) $\omega = 628 \text{ rad/sec}$ & $\omega = 2\pi f = 628$

$$f = \frac{628}{\pi} \Rightarrow 100 \text{ HZ}$$

3) $I_{rms} = I_m/\sqrt{2} = 30A$

4) $I_{av} = \frac{2I_m}{\pi} = 27A$

5) $\frac{I_{rms}}{I_{av}} = \text{form factor}$

$$\text{form factor} = \frac{30}{27} = 1.11$$

# Single -ϕ AC Circuits:-

## Introduction:-

→ Alternating supply is invariably used for domestic & Industrial Application.

→ The path for the flow of alternating current is called an A.C Circuit.

→ In DC circuits, the opposition to the flow of current is only the Resistance of the circuit

→ while in AC circuits, the opposition to the flow of current is due to Resistance (R), Inductive reactance ($X_L = 2\pi fL$) and capacitive reactance ($X_C = 1/2\pi fC$) of the circuit.

→ In AC circuits, frequency plays an important Role.

→ In these circuits, the current and voltages are Represented with magnitude and direction (phasors).

→ The voltage and current may or may not be in phase with each other depending upon the parameters (R, L & C) of the circuit.

→ In AC circuits, the currents as well as voltages are added and subtracted vectorially instead of ~~arithmetically~~ as in DC circuits.
arithmetically

## AC Circuit Containing Resistance only:- 

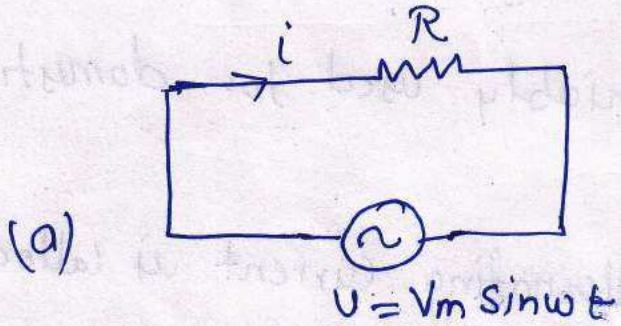



fig (1) → Circuit Containing Resistance only

→ The circuit containing a pure Resistance of R Ω is shown in fig (1).

→ Let the Alternating voltage applied across the circuit be given by the equation

$$v = V_m \sin\omega t \longrightarrow eq^n (1)$$

Then, the instantaneous value of current flowing through the Resistor will be

$$i = \frac{v}{R} = \frac{V_m}{R}\sin\omega t \longrightarrow eq^n (2)$$

The value of current will be maximum, when $\omega t = 90°$ or $\sin\omega t = 1$

$$I_m = \frac{V_m}{R}$$

substituting this value in eq$^n$ (2), we get

$$\boxed{i = I_m \sin\omega t} \rightarrow eq^n (3)$$

from eq$^n$ (1) & (3), it is clear that there is no phase difference between the applied voltage and the current flowing through pure Resistive circuit, that is phase angle between the voltage & current is zero.

The phasor diagram shown in fig 1.(b)


fig → 1-(b)
Phastor Diagram

→ Hence, in an AC circuit containing pure Resistance current is in phase with the voltage.

Power:-

$$P = vi = V_m \sin\omega t \times I_m \sin\omega t$$

$$= \frac{V_m I_m 2\sin^2\omega t}{2}$$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} (1 - \cos 2\omega t)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos 2\omega t \quad \left(\because \omega t = \frac{\pi}{2}\right)$$

$$\left(\because V_{rms} = \frac{V_m}{\sqrt{2}}, \quad I_{rms} = \frac{I_m}{\sqrt{2}}\right)$$

$$P = V_{rms} I_{rms} - 0$$

$$\boxed{P = VI}$$

# AC circuit containing pure Inductance only :- (8)

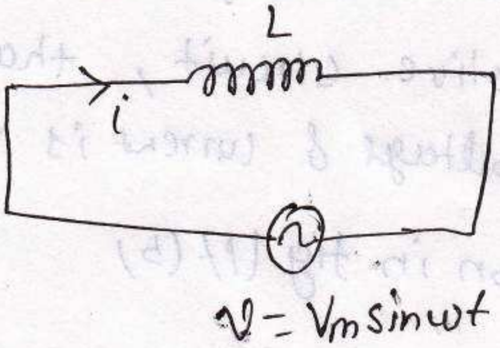



fig - 2(a)
circuit containing
Pure Inductance only

→ The circuit containing pure Inductance of L Henry is shown in fig 2(a)

→ Let the alternating voltage applied across the circuit be given by the equation

$$v = V_m \sin\omega t \quad \rightarrow eq^n \text{ (1)}$$

→ An AC i flows through the Inductance that induces an emf in it, given by the Relation.

$$e = -L\frac{di}{dt}$$

→ This induced emf is equal & opposite to the applied voltage.

$$v = -e = -\left(-L\frac{di}{dt}\right)$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t \, dt$$

Integrating Both sides

$$\int di = \int \frac{V_m}{L}\sin\omega t \, dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \text{ — eq}^n\ (2) \quad \left(\because X_L = 2\pi f L,\ \omega = 2\pi f\right)$$

the value of current will be maximum when

$\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

i.e

$$I_m = \frac{V_m}{X_L}$$

~~put~~ substituting the value in eq$^n$ (2)

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \text{ — eq}^n\ (3)$$

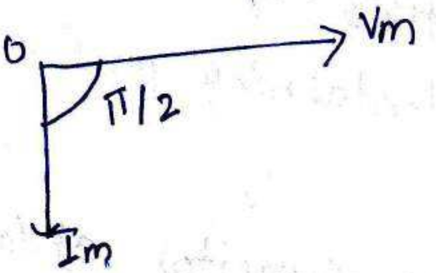



fig 2-(b)
phasor diagram.

from eq$^n$ (1) & (2), it is clear that current flowing through a pure Inductive circuit lags behind the applied voltage v by 90°. ~~show~~

The phasor diagram is shown in fig 2-(b)

Power:-



$$P = vi$$

$$P = V_m \sin\omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$P = \frac{V_m I_m 2\sin\omega t \cos\omega t}{2}$$

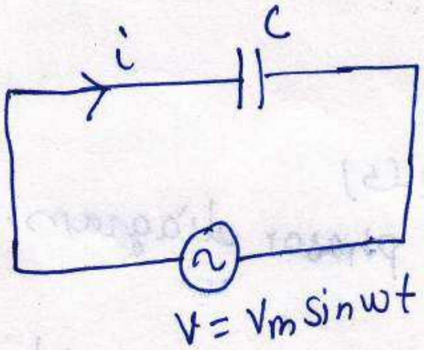
$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

$$\left(\omega t = \frac{\pi}{2}\right)$$

$$P = 0$$

Hence, Average power consumed in a pure Inductive circuit is zero.

## ⊛ AC circuit containing pure capacitor only:-




fig(3)-(a) circuit diagram containing pure capacitor only

→ The circuit containing a pure capacitor of capacitance C farad is shown in fig 3-(a)

→ Let the alternating voltage applied across the circuit be given as

$$v = V_m \sin\omega t \quad \text{— eq}^n \ (1)$$

Charge on the capacitor at any instant

$$q = CV$$

Current flowing through the circuit

$$i = \frac{d}{dt}(q) = \frac{d}{dt}(CV)$$

$$i = \frac{d}{dt} CV_m \sin\omega t = CV_m \frac{d}{dt}\sin\omega t$$

$$i = \omega C V_m \cos\omega t$$

$$i = \frac{V_m}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$\hookrightarrow$ eq$^n$ (2) $\left(\because X_C = 1/2\pi f C\right)$

the value of current will be maximum

when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

$$I_m = \frac{V_m}{X_C}$$

substituting this value is eq$^n$ (2), we get

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \rightarrow \text{eq}^n(3)$$

from eq$^n$ (1) & (3), it is clear that the current flowing through pure capacitive circuit leads the applied voltage by 90°.

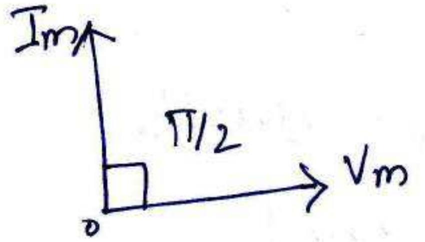



fig: phasor diagram

Power:-

$$p = Vi$$

$$p = V_m \sin\omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t \cos\omega t \times \frac{2}{2}$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

$$\left(\omega t = \frac{\pi}{\cancel{2}}\right)$$

$$p = 0$$

Average power consumed in a pure capacitive circuit is zero.

Q:- A 100 μF capacitor is connected across a 230V, 50Hz supply. Determine i) Max. Instantaneous charge on the capacitor ii) Max. Instantaneous energy stored in the capacitor.

Solⁿ i) Max. Instantaneous charge on the capacitor

$$= CV_m = (100 \times 10^{-6}) \times (230 \times \sqrt{2})$$

$$\boxed{= 32.527 \times 10^{-3}\,C}$$

ii) Max. energy stored in the capacitor

$$= \frac{1}{2} C V_m^2 = \frac{1}{2}(100 \times 10^{-6})(230 \times \sqrt{2})^2$$

$$\boxed{= 5.29\,J}$$

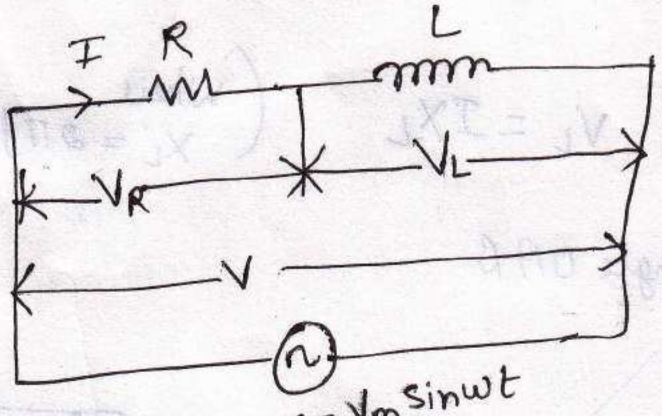
# A C series Circuits:-

→ AC circuits containing pure components such as Resistance, Inductance & capacitance.

→ AC Circuits contain two or more than two such component connected in series or parallel.

→ An AC series circuit may be
- i) R-L series circuit.
- ii) R-C series circuit
- iii) R-L-C series circuit

## ⊗ R-L Series Circuit:-




fig→ circuit containing Resistance & Inductance in series.

→ A circuit that contains a pure resistance R Ω connected in series with a coil having pure Inductance of L Henry is known as R-L series circuit.

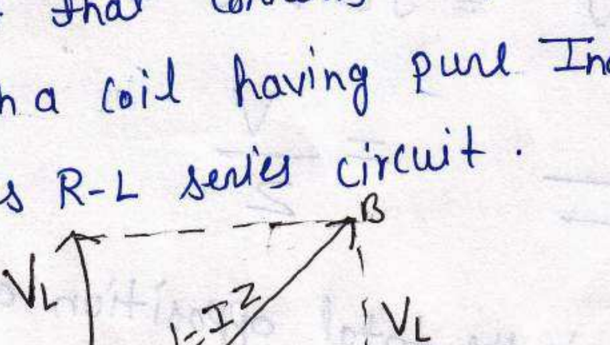



Fig → phasor diagram

An R-L series circuit and its phasor diagram are shown in fig.

→ To draw the phasor diagram, current I (rms value) is taken as the Reference vector

→ Voltage drop in Resistance $V_R (= IR)$ is taken in phase with current vector,

whereas voltage drop in inductive reactance $V_L (= IX_L)$ is taken 90° ahead of the current vector (current lags behind the voltage by 90° in pure inductive circuit).

→ The vector sum of these two voltages (drops) is equal to the applied voltage V (rms value).

Now, $V_R = IR$ & $V_L = IX_L$ (where $X_L = 2\pi f L$).

In right-angled triangle OAB

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$ is the total opposition offered to the flow of A C by an R-L series circuit and is called Impedance of the circuit. It is measured in ohms.

1) Phase angle:-

from the phasor diagram, it is clear that current in this circuit lags behind the applied voltage by an angle $\phi$ called phase angle.

from Phasor diagram, $\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$

or $\boxed{\phi = \tan^{-1} X_L/R}$ $\left(\because \tan\theta = \frac{P}{B}\right)$

2) Power:-

If the alternating voltage applied across the circuit is given by the equation.

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t - \phi)$$

$$\therefore \quad p = vi$$

$$P = \frac{V_m I_m}{2} 2\sin\omega t \cdot \sin(\omega t - \phi)$$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(2\omega t - \phi)$$

Average power consumed in the circuit over a complete cycle.

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \phi)$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi - 0$$

$$P = V_{rms} I_{rms} \cos\phi$$

$$P = VI \cos\phi$$

$\cos\phi \Rightarrow$ power factor of the circuit.

from phasor diagram

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$P = VI\cos\phi$$

$$P = IZ \cdot I \cdot \frac{R}{Z} = I^2 R$$

This show that power is actually consumed in Resistance only; Inductance does not consume any power.

③ Impedance triangle :-

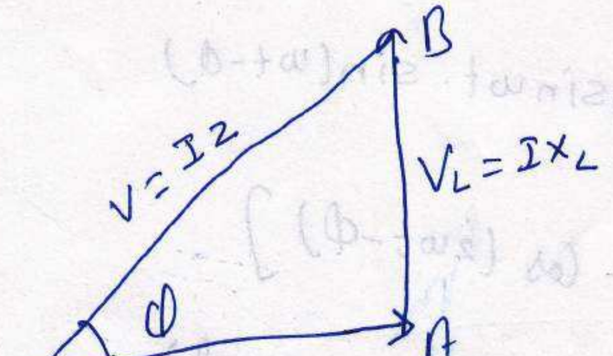



fig - ①

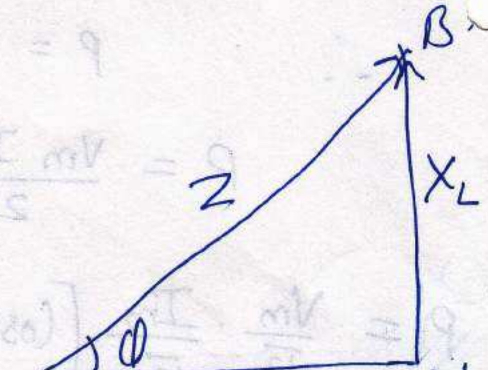



fig ②

The simplified phasor diagram of R-L series circuit is shown in fig ①. When each side of this phasor diagram is divided by a common factor I, We Get another Right-angled triangle, as shown in fig ③, whose sides represent R, $X_L$ & Z.

Such a triangle is known as impedance triangle.

The concept of impedance triangle is useful since it enables us to calculate.

i) the impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2}$$

ii) the power factor of the circuit

$$\cos\phi = R/Z$$

iii) phase angle, $\phi = \tan^{-1}\frac{X_L}{R}$

## True Power & Reactive Power:-

**True Power / Active power / Real power :-** The power that is actually consumed or utilised in an A.C circuit is called true power or active power or real power.

It has already been seen that power is consumed only in Resistance. A pure Inductor and a pure Capacitor do not consume any power.

(27) <u>Reactive Power</u>:- In a half cycle, whatever power is Received from the source by these components, the same is returned to the source.

This power flows back (i.e in both directions in the circuit) or ~~reach~~ reacts upon itself is called reactive power.

→ It does not do any useful work in the circuit.

→ It has been seen that in pure Resistive Circuit, current is in phase with the applied voltage, whereas in pure Inductive & capacitive circuit, current is 90° out of phase.

→ Therefore, it is concluded that the current in phase with the voltage produces true or active power, whereas the current 90° out of the phase with the voltage contributes to reactive power. Hence,

$$\boxed{\text{true power} = \text{Voltage} \times \text{current in phase with voltage}}$$

$$\boxed{\text{Reactive Power} = \text{voltage} \times \text{current } 90^\circ \text{ out of phase with voltage.}}$$

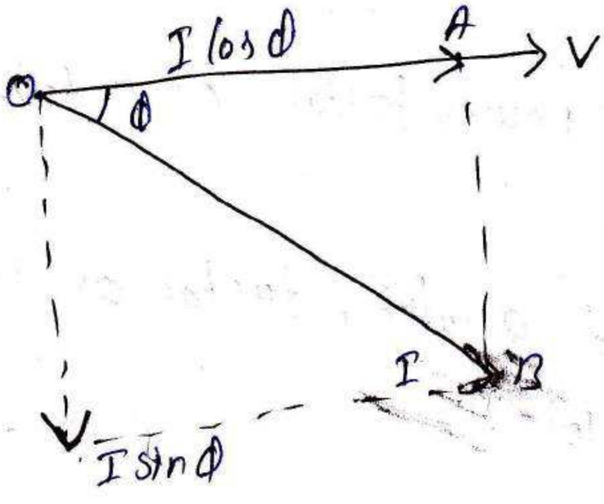



fig → ③ phasor diagram Representing active, reactive & apparent current

→ The phasor diagram for an Inductive circuit is shown in fig ③, where current $I$ lags behind the voltage $V$ by an angle $\phi$.

→ current $I$ can be Resolved into two Reactangular components that is

i) $I\cos\phi$ which is in phase with Voltage $V$ &

ii) $I\sin\phi$, which is $90^\circ$ out of phase with voltage $V$

∴
$$\boxed{\begin{array}{l} \text{True power},\ P = V\times I\cos\phi = VI\cos\phi\ \text{W} \\ \text{Reactive power},\ P_r = VI\sin\phi\ .\ \text{VAR} \\ \text{Apparent power},\ P_a = V\times I = VI\ \text{VA} \end{array}}$$

VAR → Voltage Ampere Reactive

## Q - factor

Reciprocal of power factor of a coil is known as its Q- factor.
It is also called quality factor or figure of merit of the coil.

Mathematically

$$\boxed{Q\text{- factor} = \frac{1}{P.f}} = \frac{1}{\cos\phi} = \frac{Z}{R}$$

If the value of R is very small in comparison to its inductive Reactance $X_L$, then

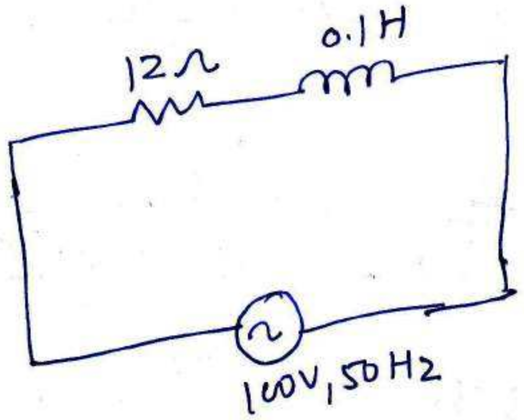
$$Q\text{ - factor} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\boxed{Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}}$$

## Numericals:-

**Q:-1** A Coil having a Resistance of $12\Omega$ and an Inductance of $0.1H$ is connected across a $100V$, $50Hz$ supply. Calculate (i) Reactance & impedance of the Coil (ii) Current (iii) phase difference b/w the Current & Applied voltage (iv) Power factor & Draw also the phasor diagram showing Voltage & Current.

**Sol$^n$** According to question, we draw the Circuit diagram.




i) Reactance, $X_L = 2\pi f L$

$$= 2\pi \times 50 \times 0.1 = 31.416\Omega$$

ii) Impedance, $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{(12)^2 + (31.416)^2}$$

$$Z = 33.63\Omega$$

ii) Current $= I = \frac{V}{Z} = \frac{100}{33.67}$

$I = 2.97 A$

iii) phase difference

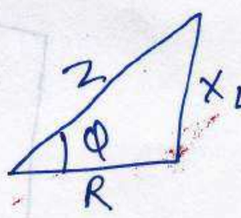
$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{31.416}{12}$

$\phi = \tan^{-1} 2.618$

$\phi = 69.1^\circ$

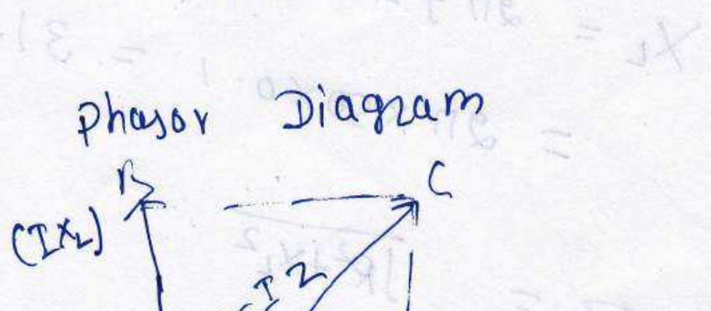
(iv) power factor, $\cos\phi = \frac{R}{Z}$

$\cos\phi = \frac{12}{33.63}$




$\cos\phi = 0.3568$

(v) phasor Diagram



(2) The voltage & current through a circuit element are

$$v = 50 \sin(314t + 55^\circ)\, V$$

$$i = 10 \sin(314t + 325^\circ)\, A$$

find the value of power drawn by the element

Sol^n Given

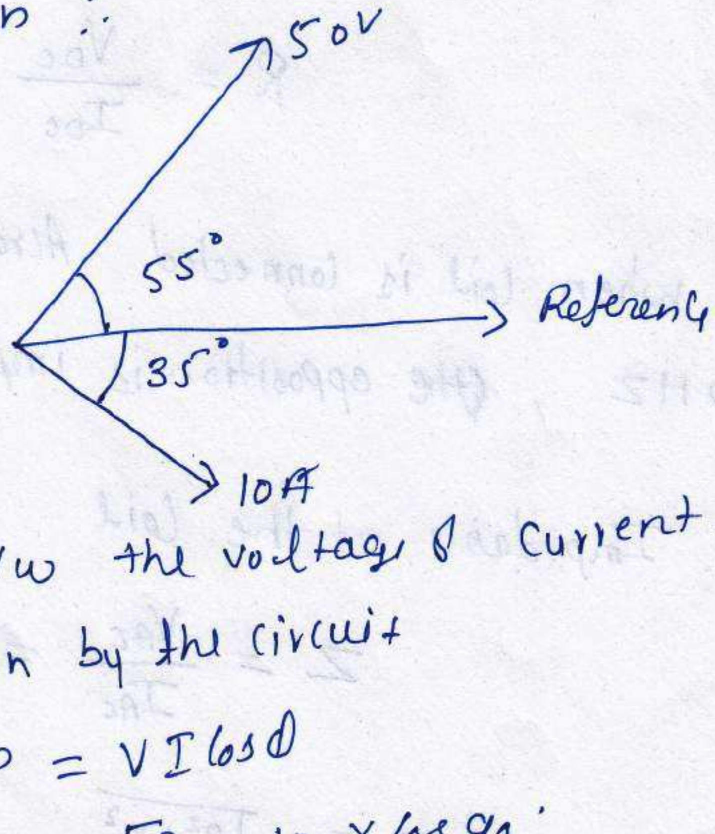
$$v = 50 \sin(314t + 55^\circ)\, V$$

$$i = 10 \sin(314t + 325^\circ)\, A$$

or

$$i = 10 \sin(314t - 35^\circ)\, A$$

phasor Representation :




phase difference b/w the voltage & current is 90°

Now, power drawn by the circuit

$$P = VI\cos\phi$$

$$= \frac{50}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}} \times \cos 90$$

$$P = 0\,W$$

This Result indicates that the element is pure Inductive

③ A Coil Connected to 100V DC supply draws 10 & 
the same coil when connected to 100 v, AC Voltage
of frequency 50HZ draws 5A. Calculate the parameters
of the coil & power factor.

Sol$^n$ Let the Resistance & Inductance of the coil
be R Ω & L Henry.

→ when coil in connected to DC supply, the opposition
is only Resistance of the coil.

∴ Resistance of the coil

$$R = \frac{V_{DC}}{I_{DC}} = \frac{100}{10} = 10\,\Omega$$

→ when coil is connected Across AC supply of 100V,
50HZ, the opposition is impedance of the coil

∴ Impedance of the coil

$$Z = \frac{V_{AC}}{I_{AC}} = \frac{100}{5} = 20\,\Omega$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z^2 = R^2 + X_L^2 \Rightarrow X_L = \sqrt{Z^2 - R^2}$$

$$X_L = \sqrt{(20)^2 - (10)^2} = \sqrt{300}$$

$$L = \frac{X_L}{2\pi f} = \frac{\sqrt{300}}{2\pi \times 50}$$

$$L = 55.13\ mH$$

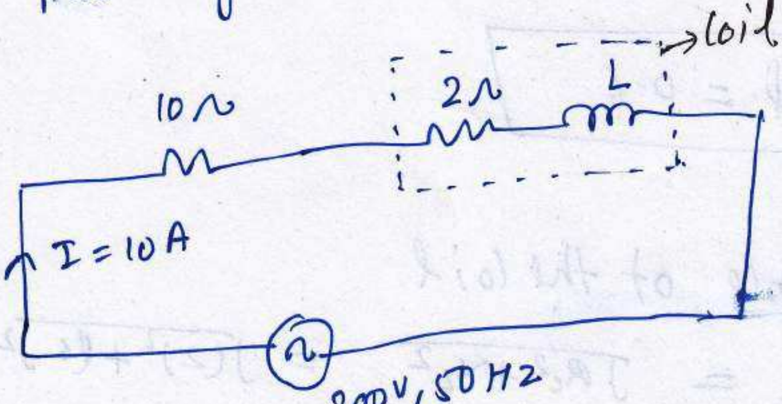
$\therefore$ parameters are $R = 10\Omega$ & $L = 55.13\ mA$

power factor

$$\cos\phi = \frac{R}{Z} = \frac{10}{20} = 0.5$$

(4) A non-Inductive Resistance of $10\Omega$ is connected in series with an Inductive coil across 200V, 50HZ AC supply. The current drawn by the series combination is 10A. The Resistance of the coil is $2\Omega$.
Determine (i) Inductance of the coil
ii) power factor iii) voltage across the coil.

Sol^n. According to Question, Draw the circuit




Total Impedance of the Circuit, $Z = \frac{V}{I}$

$$Z = \frac{200}{10} = 20\Omega$$

total Resistance of the circuit

$$\boxed{R = 10 + 2 = 12\Omega}$$

Inductive Reactance of the coil

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(20)^2 - (12)^2}$$

$$\boxed{X_L = 16\Omega}$$

i) Inductance of coil

$$2\pi f L = X_L = 16$$

$$L = \frac{16}{100\pi} = 50.93\,mH$$

$$\boxed{L = 50.93\,mH}$$

ii) Power factor of the circuit

$$\cos\phi = \frac{R}{Z} = \frac{12}{20} = 0.6$$

$$\boxed{\cos\phi = 0.6}$$

iii) Impedance of the coil

$$Z_C = \sqrt{R_C^2 + X_L^2} = \sqrt{(2)^2 + (16)^2}$$

$$Z_C = 16.124\Omega$$

Voltage across the Coil, $V_c = I Z_c$

$$V_c = 10 \times 16.124$$

$$\boxed{V_c = 161.24V}$$

Power factor of the Coil, $\cos\phi_c = \frac{R_c}{Z_c}$

$$\cos\phi_c = \frac{2}{16.124}$$

$$\boxed{\cos\phi_c = 0.124}$$

## R-C Series circuit:-

→ A circuit that contains a pure Resistance R Ω connected in series with a pure Capacitor of Capacitance C Farad is known as R-C series circuit.

→ An R-C series circuit & its phasor diagram is shown in fig (below).

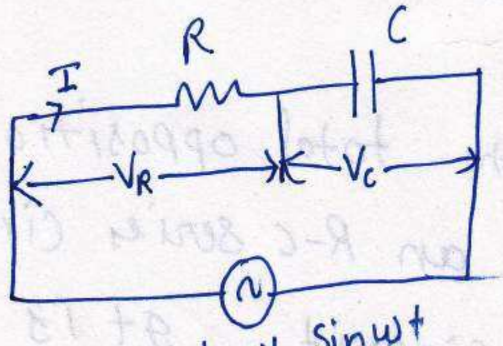



fig → Circuit containing Resistance & Capacitance in series

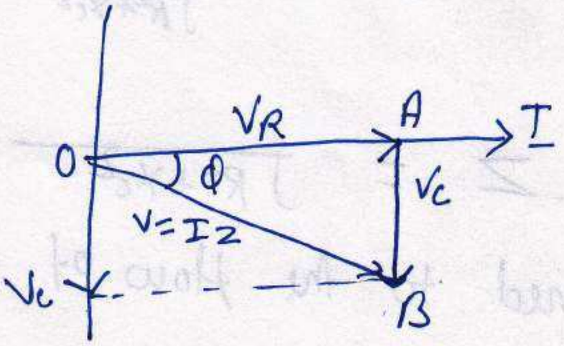



fig → Phasor diagram

→ To draw the phasor diagram, current I (rms value) is taken as Reference

→ Voltage drop in resistance $V_R = IR$ is taken in phase with Current

→ Whereas Voltage drop in Capacitive Reactance $V_C = IX_C$ is taken 90° behind the Current (current leads the voltage by 90° in pure capacitive circuit)

$$V_R = IR \quad \& \quad V_C = IX_C$$

$$\left( \text{Where } X_C = \frac{1}{2\pi fC} \right)$$

In Right-angled triangle OAB

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

$\therefore Z = \sqrt{R^2 + X_C^2}$, is the total opposition offered to the flow of AC by an R-C series circuit and is called impedance of the circuit. It is measured in ohm.

① phase angle:-

from the phasor diagram, it is clear that current in this circuit leads the applied voltage by an angle $\phi$ called phase angle.

from the phasor diagram

$$\tan\phi = \frac{V_C}{V_R} = \frac{I X_C}{I R}$$

$$\tan\phi = \frac{X_C}{R}$$

or $\boxed{\phi = \tan^{-1} X_C/R}$

② Power:-

We know that

$$v = V_m \sin\omega t$$
$$i = I_m \sin(\omega t + \phi)$$

$$\therefore \quad P = vi = V_m I_m \sin\omega t \cdot \sin(\omega t + \phi)$$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot 2\sin\omega t \sin(\omega t + \phi) \qquad \left\{ \because \cos(A-B) - \cos(A+B) \right.$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \left[\cos\phi - \cos(2\omega t + \phi)\right]$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(2\omega t + \phi)$$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P = V_{rm} I_{rms} \cos\phi$$

$$P = VI \cos\phi$$

Power factor :-

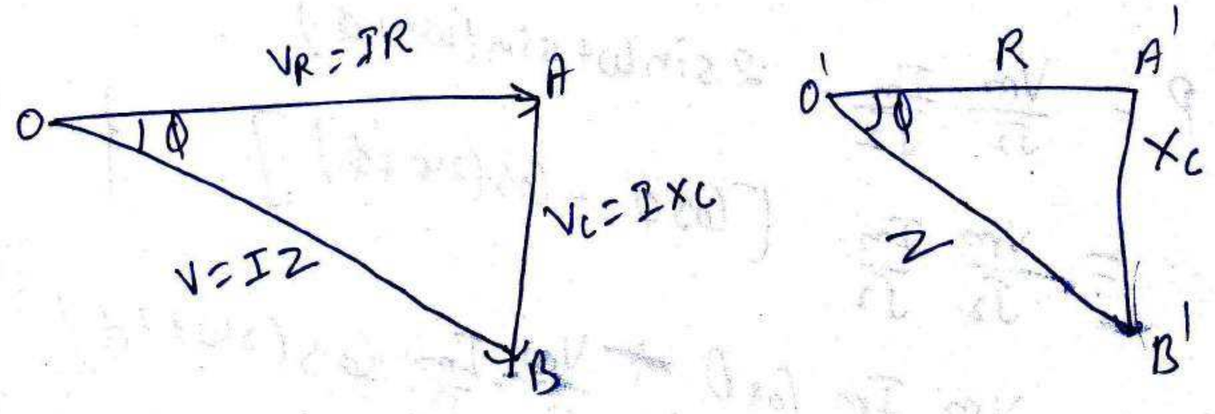
$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$ (same as in R-L series circuit)

$$P = VI\cos\phi = IZ \cdot I \cdot \frac{R}{Z}$$

$$\boxed{P = I^2 R}$$

This shows that power is actually consumed in Resistance only; capacitor does not consume any power.

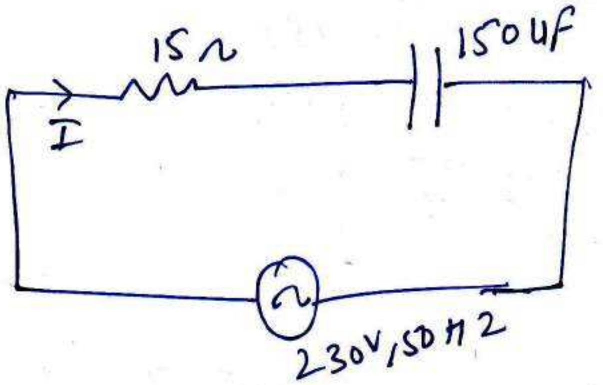
Impedance Triangle:-



## Numericals:-

① A Resistance of 15 Ω & Capacitor of 150 μF Capacitance are connected in series across a 230V, 50Hz supply. Calculate i) Impedance ii) Current iii) power factor & phase angle iv) power consumed in the Circuit

Solⁿ According to Question, Draw the circuit




i) Impedance,

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$

$$X_C = 21.22\ \Omega$$

$$R = 15\ \Omega$$

$$Z = \sqrt{(15)^2 + (21.22)^2}$$

$$\boxed{Z = 25.987\ \Omega}$$

ii) Current

$$I = \frac{V}{Z} = \frac{230}{25.987} = 8.85\,A$$

$$\boxed{I = 8.85A}$$

iii) power factor

$$\cos\phi = \frac{R}{Z} = \frac{15}{25.987}$$

$$\boxed{\cos\phi = 0.577}$$

phase angle, $\phi = \cos^{-1} 0.577$

$$\boxed{\phi = 54.75^\circ}$$

iv) Power

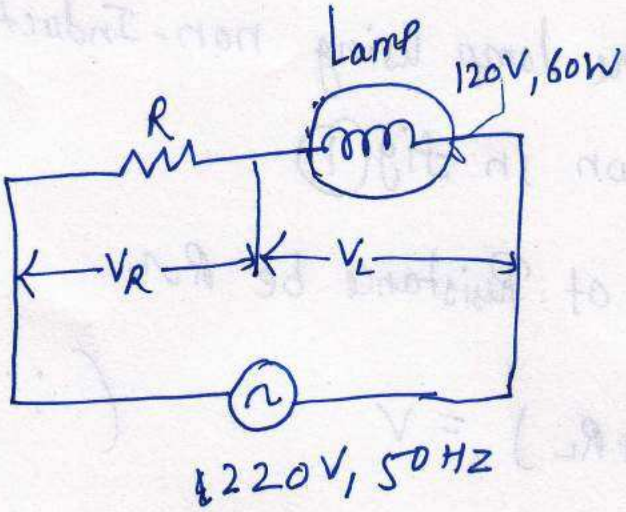
$$P = VI\cos\phi = 230 \times 8.85 \times 0.577$$

$$\boxed{P = 1174.9W}$$

(2) A 120V, 60W lamp is to be operated on 220V 50 Hz supply mains. For the lamp to operate in correct voltage, calculate the value of (37)

(i) non-Inductive Resistance (ii) pure inductance

Solⁿ According to Question, Draw the circuit



→ fig - (1) circuit as per given data

Lamp's rating = 120V, 60W

$V_s$ = supply voltage = 220V ) Given

f = 50 Hz

Resistance of the lamp

$$R_L = \frac{(120)^2}{60} \quad \left(\because P = \frac{V^2}{R}\right)$$

$$\boxed{R_L = 240\,\Omega}$$

operating Current,

$$I = \frac{60}{120} \qquad \left( \begin{array}{l} P = VI \\ \therefore I = P/V \end{array} \right)$$

$$\boxed{I = 0.5A}$$

i) for operating the lamp using non-Inductive Resistance, as shown in fig ①

Let the value of Resistance be $R \Omega$

$$\therefore \quad I(R + R_L) = V \qquad (\because V = IR)$$

$$R + R_L = \frac{220}{0.5} = 440$$

$$R = 440 - R_L$$

$$R = 440 - 240$$

$$\boxed{R = 200\,\Omega}$$

ii) for operating the lamp using pure Inductance as shown in fig ②.

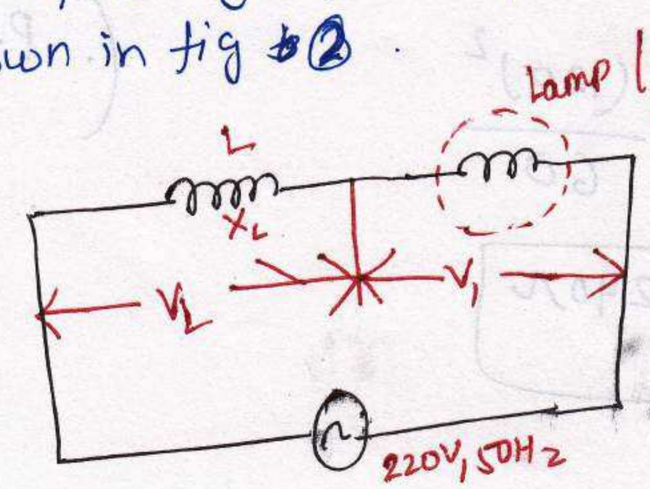



fig-②
circuit as per given data

Let the value of Inductance be L henry &

$X_L = 2\pi f L$

$V = IZ$

$Z = \frac{V}{I} = \frac{220}{0.5}$

$\boxed{Z = 440\Omega}$

$Z = \sqrt{R_L^2 + X_L^2}$

$Z^2 = R_L^2 + X_L^2$

$(440)^2 = (240)^2 + X_L^2$

$X_L = \sqrt{(440)^2 - (240)^2}$

$\boxed{X_L = 368.78\Omega}$

$X_L = 2\pi f L$

$L = \frac{368.78}{2\pi \times 50}$

$\boxed{L = 1.174\ H}$

## R-L-C Series circuit :-

→ A circuit that contains a pure Resistance of R Ω, a pure Inductance of L Henry and a pure capacitor of Capacitance C Farad, all connected in series is known as R-L-C series circuit.

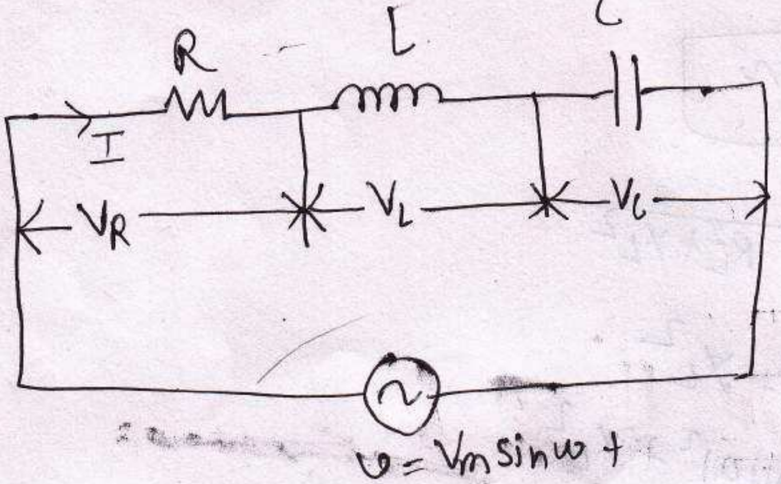



fig-①

$X_L = 2\pi f L$ & $X_C = 1/2\pi f C$

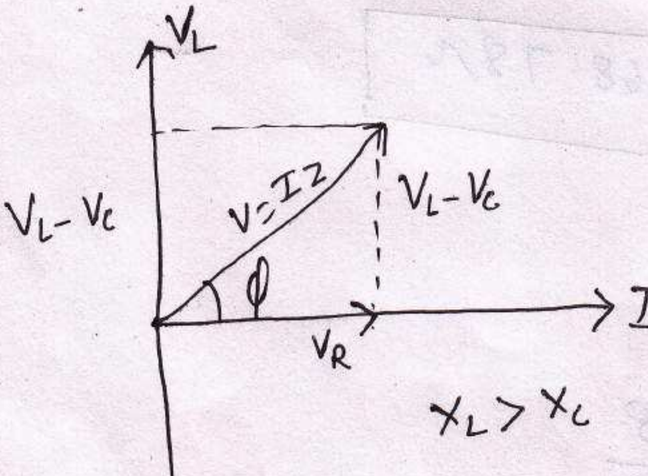



fig-② phasor Diagram

when a Resulting current I (rms value) flows through the circuit, the voltage across each component will be.

$V_R = IR$ , Voltage across R in phase with I

$V_L = IX_L$ , Voltage across L , leads I by 90°

$V_C = IX_C$ , " " C , lags by 90°

→ The phasor diagram is shown in fig ③ , where current is taken as Reference phasor. Since voltage across Inductance $V_L$ leads the current I by 90° & voltage across capacitance $V_C$ lags the current I by 90°, they act opposite to each other.

If $V_L > V_C$ , in effect the circuit behave as an Inductive circuit

→ If $V_L < V_C$ , the circuit behave as capacitive circuit.

→ Here the phasor diagram is drawn for an Inductive circuit ($\because V_L > V_C$)

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{(R)^2 + (X_L - X_C)^2}}$$

$$I = \frac{V}{Z}$$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ is the total opposition offered to the flow of AC by an R-L-C series circuit & is called impedance of the circuit.

Phase angle:-

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power:-

$$P = VI\cos\phi = I^2 R$$

$$\cos\phi = \frac{V_R}{V} = \frac{R}{Z}$$

1) $X_L > X_C$ (phase angle $\phi$ is +ve, ckt behave as an R-L series circuit.

$i = I_m \sin(\omega t - \phi)$

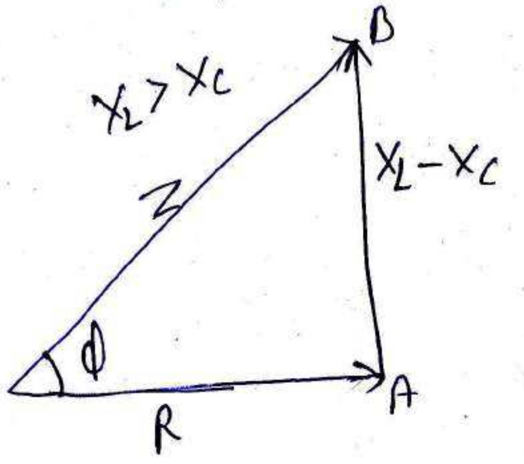
2) $X_L < X_C$ ($\phi$ is −ve, ckt behave as an R-C series circuit

$i = I_m \sin(\omega t + \phi)$

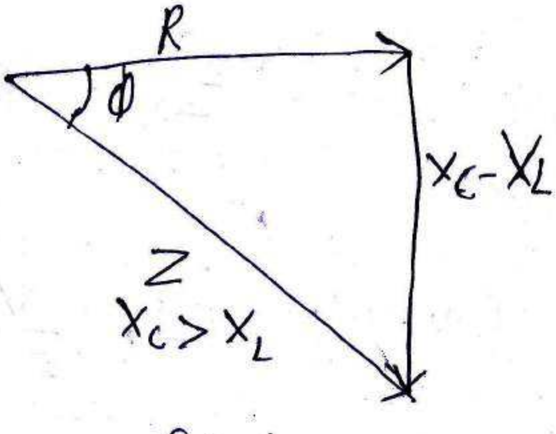
3) $X_L = X_C$ ($\phi$ is zero, ckt behave as an pure Resistive circuit.)

$i = I_m \sin\omega t$

# Impedance Triangle :-




(a)




(b)

fig → (a) Impedance triangle $(X_L > X_C)$
(b) Impedance triangle $(X_C > X_L)$

## Series Resonance :-

Resonance :- In an R-L-C series circuit, when the current is in phase with the applied voltage, the circuit is said to be in series resonance.

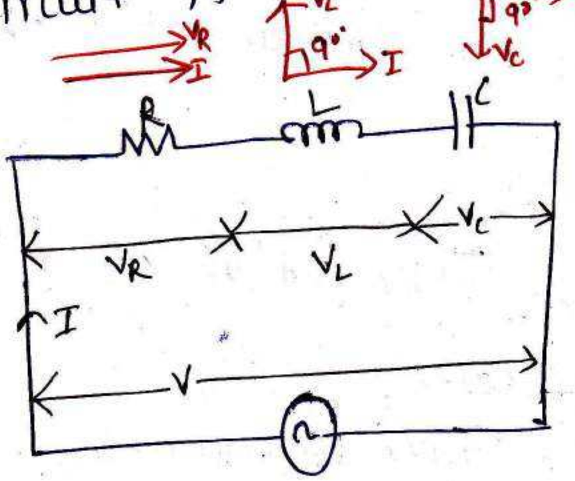



fig → R-L-C series circuit

$X_L = X_C \quad (X_L - X_C = 0)$

At Resonance $X_L - X_C = 0$ or $X_L = X_C$

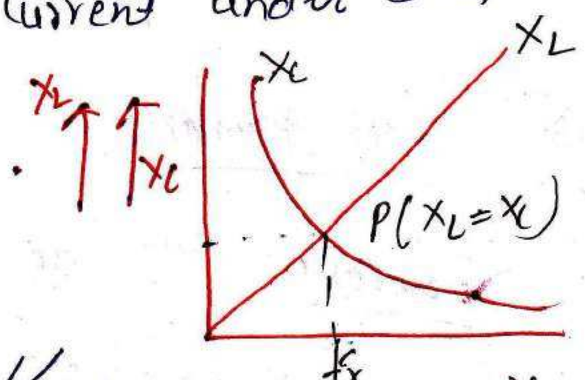
Impedance, $Z_r = \sqrt{(R^2) + (X_L - X_C)^2}$

$(\because X_L - X_C = 0$

$$\boxed{Z_r = R}$$

Current $\boxed{I_r = \frac{V}{Z_r} = \frac{V}{R}}$

→ Since at Resonance, the opposition to the flow of current is only Resistance $R$ of the circuit. The circuit draws maximum current under this condition.




Resonant frequency:-

We know that
$X_L = 2\pi f L$ & $X_C = 1/2\pi f C$

the value of $X_L$ & $X_C$ can be changed by changing the supply frequency.

When frequency increases, the value of $X_L$ increases where as the value of $X_C$ decreases.

$X_L = X_C$ (At series Resonance)

$2\pi f_r L = \frac{1}{2\pi f_r C}$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

($f_r$ → Resonant freq. in Hz.)

## Effect of Series Resonance:-

1) At Resonance $X_L = X_C$, the impedance of the circuit is maximum and is Reduced to the Resistance of the circuit

$$Z_r = R$$

ii) Since, Impedance is minimum, the circuit current is maximum at Resonance,

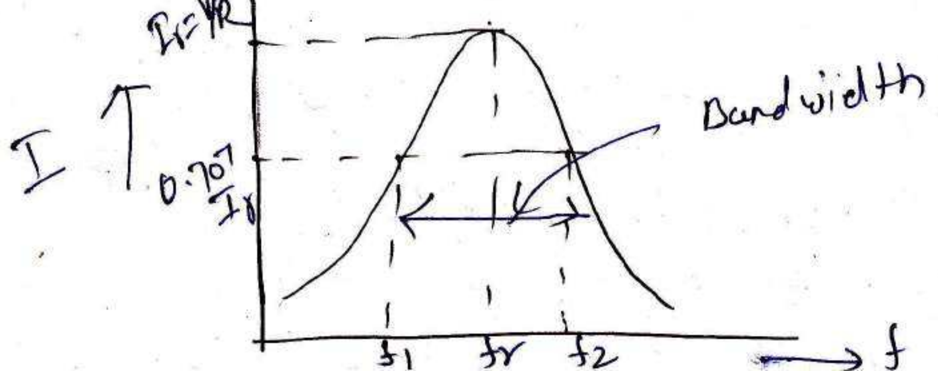
$$I_r = V/Z_r = V/R$$

iii) Power taken by the circuit is maximum as $I_r$ is maximum

$$P_r = I_r^2 R$$

(iv) As the current drawn by the circuit Resonance is very large (maximum), the Voltage drop across L & C are also very large.

**Bandwidth:-** The range of frequency over which circuit current is equal to ~~rms~~ or more than 70.7% of maximum value (i.e $I_r$ current at Resonance) is known as the Bandwidth of a series Resonant circuit.




$BW = f_2 - f_1$

$f_1 \rightarrow$ lower cut-off freq

$f_2 =$ ~~Higher~~ upper " " "

## Q - factor of Series Resonant circuit:-

$$\text{Q-factor} = \frac{\text{Voltage across L or C}}{\text{Applied voltage}}$$

$$= \frac{I_r X_L}{I_r R} = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

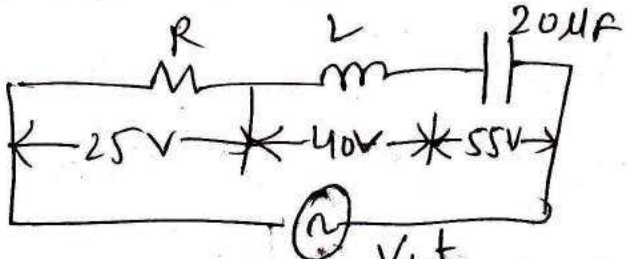
$$\omega_r = 2\pi f_r = 2\pi \times \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\left( \because f_r = \frac{1}{2\pi\sqrt{LC}} \right)$$

$$\text{Q - factor} = \frac{L}{R} \times \frac{1}{\sqrt{LC}}$$

$$\boxed{\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

The value of Q-factor depends entirely upon the design of coil (i.e R-L which is a part of R-L-C circuit.

Q:- 


find the V & power loss in the circuit shown in figure.

Sol$^n$

$$C = 20\mu f \quad , \quad I = 0.345A$$

$$V = \sqrt{(25)^2 + (40-55)^2} = 29.155V \qquad \cancel{(\sqrt{R^2 + (X_L - X_C)^2}}$$

power loss $P = V_R \times I = 25 \times 0.345 = 8.625$

# AC parallel circuits:-

## Introduction:-

→ The AC circuit in which number of branches are connected in such a manner so that voltage across each branch is the same, but current flowing through them is different are called AC parallel circuits.

→ The parallel circuits are used more frequently in AC system because of the following Reasons.

1) Almost all the electrical appliances (or devices) of different ratings are operated at the same supply voltage and are connected in parallel

2) Each device is Required to be operated independently (with a switch) without disturbing the operation of other devices. Hence they are connected in parallel.

## Methods of Solving Parallel AC Circuits:-

1) Phasor (or vector) Method
2) Admittance Method

→ The method to be applied for the solution of the problem depends upon the conditions of the problem.

## Phasor (or Vector) Method :-

To solve parallel AC Circuits by this Method, we proceed as follows :-

1) **Step I** :-

Draw the circuit as per given problem. ~~Consider~~ Here we .

{ Here, for illustration, we have considered two branches connected in parallel. One Branch contains Resistance and Inductance in series, whereas second Branch contains Resistance and Capacitance in series. The supply Voltage is V v ).

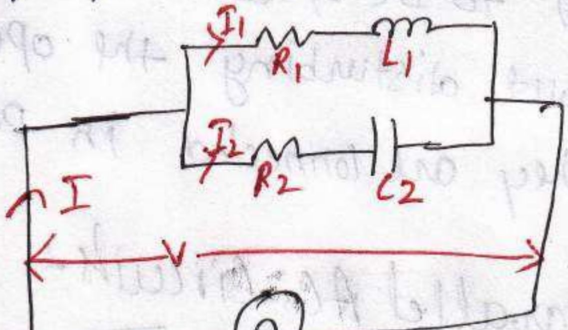



fig → (9)
Circuit diagram

**Step II** :-

Find the Impedance of each Branch of the circuit separately.

$Z_1 = \sqrt{R_1^2 + X_{L_1}^2}$ , where $X_{L_1} = 2\pi f L_1$

$Z_2 = \sqrt{R_2^2 + X_{C_2}^2}$ , where $X_{C_2} = 1/2\pi f C_2$

Step III :-

Determine the Magnitude of current and phase angle with the Voltage in each branch

$$I_1 = \frac{V}{Z_1} \; ; \; \phi_1 = \tan^{-1}\frac{X_{L_1}}{R_1} \; ; \text{(lagging)} \text{ (for Inductive Branch)}$$

$$I_2 = \frac{V}{Z_2} \; ; \; \phi_2 = \tan^{-1}\frac{X_{C_2}}{R_2} \; ; \text{(leading)} \text{ (for Capacitive Branch)}$$

Step IV :-

Draw the phasor diagram by considering Voltage as the Reference phasor.

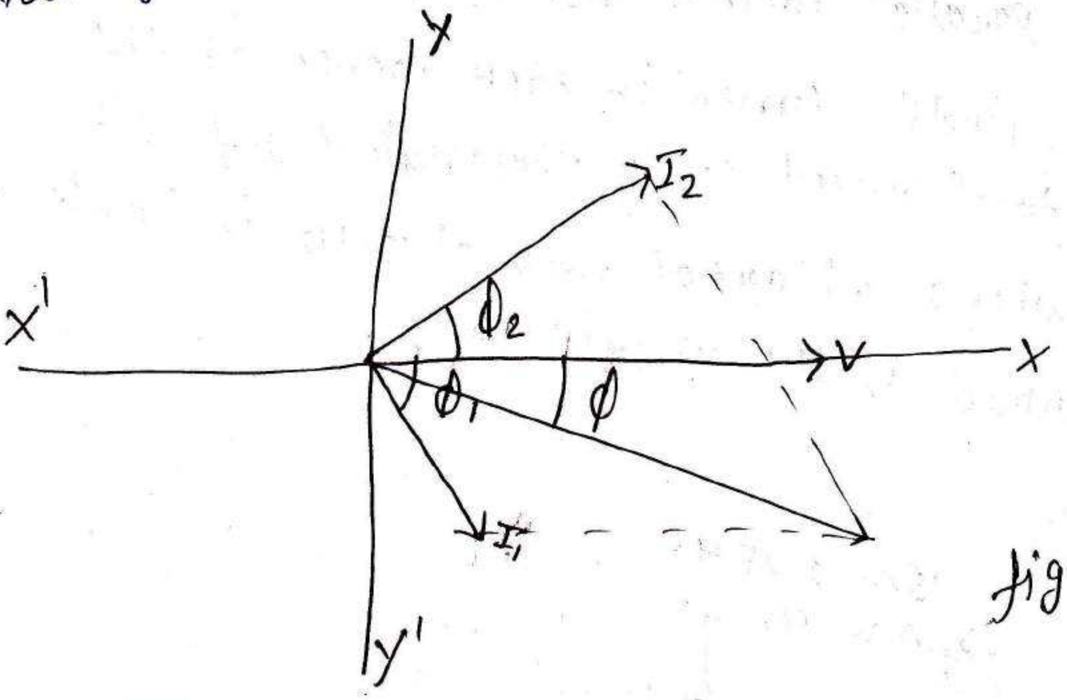



fig. phasor diagram

Step V :-

Find the phasor sum of branch currents by the Method of Components

$$I_{xx} = I_1 \cos\phi_1 + I_2 \cos\phi_2$$

$$I_{yy} = -I_1 \sin\phi_1 + I_2 \sin\phi_2$$

$$I = \sqrt{(I_{xx})^2 + (I_{yx})^2}$$

Step VI :-

Find the phase angle $\phi$ between the total current I & circuit voltage V.

$$\phi = \tan^{-1}\frac{I_{YY}}{I_{XX}} \quad \cancel{\text{lagging}}$$

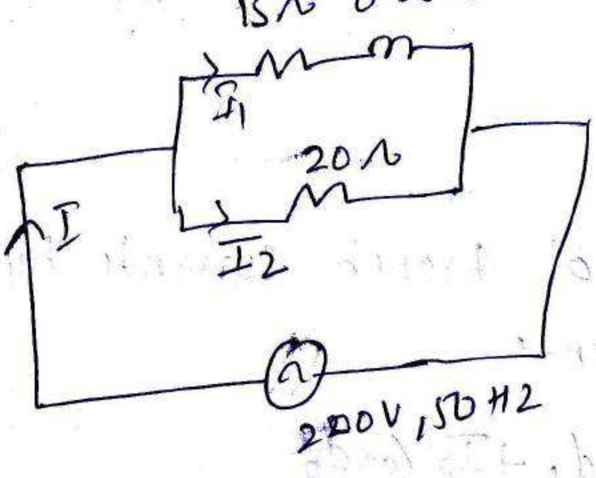
Power factor = $\cos\phi$

$$\cos\phi = \frac{I_{XX}}{I}$$

Q:- A coil of Resistance 15 Ω & Inductance 0.05 H is connected in parallel with a non-Inductive Resistance of ~~2~~ 20Ω. Find (i) current in each branch of the circuit (ii) total current (iii) phase angle & p.f of combination when a voltage of 200 V at 50 Hz is applied in the circuit.

(iv) Power consumed in the circuit.

Solⁿ step-I




$V = 200\,V$ & $f = 50\,Hz$

Step-II

$Z_1 = \sqrt{(15)^2 + (15.7)^2}$

$\left\{ \begin{aligned} X_{L_1} &= 2\pi f L_1 \\ &= 2\pi \times 50 \times 0.05 \\ &= 15.7\,\Omega \\ R_1 &= 15\,\Omega \end{aligned} \right.$

$\boxed{Z_1 = 21.72\,\Omega}$

$\boxed{R_2 = 20\,\Omega}$

Step-III :-

$I_1 = \frac{V}{Z_1} = \frac{200}{21.72}$

$\boxed{I_1 = 9.2\,A}$

$\phi_1 = \tan^{-1}\frac{X_{L_1}}{R_1} = \tan^{-1}\frac{15.7}{15}$

$\boxed{\phi_1 = 46.3^\circ \text{ (lagging)}}$

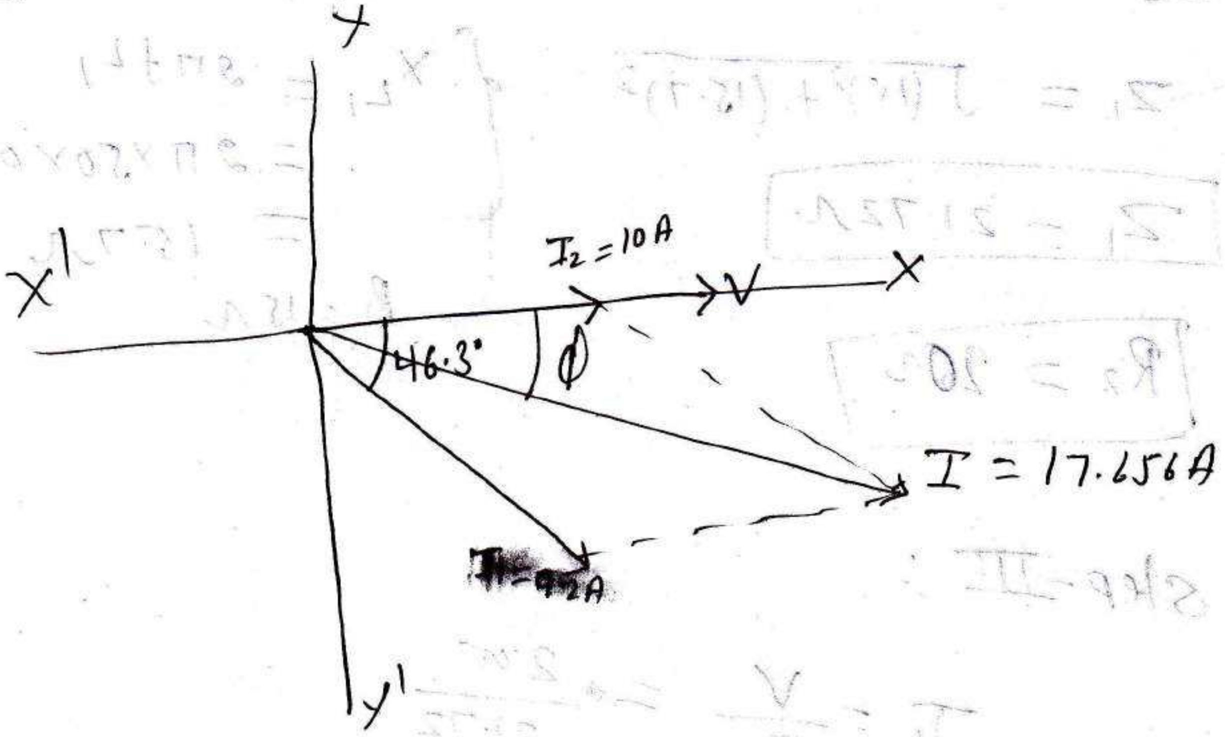
$I_2 = \frac{V}{R_2} = \frac{200}{20}$

$\boxed{I_2 = 10\,A}$

$\phi_2 = \tan^{-1}\frac{X_{L_2}}{R_2} = 0$

$\boxed{\phi_2 = 0}$ ($\because$ $I_2$ is in phase with V bcz load is Resistive)

Step- IV




Step - V

$$I_{xx} = I_1 \cos\phi_1 + I_2 \cos\phi_2$$

$$\cancel{= I_1 \cos 0° + I}$$

$$I_{xx} = 9.2 \cos 46.3° + 10 \cos 0°$$

$$\boxed{I_{xx} = 16.356A}$$

$$I_{yy} = -I_1 \sin\phi_1 + I_2 \sin\phi_2$$

$$I_{xy} = -9.2 \sin 46.3° + I_2 \sin 0°$$

$$I_{xy} = -6.65A$$

$$I = \sqrt{I_{xx}^2 + I_{xy}^2} = 17.656A$$

$$\boxed{I = 17.656A}$$

Step - VI

$$\phi = \tan^{-1} \frac{I_{yx}}{I_{xx}}$$

$$\phi = \tan^{-1}\left(-\frac{6.65}{16.356}\right) = -22.126^\circ$$

$$\boxed{\phi = -22.126^\circ}$$

$$\cos\phi = \cos(-22.126^\circ)$$

$$\boxed{\cos\phi = 0.9264 \text{ (lagging)}}$$

$$P = VI\cos\phi$$

$$\boxed{P = 3271.3\,W}$$

## Admittance Method:-

Before applying this method for the solution of parallel AC circuits, ~~the~~ we should be familiar with the following terms.

Admittance:-

The Reciprocal of Impedance of an AC Circuit is called admittance of the circuit.

→ Impedance is the total opposition to the flow of AC in an AC circuit, the admittance is the effective ability of the circuit due to which it allows the AC to flow through it. It is Represented by 'Y'

$$Z = \frac{V}{I} \quad \therefore \quad Y = \frac{I}{V} \quad \left(\because Y = \frac{1}{Z}\right)$$

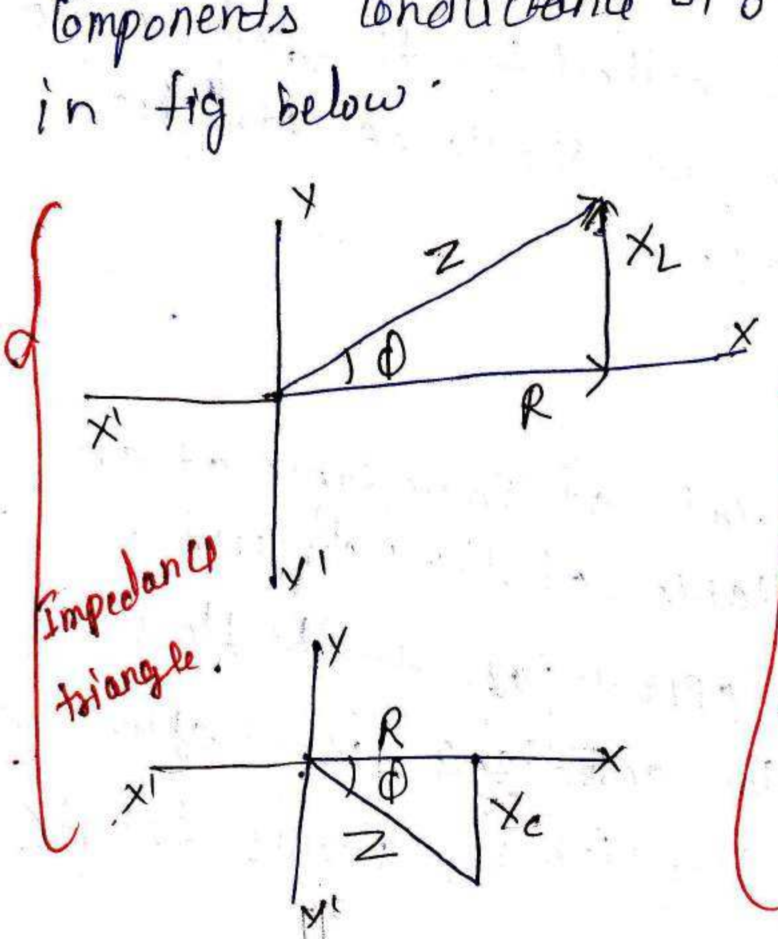
unit → mho ($\mho$)

Admittance triangle:-

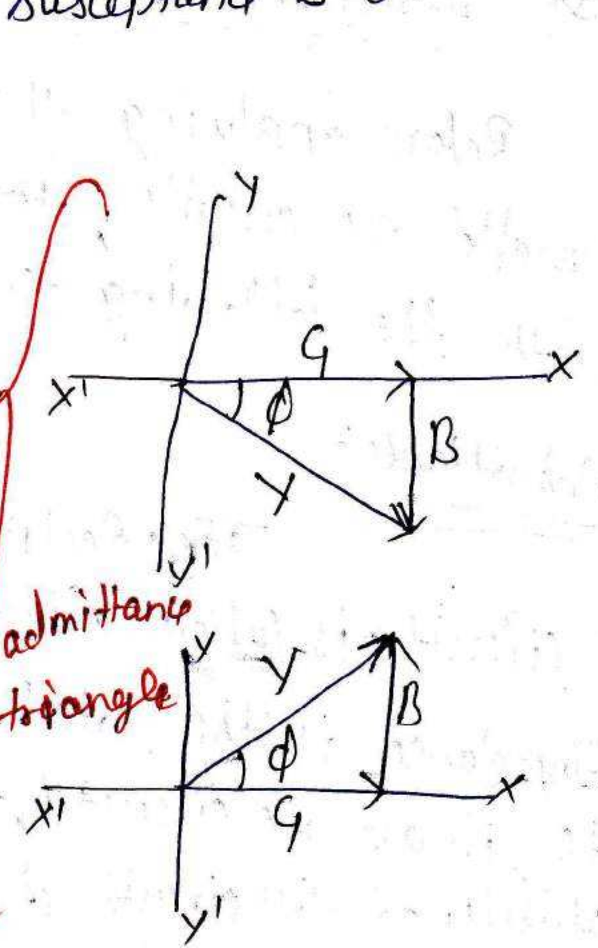
Admittance can also be Represented by a triangle similar to that of impedance.

→ Impedance Z of the circuit has two Rectangular Component Resistance R & Reactance X.

→ Similary, admittance Y also has two Rectangular Components Conductance G & susceptance B as shown in fig below.



Impedance triangle.



admittance triangle

Conductance:-

Represented by G

$$G = \frac{1}{R} \quad \text{unit} \rightarrow \text{mho } \mho$$

Base of an admittance triangle is Representing Conductance

$$G = Y\cos\phi = \frac{1}{Z}\cdot\frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2+X^2}$$

Susceptance:-

Represented by B

$$B = \frac{1}{X}$$

The perpendicular of an admittance is Representing susceptance

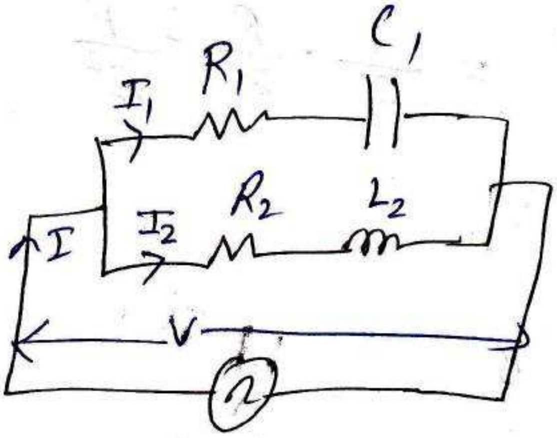
$$B = Y\sin\phi = \frac{1}{Z}\cdot\frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2+X^2}$$

Susceptance is +ve for Capacitive Reactance &
" " -ve " Inductive "

Unit of susceptance → mho ($\mho$)

# Solution of Parallel AC Circuits by admittance Method :-

**Step-I** Draw the circuit as per the given problem. (Here consider the ckt)




**Step-II**

Find the Impedance & phase angle of each branch

$$Z_1 = \sqrt{R_1^2 + X_{C_1}^2} \quad ; \quad \phi_1 = \tan^{-1}\frac{X_{C_1}}{R_1}$$

$$Z_2 = \sqrt{R_2^2 + X_{L_2}^2} \quad ; \quad \phi_2 = \tan^{-1}\frac{X_{L_2}}{R_2}$$

**Step-III** Find conductance, susceptance & admittance of each branch.

$$G_1 = \frac{R_1}{Z_1^2} \quad , \quad B_1 = \frac{X_{C_1}}{Z_1^2} \ (+ve) \ , \quad Y_1 = \sqrt{G_1^2 + B_1^2}$$

$$G_2 = \frac{R_2}{Z_2^2} \quad , \quad B_2 = \frac{X_{L_2}}{Z_2^2} \ (-ve) \ ; \quad Y_2 = \sqrt{G_2^2 + B_2^2}$$

Step- IV

find the algebric sum of conductance & susceptance

$$G = G_1 + G_2 \quad \& \quad B = B_1 - B_2$$

Step- V

find total admittance of the ckt

$$Y = \sqrt{G^2 + B^2}$$

Step- VI

find Branch currents & total currents

$$I_1 = VY_1 \; ; \; I_2 = VY_2 \; ; \; I = VY$$

Step- VII

find the phase angle & the p.f of the whole circuit

$$\phi = \tan^{-1}\frac{B}{G} \quad \text{(lagging if B is -ve)}$$

$$p.f = \cos\phi = \frac{G}{Y}$$

## J-Method:-

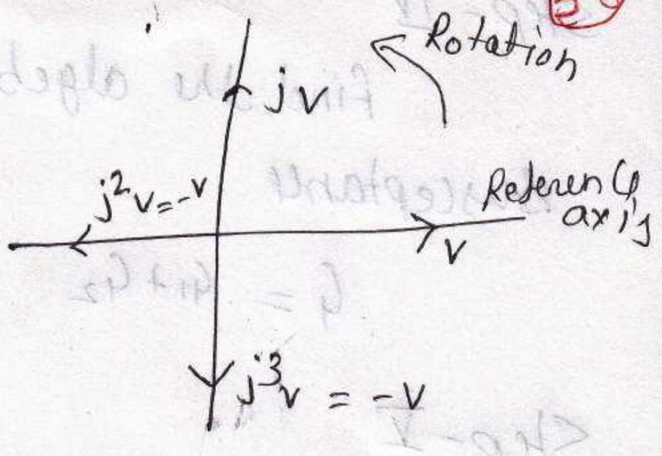


$j = \sqrt{-1}$

$j^2 = -1$

$j^3 = j^2 \cdot j = -\sqrt{-1}$

$j^4 = j^2 \cdot j^2 = +1$




$\bar{V} = a - jb$

$V = \sqrt{a^2+b^2}$ & $\theta = \tan^{-1}(-b/a)$ — Rectangular form

$\bar{V} = V\cos\theta + jV\sin\theta = V(\cos\theta + j\sin\theta)$

$\bar{V} = V\cos\theta - j\sin\theta = V(\cos\theta - j\sin\theta)$ — Trigonometric form

$\bar{V} = V\angle\theta^\circ$

$\bar{V} = V\angle-\theta$ (If $\theta$ is -ve) — Polar form

$\bar{V} = a + jb$

$= \sqrt{a^2+b^2} \angle \tan^{-1}\frac{+b}{a}$

If $\rightarrow$ $5\angle 95^\circ \rightarrow 5(\cos 95^\circ + j\sin 95^\circ)$

---

Note $\Rightarrow$ Do all the Numericals Related to these topics from J.B Gupta.