UNINFORMED SEARCH

-BFS

-DFS

-DFIS

- Bidirectional

- **State-space search** is the process of searching through a state space for a solution by making explicit a sufficient portion of an implicit state-space graph to include a goal node.
  - –Hence, initially V={S}, where S is the start node;
  - –when S is expanded, its successors are generated and those nodes are added to V and the associated arcs are added to E.
  - –This process continues until a goal node is generated (included in V) and identified (by goal test)
- During search, a node can be in one of the three categories:
  - –Not generated yet (has not been made explicit yet)
  - –**OPEN**: generated but not expanded
  - –**CLOSED**: expanded
  - –Search strategies differ mainly on how to select an OPEN node for expansion at each step of search

A General State-Space Search Algorithm


```
open := {S}; closed :={};
repeat
   n := select(open);            /* select one node from open for expansion */
        if n is a goal
            then exit with success;  /* delayed goal testing */
        expand(n)
                /* generate all children of n
                   put these newly generated nodes in open (check duplicates)
                   put n in closed (check duplicates) */
until open = {};
exit with failure
```

Some Issues

- Search process constructs a search tree, where
  - –**root** is the initial state S, and
  - –**leaf nodes** are nodes
    - not yet been expanded (i.e., they are in OPEN list) or
    - having no successors (i.e., they're "deadends")
- Some important issue that arises
  - The direction in which conduct the search(forward vs. backward reasoning)
  - How to select applicable rules(matching).
  - How to represent each node of search process(the knowledge representation problem)
  - Search tree vs. search graph

Evaluating Search Strategies

- **Completeness**
  - –Guarantees finding a solution whenever one exists
- **Time Complexity**
  - –How long (worst or average case) does it take to find a solution? Usually measured in terms of the **number of nodes expanded**
- **Space Complexity**
  - –How much space is used by the algorithm? Usually measured in terms of the **maximum size that the "OPEN" list** becomes during the search
- **Optimality/Admissibility**
  - –If a solution is found, is it guaranteed to be an optimal one? For example, is it the one with minimum cost?

Algorithm : Breadth-First Search

1. Create a variable called *NODE-LIST* and set it to the initial state.
2. Until a goal state is found or *NODE-LIST* is empty:

(a) Remove the first element from *NODE-LIST* and call it *E.* If *NODE-LIST* was empty, quit.

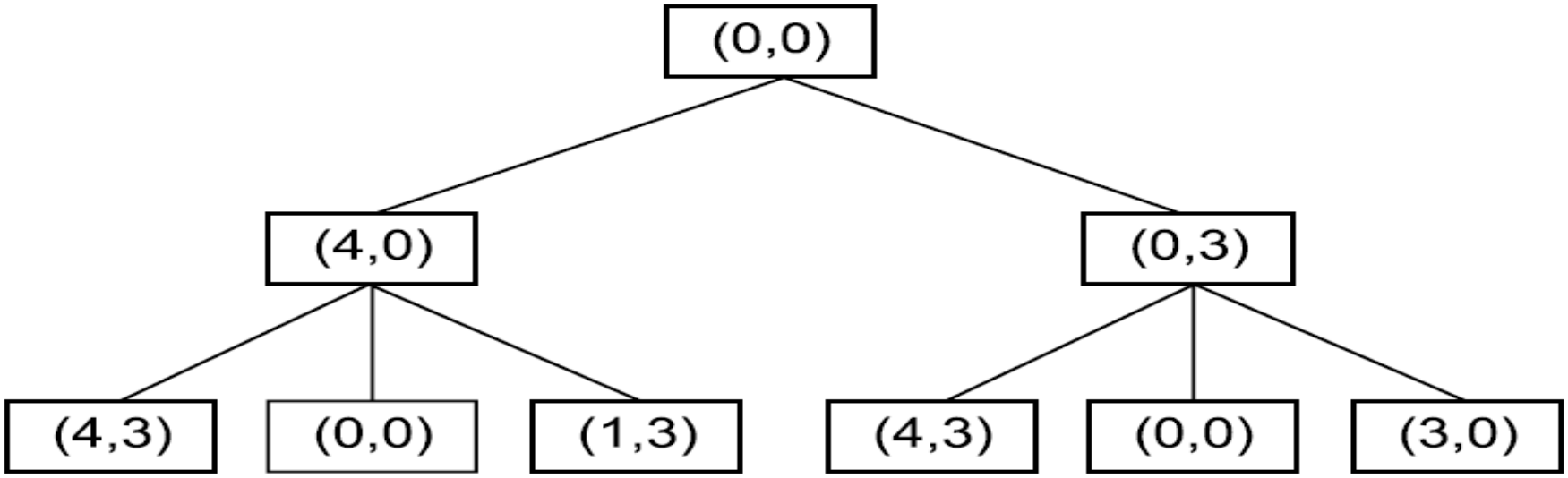
(b) For each way that each rule can match the state described in *E* do:

(i) Apply the rule to generate a new state,

(ii) If the new state is a goal state, quit and return this state.

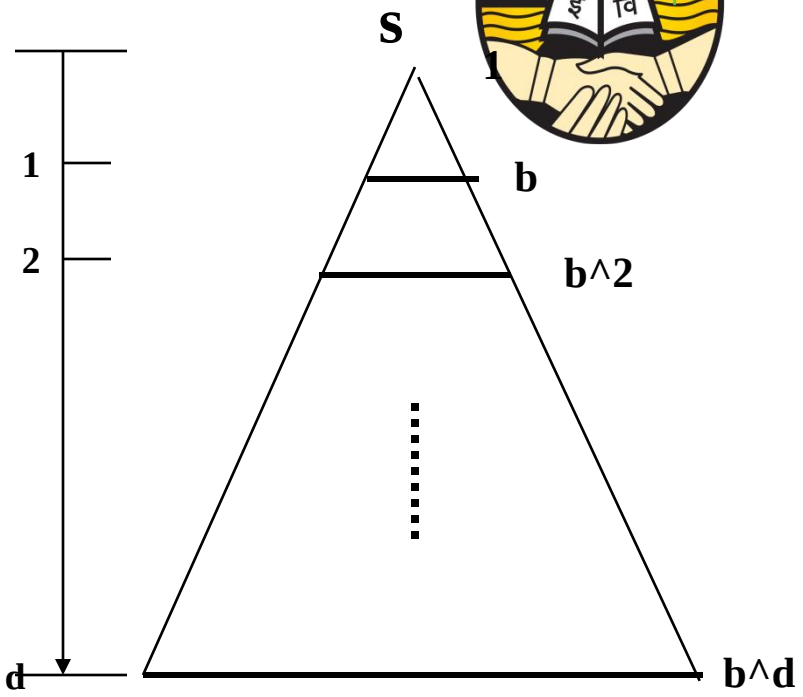
(iii) Otherwise, add the new state to the end of NODE-LIST.

Two Levels of a Breadth-First Search Tree



Breadth-First

- A complete search tree of depth d where each non-leaf node has b children, has a total of 1 + b + b^2 + ... + b^d = (b^(d+1) - 1)/(b-1) nodes
- Time complexity (# of nodes generated): O(b^d)
- Space complexity (maximum length of OPEN): O(b^d)



– For a complete search tree of depth 12, where every node at depths 0, ..., 11 has 10 children and every node at depth 12 has 0 children, there are 1 + 10 + 100 + 1000 + ... + 10^12 = (10^13 - 1)/9 = O(10^12) nodes in the complete search tree.

- BFS is suitable for problems with shallow solutions

Algorithm : Depth-First Search

1. If the initial state is a goal state, quit and return success.

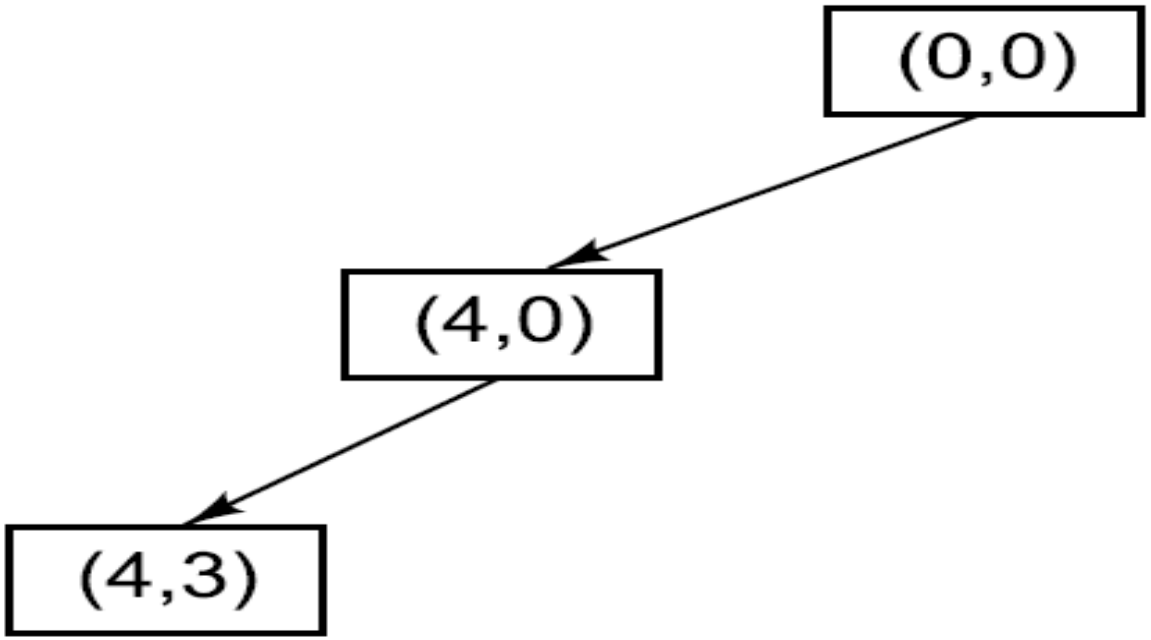
2. Otherwise, do the following until success or failure is signaled:

(a) Generate a successor, *E,* of the initial state. If there are no more successors, signal failure.

(b) Call Depth-First Search with *E* as the initial state.

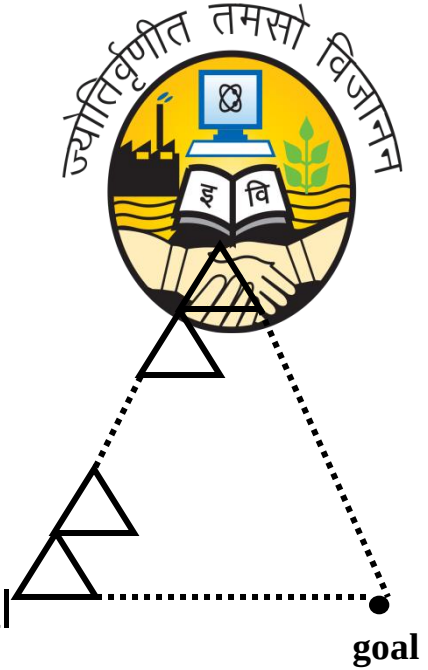
(c) If success is returned, signal success. Otherwise continue in this loop.

A Depth-First Search Tree



Depth-First (DFS)

- Algorithm outline:
  - Always select from the OPEN the node with the greatest depth for expansion, and put all newly generated nodes into OPEN
  - OPEN is organized as **LIFO** (last-in, first-out) list.
  - Terminate if a node selected for expansion is a goal



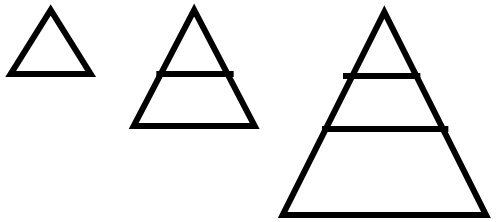
- **May not terminate** without a "depth bound," i.e., cutting off search below a fixed depth D (How to determine the depth bound?)
- **Not complete** (with or without cycle detection, and with or without a cutoff depth)
- **Exponential time**, O(b^d), but only **linear space**, O(bd), required
- Can find **deep solutions quickly** if lucky
- When search hits a deadend, can only back up one level at a time even if the "problem" occurs because of a bad operator choice near the top of the tree. Hence, only does "chronological backtracking"

Depth-First Iterative Deepening (DFID)

- BF and DF both have exponential time complexity O(b^d)
  BF is **complete** but has exponential space complexity
  DF has **linear space complexity** but is incomplete
- Space is often a **harder** resource constraint than time
- Can we have an algorithm that
  - Is complete
  - Has linear space complexity, and
  - Has time complexity of O(b^d)
- DFID by Korf in 1985 (17 years after A*)

  First do DFS to depth 0 (i.e., treat start node as having no successors), then, if no solution found, do DFS to depth 1, etc.

  until solution found do
  DFS with depth bound d
  d = d+1



Depth-First Iterative Deepening (DFID)

- **Complete** (iteratively generate all nodes up to depth d)
- **Optimal/Admissible** if all operators have the same cost. Otherwise, not optimal but does guarantee finding solution of shortest length (like BF).
- **Linear space complexity:** O(bd), (like DF)
- **Time complexity** is a little worse than BFS or DFS because nodes near the top of the search tree are generated multiple times, but because almost all of the nodes are near the bottom of a tree, the worst case time complexity is still exponential, O(b^d)

Depth-First Iterative Deepening

- If branching factor is b and solution is at depth d, then nodes at depth d are generated once, nodes at depth d-1 are generated twice, etc., and node at depth 1 is generated d times.

  Hence

  total(d) = b^d + 2b^(d-1) + ... + db

  <= b^d / (1 - 1/b)^2 = O(b^d).

  – If b=4, then worst case is 1.78 * 4^d, I.e., 78% more nodes searched than exist at depth d (in the worst case).

Bidirectional Search

- **Bidirectional search** is a graph search algorithm that finds a shortest path from an initial vertex to a goal vertex in a directed graph.
- It runs two simultaneous searches: one forward from the initial state, and one backward from the goal, stopping when the two meet in the middle.
- problem complexity in which both searches expand a tree with branching factor b, and the distance from start to goal is d, each of the two searches has complexity $O(b^{d/2})$, and the sum of these two search times is much less than the $O(b^d)$ complexity that would result from a single search from the beginning to the goal.

Thank You !!!

Heuristic search

ARD203/ARI203

Heuristic search

- Generate-and-test
- Hill climbing
- Best-first search
- Problem reduction
- Constraint satisfaction
- Means-ends analysis

Algorithm : Generate-and-Test

1. Generate a possible solution.
2. Test to see if this is actually a solution by comparing the chose point or the endpoint of the chosen path to the set of acceptable goal states.
3. If a solution has been found, quit. Otherwise, return to step 1.

GENERATE-AND-TEST

- Acceptable for simple problems.
  - Eg : 1. finding key of a 3 digit lock.
    2. 8-puzzle problem
- Inefficient for problems with large space.
- Use DFS as all possible solution generated, before they can be tested.

GENERATE-AND-TEST

- Generate solution randomly: British museum algorithm; wandering randomly.
- Exhaustive generate-and-test. : consider each case in depth
- Heuristic generate-and-test: not consider paths that seem unlikely to lead to a solution.
- Plan generate-test:
  - Create a list of candidates.
  - Apply generate-and-test to that list on the basis of constraint-satisfaction.

Ex – DENDERAL, which infers the structure of organic compounds using mass spectrogram and nuclear magnetic resonance (NMR) data.

HILL CLIMBING

- Generate-and-test + direction to move (feedback from test procedure).
- Test function + heuristic function = Hill Climbing
- Heuristic function (objective function) to estimate how close a given state is to a goal state.
- Hill climbing is often used when a good heuristic function is available for evaluating states but when no other useful knowledge is available.

SIMPLE HILL CLIMBING

- Evaluation function as a way to inject task-specific knowledge into the control process.

- Key difference between Simple Hill climbing and Generate-and-test is the use of evaluation function as a way to inject task specific knowledge into the control process.

- Better : higher value of heuristic function

  Lower value

Algorithm : Simple Hill-Climbing

1. Evaluate the initial state. If it is also a goal state, then return it and quit. Otherwise, continue with the initial state as the current state.

2. Loop until a solution is found or until there are no new operators left to be applied in the current state:

(a) Select an operator that has not yet been applied to the current state and apply it to produce a new state.

(b) Evaluate the new state.

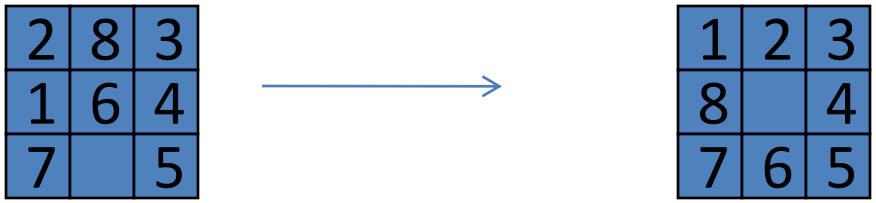
(i) If it is a goal state, then return it and quit.

(ii) If it is not a goal state but it is <u>better</u> than the current state, then make it the current state.

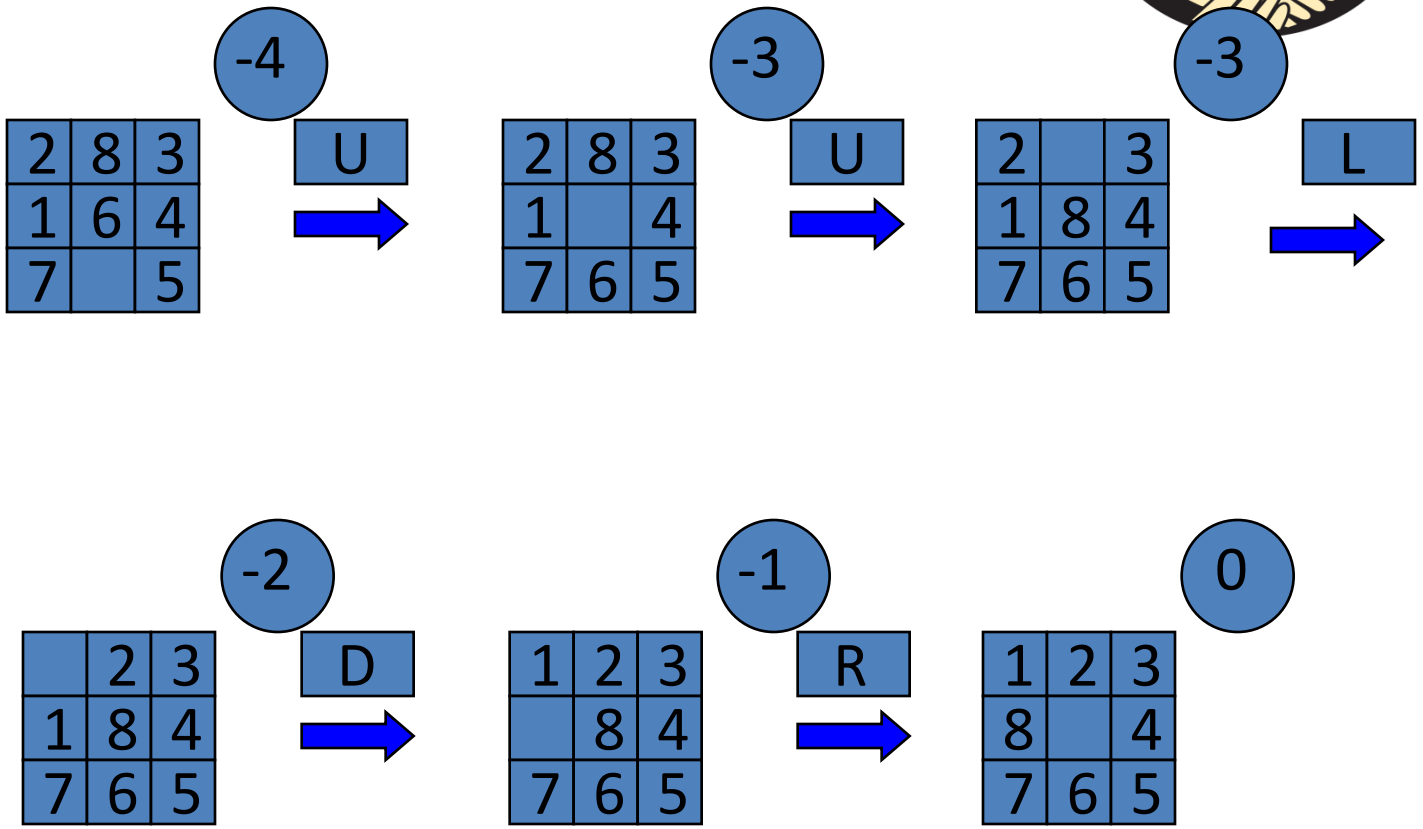
(iii) If it is not better than the current state, then continue in the loop.

EX

1) Use heuristic function as measure of how far off the number of tiles out of place.
2) Choose rule giving best increase in function.



Example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-1
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8 4
7 6 5
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STEEPEST-ASCENT HILL CLIMBING

- Considers all the moves from the current state.
- Selects the best one as the next state.
- Also known as Gradient Search.

Algorithm : Steepest-Ascent Hill Climbing or gradient search

1. Evaluate the initial state. If it is also a goal state, then return it and quit. Otherwise, continue with the initial state as the current state.
2. 2. Loop until a solution is found or until a complete iteration produces no change to current state:

(a) Let *SUCC* be a state such that any possible successor of the current state will be better than *SUCC.*

(b) For each operator that applies to the current state do:

(i) Apply the operator and generate a new state.

(ii) Evaluate the new state. If it is a goal state, then return it and quit. If not, compare it to *SUCC.* If it is better, then set *SUCC* to this state. If it is not better, leave *SUCC* alone.

(c) If the SUCC is better than current state, then set current state to SUCC.

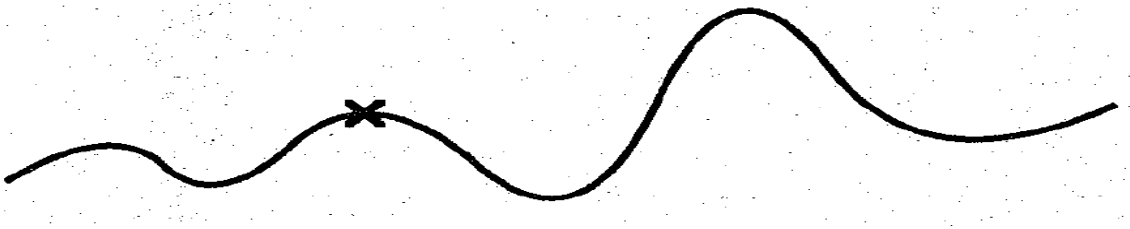
HILL CLIMBING: DISADVANTAGES

- Fail to find a solution
- Either Algorithm may terminate not by finding a goal state but by getting to a state from which no better state can be generated.

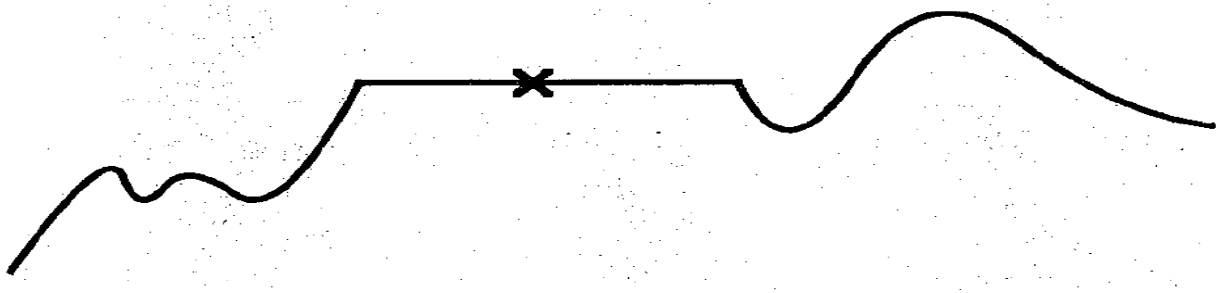
- This happen if program reached
  - Local maximum: A state that is better than all of its neighbours, but not better than some other states far away.
  - Plateau: A flat area of the search space in which all neighbouring states have the same value.
  - Ridge: Special kind of local maximum.
    The orientation of the high region, compared to the set of available moves, makes it impossible to climb up.

Hill-Climbing Dangers

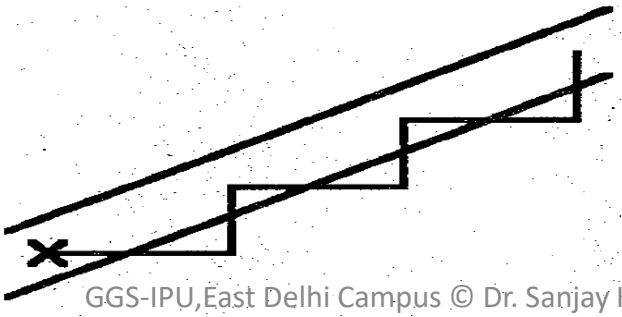
- **Local maximum**



- **Plateau**



- **Ridge**



HILL CLIMBING: DISADVANTAGES

Ways Out

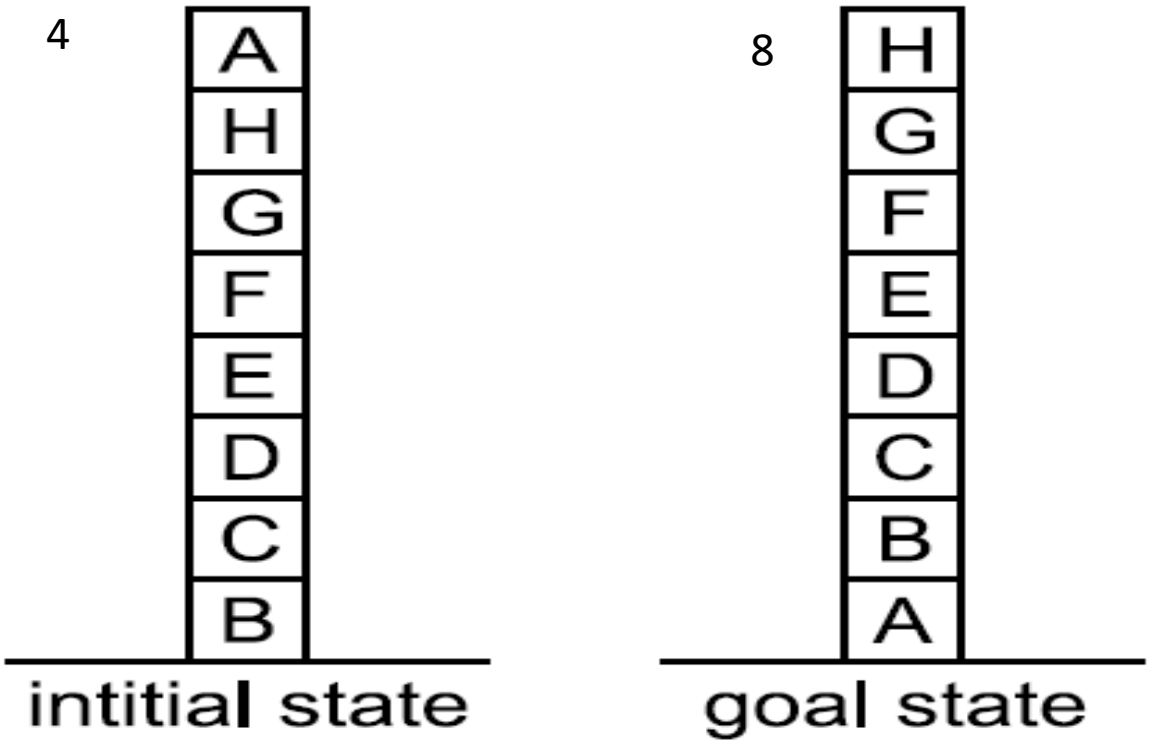
- Backtrack to some earlier node and try going in a different direction. (good way in dealing with local maxima)
- Make a big jump to try to get in a new section. (good way in dealing with plateaus)
- Moving in several directions at once. (good strategy for dealing with ridges)

HILL CLIMBING: DISADVANTAGES

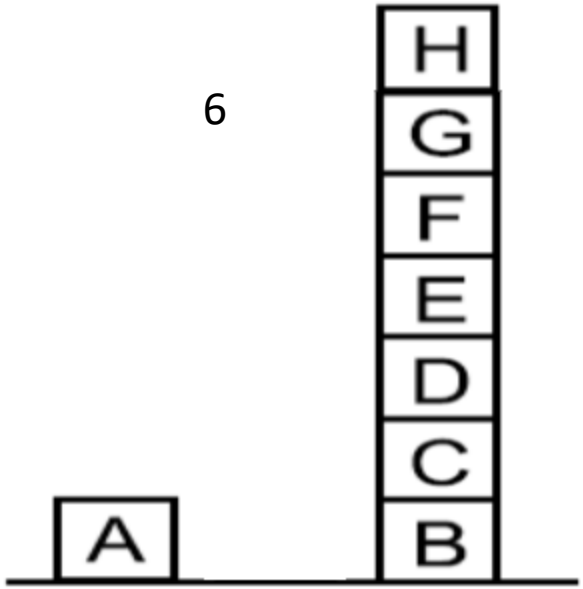
- Hill climbing is a local method:
  Decides what to do next by looking only at the "immediate" consequences of its choices rather than by exhaustively exploring all the consequences.
- Global information might be encoded in heuristic functions.

A Hill-Climbing Problem

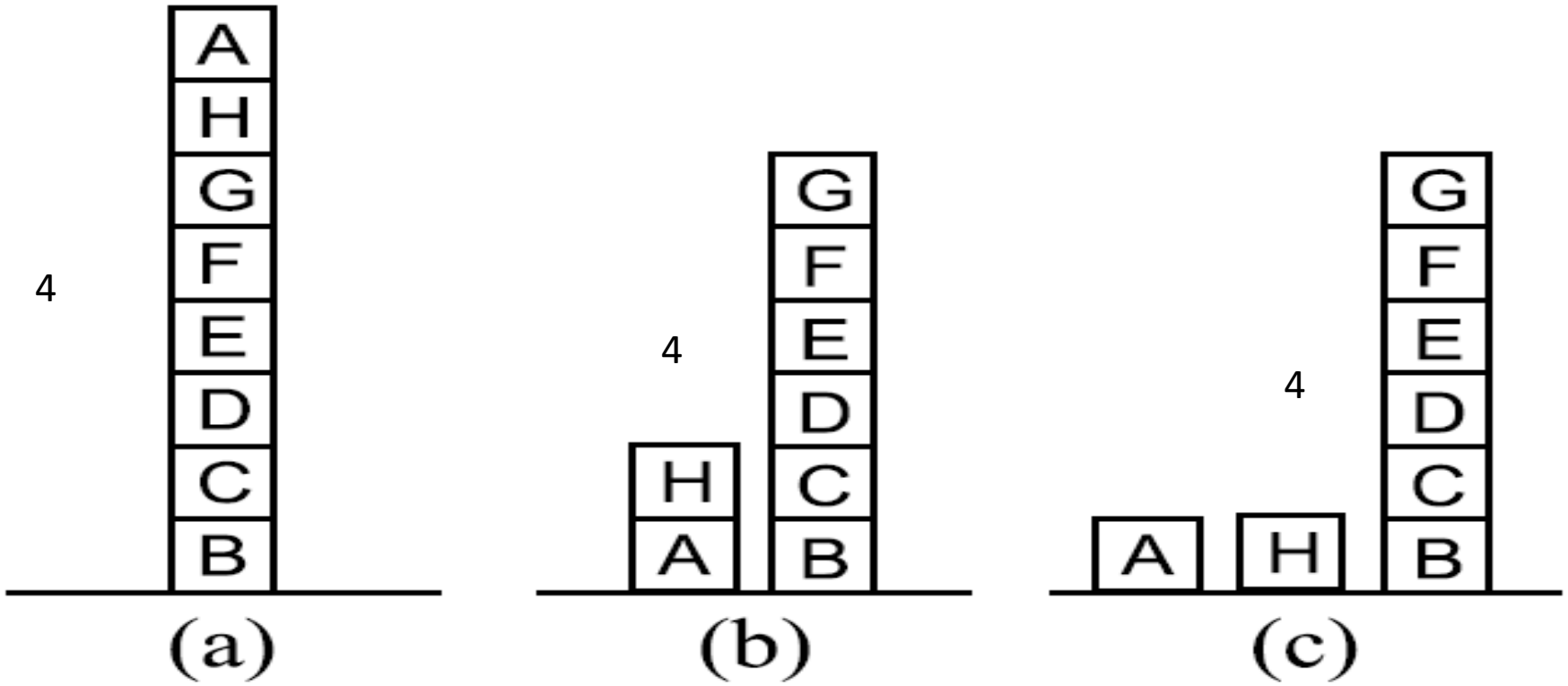
Local: Add one point for every block that is resting on thing it is supposed to be resting on.
Subtract one point from every block that is sitting on wrong thing.



One Possible Moves



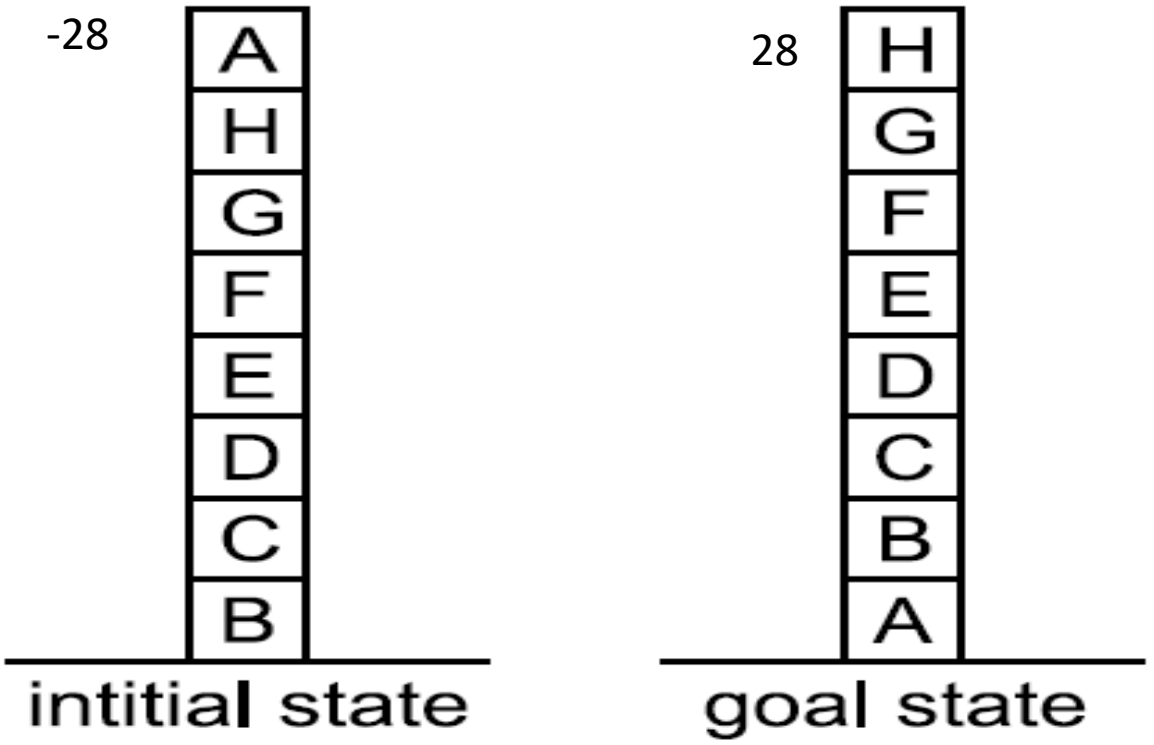
Three Possible Moves



Hill Climbing will Halt because all states have lower score than the Current state.

A Hill-Climbing Problem

Global: Add one point for every block in correct support structure, subtract one point for every block in existing support structure.



One Possible Moves

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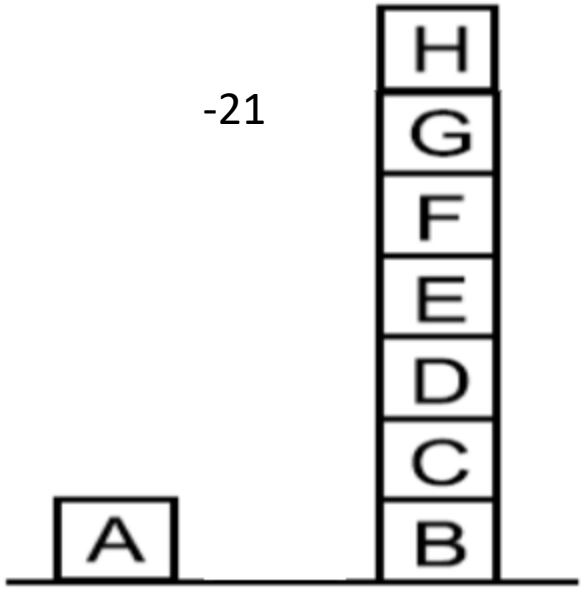
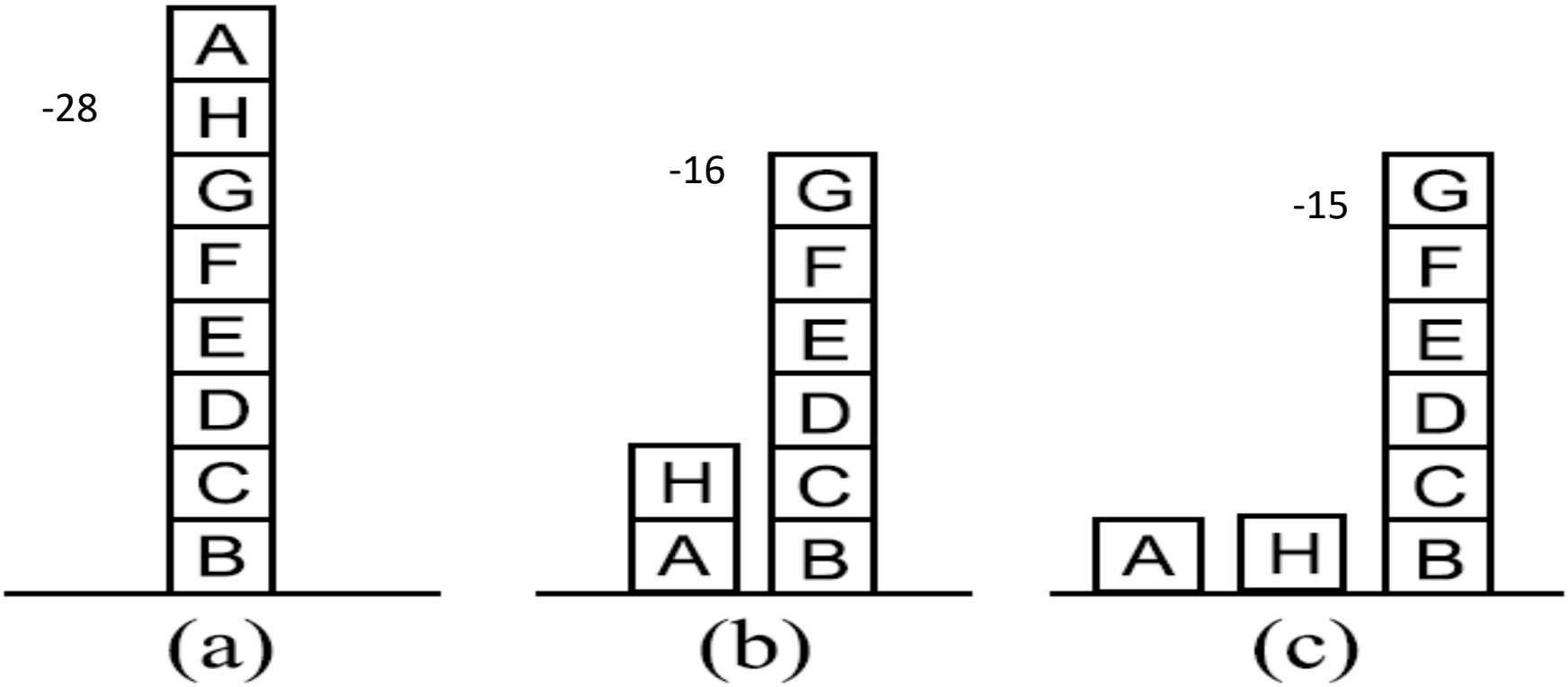


Three Possible Moves



SIMULATED ANNEALING

- A variation of hill climbing in which, at the beginning of the process, some downhill moves may be made.
- To do enough exploration of the whole space early on, so that the final solution is relatively insensitive to the starting state.
- Lowering the chances of getting caught at a local maximum, or plateau, or a ridge.
- The term **simulated annealing** derives from the roughly analogous physical process of **heating** and then **slowly cooling** a substance to obtain a strong crystalline structure.
    - The simulated annealing process lowers the temperature by slow stages until the system ``freezes'' and no further changes occur.

SIMULATED ANNEALING

- Probability of transition to higher energy state is given by function:
  - $P = e^{-\Delta E/kT}$

Where ΔE is the positive change in the energy level

T is the temperature

K is Boltzmann constant.

Suppose k=1,

$P' = e^{-\Delta E/T}$

Annealing schedule: if the temperature is lowered sufficiently slowly, then the goal will be attained.

Algorithm : Simulated Annealing

1. Evaluate the initial state. If it is also a goal state, then return it and quit. Otherwise, continue with the initial state as the current state.
2. Initialize *BEST-SO-FAR* to the current state.
3. Initialize T according to the annealing schedule.
4. Loop until a solution is found or until there are no new operators left to be applied in the current state.
   (a) Select an operator that has not yet been applied to the current state and apply it.
   (b) Evaluate the new state. Compute

$$\Delta E = (\text{value of current}) - (\text{value of new state})$$

   - If the new state is a goal state, return it and quit.
   - If it is not a goal state but is better than the current state, then make it the current state. Also set *BEST-SO-FAR* to this new state.
   - If it is not better than the current state, then make it the current state with probability p'.

   (c) Revise T according to the annealing schedule.
5. Return *BEST-SO-FAR* as the answer.

SIMULATE ANNEALING: IMPLEMENTATION

- It is necessary to select an annealing schedule which has three components:
  - Initial value to be used for temperature
  - Criteria that will be used to decide when the temperature will be reduced
  - Amount by which the temperature will be reduced.

BEST-FIRST SEARCH

- Combines the advantages of both DFS and BFS into a single method.

- Depth-first search: not all competing branches having to be expanded.

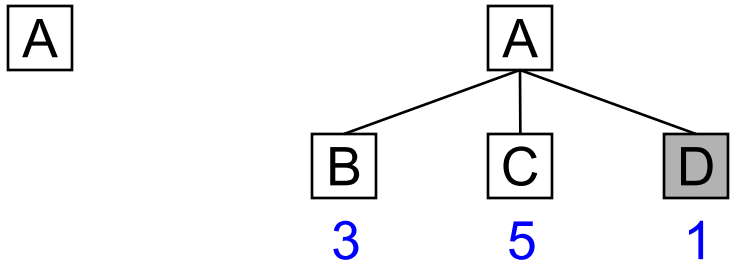
- Breadth-first search: not getting trapped on dead-end paths.

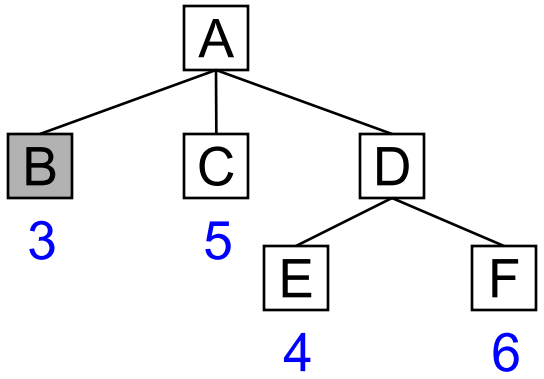
  ⇒ Combining the two is to follow a single path at a time, but switch paths whenever some competing path look more promising than the current one.

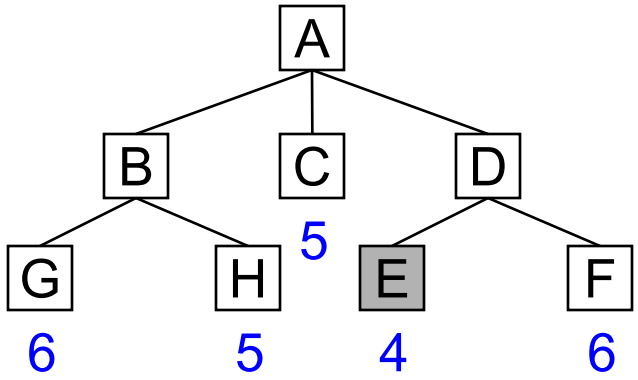
BEST-FIRST SEARCH

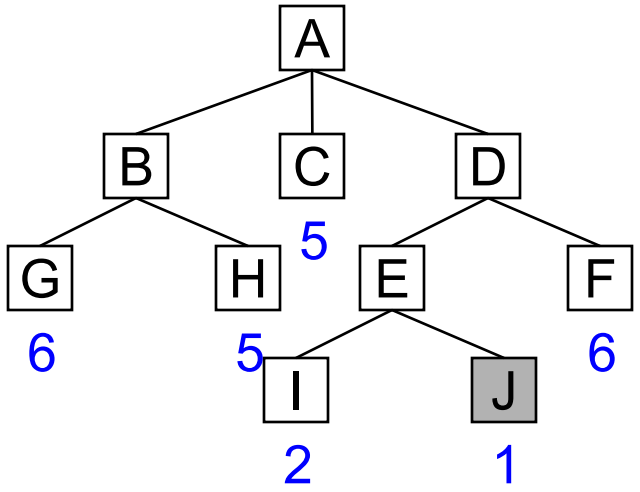
- At each step of the BFS search process, we select the most promising of the nodes we have generated so far.
- This is done by applying an appropriate heuristic function to each of them.
- We then expand the chosen node by using the rules to generate its successors
- This is called OR-graph, since each of its branches represents an alternative problem solving path

BEST-FIRST SEARCH









BEST FIRST SEARCH VS HILL CLIMBING

- Similar to Steepest ascent hill climbing with two exceptions:

  - In hill climbing, one move is selected and all the others are rejected, never to be reconsidered. In BFS, one move is selected, but the others are kept around so that they can be revisited later if the selected path becomes less promising
  - The best available state is selected in the BFS, even if that state has a value that is lower than the value of the state that was just explored. Whereas in hill climbing the progress stop if there are no better successor nodes.

BEST-FIRST SEARCH

- OPEN: nodes that have been generated, but have not examined.

  This is organized as a priority queue.

- CLOSED: nodes that have already been examined.

  Whenever a new node is generated, check whether it has been generated before.

Algorithm : Best-First Search

1. Start with *OPEN* containing just the initial state.

2. Until a goal is found or there are no nodes left on *OPEN* do:

(a) Pick them best node on *OPEN.*

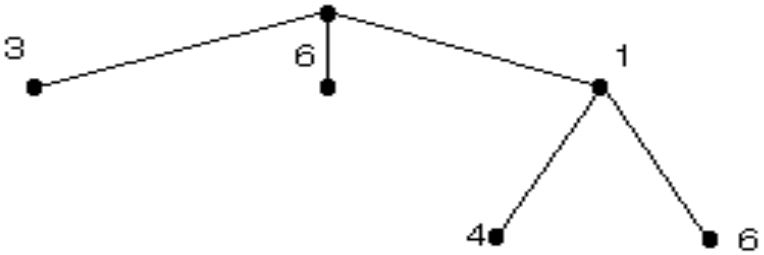
(b) Generate its successors.

(c) For each successor do:

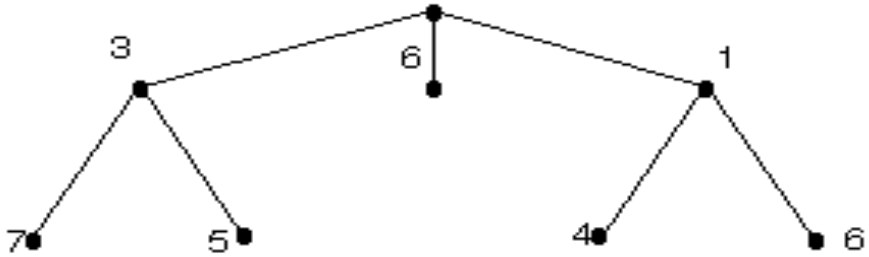
(i) If it has not been generated before, evaluate it, add it to *OPEN,* and record its parent.

(ii) If it has been generated before, change the parent if this new path is better than the previous one. In that case, update the cost of getting to this node and to any successors that this node may already, have.

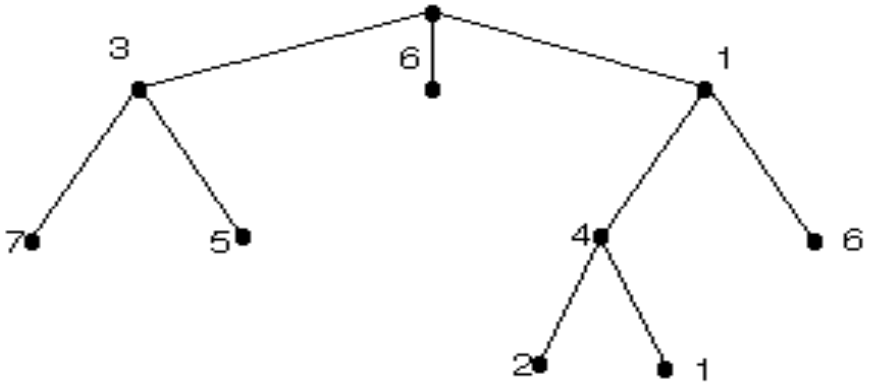
STEP 1



STEP 2



STEP 3



STEP 4

All figures indicate "cost" of move

A* ALGORITHM

- Best First Search is a simplification of A* Algorithm

- Algorithm uses:
  - f': Heuristic function that estimates the merits of each node we generate. This is sum of two components, g and h'
    - f' represents an estimate of the cost of getting from the initial state to a goal state along with the path that generated the current node.
  - g : The function g is a measure of the cost of getting from initial state to the current node.
  - h' : The function h' is an estimate of the additional cost of getting from the current node to a goal state.
  - OPEN
  - CLOSED

ALGORITHM A*

- Algorithm A* (Hart et al., 1968):

$$f(n) = g(n) + h(n)$$

$h(n)$ = cost of the cheapest path from node n to a goal state.

$g(n)$ = cost of the cheapest path from the initial state to node n.

- Algorithm A*:

$$f^*(n) = g^*(n) + h^*(n)$$

$h^*(n)$ (heuristic factor) = estimate of $h(n)$.

$g^*(n)$ (depth factor) = approximation of $g(n)$ found by A* so far.

A* ALGORITHM

1. Start with OPEN containing only initial node. Set that node's g value to 0, its h' value to whatever it is, and its f' value to h'+0 or h'. Set CLOSED to empty list.

2. Until a goal node is found, repeat the following procedure:
    1. If there are no nodes on OPEN, report failure.
    2. Otherwise pick the node on OPEN with the lowest f' value. Call it BESTNODE. Remove it from OPEN. Place it in CLOSED.
        1. See if the BESTNODE is a goal state. If so exit and report a solution.
        2. Otherwise, generate the successors of BESTNODE but do not set the BESTNODE to point to them yet.

A* ALGORITHM (CONTD….)

- For each of the SUCCESSOR, do the following:

a. Set SUCCESSOR to point back to BESTNODE. These backwards links will make it possible to recover the path once a solution is found.

a. Compute g(SUCCESSOR) = g(BESTNODE) + the cost of getting from BESTNODE to SUCCESSOR

a. See if SUCCESSOR is the same as any node on OPEN. If so call the node OLD.

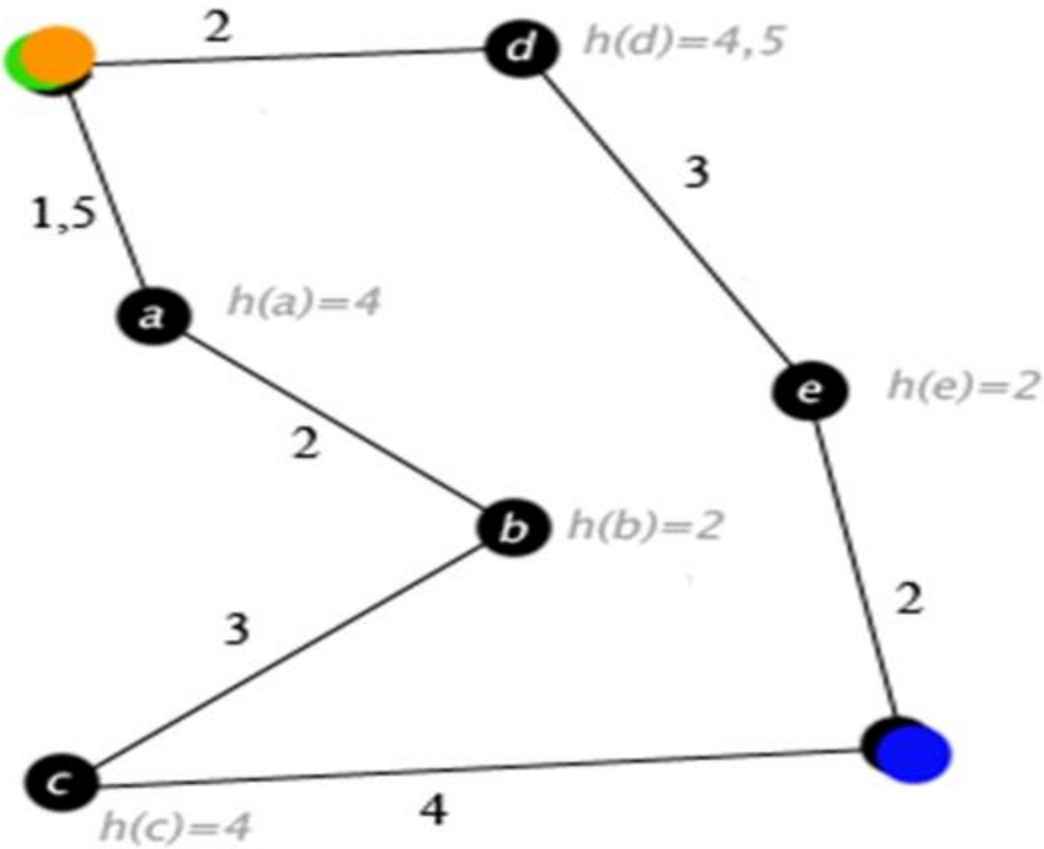
a. If SUCCESSOR was not on OPEN, see if it is on CLOSED. If so, call the node on CLOSED OLD and add OLD to the list of BESTNODE's successors.

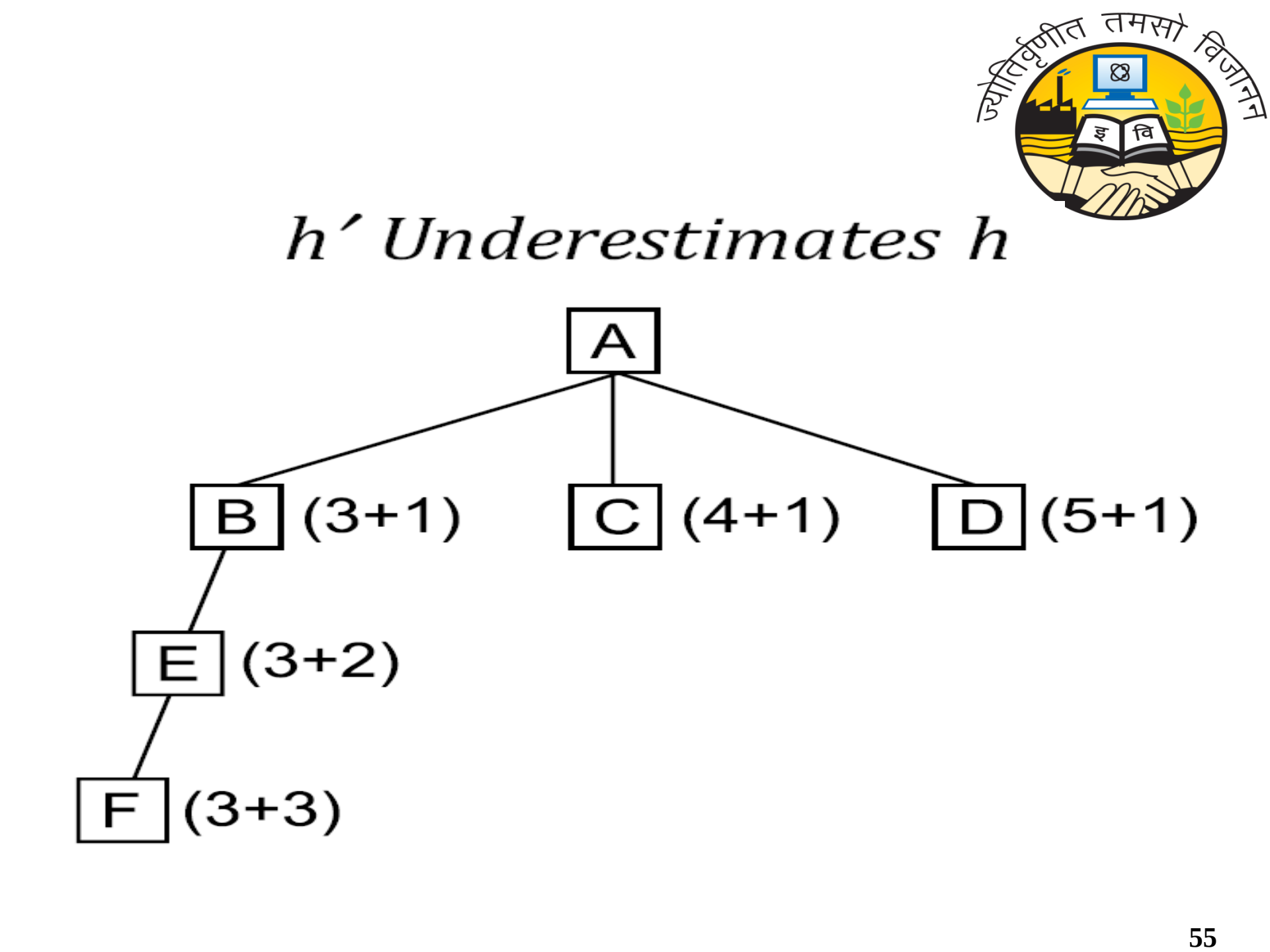
a. If SUCCESSOR was not already on either OPEN or CLOSED, then put it on OPEN and add it to the list of BESTNODE's successors. Compute f'(SUCCESSOR) = g(SUCCESSOR) + h'(SUCCESSOR)

OBSERVATIONS ABOUT A*

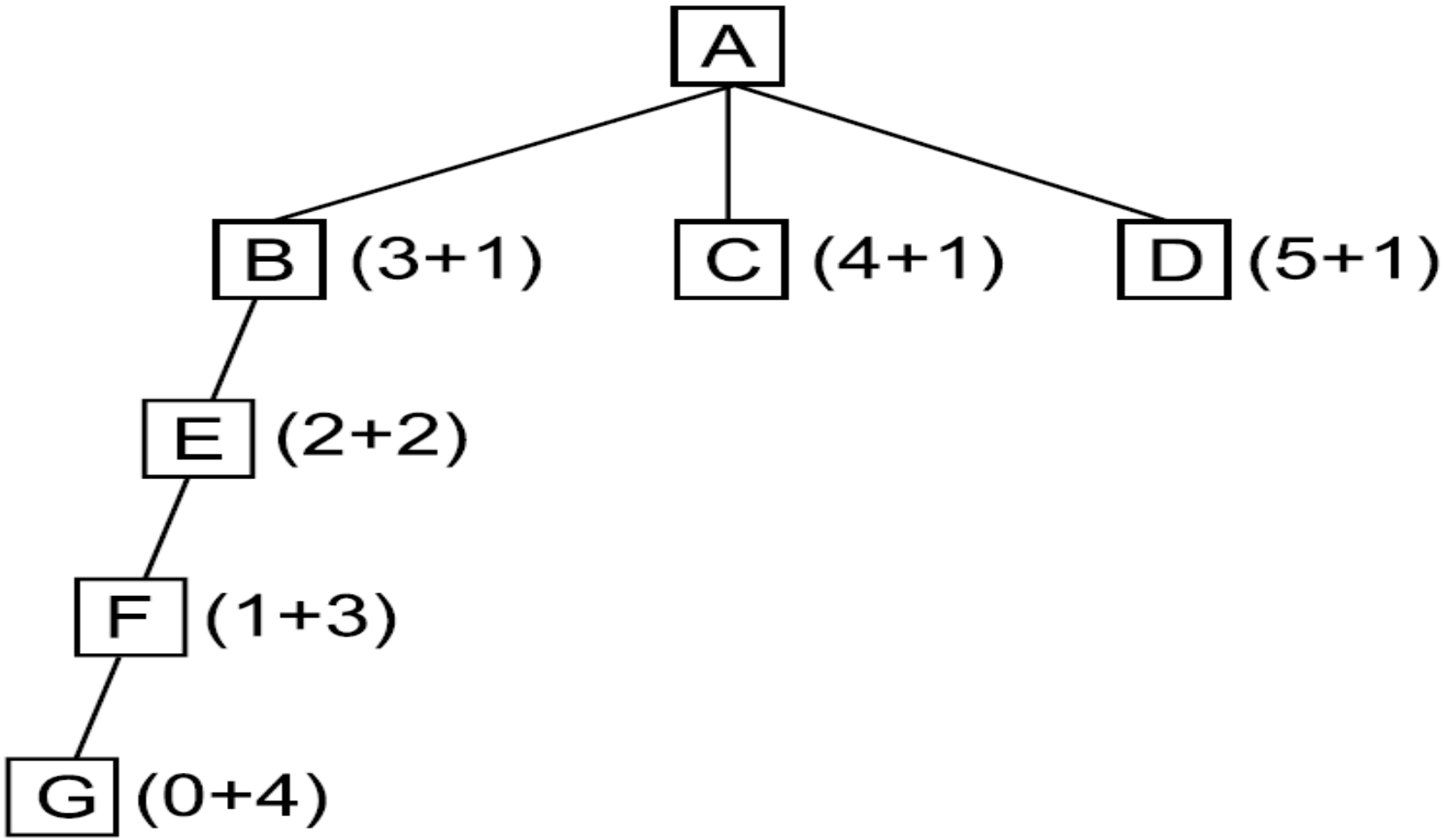
- **Role of g function:** This lets us choose which node to expand next on the basis of not only of how good the node itself looks, but also on the basis of how good the path to the node was.
- **h**, the distance of a node to the goal. If h' is a perfect estimator of h, then A* will converge immediately to the goal with no search.

Q1




h′ Underestimates h
A
B
(3+1)
C
(4+1)
D
(5+1)
E
(3+2)
F
(3+3)

h´ Overestimates h



Algorithm : Agenda-Driven Search

1. Do until a goal state is reached or the agenda is empty:

(a) Choose the most promising task from the agenda. Notice that this task can be represented in any desired form. It can be thought of as an explicit statement of what to do next or simply as an indication of the next node to be expanded.

(b) Execute the task by devoting to it the number of resources determined by its importance. The important resources to consider are time and space. Executing the task will probably generate additional tasks (successor nodes). For each of them, do the following:

(i) See if it is already on the agenda. If so, then see if this same reason for doing it is already on its list of justifications. If so, ignore this current evidence. If this justification was not already present, add it to the list. If the task was not on the agenda, insert it.

(ii) Compute the new task's rating, combining the evidence from all its justifications.

AND-OR Graphs

1. **Initialize the graph to the starting node.**

2. **2. Loop until the starting node is labeled *SOLVED* or until its cost goes above *FUTILITY*:**

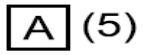
(a) Traverse the graph, starting at the initial node and following the current best path, and accumulate the set of nodes that are on that path and have not yet been expanded or labeled as solved.

(b) Pick one of these unexpanded nodes and expand it. If there are no successors, assign *FUTILITY* as the value of this node. Otherwise, add its successors to the graph and for each of them compute *f ′* (use only *h′* and ignore *g,* for reasons we discuss below). If of any node is 0, mark that node as *SOLVED.*

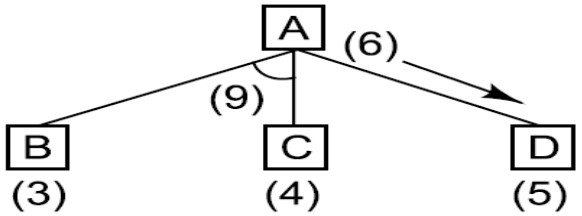
(c) Change the *f ′* estimate of the newly expanded node to reflect the new information provided by its successors. Propagate this change backward through the graph. If any node contains a successor arc whose descendants are all solved, label the node itself as *SOLVED.* At each node that is visited while going up the graph, decide which of its successor arcs is the most promising and mark it as part of the current best path.

The Operation of Problem Reduction

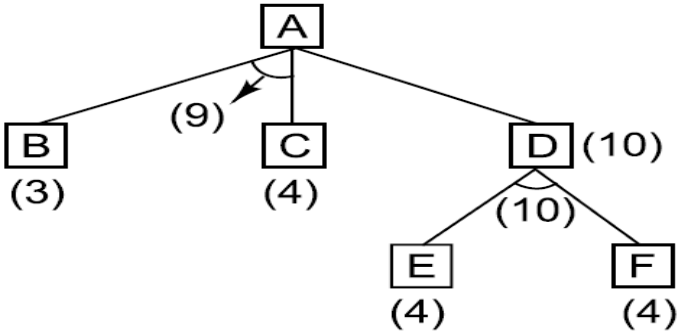
Before step 1



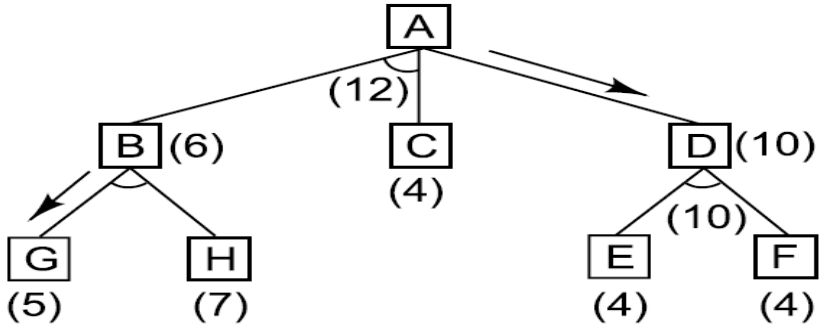
Before step 2



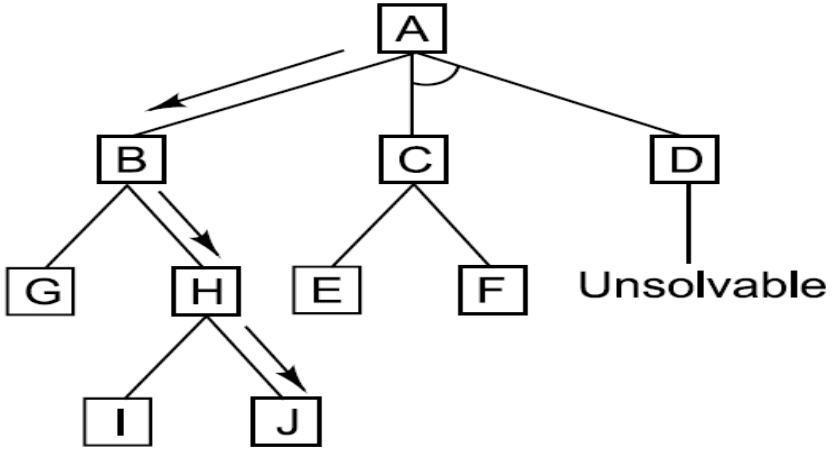
Before step 3

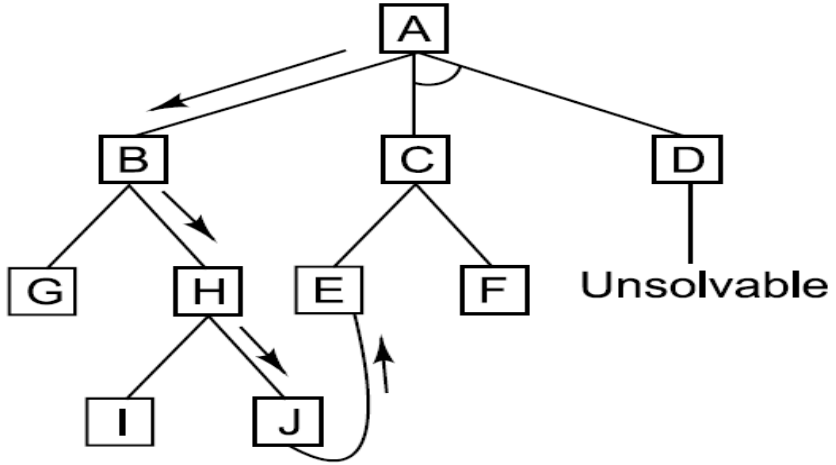


Before step 4

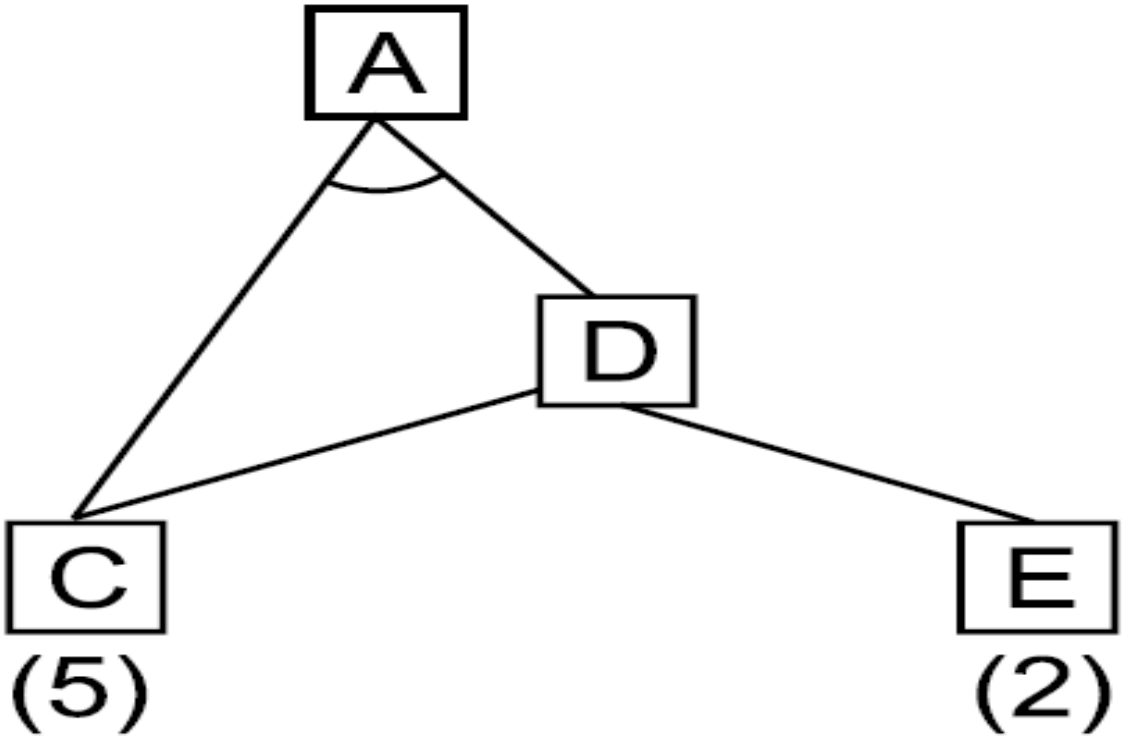


A Longer Path May Be Better





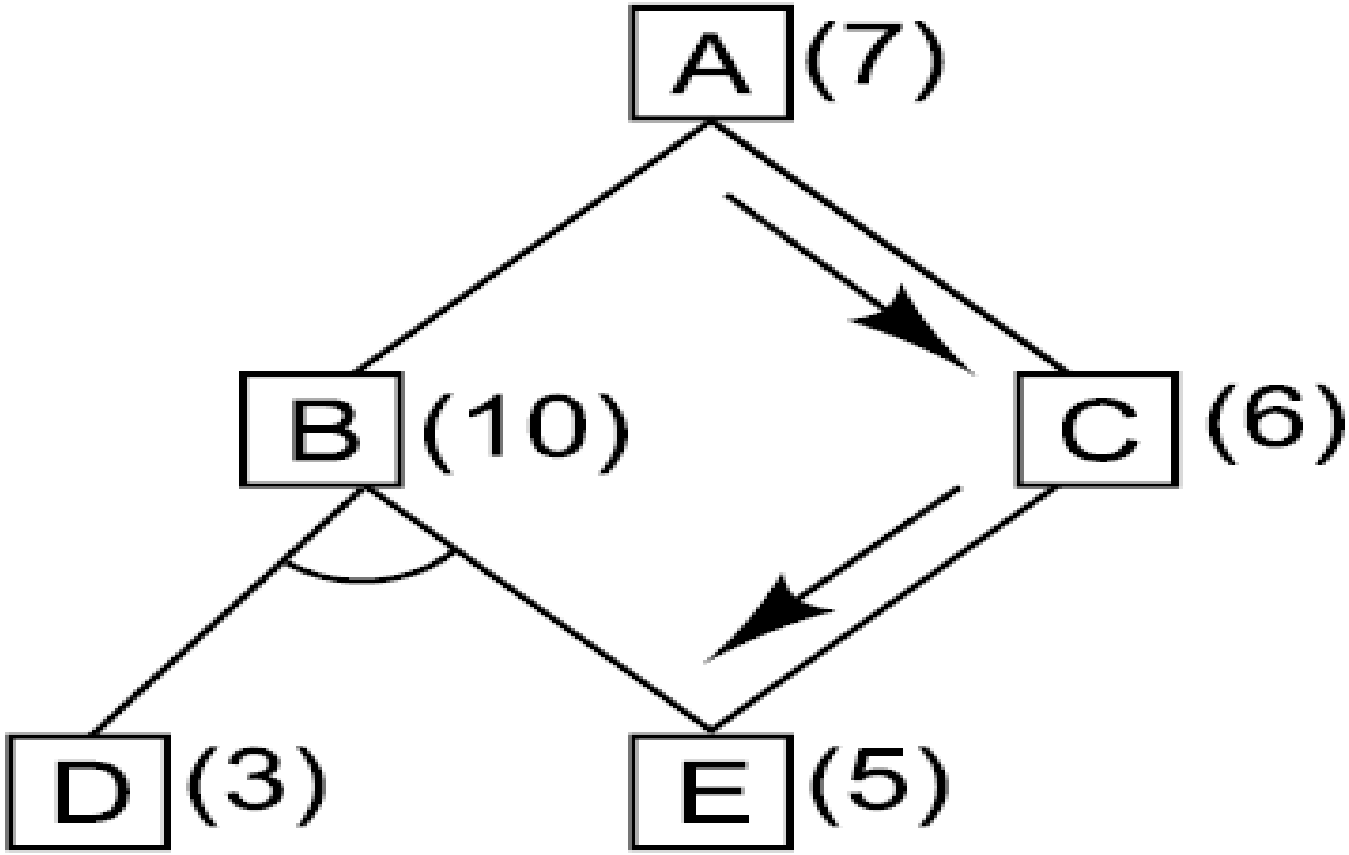
Interacting Subgoals



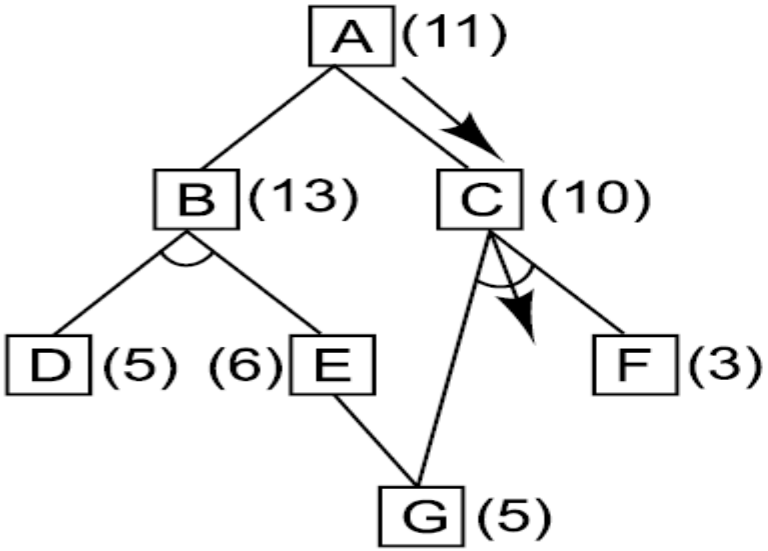
Algorithm: AO*

1. Let *GRAPH* consist only of the node representing the initial state. (Call this node *INIT.)* Compute $h'(INIT)$
2. Until *INIT* is labeled *SOLVED* or until *INIT's* h' value becomes greater than *FUTILITY,* repeat the following procedure:
   (a) Trace the labeled arcs from *INIT* and select for expansion one of the as yet unexpanded nodes that occurs on this path. Call the selected node *NODE.*
   (b) Generate the successors of *NODE.* If there are none, then assign *FUTILITY* as the h' value of *NODE.* This is equivalent to saying that *NODE* is not solvable. If there are successors, then for each one (called *SUCCESSOR)* that is not also an ancestor of *NODE* do the following:
      (i) Add *SUCCESSOR* to *GRAPH.*
      (ii) If *SUCCESSOR* is a terminal node, label it *SOLVED* and assign it an h' value of 0.
      (iii) If *SUCCESSOR* is not a terminal node, compute its h' value.
   (c) Propagate the newly discovered information up the graph by doing the following: Let *S* be a set of nodes that have been labeled *SOLVED* or whose h' values have been changed and so need to have values propagated back to their parents. Initialize 5 to *NODE.* Until *S* is empty, repeat the, following procedure:
      (i) If possible, select from *S* a node none of whose descendants in *GRAPH* occurs in *S.* If there is no such node, select any node from *S.* Call this node *CURRENT,* and remove it from *S.*
      (ii) Compute the cost of each of the arcs emerging from *CURRENT.* The cost of each arc is equal to the sum of the h' values of each of the nodes at the end of the arc plus whatever the cost of the arc itself is. Assign as *CURRENT'S* new h' value the minimum of the costs just computed for the arcs emerging from it.
      (iii) Mark the best path out of *CURRENT* by marking the arc that had the minimum cost as computed in the previous step.
      (iv) Mark *CURRENT SOLVED* if all of the nodes connected to it through the new labeled arc have been labeled *SOLVED.*
      (v) If *CURRENT* has been labeled *SOLVED* or if the cost of *CURRENT* was just changed, then its new status must be propagated back up the graph. So add all of the ancestors of *CURRENT* to *S.*

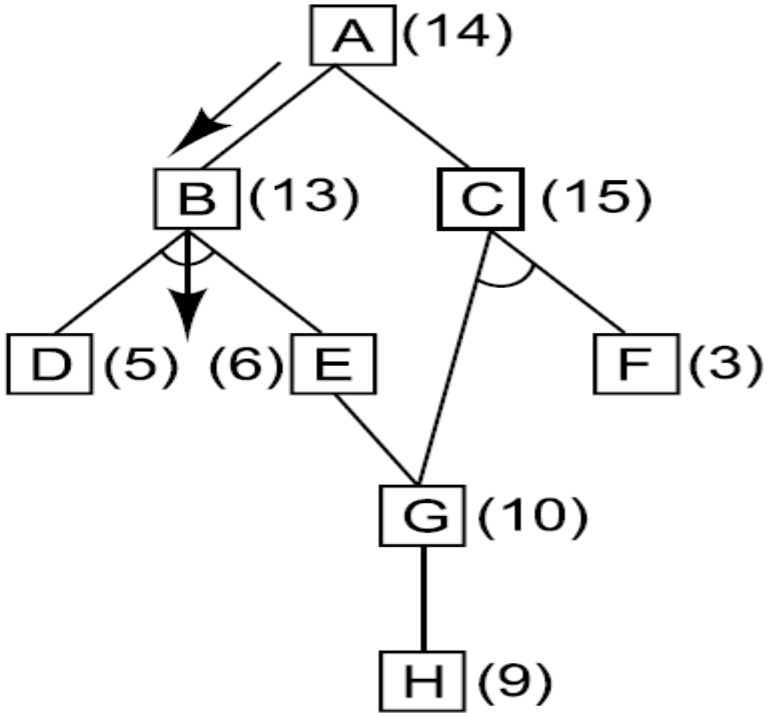
Backward Propagation



A Necessary Backward Propagation



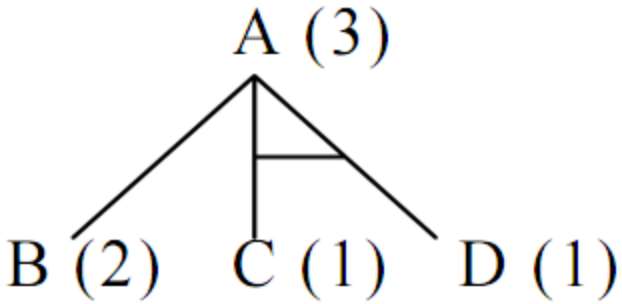
(a)



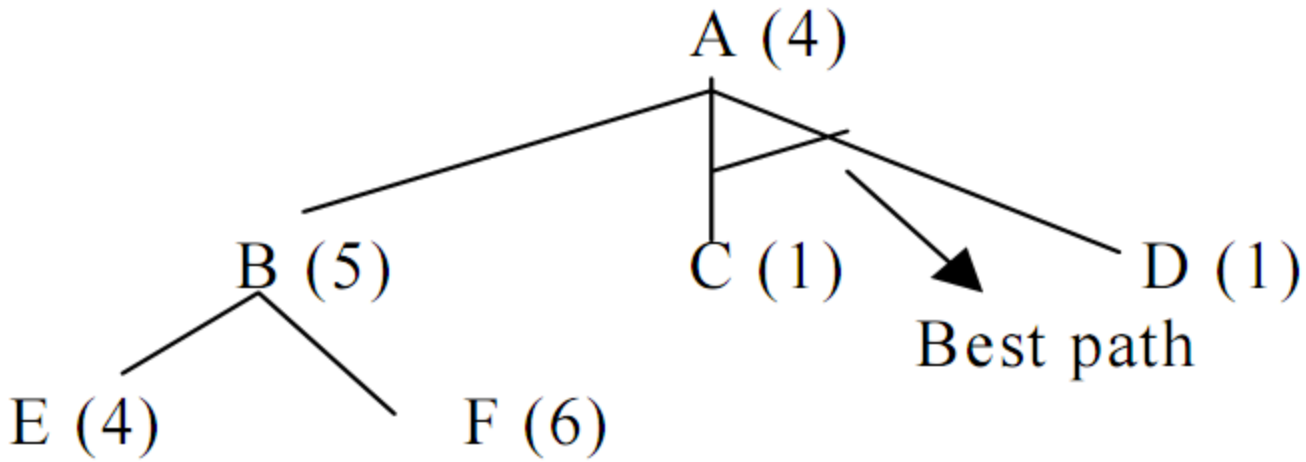
(b)

Example: Consider the following example

1. After one cycle

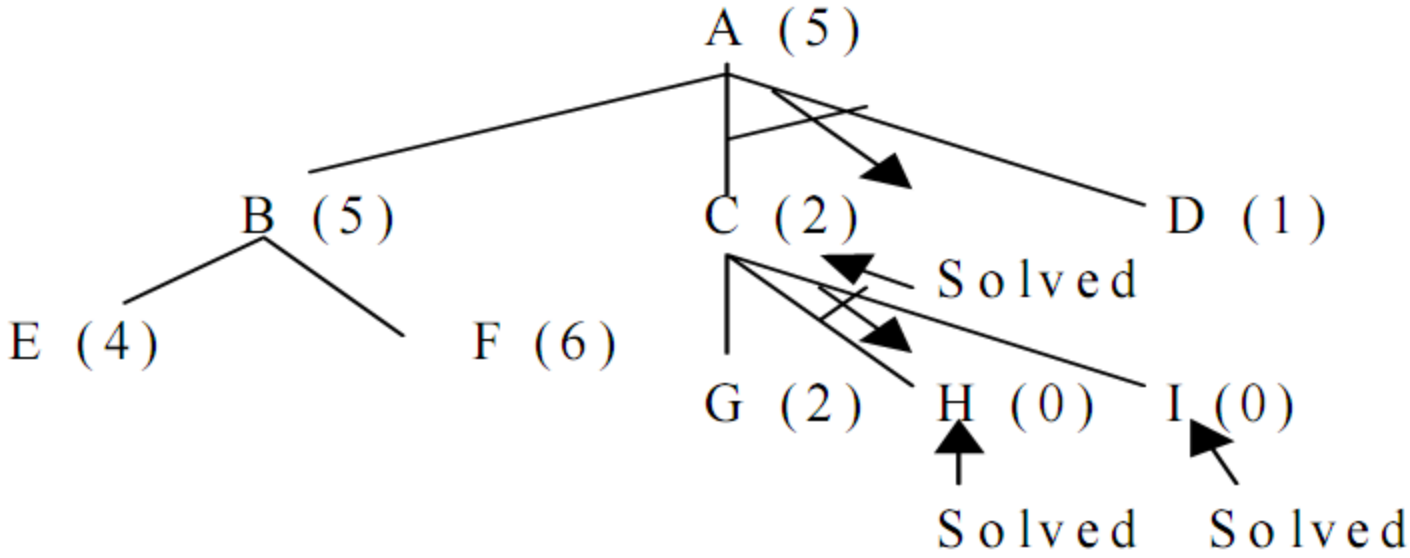


2. After two cycle



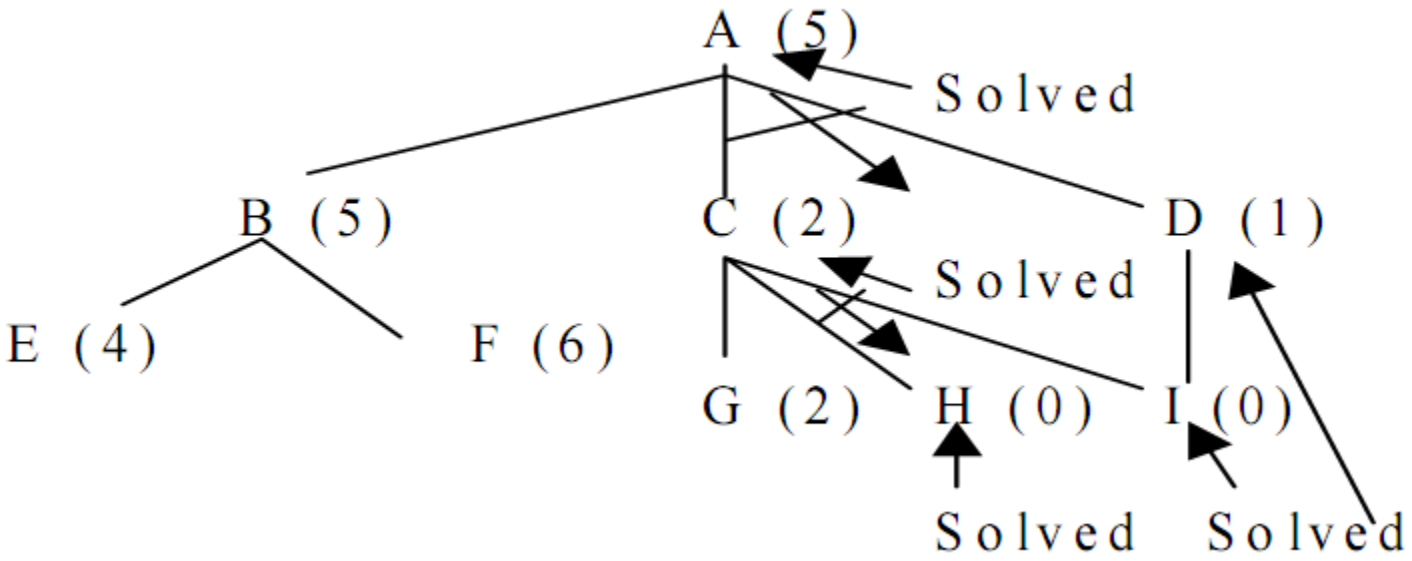
Example: Consider the following example

3. After three cycle



Example: Consider the following example

4. After four cycle



Algorithm : Constraint Satisfaction

1. Propagate available constraints. To do this, first set *OPEN* to the set of all objects that must have values assigned to them in a complete solution. Then do until an inconsistency is detected or until *OPEN* is empty:

(a) Select an object *OB* from *OPEN.* Strengthen as much as possible the set of constraints that apply to *OB.*

(b) If this set is different from the set that was assigned the last time *OB* was examined or if this is the first time *OB* has been examined, then add to *OPEN* all objects that share any constraints with *OB.*

(c) Remove *OB* from *OPEN.*

2. If the union of the constraints discovered above defines a solution, then quit and report the solution.

3. If the union of the constraints discovered above defines a contradiction, then return failure.

4. If neither of the above occurs, then it is necessary to make a guess at something in order to proceed. To do this, loop until a solution is found or all possible solutions have been eliminated:

(a) Select an object whose value is not yet determined and select a way of strengthening the constraints on that object.

(b) Recursively invoke constraint satisfaction with the current set of constraints augmented by the strengthening constraint just selected.

A Cryptarithmetic Problem

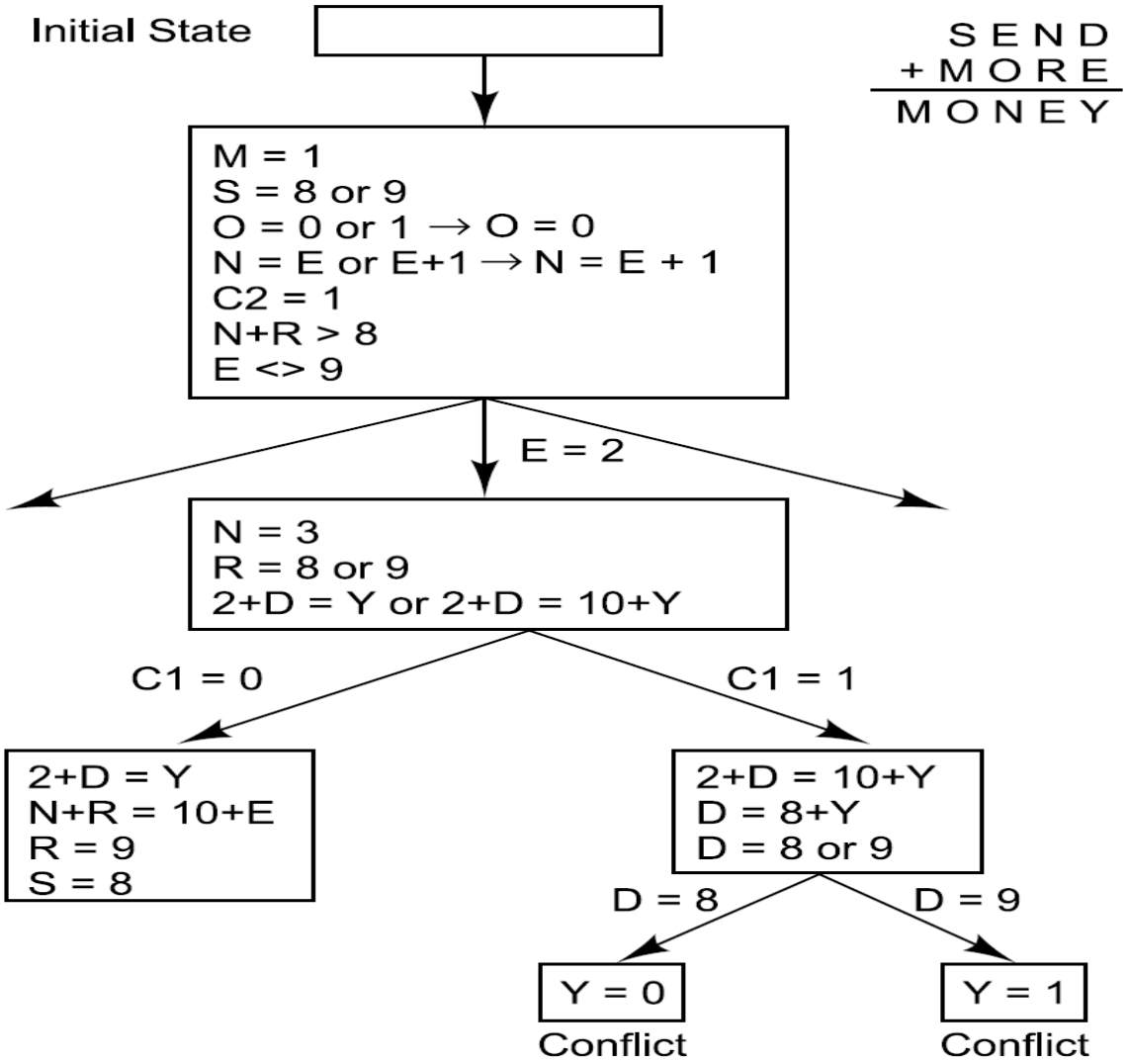
Problem:

```
  SEND
+ MORE
.....
 MONEY
```

Initial State:

No two letters have the same value.
The sums of the digits must be as shown in the problem.

Solving a Cryptarithmetic Problem



From column 5, **M = 1** since it is the only carry-over possible from the sum of two single digit numbers in column 4.
Since there is a carry in column 5, and M = 1, then **O = 0**
Since O = 0 there cannot be a carry in column 4 (otherwise N would also be 0 in column 3) so **S = 9**.
If there were no carry in column 3 then E = N, which is impossible. Therefore there is a carry and N = E + 1.
If there were no carry in column 2, then (N + R) mod 10 = E, and N = E + 1, so (E + 1 + R) mod 10 = E which means (1 + R) mod 10 = 0, so R = 9. But S = 9, so there must be a carry in column 2 so **R = 8**.
To produce a carry in column 2, we must have D + E = 10 + Y.
Y is at least 2 so D + E is at least 12.
The only two pairs of available numbers that sum to at least 12 are (5,7) and (6,7) so either E = 7 or D = 7.
Since N = E + 1, E can't be 7 because then N = 8 = R so **D = 7**.
E can't be 6 because then N = 7 = D so **E = 5** and **N = 6**.
D + E = 12 so **Y = 2**.

Means-end analysis

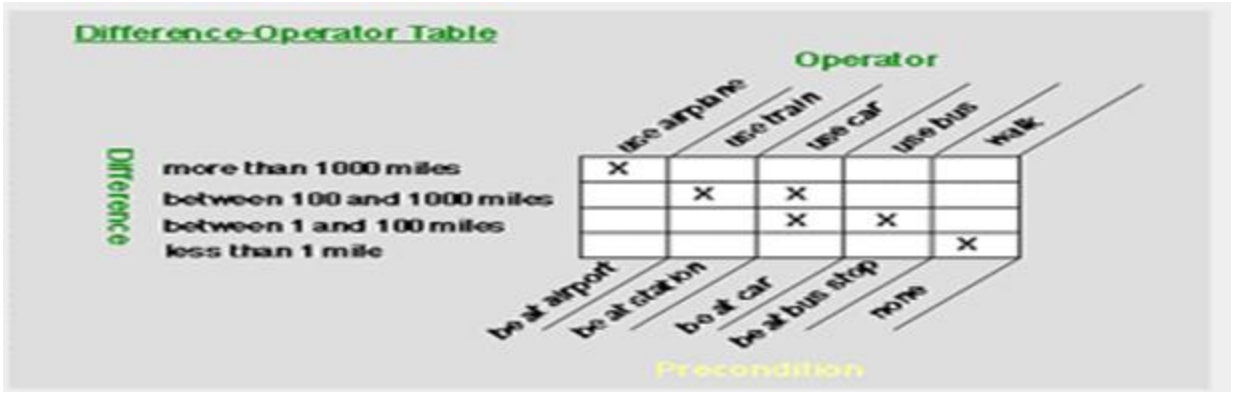
- Reducing differences between current state and goal state
  - Stop when difference is 0 (no difference)

- Subgoals
  - Intermediate goals – not your final goal-state
  - Means-end analysis is also considered a way to break up a problem into pieces (subgoals)

Means-Ends Analysis as it was employed in GPS.

- The example problem that we consider is to '*go from your house in New Brunswick to your Aunt's House in Los Angeles.*'
- GPS required knowledge about the problem domain.
- This knowledge was represented in a **Difference-Operator Table**. This table provides the knowledge needed to decompose a problem into subproblems..

Means-Ends Analysis as it was employed in GPS.



The basic idea that underlies Means-Ends Analysis is that the planner has the ability to compare two problem states and determine one or more ways in which these problem states differ from each other.

The rows of this example table lists a way in which two problem states might differ.

The labels at the top of the columns list an operator (action) that can reduce the difference.

And, the labels at the bottom of the columns list the partial state description that must be satisfied in order for the operator to be applied.

A Robot's Operators

Consider a Simple household robot domain. The available operators with preconditions And results are given bellow:

Operator	*Preconditions*	*Results*
PUSH(obj, loc)	at(robot, obj)^ large(obj)^ clear(obj)^ armempty	at(obj, loc)^ at(robot, loc)
CARRY(obj, loc)	at(robot, obj)^ small(obj)	at(obj, loc)^ at(robot, loc)
WALK(loc)	none	at(robot, loc)
PICKUP(obj)	at(robot, obj)	holding(obj)
PUTDOWN(obj)	holding(obj)	¬holding(obj)
PLACE(obj1, obj2)	at(robot, obj2)^ holding(obj1)	on(obj1, obj2)

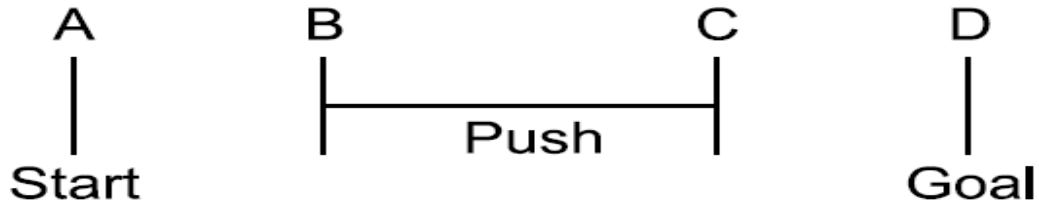
A Difference Table

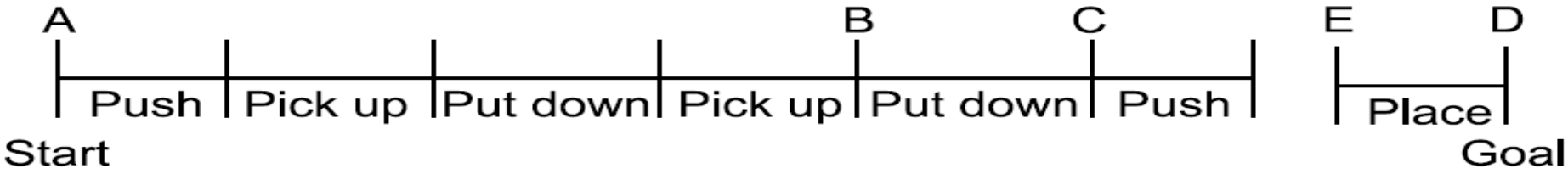
Describes that describes when each of operator is appropriate

	Push	Carry	Walk	Pickup	Putdown	Place
Move object	*	*				
Move robot			*			
Clear object				*		
Get object on object						*
Get arm empty					*	*
Be holding object				*		

Given a Problem of moving a desk with two things on it from one room to another.

The Progress of the Means-Ends Analysis Methods





Algorithm : Means-Ends Analysis

1. Compare *CURRENT* to *GOAL.* If there are no differences between them then return.

2. Otherwise, select the most important difference and reduce it by doing the following until success or failure is signaled:

(a) Select an as yet untried operator *O* that is applicable to the current difference. If there are no such operators, then signal failure.

(b) Attempt to apply *O* to *CURRENT.* Generate descriptions of two states: *O-START,* a state in which *O*'s preconditions are satisfied and *O-RESULT,* the state that would result if *O* were applied in *OSTART.*

(c) If
(FIRST-PART ← MEA(CURRENT, O-START))
and
(LAST-PART ← MEMO-RESULT, GOAL))
are successful, *then* signal success and return the result of concatenating *FIRST-PART, O,* and *LAST-PART.*

Thank You!!!