Function of Random variable:→

Let X be a Continuous random variable with Pdf $f(x)$. If the transformation $y = v(x)$ is

- Continuously differentiable and
- Either non-Increasing or non-decreasing

for all values within the range of X for which $f(x) \neq 0$ then the pdf of $y = v(x)$ is

$$g(y) = f(x(y)) \left|\frac{dx}{dy}\right|$$

Ex If $X \sim U(0,1)$ then find the distⁿ fun of $y = e^x$

Solⁿ Pdf of X is

$$f(x) = \begin{cases} 1 & ; 0 < x < 1 \\ 0 & ; o/w \end{cases}$$

$$y = e^x \Rightarrow x = \log y$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{y}$$

as $0 < x < 1 \Rightarrow 0 < \log y < 1$

$$\Rightarrow 1 < y < e$$

So $g(y) = 1 \cdot \left|\frac{1}{y}\right| = \frac{1}{y}$ as $y > 0$

∴ Pdf of y is

$$g(y) = \begin{cases} 1/y ; & 1 < y < e \\ 0 ; & o/w. \end{cases}$$

Ex If the Pdf of a Random variable X is given by

$$f(x) = \begin{cases} 2x ; & 0 < x < 1 \\ 0 ; & o/w. \end{cases}$$

Find the pdf of $Y = 8x^3$.

Solⁿ →

$$f(x) = \begin{cases} 2x; & 0 < x < 1 \\ 0; & o/w. \end{cases}$$

$$y = 8x^3 \Rightarrow x = \frac{(y)^{1/3}}{2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{2} \cdot \frac{1}{3} y^{-2/3} = \frac{1}{6} y^{-2/3}$$

$$\therefore g(y) = f(x(y)) \cdot \left|\frac{dx}{dy}\right|$$

$$= \cancel{2} \cdot \frac{(y)^{1/3}}{\cancel{2}} \cdot \frac{1}{6} \cdot y^{-2/3}$$

$$= \frac{1}{6} (y)^{-1/3} \quad ;$$

$$0 < x < 1 \Rightarrow 0 < y < 8$$

$$\therefore \text{Pdf of } y = \begin{cases} \frac{1}{6}(y)^{-1/3} & ; 0 < y < 8 \\ 0 & ; o/w. \end{cases}$$

Que If $X \sim N(0, \sigma^2)$. Find the Pdf of e^X.

Solⁿ $X \sim N(0, \sigma^2)$, the Pdf of X is

$$\Rightarrow f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-x^2/2\sigma^2}; \quad -\infty < x < \infty$$

$$y = e^x \Rightarrow x = \log y$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{y}$$

$$\because -\infty < x < \infty \Rightarrow -\infty < \log y < \infty$$

$$\Rightarrow 0 < y < \infty$$

$$\therefore g(y) = f(x(y)) \left|\frac{dx}{dy}\right|$$

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$$\Rightarrow g(y) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-(\log y)^2/2\sigma^2} \cdot \frac{1}{y} & ; 0<y<\infty. \\ 0 & ; o/w \end{cases}$$

Que If $X \sim U(0,1)$ then find the Pdf of $y = \sin x$

Sol^n Pdf of X is $f(x) = \begin{cases} 1 & ; 0<x<1 \\ 0 & ; o/w. \end{cases}$

$y = \sin x$

$\Rightarrow x = \sin^{-1} y$

$\Rightarrow \frac{dx}{dy} = \frac{1}{\sqrt{1-y^2}}$

As $0<x<1 \Rightarrow 0<\sin^{-1} y<1$

$\Rightarrow 0<y<\sin 1.$

$\therefore g(y) = f(x(y)) \left|\frac{dx}{dy}\right|$

$= 1 \cdot \left|\frac{1}{\sqrt{1-y^2}}\right| = \frac{1}{\sqrt{1-y^2}}$

So $g(y) = \begin{cases} 1/\sqrt{1-y^2} & ; 0<y<\sin 1 \\ 0 & ; o/w. \end{cases}$

H.W

Que If X is continuous random variable with Pdf

$f(x) = \begin{cases} x/12 & ; 1<x<5 \\ 0 & ; o/w. \end{cases}$

Find the Pdf of $Y = 2X-3$.

Ans'r $g(y) = \begin{cases} \frac{y+3}{48} & ; -1<y<7 \\ 0 & ; o/w. \end{cases}$

Que If $X \sim U(-1,1)$. Find the density function of

$Y = \sin\left(\frac{\pi X}{2}\right)$

Ans $g(y) = \begin{cases} \frac{1}{2\pi} \frac{1}{\sqrt{1-y^2}} ; & -1 < y < 1 \\ 0 ; & o/w. \end{cases}$

Que For a random variable X with Pdf

$$f(x) = \begin{cases} e^{-x} ; & x > 0 \\ 0 ; & o/w \end{cases}$$

Find the density function of $Y = X\sqrt{X}$.

Ans $g(y) = \begin{cases} \frac{2}{3} y^{-1/3} e^{-(y^{2/3})} ; & y > 0 \\ 0 ; & o/w \end{cases}$

Que If $X \sim Exp(\lambda)$. Find the dist^n of $Y = e^{-\lambda x}$.

Sol^n $X \sim Exp(\lambda)$. The Pdf of X is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} ; & x > 0 \\ 0 ; & o/w. \end{cases}$$

$$y = e^{-\lambda x} \Rightarrow x = \frac{-1}{\lambda} \log y$$

$$\Rightarrow \frac{dx}{dy} = \frac{-1}{\lambda y}$$

for $x > 0$; $\frac{-1}{\lambda} \log y > 0$

$$\Rightarrow \log y < 0$$

$$\Rightarrow 0 < y < 1$$

$$\Rightarrow g(y) = f(x(y)) \left| \frac{dx}{dy} \right|$$

$$= \lambda e^{\log y} \cdot \left| \frac{-1}{\lambda y} \right| ; 0 < y < 1$$

$$= \lambda y \cdot \frac{1}{\lambda y} ; 0 < y < 1$$

$$= 1$$

$$\Rightarrow g(y) = \begin{cases} 1 ; & 0 < y < 1 \\ 0 ; & o/w \end{cases} \sim U(0,1)$$

→ Let X be a discrete Random variable with p.m.f. $P(x)$.
If the transformation $y = g(x)$ is defined for all values within the range of X.
Then the Pdf of $Y = g(x)$ is obtained in following two Cases.

Case I $Y = g(x)$ is one-one.

Case II $Y = g(x)$ is not one-one.

Ex (1) Let X be a random variable with

$$f(x) = \begin{cases} 1/3 ; & x = 1,2,3 \\ 0 ; & o/w. \end{cases}$$

Find the Prob. distⁿ of $Y = 2X - 1$

Solⁿ

X	1	2	3
P(x)	1/3	1/3	1/3

$Y = 2X - 1$ is 1-1 function.

Y	1	3	5
P(Y)	1/3	1/3	1/3

So $g(y) = \begin{cases} 1/3 ; & y = 1,3,5 \\ 0 ; & o/w. \end{cases}$

(2) Let X be the no. of heads when two coins are tossed Simultaneously. Find the Prob. distⁿ of $Y = (1+x)^3$

Solⁿ

$S = \{HH, HT, TH, TT\}$

$X = 0, 1, 2$ – no. of heads

X	0	1	2
P(x)	1/4	2/4	1/4

$\Rightarrow$

Y	1	8	27
P(Y)	1/4	2/4	1/4

$(\because Y = (1+x)^3$ is 1-1.$)$

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H.W (3) Let $X \sim B(3, \frac{2}{5})$. Find the prob. distⁿ of $Y = X^2$?

~~Ans~~

(4) Let X be a geometric random variable with the Prob. distⁿ $f(x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$; $x = 1, 2, 3, \ldots$

Find the prob. distⁿ of $Y = X^2$

Solⁿ $Y = X^2$ is 1-1 function ($\because X = 1, 2, 3, \ldots$)

So $X = \sqrt{y}$

$$g(y) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{\sqrt{y}-1}; \quad y = 1, 4, 9, \ldots$$

Case II When $Y = f(x)$ is not one-one

Que Let X be the no. of heads in two tosses of a fair Coin. Find the prob. distⁿ of $Y = (X-1)^2$

Solⁿ →
$X = 0, 1, 2$
$P(x) = 1/4 \quad 2/4 \quad 1/4$

$Y = 1$ if $x = 0$
$Y = 0$ if $x = 1$ $\Rightarrow Y = (x-1)^2$ is not 1-1 function
$Y = 1$ if $x = 2$

So

Y	0	1
P(Y)	2/4	1/4 + 1/4

$\Rightarrow$

Y	0	1
P(Y)	1/2	1/2