5

DIFFRACTION

"No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them".

– Richard P. Feynman

It is matter of common experience that the sound waves or water escaping through a small hole spread out in all directions as if they have originated at the hole. After passing through the hole all the waves do not propagate in its original direction but a part of it is bent, the wavefronts for the ripples being semicircles with their centres situated at the centre of the hole as shown in Fig. 5.1. This phenomenon is called **diffraction** and is an important characteristic of wave motions.

In a similar way when a beam of light passes through a small opening, it also deviates from its rectilinear path and does bend round the corners as other types of waves do, but the amount of this bending is extremely small. The small bending in case of light is due to its very small wavelength of the order of 6000Å . The wavelength of ordinary sound waves is approximately 6 cm. Consequently the diffraction in case of light is 10^{-5} fold less, a small obstacle will be required to achieve the same effect in sound.

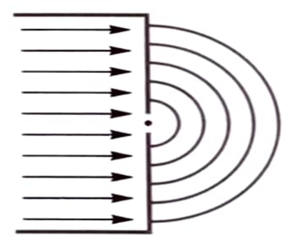


Fig. 5.1 An illustration for diffraction of light waves.

The phenomenon in the case of light was first discovered by *Francesco Grimaldi* in the *1600*. He found that with a point source of light, the shadow of an obstacle was bigger than that given by the geometrical construction and also that the shadow formed by a small object is not sharp and well defined as is expected from the rectilinear propagation of light on the basis of *corpuscular theory*. Light bends round the corners of these obstacles and spreads to some extent into the region of geometrical shadow.

The diffraction of light waves may be observed by looking through a narrow slit between two fingers at a distance light source or by looking at a street light through a cloth umbrella. Until the crack between two fingers is half a cm, the long thin light source through it would appear normal but as the crack is narrowed to about 1/10th of a mm, the source would appear spread out in a direction perpendicular to the fingers. It indicates that the light is diffracted by the opening.

"This bending of light round the corners of an obstacle (of which the size is comparable to the wavelength of light) or the encroachment of light within the geometrical shadow is called diffraction."

Although diffraction and interference occur simultaneously in nature, but they have following differences :

S.No.	Interference	Diffraction
1.	Interference is the result of superposition of secondary waves starting from two different wavefronts originating from two coherent sources.	Diffraction is the result of superposition of secondary waves starting from different parts of the same wavefront..
2.	All bright and dark fringes are of equal width.	The width of central bright fringe is much more than that of any other secondary maximum.
3.	All bright fringes are of same intensity.	Intensity of bright fringes decreases as we move away from central bright fringe on either side.
4.	Regions of dark fringes are perfectly dark. So there is a good contrast between bright and dark fringes.	Region of dark fringes are not perfectly dark. So there is a poor contrast between bright and dark fringes.
5.	At an angle λ/d, we get a bright fringe in the interference pattern of two narrow slits separated by a distance d.	At an angle of λ/d, we get the first dark fringes in the diffraction pattern of a single slit width d.

5.1 TYPES OF DIFFRACTION OF LIGHT WAVES

The diffraction phenomenon is usually divided into *two* categories

- Fresnel Type
- Fraunhofer Type

1. Fresnel's Type of Diffraction

In the Fresnel class of diffraction the source of the light and the screen are generally at a *finite distance* from the diffracting aperture as shown in Fig. 5.2. There is no lens or mirror in the optical system.

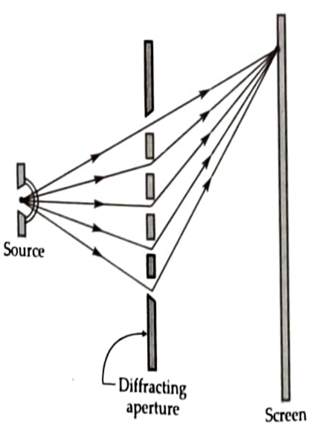


Fig. 5.2 Fresnel diffraction.

2. Fraunhofer's Type of Diffraction

In the Fraunhofer class of diffraction, the source of light and screen are at infinite distance from the diffracting aperture as shown in Fig. 5.3.

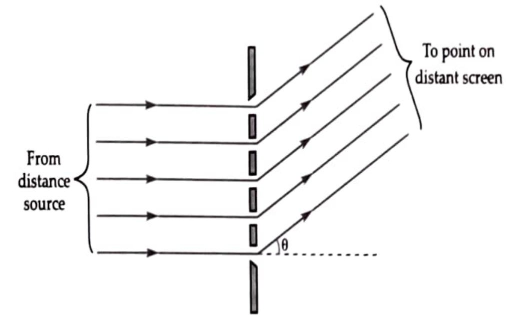


Fig. 5.3 Fraunhofer diffraction.

Fraunhofer diffraction can be easily achieved by placing the source on the focal plane of the convex lens and placing the screen on the focal plane of another convex lens. This arrangement is visualised in Fig. 5.4 on next page.

The main differences between the two types of diffraction have been summarized below :

S.No.	Fresnel diffraction	Fraunhofer diffraction
1.	The source and screen of both are at finite distance from the diffracting elements.	The distance of the source and screen from the diffracting elements is effectively infinite.
2.	The wavefronts are divergent–either spherical or cylindrical.	The wavefront incident on the aperture is plane, which is realized by using a collimating lens.
3.	Distances are important in this class of diffraction.	The angular inclinations are important in this of diffraction.
4.	No mirror or lens is used for observation.	Diffracted light is collected by a lens in a telescope.
5.	The rays proceed directly to the axial points.	There is large number of parallel rays falling on the lens corresponding to each point on the screen.
6.	The effect of several diffracted elements is not considerable.	The effect of several diffracted elements can be added together.
7.	The observed pattern is a projection of the diffracting device modified by diffracting effects and the geometry of the source.	The observed pattern is an image of the source modified by diffraction at the diffracting devices.
8.	In case of point off axis, the obliquity is different for different rays and therefore the amplitude contribution due to different zone is different.	For point off the axis, the incident rays on the lens have the same obliquity and hence the amplitude contribution due to each zone is the same.
9.	The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel zones.	The centre of the diffraction pattern is always bright for all paths parallel to the axis of the axis.
10.	Mathematical analysis is complicated and only approximated.	Mathematical analysis is rigorous and easy.

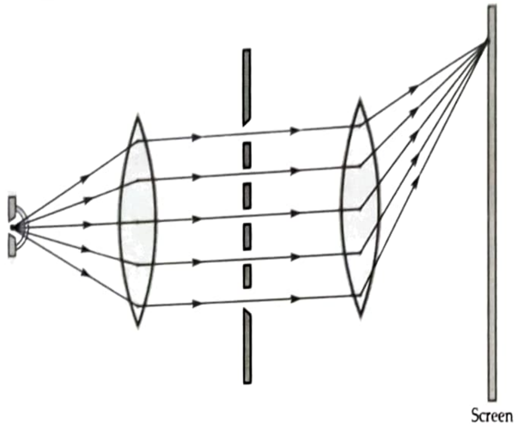


Fig. 5.4 Fraunhofer diffraction with convex lenses.

5.2 FRESNEL'S DIFFRACTION AT A CIRCULAR APERTURE

Let a narrow circular aperture *AB*(Fig. 5.5) be placed in the path of light from a point source *S*, and the diffraction pattern be obtained on a screen *XY*. The centre of the pattern *C* is bright or dark and is surrounded by alternate dark and bright diffraction rings in the illuminated region *A′B′*. The intensity in the geometrical shadow (above *A′* and below *B′*) rapidly falls to zero.

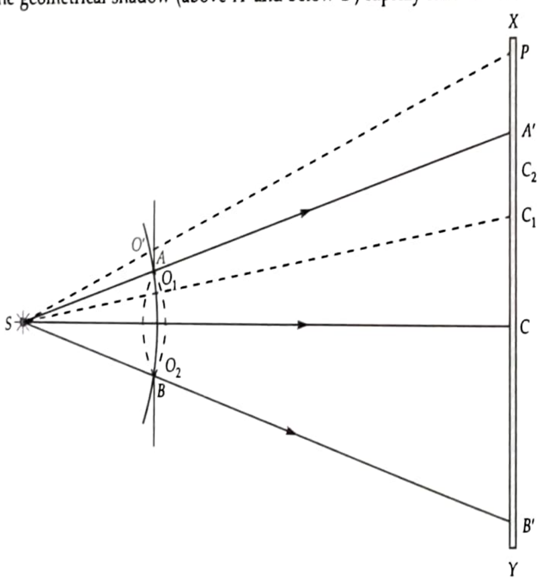


Fig. 5.5 Fresnel's diffraction at a circular aperture.

Explanation

Let us first consider all the intensity at the axial point C. Let the wavefront be divided into half period zones with respect to C. If the size of the hole is such that it allows only the first zone, the

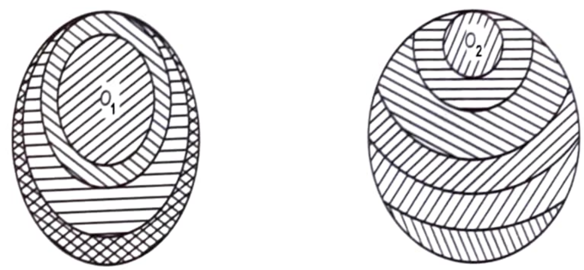
amplitude at *C* will be R_1. When the entire wavefront is unobstructed the resultant amplitude at *C* is only $R_1/2$. Thus the first zone along gives twice the amplitude or four times the intensity due to the entire wavefront. This means that a certain point on the axis the intensity of light is greater (four times) than that without the aperture.

If the aperture is wider so that it allows two half-period zones, the resultant amplitude at *C* is $(R_1 - R_2)$, which is almost zero. If three zones are allowed, the resultant amplitude is $R_1 - R_2 + R_3$, which is again large. Hence the illumination at an axial point is maximum or minimum according as the aperture allows an odd or an even number of Fresnel zones.

If the screen gradually moves towards the aperture, the width of the aperture being fixed the number of zones contained in the aperture gradually increases and becomes alternately odd and even. Hence the point *C* will be alternatively bright and dark.

Let us consider the intensity at a non-axial point such as C_1. Suppose the aperture allows three zones corresponding to the point *C* which is, therefore, bright.

As we move from *C* to C_1, the pole of the wavefront moves from *O* to O_1. Suppose in this position the first and second zones are fully exposed together with two-third of the third and about one-fifth of the fourth [Fig. 5.6(*a*)]. The first-two zones approximately cancel, leaving some light due to the effect of third and fourth zones. The intensity at C_1, is thus, less than that at *C*.



(*a*) The first and second zones are fully exposed together with two-thirds of third zone and about one-fifth of fourth zone

(*a*) First zone is fully exposed together with two-third of the second zone and one-half of the third zone, one-third of the fourth zone and a small fraction of fifth zone

Fig. 5.6 Fresnel's diffraction at a circular aperture.

For a still higher point C_2, the pole is O_2. Now first zone is fully exposed together with about two-third of the second zone, one-half of the third zone, one-third of the fourth zone, and a small fraction of the fifth zone [Fig. 5.6(*b*)]. The intensity at this point is again maximum. Thus as we move away from the axis, the intensity becomes alternately minimum and maximum. Since the circular aperture is symmetrical about the axis, these maxima and minima are concentric circles with common centre at *C*.

Let us now consider a point in the geometrical shadow such as *P*. The corresponding pole is *O′*. Clearly, only the lower half portions of a few zones send light to *P*. Therefore, there is faint illumination at *P*. As *P* moves further into the shadow, the zones sending light to *P* of higher and higher order. Hence the intensity decreases rapidly in the geometrical shadow.

5.2 FRAUNHOFER'S DIFFRACTION AT A SINGLE SLIT

A slit is a rectangular aperture[1] whose length is large compared to its breadth. Let a parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit of $AB = a$, as visualised in Fig. 5.7. Let the diffracted light be focused by a convex lens L_2. The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on either side of central bright band.

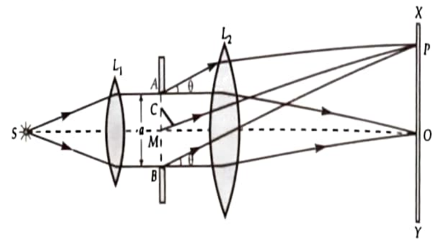


Fig. 5.7 Fraunhofer's diffraction at a single slit.

According to Huygen's theory, a plane wavefront[2] is incident normally on the slit AB and each point in AB sends out secondary wavelets in all directions. The rays proceeding in the same direction as the incident rays are focused at O; while those diffracted through an angle θ are focused at P.

Method I

Let the disturbance caused at P by the wavelet from unit width of slit at M be

$$y_0 = A\cos\omega t \qquad \text{...(5.1)}$$

Then the wavelet from width dx at C when it reaches P has the amplitude $A\,dx$ and phase $\left(\omega t + \frac{2\pi}{\lambda}x\sin\theta\right)$.

Let this small disturbance be 'dy', we have

$$dy = A\,dx\cos\left\{\omega t + \frac{2\pi}{\lambda}x\sin\theta\right\} \qquad \text{...(5.2)}$$

For the total disturbance at the point of observation at an angle θ, we get

$$y = \int_{-a/2}^{a/2} dy = \int_{-a/2}^{a/2} A\cos\left\{\omega t + \frac{2\pi x\sin\theta}{\lambda}\right\} dx$$

$$= A\cos\omega t \int_{-a/2}^{a/2} \cos\frac{2\pi x\sin\theta}{\lambda}\,dx - A\sin\omega t \int_{-a/2}^{a/2} \sin\frac{2\pi x\sin\theta}{\lambda}\,dx$$

1. **Aperture.** A measure of effective diameter (d) of a mirror or lens compared with focal distance (f); aperture $= d/f$. Thus a 50 mm camera lens may be used with an aperture diameter of 12.5 mm. Then, aperture = 12.5/50. This is usually described as an f-number. In this case the aperture diameter if $f/4$, often written as $f/4$. The transmitted light intensity depends on aperture diameter, so that I is proportional to d. However, large aperture diameter, so that I is proportional d^2. However, large aperture leads to large aberrations although diffraction effects are more serious at small apertures. In many optical instruments, iris diaphragms vary the aperture to obtain the optimum results.
2. **Wavefront.** A continuous surface associated with a wave radiation, in which all the vibrations concerned are in phase. A parallel beam has plane wavefront; the output of a point source has spherical wavefronts.

$$= \left\{A\frac{\sin\frac{\pi a\sin\theta}{\lambda}}{\frac{\pi\sin\theta}{\lambda}}\right\}\cos\omega t = Aa\left\{\frac{\sin\frac{\pi a\sin\theta}{\lambda}}{\frac{\pi a\sin\theta}{\lambda}}\right\}\cos\omega t$$

$$\Rightarrow \qquad y = A_0\left\{\frac{\sin\frac{\pi a\sin\theta}{\lambda}}{\frac{\pi a\sin\theta}{\lambda}}\right\}\cos\omega t \qquad \text{...(5.3)}$$

where $A_0 = Aa$ is the amplitude for $\theta = 0$.

Put $\qquad \frac{\pi a\sin\theta}{\lambda} = \alpha$

Therefore, Eq. (5.3) becomes $\qquad y = A_0\frac{\sin\alpha}{\alpha}\cos\omega t \qquad$ ...(5.4)

$\therefore$ Resultant amplitude, $\qquad R = A_0\frac{\sin\alpha}{\alpha} \qquad$ ...(5.5)

The resultant intensity at P is given by $I = R^2$

$$I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\Rightarrow \qquad I = I_0\frac{\sin^2\alpha}{\alpha^2} \qquad \text{...(5.6)}$$

where $I_0 = A_0^2$ represents the intensity at $\theta = 0$.

As $\alpha = \frac{\pi a\sin\theta}{\lambda}$, it is clear that α depends on the angle of diffraction θ. $\frac{\sin^2\alpha}{\alpha^2}$ gives the intensity at different value of θ.

Method II

We can consider infinite point sources of secondary wavelets on the wavefront between A and B. Let the slit AB be divided into n equal parts, each part being the source of secondary wavelets. The amplitude at the point P due to the waves obtained from each part will be equal[3], but the phase difference will be different from different parts. (The phase difference increases from 0 to $\frac{2\pi}{\lambda}a\sin\theta$ from A to B.) The phase difference between the waves obtained at point P from any two consecutive parts is

$$\varphi = \frac{1}{n}\times\frac{2\pi}{\lambda}a\sin\theta \qquad \text{...(5.7)}$$

Let the amplitude at the point P due to the waves obtained from each part be A. Thus we get n waves each of amplitude A and phase difference φ between the two consecutive waves. We can find their resultant amplitude at the point P by the vector method.

3. Since the width of each part is same and the screen is effectively at infinite distance from the slit, hence the amplitude remains independent of the distance and the angle of inclination.

For this, we draw vectors MP_1, P_1P_2, P_2P_3, ... such that the magnitude of each vector is A and the angle between two consecutive vectors is φ. These vectors form the sides of polygon. The vector MP_n joining the origin of first vector M and the terminus of last vector P_n gives the resultant vector as shown in Fig. 5.8. Let the magnitude of resultant vector MP_n is R_θ. If the centre of polygon is C, by simple geometry we can see that each vector MP_1, P_1P_2, ... subtends an angle φ at the centre C and the angle subtended by the resultant vector MP_n at the centre C is $n\varphi$. Let CX be the normal drawn from the point C on MP_1 and CY be the normal drawn on MP_n from the point C.

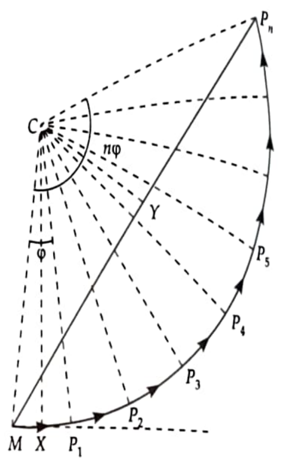


Fig. 5.8 Phasor diagram for single slit.

From ΔCXM,

$$\frac{MX}{MC} = \sin\frac{\varphi}{2} \quad \text{or} \quad MX = MC\sin\frac{\varphi}{2}$$

But $$MX = \frac{1}{2}MP_1 = \frac{A}{2}$$

$\therefore$ $$\frac{A}{2} = MC\sin\frac{\varphi}{2} \qquad ...(5.8)$$

Similarly, from ΔCYM,

$$\frac{MY}{MC} = \sin\frac{n\varphi}{2} \quad \text{or} \quad MY = MC\sin\frac{n\varphi}{2}$$

But $$MY = \frac{1}{2}MP_n = \frac{1}{2}R_\theta$$

$\therefore$ $$\frac{R_\theta}{2} = MC\sin\frac{n\varphi}{2} \qquad ...(5.9)$$

Dividing Eq. (5.9) by Eq. (5.8), we have

$$\frac{R_\theta/2}{A/2} = \frac{MC\sin(n\varphi/2)}{MC\sin(\varphi/2)} \quad \text{or} \quad R_\theta = A\frac{\sin(n\varphi/2)}{\sin(\varphi/2)}$$

Substituting the value of φ, from Eq. (5.7)

$$R_\theta = A\frac{\sin\frac{\pi a\sin\theta}{\lambda}}{\sin\frac{\pi a\sin\theta}{n\lambda}}$$

Let $\frac{\pi a\sin\theta}{\lambda} = \alpha$, then $R_\theta = A\frac{\sin\alpha}{\sin(\alpha/n)} = \frac{A\sin\alpha}{\alpha/n}$

[$\because$ n is very large, $\therefore$ α/n is very small and then $\sin(\alpha/n) = \alpha/n$]

or $$R_\theta = nA\frac{\sin\alpha}{\alpha} \qquad ...(5.10)$$

Now, if $\theta = 0$ or $\varphi = 0$ or $\alpha = 0$ (*i.e.*, if all the waves are in the same phase), then

$\therefore$ $$\lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha} = 1 \qquad \therefore \qquad R_0 = nA$$

Hence, from Eq. (5.10),

$$R_\theta = R_0\frac{\sin\alpha}{\alpha} \qquad ...(5.11)$$

Equation (5.11) gives the resultant amplitude at diffraction angle θ. Hence the resultant intensity at the point P on the screen corresponding to the angle of diffraction θ is

$$I \propto R_\theta^2 \quad \text{or} \quad I = kR_\theta^2 = k\,R_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

At $\theta = 0°$, $$I_0 = kR_0^2$$

$\therefore$ $$I = I_0\frac{\sin^2\alpha}{\alpha^2} \qquad ...(5.12)$$

Position of Central or Principal Maxima

For the central point O on the screen

$$\alpha = 0 \qquad \therefore \qquad \lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha} = 1$$

Hence intensity at O,

$$I = I_0\frac{\sin^2\alpha}{\alpha^2} = I_0$$

This is maximum as all waves reach at O in phase.

Again $\alpha = 0$ $\quad\therefore\quad \frac{\pi a\sin\theta}{\lambda} = 0$ or $\theta = 0$

This shows that the waves are travelling normal to the slit and O gives the position of *central maximum*.

Position of Minima

The intensity is minimum (zero) when

$$\frac{\sin\alpha}{\alpha} = 0 \quad \text{or} \quad \sin\alpha = 0 \qquad (\text{but } \alpha \neq 0)$$

$$\alpha = \pm\, m\pi$$

where $m = 1, 2, 3, 4, \ldots$ except zero.

As $$\alpha = \frac{\pi a\sin\theta}{\lambda}$$

$\therefore$ $$\frac{\pi a\sin\theta}{\lambda} = \pm\, m\pi \quad \text{or} \quad a\sin\theta = \pm\, m\lambda \qquad ...(5.13)$$

where $m = 1, 2, 3, \ldots$ gives the direction of first, second, third, ... minima, respectively.

Secondary Maxima

The direction of mth secondary maximum is given by

$$a \sin\theta = \pm\left(m+\frac{1}{2}\right)\lambda$$

As
$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \pm\frac{\pi}{\lambda}\left(m+\frac{1}{2}\right)\lambda = \pm\,\pi\left(m+\frac{1}{2}\right)$$

For various values of $m = 1, 2, 3, \ldots$, we get

$$\alpha = \pm\frac{3\pi}{2},\ \pm\frac{5\pi}{2},\ \pm\frac{7\pi}{2},\ \ldots$$

The intensity of the first secondary (subsidiary) maxima

$$I_1 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = I_0\left(\frac{-1}{\frac{3\pi}{2}}\right)^2 = \frac{4}{9\pi^2}I_0 = \frac{I_0}{22}$$

The intensity of second secondary maxima

$$I_2 = I_0\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = I_0\left(\frac{2}{5\pi}\right)^2 = \frac{4}{25\pi^2}I_0 = \frac{I_0}{61} \text{ and so on.}$$

Thus the intensity of secondary maxima falls off rapidly. Hence we find that secondary maxima of decreasing intensity occur on either side of the central maximum.

Thus the relative intensities of the successive maxima are in the ratio :

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \ldots \quad \text{or} \quad 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \ldots$$

To determine the position of secondary maxima differentiate the equation of intensity with respect to α and equate to zero.

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left[I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$I_0\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0 \quad \text{or} \quad \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

or
$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha \qquad \ldots(5.14)$$

This is the condition for secondary maxima. This equation can be solved by plotting the graphs for $y = \alpha$ and $y = \tan\alpha$.

The curve $y = \alpha$ is a straight line inclined to x-axis at 45°. The curve $y = \tan\alpha$ is shown in Fig. 5.9. The point of intersection of two curves gives the position of secondary maxima. The positions are $\alpha_1 = 0$, $\alpha_2 = 1.43\pi$, $\alpha_3 = 2.46\pi$, $\alpha_4 = 3.47\,\pi$, ...

Thus, it is clear that secondary maxima do not fall half way between two minima but are displaced towards the centre of the system by an amount which decreases with increase of n.

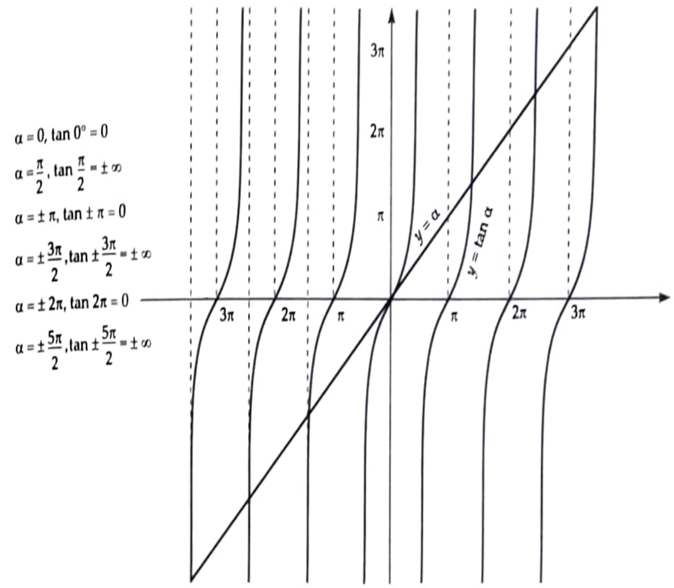


Fig. 5.9 Graphical solution for single slit pattern.

The intensity distribution curve of Fraunhofer diffraction at a single slit is shown in Fig. 5.10.

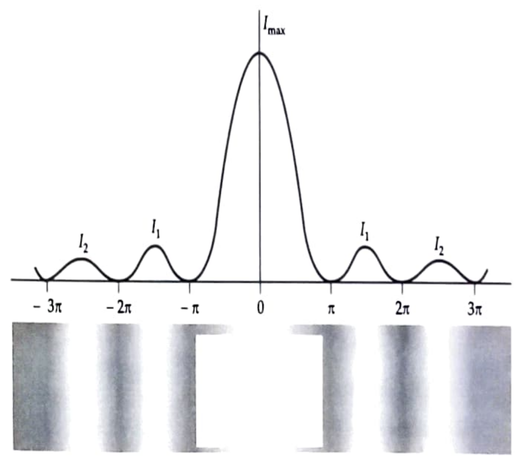


Fig. 5.10 Intensity distribution curve.

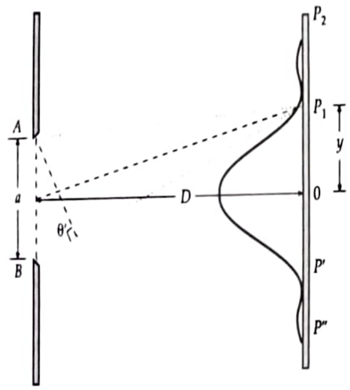
The principal (central) maxima occur at $\alpha = 0$, secondary minima occur at $\alpha = \pm\pi, \pm 2\pi, \ldots$ Secondary maxima occur at $\alpha = 1.43\,\pi, 2.46\pi, \ldots$

Width of Central Maxima

Let the distance of the first secondary minimum from the centre of the principal maximum be y.

$\therefore$ Width of central maximum $= 2y$

If the lens L lies very close to the slit or screen is very far from the lens, then distance between the slit and screen is very large. Let this distance be D as shown in Fig. 5.11.



1. Assumption of infinite source distance gives plane wave at slit so that all amplitude elements are in phase.
2. For $D >> a$ this approaches a right angle and $\theta' = \theta$.
3. Condition for minimum $a \sin\theta = n\lambda$

$$y = \frac{n\lambda D}{a}.$$

Fig. 5.11 Fraunhofer diffraction of a plane wave at a single slit.

$$\therefore \quad \tan\theta = \sin\theta = \frac{y}{D} \qquad ...(5.15)$$

For the first minima,

$$\sin\theta = \pm\frac{\lambda}{a}$$

$$\pm\frac{\lambda}{a} = \frac{y}{D}$$

$$\therefore \quad y = \pm\frac{\lambda D}{a} \qquad ...(5.16)$$

If f is the focal length of lens and the lens is very close to the slit, then $D = f$.

$$\therefore \quad y = \pm\frac{\lambda f}{a} \qquad ...(5.17)$$

$\pm$ sign indicates, on either side to central maxima.

$\therefore$ Width of central maxima $= y - (-y) = 2y$

$$= \frac{2\lambda f}{a} \qquad ...(5.18)$$

This shows that *the width of the central maximum is directly proportional to the wavelength of light (λ) and inversely proportional to the slit width (a).*

Effect of Slit Width

As $\quad \sin\theta = \pm\frac{\lambda}{a}$

If slit width a is large, then for a given wavelength of light, $\sin\theta$ is small and hence θ is small. This means that the maxima and minima lie very close to the central maximum. If slit width a is narrow, θ is large. Hence diffraction maxima and minima are quite distinct and clear.

Example 5.1 *Monochromatic light is incident on a slit of width 0.012 mm. The angular position of first bright line is 5.2°. Calculate the wavelength of incident light.*

Solution. Given : $a = 0.012$ mm $= 1.2 \times 10^{-5}$ m

$$\theta = 5.2° = \frac{5.2 \times \pi}{180} \text{ Radian}$$

$$\lambda = ?$$

We know that condition for maxima (to get bright fringes)

$$a\sin\theta = \left(n + \frac{1}{2}\right)\lambda = \frac{3}{2}\lambda$$

or $$\lambda = \frac{2a\sin\theta}{3} = \frac{2a\theta}{3} \qquad [\because \sin\theta = \theta,\ \theta \text{ is small}]$$

$$= \frac{2 \times 1.2 \times 10^{-5} \times 5.2 \times 22}{7 \times 3} = 7.257 \times 10^{-7} \text{ m} = 725.7 \text{ nm}$$

Example 5.2 *A parallel beam of light ($\lambda = 5890$ Å) is incident perpendicularly on a slit of width 0.1 mm. Calculate angular width and linear width of the central maximum formed on a screen 100 cm away.*

Solution. In single slit arrangement (Fraunhofer),

Given that : $\lambda = 5890$ Å $= 5.89 \times 10^{-7}$ m ; $a = 0.1$ mm $= 1.0 \times 10^{-4}$ m ; $D = 100$ cm $= 1$ m.

We know that, $\quad a\sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{a} = \frac{5.89 \times 10^{-7}}{1.0 \times 10^{-4}} = 5.89 \times 10^{-3} \text{ radian}$$

Since angle is very small then $\sin\theta \cong \theta$

i.e., $$\theta = 5.89 \times 10^{-3} \text{ radian}$$

Total angular width of central maximum $= 2\theta$

$$= 2 \times 5.89 \times 10^{-3} \text{ radian} = 11.78 \times 10^{-3} \text{ radian}$$

If y is the linear half width and D is the distance of the screen from the slit, then

$$y = D\theta \qquad [\text{for small } \theta]$$

$$\Rightarrow \quad y = 1 \times 5.89 \times 10^{-3} \text{ m} = 0.589 \text{ cm}$$

The linear width of the central maximum on the screen

$$= 2y = 2 \times 0.589 = 1.178 \text{ cm}$$

5.4 FRAUNHOFER'S DIFFRACTION AT TWO SLITS

The same graphic method may be applied for two fine silts as for single slit. In Fig. 5.12, AB and CD are rectangular slits. a is the width of slits AB and CD and b is the width of opaque BC. The distance between the corresponding points of two slits is equal to $(a + b)$.

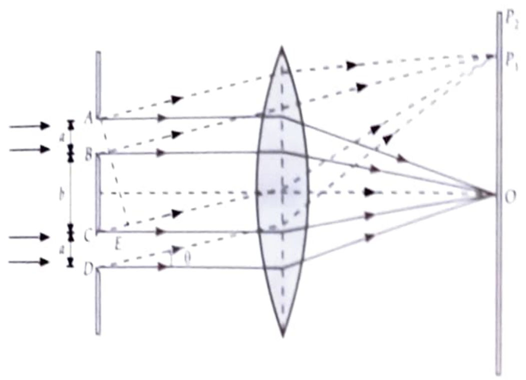


Fig. 5.12 Fraunhofer's diffraction at double slit.

Let us consider a parallel beam of monochromatic light of wavelength λ be incident upon this double slit and let each slit diffracts the beam at an angle θ.

Path difference (B and C)

$$CE = (a + b)\sin\theta$$

and corresponding phase difference

$$2\beta = \frac{2\pi}{\lambda}(a + b)\sin\theta \quad ...(5.19)$$

Here each slit can be treated as an independent small source of light and if the light from the two slits be focused by a convex lens on the screen in the focal plane of the lens as shown in Fig. 5.12. The resultant pattern will be the same as that due to the interference from two sources of light placed at the corresponding points.

If 2α is the phase difference between the extreme rays from the first slit, then

$$2\alpha = \frac{2\pi}{\lambda}a\sin\theta$$

or

$$\alpha = \frac{\pi}{\lambda}a\sin\theta \quad ...(5.20)$$

Analytically the resultant displacement y_1 due to the rays from the first slit is given by

$$y_1 = A\sin\omega t \quad ...(5.21)$$

where,

$$A = A_0\frac{\sin\alpha}{\alpha} \quad ...(5.22)$$

where A_0 is the resultant amplitude of the direct rays and A that of the rays diffracted at an angle θ from the first slit.

The resultant displacement y_2 due to the rays from the second slit is given by

$$y_2 = A\sin(\omega t + 2\beta) \quad ...(5.23)$$

Hence the resultant displacement Y due to the rays from the two slits diffracted at an angle θ is given by

$$Y = y_1 + y_2 = A\sin\omega t + A\sin(\omega t + 2\beta)$$
$$= A[\sin\omega t + \sin(\omega t + 2\beta)]$$
$$= 2A\cos\beta\sin(\omega t + \beta) \quad ...(5.24)$$

The resultant amplitude,

$$R = 2A\cos\beta \quad ...(5.25)$$

Putting $A = A_0\frac{\sin\alpha}{\alpha}$ from Eq. (5.22) in Eq. (5.24)

Then $R = 2A_0\frac{\sin\alpha}{\alpha}\cos\beta$ and intensity $I \propto R^2$

$$\Rightarrow \quad I \propto 4A_0^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Suppose constant of proportionality is one, then

$$\boxed{I = 4A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta} \quad ...(5.26)$$

Hence, the resultant intensity depends upon *two* factors :

(*i*) $\frac{\sin^2\alpha}{\alpha}$ and (*ii*) $\cos^2\beta$

$\frac{\sin^2\alpha}{\alpha}$ gives *diffraction pattern* due to each individual slit and $\cos^2\beta$ gives *interference pattern* due to diffracted light waves from the two slits.

$\frac{\sin^2\alpha}{\alpha^2}$ gives a central maximum in the direction $\theta = 0$, having alternately minima and subsidiary maxima of decreasing intensity on either side, as visualised in Fig. 5.13(*a*). The minima are obtained in the directions given by

$$\sin\alpha = 0$$

or

$$\alpha = \pm m\pi$$

as

$$\alpha = \frac{\pi a\sin\theta}{\lambda}$$

$$\therefore \quad \pi a\sin\theta = \pm m\pi\lambda$$

or

$$a\sin\theta = \pm m\lambda \quad ...(5.27)$$

where $m = 1, 2, 3, \ldots$ except zero.

The term $\cos^2\beta$ in the intensity pattern gives a set of equidistant dark and bright fringes as visualised in Fig. 5.13(*b*).

The bright fringes are obtained in the directions given by

$$\cos^2\beta = 1$$

$$\beta = \pm n\pi$$

or $$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$(a+b)\sin\theta = \pm n\lambda \qquad ...(5.28)$$

where $n = 0, 1, 2, ...$ corresponds to zero order, first order, second order, ... maxima.

The resultant intensity distribution pattern is shown in Fig. 5.13(c).

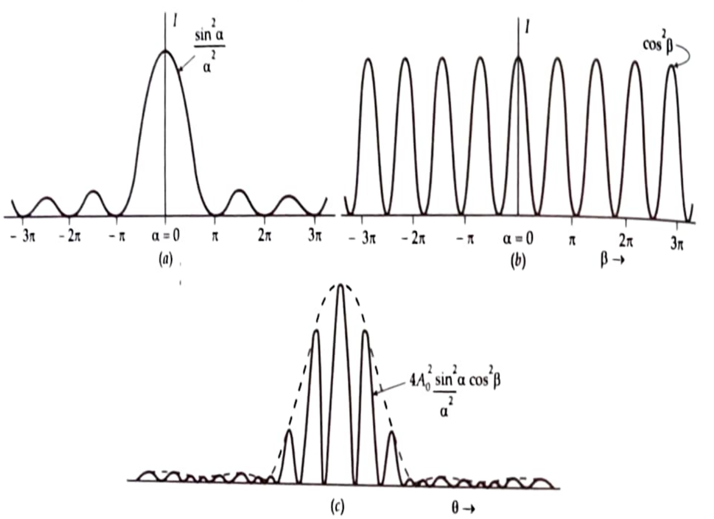


Fig. 5.13 Resultant intensity distribution pattern of double slit.

(i) Effect of increasing the slit width

On increasing the slit width a, the central peak will become sharper, but the fringe spacing remains unchanged. Hence less interference maxima falls within the central diffraction maximum.

(ii) Effect of increasing the distance between slits

On increasing the separation between slits b keeping the slit width constant, the fringes become closer together but the envelope of the pattern remains unchanged. Hence more interference maxima fall within the central envelope.

(iii) Missing order in a double diffraction pattern

We have taken the slit width as a and double slit separation b. If a is kept constant, we will observe the same diffraction pattern. However, if a is kept constant and b is varied, the spacing between interference maxima changes, depending upon the relative values of a and b. Some order of interference maxima will be missing in the resultant pattern.

We have direction for in interference maxima as

$$(a+b)\sin\theta = n\lambda \qquad ...(5.29)$$

and the direction for diffraction minima as

$$a\sin\theta = n\lambda \qquad ...(5.30)$$

where n and m are integers.

If the values of a and b are such that both Eqs. (5.29) and (5.30) are satisfied simultaneously for some value of θ, in that case position of *interference* maxima correspond to that of *diffraction* minima.

Let us take up some cases :

Case (*i*) If $$a = b$$

then $$2a\sin\theta = n\lambda \qquad ...(5.31)$$

and $$a\sin\theta = m\lambda \qquad ...(5.32)$$

$\therefore$ From Eqs. (5.30) and (5.31),

$$\frac{n}{m} = 2 \quad \text{or} \quad n = 2m$$

if $m = 1, 2, 3, ...$

then $n = 2, 4, 6, ...$

i.e., 2nd, 4th, 6th etc., orders of the interference maxima will be missing in the diffraction pattern.

Case (*ii*) If $$2a = b$$

then $$3a\sin\theta = n\lambda$$

$$a\sin\theta = m\lambda$$

$\therefore$ $$\frac{n}{m} = 3 \quad \text{or} \quad n = 3m$$

Putting $m = 1, 2, 3, ...$ etc.

Then $n = 3, 6, 9, ...$ etc.

So that, 3rd, 6th, 9th, ... etc. the interference maxima will be missing in the diffraction pattern.

Case (*iii*) If $a + b = a$ *i.e.*, $b = 0$

In that case we will have a single slit, so all interference pattern will be missing. In this case diffraction pattern observed on the screen is similar to that due to single slit with width $2a$.

Example 5.3. *A diffraction phenomenon is observed using a double slit with light of $\lambda = 5000$ Å, slit width $a = 0.02$ mm and the spacing between the two slits $b = 0.10$ mm. The distance of the screen from the slit is 1 m. Calculate (i) the distance between the central maximum and the first minimum of the fringe envelope, and (ii) distance between any two consecutive double slit dark fringes.*

Solution. Here, $a = 0.002$ cm, $b = 0.01$ cm and $\lambda = 5 \times 10^{-5}$ cm.

First minimum of the fringe envelope occurs at $\alpha = \pi$

Since, $$\alpha = \frac{\pi}{\lambda} a \sin\theta$$

$\therefore \quad \frac{\pi}{\lambda} a \sin\theta = \pi \quad$ or $\quad a\sin\theta = \lambda$

or $\quad \sin\theta = \frac{\lambda}{a} = \frac{5\times10^{-5}\text{ cm}}{0.002\text{ cm}} = 2.5\times10^{-2}$

Since θ is small

$$\sin\theta = 0 = 2.5\times10^{-2}\text{ radian.}$$

(*i*) Since distance of screen from slits = 100 cm, the distance y of the first minimum from central maximum (which is at $\theta = 0$) is given by

$$\tan\theta = \theta = \frac{y}{100} = 2.5\times10^{-2} \qquad \therefore \qquad y = 2.5\text{ cm}$$

(*ii*) Since $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$ and double slit dark fringes occur at

$$\beta = \pm(2n+1)\frac{\pi}{2} \qquad i.e., \qquad \beta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Let θ_1 be the angle corresponding to the first dark fringe and θ_2 corresponding to second dark fringe.

$\therefore$ For fringe dark fringe,

$$\frac{\pi}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta_1 \qquad \text{or} \qquad (a+b)\sin\theta_1 = \frac{\lambda}{2}$$

$\Rightarrow \quad \sin\theta_1 = \frac{\lambda}{2(a+b)} = \frac{5\times10^{-5}\text{ cm}}{2\times(0.002+0.01)\text{ cm}} = \frac{5\times10^{-5}}{0.024}$

$$\theta_1 = \frac{5\times10^{-5}}{0.024} \qquad [\text{Here } \sin\theta_1 = \theta_1] \qquad ...(i)$$

and for second dark fringe

$$\frac{3\pi}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta_2$$

$$(a+b)\sin\theta_2 = \frac{3\lambda}{2}$$

or $\quad \sin\theta_2 = \frac{3\lambda}{2(a+b)} = \frac{3\times5\times10^{-5}\text{ cm}}{0.024\text{ cm}}$

or $\quad \theta_2 = \frac{3\times5\times10^{-5}}{0.024} \qquad [\text{Here } \sin\theta_2 = \theta_2] \qquad ...(ii)$

If y_1 and y_2 be respectively the distances of the first and second minima from the centre then

$$\theta_1 = \frac{x_1}{100} = \frac{5\times10^{-5}}{0.024} \quad \Rightarrow \quad x_1 = \frac{5\times10^{-5}\times100}{0.024}$$

and $\quad \theta_2 = \frac{x_2}{100} = \frac{3\times5\times10^{-5}}{0.024} \quad \Rightarrow \quad x_2 = \frac{3\times5\times10^{-5}\times100}{0.024}$

$\therefore$ The linear separation between two minima

$$x = x_2 - x_1 = \frac{2\times5\times10^{-5}}{0.024}\times100\text{ cm} = \frac{5}{12} = 0.42\text{ cm.}$$

5.5 FRAUNHOFER DIFFRACTION AT N PARALLEL SLITS : PLANE TRANSMISSION DIFFRACTION GRATING

We know that interference of waves diffracted by individual slits determines the intensity distribution in the single slit pattern. Let us now consider the diffraction pattern produced by N slits. We use the same experimental arrangement as shown in Fig. 5.14 for N slits.

For simplicity we assume that :

(*i*) each slit is of width a and has the same length

(*ii*) all slits are parallel to each other and

(*iii*) the intervening opaque space between two successive slits is the same, equal to b.

Therefore, the distance between any two equivalent points in two consecutive slits is $(a+b)$. Let us denote it by d which we call the *grating element*. As before, we take the source of light to be in the form of a slit and adjust the length of this source slit to be vertical and parallel to the length of N slits.

"An arrangement consisting of a large number of parallel, equidistant narrow rectangular slits of the same width is known as diffraction grating."

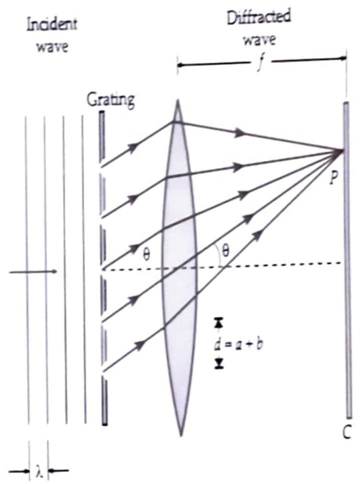


Fig. 5.14 Fraunhofer diffraction of a plane wave incident normally on a multiple slit aperture.

5.5.1 Intensity Distribution

Method I

Consider a beam of monochromatic light of wavelength λ is incident on a plane transmission grating has N parallel lines. If this has N parallel slits each of width a and opaque space in between two slits each of width b. Light diffracted from all these slits reach the screen with the same amplitude but with different phases. The superposition of N diffracted waves give rise to the intensity on the screen.

Consider point P on the screen where waves diffracted at an angle θ superimpose. Now from the theory of diffraction at single slit, we know that the disturbance originating from all points of the slit can be summed up into single wavelet of amplitude A originating from the centre of the slit, given by

$$A = A_0\frac{\sin\alpha}{\alpha} \qquad ...(5.33)$$

where $\quad \alpha = \frac{\pi}{\lambda} a\sin\theta \qquad ...(5.34)$

As seen from Fig. 5.14, the path difference between the diffracted light disturbances from the two nearby slits is given by

$$\Delta = (a+b)\sin\theta \qquad ...(5.35)$$

The corresponding phase difference is

$$\frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \qquad ...(5.36)$$

where $$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta \qquad ...(5.37)$$

To find the resultant intensity (I), we have to superimpose N waves each of amplitude A differing in phase with nearby wave by 2β. Figure 5.15 shows the phasor addition. MP_1, P_1P_2, ... $P_{N-1}P_N$ are N phasors. The angle between two successive phasors is 2β. Thus the angle between first and last phasor is $2N\beta$. The resultant phasor is MP_N. Since N is large the polygon of phasors may be assumed to an arc of circle whose centre is C. CM is the radius of the arc of this circle. From Fig. 5.15, we have for single phasor.

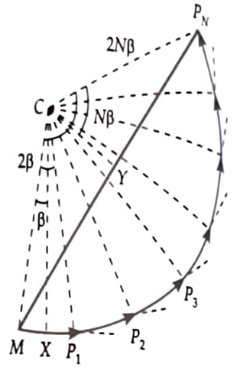


Fig. 5.15 Phasor diagram for N slits.

$$\frac{MX}{CM} = \sin\beta \quad \Rightarrow \quad MX = CM\sin\beta$$

$$MX = \frac{1}{2}MP_1$$

$\therefore$ $$MP_1 = \frac{1}{2}CM\sin\beta \qquad ...(5.38)$$

and $$\frac{MY}{CM} = \sin N\beta \quad \Rightarrow \quad MY = CM\sin N\beta$$

$\because$ $$MY = \frac{1}{2}MP_N$$

$\therefore$ $$MP_N = 2CM\sin N\beta \qquad ...(5.39)$$

Dividing Eq. (5.38) with Eq. (5.39), we have

$$\frac{MP_N}{MP_1} = \frac{\sin N\beta}{\sin\beta}$$

Now, MP_N = resultant disturbance phasor R_θ

where MP_1 is single slit disturbance phasor A

Hence $$R_\theta = A\frac{\sin N\beta}{\sin\beta}, \text{ here } A = A_0\frac{\sin\alpha}{\alpha}$$

which, on using value of A from Eq. (5.33), reduces to

$$R_\theta = A_0\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right) \qquad ...(5.40)$$

The resultant intensity is square of resultant amplitude

$$I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \quad \Rightarrow \quad I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad ...(5.41)$$

The first term in Eq. (5.41), $I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$ is the intensity due to single slit diffraction and the second term $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$, may be interpreted as interference term. Both effects combined together give intensity pattern of light diffracted by plane transmission grating.

Method II

Let a parallel beam of monochromatic light of wavelength λ be incidented normally on grating from the left. At an angle, θ on the right, we have N-light waves each of amplitude $A = A_0\frac{\sin\alpha}{\alpha}$ and successive phase difference $\varphi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$.

The resultant displacement Y is given by

$$Y = A\{\cos\omega t + \cos(\omega t+\varphi) + \cos(\omega t + 2\varphi) + ... + N \text{ terms}\}$$

$$= \text{Real part of } Ae^{i\omega t}\{1 + e^{i\varphi} + e^{2i\varphi} + ... + N \text{ terms}\}$$

$$= Ae^{i\omega t}\frac{1-e^{iN\varphi}}{1-e^{i\varphi}} \qquad ...(5.42)$$

$\therefore$ Intensity $= YY^*$

where Y^* is the complex conjugate of Y

$$I = A^2\frac{(1-e^{iN\varphi})(1-e^{-iN\varphi})}{(1-e^{i\varphi})(1-e^{-i\varphi})} = A^2\frac{1-\cos N\varphi}{1-\cos\varphi}$$

$$= A^2\frac{2\sin^2\frac{N\varphi}{2}}{2\sin^2\frac{\varphi}{2}} = A^2\frac{\sin^2\frac{N\pi(a+b)\sin\theta}{\lambda}}{\sin^2\frac{\pi(a+b)\sin\theta}{\lambda}} \qquad ...(5.43)$$

$\because$ $$A = A_0\frac{\sin\alpha}{\alpha}$$

$\therefore$ $$I = A_0^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2\frac{N\pi(a+b)\sin\theta}{\lambda}}{\sin^2\frac{\pi(a+b)\sin\theta}{\lambda}} = A_0^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta} \qquad ...(5.44)$$

where $\beta = \frac{\pi(a+b)\sin\theta}{\lambda}$

Hence the intensity distribution is product of two terms. The first term $A_0^2\frac{\sin^2\alpha}{\alpha^2}$ represents the *diffraction pattern* due to single slit. Second term $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the *interference pattern* due to N slits.

For $N=1$; $I = A_0^2 \cdot \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2\beta}{\sin^2\beta} = A_0^2\frac{\sin^2\alpha}{\alpha^2}$ [For single slits diffraction]

For $N=2$; $I = A_0^2 \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 2\beta}{\sin^2\beta} = A_0^2 \frac{\sin^2\alpha}{\alpha^2}\frac{4\sin^2\beta\cos^2\beta}{\sin^2\beta}$

$= A_0^2\frac{\sin^2\alpha}{\alpha^2}\cdot 4\cos^2\beta = 4A_0^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$ [For double slits diffraction]

For N-slits $I = A_0^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}$ [Generalised expression for N slits]

Principal Maxima

Intensity would be maximum when

$$\sin\beta = 0 \quad \text{or} \quad \beta = \pm n\pi, \text{ where } n = 0, 1, 2, 3, \ldots, \text{ also } \sin N\beta = 0.$$

Thus $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ (undefined)

So, $\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta}$ [Use L Hospital's rule]

$$= \frac{N\cos N(\pm n\pi)}{\cos\pm n\pi} = \pm N$$

So intensity at principal maxima

$$I_P = A^2\frac{\sin^2\alpha}{\alpha^2}N^2 \quad \ldots(5.45)$$

Since $\beta = \pm n\pi$

$$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi \quad \text{or} \quad (a+b)\sin\theta = \pm n\lambda \quad \ldots(5.46)$$

For $n=0$; maximum is zero order maximum

and for $n = \pm1, \pm2, \ldots$are called first, second, ... order principal maxima respectively.

Secondary Minima

When $\sin N\beta = 0$ but $\sin\beta \neq 0$, then from Eq. (5.44), $I=0$, which is minimum

or

$$N\beta = \pm m\pi$$

$$N\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\boxed{N(a+b)\sin\theta = \pm m\lambda} \quad \ldots(5.47)$$

where $m = 1, 2, 3, \ldots (N-1)$

If $m=0$ gives principal maxima and $m=N$ also gives principal maxima so $m=1, 2, 3, \ldots(N-1)$ gives minima. There are $(N-1)$ minima between two maxima.

Secondary Maxima

Since there are $(N-1)$ minima between two maxima then there must be $(N-2)$ maxima between two principal maxima. To find the position of these secondary maxima, we differentiate Eq. (5.44) with respect to β and equating it to zero.

$$\frac{dI}{d\beta} = \frac{A_0^2\sin^2\alpha}{\alpha^2}\frac{\sin N\beta}{\sin\beta}\left[\frac{N\sin\beta\cos N\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right]$$

$$\frac{dI}{d\beta} = 0 \quad \text{[condition for maxima]}$$

$$N\sin\beta\cos N\beta - \sin N\beta\cos\beta = 0$$

$$\tan N\beta = N\tan\beta$$

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}} \quad \text{[From Fig. 5.16]}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{(N^2\tan^2\beta)/(1+N^2\tan^2\beta)}{\sin^2\beta}$$

$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

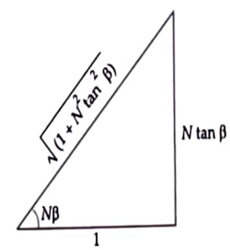


Fig. 5.16 An illustration to find the value of $\sin N\beta$.

Intensity at secondary maxima is given by

$$I_S = A_0^2\frac{\sin^2\alpha}{\alpha^2}\frac{N^2}{1+(N^2-1)\sin^2\beta} \quad \ldots(5.48)$$

From Eq. (5.45), the intensity of principal maxima $\propto N^2$.

Now we compare Eqs. (5.45) and (5.48), we get

$$\frac{\text{Intensity of secondary maxima}(I_S)}{\text{Intensity of principal maxima}(I_P)} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

If N is large, then intensity of secondary maxima is less. Since in the grating, the number of slits is very large, therefore, secondary maxima are not visible in grating spectrum.

5.5.2 Grating Spectra

Figure 5.17 shows the intensity distribution arising from factors $A_0^2\frac{\sin^2\alpha}{\alpha^2}$ and $\frac{\sin^2 N\beta}{\sin^2\beta}$ respectively. The intensity and angular spacing of secondary maxima and minima are so small in comparison to principal maxima, that they cannot be observed. Hence, there is uniform darkness between any two principal maxima.

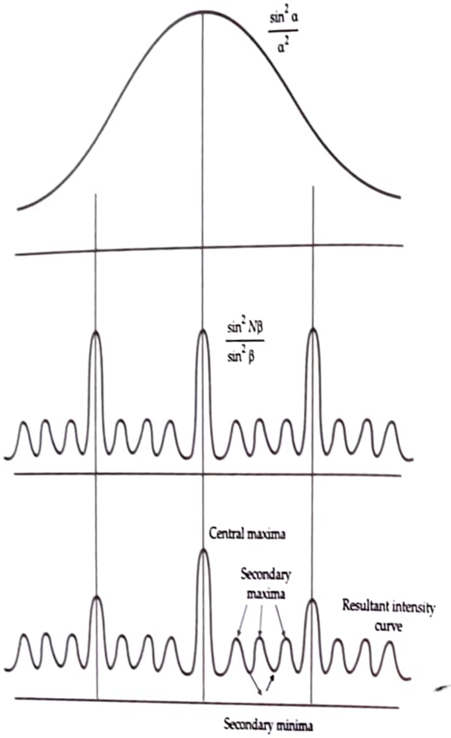


Fig. 5.17 Intensity distribution curve for diffraction grating.

5.5.3 Angular Half Width of Principal Maxima

For nth principal maxima,

$$(a+b)\sin\theta_n = \pm n\lambda \quad \text{...(5.49)}$$

Let $(\theta_n + d\theta)$ and $(\theta_n - d\theta)$ give the direction of first secondary minima on two sides of n primary maxima, then

$$(a+b)\sin(\theta_n \pm d\theta) = n\lambda \pm \frac{\lambda}{N} \quad \text{...(5.50)}$$

Dividing Eq. (5.49) by Eq. (5.50)

$$\frac{(a+b)\sin(\theta_n \pm d\theta)}{(a+b)\sin\theta_n} = \frac{n\lambda \pm \frac{\lambda}{N}}{n\lambda}$$

$$\Rightarrow \quad \frac{\sin(\theta_n \pm d\theta)}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

$$\frac{\sin\theta_n \cos d\theta \pm \cos\theta_n \sin d\theta}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

For small value of $d\theta$, $\quad \cos d\theta = 1,\ \sin d\theta = d\theta$

$$\frac{\sin\theta_n \pm \cos\theta_n\, d\theta}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

or

$$d\theta = \frac{1}{nN\cot\theta_n} \quad \text{...(5.51)}$$

Here $d\theta$ refers to half the angular width of principal maximum. $d\theta$ is inversely proportional to N *i.e.*, for large number of slits, $d\theta$ is small, then sharpness increases.

5.5.4 Absent Spectra/Missing Order Spectra

As the resultant intensity due to N-parallel slits (plane diffraction grating) is given by :

$$I = A^2 \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{...(5.52)}$$

where $\alpha = \frac{\pi a\sin\theta}{\lambda}$ and $\beta = \frac{\pi(a+b)\sin\theta}{\lambda}$

Now the direction of principal maxima in grating spectrum is given by

$$(a+b)\sin\theta = n\lambda \quad \text{...(5.53)}$$

where n = order of the maximum

The direction of minima in a single slit pattern

$$a\sin\theta = m\lambda, \quad \text{where } m = 1, 2, 3, 4, \ldots$$

If both conditions are simultaneously satisfied, a particular maximum of order n will be absent in grating spectrum, these are known as *absent spectra or missing order spectra.*

Now dividing Eq. (5.52) by Eq. (5.53)

$$\frac{(a+b)}{a} = \frac{n}{m} \quad \text{...(5.54)}$$

This is condition for the spectrum of the order n to be absent.

If $\quad b = a$, then Eq. (5.54) becomes

$n = 2m \quad$ (where $m = 1, 2, 3, 4, \ldots$)

$= 2, 4, 6, \ldots$

Hence when the width of transparencies and opacities of the grating are equal ($a = b$), then 2nd, 4th, 6th, ... order spectra are missing.

If $\quad b = 2a, \quad$ then $\quad n = 3m = 3, 6, 9, \ldots\ (m = 1, 2, 3, \ldots)$

Hence when the width of transparencies are double of the opacities *i.e.*, $b = 2a$, then 3rd, 6th, 9th, ... order spectra will be absent.

5.5.5 Dispersive Power of a Grating

The dispersive power of a grating is defined as the rate of change of the angle of diffraction with wavelength of light.

It is expressed as $\frac{d\theta}{d\lambda}$

As the grating equation is

$$(a+b)\sin\theta = n\lambda$$

Differentiating it with respect to λ, we get

$$(a+b)\cos\theta\frac{d\theta}{d\lambda} = n$$

or

$$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}} \qquad \text{...(5.55)}$$

Equation (5.55) represents the dispersive power of a grating.

From Eq. (5.55),

$$\text{Dispersive power} \propto n \text{ (order)}$$

$$\propto \frac{1}{(a+b)} \quad [(a+b) = \text{grating element}]$$

$$\propto \frac{1}{\cos\theta}$$

If θ is large, then $\cos\theta$ will be smaller, dispersive power will be high.

As θ (red) > θ (violet) in a given spectrum.

$\therefore$ Dispersion in red region > Dispersion in violet region.

If θ is small, $\cos\theta = 1$, $d\theta \propto d\lambda$.

Such spectrum is known as *normal spectrum*.

5.6 WAVELENGTH OF LIGHT BY MEANS OF DIFFRACTION GRATING

Wavelength of light can be determined using equation

$$(a+b)\sin\theta = n\lambda \qquad \text{...(5.56)}$$

This equation is valid when light is incident normally on the grating. The process of determining or measuring wavelength of light involves the following steps :

(i) Setting the grating for normal incidence

Spectrometer is set for parallel rays. The collimator and telescope are arranged in line with the slit to obtain fine image of slit on the crosswire. The telescope is now rotated by 90°. Keeping telescope in position, the grating is oriented on its stand so that reflected light is seen at the crosswire of the telescope. This shall happen when grating makes an angle 45° with incident light. The grating is now rotated by 45° or 135°. This sets grating normal to incident light. This is shown in Fig. 5.18.

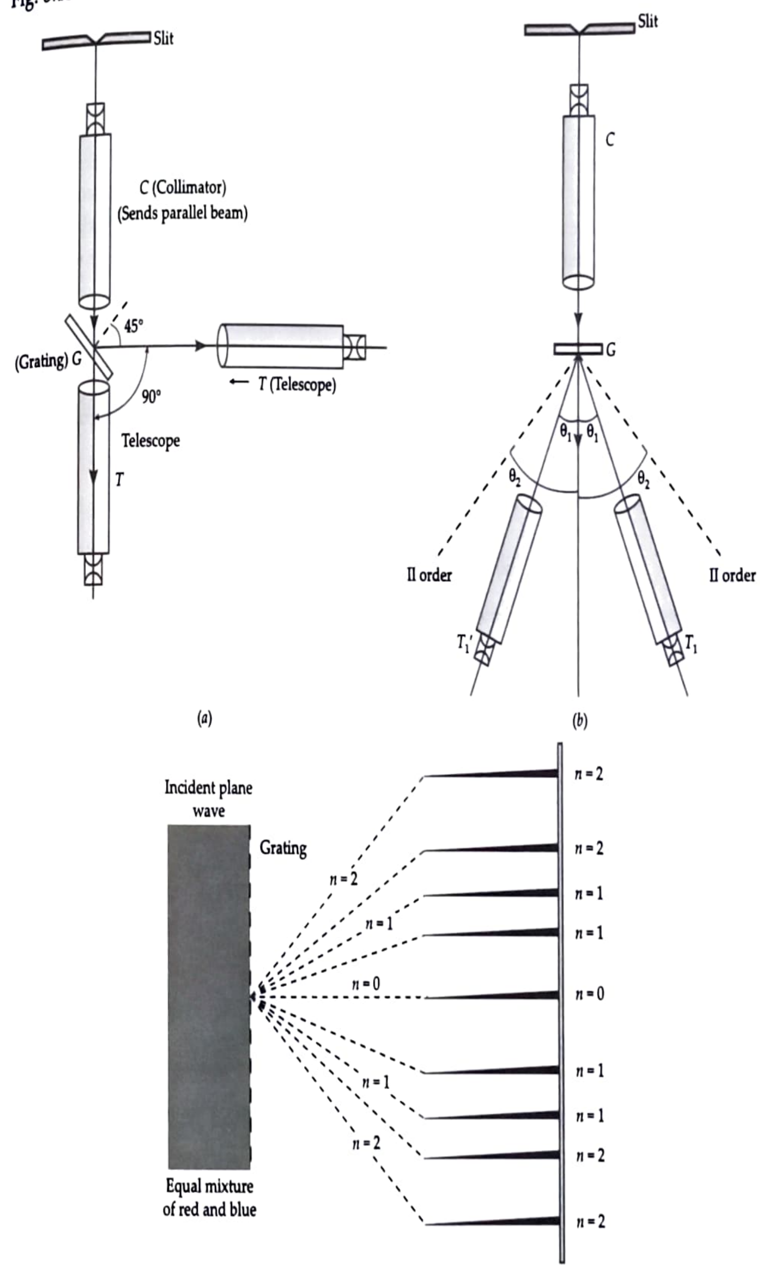


Fig. 5.18 (*a*) Arrangement to set the grating normal to incident parallel of light, (*b*) Measurement of diffraction angle θ, (*c*) Grating spectra.

(ii) Determination of grating element

On grating, number of lines per inch are marked, say N lines per inch. Then grating element is given by

$$(a+b)=\frac{2.54}{N}\text{ cm} \quad \text{...(5.57)}$$

(iii) Determination of diffraction angle (θ)

With grating locked in position and exposed to normal parallel beam of light, the telescope is rotated to find the spectral line exactly on the crosswire and reading θ_1 is noted. The telescope then rotated on the other side and the same spectral line is seen on the crosswire again. The reading θ_2 is again noted. This is shown in Fig. 5.18(*b*), the difference $(\theta_2-\theta_1)$ gives twice the diffraction angle. Knowing $(a+b)$, θ and *n*; λ may be calculated :

$$\lambda=\frac{(a+b)\sin\theta}{n} \quad \text{...(5.58)}$$

Example 5.4 *What is the highest order spectrum which may be seen with monochromatic light of wavelength 6000 Å by means of a diffraction grating with 5000 lines/cm ?*

Solution. Given : $\lambda=6000\text{ Å}=6.000\times10^{-5}\text{ cm}$; $N=5000$ lines/cm.

Then, $(a+b)=\frac{1}{N}=\frac{1}{5000}$

For highest order spectrum $\theta=90°$.

We know the grating formula,

$$(a+b)\sin\theta=n\lambda$$

or $(a+b)=n\lambda$

or $n=\frac{(a+b)}{\lambda}$

$$=\frac{10^5}{5000\times6}=\frac{100}{30}=\frac{10}{3}=3\text{ (approximately)}$$

Example 5.5 *How many orders will be visible if the wavelength of an incident radiation is 5000 Å and number of lines on the grating is 2620 per inch ?* **[GGSIPU, Dec. 2013 reappear (2 marks)]**

Solution. Given $\lambda=5000\text{ Å}=5.0\times10^{-7}$ cm,

$N=2620$ LPI, then grating element $(a+b)=\frac{2.54}{2620}$ cm,

We know grating formula $(a+b)\sin\theta=n\lambda$ (for highest order $\theta=90°$), then $(a+b)=n\lambda$

or $n=\frac{(a+b)}{\lambda}$

$$=\frac{2.54}{2620}\times\frac{1}{5.0\times10^{-5}}=19.38=19$$

Example 5.6 *Show that only first order spectra is possible if the width of grating element is less than twice the wavelength of the light.* **[GGSIPU, Dec. 2013 reappear (3 marks)]**

Solution. Given $(a+b)<2\lambda$, suppose $(a+b)=(2\lambda-x)$; then grating formula

$$(a+b)\sin\theta=n\lambda.$$

$\theta=90°$ for highest order

$$(2\lambda-x)=n\lambda.$$

$$n=\frac{2\lambda-x}{\lambda}$$

which is less than 2 or it is first order spectra.

Example 5.7 *A parallel beam of light is made incident on a plane transmission diffraction grating of 15000 lines per inch and angle of 2nd order diffraction is found to be 45°. Calculate the wavelength of light used.* **[GGSIPU, Dec. 2015 reappear (4.5 marks)]**

Solution. Given : $N=15000\text{ lines/inch}=\frac{15000}{2.54}$ lines/cm,

$$n=2,\quad \theta=45°,\quad \lambda=?$$

We know the grating formula,

$$(a+b)\sin\theta=n\lambda$$

or $\lambda=\frac{(a+b)\sin\theta}{n}$ ...(*i*)

$$(a+b)=\frac{1}{N}=\frac{2.54}{15000}\text{ cm} \quad \text{...(ii)}$$

Putting Eq. (*ii*) in Eq. (*i*), we get

$$\lambda=\frac{2.54\sin45°}{15000\times2}=5987\times10^{-5}=5987\text{ Å}$$

Example 5.8 *A plane transmission grating has 15000 lines per inch. What is the highest order of the spectra which can be observed for wavelength 6000 Å? If opaque spaces are exactly two times the transparent spaces, which order of spectra will be absent ?* **[GGSIPU, Dec. 2015 reappear (3 marks)]**

Solution. $N=15000$ lines/inch

$$(a+b)=\frac{2.54}{15000}\text{ cm} ;\quad \lambda=6000\text{ Å}=6.000\times10^{-5}\text{ cm}$$

We know the grating formula

$$(a+b)\sin\theta=n\lambda$$

For highest order, $\sin\theta=1$

$$n=\frac{(a+b)}{\lambda}=\frac{2.54\times10^5}{15000\times6}=2.8\cong3\text{ (approximately)}$$

Hence the third order is highest order visible.

The condition for the spectrum of order n to be absent

$$\frac{(a+b)}{a}=\frac{n}{m} \qquad \text{where } m=1, 2, 3, \ldots$$

Here

$$b=2a \quad \Rightarrow \quad \frac{3a}{a}=\frac{n}{m}$$

$$n=3m$$

Therefore, 3rd, 6th, 9th, etc. order of spectra will be absent.

Example 5.9 *A diffraction grating having 4000 lines per cm is illuminated normally by light of wavelength 5000 Å. Calculate its dispersive power in the third order spectrum.*

Solution. The dispersive power of grating is given by

$$\frac{d\theta}{d\lambda}=\frac{n}{(a+b)\cos\theta}$$

Also $(a+b)\sin\theta = n\lambda$.

Here $n=3$, $\lambda = 5000\text{ Å} = 5000\times10^{-8}\text{ cm} = 5.0\times10^{-5}\text{ cm}$

$$(a+b)=\frac{1}{4000}\text{ cm}$$

$$\sin\theta=\frac{3\times5.0\times10^{-5}}{1}\times4000=0.6$$

$$\cos\theta=\sqrt{(1-\sin^2\theta)}=\sqrt{1-(0.6)^2}=0.8$$

$$\therefore \quad \frac{d\theta}{d\lambda}=\frac{3\times4000}{0.8}=15000$$

5.7 RESOLVING POWER OF AN OPTICAL INSTRUMENT

When two objects are very close together they may appear as one object and it may be difficult for the naked eye to see them as separate. Similarly, there are two point sources very close together, the two diffraction patterns produced by each of them may overlap and hence it may be difficult to distinguish them as separate. To see the two objects or two spectral lines which are very close together, optical instruments like telescopes, microscopes, prisms, grating etc. are employed.

> ***The capacity of an optical instrument to show two close objects separately is called resolution, and the ability of an optical instrument to resolve the images of two close point objects is called its resolving power.***

The eye can see two objects as separate only if the angle subtended by them at the eye is greater than one minute, which is the resolving limit of the normal eye.

5.8 RAYLEIGH CRITERION FOR THE LIMIT OF RESOLUTION

By resolving power of an optical instrument we mean its ability to produce separate images of objects very close together. Using laws of geometrical optics, one designs a microscope or a

telescope. However in the final analysis it is the diffraction pattern which sets a limit on the resolving power. For example, consider two nearby sources. Only when the diffraction patterns of these two sources are separate will appear separate. When the central maxima fuse the two sources appear as one. When the central maximum of one source coincides with first minimum of the other, the resolution is marginal, a condition is called Rayleigh's criterion (Fig. 5.19).

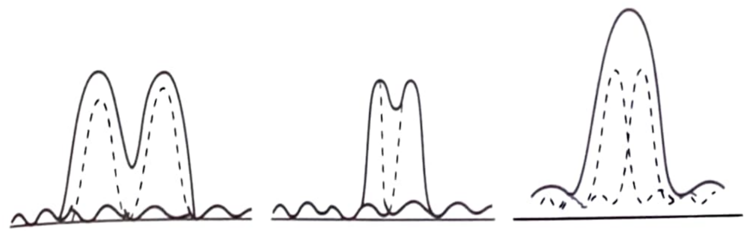


Fig. 5.19 Rayleigh criterion. Note the separation of the maxima in the left hand plot and close overlap on the right.

Therefore, the minimum angle of resolution provided by a lens of diameter D at a wavelength λ is

$$\theta_{min}=1.22\frac{\lambda}{D} \qquad \ldots(5.59)$$

5.9 RESOLVING POWER OF A PLANE DIFFRACTION GRATING

> ***The resolving power of grating is the ratio of wavelength of any spectral line to the difference in wavelengths between this line and neighbouring line such that the two lines appear to be just resolved.***

i.e., Resolving power of grating $=\dfrac{\lambda}{d\lambda}$.

Let P_1 be nth maximum of spectral line of wavelength λ and P_2 be nth primary maxima for $(\lambda+d\lambda)$ of diffraction angle $(\theta_n+d\theta)$.

According to Rayleigh, the two spectral lines will appear resolved if position of P_2 also corresponds to the first minimum at P_1.

For nth primary maximum for wavelength λ

$$(a+b)\sin\theta_n = n\lambda \qquad \ldots(5.60)$$

For nth primary maximum for wavelength $(\lambda+d\lambda)$

$$(a+b)\sin(\theta_n+d\theta)=n(\lambda+d\lambda) \qquad \ldots(5.61)$$

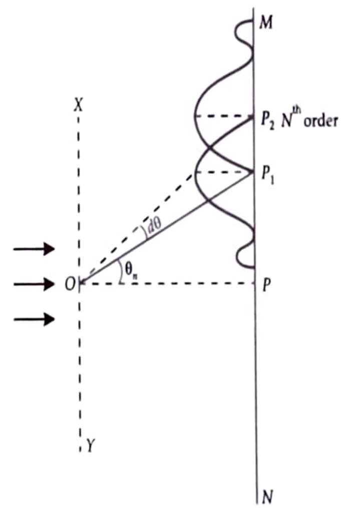


Fig. 5.20 An illustration for finding R.P. of grating

For just resolved the $(\theta_n + d\theta)$ corresponds the direction of first secondary minimum after nth primary maximum at P_1 of wavelength λ. So we introduce extra path difference, so the extra path difference $= \frac{\lambda}{N}$.

where N = number of lines on grating surface

So, $$(a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N} \quad ...(5.62)$$

From Eqs. (5.61) and (5.62),

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$\frac{\lambda}{d\lambda} = nN \quad ...(5.63)$$

Equation (5.63) is nothing but the expression for resolving power for grating. From Eq. (5.63), it is clear that *resolving power is directly proportional to order of spectrum and number of lines in grating surface.*

For central maxima $n = 0$, hence resolving power is zero. As the dispersive power of a grating is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \quad ...(5.64)$$

Therefore the resolving power of a diffraction grating may be expressed as

$$\frac{\lambda}{d\lambda} = Nn = N(a+b)\cos\theta\frac{d\theta}{d\lambda} \quad ...(5.65)$$

Resolving power of grating = Total aperture × dispersive power. ...(5.66)

Example 5.10 *What is the least separation between wavelengths that can be resolved near 640 nm in the second order, using diffraction grating that is 5 cm wide and ruled with 32 lines per millimetre.* **[GGSIPU, Oct. 2013 (2 marks)]**

Solution. Given $\lambda = 640$ nm, $n = 2$, $N = 32 \times 50 = 1600$, $d\lambda = ?$

We know resolving power of grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

$$d\lambda = \frac{\lambda}{nN}$$

$$= \frac{640 \times 10^{-9}\text{ m}}{2 \times 1600} = \frac{6400}{3200} \times 10^{-10}\text{ m}$$

$$= 2 \times 10^{-10}\text{ m} = 2\text{ Å}$$

Example 5.11 *A plane transmission grating has 40000 lines per inch. If length of diffraction grating is 2 inch then determine the resolving power in third order $(n = 3)$ for a wavelength of 5000 Å.*

Solution. The resolving power of the grating $= nN$

= Order of spectrum × No. of lines on the grating

Length of grating = 2 inches

Number of lines per inch on the grating = 40000

Total number of lines on a grating $= 40000 \times 2 = 80000$

So, resolving power $= \frac{\lambda}{d\lambda} = nN = 3 \times 80000 = 240000$

Example 5.12 *A plane transmission grating has 40000 lines in all with grating element 12.5×10^{-5} cm. Calculate the maximum resolving power for which it can be used in the range of wavelength 5000 Å.*

Solution. We know that $\frac{\lambda}{d\lambda} = nN$

Also $$(a+b)\sin\theta = n\lambda$$

$$\therefore \quad \frac{\lambda}{d\lambda} = N\frac{(a+b)\sin\theta}{\lambda}$$

Maximum resolving power $= \frac{N(a+b)}{\lambda}$ $[\because \sin\theta = 1]$

Here : $N = 40000$; $(a+b) = 12.5 \times 10^{-5}$ cm ; $\lambda = 5000\text{ Å} = 5 \times 10^{-5}$ cm

Maximum resolving power,

$$= \frac{40000 \times 12.5 \times 10^{-5}}{5 \times 10^{-5}} = 100000$$

Again, $$(n)_{max} = \frac{(a+b)}{\lambda} = \frac{12.5 \times 10^{-5}}{5 \times 10^{-5}} = 2.5 \cong 2$$

Hence the maximum number of order available with the grating = 2

Therefore, maximum resolving power of the grating

$$N(n)_{max} = 40000 \times 2 = 80000$$

Example 5.13 *Two spectral lines with average wavelength 6000 Å are resolved in second order by a grating having 500 lines per cm. The least width of the grating is 2 cm. Find the difference in the wavelength of the lines.*

Solution. Given $N = 500$ lines per cm, then actual $N = 2 \times 500$, $n = 2$,

$$\lambda = 6000\text{ Å} = 6000 \times 10^{-8}\text{ cm}, \ d\lambda = ?$$

Resolving power $\frac{\lambda}{d\lambda} = nN$

or $$d\lambda = \frac{\lambda}{nN}$$

$$\therefore \quad d\lambda = \frac{6000 \times 10^{-8}}{2 \times 1000} = 3 \times 10^{-8}\text{ cm} = 3\text{ Å}.$$

Formulae at a Glance

5.1 Fraunhofer diffraction at a single slit

(a) Resultant intensity

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

(b) Principal maximum

$$\alpha = 0$$

$$\alpha = \frac{\pi a\sin\theta}{\lambda} = 0$$

or $\quad \sin\theta = 0$

$$\theta = 0$$

$$I \propto A^2 \propto R^2$$

(c) Minimum intensity

$$\sin\alpha = 0\,;$$

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots \text{ etc.}$$

$$\frac{\pi a\sin\theta}{\lambda} = \pm m\pi$$

$$a\sin\theta = \pm m\lambda.$$

where $m = 1, 2, 3, \ldots$ etc.

(d) Secondary maxima,

$$R = \frac{na\sin\alpha}{\alpha}$$

For R_{max},

$$\frac{d\left(\frac{\sin\alpha}{\alpha}\right)}{\delta\alpha} = 0$$

$\Rightarrow \quad \tan\alpha = \alpha$

(e) $I_0 : I_1 : I_2 : I_3 : \ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \ldots$

$$= 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \ldots$$

(f) Width of central maxima,

$$\theta = \pm\frac{\lambda}{a} = \frac{2\pi}{a}$$

5.2 Fraunhofer diffraction at two slits

(a) Resultant intensity

$$I = 4A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta$$

(b) For maxima $\beta = \pm n\pi$

$$(a+b)\sin\theta = \pm n\lambda.$$

(c) For minima

$$(a+b)\sin\theta = (2n\pm 1)\frac{\lambda}{2}$$

(d) For diffracted maxima and minima

(i) For maxima

$$\sin\alpha = 1 = \sin\frac{n\pi}{2}$$

$$\frac{\pi}{\lambda}a\sin\theta = \frac{n\pi}{2}$$

or $\quad a\sin\theta = \frac{n\lambda}{2}$

(ii) For maxima,

$$\sin\alpha = 0 = \sin m\pi$$

$$\alpha = m\pi$$

$$\frac{\pi}{\lambda}a\sin\theta = m\pi$$

or $\quad a\sin\theta = m\lambda.$

5.3 Plane transmission Grating

(a) Resultant intensity

$$I = R^2 = A^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}$$

(b) Principal maxima

$$\beta = \pm n\pi,$$

where $n = 0, 1, 2, 3, \ldots$

$$(a+b)\sin\theta = \pm n\lambda.$$

(c) Secondary minima

$$\sin N\beta = 0$$

$$N\beta = \pm ma$$

$$N(a+b)\sin\theta = \pm m\lambda$$

where $n = 1, 2, 3, \ldots (N-1)$

(d) Secondary maxima,

$$I' = A^2\frac{\sin^2\alpha}{\alpha^2}\frac{N^2}{1+(N^2-1)\sin^2\beta}$$

(e) $\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{I'}{I}$

$$= \frac{1}{1+(N^2-1)\sin^2\beta}$$

5.4 Angular half width of principal maxima

$$d\theta = \frac{1}{nN\cot\theta_n}$$

5.5 Absent spectra or missing order spectra

$$\frac{a+b}{a} = \frac{n}{m}$$

5.6 Dispersive power of a grating

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

5.7 Resolving power of grating

$$\frac{\lambda}{d\lambda} = nN$$

Miscellaneous Solved Numerical Problems

Problem 5.1 *Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between first dark and next bright fringe from the axis, $\lambda = 4890$ Å.*

[GGSIPU, Sept. 2012 (3 marks) ; Sept. 2013 reappear (4 marks)]

Solution. For Fraunhofer diffraction through narrow single slit, given

$$a = 0.5 \text{ cm} = 5\times 10^{-3} \text{ m},$$

$$f = \text{focal length of lens} = 40 \text{ cm} = 0.4 \text{ m}$$

$$\lambda = 4890 \text{ Å} = 4.89\times 10^{-7} \text{ m}$$

Distance between first minima and first secondary maxima $= x_2 - x_1$.

$\because$ Condition for minima is written as $a\sin\theta = n\lambda$.

$\Rightarrow$ for $n = 1$, $\quad \sin\theta = \frac{\lambda}{a}$ and $\sin\theta = \frac{x_1}{f}$

$\therefore \quad x_1 = \frac{f\lambda}{a} = 3.912\times 10^{-5}$ m.

$\because$ Condition for secondary maxima is written as

$$a\sin\theta = (2n+1)\frac{\lambda}{2}$$

$\Rightarrow$ for $n = 1$, $\quad \sin\theta = \frac{3\lambda}{2a}$ and $\sin\theta = \frac{x_2}{f}$

Hence $\quad x_2 = \frac{3f\lambda}{2a}$

$$= \frac{3\times 0.4\times 4.89\times 10^{-7}}{2\times 5.0\times 10^{-3}} = 5.868\times 10^{-5} \text{ m}$$

$\Rightarrow \quad (x_2 - x_1) = 1.956\times 10^{-5}$ m.

Problem 5.2 *Calculate the angles at which the first dark band and next bright band are formed in the Fraunhofer's diffraction pattern of a slit 0.5 mm wide ($\lambda = 5890$ Å).*

Solution. For the Fraunhofer diffraction through narrow single slit, given that

$$a = 0.5 \text{ mm} = 5\times10^{-4} \text{ m}; \quad \lambda = 5890 \text{ Å} = 5.89\times10^{-7} \text{ m}$$

θ_1 = angle of first dark fringe = ? ;

θ_2 = angle of first secondary maxima = ?

$\because$ Condition of minima is written as

$$a\sin\theta = n\lambda$$

$\Rightarrow$ For $n=1$, $\quad \sin\theta_1 = \dfrac{\lambda}{a}$

$$\sin\theta_1 = \frac{5.89\times10^{-7}}{5\times10^{-4}}$$

$$\Rightarrow \quad \theta_1 = \sin^{-1}\left[\frac{5.89\times10^{-7}}{5\times10^{-4}}\right] = \sin^{-1}(1.18\times10^{-3}) = 0.07^\circ$$

$\because$ Condition for secondary maxima is written as

$$a\sin\theta = (2n+1)\frac{\lambda}{2}$$

$\Rightarrow$ for $n=1$, $\quad \sin\theta_2 = \dfrac{3\lambda}{2a}$

$$\theta_2 = \sin^{-1}\left[\frac{3\times5.89\times10^{-7}}{2\times5.0\times10^{-4}}\right] = \sin^{-1}(3.54\times10^{-3}) = 0.203^\circ.$$

Problem 5.3 *A parallel beam of light of wavelength 500 nm is incident normally on a narrow slit of width 0.2 mm. The Fraunhofer diffraction pattern is observed on a screen placed at the focal plane of a convex lens of focal length 20 cm. Calculate approximate distance between the first two maxima.*

Solution. $a = 0.2 \text{ mm} = 0.2\times10^{-3}$ m ; $\lambda = 500\times10^{-9}$ m, $f = 20$ cm $= 0.2$ m

For the first two maxima $\alpha_1 = 1.43\,\pi$ and $\alpha_2 = 2.46\,\pi$

Therefore, the corresponding angle of diffractions are given by

$$a\sin\theta_1 = 1.43\,\lambda \text{ and } a\sin\theta_2 = 2.46\,\lambda$$

Then $\quad \theta_1 = \sin^{-1}\left[\dfrac{1.43\times500\times10^{9}}{0.2\times10^{-3}}\right] = 3.57\times10^{-3}$ radian

Similarly $\quad \theta_2 = \sin^{-1}\left[\dfrac{2.46\times500\times10^{-9}}{0.2\times10^{-3}}\right] = 6.15\times10^{-3}$ radian

Therefore, the linear distance between the first two maximum is

$$f\times(\theta_1 \sim \theta_2) = 0.2\times(0.615\times10^{-3} - 3.57\times10^{-3})$$

$$= 0.516\times10^{-3} \text{ m} = 0.516 \text{ mm}.$$

Problem 5.4 *A parallel beam of sodium light is allowed to be incident normally a plane diffraction grating having 4250 lines per centimeter and a second order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line.*

Solution. Given $\theta = 30^\circ$, $\lambda = ?$, $N = 4250$, $(a+b) = \dfrac{1}{4250}$, $n = 2$

Now $\quad (a+b)\sin\theta = n\lambda$

$$\therefore \quad \lambda = \frac{(a+b)\sin\theta}{n} = \frac{1}{4250}\times\frac{1}{2}\times\sin30^\circ$$

$$= 5882\times10^{-8} \text{ cm} = 588.2 \text{ nm}.$$

Problem 5.5 *A diffraction grating used at normal incidence gives a line (5400 Å) in a certain superposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°, how many lines per cm are there in the grating ?*

Solution. For diffraction through grating

$$\lambda_1 = 5400 \text{ Å} = 5.4\times10^{-7} \text{ m}, \quad \lambda_2 = 4050 \text{ Å} = 4.05\times10^{-7} \text{ m}, \quad \theta = 30^\circ$$

$\because$ Condition for maxima, $(a+b)\sin\theta = n\lambda$

$\because$ λ_1 coincides with next higher order of λ_2

Hence $\quad n_2 = (n_1 + 1)$

$\Rightarrow \quad (a+b)\sin\theta = n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \quad (n_1\lambda_1) = (n_1+1)\lambda_2$

$\Rightarrow \quad (\lambda_1 - \lambda_2)n_1 = \lambda_2$

$$\Rightarrow \quad n_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4.05\times10^{-7}}{(5.40-4.05)\times10^{-7}} = \frac{4.05}{1.35} = 3$$

$$\Rightarrow \quad (a+b) = \frac{n_1\lambda_1}{\sin\theta} = \frac{3\times5.4\times10^{-7}}{\sin30^\circ} = 6\times5.4\times10^{-7} = 3.24\times10^{-3} \text{ mm} \qquad \left[\because \sin30^\circ = \frac{1}{2}\right]$$

$$N = \frac{1}{(a+b)} = 3086 \text{ per cm.}$$

Problem 5.6 *A grating is made of 200 wires per cm placed at equal distances apart. The diameter of each wire is 0.025 mm. Calculate the angle of diffraction for third order spectrum and also find the absent spectra, if any. The wavelength of the light used is 6000 Å.*

Solution. Given : $\lambda = 6000\text{Å} = 6\times10^{-7}$ m, $n = 3$, $a = 0.025 \text{ mm} = 2.5\times10^{-5}$ m

$$N = \frac{1}{a+b} = 200 \text{ wires/cm} = 200000 \text{ wires/m}$$

or $\quad a+b = \dfrac{1}{20000} = 5.0\times10^{-5}$m

$\because$ $$a+b=5.0\times10^{-5}\,\text{m}$$
$$a=2.5\times10^{-5}\,\text{m}$$
Then $$a=b=2.5\times10^{-5}\,\text{m}$$

(i) Angle of diffraction

We know that the grating equation

$$(a+b)\sin\theta=n\lambda$$
$$\sin\theta=\frac{n\lambda}{(a+b)}=\frac{3\times6\times10^{-5}}{5\times10^{-3}}$$
$$\theta=\sin^{-1}(0.036)=2.06^\circ$$

(ii) The condition for absent spectra

$$\frac{a+b}{a}=\frac{m}{n}$$

if $$a=b$$

then $$m=2n$$

i.e., second, fourth, ... order spectra will be absent.

Problem 5.7 *A plane transmission grating having 6000 lines per cm used to obtain a spectrum of light from a sodium light in the second order. Find the angular separation between the two sodium lines ($\lambda_1=5890$ Å and $\lambda_2=5896$ Å).* [GGSIPU, Dec. 2017 (5.5 marks)]

Solution. For diffraction grating,

$$(a+b)=\frac{1}{6000}\,\text{cm}=\frac{1}{6000\times100}\,\text{m},\quad \lambda_1=5.890\times10^{-7}\,\text{m},\quad \lambda_2=5.896\times10^{-7}\,\text{m}$$

$(\theta_2-\theta_1)=$ angular separation between two spectral lines $=$?

$\because$ Condition for maxima,

$$(a+b)\sin\theta=n\lambda$$
$$(a+b)\sin\theta_1=n\lambda_1$$
$$\sin\theta_1=\frac{n\lambda_1}{(a+b)}$$
$$\Rightarrow\quad \theta_1=\sin^{-1}\left[\frac{2\times5.890\times10^{-7}\times6000\times100}{1}\right]=44^\circ59'$$

and $$(a+b)\sin\theta_2=n\lambda_2$$
$$\sin\theta_2=\frac{n\lambda_2}{(a+b)}$$
$$\Rightarrow\quad \theta_2=\sin^{-1}\left[\frac{2\times5.896\times10^{-7}\times6000\times100}{1}\right]=44^\circ61'$$

Hence $\theta_2-\theta_1=2'$.

Problem 5.8 *Deduce the missing order for double slits Fraunhofer diffraction pattern, if the slit widths 0.16 mm and they are 0.8 mm apart.* [GGSIPU, Sept. 2011 (2 marks)]

Solution. Given that

$$a=0.16\text{ mm}\quad;\quad b=0.8\text{ mm}$$

If a be the slit width and b the separation between slits ; the condition of missing order spectra is given by

$$\frac{a+b}{a}=\frac{n}{m}$$
$$\frac{0.16+0.8}{0.16}=\frac{n}{m}$$
$$\frac{0.96}{0.16}=\frac{n}{m}$$
$$n=6m=6,\ 12,\ 18,\ \dots\ (m=1,2,3,\dots)$$

Thus 6th, 12th, 18th, ... orders will be missing.

Problem 5.9 *How many lines per cm are there in a grating which gives an angle of diffraction of 30° in first order spectrum of light of wavelength 6×10^{-5} cm.*

Solution. Given $\theta=30^\circ$, $n=1$, $\lambda=6\times10^{-5}$ cm

We know equation for a grating $(a+b)\sin\theta=n\lambda$

$$\therefore\quad (a+b)=\frac{1\times6\times10^{-5}}{0.5}=1.2\times10^{-4}\,\text{cm}$$

$\therefore$ No. of lines per cm

$$\frac{1}{(a+b)}=\frac{1}{1.2\times10^{-4}}=8333.$$

Problem 5.10 *How many orders will be visible if the wavelength of the incident radiation is 4800 Å and the number of lines on the grating is 2500 per inch.*

Solution. We know that $(a+b)\sin\theta=n\lambda$.

The maximum possible value of $\sin\theta=1$, hence the maximum number of orders is given by the expression

$$(a+b)=n\lambda\qquad\text{or}\qquad n=\frac{(a+b)}{\lambda}$$

Here $$(a+b)=\frac{2.54}{2500}\,\text{cm}$$
$$\lambda=4800\,\text{Å}=4.8\times10^{-5}\,\text{cm}$$
$$\therefore\quad n=\frac{2.54}{2500\times4.8\times10^{-5}}$$
$$=\frac{2.54\times10^5}{2500\times4.8}=\frac{25.4}{4.8}\times\frac{10000}{2500}>21=21\ (\text{approx.})$$

Hence the highest order of the spectrum, which can be seen is 21.

Problem 5.11 *What should be the minimum number of lines per inch in a half inch width grating to resolve the D_1 (5896 Å) and D_2 (5896 Å) lines of sodium ?*

Solution. Given : $\lambda_1 = 5896\ \text{Å} = 5.896 \times 10^{-5}$ cm and $\lambda_2 = 5890\ \text{Å} = 5.89 \times 10^{-5}$ cm

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5896 + 5890}{2}\ \text{Å}$$

$$= 5893\ \text{Å} = 5.893 \times 10^{-5}\ \text{cm.}$$

Now applying condition of just resolution,

$$\frac{\lambda}{d\lambda} = nN$$

or

$$N = \frac{1}{n}\frac{\lambda}{d\lambda} = \frac{5.893 \times 10^{-5}}{1 \times 0.006 \times 10^{-5}} \qquad [\text{For } n = 1]$$

$$= 982 \text{ lines per cm}$$

The two lines will be just resolved in the first order with minimum lines per inch.

Problem 5.12 *Find the angular separation between the two sodium lines 5890 and 5896 Å in the second order spectrum of a grating with 5000 lines/cm. The width of the grating is ½ cm. Can they be seen distinctly ?*

Solution. Given that : $\lambda_1 = 5890\ \text{Å} = 5.89 \times 10^{-5}$ cm ; $\lambda_2 = 5896\ \text{Å} = 5.896 \times 10^{-5}$ cm.

$$N = 5000 \text{ lines/cm}, \quad n = 2$$

Then $$(a+b) = \frac{1}{N} = \frac{1}{5000}\ \text{cm.}$$

For sodium line ($\lambda_1 = 5890$ Å)

From $(a+b)\sin\theta = n\lambda$

$$\sin\theta_1 = \frac{2 \times 5.89 \times 10^{-5}}{1} \times 5000$$

$\Rightarrow$ $$\sin\theta_1 = 0.589$$

$\Rightarrow$ $$\theta_1 = \sin^{-1}(0.589) = 36.09°$$

For sodium line ($\lambda_2 = 5896$ Å)

$$\sin\theta_2 = 2 \times 5.896 \times 10^{-5} \times 5000$$

$$\theta_2 = \sin^{-1}(0.5896) = 36.13°$$

Angular separation,

$$(\theta_2 - \theta_1) = 0.04° = 2.4'$$

Now applying condition for just resolution

$$\frac{\lambda}{d\lambda} = nN, \quad \lambda_2 - \lambda_1 = d\lambda$$

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = 5893 \times 10^{-8}\ \text{cm}$$

$$2 \times N = \frac{5893 \times 10^{-8}}{6 \times 10^{-8}}$$

$$N = \frac{5893}{12} = 491$$

In the given grating total number of lines $= 5000 \times 0.5 = 2500$. The two lines will be well resolved as the number of lines required is 491 and given grating has total of 2500 lines.

Problem 5.13 *A plane transmission grating of length 6 cm has 5000 lines/cm. Find the resolving power of grating and the smallest wavelength difference that can be resolved for light of wavelength 5000 Å.*

Solution. The resolving power of the grating $= nN$

= Order of spectrum × No. of lines on the grating

Length of grating = 6 cm

No. of lines per cm on the grating = 5000

Total number of lines on the grating $= 5000 \times 6 = 30000$

$\therefore$ Resolving power $\dfrac{\lambda}{d\lambda} = nN = 2 \times 30000 = 60000$

Now smallest wavelength difference ($d\lambda$) that can be resolved is given as

$$\frac{\lambda}{d\lambda} = nN$$

or

$$d\lambda = \frac{\lambda}{Nn}$$

$$d\lambda = \frac{5000 \times 10^{-8}}{60000} = \frac{5}{6} \times 10^{-9} = \frac{50}{6} \times 10^{-10} = 8.33\ \text{Å}$$

Problem 5.14 *Deduce the missing order for a double slit Fraunhofer diffraction pattern, if the slit width is 0.25 mm and they are 0.5 mm apart.*

Solution. For diffraction through double slits, given that

$$a = 0.25 \text{ mm}, \quad b = 0.5 \text{ mm}$$

missing order of spectra = ?

$\because$ Condition for missing order spectra

$$\frac{(a+b)}{a} = \frac{n}{m} \Rightarrow n = 3\,m$$

for $m = 1, 2, 3, \ldots$

$n = 3, 6, 9, \ldots$

Hence the 3, 6, 9, ... of the interference maxima will be missing from the diffraction pattern.

Conceptual Questions

5.1 *What is the difference between interference and diffraction fringes ?* [GGSIPU, Sept. 2009 (2 marks)]

Or

Explain the difference between interference and diffraction phenomenon. [GGSIPU, Oct. 2013 (2 marks)]

Or

List the five differences between interference and diffraction fringes. [GGSIPU, Dec. 2016 (2.5 marks)]

Ans. Please see the differences at page 266.

5.2 *What do you mean by diffraction of light ?*

Or

What is diffraction ? [GGSIPU, Sept. 2010 reappear (2 marks)]

Ans. If an opaque obstacle or aperture be placed between a source of light and a screen, a sufficiently distinct shadow is obtained on the screen. If the size of the obstacle or aperture is small, there is a departure from straight line propagation, and the light bends round the corners of the obstacle or aperture and enters the geometrical shadow. This bending of light is called 'diffraction'.

5.3 *Distinguish between Fresnel and Fraunhofer class of diffraction.*

[GGSIPU, Nov. 2004, Sept. 2011, 12 (2 mark) ; Dec. 2009 (4 marks)]

Or

Distinguish between Fraunhofer and Fresnel diffraction. [GGSIPU, Dec. 2013 reappear (4 marks)]

Or

Differentiate between Fresnel and Fraunhofer diffraction. [GGSIPU, Oct. 2013 (2 marks)]

Ans. Go through article 5.1 at pages 266-267.

5.4 *Why is diffraction of sound waves more evident in our daily life than that of light wave ?*

Or

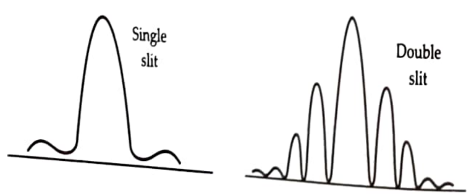
Interference and diffraction, which is more common and why.

[GGSIPU, Dec. 2008, (2 marks) ; Sept 2013 reappear (2 marks)]

Ans. To obtain a well defined diffraction pattern, the size of the obstacle or aperture should be of the same order as the wavelength. The wavelength of sound is comparable to the size of most of the obstacle or aperture, we come across in daily life. The diffraction of sound is therefore, a common experience. But wavelength of light is very small as compared to the size of the obstacles or aperture, we come across in daily life. We, therefore, do not observe diffraction of light as an everyday phenomenon.

5.5 *Distinguish between single slit and double slit diffraction patterns.* [GGSIPU, Dec. 2019 (2 marks)]

Ans. In single slit diffraction, light spreads out in a line perpendicular to the slit. No particular interesting phenomena are observed.



But in a double slit diffraction, light diffracts when passing through the slits, but the light waves coming out from those slits interfere with each other produce an interference pattern on the screen. The light is spread out in a line, like in single slit, but here there is interference, producing regions of constructive (bright fringes) and destructive (dark fringes) interference and a very bright spot at the centre of the screen, called the central maxima.

So, looking at diffraction only, there is no difference between single slit and double slit because in both cases diffraction happen but in a double slit there is diffraction as well as interference among the diffraction.

5.6 *A diffraction pattern is observed using a beam of red light. What happens if the red light is replaced by the blue light ?* [GGSIPU, Dec. 2016, (2 marks) ; Jan. 2018 (2.5 marks)]

Ans. Since, the spread of diffraction maximum on either side is given by :

$$\sin\theta = \pm\frac{\lambda}{d}$$

If λ is decreased (red to blue) ; the spread will decrease and bands will be crowded and narrower. Or In other words, the band width is directly proportional to wavelength when we go from red colour to the blue colour wavelength decreases and band width decreases *i.e.,* bands becomes narrower.

5.7 *What is the effect of increasing the number of lines per cm of the grating on the diffraction grating ?*

Or

What are the advantages of increasing the number of rulings in a grating ?

[GGSIPU, Dec. 2013 (2.5 marks)]

Ans. On increasing the number of lines per cm, decreases the grating element $(a + b)$. As a result the angle of diffraction θ increases for a given order. This results in a less number of spectra separated by large angles or an increased dispersive power.

EXERCISES

Theoretical Questions

5.1 Discuss the phenomena of Fraunhofer and Fresnel class of diffraction

[GGSIPU, Nov. 2004, 2nd counseling (2 marks)]

5.2 Show that for single slit experiment pattern the resultant amplitude on the screen is inversely proportional to the width of the slit. Also, analyse why minimum width of single slit need necessary be of one wavelength. [GGSIPU, Sept. 2007 (5 marks)]

5.3 Describe Fraunhofer diffraction of light (wavelength λ) due to a single slit of width 'b' and show that the diffraction pattern at θ the intensity is proportional to $\frac{\sin^2\beta}{\beta^2}$, where $\beta = \left(\frac{\pi}{\lambda}\right) b\sin\theta$. Hence find the angular position of maxima and minima. [GGSIPU, Sept. 2006 (6 marks)]

5.4 Describe Fraunhofer diffraction due to single slit and deduce the positions of maxima and minima. Draw the representative graph of the intensity distribution. What is the effect in diffraction pattern by changing :

(*i*) The slit width keeping the slit separation constant ?

(*ii*) The slit separation keeping the slit width constant ? [GGSIPU, Dec. 2013 (8.5 marks)]

5.5 Show graphically the distribution of intensity in Fraunhofer diffraction due to single slit. Indicate the position of maxima and minima in the figure. What are the values of relative intensities of successive maxima ? **[GGSIPU, Oct. 2013 (6 marks)]**

5.6 Discuss the phenomenon of Fraunhofer's diffraction at a single slit. **[GGSIPU, Sept. 2013 reappear (6 marks)]**

5.7 Describe Fraunhofer diffraction due to single slit and deduce the positions of maxima and minima. Show that the relative intensities of successive maxima are nearly

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \ldots$$

[GGSIPU, Sept. 2005 (6 marks)]

5.8 Discuss the phenomenon of Fraunhofer's diffraction at a single slit and show that the intensities of the successive maxima are nearly in ratio of

$$\frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \ldots$$

[GGSIPU, Sept. 2008, 2012 (7 marks) ; Nov. 2012, Nov. 04 (8 marks) ; Sept. 09 (6 marks)]

5.9 Derive an expression for intensity of diffracted light in Fraunhofer diffraction at a single slit. **[GGSIPU, Sept 2010 ; reappear (8 marks)]**

5.10 Differentiate Fresnel and Fraunhofer diffraction. Show that the intensities of the maximum in the diffraction due to a single slit are in the ratio $1 : \frac{4}{9}\pi^2 : \frac{4}{25}\pi^2 : \frac{4}{49}\pi^2 : \ldots\ldots$

[GGSIPU, Dec. 2012 ; Dec. 2015 (6.5 marks) ; Dec. 2016 (7 marks)]

5.11 Discuss Fraunhofer diffraction due to single slit. Deduce the condition for maxima and minima and show that the intensities of the maxima goes on decreasing with increase in order. **[GGSIPU, Dec. 2016 (8 marks)]**

5.12 What is the expression for intensity pattern due to Fraunhofer diffraction from a single rectangular slit of width 'b' ? From the expression, find the positions (in terms of angle) for the principal maximum and the secondary maxima. How does the plot for the intensity as a function of angular position look ? **[GGSIPU, Dec. 2015 (6 marks)]**

5.13 Derive the intensity pattern for a Fraunhofer's diffraction due to single slit using the analytical method. **[GGSIPU, Dec. 2015 (3 marks)]**

5.14 Write down the resulting intensity pattern for Fraunhofer diffraction from two slits, each of width 'b' and separated by a distance '**a**'. In what way does it differ from a single slit diffraction pattern ? What are missing orders ? **[GGSIPU, Dec. 2015 (6 marks)]**

5.15 Define. Plane transmission grating. **[GGSIPU, Sept. 2012 (1 mark)]**

5.16 Show that the intensity of light diffracted from a plane transmission grating is given by

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

where the symbols have their usual meanings.

[GGSIPU, Nov. 2012 (6 marks) ; Dec. 2011 (6.5 marks)]

5.17 In a diffraction grating how are the spectral lines affected when the rulings are made closer. **[GGSIPU, Sept. 2005 (2 marks)]**

5.18 Assuming the resultant amplitude for plane diffraction grating.

(*i*) Show that the intensity of principal maximum increases with increase in the number of slits.

(*ii*) What particular spectra would be absent if the width of the transparencies and opacities of the grating are equal ? **[GGSIPU, Sept, 2007 (3 marks)]**

5.19 What is a grating ? Deduce the expression for the dispersive power of grating. **[GGSIPU, Dec. 2008 (7.5 marks)]**

5.20 What is the condition of missing order spectra in diffraction grating ? **[GGSIPU, Sept. 2008, Sept. 2012 (2 marks)]**

5.21 For the Fraunhofer diffraction by a single slit, what is the effect of increasing : (*i*) Slit width, (*ii*) Wavelength ? **[GGSIPU, Dec. 2019 (3 marks)]**

5.22 Describe the overall effect of diffraction grating with a suitable diagram. **[GGSIPU, Dec. 2019 (3 marks)]**

5.23 Write the applications of diffraction grating. How wavelength of source is determined in laboratory using grating ? **[GGSIPU, Dec. 2018 (6 marks)]**

5.24 What is meant by a diffraction grating ? How is it useful for the determination of wavelength of a monochromatic source of light ? What is the advantage of increasing the number of lines on the grating ? **[GGSIPU, Jan. 2018 (6 marks)]**

5.25 What is meant by a diffraction grating ? How is it useful for the determination of wavelength of a monochromatic source of light ? What are the advantages of increasing the number of lines on the grating ? What is the fundamental difference in the spectra obtained by a prism to that of a diffraction grating for a white light source ? **[GGSIPU, Dec. 2016 (4 marks)]**

5.26 Show the intensity pattern due to N slits is the product of two terms the diffraction pattern due to a single slit and the interference pattern due to N slits. **[GGSIPU, Dec. 2015 (5 marks)]**

5.27 In a diffraction grating how are spectral lines affected when ruling are made closer. **[GGSIPU, Jan 2015 (2.5 marks)]**

5.28 What is a grating? Explain the spectra with theory formed by plane transmission diffraction grating. Show that intensity is not uniformly distributed over all the maxima. **[GGSIPU, Jan 2015 (7 marks)]**

5.29 What is meant by resolving power and dispersive power of an optical instrument ? **[GGSIPU, Dec. 2019 (2 marks)]**

5.30 What are the absent spectra ? **[GGSIPU, Dec. 2004 (2 marks)]**

5.31 What do you understand by missing orders spectrum ? What particular spectra would be absent if the transparencies and opacities of the grating are equal ? **[GGSIPU, Sept. 2005 (2 marks)]**

5.32 What is dispersive power ? Derive an expression for dispersive power. On what factors does it depend ? **[GGSIPU, Sept. 2010 reappear (6 marks)]**

5.33 Define dispersive power. Set up an expression for dispersive power of a prism. **[GGSIPU, Dec. 2009 (4.5 marks)]**

5.34 Describe Rayleigh's criterion of resolution. Derive an expression for resolving power of a transmission grating. **[GGSIPU, Dec. 2013 reappear (5.5 marks)]**

5.35 Show that resolving power of a transmission grating is the product of number of rulings on the grating and the order of fringe. **[GGSIPU, Dec. 2004 (4.5 marks)]**

5.36 Enunciate Rayleigh criterion of resolution. Can you think of any of its engineering applications? **[GGSIPU, Dec. 2007 (3.5 marks)]**

5.37 Enunciate Rayleigh criterion for resolution of two nearly wavelength. **[GGSIPU, Dec. 2004 (3 marks)]**

5.38 Derive an expression for resolving power and dispersive power of a diffraction grating. **[GGSIPU, Dec. 2007 (9 marks)]**

Numerical Problems

5.1 A plane wave of light of wavelength 690 nm is incident on a vertical slit of width 10^{-4} m. Sketch the intensity distribution on a screen 3 m from the slit placed parallel to the slit aperture. At what distances from the central maximum do the first two zeros occur ? **[GGSIPU, Dec. 2008 (5 marks)]**

Hint : $a\sin\theta = n\lambda$ and $\sin\theta = x/D$

$\therefore \quad x = \frac{n\lambda D}{a}$, when for $n = 1$, $x = 2.07$

for $n = 2$, $x = 4.14$ m.

5.2 In Fraunhofer's diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of central maxima, find the wavelength of light. **[GGSIPU, Oct. 2013 (2 marks)]**

Hint : $a\sin\theta = n\lambda$ or $\sin\theta = \frac{\lambda}{a}$

and $\sin\theta = \frac{y}{D}$

Hence $\lambda = \frac{ay}{D} = \frac{0.02 \times 0.5}{200}$

$= 5 \times 10^{-5}$ cm $= 5000$ Å.

5.3 A plane diffraction grating has 40000 lines. Determine the resolving power in 2nd order for a wavelength of 5000 Å. **[GGSIPU, Dec. 2019 (2.5 marks)]**

Hint : Go through Example 5.11 at page 297 and find R.P. $= 2 \times 40000 = 80000$.

5.4 In a Fraunhofer diffraction pattern experiment a monochromatic light of wavelength 5860×10^{-8} cm is incident normally on a 2 cm wide grating. The first order spectrum is produced at a angle of 20° with respect to the normal. Determine the total number of lines on the grating. **[GGSIPU, Jan. 2018 (4 marks)]**

Hint : $(a+b)\sin\theta = n\lambda$.

$\Rightarrow \quad \frac{\sin\theta}{N} = n\lambda$.

$\Rightarrow \quad N = \frac{\sin\theta}{n\lambda} = \frac{\sin 20°}{1 \times 5896 \times 10^{-8}}\ \text{cm}^{-1} = 5800\ \text{cm}^{-1}$

$\because$ 5800 lines are per cm, therefore 11600 lines will be in 2 cm.

5.5 Determine the minimum number of lines in a grating that are just able to resolve the sodium lines of wavelength 5890 Å and 5896 Å in first order spectrum. **[GGSIPU, Jan. 2018 (2.5 marks)]**

Hint : $\frac{\lambda}{d\lambda} = nN$

$\Rightarrow \quad N = \frac{1}{n} \times \frac{\lambda}{d\lambda} = \frac{5893}{6} = 982$

5.6 How many orders will be visible if the wavelength of incident radiation is 6000 nm and the number of lines on the grating is 12000 per inch ? **[GGSIPU, Dec. 2016 (2.5 marks)]**

Hint : $n = \frac{1}{\lambda N} = \frac{2.54 \times 10^{-2}}{6000 \times 10^{-10} \times 12000} \cong 3$

5.7 A slit is located 'at infinity' in front of lens of focal length 1 m and is illuminated normally with light of wavelength 600 nm. The first minima on either side of the central maximum of the diffraction pattern observed in the focal plane of the lens are separated by 6 mm. What is the width of the slit ? **[GGSIPU, Dec. 2016, 2015 (2.5 marks)]**

Hint : $\frac{y}{D} = \frac{\lambda}{a} \Rightarrow a = \frac{D\lambda}{y} = 0.1$ mm.

5.8 A grating having 15000 lines per inch produces spectra of a mercury arc. The green line of the mercury spectrum has a wavelength of 5461 Å. What is the angular separation between the first order green line and the second order green line ? **[GGSIPU, Sept. 2007 (2 marks)]**

Hint : $(a+b)\sin\theta = n\lambda$.

Here $(a+b)$ [grating element] $= \frac{2.54}{15000}$

$\lambda = 5461 \times 10^{-8}$ cm

Then $\sin\theta = \frac{5461 \times 10^{-8} \times 15000}{2.54} n$

For $n = 1$, $\theta = \theta_1$ For $n = 2$, $\theta = \theta_2$,

$\theta_1 = 18.81°$ $\theta_2 = 40.17°$

Angular separation of lines $= \theta_1 - \theta_2 = 21.36°$.

5.9 A parallel beam of monochromatic light is allowed to be incident normally on a plane grating having 1250 lines per cm and a second order spectral line is observed to be deviated through 30°. Calculate the wavelength of spectral line. **[GGSIPU, Sept. 2008 (3 marks)]**

Hint : $(a+b)\sin\theta = n\lambda$

$\Rightarrow \quad \lambda = \frac{(a+b)\sin\theta}{n} = \frac{\sin\theta}{nN}$

$= \frac{\sin 30°}{1250 \times 2} = 20{,}000$ Å. [Obtain value of λ is too large due to wrong data]

5.10 A diffraction grating has 126×10^4 rulings uniformly spread over width $W = 25.4$ mm. It is illuminated at normal incidence by blue light of wavelength 450 nm. At what angle to the central axis $d\theta$ the second maxima occur ? **[GGSIPU, Dec. 2009 (4 marks)]**

Hint : $(a+b)\sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{(a+b)} = n\lambda N$$

$$= 2\times 450\times 10^{-7}\times\frac{1.26\times 10^{4}}{2.54} = 0.4465$$

$$\theta = \sin^{-1}(0.4465) = 26°51'.$$

5.11 In a grating spectrum, which spectral line in the 4th order will overlap with 3rd order line of wavelength 5896 Å ? **[GGSIPU, Sept. 2008 (2 marks)]**

Hint : If λ_1 be the wavelength in the order n_1 coincides with wavelength λ in the order n, then

$$(a+b)\sin\theta = n\lambda = n_1\lambda_1$$

$$\Rightarrow \quad \lambda_1 = \frac{5896\times 3}{4} = 4422\,\text{Å}.$$

5.12 In a grating spectrum, which spectral line in 4th order will overlap with 3rd order line of 5461 Å ? **[GGSIPU, Dec. 2013 (4 marks)]**

Hint : $n\lambda = n_1\lambda_1$

$$\Rightarrow \quad \lambda_1 = \frac{n\lambda}{n_1} = \frac{3\times 5461}{4}\,\text{Å} = 4096\,\text{Å}.$$

5.13 Light is incident normally on a grating 0.5 cm wide with 2500 lines. Find the angle of diffraction of two sodium lines in the first order spectrum. Are two lines resolved ? **[GGSIPU, Sept. 2005 (4 marks)]**

Hint : $(a+b)\sin\theta = n\lambda$

or $\quad \sin\theta = \dfrac{\lambda}{(a+b)} \quad$ [as $n = 1$]

Here $\quad (a+b) = \dfrac{0.5}{2500} = \dfrac{5}{25000} = 2\times 10^{-4}$ cm.

Angle of diffraction for D_1 line *i.e.,* $\lambda_1 = 5890\,\text{Å}$, $\theta_1 = 17.13°$

and Angle of diffraction for D_2 line *i.e.,* $\lambda_2 = 5896\,\text{Å}$, $\theta_2 = 17.15°$

Yes, two lines will be just resolved.

5.14 A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having lines per cm and second order spectral line is found to be diffracted through 30°. Calculate wavelength of light. **[GGSIPU, Sept. 2008 (3 marks)]**

Hint : $(a+b) = \dfrac{1}{500}$, $\sin 30° = \dfrac{1}{2}$, $n = 2$

$$(a+b)\sin\theta = 2\lambda.$$

$$\Rightarrow \quad \lambda = \frac{(a+b)\sin\theta}{2} = \frac{1}{500}\times\frac{1}{2}\times\frac{1}{2} = 5000\times 10^{-8}\text{ cm} = 500\text{ nm}.$$

5.15 Deduce the missing order for a double slit Fraunhofer diffraction pattern, if the slit width is 0.25 mm and they are 0.5 mm apart.

Hint : $\dfrac{(a+b)}{a} = \dfrac{n}{m} \quad \Rightarrow \quad n = 3\text{ m}$

for $m = 1, 2, 3, \ldots$; $\quad n = 3, 6, 9, \ldots$

Hence the 3rd, 6th, 9th, ... orders of the interference maxima will be missing from the diffraction pattern.

5.16 What particular spectra of plane transmission grating would be absent if the width of the transparencies and opacities of the grating are equal ?

[GGSIPU, Sept. 2012 ; Nov. 2012 (2 marks) ; Sept. 2013 reappear (2 marks) ; Dec. 2017 (2.5 marks)]

Hint : $\dfrac{a+b}{a} = \dfrac{n}{m} \quad \Rightarrow$ here $a = b$.

Then $\quad \dfrac{2a}{a} = \dfrac{n}{m} \quad \Rightarrow \quad n = 2\text{m}$

for $m = 1,2,3,4,\ldots$ and $n = 2,4,6,8,\ldots$

5.17 In a diffraction grating the width of opacities and transparencies are in the ratio of 1 : 2. Find out the absent spectra. **[GGSIPU, Dec. 2012 (3 marks)]**

Or

A plane transmission grating with 5000 lines per cm, which order of spectra will be absent if width of opaque spaces are exactly 2.0 times with width of transparent spaces.

Hint : $\dfrac{a+b}{a} = \dfrac{n}{m} \quad \Rightarrow$ here $b = 2a$

then $\quad \dfrac{3a}{a} = \dfrac{n}{m} \quad \Rightarrow \quad n = 3m$

for $\quad m = 1, 2, 3, 4, \ldots\ldots$

$\quad n = 3, 6, 9, \ldots\ldots.$

5.18 Each slit has a width of 0.15 mm and distance between their centre is 0.75 mm. What are missing order ? **[GGSIPU, Dec. 2010 (3.5 marks)]**

Hint : Given $a = 0.15$ mm and $(a+b) = 0.75$ mm, then condition of absent spectra

$$\frac{a+b}{a} = \frac{n}{m} \quad \Rightarrow \quad n = 5\text{m}, \quad \text{so for } m = 1, 2, 3\ldots$$

$n = 5, 10, 15, \ldots\ldots$ will be absent.

5.19 In a diffraction grating the width of opacities and transparencies are in the ratio 1 : 3. Find out the absent spectra. **[GGSIPU, Dec. 2011 (3 marks)]**

Hint : $\dfrac{b}{a} = \dfrac{3}{1} \quad \Rightarrow \quad b = 3a$, then $\dfrac{a+b}{a} = \dfrac{n}{m}$

$$\frac{n}{m} = \frac{4a}{a} = 4 \quad \Rightarrow \quad n = 4m$$

For $\quad m = 1, 2, 3, \ldots$; the value of $n = 4, 8, 12, \ldots\ldots$ are absent.

5.20 A plane transmission grating has 40000 times in all with grating element 12.5×10^{-5} cm. If the maximum resolving power of the grating is 80000, find out the range of wavelength for which it can be used.

Hint : $\frac{\lambda}{d\lambda} = nN, \quad n = \frac{80000}{40000} = 2$

For maximum, $(a+b)\sin 90° = n\lambda \Rightarrow \lambda = 6500$ Å

Smallest wavelength difference,

$$\frac{\lambda}{d\lambda} = nN \Rightarrow d\lambda = \frac{\lambda}{nN} = 0.08 \text{Å}.$$

Hence $\lambda_1 = 6500$Å, $\lambda_2 = (6500 + 0.08)$Å.

5.21 D_1 and D_2 lines of sodium are 6Å apart. What should be minimum number of lines in a diffraction grating to resolve them ? **[GGSIPU, Dec. 2007 (2 marks)]**

Hint : $\lambda_1 = 5890$Å and $\lambda_2 = 5896$Å

$$d\lambda = \lambda_1 - \lambda_2 = 6\text{Å}$$

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = 5893 \text{ Å}, \quad \text{R.P.} \frac{\lambda}{d\lambda} = nN$$

$$\Rightarrow \quad \frac{\lambda}{d\lambda} = \frac{5893 \times 10^{-8}}{6 \times 10^{-8}} = 982 = nN$$

For $n = 1$, $N = \frac{982}{1} = 982.$

The two lines will be just resolved in first order with minimum of 982 lines.

5.22 A grating having 15000 lines per inch produces spectra of a mercury arc. The green line of the mercury spectrum has a wavelength of 5461 Å. What is the angular separation between the first order green line and the second order green line ? **[GGSIPU, Sept. 2007 (2 marks)]**

Hint : $(a+b)\sin\theta = n\lambda.$

Here $(a+b)$ [grating element] $= \frac{2.54}{15000}$

$$\lambda = 5461 \times 10^{-8} \text{ cm}$$

Then $\sin\theta = \frac{5461 \times 10^{-8} \times 15000}{2.54} n$

For $n = 1$, $\theta = \theta_1$, $\theta_1 = 18.81°$

For $n = 2$, $\theta = \theta_2$, $\theta_2 = 40.17°$

Angular separation of lines $= \theta_1 - \theta_2 = 21.36°$]

5.23 Light is incident normally on a grating 0.5 cm wide with 2500 lines. Find the angle of diffraction of two sodium lines in the first order spectrum. Are two lines resolved ?

[GGSIPU, Sept. 2005 (4 marks)]

Hint : $(a+b)\sin\theta = n\lambda$

or $\sin\theta = \frac{\lambda}{(a+b)}$ [as $n = 1$]

Here $(a+b) = \frac{0.5}{2500} = \frac{5}{25000} = 2 \times 10^{-4}$ cm.

Angle of diffraction for D_1 line *i.e.*, $\lambda_1 = 5890$Å, $\theta_1 = 17.13°$

and Angle of diffraction for D_2 line *i.e.*, $\lambda_2 = 5896$Å, $\theta_2 = 17.15°$

Yes, two lines will be just resolved.

5.24 A plane transmission grating of length 6 cm has 5000 lines per cm. Calculate the smallest wavelength difference that can be resolved for light of wavelength 5000 Å. **[GGSIPU, Sept. 2009 (4 marks)]**

Hint : $(a+b)\sin\theta = n\lambda \Rightarrow n_{max} = \frac{(a+b)}{\lambda} = \frac{1}{N\lambda} = 4$ and

Resolving power (R.P.) $= \frac{\lambda}{d\lambda} = nN$

or $(d\lambda)_{min} = \frac{\lambda}{n_{max} N} = 0.04$ Å

Multiple Choice Questions

5.1 In a plane transmission grating the angle of diffraction for the second order principal maxima for the wavelength 5×10^{-5} cm is 30°. The number of lines in one cm of the grating surface will be :

(*a*) 100 (*b*) 10 (*c*) 1000 (*d*) 5000

5.2 A grating resolves a given doublet in the first order and the other grating of same width resolves the same doublet in the second order. The ratio of number of lines on the two grating is :

(*a*) 1 : 2 (*b*) 1 : 4
(*c*) 2 : 1 (*d*) 4 : 1

5.3 A plane diffraction grating is 5 cm wide and has 5000 liens per cm. The resolving power of the grating in the third order spectrum is :

(*a*) 5000 (*b*) 10,000
(*c*) 25,000 (*d*) 75,000

5.4 The aperture of the objective of a microscope had diameter of 0.8 cm. The object is illuminated with visible light of wavelength 4040 Å. The maximum limit of resolution for the microscope is :

(*a*) 5.05×10^{-5} rad (*b*) 6.16×10^{-5} rad
(*c*) 4.1×10^{-5} rad (*d*) 3.2×10^{-5} rad.

5.5 Two spectral lines of a light source are coincident in the second and third order of the grating. If the wavelength of the first lines is λ, wavelength of the second line is :

(*a*) $\frac{\lambda}{2}$ (*b*) λ
(*c*) $\frac{2}{3}\lambda$ (*d*) $\frac{1}{3}\lambda$

5.6 A monochromatic beam of light of wavelength 5460 Å falls on the grating normally and gives a second order image at an angle of 45°. The grating element is of the order of :

(*a*) 10^{-2} cm (*b*) 10^{-3} cm (*c*) 10^{-4} cm (*d*) 10^{-5} cm

5.7 A zone plate behaves like a convex lens of focal length 50 cm for a light of wavelength 5000 Å. The radius of the first half period zone is :

(*a*) 5 mm (*b*) 0.5 mm (*c*) 1 mm (*d*) 1.5 mm

5.8 The power of an optical instrument by which it can form separate images of two close objects is called :

(*a*) dispersive power (*b*) magnifying power

(*c*) resolving power (*d*) diopter.

5.9 The grating formula is given by :

(*a*) $(a+b)\sin\theta = n\lambda$ (*b*) $(a+b)\cos\theta = n\lambda$

(*c*) $(a+b)\sin\theta = \left(n+\frac{1}{2}\right)\lambda$ (*d*) $(a+b)\sin\theta = 0$.

(where letters have their usual meanings)

5.10 Fresnel's half period zone differ from each other by a phase difference of :

(*a*) 2π (*b*) π (*c*) $\frac{\pi}{2}$ (*d*) $\frac{\pi}{4}$

5.11 The principal focal length of a zone plate is given by :

(*a*) $f = \frac{r_n^2}{n\lambda}$ (*b*) $f = \frac{r_n}{n\lambda}$

(*c*) $f = \frac{r_n}{n\lambda^2}$ (*d*) $f = \frac{1.22 r_n}{n\lambda}$

5.12 A plane transmission grating having 5000 lines per cm is being used under normal incidence of light. The highest order spectrum that can be seen for the light of wavelength 4800 Å.

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

5.13 In a single slit diffraction pattern, for a slit width (*d*) and wavelength (λ), the separation between central maximum and first minimum is :

(*a*) $\theta = \frac{\lambda}{d}$ (*b*) $\theta = \frac{\lambda}{2d}$

(*c*) $\theta = \frac{\lambda}{4d}$ (*d*) $\theta = \frac{\pi}{2}$.

5.14 A plane transmission grating is 5 cm wide and has 5000 lines per cm. The resolving power of the grating is second order spectrum is :

(*a*) 5000 (*b*) 10,000 (*c*) 25,000 (*d*) 50,000

5.15 In the spectrum formed by a diffraction grating, the ratio of intensity of a secondary maximum adjacent to a principal maximum is :

(*a*) between 1 and 0.01 (*b*) between 0.1 and 0.01

(*c*) between 0.01 and 0.001 (*d*) less than 0.001.

5.16 The grating element of a 2.0 cm wide plane grating, which resolves two lines, in second order, differing in wavelength by 6 Å, of mean wavelength 6000 Å, will be :

(*a*) 4×10^{-3} m (*b*) 3×10^{-3} m

(*c*) 2×10^{-3} m (*d*) 1×10^{-3} m

5.17 The angular separation, between the two lines of neon, 5882 Å and 5852 Å, in the first order spectrum of a plane transmission grating, having 6000 lines per cm, illuminated normally, will be :

(*a*) 1.7×10^{-3} rad (*b*) 1.8×10^{-3} rad

(*c*) 1.9×10^{-3} rad (*d*) 2.0×10^{-3} rad

5.18 The resolving power of a prism can be expressed as :

(*a*) $\frac{\lambda}{d\lambda} = nN$ (*b*) $\frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$

(*c*) $\frac{1}{\theta} = \frac{2\mu\sin\theta}{1.22\lambda}$ (*d*) $\frac{1}{\theta} = \frac{a}{1.22\lambda}$

Answers

5.1 (*d*)	5.2 (*c*)	5.3 (*d*)	5.4 (*b*)	5.5 (*c*)	5.6 (*d*)
5.7 (*b*)	5.8 (*c*)	5.9 (*a*)	5.10 (*b*)	5.11 (*a*)	5.12 (*d*)
5.13 (*a*)	5.14 (*d*)	5.15 (*b*)	5.16 (*c*)	5.17 (*b*)	5.18 (*b*)

TUTORIAL

5.1 A lens of focal length 100 cm forms Fraunhofer diffraction pattern of a single slit of width 0.04 cm in its focal plane. The incident light contains two wavelengths λ_1 and λ_2. It is found that fourth minimum corresponding to λ_1 and fifth minimum corresponding to λ_2 occur at the same point 0.5 cm from the central maxima. Compute λ_1 and λ_2. **[Ans.** $\lambda_1 = 500$ nm, $\lambda_2 = 400$ nm**]**

5.2 A plane transmission grating has 6000 lines per cm. Calculate the highest order of spectrum, which can be observed with light of wavelength 400 nm. **[Ans. 4]**

5.3 A parallel beam of sodium light is normally incident on a plane transmission grating having 4250 lines per cm and a second order spectral line is observed at an angle of 30°. Calculate the wavelength of the light. **[Ans.** $\lambda = 5882$ Å**]**

5.4 In a single slit diffraction pattern the intensity of the successive bright fringes fall off as we go out from the central maximum. Approximately, which fringe number has a peak intensity that is 1/2 percent of the central fringe ? (Assume Fraunhofer diffraction applies). **[Ans. 4]**

5.5 A parallel beam of wavelength 500 nm falls on a grating with 6000 lines per cm. Find the highest order of grating spectrum. **[Ans.** $n = 3$**]**

5.6 Calculate the angles at which the first dark band and the next bright band are formed in the Fraunhofer diffraction pattern of a slit 0.3 mm wide ($\lambda = 5890$ Å) **[Ans. 0.2°]**

5.7 A parallel beam of light of wavelength 5460 Å is incident at an angle of 30° on a device containing a large number of parallel slits which has 6000 lines/cm. Find the highest order spectrum that can be observed. **[Ans. 2]**