If the slit S_2 is parallel to S_1, the vibrations pass through S_2 and reach the person B with undiminished amplitude [Fig. 6.1(*a*)]. If, however S_2 is rotated to become perpendicular to S_1, the vibrations are stopped by S_2 as the displacement of particles is now at right angles to the length of S_2 [Fig. 6.1(*b*)]. The string therefore does not vibrate between S_2 and B. In the intermediate positions of the slit S_2, the vibrations will be partially transmitted and partially stopped *i.e.*, they will reach the position B with diminished amplitude. Thus we conclude that as the slit S_2 is rotated, the amplitude of vibrations 'varies'. The variation in amplitude is only due to vibrations in the string being 'transverse'. Had the vibrations been longitudinal *i.e.*, parallel to the length of the string, they would have been freely allowed by S_2 in all positions. Thus the rotation of S_2 would not have produced any change in the amplitude of emergent vibrations.

Another important point to note is that in the above experiment the vibrations of the particles of the string are taking place in *one direction only* ; namely, parallel to the slit S_1. Hence they pass through S_2 with undiminished amplitude only in one particular position of S_2, namely, when S_2 is parallel to S_1. In other positions S_2, the amplitude is diminished. If the vibrations in the string were present in all possible directions perpendicular to the length of the string, they could pass through S_2 in its all possible positions. Thus variation of amplitude, when S_2 is rotated, indicates two facts regarding the vibrations of the string.

(*i*) The vibrations are transverse.

(*ii*) The vibrations are not symmetrical about the length of the string, but confined to a *single direction* perpendicular to the length of the string.

6.1 FUNDAMENTAL CONCEPTS OF POLARISATION

6.1.1 Polarisation of Light

When ordinary light is incident normally on a pair of parallel tourmaline crystal plates A and B (Fig. 6.2) cut parallel to their crystallographic axis[1] the emergent light shows a variation as B is rotated. The intensity is a maximum when the axis of B is parallel to that of A as shown in Fig. 6.2(*a*), and minimum when at right angles as shown in Fig. 6.2(*b*). This shows that the light emerging from A is not symmetrical about the direction of propagation of light, but its vibrations are confined only a *single line* in a plane perpendicular to the direction of propagation. Such light is called '*plane polarised*' or '*linearly polarised*' light.

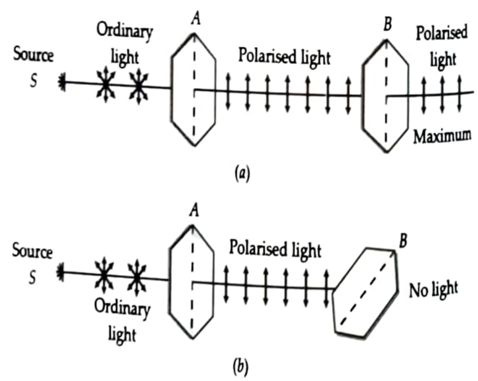


Fig. 6.2 Experiment with tourmaline crystals for polarisation.

1. The crystallographic axis is not a fixed line in the crystal but merely a direction and any line parallel to it may be taken as the axis.

According to the 'electromagnetic theory of light', a light wave consists of electric and magnetic vectors vibrating in mutually perpendicular planes, both being perpendicular to the direction of propagation of light. The electric vector acts as the *light vector*. Hence the plane polarised light is the light in which the light vector vibrate along a fixed straight line in a plane perpendicular to the direction of propagation.

6.1.2 Plane of Vibration and Plane of Polarisation

The properties of plane polarised beam differ with respect to two planes, one containing the vibrations and other perpendicular to it. The plane in which the vibrations take place *i.e.*, the plane containing the direction of vibration and the direction of propagation, is called *plane of vibration*. The plane *PQRS* is the *plane of vibration* as shown in Fig. 6.3.

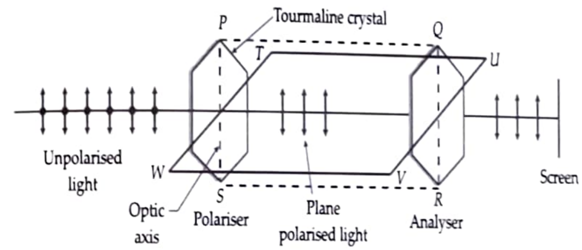


Fig. 6.3 Production of plane polarised light.

A plane perpendicular to the plane of vibration is called the *plane of polarisation*. Thus the plane of polarisation is the plane passing through the direction of propagation and containing no vibrations. The plane *TUVW* in Fig. 6.3 is the *plane of polarisation*.

6.1.3 Pictorial Representation of Light Vibrations

Ordinary light from a source, also called unpolarised light, consists of a very large number of wavelengths with vibrations in all possible planes with equal probability. Hence its pictorially represented end view would be as shown in Fig. 6.4(*a*). Now the vibrations may be linear, circular or elliptical. The later two may be regarded as made up to two linear rectangular vibrations at right angles to each other. Thus a beam of ordinary light can be considered as consisting of two sets of vibrations. One set vibrating in one plane and the other perpendicular to it. If the direction of propagation of the beam is in the plane of paper, these two sets of waves vibrating perpendicular to each other. One in the plane of incidence and other at right angled to it, may be represented respectively by arrows and dots as shown in Fig. 6.4(*b*). Hence Fig. 6.4(*b*) can be taken to represent an ordinary beam of light.

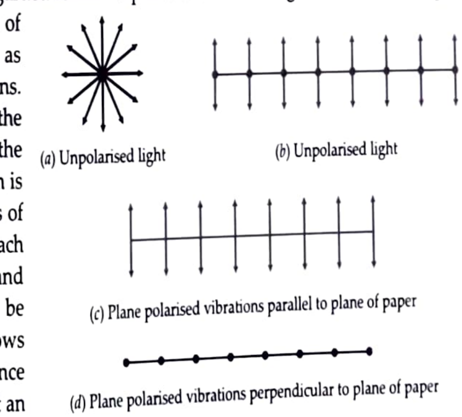


Fig. 6.4 Pictorial representation of light vibrations.

In a plane polarised beam of light, the vibrations are along a single straight line. When the plane polarised light has got vibrations in the plane of the paper, they are represented by arrows as shown in Fig. 6.4(*c*). When the vibrations lie in a direction perpendicular to the plane of the paper, they are represented by dots as visualised in Fig. 6.4(*d*).

6.2 TYPES OF POLARISATION

Polarisation may be categorized into *three* :

- Plane polarised light
- Circularly polarised light
- Elliptically polarised light.

(i) Plane Polarised Light

If in a polarised light, the electric vector vibrates in a fixed straight line perpendicular (normal) to the direction of propagation of light, it is said to be plane polarised light (PPL). The transverse electric field wave is accompanied by a magnetic field wave as illustrated in Fig. 6.5.

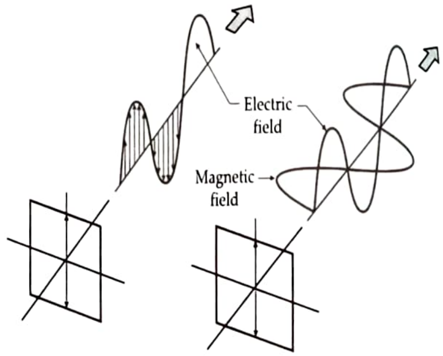


Fig. 6.5 Plane polarised light.

(ii) Circularly Polarised Light

When two plane polarised light waves are superimposed then under certain conditions, the resultant *light vector* rotates with a constant magnitude in a plane perpendicular (normal) to the direction of propagation of light ; the tip of the vector space traces a *circle* and the light is said to be *circularly polarised light*. Circularly polarised light consists of two perpendicular electromagnetic plane waves of equal amplitude and with 90° difference in phase. The light illustrated in Fig. 6.6 is right *circularly polarised*.

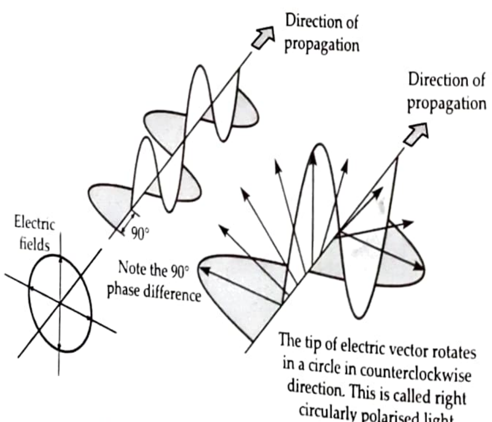


Fig. 6.6 Right-circularly polarised light.

(iii) Elliptically Polarised Light

When two plane polarised light waves are superimposed, then under certain conditions, the resultant light vector rotates the *a* plane perpendicular (normal) to the direction of propagation of light ; the tip of vector traces an *ellipse* and the light is said to be *elliptically polarised light*. Elliptically polarised light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°.

Figure 6.7 shows the right *elliptically polarised light*.

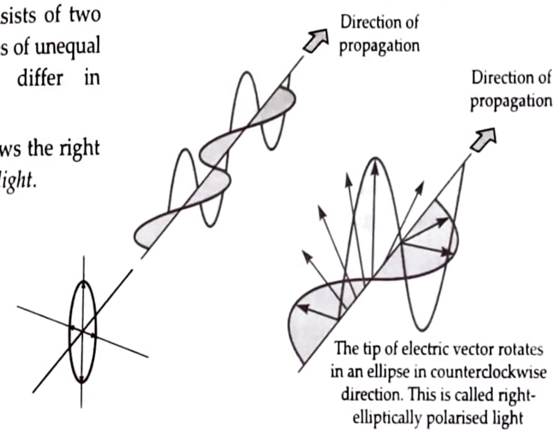


Fig. 6.7 Right-elliptically polarised light.

The three states of polarisation are shown in Fig. 6.8.

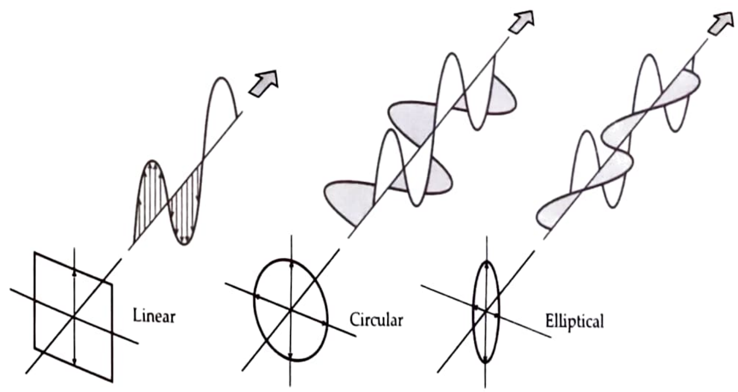


Fig. 6.8 State of polarisation – Linear, circular, and elliptical.

6.3 METHODS TO PRODUCE PLANE POLARISED LIGHT

Various methods to produce plane polarised light may be broadly summarized as below :

1. Polarisation by reflection (*e.g.*, Biot Polariscope)
2. Polarisation by refraction (*e.g.*, Piles of Plates Method)
3. Polarisation by double refraction (*e.g.*, Nicol prism and double image prisms)
4. Polarisation by scattering
5. Polarisation by selective absorption by crystals (*e.g.*, Polaroids)

These are discussed below :

6.3.1 Polarisation by Reflection

In 1808, Etienne Malus discovered that ordinary light when reflected from a plane sheet of glass gets partially polarised. The degree of polarisation varies with angle of incidence. At a particular angle (i_p), known as *angle of polarisation* (i_p), the percentage of polarisation is maximum (Fig. 6.9). The angle of polarisation slightly depends upon the nature of reflecting surface and wavelength of light.

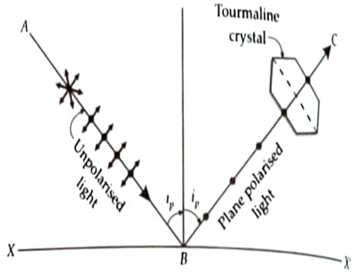


Fig. 6.9 Polarisation by reflection.

Brewster's Law

In 1811, David Brewster performed number of experiments to study the polarisation of light by reflection. He found that at a particular angle i_p the light is completely polarised in the plane of incident ray. He also found that the value of i_p depends upon the refractive index of the reflecting medium. Brewster discovered a relation

$$\mu = \tan i_p \quad \text{...(6.1)}$$

which is known as *Brewster's law*. It also came into picture that reflected and refracted rays are perpendicular to each other.

Suppose a beam AB of the unpolarised light is incident on the glass surface at polarising angle i_p. It is reflected along BC and refracted along BD as shown in Fig. 6.10.

Then from Brewster's law

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p}$$

From Snell's law

$$\mu = \frac{\sin i_p}{\sin r}$$

$$\therefore \quad \frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r}$$

or

$$\frac{\sin i_p}{\sin\left(\frac{\pi}{2} - i_p\right)} = \frac{\sin i_p}{\sin r}$$

or

$$\frac{\pi}{2} - i_p = r$$

or

$$i_p + r = \frac{\pi}{2} \quad \text{...(6.2)}$$

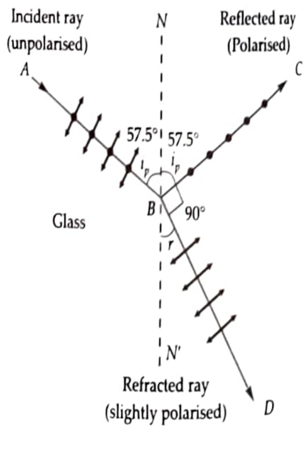


Fig. 6.10 An illustration for Brewster's law.

As $\angle NBN' = \pi$

$i_p + \angle CBD + r = \pi$ or $\angle CBD = \pi - (i_p + r) = \frac{\pi}{2}$

i.e., the polarising angle, the reflected ray is at right angle to the refracted ray.

Example 6.1 *When the angle of incidence on a certain material is 60°, the reflected light is completely polarised. Find the refractive index for the material and also the angle of refraction.*

Solution. By using Brewster's law

$$\mu = \tan i_p$$

Here $\quad i_p = 60°$

$\therefore$ Refractive index of the material

$$\mu = \tan 60° = \sqrt{3} = 1.732$$

Angle of refraction $= 180° - (90° + 60°) = 30°$

Example 6.2 *A glass plate is to be used as a polariser. Find the angle of polarisation. Also find the angle of refraction. Given μ for glass = 1.54.* **[GGSIPU, Sept. 2008 (2 marks) ; Dec. 2013 reappear (4 marks)]**

Solution. We know that from Brewster's law,

$$\tan i_p = \mu \quad \text{or} \quad i_p = \tan^{-1}(\mu) = \tan^{-1}(1.54) = 57°$$

If r is the angle of refraction, then from Brewster's law

$$i_p + r = \frac{\pi}{2}$$

$$r = \frac{\pi}{2} - i_p = 90° - 57° = 33°.$$

NOTE

r may alternately be found by using Snell's law $\frac{\sin i_p}{\sin r} = \mu$.

Example 6.3 *Find the angle of incidence for which we get plane polarised light by reflection from a glass plate (μ = 1.5697).* **[GGSIPU, Sept. 2013 reappear (2 marks)]**

Solution. Given $\mu = 1.5697$

From Brewster's law, $\mu = \tan i_p$

or $\quad i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5697) = 57.5°$

6.3.2 Polarisation by Refraction

It is found that when ordinary light gets refracted through any transparent medium, the refracted ray is partial polarised. In order to obtain completely polarised light, it is refracted through piles of plates which consists of adequate number of glass plates separated by airgaps. After multiple refraction through this arrangement the emerging light gets completely polarised.

Malus Law

In 1809, French army engineer E.L. Malus discovered a law regarding the intensity of light transmitted by the analyser.

According to **Malus :**

"When a completely plane polarised light is incident on an analyser, the intensity of the emergent light varies as the square of the cosine of the angle between the planes of transmission of the analyser and the polariser".

This law holds good for combination of reflecting surfaces, polarising and analysing tourmaline crystals, Nicol prism etc., but fails when the light is completely polarised.

Proof. Let $OP = a$ (Fig. 6.11) be the amplitude of the incident plane polarised light from a polariser and θ is the angle between plane of polariser and plane of analyser.

The amplitude of incident plane polarised light can be resolved into two components.

(i) One parallel to the plane of transmission of analyser $(a\cos\theta)$

(ii) The other perpendicular to it $(a\sin\theta)$

The component $(a\cos\theta)$ is transmitted through the analyser.

$\therefore$ Intensity of the transmitted light through analyser

$$I = (a\cos\theta)^2$$

$$= a^2\cos^2\theta \quad [\because \text{Intensity} \propto (\text{amplitude})^2]$$

If I_0 be the intensity of incident polarised light, then

$$I_0 = a^2$$

$$\therefore \quad I = I_0\cos^2\theta$$

or
$$I \propto \cos^2\theta \qquad \text{...(6.5)}$$

(i) When $\theta = 0$, *i.e.*, two planes are parallel.

$$I = I_0 \qquad [\text{as } \cos\theta = 1]$$

(ii) When $\theta = \pi/2$, *i.e.*, two planes are perpendicular

$$I = 0$$

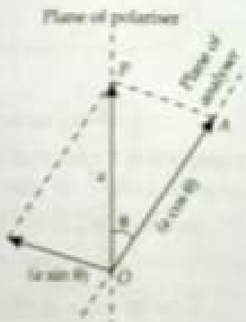


Fig. 6.11 Malus law.

The above results are experimentally observed in case of two tourmaline crystals.

Example 6.4 *Two polarising sheets have their directions parallel so that the intensity of transmitted light is maximum. Through what angle must either sheet be turned so that the intensity becomes one half of the initial value.*

Solution. Given : $I = I_0/2$.

We know that Malus law, $I = I_0\cos^2\theta$

Hence $\quad \dfrac{I_0}{2} = I_0\cos^2\theta \quad \therefore \quad \cos\theta = \pm\dfrac{1}{\sqrt{2}} \quad$ or $\quad \theta = 45°$ or $135°$

Example 6.5 *Intensity of light through a polariser and analyser is maximum, when their principal planes are parallel. Through what angle the analyser must be rotated so that the intensity gets reduced to 1/4 of the maximum value.*

Solution. Given $I = I_0 - \dfrac{I_0}{4} = \dfrac{3}{4}I_0$

Then from Malus law, $I = I_0\cos^2\theta$

Hence $\quad \cos^2\theta = \dfrac{3I_0/4}{I_0} = \dfrac{3}{4} \Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2} \quad$ or $\quad \theta = 30°$.

6.3.3 Polarisation by Double Refraction

(i) Doubly Refracting Crystals

There are certain crystals which split a ray of light incident upon them into two refracted rays. Such crystals are called '*doubly refracting crystals*'. They are of two types : 'uniaxial' and 'biaxial'.

In uniaxial crystals there are one direction, called 'optic axis' along which the two refracted rays travel with the same velocity. Examples : calcite, tourmaline and quartz. In *biaxial crystals* there are two optic axes. *Examples* : topaz, aragonite, mica and cane sugar.

(a) Geometry of Calcite Crystal (Uniaxial)

The calcite crystal, also known as Iceland spar $(CaCO_3)$, is a colourless crystal, transparent to visible as well as to ultraviolet light. It occurs in nature in various forms ; all of which readily break up into simple rhombohedron as shown in Fig. 6.12. Each face of the crystal is a parallelogram having angles 102° and 78°. At the two diametrically opposite corners A and B, three obtuse angles meet.

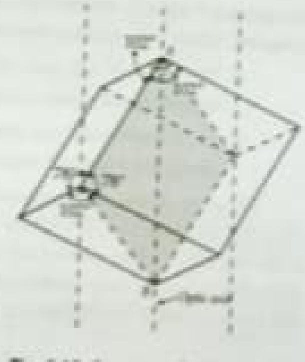


Fig. 6.12 Geometry of calcite crystal.

These are called the 'blunt' corners of the crystals. At the rest of the six corners ; one angle is obtuse and two are acute.

(b) Optic Axis of the Calcite Crystal

A line passing through any one of the blunt corners and making equal angles with the three faces which meet there, is the direction of the 'optic axis' of the crystal. Optic axis is a direction and not a line. Hence any line parallel to the one described above (Fig. 6.12) represents the optic axis.

(c) Principal Section of the Calcite Crystal

A plane containing the optic axis and perpendicular to the pair of opposite faces of the crystal is called the 'principal section' of the crystal for that pair of faces. Thus there are principal sections passing through any point inside the crystal, one corresponding to each pair of opposite faces. A principal

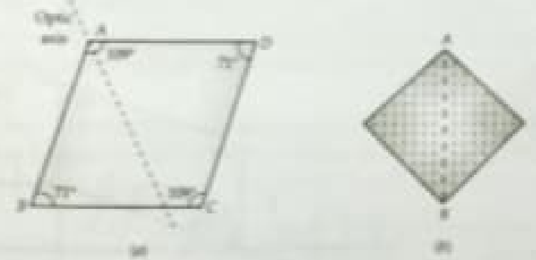


Fig. 6.13 Principal section of the crystal.

section always cuts surfaces of the calcite crystal in a parallelogram having angle 71° and 109° as shown in Fig. 6.13(*a*). An end view of principal section perpendicular to a pair of opposite faces of the crystal cuts these faces in a line parallel to the shorter diagonal of these faces. In Fig. 6.13(*b*) is shown a face of the crystal in which the dotted line AB represents the end views of the other principal sections parallel to that in Fig. 6.13(*b*).

In 1669 Erasmus Bartholinus, while studying the polarisation phenomenon through different types of crystals, discovered that when a beam of ordinary light is passed through a calcite crystal, the refracted light is splitted up into two refracted rays. The phenomenon is said to be double refraction. These two splited rays are known as *ordinary* and *extraordinary rays* respectively.

- *The ordinary ray*, one which always obeys the ordinary laws of refraction and having vibrations perpendicular to principal section is known as *ordinary ray (o-ray)*.
- *The extraordinary ray*, one which does not obey the laws of refraction and have vibrations along the principal section is known as *extraordinary ray (e-ray)*.

The *o*-ray and *e*-ray have been shown by dots and lines with arrow heads respectively. Since *o*-ray travels in the crystal with the same speed in all the directions *i.e.*, refractive index for the *o*-ray is having single value while *e*-ray travels in the crystal with a speed that varies with direction *i.e.*, refractive index for *e*-ray varies with the direction. The difference between the refractive indices for *o*- and *e*-rays is called *birefringence i.e.*,

$$\mu_o - \mu_e = \text{birefringence}$$

Both *o*- and *e*-rays travel with same speed along optic axis. In uniaxial crystals the wavefront of *o*-ray is spherical while the wavefront of *e*-ray is ellipsoidal in nature.

Figure 6.14(*b*) represents the phenomenon of double refraction. A ray of light is incident normally on a crystal, a principal section of which is visualised. The ray is split up into two rays *o*- and *e*-rays. The *o*-ray passes through the crystal undeviated while *e*-ray is refracted at some angle. As the opposite faces of the crystal are parallel, the ray emerge parallel to the incident ray, but relatively displaced by a distance proportional to the thickness of the crystal. Inside the crystal the *o*-ray always lies in the plane of incident, while the *e*-ray does not. Only when the plane of incident is a principal section, the *e*-ray also lie in the plane of incidence.

If the face *AB* of the crystal is placed on a dot made on a paper, on looking through the opposite face, two images *o* and *e*, corresponding to *o*- and *e*-rays are observed as shown in Fig. 6.14(*c*). The

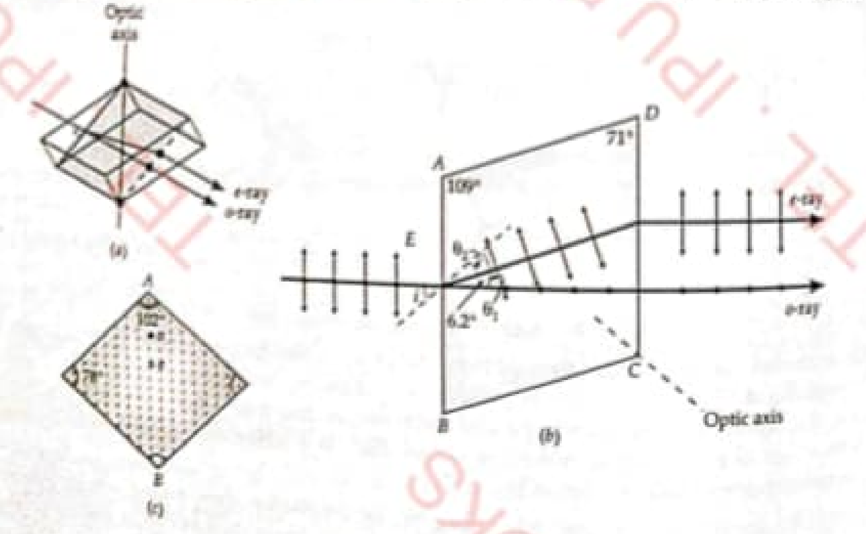


Fig. 6.14 Extraordinary and ordinary rays.

line joining them is in a principal section of the crystal and lies either along the shorter diagonal *AB* or along a line parallel to *AB*. *When the crystal is rotated about the o-image, the e-image remains stationary while the e-image moves round it in a circle remaining always on the shorter diagonal.*

For ordinary ray the refractive index $\mu_o = \dfrac{\sin i}{\sin \theta_1}$ is constant, while for extraordinary ray $\mu_e = \dfrac{\sin i}{\sin \theta_2}$ is not constant, but varies with the angle of incident *i*.

- In calcite crystal $\theta_2 > \theta_1$, therefore $\mu_o > \mu_e$, hence $v_o < v_e$ that is, inside calcite crystal the *e*-ray travels faster as compared to *o*-ray. Such crystals are called ***uniaxial negative crystal***.
- In quartz crystal $\theta_2 < \theta_1$, therefore $\mu_e > \mu_o$, hence $v_o > v_e$ *i.e.*, inside the quartz crystal the *o*-ray travels faster as compared to *e*-ray. Such crystals are called ***uniaxial positive crystals***.

(i) *Huygen's Theory of Double Refraction in Uniaxial Crystals*

Huygen explained the phenomenon of double refraction on the basis of wave theory. We know that each point on a wavefront is a source of the secondary wavelets. When a wavefront strikes, an isotropic medium like glass, secondary wavelets travel inside it with a velocity equal to the velocity of light in glass. The envelope of all these secondary wavelets at any instant represents a wavefront which is the locus of particles vibrating in the same phase. S is a source of light in uniaxial crystal.

In order to explain the phenomenon of double refraction in uniaxial crystals, Huygen extended his theory of secondary wavelets. The postulates of the theory are given below :

1. When light waves are incident on a doubly refracting crystal, every point of it becomes a source of secondary wavelets and sends out not one but two wavefronts, one for *o*-ray and other for *e*-ray.
2. For the *o*-ray, the crystal is isotropic and homogeneous. Hence *o*-ray travels with the same velocity in all direction and the wavefront (or wave-surface) corresponding to it is *spherical*.
3. For the *e*-ray, the crystal is anisotropic (not having identical properties in different directions). Hence its velocity varies with the directions and the extraordinary wave surface cannot be sphere, it is a *spheroid* or *ellipsoid of revolution*.

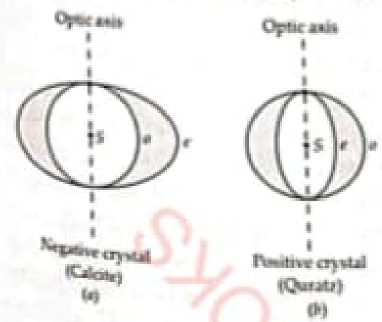


Fig. 6.15 Uniaxial crystals.

4. The spherical wavefront corresponding to the *o*-ray and the ellipsoid of revolution corresponding to the *e*-ray touch each other at two points. The direction of the line joining these two points is the optic axis.
5. In a negative uniaxial crystal (like calcite) the sphere lies inside the ellipsoid [Fig. 6.15(*a*)], while in a positive uniaxial crystal (like quartz) the ellipsoid lies inside the sphere [Fig. 6.15(*b*)].

(iii) Difference between Negative and Positive Uniaxial Crystals

S.No.	Negative crystal	Positive crystal
1.	The wavefronts surrounding a point source S in such a crystal is shown in Fig. 6.15(*a*)	The wavefront surrounding a point source S in such a crystal is shown in Fig. 6.15(*b*).
2.	Ordinary wave surface lies inside the extra-ordinary wave surface.	The ordinary wave surface lies outside the extraordinary wave surface.
3.	Velocity of *o*-ray is constant in all directions.	The velocity of *o*-ray is constant in all direction.
4.	The velocity of *e*-ray varies with the direction. It is minimum and equal to the velocity of the ordinary ray along the optic axis. It is maximum in a direction perpendicular to the direction of the optic axis.	The velocity of *e*-ray varies with the direction. It is maximum and equal to the velocity of the *o*-ray along the optic axis. It has a minimum value in a direction perpendicular to the optic axis.
5.	The refractive index for the *e*-ray is less than the refractive index for the *o*-ray *i.e.,* $\mu_e < \mu_0$	The refractive index for *e*-ray is more than the refractive index for the *o*-ray *i.e.,* $\mu_e > \mu_0$
6.	*Example.* Calcite	*Example.* Quartz

6.4 NICOL PRISM

Nicol prism is an optical device, invented by William Nicol in 1826 for producing and analysing plane polarised light.

Principle. It is based on the phenomenon of double refraction. We know that when an unpolarised ray is passed through a doubly refracting uniaxial crystal, it is broken up into *two* rays :

➢ Ordinary ray and

➢ Extraordinary ray

Both are polarised, having their vibrations at right angle to each other. *Now if by any suitable mean one of the two rays is eliminated, the remaining ray coming out from the crystal will be plane polarised. In case of Nicol prism the ordinary ray is eliminated using the phenomenon of total internal reflection.*

Construction. Nicol prism is constructed from a calcite crystal whose length is nearly three times of its width ($l : b = 3 : 1$). The end faces of the crystal are cut down so as to reduce the angles of the principal section to a more acute angle of 68° instead of 71°. The crystal is then cut along a diagonal and the two cut surfaces after polishing , cemented back together with a special cement called Canada balsam, which is a transparent substance. It is optically more denser than calcite for the *e*-ray and less denser for *o*-ray [for sodium light $\mu_o = 1.65836$, $\mu_{cb} = 1.55$, $\mu_e = 1.48641$].

Action. A ray of light *SM* is incident nearly parallel to *BD′* on the face *A′B* of the Nicol prism, it splits into *e*-ray and *o*-ray whose vibrations are respectively, perpendicular and parallel to the principal section of the Nicol prism as shown in Fig. 6.16. The *o*-ray suffers total internal reflection at the Canada balsam surface for nearly normal incidence, because Canada balsam is optically more denser than calcite for the *e*-ray and less denser than calcite for the *o*-ray.

The *e*-ray is refracted through Canada balsam and is transmitted but *o*-ray, moving from a denser calcite medium to the rarer Canada balsam medium, is totally internally reflected for angle of incidence greater than the critical angle.

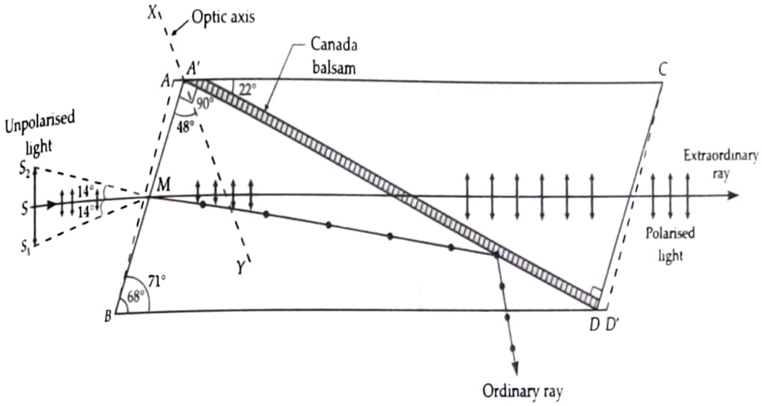


Fig. 6.16 Ray diagram depicting the polarisation of e.m. waves by a Nicol Prism.

The value of critical angle i_c for the *o*-ray for calcite to Canada balsam is

$$i_c = \sin^{-1}\frac{1.550}{1.65836} = 69°$$

Thus starting with ordinary unpolarised light, a Nicol prism transmits only the plane polarised light.

Limitation. The Nicol prism works only when the incident ray is slightly convergent or slightly divergent.

➢ If the incident ray makes an angle much smaller than $\angle BMS$ with the surface *A′B*, the *o*-ray will strike the Canada balsam layer at an angle less than the critical angle and hence will be transmitted and the light emerging from the Nicol prism will not be plane polarised.

➢ If the incident ray makes an angle greater than $\angle BMS$, the *e*-ray become more and more parallel to the optic axis and hence its refractive index will become nearly equal to that of calcite for the *o*-ray. This will suffer total internal reflection like *o*-ray. Hence no light will emerge out of Nicol prism.

Production of Plane Polarised Light by Nicol Prism

To get the plane polarised light, the angle between the extreme rays of the incoming beam ($\angle S_1MS_2$) is limited to about 28°. The Nicol prism can be used as a *'polariser'* and as an *'analyser'*.

(*i*) When an unpolarised ray of light is incident on a Nicol prism *P* as shown in Fig. 6.17(*a*), the ray emerging from *P* is plane polarised with vibrations in the principal section of *P*. If this ray falls on a second Nicol prism *A*, whose principal section is parallel to that of *P*, its vibrations will in principal section of *A*. Hence the ray will behave as *e*-ray in the prism *A* and will be completely transmitted. The intensity of the emergent light will be a maximum.

(*ii*) Now if the Nicol A is rotated such that its principal section becomes perpendicular to that of P as shown in Fig. 6.17(*b*), the vibrations in the plane polarised ray incident on A will be perpendicular to the principal section of A. Hence the ray will behave as *o*-ray inside A and will be lost by total internal reflection at the calcite balsam surface. Therefore no light will emerge from A. In this position the two Nicols are said to be crossed.

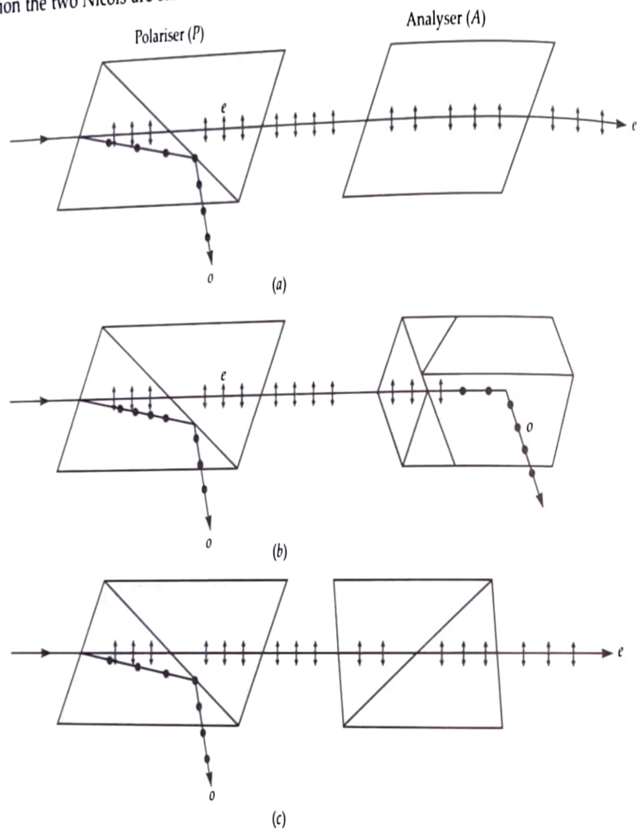


Fig. 6.17 Production of plane polarised light by Nicol prism.

(*iii*) If the Nicol A be further rotated to have its principal section again parallel to that of P as shown in Fig. 6.17(*c*), the intensity of emergent light will again be maximum.

The prism P is called '*polariser*' and prism A is called '*analyser*'.

These facts can be used for analysing plane polarised light. If the given light on viewing through a rotating Nicol shows variation in intensity with minimum intensity zero, the given light is plane polarised.

A similar prism prepared from quartz would not serve a similar purpose although it is doubly refracting. To use quartz for plane polarised light, we have to use either a Rochon prism or a Wollaston prism, which are known as '*double image prisms*'.

6.5 THEORY OF PRODUCTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

When two plane polarised light waves are superimposed on each other, under certain conditions the *resultant light vector* may rotate. When the magnitude of the light vector remains unchanged while its orientation changes its state continuously *i.e.*, when the tip of the light vector traces the geometry of *circle*, the light so produced is known as *circularly polarised light*. However, when both the magnitude and its orientation continuously changes such that the tip of the light vector traces an *ellipse*, then the light is called *elliptically polarised light*.

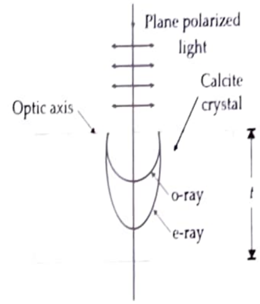


Fig. 6.18 An illustration for elliptically and circularly polarised light.

Let us allow a plane polarised light to fall on a calcite crystal in a normal direction with the crystal cut with its faces parallel to optic axis as shown in Fig. 6.18.

As it enters the calcite crystal, the plane polarised light gets split into *e*-ray and *o*-ray. Both these ray travel along the same direction with different velocities. As the velocity of *e*-ray is faster than that of the *o*-ray, when these two rays emerge out of the crystal a phase φ is introduced between them, depending on the thickness t of the crystal.

Theory. Plane polarised and circularly polarised light are the special cases of elliptically polarised light. Let $P = A\sin\omega t$ be the incident plane polarised light (PPL) on the calcite crystal making an angle ϕ with the direction of optic axis is shown in Fig. 6.19. This plane polarised light can be resolved into two components $A\cos\phi$ along PE, the optic axis and $A\sin\phi$ along PO, *i.e.*, perpendicular to the optic axis. The component $A\cos\phi$ having vibrations parallel to the optic axis forms the *e*-ray while the component $A\sin\phi$ having vibration perpendicular to the optic axis forms the *o*-ray.

If a phase shift φ is introduced between the two rays upon emerging from the crystal of thickness t, the equations representing *e*-ray and *o*-ray are :

$$x = A\cos\phi\sin(\omega t + \varphi) \quad \text{(for e-ray)} \qquad ...(6.4)$$

and

$$y = A\sin\phi\sin\omega t \quad \text{(for o-ray)} \qquad ...(6.5)$$

Let us consider,

$$A\cos\phi = a \text{ and } A\sin\phi = b$$

Equations (6.4) and (6.5) reduce to

$$\frac{x}{a} = \sin(\omega t + \varphi) \qquad ...(6.6)$$

and

$$\frac{y}{b} = \sin\omega t \qquad ...(6.7)$$

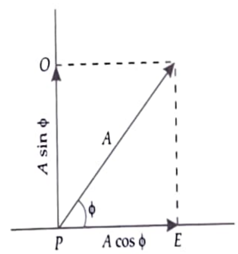


Fig. 6.19 Components of plane polarised light.

Equation (6.6) may be written as

$$\frac{x}{a} = \sin \omega t \cos \varphi + \cos \omega t \sin \varphi \qquad ...(6.8)$$

From Eq. (6.7), $\cos \omega t = \sqrt{1 - \frac{y^2}{b^2}}$ ...(6.9)

Putting the value of $\sin \omega t$ and $\cos \omega t$ in Eq. (6.8), we have

$$\frac{x}{a} - \frac{y}{b} \cos \varphi = \sqrt{1 - \frac{y^2}{b^2}} \sin \varphi \qquad ...(6.10)$$

Squaring and rearranging Eq. (6.10),

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \varphi + \frac{y^2}{b^2} = \sin^2 \varphi \qquad ...(6.11)$$

Equation (6.11) is the *most general equation of an ellipse.*

Special Cases

Case (*i*) : When $\varphi = 0°, \sin \varphi = 0$ and $\cos \varphi = 1$

Equation (6.11), will get from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0 \quad \text{or} \quad \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

i.e.,

$$y = \frac{b}{a} x \qquad ...(6.12)$$

Equation (6.12) is an equation of *straight line*. Thus the emergent ray is *plane polarised* with vibrations in the same plane as that of incident light as shown in Fig. 6.20(*a*).

Case (*ii*) : When $\varphi = \pi, \cos \varphi = -1$ and $\sin \varphi = 0$

Equation (6.11) reduces as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0 \quad \text{or} \quad \left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$$

i.e.,

$$y = -\frac{b}{a} x \qquad ...(6.13)$$

This is also an equation for *a straight line with negative slope*. Thus the emergent ray will also be polarised with direction of vibration making an angle $2\varphi = 2 \tan^{-1}\left(\frac{b}{a}\right)$ with that of the incident ray as shown in Fig. 6.20(*b*).

Case (*iii*) : When $\varphi = \frac{\pi}{2}$ and $a = b, \cos \varphi = 0, \sin \varphi = 1$

Equation (6.11) becomes $x^2 + y^2 = a^2$ ...(6.14)

This is an equation of a *circle*. Thus the emergent ray is *circularly polarised*. It is clear that the resultant of two plane polarised light beams with equal amplitudes and a phase difference of $\frac{\pi}{2}$ is circularly polarised light as shown in Fig. 6.20(*c*).

Case (*iv*) : When $\varphi = \frac{\pi}{2}$ and $a \neq b, \cos \varphi = 0, \sin \varphi = 1$

Then Eq. (6.11) becomes $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(6.15)

This is an equation of *an ellipse*. Thus the emergent light is *elliptically polarised* as long as $a \neq b$ as shown in Fig. 6.20(*d*). It is seen that the resultant of two plane polarised light beams with unequal amplitude and phase difference of $\pi/2$ is an elliptically polarised light.

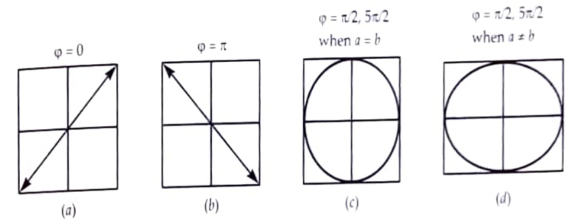


Fig. 6.20 Various types of polarised lights.

Thus, we conclude that plane polarised and circularly polarised light are the special cases of elliptically polarised light.

6.6 RETARDERS/WAVE PLATES : QUARTER AND HALF WAVE PLATES

Retardation Plates

The simplest device for producing and detecting circularly and elliptically polarised light is known as retardation plate. A plate cut from a doubly refracting crystal so as to produce a definite value of path difference or phase difference between *e*- and *o*-rays is known as retardation plate.

Generally, a retarding plate is cut from a doubly refracting crystal with its face parallel to the optic axis. There are two types of retarding plates, namely (*i*) quarter wave plate, (*ii*) half wave plate.

Quarter Wave (or λ/4) Plate [QWP]

A plate of doubly refracting uniaxial crystal cut with its optic axis parallel to the refracting faces and capable of producing a path difference of λ/4 or a phase difference of π/2 between the ordinary and extraordinary waves is called a 'quarter wave plate' or λ/4 plate.

When a beam of monochromatic light of wavelength λ is incident normally on such a plate as shown in Fig. 6.21, it is broken up into *o*- and *e*-waves inside the plate. Both of these waves travel in the same direction (perpendicular to the faces) but with different velocities.

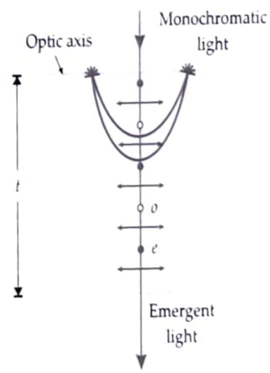


Fig. 6.21 QWP.

In case of negative crystal such as calcite, the e-ray travels faster than o-ray, so that $\mu_o > \mu_e$ where μ_o and μ_e are the principal refractive indices of the crystal for o- and e-rays respectively.

If t is the thickness of the plate, then path t in the crystal plate is equivalent to $\mu_o t$ and $\mu_e t$ in air for o- and e-rays respectively. Hence the path difference between the two waves on emerging in case of negative crystal is given by

$$\Delta = (\mu_o - \mu_e)t \qquad ...(6.16)$$

If the plate acts as quarter wave plate, the path difference (Δ) must be equal to $\frac{\lambda}{4}$ i.e.,

$$\Delta = \frac{\lambda}{4}$$

$$\therefore \quad \frac{\lambda}{4} = (\mu_o - \mu_e)t$$

or

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} \quad \text{(for negative crystal)} \qquad ...(6.17)$$

For positive crystal such as quartz $\mu_e > \mu_o$

$$\Delta = (\mu_e - \mu_o)t = \frac{\lambda}{4} \quad \text{or} \quad t = \frac{\lambda}{4(\mu_e - \mu_o)} \qquad ...(6.18)$$

A quarter wave plate is used in the production of circularly and elliptically polarised light. If the angle of incidence of the plane polarised light is 45° with optic axis, the emergent light is *circularly polarised*. On the other hand, if the incident plane polarised light meets the optic axis at an angle not equal to 45°, then the emergent ray is *elliptically polarised*.

Half Wave (or $\lambda/2$) Plate [HWP]

A plate of doubly refracting uniaxial crystal cuts with its optic axis parallel to the refracting faces and capable of producing a path difference of $\lambda/2$ or phase difference of π between o and e-rays is called a "*half wave plate or $\lambda/2$ plate*".

If t is the thickness of such a plate, then in case of *negative crystal* such as calcite ($\mu_o > \mu_e$), the path difference between the o-ray and e-ray is given by

$$\Delta = (\mu_o - \mu_e)t$$

If λ is the wavelength of light used, then for a half wave plate, the path difference (Δ)

$$\Delta = \frac{\lambda}{2}$$

$$\therefore \quad \frac{\lambda}{2} = (\mu_o - \mu_e)t$$

or

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} \quad \text{(for negative crystal)} \qquad ...(6.19)$$

For positive crystal such as quartz $\mu_e > \mu_o$.

$$\therefore \quad t = \frac{\lambda}{2(\mu_e - \mu_o)} \qquad ...(6.20)$$

If plane polarised light is incident upon a half wave plate such that it makes an angle ϕ with the direction of the optic axis, the emergent light is also plane polarised with vibrations inclined at an angle 2ϕ. Therefore, a $\lambda/2$ plate is used in the construction of Laurent's half shade device used in a polarimeter.

Difference Between Half Wave Plate and Quarter Wave Plate

Half wave plate (HWP) rotates the plane of polarisation of plane polarised light but quarter wave plate (QWP) converts it either to circular or elliptic polarisations. Thus, the monochromatic light after passing through Nicol is allowed to fall on these plates one by one by making an angle $\theta\left(\neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right)$ and examined by rotating Nicol, the following shall be observed in one complete rotation.

$$\text{HWP} \rightarrow I_{max} \rightarrow I_{min} = 0 \rightarrow I_{max} \rightarrow I_{min} = 0$$

$$\text{QWP} \rightarrow I_{max} \rightarrow I_{min} \neq 0 \rightarrow I_{max} \rightarrow I_{min} \neq 0$$

The HWP and QWP are thus distinguished.

Disadvantages of Retardation Plates

The retardation plates are true for one particular wavelength because their thickness depends upon λ. Thus the same will not be equally effective for all the colours.

Example 6.6 *A retardation plate of thickness 8.56×10^{-7} m introduces a phase difference in the path of polarised light of wavelength 5890 Å. The principal refractive indices are $\mu_o = 1.658$, $\mu_e = 1.486$. Find the nature of retardation plate.*

Solution. Given $t = 8.56 \times 10^{-7}$ m, $\lambda = 5890$ Å $= 5.89 \times 10^{-7}$ m, $\mu_o = 1.658$ and $\mu_e = 1.486$.

We know that path difference in case of retardation plates is $\Delta = (\mu_o - \mu_e)t$

Then phase difference $= \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda}(\mu_o - \mu_e)t$

$$= \frac{2 \times \pi \times (1.658 - 1.486) \times 8.56 \times 10^{-7}}{5.89 \times 10^{-7}} = \frac{\pi}{2} \text{ radian}$$

$\therefore$ Phase difference between extraordinary ray and ordinary ray is $\frac{\pi}{2}$, then path difference $\frac{\lambda}{4}$.

So particular plate will be quarter wave plate.

Example 6.7 *What is the optical thickness of a quarter wave plate for the light of wavelength 6000 Å, the birefringence of the plate ($\mu_e \sim \mu_o$) being 0.172 ?*

Solution. *In quarter wave plate* : Given $\lambda = 600$ nm $= 6000 \times 10^{-10}$ m ; $(\mu_e - \mu_o) = 0.172$

We know that optical thickness of quarter wave plate

$$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$= \frac{6.000 \times 10^{-10}}{4 \times 0.172} \text{ m} = \frac{6000}{4 \times 172} \times 10^{-7} \text{ m} = 8.720 \times 10^{-7} \text{ m}.$$

Example 6.8 *Calculate thickness of Quartz half wave plate, given μ_0 and μ_e are 1.5442 and 1.5533 respectively, where wavelength of light used is 5890 Å.* **[GGSIPU, Oct. 2013 (2 marks) ; Nov. 2014 (3 marks)]**

Solution. Given $\mu_0 = 1.5442$, $\mu_e = 1.5533$ and $\lambda = 5890$ Å $= 5890 \times 10^{-7}$ m

Thickness of quartz half wave plate

$$t = \frac{\lambda}{2(\mu_e - \mu_0)}$$

$$= \frac{5.89 \times 10^{-7}}{2 \times (1.5533 - 1.5442)} = \frac{5.89 \times 10^{-7}}{2 \times 0.0091}$$

$$= 323.62 \times 10^{-7} = 32.35 \ \mu\text{m}.$$

Example 6.9 *A half wave plate is constructed for a wavelength of 6000 Å. For what wavelength does it work as a quarter wave plate ?* **[GGSIPU, Dec. 2013 (2.5 marks)]**

Solution. The thickness of a half wave plate for a given crystal is

$$t = \frac{\lambda}{2(\mu_0 \sim \mu_e)}$$

or

$$t = \frac{2\lambda}{4(\mu_0 \sim \mu_e)} = \frac{\lambda'}{4(\mu_0 \sim \mu_e)}$$

which is expression for thickness of a quarter wave plate for wavelength λ'. Hence the half wave plate for λ will behave as a quarter wave plate for $\lambda' = 2\lambda$ (2×600 Å $= 12000$ Å) provided the variation of μ with λ is neglected.

6.7 CONVERSION OF DIFFERENT TYPES OF POLARISED LIGHT

(i) Conversion of a Left Handed Circularly Polarised Light into Right Handed Circularly Polarised Light

A circularly polarised light is a combination of two mutually plane polarised vibration of equal amplitude having the same time period, but with a phase difference of $\pi/2$. As already explained when $\varphi = \pi/2$ the general equation of combination of two mutually perpendicular vibrations of unequal amplitude is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (\omega t = 2\pi \nu t)$$

where $y = b \sin 2\pi\nu t$ [ν = frequency of either particle]

and $x = a \sin(2\pi\nu t + \varphi) = a \cos 2\pi\nu t$ $\left[\because \varphi = \frac{\pi}{2}\right]$

As a special case when φ is 45° in addition to $\varphi = (2n+1)\frac{\pi}{2}$ *i.e.*, an odd multiple of $\frac{\pi}{2}$, the ellipse reduces to a circle

$$x^2 + y^2 = a^2 \qquad (\because a = b)$$

The emergent light is circularly polarised and the optical vector rotates with uniform angular velocity without change of speed.

If $\varphi = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ the circle is describe counter clockwise with respect to an observer towards whom the wave travels as in Fig. 6.22(*a*).

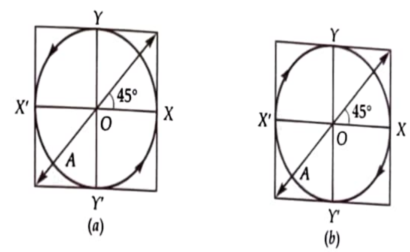


Fig. 6.22 (*a*) Left handed circularly polarised light. (*b*) Right handed circularly polarised light.

If $\varphi = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$ the components of the circular motion along axis are

$$x = -a \cos 2\pi\nu t \quad \text{and} \quad y = a \sin 2\pi\nu t$$

The circle is now described in the clockwise direction and the light is spoken as *right handed circularly polarised.*

Thus to convert left handed circularly polarised light into right handed circularly polarised light, it is made to fall on a half wave plate (HWP) normally such that its vibrations make an angle $\theta = 45°$ with the direction of optic axis. The outgoing light will be right handed circularly polarised light as shown in Fig. 6.22(*b*).

(ii) Conversion of Left Handed Elliptically Polarised Light into Right Handed Elliptically Polarised Light

A plane polarised light on entering a doubly refracting crystal cut with its faces parallel to optic axis, normal to the surface and making an angle ϕ with the direction of optic axis, breaks up into *e*- and *o*-components. The *e*-component has vibrations along the direction of optic axis and *o*-component perpendicular to the direction of optic axis. These components travel along the same direction but with different velocities.

In a negative crystal like calcite *e*-component travels faster than the *o*-component, but reverse is the case for positive crystal like quartz. Hence on coming out the crystal a phase difference φ is introduced between them depending upon the thickness of the plate.

If *A* is the amplitude of the incident vibration, then the amplitude *a* of *e*-vibration along the X-axis is given by

$$a = A \cos\phi \qquad \text{...(6.21)}$$

The amplitude *b* of *o*-vibration

$$b = A \sin\phi \qquad \text{...(6.22)}$$

If ν is the frequency of either particles, then displacement of *o*-vibration along the Y-axis.

$$y = b\sin(2\pi\nu t) \qquad ...(6.23)$$

and that of *e*-vibration along the X-axis

$$x = a\sin(2\pi\nu t + \phi) \qquad ...(6.24)$$

The resultant of these vibrations is obtained as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi = \sin^2\phi \qquad ...(6.25)$$

This is the general equation of an *ellipse*.

If $\phi = \frac{\pi}{2}$, then $\cos\phi = 0$ and $\sin\phi = 1$.

$$\therefore \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad ...(6.26)$$

This is the equation of an *ellipse*. Hence the outgoing light will be elliptically polarised.

The ellipse is described in counter-clockwise with respect to the observer towards whom the light travels. The light is said to be left handed elliptically polarised as shown in Fig. 6.23(*a*).

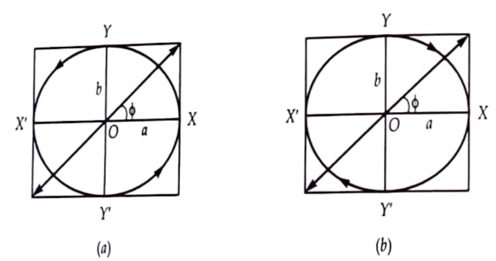


Fig. 6.23 (*a*) Left handed elliptically polarised light. (*b*) Right handed elliptically polarised light.

If $\phi = \frac{3\pi}{2}, \frac{7\pi}{2}, ...$ the displacement of components of elliptical vibration along the major and minor axis are :

$$x = -a\cos 2\pi\nu t \quad \text{and} \quad y = a\sin 2\pi\nu t$$

The ellipse is now described in the clockwise direction and the polarised light is known as *right handed elliptically polarised light*.

To convert left handed elliptically polarised light into right handed elliptically polarised light, it is made to fall on a half wave plate normally such that its vibrations make an angle ϕ, other than 0°, 45°, 90° with the optic axis. To outgoing light will be right handed elliptically polarised light as explained, as shown in Fig. 6.23(*b*).

6.8 PRODUCTION OF CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

(a) Circularly Polarised Light

Figure 6.24 depicts the basic arrangement for the production of circularly polarised light. When unpolarised light is passed through the Nicol prism, plane polarised light is obtained. When this plane polarised light falls normally on a quarter wave plate such that the vibration in the incident plane polarised light makes an angle of 45° with the optic axis of QWP, the emergent light from quarter wave plate (QWP) is the *circularly polarised*.

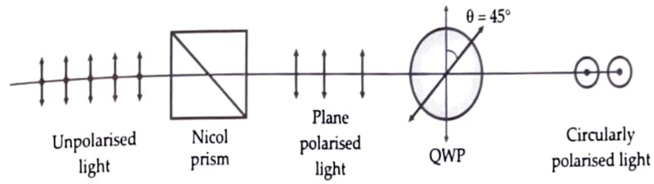


Fig 6.24 Production of circularly polarised light.

(b) Elliptically Polarised Light

When unpolarised light passes through a Nicol prism, plane polarised light makes an angle $\theta \neq 0°, 45°$ and 90°, then the plane polarised light will be split into *e*-ray and *o*-ray inside the quarter wave plate (QWP), having unequal amplitudes. When they emerge out of quarter wave plate (QWP), a path difference of $\lambda/4$ exists between them and they combine to form *elliptically polarised light* as shown in Fig. 6.25.

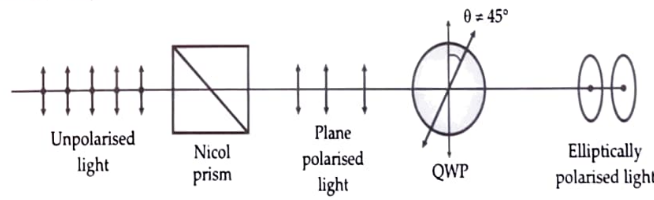


Fig 6.25 Production of elliptically polarised light.

6.9 DETECTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

(a) Plane Polarised Light

A plane polarised light produced by a Nicol prism (Polariser-*P*) is allowed to fall on another Nicol prism (Analyser-*A*) as illustrated in Fig. 6.26. *When the analyser is rotated once with respect to the polariser and if the light is extinguished completely twice*, then the light coming out of the polariser is plane polarised light.

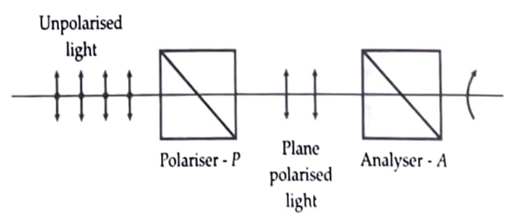


Fig. 6.26 Detection of plane polarised light.

(b) Circularly Polarised Light

When a circularly polarised light is viewed with a rotating analyser, there is no variation in intensity. An unpolarized light will also show no variation in intensity with a rotating analyser. However, to distinguish circularly polarised light, a QWP is used as shown in Fig. 6.27. When circularly polarised light falls on the QWP as shown in Fig. 6.27(*a*), it is converted into plane polarised light and with a rotating analyser extinguishment occurs twice. However, when unpolarised light passes through the QWP as shown in Fig. 6.27(*b*), it remains unpolarised light. Thus, with a rotating analyser, it can not be extinguished.

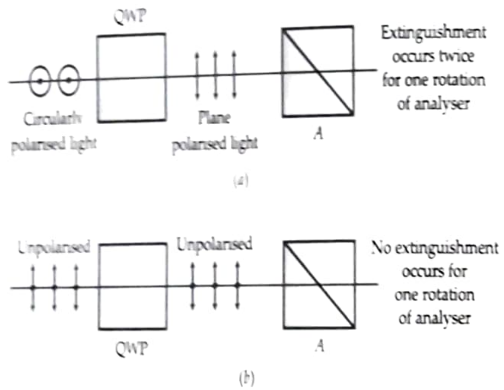


Fig. 6.27 Detection of circularly polarised light.

(c) Elliptically Polarised Light

When elliptically polarised light is viewed through a rotating analyser, a variation in intensity between maxima and minima is noted. With partially plane polarised light, a similar variation in the intensity of light is also observed with a rotating analyser. To distinguish the elliptically polarised light from the partially polarised light, a QWP is used as shown in Fig. 6.28.

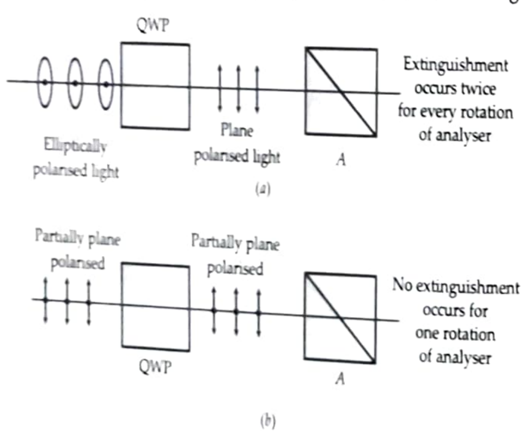


Fig. 6.28 Detection of elliptically polarised light.

When elliptically polarised light falls on QWP as shown in Fig. 6.28(*a*), the emergent light is plane polarised and this light extinguishes twice for every one rotation of the angular. However, when partially plane polarised light is incident on QWP as shown in Fig. 6.28(*b*), the emergent beam is still partially plane polarised. Thus when this is examined with a rotating analyser the intensity varies between maxima and minima.

6.10 ANALYSIS OF POLARISED LIGHT

Suppose a ray of light is coming through a slit and we want to identify the nature of polarisation associated with it. That is to say whether the incoming light is

- unpolarised
- circularly polarised
- partially plane polarised
- elliptically polarised or
- plane polarised

Using a Nicol prism (analyser) and a QWP (quarter wave plate) the status of polarisation can be identified as follows :

Case 1. When the light is investigated by a rotating analyser and if the intensity changes from maximum-zero-maximum-zero-maximum, then light is *plane polarised.*

Case 2. When the analyser is rotated once and if the intensity of light remains uniform, then the incoming light may be either *circularly polarised or unpolarised.* To distinguish circularly polarised light, the incoming light is allowed to go through QWP before falling on the rotating analyser [Fig. 6.27].

In case of circularly polarised light on passing through a QWP it becomes plane polarised and hence with the rotating analyser the extinguishment occurs twice. However unpolarised light on passing through a QWP produces only unpolarised light and this when inspected with a rotating analyser will give no variation in intensity of light.

Case 3. When the incoming light is allowed to go through the rotating analyser, the intensity changes through maxima-minima (but not zero). The incoming light may then be either *elliptically* or *partially plane polarised.*

To distinguish elliptically polarised light from partially plane polarised light, the incoming light is allowed to fall initially on a quarter wave plate (QWP) [Fig. 6.28] before being investigated using a rotating analyser.

If the incoming light is elliptically polarised light, then on passing through a QWP it becomes plane polarised light. When this light is studied with a rotating analyser, the intensity of light shows two maxima and two extinguishments.

On the other hand, if the incoming light rays are partially polarised, on passing through a quarter wave plate, it still retains its partially plane polarised state. When viewed through a rotating analyser two maxima and two minima (not zero) in intensity variation are noted. The entire details are visualised by Fig. 6.29.

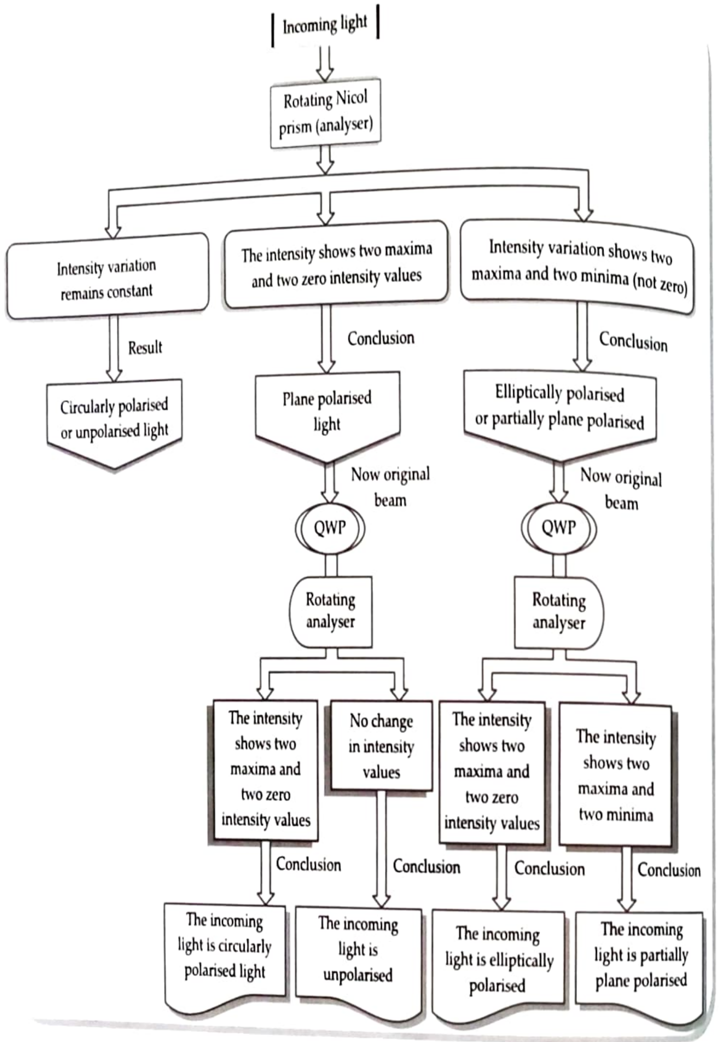


Fig. 6.29 Analysis of the light for the state of polarisation.

6.11 ROTATORY POLARISATION (OPTICAL ACTIVITY)

If monochromatic light is passed through two Nicol prisms P (polariser) and A (analyser) placed in the crossed position, no light will emerge out of them. Nicol prism P renders the beam plane polarised with vibration in its principal plane and these vibrations are not allowed to pass through Nicol prism A because its principal plane is perpendicular to the direction of these vibrations. However, if we introduce a plate of positive doubly refracting crystal *i.e.*, of quartz in between these two prisms, it is found that some light emerges out Nicol prism A and to get

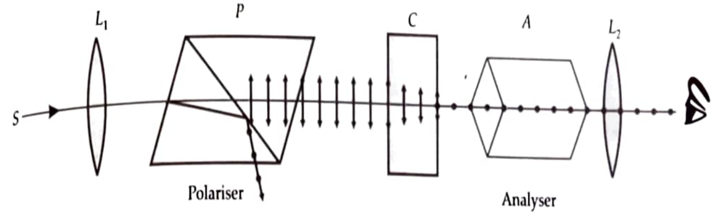


Fig. 6.30 An illustration for optical activity.

complete darkness, Nicol A has to be turned through some angle as shown in Fig. 6.30. This indicates that light coming through the quartz is still plane polarised but the plane of polarisation is rotated through an angle equal to the angle through which Nicol prism A has to be rotated to restore the original condition.

This property of rotating the plane of polarisation about the direction of propagation of light is called optical activity and the substances having this property are called optically active substances.

Some of them rotate the plane clockwise called *right handed* or *dextrorotatory* while others doing so in anticlockwise direction are called *left handed* or *laevorotatory*. Cane sugar is dextrorotatory while the fruit sugar is laevorotatory.

6.11.1 Biot's Laws for Rotatory Polarisation

Biot, in 1815, studied the phenomenon of optical activity in detail and gave following laws :

1. The angle of rotation is directly proportional to the thickness of the substance transversed.

   i.e., $\theta \propto l$

2. The angle of rotation is directly proportional to the concentration of solution.

   i.e., $\theta \propto C$

3. The angle of rotation is inversely proportional to the square of the wavelength of light passing through it.

   i.e., $\theta \propto \dfrac{1}{\lambda^2}$

4. Rotation follows the law of addition if number of substances are used.

   i.e., $\theta_{resultant} = \Sigma\theta_i$

5. Rotation is also a function of temperature (T).

   i.e., $\theta = f(T)$

6.11.2 Specific Rotation

For a comparison of optical activity, a term specific rotation or specific rotatory power is adopted, which is usually denoted by 'S'. It is defined in different ways for solids, liquids and solutions.

(i) For solids

In case of solids, it is defined by the equation

$$\theta = S \times l \qquad ...(6.27)$$

where θ in the rotation produced in degrees by 1 millimeter length of the solid. Thus for solids, the specific rotation is the angle of rotation produced by 1 mm thickness of the solid in the direction of optic axis.

(ii) For liquids

In case of liquids, it is defined by the equation

$$\theta = S \times l \times \rho \qquad ...(6.28)$$

where the path length l is measured in decimeter (= 10 cm) and ρ is the density in gm/cc. Thus for liquids, the specific rotation of the plane of polarisation produced by 1 decimeter column of the pure liquid divided by its density.

(iii) For solutions

In case of solutions e.g., sugar dissolved in distilled water, the specific rotation is defined by the equation

$$\theta = S \times l \times C \qquad ...(6.29)$$

where θ is the rotation produced in degrees, l is the length of the solution in decimeter.

Thus specific rotation for solutions is defined as the rotation (in degrees) of plane of polarisation by 1 decimeter length of solution, when its concentration in 1 gm/cc. Thus

$$S = \frac{\theta}{l \times C} \qquad ...(6.30)$$

or

$$\text{Specific rotation} = \frac{\text{Rotation in degrees}}{\text{Length in decimeter} \times \text{Concentration in g/cc}}$$

and

$$S = \frac{\theta}{l \times \left(\frac{m}{V}\right)} = \frac{\theta V}{lm} \qquad ...(6.31)$$

[in degree (decimeter^{-1}) (gm/cc)$^{-1}$]

where m is mass of active substance in gm in V c.c. solution, it is to be noted here that specific rotation is not a constant but it varies with wavelength of light, the nature of active solvent, the concentration of the solution and its temperature.

The product of specific rotation and molecular weight of the optically active substance is known as molecular rotation i.e.,

Molecular rotation = Specific rotation × Molecular weight.

Example 6.10 *The plane of plane polarised light is rotated through an angle of 13.5°, when light passes through a 10 cm cube containing 20% sugar solution. Calculate the specific rotation.*

Solution. *Given* : $\theta = 13.5°$, $l = 10$ cm = 1 dm, $C = 20\% = \frac{20}{100}$ gm/cc

The specific rotation is given by

$$[S]_T^{\lambda} = \frac{\theta}{l \times C} = \frac{13.5 \times 100}{1 \times 20} = 67.5 \text{ /dm/gm/cm}^3$$

Example 6.11 *A 5% solution of cane sugar placed in a tube of length 40 cm, causes the optical rotation of 20°. How much length of 10% solution of the same substance will causes 35° rotation ?*

[GGSIPU, Sept. 2012 (3 marks)]

Solution. Given : $C_1 = 5\% = 0.05$; $l_1 = 40$ cm ; $\theta_1 = 20°$

$C_2 = 10\% = 0.1$; $l_2 = ?$; $\theta_2 = 35°$

We know the specific rotation $[S]_T^{\lambda} = \frac{\theta}{l \times C}$

According to problem,

$$S = \frac{\theta_1}{l_1 \times C_1} = \frac{\theta_2}{l_2 \times C_2}$$

or

$$l_2 = \left(\frac{\theta_2}{\theta_1}\right) \times \left(\frac{C_1}{C_2}\right) \times l_1 = \frac{35}{20} \times \frac{0.05}{0.10} \times 40 \text{ cm} = 35 \text{ cm.}$$

6.12 POLARIMETERS

The polarimeters are the instruments designed to measure the angle of rotation produced by substance. When they are calibrated to read directly the percentage of cane sugar solution, they are known as *sacharimeters*. Polarimeters can be used to find the specific rotation of sugar solution or if the specific rotation is known, they can be used to find its concentration.

There are *four* types of polarimeters :

1. Laurent half shade polarimeter
2. Biquartz polarimeter
3. Lippich polarimeter
4. Soleil compensated biquartz polarimeter.

Here we discuss only Laurent half shade polarimeter.

6.12.1 Laurent's Half Shade Polarimeter

This instrument is used to measure the optical rotation of certain solutions. For known specific rotation the concentration of sugar solution can be determined.

Construction. The Laurent's half shade polarimeter is illustrated in Fig. 6.31. It consists of two Nicol prisms – one serves the purpose of polariser while other works as analyser. These polariser and analyser are capable to rotate about an axis. A glass tube is provided which is filled with the solution of an optically active substance. A half shade device is placed between the polariser and glass tube. The ends of this glass tube are covered firmly with two glass plates. This tube is placed between polariser and analyser.

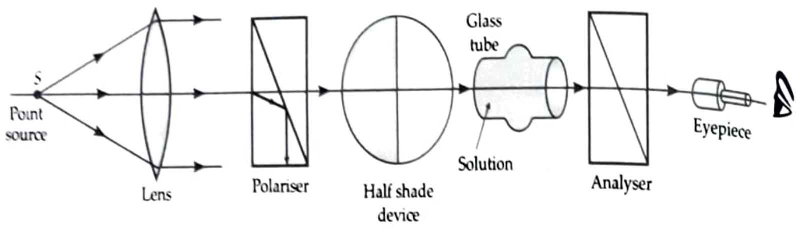


Fig. 6.31 Experimental set up for determination of specific rotation.

Function of Half-shade Device

The half-shade device is a circular plate. Its half portion is made up of quartz while the other half is made of glass. The thickness of quartz half is taken such that it produces a path difference of $\lambda/2$ or phase difference of π between ordinary and extraordinary rays *i.e.*, it serves the purpose of half wave plate. The thickness of the glass plate is so chosen as to absorb the same light to that of quartz half.

Working. Let the plane polarised light emerging from the polariser has the vibrations along OA. Now from the glass plate the emergent light will have same direction of vibration *i.e.*, along OA as shown in Fig. 6.32.

In case of quartz plate the incident light splits up into *o*- and *e*-rays. The *e*-rays are parallel to optic axis while *o*-rays are perpendicular to it. Now since for quartz the *o*-rays travel faster, they gain a phase difference of π or path difference of $\lambda/2$ on emergence from the quartz plate. Therefore, the emergent light from quartz plate will have the vibrations along OX' (*o*-ray) and along OX (*e*-ray). Thus, from quartz plate resultant will be along OB. Now when the principal plane of the analyser is parallel to BOB' the light from the quartz plate will pass through completely, while from glass plate will transmit partly. Therefore, the half part at the left side will be brighter as compared to the right half. When the principal plane of analyser is parallel to AOA' the right half will be brighter. But when the principal plane of analyser is set parallel to optic axis of half shade device, *i.e.*, along XOY, the two halves will be equally bright. This position of analyser is recorded. With the help of this recorded value of specific rotation of the solution may be determined by adopting procedure given below :

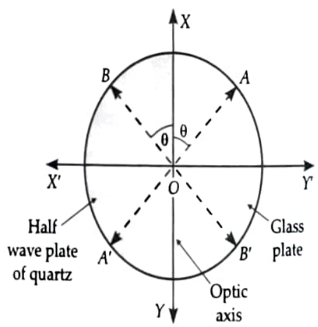


Fig. 6.32 Working of half shade device.

Determination of Specific Rotation of Sugar Solution

The specific rotation of sugar solution may be measured by adopting the following procedure :

- The polarimeter tube or glass tube is first filled with water and the reading of the analyser is recorded when two halves become equally bright.

- Now the water is removed and sugar solution of known concentration is filled in the tube. The analyser is again rotated till the two halves are equally bright and reading is noted.
- The difference in readings obtained by above two steps gives the angle of rotation θ of the plane of polarisation for that concentration of the solution.
- Now the same procedure is repeated with the solutions of different concentrations and the corresponding values of θ are determined.

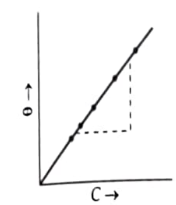


Fig. 6.33 Concentration versus angle of rotation.

- A graph (Fig. 6.33) is plotted between the angle of rotation (θ) and concentration of solution (C). This graph is a straight line.
- The slope of the graph gives the value of θ/C and therefore, the specific rotation of sugar is calculated with the help of expression

$$[S]_T^\lambda = \frac{\theta}{lC} \qquad ...(6.32)$$

where l is the length of the tube in decimeter and temperature and wavelength are constant throughout the whole experiment.

Formulae at a Glance

6.1 Brewster's law, $\mu = \tan i_p$

where

μ = refractive index of the reflecting medium and

i_p = polarising angle

6.2 Malus law,

$$I = a^2\cos^2\theta \text{ or } I = I_0\cos^2\theta$$

where

I = intensity of transmitted light through analyser

I_0 = Intensity of incident polarised light

a = Amplitude of incident polarised light

6.3 General equation of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\varphi = \sin^2\varphi$$

(*i*) $\varphi = 0$ $y = \frac{b}{a}x$, straight line

(*ii*) $\varphi = \pi$, $y = -\frac{b}{a}x$, straight line

(*iii*) $\varphi = \frac{\pi}{2}$, $a = b$, $x^2 + y^2 = a^2$, circle

(*iv*) $\varphi = \frac{\pi}{2}$, $a \neq b$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ellipse

6.4 (*a*) QWP formula : $t = \frac{\lambda}{4(\mu_e \sim \mu_o)}$

(*b*) HWP formula : $t = \frac{\lambda}{2(\mu_e \sim \mu_o)}$

where t = thickness of half or quarter wave plate, μ_e = refractive index of *e*-ray

μ_0 = refractive index of *o*-ray

6.5 Specific rotation $[S]_T^\lambda = \frac{\theta}{lC} = \frac{\theta V}{lm}$

where θ = rotation in degrees

l = length in decimeter

c = concentration in *g/cc*

m = mass of solution in *g*

V = volume of solution in *cc*

Miscellaneous Solved Numerical Problems

Problem 6.1 *Find the polarising angle for light incident from*

(*i*) *air to glass* (*ii*) *glass to air* (*iii*) *water to glass*

(*iv*) *glass to water* (*v*) *air to water, and* (*vi*) *water to air*

[*Given* μ *for glass* = 1.54 ; μ *for water* = 1.33 ; μ *for air* = 1.00]

Solution. (*i*) When light is incident from air to glass

$$\mu = \frac{\text{refractive index of glass}}{\text{refractive index of air}} = \frac{\mu_2}{\mu_1} = \frac{1.54}{1} = 1.54,$$

Now using Brewster's law $\tan i_p = \mu$

or polarising angle $i_p = \tan^{-1}(\mu) = \tan^{-1}(1.54) = 57°$

(*ii*) When light is incident from glass to air

$$\mu = \frac{\text{refractive index of air}}{\text{refractive index of glass}} = \frac{1}{1.54} = 0.6494,$$

$$\therefore \quad i_p = \tan^{-1}(\mu) = \tan^{-1}(0.6494) = 33°$$

(*iii*) When light is incident from water to glass

$$\mu = \frac{\text{refractive index of glass}}{\text{refractive index of water}} = \frac{1.54}{1.33} = 1.158,$$

$$\therefore \quad i_p = \tan^{-1}(\mu) = \tan^{-1}(1.158) = 49°12'$$

(*iv*) When light is incident from glass to water

$$\mu = \frac{\text{refractive index of water}}{\text{refractive index of glass}} = \frac{1.33}{1.54} = 0.8636,$$

$$\therefore \quad i_p = \tan^{-1}(\mu) = \tan^{-1}(0.8636) = 40°49'$$

(*v*) When light is incident from air to water

$$\mu = \frac{\text{refractive index of water}}{\text{refractive index of air}} = \frac{1.33}{1} = 1.33,$$

$$\therefore \quad i_p = \tan^{-1}(1.33) = 53°4'$$

(*vi*) Similarly, when light is incident from water to air

$$\mu = \frac{\text{refractive index of air}}{\text{refractive index of water}} = \frac{1}{1.33} = 0.7519,$$

$$\therefore \quad i_p = \tan^{-1}(0.7519) = 36°57'$$

Problem 6.2 *A Nicol prism polaroid is used to examine two adjacent polarised beams A and B, whose planes of polarisation are mutually perpendicular. The Nicol is initially oriented so that the beam B gives zero intensity with it. When Nicol is rotated by 30° to initial direction both the beams give equal intensity. What is the ratio of intensities* (I_A / I_B) *of two beams ?*

Or

A polaroid examines two adjacent plane polarised beams A and B, whose plane of polarisation are mutually perpendicular. In one position of the analyser the beam B shows zero intensity. From this position, a rotation of 30° shows that two beams have equal intensity. What is the ratio of intensity of two beams ?

Solution. Given, when analyser is rotated through an angle 30° from the direction, beam B gives no intensity, then angle made by the vibrating electric field vector with B and A are respectively.

$$90° - 30° = 60°$$

and

$$90° - 60° = 30°$$

Then by means of Malus law $I = I_0 \cos^2\theta$

i.e.,

$$I_A \cos^2 30° = I_B \cos^2 60°$$

or

$$\frac{I_A}{I_B} = \frac{\cos^2 60°}{\cos^2 30°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$$

or

$$I_A : I_B = 1 : 3.$$

Problem 6.3 *Unpolarised light of intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with first. If the intensity of the emerging light is 3 Wm^{-2}, what is the angle between the transmission axis of the first two polarisers ? At what angle will be transmitted intensity be maximum ?*

Solution. Let P_1, P_2, P_3 be the three polarisers and θ be the angle between the transmission axes of P_1 and P_2. As P_1 and P_3 are crossed, the angle between P_2 and P_3 is $(90° - \theta)$.

Suppose I_0 be the intensity of unpolarised light falling on P_1. Then the intensity of light emerging from P_1 will be

$$I_1 = \frac{1}{2} I_0$$

By Malus law, the intensity of light emerging from P_2 is

$$I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$$

The intensity of light emerging from P_3 is

$$I_3 = I_2 \cos^2(90° - \theta) = \frac{1}{2} I_0 \cos^2\theta \sin^2\theta = \frac{1}{8} I_0 \sin^2 2\theta$$

$$\therefore \quad \sin^2 2\theta = \frac{8I_3}{I_0} = \frac{8 \times 3}{32} = \frac{3}{4}$$

or

$$\sin 2\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow 2\theta = 60° \quad \Rightarrow \theta = 30°$$

As

$$I_3 = \frac{1}{8} I_0 \sin^2 2\theta$$

$\therefore$ I_3 will be maximum, when

$$\sin^2 2\theta = 1 \text{ or } \sin 2\theta = 1 = \sin 90° \text{ or } \theta = \mathbf{45°}$$

Problem 6.4 *Compute the minimum thickness of a quarter wave plate of calcite for $\lambda = 5460$ Å. The principal indices of calcite are 1.652 and 1.488.*

Solution. Given, $\lambda = 5460\times10^{-10}$ m, $\mu_o = 1.652$ and $\mu_e = 1.488$

The minimum thickness of a quarter wave plate of calcite (for which $\mu_o > \mu_e$) is given by

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

$$\therefore \quad t = \frac{5460\times10^{-10}}{4(1.652-1.488)}$$

$$= \frac{5460\times10^{-10}}{4\times0.161} = 8.32\times10^{-7} \text{ m.}$$

Problem 6.5 *A plane polarised light is incident on a quartz plate cut parallel to the axis. Calculate the least thickness of the plate for which the o- and e-rays combine to form plane polarised light. Assume that $\mu_e = 1.5533$ and $\mu_o = 1.5442$ and $\lambda = 5.4\times10^{-5}$ cm.* **[GGSIPU, Dec. 2015 (2 marks)]**

Solution. In this case the quartz plate must act as half wave plate. Thus if t be the required thickness then we have

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

or

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

Putting the given values, we get

$$t = \frac{5.4\times10^{-5}\text{ cm}}{2(1.5533-1.5442)}$$

$$= \frac{5.4\times10^{-5}\text{ cm}}{2\times0.0091} = 3\times10^{-3}\text{ cm.}$$

Problem 6.6 *Deduce the speeds of o-and e-ray in calcite crystal : (i) in a plane perpendicular to the optic axis (ii) along the optic axis. Hence find the relative phase difference between the rays for light of wavelength 6000 Å in travelling a distance 3×10^{-3} cm at right angles to the optic axis.*

[Given that : $\mu_o = 1.658$ and $\mu_e = 1.486$]

Solution. (i) Velocity of ordinary ray

$$v_o = \frac{c}{\mu_o} = \frac{3\times10^8}{1.658} = 1.8\times10^8 \text{ m/s}$$

Velocity of e-ray $\quad v_e = \dfrac{c}{\mu_e} = \dfrac{3\times10^8}{1.486} = 2.02\times10^3$ m/s

Along the optic axis, the two rays travel with the same velocity and its value is equal to the o-ray. Hence, $v_e = v_o = 1.8\times10^8$ m/sec.

Now the time difference in travelling a distance x

$$\Delta t = \frac{x}{v_o} - \frac{x}{v_e} = \frac{x}{c}\left[\frac{c}{v_o} - \frac{c}{v_e}\right] = \frac{x}{c}[\mu_o - \mu_e]$$

Hence, path difference $= c\times\Delta t = x[\mu_o - \mu_e]$

$$\therefore \quad \text{Phase difference} = \frac{2\pi}{\lambda}\times\text{path difference} = \frac{2\pi}{\lambda}\times x[\mu_o - \mu_e]$$

$$= \frac{2\times3.14}{6000\times10^{-10}}\times3\times10^{-3}[1.658-1.486] = 54 \text{ radian.}$$

Problem 6.7 *A beam of linearly polarised light is changed into circularly polarised light by passing it through a slice of crystal 0.003 cm thick. Calculate the difference in the refractive indices for the two rays in the crystal assuming this to be of minimum thickness that will produce the effect and that the wavelength is 6×10^{-7} m.*

Solution. If the vibrations in the incident plane polarised light make an angle $\phi = 45°$ with the optic axis of the crystal plate and the thickness of the plate in such that the least phase difference between the emergent o- and e-rays is $\frac{\pi}{2}$, then these rays combine to form circularly polarised light. A phase difference of $\frac{\pi}{2}$ is equivalent to the path difference of $\frac{\lambda}{4}$ and thus linearly polarised light is converted into circularly polarised light by means of a quarter wave plate. Hence the given crystal slice is quarter wave plate.

Now for a quarter wave plate,

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

Given that $t = 0.003$ cm $= 3\times10^{-5}$ m and $\lambda = 6\times10^{-7}$ m

$$\therefore \quad (\mu_o - \mu_e) = \frac{\lambda}{4t} = \frac{6\times10^{-7}}{4\times3\times10^{-5}} = 5\times10^{-3} = 0.005.$$

Problem 6.8 *A given calcite plate behaves as a half wave plate for a particular wavelength λ. Assuming variation of refractive index with λ to be negligible, how would the above plate behave for another light of wavelength 2λ ?*

Solution. The thickness of a half wave plate for a negative calcite crystal ($\mu_0 > \mu_e$) is given by

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

or

$$t = \frac{2\lambda}{4(\mu_o - \mu_e)} = \frac{\lambda'}{4(\mu_o - \mu_e)}$$

which is expression for thickness of a quarter wave plate for wavelength λ'. Hence the half wave plate for λ will behave as a quarter wave plate for $\lambda' = 2\lambda$ provided the variation of μ with λ is neglected.

Problem 6.9 *A beam of linearly polarised light of wavelength 5×10^{-5} cm inclined at 45° to the optic axis is changed into circularly polarised light by passing it through a negative crystal 0.0025 cm thick. Calculate the difference in the refractive index of the two rays in the crystal assuming this to be minimum thickness that will produce the effect.*

Solution. If the least phase difference between the ordinary and extraordinary rays on emergence be $\pi/2$, they will combine to a circularly polarised light. The crystal plate must then be a quarter wave plate

$$\frac{\lambda}{4} = t(\mu_o - \mu_e) \quad \text{or} \quad (\mu_o - \mu_e) = \frac{\lambda}{4t} = \frac{5\times10^{-5}}{4\times0.0025} = 0.005.$$

Problem 6.10 *Calculate the thickness of a half wave and quarter wave plate for a light of wavelength 5892 Å when μ_0 differ from μ_e by 0.01.*

Solution. Given $\lambda = 5892$ Å $= 5.892\times10^{-7}$ m, $\mu_o \sim \mu_e = 0.01$

For Half Wave plate,

Thickness,

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} = \frac{5.892\times10^{-7}}{2\times0.01} = 2.946\times10^{-5}\ \text{m}$$

For Quarter Wave plate,

Thickness,

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} = \frac{5.892\times10^{-7}}{4\times0.01} = 1.473\times10^{-5}\ \text{cm}$$

Problem 6.11 *Calculate the thickness of (i) quarter wave plate (ii) half wave plate. Given that : $\mu_o = 1.54$, $\mu_e = 1.533$ and $\lambda = 5500$ Å*

Solution. Given : $\mu_o = 1.544$, $\mu_e = 1.533$, $\lambda = 5500$ Å $= 5.5\times10^{-7}$ m,

$$\mu_o - \mu_e = 1.544 - 1.533 = 0.011$$

(i) *For Quarter Wave plate*

Thickness,

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} = \frac{5.5\times10^{-7}}{4\times0.011} = \frac{55}{44}\times10^{-5}\ \text{m} = 1.25\times10^{-5}\ \text{m}$$

(ii) *For Half Wave plate,*

Thickness,

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} = \frac{5.5\times10^{-7}}{2\times0.011} = \frac{55}{22}\times10^{-5}\ \text{m} = 2.5\times10^{-5}\ \text{m}.$$

Problem 6.12 *Calculate thickness of half wave plate made of quartz for sodium light ($\lambda = 5890$ Å) given that $\mu_e = 1.553$ and $\mu_o = 1.544$.*

Solution. Given : $\mu_e = 1.553$, $\mu_o = 1.544$,

$\lambda = 5890$ Å $= 5.89\times10^{-7}$ m, $\mu_e - \mu_o = 0.009$

For half wave plate,

$$t = \frac{\lambda}{2(\mu_e - \mu_o)} = \frac{5.89\times10^{-7}}{2\times0.009} = 3.27\times10^{-5}\ \text{m}.$$

Problem 6.13 *80 gm of impure sugar when dissolved in a litre of water, gives an optical rotation of 9.9°, when placed in a tube of length 200 mm. If the specific rotation of surfar is 66 degree/dm/(gm/cc), find the percentage purity of sugar sample.*

Solution. Given $\theta = 9.9°$, $l = 200$ mm $= 2.0$ dm and $S = 66°$/dm/(gm/cc)

We know that, the strength of the solution is given by

$$C = \frac{\theta}{l\times S} = \frac{9.9°}{2.0\times66°}\ \text{g/cc} = 0.075\ \text{g/cc} = 75\ \text{g/litre}$$

The sugar sample dissolved in a litre of water 80 g in which 75 g is pure sugar.

Therefore purity is $\frac{75}{80}\times100 = 93.75\%$.

Problem 6.14 *A 20 cm long tube containing sugar solution is placed between crossed Nicols and illuminated with light of wavelength 6×10^{-5} cm. If the specific rotation is 60°/dm/gm/cm^3 and optical rotation produced is 12°, determine the strength of the solution.*

[GGSIPU, Sept. 2011 (2 marks) ; Jan 2015 (3 marks)]

Solution. The specific rotation S of a solution is given by

$$[S]_T^{\lambda} = \frac{\theta}{l\times C}$$

Here, $\theta = 12°$, $l = 2.0$ dm and $S = 60$/dm/gm/cm^3

$$\therefore \quad C = \frac{12}{2.0\times60} = 0.1\ \text{gm/cc} = 10\%.$$

Conceptual Questions

6.1 *What do you mean polarisation of light ?* **[GGSIPU, Sept. 2011 reappear (1 marks)]**

Ans. Maxwell predicted that the light is a transverse nature of light has been established by the polarisation experiments. In transverse wave, the particles of the medium execute periodic oscillations in a direction perpendicular to the direction of propagation of light wave. In electromagnetic wave, the light emitting atoms oscillate at random and emit individual wave trains in all possible directions. The corresponding light wave is called *natural light* or unpolarised.

If, however, the oscillations are confined to only one direction, then we get a plane polarised light. It electric vector of transversed e.m. wave is identified as *light vector*. Hence, in plane polarised light, the light vectors (electric vector) vibrate in a single plane called the *plane of vibration* and the plane perpendicular to it is the *plane of polarisation*.

Polarised light is of three types *viz*.

(*i*) plane polarised light (*ii*) circularly polarised light and (*iii*) elliptically polarised light

6.2 *Light is generally characterized by electric vector, although it also possesses magnetic vector. Explain.*

Ans. Light is form of electromagnetic wave motion. The e.m. wave has electric and magnetic fields vectors oscillating in phase but perpendicular to each other. The interaction of e.m. waves with matter can be explained either with the help of electric vector or with magnetic vector. But peak value of electric vector is much higher than that of magnetic vector. Moreover, our eyes are more sensitive to electric vector than to magnetic vector. Hence polarisation and other optical phenomenon are preferably described in terms of electric vector. This is why light is characterized by electric vector although it also possess magnetic vector.

6.3 *The axis of polarizer and analyzer are oriented at 30° to each other.*

(i) *If un-polarized light of intensity I_0 is incident on them, what is the intensity of the transmitted light?*

(ii) *Polarized light of intensity I_0 is incident on this polarizer-analyzer system. If the amplitude of the light makes an angle of 30° with the axis of the polarizer, what is the intensity of the transmitted light?*

[GGSIPU, Dec. 2015 (2.5 marks)]

Ans. (i) The light incident on the first polarizer is unpolarised, so the angle is irrelevant the intensity is reduced by a factor of 2.

$$I_1 = \frac{1}{2}I_0$$

(ii) When it emerges from first polarizer, the light is incident at 30° with the axis of polarizer

$$I_2 = I_1\cos^2 30° = \frac{3}{4}I_1$$

6.4 *Prove that the maximum transmission through polariser is half of the maximum even if the polariser is ideal.*

Ans. Let the intensity of the incident on the polariser is I_0. Let θ be the angle between the polarising direction of the polariser and the direction of vibration of the electric vector in the incident light.

Therefore, applying Malus law, the intensity of the light transmitted light through the polariser (say I) will be $I = I_0\cos^2\theta$.

The incident light is unpolarised one. So it can have any direction of vibration perpendicular to the direction of propagation of light. So, to compute the intensity I, we have to take the average value of I. Now average value of $\cos^2\theta$ is $\frac{1}{2}$.

Therefore $\quad I_{max} = I_0\cos^2\theta = \frac{1}{2}I_0$

an ideal polariser will transmit this maximum quantity which is equal to the half of the intensity of incident one. This implies that even an ideal polariser can at best have a maximum transmission of 50%.

6.5 *If unpolarised light falls on a system of two crossed polarised sheets, no light is transmitted. If third polarising sheet is placed between them will light be transmitted. Explain.* **[GGSIPU, Sept. 2009 (2 marks)]**

Ans. Let A and B are two crossed polarising sheets and I_0 be the intensity of the light incident on A. Then the intensity of light transmitted through A is

$$I_A = \frac{1}{2}I_0$$

when the third sheet C is inserted between A and B (but not parallel to either), then the intensity of light transmitted by C is

$$I_C = I_A\cos^2\theta = \frac{1}{2}I_0\cos^2\theta$$

Since A and B are crossed the angle between C and B is (90°−θ), so the intensity of light transmitted by B will be

$$I_B = I_C\cos^2(90°-\theta) = I_C\sin^2\theta = \frac{1}{2}I_0\cos^2\theta\sin^2\theta$$

Now if θ = 90°, $I_B = 0$ and if θ = 0°, $I_C = 0$, but when θ = 45°

$$I_B = \frac{1}{2}I_0\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{I_0}{8}$$

So introduction of third sheet can give transmitted part of the incident light even if the two polarising sheets are perpendicular to each other.

6.6 *A transparent plate is given. How you will distinguish whether the given plate is a quarter wave plate or a half wave plate or a simple glass plate?* **[GGSIPU, Jan. 2015 (2.5 marks)]**

Ans. (i) (a) If linearly polarised light is incident on a transparent plate at 45° to optic axis. If the light is divided into two equal electric field components and one of these is retarded by a quarter wavelength. Then this plate will produce circularly polarised light. Then we can say that this plate is quarter wave plate (QWP). (b) If circularly polarised light is incident on QWP at 45° to the optic axis, then it produces linearly polarised light. (c) If linearly polarised light is incident on QWP other than 45° to the optic axis then it produces elliptical polarised light.

(ii) If the given plate is half wave plate (HWP), then it can rotate the polarisation of linearly polarised light to twice the angle between the optic axis and the plane of polarisation. Placing the optic axis of a HWP at 45° to the polarisation plane results in a polarisation rotation of 90° to its original plane.

(iii) If given plate is simple glass plate then it will on principle of Brewster's law (see section 6.3.1 at page 324).

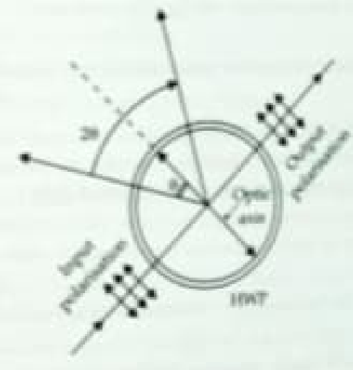


Fig. 6.34 Half wave plate.

6.7 *What do you mean by half shade device or Laurent plate?*

Ans. Laurent's half shade device consists of two semi-circular plates, one of quartz and another of glass. Quartz plate serves the purpose of half wave plate, *i.e.*, its thickness is such that it introduces a phase change of π between extraordinary and ordinary rays. The thickness of glass plate is chosen such that it absorbs the same amount of light as quartz plate. The quartz plate is cut parallel to optic axis and connected along diameter with the glass plate.

EXERCISES

Theoretical Questions

6.1 Obtain the Brewster's law for the polarising angle. **[GGSIPU, Dec. 2008 (3.5 marks)]**

6.2 Explain Brewster's law. Show from this law that when light is incident on a transparent substance at the polarising angle, the reflected and refracted rays are at right angles. **[GGSIPU, Sept. 2005 (5 marks)]**

6.3 State Brewster's law. Show that when a ray is incident at Brewster angle, the reflected ray is perpendicular to refracted ray. **[GGSIPU, Sept. 2011 (3 marks), Sept. 2013 reappear (7 marks)]**

6.4 State and explain Brewster's law. Use it to prove that when light is incident on a transparent substance at the polarising angle, the reflected and the refracted rays are at right angle to each other. **[GGSIPU, Sept. 2010 (3 marks) ; Dec. 2010 (6 marks)]**

6.5 What is Brewster's law ? Show that when light is incident on a transparent surface at a polarising angle, the reflected and the refracted rays are at right angles to each other. **[GGSIPU, Dec. 2011 (4 marks) ; Dec. 2012 (6.5 marks)]**

6.6 (*a*) Enunciate Malus law. **[GGSIPU, Dec. 2009 (1 marks)]**

(*b*) State Malus Law. **[GGSIPU, Dec. 2007 ; Dec. 2011, 12 (1 mark)]**

(*c*) State and prove Malus law of polarisation. **[GGSIPU, Dec. 2009 (4 marks)]**

6.7 State the law of Malus. **[GGSIPU, Oct. 2013 (2 marks)]**

6.8 If unpolarised light falls on a system of two cross polarised sheets, no light is transmitted. If third polarising sheet is placed between them will light be transmitted ? Explain. **[GGSIPU, Sept. 2009 (2 marks)]**

6.9 Describe in brief the various methods for producing linearly polarised light. **[GGSIPU, Dec. 2007 (2.5 marks)]**

6.10 Differentiate the uniaxial and biaxial crystals. **[GGSIPU, Dec. 2015 (3 marks)]**

6.11 Explain the phenomenon of double refraction. Describe the working principle of a Nicol prism. How is a Nicol prism used to produce circularly polarised light ? **[GGSIPU, Dec. 2016 (6 marks)]**

6.12 Explain the phenomenon of double refraction with one application. **[GGSIPU, Dec. 2018 (3.5 marks)]**

6.13 What do you mean by double refraction ? What are ordinary and extraordinary rays ? **[GGSIPU, Jan 2015 (3.5 marks)]**

6.14 Explain the phenomenon of double refraction in Calcite crystal. Give the construction and theory of (*i*) quarter wave plate and (*ii*) half wave plate. Where is quarter wave plate used ? **[GGSIPU, Dec. 2019 (5 marks)]**

6.15 Define optic axis. **[GGSIPU, Dec. 2013 reappear (2.5 marks)]**

6.16 Define (*i*) optic axis, (*ii*) principal section of a crystal. **[GGSIPU, Sept. 2010 ; Dec. 2011 (2 marks)]**

6.17 Explain the phenomenon of double refraction with the help of a neat diagram. Also compare the properties of ordinary and extraordinary rays. **[GGSIPU, Sept. 2010 (5 marks)]**

6.18 Describe in brief the phenomenon of double refraction. Compare the properties of ordinary and extraordinary rays. **[GGSIPU, Dec. 2013 reappear (6 marks)]**

6.19 Explain Hurygen theory of double refraction with ray diagram. **[GGSIPU, Dec. 2010 (3 marks)]**

6.20 Explain Hurygen's theory of double refraction. Explain with ray diagram. How a Nicol prism is used as polariser ? **[GGSIPU, Sept. 2011 (5 marks)]**

6.21 Explain the phenomenon of double refraction in calcite crystal. **[GGSIPU, Sept. 2011 reappear (4 marks)]**

6.22 What is the phenomenon of double refraction ? Explain with proper ray diagram how the Nicol prism is used both as a polariser and an analyser. **[GGSIPU, Sept. 2007, Sept 2012 (5 marks)]**

6.23 Explain the construction and working of a Nicol prism. **[GGSIPU, Dec. 2008 (5 marks)]**

6.24 Illustrate with a series of neat scientific, well-labeled diagrams' the formation of a Nicol prism from a doubly refracting crystals. **[GGSIPU, Dec. 2015 (2 marks)]**

6.25 Give the construction and working of a Nicol prism . How would you distinguish between quarter wave plate and half wave plate ? **[GGSIPU, Dec. 2018 (4 marks)]**

6.26 Describe the construction of a Nicol prism and show how it can be used as a polarizer or as an analyzer. **[GGSIPU, Dec. 2017 (3.5 marks)]**

6.27 Discuss with a neat diagram the principle and working of a Nicol prism. What are its limitations ? **[GGSIPU, Dec. 2012 ; Sept. 2010 reappear (8 marks)]**

6.28 Write short note on Nicol prism. **[GGSIPU, Sept. 2011 reappear (4 marks)}**

6.29 Explain the principle, construction and working of a Nicol prism. **[GGSIPU, Dec. 2011 (5 marks) ; Dec. 2012 (6 marks)]**

6.30 What do you mean by polarisation of light ? Describe the construction of Nicol prism and show how it can be used as a polariser and analyser. **[GGSIPU, Dec. 2013 (4.5 marks)]**

6.31 Draw a labelled ray diagram for Nicol prism. **[GGSIPU, Dec. 2013 reappear (2 marks)]**

6.32 Describe the construction and action of Nicol prism. **[GGSIPU, Nov. 2012 (4 marks)]**

6.33 Explain with proper ray diagram how the Nicol prism is used both as a polariser and an analyser. **[GGSIPU, Nov. 2012 reappear (7 marks)]**

6.34 What is meant by plane polarized, circularly polarized and elliptically polarized light ? Show that the plane polarized and circularly polarized lights are special cases of elliptically polarized light. **[GGSIPU, Dec. 2019 (4 marks)]**

6.35 Explain the superposition of polarized light. Hence, differentiate between plane polarized, circularly polarized and elliptically polarized lights. **[GGSIPU, Dec. 2015 (5 marks)]**

6.36 Explain with a series of neat scientific well-labeled diagrams the functioning of the retarding plates : (*i*) Half wave plate (*ii*) Quarter wave plate. **[GGSIPU, Dec. 2016 (4 marks)]**

6.37 List one important limitation and use of half wave plate. **[GGSIPU, Sept. 2011 (2 marks)]**

6.38 What is a half wave plate and how is it used ? **[GGSIPU, Sept. 2012 (2 marks)]**

6.39 Distinguish between linearly, circularly and elliptically polarised light. Explain their production with the help of mathematical equations. **[GGSIPU, Oct. 2013 (7 marks)]**

6.40 How would you distinguish between plane, circularly and elliptically polarised light ? **[GGSIPU, Dec. 2013 (4 marks)]**

6.41 How can we convert a right circularly polarised light into left circularly polarised light. [GGSIPU, Dec. 2007 (6 marks)]

6.42 How would you differentiate unpolarised light from circularly polarised light ? [GGSIPU, Dec. 2010 (2 marks)] [GGSIPU, Dec. 2013 reapper (2 marks)]

6.43 Define optical activity. [GGSIPU, Dec. 2009 (4 marks)]

6.44 Define specific rotation. Is it a temperature dependent property ? [GGSIPU, Dec. 2010 (2 marks)]

6.45 Is specific rotation a temperature dependent ? Explain. [GGSIPU, Dec. 08 (1 mark)]

6.46 Is specific rotation a constant for particular optically active solution ?

6.47 What is the specific rotation ? Describe the construction and working of Laurent's half shade device. [GGSIPU, Sept. 2008 (7 marks)]

6.48 What is specific rotation ? Explain the role of half wave plate in Laurent's half shade polarimeter. [GGSIPU, Sept. 2011 reappear (6 marks)]

6.49 What is specific rotation ? Describe the working of a Laurent's half shade polarimeter. How will you use it to find the specific rotation of sugar ? [GGSIPU, Dec. 2017 (6 marks)]

6.50 Define specific rotation. Explain the construction and working of Laurent's half shade polarimeter. [GGSIPU, Jan. 2015 (6 marks)]

6.51 Give construction and working of Laurent's half shade polarimeter. [GGSIPU, Nov. 2012 (4 marks)]

6.52 Explain the role of Laurent's half wave plate in Laurent's half shade polarimeter. [GGSIPU, Sept. 2009 (4 marks)]

6.53 Describe Laurent half shade polarimeter. How it can be used to find the specific rotation of an optically active substance ? [GGSIPU, Nov. 2007, 2006 (4 marks)]

6.54 Define specific rotation. Describe the construction and working of Laurent's half shade polarimeter, explaining fully the action of half shade device. How would you use it to determine the specific rotation of cane sugar solution ? [GGSIPU, Sept. 2005 (7 marks)]

6.55 Describe Laurent's half shade polarimeter [GGSIPU, Dec. 2017 (3.5 marks)]

Numerical Problems

6.1 The velocity of light in water is 2.2×10^8 ms^{-1}. What is the polarizing angle of incidence for water surface for water surface ? (Given the speed of light in free space = 3×10^8 ms^{-1}). [GGSIPU, Dec. 2016 (2.5 marks)]

Hint : $\mu_{water} = \frac{c}{v} = \frac{3\times10^8}{2.2\times10^8} = 1.36 \Rightarrow \mu = \tan i_p \Rightarrow i_p = \tan^{-1}\mu = \tan^{-1}(1.36) = 53.74°$.

6.2 Calculate the Brewster angle for ethyl alcohol ($\mu = 1.46$). [GGSIPU, Dec. 2013 reappear (2 marks)]

Hint : $\mu = \tan i_p$ or $i_p = \tan^{-1}(\mu) \Rightarrow i_p = \tan^{-1}(1.46) = 56°$.

6.3 At what angle the light should be incident on a glass plate ($\mu = 1.5697$) to get a plane polarised light by reflection ? [GGSIPU, Sept. 2012 (2 marks)]

Hint : $\mu = \tan i_p$ Here $\mu = 1.5697$

Then $i_p = \tan^{-1}(1.5697) = 57.5°$.

6.4 Light travelling in water ($\mu = 1.33$) is incident on a plate of glass ($\mu = 1.53$). At what angle of incidence will the light be fully polarised ? [GGSIPU, Dec. 2009 (4 marks)]

Hint : ${}_a\mu_\omega = 1.33$ and ${}_a\mu_g = 1.53$ then ${}_\omega\mu_g = \frac{{}_a\mu_g}{{}_a\mu_\omega} = \frac{1.53}{1.33} = 1.15$

$${}_\omega\mu_g = \tan i_p \Rightarrow i_p = \tan^{-1}(1.15) = 49° 12'.$$

6.5 What will be the Brewster's angle for a glass slab ($\mu = 1.5$) immersed in water ? [GGSIPU, Dec. 2007 (4 marks)]

Hint : $\mu = \tan i_p$ $i_p = d \tan^{-1} 15 = 56.7°$.

6.6 What will be the Brewster's angle for a glass slab ($\mu = 1.5$) immersed in water ($\mu = 1.33$) ? [GGSIPU, Sept., 2010 ; Sept 2012 (2 marks)]

Hint : ${}_a\mu_g = 1.5$, ${}_a\mu_\omega = 1.33$ then ${}_w\mu_g = \frac{1.5}{1.33} = 1.128$

then ${}_w\mu_g = \tan i_p \Rightarrow i_p = \tan^{-1}({}_w\mu_g) = \tan^{-1}(1.128) = 48.5°$.

6.7 What is the polarising angle for glass whose refractive index for light used in 1.6827 ? [GGSIPU, Sept. 2011 Reappear (2 marks)]

Hint : Using Brewster's law, we know that

$$\tan i_p = \mu \Rightarrow i_p = \tan^{-1}(1.6827) = 59°.$$

6.8 Refractive index of water is 1.33. Calculate the angle of polarisation of light reflected from the surface of the pond. [GGSIPU, Nov. 2012 (2 marks)]

Hint : $\mu = \tan i_p \Rightarrow i_p = \tan^{-1}(1.33) = 53.06°$.

6.9 If the plane of vibration of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light.

Hint : $\frac{I_e}{I_o} = ?$, where I_e and I_o are the intensity of e-ray and o-ray respectively.

$$\frac{I_e}{I_o} = \frac{\cos^2\theta}{\sin^2\theta} = \frac{3}{1}.$$

6.10 Determine the state of polarization (SOP) of the following set of equations as :

$\vec{E} = E_0 \hat{x} \cos(kz - \omega t)$ and $\vec{E} = E_0 \hat{y} \cos(kz - \omega t + \phi)$ when (a) $\phi = \frac{\pi}{2}$ and (b) $\phi = \pi$. [GGSIPU, Dec. 2018 (3 marks)]

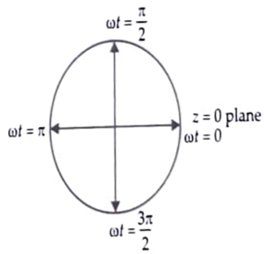
Hint : Case (a)

When $\phi = \frac{\pi}{2}$, $\vec{E} = E_0$ and consider a plane $z = 0$.

at time $\omega t = 0$ $\vec{E} = E_x \hat{x}$ and $\vec{E} = 0\hat{y}$

at time $\omega t = \frac{\pi}{2}$ $\vec{E} = 0\hat{x}$ and $\vec{E} = E_o \hat{y}$

at time $\omega t = \frac{3\pi}{2}$ $\vec{E} = 0\hat{x}$ and $\vec{E} = -E_o \hat{y}$



Case (b). When $\phi = \pi$, contribution of both component aligned in same direction irrespective SOP will be linearly polarized life.

6.11 Two Nicols are oriented with their planes making an angle of 60°. What percentage of incident unpolarised light will pass through the system ?
[GGSIPU, Sept. 2004 (5 marks), Sept. 2008, Oct. 2013 (3 marks) ; Dec. 2017 (3 marks)]

Hint : If unpolarised light incidents on a polariser, the intensity of light transmitted through the polariser.

$$\frac{I_0}{2} = I_1 \quad \text{and} \quad I = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2 60° = 0.125\, I_0 = 12.5\% \text{ of } I_0.$$

6.12 A polariser and an analyser are oriented so that maximum amount of light is transmitted. To what fraction of its maximum value is the intensity of transmitted light reduced when the analyser is rotated through 22.5° ? **[GGSIPU, Dec. 2008 (4 marks)]**

Hint : According to Malus law, intensity of transmitted light through analyser is given by

$$I = I_0 \cos^2\theta = I_0 \cos^2 22.5 = 0.85\, I_0$$

Thus its maximum intensity is reduced to 85% of the maximum.

6.13 What will be the state of polarisation of the emerging light when

(*i*) A beam of circularly polarised light is passed through a quarter wave plate (QWP) ;
[GGSIPU, Sept. 2012 (2 marks)]

(*ii*) A beam of plane polarised light is passed through a quarter wave plate ;

(*iii*) A beam of elliptically polarised light is passed through quarter wave plate ; and

(*iv*) A beam of plane polarised light is passed through a QWP such that the plane polarised light falling on QWP makes an angle of 45° with optic axis ?

Hint : (*i*) Plane/linearly polarised light. (*ii*) Circularly or elliptically or plane polarised light.
(*iii*) Plane/linearly polarised light. (*iv*) Circularly polarised light.

6.14 Calculate the thickness of (*i*) quarter wave plate and (*ii*) half wave plate, given $\lambda = 5000\,\text{Å}, \mu = 1.544$, $\mu_e = 1.553$. **[GGSIPU, Dec. 2019 (2.5 marks)]**

Hint : (*i*) QWP $t = \dfrac{\lambda}{4(\mu_e - \mu_0)}$

$$t = \frac{5000\times10^{-10}}{4\times(1.553-1.544)} = \frac{5000\times10^{-10}}{4\times0.009}$$

$$= \frac{50}{36}\times10^{-5}\,\text{m} = 1.39\times10^{-5}\,\text{m} = 0.139\,\mu\text{m}$$

(*ii*) HWP $t = \dfrac{\lambda}{2(\mu_e-\mu_0)} = 0.278\,\mu\text{m}$

6.15 Calculate the thickness of (*i*) a quarter wave plate and (*ii*) a half wave plate given that $\mu_e = 1.553$, $\mu_o = 1.544$ and $\lambda = 6328\,\text{Å}$. **[GGSIPU, Dec. 2018 (2 marks)]**

Hint : (*i*) QWP $t = \dfrac{\lambda}{4(\mu_e-\mu_0)} = \dfrac{6328\times10^{-10}}{4\times(1.553-1.544)} = 0.176\,\mu\text{m}$

(*ii*) HWP $t = \dfrac{\lambda}{2(\mu_e-\mu_0)} = \dfrac{6328\times10^{-10}}{2\times(1.553-1.544)} = 0.352\,\mu\text{m}$

6.16 A beam of linearly polarised light is changed into circularly polarised light by passing it through a sliced crystal of thickness 0.03 cm. Calculate the difference in refractive indices of the two rays in the crystal and using their to be of minimum thickness that will produce the effect. The wavelength of light used is 6×10^{-7} m.

Hint : For quarter wave plate

$$t = \frac{\lambda}{4(\mu_e-\mu_0)}$$

So $(\mu_e-\mu_0) = \dfrac{\lambda}{4t} = \dfrac{6\times10^{-7}}{4\times3\times10^{-4}} =$ [illegible] $\times10^{-3} = 5.0\times10^{-4}$.

6.17 Calculate the minimum thickness of a quarter wave plate for the light of wavelength 5893 Å . Given $\mu_0 = 1.57$ and $\mu_e = 1.526$. **[GGSIPU, Sept. 09, 10 reappear, 12 reappear, 2013 reappear (2 marks)]**

Hint : $t = \dfrac{\lambda}{4(\mu_0-\mu_e)} = 3.35\,\mu\text{m}$.

6.18 Calculate the minimum thickness of a calcite plate which would convert plane polarised light into circularly polarised light. The principal refractive indices for the ordinary and extraordinary rays are 1.658 and 1.486 respectively at wavelength 5890 Å.
[GGSIPU, Nov. 2012 reappear ; Dec. 2011 (3.5 marks)]

Hint : This will be QWP and formula for thickness is

$$t = \frac{\lambda}{4(\mu_2-\mu_a)} = \frac{5890\times10^{-8}}{4\times0.172} = 0.86\,\mu\text{m}.$$

6.19 Linearly polarised light is changed into circularly polarised light after passing through a slice of the crystal 2.5×10^{-5} m thick. Find the wavelength of light used, if the difference in refractive indices for ordinary and extraordinary rays is 0.005. **[GGSIPU, Nov. 2012 (2 marks)]**

Hint : Given $t = 2.5\times10^{-5}$ m and $(\mu_0-\mu_e) = 0.005$.

This plate is QWP, then

$$t = \frac{\lambda}{4(\mu_0-\mu_e)} \quad \text{or} \quad \lambda = 4t(\mu_0-\mu_e)$$

$$\Rightarrow \quad \lambda = 4\times2.5\times10^{-5}\times0.005 = 5000\times10^{-10}\,\text{m} = 5000\,\text{Å}$$

6.20 Calculate the thickness of half wave plate for sodium light ($\lambda = 5893\,\text{Å}$), if $\mu_0 = 1.54$ and ratio of velocity of ordinary and extraordinary waves is 1.007. Is this crystal is positive or negative ?
[GGSIPU, Dec. 2010 (3.5 marks)]

Hint : Given $\lambda = 5893\,\text{Å},\ \mu_0 = 1.54,\ v_e/v_0 = 1.007$

We know that $\dfrac{\mu_0}{\mu_e} = \dfrac{v_e}{v_0} \Rightarrow \mu_e = 1.54\times1.007 = 1.551$

$\because\ \mu_e > \mu_0$, so crystal is positive, then $t = \dfrac{\lambda}{2(\mu_e-\mu_0)} = 2.733\times10^{-5}$ m.

6.21 The plane of polarization gets rotated through 23.4° as light travels through an 18 cm long column of 20% sugar solution. Determine the specific rotation of solution. **[GGSIPU, Dec. 2016 (2.5 marks)]**

Hint : $[s]_T^{\lambda} = \dfrac{\theta}{l\times C} = \dfrac{23.4\times100}{1.8\times20} = 65°/\text{dm}/\text{gm}/\text{cm}^3$.

6.22 A 10 cm long tube contains 10% sugar solution and produces an optical rotation of 13.2°. Calculate the specific rotation. **[GGSIPU, Dec. 2009 (4 marks)]**

Hint : $[S]_T^{\lambda} = \dfrac{\theta}{l\times C} = \dfrac{13.2\times10}{1\times1} = 132°/\text{dm}/\text{gm}/\text{cm}^3$.

6.23 A sugar solution in a tube of length 20 cm produces optical rotation of 13°. The solution is then diluted to one-third of its previous concentration. Find optical rotation produced lay 30 cm long tube containing the dilute solution. **[GGSIPU, Dec. 2013 (4 marks)]**

Hint : Given $l_1 = 20$ cm, $\theta_1 = 13°$, $C_1 = C$ and $l_2 = 30$ cm, $\theta_2 = ?$ $C_2 = \frac{1}{3}C$

Now, as per question $s = \frac{\theta_1}{l_1 \times C_1} = \frac{\theta_2}{l_2 \times C_2}$

$$\theta_2 = \frac{\theta_1 \times l_2 \times C_2}{l_1 \times C_1} = \frac{13° \times 30 \times C}{20 \times C \times 3} = 6.5°$$

Multiple Choice Questions

6.1 Two Nicol prisms are first crossed and then one of them is rotated through 60°. The percentage of incident light transmitted is :

(*a*) 12.5 (*b*) 25.0 (*c*) 37.5 (*d*) 50.0

6.2 The thickness of a $\frac{\lambda}{4}$-plate for light of wavelength 6000 Å will be (in cms) :

(*a*) 1.5×10^{-4} (*b*) 1.54×10^{-4} (*c*) 1.55×10^{-4} (*d*) 1.5×10^{-3}

[For a plate $\mu_o = 1.54$ and $\mu_e = 1.55$]

6.3 A quarter wave plate is designated for 6000 Å. If change in refractive index with wavelength is negligible, for 4500 Å the phase retardation will be :

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{2\pi}{3}$ (*c*) π (*d*) $\frac{\pi}{3}$

6.4 When an unpolarised light is incident on a calcite crystal, it splits two refracted rays. The phenomenon is known as :

(*a*) scattering (*b*) dispersion
(*c*) double refraction (*d*) diffraction.

6.5 The phenomenon of rotating the plane of vibration of a polarised light is known as :

(*a*) polarisation (*b*) optical activity
(*c*) double refraction (*d*) Kerr effect.

6.6 When light is incident on a plane of a transparent material at the angle of polarisation, the reflected and refracted beams are :

(*a*) parallel (*b*) inclined by 45°
(*c*) inclined by 60° (*d*) perpendicular to each other.

6.7 A plane polarised beam of light is passed through a quarter wave plate. The transmitted light beam analysed using a Nicol shows no variation in intensity as the Nicol is turned through 360°. It means that the transmitted light is :

(*a*) circularly polarised (*b*) elliptically polarised
(*c*) unpolarised (*d*) mixture of plane polarised and unplolarised.

6.8 When a beam of light is incident on a glass plate at polarising angle, then the angle between reflected and refracted beam is :

(*a*) 0° (*b*) 45° (*c*) 60° (*d*) 90°

6.9 A plane polarised beam of light is passed through a quarter wave plate. The transmitted beam is analysed using a Nicol shows no variation in intensity as Nicol is turned through 360°. The quarter wave plate is turned through 90° and the transmitted light is again analysed similarly. Now we will observe :

(*a*) two zero intensity minima (*b*) no change in intensity
(*c*) two non-zero minima (*d*) four zero intensity minima.

6.10 An unpolarised beam is incident at an angle of 60° on a glass surface and after reflection it, is linearly polarised. The approximate refractive index of the glass is :

(*a*) 1.4 (*b*) 1.5 (*c*) 1.7 (*d*) 1.6.

6.11 In a Nicol prism, the refractive index of calcite crystal for ordinary ray is 1.66 and that for extra ordinary ray is 1.49. The refractive index of Canada balsam is :

(*a*) 1.49 (*b*) more than 1.66
(*c*) betweeen 1.49 and 1.66 (*d*) less than 1.49.

Answers

6.1 (*b*)	6.2 (*d*)	6.3 (*b*)	6.4 (*c*)	6.5 (*b*)	6.6 (*d*)
6.7 (*a*)	6.8 (*d*)	6.9 (*a*)	6.10 (*c*)	6.11 (*c*)	

TUTORIAL

6.1 A glass plate is to be used as a polariser. Find the angle of polarisation for it. Also find the angle of refraction (μ for glass = 1.54). [**Ans.** $i_p = 57°$, $r = 33°$]

6.2 At what angle the light should be incident on glass plate (μ = 1.5697) to get a plane polarised light by reflection ? [**Ans.** 57.6°]

6.3 Unpolarised light falls on two polarising sheets placed one on the top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of transmitted light is sheets if the intensity of transmitted light is :

(**a**) one third the maximum intensity of the transmitted beam ?

(**b**) one third the intensity of incident beam ? [**Ans.** (*a*) ± 55° (*b*) ± 35°]

6.4 Discuss the state of polarisations of the following waves :

(*i*) $\vec{E} = \hat{j}\, A\cos(kx - \omega t) + \hat{k}\, 2A\cos(kx - \omega t + \pi/4)$

(*ii*) $\vec{E} = \hat{j}\, A\cos(kx - \omega t) + \hat{k}\, A\sin(kx - \omega t)$

[**Ans.** (*i*) E_y remains same but E_z increases. Hence polarisation is left handed EPL.
(*ii*) E_y remains same but E_z decreases. Hence polarisation is right handed CPL.]

6.5 Plane polarised light passes through a quartz plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam will be :

(*i*) plane polarised (*ii*) circularly polarised [**Ans.** (*i*) $t = 27.5\ \mu$m (*ii*) $t = 3.17\ \mu$m]

6.6 A beam of linearly polarised light is changed into circularly polarised light by passing it through a sliced crystal of thickness 0.03 cm. Calculate the difference in refractive indices of the two rays in the crystal on using their to be of minimum thickness that will produce the effect. The wavelength of light used is 6×10^{-7} m. [**Ans.** 5.0×10^{-2}cm]

6.7 Calculate the thickness of (*i*) a quarter wave plate and (*ii*) a half wave plate given that $\mu_e = 1.553$ and $\mu_0 = 1.544$ and $\lambda = 5000$ Å. [**Ans.** $QWP \rightarrow 0.13 \times 10^{-2}$cm and $HWP \rightarrow 0.27 \times 10^{-2}$ cm]

6.8 Calculate the thickness of calcite plate required to change plane polarised light into circularly polarised light. For calcite, $\mu_o = 1.658$, $\mu_e = 1.486$ and $\lambda = 589$ nm.

[**Ans.** $t = 0.856$ μm, 4.28 μm, 7.7 μm,...]

6.9 A phase retardation plate of calcite acts as a half wave plate for the wavelength λ_1 and as a quarter wave plate for the wavelength λ_2. Find the ratio of λ_1 and λ_2.

$$\left[\textbf{Ans. } \frac{\lambda_1}{\lambda_2} = \frac{4m+1}{4n+2} = \frac{1}{2}, \frac{5}{6}, \frac{9}{10}, \ldots\right]$$

6.10 A partially polarised beam is passed through a Nicol. In 90° rotation of Nicol, the intensity changes by 75%. Calculate degree of polarisation. [**Ans.** 60%]

6.11 If 20 cm length of a certain solution causes right handed rotation of 42° and 30 cm of another solution causes left handed rotation of 27°. What optical rotation will be caused by 30 cm length of a mixture of the solutions in the volume ratio of 1 : 2 ? The solutions are not chemically reactive.

[**Ans.** $\theta = \theta_1 + \theta_2 = 3°$]

6.12 A sugar solution in a glass tube of 20 cm length produces an optical rotation of 13°. The solution is then diluted to one-third of its previous concentration. Estimate the optical rotation produced by a 30 cm long glass tube containing the diluted solution. [**Ans.** 6.5°]

6.13 Calculate the specific rotation for sugar solution using the following data :

Length of the tube = 20 cm ; Volume of the tube = 120 cm^3 ; Quantity of sugar dissolved = 6 g

Angle of rotation of the analyser for reforming equal intensity = 6.6°. [**Ans.** 66 deg/{dm × (g/cc)}]

6.14 A 200 mm long tube and containing 48 cm^3 of sugar solution produces an optical rotation of 11° when placed in a polarimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of a solution. [**Ans.** 4 g]

6.15 A certain length of 5% solution causes the optical rotation of 20°. How much length of 10% solution of the same substance will cause 35° rotation ? [**Ans.** $l_2 = 7l_1 / 8$]