

1<sup>st</sup> Null {Stack} linear data structure  
- one predecessor and one successor only



$\nwarrow$

LIFO - last In first Out

Applications - Recursions

Memory Management

gmp ✓ Expression Evaluation

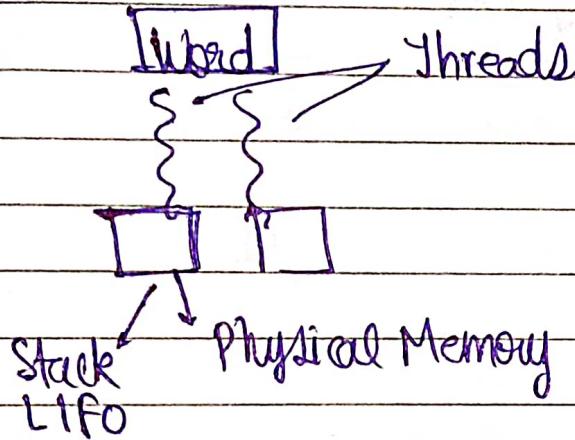
Paranthesis Matching

Backtracking

to check parentheses matching  
in an expression

- can control memory allocation and deallocation
- helps to clean up the objects

① Memory Management



② Expression Evaluation

$$\{ \text{Top} = \text{Size} - 1 \}$$

$$(2+3)+x-y$$

→ Overflow

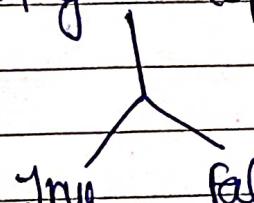
3	50
2	20
1	10
0	8

Top  
{Current Position}

1. Push (Insert at last)

2. Pop → isEmpty      top = 1

3. Traverse



"Under flow"

E Push } → top = top + 1

insert

"top" should start from -1

int top = -1

int str. array [4], x;

void push () {

if (top == size - 1) {

printf ("Overflow") ;

else {

top += 1

str.array [top] = x

}

void main () {

push();

}

size = 5



2nd

Now (Pop)

int IsEmpty (struct Stack \*pfr)

→ Check whether the array contain any element or not

```

if (top == -1) {
    printf ("Underflow");
} else {
    printf ("%d", str_array [top]);
    top = top - 1;
}

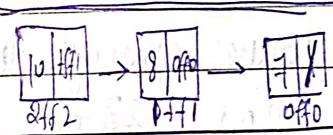
```

### Stack Implementation with Linked List

↳ To use of top in case of stack implementation



Note



first element of

→ Always push and pop from the beginning. stack will be last of the linked list

Note - Stack need to implemented in array and linked list

9<sup>th</sup> Nov"Types of parsing"Expression Evaluation

→ Infix      2+3

→ Prefix      + 2 3

→ Postfix      2 3 +

(operator प्रति)

(operator विपरीत)

- Conversions of these three possible

 $a + (b + c) - e$ 

operands    operators

Infix

- two 1 operator in between 2 operand

- eg 2+ a+b (2+3)

Prefix

→ operator comes first and then operand

eg + a b (+ 2 3)

Postfix

- operator comes after the operand

eg ab+ (23+)

Application - When we do compiler design

- Precedence of \*, / remain same

Evaluate

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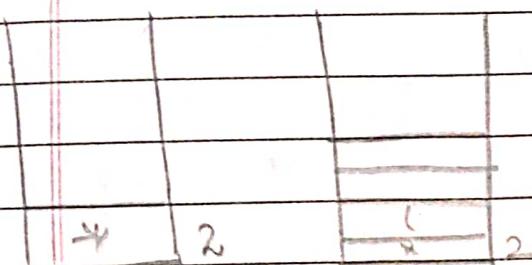
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Q) Solve the given Infix operations using Stack

D) Stack के अंदर operator using precedence (LIFO)

$$\Rightarrow 2 * (5 + (3+6)) / 15 - 2$$

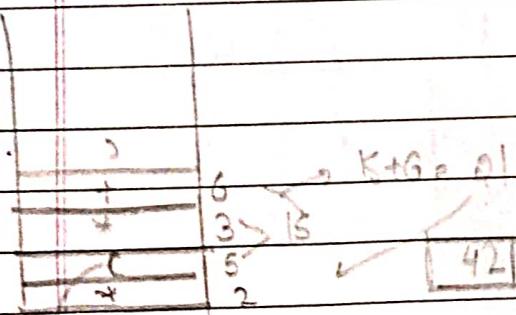
- Precedence
  - 1) ^
  - 2) \* /
  - 3) + - %



- Highest priority  $\frac{3}{10} \text{ & } 2$   
 $\frac{3}{10} \text{ & } 2$  at lowest priority  
can not go

$$\therefore 2 * (5 * 3 + 6)$$

- If opening bracket their then no need to check highest / lowest priority outside the bracket



- Ignore the bracket need to check the precedence

no need to check precedence since bracket is open

- If same precedence in bracket then use LIFO

- At end, we get an empty stack of operator and an element in operand stack (which will be the answer)

$$2 * (5 * (3+6)) / 15 - 2$$

+ -

+

6

3

5

2

18

40

5

45

2

Add can be their own  
 there is high priority (\*)  
 because we have opening  
 bracket

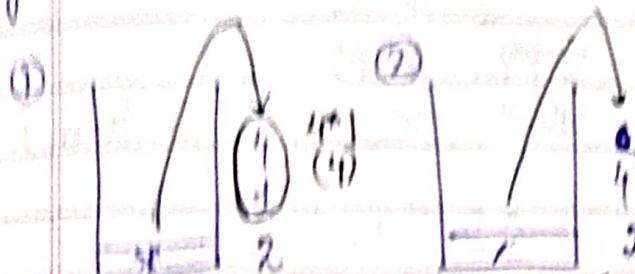
# Right to left Pseudo Move

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eg:  $4 \cdot 8 / * 2 2 1 / * 4 1 2$

① small right to left

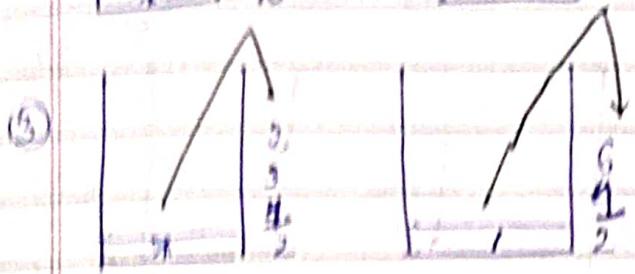


$arb \rightarrow arb$

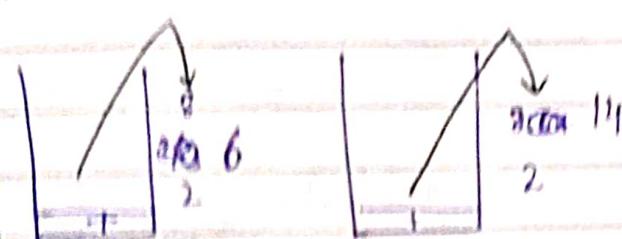
②

$1 2 1$

out after right side  
3 1 2 at left side  
a+b



③ No checking procedure



$\Rightarrow 1000016$

Pulse	Rush	Pop	Value
2	2		
4	4		
6	4	*	4
1	1	*	2
3	3		
2	2		
4	*	*	6
1	1	*	6
8	8	*	
+	+	*	14
+	+	*	16

↗ Parsing from right to left

→ ALU (Arithmetic Logic Unit) can do only postfix

→ persist remain same in infix, prefix and postfix

Prefix  $+23 \Rightarrow 5$

$2+3$

$\frac{1}{+9} = 26$

Main  
environment

① Parse right to left

↓ Priority

② (Precedence taken)

$62 * 9 + - \text{reverse}$

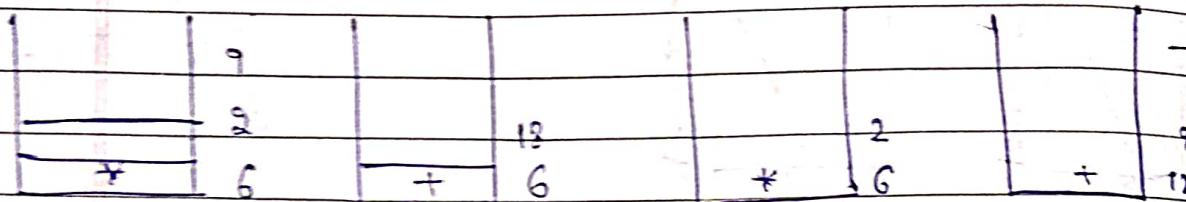
Method 2

~~4+2~~ → right to left

Prefix (Precedence not taken)

$62 + 9 * - \text{reverse}$

↓ P.L.



124 ||

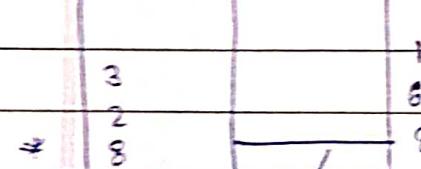
Postfix i. left to right parse

$a+b$

$a+b$

①

②



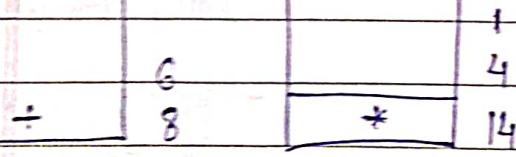
\* Postfix

$a+b$

$a+b$

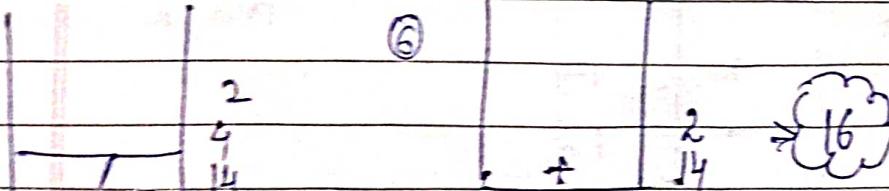
③

④



⑤

⑥



Precedence ↗ Associativity left to right  
Imp

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Postfix to Prefix

→ Note

$$A + B * (C + D) / F + D * E$$

↓ operand

\* Prefix → Right to Left

$$A + B * \underline{+ C D} / F + D * E$$

\* Postfix / Infix → left to Right

$$A + \cdot * B + C D, / F + D * E$$

\* Infix - Precedence in account

$$A + \underline{/} * B + C D F, + D * E$$

Postfix/- not in account  
Prefix

$$A + \underline{/} * B + C D F, + \underline{* D E}$$

$$\underline{+ A / B} * B + C D F + \underline{* D E}$$

$$+ + A / B * B + C D F * D E$$

All a ✓



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## Queue (FIFO)

Insertion

Overflow - Rear = Size - 1

Underflow → Rear = front = -1      (When created the queue)  
Front > Rear

Deletion

{ Dequeue → Rear }  
Dequeue → front }

Insertion ① Check overflow

② Increment rear then value

Insertion

→ If (rear == (Max - 1))  
    printf ("Overflow");

→ If rear = front - 1;      { When the queue is inserted first }  
    Front = 0;                  time  
    Rear = 0;

→ else;

Rear = Rear + 1

Queue [Rear] = data;

Dequeue → If (front = rear == -1) // (front > rear) → Maintaining playlist  
    printf ("Underflow");

else

val = queue [Front]

Front = Front + 1

Application

→ Data Packets in  
Data Network flow  
(Handling website  
traffic)

→ Maintaining playlist  
in Media-Players  
→ CPU scheduling and  
disk

## linked list

1. Insertion from end  $\Rightarrow$  Enqueue
2. Deletion from front  $\Rightarrow$  Dequeue

```
struct node * {  
    int data;  
    struct node * next;  
};
```

```
void enqueue (int data, struct node * head) {  
    struct node * new_node = (struct node *) malloc  
        (sizeof (struct node));  
    new_node->data = data;  
    new_node->next = NULL;  
    while (head->next != NULL) {  
        head = head->next;  
    }  
    head->next = new_node;  
}
```



## Circular Queue

इसमें rear से front 42 तकी है।  
In circular no rear / front but  
it follows FIFO

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• int isfull {

if ((front == rear + 1) || (front == 0 && rear == size - 1))

return 1

else

return 0

int isempty () {

if (front == -1)

return 1

else

return 0

• void enqueue (int item) {

if (isfull ())

printf ("Overflow")

else

if (front == -1)

front = 0

else

rear = (rear + 1) % size

CircularQueue [rear] = item

3

eg [10 | 7 | 6 | 3] 1%5

int dequeue () {

if (isempty ())

printf ("Underflow")

else

item = Cir\_Queue [front]

if (front == rear)

front = -1

rear = -1

else

return front = (front + 1) % size

→ Doubly-ended.

Insertion from Rear + Front  
Deletion from Front + Rear

## Deque / DE - Queue

Input  
restricted  
queue

Output  
restricted  
queue

→ do not follow FIFO

→ called queue because can add/insert both from front/end but not from the middle

→ Enqueue  
- Rear  
→ Dequeue

Enqueue -  
Rear / front

Enqueue - front, Rear  
Dequeue - front, Rear

Rear /  
front

Dequeue -  
front

{ Insertion from  
front not  
allowed }

{ Deletion from rear }  
not allowed

Enqueue

Enqueue - front ()

if (front < 1)

front = n - 1

IF  
DR

else

front = front - 1

dequeue [front] = item;

Enqueue - rear  
and  
dequeue - front

Application

Right to left

Dequeue - rear ()

① front = -1

Underflow

Right to left

2. front != rear

front = -1

rear = -1

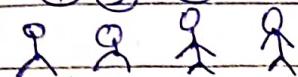
3. rear == 0

rear = n - 1

④ rear = rear - 1

• Queue  $\Rightarrow$  FIFO

$\Rightarrow ④ ③ ② ①$



Insertion  
End

Deletion  
End

Queue ADT

$\Rightarrow$  Storage

In. End

Del. End

Methods - Enqueue

- Dequeue

- first val

- last val

- peak

- isEmpty

- isFull

Queue  $\rightarrow$

Application

Stack  $\rightarrow$  Increment back at end ] O(1)  
 (queue)  $\rightarrow$  Insert at back

dequeue  $\rightarrow$  (1) Remove element at 0  
 (2) Shift all elements

[ O(n) ] Thus we find other alternative  
 maintain a front index  
 and it will not contain index  
 of first element but index of  
 prior to first element

Initially Front and back = -1

Queue empty  $\Rightarrow$  front = back

Queue full  $\Rightarrow$  front = size - 1

## (Merge Sort)

$\rightarrow O(n \log n)$

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- work on "divide and conquer" strategy

- Recursive Function used

↳ call itself

↳ starting function - base function

↳

eg sum (int n)

{ if ( $n=0$ )

    return n;

else

{ return  $n + \text{sum}(n-1)$

}

Divide and Conquer

- ① ⑤ sum(5) ⑪ ↗

② 5 + sum(5-1) ⑪ ↗

③ 5 + 4 + sum(4-1) ⑪ ↗

④ 5 + 4 + 3 + sum(3-1) ⑪ ↗

⑤ 5 + 4 + 3 + 2 + sum(2-1) ⑪ ↗

⑥ 5 + 4 + 3 + 2 + 1 + sum(1-1) ⑪ ↗

base condition

\* ⑪ Base condition

Note - divide till atomic problem

eg

1<sup>st</sup> [ 9 | 7 | 3 | 6 | 2 ]

2<sup>nd</sup> [ 9 | 7 | 3 ]     [ 6 | 2 ]

3<sup>rd</sup> [ 9 | 7 ]     [ 3 ]

4<sup>th</sup> [ 9 ]     [ 7 ]

\*  $m = (l+r)/2$

\* division = recursive  
merge  $\Rightarrow$  need algo

→ heap  
Flowing

1. After first pass, pivot element reaches right position

2

35	50	15	25	30	20	40	25
PIVOT	↑ left	↓				↑ right	

Final pivot  
at 4<sup>th</sup> element

3

QuickSortRamu  
Date

- int pivot-position (arr, first, last)
  - { pivot = arr[first]
  - left = first + 1
  - right = last
  - while (True) {
    - left and right  $\rightarrow$  while ( $left \leq right \& arr[left] \leq pivot$ ) || move left  
not crossing each other
      - left = left + 1
      - while ( $left \leq right \& arr[right] \geq pivot$ )
      - right = right - 1 || move right
    - right and left cross each other
      - if ( $right < left$ )
        - break || BE if at break
        - else
    - left and right swap
      - swap ( $arr[left], arr[right]$ )
      - swap ( $arr[first], arr[right]$ )
      - return right

void quicksort (arr, first, last) {

p = pivot-position (arr, first, last)

quicksort (arr, first, p-1)

quicksort (arr, p+1, last)

}

Difference → ~~Input / Output~~  
 → Input / Output  
 → time complexity  
 → Space complexity

## Time Complexity

Sorting Algo

Best  
( $\Theta$ )

Average  
( $\Theta$ )

Worst  
( $\Theta$ )

Space Complexity

Bubble

$n$

$n^2$

$n^2$

$O(1)$

Selection

$n^2$

$n^2$

$n^2$

$O(1)$

Insertion

$n$

$n^2$

$n^2$

$O(1)$

Merge

$n \log n$

$n \log n$

$n \log n$

$O(n)$

Quick Sort

$n \log n$

$n \log n$

$n^2$

$O(n)$

→ Here need to specify that best case in Quick Sort is when pivot element is in middle

Note

- When data not large, then need to use

Bubble  
Selection  
Insertion

Work on  
Divide and  
Conquer

Merge  
Quick

- Sorted array best case for all the last except Quick sort

In Quick sort

- Worst case when array sorted  $O(n^2)$
- Best case when pivot element come in middle  $O(n \log n)$

- When data large then need to use

Merge  
Quick

Request / Hit  
Miss

Cache - much faster than RAM  
- directly accessible by CPU

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- When data not large and sorted array present then which sorting algo to use?

Bubble Sort X  $\Theta(n^2)$   
Insertion Sort ✓  $\Theta(n)$   
(Why?)



Insertion Sort  $\Rightarrow$  Cache Miss ↓  $\Rightarrow$  searching at end of a large arr.

Bubble Sort  $\Rightarrow$  Cache Miss ↑  $\Rightarrow$  searching takes more frequently

- Q When data unsorted can not find first element and data very large  $\Rightarrow$  Merge Sort

Whole

Note - Data not transferred to RAM / Cache when program started.  
Small chunk of it only transferred.

When data required then CPU checks in cache if it find that data it is called cache hit and if data not transferred then it is called cache miss.

If cache miss then CPU alert and tell cache to upload more data.

{ concept of page segmentation }

→ Baluvalaya

24th Dec

**Non Linear Data Structure**

**Trees**  
and

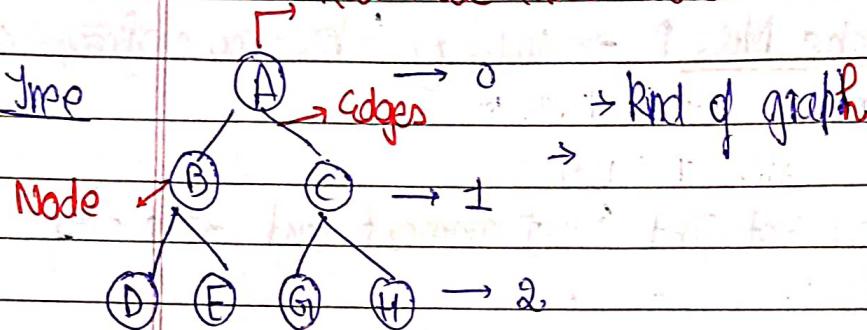
(more than one predecessor or  
more than one successor)  
**special kind of graph**

**Graph**

General Tree  
Binary Tree  
Avalant Tree

\* generally through linked list

**Root Node (first Node)**



Successor = 2 } Hence  
Predeccesor = 1 } Non  
{}  
Data Structure

### Terminology

1) Node

2) Edge

3) Root (first element)

4) Parent Node

5) Child Node

6) Sibling { One parent of two  
eg. B, C }

7) Leaf  
Node

{ having no child }  
eg. D, E, G, H

8)

$4n - 3t + 2$  total edges

Degree

out - after 2 off edge

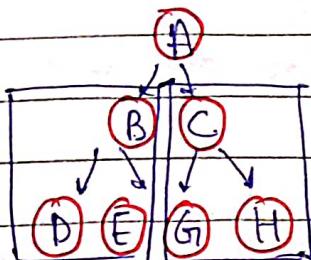
of B  $\rightarrow$  In degree = 1

Out degree = 2

- No. of edges from root node to given node = Depth of that Node

g) Level - Root Node Start with level 0<sup>th</sup>

h) Subtree → When a tree can be broken down into subsequent trees



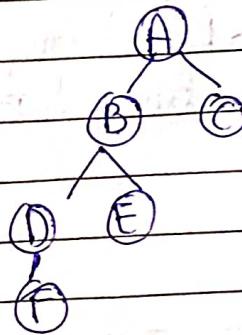
Application ① Compiler Design - Parsing, Parenthesis Management can be done in more optimized way as compared to stack

② Networking - shortest path using tree and graph

③ Cloud Computing → cloud datacentre's data store, in cloud computing fashion

④ Deep Learning / Machine → learning

Properties → a) No. of nodes = N  
of Tree  
No. of edges = N-1



b) Tree kind of graph with no loops / circuit

c) Depth = Pathway to node (no of edges from root node to given node)  
→ e.g. D → 2 → Depth

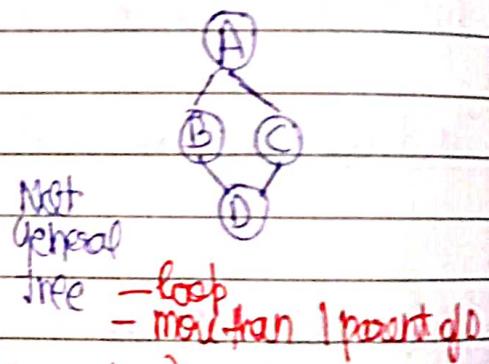
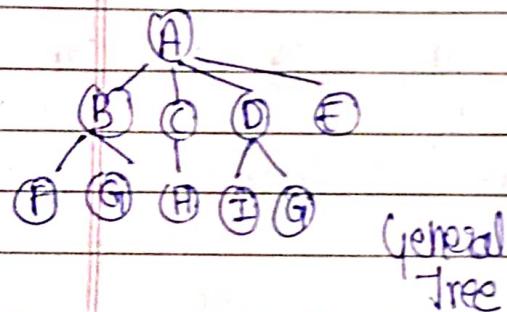
d) Height - longest possible path of the graph tree  
Height = 4 (A < B < D < F)

5) Degree of Tree → max no of child of that particular parent  
 → max. no of child node in tree  
 eg In given tree = 2  
 → generally in tree In degree = 1 - less significant

Type of Tree

- ① General Tree - (no condition on no of child, leaf node root node)
- ②

→ child can be any no. of times but parent need to be one



② Binary Tree

• Atmost two children

(0,1,2) ✓

• Parent need to be one

Binary Tree

full Binary Tree (Strict Binary Tree)

Complete (Proper) Binary Tree

Almost Complete Binary Tree (ACBT) related to Heaps

Binary Search Tree (BST)

Not Binary Tree

→ A

→ A

→ B

→

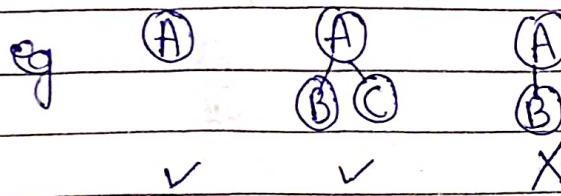
→ A

→ B

→ C

→ D

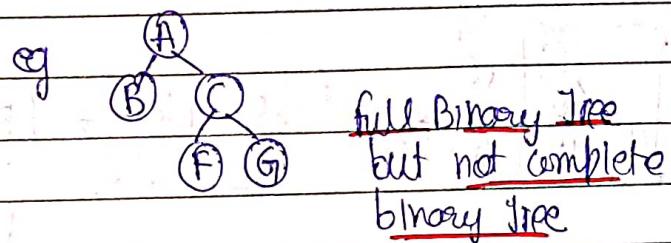
full Binary tree → या तो 0 children OR 2 children (should not be 1 child)



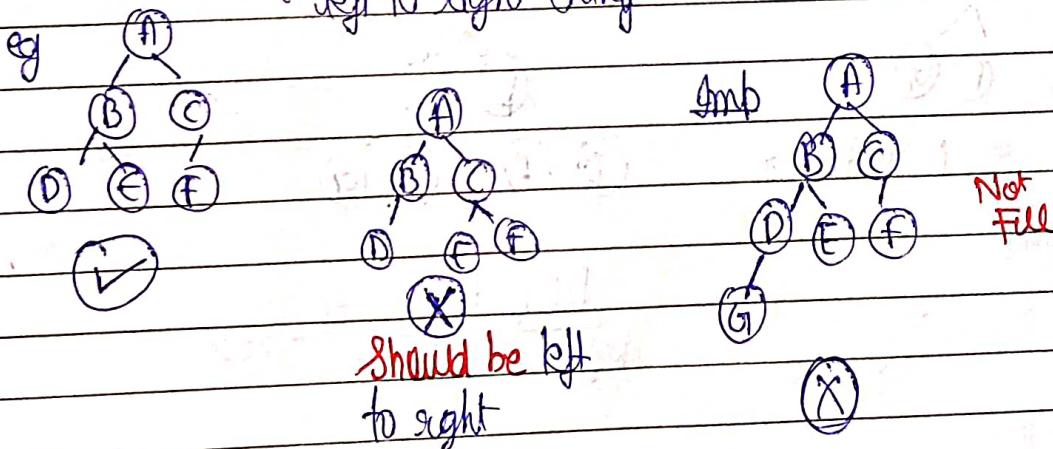
Complete Binary tree - एक level से छह सेरे level जांच के बिना पहला नीं होना  
- एक level पूरा होना चाहिए

- left to right fill and when one level completely filled then move to next

eg Left to right level fill  
complete  
and fill  
Binary Tree

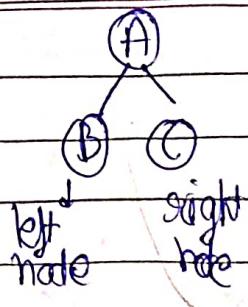


Almost complete binary tree • mostly implemented in Heap  
• level पूरा होना अस्फरी नहीं  
• left to right entry



## Traversal of Binary Tree

① Preorder Root, left, right

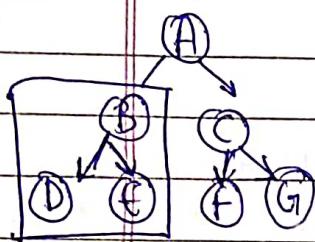


② Inorder left, Root, right

③ Postorder left, right, root

Note Depend on ~~the~~ positioning of ~~Binary~~ ~~Root~~

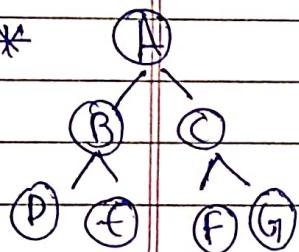
\*



\* {Preorder}  $\rightarrow$  ABDECFG  
Traversal

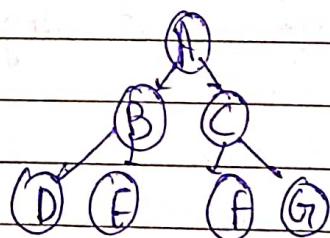
full left traversal

\*



\* Post Order

In Post Order Traversal



= DBEAFCG

DEB      F G C    A  
left      right      Root

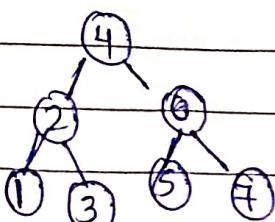
Binary Search Tree

• left < Root < Right  
Value      Value      Value

• Type of binary tree

eg from BT

4 2 3 6 5 7 1 (Need to make ACB)



Inorder • left root right

Traversal

20 23 34

1 2 3 4 5 6 7

EBST Inorder ascending

Note → Binary Tree →  $O(n)$  → No restriction

→ Binary Search Tree →  $O(\log n)$  →  $\text{left} < \text{root} < \text{right}$

29<sup>th</sup> Dec 2022

Inorder → Tree

- minimum two combination given
- In → DBE AFC {Use to find left and right element}
- Pre → ABD ECF {Use for root element}

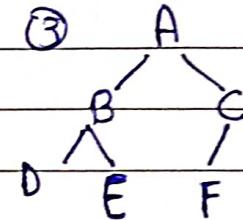
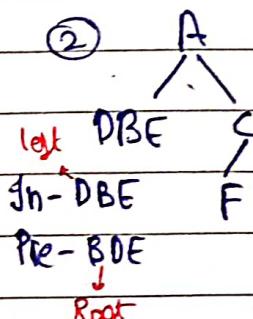
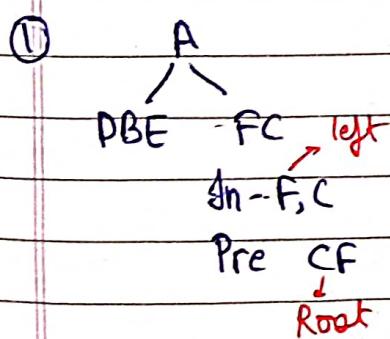
Q1 first to draw the graph then find post order DEBFCA

↓ Root

① In - DBE AFC (L R R)

Pre - ABD ECF (R L R)

↓ Root



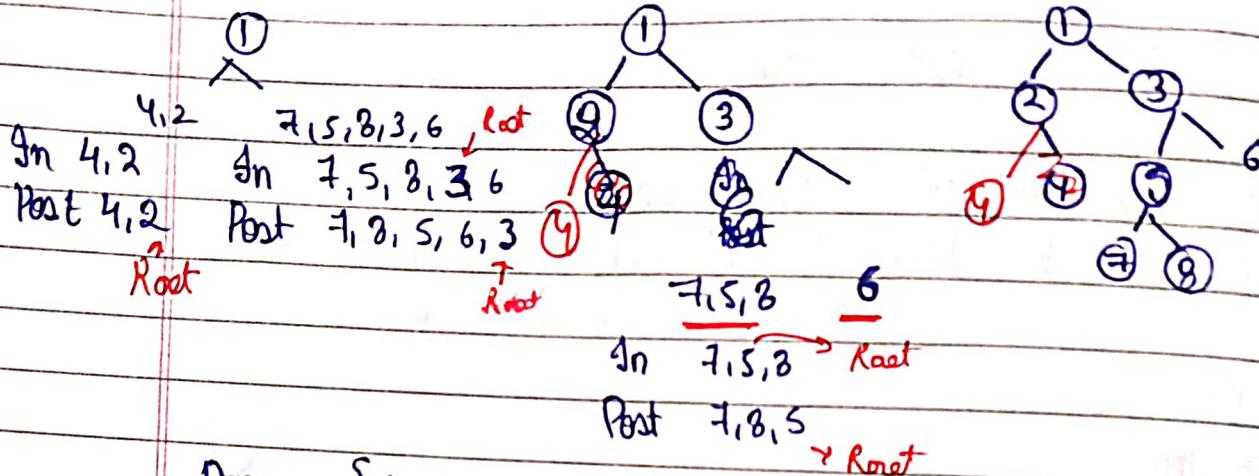
In - Root Left /  
 Post - Right / Left Root

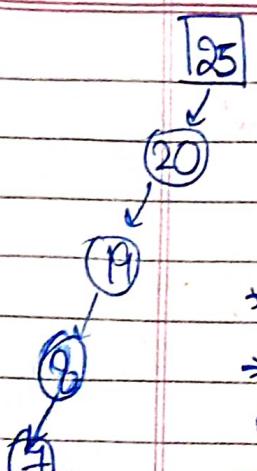
(Q2)

In = 4, 2, 7, 5, 8, 3, 6 (L R R)

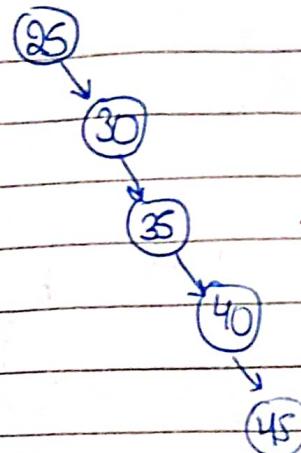
Post = 4, 2, 7, 8, 5, 6, 3, 1 (L R R)

Left      Right      Root





- Left-skewed
- Complexity does not decrease
- Height imbalance a big problem

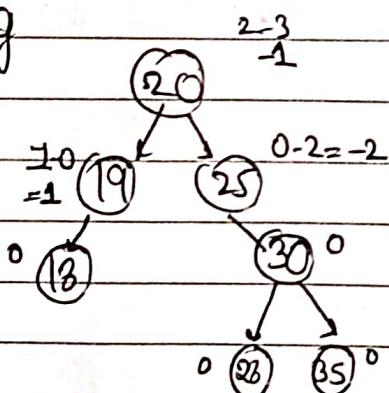


- right skewed
- does not get the properties of Binary tree

### AVL Tree

① Balance  $\Rightarrow$  Height of factor (LST) - Height of factor (RST)

So every node eg



② BF  $\Rightarrow$  0, 1, -1

(Balance factor)

If BF of all node in {0, 1, -1} then tree balanced

(neatness of AVL)

Rotation Rules

1. left to right move
2. Check if each step

Imbalance

RR  $\Rightarrow$  Left

LR  $\Rightarrow$  R

RL  $\Rightarrow$  L

Interchange

Note After rotation, tree should be balanced and follow BST

If any node left out in between rotation check with help of BST

Name

→ always in linked list

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e-Tree Implementation?

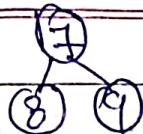
struct node {

int data,

struct node \* left

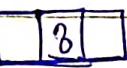
struct node \* right }

\* Root



\* Second

\* First



struct node \* Root

struct node \* left;

struct node \* right;

{Memory  
Allocate}

Root → left = first

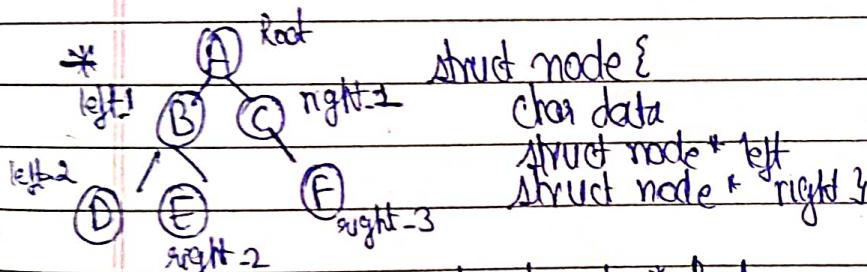
Root → right = second

first → left = NULL

first → right = NULL

second → left = NULL;

second → right = NULL;



struct node \*

char data

struct node \* left

struct node \* right;

struct node \* Root

struct node \* left\_1

struct node \* right\_1

struct node \* left\_2

struct node \* right\_2

struct node \* right\_3

Root → data = 'A'

Root → left = left\_1

Root → right = right\_1

right\_1 → data = 'C'

right\_1 → left = NULL

right\_1 → right = right\_3

left\_1 → data = 'B'

left\_1 → left = left\_2

left\_1 → right = right\_2

left\_2 → data = 'D'

→ left = NULL

→ right = NULL

10th Jan

'Heap Sort'

→ ~~Max~~ Heap → implementation in graph (spanning tree)

Min  
HeapMax  
Heap

→ Uses ACBT

→ Common operation

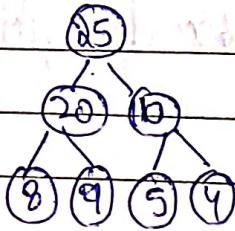
- (i) Insertion      (iv) Deletion
- (ii) Sort

Min Heapfollow ACBT  
max 2 child(left to right)  
one level  
complete

→ not a type of BST

P (left can bigger than right)  
no relation in sibling→ Value of parent < Value of child  
(Minimum)Max Heap → Value of parent > Value of child  
(Maximum)

eg



Insertion

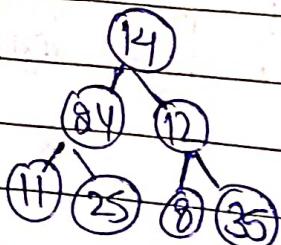
- ① One by one
- ② Heapsify (Building Heap)

eg

14, 24, 12, 11, 25, 8, 35

①

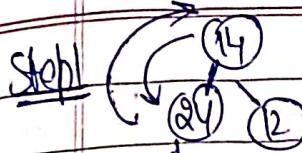
Simple ACBT



# 'One by One Method'

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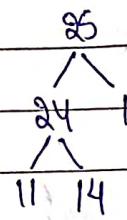
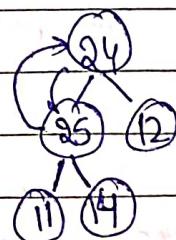
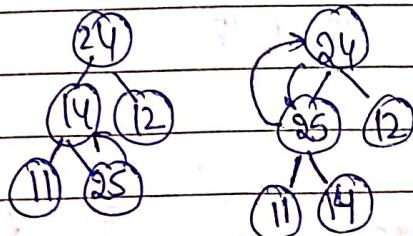


Check whether follow min heap

left  
min of the  
child node

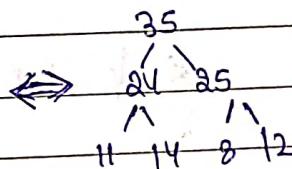
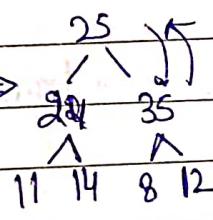
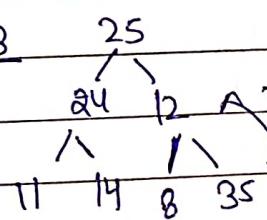


Step 2



Not min heap

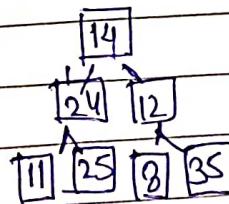
Step 3



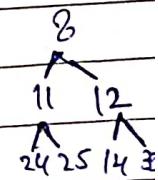
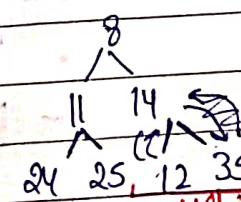
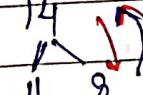
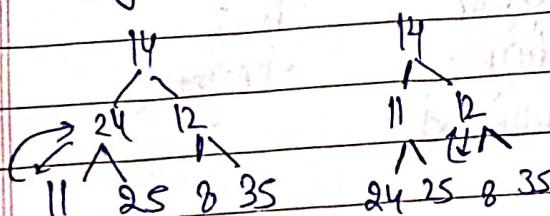
\* Need to swap  
only data not  
node, because  
if swap node then,  
left and right will  
change

Min Heap

Step 1 Create {AEBF}



Step 2 Bottom to top  
left to right

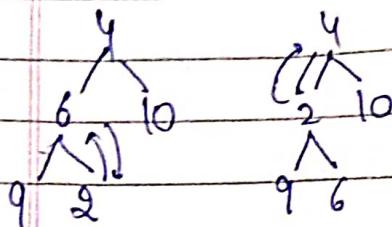


{More to next}  
level

Not in  
Min  
Heap

HeapSort

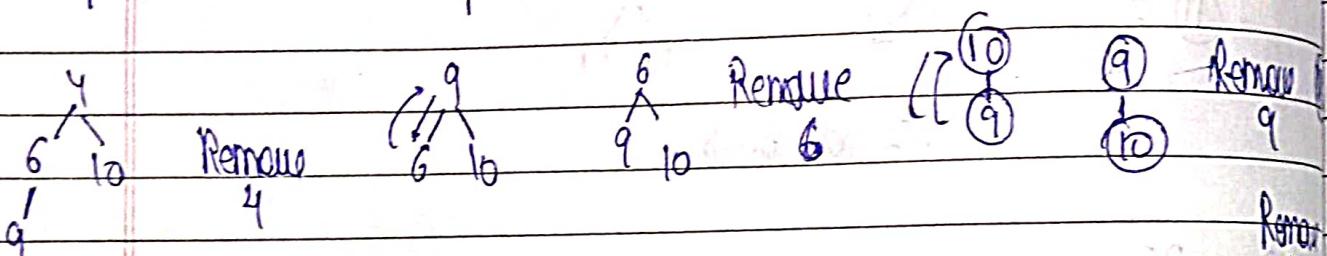
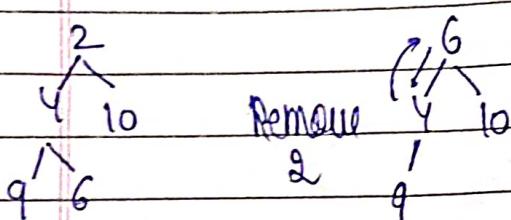
- 4, 6, 10, 9, 2



Step1 Create min heap  
Step2 Then delete the root element after sorting

→ we will replace by left child  
sight node to node

(so that ACBT remains true  
and then again make in  
min heap)

M-tree

[5]



\* children are uncertain

\*



Graph → non linear data structure

- used to find shortest path.
- very much use in blockchain.
- used in computer design

G(V, E)

- finite
- infinite
- Connected
- Disconnected
- Terminal
- Cyclic
- Acyclic
- Directed
- Simple
- Multi
- Weighted

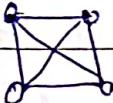
(Highly Preferable)

Infinite



{Complete} - P2P connection  
 Graph - every node has 'n-1' degree

Connected



\* Tree type of graph without cycle

Triival

• (without edge only 1 vertex)

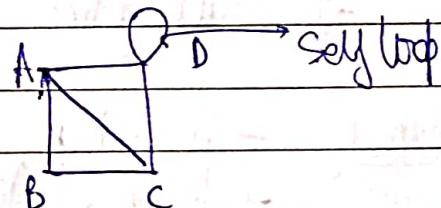
Cyclic • If even one cycle present then cyclic

Simple - Only 1 edge b/w two vertex

Weighted  $\rightarrow$  if each vertex is assigned with a weight (either cost)

Adjacency Matrix

	A	B	C	D
A	0	1	1	1
B	1	0	1	0
C	1	1	0	1
D	1	0	1	1



If directed then

$\rightarrow$  If weighted graph then in place of 0 and 1 we will write weighted value.

Spanning Tree → shortest distance path

① Need to take out a subgraph

such that

(a) include all the vertices

(b) No of edges =  $V - 1$

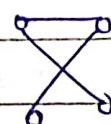
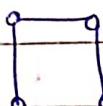
(c) connected

(d) no cycles (<sup>tree</sup> cannot be cycle)

e.g.



Spanning Tree



Note - Without a graph, we can generate many no of spanning tree  
 $= n^{n-2}$  { $n = \text{no. of vertices}$ }

# Spanning tree with least cost will be best tree

Min Spanning Tree → Prim's Algo

work on

→ Krushal Algo

Greedy Approach

Prim's - start with any one vertex

Algo - min cost remain same (structure of min. spanning tree can be different)

Min Spanning Tree → broadcast a net packet in network  
 Use → water supply

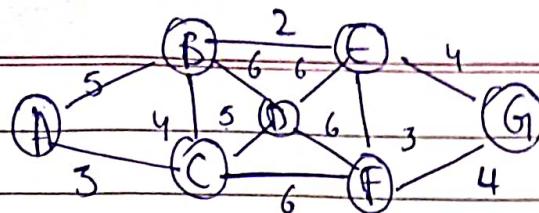
Interconnected graph can be disconnected

Min spanning tree  $\rightarrow$  min cost guaranteed

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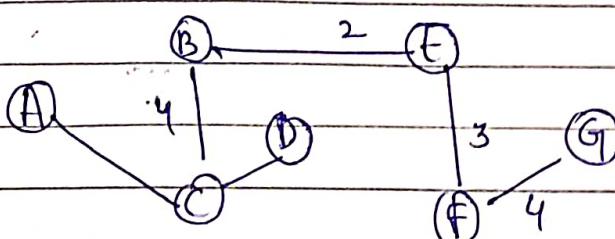
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Kruskal Algo.



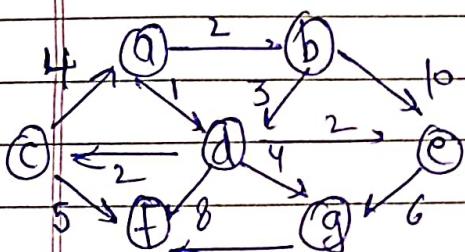
Sort all the edges

- (1) Draw all vertex
- (2) Then choose min weighted edge



Dijkstra  $\rightarrow$

- Unexplored  $\rightarrow$
- explored  $\rightarrow$  which we have explored



\* let  $a$  be my source

Explored

1. a

Unexplored

a, b, c, d, e, f, g, h

2. a

b, c, d, e, f, g, h

\*  $l_a < l_d$

3. a d

b, c, e, f, g, h  
b, c, e, f, g, h

4. a d b

c, e, f, g, h

\* can choose any c/e

a d b c

e, f, g, h

a d b c e

f, g, h

a d b c e g

f, g, h

b

$$a \rightarrow b = 2$$

c

$$a \rightarrow d \rightarrow c = 3$$

d

$$a \rightarrow d = 1$$

e

$$a \rightarrow d \rightarrow c = 3$$

f

$$a \rightarrow d \rightarrow g \rightarrow f = 6$$

g

$$a \rightarrow d \rightarrow g = 5$$