University School of Automation and Robotics
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Engineering Mechanics

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- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates

- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.

- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.

- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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KINETICS OF RIGID BODY FORCE AND ACCELERATION

- For a particle moving in a plane, relation between forces acting on a particle and resulting translatory motion is given by a single vector equation,

$$\sum \boldsymbol{F} = \boldsymbol{ma}$$

- The corresponding scalar equations are.

$$\sum \boldsymbol{F_x} = \boldsymbol{ma_x} \; and \sum \boldsymbol{F_y} = \boldsymbol{ma_y}$$

- Concept of idealizing a body by a particle is an assumption designed to simplify the analysis.
- For a rigid body in plane motion, its rotary motion is also to be considered which results in an increase in one equation of motion to account for this rotary motion.
- The equation for the rotary motion is similar in form to that of translatory motion.

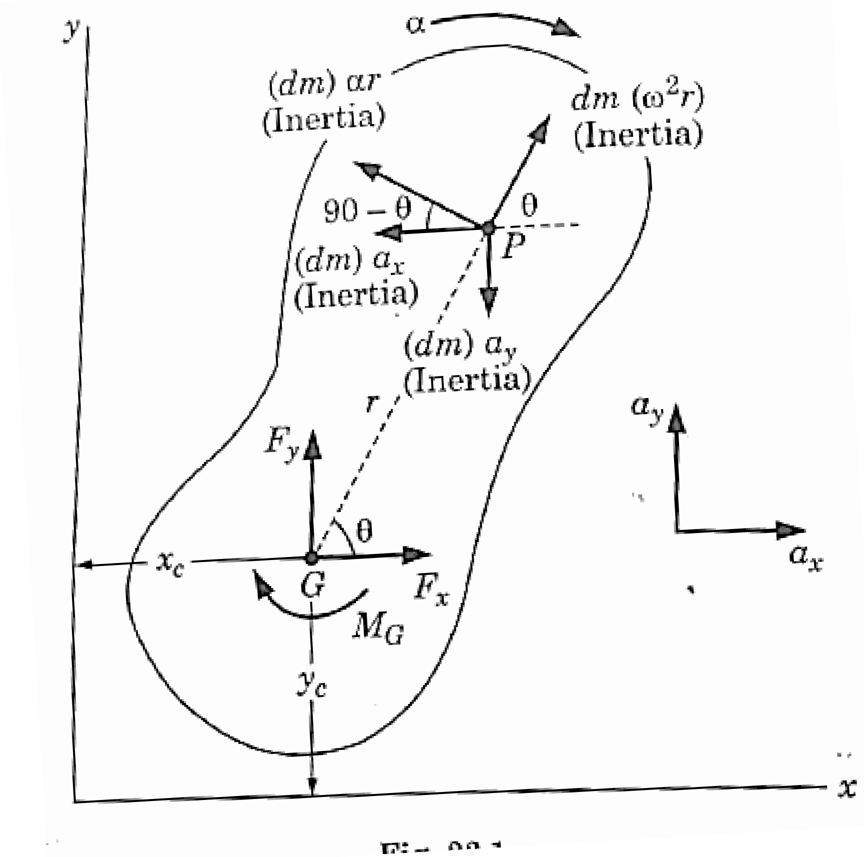
$$\boldsymbol{M} = \boldsymbol{I\alpha}$$

KINETICS OF RIGID BODY FORCE AND ACCELERATION

- Comparing $M = I\alpha$ with equation (**ΣF=ma**) it can be observed that:
  - ❑ Force has been replaced by the moment of the force,
  - ❑ Linear acceleration by the angular acceleration, and
  - ❑ Mass of the body by the moment of inertia of the body about the axis of rotation.
- Thus, to describe the motion of a rigid body in plane motion three equations of the motion are needed.
- Further, the motion of a rigid body can be described by the motion of any convenient point located in the rigid body.
- The mass centre of the body is usually chosen for this purpose.

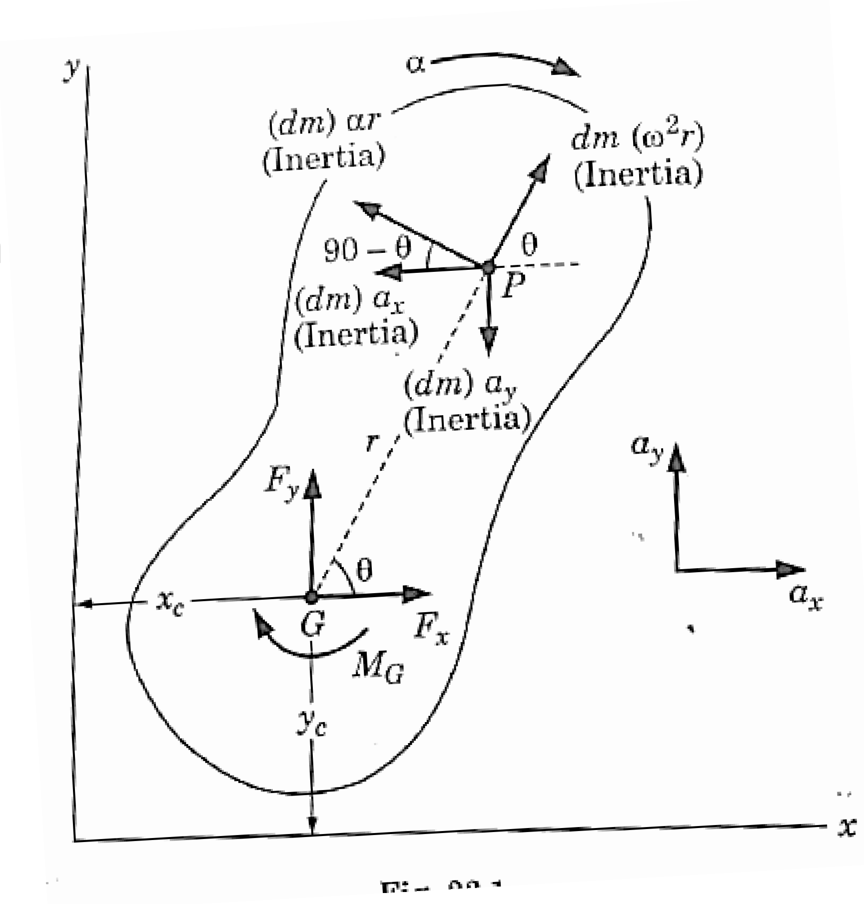
PLANE MOTION OF A RIGID BODY: EQUATIONS OF MOTION

- Consider a rigid body of mass m in plane motion under the action of the applied external forces.
- Let the resultant of these external forces be reduced to:
  - Force F_x in the x direction,
  - Force F_y in the y direction, and
  - Moment M_G about the mass centre.
- Let the mass centre G of the body move parallel to the x-y plane under the action of applied forces.
- Consider the motion of any particle P of mass dm situated at a distance r from the axis passing through G and normal to the plane of motion x-y.



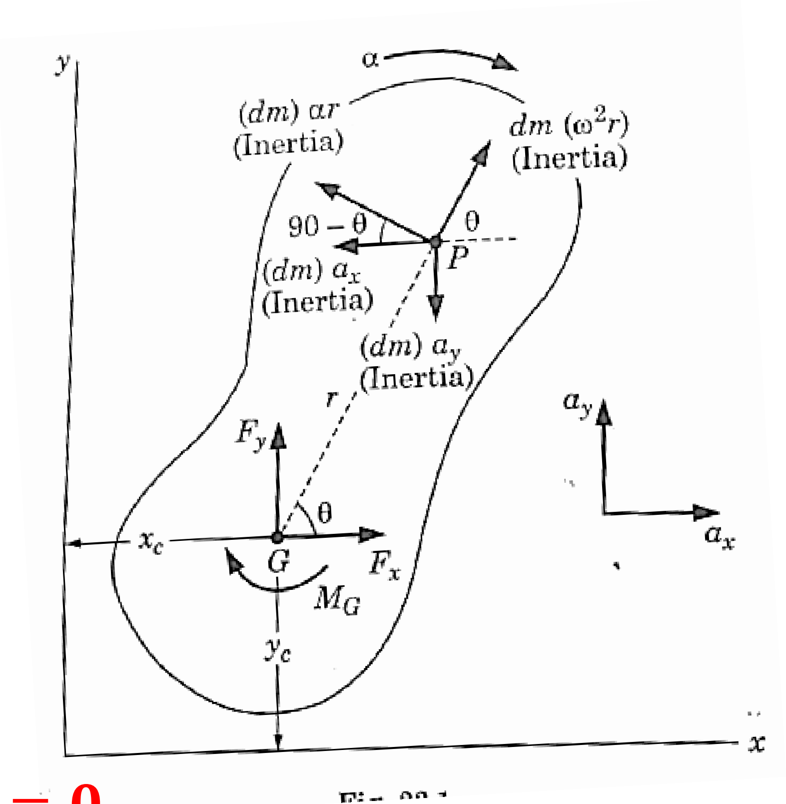
PLANE MOTION OF A RIGID BODY: EQUATIONS OF MOTION

- Choose mass-center G as pole and its coordinates be x_c, y_c.
- Let angle which line PG makes with the x-axis be θ.
- Motion of point P can be considered to be sum of translation of the pole G and rotation of point P about the axis passing through G and normal to the plane of motion.
- Let linear acceleration of pole G and Particle P be a_x in the x direction and a_y in the y-direction.
- Inertia forces acting on particle P corresponding to linear acceleration are:
  - ❑ $(dm)a_x$ acting along x-axis opposite to direction of a_x, and
  - ❑ $(dm)a_y$, acting along y--axis opposite to direction of a_y.



PLANE MOTION OF A RIGID BODY: EQUATIONS OF MOTION

- Let angular velocity and acceleration of body be ω and α.
- As particle P rotates about an axis through G and at a distance r from it, inertia forces acting on it are:
  - ❑ $dm(\omega_2 r)$, acting along GP (Normal component) and
  - ❑ $dm(\alpha r)$, acting normal to GP (Tangential component).
- Inertia forces acting on entire rigid body can be calculated by integrating) inertia forces acting on all particles of the body.
- Equations of dynamic equilibrium of body can be written as,



$$\sum F_x = 0:\ F_x - a_x \int dm - \alpha \int r\cos(90-\theta)\, dm + \omega^2 \int r\cos\theta dm = 0$$

$$\sum F_y = 0:\ F_y - a_y \int dm + \alpha \int r\sin(90-\theta)\, dm + \omega^2 \int r\sin\theta dm = 0$$

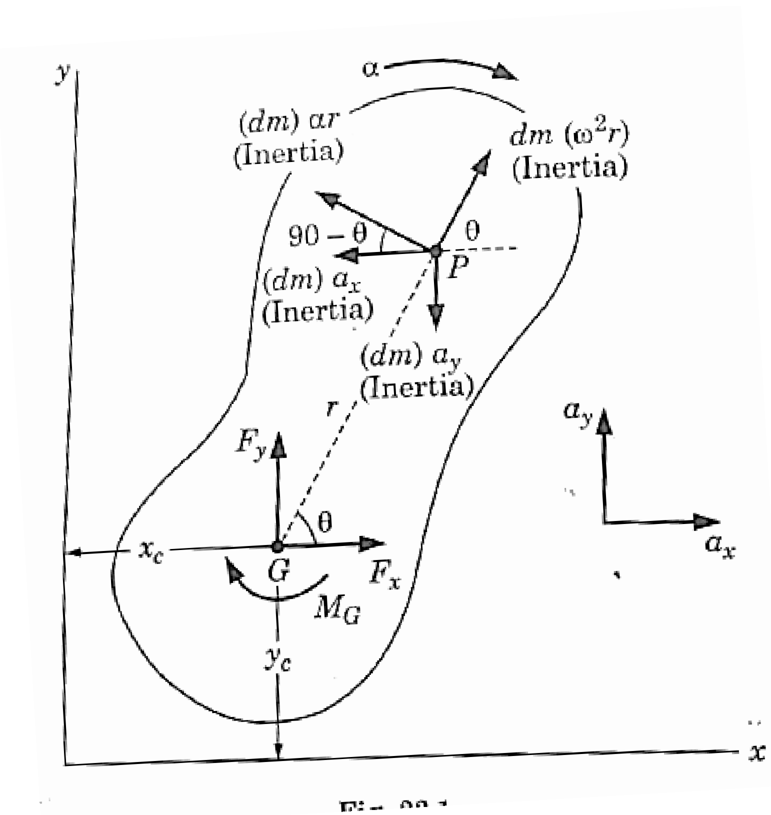
$$\sum M_G = 0:\ M_G + a_y \int r\cos\theta dm - a_x \int r\sin\theta dm - \alpha \int r^2 dm = 0$$

PLANE MOTION OF A RIGID BODY: EQUATIONS OF MOTION

- But, $\int dm = m, \int r\sin\theta dm = 0, \int r\cos\theta dm = 0, \int r^2 dm = I_G$
- I_G being moment of inertia of body about the axis through mass centre G and normal to plane of motion x-y.
- Therefore the above equations become,
- $F_x - ma_x = 0$ **or** $F_x = ma_x$
- $F_y - ma_y = 0$ **or** $F_y = ma_y$
- $M_G - I_G\alpha = 0$ **or** $M_G = I_G\alpha$

Equations of Motion of a Rigid Body

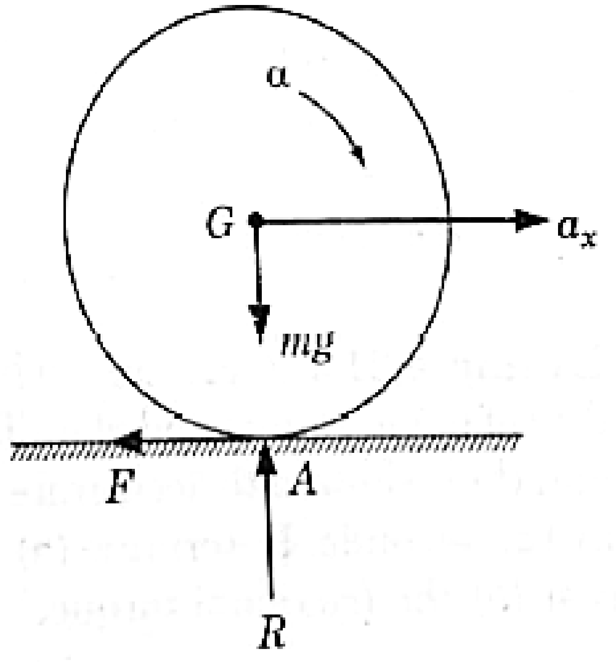
- From above equation it can be concluded that for a rigid body in plane motion we can write three equations of motion as:-
  - Two equations for translatory motion of its mass centre due to external forces F_x and F_y. $(F_x = ma_x$ **and** $F_y = ma_y)$
  - One equation for rotary motion of body, due to moment M_G of external forces about the axis through mass centre and perpendicular to plane of motion.



F_x, F_y and M_G represent resultant of applied external forces.

RELATION BETWEEN THE TRANSLATORY MOTION AND ROTARY MOTION OF A BODY IN PLANE MOTION

- Consider specific case of plane motion of a right circular cylinder on a horizontal surface as shown in figure.
- Let **mass of cylinder = m, R = normal reaction and F = force of friction**.
- **Acceleration** of its mass centre G be a_x and **angular acceleration α**.
- The cylinder may:



1. **Roll without slipping:-** Here, forward motion of mass centre is related to its angular motion. Distance travelled and angle turned by the body are related as,

$$x = \theta r;$$

$$\dot{x} = \dot{\theta} r = \omega r;$$

$$\ddot{x} = \ddot{\theta} r = \alpha r \qquad \textbf{or} \qquad a_x = \alpha r$$

   And frictional force F is such that, $F \leq \mu R$

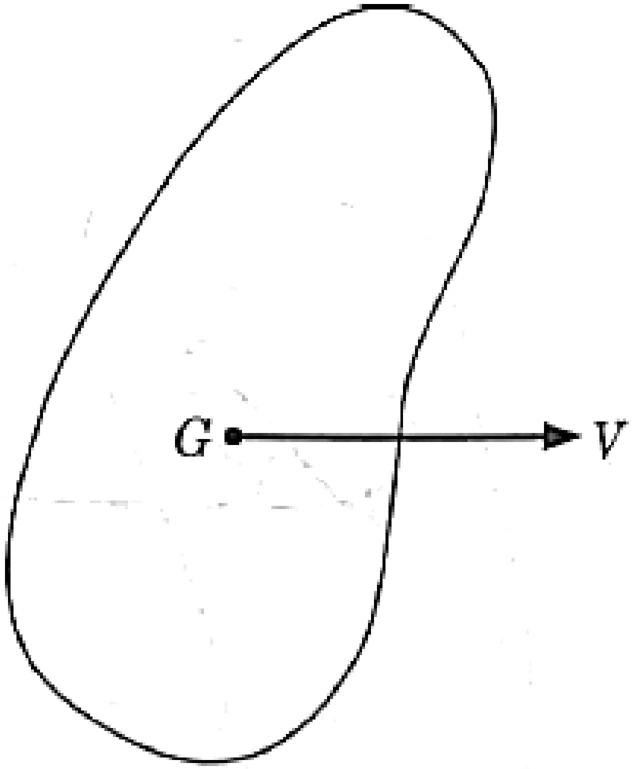
2. **Slips without rolling:-** There is no rotary motion involved $(F = \mu R)$
3. **Rolls as well as slips:-** In this case translatory and rotary motions are independent of each other $(F \leq \mu R\ )$

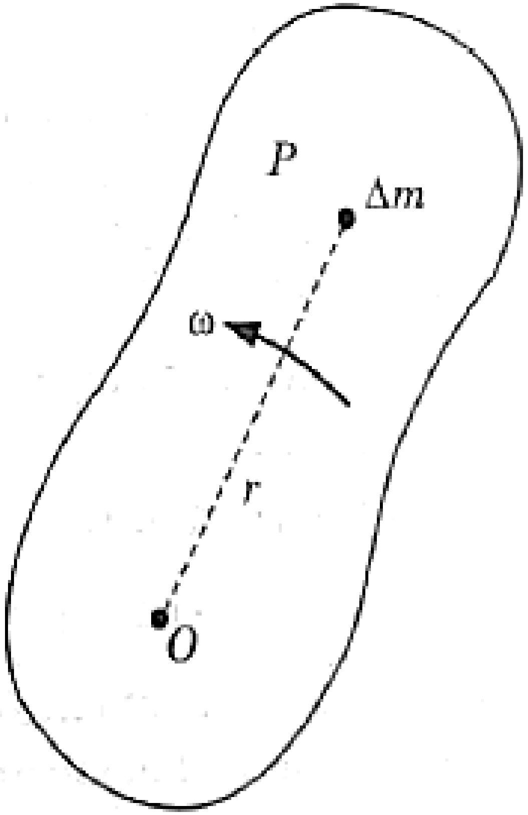
ANALOGY BETWEEN RECTILINEAR MOTION AND ROTATIONAL MOTION

linear	**rotational**
mass, m	moment of inertia, I
displacement, s	angle, θ
velocity, $v = \frac{\Delta s}{\Delta t}$	angular velocity, $\omega = \frac{\Delta \theta}{\Delta t}$
acceleration, $a = \frac{\Delta v}{\Delta t}$	angular acceleration, $\alpha = \frac{\Delta \omega}{\Delta t}$
linear momentum $= m v$	angular momentum $= I \omega$
$KE = \frac{1}{2} m v^2$	$KE = \frac{1}{2} I \omega^2$
$F = m a = \frac{\Delta (m v)}{\Delta t}$	$T = I \alpha = \frac{\Delta (I \omega)}{\Delta t}$
$W = F s$	$W = T \theta$
$v = u + a t$ $v^2 = u^2 + 2 a s$ $s = u t + \frac{1}{2} a t^2$	$\omega = \omega_0 + \alpha t$ $\omega^2 = \omega_0^2 + 2 \alpha \theta$ $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
$s = \frac{1}{2} (u+v) t$	$\theta = \frac{1}{2} (\omega_0 + \omega) t$
conservation of linear momentum if no external forces act	conservation of angular momentum if no external torques act

KINETIC ENERGY OF A RIGID BODY

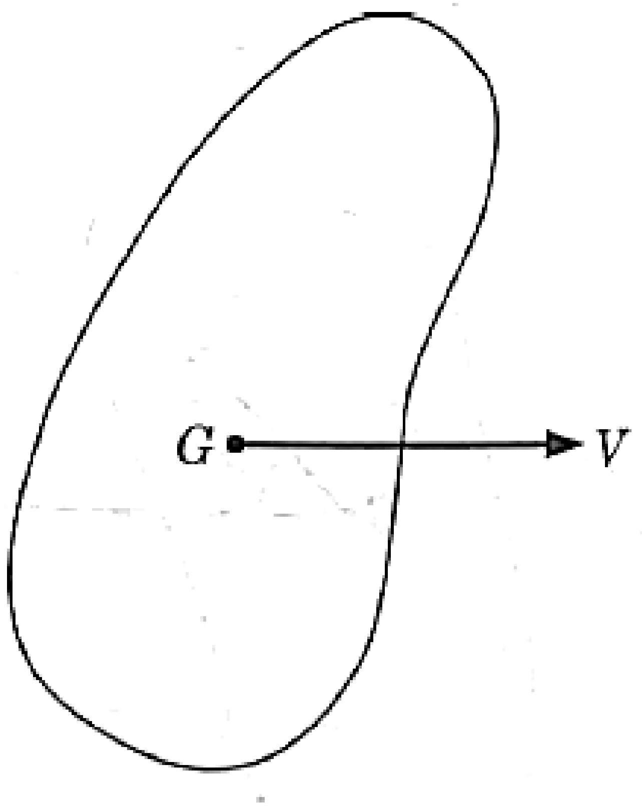
- Consider a rigid body in plane motion.
- Let body be made up of a large number of particles each of mass **Δm**.
- Mass **m** of body is sum of masses of its particles therefore, **m=ΣΔ m**.
- Kinetic energy of rigid body would be equal to K.E. of all its particles.
- Plane motion of a rigid body consists of a translatory and a rotary motion.
- Therefore kinetic energy of a rigid body is sum of K.E. in translation and the K.E. in rotation of all its particles.



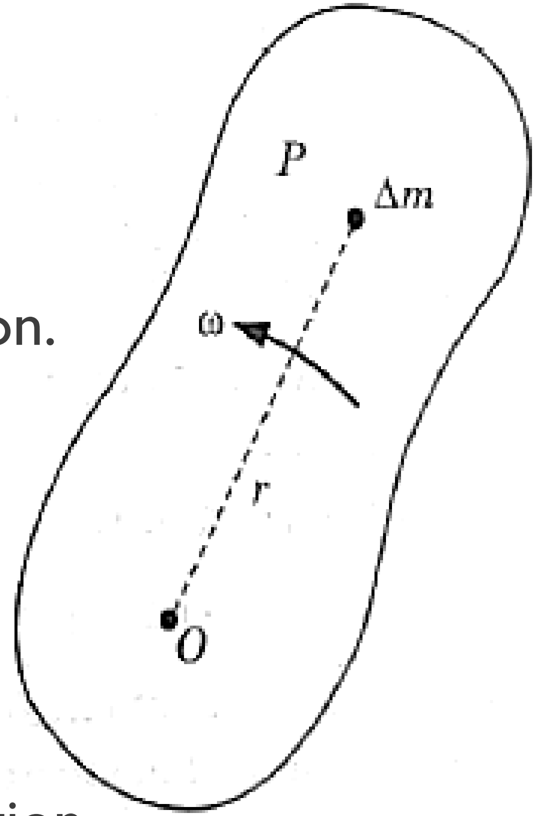


KINETIC ENERGY IN TRANSLATION

- Let all particles of rigid body be moving with same velocity as velocity v of its mass centre **G**.
- Kinetic Energy of particle of mass, Δm, = $\frac{1}{2}\Delta m v^2$
- Kinetic energy of body = $\frac{1}{2}\left(\sum \Delta m\right) v^2$
- Kinetic energy of body in Translation = $\frac{1}{2} m v^2$



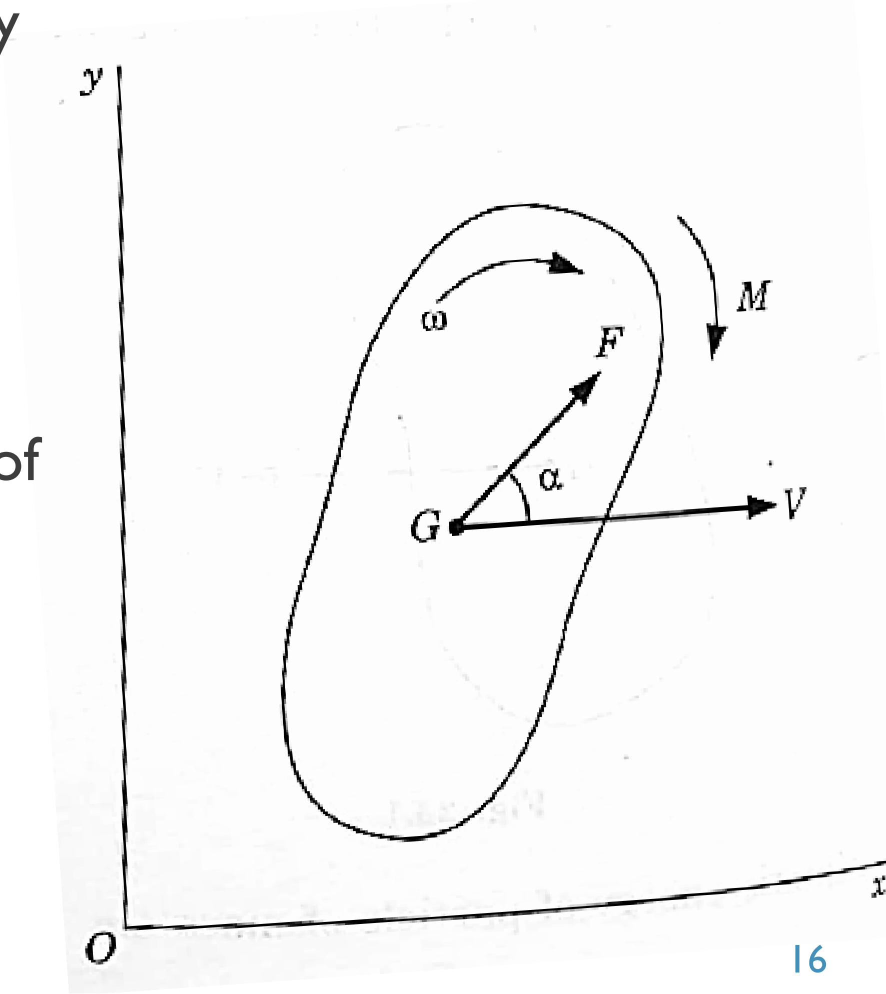
KINETIC ENERGY IN ROTATION



- Consider a rigid body rotating at an angular velocity ω about a fixed axis intersecting plane of motion at O as shown in figure.
- Consider a particle P of mass **Δm** situated at **r** distance from axis of rotation.
- Velocity of Particle= **ωr,**
- Kinetic energy of rotation of particle, = $\frac{1}{2}\Delta\mathrm{m}(\omega r)^2$
- Kinetic energy of body is (made up of similar particles), = $\frac{1}{2}\omega^2\sum\Delta\mathrm{m}r^2$
- $\sum\Delta\mathrm{m}r^2$ represents moment of inertia (I) of whole body about axis of rotation.
- Kinetic energy of rotation of body, = $\frac{1}{2}\mathrm{I}(\omega)^2$
- Total kinetic energy of the body, = $\frac{1}{2}\mathrm{m}(v)^2$ (translation) $+\frac{1}{2}\mathrm{I}(\omega)^2$ (rotation)

WORK OF FORCES ACTING ON A RIGID BODY

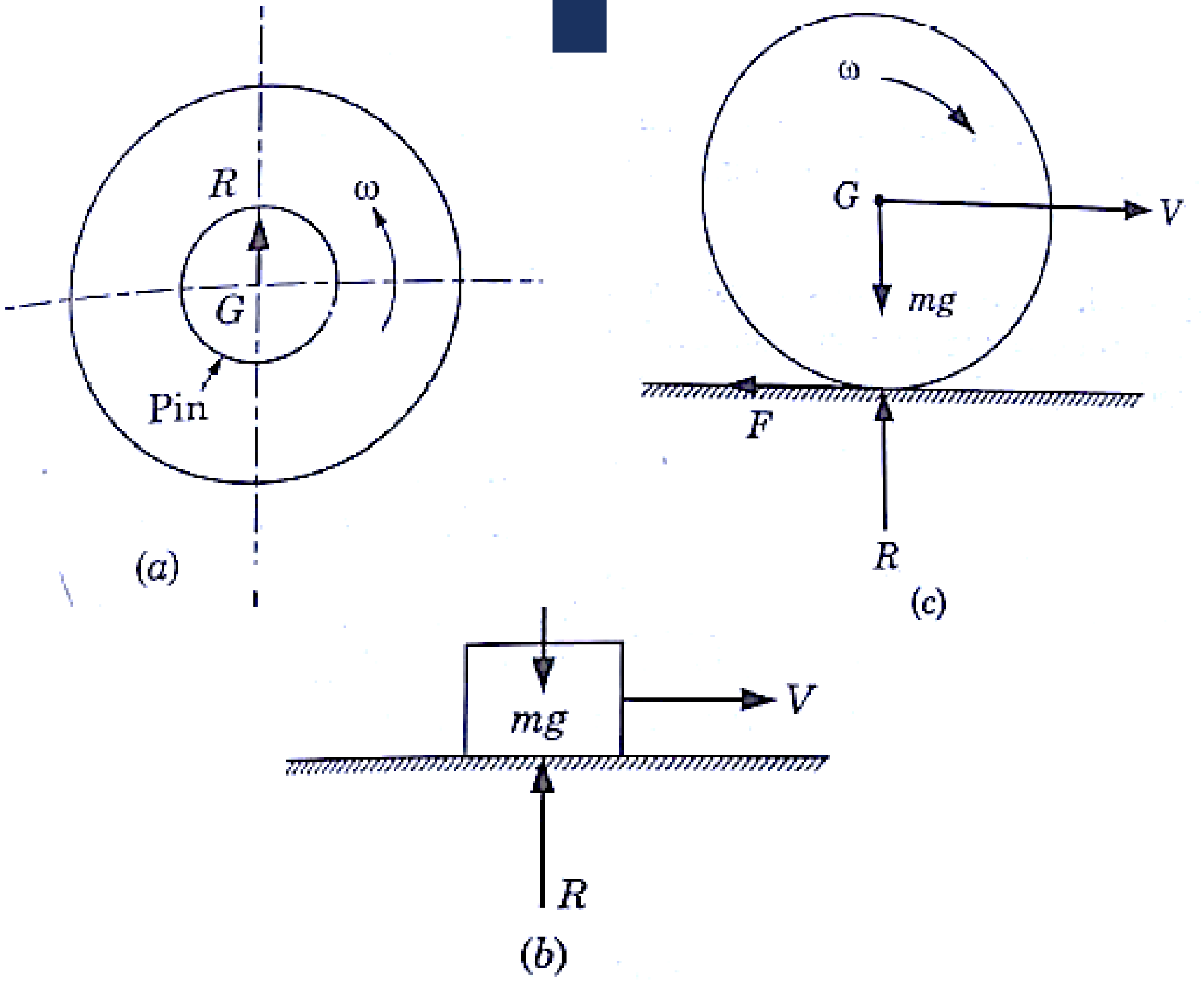
- Consider a rigid body of mass m in plane motion parallel to x-y plane.
- Let, velocity of mass centre G be v, angular velocity of body ω, resultant force acting on body be F and moment of resultant couple be M.
- If resultant force F makes an angle α with direction of motion of its mass centre then,
- Work done on rigid body, $U_{1-2} = \int(F\cos\alpha)ds + \int M d\theta$
- **Case 1:** When constant force of magnitude F and constant couple of moment M move body from position 1 to position 2 causing displacement x in direction of force and angular displacement θ, then $U_{1-2} = F_x + M\theta$



WORK OF FORCES ACTING ON A RIGID BODY

- There are some forces which do no work during motion of a rigid body. Examples are:

1. Reaction R of pin on body about which body is rotating (figure a).
2. Reaction R of surface on body when it moves horizontally and surface is frictionless (figure b).
3. Friction force F acting at point of contact when body rolls without sliding on a fixed horizontal or inclined surface (figure c).



PRINCIPLE OF WORK AND ENERGY FOR A RIGID BODY

- Principle for a rigid body can be written in same form as that for a particle
- i.e., **Work done = change in kinetic energy**

$$U_{1-2} = (T_2 - T_1)$$

- In a rigid body, Work done = Work resulting due to linear displacement from initial position s_1 to final position s_2 + work resulting due to angular displacement from position θ_1 to θ_2.

$$U_{1-2} = \int_{s_1}^{s_2} (F\cos\alpha)ds + \int_{\theta_1}^{\theta_2} Md\theta$$

$U_{1-2} = F_s + M\theta$ (When force F and moment M are constant)

- $(T_2 - T_1)$ is change in K.E. of body resulting from translation and rotation between initial and final positions.
- Change in kinetic energy = $(T_2 - T_1) = \left(\frac{1}{2}\mathrm{m}v_2^2 - \frac{1}{2}\mathrm{m}v_1^2\right) + \left(\frac{1}{2}\mathrm{I}\omega_2^2 - \frac{1}{2}\mathrm{I}\omega_1^2\right)$
- Or, Change in K.E. = change in K.E. in translation + change in K.E. in rotation

PRINCIPLE OF CONSERVATION OF ENERGY

- Sum of kinetic energy and potential energy of a rigid body under action of conservative forces remains constant.

$$Work\ done = \ change\ in\ kinetic\ energy$$

$$U_{1\text{-}2} = K.E._2 - K.E._1$$

- If a particle moves under action of a conservative force, work done is stored as potential energy,

$$U_{1\text{-}2} = -\ (V_2 - V_1)$$

- Work done= -(negative change in potential energy)
- Combining above two equations: $K.E._2 - K.E._1 = -(V_2 - V_1)$
- Or, $K.E._2 + V_2 = K.E._1 + V_1$

$$K.E._1 + P.E._1 = K.E._2 + P.E._2$$

- In case of a rigid body K.E. means sum of K.E. in translation and rotation.