Loss of Kinetic energy

During collision, the kinetic energy is lost due to imperfect elastic action. Energy is also lost due to:

a) heat generated,
b) sound generated, &
(c) Vibration of colliding bodies.

The loss of kinetic energy can be found by finding kinetic energy before impact and after impact. Let u_1, u_2 be initial velocities and v_1, v_2 be final velocities of two bodies colliding in the line of impact, their weights being W_1 & W_2.

Then

Initial $K.E = \frac{W_1}{2g} u_1^2 + \frac{W_2}{2g} u_2^2$

Final $K.E = \frac{W_1}{2g} v_1^2 + \frac{W_2}{2g} v_2^2$

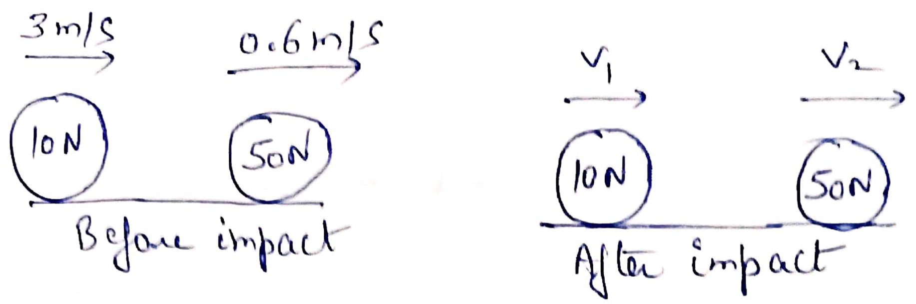
$\therefore$ Loss of K.E

$= \text{Initial } K.E - \text{Final } K.E$

$$= \left(\frac{W_1}{2g} u_1^2 + \frac{W_2}{2g} u_2^2\right) - \left(\frac{W_1}{2g} v_1^2 + \frac{W_2}{2g} v_2^2\right)$$

$$= \frac{W_1}{2g}(u_1^2 - v_1^2) + \frac{W_2}{2g}(u_2^2 - v_2^2)$$

Q/ A sphere of weight 10N moving at 3 m/s collides with another sphere of weight 50N moving in the same line at 0.6 m/s. Find the loss of kinetic energy during impact & show that the direction of motion of the first sphere is reversed after the impact. Assume coefficient of restitution as 0.75.



VAT
VAT =
Mar.

Soln Let velocities after impact be V_1 & V_2

$u_1 = 3\,m/s$, $u_2 = 0.6\,m/s$

From principles of conservation of momentum.

$$\frac{10}{g}(3) + \frac{50}{g}(0.6) = \frac{10}{g}V_1 + \frac{50}{g}V_2$$

$$V_1 + 5V_2 = 6 \qquad \text{——(1)}$$

From the definition of coefficient of restitution, we have.

$$0.75(3-0.6) = V_2 - V_1 \quad \text{or} \quad V_2 - V_1 = 1.8 \qquad \text{——(2)}$$

from eqn (1) & (2), $V_2 = 1.3\,m/s$

& $V_1 = 6 - 1.3 \times 5 = -0.5\,m/s$

Thus, the velocity of first ball is reversed after impact.

Loss of K.E = Initial K.E − Final K.E

$$= \frac{10}{2g}(3^2) + \frac{50}{2g} \times 0.6^2 - \left\{\frac{10}{2g} \times (0.5)^2 + \frac{50}{2g} \times 1.3^2\right\}$$

$$= \frac{10}{2 \times 9.81}(9 + 18 - 0.25 - 8.45)$$

$$= 1.07 \text{ Joules}$$

Q Find the loss of kinetic energy in example, if the weight of each ball is 10 N.

Soln Component of velocities in y-direction is not affected. Hence no change in K.E. due to the components of velocities in y-direction. In x-direction

$U_{Ax} = 7.79\, m/s \qquad U_{Bx} = -6\, m/s$

$V_{Ax} = -5.31\, m/s \qquad V_{Bx} = 7.104\, m/s$

Mass of the ball $= \frac{10}{g} = \frac{10}{9.81}$

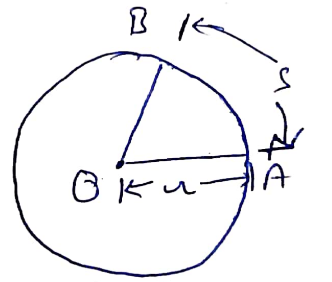
$\therefore$ Loss of K.E $= \frac{10}{2\times 9.81}\left[U_{Ax}^2 + U_{Bx}^2 - V_{Ax}^2 - V_{Bx}^2\right]$

$= \frac{10}{2\times 9.81}\left[7.79^2 + (-6)^2 - (-5.31)^2 - 7.104^2\right]$

$= 18.02\, J$

Unit-4

Angular Motion



fig(i)

The rate of change of angular displacement with time is called angular velocity and is denoted by ω. Thus

$$\omega = \frac{d\theta}{dt} \quad \text{—— (i)}$$

the rate of change of angular velocity with time is called angular acceleration and is denoted by α.

Thus $$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad \text{—— (ii)}$$

the angular acceleration may be expressed in another useful form.

Now, $$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

Thus $$\alpha = \omega \frac{d\omega}{d\theta} \quad \text{—— (iii)}$$

Relationship between angular motion & linear motion

when the particle moves from A to B (ref fig(i)), the distance travelled by it is s. if r is the distance of the particle from the centre of rotation then

$$s = r\theta \qquad \text{— (iv)}$$

The tangential velocity of the particle is called as linear velocity. A is denoted by V.

$$V = \frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \text{— (v)}$$

The linear acceleration of the particle in tangential direction α_t is given by.

$$\alpha_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2} \qquad \text{— (vi)}$$

while treating the curvilinear motion, it has been shown that, if V is the tangential velocity, then there is radial acceleration V^2/r

Denoting radial acceleration by α_n, we get

$$\alpha_n = \frac{V^2}{r} = r\omega^2. \qquad \text{— (vii)}$$

<u>Uniform angular velocity</u>

If the angular velocity is uniform, the angular distance moved in t second by a body having angular velocity ω radian/second is given by

$$\theta = \omega t \text{ radian} \qquad \text{— (viii)}$$

Many a times the angular velocity is given in terms of number of revolution per minute (rpm). Since there are 2π radians in one revolution and 60 seconds in one minute the angular acceleration ω is given by.

$$\omega = \frac{2\pi N}{60} \text{ rad/s} \qquad \text{— (ix)}$$

where, N is in rpm.

Since angular velocity is uniform, the time taken for one revolution T is given by Eqn — (viii)

cineti

$$2\pi = \omega T$$

or $T = 2\pi/\omega$ —— (X)

Kinetics of rigid body rotation

Consider the wheel shown in fig (i) rotating about its axis in clockwise direction with an acceleration α. Let δm be mass of an element at a distance r from the axis of rotation. If δP be the resulting force on this element

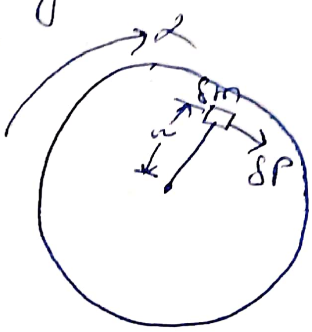


fig (i)

$\delta P = \delta m \times a$ where, a is tangential acceleration

But $a = r\alpha$ where, α is angular acceleration

$\therefore \delta P = \delta m\, r\alpha$

Rotational moment δM_t due to this force δP is given by

$$\delta M_t = \delta P \times r$$
$$= \delta m\, r^2 \alpha$$
$$M_t = \Sigma \delta M_t = \Sigma \delta m\, r^2 \alpha$$
$$= \alpha \Sigma \delta m\, r^2$$
$$= \alpha I$$

where I is mass moment of inertia of rotating body

Thus $M_t = I\alpha$

Note the similarity between expressions $M_t = I\alpha$ & $F = ma$ used in linear motion.

The product of the mass moment of inertia & the angular velocity of a rotating body is called angular momentum.

Thus: Angular Momentum $= I\omega$

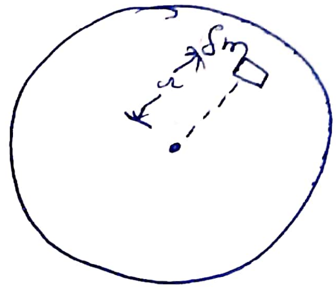
Kinetic energy of rotating bodies

Consider the rotating body shown in fig (ii) with angular velocity ω. Let δm be mass of an element which is at a distance r from the axis of rotation. Hence, if V is the linear velocity of the element.

reta

ω

$$V = r\omega$$



Now, kinetic energy of the elements mass

$$= \frac{1}{2}\delta m V^2$$

$\therefore$ K.E of rotating body

$$= \sum \frac{1}{2}\delta m V^2$$

$$= \sum \frac{1}{2}\delta m r^2 \omega^2$$

$$= \frac{1}{2}\omega^2 \sum \delta m r^2$$

But from the definition of mass moment of inertia

$$I = \sum \delta m r^2$$

$$\therefore \quad K.E = \frac{1}{2} I \omega^2$$

Q. A flywheel weighing 50 kN and having radius of gyration 1 m loses its speed from 400 rpm to 280 rpm in 2 minutes. Calculate:

(i) the retardation torque acting on it.

(ii) change in its kinetic energy during the above period, &

(iii) change in its angular momentum during the same period.

Sol^n

$$\omega_0 = 400\,rpm = \frac{400 \times 2\pi}{60} = 41.888\ rad/s$$

$$\omega = 280\,rpm = \frac{280 \times 2\pi}{60} = 29.322\ rad/s$$

$$t = 2\ min = 120\ seconds$$

but,

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{29.3224 - 41.888}{120}$$

$= -0.1047\ rad/s^2$

retardation is $0.1047\ rad/s^2$

weight of flywheel = 50 kN = 50,000 N

Radius of gyration $k = 1m$

$\therefore\ I = mk^2 = \frac{50,000}{9.81} \times 1^2 = 5096.84$

(i) Retarding Torque Acting on the Flywheel.

$= I\alpha = 5096.84 \times 0.1047$

$= 533.74\ N\text{-}m$

(ii) change in kinetic energy

= Initial K.E − Final K.E

$= \frac{1}{2}I\omega_0^2 - \frac{1}{2}I\omega^2$

$= \frac{1}{2} \times 5096.84\ (41.888^2 - 27.322^2)$

$= 2280442.9\ N\text{-}m$

(iii) change in its Angular Momentum

$= I\omega_0 - I\omega$

$= 5096.84\ (41.888 - 29.322)$

$= 64048.93\ Ns$

Q/ A pulley of weight 400N has a radius of 0.6m. A block of 600N is suspended by a light rope wound round the pulley, the other end being attached to the pulley as shown in fig. Determine the resulting acceleration of the weight & the tension in the rope.

Sol^n Let a be the resulting acceleration and T be the tension in the rope. Hence angular acceleration of pulley,

$\alpha = a/r = a/0.6 = 1.667a\ rad/s^2$ — (1)

An intertia force of $(600/g)\,a$ may be considered and the dynamic equilibrium condition can be written the block as:

$$T = \left(600 - \frac{600}{9.81}a\right) \quad \text{---(ii)}$$

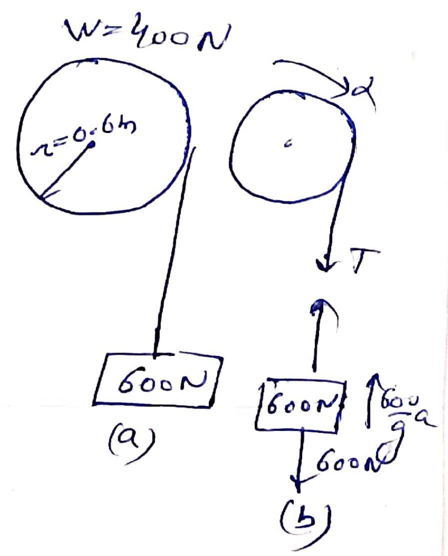
From kinetic equation for pulley, we get

$$M_t = I\alpha \text{ i.e. } T(0.6) = I \times 1.667a \quad \text{---(iii)}$$

But $$I = \frac{Mr^2}{2} = \frac{400}{9.81} \times \frac{0.6^2}{2}$$

from eq (iii), $$T = \frac{400}{9.81} \times \frac{0.6^2}{2} \times \frac{1.667a}{0.6}$$

$$= \frac{200}{9.81}\,a \text{ N-m}$$



Substituting it in eqn (ii), we get

$$\frac{200}{9.81}a = 600 - \frac{600}{9.81}a$$

$$a = \frac{600 \times 9.81}{800}$$

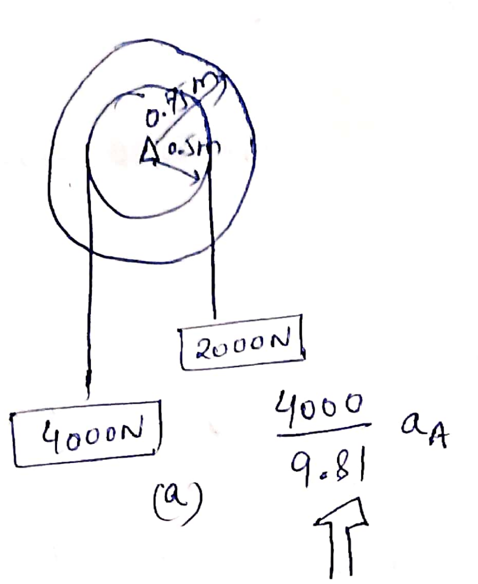
$$= \cancel{7}.358 \text{ m/sec}^2$$

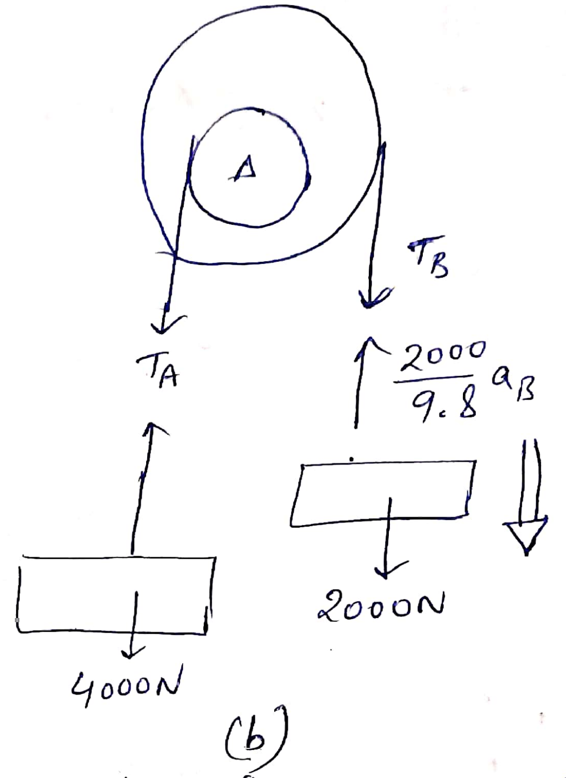
$\therefore$ $$T = \frac{200}{9.81} \times 7.358$$

$$= 150 \text{ N}$$

Q/ The composite pulley shown in fig (i) weighs 800N & has a radius of gyration of 0.6m. the ~~20~~ 2000N & 4000N blocks are attached to the pulley by inextensible strings as shown in figure.

Neglecting weight of the strings, determine the tension in the strings & angular acceleration of the pulley.





Sol^n Since the moment of 4000 N block is more than that of 2000 N block about the axis of rotation, the pulley rotates in anticlockwise direction as shown in fig (b)

Let a_A be acceleration of 4000 N block A a_B that of 2000 N block. Then

$$a_A = 0.5\alpha \text{ \& } a_B = 0.75\alpha$$

where, α - angular acceleration of pulley.

writing dynamic equilibrium equation for two blocks, we get

$$T_A = 4000\left(1 - \frac{a_A}{9.81}\right) = 4000\left(1 - \frac{0.5}{9.81}\alpha\right)$$

$$T_B = 2000\left(1 + \frac{a_B}{9.81}\right) = 2000\left(1 + \frac{0.75}{9.81}\alpha\right)$$

from kinetic equation of pulley, we have

$$M_t = I\alpha$$

i.e $T_A \times 0.5 - T_B \times 0.75 = \frac{800}{9.81} 0.6^2 \alpha$ $\quad M_t = P \times r$

$$4000\left(1 - \frac{0.5}{9.81}\alpha\right)0.5 - 2000\left(1 + \frac{0.75}{9.81}\alpha\right)0.75 = \frac{800}{9.8}(0.6)^2\alpha$$

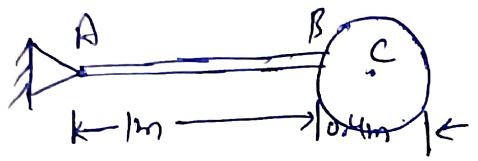
$$245.97\alpha = 500$$

$$\alpha = 2.033 \text{ rad/s}^2$$

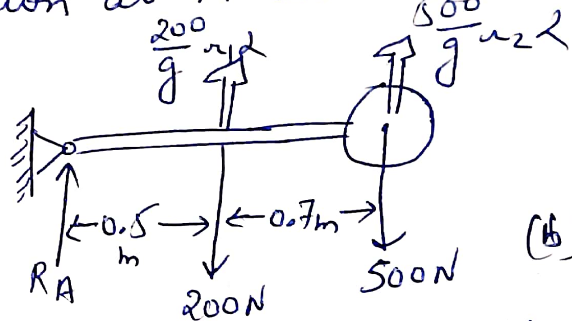
$\therefore \quad T_A = 4000\left(1 - \frac{0.5}{9.81} \times 2.033\right) = 3585.58\,N$

$T_B = 2000\left(1 + \frac{0.75}{9.81} 2.033\right) = 2310.83\,N$

Q. A cylinder weighing 500N is welded to a 1m long uniform bar of 200N as shown in fig. Determine the acceleration with which the assembly will rotate about point A, if released from rest in horizontal position. Determine the reaction at A at this instant.



(a)



(b)

Sol^n Let α be the angular acceleration with which the assembly will rotate. Let I be the mass moment of Inertia of the assembly about axis of rotation A. Using the transfer formula $I = I_g + Md^2$, we can assemble mass moment of inertia about the axis of rotation through A.

Mass moment of inertia of the bar about A.

$$= \frac{1}{12} \times \frac{200}{9.81} \times 1^2 + \frac{200}{9.81} \times (0.5)^2 = 6.7968$$

Now moment of inertia of the cylinder about A

$$= \frac{1}{2} \times \frac{500}{9.81} \times 0.2^2 + \frac{500}{9.81} \times 1.2^2 = 74.414$$

$\therefore$ Mass moment of inertia of system about A

$$I = 6.7958 + 74.41 = 81.2097$$

Rotational moment about A (fig b)

$$M_t = 200 \times 0.5 + 500 \times 1.2 = 700\,Nm$$

...ating it to $I\alpha$, we get,

$$81.2097\alpha = 700 \quad \text{or} \quad \alpha = 8.6197 \text{ rad/s}$$

Instantaneous acceleration of rod AB is vertical & its magnitude is given by

$$= r_1\alpha = 0.5 \times 8.6197$$
$$= 4.310 \text{ m/s}$$

Similarly the instantaneous acceleration of the cylinder is also vertical & is equal to

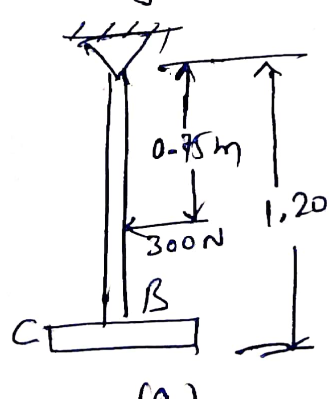
$$r_2\alpha = 1.2 \times 8.6197 = 10.344 \text{ m/s}$$

Applying D'Alembert's dynamic equilibrium equation to the system of forces shown in fig (b).

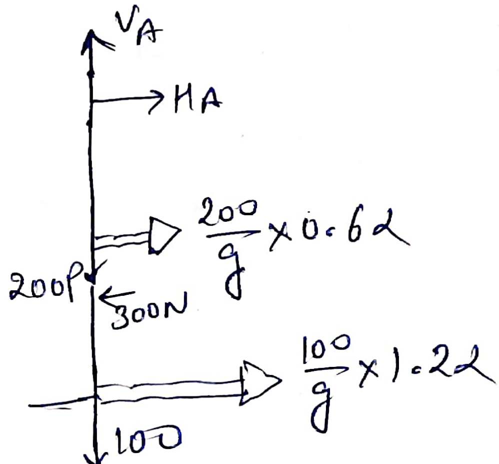
$$R_A = 200 + 500 - \frac{200}{9.81} \times 4.3100 - \frac{500}{9.81} \times 10.344$$

$$R_A = 84.934 \text{ N}$$

Q/ Rod AB, weighing 200 N is welded to the rod CD weighing 100 N as shown in fig (a). The assembly is hinged at A & is freely held. Determine the instantaneous vertical & horizontal reactions of A when a horizontal force of 300 N acts at a distance of 0.75 m from A



(a)



(b)

Soln Mass moment of inertia of AB about axis of rotation

$$A = \frac{1}{12} \times \frac{200}{9.81} \times 1.2^2 + \frac{200}{9.81} \times 0.6^2 = 9.786$$

Mass moment of inertia of rod CD about A.

$$= \frac{1}{12} \times \frac{100}{9.81} \times 0.6^2 + \frac{100}{9.81} \times 1.2^2 = 147.0$$

∴ Total mass moment of the system about A.

$$9.786 + 147.0 = 156.78$$

Let α be the instantaneous angular acceleration. writing the kinetic equation for the rotation about A, we get

$$I\alpha = M_t$$

$$156.786\alpha = 300 \times 0.75$$

$$\alpha = 1.4351 \text{ rad/s}$$

At the instant 300N force is applied, the linear acceleration of AB & CD are horizontal and are equal to 0.6α & 1.2α respectively. Hence the inertia forces are shown in fig (b) by dotted arrows. Let V_A & H_A be the vertical & horizontal reactions at A. Writing the dynamic equilibrium conditions, we get,

$$V_A = 200 + 100 = 300N$$

&

$$H_A = 300 - \frac{200}{9.81} \times 0.6\alpha - \frac{100}{9.81} \times 1.2\alpha$$

Substituting the value of α, we get

$$H_A = 264.891 \text{ N}$$

& the geometric ...

...ve the relationship between the linear motion of geometric centre & angular motion of a wheel rolling without slipping.

Soln Consider a wheel of radius r rolling with angular velocity ω & angular acceleration α. In fig the dotted line shows the original position and solid line shows the position after rotating through angular distance θ.

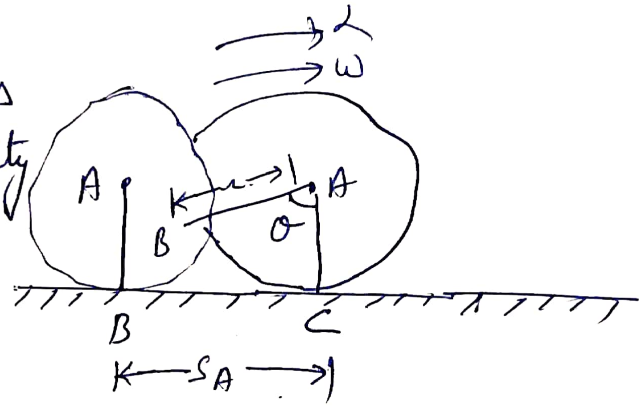


fig (ii)

Since there is no slip, linear distance $BC = S_A$ must be equal to angular distance $BC = r\theta$,

i.e $\quad S_A = r\theta \quad$ —— (1)

$$\frac{dS_A}{dt} = r\frac{d\theta}{dt} \quad \text{i.e } V_A = r\omega \quad \text{—— (2)}$$

& $$\frac{d^2S_A}{dt^2} = r\frac{d^2\theta}{dt^2} \quad \text{i.e } a_A = r\alpha \quad \text{—— (3)}$$

Thus when wheel rotates without slip, the relationship between motion of its geometric centre and its angular motion are.

$$\left.\begin{aligned} S_A &= r\theta \quad \text{—— (i)} \\ V_A &= r\omega \quad \text{—— (ii)} \\ a_A &= r\alpha \quad \text{—— (iii)} \end{aligned}\right\}$$

22.3 INSTANTANEOUS AXIS OF ROTATION

In the previous article it is shown that plane motion of a point (v_B) can be split into translation of another point (v_A) and rotation of about that point ($r\omega = v_{B/A}$). At any instant it is possible to locate a point in the plane which has zero velocity and hence plane motion of other points may be looked upon as pure rotation about this point. Such point is called **Instantaneous Centre** and the axis passing through this point and at right angles to the plane of motion is called **Instantaneous Axis of Rotation.**

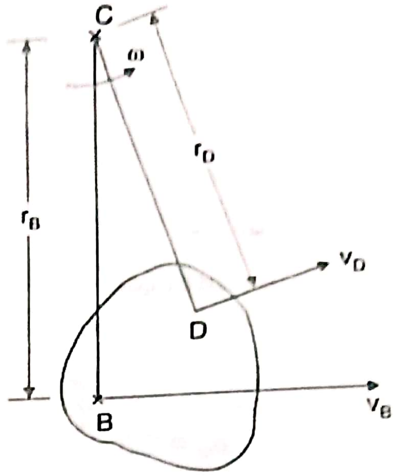


Fig. 22.7

Consider the rigid body shown in Fig. 22.7 which has plane motion. Let B be a point having velocity v_B at the instant considered. Now locate a point C on perpendicular to the direction v_B at B at a distance r_B from the axis of rotation. Now plane motion of the B can be split into translation of C and rotation about C.

$\therefore \quad v_B = v_C + r_B\,\omega$ and the direction of the velocity is at right angles to CB. If we write $r_B = v_B/\omega$, then from the above relation we get

$$v_B = v_C + \frac{v_B}{\omega}\cdot\omega \quad i.e., \quad v_C = 0.$$

Thus if point C is selected at a distance v_B/ω along the perpendicular to the direction of velocity at B, the plane motion of B reduces to pure rotation about C.

Hence C is instantaneous centre.

If D is any other point on the rigid body, its velocity will be given by

$$v_D = v_C + r_D\omega$$
$$= r_D\omega \quad \text{since } v_C = 0,$$

and it will be at right angles to CD.

Thus, if the instantaneous centre is located, motion of all other points at that instant can be found by pure rotation about C.

Methods of Locating Instantaneous Centre

Instantaneous centre can be located by any one of the following two methods:

(*i*) If the angular velocity ω and linear velocity v_B are known, instantaneous centre is located at a distance v_B/ω along the perpendicular to the direction of v_B at B.

(*ii*) If the linear velocities of two points of the rigid body are known, say v_B and v_D, drop perpendiculars to them at B and D. The intersection point is the instantaneous centre.

Use of Instantaneous Centre

If instantaneous centre is located, the velocities of all other points on the rigid body at that instant can be determined by treating the plane motion as pure rotation about the instantaneous centre. It may be noted that point C located as instantaneous centre, has zero velocity only at that instant. If it has zero velocity at time t, at time $t + dt$ it has some velocity. It means, the instantaneous centre is not having zero acceleration. Hence for the acceleration calculations we can not treat the plane motion as pure rotation about instantaneous centre. The plane motion can be treated as a case of pure rotation about instantaneous centre only for velocity calculations.

Example 22.5. *Determine the velocities of the points B and D given in example 22.2 by instantaneous centre method. Given*

Angular velocity $\omega = 5$ *rad/s.*

Wheel radius = 1 m.

Solution.

$\therefore$ Velocity of geometric centre A, $v_A = r\omega = 1 \times 5 = 5$ m/s and it is horizontal in the rightward direction. Hence instantaneous centre is in the vertically downward direction at a distance

$$= v_A/\omega = 5/5 = 1\text{m}, \quad i.e., \quad \text{at point } C,$$

which is in contact with floor (see Fig. 22.8).

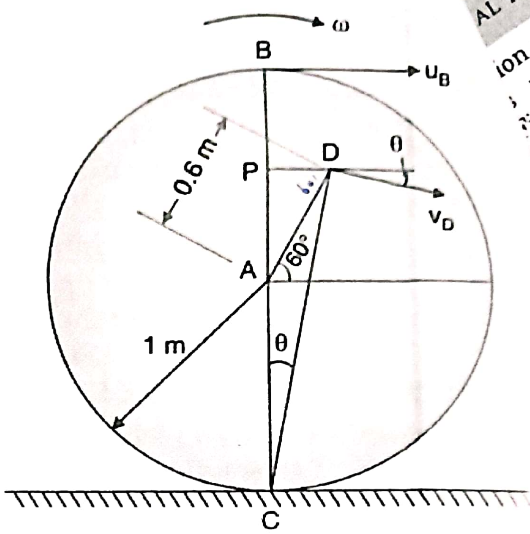


Fig. 22.8

$$\therefore \quad v_B = CB \times \omega = 2 \times 5 = 10 \text{ m/s.}$$

$$v_D = CD \times \omega$$

Now $CP = CA + AP$ where P is foot of $\perp^r$ of D on CB.

$$\therefore \quad CP = 1 + 0.6 \times \sin 60° = 1.520 \text{ m}$$

$$PD = 0.6 \times \cos 60° = 0.3 \text{ m}$$

AP/AD = sin 60
AP = 0.6 · sin 60

$$\therefore \quad CD = \sqrt{CP^2 + PD^2} = \sqrt{1.520^2 + 0.3^2} = 1.549 \text{ m}$$

$$\therefore \quad v_D = CD \times \omega = 1.549 \times 5.0$$

i.e., $\quad v_D = \mathbf{7.745}$ **m/s** **Ans.**

Its direction is at right angle to CD. Its inclination to horizontal, θ is given by

$$\theta = \angle PCD = \tan^{-1}\frac{0.3}{1.520}$$

i.e., $\quad \theta = \mathbf{11.17°}$ **Ans.**

Example 22.6. *Find the velocity of B in example 22.3 by instantaneous centre method.*

Solution. Since velocity at A (v_A) is horizontal and velocity at B (v_B) is vertical, instantaneous centre is point C, which is obtained by dropping perpendicular to the directions v_A and v_B at points A and B respectively. Now,

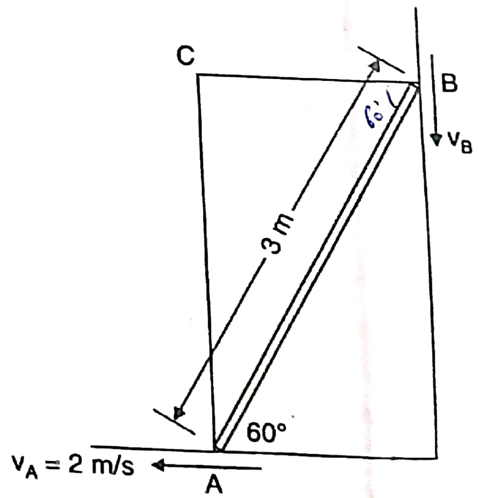


Fig. 22.9

$$v_A = AC \times \omega$$

i.e., $\quad 2 = 3 \sin 60° \times \omega$

and $\quad \omega = 0.770$ rad/s

i.e., $\quad v_B = BC \times \omega = 3 \cos 60° \times 0.770$

$$v_B = \mathbf{1.555 \text{ m/s}} \quad \textbf{Ans.}$$

22.4 KINETICS OF ROLLING BODIES

It is already known that the plane motion of a rolling body can be split into linear motion and rotation about ...