Unit - 1

$z = x + iy$

Polar form of Complex number $\Rightarrow z = x + iy \quad r = \sqrt{x^2+y^2}$

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1} y/x$

Euler formula $\Rightarrow e^{i\theta} = \cos\theta + i\sin\theta$

Power of z $\Rightarrow z^n = r^n(\cos n\theta + i\sin n\theta)$ [De Moivre's]

Cauchy Rieman eq^n $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Where $u(x,y) + iv(x,y)$

Exponential form of CR eq^n

$f(z) = e^z$

$= e^{x+iy}$

$= e^x(e^{iy})$

$f(z) = e^x(\cos y + i\sin y)$

Polar form of Complex numbers

$z = r\cos\theta + i\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= re^{i\theta}$

Milne Thomson method

$f(z) = u(x,y) + iv(x,y) \qquad x = \frac{z+\bar{z}}{2}, \; y = \frac{z-\bar{z}}{2}$

$f(z) = u(z,0) + iv(z,0)$

Cauchy Integral theorem

$\int_C f(z)\,dz = 0$

Cauchy Integral formula

$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$

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Derivative of analytic function

$$f^{n}(a) = \frac{n!}{2\pi i}\int \frac{f(z)}{(z-a)^{n+1}}\,dz$$

Arguement of $|z|$ = ~~[illegible]~~ $\pi - \theta$

$$\theta = \tan^{-1} y/x$$

Polar Co-ordinates analytic $\Rightarrow \dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$

$$\frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$$

Laplace Eqn $\Rightarrow \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ [Harmonic funn]

Derivate given eqn 2 times ans $\Rightarrow 0$

$$e^x = 1 + x + \frac{x^2}{2!} \dots$$

Quadratic formula $\quad z = \dfrac{-B \pm \sqrt{B^2 - 4ac}}{2A}$

Taylor's theorem $\quad \dfrac{f^{n}(a)}{n!}(z-a)^n$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \dots$$

$$(1-x)^{-1} = 1 + x - x^2 + x^3 - x^4 \dots$$

Taylor theorem Eg

$$f(z) = \frac{1}{(z-1)(z-3)}$$

$$f(z) = \frac{1}{2}\left[\frac{1}{z-1} - \frac{1}{z-3}\right]$$

$$f'(z) = \frac{1}{2}\left[\frac{1}{(z-1)^2} - \frac{1}{(z-3)^2}\right]$$

$Re(z) = x = \frac{1}{2}(z+\bar{z})$

$Im(z) = y = \frac{1}{2i}(z-\bar{z})$

Cubes of unity $1, \omega, \omega^2 \ldots$

If $z^n = 1$ then $z^n = 1^{1/n}$

$(\cos 2k\pi + i \sin 2k\pi)^{1/3}$

$\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3}$

Hyperbolic fun^n

$$\sinh z = \frac{e^z - e^{-z}}{2}$$

$$\cosh z = \frac{e^z + e^{-z}}{2}$$

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} \ldots$$

$$e^{-z} = 1 - \frac{z}{1!} + \frac{z^2}{2!} - \frac{z^3}{3!} + \frac{z^4}{4!} \ldots$$

$$\cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} \ldots$$

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} \ldots$$