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Set :→ A well defined Collection of objects.

Eg. $A = \{a, e, i, o, u\}$

$A = \{$Set of all prime no.s$\}$.

Empty Set :→ A Set having no Element ie. $A = \phi$
↳ Null / void / Empty

Singleton Set :→ A Set Consisting of only one Element

E.g. $A = \{a\}$, $A = \{i\}$.

Finite Set :→ If it is Either void set or elements can be Counted by natural no.s 1, 2, 3 --- And this Counting ends at Certain natural no. n.

Eg. $A = \{1, 2, \dots n\}$

$A = \{1, 2, 3 \dots\}$ − $\mathbb{N}$ − Infinite Set

Cardinality of Set / order of Set :→ If A is finite then no. of Elements in A is called order / Cardinality of A.

denoted by $|A|$ or $n(A)$.

→ For Infinite Set,

Subset :→ Let A and B be two sets. If $\forall\, a \in A,\ a \in B$

Then $A \subseteq B$.

R

→ Every Set is a Subset of itself

$\because \forall\, a \in A,\ a \in A \Rightarrow A \subseteq A$.

→ Empty set is a Subset of Every set.

$\because \phi \subseteq A$ as ϕ has no Element.

} Improper Subsets.

→ A Subset B of A is Called Proper Subset If $B \neq \phi$, $B \neq A$.

→ **Power Set :→** let A be a set.

Then collection of all subsets of A is called Power set of A.

$P(A) = \{B \mid B \subset A\}$.

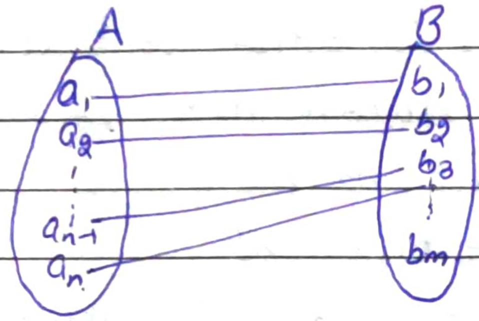
→ **# of Elements in $P(A)$:→** If A is finite set

let $|A| = n$

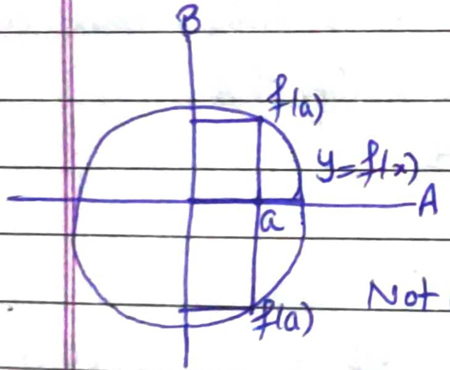
$\cancel{2c_0 + 2c_1}$ $\quad {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = (1+1)^n$
$= 2^n$

($\because$ no. of subsets of A having r elements $= {}^nC_r$)

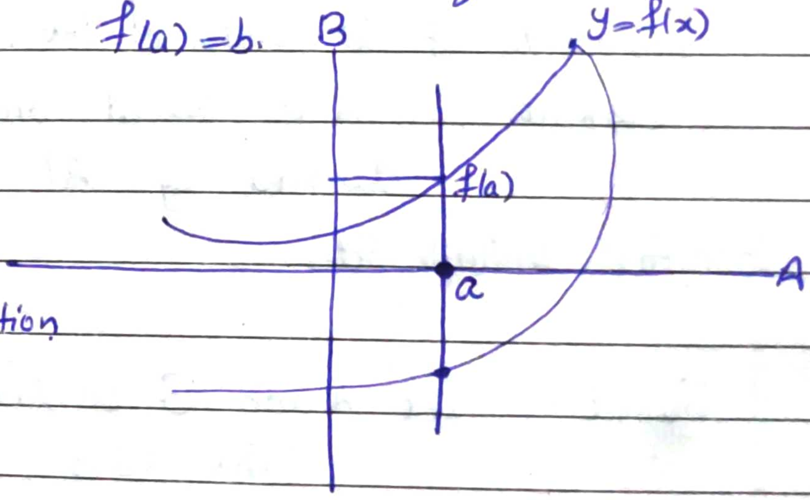
Function :→



let $A, B \neq \phi$ sets. If $\forall\, a \in A$, $\exists$ unique $b \in B$ s.t.:
$f(a) = b$.



Not a function

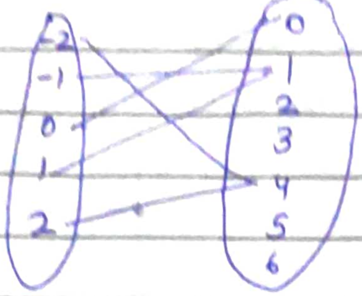


→ For every line passing through x-axis, $\parallel$ to y-axis, it cuts the curve $y = f(x)$ exactly once.

→ A - Domain of f
B - Co-Domain of f.

Ex $f: A \to B$; $A = \{-2, -1, 0, 1, 2\}$

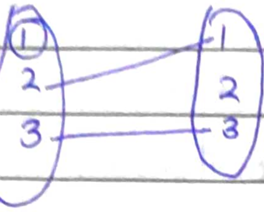
$f(x) = x^2$ $B = \{0, 1, 2, 3, 4, 5, 6\}$.



function

Ex $f: A \to B$

$f(x) = 2x - 3$; $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$



Not a function.

→ # of functions from $A \to B$ where $|A| = n$, $|B| = m$.

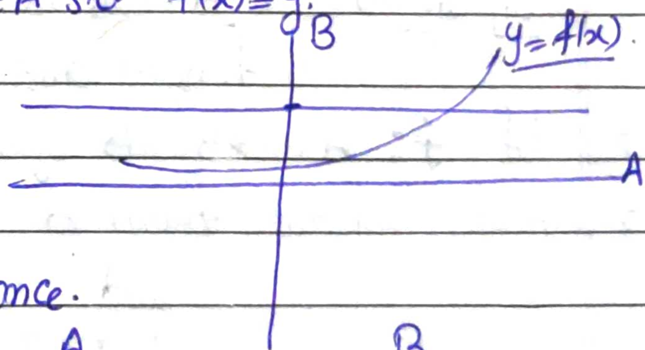
$= m^n$. $\because \underbrace{m \times m \times \dots \times m}_{n \text{ times}}$

→ $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$

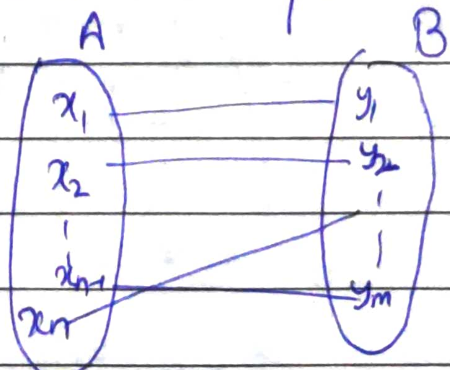
# of fun. from $A \to B$ s.t. $f(2) = c$, $f(x) \neq b$ $\forall x \in$

→ Onto functions :→ $f: A \to B$

$\forall y \in B, \exists x \in A$ s.t $f(x) = y$.



Geometrically — Every line passing through y-axis ∥ to x-axis cuts the curve $y = f(x)$ atleast once.



→ If $|B| > |A|$ Then $\nexists$ onto fun. from $A \to B$.

$f: A \to B$ onto $\Rightarrow$

$|A| \geq |B|$.

→ If $|A| \geq B \Rightarrow \exists f: A \to B$ onto fun.

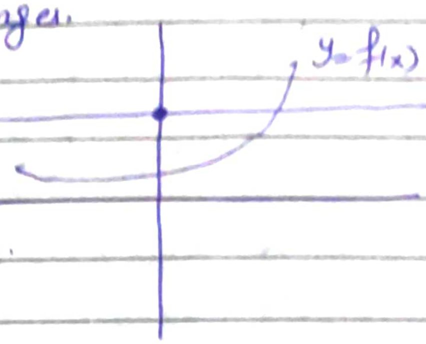
→ **1-1 function** :→ $f: A \to B$

$\forall x_1 \neq x_2, \ f(x_1) \neq f(x_2)$

or If $f(x_1) = f(x_2)$ Then $x_1 = x_2$.

i.e. Distinct Elements have distinct Images.

Graphically $\forall$ line passing through y-axis, $\parallel$ to x-axis Cuts the Curve $y = f(x)$ atmost once.



→ $f: A \to B$ is 1-1 $\Rightarrow |A| \leq |B|$

→ If $|A| \leq |B|$ Then $\exists \ f: A \to B$ s.t. f is 1-1.

→ If $|A| > |B|$ Then $\nexists \ f: A \to B$ s.t. f is 1-1.

→ f, 1-1, g 1-1 $\Rightarrow$ $f \circ g$ 1-1

→ f, g onto $\Rightarrow$ $f \circ g$ onto.

→ f, g 1-1, onto $\Rightarrow$ $f \circ g$ 1-1, onto.

→ $f \circ g$ 1-1 onto $\Rightarrow$ g 1-1 & f onto.

→ $f \circ g$ 1-1 $\Rightarrow$ g is 1-1

→ $f \circ g$ is onto $\Rightarrow$ f is onto.

$f: A \to B, \ g: B \to C$

$g \circ f: A \to C$

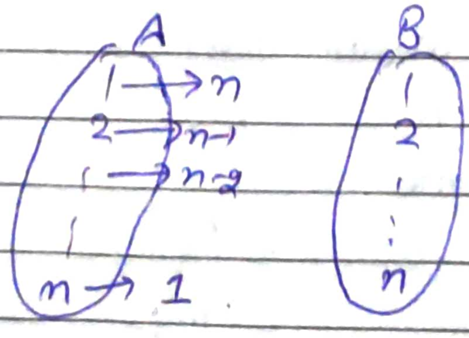
$R(f) \subseteq D(g)$

$f \circ g$ does not Exist.

Bijection :→ $f: A \to B$ is Called Bijection If f is 1-1 and onto.

→ If $f: A \to B$ is bijection $\Leftrightarrow |A| = |B|$

→ # of bijection from $A \to B$ is $n!$



$= \underline{\lfloor n}$ Choices.

Real function:→ If the domain and Co-domain of a function are subsets of $\mathbb{R}$ (set of real nos)

Range of function $f: A \to B$ is a fun.

$R(f) = \{f(x) \mid x \in A\}$.

$R(f) \subset B$

If $R(f) = B$ Then f is onto.

Domain of Real function:→ Dom of f is the set of all real nos x for which f(x) is real no.

Ex find dom of fun. $f(x) = \dfrac{1}{\sqrt{9-x^2}}$

$9 - x^2 > 0 \Rightarrow x^2 - 9 < 0$

$\Rightarrow x^2 < 9$

$\Rightarrow x \in (-3, 3)$

Even function → If $f(-x) = f(x) \ \forall x$

odd function → If $f(-x) = -f(x) \ \forall x$

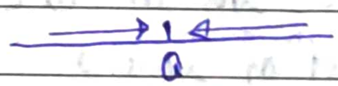
Ex $f(x) = x^2$ Even fun

$f(x) = x^3$ odd fun.

Ex S.T. $f(x) = \log(x + \sqrt{x^2+1})$ is an odd fun.

Limits:→

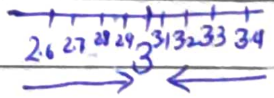
$f(x)$



$x \to a$ from left then $f(x)$

$x \to a$ from right then $f(x)$

$\lim_{x \to 3^-} f(x) = LHL$



$\lim_{x \to 3^+} f(x) = RHL$

If LHL = RHL

Then limit Exists.

$\lim_{x \to a^-} f(x)$

LHL Put $x = a-h$, then $\lim_{h \to 0} f(a-h)$

RHL $\lim_{x \to a^+} f(x)$

Put $x = a+h$

then $\lim_{h \to 0} f(a+h)$

Ex
$$f(x) = \begin{cases} \frac{x-|x|}{x} & ; x \neq 0 \\ 2 & ; x = 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(-h) = \lim_{h \to 0} \frac{-h - |-h|}{-h} = \lim_{h \to 0} \frac{-h-h}{-h}$$
$$= +2$$

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(h) = \lim_{h \to 0} \frac{h-|h|}{h} = 0$$

$\lim_{x \to 0} f(x)$ doesn not Exist

Neighbourhood of a pt:→

$a-\delta \quad a \quad a+\delta$

Set of real nos lying b/w $(a-\delta, a+\delta)$ is called nbd of a of radius δ, denoted by $N_\delta(a)$.

Deleted nbd of a :→ $N_\delta(a) - \{a\}$

So $(a-\delta, a)$ left nbd of a

$(a, a+\delta)$ right nbd of a.

x lies in $(a-\delta, a+\delta) \Rightarrow a-\delta < x < a+\delta \Rightarrow |x-a| < \delta$.

Def^n of limit:→ $\lim_{x \to a} f(x) = l$

$$|f(x) - l| < \varepsilon \text{ if } |x-a| < \delta$$

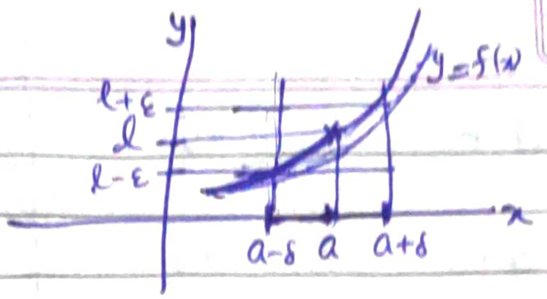
if $\forall \ \varepsilon > 0 \ \exists \ \delta > 0$ s.t.

$$|f(x) - l| < \varepsilon \text{ ~~if~~ when } |x-a| < \delta.$$

So l is the lt of $f(x)$ as x tends to a

if $\forall \ \varepsilon > 0 \ \exists \ \delta > 0$ s.t.

$|f(x) - l| < \varepsilon$ when $|x-a| < \delta$.



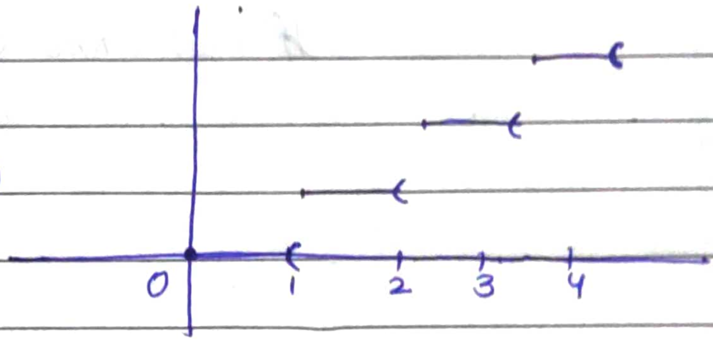
→ $f(x)$ is cts at a If $\lim\limits_{x\to a^-} f(x) = f(a) = \lim\limits_{x\to a^+} f(x)$

→ $f(x)$ is cts at $x=a$ If $f(a)$ is defined

$\lim\limits_{x\to a} f(x)$ Exist

And $\lim\limits_{x\to a} f(x) = f(a)$

Ex $f(x) = [x]$



$$\lim_{x\to n^-} f(x) = \lim_{h\to 0} f(n-h) = \lim_{h\to 0} [n-h] = n-1$$

$$\lim_{x\to n^+} f(x) = \lim_{h\to 0} f(n+h) = \lim_{h\to 0} [n+h] = n.$$

$\lim\limits_{x\to n^+} f(x) \neq \lim\limits_{x\to n^-} f(x) \Rightarrow$ lt does not Exist.

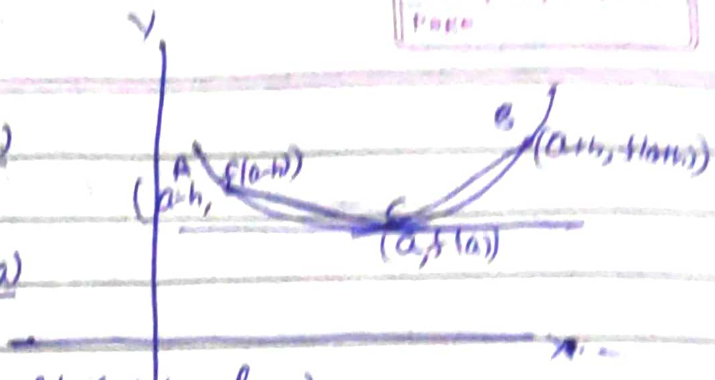
→ Differentiability at a point → $\lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a} = f'(a)$

f is diff. at a Iff $\lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a}$ Exist finitely.

or $\lim\limits_{x\to a^-} \dfrac{f(x)-f(a)}{x-a} = \lim\limits_{x\to a^+} \dfrac{f(x)-f(a)}{x-a}$

or $\underbrace{\lim\limits_{h\to 0} \dfrac{f(a-h)-f(a)}{-h}}_{LHD} = \underbrace{\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}}_{RHD}$

Slope of AC = $\dfrac{f(a-h)-f(a)}{-h}$



Slope of BC = $\dfrac{f(a+h)-f(a)}{h}$

$$\lim_{h\to 0} \frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0} (\text{slope of } AC)$$

LHD $= \lim_{A\to C} (\text{Slope of } AC)$

= Slope of tangent line at C.

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \text{slope of tangent line at } C.$$

RHD

If LHD = RHD $\iff$ There is ! tangent line at C.

$\iff$ no sharp Edge.