Chapter 19

PERSPECTIVE PROJECTION

19-1. INTRODUCTION

Perspective projection or perspective drawing is the representation of an object on a plane surface, called the *picture plane*, as it would appear to the eye, when viewed from a fixed position.

It may also be defined as the figure formed on the picture plane when visual rays from the eye to the object cut the picture plane. Perspective is mainly used in architecture. By means of perspective, the architect is able to show how an object would appear when constructed.

It is essential to have full knowledge of the principles of orthographic projection (third-angle method) before the theory of perspective drawing can be studied.

In this chapter, we shall deal with the topics of perspective drawing as follows:

1. Principle of perspective projection
2. Definitions of perspective elements
3. Station point
4. Angle of vision
5. Picture plane
6. Methods of drawing perspective view
7. Types of perspective
8. Distance points
9. Measuring line or line of heights
10. Perspectives of circles and solids.

19-2. PRINCIPLE OF PERSPECTIVE PROJECTION

In perspective projection, the eye is assumed to be situated at a definite position relative to the object. The vertical plane, which (in perspective) is called the picture plane, is placed between the object and the eye.

Visual rays from the eye to the object converge to a point in the eye and are, therefore, inclined to the picture plane. The rays pierce the picture plane and form an image on it. This image is the perspective of the object.

19-3. DEFINITIONS OF PERSPECTIVE ELEMENTS

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 45 for the definitions of perspective elements.

Various elements used in obtaining the perspective view are defined below. Refer fig. 19-1.

(1) **Ground plane** (GP): It is a horizontal plane on which the object is assumed to be situated.

(2) **Station point** (S): It is the point where the eye of the observer is located while viewing the object.

(3) **Picture plane** (PP): It is a vertical transparent plane located between the station point and the object which is to be viewed. It is the plane on which the perspective is formed. The front view of perspective elements and of the object (if necessary) is also projected on this plane.

(4) **Horizontal plane** (HP): This imaginary plane is at the level of the eye, i.e. the station point. It is a horizontal plane, above the ground plane and at right angles to the picture plane.

(5) **Auxiliary ground plane** (AGP): It is a horizontal plane placed above the horizon plane. The top view of the object and of the perspective elements is projected on this plane.

(6) **Ground line** (GL): The line of intersection of the picture plane with the ground plane is called the ground line.

(7) **Horizon line** (HL): It is the line in which the horizon plane intersects the picture plane. It is parallel to the ground line.

(8) **Perpendicular axis** (PA): It is the line drawn through the station point, perpendicular to the picture plane. It is, sometimes called the *Line of sight or Axis of vision.*

(9) **Centre of vision** (C): The point in which the perpendicular axis pierces the picture plane is called the centre of vision. It lies on the horizon line.

(10) **Central plane** (CP): It is an imaginary vertical plane, which passes through the station point and the centre of vision. It contains the perpendicular axis. It is perpendicular to both, the picture plane and the ground plane.

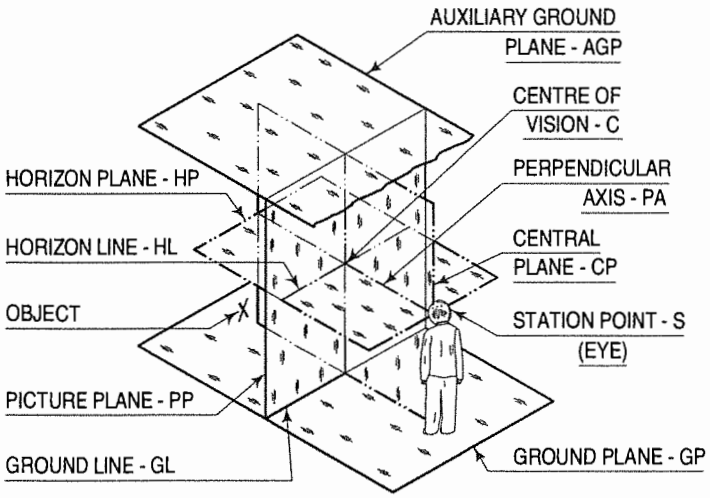

FIG. 19-1

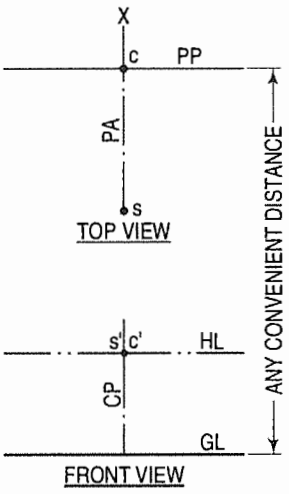

FIG. 19-2

Fig. 19-2 shows the projections of the perspective elements. In the top view, the picture plane PP is seen as a horizontal line. The object *X* is above PP, while S the station point is below PP. The line SC is the perpendicular axis and represents the central plane also. The ground plane, the horizon plane and the auxiliary ground plane will be seen as rectangles, but are not shown. In the front view, lines GL (ground line) and HL (horizon line) represent respectively the ground plane and the horizon plane. The station point *s'* and the centre of vision *c'* coincide with each other on HL. The central plane CP is seen as a vertical line through *s'*. The picture plane will be seen as a rectangle, but is not shown. The perspective view (when drawn) will be seen above or around GL.

19-4. STATION POINT

The position of the station point is of great importance. Upon its position, the general appearance of the perspective depends. Hence, it should be so located as to view the object in the best manner.

For large objects such as buildings, the station point is usually taken at the eye level of a person of normal height as shown in fig. 19-1 i.e. about 1.8 metres. For small objects, the station point should be fixed at such a height as would give a good view of the top surface as well as side surfaces.

The distance of the station point from the picture plane, when taken equal to about twice the greatest dimension of the object, usually gives good view in the perspective.

For objects having heights and widths more or less equal, the location of the station point may be so fixed that the angle between the visual rays from the station point to the outer-most boundaries of the object is approximately 30°.

The station point should be so situated in front of the object that the central plane passes through the centre of interest of the object. It may not, necessarily, be placed in front of the exact middle of the object. Refer fig. 19-1.

19-5. ANGLE OF VISION

Angle of vision is angle subtended by eye in horizontal or vertical direction in which one can visualize the things clearly. Horizontal and vertical angle of vision is generally 60° and 45° respectively. Refer fig. 19-3.

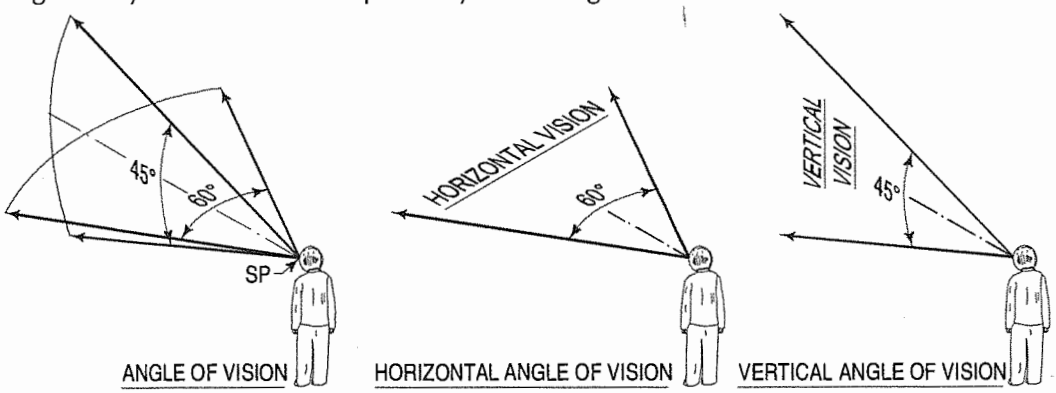

FIG. 19-3

19-6. PICTURE PLANE

The position of the picture plane relative to the object, determines the size of the perspective view. The perspective will show the object reduced in size when it is placed behind the picture plane. If the object is moved nearer the picture plane, the size of the perspective will increase. When the picture plane coincides with the object, the perspective of the object will be of its exact size. When the object is placed in front of the picture plane, its perspective, when projected back, will show the object enlarged in size.

In fig. 19-4(i), the line *AB* is behind the picture plane. Its perspective *A'B'* is shorter than *AB*. In fig. 19-4(ii), *AB* is in the picture plane; its perspective *A'B'* is equal to *AB* and coincides with it. Fig. 19-4(iii) shows the line *AB* placed in front of the picture plane; when projected back on the picture plane, its perspective *A'B'* is longer than *AB*.

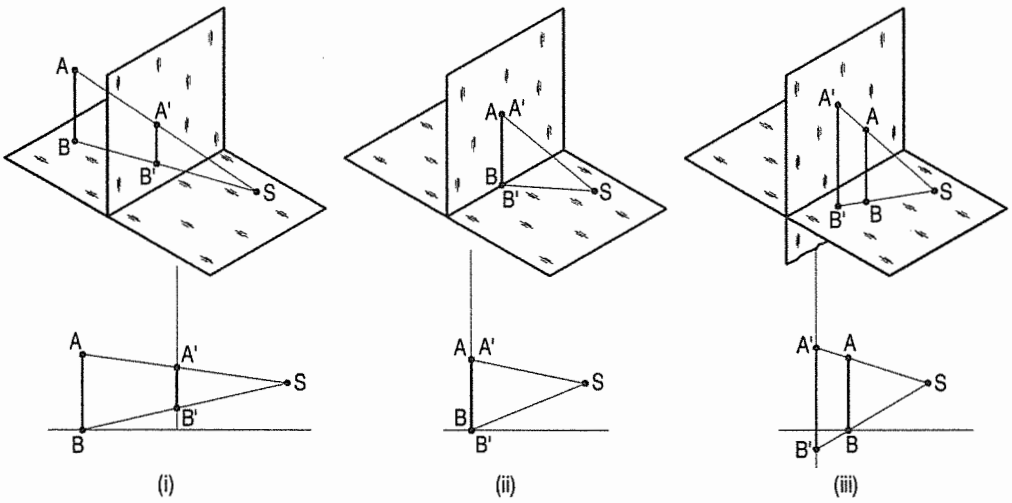

FIG. 19-4

19-7. METHODS OF DRAWING PERSPECTIVE VIEW

The perspective view of an object may be obtained by either

(1) Visual-ray method

(2) Vanishing-point method.

In the visual-ray method, points on the perspective are obtained by projecting

(i) the top view and

(ii) either the front view or the side view of the visual rays.

Vanishing-point method is comparatively simple. In addition to the top view of the visual rays, use of vanishing points of straight lines is made in this method. A front view or a side view of the object is also required to be drawn, for determining the heights.

19-7-1. VISUAL-RAY METHOD

This method is explained by means of the following three illustrative problems:

Problem 19-1. (fig. 19-5): *A point A is situated 15 mm behind the picture plane and 10 mm above the ground plane. The station point S is 25 mm in front of the picture plane, 20 mm above the ground plane and lies in a central plane 10 mm to the right of the point. Draw the perspective view of the point A.*

The pictorial view of the ground plane, the picture plane, the given point and the station point in their respective positions is given in fig. 19-5(i). The visual ray *AS* from the station point *S* to the point *A* is also shown. It passes through the picture plane. To mark the perspective of *A*, the point *A'* at which *AS* pierces the picture plane should be located.

In fig. 19-5(ii), an auxiliary ground plane (AGP) is shown placed above the point *A*, and the visual ray *AS* is shown projected on it, *as* is the top view of *AS* and a_1 is the top view of the point *A'* at which *AS* pierces the picture plane. a_1 shows the position of the point *A'* along the length of the picture plane. When the auxiliary ground plane is revolved and brought in the same plane as that of the picture plane, the view will be as shown in fig. 19-5(iii).

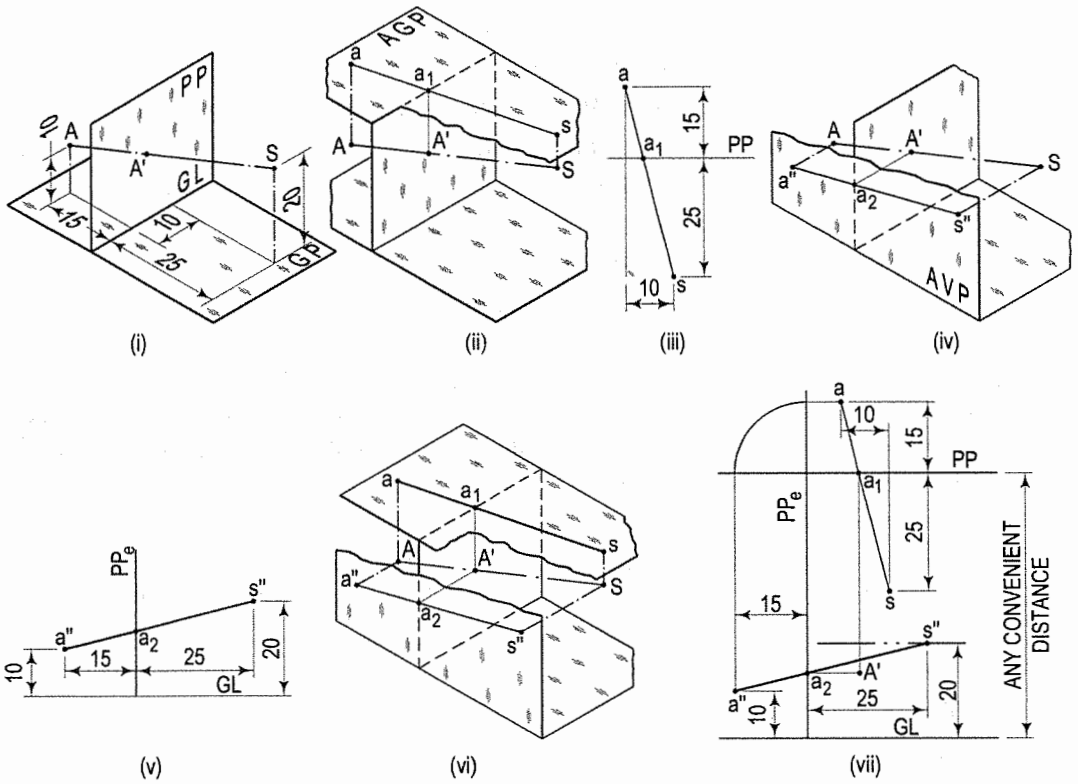

FIG. 19-5

To obtain the height of *A'* above the ground plane, an auxiliary vertical plane (AVP) perpendicular to both the picture plane and the ground plane is placed to the left of *A* and the side view of *AS* is projected on it [fig. 19-5(iv)]. *a" s"* is the side

view of *AS* and a_2 is the side view of *A'*. It shows the height of *A'* above the ground plane. Fig. 19-5(v) shows the orthographic view (side view) when *AVP* is revolved and brought in the same plane as that of the picture plane.

Fig. 19-5(vii) shows the top view [fig. 19-5(iii)] and the side view [fig. 19-5(v)] combined together. A horizontal line drawn through a_2 and intersecting the vertical line through a_1 gives the point *A'* which is the perspective view of the point *A*. It is quite clear from the pictorial view [fig. 19-5(vi)] that *A'* lies in the picture plane on the line *AS*.

Steps in drawing the perspective view of the point A [fig. 19-5(vii)]**:**

(i) Draw a horizontal line PP representing the picture plane in the top view.

(ii) Mark *a*, the top view of *A*, 15 mm above PP.

(iii) Draw a line (representing the central plane) perpendicular to PP and 10 mm to the right of *a*. On this line, mark s, the top view of the station point, 25 mm below PP.

(iv) Draw a line joining *a* with *s* and intersecting PP at a point a_1.

(v) At any convenient distance below PP, draw a horizontal line GL. It is the ground line and also represents the ground plane in the front view.

(vi) Draw a line HL parallel to and 20 mm above GL. It is the horizon line and also represents the horizon plane in the front view.

(vii) At any point on GL and to the left of *a*, draw a vertical line PP_e (representing the picture plane in the side view).

(viii) Mark *a"*, the side view of *A*, 10 mm above GL and 15 mm to the left of PP_e.

(ix) Mark *s"*, the side view of the station point, on HL and 25 mm to the right of PP_e.

(x) Draw a line joining *a"* with *s"* and intersecting PP_e at a point a_2.

(xi) Through a_1, draw a vertical line. Through a_2, draw a horizontal line intersecting the vertical line at a point *A'*.

Then *A'* is the perspective view of the point *A*.

Alternative method: Instead of the side view of *AS*, its front view *a's'* may be projected on the picture plane (considering it as a vertical plane of projection) as shown in fig 19-6(i). The point *A'* must lie on this line *a's'*. It can be located by combining the top view and the front view as shown in fig. 19-6(ii) and fig. 19-6(iii), and as described below.

(i) Draw the line as in the top view [steps (i) to (iv)].

(ii) Draw the ground line GL at any convenient distance below PP and mark *a'*, the front view of *A*, 10 mm above GL and in projection with *a*.

(iii) Draw the horizon line HL, 20 mm above GL and on it, mark *s'*, the front view of *S*, in projection with *s*.

(iv) Draw a line joining *a'* with *s'*.

(v) Draw a vertical line through a_1 intersecting *a's'* at a point *A'*.

Then *A'* is the perspective view of the point *A*.

This method is comparatively simple and is generally adopted. In case of large objects, the perspective view often partly overlaps the front view. This, sometimes, causes confusion.

Perspective view of a straight line by the visual-ray method is drawn by first marking the perspectives of its ends which are points and then joining them.

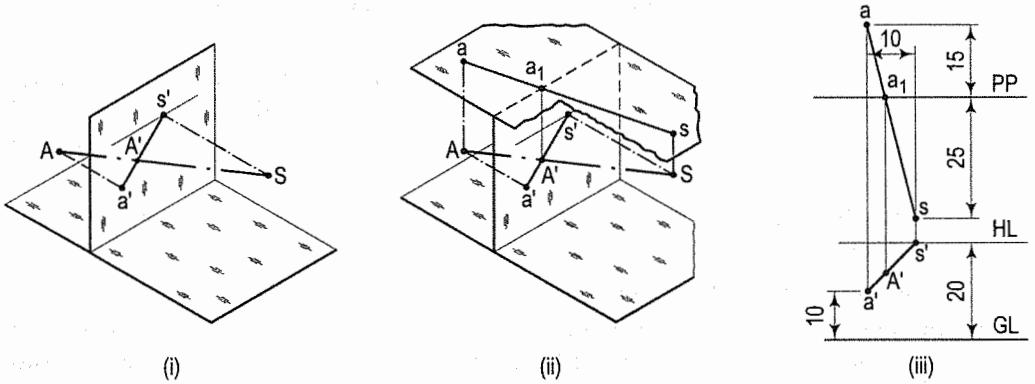

FIG. 19-6

Problem 19-2. (fig. 19-7): *A straight line AB, 40 mm long, is parallel to and 15 mm above the ground plane, and inclined at 30° to the picture plane. The end A is 20 mm behind the picture plane. The station point is 40 mm above the ground plane, 50 mm in front of the picture plane and lies in a central plane which passes through the mid-point of AB. Draw its perspective view.*

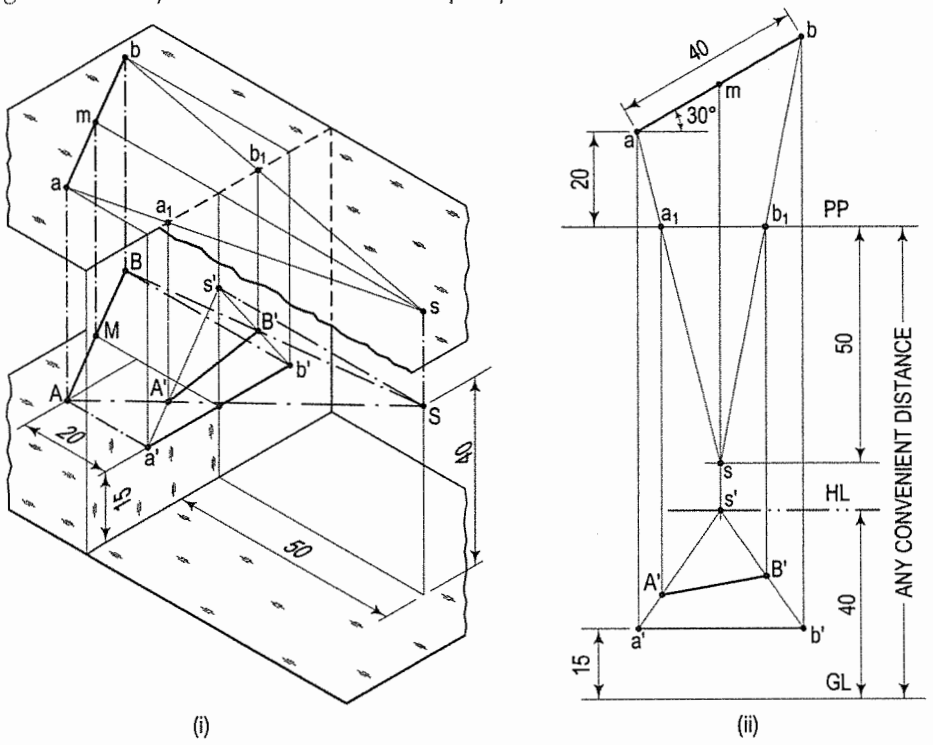

FIG. 19-7

(i) Draw a horizontal line PP. As *AB* is parallel to the ground plane, its top view will show its true length. Therefore, draw a line *ab*, 40 mm long, inclined at 30° to PP and the end *a*, 20 mm above PP.

(ii) Draw a vertical line through m, the mid-point of ab and on it mark s, the top view of the station point, 50 mm below PP.

(iii) Draw lines joining s with a and b, and intersecting PP at points a_1 and b_1 respectively.

(iv) Draw the ground line GL at any convenient distance below PP. Draw the horizon line HL parallel to and 40 mm above GL. Project s', the front view on HL.

(v) From ab, project the front view $a'b'$, parallel to and 15 mm above GL. Draw lines joining s' with a' and b'.

(vi) Through a_1 and b_1, draw verticals to intersect $a's'$ and $b's'$ at points A' and B' respectively.

(vii) Join A' with B'. Then $A'B'$ is the required perspective view of AB.

The perspective can also be obtained with the aid of the side view instead of the front view. Perspective view of any solid (by visual-ray method) can similarly be drawn by first obtaining the perspective of each corner and then joining them in correct sequence, taking care to show the hidden edges by dashed lines.

Problem 19-3. (fig. 19-8 and fig. 19-9): *A rectangular pyramid, base 30 mm × 20 mm and axis 35 mm long, is placed on the ground plane on its base, with the longer edge of the base parallel to and 30 mm behind the picture plane. The central plane is 30 mm to the left of the apex and the station point is 50 mm in front of the picture plane and 25 mm above the ground plane. Draw the perspective view of the pyramid.*

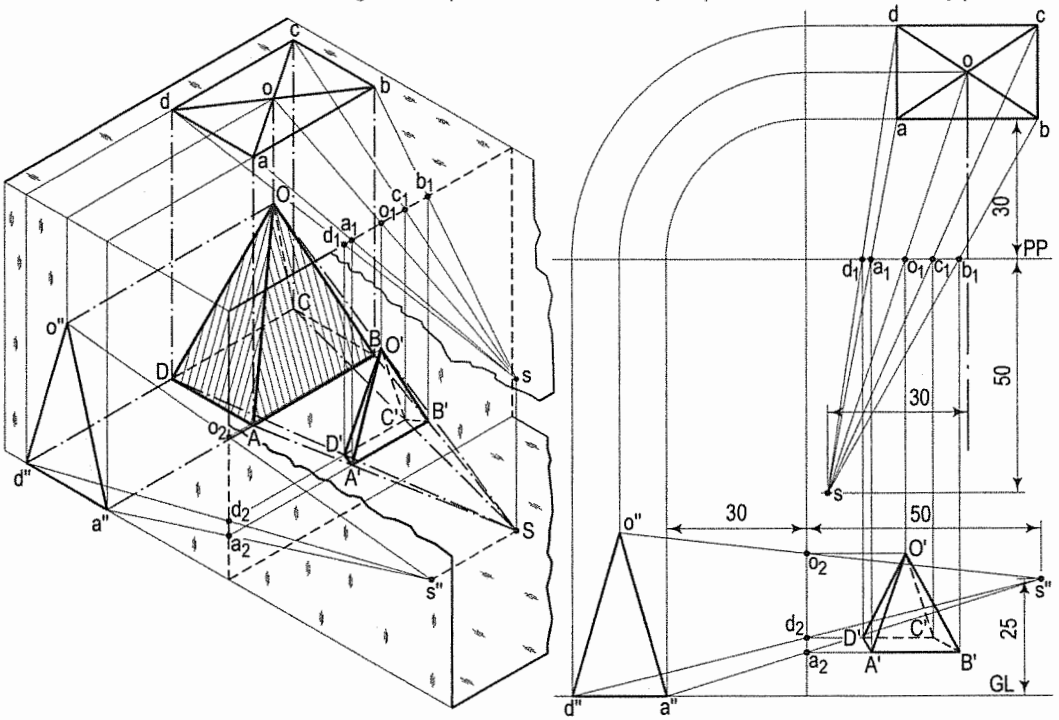

FIG. 19-8

Fig. 19-8 shows the perspective view of the pyramid obtained by means of its top view and the side view. The pictorial view shows clearly that points on the perspective lie in the picture plane on respective visual rays.

In fig. 19-9, the perspective view is drawn by means of its top view and the front view. It partly overlaps the front view.

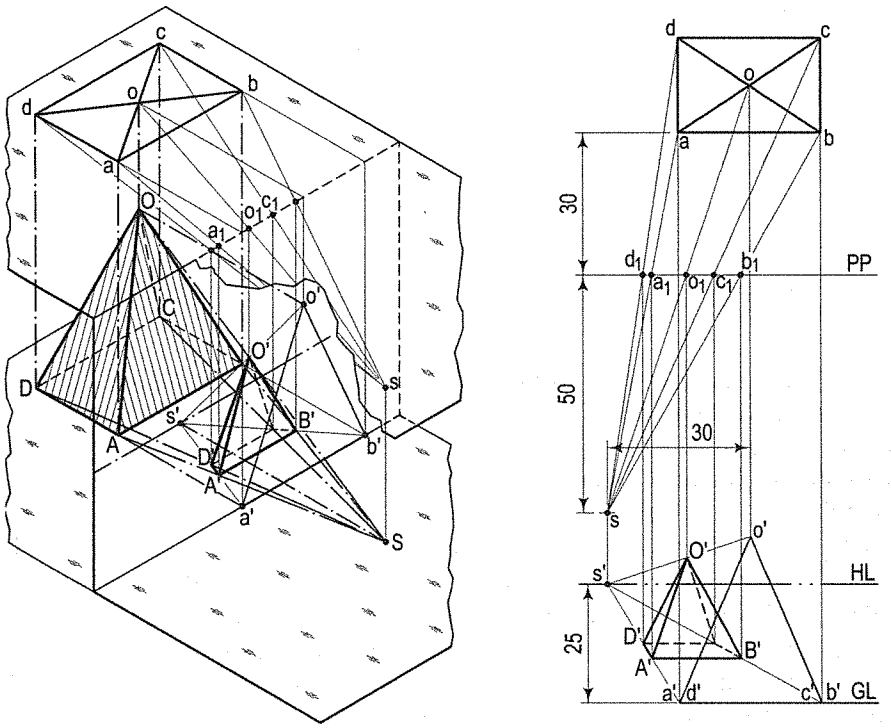

FIG. 19-9

19-7-2. VANISHING-POINT METHOD

Vanishing points: These are imaginary points infinite distance away from the station point. In practice, the point at which the visual ray from the eye to that infinitely distant vanishing point pierces the picture plane is referred to as the vanishing point.

If we stand between the rails of a long straight stretch of a railway track, it would appear as if the rails meet very far away at a point just at the level of the eye, i.e. on the horizon line. Even the telegraph and telephone wires running along the track at the sides of the track appear to meet at the same point. This point is a vanishing point.

In fig. 19-10, *ab* is the top view of a line *AB* lying on the ground plane and inclined at angle θ to the picture plane. When viewed from the station point *s*, its intercept on PP is a_1b_1.

If the line is moved along the ground to the right, keeping the same inclination θ with the picture plane, its intercept will go on decreasing. The intercept becomes zero, or the line vanishes in a point at *v* when *ab* and the visual ray fall in a straight line. The point *v* is the top view position of the vanishing point for the horizontal line *AB* and for all lines parallel to *AB*, irrespective of their positions. The front-view position *V* of the vanishing point is obtained by projecting *v*, vertically on the horizon line.

Therefore, the vanishing point for any horizontal line is found by drawing a line parallel to the top view of that line from the top view of the station point. The point at which this line intersects the top view of the picture plane is then projected

on the horizon line. This point on the horizon line is the front-view position of the vanishing point.

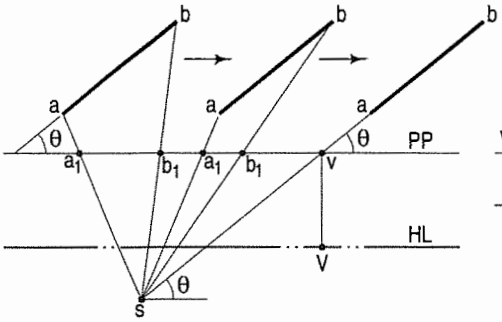

FIG. 19-10

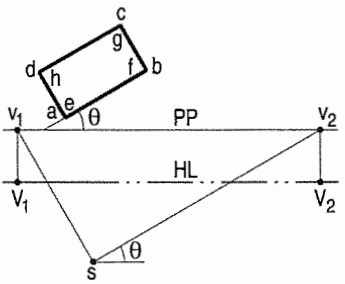

FIG. 19-11

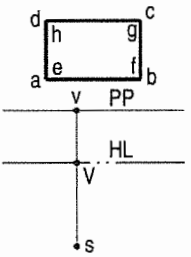

FIG. 19-12

In fig. 19-11, *abcd* is the top view of a rectangular block placed on the ground plane so that a vertical face is inclined at angle θ to the picture plane.

The vanishing point for the line *ab* and for lines *cd*, *ef* and *gh* (which are parallel to *ab*) is obtained by drawing a line through *s*, parallel to *ab* and intersecting PP at a point v_2. Through v_2, a vertical line is drawn to meet HL at a point V_2.

Then V_2 is the front-view position of the vanishing point. In perspective view of the block, edges *AB*, *CD*, *EF* and *GH* will converge to this point V_2.

Similarly, V_1 is the vanishing point to which edges *AD*, *BC*, *EH* and *FG* will converge.

Thus, perspectives of all horizontal lines, if produced, pass through their respective vanishing points on the horizon line. Perspectives of all horizontal parallel lines converge to a vanishing point on the horizon line.

Vanishing point for lines perpendicular to the picture plane is obtained by drawing a line through the top view of the station point, and perpendicular to the picture plane. It lies on the horizon line and coincides with the centre of vision. It is the front-view position of the station point.

In fig. 19-12, *V* is the front-view position of the station point and the vanishing point, at which perspectives of lines *AD*, *BC*, *EH* and *FG* will converge. Thus, perspectives of all lines perpendicular to the picture plane converge to the centre of vision on the horizon line.

Lines which are parallel to the picture plane will have no vanishing points. They vanish at infinity. Therefore, perspectives of vertical lines are vertical; perspectives of horizontal lines which are parallel to the picture plane, remain horizontal; and perspectives of lines inclined to the ground plane and parallel to the picture plane will be inclined in the same direction (see fig. 19-13).

19-8. TYPES OF PERSPECTIVE

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 46 for the parallel or one point perspective.

(1) Parallel perspective or one point perspective: When an object has its one or more faces parallel to the picture plane, its perspective is called *parallel perspective* also called *one point perspective* as the edges converge to a single vanishing point of the parallel faces.

Fig. 19-13 shows the perspective view of a hut having its front face in the picture plane. The front face is seen in its true size and shape, while the back parallel face is of the same shape but reduced in size. As the lines *AF*, *BG*, *CH*, *DJ* and *EK* are perpendicular to the picture plane, their perspectives *A' F'*, *B' G'* etc. converge to the centre of vision *c'* on HL. Note that vertical lines *AE*, *CD* etc. remain vertical in perspective. Similarly, horizontal lines *ED* and *KJ*, and sloping lines *AB*, *BC*, *FG* and *GH* (which are all parallel to the picture plane) remain respectively horizontal and sloping in perspective.

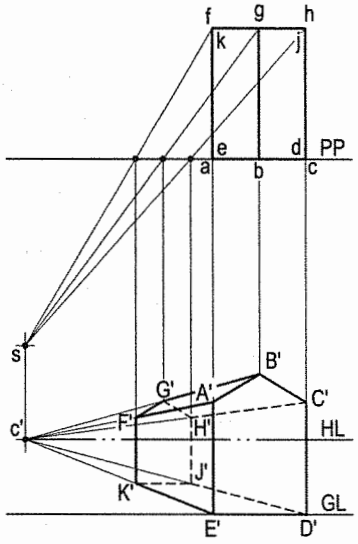

FIG. 19-13

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 47 for the angular or two point perpective.

(2) Angular perspective or two point perpective: When an object has its two faces inclined to the picture plane, its perspective is called *angular perspective* also called *two point perspective* as the edges of the object converge to two vanishing points.

Problem 19-4. (fig. 19-14): *A rectangular block, 30 mm × 20 mm × 15 mm, is lying on the ground plane on one of its largest faces. A vertical edge is in the picture plane and the longer face containing that edge makes an angle of 30° with the picture plane.*

The station point is 50 mm in front of the picture plane, 30 mm above the ground plane and lies in a central plane which passes through the centre of the block. Draw the perspective view of the block.

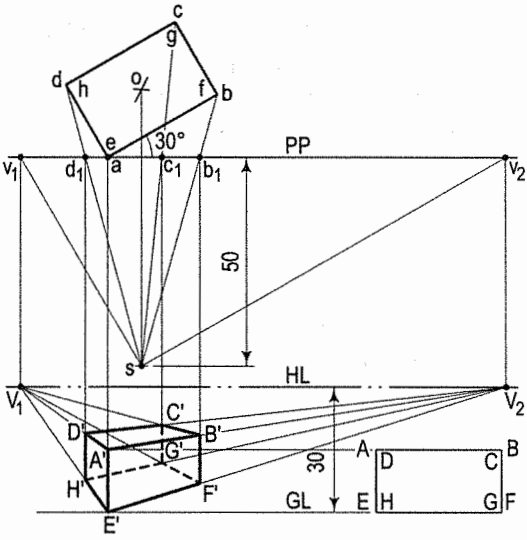

FIG. 19-14

(i) Draw the top view *abcd* with *a* in PP and the longer edge *ab* inclined at 30° to PP. Mark its centre *o*. Mark *s*, the top view of the station point, on a vertical line through *o* and 50 mm below PP.

(ii) Draw lines joining s with corners *b*, *c* and *d*, and intersecting PP at points b_1, c_1 and d_1.

(iii) Draw the ground line GL at any distance below PP and the horizon line HL, 30 mm above GL.

(iv) Through s, draw lines parallel to ad and ab cutting PP at points v_1 and v_2 respectively. Project v_1 to V_1 and v_2 to V_2 on HL. V_1 and V_2 are the vanishing points. Perspectives of edges AD, EH, BC and FG will converge to V_1 and those of edges AB, CD, EF and GH will converge to V_2. Perspectives of vertical edges AE, BF, CG and DH will remain vertical.

(v) As AE is in the picture plane, its perspective will be equal to the true length and the end E will lie on GL. Therefore, through a, draw a vertical line to a point E' on GL and on it, mark A' so that $A'E' = AE$.

(This length may be measured directly or may be projected from the front view as shown.)

(vi) Draw lines joining A' and E' with V_1 and V_2. Through b_1, draw a vertical line to intersect $A'V_2$ at B' and $E'V_2$ at F'. Similarly, draw a vertical through d_1 and obtain points D' and H'.

(vii) Draw lines joining B' and F' with V_1 and D' and H' with V_2, intersecting at points C' and G' respectively. They must lie on the vertical line through c_1. Note that lines meeting at G' are all hidden and therefore, shown dotted.

In fig. 19-15, the ground line GL has been so drawn that HL coincides with PP. Hence, V_1 and V_2 coincide with v_1 and v_2 respectively on PP. The perspective view is obtained in the same manner as described above.

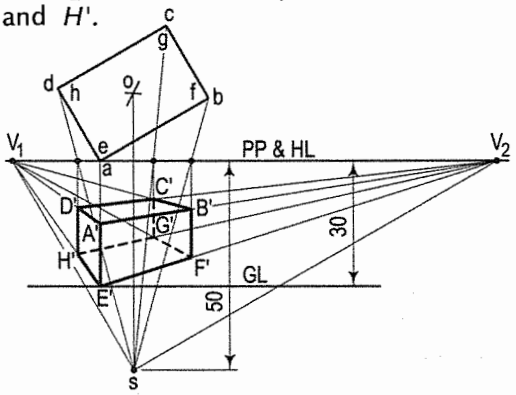

FIG. 19-15

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 48 for the oblique or three point perspective.

(3) Oblique perspective or three point perspective: When an object has its three faces inclined to the picture plane, its perspective is called *oblique perspective* also called *three point perspective* as edges of the object converge to three vanishing points, as shown in fig. 19-16.

Problem 19-5: *Draw the perspective view of a cube of 80 mm side having its one corner of the edge on the ground plane and the other corner of the edge resting on the picture plane such that the edge is inclined at 30° to the picture plane. The other two edges of the corner are equally inclined with the picture plane. The station point is 100 mm in front of the picture plane, 150 mm above the ground plane and lies in a central plane which passes through the centre of the cube.*

Refer fig. 19-16.

(i) Draw projection plane lines PP and PP_e, perpendicular to each other.

(ii) Draw side view on PP_e from auxiliary top view.

(iii) Draw top view on PP by taking projection from side view.

(iv) Mark the station point s for F.V. and s_1 for S.V. at a distance of 100 from PP and PP_e respectively and 150 from G.L.

(v) Obtain the three vanishing points V_1, V_2 and V_3 and complete the perspective view as shown.