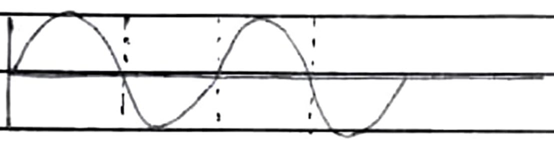
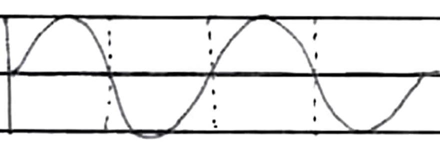
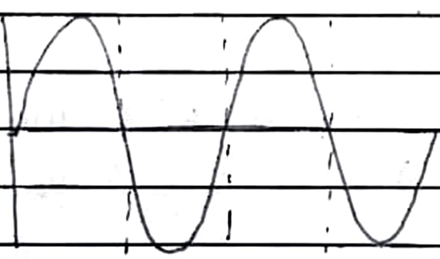
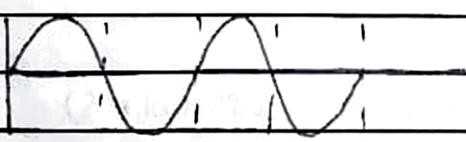
Interference - It is a phenomena in which two or more waves superimpose to form a resultant wave of higher amplitude or lower amplitude.
Basic condition for the interference is both the waves are generated from the coherent source.

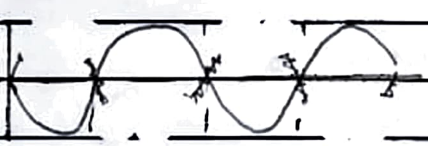
Coherent source. Two sources are said to be coherent if they emit the light waves of same frequency nearly in same amplitude & in same phase.
That means light emitted from this source are of same colour.

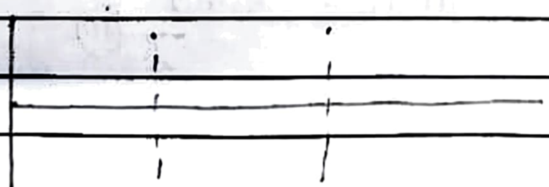
I  Constructive interference

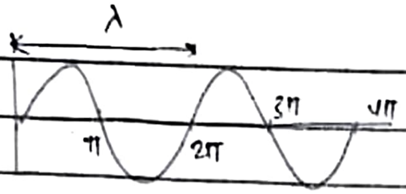
II 

 Resultant wave.

I  Destructive interference

II 

 R.W

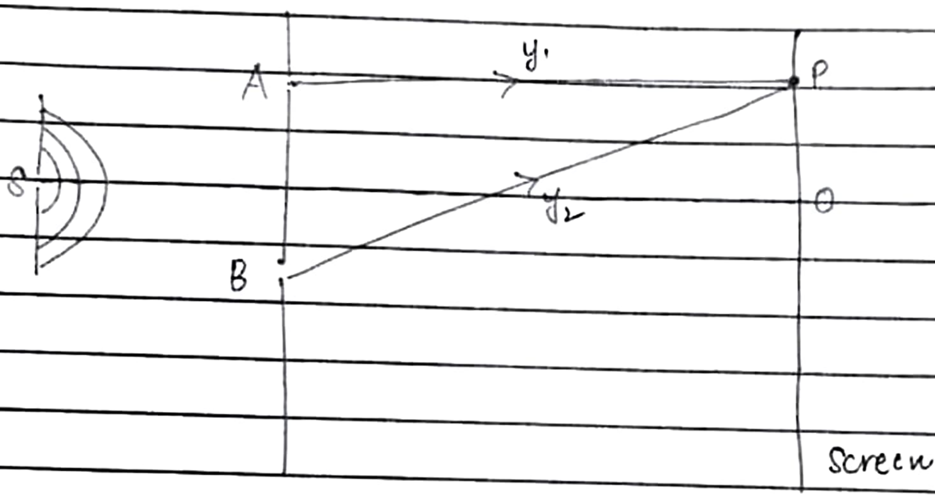


$\lambda \rightarrow 2\pi$

$$\boxed{\delta = \frac{2\pi}{\lambda} x}$$

$$\boxed{\text{Phase diff.} = \frac{2\pi}{\lambda} \cdot \text{Path diff.}}$$

Analytical treatment of Interference



Lets consider the amplitude of the wave is 'a' & the phase diff. b/w two waves is 'δ'. If y_1 & y_2 are displacement of two waves.

$y_1 = a \sin \omega t$

$y_2 = a \sin(\omega t + \delta)$

$y = y_1 + y_2 \Rightarrow y = a \sin \omega t + a \sin(\omega t + \delta)$

$y = a \sin \omega t + a \sin \omega t \cos\delta + a \cos \omega t \sin\delta$

$y = a \sin \omega t (1 + \cos\delta) + a \cos \omega t \sin\delta$ ——— ①

Let $a(1 + \cos\delta) = R\cos\theta$ —(i), $a \sin\delta = R\sin\theta$ ——(ii)

from ①, $y = R \sin \omega t \cos\theta + R \cos \omega t \sin\theta$

$y = R \sin(\omega t + \theta)$

squaring & adding (i) & (ii)

$$a^2(1+\cos\delta)^2 + a^2\sin^2\delta = R^2(\cos^2\theta + \sin^2\theta)$$

$$a^2(1+\cos^2\delta + 2\cos\delta + \sin^2\delta) = R^2$$

$$a^2(2+2\cos\delta) = R^2$$

$$2a^2(1+\cos\delta) = R^2$$

$$2a^2\left(2\cos^2\frac{\delta}{2}\right) = R^2$$

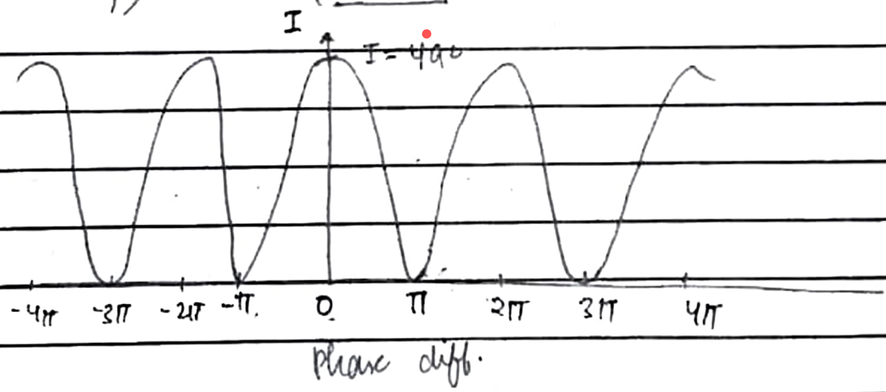
$$4a^2\cos^2\left(\frac{\delta}{2}\right) = R^2 = I$$

$$I = R^2$$

$$\boxed{I = 4a^2\cos^2\left(\frac{\delta}{2}\right)}$$

case -(i) Phase diff $0, 2\pi, 4\pi \ldots 2n\pi$
Path diff $0, \lambda, 2\lambda \ldots n\lambda$
(maxm intensity) $\boxed{I = 4a^2}$

Case (ii) Phase diff. $\pi, 3\pi, 5\pi \ldots (2n+1)\pi$
Path diff. $\lambda/2, 3\lambda/2, 5\lambda/2 \ldots (2n+1)\lambda/2$
(minm intensity) $\boxed{I = 0}$



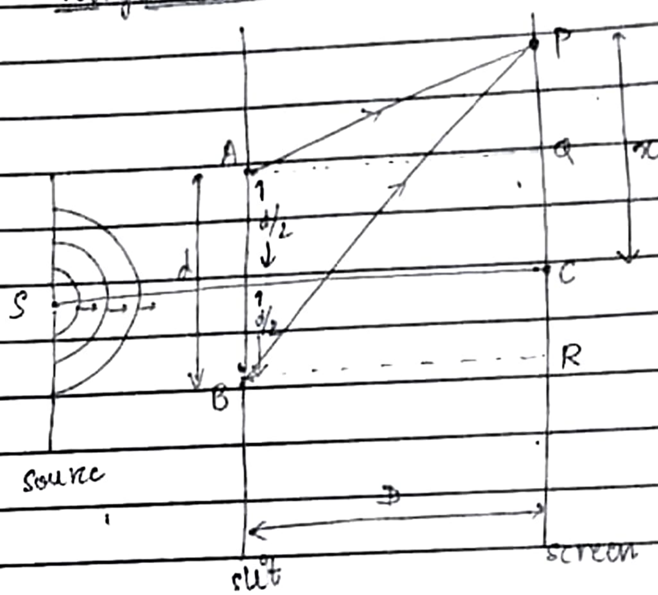
Types of Interference -

There are two types of interference -

i) Division of wavefront
ii) Division of amplitude

→ **Division of wavefront**

- Young's double slit Experiment.



$PQ = x - d/2$, $PR = x + d/2$

In ΔBPR,

$$(BP)^2 = D^2 + (x + d/2)^2$$

$$BP = \left(D^2 + (x+d/2)^2\right)^{1/2} = D\left(1 + \frac{(x+d/2)^2}{D^2}\right)^{1/2}$$

$$\left[(1+x)^n \quad x <<< 1\right] = \underline{1 + nx}$$

$$BP = D\left[1 + \frac{(x+d/2)^2}{2D^2}\right] \quad \text{------} \quad (1)$$

In ΔAPQ, $(AP)^2 = D^2 + (x - d/2)^2$

$$AP = \left[D^2 + (x - d/2)^2\right]^{1/2}$$

$$AP = D\left[1 + \frac{(x-d/2)^2}{D^2}\right]^{1/2}$$

$$AP = D\left[1 + \frac{(x - d/2)^2}{2D^2}\right] \quad \ldots (2)$$

Now taking path diff. (1) - (2)

$$BP - AP = D\left[1 + \frac{(x + d/2)^2}{2D^2} - 1 - \frac{(x - d/2)^2}{2D^2}\right]$$

$$= D\left[\frac{x^2 + d^2/4 + xd - x^2 - d^2/4 + xd}{2D^2}\right]$$

$$= D\left[\frac{xd}{D^2}\right] = \frac{xd}{D}$$

$$\boxed{\text{Path diff} = \frac{xd}{D}}$$

Case I → For bright fringe.

If the path diff. is the integral multiple of $n\lambda$ where $n = 0, 1, 2, 3 \ldots$, then,

$$\frac{xd}{D} = n\lambda \rightarrow \boxed{x = \frac{n\lambda D}{d}}$$

[B.F → Bright Fringe]

F.B.F $n = 1$, $x = \frac{\lambda D}{d}$

S.B.F $n = 2$, $x = \frac{2\lambda D}{d}$

T.B.F $n = 3$, $x = \frac{3\lambda D}{d} \ldots$

→ The diff. b/w two consecutive bright fringe, [constant i.e. $\frac{\lambda D}{d}$]

$$x_2 - x_1 = \frac{\lambda D}{d}$$

$$x_3 - x_2 = \frac{\lambda D}{d}$$

$\vdots$

$$\boxed{\text{Fringe width} = \beta = \frac{\lambda D}{d}}$$

- Case II → For dark fringe :

Path diff. = $(2n+1)\lambda/2$

$$\frac{xd}{D} = \frac{(2n+1)\lambda}{2}$$

$$\boxed{x = \frac{(2n+1)\lambda D}{2d}} \qquad [n = 0, 1, 2, 3 \ldots]$$

$n=0$, $x_0 = \frac{\lambda D}{2d}$ | $x_2 = \frac{5\lambda D}{2d}$ $(n=2)$

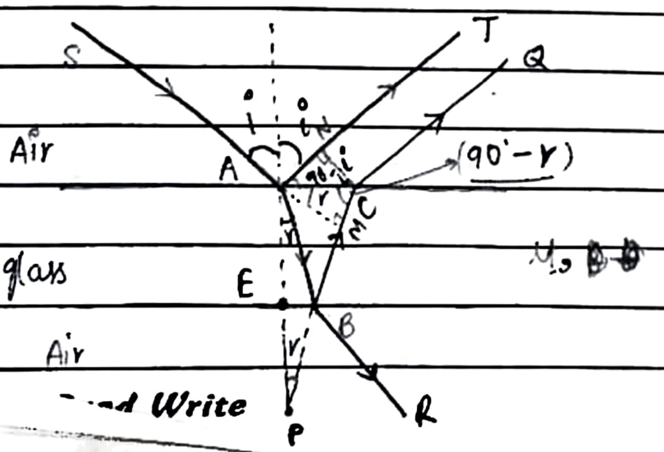
$n=1$, $x_1 = \frac{3\lambda D}{2d}$ | ⋮

$$(x_1 - x_2) = \frac{\lambda D}{d}$$ (Path diff. b/w two ~~consicu~~ ~~cose~~ consecutive dark fringes is also constant. i.e. $\frac{\lambda D}{d}$)

- The distance b/w two consecutive bright or dark fringe is $\frac{\lambda D}{d}$. This is called fringe width.

→ Division of amplitude

- Interference due to reflected light (Thin film)



~~μ, t (thickness)~~

From the figure $AB = BC$

Optical path diff $x = \mu(AB + BC) - AN$ —— (1)

$$\mu = \frac{\sin i}{\sin r} = \frac{AN/AC}{CM/AC} = \frac{AN}{CM}$$

$$AN = \mu CM$$

From (1), $x = \mu(AB + BC - CM)$

$x = \mu(2BC - CM)$

$x = \mu(PC - CM)$

$x = \mu PM$ ——→ (2)

In ΔAPM $\cos r = PM/AP$

$PM = AP \cos r = 2t\cos r$

From (2), $\boxed{x = 2\mu t\cos r}$

This path diff is of apparent path diff. On the basis of electromagnetic theory if a ray reflected from any optically denser medium there is a phase change π or path diff $\lambda/2$. So the correct optical path diff.

$$\boxed{x = 2\mu t\cos r - \lambda/2}$$

- **Case I →** For bright fringe

$$2\mu t\cos r - \lambda/2 = n\lambda \quad [n = 0, 1, 2 \ldots]$$

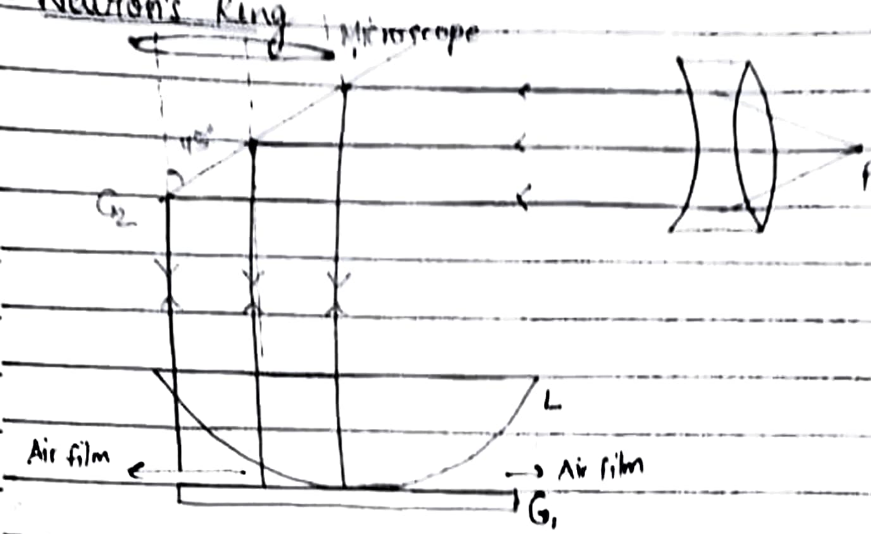
$$\boxed{2\mu t\cos r = n\lambda + \lambda/2 = (2n+1)\lambda/2}$$

- **Case II →** For dark fringe

$$2\mu t\cos r - \lambda/2 = (2n+1)\lambda/2$$

$$\boxed{2\mu t\cos r = (n+1)\lambda}$$

- Newton's Ring



when we place a plano-convex lens of long focal length on a plane glass plate, a thin film of air is enclosed In between upper part of the glass & the lower part of the lens The thickness of the ~~air~~ air film is very small at the pt of contact & ~~gradual~~ gradually increases when we move from centre outwards.
The fringes we used are of monochromatic light, are circular, cocentric & uniform in thikness.

From the figure,

$BC = R$, $OQ = r = AB$

$BQ = AO = t$

$AC = (R-t)$

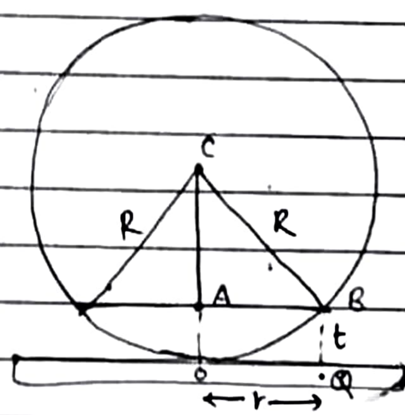
ΔABC, $BC^2 = AC^2 + AB^2$

$R^2 = (R-t)^2 + r^2$

$R^2 = R^2 + \cancel{t^2} - 2Rt + r^2$

$2Rt = r^2$

$$\boxed{t = \frac{r^2}{2R}}$$



$t <<< r$

Interference is formed due to reflected light, so the cond^n for the bright fringe—

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$$2\mu t\cos\theta = (2n+1)\lambda/2$$

This condⁿ is valid only when ~~we~~ centre is bright.
When centre is dark,

$$n \longrightarrow n-1$$

$$2\mu t\cos\theta = (2n-1)\frac{\lambda}{2}$$

$\mu = 1$ for air

If θ is very small, $\cos\theta \approx 1$

$$2t = (2n-1)\lambda/2$$

$$\frac{2r^2}{2R} = (2n-1)\lambda/2$$

$$\boxed{r = \sqrt{\frac{(2n-1)\lambda R}{2}}}$$ condⁿ for bright fringe.

$r \propto \sqrt{\lambda}$, $r \propto \sqrt{R}$

For dark ring, $2\mu t\cos\theta = (n+1)\lambda$

$$n \longrightarrow n-1$$

$$2\mu t\cos\theta = n\lambda$$

$\mu = 1$, $\theta \approx$ small, $\cos\theta \approx 1$

$$2t = n\lambda$$

$$\frac{2r^2}{2R} = n\lambda$$

condⁿ for dark fringe,

$$\boxed{r = \sqrt{n\lambda R}}$$

$r \propto \sqrt{n}$
$r \propto \sqrt{\lambda}$
$r \propto \sqrt{R}$

$n = 0$, $\boxed{r = 0} \longrightarrow$ dark

Determination of wavelength using Newton's Ring –

$r_n = \sqrt{n\lambda R}$

$r_n^2 = n\lambda R$ $\qquad\qquad r_n = \frac{D_n}{2}$

$\frac{D_n^2}{4} = n\lambda R$

$D_n^2 = 4n\lambda R$ —— (1)

Let the diameter of $(n+m)$th ~~bright fringe~~ dark ring is D_{n+m}.

$D_{n+m}^2 = 4(n+m)\lambda R$ — (2)

eqn (2) – eqn (1)

$D_{n+m}^2 - D_n^2 = 4m\lambda R$

$$\boxed{\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}}$$

Q. In a Newton's ring exp, the diameter of the 5th ring was 0.3 cm & the diameter of 25th ring was 0.8 cm. If the radius of curvature of the planoconvex lens is 1m, calculate the wavelength of light used

$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$ $\qquad m = 20$, $n = 5$, $R = 100$ cm

$\lambda = \frac{\cancel{D_{20}}\ D_{25}^2 - D_5^2}{4 \times 20 \times 100}$

$= \frac{0.8 \times 0.8 - 0.3 \times 0.3}{4 \times 20 \times 100} = \frac{0.55}{8 \times 10^3} =$

$= 68800$ Å

$0.64 - 0.9$

2) In Newton's ring exp., the diameters of 4th & 12th dark ring are 0.4 & 0.7 cm. resp. Calculate the diameter of 20th dark ring.

$n = 8$
$m = 4$

$$\lambda = \frac{D_{12}^2 - D_4^2}{4 \times 8 \times R}$$

$$= \frac{0.49 - 0.16}{16 \times R} \quad 0.01 \quad = \frac{0.48}{R} \text{ cm} \quad \frac{0.12}{R} \text{ cm}$$

0.48×10^{-2} m

$48 \times 10^{-4} \times 10^{10}$

48×10^{6}

$$r_{20} = \sqrt{\frac{20 \times 0.12}{R} \times R} = \sqrt{20 \times 48 \times 10^{6}}$$

$= \cancel{4 \times 10^{8}}$

$= \sqrt{2.4} \quad \cancel{0.12}$

$\frac{48}{16}$

12

$4 \times 2 \times 4 \times 2$

$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda R$

$$\lambda = \frac{D_{20}^2 - D_{12}^2}{4 \times 8 R} \Rightarrow D_{20}^2 - D_{12}^2 = 4 \times 8 \times \lambda R$$

$D_{20}^2 = 2D_{12}^2 - D_4^2 = 2 \times 0.49 - 0.16$

$D_{20}^2 = 0.86$ cm

Q- In a Newton's ring exp. the diameter of the 5th ring was 0·3 cm & the diameter of 25th ring was 0·8 cm. If the radius of curvature of the planoconvex lens is 1m, calculate the wavelength of light used

$$\lambda = \frac{D^2_{n+m} - D_n^2}{4mR}$$

$m = 20$

$n = 5$

$R = 100\,cm$

$$\lambda = \frac{D^2_{25} - D_5^2}{4 \times 20 \times 100}$$

$$= \frac{0.8 \times 0.8 - 0.3 \times 0.3}{4 \times 20 \times 100} = \frac{0.55}{8 \times 10^3}$$

$= 68800\ \text{Å}$

0.64 - 0.9

Good Write

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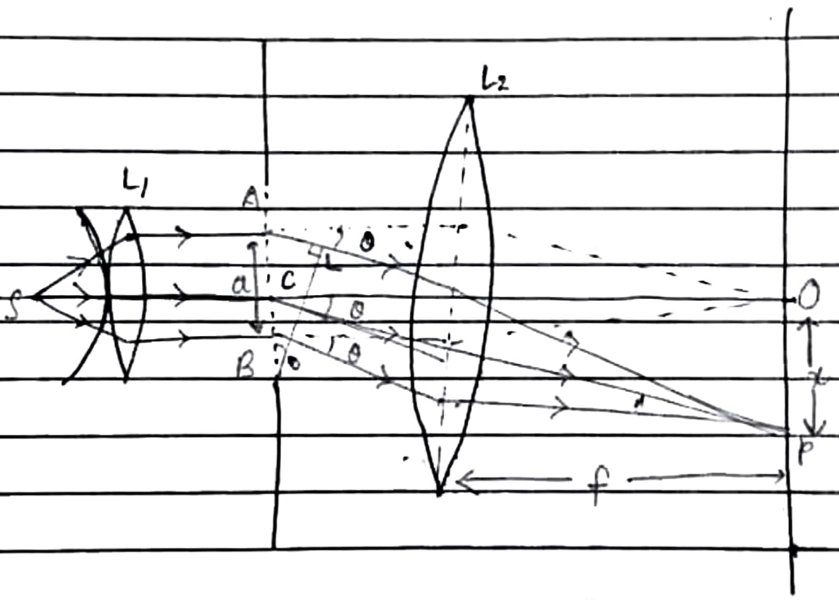
DIFFRACTION

Two types-
1) Fresnel diff.
2) Fraunhaffer diff.

In fresnel diff., either the source or screen or both at finite distance from the slit. There is no lenses are used for making the light parallel or ~~no~~ converge the light on the screen. The incident wavefront is either spherical or cylindrical.
In Fraunhaffer diff., either the source or screen at infinite distance from the slit. We use lenses for making the light parallel or converging the light on the screen. The incident ray is a plane wavefront.

FRAUNHAFFER DIFFRATION FOR SINGLE SLIT



At pt. O, the form central maxima
Secondary waves passes normally from the slit focused at 'O'
Since there is no path diff. b/w these 2 rays so, at 'O' they form central bright maxima.

If the secondary waves passes at angle 'θ' from the slit is focused at 'P', so the path diff. b/w these two rays is AL.

In Δ ABL, $\sin\theta = \frac{AL}{AB}$ $\quad AL = a\sin\theta$ $\quad [\because AB = a]$

If we consider, path diff.

$\boxed{a\sin\theta = \lambda}$ secondary minima

then the slit AB divided in two parts AC & BC.
AC generate $\lambda/2$ path diff. & BC generate $\lambda/2$ path diff.
If the path diff. is $\lambda/2$, the form destructive interference & minima will be formed at 'P'.
If $a\sin\theta = 2\lambda$, the slit AB is divided by 4 equal parts of $\lambda/2$ & they form minima.
Similarly, If $\boxed{a\sin\theta = n\lambda}$ minima condn.

For first minima. $a\sin\theta = \lambda$

If $\theta \approx$ small, $\theta = \lambda/a$

If the lens L_2 is very close to the slit or the distance of screen from the slit is very large.

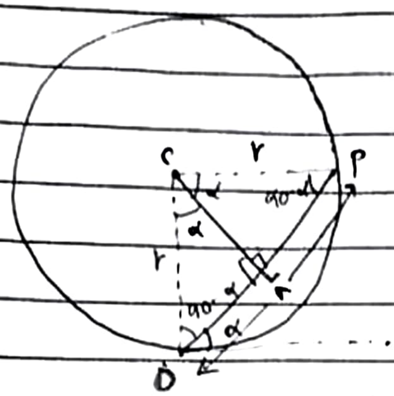
$\theta = \frac{x}{f} \Rightarrow x = \frac{\lambda f}{a}$

width of the central maxima,

$$\boxed{2x = \frac{2\lambda f}{a}}$$

for maxima,

$$\boxed{a\sin\theta = (2n+1)\frac{\lambda}{2}}$$ condn for maxima



Phase diff. $= 2\alpha$

Path diff. $= a\sin\theta$

$2\alpha = \frac{2\pi}{\lambda} \times a\sin\theta \Rightarrow \boxed{\alpha = \frac{\pi}{\lambda} a\sin\theta}$

change from 0 to $\frac{\pi}{\lambda} a\sin\theta$

In ΔOLC, $\sin\alpha = \frac{OL}{OC} = \frac{OL}{r}$

$OL = r\sin\alpha$, $OP = 2(OL) = 2r\sin\alpha$ — (2)

$OP = A$ (resultant amplitude)

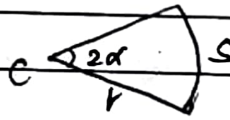
$A = 2r\sin\alpha$ — (1)

Arc length is directly proportional to the slit width.

$S \propto a$

$S = ka = A_0$ (say)

solid angle $2\alpha = \frac{S}{r}$



$2r = \frac{S}{\alpha} = \frac{A_0}{\alpha}$

from (1) $\boxed{A = \frac{A_0 \sin\alpha}{\alpha}}$

$I = A^2$, $I = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \Rightarrow I = \left(\frac{A_0 \sin\alpha}{\alpha}\right)^2$

If $I_0 = A_0^2 \Rightarrow \boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2}}$

α is related to the diffraction angle θ, so I also depends on the value of θ.

i) Principal Maxima

$$\theta = 0 , \quad \alpha = 0$$

$$I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \frac{\sin\alpha}{\alpha} = \frac{0}{0}$$

applying L-hospital,

$$\frac{\frac{d}{d\alpha}\sin\alpha}{\frac{d}{d\alpha}\alpha} = \frac{\cos\alpha}{1} = 1$$

$\boxed{I = I_0}$ cond^n for central maxima

minima

For minima, $\alpha = \frac{\pi}{\lambda} a\sin\theta \Rightarrow \alpha = \frac{\pi}{\lambda} \times n\lambda$

$$a\sin\theta = n\lambda, \qquad \alpha = n\pi,$$

$$\alpha = 0, \quad \boxed{I = 0}$$

ii) Secondary maxima

$$a\sin\theta = (2n+1)\lambda/2$$

$$\alpha = \frac{\pi}{\cancel{\lambda}} \times (2n+1)\frac{\lambda}{2} \Rightarrow \underbrace{\alpha = (2n+1)\frac{\pi}{2}} \qquad n = 1, 2, 3 \ldots$$

$n=1$, $\alpha_1 = \frac{3\pi}{2}$

$n=2$, $\alpha_2 = \frac{5\pi}{2}$

$\ldots \frac{7\pi}{2}, \frac{9\pi}{2} \ldots$

for $n=1$,
$$I = I_0 \left(\frac{\sin\alpha_1}{\alpha_1}\right)^2$$

$$= I_0 \left(\frac{\sin\frac{3\pi}{2}}{3\pi/2}\right)^2$$

$$= I_0 \frac{4}{9\pi^2} = \frac{4I_0}{9\pi^2}$$

for $n=2$,
$$I = I_0 \left(\frac{\sin\frac{5\pi}{2}}{5\pi/2}\right)^2 \Rightarrow I = \frac{4I_0}{25\pi^2}$$

$$n=3, \quad I = \frac{4I_0}{49\pi^2}$$

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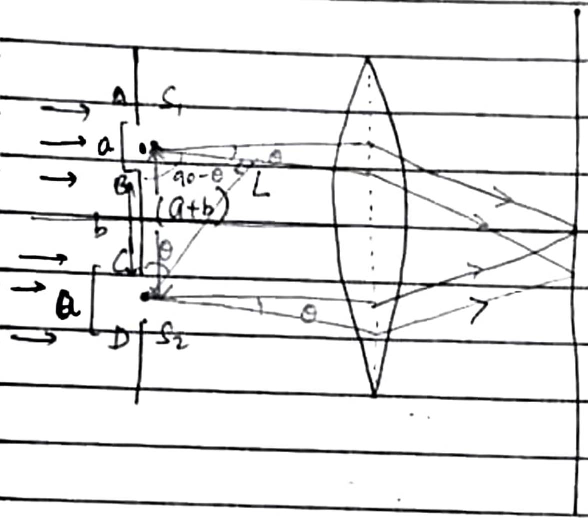
Q. Discuss the Fraunhoffer diffraction at single slit & show that the relative intensities of the maxima are nearly in the ratio of $1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \dots$

→ Fraunhoffer diffraction at double slit



At O, they form central maxima. The distance b/w two slits S_1 & S_2 from the centre $(a+b)$

In ΔS_1S_2L,

$$\sin\theta = \frac{AL}{S_1S_2} = \frac{AL}{(a+b)}$$

$AL = (a+b)\sin\theta$ (optical path diff.)

For minima,

$$(a+b)\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{(a+b)}$$

For maxima,

$$(a+b)\sin\theta = (2n+1)\frac{\lambda}{2} \quad (n = 1, 2, 3 \dots$$

$$\sin\theta = \frac{(2n+1)\lambda}{2(a+b)}$$

For $n = 1$, $\sin\theta = \frac{3\lambda}{2(a+b)}$

$n = 2$, $\sin\theta_2 = \frac{5\lambda}{2(a+b)}$

$$\left.\sin\theta_2 - \sin\theta_1 = \frac{\lambda}{(a+b)}\right\} \text{fringe width}$$

The angular seperation of any pair of consecutive maxima or minima will be found to be same.

Intensity distribution: for double slit

Path diff. $= (a+b)\sin\theta$

phase diff. $2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$

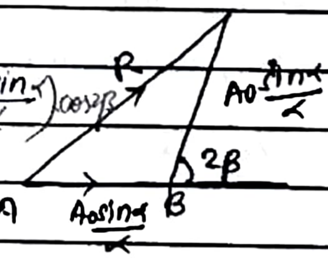
We know the resultant amplitude of the single slit.

$$A = A_0 \frac{\sin\alpha}{\alpha}$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

that means, all the secondary waves coming from the slit AB & CD have the same amplitude & same initial phase 'α', but they ~~arri~~ arrive at pt 'P' in different phase (2β) because of their optical path difference. So, the resultant amplitude can be calculated with the help of vector sum property.

$$R^2 = \left(\frac{A_0\sin\alpha}{\alpha}\right)^2 + \left(\frac{A_0\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A_0\sin\alpha}{\alpha}\right)\left(\frac{A_0\sin\alpha}{\alpha}\right)\cos 2\beta$$



$$= 2\left(\frac{A_0\sin\alpha}{\alpha}\right)^2 \left[1 + \cos 2\beta\right]$$

$$= 2\left(\frac{A_0\sin\alpha}{\alpha}\right)^2 \left[1 + 2\cos^2\beta - 1\right]$$

$$= \frac{4A_0^2\sin^2\alpha}{\alpha^2}\cos^2\beta \rightarrow \boxed{I = \frac{4I_0\sin^2\alpha}{\alpha^2}\cos^2\beta}$$

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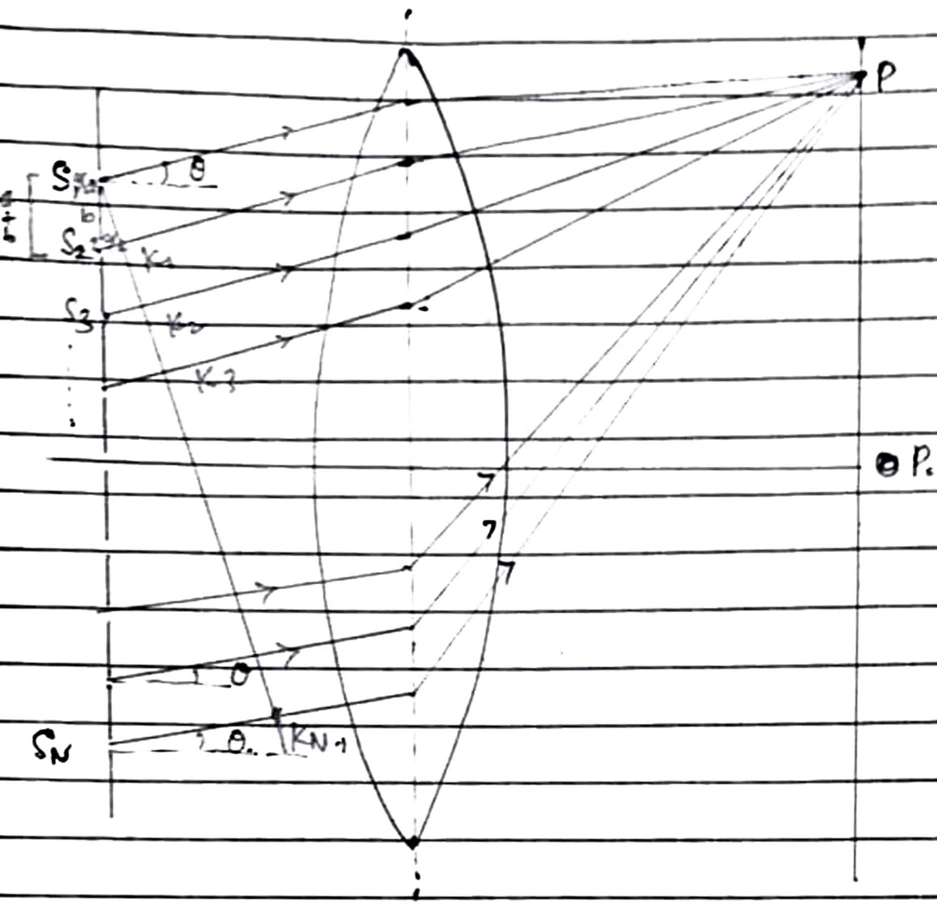
Diffraction in N slits-



If there are 'N' parallel slits each of width 'a' & seperated by a distance 'b'. At P_0, they form central bright maxima, so the resultant amplitude due to single slit.

$$A = \frac{A_0 \sin \alpha}{\alpha}$$

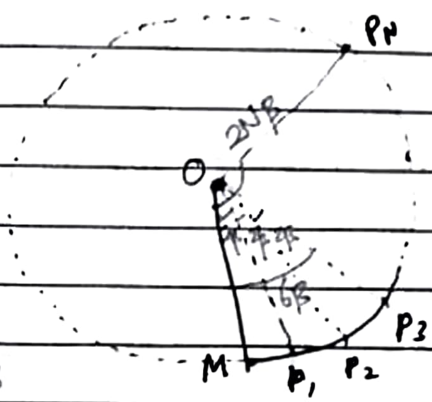
where $\alpha = \frac{\pi}{\lambda} a \sin\theta$

If there are $S_1, S_2, S_3 \dots S_N$ no. of slits, so we have 'N' diffracted parallel rays one each from the middle point of S_1, S_2, S_3 to S_N, so path diff. b/w S_1 & S_2

$$S_2K_1 = (a+b)\sin\theta$$

$$2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$$

For N slits, we use the graphical method. If we consider the initial draw N equal lengths ~~NP~~ $MP_1, P_1P_2, P_2P_3 \ldots\ldots P_{N-1}P_N$ represent equal amplitude 'A' & if 'O' is the centre of the polygon



$$\sin 2\beta = \frac{MP_1}{OM}$$

$MP_1 = OM\sin 2\beta = OM\,2\sin\beta\cos\beta$

If $\beta \approx$ small

$\cos\beta \approx 1$

$MP_1 = 2OM\sin\beta$ — (1)

$MP_2 = OM\sin 4\beta = OM\,2\sin 2\beta\cos 2\beta = OM\,4\sin\beta\cos\beta\cos 2\beta$

$\cdot\ \beta \approx$ small

$MP_2 = 4OM\sin\beta = 2OM\sin 2\beta$

$MP_3 = 2OM\sin 3\beta$

$MP_N = 2OM\sin N\beta$

from (1), $OM = \dfrac{MP_1}{2\sin\beta}$

from (2), $MP_N = \dfrac{2MP_1}{2\sin\beta}\sin N\beta = \dfrac{MP_1\sin N\beta}{\sin\beta}$

$$MP_N = \frac{A\sin N\beta}{\sin\beta}$$

$$\boxed{R = \frac{A_0\sin\alpha}{\alpha}\,\frac{\sin N\beta}{\sin\beta}}$$

$$R^2 = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\boxed{I = I_0 \underbrace{\frac{\sin^2\alpha}{\alpha^2}}_{\text{distribution due to single slit}} \cdot \underbrace{\frac{\sin^2 N\beta}{\sin^2\beta}}_{\text{due N diffracted rays from N slits}}}$$

~~intensity~~ distribution due to single slit

~~intensity distribution~~ due N diffracted rays from N slits

For pulpciple maxima.

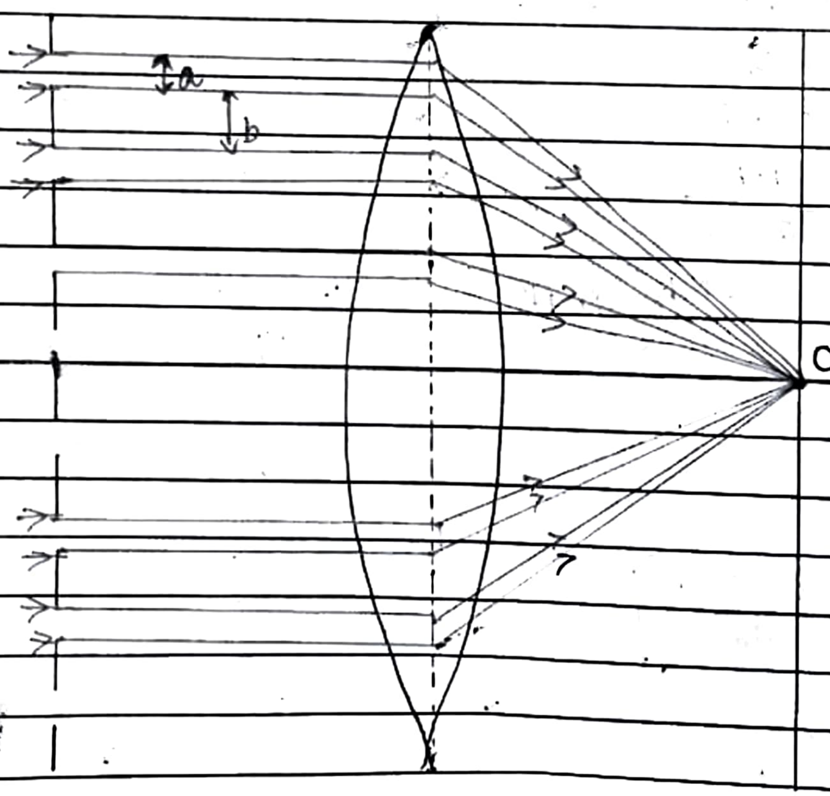
$\sin\beta = 0$

$\beta = \pm n\pi$

$$\lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta} \sin N\beta}{\frac{d}{d\beta} \sin\beta} = \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

$$\boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2} N^2}$$

Diffraction Grating (Transmission grating)



when a wavefront is incident on a grating surface, light is transmitted through the slit & obstructed by the opaque portion. Such a grating is called transmission grating. If the spacing b/w the lines is of the order of wavelength of light then appreciable deviation of the light is produced. Grating is used for the study of the visible region of the spectrum. contains 10,000 to 18,000 lines per inch.

From the figure, $(a+b)$ is called the grating element, for a grating width 15000 lines per inch

$$(a+b) = \frac{2.54}{15000} \text{ cm}.$$

Resolving power of a plane diffraction grating

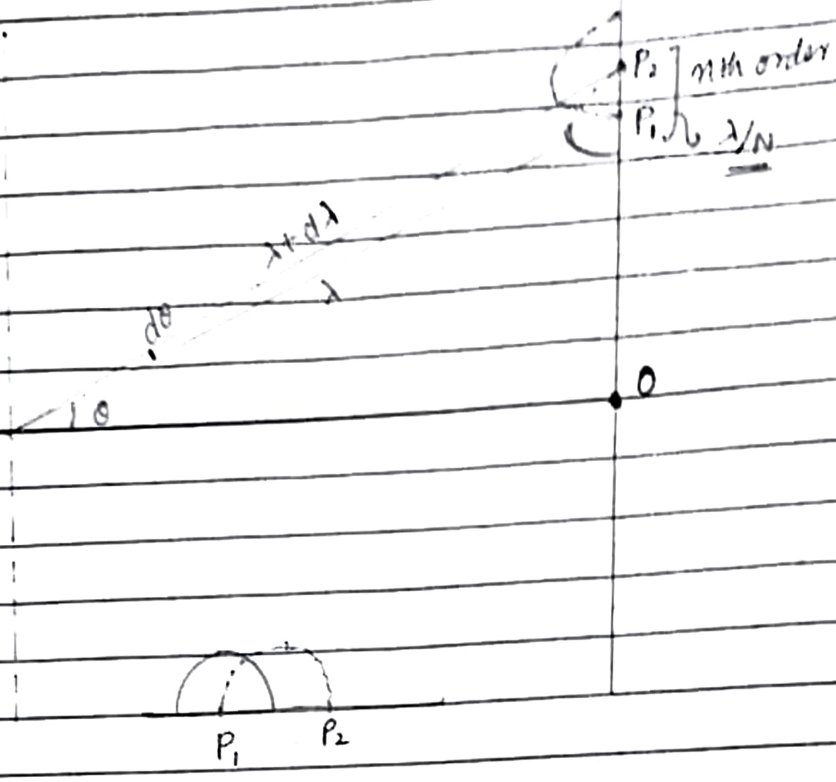
The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference b/w this line & a neighbouring line such that the two lines appear to be just resolved. So, the resolving power of a grating is $\lambda / d\lambda$.

$5890 \text{ Å} \longrightarrow \lambda$

$5896 \text{ Å} \longrightarrow \lambda + d\lambda$

$d\lambda = 6 \text{ Å}$

$$\boxed{R.P = \frac{\lambda}{d\lambda}}$$



For nth maxima, ~~at~~

$$(a+b)\sin\theta_n = n\lambda \quad \text{— (1)}$$

$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda)$$

The two lines will appear just resolved, if the extra path difference is $\frac{\lambda}{N}$ where N is the total number of lines on the grating surface.

$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda)$$

$$(a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N}$$

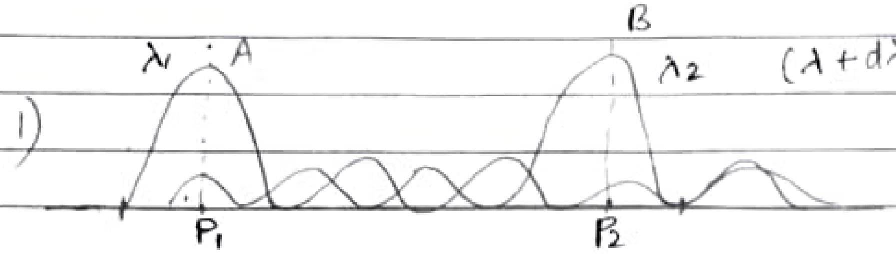
$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

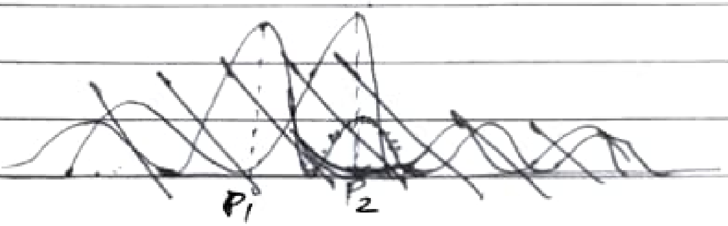
$$n\,d\lambda = \frac{\lambda}{N}$$

$$\boxed{R.P = \frac{\lambda}{d\lambda} = nN}$$

Rayleigh criteria for diffraction grating

1)





According Rayleigh, two nearby images are said to be just resolved if the position of the central maxima of one coincides with the first secondary minima of the other & vice versa.

1) The difference in the angle of diffraction then the two images can be seen as seperate image.

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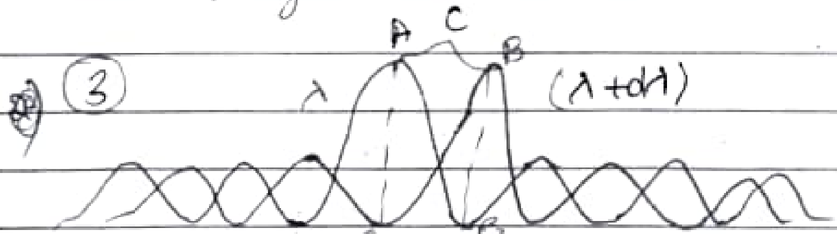

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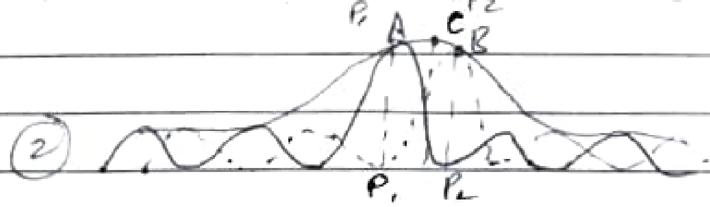

If the central maxima, corresponding to the wavelength $\{\lambda_1 + \lambda_2 [\lambda + d\lambda)]$ (are very close), so these two images overlap & they cannot be distinguished as seperate image. The resultant intensity curve with ~~with~~ a maximum at C.

3) when maxima of A coincide with first wavelength minima of B & maxima of B coincide with the first minima of A, then this condition is called just resolved cond" & the resultant intensity curve show a dip at C.

Q light wavelength 5000 Å is incident normally on a plane transmission grating of width 3 cm & 15000 lines. Calculate the angle of diffraction in first order.

$n = 1, \quad \lambda = 5000 Å, \quad N = 15000$

$$\frac{\lambda}{d\lambda} = nN \Rightarrow \quad d\lambda = \frac{5000}{1 \times 15000} = \cancel{0.3}\ \frac{1}{3}$$

$(a+b) \sin\theta_n = n\lambda$

$\cancel{\frac{3}{15000} \times \sin\theta_n = 1 \times 5000}$

$\sin\theta_n =$

$\cancel{\frac{3}{15000}}$

$\cancel{\frac{3}{15000} \times \sin\theta_n = 1 \times 5000 \times 10^{-8}}$

$\cancel{\sin\theta_n = 25 \times 10^{-2}}$

$$\sin\theta_n = \frac{\lambda}{(a+b)} = \frac{5000 \times 10^{-8}}{3} \times 15000$$

$$= \frac{75 \times 10^{-2}}{3} = \frac{0.75}{3}$$

$$\sin\theta = \frac{1}{4} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{4}\right) = 14°29'$$