

# An introduction to Molecular Orbital Theory

6 Lecture Course

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## Objectives of the course

- Wave mechanics / Atomic orbitals (AOs)
  - The basis for rejecting classical mechanics (the Bohr Model) in treating electrons
  - Wave mechanics and the Schrödinger equation
  - Representation of atomic orbitals as wave functions
  - Electron densities and radial distribution functions
  - Understanding the effects of shielding and penetration on AO energies
- Bonding
  - Review VSEPR and Hybridisation
  - Linear combination of molecular orbitals (LCAO), bonding / antibonding
  - Labelling of molecular orbitals (MOs) ( $\sigma$ ,  $\pi$  and g, u)
  - Homonuclear diatomic MO diagrams – mixing of different AO's
  - More complex molecules (CO, H<sub>2</sub>O ....)
  - MO diagrams for Inorganic complexes

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## Lecture schedule

Lecture 1	Revision of Bohr model of atoms
Lecture 2	Schrödinger equation, atomic wavefunctions and radial distribution functions of s orbitals
Lecture 3	More complex wavefunctions and radial distribution functions and electron shielding
Lecture 4	Lewis bonding, Hybridisation, and molecular orbitals
Lecture 5	Labelling MO's. 1 <sup>st</sup> row homonuclear diatomics
Lecture 6	MO approach to more complex molecules and CO bonding in transition metals complexes

## Literature

- Book Sources: all titles listed here are available in the Hamilton Library
  - 1. Chemical Bonding, M. J. Winter (Oxford Chemistry primer 15) Oxford Science Publications ISBN 0 198556942 – condensed text, excellent diagrams
  - 2. Basic Inorganic Chemistry (Wiley) F.A.Cotton, G. Wilkinson, P. L. Gaus – comprehensive text, very detailed on aufbau principle
  - 3. Inorganic Chemistry (Prentice Hall) C. Housecroft, A. G. Sharpe – comprehensive text with very accessible language. CD contains interactive energy diagrams
- Additional sources:  
<http://winter.group.shef.ac.uk/orbitron/> - gallery of AOs and MOs

## Tutorials

- Expectation
  - Tutorials are to go through problems that students are having with the course
  - Tutorials are NOT for the lecturer to give you the answers to the questions – or to give you another lecture.
  - All student must BEFORE the tutorial
    - Look at the notes for the course and try to understand them
    - Attempt the questions set – and hence find out what you can not do!
    - Bring a list of questions relating to aspects of the course which you could not understand (either from looking at the notes or attempting the questions)
- It is a waste of both the lecturers and students time if the tutorial ends up being a lecture covering questions.

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## An introduction to Molecular Orbital Theory

### Lecture 1 The Bohr Model

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## Adsorption / Emission spectra for Hydrogen

Johann Balmer (1885) measured line spectra for hydrogen  
364.6 nm (uv), 410.2 nm (uv), 434.1 nm (violet), 486.1 nm (blue), and 656.3 nm (red).



Balmer discovered these lines occur in a series - both absorption and emission - where  $\mathfrak{R}$  is the Rydberg constant ( $3.29 \times 10^{15} \text{ Hz}$ )

$$\nu = \mathfrak{R} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Balmer series  $n_1=2$  and  $n_2=n_1+1, n_1+2, n_1+3 \dots$

Other series for  $n_1=1$  (Lyman – UV),  $n_1=3$  (Paschen – IR) etc.

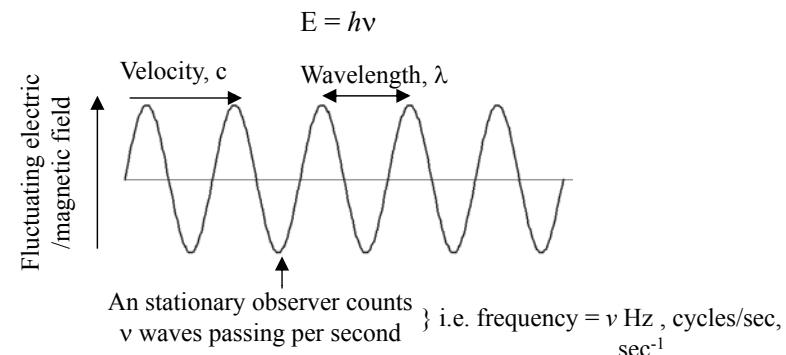
Electrons must have specific energies – no model of the atom could explain this

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Bohr model of the atom (1913)  
<http://www.youtube.com/watch?v=R7OKPaKr5QM>

### Assumptions

- 1) Rutherford (1912) model of the atom (Planetary model with central nucleus + electrons in orbit)
- 2) Planck (1901), Einstein (1905) – the energy electromagnetic waves is quantised into packets called photons (particle like property).



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## Bohr model of the atom

Speed of electromagnetic waves ( $c$ ) is constant ( $v$  and  $\lambda$  vary)

$$c = v \lambda, v = c / \lambda, E = h v, E = h c / \lambda$$

As frequency increases, wavelength decreases. Given  $\lambda \rightarrow v$

e.g. radiowaves:	$\lambda = 0.1 \text{ m}$	X-rays:	$\lambda = 1 \times 10^{-12} \text{ m}$
	$v = 3 \times 10^9 \text{ Hz}$		$v = 3 \times 10^{20} \text{ Hz}$
	$E = 2 \times 10^{-24} \text{ J}$		$E = 2 \times 10^{-13} \text{ J}$

$E$  – energy (J),  $h$  – Planck's constant (J s),  $v$  – frequency (Hz),  
 $c$  – speed of light ( $\text{m}^{-1}$ ),  $\lambda$  – wavelength (m)

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## Bohr model

Electron travelling around nucleus in circular orbits – must be a balance between attraction to nucleus and flying off (like a planet's orbit)

Electron feels two forces – must be balanced

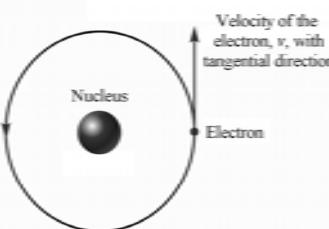
1) Centripetal (electrostatic)  $F = \frac{-Ze^2}{4\pi\epsilon_0 r^2}$   $PE = \frac{-Ze^2}{4\pi\epsilon_0 r}$

2) Centrifugal  $F = \frac{mv^2}{r}$   $KE = \frac{1}{2}mv^2$

Equalize magnitude of forces Resulting energy

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2} \Rightarrow mv^2 = \frac{Ze^2}{4\pi\epsilon_0 r} \quad E = \frac{1}{2}mv^2 + \frac{-Ze^2}{4\pi\epsilon_0 r} = \frac{-Ze^2}{8\pi\epsilon_0 r} = -\frac{1}{2}mv^2$$

$-Ze$  – nuclear charge,  $e$  – electron charge,  $\epsilon_0$  – permittivity of free space,  
 $r$  – radius of the orbit,  $m$  – mass of electron,  $v$  – velocity of the electron



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## Bohr model of the atom

- 3) Electron assumed to travel in circular orbits.
- 4) Apply quantisation to orbits – only orbits allowed have quantised angular momentum (comes from observation of spectra)

$$mv r = n \left( \frac{h}{2\pi} \right)$$

- 5) Classical electrodynamical theory rejected (charged particles undergoing acceleration must emit radiation)
- 6) Radiation adsorbed or emitted only when electrons jump from one orbit to another

$$\Delta E = E_a - E_b$$

where  $a$  and  $b$  represent the energy of the start and finish orbits

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## Bohr model – calculating the energy and radius NOT EXAMINABLE

- Energy

$$\frac{-Ze^2}{8\pi\epsilon_0 r} = -\frac{1}{2}mv^2$$

- Quantised angular momentum

$$mv r = n \left( \frac{h}{2\pi} \right)$$

- Combining the two

$$\frac{-Ze^2}{8\pi\epsilon_0 r} = -\frac{1}{2}mv^2 = \frac{-(mv)^2}{2mr^2} = \frac{-n^2h^2}{8\pi^2 mr^2}$$

- Rearranging to give  $r$

$$\frac{r^2}{r} = \frac{-n^2h^2}{8\pi^2 m} \frac{8\pi\epsilon_0}{(-Ze^2)} \quad r = \frac{n^2h^2\epsilon_0}{\pi m Ze^2}$$

- Substitute  $r$  into energy gives

$$\frac{-Ze^2}{8\pi\epsilon_0 r} = \frac{-mZ^2e^4}{8n^2h^2\epsilon_0^2}$$

- Energy is dependent on  $n^2$  and  $Z^2$  (2s and 2p the same – only true for 1 electron systems)

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## Energy levels of Hydrogen

Substitute quantised momentum into energy expression and rearrange in terms of  $r$  (radius) (see previous slide)

$$r = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} = \frac{n^2 a_0}{Z}$$

$a_0$  (Bohr) radius of the 1s electron on Hydrogen 52.9 pm ( $n=1, Z=1$ )

Radius ( $r$ ) depends on  $n^2$  and  $\frac{1}{Z}$

Substitute  $r$  back into energy expression gives

$$E_n = \frac{-m Z^2 e^4}{8 n^2 h^2 \epsilon_0^2} = \frac{13.6056 \times Z^2}{n^2} \text{ (in eV)}$$

Energy of 1s electron in H is 13.6056 eV = 0.5 Hartree ( $1\text{eV} = 1.602 \times 10^{-19} \text{ J}$ )

Energy ( $E$ ) depends on  $\frac{1}{n^2}$  and  $Z^2$

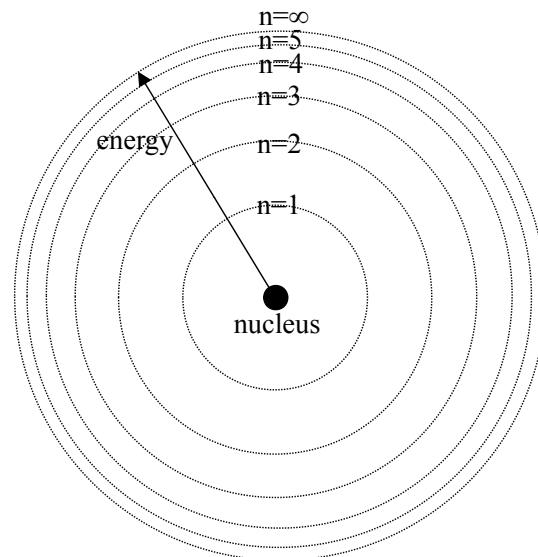
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## Energy levels of Hydrogen

For hydrogen ( $Z=1$ )

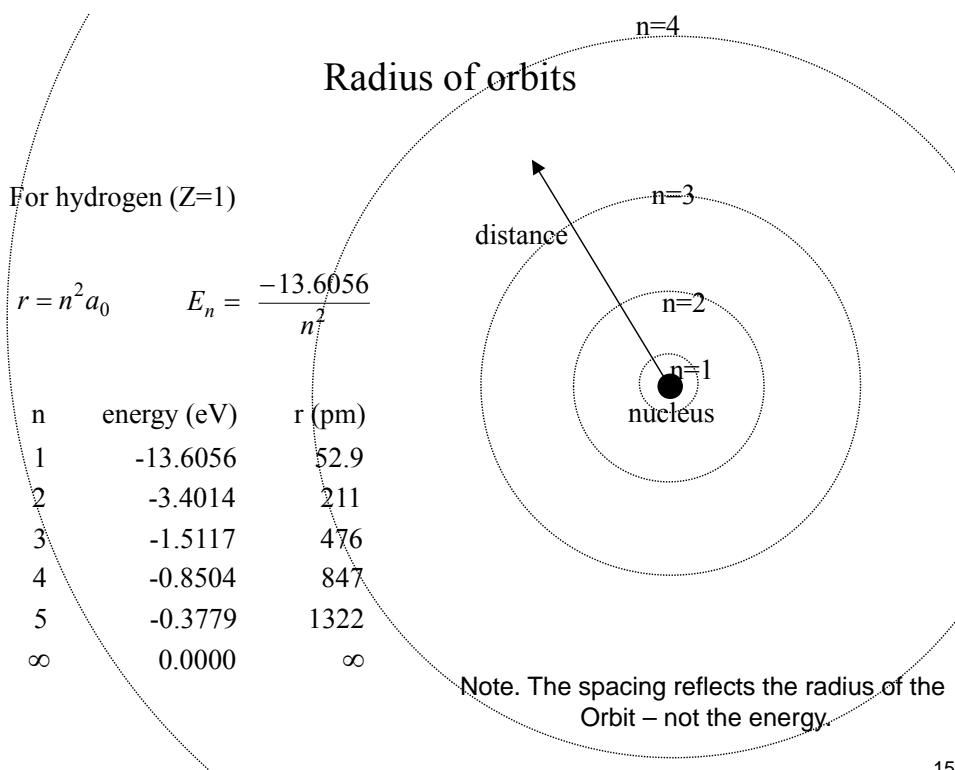
$$r = n^2 a_0 \quad E_n = \frac{-13.6056}{n^2}$$

n	energy (eV)
1	-13.6056
2	-3.4014
3	-1.5117
4	-0.8504
5	-0.3779
$\infty$	0.0000



Note. The spacing reflects the energy not the radius of the orbit.

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## Emission spectra

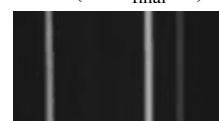
(<http://www.youtube.com/watch?v=5z2ZfYVzefs>)

Energy of emission is  $E_{\text{initial}} - E_{\text{final}} =$

$$\Delta E = 13.6056 \left( \frac{1}{n_{\text{initial}}^2} - \frac{1}{n_{\text{final}}^2} \right)$$

Same form as fitted to emission spectra

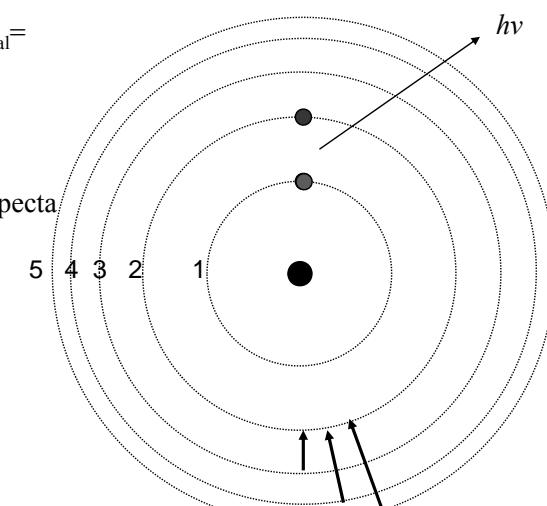
Balmer series ( $\rightarrow n_{\text{final}}=2$ )



$$n=3 \rightarrow n=2 \rightarrow \lambda = 656 \text{ nm}$$

$$n=4 \rightarrow n=2 \rightarrow \lambda = 486 \text{ nm}$$

$$n=5 \rightarrow n=2 \rightarrow \lambda = 434 \text{ nm}$$



Note. The spacing reflects the energy not the radius of the orbit.

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$$\mathfrak{N} = 13.6056 \text{ eV} / c = 3.29 \times 10^{15} \text{ Hz}$$

## Problems with the Bohr Model

- Only works for 1 electron systems
  - E.g. H, He<sup>+</sup>, Li<sup>2+</sup>
- Can not explain splitting of lines in a magnetic field
  - Modified Bohr-Sommerfield (elliptical orbits - not satisfactory)
- Can not apply the model to interpret the emission spectra of complex atoms
- Electrons were found to exhibit wave-like properties
  - e.g. can be diffracted as they pass through a crystal (like x-rays)
  - considered as classical particles in Bohr model

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## Wave / particle duality

[http://www.youtube.com/watch?v=IsA\\_oIXdF\\_8](http://www.youtube.com/watch?v=IsA_oIXdF_8)

### de Broglie (1923)

By this time it was accepted that EM radiation can have wave and particle properties (photons)

de Broglie proposed that particles could have wave properties (wave / particle duality). Particles could have an associated wavelength ( $\lambda$ )

$$E = mc^2, \quad E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{h}{mc}$$

No experimental at time.

1925 Davisson and Germer showed electrons could be diffracted according to Braggs Law (used for X-ray diffraction)

Numerically confirm de Broglie's equation

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## Wave Mechanics

- For waves: it is impossible to determine the position and momentum of the electron simultaneously – Heisenberg ‘Uncertainty principle’
- Use probability of finding an electron from  $\psi^2$  (actually  $\psi^*\psi$  – but functions we will deal with are real)

Where  $\psi$  is a wave function and a solution of the Schrödinger equation (1927). The time-independent form of the Schrödinger equation for the hydrogen atom is:

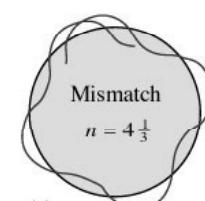
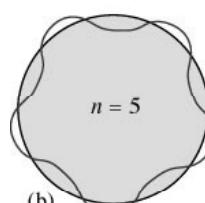
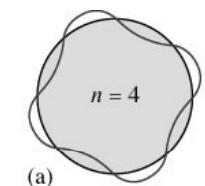
$$\left[ \frac{-\hbar^2}{8\pi^2 m} \nabla^2 \right] \Psi + \left[ \frac{-e^2}{4\pi\epsilon_0 r} \right] \Psi = E \Psi \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Kinetic energy      Potential energy      Total energy

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## Wave mechanics and atoms

- What does this mean for atoms
- Electrons in “orbits” must have an integer number of wavelengths
- E.g. n=4 and n=5 are allowed
  - These create continuous or standing waves (like on a guitar string)
- E.g. n=4.33 is not allowed
  - The wavefunction is not continuous
- The wave nature of electrons brings in the quantized nature of the orbital energies.



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## Atomic solutions of the Schrödinger equation for H

- Schrödinger equation can be solved exactly for 1 electron systems
  - Solved by trial and error manipulations for more electrons
- 3 quantum numbers describing a three dimensional space called an atomic orbital:  $n, l, m$  (and spin quantum number describing the electron s)

$n$  = principal quantum number, defines the orbital size with values 1 to  $\infty$

$l$  = azimuthal or angular momentum quantum number, defines shape.  
For a given value of  $n$ ,  $l$  has values 0 to  $(n-1)$ .

$m_l$  = magnetic quantum number, defines the orbital orientation.  
For a given value of  $l$ ,  $m_l$  has values from  $+l$  through 0 to  $-l$ .

## Solution of the Schrödinger equation for H

$l$  has values 0 to  $(n-1)$        $m$  has values from  $+l$  through 0 to  $-l$

$n$	1	2	2	2	2
$l$	0	0	1	1	1
$m_l$	0	0	-1	0	1
Orbital	1s	2s	2p	2p	2p

$n$	3	3	3	3	3	3	3	3	3
$l$	0	1	1	1	2	2	2	2	2
$m_l$	0	-1	0	1	-2	-1	0	1	2
Orbital	3s	3p	3p	3p	3d	3d	3d	3d	3d

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## An introduction to Molecular Orbital Theory

Lecture 2 – Representing atomic orbitals - The Schrödinger equation and wavefunctions.

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## Last Lecture

- Recap of the Bohr model
  - Electrons
  - Assumptions
  - Energies / emission spectra
  - Radii
- Problems with Bohr model
  - Only works for 1 electron atoms
  - Can not explain splitting by a magnetic field
- Wave-particle duality
- Wave mechanics
  - Schrödinger
  - Solutions give quantum number  $n, l, m_l \rightarrow$  atomic orbitals

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## Representations of Orbitals:

For an atomic system containing one electron (e.g. H, He<sup>+</sup> etc.)

The wavefunction,  $\Psi$ , is a solution of the Schrödinger equation

It describes the behaviour of an electron in region of space called an atomic orbital ( $\phi$  - phi)

Each orbital wavefunction ( $\phi$ ) is most easily described in two parts

radial term – which changes as a function of distance from the nucleus  
angular terms – which changes as a function of angles

$$\varphi_{xyz} = \varphi_{\text{radial}}(r) \varphi_{\text{angular}}(\phi, \theta) = R_{nl}(r) Y_{lm}(\phi, \theta)$$

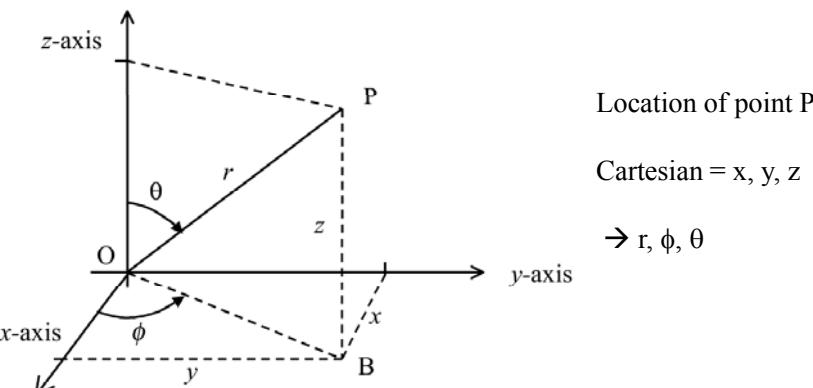
Orbitals have

- SIZE determined by  $R_{nl}(r)$  - radial part
- SHAPE determined by  $Y_{lm}(\phi, \theta)$  - angular part (spherical harmonics)
- ENERGY determined by the Schrödinger equation

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## Polar Coordinates

- To describe the wavefunction of atomic orbitals we must describe it in three dimensional space
- For an atom it is more appropriate to use spherical polar coordinates:



Location of point P

Cartesian = x, y, z

$\rightarrow r, \phi, \theta$

java applet on polar coordinates at <http://qsad.bu.edu/applets/SPCExp/SPCExp.html>

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## Wavefunctions for the AO's of H

General hydrogen like orbitals

$R_{nl}(r)$	$Y_{lm}(\phi, \theta)$
$1s \quad 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-\rho r/2}$	$\left(\frac{1}{4\pi}\right)^{1/2} \rho = \left(\frac{2Z}{na_0}\right)$

For hydrogen this simplifies as Z=1 and  $a_0=1$  (in atomic units) and thus  $\rho = 2$ .

Hence

$R_{nl}(r)$	$Y_{lm}(\phi, \theta)$
$1s \quad 2e^{-r}$	$\frac{1}{2\sqrt{\pi}}$

Angular component is a constant  
 $\rightarrow$  Spherical

**Normalisation**  
Constants are such that

$$\int \varphi^2 d\tau = 1$$

that is the probability of the electron in an orbital must be 1 when all space is considered

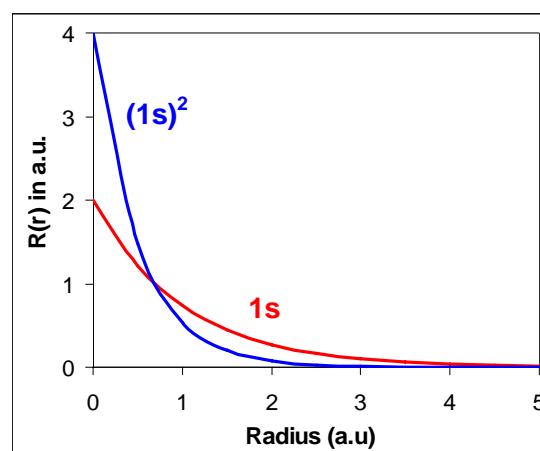
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## Radial Wavefunction

- $R(r)$  of the 1s orbital of H

it decays exponentially with r  
it has a maximum at  $r = 0$

- $R(r)$  has no physical meaning
- Probability depends on  $R(r)^2$ 
  - Misleading – does not take into account the volume
  - $R(r)^2$  increases toward  $r = 0$
  - Volume very small so probability of being at small  $r$  is small



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## Radial distribution functions (RDF)

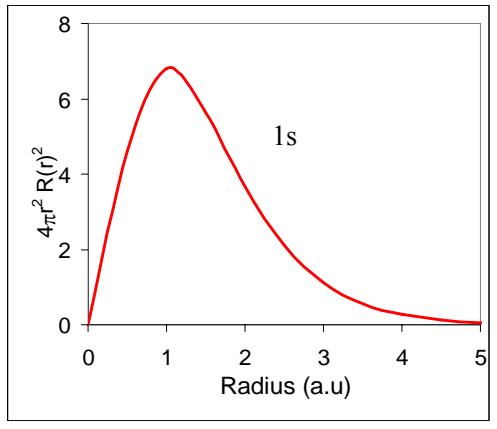
- Probability of an electron at a radius  $r$  (RDF) is given by probability of an electron at a point which has radius  $r$  multiplied by the volume at a radius of  $r$
- Consider a sphere – volume as we move at a small slice is  $4\pi r^2 \delta r$

– By differentiation,

$$\text{the volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{so } \frac{\partial V}{\partial r} = 4\pi r^2$$

- $\text{RDF}(r) = 4\pi r^2 R(r)^2$
- Maximum for 1s at  $a_0$  (like Bohr!)



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## Wave functions of Hydrogen 2s and 3s orbitals

$$\text{For H } 2s(r) = \frac{1}{2\sqrt{2}}(2-r)e^{(-r/2)}$$

$$\text{For H } 3s(r) = \frac{1}{9\sqrt{3}}\left[6 - 4r + \left(\frac{2}{3}r\right)^2\right]e^{(-r/3)}$$

exponential decreases more slowly than 1s,  $e^{(-r/n)}$  → more diffuse orbital

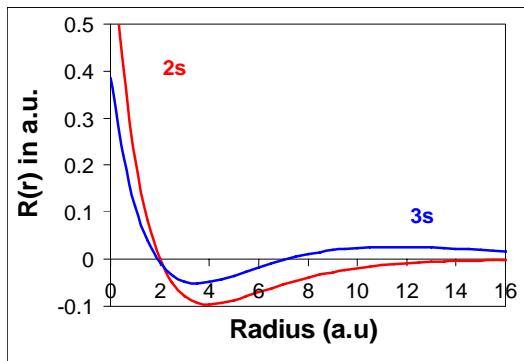
Wavefunctions changes sign

$$R(r) = 0 \rightarrow \text{RADIAL NODE}$$

2s at  $(2-r) = 0$  (i.e.  $r=2$  a.u.)

3s changes sign twice with two nodes ( $r = 1.9, 7.1$  a.u.)

→ Caused by the order of the polynomial !



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## Wave functions of 2s and 3s orbitals

	2s(r)	3s(r)
General	$\frac{1}{2\sqrt{2}}\left(\frac{Z}{a_0}\right)^{3/2}(2-\rho r)e^{(-\rho r/2)}$ $\rho = \left(\frac{2Z}{na_0}\right)$	$\frac{1}{9\sqrt{3}}\left(\frac{Z}{a_0}\right)^{3/2}(6-6\rho r+(\rho r)^2)e^{(-\rho r/2)}$ $\rho = \left(\frac{2Z}{na_0}\right)$
For H	$2s \rightarrow Z=1, n=2, \rho=1$ $\frac{1}{2\sqrt{2}}(2-r)e^{(-r/2)}$	$3s \rightarrow Z=1, n=3, \rho=2/3$ $\frac{1}{9\sqrt{3}}\left[6 - 4r + \left(\frac{2}{3}r\right)^2\right]e^{(-r/3)}$

The form of the wave functions is the important concept – not the precise equation

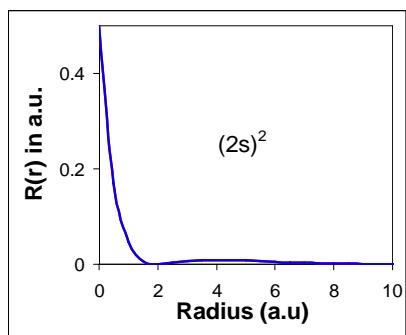
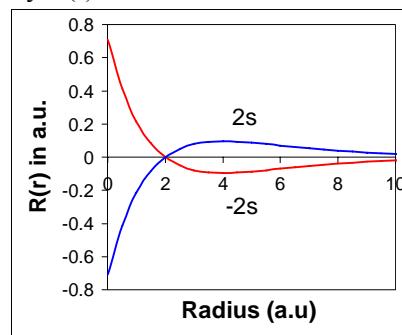
Note  $R(r)$  has functional form

Normalisation constant \* polynomial (increasing order with n) \* exponential  $(-r/n)$

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## What does a negative sign mean

- The absolute sign of a wave function is not important.
  - The wave function has NO PHYSICAL SIGNIFICANCE
  - the electron density is related to the square of the wave function
  - This is the same irrespective of the sign
- Wavefunction signs matter when two orbitals interact with each other (see later)
- Some books have the 2s as opposite sign – you can see that the electron density  $R(r)^2$  is the same



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## Radial Nodes

- Number of radial node =  $n - l - 1$

$$1s = 1 - 0 - 1 = 0$$

$$2s = 2 - 0 - 1 = 1 \quad 2p = 2 - 1 - 1 = 0$$

$$3s = 3 - 0 - 1 = 2 \quad 3p = 3 - 1 - 1 = 1 \quad 3d = 3 - 2 - 1 = 0$$

- Why are there radial nodes ?

- Pauli exclusion principle – no two electrons can have the same set of QN's
- Actually – no two electron can overlap (i.e. occupy same space)
- Overlap integral =  $\int \varphi_A^* \varphi_B d\tau = 0$  (analogous to normalisation)  
AO's are said to be Orthogonal
- Satisfied for AO's with same  $l$  by having change(s) in the wave function sign
- Satisfied for AO's with different  $l$  angular component ensures no overlap

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## RDF's of $ns$ orbitals

1s – 1 peak.

2s – 2 peaks

3s – 3 peaks

Maximum at  $r = a_0$

Maximum at  $r \approx 5 a_0$

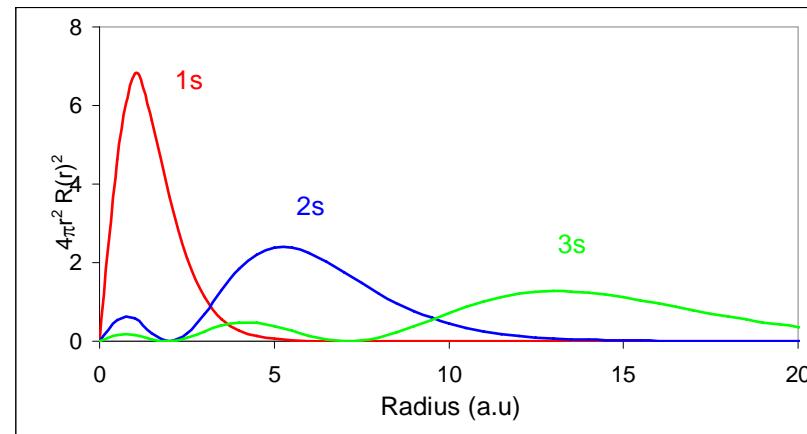
Maximum at  $r \approx 13 a_0$

- Bohr Model  $\rightarrow$  radius of  $a_0$

- Bohr Model  $\rightarrow$  radius of  $4 a_0$

- Bohr Model  $\rightarrow$  radius of  $9 a_0$

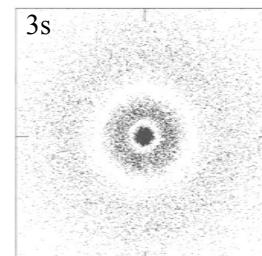
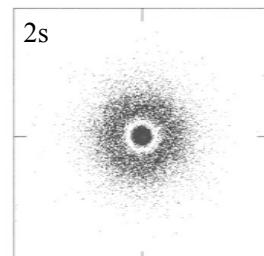
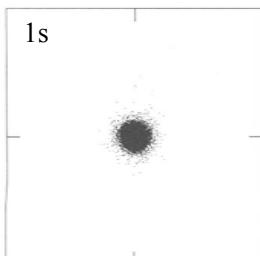
Shape important for orbital energies



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## Representing atomic orbitals

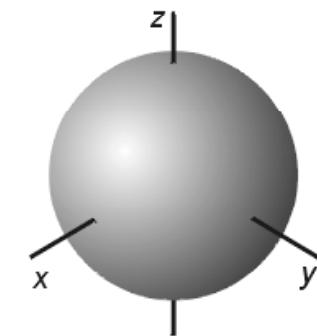
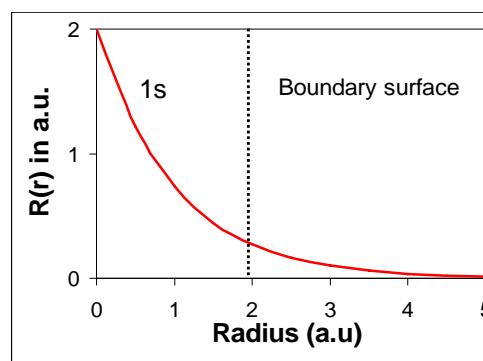
- Represent orbitals, so far radial and angular terms
- In 2D we can use dot diagrams to look at the whole wave function
  - s orbitals have no angular component – spherical symmetry
  - Dot diagrams show electron density within a plane – no sign
  - Can see where density goes to zero – nodes
  - Can see how greater volume as  $r$  increases makes most probable distance.



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## Boundary surface

- Represent the wave function in 3D
  - Draw a 3D contour at a given value of  $\varphi$
  - Alternatively can define contour such that it encloses a space which the electron spends most of its time
  - Shows the shape and size of the orbital
  - Can not see the inner structure of the wave function



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## p orbitals - wavefunctions

- There are three  $p$  orbitals for each value of  $n$  ( $p_x, p_y, p_z$ )
  - The radial function is the same for all  $np$  orbitals
  - The angular terms are different → different shapes (orientations)
  - Angular terms are the same for different  $n \rightarrow 2p_x, 3p_x, 4p_x$
- Wave function for  $2p$  and  $3p$  orbitals

$$R(r)$$

$$R(2p) = \frac{1}{2\sqrt{6}} \left(\frac{Z}{a_0}\right)^{3/2} \rho r e^{(-\rho r/2)}$$

$$R(3p) = \frac{1}{9\sqrt{6}} \left(\frac{Z}{a_0}\right)^{3/2} (4 - \rho r) \rho r e^{(-\rho r/2)}$$

$$Y(\theta \phi)$$

$$Y(p_z) = \left(\frac{3}{4\pi}\right)^{1/2} \cos(\theta)$$

$$Y(p_x) = \left(\frac{3}{4\pi}\right)^{1/2} \sin(\theta) \cos(\phi)$$

$$Y(p_y) = \left(\frac{3}{4\pi}\right)^{1/2} \sin(\theta) \sin(\phi)$$

$$\rho = \left(\frac{2Z}{na_0}\right)$$

Note the functional form of  $R(r) \rightarrow$  Constant \* polynomial \*  $r$  \* exponential

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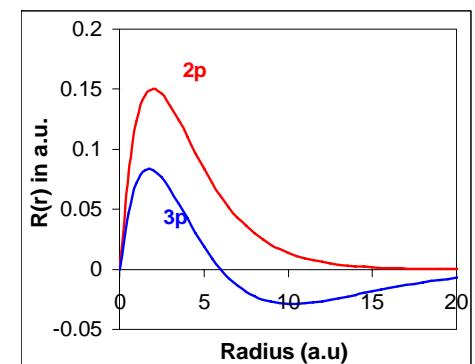
## p orbitals – radial functions

- Radial wave function for hydrogen  $p$  orbitals ( $Z=1$ )
  - for  $2p$   $n = 2 \rightarrow \rho = 1$
  - for  $3p$   $n = 3 \rightarrow \rho = 2/3$

$$R(2p) = \frac{1}{2\sqrt{6}} r e^{(-r/2)}$$

$$R(3p) = \frac{1}{9\sqrt{6}} \left(4 - \frac{2r}{3}\right) \frac{2r}{3} e^{(-2r/3)}$$

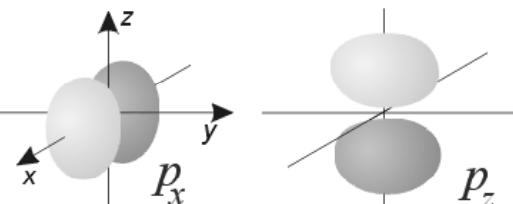
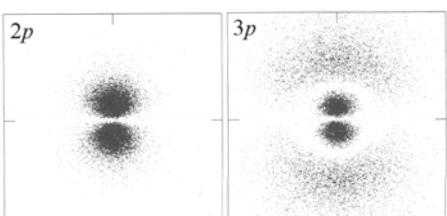
- Polynomial → nodes
  - Equation for no. of radial nodes
  - $n - l - 1 \rightarrow 2p = 0, 3p = 1$
  - Ensures  $2p$  and  $3p$  orthogonal
- All  $p$  orbitals are multiplied by  $r$ 
  - $R(r) = 0$  at  $r = 0$
- Required to match the angular function
  - angular node



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## p orbitals – angular functions boundary surfaces

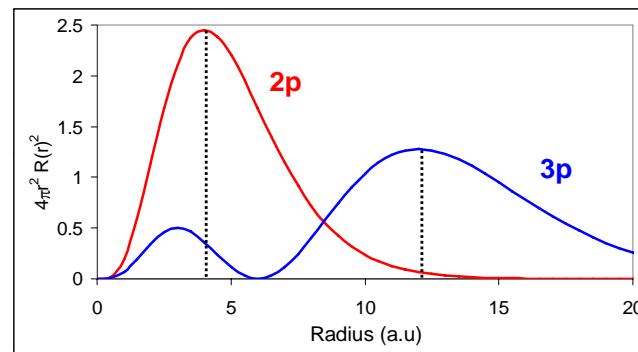
- All  $p$  orbitals have the same shape
- Angular function give rise to direction
- Can represent  $p$  orbital as dot diagrams or boundary surfaces
- 1 angular nodal plane  $p_x$  (yz plane),  $p_y$  (xz plane)  $p_z$  (xy plane)
  - Ensures that  $p$  orbitals are orthogonal to  $s$  orbitals



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## p orbitals – RDF's

- Radial distribution function show probability at a given radius
- $2p$  function – no nodes, maximum at  $r = 4 a_0$  (same as  $n=2$  for Bohr model)
- $3p$  function – two peaks, maximum at  $r \approx 12 a_0$  (not the same as Bohr)



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# An introduction to Molecular Orbital Theory

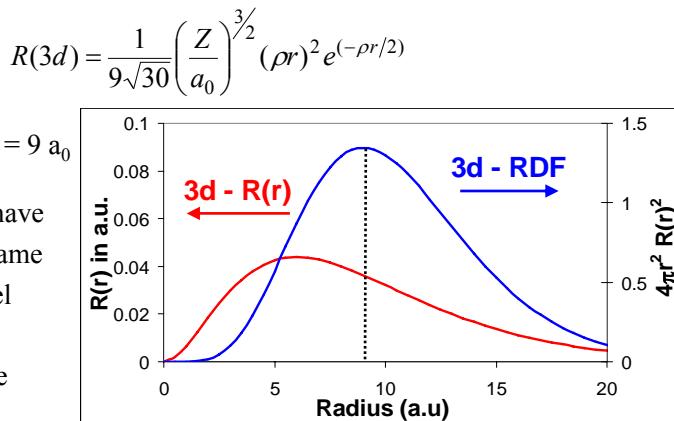
Lecture 3 – More complex wave functions, radial distribution functions and electron shielding

Revision of Lewis bonding and hybridization

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## d orbitals – wave functions

- Five d orbitals for each value of n ( $n \geq 3$ )  $\rightarrow l = 2, m_l = -2, -1, 0, 1, 2$
- Wave functions slightly more complicated (constant \* polynomial \*  $r^2$  \* exp)
  - Radial wave functions same for all 3d orbital
- Max probability at  $r = 9 a_0$
- AO's with 0 nodes have max probability at same radius as Bohr model
- 4d orbital has 1 node



Note the functional form of  $R(r) \rightarrow \text{Constant} * \text{polynomial} * r^2 * \text{exponential}$

## Last week

- Solutions of the Schrödinger equation for atoms
  - Atomic orbitals ( $\varphi$ )
  - Defined three quantum number ( $n, l, m_l$ )
- Defined polar coordinates  $\rightarrow$  radial and angular terms
- Examined wavefunctions of the s orbitals
  - Angular term constant for s orbitals
  - Wavefunction as constant \* polynomial \* exponential
  - Decays as  $e^{(-r/n)}$   $\rightarrow$  the larger  $n$  the more diffuse the orbital
  - Defined radial nodes and examined their number (polynomial  $\rightarrow n - l - 1$ )
  - Discussed the requirement for radial nodes  $\rightarrow$  Pauli exclusion principle
- p orbitals
  - Radial functions similar to s orbital (except additional  $r$ )  $\rightarrow R(0) = 0$
  - Angular terms define shapes  $p_x, p_y$  and  $p_z$  – same for different  $n$
  - Radial distribution function for p orbitals

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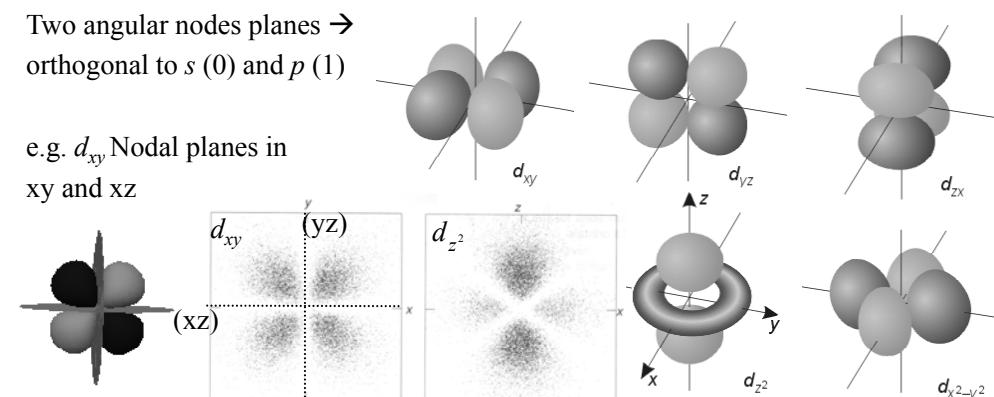
## d orbitals – angular functions

- Angular functions same for  $d_{xy}, d_{yz}, d_{xz}, d_{z^2}, d_{x^2-y^2}$  irrespective of  $n$   
 $\rightarrow$  same shape for 3d, 4d, 5d orbitals using boundary surfaces

Five different angular function e.g.  $Y(d_{xz}) = \left(\frac{15}{16\pi}\right)^{1/2} \sin(\theta)\cos(\theta)\cos(\phi)$

Two angular nodes planes  $\rightarrow$  orthogonal to s (0) and p (1)

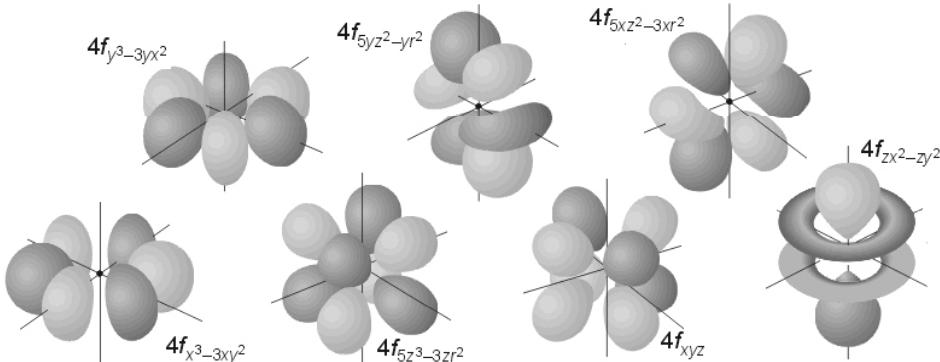
e.g.  $d_{xy}$  Nodal planes in xy and xz



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## f orbitals

- Almost no covalent bonding → shape not really important
- $l = 3 \rightarrow$  Seven different angular function for each  $n$  ( $n \geq 4$ )
  - $f$  block is 14 element wide, same shape for  $4f$ ,  $5f$  etc
  - Radial functions same for all  $nf$  orbitals
  - Three angular nodes (nodal planes) → orthogonal to  $s$ ,  $p$  and  $d$  orbitals

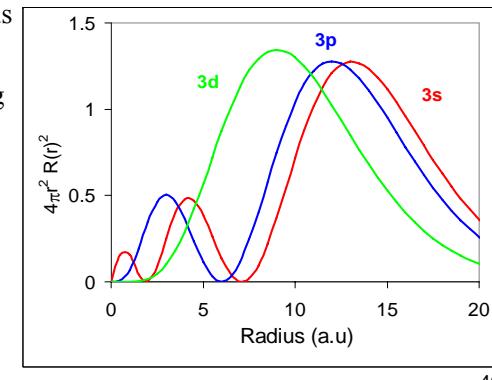


Note the functional form of  $R(r) \rightarrow \text{Constant} * \text{polynomial} * r^{3*} \text{exponential}$

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## Penetration

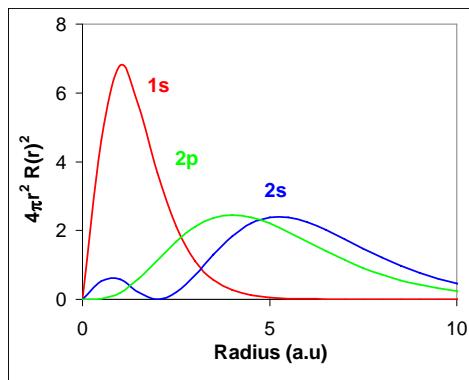
- The RDF's of AO's within a given principle QN ( $n$ ) have different shapes
- Number of nodes  $n - l - 1$ 
  - $n = 3 \quad 3s \rightarrow 2$  nodes     $3p \rightarrow 1$  node     $3d \rightarrow 0$  nodes
  - $3s$  has a peak very close to the nucleus
  - $3p$  has a peak close to the nucleus
- These close peaks have a very strong interaction with the nucleus
- $3s$  is said to be the most penetrating
- Penetration  $3s > 3p > 3d$



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## Multi electron atoms

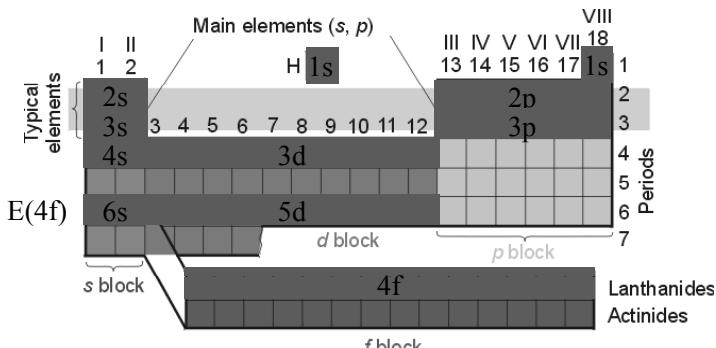
- Can not analytically solve the Schrödinger equation for multi-electron atoms
  - We assume hydrogen like wave functions for multi-electron atoms
    - Nuclear charge increases with atomic No.
    - electrons repel each other and shield the nucleus from other electrons
  - Effective nuclear charge  $Z_{\text{eff}} = Z - S$   
S = a screening or shielding constant
- E.g. Li atom – why is the electronic configuration  $1s^2 2s^1$  and not  $1s^2 2p^1$ ?
  - 1s electrons shields the valence electrons from the nuclear charge
  - 2s penetrates more effectively → feels a greater nuclear charge
  - 2p penetrates less effectively
  - 2s is filled first
- $E(1s) < E(2s) < E(2p)$
- $E(ns) < E(np) < E(nd)$



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## Periodic table

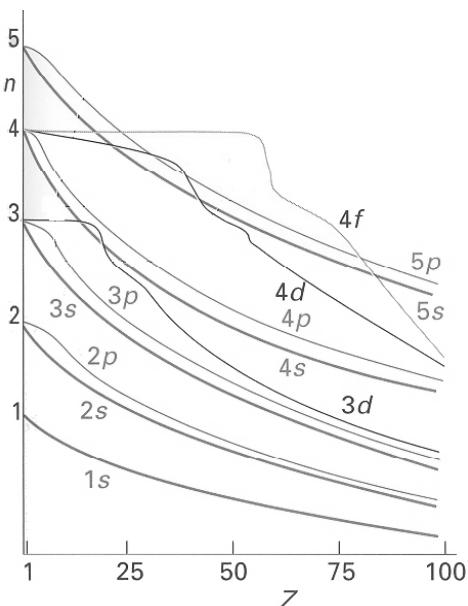
- Shielding and penetration →  $E(ns) < E(np) < E(nd) < E(nf)$
- This gives rise to electronic configuration of atoms and the order of elements in the periodic table
- Electrons are filled in increasing energy (Aufbau principle) and electrons fill degenerate (same energy) levels singularly first to give maximum spin (Hund's rule)
- $E(4s) < E(3d)$   
K, Ca
- $E(6s) < E(5d) \approx E(4f)$   
La [Xe]  $6s^2 5d^1$   
Ce [Xe]  $6s^2 4f^2$



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## More complex results of penetration and shielding Energy levels vs atomic number

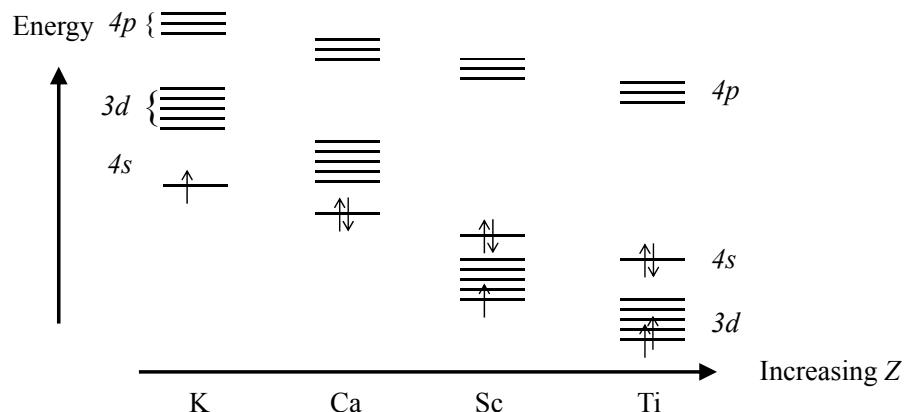
- For H ( $Z=1$ ) all orbitals within a principle QN have same energy
- For multi electron atoms penetration follows  $s > p > d > f$
- $3d$  shielded very effectively by orbitals of  $n \leq 3$  and  $3d$  almost does not change in energy with  $Z$  until  $Z = 19$
- $4s$  filled before  $3d$
- However  $n = 4$  does not shield  $3d$  effectively → energy drops
- Similar pattern for  $4d$  and  $4f$



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## The energy of the 4s and 3d orbitals

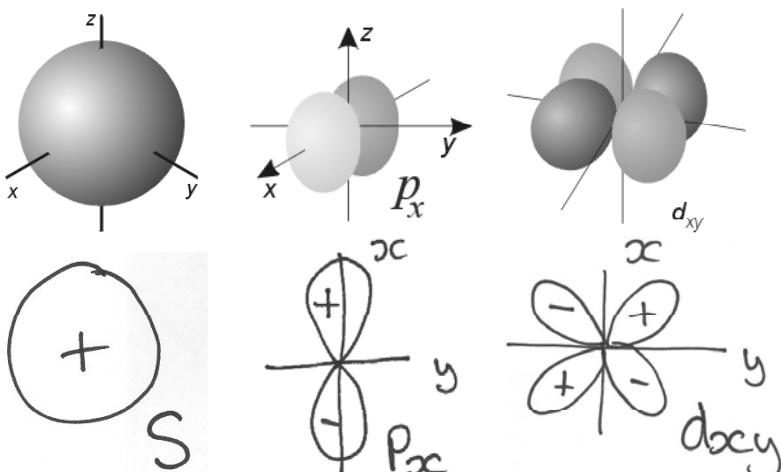
- For K and Ca the  $E(3d) > E(4s)$ , Sc on the  $E(3d) < E(4s)$  (but close)
  - If  $4s$  electron go into  $3d$  orbital the extra e-e repulsion and shielding cause the  $3d$  to rise above  $4s$  again – hence the strange energy level diagram
  - Result is that TM's loose  $4s$  electrons first when ionized



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## Drawing representations of AO's

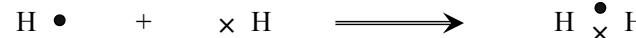
- Need to be able to draw AO's when considering their interactions in MO's
  - So far diagrams have been to help visualise the 3D nature of AO's
  - Simple drawings are all you need !!!!!



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## Making Bonds Localised Bond Pictures Revision of JF of Lewis Bonding / VSEPR

- Localised view of bonding
  - Views covalent bonds as occurring between two atoms
  - Each bond is independent of the others
  - Each single bond is made up of two shared electrons
  - One electron is usually provided by each atom
  - Each 1<sup>st</sup> and 2<sup>nd</sup> row atom attains a noble gas configuration (usually)
  - Shape obtained by VSEPR (Valence Shell Electron Pair Repulsion)

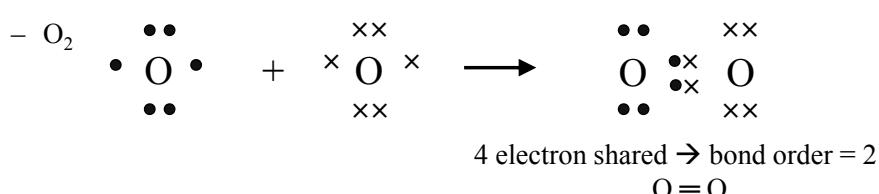
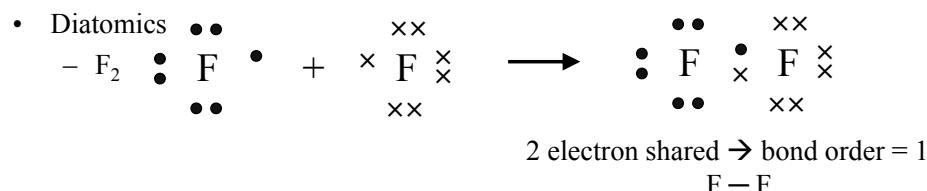
e.g.  $H_2$ 

Each H has a share of 2 electrons →  $H - H$

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## Lewis bonding

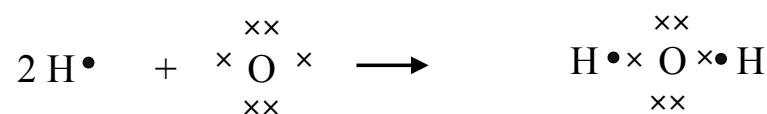
- Octet rule for main group elements / 18 electron rule for transition metals
  - All atoms have (or a share of) 8 electrons (main) or 18 electrons (TM)
  - Gives rise to noble gas configuration
  - Stability since all valence levels filled



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## Lewis bonding – polyatomics ( $\text{H}_2\text{O}$ )

- Oxygen atom has 6 valence electrons and each hydrogen has 1 electron



- Lewis bonding in  $\text{H}_2\text{O}$

- Oxygen has 8 electrons, hydrogen has 2 electrons  $\rightarrow$  noble gas config.
- Oxygen – hydrogen interactions share 2 electrons  $\rightarrow$   $\text{H}-\text{O}$
- Oxygen also has two lone pairs

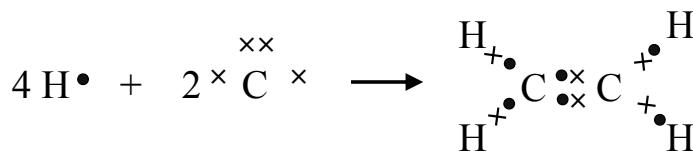
- Shape – VSEPR

- Electrons repel each other (lone pairs repulsion > than bonding pairs)
- Oxygen has 2 bond pairs and 2 lone pairs  $\rightarrow$  4 directions to consider
- Accommodate 4 directions  $\rightarrow$  Tetrahedral shape
- $\text{H}_2\text{O}$  is bent with  $\text{H}-\text{O}-\text{H}$  angle of  $104.5^\circ$
- Compares with a perfect tetrahedral of  $109.45^\circ$   $\rightarrow$  lone pair repulsion

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## Lewis bonding – polyatomics (ethene)

- Used different symbols for electrons on adjacent atoms



Carbon atoms share 4 electron  $\rightarrow$  bond order = 2  $\rightarrow$   $\text{C}=\text{C}$   
 Carbon –hydrogen interactions share 2 electrons  $\rightarrow$   $\text{C}-\text{H}$

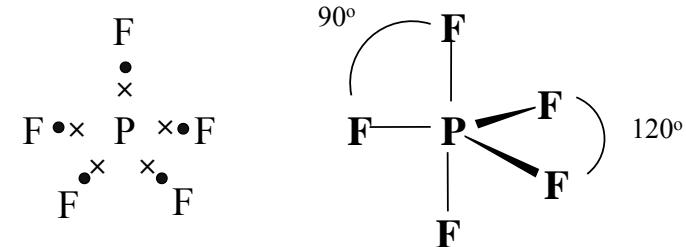
- Shape – VSEPR
  - Electrons repel each other
  - Carbon atoms have 3 directions – bond to C and two bonds to H
  - Accommodate 3 bond direction  $\rightarrow$   $120^\circ$  in a plane (molecule is flat)

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## Lewis structures – breaking the octet rule

- Some structures do not obey the 8 electron rule.

- e.g.  $\text{PF}_5$



(only the electrons round the P are shown for clarity)

- F atoms have 3 lone pairs (6 electrons) + 2 in each bond  $\rightarrow$  8
- P atom has 5 bond pairs with 2 electrons each  $\rightarrow$  10 electrons!

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## TUTORIAL 1

1. What is the relationship between the possible angular momentum quantum numbers to the principal quantum number?
2. How many atomic orbitals are there in a shell of principal quantum number n ?
3. Draw sketches to represent the following for 3s, 3p and 3d orbitals.
  - a) the radial wave function
  - b) the radial distribution
  - c) the angular wave function
4. Penetration and shielding are terms used when discussing atomic orbitals
  - a) Explain what the terms penetration and shielding mean.
  - b) How do these concepts help to explain the structure of the periodic table
5. Sketch the d orbitals as enclosed surfaces, showing the signs of the wavefunction.
6. What does the sign of a wavefunction mean ?

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## An introduction to Molecular Orbital Theory

Lecture 4 Revision of hybridisation  
Molecular orbital theory and diatomic molecules

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### Last lecture

- d orbitals
  - Radial wavefunctions, nodes and angular wavefunctions (shapes)
- f orbitals
  - Radial wavefunctions, nodes and angular wavefunctions (shapes)
- Multielectron atoms
  - Penetration and shielding
  - Atomic orbital energies, filling and the periodic table
- Valence bond theory (localised electron pairs forming bonds)
  - Lewis structures → number of electron pairs
  - VSEPR → bond order (electrons shared divided by 2)
  - VSEPR → repulsion of electron pairs (BP and LP)
  - molecular shape

### Valence bond theory and hybridisation

- Valence bond theory (Linus Pauling)
  - Based on localised bonding
  - Hybridisation to give a geometry which is **consistent** with experiment.
  - Hybridisation constructs new hybrid atomic orbitals from the AO's
- Use Lewis model (number of electron pairs) → hybridisation → shape.
  - E.g. BeH<sub>2</sub>, Be – 1s<sup>2</sup> 2s<sup>2</sup>



- Correctly predicted by VSEPR to be linear – can we explain it using AO's
  - Mix S with p<sub>z</sub> orbital → 2 sp hybridized orbitals

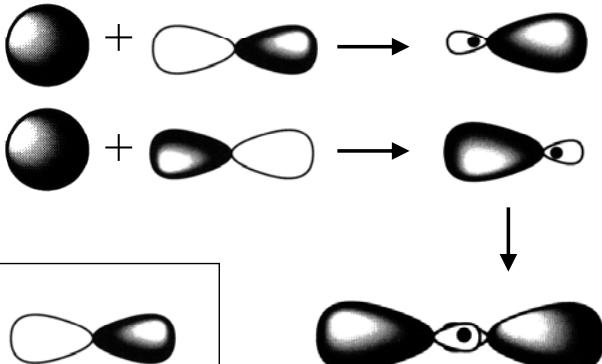
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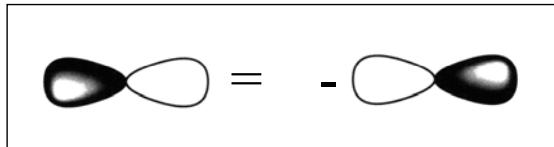
## sp hybridisation

- sp hybridisation
  - Mix an s and a p orbital – two combinations  $s + p_z$  and  $s - p_z$
  - Two AO's → two hybrid AO's
  - Relative sign is important when mixing orbitals
  - sp therefore means that the hybrid orbital is 50% s and 50% p

$$\psi_{sp} = \frac{1}{\sqrt{2}}(\phi_{2s} + \phi_{2p_z})$$



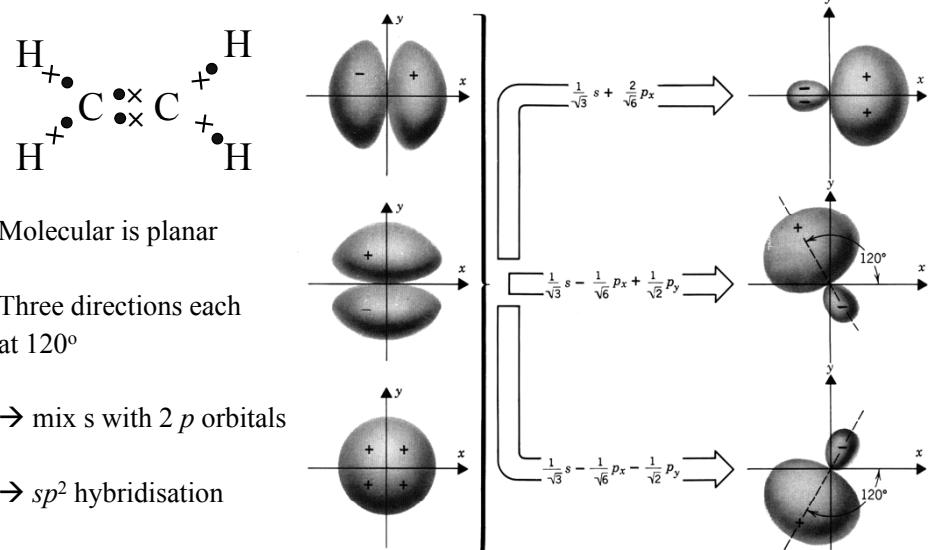
$$\psi_{sp} = \frac{1}{\sqrt{2}}(\phi_{2s} - \phi_{2p_z})$$



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## Hybridisation – $sp^2$ hybridisation

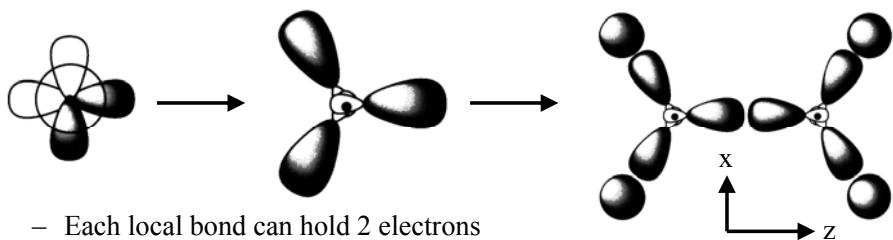
- Lewis structure → 3 directions



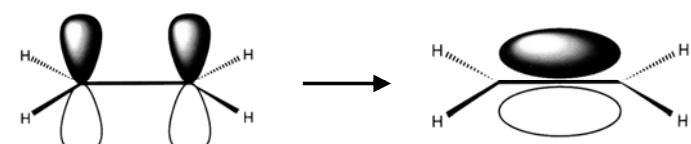
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## Hybridisation – $\pi$ bonds

- For ethene  $sp^2$  hybridisation → bonding in three directions



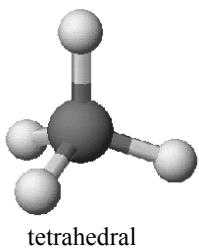
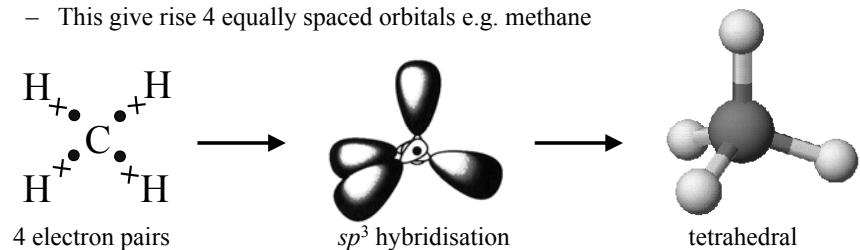
- Each local bond can hold 2 electrons
- Have not accounted for the second pair of electron shared by the C atoms
- Creates a  $\pi$  bond above and below the plane of the molecule
  - Could think of the C as going from  $s^2 p^2 \rightarrow (sp^2)^3 p_x^1$



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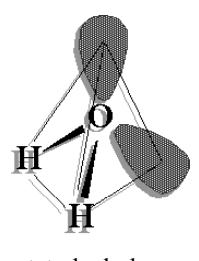
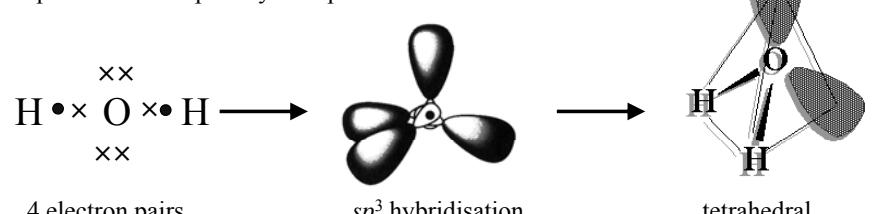
## Hybridisation – $sp^3$

- For tetrahedral molecules we have to mix the s with all the p orbitals ( $sp^3$ )
  - This give rise 4 equally spaced orbitals e.g. methane



tetrahedral

- $H_2O$  can also be thought of like this with two of the  $sp^3$  orbitals occupied by lone pairs.

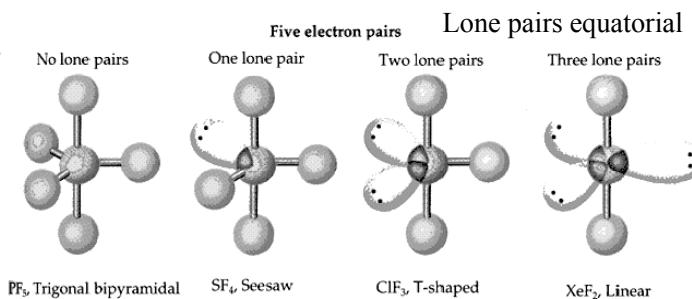
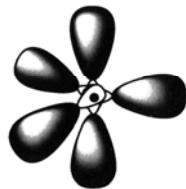


tetrahedral

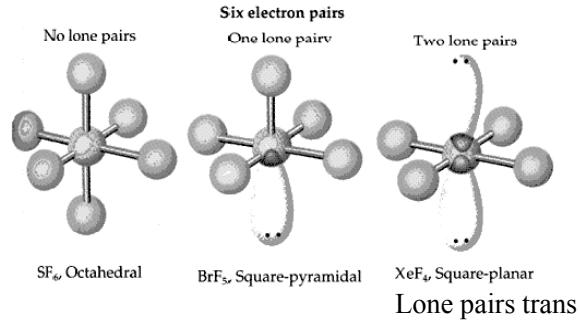
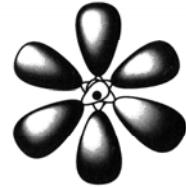
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## Hybridisation – $d$ orbitals

Trigonal Bipyramidal  
 $sp^3d \rightarrow 5$  electron pairs  
 $(s + px + py + pz + dz^2)$



octahedra  
 $sp^3d^2 \rightarrow 6$  electron pairs  
 $(s + px + py + pz + dz^2 + dx^2 - y^2)$



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## Hybridisation – summary

Hybridisation	Atomic orbitals that are mixed	Geometry	General formula	Examples
$sp$	$s + p$	linear	$\text{AB}_2$	$\text{BeH}_2$
$sp^2$	$s + p_x + p_y$	trigonal planar	$\text{AB}_3$	$\text{BF}_3, \text{CO}_3^{2-}, \text{C}_2\text{H}_4$
$sp^3$	$s + p_x + p_y + p_z$	tetrahedral	$\text{AB}_4$	$\text{SO}_4^{2-}, \text{CH}_4, \text{NH}_3, \text{H}_2\text{O}$
$sp^3d$	$s + p_x + p_y + p_z + dz^2$ $s + p_x + p_y + p_z + dx^2 - y^2$	Trigonal Bipyramidal square pyramidal	$\text{AB}_5$	$\text{PCl}_5, \text{SF}_4$
$sp^3d^2$	$s + p_x + p_y + p_z + dz^2 + dx^2 - y^2$	octahedral	$\text{AB}_6$	$\text{SF}_6, [\text{Ni}(\text{CN})_4]^{2-}, [\text{PtCl}_4]^{2-}$

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## Molecular orbital theory

- Molecule orbital theory (Robert Mullikan)
- Electrons are delocalised
  - Different to Lewis and hybridisation (these are not MO)
  - Molecular orbitals are formed which involve all of the atoms of the molecule
  - Molecular orbital are formed by addition and subtraction of AO's
- Linear Combination of Atomic Orbitals (LCAO)
- like hybrid AO's but the MO involves the whole molecule

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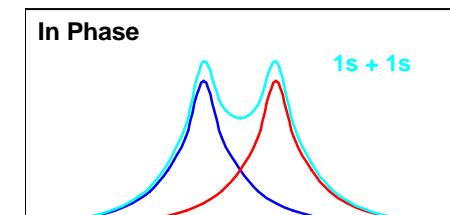
## Molecular orbital theory of $\text{H}_2$ - bonding

- $\text{H}_2$  molecule – interaction of two hydrogen 1s orbitals ( $\varphi_a$  and  $\varphi_b$ )

In phase interaction (same sign)

$$\psi_1 = (\varphi_a + \varphi_b)$$

→ Constructive interference



Animation shows the in phase interaction of the s orbitals as they are brought together



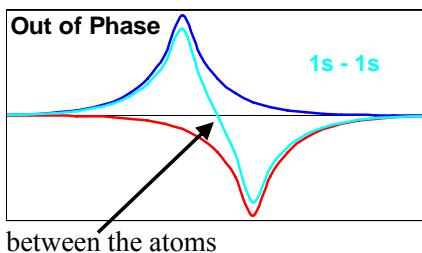
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## Molecular orbital theory of H<sub>2</sub> - antibonding

- H<sub>2</sub> molecule – interaction of two hydrogen 1s orbitals ( $\varphi_a$  and  $\varphi_b$ )

Out of phase interaction (opposite sign)

$$\psi_2 = (\varphi_a - \varphi_b) \\ \rightarrow \text{Destructive interference}$$



Animation shows the out of phase interaction (different colours) of the s orbitals as they are brought together

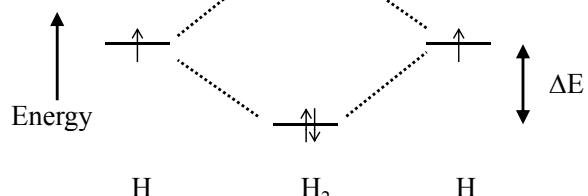


Interaction of 2 AO → 2 MO's – A general rule is that n AO → n MO's

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## Energy level diagram for H<sub>2</sub>

- Interference between AO wave functions → bonding
  - Constructively → bonding interaction
  - Destructively → anti-bonding interaction
- Energy level diagram represents this interaction
  - Two s orbitals interact to create a low energy bonding and high energy anti-bonding molecular orbital
  - Electrons fill the lowest energy orbital (same rules as for filling AO's)
  - Bonding energy =  $2 \Delta E$



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## Charge density associate with MO's in H<sub>2</sub>

- In phase interaction - charge density given by  $\psi^2$

$$\psi_1^2 = (\varphi_a + \varphi_b)^2 \quad \Rightarrow \quad \psi_1^2 = [\varphi_a]^2 + [\varphi_b]^2 + 2[\varphi_a\varphi_b]$$

- This gives an enhanced density where the AO's overlap between the atoms
- referred to as positive overlap and pull the atoms together ( $\sigma$  bonding)  $\psi_1 = \psi_\sigma$

- Out of phase interaction

$$\psi_2^2 = (\varphi_a - \varphi_b)^2 \quad \Rightarrow \quad \psi_2^2 = [\varphi_a]^2 + [\varphi_b]^2 - 2[\varphi_a\varphi_b]$$

- This leads to reduced density between the atoms
- referred to as negative overlap and pushes the atoms apart ( $\sigma^*$  anti-bonding)  $\psi_2 = \psi_{\sigma^*}$

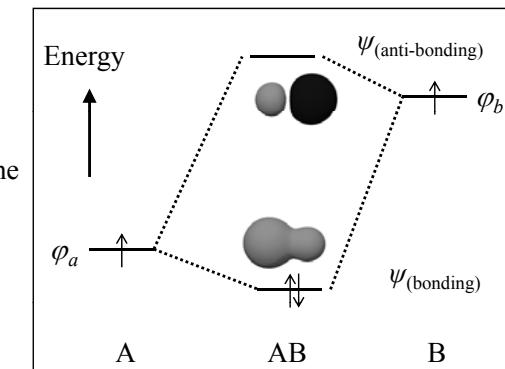
- Can not create electrons → New wave functions must be normalised to ensure probability in 1 !

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## What happens when the AO's have different energies?

- Hypothetical molecule where the two s orbitals have different energies  $E(\varphi_a) < E(\varphi_b)$
- What would the MO's be like
  - Bonding MO will be much more like the low energy orbital  $\varphi_a$
  - Anti-bonding MO will be much more like high energy orbital  $\varphi_b$
- We can say that the bonding MO is
 
$$\psi_\sigma = (C_a^\sigma \varphi_a + C_b^\sigma \varphi_b)$$
- Where the coefficients C, indicate the contribution of the AO to the MO

So for  $\psi_\sigma \quad C_a^\sigma > C_b^\sigma$



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## Linear Combination of Atomic Orbitals - LCAO

- We wrote an equation using coefficients for the contribution of AO's to the bonding MO, we can do the same for the anti-bonding MO

$$\psi_{\sigma} = (C_a^{\sigma} \varphi_a + C_b^{\sigma} \varphi_b)$$

$$\psi_{\sigma^*} = (C_a^{\sigma^*} \varphi_a - C_b^{\sigma^*} \varphi_b)$$

where the coefficients are different are reflect the contribution to each MO



$$C_a^{\sigma} > C_b^{\sigma}$$

$$C_a^{\sigma^*} < C_b^{\sigma^*}$$



- The sign can be absorbed into the coefficient and we can write all of the MO's in a general way

$$\psi_n = (C_a^n \varphi_a + C_b^n \varphi_b)$$

n = 1       $\psi_1 = (C_a^1 \varphi_a + C_b^1 \varphi_b)$        $C_a^1 = 0.8, C_b^1 = 0.2$

n = 2       $\psi_2 = (C_a^2 \varphi_a + C_b^2 \varphi_b)$        $C_a^2 = 0.2, C_b^2 = -0.8$

- The coefficients contains both phase (sign) of the AO's and how big their contribution (size) is to a particular MO

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## LCAO

- Generally we can write

$$\psi_n = \sum_{x=a...}^{NoAO's} C_x^n \varphi_x$$

x = a,b,c .... (all of the AO's in the molecule) n = 1,2,3.....(the resulting MO's)

- So MO(1) =  $\psi_1 = C_a^1 \varphi_a + C_b^1 \varphi_b + C_c^1 \varphi_c + C_d^1 \varphi_d + \dots$

$$MO(2) = \psi_2 = C_a^2 \varphi_a + C_b^2 \varphi_b + C_c^2 \varphi_c + C_d^2 \varphi_d + \dots$$

$$MO(3) = \psi_3 = C_a^3 \varphi_a + C_b^3 \varphi_b + C_c^3 \varphi_c + C_d^3 \varphi_d + \dots$$

$C_x^1$  - coefficients for MO(1),

$C_x^2$  - coefficients for MO(2) etc.

- And an examination of the coefficients tells us the bonding characteristics of the MO's

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## What interactions are possible

- We have seen how s orbitals interact – what about other orbitals

- If you have positive overlap reversing the sign  $\rightarrow$  negative overlap

E.g.  $s + s$  and  $p_x + p_x \rightarrow +ve$   
 $s - s$  and  $p_x - p_x \rightarrow -ve$

- Must define orientation and stick to it for all orbitals.

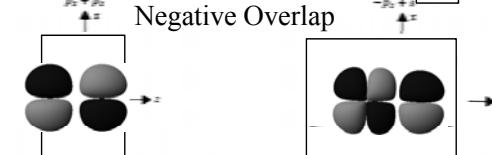
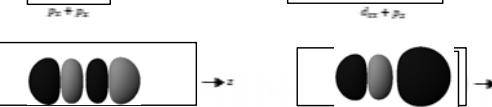
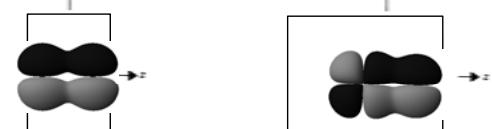
Thus

$$p_z + p_z \rightarrow -ve$$

$$p_z - p_z \rightarrow +ve$$

i.e. for sigma bond between

P orbital need opposite sign coefficients !

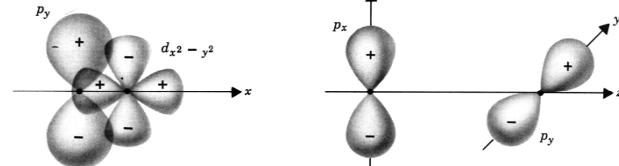
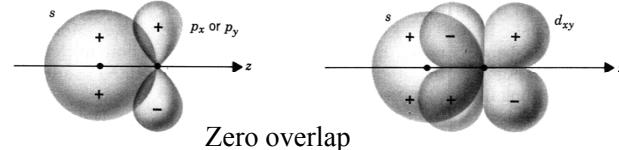


## What interactions are NOT possible

- Some orbital can not interact – they give rise to zero overlap

- Positive overlap (constructive interference) on one side is cancelled by negative overlap (destructive interference) on the other

- $s + p_x$  positive overlap above the axis is cancelled by negative overlap below  
– Same is true for the other interactions below



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# An introduction to Molecular Orbital Theory

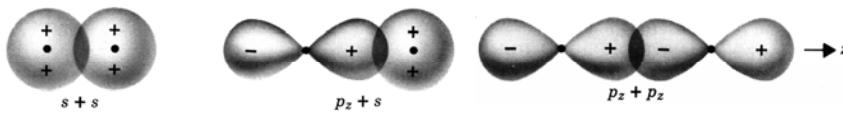
Lecture 5 Labelling MO's. 1<sup>st</sup> row homonuclear diatomics

Prof G. W. Watson  
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watsong@tcd.ie

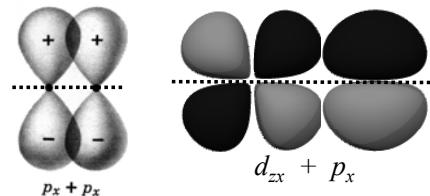
## Labelling molecular orbitals

### 1) Symmetry of bonding

$\sigma$  = spherical symmetry along the bond axis - same symmetry as s orbital no nodes pass through the bond axis (can be at right angles  $\rightarrow \sigma^*$ )



$\pi$  = one nodal plane which passes through the bond axis



$\delta$  = two nodal planes which pass through the bond axis

(end on  $d_{xy}$  or  $d_{x^2-y^2}$ )

## Last lecture

- Hybridisation  $\rightarrow$  combining AO's on one atom to  $\rightarrow$  hybrid orbitals  $\rightarrow$  hybridisation made consistent with structure

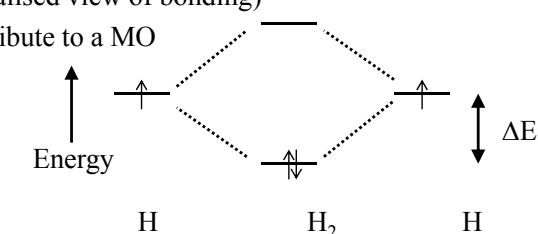
- Molecular orbital theory (delocalised view of bonding)

- LCAO – all AO's can contribute to a MO

- n AO's  $\rightarrow$  n MO's

- Filled in same way as AO's

- Example of  $H_2$



- Molecular orbitals for AO's of different energy

- Linear Combination of Atomic Orbitals (LCAO)

- Use of coefficient to describe (i) phase of interaction and (ii) size of contribution of a given AO

$$\psi_n = \sum_{x=a...}^{\text{No AO's}} C_x^n \phi_x$$

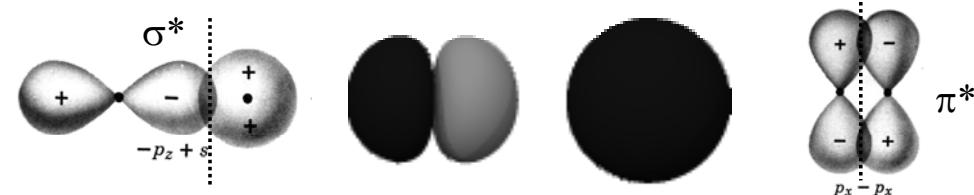
## Labelling molecular orbitals

### 2) bonding and anti-bonding (already met this label)

- Nothing if bonding (no nodes between bonded atoms)



- Additional \* if a nodal plane exists between the atoms, that is if the wavefunction changes sign as you go from one atom to the other.



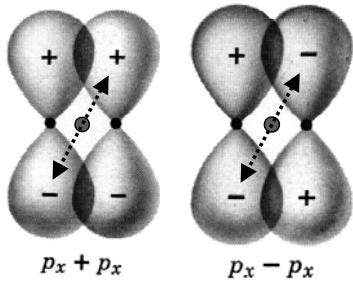
## Labelling molecular orbitals

- 3) Is there a centre of inversion? i.e. is it Centrosymmetric?

– The final label indicated whether the MO has a centre of inversion symmetry

$p_x + p_x$

As you go from one side of wave function through the centre of the bond the sign of the wavefunction reverses  
 $\rightarrow$  not centrosymmetric  
 $\rightarrow u = \text{ungerade or odd}$



$p_x - p_x$   
 Wave function does not change sign  
 $\rightarrow$  centrosymmetric  
 $\rightarrow g = \text{gerade or even}$

$\pi_g^*$

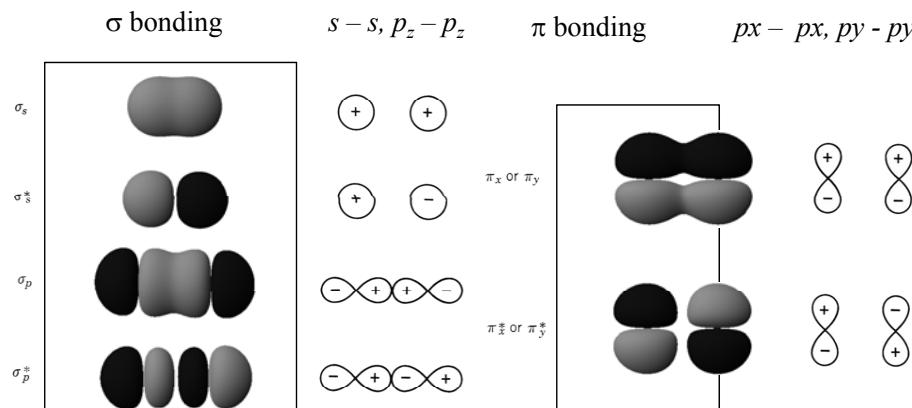
$\pi_u$

- MO's sometimes labelled with the type of AO forming them e.g.  $\sigma_s$  or  $\sigma_p$

## 2<sup>nd</sup> row homonuclear diatomics

- Li-Li  $\rightarrow$  Ne-Ne

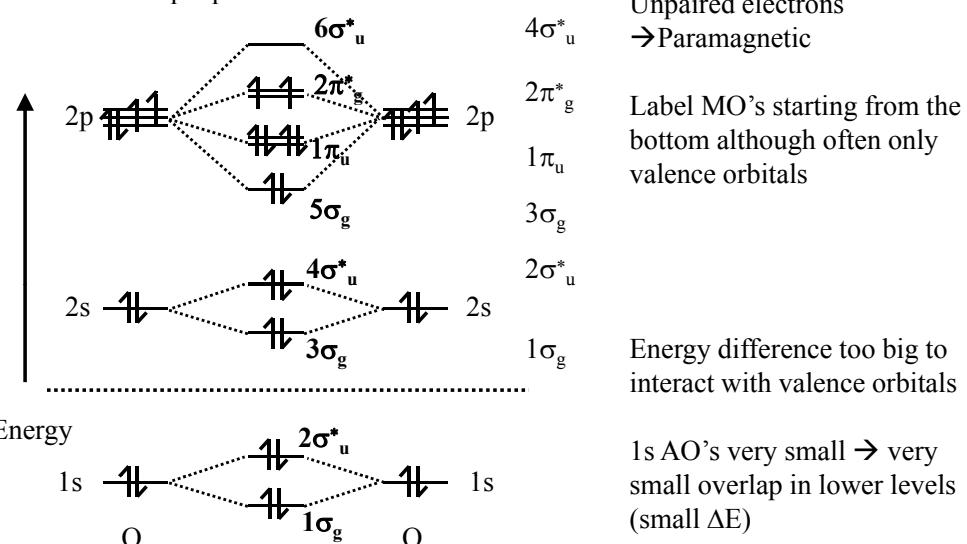
– Possible interactions between 1s, 2s and 2p



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## Energy level diagram for O<sub>2</sub>

- 2s and 2p energies sufficiently spaced to give little interaction
  - Simple picture of the MO



Unpaired electrons  
 $\rightarrow$  Paramagnetic

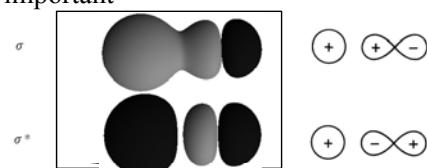
Label MO's starting from the bottom although often only valence orbitals

Energy difference too big to interact with valence orbitals  
 1s AO's very small  $\rightarrow$  very small overlap in lower levels (small  $\Delta E$ )

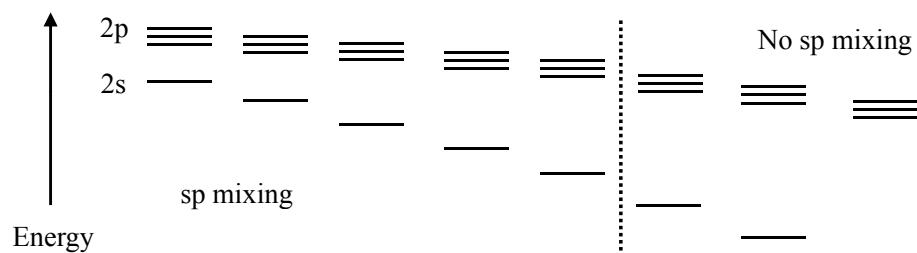
## Other possible interactions

- Will  $\sigma$  interactions between  $s$  and  $p_z$  be important

– Depends on energy difference between  $s$  and  $p_z$   
 – If large then no effect



- How does the energy of the 2s and 2p vary with Z (shielding / penetration)



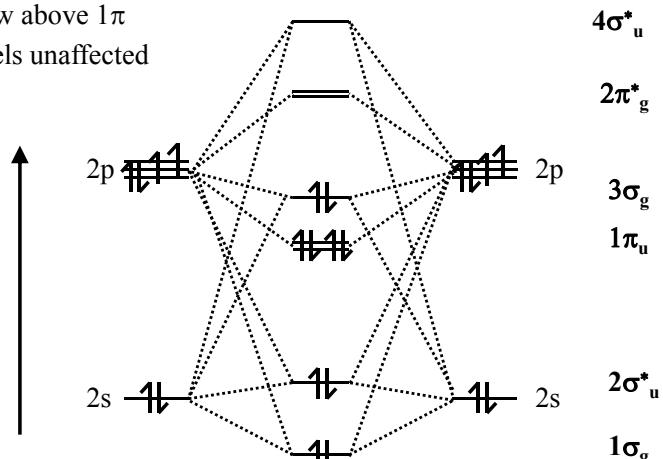
83

- Gap increases – 2p more effectively shielded - critical point between O and N

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## MO diagram for N<sub>2</sub>

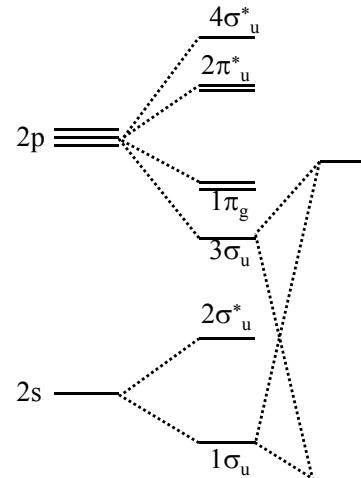
- 2s and 2p energies sufficiently close for interaction → more complex
  - 1σ and 2σ shift to lower energy
  - 3σ shifted 4σ shifted to high energy
  - 3σ now above 1π
  - π levels unaffected



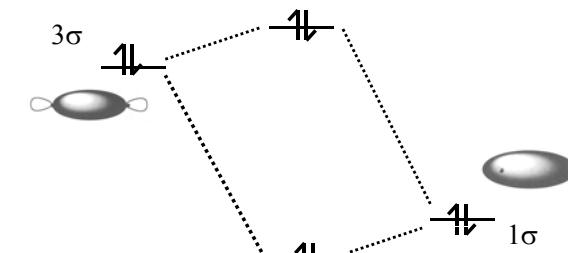
85

## Making sense of N<sub>2</sub>

- Take basic model for oxygen – no s p interaction
  - Examine how the MO's can interact
  - π and σ can not interact – zero overlap → π level remain the same
  - Examine σ – σ interactions



Bonding interactions can interact with each other 1σ<sub>u</sub> and 3σ<sub>u</sub>

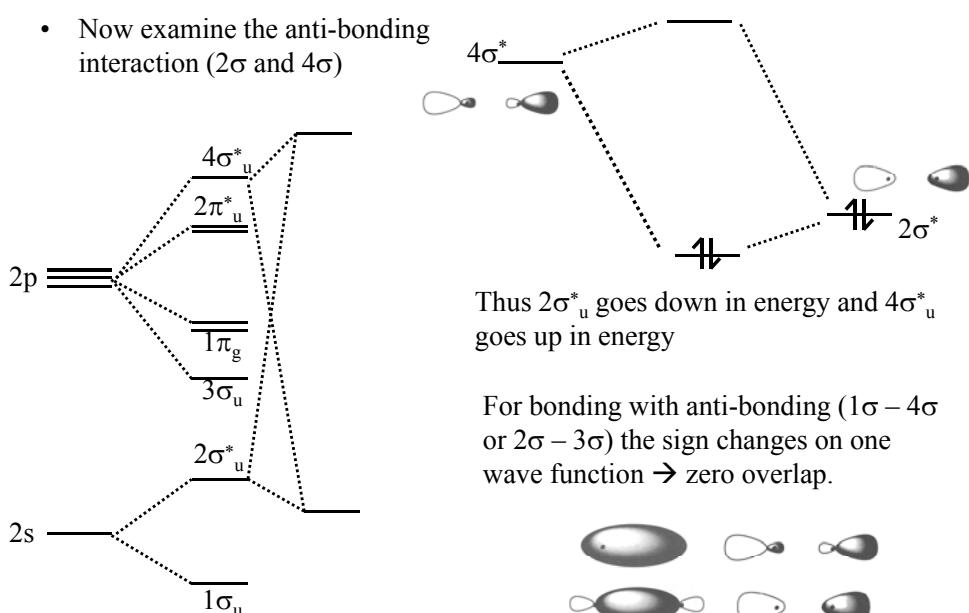


Thus 1σ<sub>u</sub> goes down in energy and 3σ<sub>u</sub> goes up in energy

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## Making sense of N<sub>2</sub>

- Now examine the anti-bonding interaction (2σ and 4σ)



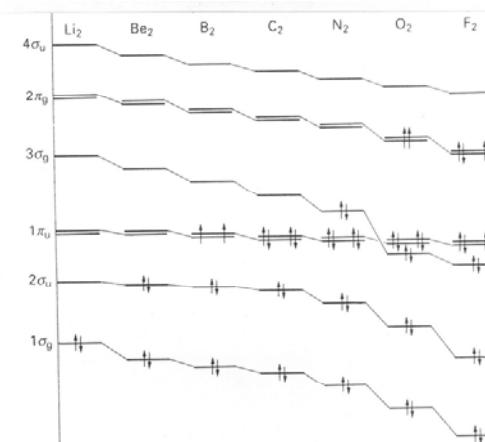
Thus 2σ\*<sub>u</sub> goes down in energy and 4σ\*<sub>u</sub> goes up in energy

For bonding with anti-bonding (1σ – 4σ or 2σ – 3σ) the sign changes on one wave function → zero overlap.

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## MO diagrams for 2<sup>nd</sup> row diatomics

- The effect of the overlap between 2s and 2p is greatest for the Li. The MO diagram changes systematically as you go across the periodic table



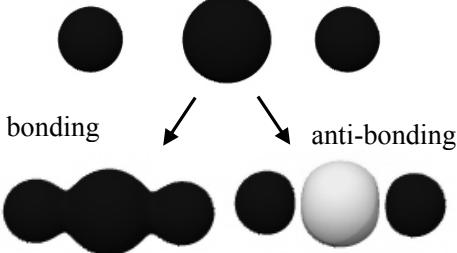
- s – p mixing → B<sub>2</sub> – paramagnetic and C<sub>2</sub> diamagnetic

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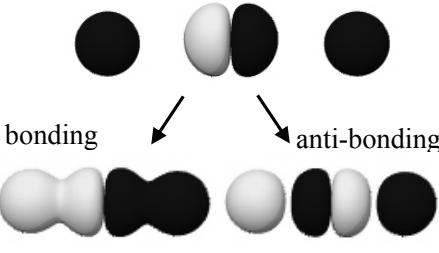
## MO treatment of BeH<sub>2</sub>

- VSEPR → linear molecule,
  - Be – 1s<sup>2</sup> 2s<sup>2</sup> 2p<sup>0</sup> H – 1s<sup>1</sup>
  - Examine interaction of 6 AO with each other
  - 2 H 1s, Be 2s and Be 2p<sub>x</sub>, Be 2p<sub>y</sub>, Be 2p<sub>z</sub> → 6 MO's

Interaction between H 1s and Be 2s



Interaction between H 1s and Be 2p<sub>z</sub>

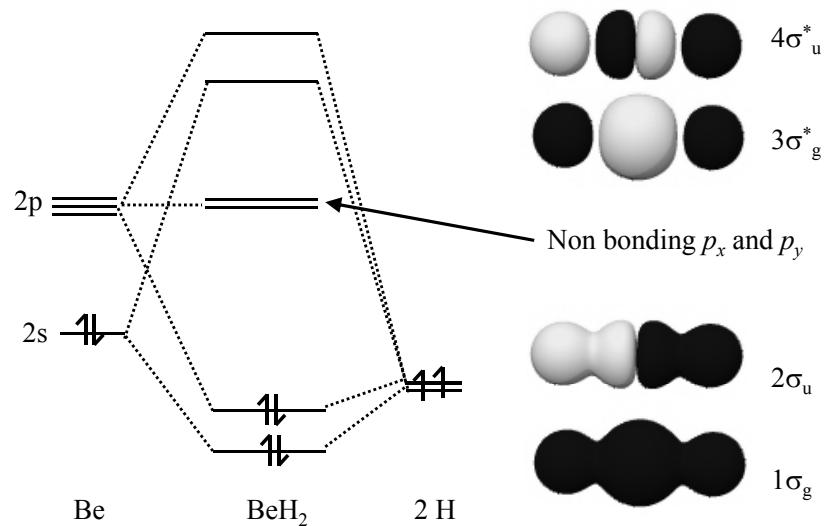


Each of these is delocalised over three atoms and can hold up to two electrons

p<sub>x</sub> and p<sub>y</sub> have zero overlap → non bonding

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## Energy level diagram for BeH<sub>2</sub>



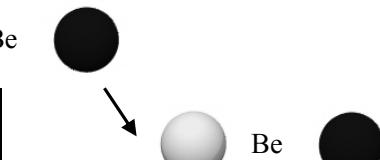
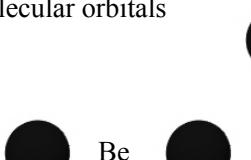
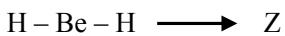
Compare these two MO's with no sp mixing with the localised model of two Equivalent bonds formed via sp hybridization

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## Alternative approach

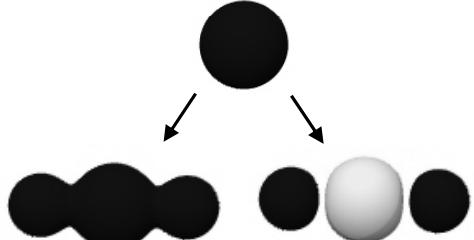
- Step wise approach (ligands first)

Mix hydrogen 1s orbitals first  
→ two Molecular orbitals

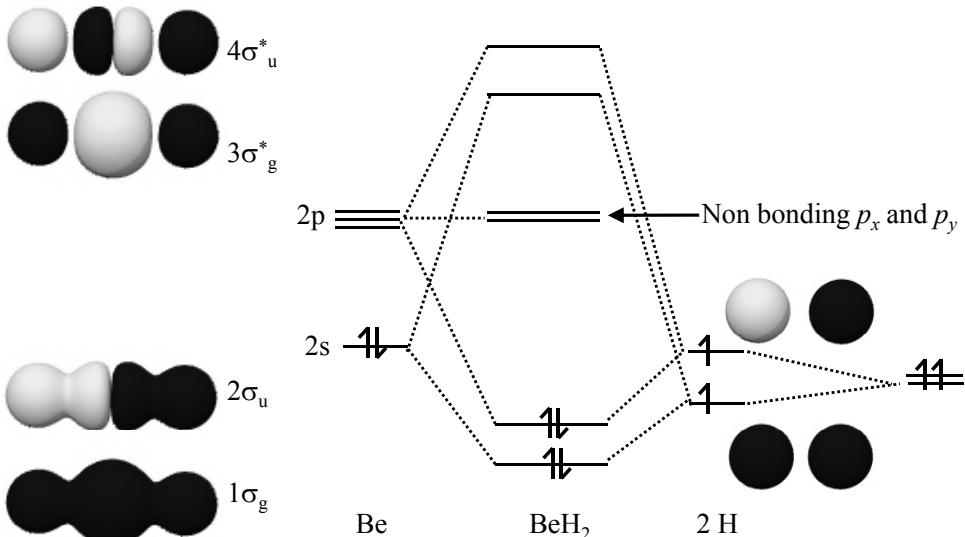


Then mix with s (zero overlap with p<sub>z</sub>)

Then mix with p<sub>z</sub> (zero overlap with s)



## Alternative energy level diagram for BeH<sub>2</sub>



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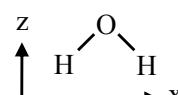
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# An introduction to Molecular Orbital Theory

Lecture 6 More complex molecules and CO bonding  
in transition metal complexes

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## MO treatment of H<sub>2</sub>O



- H<sub>2</sub>O is not linear – but why ?
    - We will examine the MO's for an non linear tri-atomic and find out.
    - What orbitals are involved – 2 H 1s + O 2s O 2p<sub>x</sub> O 2p<sub>y</sub> and O 2p<sub>z</sub>
  - Lets start by creating MO's from the hydrogen 1s orbitals.
- 
- Taking the in phase pair first- it will interact with the O 2s and O 2p<sub>z</sub> (zero overlap with O 2p<sub>x</sub> and O 2p<sub>y</sub>)
- 
- Problem - This is mixing three orbitals → must produce three orbital

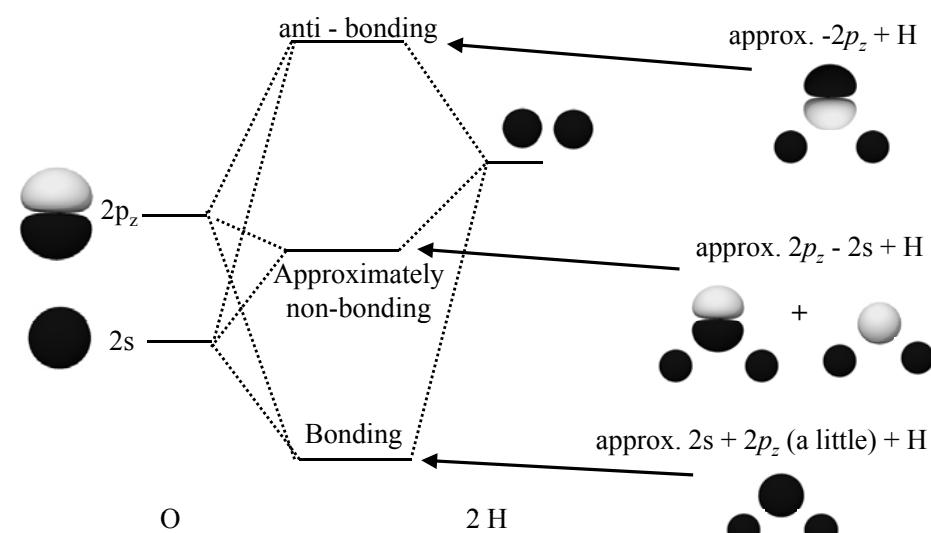
## Last lecture

- LCAO
  - Interaction of AO's with different energy → lower AO has bigger contribution
  - Representing contribution as coefficient
- AO interactions that were possible → MO's
  - positive, negative and zero overlap
  - labelling of MO's ( $\sigma$  /  $\pi$ , \*, g / u)
- 2<sup>nd</sup> row homo nuclear diatomics
  - 2s – 2p mixing occurs up to N → energy different too big after this (O<sub>2</sub>, F<sub>2</sub>)
  - Difference in MO diagram for N<sub>2</sub> and O<sub>2</sub>
- Molecular orbital treatment of BeH<sub>2</sub>

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## MO's of H<sub>2</sub>O

- Three orbitals → three MO's

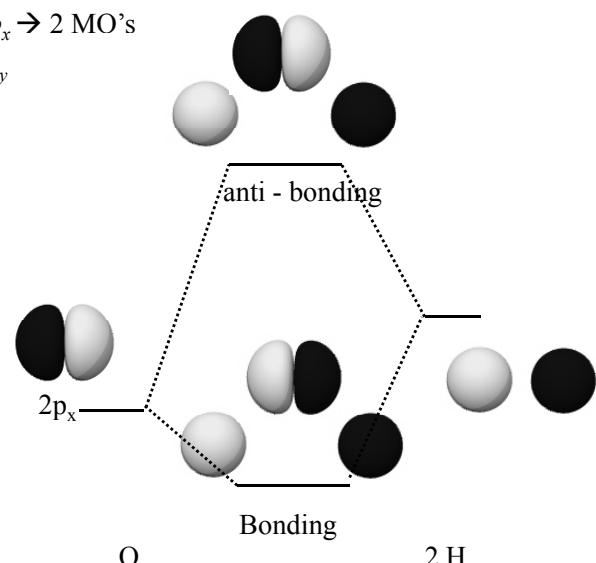


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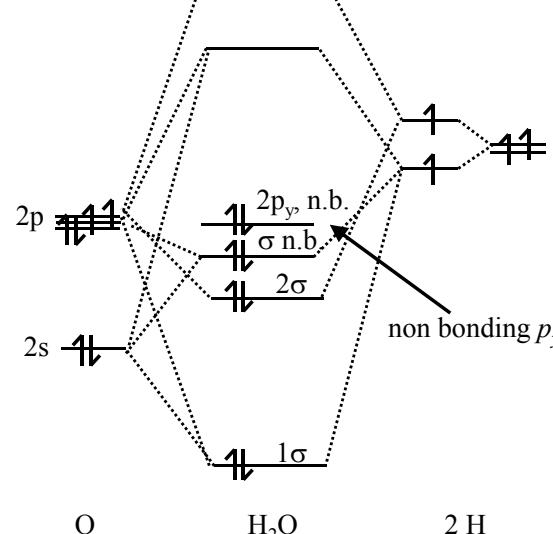
## MO's of $\text{H}_2\text{O}$

- Out of phase H  $1s$  orbitals
  - Only interact with  $p_x \rightarrow 2$  MO's
  - Zero overlap with  $p_y$



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## Energy level diagram for $\text{H}_2\text{O}$



There are not two lone pairs !



Very different to the VB concept of two identical  $\text{sp}^3$  filled orbital

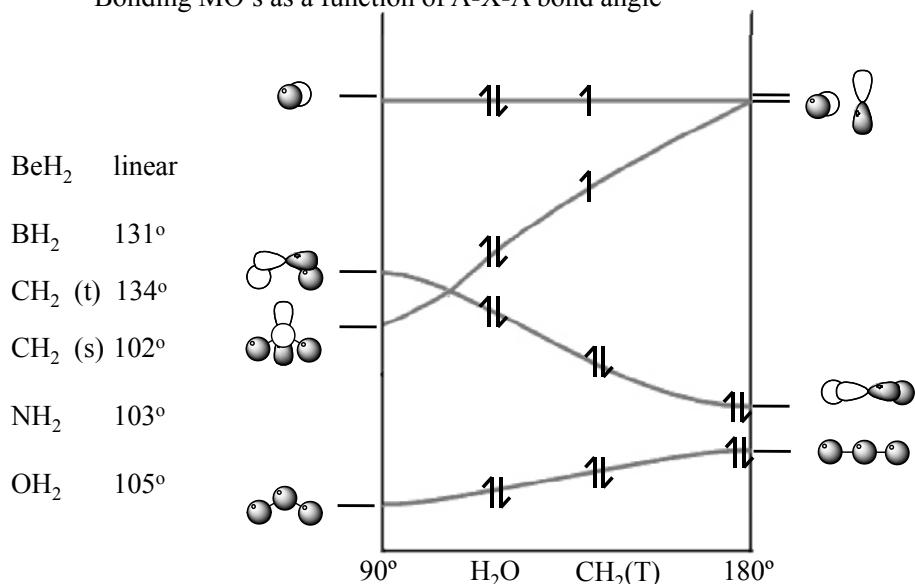


MO theory correct.

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## Comparison of $\text{H}_2\text{O}$ and $\text{BeH}_2$

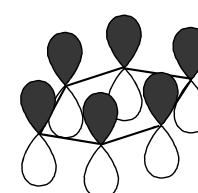
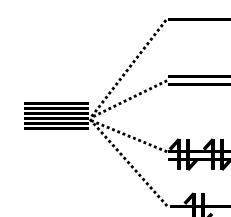
- Both cases of  $\text{XA}_2 \rightarrow$  same MO – different No of electrons
  - Bonding MO's as a function of A-X-A bond angle



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## $\pi$ MO's of Benzene

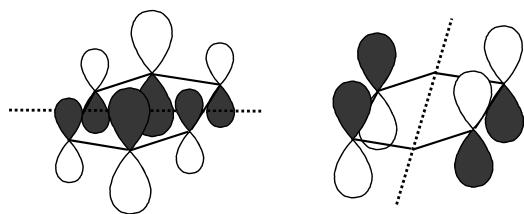
- $\pi$  bonding is more important for reactivity – independent of  $\sigma$  (zero overlap)
  - six  $p_x$  orbitals  $\rightarrow$  combine to form six MO's
- Different ways of arranging six  $p_x$  orbitals on a ring
  - Lowest energy – all in phase
  - Degenerate levels (1 nodal plane  $\rightarrow$  2 nodes)
  - Degenerate levels (2 nodal planes  $\rightarrow$  4 nodes)
  - Highest energy – all out of phase (3 nodal planes  $\rightarrow$  6 nodes)
  - Energy increases with number of nodes – as in AO's
  - Also the number of nodes on a ring must be even  $\rightarrow$  continuous wavefunction
- Lowest energy all in phase
  - All coefficients the same



100

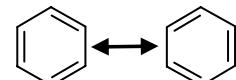
## $\pi$ MO's of Benzene

- Next occupied degenerate pair  $\rightarrow$  1 nodal plane
  - Two ways of doing this – between atoms and through a pair of atoms
- As the wavefunction goes through 0 (at the node) the smooth wavefunction has smaller coefficients next to the node zero at the node
- 2 electron per MO spread over 6 atoms
  - Compare with Lewis structure with individual double bonds
  - With local bonding have to resort to resonance structures to explain benzene



– As the wavefunction goes through 0 (at the node) the smooth wavefunction has smaller coefficients next to the node zero at the node

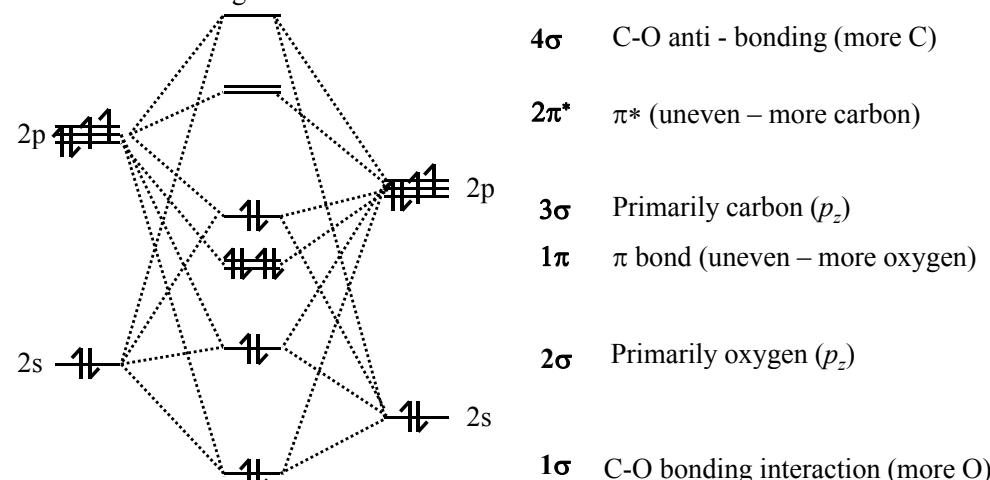
- 2 electron per MO spread over 6 atoms
  - Compare with Lewis structure with individual double bonds
  - With local bonding have to resort to resonance structures to explain benzene



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## MO diagram for CO

- Same orbitals as homo nuclear diatomics
  - different energies give rise to significant  $2s - 2p$  mixing
  - confusing set of orbitals



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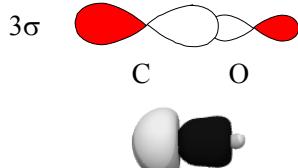
## The HOMO and LUMO of CO

- For chemical reaction the HOMO (Highest Occupied Molecular Orbital) and the LUMO (Lowest unoccupied Molecular Orbital) are the most important.

HOMO –  $3\sigma$

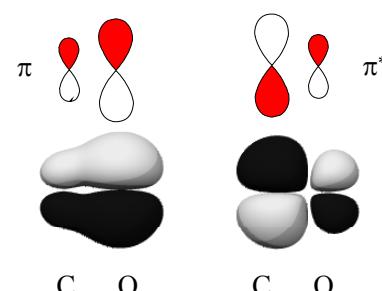
low energy Oxygen orbitals  
makes  $2\sigma \rightarrow$  mainly O p<sub>z</sub>  
 $\rightarrow$  in  $3\sigma$  mainly C p<sub>z</sub>

Some anti-bonding mixes in  
due to sp mixing



LUMO –  $2\pi^*$

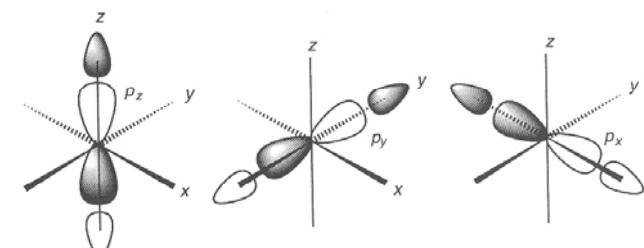
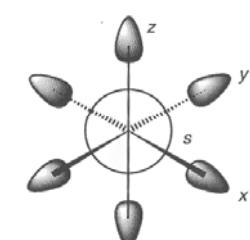
Comes from standard  $\pi$  interaction  
however lower oxygen orbital  
means  $\pi$  has more oxygen and  
 $\pi^*$  more carbon



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## Interaction of the CO $3\sigma$ with $d$ orbitals

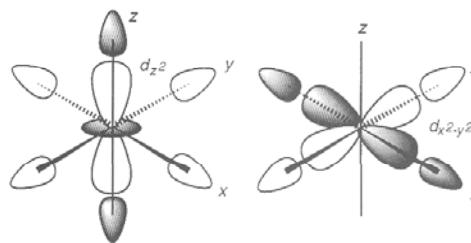
- Three sets of interaction based on symmetry of ligand AO's
  - $a_{1g}$  all ligand AO's in phase
    - Interaction with s orbital  $\rightarrow$  1
  - $t_{1u}$  ligands in one axis contribute
    - With opposite phase – one nodal plane
    - Interaction with p orbitals  $\rightarrow$  3



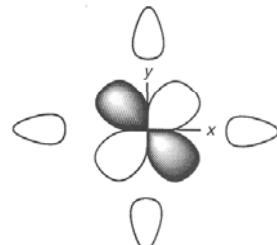
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## Interaction of the CO $3\sigma$ with $d$ orbitals

- $e_g$  ligand phases have two nodal planes
  - Interact with  $d_{z^2}$ ,  $d_{x^2-y^2}$   $\rightarrow 2$



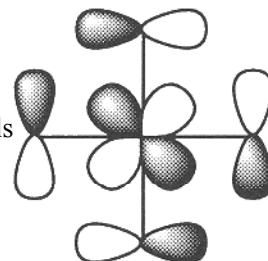
- Three remaining  $d$  orbitals point between ligands
  - zero overlap ( $t_{2g}$ )



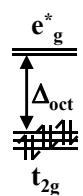
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## $\pi$ interactions with TMs

- Orbitals with  $\pi$  character can interact with the  $t_{2g}$  d orbitals
  - Must be correct symmetry ( $t_{2g}$ )  $\rightarrow 3$  arrangements possible using  $d_{xy}$ ,  $d_{yx}$ ,  $d_{xz}$
- Two extreme situations
  - Ligand orbitals are low energy and filled (e.g. F)
  - Ligand orbitals are high energy and empty (e.g. CO)



Interaction with high energy empty ligand orbitals



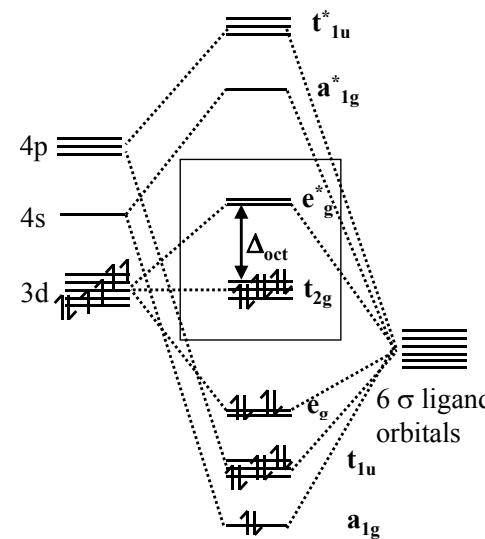
Interaction with low energy filled ligand orbitals



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## MO diagram for Tm ( $\sigma$ -L)<sub>6</sub>

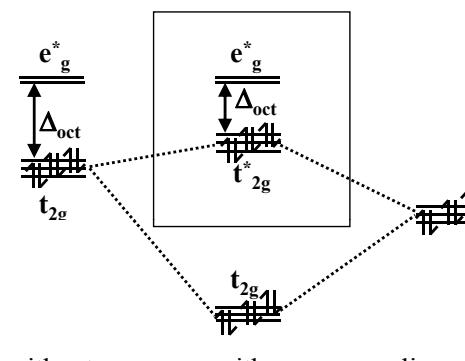
- Electrons from filled  $\sigma$  orbitals on the ligands fill all the bonding orbitals
- $d$  electrons fill  $t_{2g}$  (non bonding) and  $e_g^*$  (antibonding)
- Example is  $d^6$  – e.g.  $\text{Co}^{3+}$
- These are the orbitals considered in ligand field theory. Note the  $e_g^*$  is anti-bonding !
- What really decided  $\Delta_{\text{oct}}$  is the  $\pi$  interaction



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## Low ligand field situation

- Ligand orbitals are low energy and filled (e.g. F)
  - Filled orbitals interact in a  $\pi$  fashion
  - Bonding combinations are reduced in energy and filled (like ligand orbitals)
  - Antibonding combination are raised in energy and filled (like d orbitals)

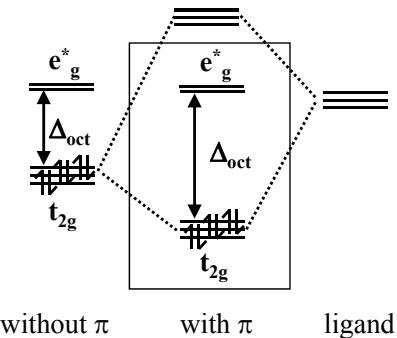


- Strong interaction with filled orbitals with  $\pi$  interaction leads to reduction in  $\Delta_{\text{oct}}$  (box shows the orbitals considered in ligand field theory)

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## High ligand field situation

- Ligand orbitals are high energy and empty (e.g. CO  $2\pi^*$ )
  - Filled orbitals interact in a  $\pi$  fashion
  - Bonding combinations are reduced in energy (like d orbitals)
  - Antibonding combination are raised in energy and empty (like ligand orbitals)



- Strong interaction with empty orbitals with  $\pi$  interaction leads to increase in  $\Delta_{\text{oct}}$  (box shows the orbitals considered in ligand field theory)

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## Tutorial 2 – part a

- Explain the MO approach for the interaction of
  - two s orbitals of identical energy
  - two s orbitals of slightly different energy
  - two s orbital of very different energy.
- Consider the bonding in the molecule O<sub>2</sub>
  - Draw a Lewis structure for O<sub>2</sub>
  - Determine the hybridization
  - Perform an MO treatment of O<sub>2</sub>
    - What orbitals are involved?
    - what interactions are possible?
    - what do the resulting MO's look like?
    - sketch an MO energy level diagram.
  - What difference are there in the details of the bonding diagram between the Lewis and MO treatments

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## Tutorial 2 – part b

- Consider the molecule BeH<sub>2</sub>
  - Draw a Lewis structure
  - Determine the hybridization
  - Perform an MO treatment of O<sub>2</sub>
    - What orbitals are involved.
    - Generate appropriate 'ligand' MO's and interactions with the central atom
    - what do the resulting MO's look like?
    - sketch an MO energy level diagram.
  - What difference are there in the details of the bonding diagram between the Lewis and MO treatments
- Perform the same analysis for BeH<sub>2</sub>, HF, BH<sub>3</sub>, and CH<sub>4</sub>
- Use molecular orbital theory to explain
  - The splitting of the d orbitals by sigma interactions with ligands
  - The effect of  $\pi$  interaction on the ligand field strength.

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**THE END**

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