# ENGINEERING MECHANICS

## ↳ INTRODUCTION AND VECTORS

Branch of science which deals with force and its effects is called mechanics.

SOLID MECHANICS

- RIGID BODY MECHANICS
- DEFORMABLE BODY ANALYSIS

Eg:→ [20kg] → 10N

10N ← [20kg] → 10N

10N ← [20kg] → 30N

10N ← [20kg] → 10N → 20N

RIGID + DEFORMABLE

### RIGID BODY ANALYSIS

- If the distance b/w two points within the body before and after the application of load remains constant that is R.B.A.
- And the branch of Engineering which deals with Rigid body Analysis is Engineering Mechanics.

### DEFORMABLE BODY ANALYSIS

- If the distance b/w two points within the body before & after changes is Deformable Body Analysis.
- And the branch which deals with deformable body ANA is Strength of material.

# Types of Physical Quantity

- Fundamental
  - → L, M, T, E, T, L, A
    - L → Length
    - M → Mass
    - T → Temp
    - E → Electric current
    - T → Time
    - L → Luminous Intensity
    - A → Amount of Substance
- Derived

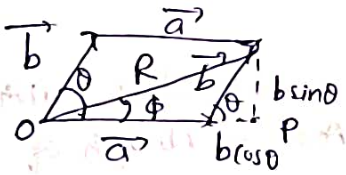
Types of Physical Quantity
- → SCALAR (Magnitude)
- → VECTOR (Magnitude + Direction) + obeys parallelogram law of addition

↓

E.g => current have direction but do not obey ||gm law of add. And it is a scalar quantity.

# In mechanics we only study about force in vector.

# RESULTANT OF VECTORS




$$R^2 = (b\sin\theta)^2 + (a+b\cos\theta)^2$$

$$R^2 = b^2\sin^2\theta + a^2 + b^2\cos^2\theta + 2ab\cos\theta$$

$$R^2 = a^2 + b^2 + 2ab\cos\theta$$

$$\boxed{R = \sqrt{a^2+b^2+2ab\cos\theta}}$$

$$\boxed{\tan\phi = \frac{b\sin\theta}{a+b\cos\theta}}$$

# Addition of vectors

(i) without $\hat{i}, \hat{j}$ direction

$\vec{OA} = 5 @ -36.86°$

$\vec{OB} = 5 @ 53.13°$

$\vec{OA} + \vec{OB} = \vec{OR}$

$|\vec{OR}| = \sqrt{(\vec{OA})^2 + (\vec{OB})^2 + 2(\vec{OA})(\vec{OB})\cos\theta}$

$= \sqrt{25 + 25 + 2 \times 5 \times 5 \times \cos 90°}$

$\boxed{|\vec{OR}| = \sqrt{50}}$

(ii) with $\hat{i}, \hat{j}$.

~~$\vec{OA} = 3\hat{i} + 4\hat{j}$~~

~~$\vec{OB} = 4\hat{i} - 3\hat{j}$~~

~~$\vec{OR} = \vec{OA} + \vec{OB}$~~

$\boxed{\cancel{\vec{OR} = 7\hat{i} + \hat{j}}}$

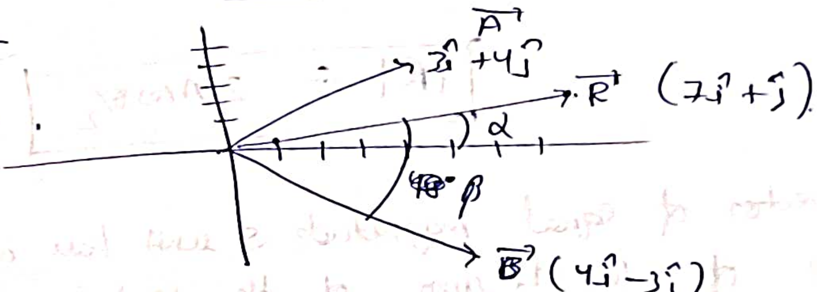
~~$|\vec{OR}| = \sqrt{(7)^2 + (1)^2}$~~

$\boxed{\cancel{|\vec{OR}| = \sqrt{50}}}$

~~$\tan\alpha = 1/7$~~

~~$\alpha = \tan^{-1}(1/7)$~~

$\boxed{\cancel{\alpha = 8.13°}}$




$\vec{OR} = 7\hat{i} + 3\hat{j} = \vec{OA} + \vec{OB}$.

$\boxed{\cancel{\tan^{-1}(3/4) = 36.86}}$ → Direction

$\boxed{\alpha = \tan^{-1}(1/7)}$

$\beta = \tan^{-1}(3/4)$

$\boxed{\beta = 36.86}$

# DOT PRODUCT (SCALAR PRODUCT)

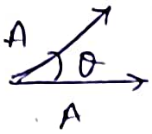
$$\boxed{\vec{a}\cdot\vec{b} = |a|\,|b|\cos\theta}$$

# CROSS PRODUCT (Vector PRODUCT)

$$\boxed{\vec{a}\times\vec{b} = |a|\,|b|\sin\theta\ \hat{n}.}$$

Q1) Two same vectors of magnitude A with an angle θ. find the magnitude and direction of the resultant.

Ans)




$$R = \sqrt{A^2 + A^2 + 2A.A\cos\theta}$$

$$= \sqrt{2A^2 + 2A^2\cos\theta}$$

$$= A\sqrt{2(\cos\theta + 1)}$$

$$= \sqrt{2}A\sqrt{\cos\theta + 1}$$

$[\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\quad = \cos^2\theta - (1 - \cos^2\theta)$

$\quad = 2\cos^2\theta - 1]$

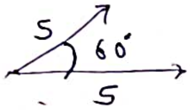
$\left[\cancel{\dfrac{\cos 2\theta + 1}{2} = \cos 2\theta + 1}\right]$

$[\cos 2\theta + 1 = 2\cos^2\theta]$

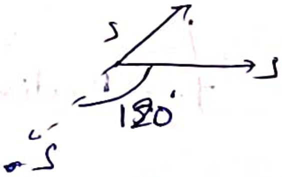
$$\boxed{|R| = 2A\cos\theta/2}$$

Q2) Two vector of equal magnitude 5 unit have an angle 60° b/w them. find the magnitude of (a) The sum of the vectors and (b) The difference of the vectors.

Ans)




(a)




$$|\text{sum}| = \sqrt{25 + 25 + 2(5)(5)\cos 60°}$$

$$= \sqrt{75}$$

$$= 5\sqrt{3}$$

$$|\text{Diff}| = \sqrt{25 + 25 + 2(5)(5)\cos 120°}$$

$$= \sqrt{25 + 25 - 25}$$

$$= 5$$

↳ FORCE AND MOMENT - EQUILLIBRIUM

## # Degree of freedom in 2-D

In 2-D if any motion is out of the plane then it is not counted.

$\Delta \Rightarrow$ Moment across axis plane (Left & Right), $\theta \Rightarrow$ Moment across own axis rotation.

X-Y (PLANE) $\longrightarrow$ $\Delta_x, \Delta_y$ & $\theta_z$

Y-Z $\longrightarrow$ $\Delta_y, \Delta_z$ & $\theta_x$

Z-X $\longrightarrow$ $\Delta_z, \Delta_x$ & $\theta_y$.

## # Degree of freedom in 3-D

All six rotation allow

$\Delta_{x,y,z}$ & $\theta_{x,y,z}$.

## # $Eq^n$ of static equilibrium (2-D)

To prevent,

$$\Delta_x \longrightarrow \Sigma F_x = 0$$

$$\Delta_y \longrightarrow \Sigma F_y = 0$$

$$\theta_z \longrightarrow \Sigma M_z = 0.$$

## # $Eq^n$ of static equilibrium (3-D)

To prevent,

$$\Delta x \longrightarrow \Sigma F_x = 0$$

$$\Delta y \longrightarrow \Sigma F_y = 0$$

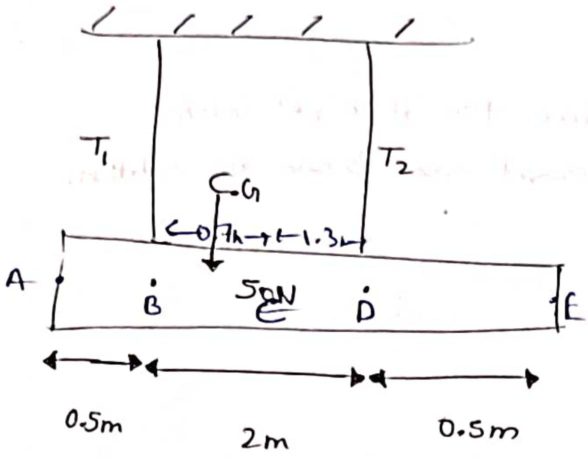
$$\Delta z \longrightarrow \Sigma F_z = 0$$

$$\theta_x \longrightarrow \Sigma M_x = 0$$

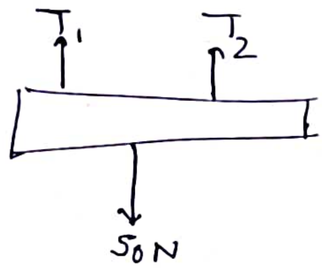
$$\theta_y \longrightarrow \Sigma M_y = 0$$

$$\theta_z \longrightarrow \Sigma M_z = 0.$$

Q1) Find Tension of both Cables i.e $T_1$ and $T_2$.




Ans) FBD




$\Sigma F_x = 0$ (No force)

$\Sigma F_y = 0 \Rightarrow \quad T_1 + T_2 - 50 = 0$

$$\boxed{T_1 + T_2 = 50}$$

$\Sigma M_z = 0 \Rightarrow$ Taking point B as reference

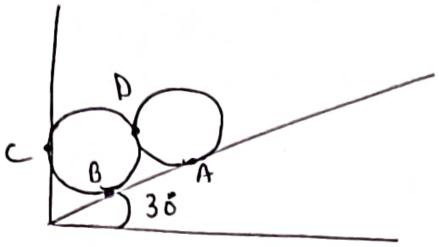
$$\cancel{T_1 \times (0.5)} \quad (T_1 \times 0) - (50 \times 0.7) + (T_2 \times 2) = 0$$

$$T_2 = \frac{25 \times 0.7}{10}$$

$$\boxed{T_2 = 32.5}$$

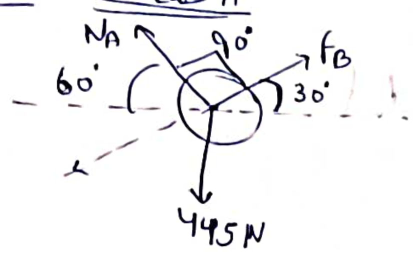
And $\boxed{T_1 = 17.5}$

Q2) Two identical rollers each of weight $Q = 445N$ are supported by an inclined plane & a vertical wall as shown. Assuming smooth surfaces. Find the reactions induced at the points of support A B & C.




Ans)

BLOCK A




$\Sigma F_x = 0$

$-N_A \cos 60° + F_B \cos 30° = 0$

$F_B \cos 30° = N_A \cos 60°$

$\frac{F_B}{N_A} = \frac{\cos 60°}{\cos 30°} = \frac{\sin 30°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = 1/\sqrt{3}$

$\boxed{F_B = \frac{N_A}{\sqrt{3}}}$

$\Sigma F_y = 0$

$N_A \sin 30° + F_B \sin 60° - 445 = 0$

$\frac{N_A}{2} + \frac{F_B\sqrt{3}}{2} = 445$

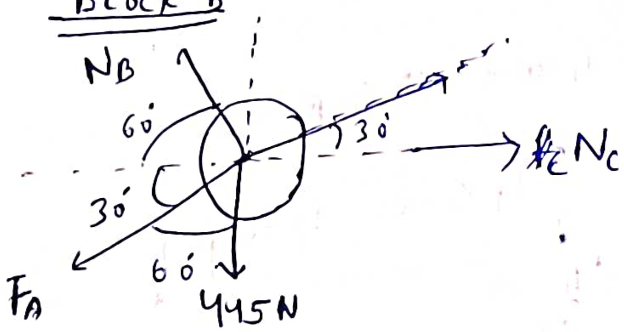
$\boxed{N_A + F_B\sqrt{3} = 890}$

$2F_B\sqrt{3} = 890$

$\boxed{F_B = 445/\sqrt{3}}$

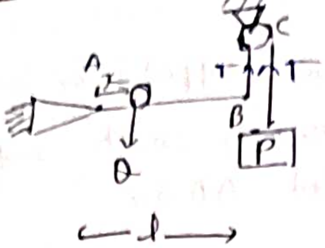
$\boxed{N_A = 445}$

BLOCK B




$\Sigma F_x = 0 \Rightarrow -N_B \cos 60° - F_A \cos 30° + N_C = 0$

$\Sigma F_y = 0 \Rightarrow N_B \sin 60° - F_A \cos 60° - 445 = 0$

Q3) 


Find x? If system remains in equilibrium.

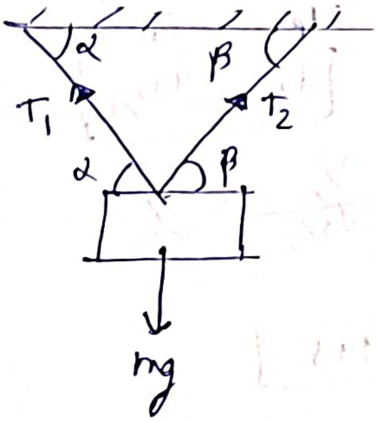
Ans) Taking Point A as refrence

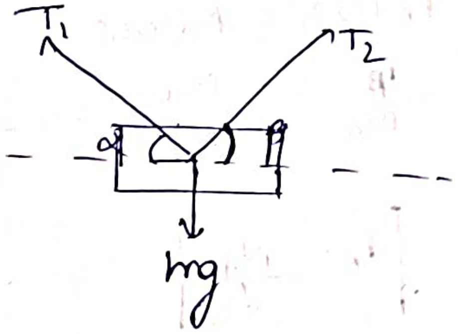
$$\Sigma M_z = 0$$

$$(Q \times x) - P(l) = 0$$

$$\boxed{x = \frac{Pl}{Q}} \qquad \boxed{T = P}$$

Q4) A body of mass m is suspended by two strings making angles $\alpha$ and $\beta$ with horizontal. Tension in two strings are

Ans) 


$$\Sigma F_x = 0$$

$$-T_1 \cos\alpha + T_2 \cos\beta = 0$$

$$\boxed{T_1 \cos\alpha = T_2 \cos\beta}$$

$$\Sigma F_y = 0$$

$$T_1 \sin\alpha + T_2 \sin\beta - mg = 0$$

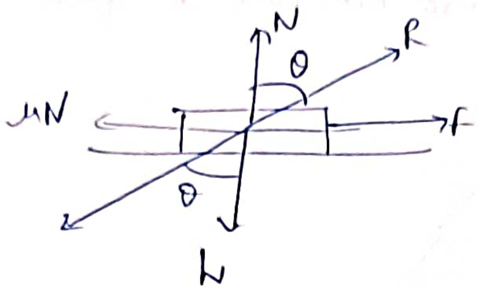
$$T_1 \sin\alpha + T_2 \sin\beta = mg$$

$$\frac{T_1}{T_2} \sin\alpha + \sin\beta = \frac{mg}{T_2}$$

$$\boxed{\frac{\cos\beta}{\cos\alpha} \sin\alpha + \sin\beta = \frac{mg}{T_2}}$$

↳ FRICTION

## Angle of friction




θ ⇒ Angle of friction

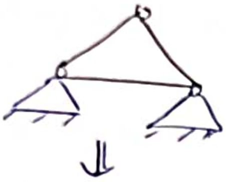
$$\tan\theta = \frac{\mu \cancel{N}}{\cancel{N}} = \mu$$

$$\boxed{\theta = \tan^{-1}\mu}$$

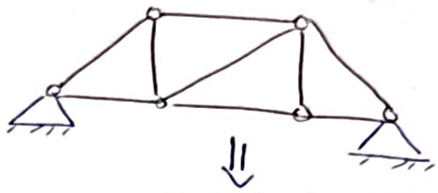
## Angle of Repose

# TRUSSES-1

Truss is a assembly of members which are connected with joints to form a rigid structure.



SIMPLE TRUSS



RIGID TRUSS

## # Assumption

1) All the joints are pin/hinge connected
2) All the loadings must be applied at joints only.
3) Self weight of the member is to be ignored.
4) Size of the member should be perfect.

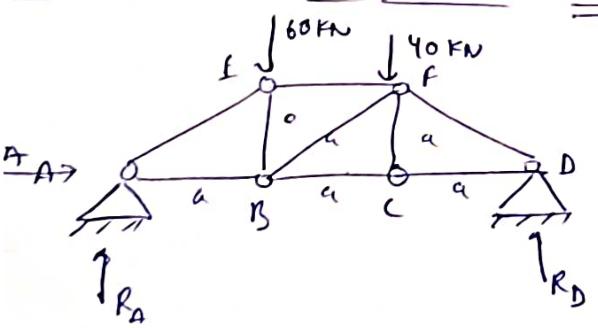
All the above assumption are used to ignore bending of members & load should be applied axial. force

## # STABILITY OF TRUSSES

$\boxed{m = 2J - 3}$ $(J \geq 3)$ ⇓ for 2-D

$\boxed{m = 3J - 6}$ $(J \geq 4)$ ⇓ for 3-D

## # CALCULATION OF EXTERNAL REACTION




$\Sigma F_x = 0$

$\boxed{H_A = 0}$

$\Sigma F_y = 0$

$R_A + R_D - 60 - 40 = 0$

$\boxed{R_A + R_D = 100}$

$\Sigma M_{z(A)} = 0$

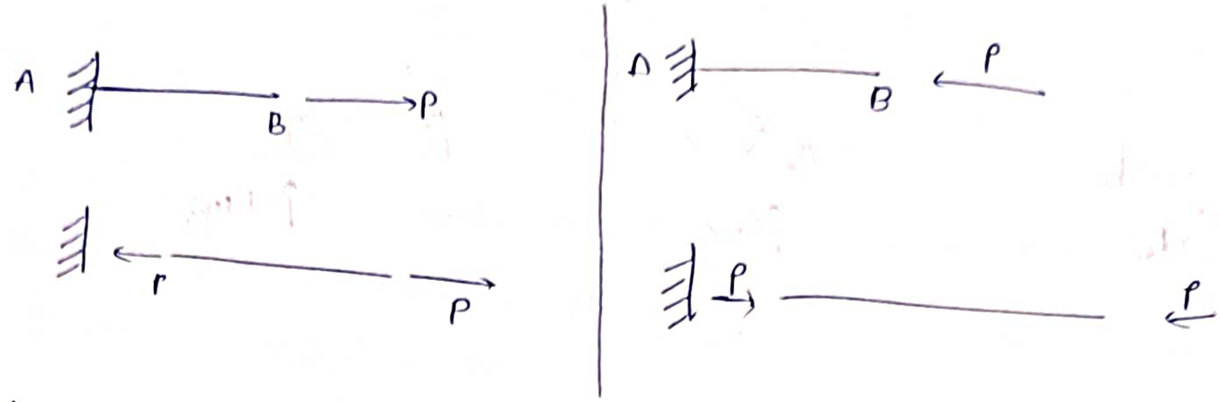
$(60 \times a) + (40 \times 2a) - (R_D \times 3a) = 0$

$60a + 80a - 3R_D a$

$140\cancel{a} = 3R_D \cancel{a}$

$\boxed{R_D = 140/3}$

# SIGN CONVENTION




NOTE ⇒ All the tensile forces are going away from joint.
All the compressive forces are coming towards the joint.

# ANALYSIS OF TRUSSES

1) Method of joints
2) Method of Section.

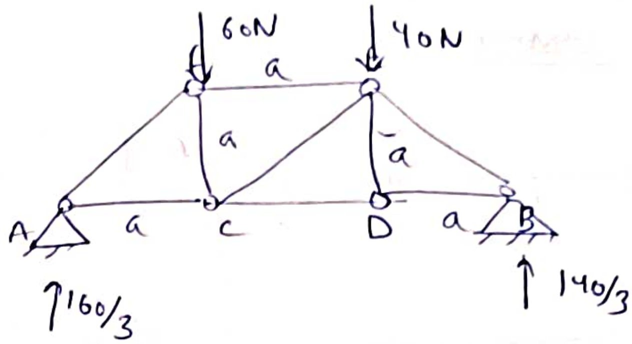
↳ METHOD OF JOINTS

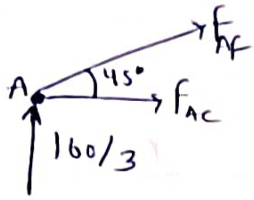
1) Find the external reactions by using static force & moment equilibrium.
2) Identify the joint which has at most two unknown forces.
3) Draw the free body diagram of identified joint & apply $\Sigma F_x = 0$ & $\Sigma F_y = 0$ in order to find unknown forces.
4) Repeat the procedure till all unknown forces are obtained.

NOTE :-

- ~~If~~ Take all forces as Tensile by default.
- If after applying joint equilibrium condition, force comes negative It means final force is compression instead of tension.

FBD of Joint A




$\sum f_x = 0$

$F_{AC} + F_{AF}\cos 45^\circ = 0$

$F_{AC} = -F_{AF}\cos 45^\circ$

$= \frac{160\cancel{\sqrt{2}}}{3} \times \frac{\cancel{\sqrt{2}}}{\sqrt{2}}$

$\boxed{F_{AC} = 160/3}$

$\boxed{F_{AC} = 53.33}$

$\sum f_y = 0$

$160/3 + F_{AF}\sin 45^\circ = 0$

$\frac{F_{AF}}{\sqrt{2}} = \frac{-160}{3}$

$\cancel{\boxed{F_{AF} = \frac{-320}{3}}}$

$\boxed{F_{AF} = \frac{-160\sqrt{2}}{3}}$

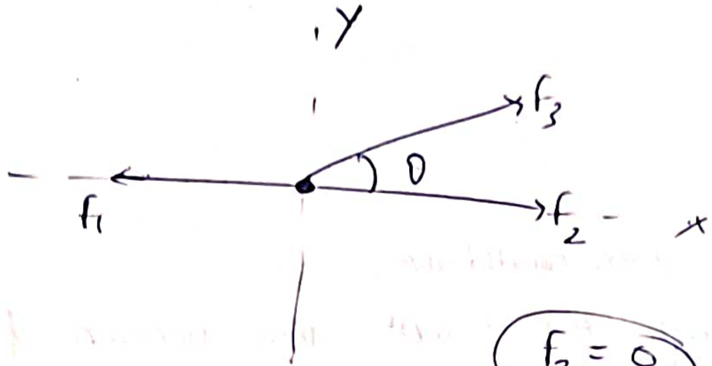
$\boxed{F_{AF} = -75.42}$

Similarly find other forces on other joints.

# TRUSSES - 2

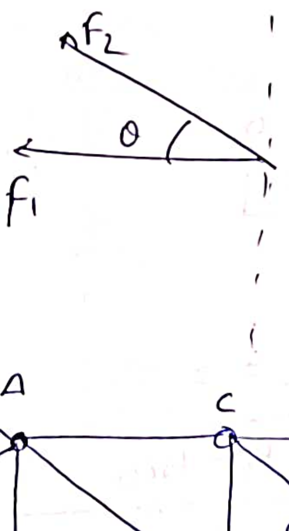
## # Zero forces Members

1) If the joint has 3 members out of which 2 are collinear and then the third member (non-collinear) will carry zero force if no external forces is applied at that joint.



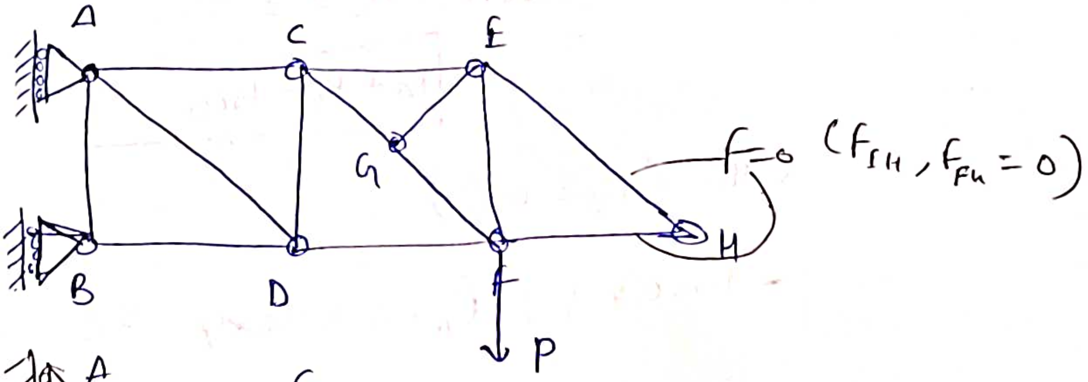

$\boxed{f_3 = 0}$

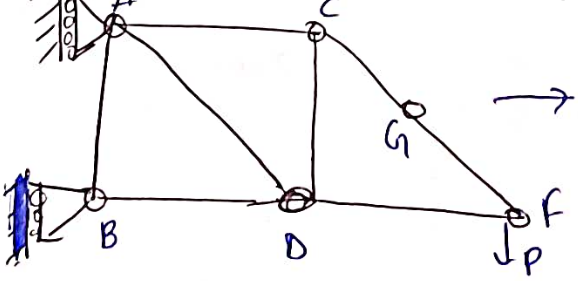
2) If the joint has 2 members (non-collinear) & no external force is applied at that joint then both the members will carry zero forces members




$\boxed{f_1, f_2 = 0}$

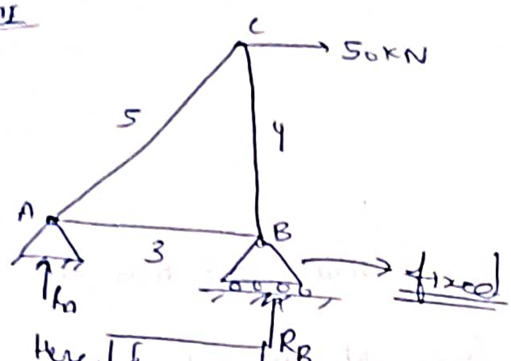
E.g =>




→ After removing zero force member.

NOTE



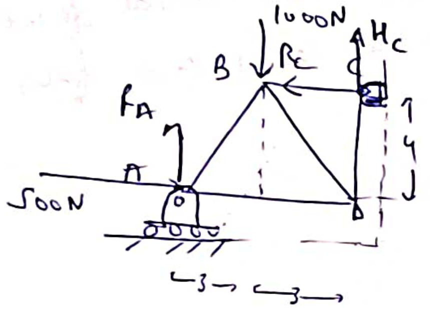

Here, $\boxed{F_{BA} = 0}$

→ **METHOD OF SECTION**

STEPS

Find the external reactions by using static equilibrium.
Cut the truss into two sections such that at most three unknown forces
Draw the FBD of either part & apply $\Sigma F_x = 0$, $\Sigma F_y = 0$ & $\Sigma M_z = 0$ in order to find unknown forces

Q) Calculate force in each member of truss




(i) $\Sigma F_x = 0$

$500N - R_C = 0$

$\boxed{R_C = 500N}$

(ii) $\Sigma F_y = 0$

$R_A - 1000N + H_C = 0$

$\boxed{R_A + H_C = 1000}$

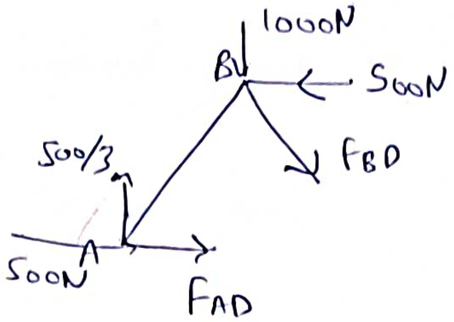
(iii) $\Sigma M_z = 0$ (C)

$-1000 \times 3 + R_A \times 6 + 500 \times 4 = 0$

$= 3000 - 2000$

$R_A = \frac{\cancel{1000}\ 500}{\cancel{6}\ 3}$

$\boxed{R_A = 166.6}$




~~$\Sigma F_x = 0$~~

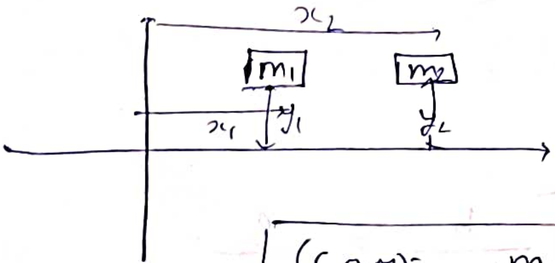
~~$F_{AD} + 500 - 500 + F_{BD}\,3/5$~~

$\Sigma M_z = 0 \; (B)$

$- 500/3 \times 3 + 500 \times 4 + F_{AD} \times 4 = 0$

## CENTRE OF MASS (COM)

C.O.M is the point or distance of a point from axis where all the mass of object is centered or situated.

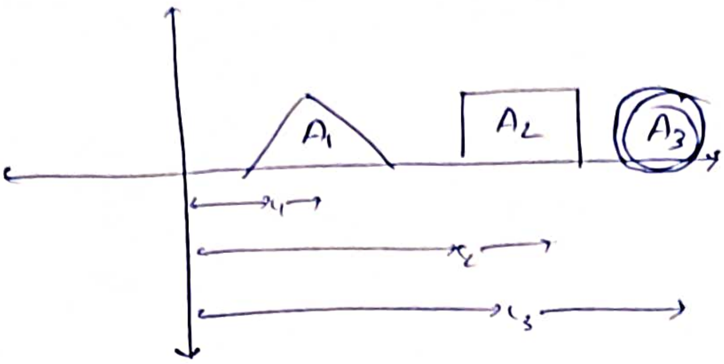



$$\boxed{(C.O.M)_x = \frac{m_1x_1 + m_2x_2}{m_1+m_2}}$$

$$\boxed{(C.O.M)_y = \frac{m_1y_1 + m_2y_2}{m_1+m_2}}$$

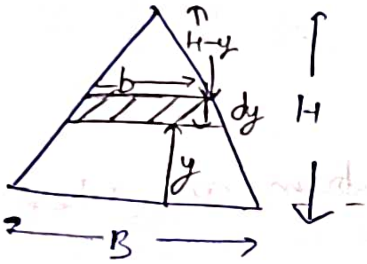
$$\boxed{C.O.M = \left((C.O.M)_x, (C.O.M)_y\right)}$$

↳ II Area is continous




$$\boxed{\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1+A_2+A_3}}$$

#




$$\boxed{A(C.o.M) \int A\, d_{x/y}}$$

$$\left[\frac{b}{H-y} = \frac{B}{H}\right]$$

$$\left[b = \frac{B(H-y)}{H}\right]$$

$$A\bar{Y} = \int \underbrace{b\,dy}_{\text{Area}}\, \underbrace{y}_{\text{Distance}}.$$

$$\bar{Y} = \frac{\int \frac{B(H-y)}{H}\, dy\, y}{A}$$

$$\boxed{\bar{Y} \int_0^H \frac{B(H-y)\,y\,dy}{H\ \frac{1}{2}B\times H} = H/3}$$

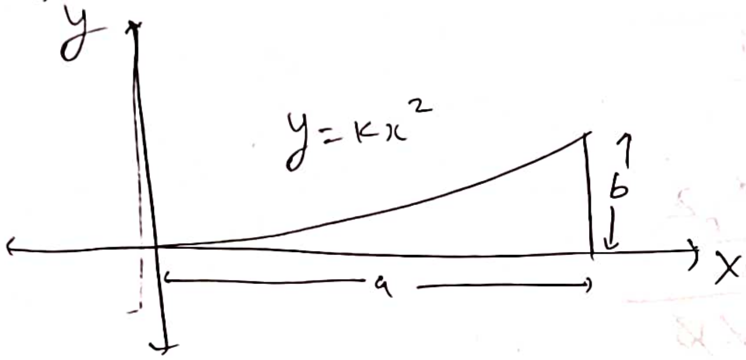
# Different formula of C.O.M for continuous mass, distance

$$\boxed{\bar{X} = \frac{\sum m_i x_i}{\sum m_i}}$$

$$\boxed{\bar{X} = \frac{\sum A_i x_i}{\sum A_i}}$$

$$\boxed{\bar{X} = \frac{\int dA.x}{A}}$$

**Q1)** Determine by direct integration the location of centroid of a parabolic spandel.




**Ans)**

$$\bar{X} = \frac{\int A.\cancel{Ax}x}{\int A}$$

$$= \frac{\int y\,dx\,x}{\int y\,dx}$$

$$= \frac{\int kx^3\,dx}{\int kx^2\,dx} = \frac{\int_0^a \frac{kx^4}{4}}{\int_0^a \frac{kx^3}{3}}$$

$$\bar{X} = \frac{ka^4/4}{ka^3/3} = 3a/4.$$

$$\bar{y} = \frac{\int x\,dy\,y}{\int x\,dy}$$

$$y = kx^2$$

$$\frac{dy}{dx} = 2kx$$

$$dy = 2kx\,dx$$

$$= \frac{x(kx^2)\,2kx\,dx}{\int x\,2kx\,dx}$$

$$\frac{\int 2k^2x^4\,dx}{\int 2kx^2\,dx}$$

$$= \frac{\int_0^b \frac{2k^2x^5}{5}}{\int_0^b \frac{2kx^3}{3}}$$

$$= \frac{\frac{\cancel{2}k^{\cancel{2}}b^{\cancel{2}}}{5}}{\frac{\cancel{2}\cancel{k}\cancel{b^3}}{3}}$$

$$\boxed{\bar{y} = \frac{3kb^2}{5}}$$