Date :

Mechanics

* Plane Trusses.

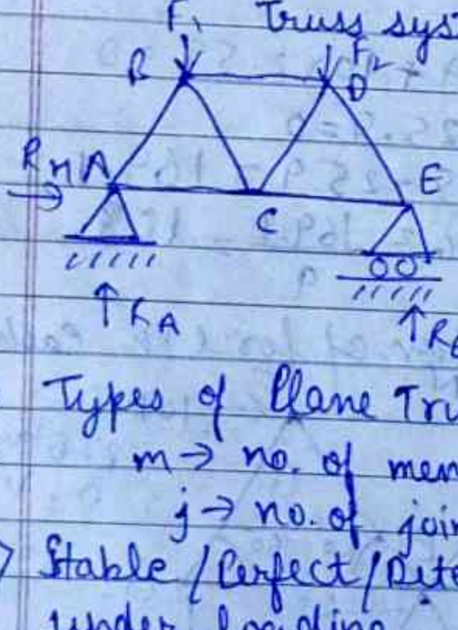
- Truss :- It is a rigid structure in which all the members are subjected to either tensile or axial compressive load only
- Binding Moment is zero

→ Conditions of Truss.

- All the members should be pin jointed or hinged only
- Load should be applied at joints only
- Only concentrated point should be applied.

NOTE :- A member should make a joint at its ends only.

Truss system = members + Pin or Joints



$\Sigma \vec{F}_H = 0 \Rightarrow R_H = 0$

$\Sigma F_V = 0$

$R_A + R_E = F_1 + F_2$

→ Types of Plane Trusses.

$m \rightarrow$ no. of members,

$j \rightarrow$ no. of joints

1) Stable / Perfect / Determine :- Truss which does not collapse under loading. $m = 2j - 3$

2) Collapsible / Destructible / Unstable :- Truss which collapse under loading. $m < 2j - 3$

3) Indeterminate / Indestructible :- if $m > 2j - 3$

Anticlockwise = +ve
Clockwise = −ve

Q1 Find magnitude and direction of force in each member.

Ans)

$\tan 60° = \frac{h}{B}$

$h = 1.5 \tan 60°$
$= 2.59$

Sol:

$R_H = 10\,kN$, $R_A + R_G = 40 + 30\,kN = 70\,kN$

$\Sigma M_A = 0$

$40 \times 1.5 + 30 \times 4.5 - R_G \times 9 - 10 \times 2.59 = 0$

$60 + 135 - 9R_G - 25.9 = 0$

$9 R_G = \frac{169.1}{9} = 18.8$

$R_A + R_G = 70$, $R_A = 70 - 18.8 = 51.2$

$\Sigma M_A = 0$

$-40 \times 1.5 - 30 \times 4.5 + R_G \times 9 + 10 \times 2.59 = 0$

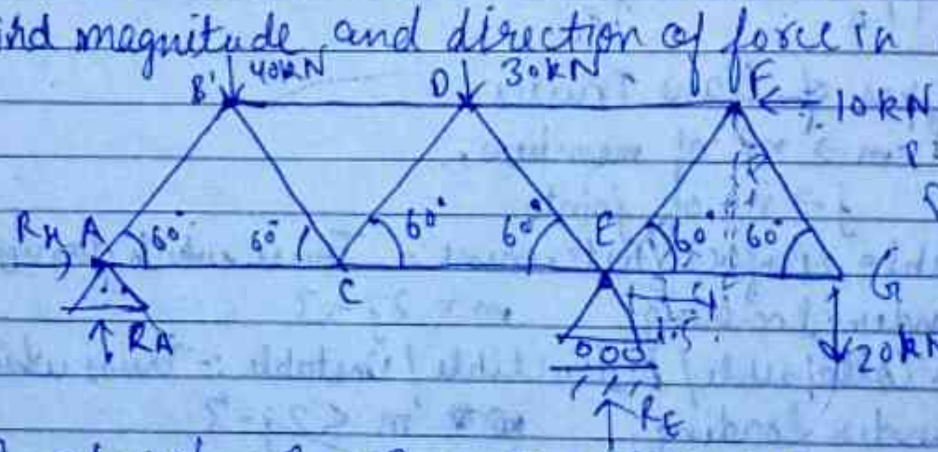
$-60 - 135 + 9R_G + 25.9 = 0$

$9R_G = 195 - 25.9 = 169.1$

$R_G = \frac{169.1}{9} = 18.8$

Q1 Find magnitude and direction of force in each member

Ans)



$P = B \tan 60°$
$P = 1.5 \tan 60°$
$= 2.59$

$R_H = 10\,kN$, $R_E + R_A = 90\,kN$

$\Sigma M_A = 0$

$-40 \times 1.5 - 30 \times 4.5 + R_E \times 6 + 10 \times 2.59 - 20 \times 9 = 0$

~~$F_{AB}\sin 60^\circ = -31.9$~~

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$$-60 - 135 + 6R_E + 25.9 - 180 = 0$$

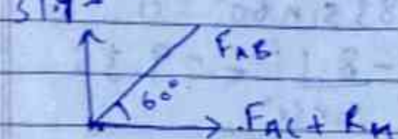
$$6R_E = 375 - 25.9$$

$$R_E = 58.18$$

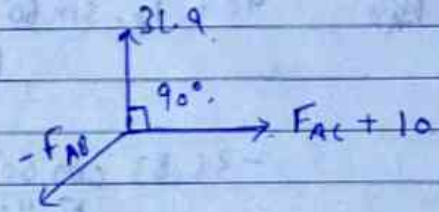
$$R_E + R_A = 90\,kN$$

$$R_A = 90 - 58.18 = 31.9$$

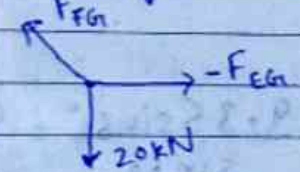
FBD of A



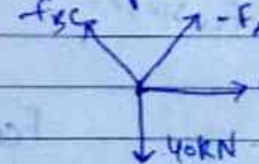
=



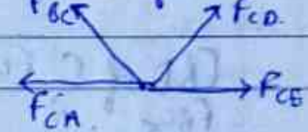
FBD of G



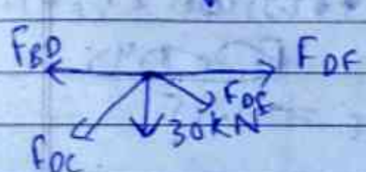
FBD of B



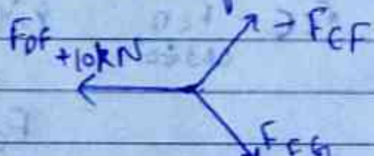
FBD of C



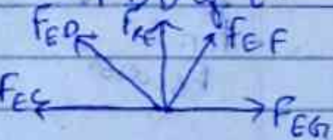
FBD of D



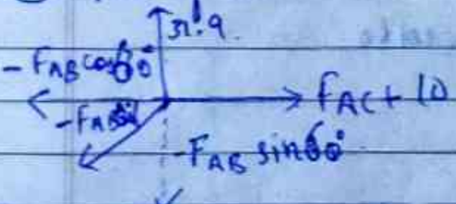
FBD of F



FBD of E



(1) FBD of A



~~$-F_{AB}\sin 30^\circ = 31.9$~~

~~$F_{AB} = -\frac{31.9}{\sin 30^\circ} = -63.8$~~

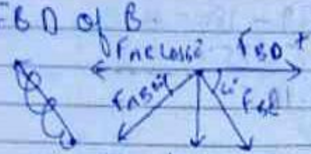
~~$F_{AC} + 10 = -(-63.8)\cos 30^\circ$~~

$$-F_{AB}\sin 60^\circ = 31.9, \quad F_{AB} = \frac{-31.9}{\sin 60^\circ} = -36.83$$

$$36.83\cos 60^\circ = F_{AC} + 10$$

$$F_{AC} = 8.48\ kN$$

FBD of B



$F_{AB} = \frac{40}{\sin 60}$

$$40 + F_{BC}\sin 60° - 36.83\sin 60° = 0$$

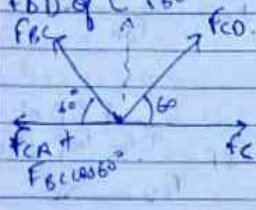
$$F_{BC} = \frac{-8.1}{\sin 60°} = -9.35$$

$$-36.83\cos 60° = F_{BD} - 9.35\cos 60°$$

$$-18.415 = F_{BD} - 4.675$$

$$F_{BD} = -13.74$$

FBD of C



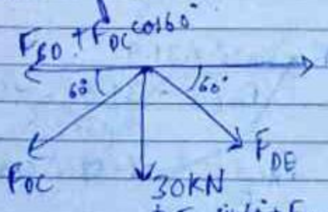
$$F_{CD} = \frac{9.35\sin 60°}{\sin 60°} = 9.35$$

$$F_{CA} + F_{BC}\cos 60° = F_{CE} + F_{CD}\cos 60°$$

$$8.4 + 9.35\cos 60° = F_{CE} + 9.35\cos 60°$$

$$F_{CE} = -0.95$$

FBD of D



$$\frac{9.35\sin 60° + 30}{\sin 60°} = -F_{DE}$$

$$F_{DE} = -43.87$$

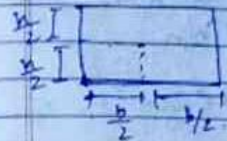
$$F_{BD} + F_{DC}\cos 60° = F_{DF} + F_{DE}\cos 60°$$

$$-13.74 + 9.35\cos 60° = F_{DF} - 43.87\cos 60°$$

$$F_{DF} = 12.87$$

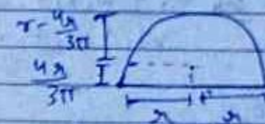
* Centroid :- The point where max. area is constituted

1) Rectangle



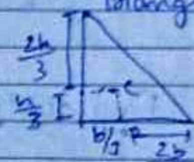
$\boxed{A = b \times h}$

2) Semi Circle



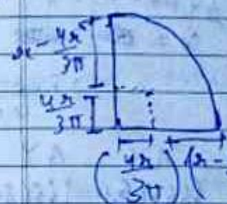
$\boxed{A = \frac{\pi r^2}{2}}$

3) Right angle triangle



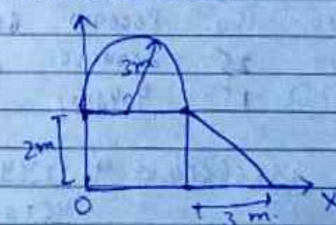
$\boxed{A = \frac{1}{2} \times b \times h}$

4) Quarter circle



$$A = \frac{\pi r^2}{4}$$

Q2 Find centroid of



Shape	Area	X	Y	Ax	Ay
2m, 6m (rectangle)	12	3	1	36	[illegible]
2m, 3m (triangle)	3	7	0.666	21	[illegible]
3m (semicircle)	14.137	3	3.273	42.41	[illegible]
	29.137			99.41	60.268

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{99.411}{29.137} = 3.411\,m$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{60.268}{29.137} = 2.068\,m$$

So, Centroid is $(\bar{x}, \bar{y}) = (3.411, 2.068)$

* Centre of Mass

$$x = \frac{m_1x_1 + m_2x_2 + m_nx_n}{m_1 + m_2 + m_3 ... + m_n}$$

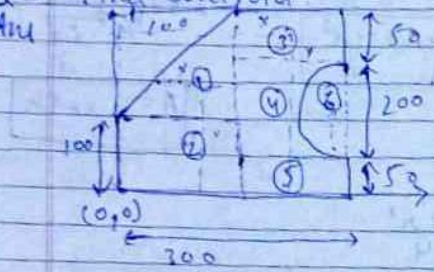
$$y = \frac{m_1y_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_nx_n}$$

→ Centre of Gravity :-

$$x = \frac{W_1x_1 + W_2x_2 + ... + W_nx_n}{W_1 + W_2 + ... + W_n}$$

Q3 Find centroid

Ans



$r = \frac{4r}{3\pi}$; $\frac{h}{3} = \frac{200}{3}$

$\frac{100 - 400}{1 \cdot 3\pi} + 200$; $\frac{\pi r^2}{2} = \frac{\pi(10000)}{2}$

$= \frac{300\pi - 400 + 600}{3\pi}$

$\frac{1}{2} \times 100 \times 200$

$x = 2426$; $\frac{2826 - 400}{3\pi}$

Shapes	A	X	Y	AX	AY
(1)	10000	$\frac{200}{3}$	$\frac{500}{3}$	666666.66	1666666
(2)	10000	50	50	500000	500000
(3)	10000	200	275	2000000	2750000
(4)	40000	200	150	8000000	6000000
(5)	10000	200	25	2000000	250000
(6)	15700	257.53	150	4043221	2355000

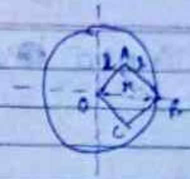
$$\frac{\Sigma Ax - \Sigma A_6x_6}{\Sigma A - A_6} = \frac{9123445.66}{46300}$$

$= 197.05$

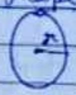
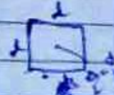
$\Sigma A_y = 11081666.66$

$= 238.91$

Q4



Ans 4)

Shape	A	X	AX
	$\frac{\pi d^2}{(2)^2}$	$\frac{d}{2}$	
	$\frac{d^2}{(2\sqrt{2})^2}$	$\frac{d}{4}$	

$l^2 + l^2 = r^2$

$2l^2 = r^2$

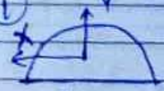
$l = \frac{r}{\sqrt{2}}$

$l = \frac{d}{2\sqrt{2}}$

$$x = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{\frac{\pi d^2}{4}\left(\frac{d}{2}\right) - \frac{d^2}{8}\left(\frac{d}{4}\right)}{\frac{\pi d^2}{4} - \frac{d^2}{8}}$$

* Moment of Inertia

(1)

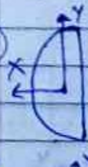


$I_x = 0.11r^4$

$I_y = 0.392r^4$

$I_y = \frac{\pi}{8}r^4$

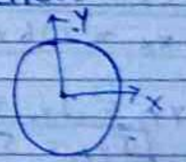
(2)



$I_x = 0.392r^4$

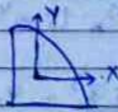
$I_y = 0.11r^4$

(3) Circle



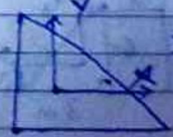
$I_x = I_y = 0.049d^4 = \frac{\pi}{64}d^4$

(4)



$I_x = I_y = 0.055r^4$

(5)



$I_x = \frac{bh^3}{36}$

$I_y = \frac{hb^3}{36}$

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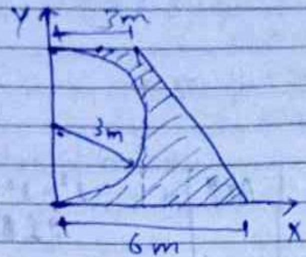
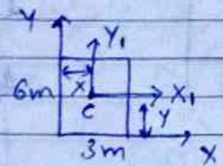
Q4 Find centroid and Moment of Inertia of shaded region

Ans 4)



(1) Rectangle



$A = 18\ m^2,\ x = 1.5m,\ y = 3m$

$I_{x_1} = \frac{bh^3}{12} = \frac{3 \times 6^3}{12} = 54 m^4$

$I_{y_1} = \frac{hb^3}{12} = \frac{6 \times 3^3}{12} = 13.5 m^4$

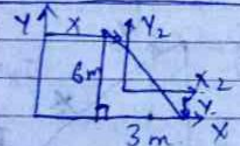
$I_{xx_1} = I_{x_1} + Ay^2 = 54 + 18 \times 3^2$

$\boxed{I_{xx_1} = 216\ m^4}$

$I_{yy_1} = I_{y_1} + Ax^2 = 13.5 + 18 \times 5^2$

$\boxed{I_{yy_1} = 54 m^4}$

(2)



$A = 9m^2,\ X = 4m,\ y = 2$

$I_{x_2} = \frac{bh^3}{36} = \frac{3 \times 6^3}{36} = 18\ m^4$

$I_{y_2} = \frac{hb^3}{36} = \frac{6 \times 3^3}{36} = 4.5 m^4$

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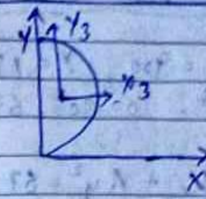
$I_{xx_2} = I_{x_2} + Ay^2 = 18 + 9 \times 2^2$

$\boxed{I_{xx_2} = 54 m^4}$

$I_{yy_2} = I_{y_2} + Ax^2 = 4.5 + 9 \times 4^2$

$\boxed{I_{yy_2} = 148.5 m^4}$

(3)



$A = 14.137 m^2,\ X = 1.273 m,\ Y = 3m$

$I_{x_3} = 0.392 r^4 = 0.392 \times 3^2$

$\boxed{I_{x_3} = 31.752 m^4}$

$I_{y_3} = 0.11 r^4 = 0.11 \times 3^4$

$\boxed{I_{y_3} = 8.91 m^4}$

$I_{xx_3} = I_{x_3} + Ax^2 = 31.752 + 14.137 \times 3^2$

$\boxed{I_{xx_3} = 158.985 m^4}$

$I_{yy_3} = I_{y_3} + Ay^2 = 8.91 + 14.137 \times 1.273^2$

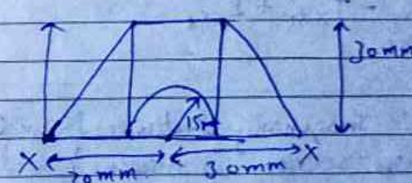
$\boxed{I_{yy_3} = 31.819\ m^4}$

$I_x = I_{xx_1} + I_{xx_2} - I_{xx_3} = 111.015 m^4$

$I_y = I_{yy_1} + I_{yy_2} - I_{yy_3} = 170.681 m^4$

Q5

Ans 5)



$\frac{1}{2} \times 15 \times 30^{15}$ $\frac{2b}{3}$

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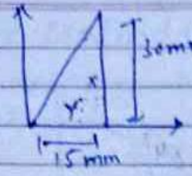
① 

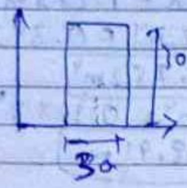
$A = 225$, $x = 10$, $y = 10$

$I_{x_1} = \frac{bh^3}{36} = \frac{15 \times 30^3}{36}$

$I_{x_1} = 11250$

$I_{xx_1} = I_{x_1} + Ay^2 = 11250 + 225 \times 10^2$

$33750 = \cancel{33500}\ mm^4$

② 

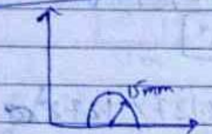
$A = 30 \times 30 = 900$; $y = 15$

$I_{x_2} = \frac{bh^3}{12} = \frac{30 \times 30^3}{12} = 67500$

$I_{xx_2} = I_{x_2} + Ay^2 = 67500 + 900 \times 15^2$

$= \cancel{157500}$

$= 270000$

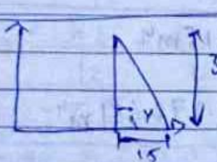
③ 

$A = \frac{\pi r^2}{2} = 353.25$, $y = \frac{60}{3\pi} = 6.36$

$I_{x_3} = 0.11 r^4 = 0.11 \times 15^4 = 5568.75$

$I_{xx_3} = I_{x_3} + Ay^2 = 5568.75 + 353.25 \times (6.36)^2$

$= 19857.57$

④ 

$A = 225$, $y = 10$

$I_{x_4} = 11250$

$I_{xx_4} = 33750$

$I_x = I_{xx_1} + I_{xx_2} - I_{xx_3} + I_{xx_4}$

$= 317642.43$

* Radius of gyration :- Imaginary distance from Centroid at which area of cross-section is imagined to be focused as at a point in order to obtain same MOI

$I = mk^2$ k = Radius of gyration

$k = \sqrt{\frac{I}{m}}$

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Mechanics

* Force :- Force can be thought of as a push or pull of an object
* Principle of transimissibility :- It states that the point of application of a force can be moved anywhere along its line of action without changing external reaction forces on a rigid body.

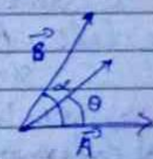
When forces of same magnitude and direction donot share same line of action they will always produce dissimilar results and they are not interchangeable.

* System of forces :- When two or more than two forces act on a body, they are said to form a system of forces

1) Coplanar :- Forces, whose line of action lie on same plane
2) Concurrent :- Forces, which meet at one point
3) Copalanar concurrent :- which meet at one point & their line of action (LOF) also lie on same plane.
4) Coplanar Non concurrent :- which donot meet at one point but their lines of action lie on same plane.
5) Non coplanar concurrent forces :-
6) Non planar non concurrent forces :-

$R = \sqrt{A^2 + B^2 + 2AB\cos\alpha}$

$\tan\theta = \frac{B\sin\alpha}{A + B\cos\alpha}$



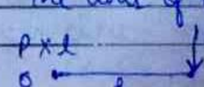
$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$

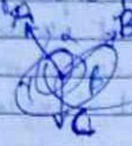
* Moment of a force is equal to the product of the force and the perpendicular distance of the point, about which moment is required and the line of action of force.

The unit of moment is N-m

$|M| = F \times d$



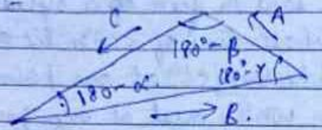
* Lami's Theorem :- It states that "If three concurrent forces act on a body keeping it in equillibrium, then each force is proportional to sine of angle b/w 2 forces

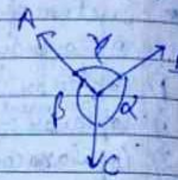


$$\frac{A}{\sin\beta} = \frac{B}{\sin\gamma} = \frac{C}{\sin\alpha}$$

- It is not applicable for parallel and general force system.
- It is applicable only when three forces acting at a point are in equillibrium

Proof :-





$F_A + F_B + F_C = 0$.

Using Δ law of vector addition & by applying sine rule.

$$\frac{A}{\sin(180-\alpha)} = \frac{B}{\sin(180-\beta)} = \frac{C}{\sin(180-\gamma)}$$

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}.$$

* Varignon's Theorem :- It states that if a number of coplanar forces acting on a particle are in equillibrium, then the algebric sum of their moments about any point is equal to the moment of their resultant force about same point

$$\Sigma M_A^F = M_A^R$$

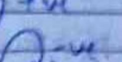
or

Moment of all forces from point A is equal to the moment of resultant from point A

* Couple (Parallel Forces) :- Two parallel forces of equal magnitude and opp forms a couple.

→ Effect of a couple is to rotate the body on its axis.

$M = F \times d$.



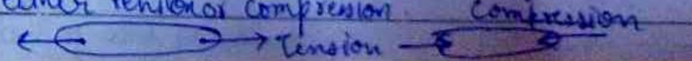
→ Magnitude of rotation is known as moment of couple

* Props. of couple
1) Causes rotation of body about axis ⊥ to plane containing 2 parallel forces
2) $M = F \times d$
3) Resultant force of couple system is 0
4) To balance a system whose resultant is couple, another couple of opposite direction is required
5) To shift a force to a new parallel position, a couple is to be added to system

* Equilibrium:- If the resultant of forces system happen to be zero, system is said to be in state of equilibrium

→ Conditions of equilibrium.
1) Sum of all forces should be zero i.e. $\Sigma \vec{F} = 0$.
$\Sigma F_y = 0$, $\Sigma F_x = 0$.
2) Sum of moment of all forces should be 0 i.e. $\Sigma \vec{M} = 0$

→ Two force member :- It is a body that has forces acting on it in only two locations. In order to have a two force member in static equillibrium, net force at each location must be equal, opposite and collinear. This will result in all two force members being in either tension or compression.



→ Three-force members :- If three non-parallel act on a body in equilibrium, it is known as a three force members

* Center of gravity :- The point through which the whole mass of body acts, irrespective of the position of body is known as centre of gravity.

* Centre of mass :- Point at which whole mass of body or all masses of a system of particle appear to concentrated.

* Parallel Axis theorem :- MOI of a body about any axis is equal to sum of its moment of inertia about a parallel axis through its center of mass and product of mass of body and square of perpendicular distance b/w the two axes

Dry Friction / Coulomb Friction

* Whenever a body moves or tends to move tangentially wrt the surface on which it rests, then there is a opposing force, which acts in opp. direction of movement of the body. This opposing force is called force of friction

Friction → 1) Static Friction (f_s)
→ 2) Kinetic Friction (f_k)

1) Static friction → The friction experienced by a body when it is at rest and has tendency of motion.

2) Kinetic friction :- If friction experienced by a body when it is in motion.

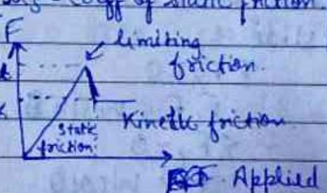
- Sliding :- slides over another body
- Rolling :- rolls over another body.

$0 < f_s \leq (f_s)_{max}$

$(f_s)_{max} \propto N$

$\boxed{(f_s)_{max} = \mu_s N}$

μ_s = coeff of static friction.



$\boxed{f_k = \text{constant} = \mu_k N}$

$\therefore$ μ_k = coeff of Kinetic friction

$(f_s)_{max} > f_k$

$\boxed{\mu_s > \mu_k}$

1) If A.F $\leq (f_s)_{max}$, then static friction

2) $(f_s)_{max}$ is also called limiting friction because this is max. value of friction.

* Laws of Dry Friction :-

1) It depends upon the type of surface in contact.
2) It always acts tangential to the areas in contact & opposes motion.
3) Independent of area in contact

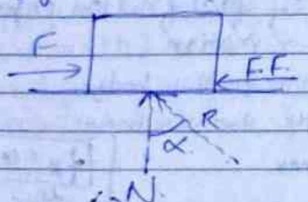
4) Directly proportional to the normal reaction
5) Magnitude of force of friction is exactly equal to the force, which tends the body to move

* Coefficient of friction (μ) :- Ratio of limiting friction to the Normal reaction b/w 2 bodies

$$\mu = \frac{F.F}{N}$$

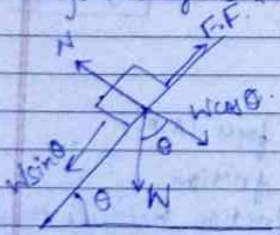
* Angle of friction :- Resultant of friction and normal makes an angle with normal, this angle is called angle of friction (α)

$$\tan\alpha = \frac{F.F}{N} = \mu$$



$$\boxed{\alpha = \tan^{-1}\mu}$$

* Angle of Repose (θ) :- Minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide.



$\Sigma F_x = 0$

$F.F = W\sin\theta = \mu N$

$\Sigma F_y = 0$

$N = W\cos\theta$

$\cancel{W}\sin\theta = \mu \cancel{W}\cos\theta$

$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$$\boxed{\theta = \tan^{-1}\mu}$$

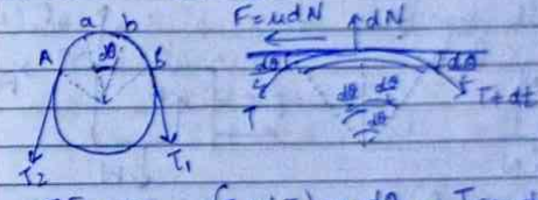
If ~~angle~~ $\theta < \tan^{-1}\mu$, then no slipping

$\theta > \tan^{-1}\mu$, then slipping

* Cone of friction - If resultant reaction is ~~rotation~~ rotated about normal reaction force, it will form a cone known as cone of friction

* Frictional lock - It is a type of lock which uses friction to prevent the rotation or movement of a component or object. It is created by increasing contact pressure b/w 2 surfaces in contact, by tightening a screw or bolt

* Friction in Flat Belts



$\Sigma F_x = 0 \quad (T+dT)\cos\frac{d\theta}{2} = T\cos\frac{d\theta}{2} + \mu dN$ — (1)

$\Sigma F_y = 0 \quad (T+dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2} = dN$ — (2)

$\theta \Rightarrow$ angle of contact

$d\theta$ is very small to $\frac{d\theta}{2} \to 0$ $\therefore \cos\frac{d\theta}{2} = 1$ & $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$

$T + dT = T + \mu dN$

$dT = \mu dN$ — (3)

$T\frac{d\theta}{2} + dT\frac{d\theta}{2} + T\frac{d\theta}{2} = dN$

$T d\theta = dN$ — (4) ($\because dT$ is very small)

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From eq. ③ and ④.

$$dT = \mu T \, d\theta$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu \, d\theta$$

$$[\log T]_{T_2}^{T_1} = \mu\theta$$

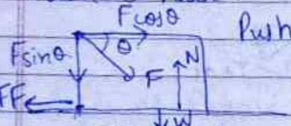
$$\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Power transfer increases if angle of contact is more. To increase angle of contact, we use a cross belt over the pulley. Here, friction has +ve application as power increases with increase in μ.

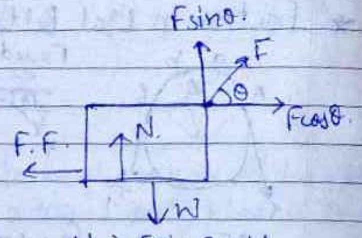
* Push Vs Pull

Push



$F.F = \mu N$

$N = W + F\sin\theta$



$N + F\sin\theta = W$

$F.F = F\cos\theta = \mu N$

*

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Unit - 3

* Kinematics

→ Rectillinear Motion :- When a particle moves along a path which is a straight line, it is called rectilinear motion.

→ Curvillinear motion :- When a particle moves along a curved path is called a curvillinear motion. If the curved path lies in a line, it is called plane curvillinear motion.

* Rectlinear motion.

1) Uniform motion [v = constant, $a = 0$]

$$s = ut$$

2) Uniformly accelerated motion.

$$v = u + at$$

$$v^2 = u^2 + 2as$$

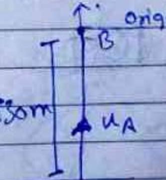
$$s = ut + \frac{1}{2}at^2$$

3) Motion with var. acceleration [$a \neq$ constant]

→ Displacement = Final position - Initial position

→ Dist. travelled = Actual length of path travelled by particle during motion.

→



Single body.
$y_0 = 0$, $y = -130$ m
time = t sec

Two bodies.

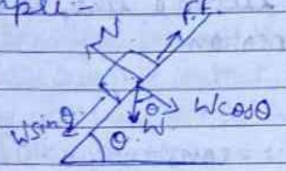
Body B	Body A
$y_0 = 0$	$y_0 = -130$
time = t sec	time = t-2

Kinetics.

* D'Alemberts :- It states that the resultant force ~~or~~ acting on the body together with the reversed effective or inertial force are in equillibrium

$$\Sigma F_{system} - \underbrace{ma}_{\text{Inertial Force}} = 0$$

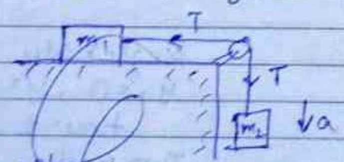
Example :-

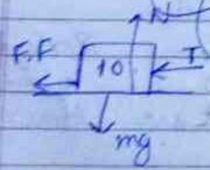


By NLM 2nd Law	D'Alemberts.
$\Sigma F_x = ma$	Along x axis
$W\sin\theta - \mu_k N = ma$	$W\sin\theta - \mu_k N - ma = 0$.
$\Sigma F_y = 0$.	$\Sigma F_y = 0$.
$N - W\cos\theta = 0$	$N - W\cos\theta = 0$.

Q1 Two blocks of masses M_1 and M_2 are connected by a flexible but inextensible string as shown in figure. Assuming μ, Find the acceleration of masses and tension in string. $\mu = 0.25$, $M_1 = 10kg$, $M_2 = 5kg$

Ans)

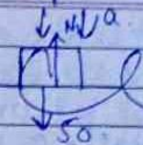




$N = 100$

$F.F = \mu N = 0.25 \times 100$.

$T = 25$



$\Sigma F_y = 0$.

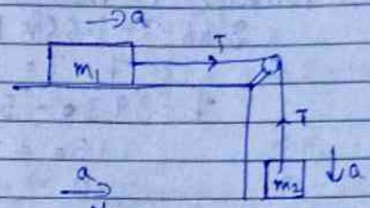
$N - T + ma - 50 = 0$

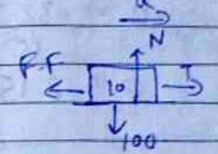
$N - 25 - 5a - 50 = 0$

$100 - 75 - 5a = 0$

$a = 5\ m/s^2$

Ans1)





$\Sigma F_y = 0$.

$N = 100$.

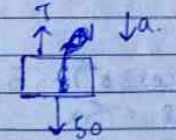
$\Sigma F_x = ma$.

$F.F - T - ma = 0$

$0.25 \times 100 - T - 10a = 0$.

$25 = T + 10a$

$T - 25 = 10a$



$50 - T = 5a$

$50 - 25 - 10a = 5a$

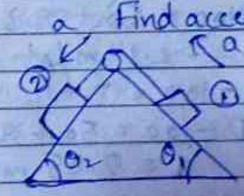
$25 = 15a$

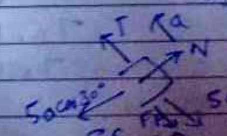
$a = \frac{25}{15} = 1.6\ m/s^2$

$T = 16 + 25 = 41$

Q2 Find acceleration $m_1 = 5kg$, $m_2 = 10kg$, $\theta_1 = 30°$, $\theta_2 = 60°$, $\mu = 0.33$

Ans2)





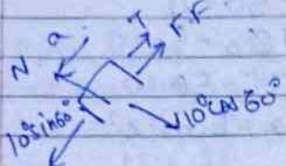
$\Sigma F_y = 0$.

$N = 50\cos30° = 43.3$

$\Sigma F_x = 5a$,

$T - 50\sin30° - 14.3 = 5a$

$T - 39.3 = 5a$



$\Sigma F_y = 0$

$N = 10\cos 60^\circ = 5$

$10\sin 60^\circ - T - 1.65 = ma$

$8.66 - T - 1.65 = 10a$

$7 - T = 10a$

$7 - 39.3 - 5a = 10a$

$15a = -32.3$

$a = \frac{-32.3}{15}$

$a = 2.15\ m/s^2$

Block 1

$\Sigma F_y = 0$

$N = 50\cos 30^\circ$

$\Sigma F_x = 5a$

$T - 50\sin 30^\circ - 0.33(50\cos 30^\circ) = 5a$

$T - 39.28 = 5a$

Block 2

$\Sigma F_y = 0$

$N = 10\cos 60^\circ$

$100\sin 60^\circ - T - 0.33(10\cos 60^\circ) = 10a$

$\cancel{-T + 7.16 = 10a}$

$\cancel{7.16 - 10a = T}$

$\cancel{7.16 - 10a = 5a + 39.3}$

$\cancel{a = -2.14\ m/s^2}$

$70.1 - 10a = T$

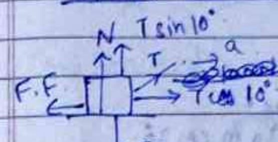
$70.1 - 10a = 5a + 39.3$

$a = 2.05\ m/s^2$

Q3 Two blocks A and B of masses 5kg and 20kg are connected by an inclined string. A horizontal force P of 100N is applied to block B as shown in figure. Calculate tension and the acceleration of system. μ_A of A = 0.5 & μ_B of B = 0.25 resp.

Ans)

$F = 100N$ $m_A = 5kg$ $m_B = 20kg$



$\Sigma F_y = 0$

$N + T\sin 10^\circ = 50$

$\Sigma F_x = ma$

$100 + T\cos 10^\circ - \mu_A N = 5a$

$100 + T\cos 10^\circ - 0.5(50 - T\sin 10^\circ) = 5a$

$100 + T\cos 10^\circ - 25 + T\sin 10^\circ = 5a$

$75 + T(\sin 10^\circ + \cos 10^\circ) = 5a$

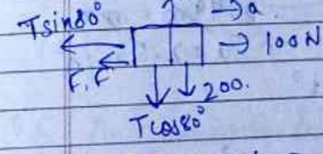
$T\cos 10^\circ - 0.5N = 5a$

$T\cos 10^\circ - 0.5(50 - T\sin 10^\circ) = 5a$

$T\cos 10^\circ - 25 + 0.5T\sin 10^\circ = 5a$

$0.98T + 0.086T = 5a - 25$

$1.066T + 25 = 5a$



$\Sigma F_y = 0$

$N = T\cos 80^\circ + 200$

$N = 0.17T + 200$

$\Sigma F_x = 200a$

$100 - T\sin 80 - 0.25(0.17T + 200) = 20a$

$100 - 0.98 - 0.04T - 50 = 20a$

$49.02 - 0.04T = 20a$

$20a + 0.04T = 49.02 \times 5$

$5a - 1.066T = 25 \times 20$

$100a + 0.2T = 245.1$

$100a - 21.32T = 500$

$+ \quad - \quad 18$

$200a + 0.04T = 49.02 \times 5$

$5a - 1.066T = 25 \times 200$

$1000a + 0.2T = 245.1$

$1000a - 213.2T = 5000$

$- \quad + \quad -$

$T = \frac{-4754.9}{213.4}$

$= -22.28$

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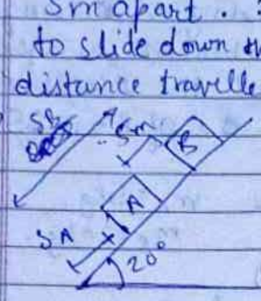
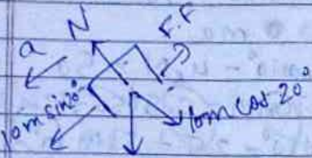
Q4 Two blocks A and B are held on an inclined plane 5m apart. $\mu_A = 0.2$, $\mu_B = 0.1$. If blocks begin to slide down the plane. Calculate time and distance travelled by each block before collision

Ans) 

$\mu_A = 0.2$, $\mu_B = 0.1$

$t_A = t_B$



$\Sigma F_y = 0$

$N = 10m \cos 20°$

$\Sigma F_x = ma$

$10m \sin 20° - F.F = ma$

$3.42m - 1.87m = ma$

$1.5 m = ma$

$a = 1.5 \, m/s^2$

~~10m sin 20~~

$3.42m - 0.93m = ma$

$2.49m = ma$

$a = 2.5 \, m/s^2$

$S_A = ut + \frac{1}{2} at^2$

$S_A = 0 + \frac{1}{2} \times 1.5 \times t^2$

$S_A = 0.75 t^2$

$S_A - 5 = \frac{1}{2} \times 2.5 \times t^2$

$S_A - 5 = 1.25 t^2$

$\frac{S_A}{S_A - 5} = \frac{0.75 t^2}{1.25 t^2}$

$1.25 S_A = 0.75 (S_A - 5)$

~~$S_A - S_A = -15$, $2S_A = 15$, $S_A = 7.5$~~

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$S_B - S_A = 5$

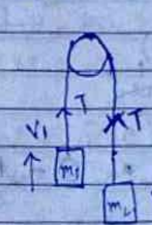
$\frac{2.5}{2} t^2 - \frac{1.5 t^2}{2} = 5$

$\frac{t^2}{2} = 5, \quad t^2 = \sqrt{10} = 3.16$

$S_B = \frac{1}{2} (2.5)(3.16)^2 = 8.22 \, m/s$

$S_A = \frac{1}{2} (1.5)(3.16)^2 = 13.22 \, m/s$

★ Constrained motion :- $P_{net} = 0$, $P = F.V \cos\theta$



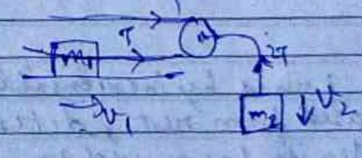
$P_{net} = 0$

$P_1 + P_2 = 0$

$TV_1 - TV_2 = 0$

$V_1 = V_2$

$a_1 = a_2$



$P_{net} = 0$

$P_1 + P_2 = 0$

$TV_1 - 2TV_2 = 0$

$V_1 = 2V_2$

Q5 An automobile of mass 1500kg is driven down a 5° incline at a speed of 100 km/h. Braking force = 8000N. Determine distance travelled by automobile as it comes to stop.

Ans) $m = 1500 \, kg$, $F = 8000 N$, speed $= 100 \, km/h$

$F = ma$

$\frac{8000}{1500} = a$, $a = 5.3 \, m/s^2$

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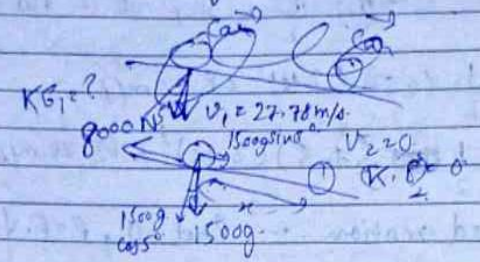
$m = 1500\,kg,\ \theta = 5^\circ,\ v_{car} = 100 \times \frac{5}{18} = 27.7\,m/s$

$F_{braking} = 8000\,N$.



$W = (K.E)_2 - (K.E)_1$

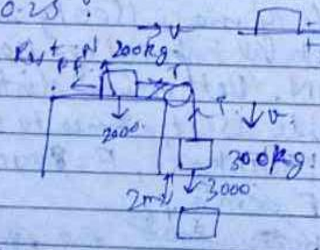
$= 0 - \frac{1}{2} m v_1^2$

$(1500 g \sin 5)\,x - (8000)x = 0 - \frac{1}{2} \times 1500 \times 27.78^2$

$x = 86.18\,m$.

Q6 Two blocks are joined by inextensible cable. If system is released from rest, determine velocity of block A after it has moved 2m. Assume $\mu_A = 0.25$.

Ans



$T = 3000$.

$F.F = T$

$T = 490.5\,N$

Block A - $W_{12} = T_2 - T_1$

$T \times 2 - (490.5 \times 2) = \frac{1}{2} \times 200 \times v^2$.

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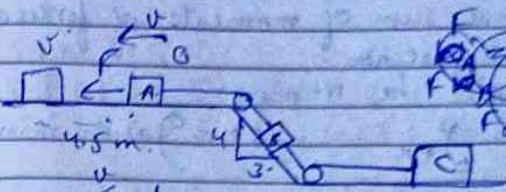
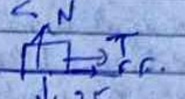
Q7 Determine the constant F required to give the system of three blocks A, B and C as shown in figure a velocity of 3 m/s after 4.5 m from rest. The $\mu = 0.3$.



$m_A = 25$
$m_B = 100$
$m_C = 80$
$F_A = 250\,N$
$F_B = 1000\,N$
$F_C = 500\,N$

Ans 7)



$F.F = \mu N = 0.3 \times 25 = 7.5$

$T + F.F - F$

$W = (K.E)_2 - (K.E)_1$

$(T + 7.5 - F)x = \frac{1}{2} m v^2$

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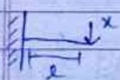
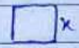
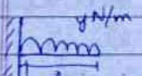
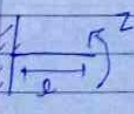
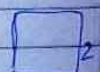
* Shear force & bending moment
→ Shear force :- Defined as algebric sum of forces acting either on ~~a~~ LHS or RHS of the section
* It's ~~unit~~ unit will be N -
→ BM :- Algebric sum of moments of forces acting on LHS or RHS of section.
Its unit will be N - mm

S.F. = ↻ +ve   B.M = ↻ +ve in right

1) Length of S.F.D & B.M must be equal to the span of beam
2) S.F.D is drawn below loaded beam & B.M.D is below SFD.
3) For simply supported beam, B.M is 0 at supports.
4) For cantilever beam, B.M will be zero at free end.
Calculate SF & B.M at all critical points
If no load is present b/w 2 points, then S.F will be constant

Load	Shear Force	B.M.	Point of action.	S.F	B.M
	x	xl	Point of Application		
	yL	$yL \times \frac{l}{2}$	$\frac{l}{2}$ from point A	 1st order	 2nd order
	$\frac{1}{2}yL$	$\frac{1}{2}yL \times \frac{l}{3}$	$\frac{l}{3}$ from A.	 2nd order.	 3rd order
	No shear force (-)	Z	X	—	

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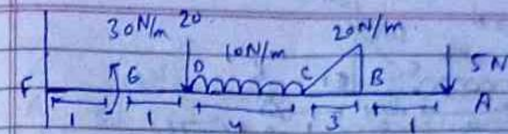

S.F
S.F at A = 5 N
S.F at B = 5×1 = 5 N
S.F at C = $5 + \frac{1}{2} \times 20 \times 3$ = ~~50~~ 35 N.

S.F. at D = $35 + 10 \times 4$ +20 = 95 N
S.F at E = 95 N
S.F at F = 95 N

B.M.
B.M. at A = 0 N
B.M at B = $-5 \times 1 = -5$ N
B.M at C = $-(5\times4) - \frac{1}{2} \times 20 \times 3 \times \frac{2\times 3}{3}$ = -80

B.M at D = $-(5\times8) - \frac{1}{2}\times 20\times 3 \times \left(4+\frac{2\times3}{3}\right) - 10\times4\times\frac{4}{2}$ =

B.M at E = $-(5\times9) - \frac{1}{2}\times20\times3\times\left(5+\frac{2\times3}{3}\right) - \left(10\times4\times\left(1+\frac{4}{2}\right)\right) - 20\times1$

B.M at F = $-5(10) - \frac{1}{2}\times20\times3\times\left(6+\frac{2\times3}{3}\right) - \left(10\times4\times\left(2+\frac{4}{2}\right)\right) -$
$20(1) + 30.$

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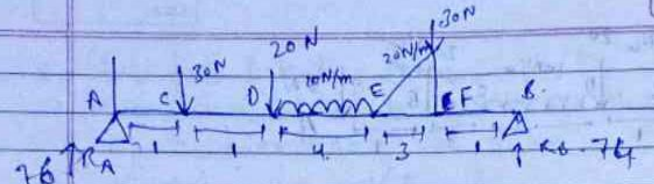

$\Sigma M_A = 0$ like B.M

$-30\times1 - 20\times2 - (10\times4)\left(2+\frac{4}{2}\right) - 30\times9 - \frac{1}{2}\times20\times3\times\left(6+\frac{2\times3}{3}\right)$

$+10R_B = 0$

$R_B = \frac{740}{10} = 74$

$R_A + R_B = 30+20+30+10\times4+\frac{1}{2}\times20\times3$

$R_A = 150-74 = 76$

S.F.

S.F at B = -74 N

S.F at F = $-74\times1 = -74$ N

S.F at E = $-74 + 30 + \frac{1}{2}\times20\times3 = -14$ N

S.F. at D = $-14 + 10\times4 + 20 = 46$ N

S.F at C = $46 + 30 = 76$ N

S.F. at A = $76 - 76 = 0$ N

B.M

B.M at B = 0

B.M at F = $74\times1 = 74$ N/m

B.M at E = $74\times4 - 30\times3 - \frac{1}{2}\times20\times3\left(\frac{2\times3}{3}\right) = 146$

B.M at D = $74\times8 - 30\times7 - \frac{1}{2}\times20\times3\left(4+\frac{2\times3}{3}\right) - 10\times4\times\frac{4}{2}$

$= 122$

B.M at C = $74\times9 - 30\times8 - \frac{1}{2}\times20\times3\left(5+\frac{2\times3}{3}\right) - 10\times4\times\left(1+\frac{4}{2}\right) - 20$

$= 76$

B.M at A = 0

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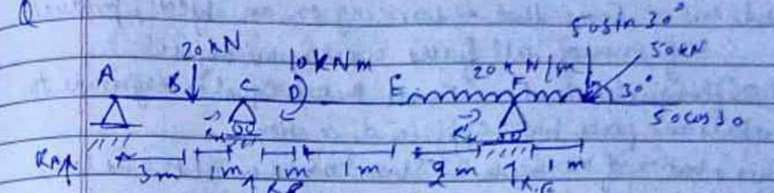
Q

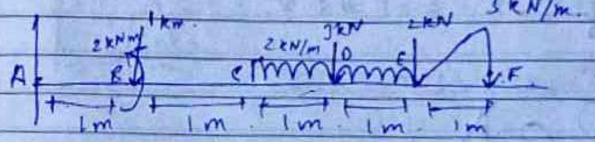


$\Sigma M_A = 0$

$-20\times3 - 10 - 20\times2\times\left(6+\frac{2}{2}\right) - 20\times1\times\left(8+\frac{1}{2}\right) - 50\sin30^\circ(9)$

$- R_C(4) - R_F(8) = 0$ $\quad 2R_H = 50\cos30$

4)



$-1.5 - 5 - 6 - 8$

B.M at B = $-\frac{1}{2}\times3\times1\left(3+\frac{2}{3}\right) - 2\times3$
$- 2\times\left(2+\frac{1}{2}\right) - 3(2) - \left(2\times\left(2+\frac{1}{2}\right)\right)$

S.F

S.F at F = 0 N

S.F at E = $\frac{1}{2}\times3\times1 + 2 = 3.5$

S.F at D = $\frac{1}{2}\times3\times1 + 2 + 2(1) + 3 = 8.5$

S.F at C = $8.5 + 2 = 10.5$

S.F at B = 11.5

B.M

B.M at F = 0

B.M at E = $-\frac{1}{2}\times3\times1\times\left(2\times\frac{1}{3}\right) = -1$ kN m

$-2.5 - 2 - 1$

B.M at D = $-\frac{1}{2}\times3\times1\left(1+\frac{2\times1}{3}\right) - 2\times1 - \left(2\times\frac{1}{2}\right) = -5.5$

B.M at C = $-\frac{1}{2}\times3\times1\left(2+\frac{2\times1}{3}\right) - 2\times2 - 2\times\left(1+\frac{1}{2}\right) - 3\times1 - 2\times\frac{1}{2}$

$= -15$

- Resultant :- Force that is working on an object, product of sum of all forces working on object

→ ~~Oracle~~ Equilibriant :- Force that is equal in magnitude to resultant force but opp. in direction

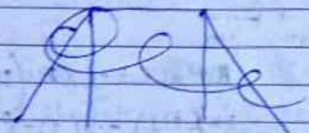
→ Conservation of Momentum - It is based on NLM 2nd law, which states that in an isolated system, total momentum remains same

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

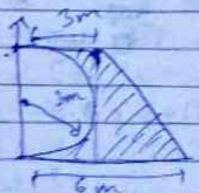
Momentum is neither created nor destroyed but only changed through action of forces

→ ~~Principle of~~

Q



Q



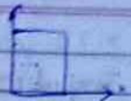
~~Here~~ $x = \frac{12}{3\pi} = \frac{4}{\pi}$, $y = \frac{6}{2} = 3$

~~$I_x = 0.11(3)^4$, $I_y = 0.39(3)^4$~~

$I_x = 0.392(3)^4$, $I_y = 0.11(3)^4$

$I_{xx} = I_x + Ay^2 = 158.985$

$I_{yy} = I_y + Ax^2 = 31.819$

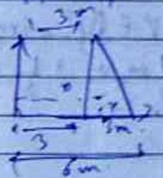


$x = \frac{3}{2}$, $y = \frac{6}{2}$

$I_x = \frac{bh^3}{12} = \frac{3 \times 6^3}{12} = 54$, $I_y = \frac{hb^3}{12} = \frac{6 \times 3^3}{12} = 13.5$

$I_{xx} = I_x + Ay^2 = 54 + 18(3)^2 = 216$

$I_{yy} = I_y + Ax^2 = 13.5 + 18(3)^2 = 54$



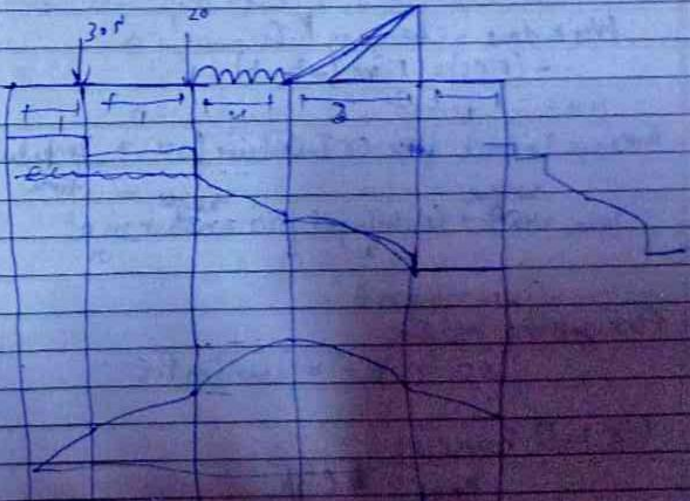
~~So~~ $x = 3 + \frac{3}{3} = 4$, $y = 2$

~~Here~~ $I_x = \frac{bh^3}{36} = 18$, $I_y = \frac{hb^3}{36} = 4.5$

$I_{xx} = I_x + Ay^2 = 18 + 9(2)^2 = 54$

$I_{yy} = 148.5$

$I_{xx_0} = -I_{xx_1} + I_{xx_2} + I_{xx_3} = -158.985 + 216 + 54$



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* Impulse and momentum
- $F = ma$, $a = \frac{dV}{dt} = \frac{v\,dv}{dx}$, $P = mv$, Impulse = Change in momentum
- Impule = $F \times t = mv_2 - mv_1$
- Impact = Collision b/w 2 bodies
- Principle of conservation of momentum
  Moment b/impact = moment after impact
  $$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$
- Principle of conservation of energy
  K.E. before Impact = K.E After impact
  $$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$
- $K.E = \frac{1}{2} mv^2$, $P.E = mgh$
- Loss of K.E impact = K.E before impact − K.E after impact
  $$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)$$
- Coefficient of restitution, $e = \frac{v_2 - v_1}{u_1 - u_2}$ $(e = 0 \text{ to } 1)$
- Work done = change in K.E
  $$-(F \times S) = \frac{1}{2} m(v^2 - u^2)$$
- Average Impact force or Impulsive force = $\frac{\text{Impulse}}{\text{time}} = \frac{m(v-u)}{t}$
  T_{max} = Weight + Centrifugal force = Weight + $\frac{mv^2}{r}$

  $T_{min} = mg\cos\theta$
- For getting height
  Loss in K.E = Loss in P.E

For ball bouncing
$$h_{n+1} = e^2 h_n$$

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Q Body A & B is moving in same direction $v_A = 5\,m/s$, $v_B = 3\,m/s$, $m_A = 7kg$, $m_B = 5kg$. If $e = 0.6$. Determine velocities of 2 bodies after impact.

Ans
$$e = \frac{v_2 - v_1}{u_1 - u_2} \qquad e = 0.6$$
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$
$$0.6 = \frac{v_2 - v_1}{5 - 3}$$
$$v_2 - v_1 = 1.2$$
$$v_2 = 1.2 + v_1$$
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$
$$35 + 15 = 7v_1 + 5(1.2 + v_1)$$
$$50 - 6 = 12 v_1$$
$$v_1 = \frac{44}{12} = 3.6\ m/s^2$$
$$v_2 = 4.8\ m/s^2$$

Q Above same ques but v in opp. direction
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$
$$0.6 = \frac{v_2 - v_1}{5 - (-3)}, \quad v_2 - v_1 = 4.8$$
$$m_1 u_1 - m_2 u_2 = m_1 v_1 + m_2 v_2$$
$$35 - 15 = 7v_1 + 5(4.8 + v_1)$$
$$20 = 12 v_1 + 24$$
$$v_1 = -0.33$$

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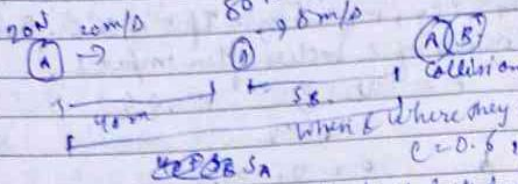
Q

20^N 20 m/s   80 N → 8 m/s



When & where they will collide
$e = 0.6$, find velocity after impact
If impact lasts for 0.02 sec, find avg impact force

Ans

For ball B

$S_B = 8t$ m.

Ball A

$S_A = 20t$ m.

$40 + S_B = 20t$ m.

$t = \frac{40}{12} = 3.33$ sec

$S_B = 8(3.33) = 26.64$

$S_A = 20(3.33)$.

$0.6 = \frac{v_2 - v_1}{20 - 8}$

$v_2 - v_1 = 7.2$

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$400 + 640 = 20 v_1 + 80(7.2 + v_1)$

$1040 = 100 v_1 + 576$

$v_1 = 4.64$ m/s

$v_2 = 11.84$ m/s.

Avg Impact force for A $= \frac{m(v_A - u_A)}{t} = \frac{20(4.64 - 20)}{0.02}$

$= -15656.74$

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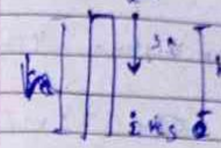
Q 6

a)



During last one second,

$h_s = \frac{1}{7} h$

$s_1 = ut - \frac{1}{2} \times 10 t^2$

$s_1 = -5t^2$

$h = s_1 - s_2$

$h = -5t^2 - (ut - \frac{1}{2} at^2)$

$h = -5t^2 - (0 - \frac{1}{2} a (t-1)^2)$

$= -5t^2 + 5t^2 - 10t + 5$

$h = 5 - 10t$

$-5t^2 + 10t - 5 = \frac{1}{7}(5 - 10t)$

$-35t^2 + 70t - 35 = 5 - 10t$

$0 = 35t^2 - 65t + 25$

$\frac{h}{7} =$

$S_1 = -\frac{1}{2} \times 10 \times t^2 = -5t^2$

$\frac{h}{7} = -5t^2$

$h = h_c + h_s$

$h_s = h - h_c$

$\frac{h}{7} = -\frac{h}{2} - (-\frac{1}{2} \times 10 \times (t-1)^2)$

$\frac{6h}{7} = 5t^2 - 10t + 5$

$-30t^2 = 35t^2 - 70t + 35$

$65t^2 - 70t + 35$

$13t^2 - 14t + 7 = 0$

$\frac{6}{7} h = \frac{1}{2} 10 t^2$

$h = \frac{7}{6} \times 5 = \frac{35}{6}$

875, 175, 35, 7, 1

2275, 455, 91, 7, 1

2625