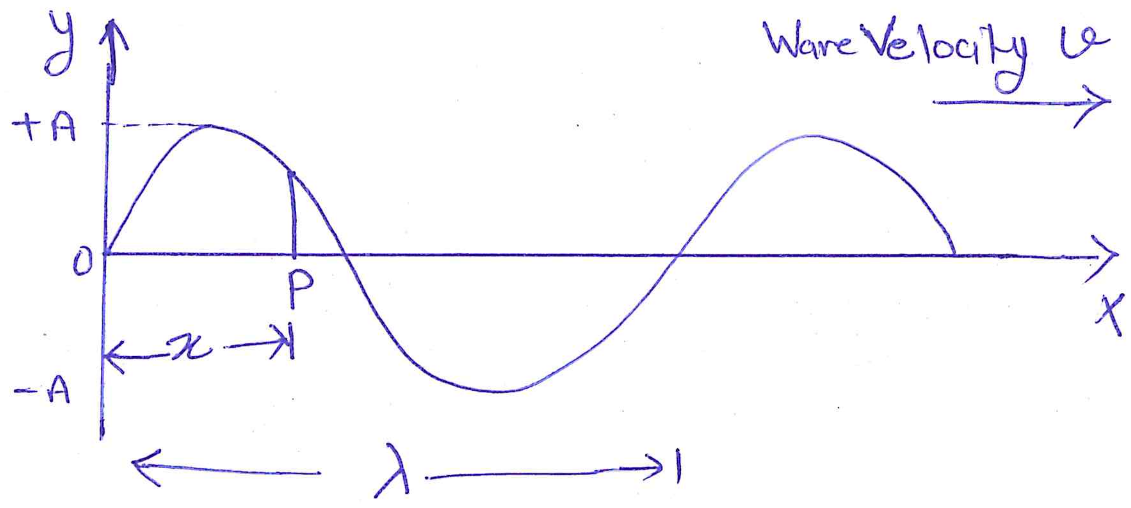
Wave Motion — Wave motion is a kind of ~~distur~~ disturbance which travels through a medium due to repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next particle. In a wave, both information and energy propagate from one point to another but there is no motion of matter as a whole through a medium.

Progressive Wave — A wave that travels from one point of medium to another is called a progressive wave, it may be transverse or longitudinal wave.

Transverse Wave — Wave motion in which particle of medium vibrate about their mean position at right angles to the direction of ~~prog~~ propagation of wave. e.g light waves, water waves

Longitudinal Wave — Wave motion in which the particles of the medium vibrate about their mean position along the same line as propagation of wave. e.g. Sound waves.

Equation of Plane progressive Wave



The displacement of the particle at the origin 'o' at any instant 't' is given by

$$y(0,t) = A\sin\omega t \quad \text{——— (1)}$$

Consider a particle P on X-axis at a distance x from origin O, so the time taken by the particle to reach P is $\frac{x}{v}$ sec. This means the particle P will start vibration $\frac{x}{v}$ seconds later than the particle at O.

Therefore

Displacement of the particle at P at any instant 't'
= Displacement of the particle at 'O' at time $(t-\frac{x}{v})$

Thus the displacement of the particle at P at any instant time 't' can be obtained by replacing t by $(t-x/v)$ in eqn (1)

$$y(x,t) = A\sin\omega\left(t-\frac{x}{v}\right)$$

$$y(x,t) = A \sin\left(\omega t - \frac{\omega}{v} x\right)$$

But $\frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k$ $\left\{ \begin{array}{l} \text{Wave Velo. } v = \frac{\lambda}{T} \\ v = \nu\lambda \\ \frac{\nu}{v} = \frac{1}{\lambda} \end{array} \right\}$

k is called angular wave number or propagation constant.

So $y(x,t) = A \sin(\omega t - kx)$ —— (2)

This eqn represents a harmonic wave travelling along + X-axis. It can also be written in the following forms:

$$y(x,t) = A \sin\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)$$

$$y(x,t) = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{—— (3)}$$

Also can written

$$y(x,t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{\lambda} T\right)$$

$$y(x,t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right)$$

$$y(x,t) = A \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{T} t - x\right)$$

$$y(x,t) = A \sin \frac{2\pi}{\lambda}\left(vt - x\right)$$

UNIT-2 Wave and Oscillations

Periodic Motion — Any motion that repeats itself over and over again at regular ~~interval~~ intervals of time is called periodic or harmonic motion.
For Example; motion of any planet around the sun in an elliptical orbit is periodic and the motion of moon around the earth is periodic.

Oscillatory or Harmonic motion —

If a body moves back and forth repeatedly about its mean position, its motion is said to be Oscillatory or harmonic motion. Such a motion repeats itself over and over again about a mean position such that it remains confined within well defined limits on either side of the mean position.
for Example; The oscillations of a mass suspended from a spring. and the swinging motion of the pendulum of a wall clock.

Simple Harmonic motion —

A particle is said to execute simple harmonic motion if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from the mean position and is always directed towards the mean position.

If the displacement of the oscillating body from the mean position is small, then

Restoring force $\propto$ Displacement

$$F \propto x$$

$$F = -kx$$

k is force constant which is defined as the restoring force produced per unit displacement.

According to Newton's second law of motion

$$F = ma$$

therefore $ma = -kx$

$$a = -\frac{k}{m}x$$

ie

$$a \propto x$$

Simple harmonic motion may also be ~~fan~~ defined as

A particle is said to possess simple harmonic motion if it is moves to and fro about a mean position under an acceleration which is directly proportional to its displacement from the mean position and always directed towards that position.

Differential Equation for Simple Harmonic motion

We know that in S.H.M. the restoring force acting on the particle is proportional to its ~~displacem~~ displacement

$$F = -kx \qquad \text{--- (1)}$$

negative sign shows that F and x are oppositely directed.

By Newton's Second law

$$F = ma = m\frac{d^2x}{dt^2} \qquad \text{--- (2)}$$

from eqn (1) and eqn (2)

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

let $\frac{k}{m} = \omega^2$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Here ω is angular frequency

or $$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \text{--- (3)}$$

This is the differential eqn of SHM

let the possible solution of differential eqn (3) is

$$x = A\cos(\omega t + \phi_0)$$

Then $\frac{dx}{dt} = -\omega A \sin(\omega t + \phi_0)$

and $\frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi_0)$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

or $\frac{d^2x}{dt^2} + \omega^2 x = 0$

which is same as eqn ③ Hence the solution of eqn ③ is

$$x = A\cos(\omega t + \phi_0) \quad \text{———} \quad ④$$

This gives displacement of the harmonic oscillator at any instant 't'.

Now the velocity of particle

$$v = \frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dx}{dt}\cdot\frac{dv}{dx}$$

$$\frac{d^2x}{dt^2} = \frac{v\,dv}{dx}$$

$$-\omega^2 x = \frac{v\,dv}{dx}$$

$$v\,dv = -\omega^2 x\,dx$$

Integrating both side.

$$\int v\,dv = \int -\omega^2 x\,dx$$

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C_1$$

$v = v_0$ at $x = 0$

So $$C_1 = \frac{v_0^2}{2}$$

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{v_0^2}{2}$$

$$v^2 = -\omega^2 x^2 + v_0^2$$

$$v^2 = v_0^2 - \omega^2 x^2$$

$$v = \sqrt{v_0^2 - \omega^2 x^2} \qquad \text{——— } (5)$$

The Speed of particle ~~decr~~ decreases as it moves away from the origin

Energy of Oscillation —

The Energy of harmonic Oscillator is partly Kinetic and partly potential. When a ~~bo~~ body is displaced from its Equilibrium position by doing work upon it, it acquires potential Energy. When the body is released, it begins to move back with a Velocity, thus ~~acquiring kinetic~~ acquiring Kinetic Energy.

1) Kinetic Energy — At any instant, the displacement of a particle executing Simple Harmonic motion is given by

$$x = A\cos(\omega t + \phi_0)$$

Therefore Velocity, $v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi_0)$

The Kinetic Energy of a particle at any displacement x is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi_0)$$

$$K = \frac{1}{2}m\omega^2 A^2[1 - \cos^2(\omega t + \phi_0)]$$

$$= \frac{1}{2}m\omega^2[A^2 - A^2\cos^2(\omega t + \phi_0)]$$

$$= \frac{1}{2}m\omega^2[A^2 - x^2]$$

$$K = \frac{1}{2}k(A^2 - x^2)$$

$$\left\{\frac{k}{m} = \omega^2\right\}$$

$$k = m\omega^2$$

force constant

$$K = \frac{1}{2}k(A^2 - x^2)$$

Kinetic Energy

Potential Energy —

When the displacement of a particle from its equilibrium position is x, the restoring force acting on it is

$$F = -kx$$

If we displace the particle further through a small distance dx then the work done against the restoring force is

$$dW = -F\,dx$$

$$dW = kx\,dx$$

So, the total work done in moving the particle from mean position $(x=0)$ to displacement x is

$$W = \int dW = \int_0^x kx\,dx$$

$$W = \frac{1}{2}kx^2$$

This work done against the restoring force is stored as the potential energy of a particle.

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \qquad \left\{ \frac{k}{m} = \omega^2 \right.$$

$$U = \frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi_0)$$

So, total Energy $= K + U$

$$= \frac{1}{2}k(A^2-x^2) + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kA^2$$

$$E = \frac{1}{2}m\omega^2A^2$$

Graphical representation —

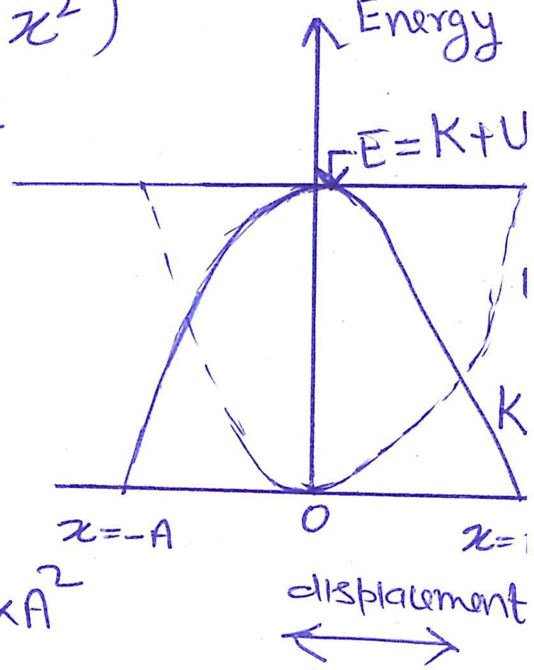
1) At the mean position, $x = 0$

Kinetic Energy $K = \frac{1}{2} k (A^2 - x^2)$

$$K = \frac{1}{2} k A^2$$

Potential Energy $U = \frac{1}{2} k x^2$

$$U = 0$$



So, total Energy at $x=0$ is

$$E = K + U = \frac{1}{2} k A^2$$

Hence at mean position, the Energy is all Kinetic.

2) At Extreme positions, $x = \pm A$

Kinetic Energy $K = \frac{1}{2} k (A^2 - x^2) = \frac{1}{2} k (A^2 - A^2)$

$$K = 0$$

and potential Energy $U = \frac{1}{2} k A^2$

Hence, at the two extreme positions, the Energy is all potential.

The figure shows the variations of Kinetic energy K, potential Energy and total energy E with displacement. The graph for K and U are parabolic while that for E is a straight line parallel to the displacement ~~x~~ axis. At $x=0$ the energy is all Kinetic and for $x = \pm A$ the Energy is all potential.

Superposition of waves —

According to the principle of Superposition, when a medium is disturbed Simultaneously by any numbers of waves, the Instantaneous resultant displacement of the medium at an instant is the ~~algebraic~~ algebraic Sum of the displacement of the medium due to individual waves in the absence of others.

Let $y_1, y_2, y_3 \cdots y_n$ be the displacements due to the wave $1, 2, 3 \cdots n$.

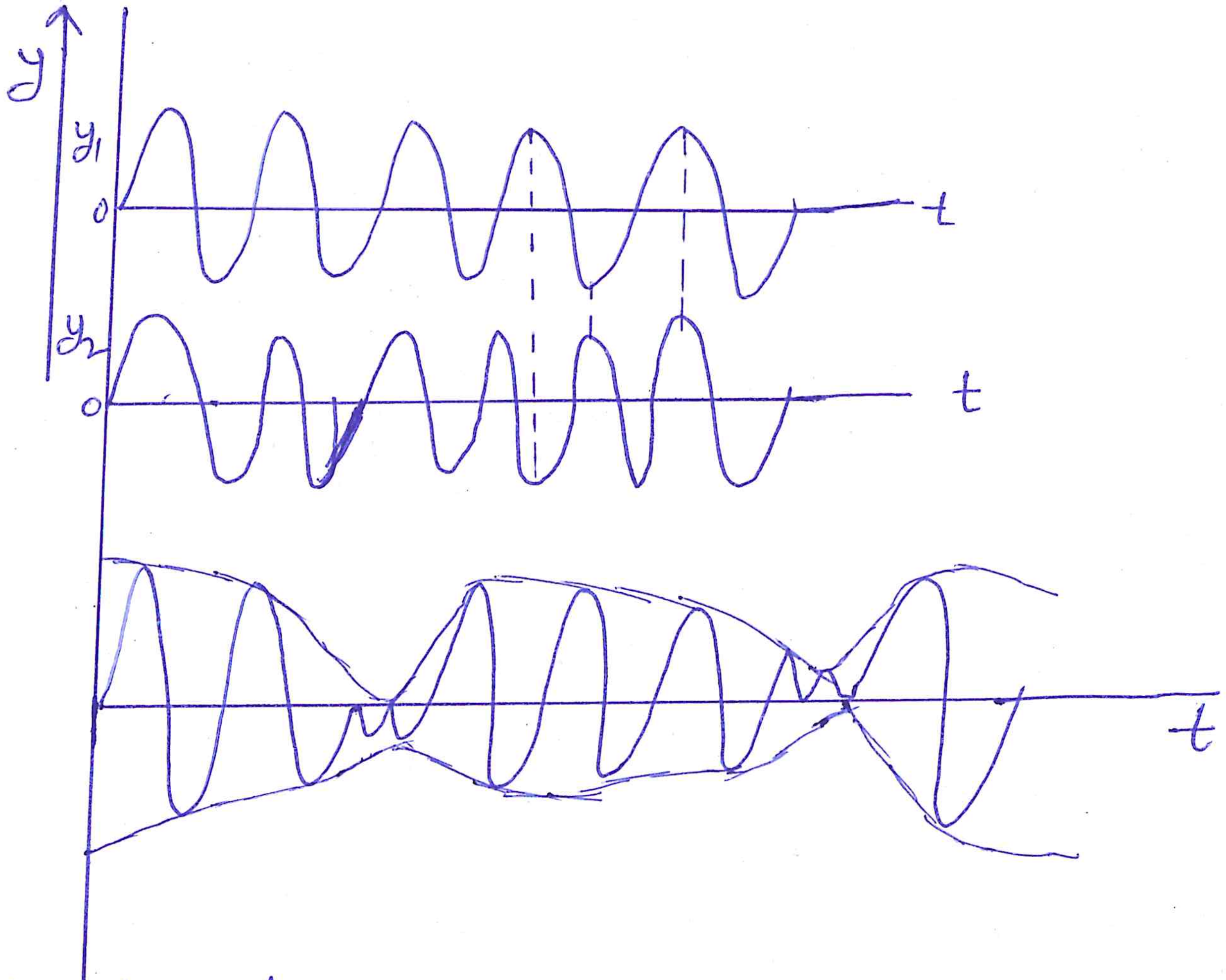
Then $$y = y_1 + y_2 + y_3 \cdots y_n$$

1) Two waves of Same frequency moving in the Same direction give rise to "interference of waves".

2) Two waves of slightly different frequency moving in the Same direction give rise to "beats."

3) Two waves of Same frequency moving in the opposite direction produces Stationary waves.

<u>Beats</u> : Two waves of Slightly different frequencies moving in the Same direction produces Waxing and Waning of Sound Called beats.



<u>Mathematical Analysis</u> ——

Let y_1 and y_2 be the displacements of the two waves from the mean position having frequency n_1 and n_2.

$$y_1 = a \sin 2\pi n_1 t \quad \text{——} (1)$$

$$y_2 = a \sin 2\pi n_2 t \quad \text{——} (2)$$

The According to Superposition

$$y = y_1 + y_2$$

$$y = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t$$

$$= \left[2a \cos 2\pi \left(\frac{n_1 - n_2}{2}\right)t\right] \times \sin 2\pi \left(\frac{n_1 + n_2}{2}\right)t$$

$$= A \sin 2\pi \left(\frac{n_1 + n_2}{2}\right)t$$

Where $A = 2a \cos 2\pi \left(\frac{n_1 - n_2}{2}\right)t$

A = amplitude

For A to be maximum

$$\cos 2\pi \left(\frac{n_1 - n_2}{2}\right)t = 1$$

Therefore, $2\pi \left(\frac{n_1 - n_2}{2}\right)t = k\pi$

where $k = 0, 1, 2, \ldots$

So $\boxed{t = \frac{k}{n_1 - n_2}}$

for $k = 0$

$$t_0 = 0$$

$k = 1$, $t_1 = \frac{1}{n_1 - n_2}$ and $k = 2$ $t_2 = \frac{2}{n_1 - n_2}$

So, the time interval between two successive maxima is

$$\boxed{\Delta t = \frac{1}{n_1 - n_2} \text{ sec.}}$$

For minima

$$\cos 2\pi \left(\frac{n_1-n_2}{2}\right)t = 0$$

or $$2\pi \left(\frac{n_1-n_2}{2}\right)t = \frac{(2k+1)\pi}{2}$$

$$t = \frac{(2k+1)}{2(n_1-n_2)} \qquad \text{where } k = 0, 1, 2, \ldots$$

For $k=0$

$$t_0 = \frac{1}{2(n_1-n_2)}$$

for $k=1$

$$t_1 = \frac{3}{2(n_1-n_2)}$$

$k=2$

$$t_2 = \frac{5}{2(n_1-n_2)}$$

So the time interval between two successive minima is

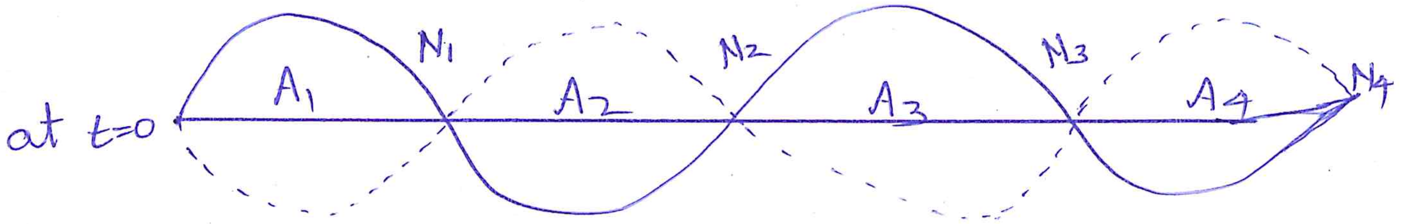
$$\boxed{\Delta t = \frac{1}{n_1-n_2} \text{ sec}}$$

Therefore, frequency for maxima and minima is same.

because $$\boxed{\Delta t_{max} = \Delta t_{min}}$$

Stationary waves —

When two identical progressive waves travel through a medium along the same line in opposite directions with same velocities. They superimpose over each other and produce a new type of wave called Stationary waves.



There are certain points where displacement is minimum called nodes, and the points where displacement is maximum called Antinodes.

Analytical treatment —

Let the equation of simple harmonic wave travelling along positive x-axis be

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)$$

The equation of the reflected wave i.e. ($-x$ axis) be

$$y_2 = a \sin \frac{2\pi}{\lambda}(vt + x)$$

By principle of superposition

$$y = y_1 + y_2$$

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x)$$

$$\boxed{y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}}$$ → Eqn of Stationary wave

$$\boxed{\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}}$$

Introduction to Electromagnetic Theory

Maxwell's Equations —

Differential form

1) $\nabla \cdot \vec{D} = \rho$

or $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

2) $\nabla \cdot \vec{B} = 0$

3) $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

4) $\nabla \times \vec{H} = J + \epsilon_0 \frac{\partial E}{\partial t}$

Integral form

① $\oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \int \rho \, dv$

2) $\oint \vec{B} \cdot d\vec{S} = 0$

3) $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$

4) $\oint \vec{H} \cdot d\vec{l} = \int \left(\vec{J} + \epsilon_0 \frac{\partial E}{\partial t} \right) \cdot d\vec{S}$

Physical Significance of Maxwell's Equations —

Maxwell's first Eqn $\nabla \cdot \vec{E} = \rho / \epsilon_0$ or $\text{div}\, \vec{E} = \rho / \epsilon_0$ represent the Gauss's Law in Electrostatic for static Charges, which state that the electric flux through any Closed hypothetical Surface is equal to $1/\epsilon_0$ times the total Charge enclosed by the Surface.

Maxwell's Second Equation ($\mathrm{div}\, B = 0$) Expresses Gausses Law for magnetism. It State that the net magnetic flux through any Closed Surface is zero. Since a magnetic monopole does not exist, any Closed Volume always Contains Equal and opposite magnetic poles (N-pole and S-pole) leading to a net magnetic pole Strength zero." It also Signifies that the number of magnetic lines of the flux entering into any region is Equa to line of flux leaving it or magnetic lines of flux are Continuous."

Third Maxwell's Equation ($\nabla \times E = -\partial B / \partial t$) is a faradays Law ~~of~~ in Electromagnetic induction. It State that induced electromotive force around any Closed Surface is Equal to the negative time rate of charge of magnetic flux through the path ~~enclo~~ enclosing the Surface. Therefore "It Signifies that an electric field is produced by a changing magnetic flux"!

Fourth Maxwell's Equation ($\nabla \times H = J + \epsilon_0 \frac{\partial E}{\partial t}$) represents the generalized form of Ampere's Law as extended by Maxwell for the ~~inclusing~~ inclusion of time Varing Electric fields. It state that magnetomatic force around a Closed path is Equals to the sum of Conduction Current and displacement current through the Surface bounded by that path." This Signifies that a Conduction Current as well as changing Electric flux, produce a magnetic field!"

Maxwell's first Equation —

When a thin dielectric sample is placed in a Uniform Electric field its molecules get polarised. Thus, a dielectric Sample in an electric field contains free charge.

If ρ and ρ_p are the free and bound Charge densities respectively at a point in a small Volument element dV, Then for such medium Gauss's law may be expressed as

$$\int_S E\,ds = \frac{1}{\epsilon_0}\int_V (\rho + \rho_p)\,dV \quad \text{——— (1)}$$

We know that the polarisation Charge density

$$\rho_p = -\operatorname{div}\vec{P}$$

$\vec{P} \rightarrow$ Electric polarisation

Substituting the value of ρ_p in eqn (1) we get

$$\int \vec{E}\,\vec{ds} = \frac{1}{\epsilon_0}\int_V (\rho - \operatorname{div}\vec{P})\,dV$$

$$\int \epsilon_0 \vec{E}.\vec{ds} = \int_V \rho\,dV - \int_V \operatorname{div}P\,dV$$

According to Gauss divergence theorem, Surface integral Can be change into Volume Integral.

$$\int_V \operatorname{div}(\epsilon_0 E)\,dV = \int_V \rho\,dV - \int_V \operatorname{div}P\,dV$$

$$\int_V \operatorname{div}(\epsilon_0 E + P)\,dV = \int_V \rho\,dV \quad \text{——— (2)}$$

But $\epsilon_0 E + P = D$ (Electric displacement Vector)

$$\int_V \operatorname{div} D\,dV = \int_V \rho\,dV$$

$$\int_V (\operatorname{div} D - \rho)\, dv = 0 \qquad \text{--- (3)}$$

Since Eqn (3) is true for all Volumes, hence the integrand, $\operatorname{div} D - \rho$ must vanish i.e.

$$\operatorname{div} D - \rho = 0$$

$$\operatorname{div} D = \rho$$

$$\boxed{\begin{aligned} \nabla \cdot D &= \rho \\ \nabla \cdot E &= \frac{\rho}{\epsilon_0} \end{aligned}}$$

This is required maxwells first equation

In free space Volume charge density is zero $(\rho = 0)$

$$\nabla \cdot D = 0$$

or $\nabla \cdot E = 0$

Maxwell's Second Equation $\mathrm{div}\,B = 0$ or $\nabla \cdot B = 0$

The number of magnetic lines of force entering any arbitary closed surface enclosing a volume is exactly the same as that leaving it. i.e. the net magnetic flux of magnetic induction $\vec{B}$ through any closed Gaussian surface is always zero.

$$\phi_B = \oint \vec{B}\cdot d\vec{S} = 0 \quad \text{——— (1)}$$

According to Gauss divergence theorem, surface integral can be change into volume integral

$$\int_S \vec{B}\cdot d\vec{S} = \cancel{\oint_V \mathrm{div}\, B} \int_V \mathrm{div}\,\vec{B}\, dV = 0 \quad \text{—— (2)}$$

or $$\int_V \mathrm{div}\,\vec{B}\, dV = 0 \quad \text{——— (3)}$$

Since Eqn (3) is true for all volumes, therefor the integrand $\mathrm{div}\,\vec{B}$ must vanish,

$$\boxed{\mathrm{div}\,\vec{B} = 0}$$

$$\boxed{\nabla \cdot B = 0}$$

3- Maxwell's Third Equation or Faraday's Law of Electromagnetic Induction

$$\text{Curl } \vec{E} = \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

According to Faraday's Law of Electromagnetic induction induced e.m.f. around a closed circuit is equal to the negative time rate of change of magnetic flux linked with the circuit. i.e.

$$e = -\frac{d\phi_B}{dt} \quad \text{———} \quad (1)$$

But $\phi_B = \int \vec{B} \cdot \vec{ds}$ ——— (2)

$$e = -\frac{d}{dt} \int \vec{B} \, \vec{ds}$$

$$e = -\int \frac{d\vec{B}}{dt} \vec{ds} \quad \text{———} \quad (3)$$

But changing magnetic flux induces an electric field around a circuit.

So, $e = \int_C \vec{E} \, d\vec{l}$ ——— (4)

$$\int \vec{E} \, d\vec{l} = -\int \frac{d\vec{B}}{dt} \vec{ds}$$

According to Stoke's theorem the line integral may be transform into surface integral

$$\int_C \vec{E} \, d\vec{l} = \int_S \text{curl } \vec{E} \, \vec{ds}$$

$$\int_S \text{Curl}\,\vec{E}\,d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t}\,d\vec{S}$$

$$\int \left(\text{Curl}\,\vec{E} + \frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S} = 0 \quad \text{——— (5)}$$

Eqn (5) is true for all surface, therefore the integrand must be vanish i.e.

$$\text{Curl}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\boxed{\begin{aligned}\text{Curl}\,\vec{E} &= -\frac{\partial \vec{B}}{\partial t}\\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t}\end{aligned}}$$

4. Maxwell's fourth Equation $\text{Curl}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

In a closed path, the total current I is the flux of current density J through the surface bounded by closed loop.

$$I = \int_S \vec{J}\,d\vec{S} \quad \text{——— (1)}$$

But according to Ampere's Circuital law

$$\oint H\,dl = I \quad \text{——— (2)}$$

Therefore

$$\int \vec{H}\,d\vec{l} = \int \vec{J}\,d\vec{S}$$

According to stoke's theorem, line integral may be transform into surface integral

$$\int \vec{H}\,d\vec{l} = \int \text{Curl}\,\vec{H}\,d\vec{S}$$

$$\int \text{Curl}\,\vec{H}\,d\vec{s} = \int \vec{J}\,d\vec{s}$$

$$\int (\text{Curl}\,\vec{H} - \vec{J})\,d\vec{s} = 0 \qquad \text{—— (3)}$$

As the surface is arbitrary, integrand of eqn (3) must vanish

$$\text{Curl}\,\vec{H} - \vec{J} = 0$$

$$\text{Curl}\,\vec{H} = \vec{J} \qquad \text{—— (4)}$$

Let us examine eqn (4) for time varying field

taking div both side of eqn 4

$$\text{div curl}\,\vec{H} = \text{div}\,\vec{J}$$

So $\text{div}\,\vec{J} = 0$ $\qquad \{\text{div Curl } H = 0\}$

But

$$\text{div}\,J + \frac{\partial \rho}{\partial t} = 0 \qquad \text{for time varying field}$$

$$\text{div}\,J = -\frac{\partial \rho}{\partial t}$$

So eqn (4) holds only for steady state condition. So as to make the equation consistent with the equation of continuity, Maxwell added a current density J_d to the ~~ord~~ original current density J.

So $\text{Curl}\,H = J + J_d$

$$\text{div Curl}\,H = \text{div}\,(J + J_d)$$

$$\text{div}(J + J_d) = 0$$

$$\text{div}\,J + \text{div}\,J_d = 0$$

$$\text{div } J_d = -\text{div } J \qquad \left\{ \text{div } J = -\frac{\partial \rho}{\partial t} \right\}$$

$$\text{div } J_d = -\left(-\frac{\partial \rho}{\partial t}\right)$$

$$\text{div } J_d = \frac{\partial \rho}{\partial t}$$

But $\text{div } D = \rho$ from maxwell's first eqn

So $$\text{div } J_d = \frac{\partial(\text{div } D)}{\partial t}$$

$$\text{div } J_d = \text{div}\left(\frac{\partial D}{\partial t}\right)$$

$$J_d = \frac{\partial D}{\partial t}$$

And $\text{Curl } H = J + J_d$

$$\boxed{\begin{aligned} \text{Curl } \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \nabla \times H &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned}}$$

Poynting Theorem and Poynting Vector —

Poynting theorem States that the Work done on the Charge by an electromagnetic force is equal to the decrease in energy stored in the field, less than the Energy which Flowed out through the Surface. It also Called the energy Conservation law in electrodynamics.

The Energy per Unit time, per unit area transported by electromagnetic field is Called the poynting Vector and is represented by

$$\vec{S} = (\vec{E} \times \vec{H})$$

$$\text{or } S = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$$\boxed{\int_v (\vec{E}\cdot\vec{J})\,dv = -\frac{\partial}{\partial t}\int_v \left(\frac{\mu}{2}H^2 + \frac{\epsilon}{2}E^2\right)dv - \int (\vec{E}\times\vec{H})\cdot ds}$$

$\int_v (\vec{E}\cdot\vec{J})\,dv \rightarrow$ represent the total power dissipated in a Volume V or Work done per unit time, per unit Volume By the Electromagnetic field.

$\left(\frac{\mu}{2}H^2 + \frac{\epsilon}{2}E^2\right) \rightarrow$ represent the Energy Stored in magneti field and Electric field or total Energy Stored in Electromagnetic field.

relation may be obtained through maxwell's Eqn

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \left\{ \begin{matrix} D = \epsilon E \\ B = \mu H \end{matrix} \right\}$$

$$\nabla \times H = J + \epsilon \frac{\partial E}{\partial t}$$

So $$J = \nabla \times H - \epsilon \frac{\partial E}{\partial t} \quad \text{——— (1)}$$

This is the Equation of Current density, when multiplied by E, this will be result in a relation between the quantities which has the dimension of power

$$E.J = E.(\nabla \times H) - \epsilon E \frac{\partial E}{\partial t} \quad \text{——— (2)}$$

We have Vector identity

$$\nabla.(E \times H) = H.(\nabla \times E) - E(\nabla \times H)$$

from Eqn (2) we have

$$E.J = H(\nabla \times E) - \nabla.(E \times H) - \epsilon E \frac{\partial E}{\partial t} \quad \text{——— (3)}$$

from maxwell's third Eqn

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$

We have

$$E.J = -H.\mu \frac{\partial H}{\partial t} - \epsilon E \frac{\partial E}{\partial t} - \nabla.(E \times H)$$

$$E.J = -\mu H \frac{\partial H}{\partial t} - \epsilon E \frac{\partial E}{\partial t} - \nabla.(E \times H)$$

$$H \cdot \frac{\partial H}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} H^2$$

and $$E \cdot \frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} E^2$$

So $$E \cdot J = -\frac{\mu}{2}\frac{\partial}{\partial t} H^2 - \frac{\epsilon}{2}\frac{\partial}{\partial t} E^2 - \nabla \cdot (E \times H) \quad \text{--- (4)}$$

Integrating over Volume V, We have

$$\int (E \cdot J)\, dV = -\frac{\partial}{\partial t} \int \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2\right) dV - \int \nabla \cdot (E \times H)\, dV$$

Now using the divergence theorem

$$\int \nabla \cdot (E \times H)\, dV = \int (E \times H) \cdot ds$$

$$\int (E \times H) \cdot ds = -\frac{\partial}{\partial t} \int \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2\right) dV - \int (E \cdot J)\, dV \quad \text{--- (5)}$$

Eqn (5) is Called the poynting theorem and $S = E \times H$ is Called poynting vector

Electromagnetic Waves —

The coupled oscillating electric and magnetic fields that moves with the speed of light and exhibit wave behaviour is called Electromagnetic wave. The Electric and magnetic components of plane electromagnetic wave are perpendicular to each other and to the direction of propagation. Such waves are also called plane polarised electromagnetic wave.

Wave Equation —

If we consider a linear medium having permittivity ε, permeability μ and conductivity σ

$$D = \varepsilon E, \quad B = \mu H, \quad J = \sigma E \text{ (current density)}$$

and charge density $\rho = 0$

With these condition Maxwell's eqn reduce to

$$\nabla \cdot D = \rho \Rightarrow \nabla \cdot E = 0 \quad \text{—— (1)}$$

$$\nabla \cdot B = 0 \Rightarrow \nabla \cdot H = 0 \quad \text{—— (2)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow \nabla \times E = -\mu \frac{\partial H}{\partial t} \quad \text{—— (3)}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \Rightarrow \nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \quad \text{—— (4)}$$

Now taking the curl of eqn (3)

$$\text{Curl Curl } E = -\mu \frac{\partial}{\partial t}(\text{Curl } H)$$

$$\text{Curl Curl } E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\text{Curl Curl } E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{——— (5)}$$

Similarly, $\text{Curl Curl } H = -\sigma\mu \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2}$ ——— (6)

We know that

$$\text{Curl Curl } A = \text{grad div } A - \nabla^2 A$$

But $\text{div } E = \text{div } H = 0$

therefore Eqn (5) and Eqn (6) may be

$$-\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

or $\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$ ——— (7)

and $\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0$ ——— (8)

Eqn (7) and Eqn (8) are the wave Eqns.

Plane Electromagnetic waves in free Space—

Maxwell's Eqns are

$$\nabla\cdot D = \rho$$

$$\nabla\cdot B = 0$$

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

$$\nabla\times H = J + \frac{\partial D}{\partial t}$$

Here $B = \mu H$, $D = \epsilon E$ and $J = \sigma E$

for free Space $\rho = 0$, $\sigma = 0$, $\mu = \mu_0$ and $\epsilon = \epsilon_0$

therefore

$$\nabla\cdot E = 0 \quad \text{———— (1)}$$

$$\nabla\cdot H = 0 \quad \text{———— (2)}$$

$$\nabla\times E = -\mu_0\frac{\partial H}{\partial t} \quad \text{———— (3)}$$

$$\nabla\times H = \sigma E + \frac{\partial \epsilon_0 E}{\partial t}$$ Here $\sigma = 0$

$$\nabla\times H = \epsilon_0\frac{\partial E}{\partial t} \quad \text{———— (4)}$$

taking the Curl of Eqn (3) and Eqn (4)

$$\text{Curl Curl } E = -\mu_0\frac{\partial}{\partial t}\text{Curl } H$$

$$= -\mu_0\frac{\partial}{\partial t}\left(\epsilon_0\frac{\partial E}{\partial t}\right)$$

$$\text{Curl Curl } E = -\mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\text{Curl Curl } E = \text{grad div } E - \nabla^2 E$$

$$\text{Curl Curl } E = -\nabla^2 E \qquad \{\text{div} E = 0\}$$

So

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{———} \quad (5)$$

Similarly

$$\nabla^2 H - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{———} \quad (6)$$

Eqn (5) and eqn (6) are the wave eqn in free space

or Eqn (5) and Eqn (6) can be written as also

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{———} \quad (7)$$

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{———} \quad (8)$$

where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/sec.

Let the solⁿ of eqn (7) and Eqn (8) are

$$E(r,t) = E_0 e^{ik \cdot r - i\omega t} \quad \text{———} \quad (9)$$

$$H(r,t) = H_0 e^{ik \cdot r - i\omega t} \quad \text{———} \quad (10)$$

Where E_0 and H_0 are Complex amplitudes which are Constant in Space and $\hat{k} = k\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\nu}{c}\hat{n} = \frac{\omega}{c}\hat{n}$

$\hat{n}$ is a unit vector in the direction of Wave propagati

In order to apply the Condition

$$\nabla . E = 0$$
$$\nabla . H = 0$$

$$\nabla . E = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) E_0 e^{ik \cdot r - i\omega t}$$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot \left[(i E_{0x} + j E_{0y} + k E_{0z})\right] e^{i(k_x x + k_y y + k_z z) - i\omega t}$$

$$= i(i k_x + j k_y + k k_z) \cdot (i E_{0x} + j E_{0y} + k E_{0z}) e^{ik \cdot r - i\omega t}$$

$$= i k \cdot E_0 e^{ik \cdot r - i\omega t}$$

$$\nabla . E = ik . E$$

Similarly $\nabla . H = ik . H$

Therefore $k . E = 0$

$k . H = 0$

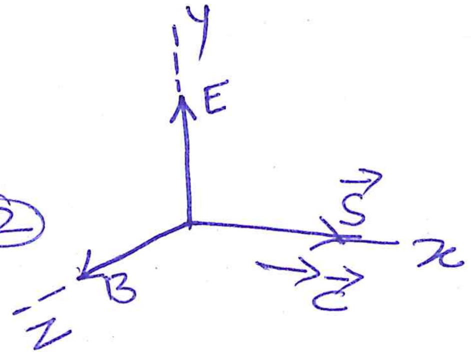
Which means E and H are both perpendicular to the direction of propagation k i.e. Electromagnetic Waves are transverse in character.

Energy Carried by Electromagnetic Waves —

The rate of flow of Energy in an Electromagnetic wave is described by a vector S, Called pointing Vector, Which is defined by the Expression

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \quad \text{———} \ (1)$$

The magnitude of $\vec{S}$ represents power per Unit area. The direction of vector is along the direction of wave propagation.



$$|\vec{S}| = |\vec{E} \times \vec{B}| = EB$$

$$S = \frac{EB}{\mu_0} \quad \text{———} \ (2)$$

But $B = \frac{E}{c}$

So, $$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \quad \text{———} \ (3)$$

This Eqn represent the instantaneous rate at which Energy is passing through a unit Area.

The time average of S over one or more cycle of Sinusoidal plane Electromagnetic wave is called wave intensity I.

**[We obtain an expression involving the time average of $\cos^2(kx-\omega t)$ in the case of intensity of Sound wave ~~which~~ which is Equals to $\frac{1}{2}$]

Hence the average value of 'S' is

$$I = S_{avg} = \frac{E_{max} B_{max}}{2 \mu_0}$$

$$= \frac{E_{max}^2}{2 \mu_0 c}$$

$$S_{avg.} = \frac{c B_{max}^2}{2 \mu_0} \quad \text{——— (4)}$$

The instantaneous energy density u_E associated with an Electric field is

$$u_E = \frac{1}{2} \epsilon_0 E^2 \quad \text{——— (5)}$$

and instantaneous Energy density u_B associated with magnetic field is

$$u_B = \frac{1}{2} \frac{B^2}{\mu_0} \quad \text{——— (6)}$$

Because E and B vary with time for an Electromagnetic wave, the Energy densities also vary with time.

Using the relationship $B = E/c$ and $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

The Expression for u_B becomes

$$u_B = \frac{(E/c)^2}{2 \mu_0}$$

$$= \frac{\mu_0 \epsilon_0}{2 \mu_0} E^2$$

$$u_B = \frac{1}{2} \epsilon_0 E^2 \quad \text{——— (7)}$$

Comparing Eqn 5 and 7

$$u_B = u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

the instantaneous energy density associated with the magnetic field of an electromagnetic wave equals the instantaneous energy density associated with the electric field.
So, in a given volume, the energy is equally shared by the two fields.

The total instantaneous energy density 'u' is equal to the sum of energy density associated with the electric and magnetic field.

$$u = u_E + u_B$$

$$\boxed{u = \epsilon_0 E^2}$$

also $$\boxed{u = \frac{B^2}{\mu_0}}$$

The total average energy per unit volume is

$$u_{avg} = \epsilon_0 (E^2)_{avg}$$

$$u_{avg} = \frac{1}{2}\epsilon_0 E_{max}^2$$

also $$u_{avg} = \frac{B_{max}^2}{2\mu_0}$$