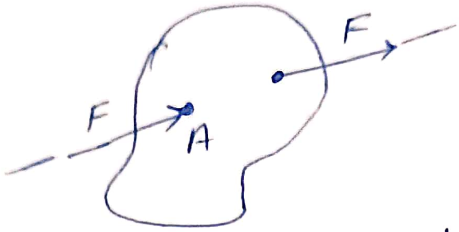
Principle of Transmissibility of forces

It states, "If a force acts at any point on a rigid body, it may also be considered to act at anyother point on its line of action, provided this point is rigidly connected with the body. That means the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body.

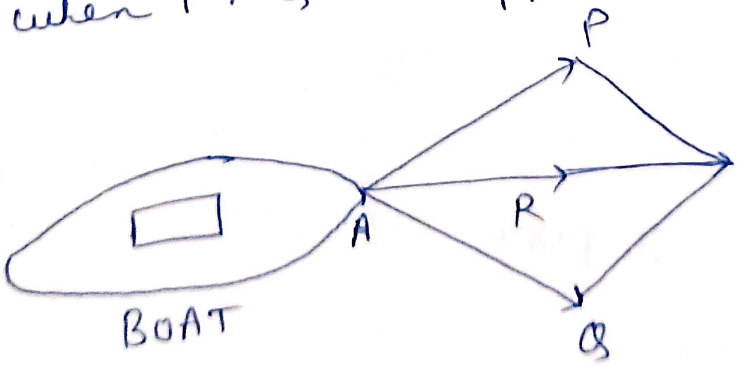


Hence force at point A = force at B (the magnitude of force in the body at any point along the line of action are same).

Principle of superposition of forces:

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

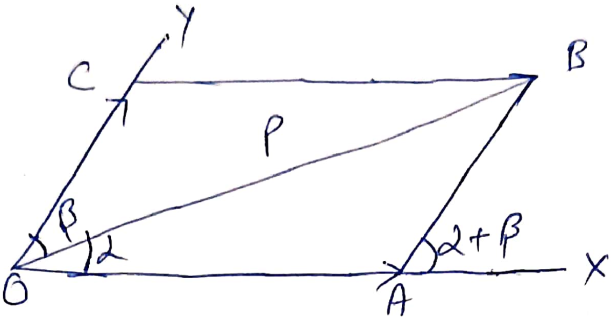
Consider two forces P & Q acting at A on a boat as shown in fig. Let R be the resultant of these two forces P & Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P & Q are applied simultaneously.



Resolution of a given force into two components in two assigned Direction

Let P be the given force represented in magnitude & direction by OB as shown in fig.

Also let OX & OY be two given direction along which the components of P are to be found out.



Let $\angle BOX = \alpha$ & $\angle BOY = \beta$

From B, lines BA & BC are drawn parallel to OY & OX.

Then the required components of the given force P along OX & OY are represented in magnitude & direction by OA & OC respectively. Since AB is parallel to OC,

$$\angle BAX = \angle AOC = \alpha + \beta$$

$$\angle AOB = 180°$$

$$\angle OAB = 180 - (\alpha + \beta)$$

Now, in ΔOAB

$$\frac{OA}{\sin OBA} = \frac{AB}{\sin AOB} = \frac{OB}{\sin OAB}$$

or $$\frac{OA}{\sin\beta} = \frac{AB}{\sin\alpha} = \frac{OB}{\sin[180 - (\alpha+\beta)]}$$ $[\sin(180 - \theta) = \sin\theta$

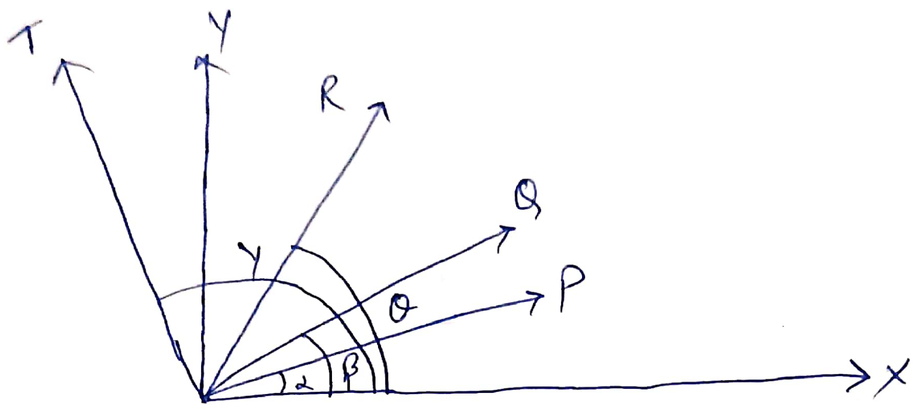
$$\frac{OA}{\sin\beta} = \frac{AB}{\sin\alpha} = \frac{P}{\sin(\alpha+\beta)}$$

$$OA = \frac{P\sin\beta}{\sin(\alpha+\beta)} \quad \& \quad AB = \frac{P\sin\alpha}{\sin(\alpha+\beta)}$$

But $AB = OC$

$$OC = \frac{P\sin\alpha}{\sin(\alpha+\beta)}$$

Analytical method of determining the resultant of any number of co-planar concurrent forces.



Resolving forces horizontally.

Then, $\Sigma X = P\cos\alpha + Q\cos\beta + T\sin(\gamma - 90)$

$= P\cos\alpha + Q\cos\beta - T\cos\gamma$

$\Sigma Y = P\sin\alpha + Q\sin\beta + T\cos(\gamma - 90)$

$P\sin\alpha + Q\sin\beta + T\sin\gamma$

$\Sigma X = R\cos\theta$ & $\Sigma Y = R\sin\theta$

$(\Sigma X)^2 + (\Sigma Y)^2 = R^2\cos^2\theta + R^2\sin^2\theta$

$= R^2(\cos^2\theta + \sin^2\theta)$

$= R^2$

$$\boxed{R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}} \Rightarrow \text{Resultant}$$

$$R = \sqrt{(\Sigma H)^2 + \Sigma V)^2}$$

$$\frac{R\sin\theta}{R\cos\theta} = \frac{\Sigma Y}{\Sigma X} \left(= \frac{\Sigma V}{\Sigma H}\right)$$

$$\tan\theta = \frac{\Sigma Y}{\Sigma X} \left(= \frac{\Sigma V}{\Sigma H}\right)$$

(4)

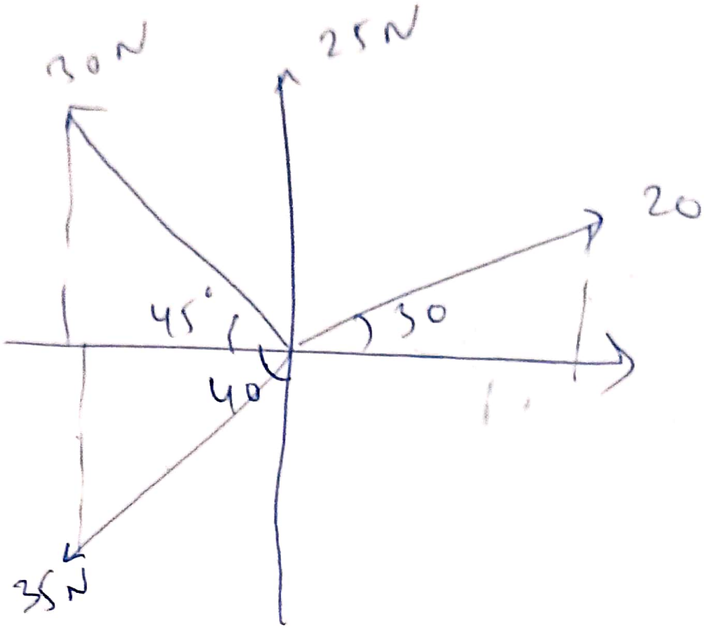
Example 1.3 A

exampl 1.7

$$\begin{bmatrix} \cos 0 = 1 \\ \cos 90 = 0 \end{bmatrix}$$

$\sin 0 = 0$

$\sin 90 = 1$



$\Sigma H = 20\cos 30 + 0 - 30\cos 45 - 35\cos 40$

$= -30.7$

$\Sigma V = 20\sin 30 + 25 + 30\sin 45 - 35\sin 40$

$= 33.7\,N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\tan\theta = \Sigma V / \Sigma H = \dfrac{33.7}{-30.7} = -1.098$

or $\theta = 47.7'$

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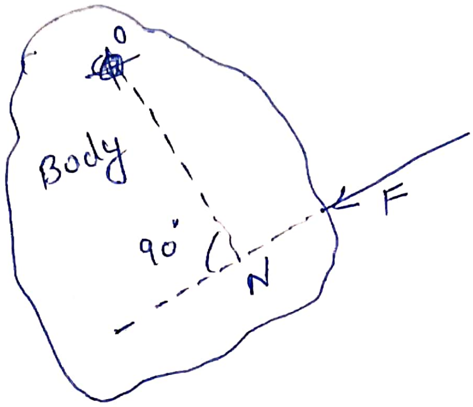
Moment of force

$M = P \times I$

when P = Force acting on the body

I = Perpendicular distance between the point, about which the moment is required and the line of action of force

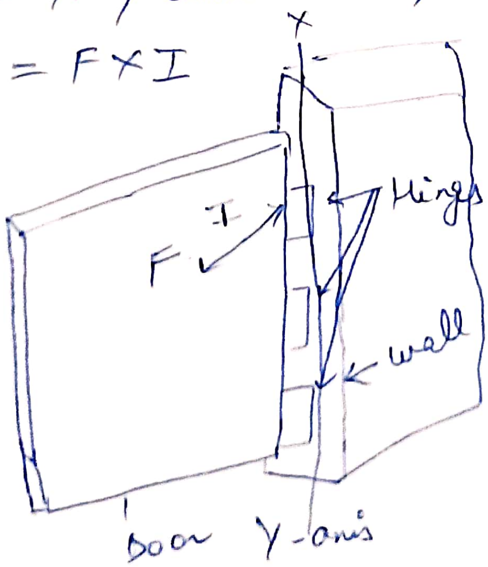
moment of a force about a point is the product of the force & the perpendicular distance of the point from the line of action of the force.



The moment of P about the point O in the body is = F×ON

Moment of force about an axis

let a force F be applied to the door leaf at right angles to its plane and at a perpendicular distance of I from the x-y axis. Then, moment of the force F about xy-axis = F×I

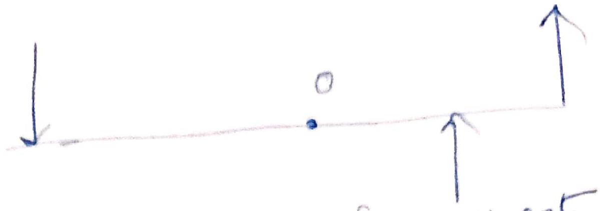


Unit of Moment

Nm = Force × distance

Types of Moments

(a) clockwise moments



(b) Anti clockwise moments

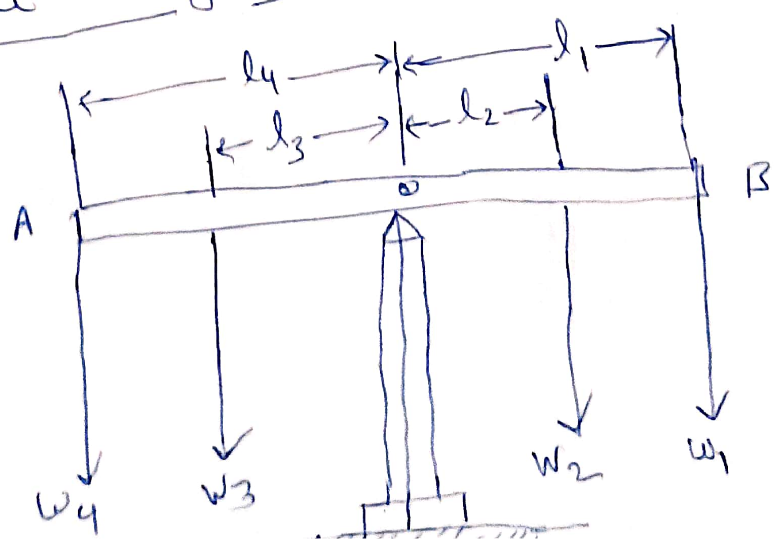
Graphical representation of the moment of the force about a point

Let a force F represented in magnitude & direction by AB be acting on a body & let O be any point in the plane of the force F as shown in fig.

From O, perpendicular OM is drawn on the line of action of F. Then, moment of F about O

$$= F \times OM$$
$$= 2 \times \frac{1}{2} F \times OM$$
$$= 2 \times \frac{1}{2} AB \times OM$$
$$= 2 \times \text{Area of } \Delta AOB$$

Positive Moment & Negative Moment



Algebraic sum of moments

moment due to W_1 about point $O = W_1 l_1$

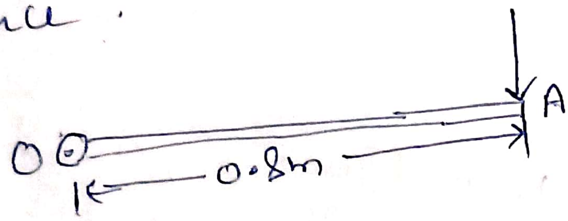
W_2 " " $O = W_2 l_2$

W_3 " " $O = W_3 l_3$

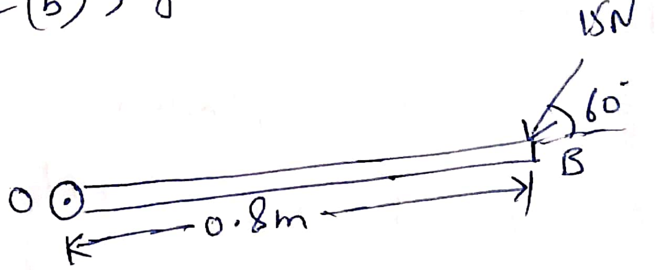
W_4 " " $O = W_4 l_4$

Hence algebraic sum of moments of W_1, W_2, W_3, W_4 about $O = W_3 \times l_3 + W_4 l_4 - W_2 l_2 - W_1 . l_1$

Q 1.12A A force of 15N is applied perpendicular to the edge of a door 0.8m wide as shown in fig (a). Find the moment of the force about the hinge. If this force is applied at an angle of 60° to the edge of the same door, as shown in fig (b), find the moment of the force.



(a)

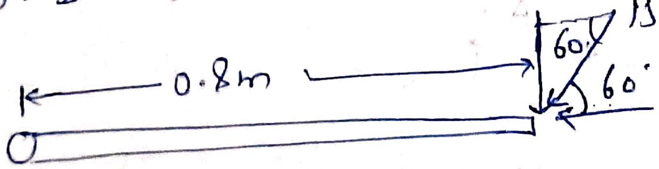


(b)

Sol^n Moment of force about the hinge

(a) $= P \times l$
$= 15 \times 0.8 = 12.0$ N-m

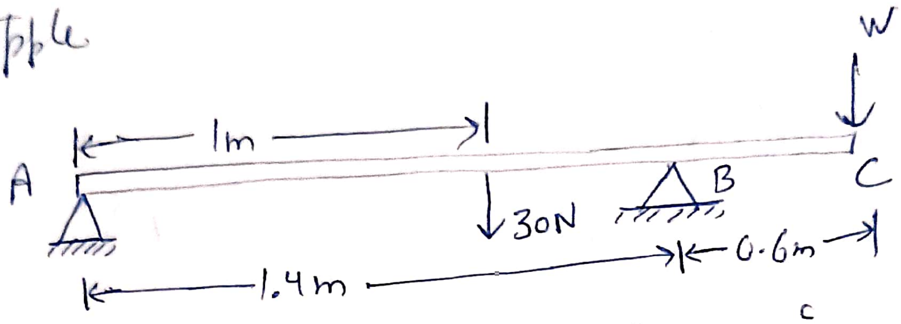
(b)



Vertical component of force, (Resolving forces vertically)
$= 15 \sin 60° = 15 \times 0.866 = 13.0$ N

$\therefore$ Moment $= 13 \times 0.8 = 10.4$ N-m

Q 1.13A A uniform plank ABC of weight 30N long is supported at one end A & at a point 1.4m from A as shown in fig. Find the maximum weight W, that can be placed at C, so that plank does not topple



Sol^n

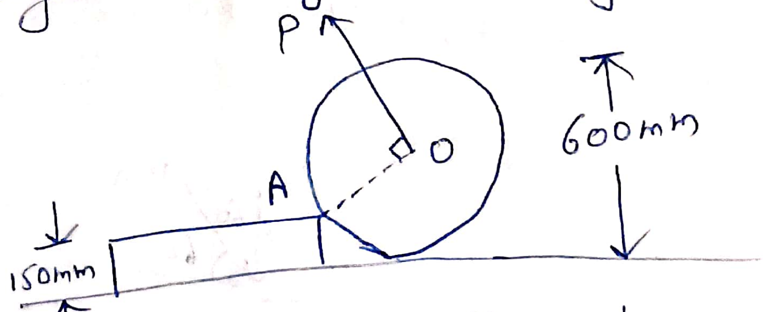
we know that if the plank is not to topple, then the reaction at A should be zero for the maximum weight at C

Now taking moments about B & equating the same.

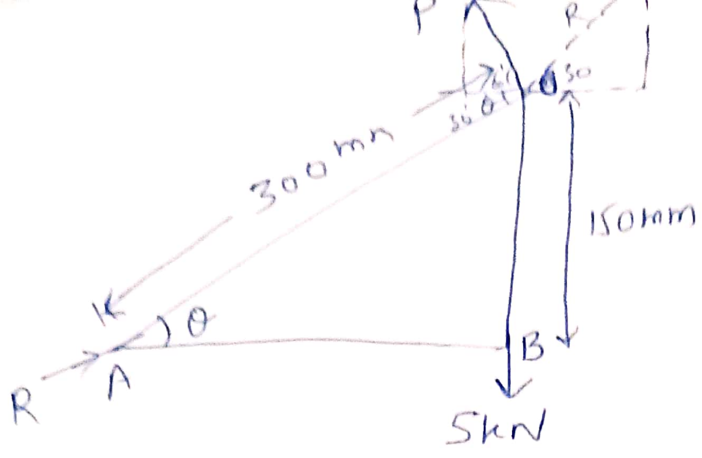
$$30 \times 0.4 = W \times 0.6$$

$$W = 12/0.6 = 20\,N$$

Q 1.14A A uniform wheel of 600mm diameter, weighing 5kN rests against a rigid rectangular block of 150mm height.



Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.



$\sin\theta = \frac{150}{300} = 0.5$

$\theta = 30°$

$AO^2 = AB^2 + BO^2$

$AB^2 = AO^2 - BO^2$

$= \sqrt{(300)^2 - (150)^2} = 260\,mm$

Now taking the moment about A & equating the same

$P \times 300 = 5 \times 260$

$P = \frac{1300}{300} = 4.33\,kN$

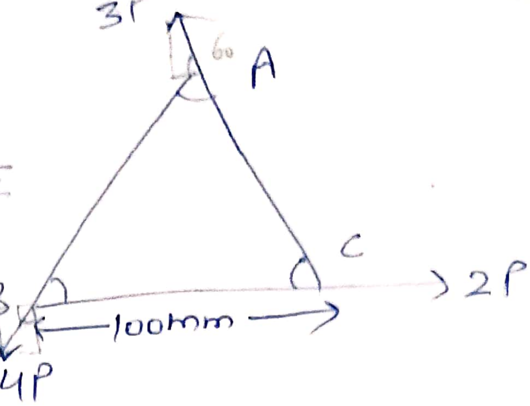
Resolving forces horizontally & equating the same

$R\cos 30 = P\cos 60°$

$R = \frac{P\cos 60}{\cos 30} = \frac{4.33 \times 0.5}{0.866} = 2.5\,kN$

Q1.15 Three forces of 2P, 3P & 4P act along the three sides of an equilateral triangle of side 100 mm taken in order. Find the magnitude & position of the resultant force.

Sol^n



Magnitude of the resultant force

Resolving all the forces horizontally

$$\Sigma H = 2P - 3P\cos 60 - 4P\cos 60^\circ$$

$$= 2P - 3P\times 0.5 - 4P\times 0.5$$

$$= -1.5P \quad \text{—— (i)}$$

& now resolving all the forces vertically

$$\Sigma V = 3P\sin 60 - 4P\sin 60^\circ$$

$$= (3P\times 0.866) - (4P\times 0.866)$$

$$= -0.866P \quad \text{---- (ii)}$$

we know that magnitude of resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-1.5P)^2 + (-0.866P)^2}$$

$$= 1.732P$$

Position of the resultant force

Let x = perpendicular distance between B & the line of action of the resultant force. Now taking moments of the resultant force about B & equating the same.

$$1.732P \times x = 3P \times 100\sin 60^\circ$$

$$x = \frac{259.8}{1.732} = 150\text{ mm}$$

Equilibrium

A little consideration will show, that if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

A body can be said to be in equilibrium when all the force acting on a body each other or in other word there is no net force acting on the body.

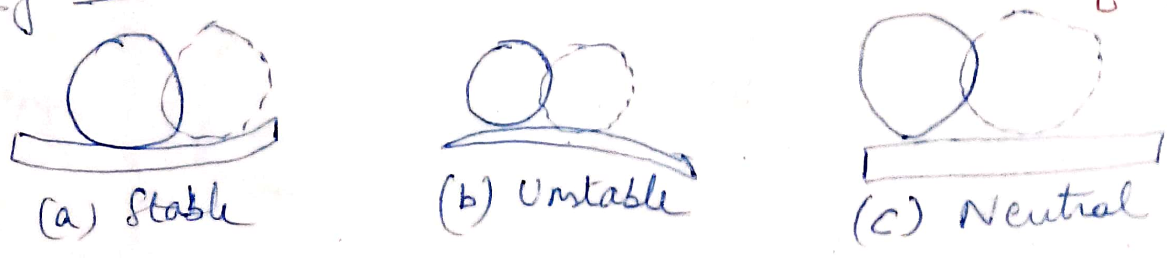
Equilibrium of a body is a state in which all the forces acting on the body are balanced (cancelled out), and the net force acting on the body is zero.

i.e $\Sigma F = 0$

Principles of Equilibrium

1. Two force principle: As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite & collinear.
2. Three force principle: As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. Four force principle: As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite & collinear with the resultant of the other two forces.

Types of Equilibrium



Stable equilibrium :

A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest. This happens when some additional force sets up due to displacement and brings the body back to its original position.

Unstable equilibrium :

A body is said to be in an unstable equilibrium, if it does not return back to its original position, & heels farther away, after slightly displaced from its position of rest.

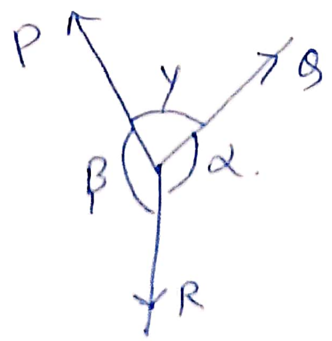
Neutral equilibrium :

A body is said to be in neutral equilibrium, if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rests.

Lami's Theorem

It states, "if three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two"

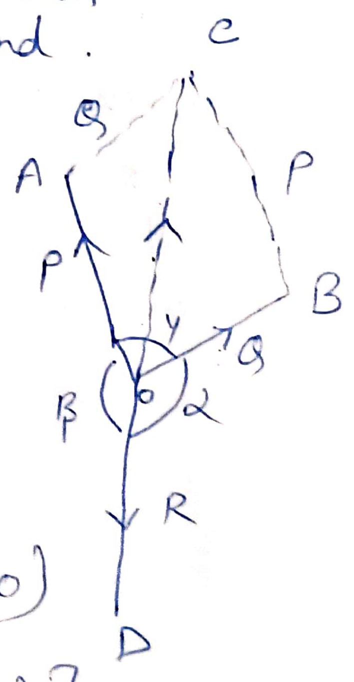
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



fig

where, P, Q and R are three forces & α, β, γ are the angles as shown in fig. ③

Proof :
Consider three coplanar forces P, Q & R acting at point O.
Let the opposite angles to three forces be α, β & γ as shown in fig.
Now let us complete the parallelogram OACB with OA & OB as adjacent sides a shown in the figure.
we know that resultant of two forces P & Q will be given by diagonal OC both in magnitude & direction of the parallelogram OACB
Since these forces are in equilibrium, therefore the resultant of the forces P & Q must be in line with OD and equal to R, but in opposite direction.
From the geometry of the figure, we find.



$BC = P$ & $AC = Q$

$\angle AOC = 180 - \beta$

$\angle ACO = \angle BOC = 180 - \alpha$

$\angle CAO = 180 - (\angle AOC + \angle ACO)$

$= 180 - [(180 - \beta) + (180 - \alpha)]$

$= 180 - 180 + \beta - 180 + \alpha$

$= \alpha + \beta - 180°$

But $\alpha + \beta + \gamma = 360°$

Subtracting $180°$ from both the sides of the above equ.

$$(\alpha+\beta-180°)+\gamma = 360°-180$$
$$= 180°$$

Or $\angle CAO = 180-\gamma$

we know that in traingle AOC

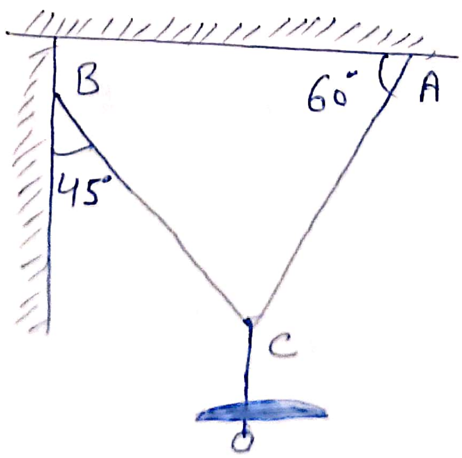
$$\frac{OA}{\sin(\angle ACO)} = \frac{AC}{\sin(\angle AOC)} = \frac{OC}{\sin(\angle CAO)}$$

$$\frac{OA}{\sin(180-\alpha)} = \frac{AC}{\sin(180-\beta)} = \frac{OC}{\sin(180-\gamma)}$$

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Notes:

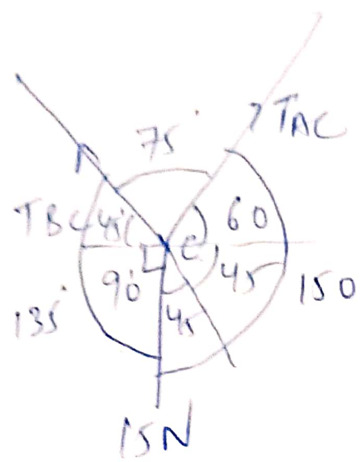
Q) An electric light fixture weighting 15N hangs from a paint C, by two strings AC & BC. The string AC is inclined at $60°$ to the horizontal and BC at $45°$ to the vertical as shown in fig. using Lami's theorem, or otherwise, determine the forces in the string AC & BC

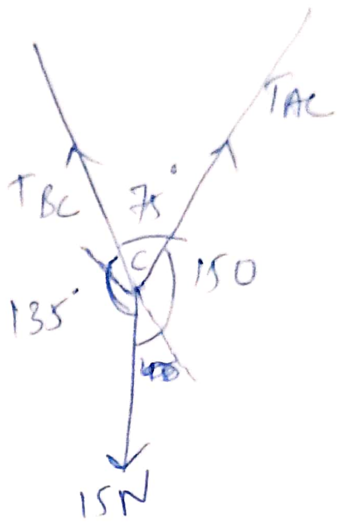


i) weight at C = 15N

T_{AC} = force in the string ACA

T_{BC} = force in the string BC



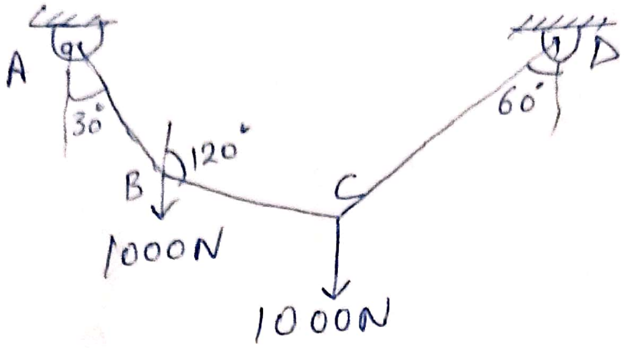


$$\frac{15}{\sin 75} = \frac{T_{AC}}{\sin 135} = \frac{T_{BC}}{\sin 150'}$$

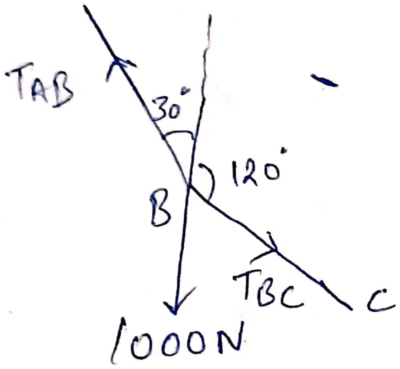
$$\frac{15}{\sin 75} = \frac{T_{AC}}{\sin(180-45)} = \frac{T_{BC}}{\sin(180-30)}$$

$$\frac{15}{\sin 75} = \frac{T_{AC}}{\sin 45} = \frac{T_{BC}}{\sin 30}$$

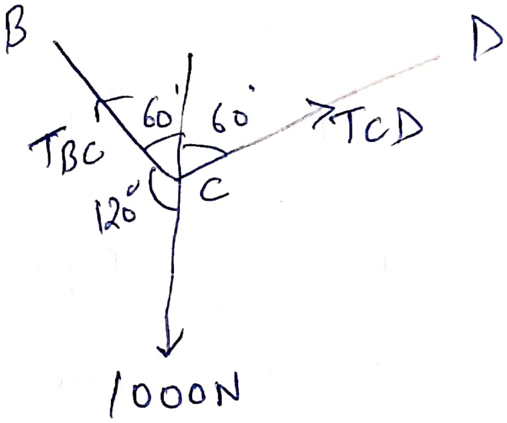
Q/ A string ABCD, attached to fixed points A A D has two equal weights of 1000N attached to it at B A C the weight rest with the portions AB A CD inclined at angles as shown in fig. Find the tensions in the portions AB, BC A CD of the string. If the inclination of the portion BC with the vertical is 120°



Sol^n



at Joint B
(a)



(b) Joint C

Applying Lami's equation at joint B

$$\frac{T_{AB}}{\sin 60'} = \frac{T_{BC}}{\sin 150} = \frac{1000}{\sin 150'}$$

$$\frac{T_{AB}}{\sin 60'} = \frac{T_{BC}}{\sin(180-30)} = \frac{1000}{\sin(180-30)}$$

$$\frac{T_{AB}}{\sin 60} = \frac{T_{BC}}{\sin 30} = \frac{1000}{\sin 30}$$

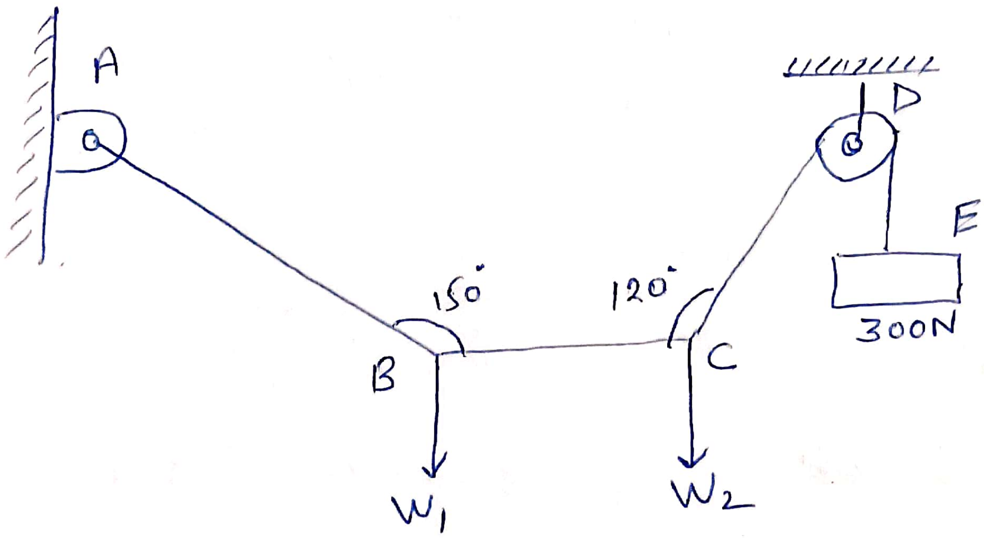
$$T_{AB} = \frac{1000 \sin 60'}{\sin 30} \quad \sin 30 \qquad , \quad T_{BC} = \frac{1000 \sin 30'}{\sin 30'}$$

plying Lami's theorem at Joint C.

$$\frac{T_{BC}}{\sin 120} = \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\cancel{T_{BC}} \; T_{CD} = \frac{1000 \sin 120^\circ}{\sin 120^\circ} = 1000\,N$$

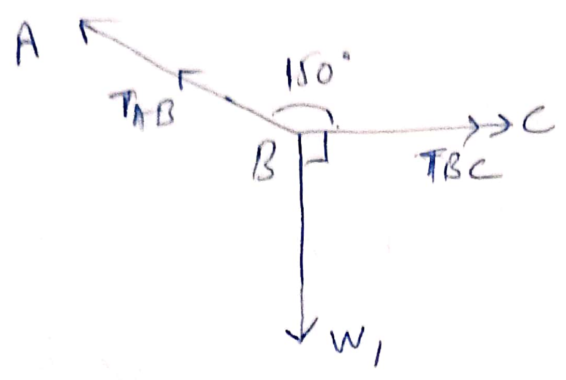
Q. A light string ABCDE whose extremity A is fixed, has weights W_1 & W_2 attached to it at B & C. It passes round a small smooth peg at D carrying a weight of 300N at the free end E as shown in figure. If in the equilibrium position, BC is horizontal & AB & CD make 150° & 120° with BC. find (i) Tensions in the portion AB, BC & CD of the string and (ii) Magnitudes of W_1 & W_2.



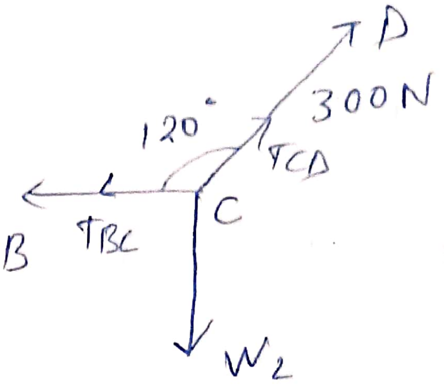
Soln Tensions in the portion AB, BC & CD of string

$$T_{CD} = T_{DE} = 300\,N$$

the system of forces at Joint B & C. at fig (a)



a) Joint B



(b) Joint C

(ii) at Joint B

$$\frac{T_{AB}}{\sin 90} = \frac{T_{BC}}{\sin 120'} = \frac{W_1}{\sin 150} \quad \text{Or} \quad \frac{T_{AB}}{1} = \frac{150}{\sin 60°} = \frac{W_1}{\sin 30}$$

$$T_{AB} = \frac{150}{0.866} = 173.2N$$

$$W_1 = \frac{150 \sin 30}{\sin 60°} = \frac{150 \times 0.5}{0.866} = 86.6$$

(i) at Joint C

$$\frac{T_{BC}}{\sin 150} = \frac{T_{CD}}{\sin 90'} = \frac{W_2}{\sin 120'}$$

$$\frac{T_{BC}}{\sin 150} = \frac{300}{\sin 90} = \frac{W_2}{\sin 120'}$$

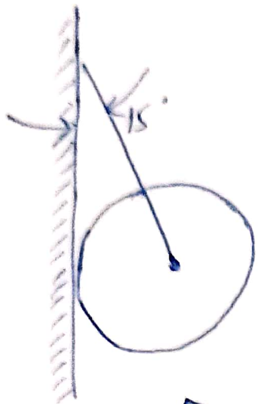
$$\frac{T_{BC}}{\sin 30'} = \frac{300}{\sin 90} = \frac{W_2}{\sin 60'}$$

$$T_{BC} = \frac{300 \sin 30}{\sin 90} = 300 \times 0.5 = 150N$$

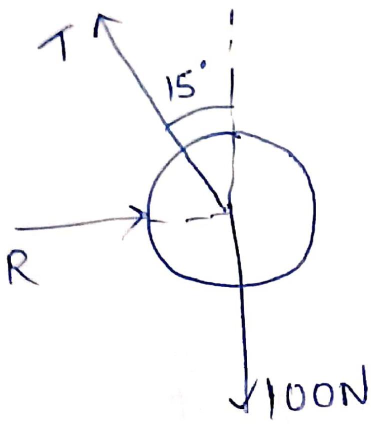
$$W_2 = 300 \times \sin 60 = 300 \times 0.866 = 259.8N$$

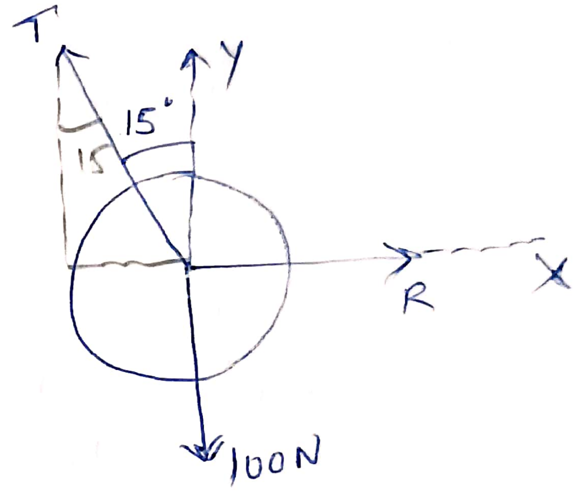
8)

*Q. A sphere weighing 100 N is tied to a smooth wall by a string as shown in fig. Find the tension T in the string and the reaction R from the wall. (9)



Soln





(i) method

$$\frac{R}{\sin 165} = \frac{T}{\sin 90^\circ} = \frac{100}{\sin 105}$$

$R = 26.8 N$

$T = 103.5 N$

(ii) Method

$$\Sigma F_y = 0$$

$$T \cos 15 - 100 = 0$$

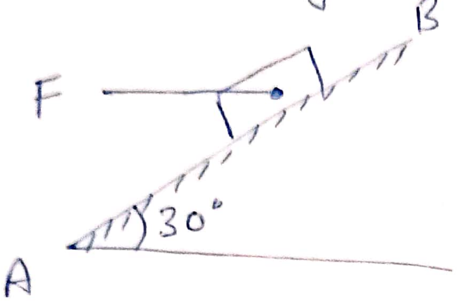
$$T = 103.5 N$$

$$\Sigma F_x = 0$$

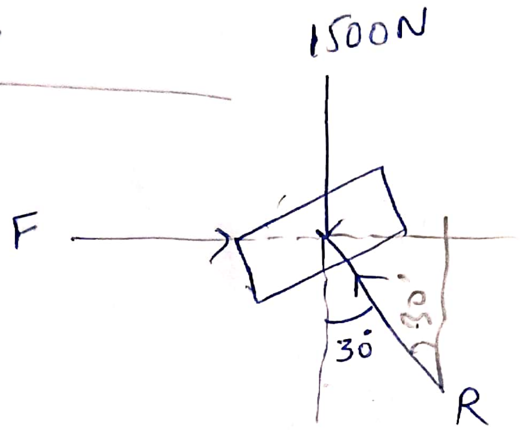
$$R - T \sin 15 = 0$$

$$R = 103.5 \sin 15^\circ = 26.8 N$$

Q/ Determine the horizontal force F to be applied to the block weighing 1500 N to hold in position on a smooth inclined plane AB which makes an angle of 30° with the horizontal.



Soln



Soln

$$\Sigma F_y = 0$$

$$R\cos 30^\circ - 1500 = 0$$

$$R = \frac{1500}{\cos 30} = 1732.0\,N$$

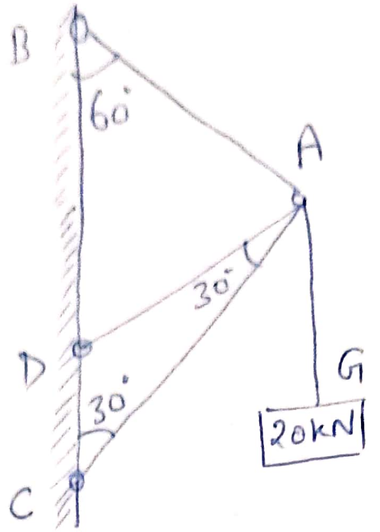
$$\Sigma F_x = 0$$

$$F - R\sin 30^\circ = 0$$

$$F = 1732\sin 30^\circ$$

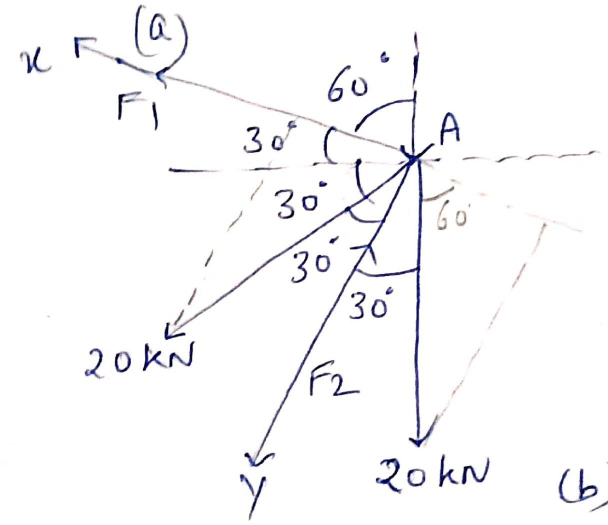
$$F = 866\,N$$

Q the frictionless pulley A shown in figure is supported by two bars AB & AC which are hinged at B & C to a vertical wall. The flexible Cable hinged at D, Goes over the pulley & Supports a load of 20kN at G. The angle made by various members of the system are as shown in figure. Determine the forces the bars AB & AC. Neglect the size of the pulley. (11)



(a)

Soln



(b)

Soln Since pulley is frictionless, same force exists throughout in flexible cable. Hence the force AD is also 20kN as shown in fig (b). From the figure it may be observed that AC is perpendicular to AB. Selecting AB & AC as Cartesian x & y axes

$$\Sigma F_x = 0$$

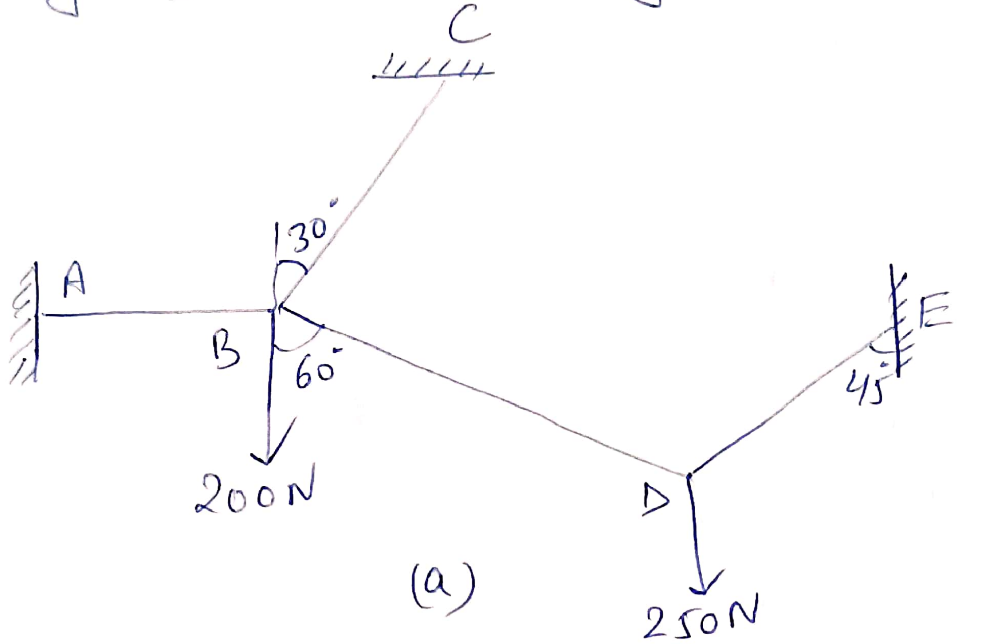
$$F_1 + 20\cos 60° - 20\cos 60° = 0$$

$$F_1 = 0$$

$$\Sigma F_y = 0$$

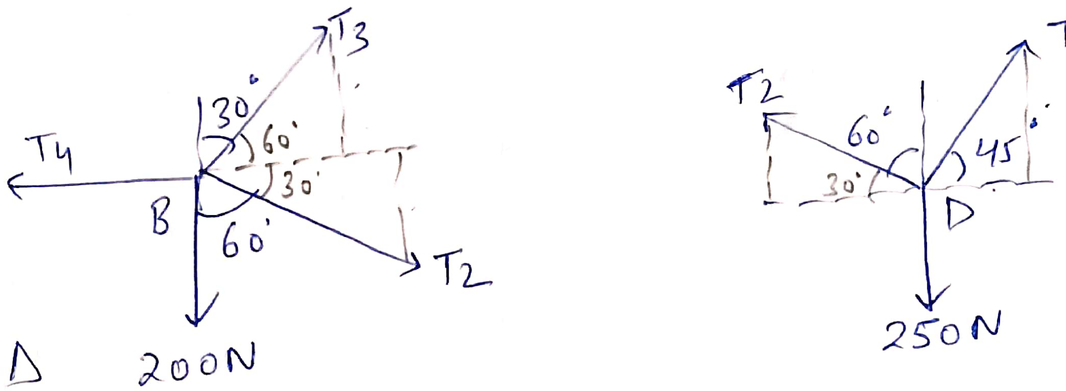
$F_2 - 20\sin 60° - 20\sin 60° = 0$ & $F_2 = 40\cos 30° = 34.64\,N$

Q. A system connected to flexible Cables shown in fig (a) is Supporting two vertical forces 200N & 250N at points B & D. Determine the forces in various segments of the Cable.



(a)

Soln



(b)

At point D

wrong method

$\Sigma F_x = 0$

$T_1 \cos 45 - T_2 \cos 30° = 0$ —(i)

$\Sigma F_y = 0$

$T_1 \sin 45 + T_2 \sin 30 - 250 = 0$

$T_1 \sin 45 + T_2 \sin 30 = 250$

$T_1 = \dfrac{250 - T_2 \sin 30}{\sin 45}$ (ii)

$\dfrac{(250 - T_2 \sin 30°)\cos 45}{\sin 45} - T_2 \cos 30° = 0$ or $250 - T_2 0.5 - T_2 0.986$

$250 = T_2 \times 0.4986$

$250 - T_2 \times 0.5 - 0.866025404\, T_2 = 0$

$\frac{250}{1.3660254} = T_2$

$\boxed{T_2 = 183\,N}$

from (i)

$T_1 \cos 45 - 183 \cos 30 = 0$

$T_1 \times 0.707106781 - 183 \times 0.866025404 = 0$

$T_1 = \frac{158.482649}{0.707106781}$

$\boxed{= 224.128312\,N}$

At point B

$\Sigma F_x =$

$-T_4 + T_3 \cos 60^\circ + T_2 \cos 30^\circ = 0$

$-T_4 + T_3 \cos 60^\circ + 183 \times 0.866025404 = 0$

$-T_4 + T_3 \cos 60 + 158.482649 = 0$

$T_3 \cos 60 - T_4 + 158.482649 = 0$

ΣF_y

$200 + T_2 \sin 30^\circ - T_3 \sin 60^\circ = 0$

$200 + 183 \times 0.5 - T_3 \times 0.866025404 = 0$

$\frac{291.5}{0.866025404} = T_3$ or $\boxed{T_3 = 336.6\,N}$

Scanned with OKEN Scanner

T_4

$336.6 \times 0.5 - T_4 + 158.482649 = 0$

$\boxed{326.78\,N = T_4}$

5/10/23 Q A wire rope is fixed at two points A & D as shown in figure (a). weights 20 kN & 30 kN are attached to it at B & C respectively. the weights rest with portions AB & BC inclined at 30° & 50° respectively, to the vertical as shown in figure. Find the tension in segments AB, BC & CD of wire. Determine the inclination of the segment CD to vertical.

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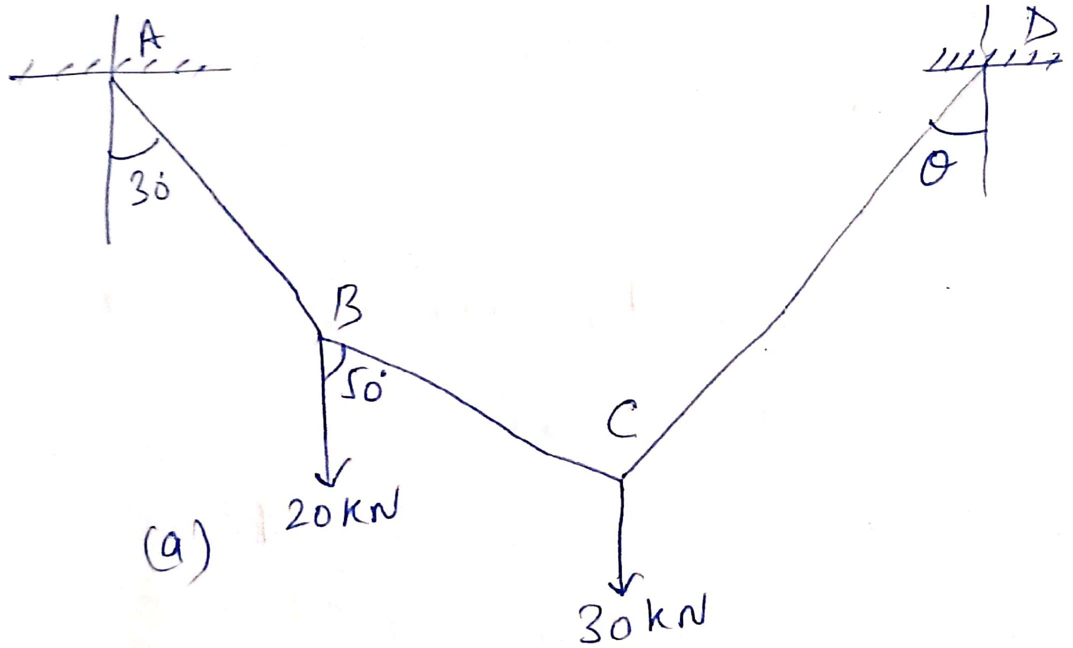
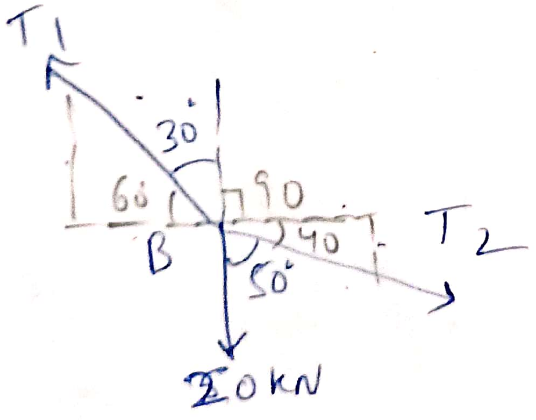
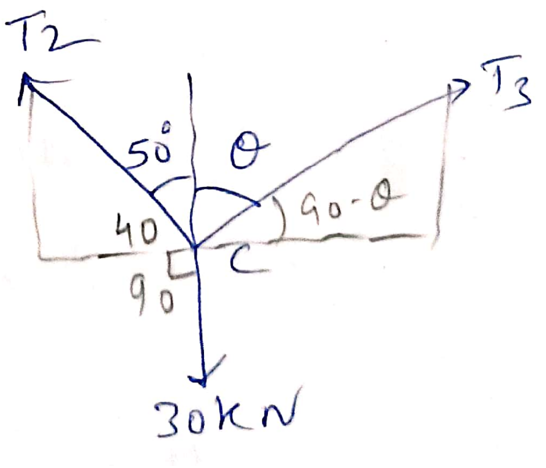


(a)





At B

$\Sigma F_x = 0$

$T_2 \cos 40^\circ - T_1 \cos 60^\circ = 0$

$$T_2 = \frac{T_1 \cos 60}{\cos 40} = \frac{T_1 \times 0.5}{0.766044443}$$

$T_2 = T_1 \times 0.652703645$ — (i)

$\Sigma F_y = 0$

$20 + T_2 \sin 40^\circ - T_1 \sin 60 = 0$ — (ii)

put (i) in (ii)

$20 + T_1 \times 0.652703645 \times 0.64278761 - T_1 \times 0.866025404 = 0$

$\cancel{T_p = \frac{20}{0.21332175}}$

$20 + T_1 \times 0.419549816 - T_1 \times 0.866025404 = 0$

$20 - T_1 \times 0.446475588 = 0$

$$T_1 = \frac{20}{0.446475588}$$

$T_1 = 44.80$ or 48 N

$T_2 = 44.80 \times 0.652703645$

$= 29.24$ N

At C.

$\Sigma F_x = 0$

$T_3 \cos(90-\theta) - T_2 \cos 40^\circ = 0$

$T_3 \cos(90-\theta) - T_2 \cos 40^\circ = 0$ $\qquad \cos(90-\theta) = \sin\theta$

$T_3 \sin\theta - T_2 \cos 40^\circ = 0$

$\Sigma F_y = 0$ $\qquad$ or $T_3 \sin\theta = T_2 \cos 40^\circ$ — (i)

$30 - T_3 \sin(90-\theta) - T_2 \sin 40^\circ = 0$

$\cos\theta$.

$30 - T_3 \cos\theta - T_2 \sin 40 = 0$ $\qquad \sin(90-\theta)$ $\cos\theta$.

— (ii)

~~put value~~ from of (I) in (II)

$T_3 \cos\theta = 30 - T_2 \sin 40$

$$\frac{T_3 \sin\theta}{T_3 \cos\theta} = \frac{T_2 \cos 40}{30 - T_2 \sin 40^\circ}$$

$$\tan\theta = \frac{29.24 \times \cos 40}{30 - 29.24 \times \sin 40} = \frac{22.399139}{30 - 18.795192}$$

$$= \frac{22.3991395}{11.2048908} = 2$$

or $\theta = \tan^{-1} 2 = 63.43^\circ$

A 400N 30° (a) ig new

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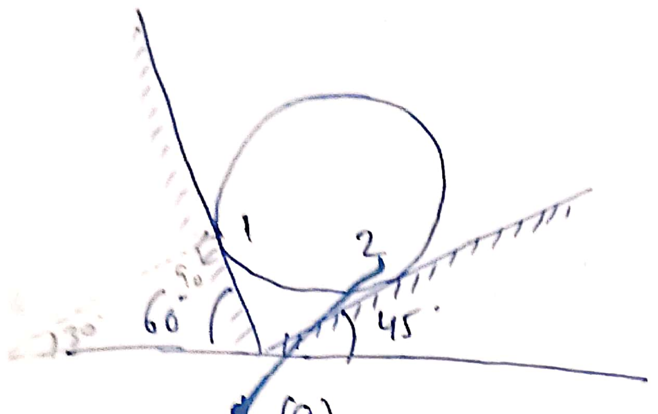
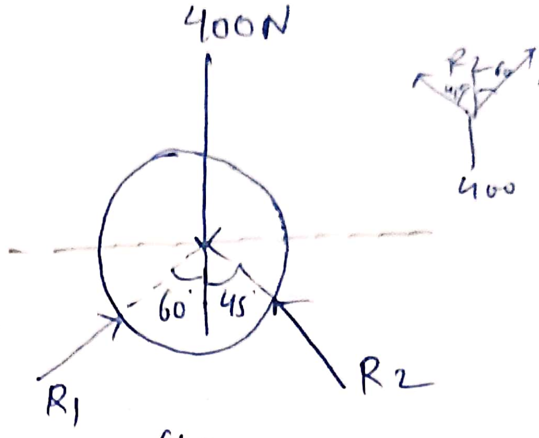
(17)

A 400N sphere is resting in a trough a shown in fig (a). Determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth.



(a)



(b)

Solⁿ Applying Lami's theorem to the system of forces, we get

$$\frac{R_1}{\sin(180-45)} = \frac{R_2}{\sin(180-60)} = \frac{400}{\sin(60+45)}$$

$\therefore$ $R_1 = 292.8N$ & $R_2 = 358.6N$

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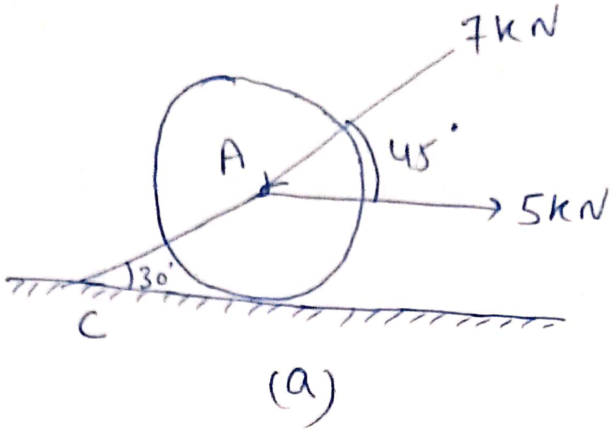
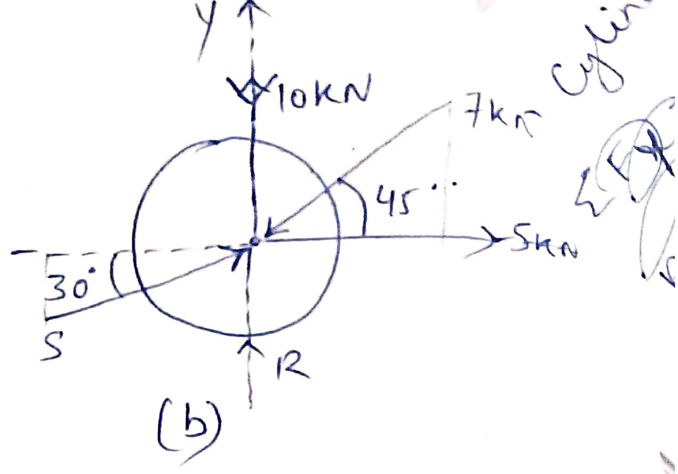
Q/ A roller weighing 10 kN rests on a smooth horizontal floor & is connected to the floor by the bar AC as shown in fig (a). Determine the force in the bar AC & reaction from the floor, if the roller is subjected to a horizontal force of 5 kN & an inclined force of 7 kN as shown in the figure





Soln

$$\Sigma F_x = 0$$

$$5 - 7\cos 45 + S\cos 30^\circ = 0 \quad \text{or} \quad S = -0.058\,kN$$

$$\Sigma F_y = 0$$

$$R - 10 - 7\sin 45 + S\sin 30 = 0$$

$$R = 14.980\,kN.$$

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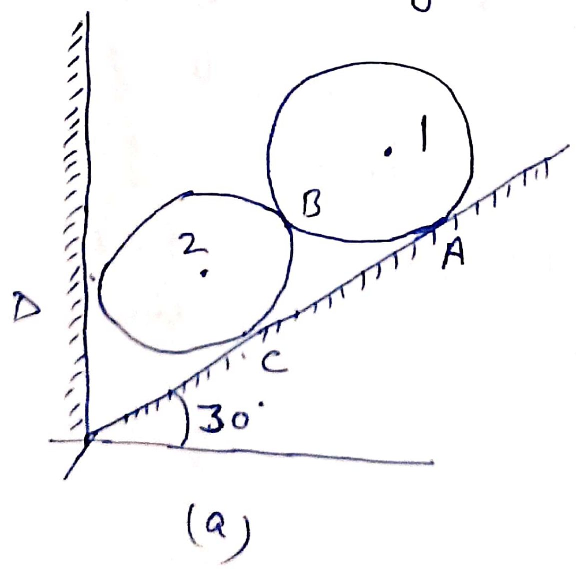
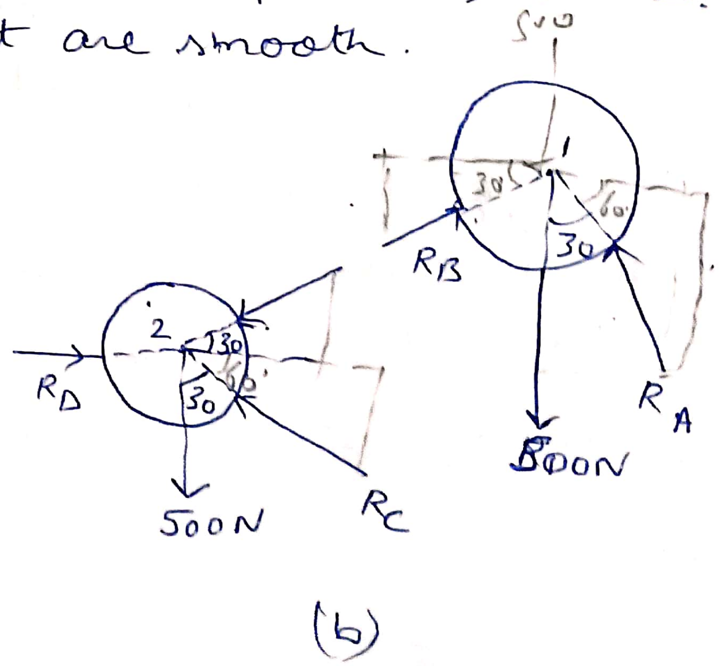
Q. Two identical cylinders, each weighing 500N are placed in a trough as shown in fig a. Determine the reactions developed at contact points A, B, C & D. Assume all points of contact are smooth.

Soln





In cylinder 1.

$\sum F_x = 0$

$500 \quad R_A \cos 60^\circ - R_B \cos 30^\circ = 0$

or $\quad R_A = \dfrac{R_B \cos 30}{\cos 60^\circ}$

$\sum F_y = 0$

$-500 + R_A \sin 60 + R_B \sin 30^\circ = 0$

$R_A \sin 60 + R_B \sin 30^\circ = 500$

$R_B \dfrac{\cos 30 \times \sin 60}{\cos 60} + R_B \sin 30^\circ = 500$

$R_B \times \dfrac{(0.866025404)^2}{0.5} + R_B \, 0.5 = 500$

$R_B \, 1.5 + R_B \, 0.5 = 500$

$R_B = \dfrac{500}{2} = 250\,N$

$R_A = \dfrac{250 \times 0.866025404}{0.5} = 433\,N$

In cylinder 2

$\sum F_x = 0$

$R_C \cos 60^\circ + R_B \cos 30 - R_D = 0$

$R_C \cos 60 + 250 \times 0.866025404 - R_D = 0$

$R_C \times 0.5 - R_D + 216.506351 = 0 \quad — (1)$

$\sum F_y = 0$

$500 - R_C \sin 60 + R_B \sin 30^\circ = 0$ or $500 - R_C \sin 60 + 125 = $

$R_C = \dfrac{625}{\ldots}$

$R_C = 721.7 N$ — (11)

put in (1)

$721.7 \times 0.5 - R_D + 216.50635) = 0$

$R_D = 577.4 N$

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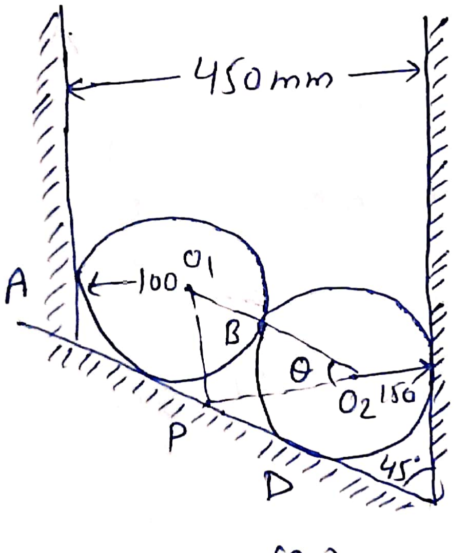
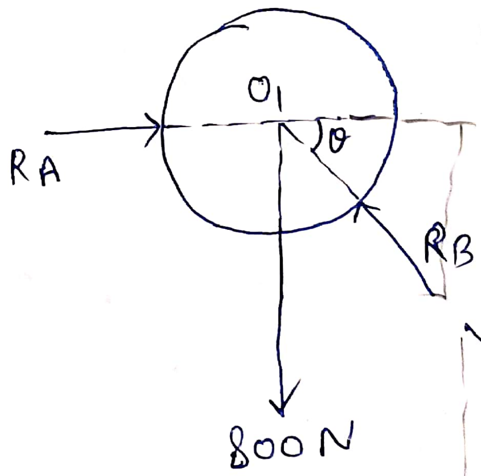
Q/ Cylinder 1 of diameter 200mm and cylinder 2 of diameter 300mm are placed in a trough as shown in fig a. If cylinder 1 weights 800 N and cylinder 2 weighs 1200 N, determine the reactions developed at contact surfaces A, B, C & D. Assume all contact surfaces are smooth.

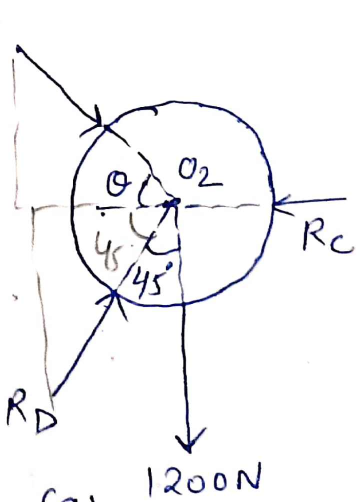
Sol



(a)



(b)



(c)

$$\cos\theta = \frac{O_2 P}{O_1 O_2} = \frac{450-100-150}{100+150} = 0.8$$

$$\theta = 36.87^\circ$$

Consider the equilibrium of cylinder 1

$$\Sigma F_y = 0$$

$$R_B \sin 36.87 - 800 = 0$$

$$R_B = 1333.3\,N$$

$$\Sigma F_x = 0$$

$$R_A - R_B \cos 36.87 = 0$$

$$R_A = 1333.3 \cos 36.87$$

$$= 1066.7\,N$$

Consider the equilibrium of cylinder 2

$$\Sigma F_y = 0$$

$$1200 + R_B \sin 36.87 - R_D \sin 45 = 0$$

$$1200 + 1333.3 \times 0.600001429 - R_D \sin 45 = 0$$

$$\frac{1999.98191}{0.707106781} = R_D$$

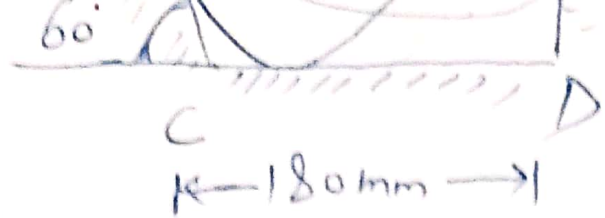
$$R_D = 2828.4\,N$$

$$\Sigma F_x = 0$$

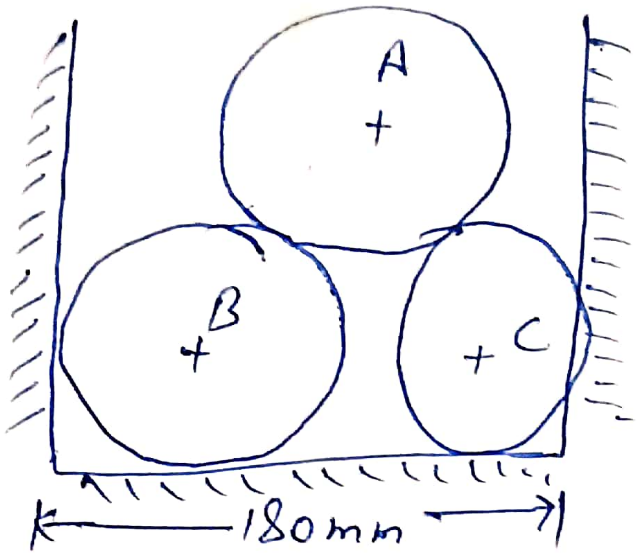
$$-R_B \cos 36.87 - R_D \cos 45 + R_C = 0$$

$$-1333.3 \times 0.79999892 8 - 2828.4 \times 0.707106781 + R_C = 0$$

$$R_C = 3066.7\,N$$



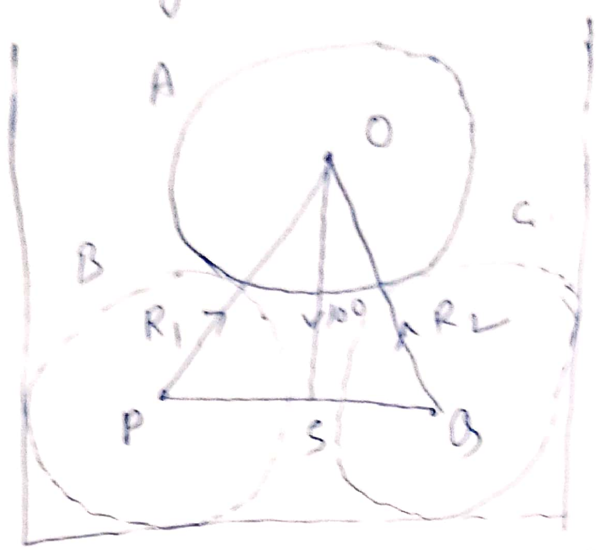
Q) Three cylinders weighting 100 N each and of 80mm diameter are placed in a channel of 180mm width as shown in fig(a). Determine the pressure exerted by (i) the cylinder A on B at the point of contact (ii) Cylinder B on the base (iii) The cylinder B on the wall.



(a)

Free body diagram

for Cylinder A



(b)

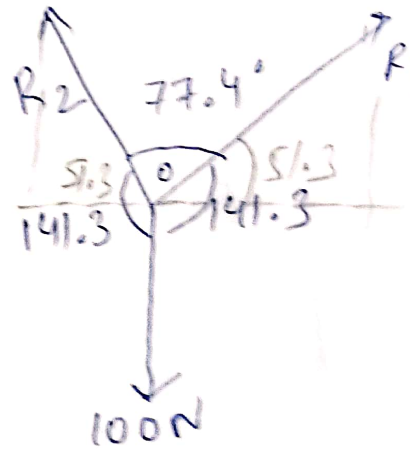
from the geometry of the traingle OPS, we find

$$OP = 40 + 40 = 80\,mm$$

$$PS = 50 = (90 - 40)$$

$$\sin \angle POS = \frac{PS}{OP} = \frac{50}{80} = 0.625$$

$$\angle POS = 38.7^\circ$$



(C) force diagram

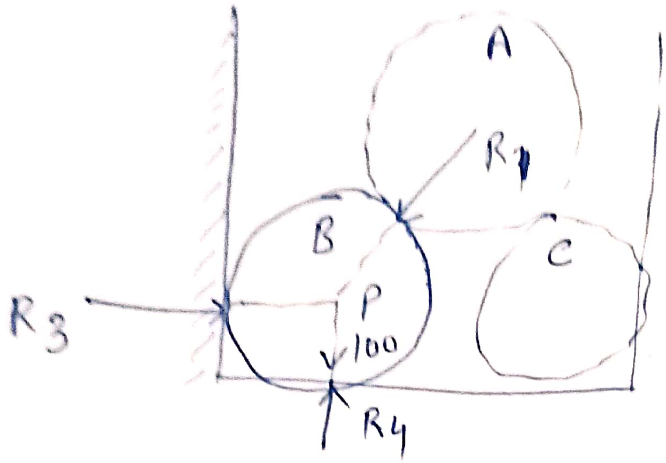
applying Lami's theorem

$$\frac{R_1}{\sin 141.3} = \frac{R_2}{\sin 141.3} = \frac{100}{\sin 77.4}$$

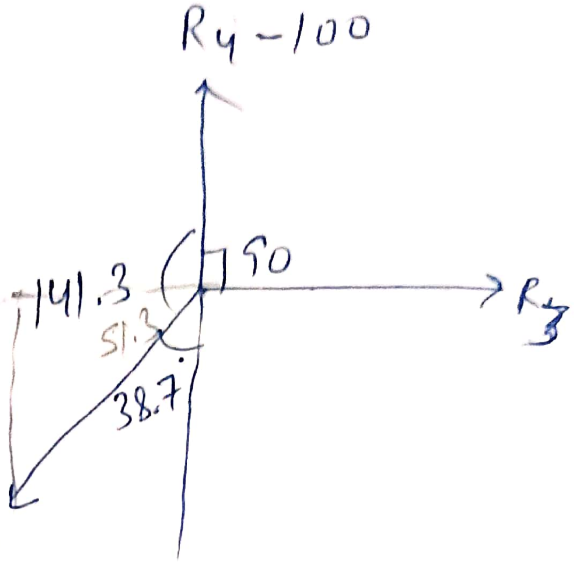
$$R_1 = 64N$$

$$R_2 = R_1 = 64N$$

For cylinder B



a) free body diagram



b) force diagram

$$\frac{64}{\sin 90} = \frac{R_3}{\sin 141.3} = \frac{R_4 - 100}{\sin(90 + 38.7)}$$

$R_4 = 150\,N$ $\quad R_3 = 40\,N$

IInd method

for cylinder A

$$\Sigma F_y = 0$$

$$100 - R_2 \sin 51.3 - R_1 \sin 51.3 = 0 \quad \text{—(i)}$$

$$\Sigma F_x = 0$$

$$R_1 \cos 51.3 + R_2 \cos 51.3 = 0$$

$$R_1 = \frac{R_2 \cos 51.3}{\cos 51.3} \quad \text{—(ii)}$$

put value of R_1 in (1)

$$100 - R_2 \sin 51.3 - \frac{R_2 \cos 51.3}{\cos 51.3} \cdot \sin 51.3 = 0$$

$$2 R_2 \sin 51.3 = 100$$

$$R_2 = \frac{50}{\sin 51.3}$$

$$R_2 = 64.06 N$$

$$\therefore R_1 = 64.06 N$$

for cylinder B

$$\Sigma F_y = 0$$

$$R_4 - 100 - R_1 \sin 51.3 = 0$$

$$R_4 - 100 - 64 \times \sin 51.3 = 0$$

$$R_4 = 100 + 64 \times 0.78043042$$

$\Sigma F_x = 0$

$$R_3 - R_1 \cos 51.3 = 0 \qquad = 150 N$$

$$R_3 - 64 \times 0.625242656 = 0$$

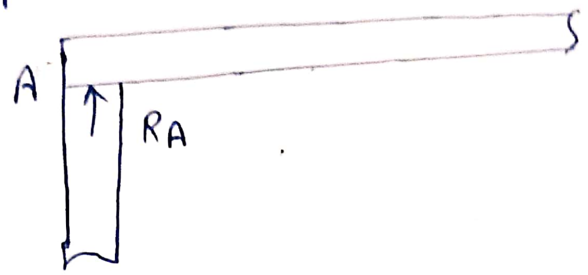
$$R_3 = 40 N$$

Application to Beam problems

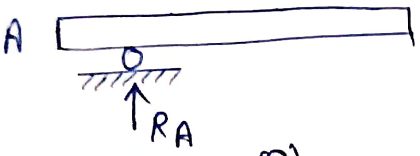
A beam is a structural element which has one dimension (length) considerably larger than the other two dimensions in the cross-section and is supported at few points.

Types of Supports

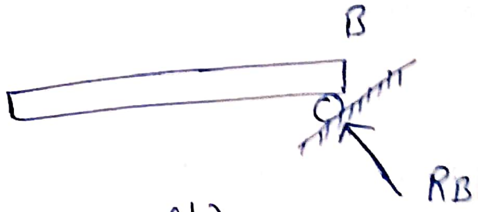
(i) Simple Support.



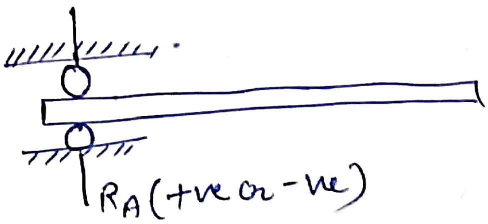
(ii) Roller support



(a)

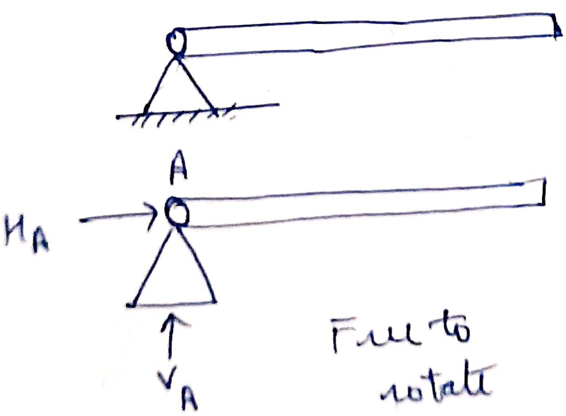


(b)



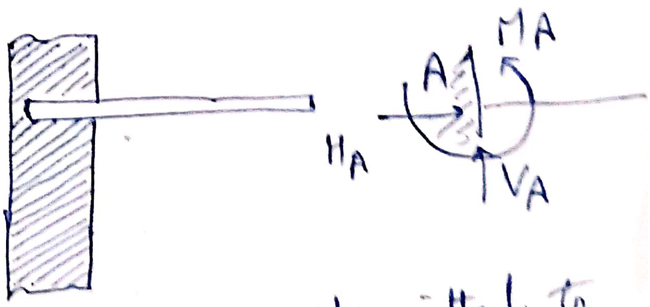
Ends are free to rotate

(iii) Hinged or pinned support.



Free to rotate

(iv) Fixed Support



Not permitted to move nor allowed to rotate

Types of Beams

Depending upon the types of supports, beams may be classify as following:

(i) Cantilever
(ii) Simply supported
(iii) One end hinged and other on roller
(iv) overhanging
(v) Both ends hinged
(vi) Propped Cantilever
(vii) Continous

(i) **Cantilever:** If a beam is fixed at one end and free at other end, it is called a cantilever beam. In this there are three reaction components at fixed end [V_A, H_A, M_A] and no reaction component at free end. [fig (a)]

(ii) **Simply Supported Beam:** In this type of beam both ends are simply supported as shown in fig 3.32. There is one reaction component at each end (R_A & R_B) fig (b)

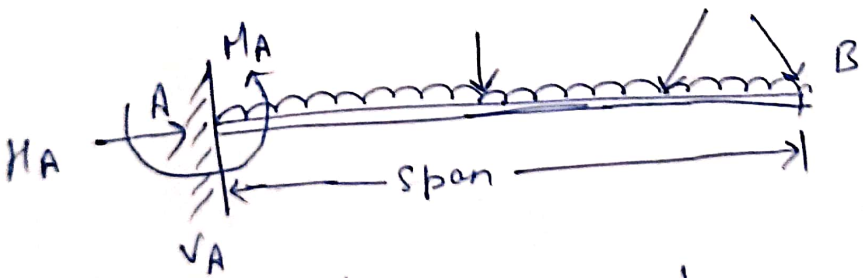


fig (a)

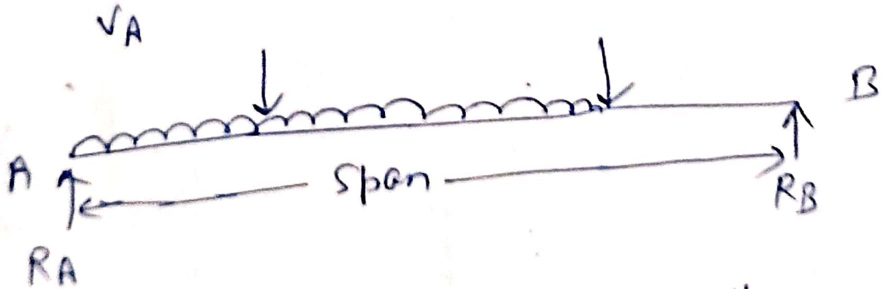
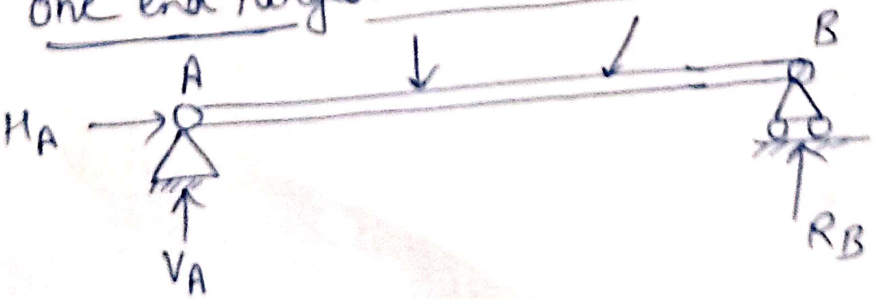
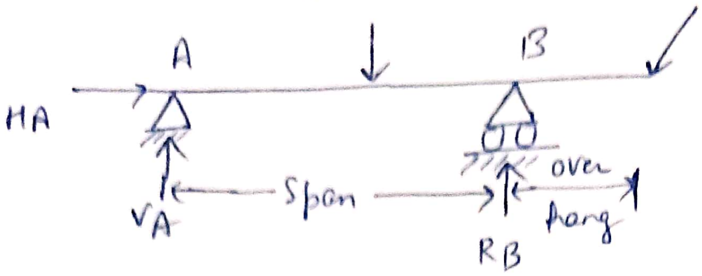


fig (b)

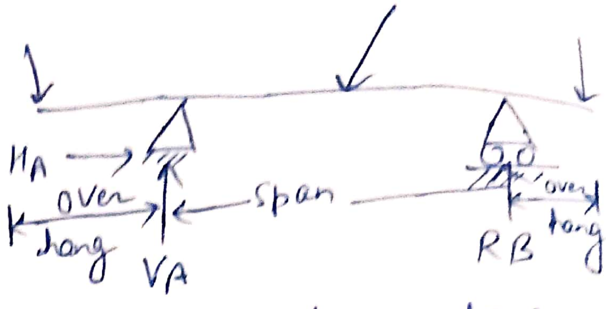
(iii) **One end Hinged and other on roller**



(iv) Overhanging beam

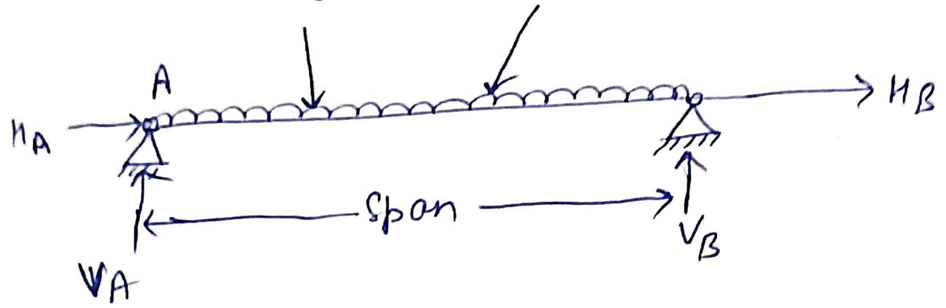


a) Single overhang

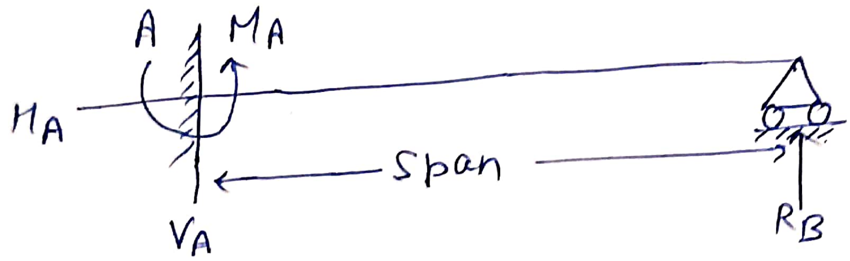


(b) Double overhang

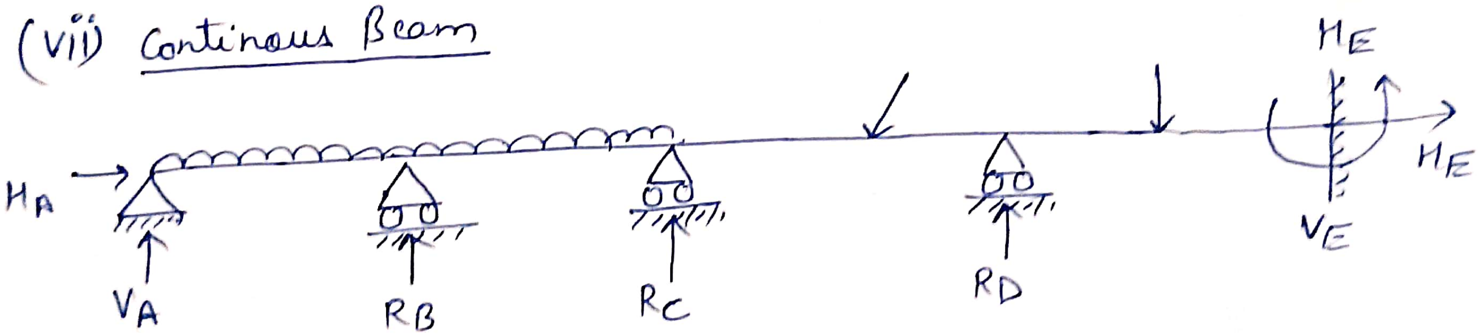
(v) Both ends Hinged



(vi) Propped Cantilever.

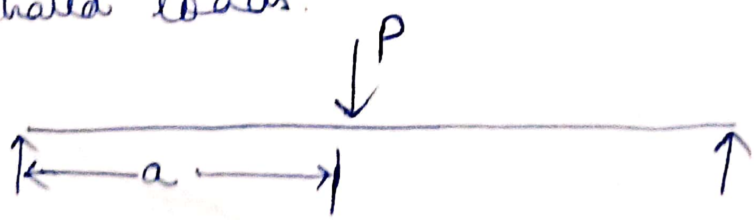


(vii) Continous Beam

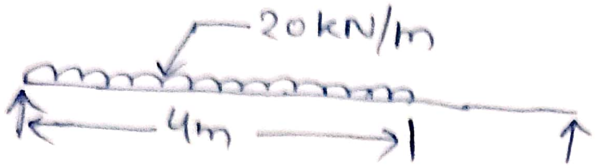


Types of loading

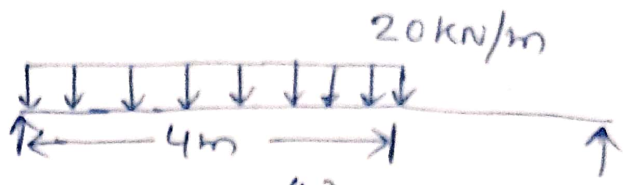
(i) Concentrated loads.



Uniformly distributed load (UDL)



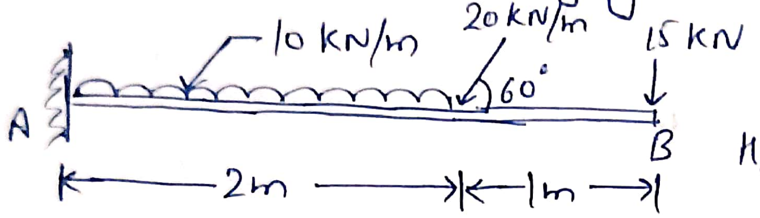
(a)



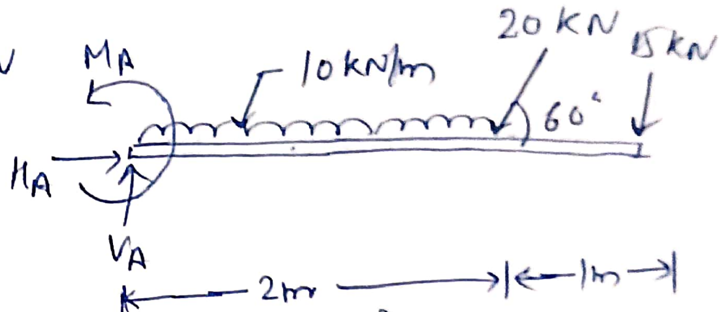
(b)

Total load = Intensity × length

Q1 Determine the reactions developed in the cantilever beam as shown in fig (a)



(a)



(b)

Soln let the reactions developed at fixed support A be V_A, H_A & M_A as shown in fig (b).

Σ Forces in vertical direction = 0, gives

$$V_A - 10 \times 2 - 20 \sin 60 - 15 = 0$$

$$V_A = 52.32 \text{ kN}$$

Σ Forces in horizontal direction = 0, gives

$$H_A - 20 \cos 60 = 0 \quad \therefore H_A = 10 \text{ kN}$$

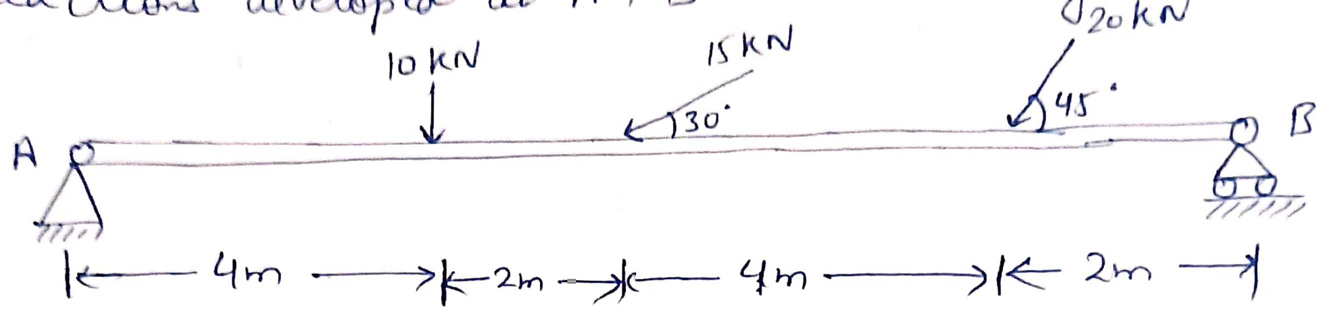
Σ Moment about A = 0

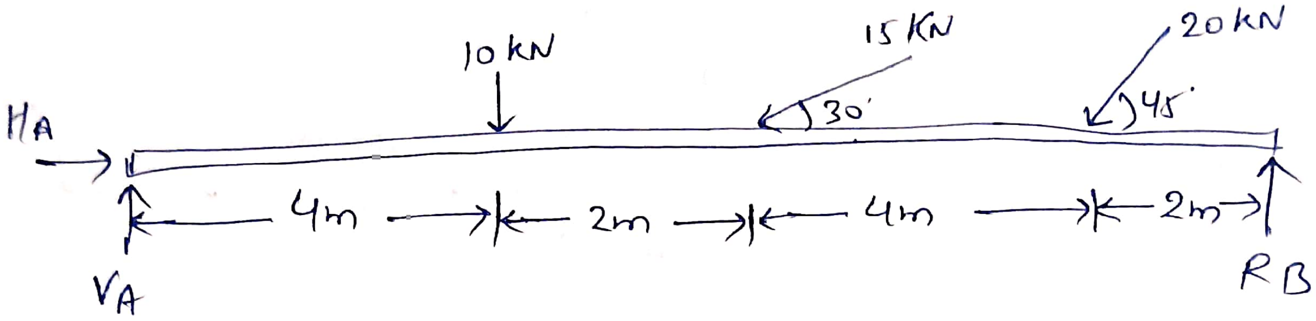
centre of gravity

$$-M_A + 10 \times 2 \times 1 + 20 \sin 60 \times 2 + 15 \times 3$$

$$M_A = 99.64 \text{ kNm}$$

Q2 The beam AB of span 12m as shown in fig is hinged at A & is on rollers at B. Determine the reactions developed at A & B due to loading as shown.





Soln

$\Sigma H = 0$, gives

$H_A - 15\cos 30 - 20\cos 45 = 0$

$H_A = 27.13\,kN$

$\Sigma M_A = 0$, gives

$-R_B \times 12 + 10 \times 4 + 15\sin 30 \times 6 + 20\sin 45 \times 10 = 0$

$\therefore\quad R_B = 18.87\,kN$

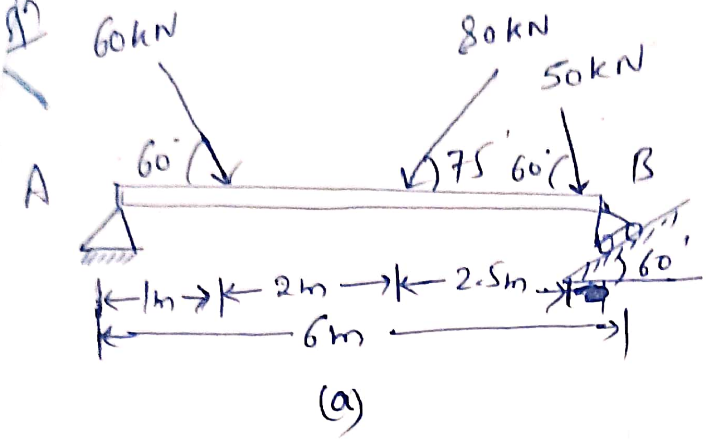
$\Sigma V = 0$, gives

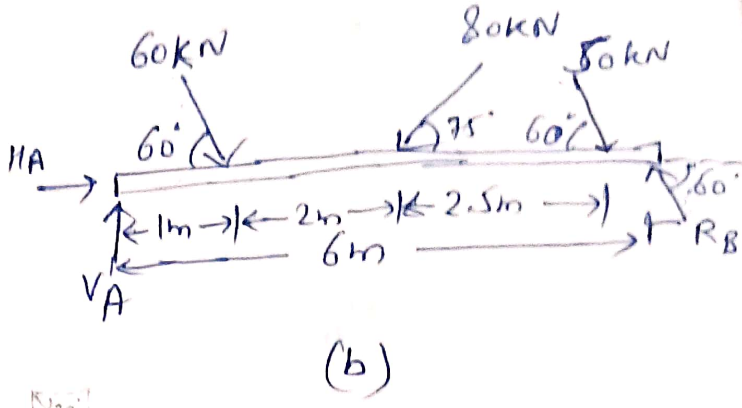
$V_A + R_B - 10 - 15\sin 30 - 20\sin 45 = 0$

$V_A + 18.87 - 10 - 15\sin 30° - 20\sin 45 = 0$

$V_A = 12.77\,kN$

Find the magnitude and direction of reactions at supports A & B in the beam AB as shown in fig. (6)





$\Sigma M_A = 0$

$60 \sin 60 \times 1 + 80 \sin 75 \times 3 + 50 \sin 60 \times 5.5 - R_B \sin 60 \times 6 = 0$

$R_B = 100.45 \text{ kN}$

$\Sigma H = 0$

$H_A + 60 \cos 60 - 80 \cos 75 + 50 \cos 60 - R_B \cos 60 = 0$

$H_A = -60 \cos 60 + 80 \cos 75 - 50 \cos 60 + 100.45 \cos 60$

$H_A = 15.93 \text{ kN}$

$\Sigma V = 0$

$V_A - 60 \sin 60 - 80 \sin 75 - 50 \sin 60 + R_B \sin 60 = 0$

$\therefore V_A = -100.45 \sin 60 + 60 \sin 60 + 80 \sin 75 + 50 \sin 60$

$V_A = 85.54 \text{ kN}$

$\therefore R_A = \sqrt{H_A^2 + V_A^2} = \sqrt{15.93^2 + 85.54^2}$

i.e. $R_A = 87.02 \text{ kN}$

$\alpha_A = \tan^{-1} \dfrac{85.54}{15.93} = 79.45°$

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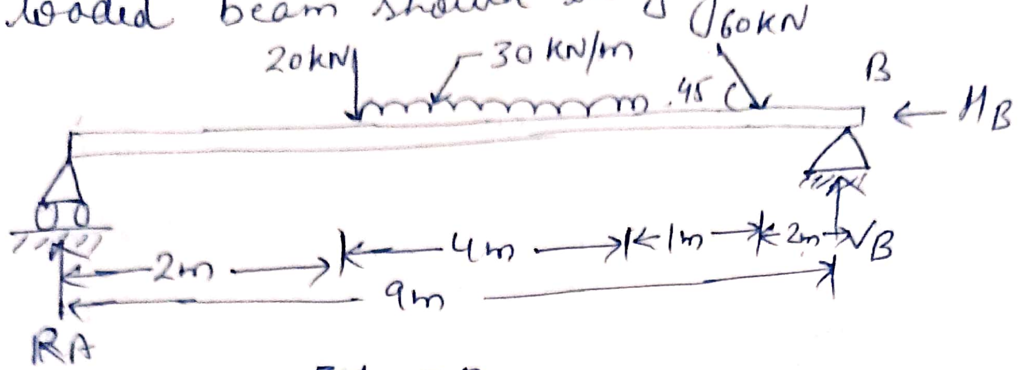
Q4 Find the reactions developed at supports A & B loaded beam shown in fig.



Sol^n $\Sigma M_B = 0$.

$$R_A \times 9 - 20 \times 7 - 30 \times 4 \times 5 - 60 \sin 45 \times 2 = 0$$

$$R_A = 91.65 \text{ kN}$$

$$\Sigma H_B = 0$$

$$H_B - 60 \cos 45 = 0$$

$$H_B = 42.43 \text{ kN}$$

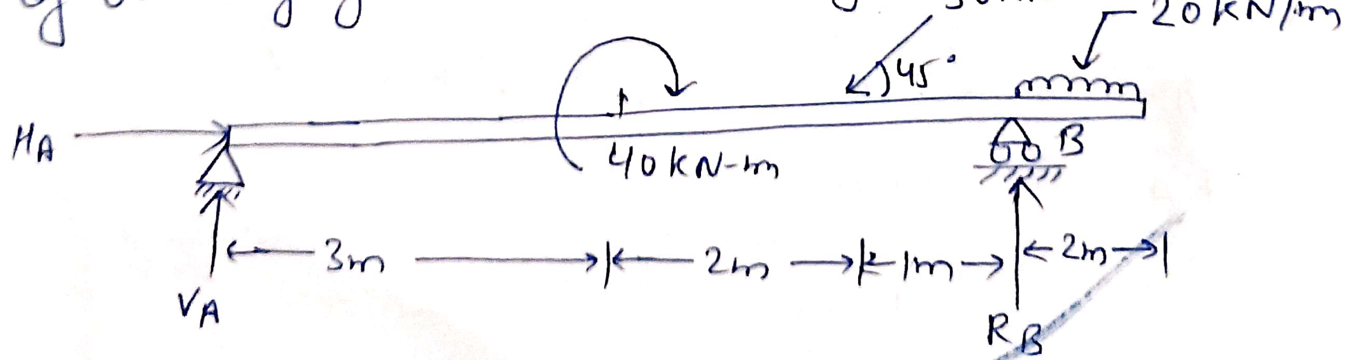
$$\Sigma V = 0$$

$$V_B - 60 \sin 45 - 30 \times 4 \times 2 - 20 + R_A = 0$$

i.e. $V_B = 20 + 30 \times 4 + 60 \sin 45 - 91.65$

$\therefore$ $V_B = 90.78$ kN

Q5 Determine the reactions developed at support A & B of overhanging beam shown in figure.



$\Sigma M_A = 0$

$40 + 30 \sin 45 \times 5 + 20 \times 2 \times 7 - R_B \times 6.$

$\therefore \quad R_B = 71.01\,kN$

$\Sigma F_H = 0$

$H_A - 30 \cos 45 = 0$

$\therefore \quad H_A = 21.21\,kN$

$\Sigma F_V = 0$

$V_A - 30 \sin 45 - 20 \times 2 + R_B = 0$

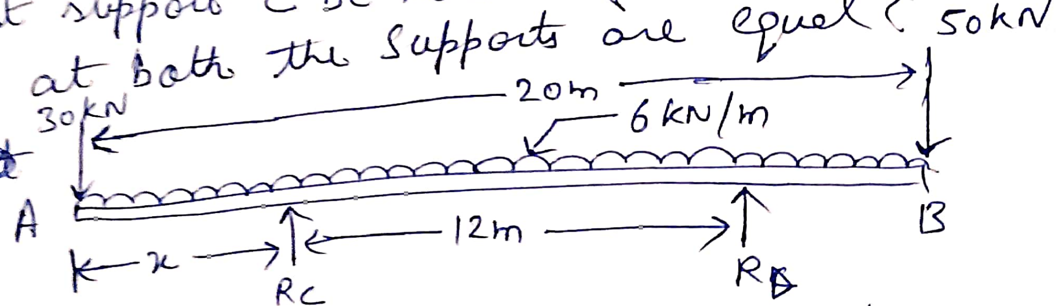
$V_A = 30 \sin 45 - 71.01 + 40$

$\therefore \quad V_A = -9.79\,kN$

$V_A = 9.79 \downarrow kN$

Q6 A beam of 20m long supported on two intermediate supports, 12m apart, carries a udl of 6 kN/m & two concentrated loads of 30kN at left end A and 50kN at the right end B as shown in fig. How far away should the first support C be located from the end A so that the reactions at both the supports are equal?

Soln Let



Soln Let the support C be at a distance x metres from end A

Now, it is given that $R_C = R_D$

$\Sigma F_V = 0$

$R_C + R_D - 30 - 50 - 6 \times 20 = 0$

$2R_C = 30 + 120 + 50 \qquad$ Since $R_D = R_C$

$R_C = 100\,kN$

$R_D = 100\,kN$

$\Sigma M_A = 0$ (9)

$x R_C + (12+x) R_D - 6 \times 20 \times 10 - 50 \times 20 = 0$

$100x + 100(12+x) - 1200 - 1000 = 0$

Sin $R_C = R_D = 100$ kN

$\therefore \quad x = 5m$

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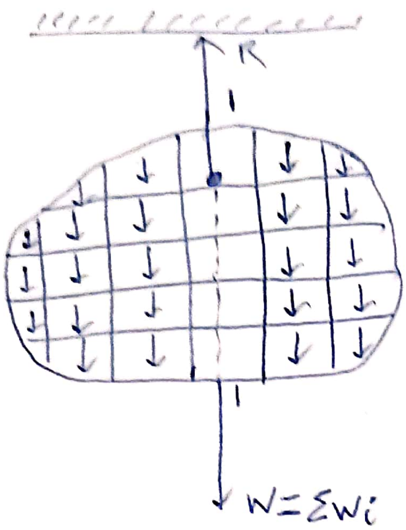
Centre of Gravity

The Centre of gravity can be defined as the point through which resultant of force of gravity (weight) of the body acts.



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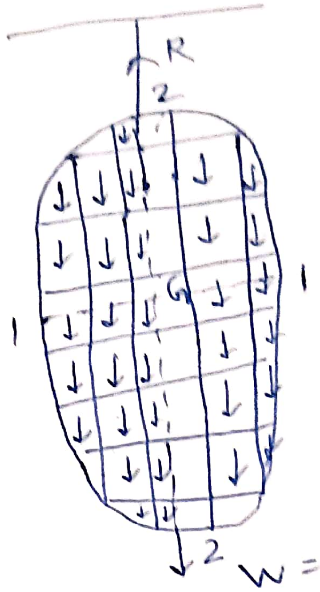
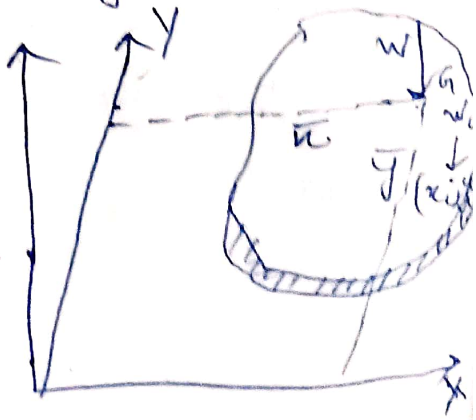


Centre of gravity of a flat plate

Consider a flat plate of thickness t as shown in fig.
let w_i be the weight of any elemental portion acting at a point (x_i, y_i).
let W be the total weight of the plate acting at the point $(\bar{x}, \bar{y})$. According to definition of Centre of gravity, the point $(\bar{x}, \bar{y})$ is the centre of gravity now.

Total weight, $W = \Sigma w_i$

Taking moment about x-axis and equating moment of resultant to moment of component forces, we get



$W\bar{y} = w_1 y_1 + w_2 y_2 + w_3 y_3 + \ldots$

(force × distance) $= \Sigma w_i y_i$ ——— ①

$\therefore \bar{y} = \dfrac{\Sigma w_i y_i}{W}$

Similarly, taking moment about y-axis, we get

$W\bar{x} = w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots$

$= \Sigma w_i x_i$ ——— ②

$\bar{x} = \dfrac{\Sigma w_i x_i}{W}$

Centroid :

Let

A_i = area of i^{th} element of plate of uniform thickness in the plate. ~~If~~.

If γ is the unit weight of the material of plate and t its uniform thickness, then

$$W_i = \gamma A_i t$$

$\therefore$ Total weight, $W = \Sigma \gamma A_i t = \gamma t \Sigma A_i$

$$= \gamma t A$$

where, $A = \Sigma A_i$ is total area.

from eq^n (1) & (2)

$$\bar{y} = \frac{\Sigma A_i \gamma t y_i}{\gamma t A} = \frac{\Sigma A_i y_i}{A} \quad \text{—— (3)}$$

$$\bar{x} = \frac{\Sigma A_i \gamma t x_i}{\gamma t A} = \frac{\Sigma A_i x_i}{A} \quad \text{—— (4)}$$

Since γ & t are constants.

Difference between Centre of gravity & Centroid

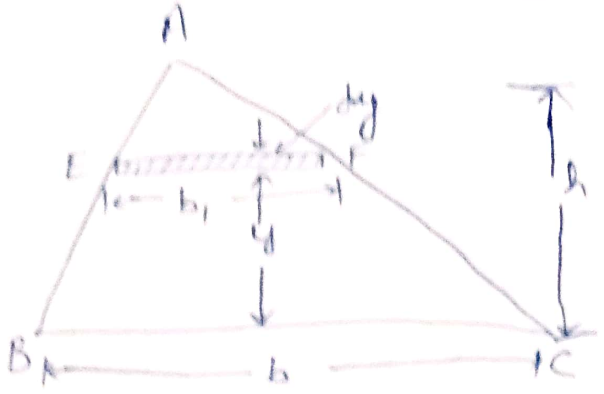
The term Centre of gravity applies to bodies with mass & weight and centroid applies to plane areas.

Determination of centroid of simple figures from first principle

For traingle & semicircle, we can write the general expression. the eq^n (3) & (4) becomes.

$$\bar{y} = \frac{\int y \, dA}{A} \quad \text{—— (5)}, \quad \bar{x} = \frac{\int x \, dA}{A} \quad \text{—— (6)}$$

Centroid of Triangle



Consider triangle ABC of base width b and height h as shown in fig. Let b_1 be the width of elemental strip of thickness dy at a distance y from the base. Since ΔAEF and ΔABC are similar triangles, we can write:

$$\frac{b_1}{b} = \frac{h-y}{h} \quad \text{or} \quad b_1 = \left(\frac{h-y}{h}\right)b = \left(1-\frac{y}{h}\right)b$$

$\therefore$ Area of element $= dA = b_1\,dy$

$$= \left(1-\frac{y}{h}\right)b\,dy$$

Area of traingle, $A = \frac{1}{2}bh$

from eqn ⓒ

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\int y\,dA}{A}$$

Now, $$\int y\,dA = \int_0^h y\left(1-\frac{y}{h}\right)b\,dy = \int_0^h \left(y-\frac{y^2}{h}\right)b\,dy$$

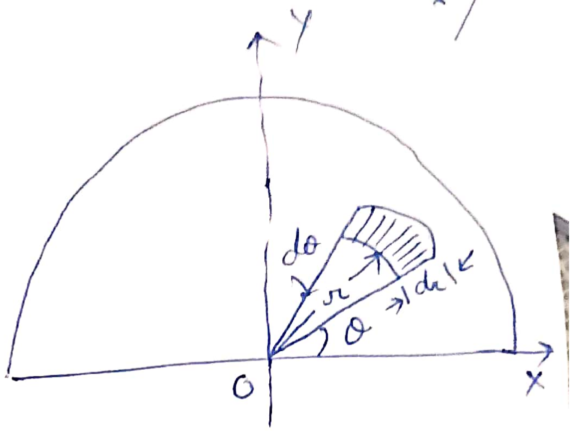
$$= b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h = \frac{bh^2}{6}$$

$$\therefore \bar{y} = \frac{\int y\,dA}{A} = \frac{bh^2}{6} \times \frac{1}{\frac{1}{2}bh}$$

$$\bar{y} = \frac{h}{3}$$

Thus the centroid of a traingle is at a distance $h/3$ from the base of the traingle where h is the height of traingle

Centroid of semicircle



Consider a semicircle of radius R as shown in fig. Due to symmetry, Centroid must lie on y-axis. Let its distance from diamethal axis be $\bar{y}$.
To find $\bar{y}$, consider an element at a distance r from the Centre O of the Semicircle, radial width being dr & bound by radii at θ.

Area of element $= r\,d\theta\,dr$ (arc length × b)

Its moment about diamethal axis x is given by:

$$r\,d\theta \times dr \times \underbrace{r\sin\theta}_{\perp\text{distance}} = r^2 \sin\theta\, dr\, d\theta$$

$\therefore$ Total moment of area about diamethal axis

$$= \int_0^{\pi}\int_0^{R} r^2 \sin\theta\, dr\, d\theta$$

$$= \int_0^{\pi} \left[\frac{r^3}{3}\right]_0^R \sin\theta\, d\theta$$

$$= \frac{R^3}{3}\left[-\cos\theta\right]_0^{\pi} = \frac{R^3}{3}[1+1] = \frac{2R^3}{3}$$

Area of Semicircle, $A = \frac{1}{2}\pi R^2$

$$\therefore \bar{y} = \frac{\text{Moment of Area}}{\text{Total area}} = \frac{\frac{2R^3}{3}}{\frac{1}{2}\pi R^2} = \frac{4R}{3\pi}$$

Thus, the Centroid of the Semicircle is at a distance $\frac{4R}{3\pi}$ from the diamethal axis.

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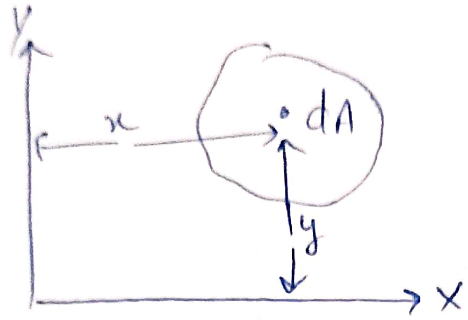
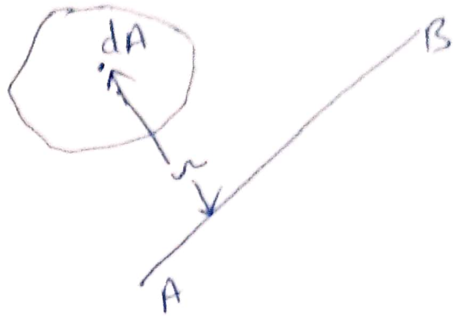
~~oid of Sector of a circle : HW~~
~~troid of parabolic Spandrel : HW~~

Moment of Inertia



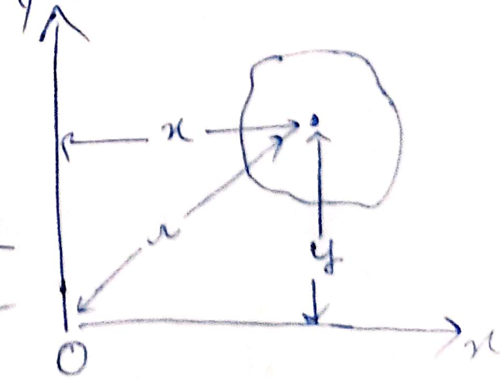


If dA is an elemental area with coordinates as x & y. The term $\Sigma y^2 dA$ is called moment of inertia of the area about x-axis & is denoted by I_{xx}. Similarly moment of inertia about y-axis is $I_{yy} = \Sigma x^2 dA$.

In general, if r is the distance of elemental area dA from the axis AB, the sum of the terms $\Sigma r^2 dA$ to cover the entire area is called moment of inertia of the area about the axis AB.

$$I_{AB} = \Sigma r^2 dA = \int r^2 dA$$

Polar moment of Inertia



Moment of inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia. It may be denoted as J or I_{zz}.

Thus, the moment of inertia about an axis perpendicular to the plane of the area at O is called polar moment of inertia at point O, $I_{zz} = \Sigma r^2 dA$

Radius of gyration

It is mathematical term defined by the relation

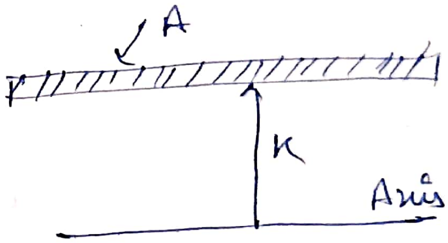
$$k = \sqrt{\frac{I}{A}} \quad \text{--- (7)}$$

where k = radius of gyration
I = moment of inertia
A = Cross-sectional area

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}}, \quad k_{yy} = \sqrt{\frac{I_{yy}}{A}}, \quad k_{AB} = \sqrt{\frac{I_{AB}}{I}}$$

from (7)

$$I = kA^2$$



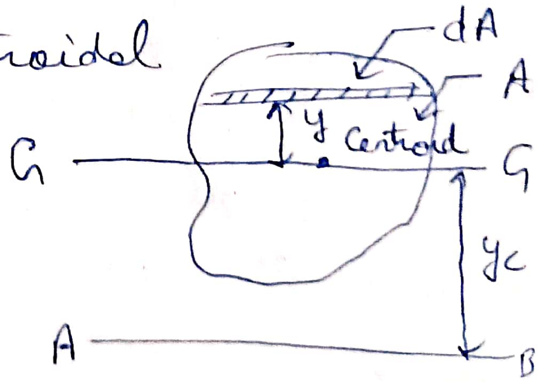
theorems of Moment of Inertia

1. Parallel Axis theorem.

Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis & the product of area & square of the distance between the two parallel axes. Ref fig, the above theorem means:

$$I_{AB} = I_{GG} + Ay_c^2$$

where, I_{AB} = moment of inertia about axis AB
I_{GG} = moment of inertia about centroidal axis GG parallel to AB
A = the area of plane figure given
y_c = the distance between the axis AB & parallel centroidal axis GG.



...elation: Consider an elemental parallel strip dA at a distance y from the centroidal axis.

...n, $I_{AB} = \Sigma(y + y_c)^2 dA = \Sigma(y^2 + 2yy_c + y_c^2)\,dA$

$= \Sigma y^2 dA + \Sigma 2yy_c dA + \Sigma y_c^2 dA$

Now,

$\Sigma y^2 dA$ = Moment of inertia about the axis $GG = I_{GG}$

$\Sigma 2yy_c dA = 2y_c \Sigma y\,dA$

$= 2y_c A \cdot \dfrac{\Sigma y\,dA}{A}$

In above term $2y_c A$ is constant and $\dfrac{\Sigma y\,dA}{A}$ is the distance of centroid from the reference line GG. Since GG is passing through the centroid itself $\dfrac{y\,dA}{A}$ is zero & hence the term $\Sigma 2yy_c dA$ is zero.

Now, third term.

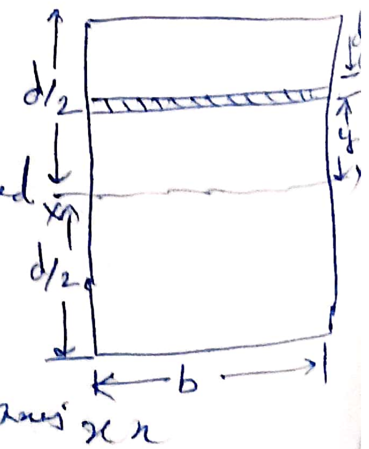
$\Sigma y_c^2 dA = y_c^2 \Sigma dA = A y_c^2$

$\therefore I_{AB} = I_{GG} + Ay_c^2$

<u>Moment of Inertia from first principle</u>

1. <u>Moment of Inertia of a Rectangle about the Centroidal Axis</u>

Consider a rectangle of width b and depth d. Moment of inertia about the centroidal axis $x-x$ parallel to the short side is required. Consider an elemental strip of width dy at a distance y from the axis. Moment of inertia of the elemental strip about centroidal axis $x\,x$



$= y^2 dA$

$= y^2 b\,dy$

$\therefore \; I_{xx} = \int_{-d/2}^{d/2} y^2 b\,dy$

$= b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$ or $b\left[\frac{d^3}{24}+\frac{d^3}{24}\right]$

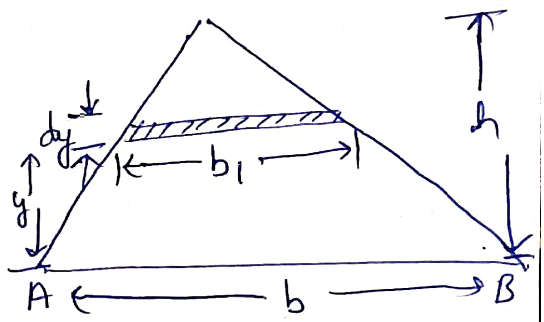
$$I_{xx} = \frac{bd^3}{12}$$

2. Moment of Inertia of a traingle about its base

Both are similar triangles

width of strip

$b_1 = \frac{(h-y)}{h} \times b$



Moment of inertia of this strip about AB

$= y^2 dA = y^2 b_1\,dy$

$= y^2 \frac{(h-y)}{h} \times b \times dy$

$\therefore$ Moment of inertia of the traingle about AB

$I_{AB} = \int_0^h \frac{y^2(h-y)b\,dy}{h} = \int_0^h \left(y^2 - \frac{y^3}{h}\right) b\,dy$

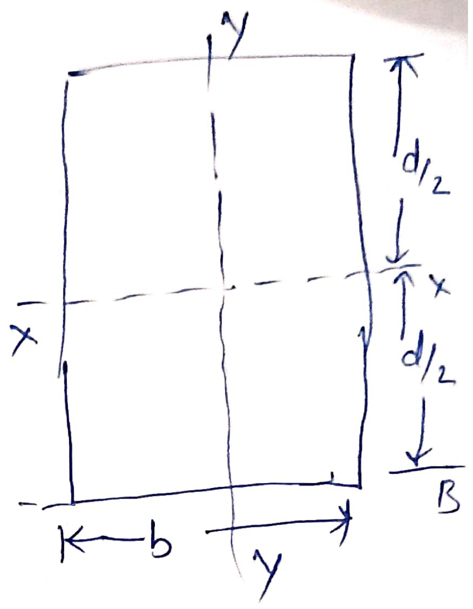
$= \left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h b = b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$

$$I_{AB} = \frac{bh^3}{12}$$

...nt of Inertia of circle about its diametral Axis ← HW

...ment of Inertia of Standard Sections

Rectangle:



a) $I_{xx} = \frac{bd^3}{12}$

b) $I_{yy} = \frac{db^3}{12}$

c) About a base AB, from parallel axis theorem

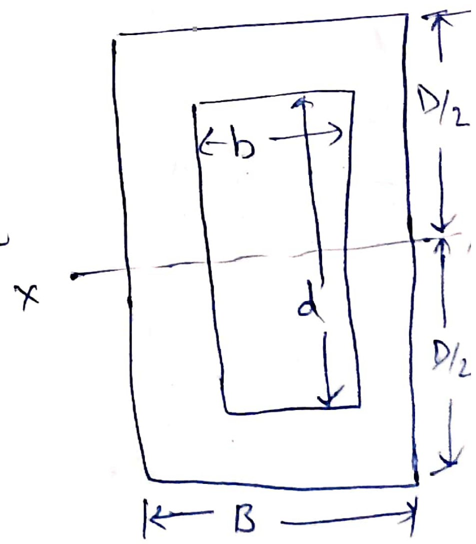
$$I_{AB} = I_{xx} + Ay_c^2$$

$$= \frac{bd^3}{12} + bd\left(\frac{d}{2}\right)^2, \text{ since } y_c = \frac{d}{2}$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

$$= \frac{bd^3}{3}$$

Hollow rectangular section



Moment of Inertia I_{xx} = Moment of inertia of larger rectangle − Moment of inertia of hollow portion, That is,

$$= \frac{BD^3}{12} - \frac{bd^3}{12}$$

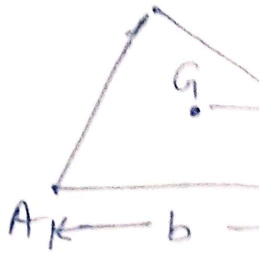
$$= \frac{1}{12}(BD^3 - bd^3)$$

Traingle:

a) about Base, $I_{AB} = \frac{bh^3}{12}$

b) About centroidal axis, x-x parallel to base of

from parallel axis theorem



$$I_{AB} = I_{xx} + Ay_c^2$$

Now, y_c, the distance between the non-centroidal axis AB and centroidal axis x-x, is equal to $h/3$.

$$\therefore \frac{bh^3}{12} = I_{xx} + \frac{1}{2}bh\left(\frac{h}{3}\right)^2 = I_{xx} + \frac{bh^3}{18}$$

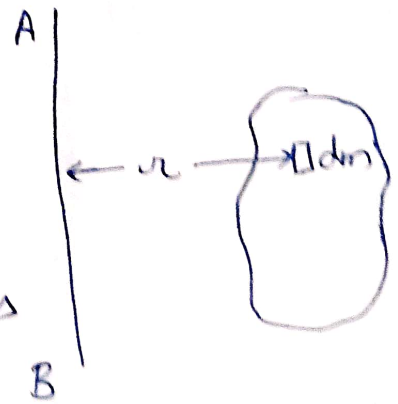
$$\therefore I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$

Moment of Inertia of circle, hollow circle, Semicircle, Quarter of circle — HW

Mass Moment of Inertia

Mass moment of inertia of a body about an axis is defined as the sum total of product of its elemental masses and square of their distance from the axis.

Thus the mass moment of body as shown in fig about axis AB is given by



$$I_{AB} = \sum dm\, r^2 = \int r^2\, dm$$

where r is the distance of element of mass dm from AB ius of Gyration

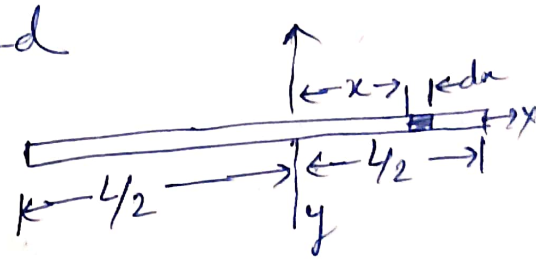
adius of gyration is that distance which when squared and multiplied with total mass of the body gives the mass moment of inertia of the body. Thus if I is moment of inertia of a body of mass M about an axis, then its radius of gyration k about that axis is given by the relation

$$I = Mk^2 \quad \text{or} \quad k = \sqrt{\frac{I}{M}}$$

Determination of Mass Moments of Inertia from first principles

Q/ Determine the mass moment of inertia of a uniform rod of length L about axis normal to it at its.
(a) Centroid (b) end.

Sol^n a) about Centroid Axis normal to rod



Consider an elemental length dx at a distance x from Centroidal axis $y-y$ as shown in fig. Let the mass of rod be m per unit length.
Then mass of the element $dm = m\,dx$ (mass/load × length)

$$I = \int_{-L/2}^{L/2} x^2 \times dm = \int_{-L/2}^{L/2} x^2 \times m\,dx$$

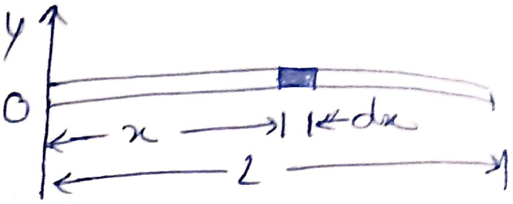
$$= m\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \frac{mL^3}{12}$$

i.e $I = \dfrac{ML^2}{12}$

where $M = mL$ is total mass of rod.

(b) About Axis at the end of the rod & normal to it

Consider an element of length dx at a distance x from end as shown in fig.



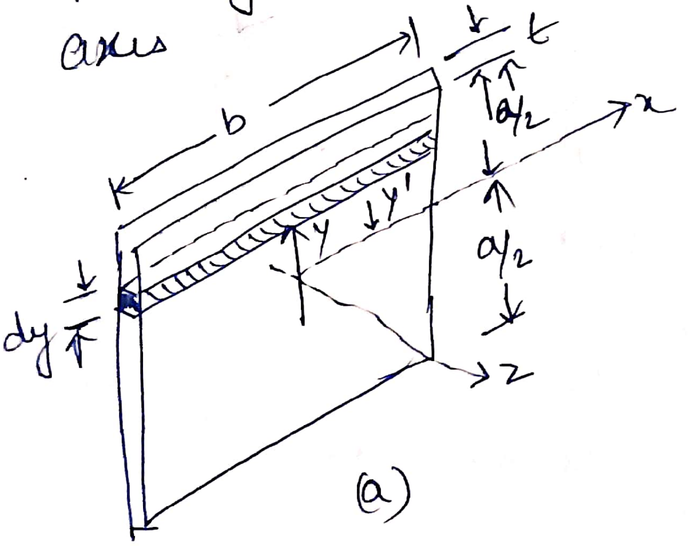
Moment of inertia of the rod about y-axis

$$I = \int_0^L x^2 \, m \, dx$$

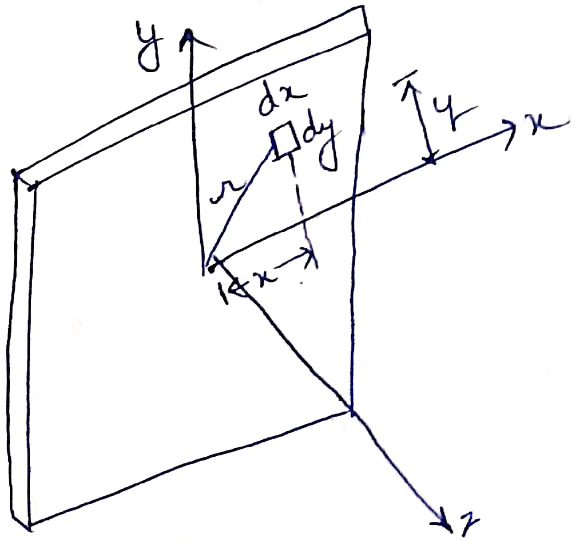
$$= m\left[\frac{x^3}{3}\right]_0^L = m L^3$$

$$I = \frac{ML^2}{3}$$

Q/ Determine the moment of inertia of a rectangular plate of size a×b & thickness t about its Centroidal axis



(a)



(b)

Sol^n To find I_{xx}

Consider an elemental strip of width dy at a distance y from x-axis as shown in fig (a). Mass of the element

$$dm = \rho \, b \times t \times dy$$

(ρ is the unit mass of the material)

$$I_{xx} = \int_{-a/2}^{a/2} y^2\,dm = \int_{-a/2}^{a/2} y^2 \rho b t\,dy$$

$$= \rho b t \left[\frac{y^3}{3}\right]_{-a/2}^{a/2}$$

$$= \frac{\rho b t a^3}{12}$$

But mass of the plate $M = \rho b t a$

$$\therefore \quad I_{xx} = \frac{M a^2}{12}$$

To find I_{yy}:

Taking an elemental strip parallel to y-axis, it can be easily shown that:

$$I_{yy} = \frac{M b^2}{12}$$

To find I_{zz}:

Consider an element of size $dx\,dy$ & thickness t as shown in fig b

Now

$$r^2 = x^2 + y^2$$

$$I_{zz} = \int r^2\,dm = \int (x^2+y^2)\,dm$$

$$= \int x^2\,dm + \int y^2\,dm = I_{xx} + I_{yy}$$

$$= \frac{M a^2}{12} + \frac{M b^2}{12}$$

$$I_{zz} = \frac{1}{12} M (a^2+b^2)$$

Q/ find the moment of inertia of circular plate of radius R and thickness t about its centroidal

Q/ Determine the mass moment of inertia of circular ring of uniform cross-section

Q/ find the mass moment of inertia of the solid cone of height h & base radius R about:

(i) its axis of rotation &

(ii) an axis through vertex normal to the axis of rotation

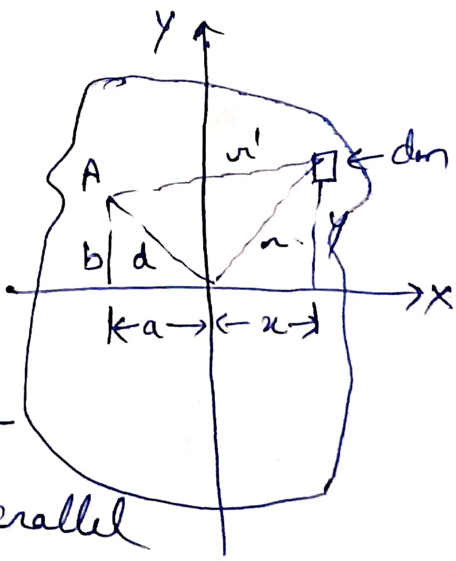
Q/ Determine the moment of inertia of a solid plate of radius R about its diametral axis.

Q/ using the moment of inertia expression for plates, find the expressions for moment of inertia of:

a) parallelopiped &

b) circular cylinder about Z axis as shown in fig.

Parallel axis theorem



The moment of inertia of a body about an axis at a distance d & parallel to a centroidal axis is equal to sum of moment of inertia about centroidal axis and product of mass & square of distance of parallel axis.

If I_g is the moment of inertia of the body of mass M about a centroidal axis & I_A is the moment of inertia about a parallel axis through A which is at a distance d from centroidal axis, then

$$I_A = I_g + Md^2$$

Proof: Let dm be an element at a distance r from centroidal axis Z through centre of gravity as shown in fig.

$$I_g = \int r^2 dm$$

$$= \int (x^2+y^2)\, dm$$

Now, moment of inertia about Z-axis through A is,

$I_A = \int r'^2 dm$, where r' is the distance of element A

$$= \int [(x+a)^2+(y-b)^2]\, dm$$

$$= \int (x^2+2ax + a^2+y^2-2yb+b^2)\, dm$$

$$= \int (x^2+y^2)\,dm + \int (a^2+b^2)\,dm + \int 2ax\,dm - \int 2yb\,dm$$

$$= \int r^2 dm + \int d^2 dm + \int 2ax\,dm - \int 2by\,dm$$

Since $x^2+y^2=r^2$ & $a^2+b^2=d^2$

But, $\int r^2 dm = I_g$

$\int d^2 dm = d^2\int dm = Md^2$

$$\int 2ax\,dm = 2a\int x\,dm = 2aM\bar{x}$$

& $$\int 2by\,dm = 2b\int y\,dm = 2bM\bar{y}$$

where $\bar{x}$ & $\bar{y}$ are distances of Centre of gravity from the reference axis. In this case $\bar{x}$ & $\bar{y}$ are zero since reference axis contains centroid. Thus $\int 2ax\,dm = \int 2by\,dm = 0$

Hence, $$I_A = I_g + Md^2$$

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