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Applied Mathematics - I

Assignment

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BTech CSE
Sec - B

Ans1

$$\frac{\partial u}{\partial x} = \frac{\sec^2 x}{(\tan x + \tan y + \tan z)}$$

$$\sin 2x \frac{\partial u}{\partial x} = \frac{2\sin x \cos x}{\cos^2 x}\left[\frac{1}{\tan x + \tan y + \tan z}\right]$$

$$= \frac{2\tan x}{\tan x + \tan y + \tan z}$$

Similarly,

$$\sin 2y \frac{\partial u}{\partial y} = \frac{2\tan y}{\tan x + \tan y + \tan z}$$

$$\sin 2z \frac{\partial u}{\partial z} = \frac{2\tan z}{\tan x + \tan y + \tan z}$$

LHS.

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z}$$

$$= 2\left[\frac{\tan x + \tan y + \tan z}{\tan x + \tan y + \tan z}\right]$$

$$= 2$$

LHS = RHS

Hence Proved

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Ans 2
$$\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial z}\left(\log\left(x^2+y^2+z^2\right)\right)\right]$$

$$= \frac{\partial}{\partial y}\left[\frac{2z}{x^2+y^2+z^2}\right]$$

$$= \frac{-2z(2y)}{(x^2+y^2+z^2)}$$

$$= \frac{-4yz}{(x^2+y^2+z^2)}$$

$$\frac{\partial^2 u}{\partial z \partial x} = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial x}\left\{\log\left(x^2+y^2+z^2\right)\right\}\right]$$

$$= \frac{\partial}{\partial z}\left[\frac{2x}{x^2+y^2+z^2}\right]$$

$$= \frac{-4xz}{(x^2+y^2+z^2)}$$

LHS

$$x\frac{\partial^2 u}{\partial y \partial z} = \frac{-4xyz}{(x^2+y^2+z^2)}$$

RHS =

$$y\frac{\partial^2 u}{\partial z \partial x} = \frac{-4xyz}{(x^2+y^2+z^2)}$$

LHS = RHS

Hence Proved

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Ans 3 $\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{du}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$

$= 2x[e^t] + 2y[e^t \cos t - e^t \sin t] + 2z[e^t \sin t + e^t \cos t]$

$= 2xe^t + 2ye^t(\cos t - \sin t) + 2ze^t(\sin t + \cos t)$

$= 2e^{2t} + 2e^{2t}(\cos^2 t - \sin^2 t)$

$= 2e^{2t} + 2e^{2t}$

$= 4e^{2t}$

Ans 4 let $u = f(r, s, t)$

where $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial x} + \frac{\partial u}{\partial t} \cdot \frac{\partial t}{\partial x}$

$= \frac{\partial u}{\partial r} \frac{1}{y} + 0 + \frac{\partial u}{\partial t} \cdot \frac{(-z)}{x^2}$

$= \frac{\partial u}{\partial r}\left(\frac{1}{y}\right) - \frac{\partial u}{\partial t} \frac{z}{x^2}$

$x \frac{\partial u}{\partial x} = \frac{x}{y} \frac{\partial u}{\partial r} - \frac{\partial u}{\partial t} \frac{z}{x}$

Similarly

$y \frac{\partial u}{\partial y} = \frac{-x}{y} \frac{\partial u}{\partial r} + \frac{y}{z} \frac{\partial u}{\partial s}$

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$$z\frac{\partial u}{\partial z} = -\frac{y}{z}\frac{\partial u}{\partial s} + \frac{z}{x}\frac{\partial u}{\partial t}$$

LHS

$$= \frac{x}{y}\frac{\partial u}{\partial r} - \frac{z}{x}\frac{\partial u}{\partial t} - \frac{x}{y}\frac{\partial u}{\partial r} + \frac{y}{z}\frac{\partial u}{\partial s} - \frac{y}{z}\frac{\partial u}{\partial s} + \frac{z}{x}\frac{\partial u}{\partial t}$$

$= 0$

LHS = RHS

Hence Proved

Ans $$\frac{du}{dx} = \frac{\partial u}{dx}\frac{dx}{} + \frac{\partial y}{\partial y}\frac{dy}{}$$

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial y}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$a^2x^2 + b^2y^2 = c^2$$

diff both sides

$$a^2(2x) + b^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{a^2(2x)}{b^2(2y)}$$

$$= \frac{-a^2x}{b^2y}$$

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$$\frac{\partial u}{\partial x} = \cos(x^2+y^2)\,2x$$

$$\frac{\partial u}{\partial y} = \cos(x^2+y^2)\,2y$$

$$\frac{du}{dx} = \cos(x^2+y^2)\,2x - \cos(x^2+y^2)\,2y \cdot \frac{a^2x}{b^2y}$$

$$= 2x\cos(x^2+y^2)\left[1-\frac{a^2}{b^2}\right]$$

6) Volume of parallelopiped $= 2x \cdot 2y \cdot 2z$

$$= 8xyz$$

Using Lagrange's multiplier method

$$f(x,y,z) = 8xyz$$

$$\phi(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0 \quad \text{— (1)}$$

$$F(x,y,z) = 8xyz + \lambda\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right]$$

$$df = \left(8yz + \frac{\lambda 2x}{a^2}\right)dx + \left(8xz + \frac{\lambda 2y}{b^2}\right)dy + \left(8xy + \frac{\lambda 2z}{c^2}\right)dz$$

For ~~stibrau~~ stationary pts

$$8yz + \frac{\lambda 2x}{a^2} = 0$$

$$\frac{1}{a^2} = \frac{-8yz}{2\lambda x} \quad \text{— (i)}$$

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$$8xz + \frac{\lambda 2y}{b^2} = 0$$

$$\frac{1}{b^2} = -\frac{8xz}{2\lambda y} \quad \text{— (ii)}$$

$$8xy + \frac{\lambda 2z}{c^2} = 0$$

$$\frac{1}{c^2} = -\frac{8xy}{2\lambda z} \quad \text{— (iii)}$$

Putting values in (1)

$$-\frac{8xyz}{2\lambda} + \left(-\frac{8xyz}{2\lambda}\right) + \left(-\frac{8xyz}{2\lambda}\right) = 1$$

$$-12xyz = \lambda$$

$$xyz = -\frac{\lambda}{12}$$

Puting $\lambda = -12xyz$ into (i)

$$\frac{1}{a^2} = \frac{-8yz}{-24x^2y^2}$$

$$x^2 = \frac{a^2}{3}$$

$$x = \frac{a}{\sqrt{3}}$$

similarly

$$y \ \cancel{b} = \frac{b}{\sqrt{3}}$$

$$z = \frac{c}{\sqrt{3}}$$

$\therefore$ volume $= 8xyz$

$$= \frac{8abc}{3\sqrt{3}}$$

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$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 1 & 1 & 1 \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \\ 2x-y-z & 2y-x-z & 2z-y-x \end{vmatrix}$$

$$= \left[(3y^2-3z)(2z-y-x) - (3z^2-3xy)(2y-x-z)\right]$$
$$- (3x^2-3yz)(2y-x-z) + (3z^2-3xy)(2x-y-z)$$
$$+ \left[(3x^2-3yz)(2y-x-z) - (3y^2-3xz)(2x-y-z)\right]$$

$$= 0$$

Hence the relation exists

$$v = \cancel{3}\, x^3 + y^3 + z^3 - 3xyz$$
$$(x+y+z)\left[x^2+y^2+z^2-(xy+yz+zx)\right]$$

$$= u\left[x^2+y^2+z^2-xy-yz-xz\right]$$

$$= uw$$

$$\therefore v = uw$$

Functions are functionalley dependent by the relation $v = uw$

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Ans 8) $f(x,y) = \sin(x) + \sin(y) + \sin(x+y)$

$p = \cos x + \cos(x+y)$
$q = \cos y + \cos(x+y)$
$r = -\sin x - \sin(x+y)$
$t = -\sin y - \sin(x+y)$

for stationary points

when $p=0$
$\cos x + \cos(x+y) = 0$
$\cos x = -\cos(x+y)$
$\cos x = \cos y$
$-\cos x = -\cos y$
$x = y$

when $p=0$
$\cos y + \cos(x+y) = 0$
$\cos y = -\cos(x+y)$

$\cos 2x = \cos(x+y) = -\cos x$
$2x = \pi - x$
$3x = \pi$.
$x = \frac{\pi}{3}$

$y = x = \frac{\pi}{3}$

$\Rightarrow$ critical point is $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

now $rt - s^2$
$= \{[-\sin(\frac{\pi}{3}) + \sin(2\frac{\pi}{3})] \cdot [\sin(\frac{\pi}{3}) + \sin(2\frac{\pi}{3})]\}$

$- \left[-\sin \frac{2\pi}{3}\right]^2$

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$r = (-\sqrt{3})$

$(-\sqrt{3}) < 0$

$f(x,y)$ attains max at $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

max value

$$= \sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \sqrt{3}$$

$$= \frac{3\sqrt{3}}{2}$$

Ans 9) $x = uv$

$$\frac{\partial(u,v)}{\partial(x,y)} = 1 \Big/ \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \quad -(i)$$

$$\frac{\partial x}{\partial u} = v \quad -(ii)$$

$$\frac{\partial x}{\partial v} = u \quad -(iii)$$

$$\frac{\partial y}{\partial u} = \frac{(u-v) - (u+v)}{(u-v)^2} \quad -(iv) = \frac{-2v}{(u-v)^2} \quad -(iv)$$

$$\frac{\partial y}{\partial v} = \frac{(u-v) + (u+v)}{(u-v)^2} = \frac{2u}{(u-v)^2} \quad -(v)$$

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From (i) (ii) (iii) (iv) (v)

$$\frac{\partial(u,v)}{\partial(x,y)} = 1 \Big/ \begin{vmatrix} v & u \\ \frac{-2v}{(u-v)^2} & \frac{2u}{(u-v)^2} \end{vmatrix}$$

$$= 1 \Big/ \left[\frac{2uv}{(u-v)^2} + \frac{2uv}{(u-v)^2} \right]$$

$$= 1 \Big/ \frac{4uv}{(u-v)^2}$$

$$= \frac{(u-v)^2}{4uv}$$

Ans 10) $f(x,y) = x^3 + y^3 - 3axyz$

$p = 3x^2 - 3ayz$

$q = 3y^2 - 3axz$

$r = 6x$

$s = -3az$

$t = 6y$

For stationary pts

when $p = 0$

$3x^2 - 3ayz = 0$

$x^2 = ayz$

$x = az$

$y = az$

$r = 6az$

$t = 6az$

$s = -3az$

When $q = 0$

$3y^2 - 3axz = 0$

$y^2 = axz$

Extreme point at (az, az)

$r = 6az > 0$

minima at (az, az)