Digital Electronics

Electronics refers to a study of flow of electrons and its dynamics.

Electronics

Analog Electronics | Digital electronics

(0, 1).

1 → True → ON → Activate
0 → False → OFF → Deactivate

George Boole introduced Boolean Algebra

Boolean Algebra consist of
- Basic laws
- Theorems.

Basic laws Of Boolean Algebra

$A + A = A$] idempotency law
$A \cdot A = A$
$A + 1 = 1$ $\qquad \bar{\bar{A}} = A$ (double inversion)
$A + 0 = A$
$A + \bar{A} = 1$ (Full set)
$A \cdot \bar{A} = 0$ (Null set)

Boolean thearums

~~Boolean theo~~

~~Boolean theorum works~~

1) Demorgan's thearum

Demorgan's thearum works on the Principle to (change the sign and break the bar)

(i) $\overline{AB} = \overline{A} + \overline{B}$

(ii) $\overline{A+B} = \overline{A} \cdot \overline{B}$

2) Transposition thearum

$(A+B)(A+C) = A+(B \cdot C)$

3) Distribution thearum

$A+(B \cdot C) = (A+B)(A+C)$

$A(B+C) = A \cdot B + A \cdot C$

4) Consensus Thearum

→ It is used for elimination of redundant variable.

→ It contains 3 variable.

→ Only one variable is in complemented or uncomplemented form.

→ Then the related terms to that complemented or uncomplemented variable is the answer.

E.g.

$Y(A,B,C) = AB + \bar{B}C + AC$

$= AB + \bar{B}C$

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e.g. $Y(A,B,C) = (A+\bar{B})(B+C)(A+C)$
$= (A+\bar{B})(B+C)$

e.g. $Y(A,B,C) = AB + \bar{A}C + BC$
$= AB + \bar{A}C$

Qn
(i) $Y = \bar{A}B + AB + \bar{A}\bar{B}$

$Y = B(\bar{A}+A) + \bar{A}\bar{B}$
$Y = B + \bar{A}\bar{B}$

(ii) $Y(A,B,C) = ABC + A\bar{B}C + AB\bar{C}$
$= AC(B+\bar{B}) + AB\bar{C}$
$= AC + AB\bar{C}$
$= A(C + B\bar{C})$

(iii) $Y = A + \bar{A}B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$
$\cancel{= (A+\bar{A})(A+B) + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D}$
$\cancel{= A + B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D}$
$\cancel{= A + B + \bar{A}}$

Sol
$= A + \bar{A}(B + \bar{B}C + \bar{B}\bar{C}D)$
$= A + \bar{A}\left[(B+\bar{B})(B+C) + \bar{B}\bar{C}D\right]$
$= A + \bar{A}\left[B + C + \bar{B}\bar{C}D\right]$
$= A + \bar{A}\left[(B+\bar{B})\cdot(B+\bar{C}D) + C\right]$
$= A + \bar{A}\left[B + C + \bar{C}D\right]$
$= A + \bar{A}\left[B + (C+\bar{C})(C+D)\right]$
$= A + \bar{A}(B + C + D)$

Qn $Y = \overline{(AB + \bar{C})(\overline{A+B} + C)}$

$Y = \overline{(AB + \bar{C})(\bar{A}\cdot\bar{B} + C)}$

$Y = \overline{AB\cdot\bar{A}\cdot\bar{B} + ABC + \bar{A}\bar{B}\bar{C} + C\bar{C}}$

$Y = \bar{A}\bar{B}\bar{C} + ABC$

$Y = \overline{ABC + \bar{A}\bar{B}\bar{C}}$

$Y = \overline{ABC}\cdot\overline{\bar{A}\bar{B}\bar{C}}$

$Y = (\bar{A} + \bar{B} + \bar{C})\cdot(\bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}})$

$Y = (\bar{A} + \bar{B} + \bar{C})\cdot(A + B + C)$

Qn $Y = \bar{A}C[\overline{\bar{A}BD}] + \bar{A}BC\bar{D} + A\bar{B}C$

$Y = \bar{A}C[\bar{\bar{A}} + \bar{B} + \bar{D}] + \bar{A}BC\bar{D} + A\bar{B}C$

$Y = \bar{A}C[A + \bar{B} + \bar{D}] + \bar{A}BC\bar{D} + A\bar{B}C$

$Y = \bar{A}AC + \bar{A}\bar{B} + \bar{A}\bar{D}C + \bar{A}BC\bar{D} + A\bar{B}C$

$Y = \bar{A}AC + \bar{A}\bar{B} + \bar{A}\bar{D}C(1 + B) + A.\bar{B}C$

$Y = \bar{A}\bar{B}C + \bar{A}\bar{D}C + A\bar{B}C$

$Y = \bar{B}C(\bar{A} + A) + \bar{A}\bar{D}C$

$Y = \bar{B}C + \bar{A}\bar{D}C$

$Y = C(\bar{B} + \bar{A}\bar{D})$

Q. Proof the following :-

$BCD + A\bar{C}\bar{D} + ABD = BCD + A\bar{C}\bar{D} + AB\bar{C}$

Sol. $L.H.S = BCD + A\bar{C}\bar{D} + ABD$

L.H.S

$BCD + A\bar{C}\bar{D} + ABD \cdot 1$

$= BCD + A\bar{C}\bar{D} + ABD(C + \bar{C})$

$= BCD + A\bar{C}\bar{D} + ABCD + AB\bar{C}D$

$= BCD(1 + A) + A\bar{C}(\bar{D} + BD)$

$= BCD + A\bar{C}[(\bar{D} + B) \cdot (\bar{D} + D)]$

$= BCD + A\bar{C}\bar{D} + A\bar{C}B$

$= R.H.S$

Hence Proved

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Q^n If $A\bar{B} + \bar{A}B = C$, show that $A\bar{C} + \bar{A}C = B$.

Given :- $A\bar{B} + \bar{A}B = C$

To Prove :- $A\bar{C} + \bar{A}C = B$

$$L.H.S = A\bar{C} + \bar{A}C$$

$$= A\left[\overline{(A\bar{B} + \bar{A}B)}\right] + \bar{A}\left[A\bar{B} + \bar{A}B\right]$$

$$= A\left[\overline{A\bar{B}} \cdot \overline{\bar{A}B}\right] + \cancel{\bar{A}A}\bar{B} + \bar{A}\bar{A}B$$

$$= A\left[(\bar{A} + \bar{\bar{B}})(\bar{\bar{A}} + \bar{B})\right] + \bar{A}B$$

$$= A\left[(\bar{A} + B)(A + \bar{B})\right] + \bar{A}B$$

$$= A\left[\cancel{\bar{A}A} + \bar{A}\bar{B} + AB + \cancel{B\bar{B}}\right] + \bar{A}B$$

$$= A\left[\bar{A}\bar{B} + AB\right] + \bar{A}B$$

$$= \cancel{A\bar{A}\bar{B}} + AAB + \bar{A}B$$

$$= AB + \bar{A}B$$

$$= B(A + \bar{A})$$

$$= B$$

$$= R.H.S$$

Hence Proved

Ques. for Practice

Simplify the following expression using the Boolean Algebra :-

1. $Y = \bar{A}B + AB + \bar{A}\bar{B}$
2. $Y = ABC + A\bar{B}C + AB\bar{C}$
3. $Y = (AB+\bar{C})\,(\overline{A+B} + C)$

Boolean Funcⁿ Representation

Boolean Funcⁿ Representation → Std. form (Minimized form) ; Cannonical form

Std. form (Minimized form)
↓
All the terms donot have each variables (literals)

e.g → $F(A,B,C) = A + BC + A\bar{B}C$

→ SOP ; POS

Cannonical form
↓
All the terms contains each variables (literals)

e.g → $F(A,B,C) = \bar{A}BC + ABC + \bar{A}B\bar{C}$

→ Cannonical SOP form (minterm SOP form) — Sum of Product
→ Cannonical POS form (maxterm POS form) — Product of Sum

~~Canonical~~ SOP Form

The SOP expression usually takes the form of two or more variables ANDed together

e.g → $Y = \bar{A}BC + A\bar{B} + AC$

$Y = \bar{B}\bar{C}D + A\bar{B}D + CD$

→ SOP form is used for construcⁿ of Truth tables and timing diagrams.

→ SOP circuits can be constructed using a special gate called "AND-OR-Inverter" gate.

→ Cannonical SOP form

1. In this form, each terms contains all the binary variables.
2. Each term of this form is called min-term
3. In minterm, we assign '1' to uncomplemented variable, and '0' to each complemented variable *
4. n- binary variables have 2^n possible combination and each of these combination is called minterm
5. Minterms are represented by Σm (------)
6. This form are used to write the logical expression only for those outputs which are "1"

in Truth table

No. of Possible combination of n-variables is 2^n

e.g → $Y(A, B, C)$

No. of Possible combination = $2^3 = 8$

A	B	C	Minterm
0	0	0	$\bar{A}\bar{B}\bar{C} \rightarrow m_0$
0	0	1	$\bar{A}\bar{B}C \rightarrow m_1$
0	1	0	$\bar{A}B\bar{C} \rightarrow m_2$
0	1	1	$\bar{A}BC \rightarrow m_3$
1	0	0	$A\bar{B}\bar{C} \rightarrow m_4$
1	0	1	$A\bar{B}C \rightarrow m_5$
1	1	0	$AB\bar{C} \rightarrow m_6$
1	1	1	$ABC \rightarrow m_7$

Q^n Find the minterm corresponding to decimal no. 5 and 13

Sol^n $5 \rightarrow 101 \rightarrow A.\bar{B}C$

$13 \rightarrow 1101 \rightarrow AB\bar{C}D$

Q^n obtain the Cannonical SOP form for the given Boolean func^n.

$Y(A, B, C) = A + BC$

Find the total no. of minterms.

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Solⁿ $Y(A, B, C) = A + BC$

$= A.1.1 + 1.BC$

$= A(B+\bar{B})(C+\bar{C}) + (A+\bar{A})BC$

$= A(BC + B\bar{C} + \bar{B}C + \bar{B}\bar{C}) + ABC + \bar{A}BC$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC$

$= AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC$

110 101 100 111 011

∴ Total no. of minterms = 5 Ans

$Y(A, B, C) = \sum m(6, 5, 4, 7, 3)$

$Y(A, B, C) = \sum m(3, 4, 5, 6, 7)$

Qⁿ Find the no. of min-terms in Cannonical SOP form for the given Boolean funcⁿ :-

$Y(A, B, C) = A + \bar{B}C$

$Y = A(\bar{B}+B)(\bar{C}+C) + (\bar{A}+A)\bar{B}C$

$Y = A(\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC) + (\bar{A}+A)\bar{B}C$

$Y = A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}\bar{B}C + A\bar{B}C$

$Y = A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}\bar{B}C$

$Y = 100 + 101 \quad 110 \quad 111 \quad 001$

$Y = \sum m(4, 5, 6, 7, 1)$

$A + A = A$

$A \cdot A = A$

Q^n Find the minimized SOP expression for the given Truth Table.

A	B.	Y
0	0	1 ✓
0	1	0
1	0	1 ✓
1	1	0

Sol^n

$$Y(A, B) = \bar{A}\,\bar{B} + A\bar{B}$$

$$= \bar{B}\,(\bar{A} + A)$$

$$= \bar{B}$$

Q^n Find the minimized SOP expression for the given truth table of 3-Variable Boolean func^n

Inputs			output
A	B	C	Y
0	0	0	
0	1	0	
0	1	0	
0	1	1	
1	0	0	
1	1	0	
1	1	1	

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I/P			O/P
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

3, 5, 6, 7

$$Y = \bar{A}BC + A\bar{B}C + AB(\bar{C} + C)$$

$$Y = \bar{A}BC + A\bar{B}C + AB$$

$$\cancel{Y = (\bar{A}B + A\bar{B})C + AB}$$

$$= \bar{A}BC + A[B + \bar{B}C]$$

$$= \bar{A}BC + A[(B + \bar{B})(B + C)]$$

$$= \bar{A}BC + A(B + C)$$

$$= \bar{A}BC + AB + AC$$

$$= B[A + \bar{A}C] + AC$$

$$= B[(A + \bar{A})(A + C)] + AC$$

$$= B(A + C) + AC$$

$$= AB + BC + AC$$

Product of Sum (POS) form

The POS expression usually takes the form of two or more ORed variables within parenthesis ANDed with two or more other variables within parenthesis.

e.g. $Y(A, B, C, D, E) = (A+\bar{C}).(\bar{B}+E).(B+C)$

→ Each individual term in Canonical POS form is called Maxterm

→ POS forms are used to write logical expression for o/p becoming logic '0'.

→ In maxterm, we assign '0' to uncomplemented variable and '1' to complemented variables.

→ Maxterms are represented by $\pi M(----)$

Q^n Find the max term corresponding to decimal no. 5 and 13

Sol^n $5 \longrightarrow 1\ 0\ 1 \longrightarrow (\bar{A}+B+\bar{C})$

$13 \rightarrow 1101 \longrightarrow (\bar{A}+\bar{B}+C+\bar{D})$

Principle of duality

Q^n obtain the Cannonical POS form for the given expression :-

$$Y(A,B,C) = (A+\bar{B})(B+C)(A+\bar{C})$$

Sol^n $(A+\bar{B}+0)\ (0+B+C)\ (A+0+\bar{C})$

$= (A+\bar{B}+C.\bar{C})\ (A.\bar{A}+B+C)\ (A+B.\bar{B}+\bar{C}$

$= (A+\bar{B}+C.\bar{C})\ (B+C+A.\bar{A})\ (A+C+B.\bar{B})$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})\ (B+C+A)\ (B+C+\bar{A})$

$(A+C+B)\ (A+C+\bar{B})$

$= (A+\bar{B}+C)\ (A+\bar{B}+\bar{C})\ (A+B+C)\ (\bar{A}+B+C) +$
$(A+B+C)\ (A+\bar{B}+C)$

$= (A+\bar{B}+C)\ (A+\bar{B}+\bar{C})\ (A+B+C)\ (\bar{A}+B+C)$

010 011 (000) (100)

No. of Max Terms = 4

$$Y(A,B,C) = \Pi M(2,6,0,4)$$

Total no. of Terms = Min Terms + Max terms

$$= \Sigma m(1,3,5,7)$$

Q^n Express the Boolean func^n

$$Y(A,B,C) = A + \bar{B}C$$

in Cannonical POS form

$$Y(A,B,C) = (A + 0 + 0) + (0 + \bar{B}C)$$

$$Y(A,B,C) = (A + B\cdot\bar{B} + C\cdot\bar{C}) + (A\cdot\bar{A} + \bar{B}C)$$

$$Y(A,B,C) = ((A+B)(A+\bar{B})((A+C)(A+\bar{C}))$$

$$Y(A,B,C) = A + \bar{B}C$$

$$Y = (A+\bar{B})(A+C)$$

$$Y = (A + \bar{B} + 0)(A + 0 + C)$$

$$Y = (A+\bar{B} + C\cdot\bar{C})(A + B\cdot\bar{B} + C)$$

$$Y = (A + \bar{B} + C)(A + \bar{B} + \bar{C})(A + B + C)(A + \bar{B} + C)$$

$\underline{Y(A,B,C) = \Pi M}$

$$Y = (A + \bar{B} + C)(A + \bar{B} + \bar{C})(A + B + C)$$

$$Y = \pi M(\underset{010}{2}, \underset{011}{3}, \underset{000}{0})$$

$(A + \bar{B} + C) \longrightarrow \bar{A}B\bar{C}$

$(A + \bar{B} + \bar{C}) \longrightarrow (\bar{A}BC)$

$(A + B + C) \longrightarrow (\bar{A}\bar{B}\bar{C})$

Q. For the given truth table, obtained the minimi[zed] POS form expression

A	B	Y
0	0	1
0	1	0 ✓
1	0	1
1	1	0 ✓

Sol.

$$Y(A,B) = (A+\bar{B}) \cdot (\bar{A}+\bar{B})$$

$$= \cancel{A\bar{A}}^{\,0} + A\bar{B} + \bar{A}\bar{B} + \bar{B}.\bar{B}$$

$$= A\bar{B} + \bar{A}\bar{B} + \bar{B}$$

$$= \bar{B}(\bar{A}+1) + A\bar{B}$$

$$= \bar{B} + A\bar{B}$$

$$= \bar{B}(1+A)$$

$$= \bar{B}$$

Q. Find the no. of MAX Terms for the given Truth Table.

$$Y(A,B,C) = \Pi M(\ ?\ )$$

I/P			O/P
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$Y(A,B,C) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$$

$$Y(A,B,C) = \pi M(0,1,2,4)$$

K-Maps

→ It is the graphical method for simplification of Boolean func^n.

→ The func^n must be given in Cannonical form to obtain the K-Map,

→ K-Map can be constructed either for SOP or POS form.

→ If a truth table is given then only those output will be considered for which the value is 1.

→ For 'n' no. of variables, the no. of cells in the K-Map is always 2^n.

→ The value of each cell is obtained by gray code.

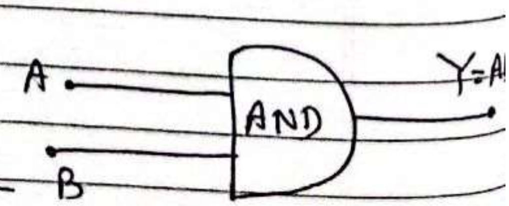
$\boxed{\text{Two-Variable K-Map}}$

Ex $Y(A, B) =$ $\bar{B}$ B

2^n

A \ B	0 ($\bar{B}$)	1 (B)
$\bar{A}$ ← 0	00 $\bar{A}\bar{B}$ 0	01 $\bar{A}B$ 1
A ← 1	10 $A\bar{B}$ 2	11 AB 3

e.g → AND Gate



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Three-Variable K-Map

$Y(A, B, C)$

No. of Cells $= 2^3 = 8$

AB \ C	0	1
00	000 (0)	001 (1)
01	010 (2)	011 (3)
11	110 (6)	111 (7)
10	100 (4)	101 (5)

e.g. → Minimize the given Boolean func^n using K-Map

$$Y(A, B, C) = A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$

$$(100 + 000 + 001 + 110)$$

AB \ C	0	1
00	1	1
01		
11	1	
10	1	

→ $\bar{A}\bar{B}$

$A\bar{C}$

$$Y = \bar{A}\bar{B} + A\bar{C}$$

Q. Simplify the Boolean Expression —

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$$

AB \ C	0	1
00	1	
01	1	1
10	1	ϕ
11	1	

$\bar{A}\bar{C} + \bar{A}B +$

AB \ C	0	1
00	1	
01	1	1
11		
10	1	

$\bar{A}\bar{C} + \bar{B}$

$\cancel{A + C + B}$

$\bar{A}B + \bar{A}\bar{C} + \bar{B}\bar{C}$

Four-Variable K-Map

$Y(A, B, C, D)$

No. of cells $= 2^4 = 16$

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AB \ CD	00	01	11	10
00	0000 (0)	0001 (1)	0011 (3)	0010 (2)
01	0100 (4)	0101 (5)	0111 (7)	0110 (6)
11	1100 (12)	1101 (13)	1111 (15)	1110 (14)
10	1000 (8)	1001 (9)	1011 (11)	1010 (10)

Q Minimize the logic func^n using K-Map

$$Y = ACD + ABD + B\bar{C}D$$

Sol^n

$$Y = A(B+\bar{B})CD + AB(C+\bar{C})D + (A+\bar{A})B\bar{C}D$$

$$= A(BCD + \bar{B}CD) + AB(CD + \bar{C}D) + AB\bar{C}D + \bar{A}B\bar{C}D$$

$$= ABCD + A\bar{B}CD + \cancel{ABCD} + AB\bar{C}D + \cancel{AB\bar{C}D} + \bar{A}B\bar{C}D$$

$$Y = \underset{\substack{(1111)\\15}}{ABCD} + \underset{\substack{(1011)\\11}}{A\bar{B}CD} + \underset{\substack{(1101)\\13}}{AB\bar{C}D} + \underset{\substack{(0101)\\5}}{\bar{A}B\bar{C}D}$$

$$Y = \sum m(5, 11, 13, 15)$$

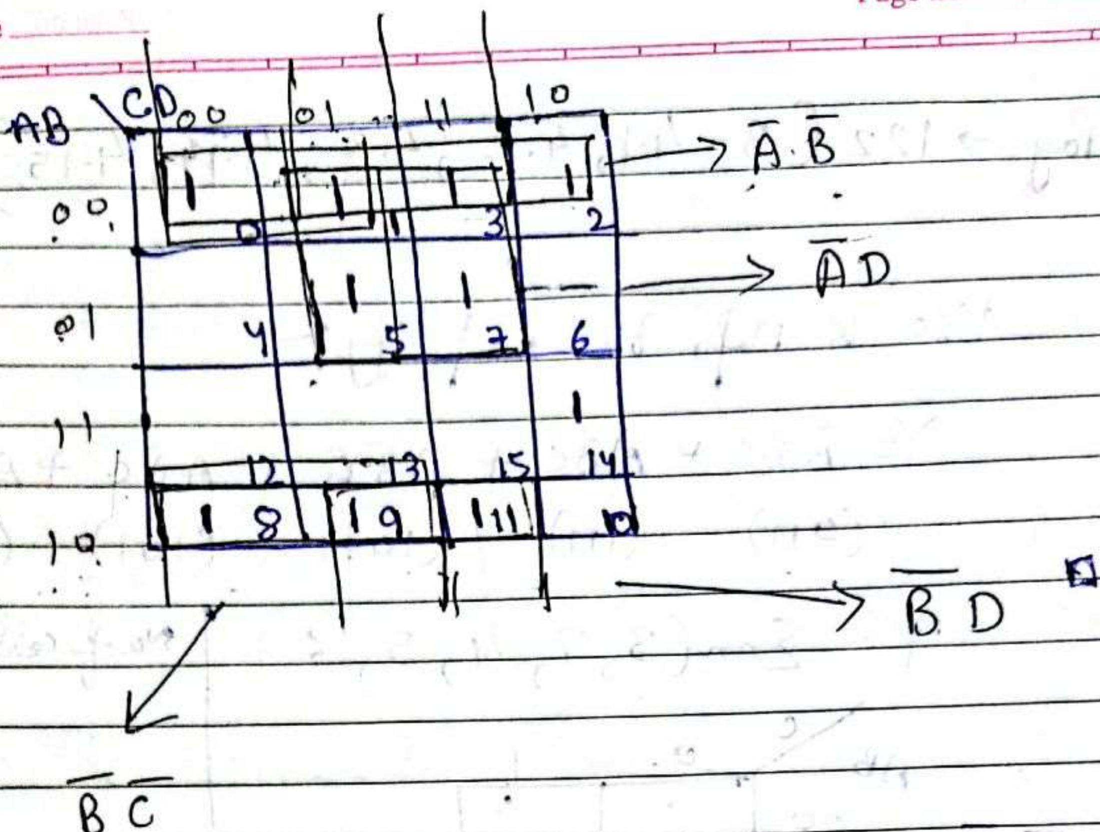
AB \ CD	00	01	11	10
00	0	1	3	2
01	4	1 5	7	6
11	12	1 13	1 15	14
10	8	9	1 11	10

$Y = B\bar{C}D + ABD + ACD$

Q) Minimize the logic func^n

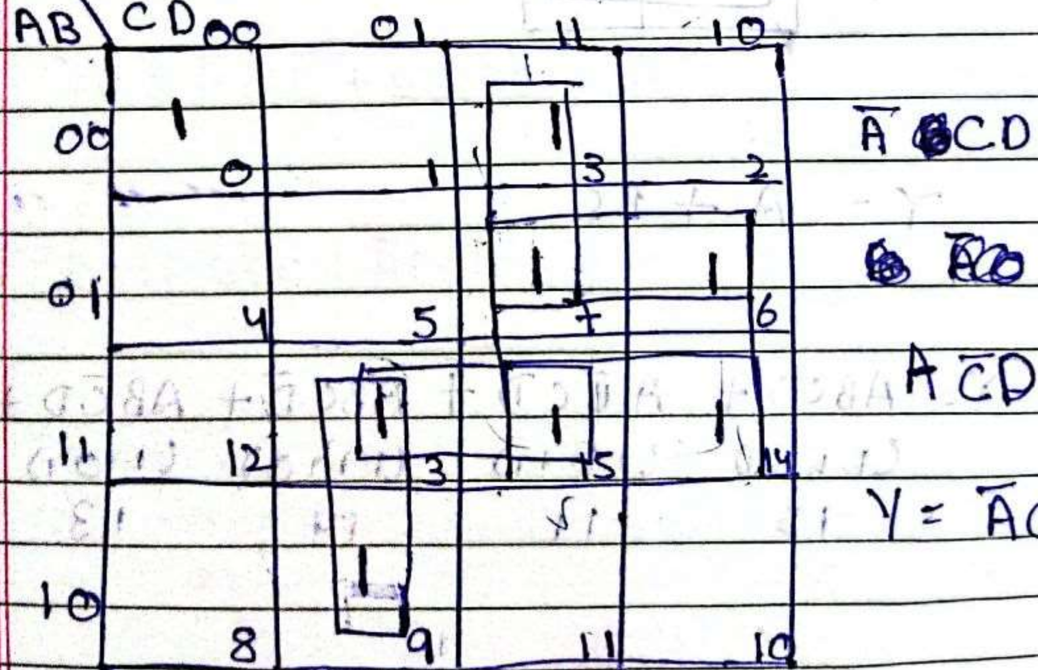
$Y(A, B, C, D) = \sum m\,(0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$

AB \ CD				
	1 0	1	1 3	1 2
	4	1 5	1 7	6
	12	13	15	1 14
	1 8	1 9	1 11	10



$$Y = \bar{A}\bar{B} + \bar{A}D + \bar{B}D + \bar{B}\bar{C}$$

Q. $f(A,B,C,D) = \sum m(0,3,6,7,9,13,14,15)$



$Y = \bar{A}CD + BC + A\bar{C}$

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Page → 122 (Q - 4.1, 4.2, 4.3, 4.14, 4.15)

4.1 Use K-Map to simplify :-

$$Y = \bar{A}BC + ABC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

(011) (111) (100) (101) (110)

Solⁿ $Y = \sum m(3, 7, 4, 5, 6)$ No. of cells $= 2^3 = 8$

AB \ C	0	1
00		
01		1
11	1	1
10	1	1

$$Y = A + BC$$

4.2 $Y = ABCD + A\bar{B}CD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$

(1,1,1,1) (1 0 1 1) (1 1 1 0) (1 1 0 1) (1 1 0 0)

15 11 14 13 12

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	1 (12)	1 (13)	1 (15)	1 (14)
10	8	9	1 (11)	10

$Y = AB + ACD$

Q. Using Karanaugh map, simplify the given expression

$$Y = \bar A \bar B \bar C \bar D + A \bar B \bar C \bar D + \bar A B C + \bar A C \bar D + \bar A \bar C \bar D$$

$$Y = \bar A \bar B \bar C \bar D + A \bar B \bar C \bar D + \bar A BC(D + \bar D) + \bar A (B + \bar B) C \bar D + \bar A (B + \bar B) \bar C \bar D$$

$$Y = \bar A \bar B \bar C \bar D + A \bar B \bar C \bar D + \bar A BCD + \bar A BC \bar D + \cancel{\bar A B C \bar D} + \bar A \bar B C \bar D + \bar A B \bar C \bar D + \bar A \bar B \bar C \bar D$$

$$Y = \bar A \bar B \bar C \bar D + A \bar B \bar C \bar D + \bar A BCD + \bar A BC \bar D + \bar A \bar B C \bar D +$$

(0000) (1000) (0111) (0110) (0010)

0 8 7 6 2

$\bar A B \bar C \bar D$

(0100)

4

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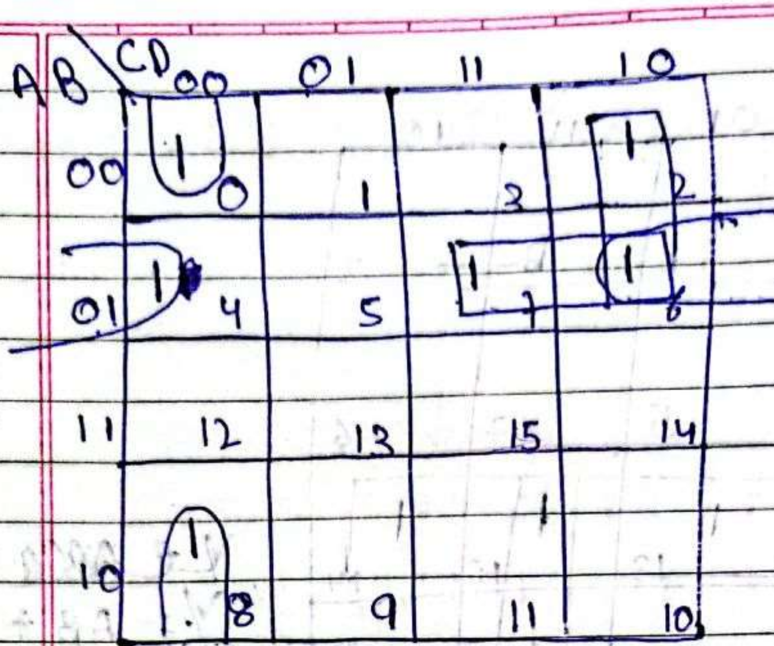


$$Y = \bar{B}\bar{C}\bar{D} + \bar{A}BC + \bar{A}C\bar{D} + \bar{A}BD$$

Q. Find the minimized SOP expression for the given truth table :-

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

Soln

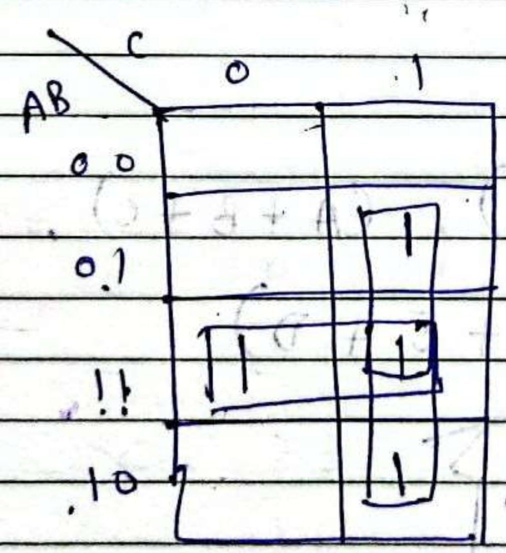
$$Y = \bar{B}$$

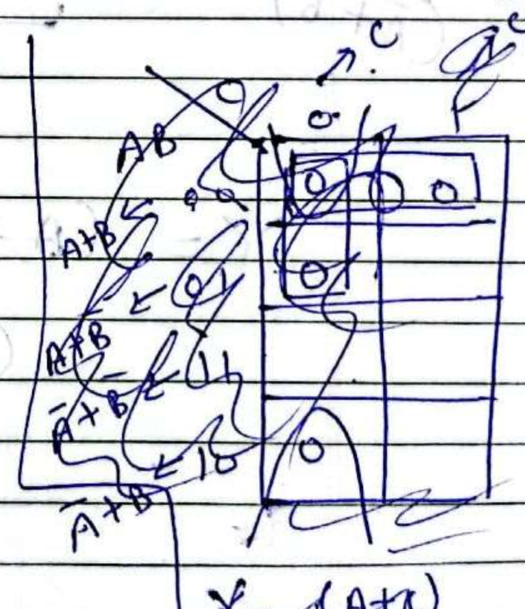
Q. Find the minimized SOP expression for the given truth table :-

Input (3 Variables) | | | Output

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Sol





$Y = (A + C)$

$$Y = BC + AB + AC$$

$$Y = (\bar{B} + \bar{C})(\bar{A} + \bar{B})(\bar{A} + \bar{C})$$

Q Simplify the Boolean func^n

$$Y = \Pi M(0, 1, 3, 5, 6, 7, 10, 14, 15)$$

$$= \cancel{\Sigma m(2, 4, 8, 9, 11, 12, 13)}$$

Sol^n

AB \ CD	00 $(C+D)$	01 $(C+\bar{D})$	11 $(\bar{C}+\bar{D})$	10 $(\bar{C}+D)$
$(A+B)$ ← 00	0 (0)	0 (1)	0 (3)	(2)
$(A+\bar{B})$ ← 01	(4)	0 (5)	0 (7)	0 (6)
$(\bar{A}+\bar{B})$ ← 11	(12)	(13)	0 (15)	0 (14)
$(\bar{A}+B)$ ← 10	(8)	(9)	(11)	0 (10)

$$Y = (A+\bar{D}) \cdot (A+B+C) \cdot (\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{C}+D)$$

⇕

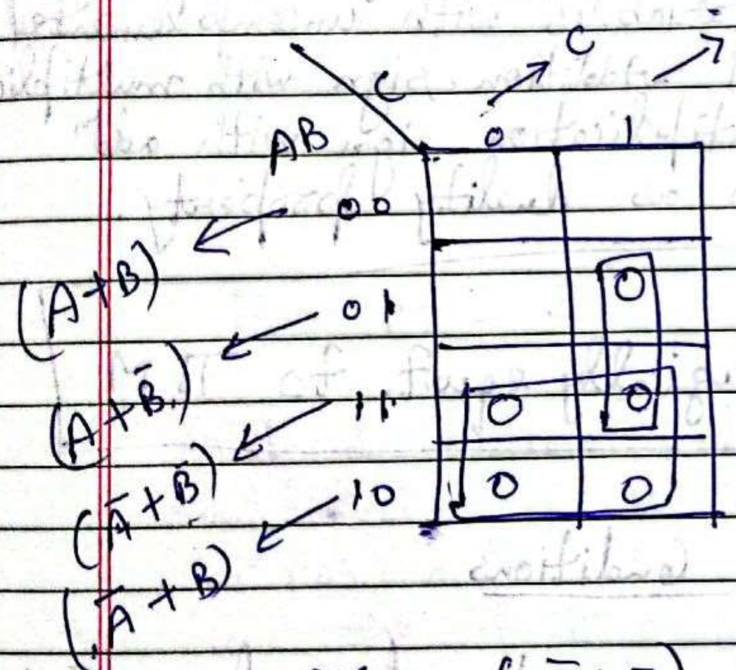
$$= \bar{A}D + \bar{A}\bar{B}\bar{C} + BC + AC\bar{D}$$

Q. Use K-Map to simplify the Boolean funcⁿ using POS form.

$$Y = \bar{A}BC + ABC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$
$$\quad 3 \qquad 7 \qquad 4 \qquad 5 \qquad 6$$

Solⁿ

$$Y = \underset{3}{(A+\bar{B}+\bar{C})}\ \underset{7}{(\bar{A}+\bar{B}+\bar{C})}\ \underset{4}{(\bar{A}+B+C)}$$
$$\underset{5}{(\bar{A}+B+\bar{C})} \cdot \underset{6}{(\bar{A}+\bar{B}+C)}$$



$$Y = (\bar{B}+\bar{C}) \cdot \bar{A}$$

$$\Downarrow$$

$$Y = BC + A$$

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Duality Theorem

→ Any Boolean funcⁿ can be represented either in SOP or POS form. If it is represented in SOP form, then each term is known as min-term and if it is represented in POS form, then each term is known as max-term.

→ The POS form can be easily obtained with the help of SOP form by replacing complemented variables with uncomplemented variables and Addition sign with multiplicati sign and multiplication sign with addⁿ. It is known as duality property.

$$\boxed{\sum m \text{ logically equit. to } \prod M}$$

Don't care Conditions

→ Some logic circuits can be designed so that there are certain i/p condⁿs for which they are no specified levels, usually these i/p condⁿs will never occur.

→ A circuit designer is free to make the o/p for any don't care condⁿ either a 0 or 1 in order to produce the simplest o/p expression.

Qn Simplify the given Boolean funcn

$$F(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + \sum d(0, 2, 5)$$

Soln

AB \ CD	00	01	11	10
00	X (0)	1 (1)	1 (3)	X (2)
01	(4)	X (5)	1 (7)	(6)
11	(12)	(13)	1 (15)	(14)
10	(8)	(9)	1 (11)	(10)

Ungrouped

$$F = CD + \bar{A}\bar{B}$$

Note : → ① The don't care Condns are only used for simplification of K-Map by increasing 0's (for POS) and 1's (for SOP). ∵ we don't Care the funcn o/p for such combinations. These combns can be plotted on a map to provide further simplificn of the funcn.

② Due to diff. groupings, the simplified expression for any given funcn may not be unique but all the expressions are logically equivalent since they have same value for every combination of Input Variable.

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Q. Simplify the following expression :-

(i) $Y = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

(ii) $Y = \Sigma m(7, 9, 10, 11, 12, 13, 14, 15)$

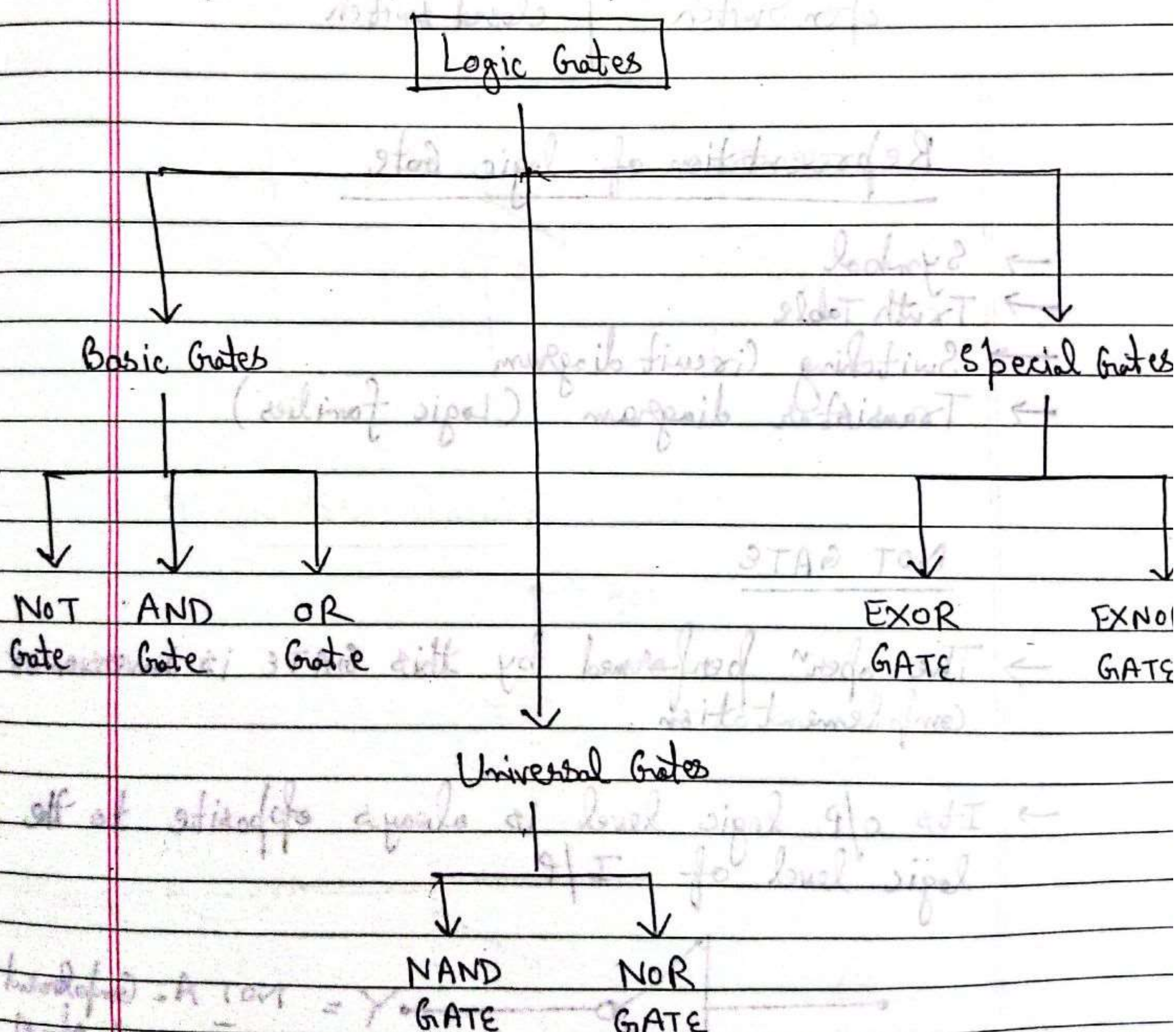
(iii) $Y = ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C + AB$

(iv) $Y = \Sigma m(0, 2, 3, 6, 7) + \Sigma d(8, 10, 11, 15)$

Logic Gates

→ They are the most fundamental digital circuits that can be constructed from diodes, Transistors and Resistors.

→ It is simply a device that has two or more inputs and one output.



Boolean '0' and '1' represents the logic level.

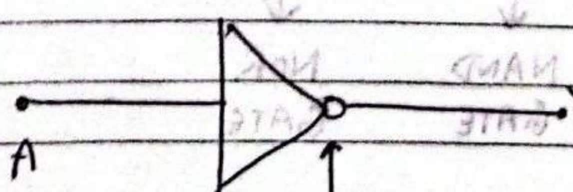
Logic '0'	Logic '1'
FALSE	TRUE
OFF	ON
LOW	HIGH
No	YES
open Switch	closed switch

Representation of logic Gate

- Symbol
- Truth Table
- Switching Circuit diagram
- Transistor diagram (Logic families)

NOT GATE

- The oper^n performed by this GATE is inversion or complementation.
- Its o/p logic level is always opposite to the logic level of I/P



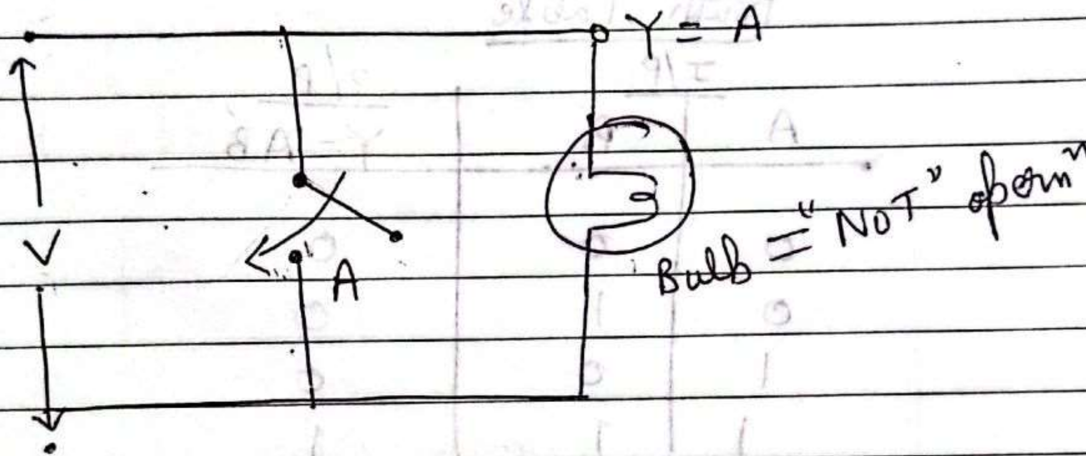
Y = NOT A = $\bar{A}$ = A' — Complement of A

Presence of this small bubble always denotes inversion

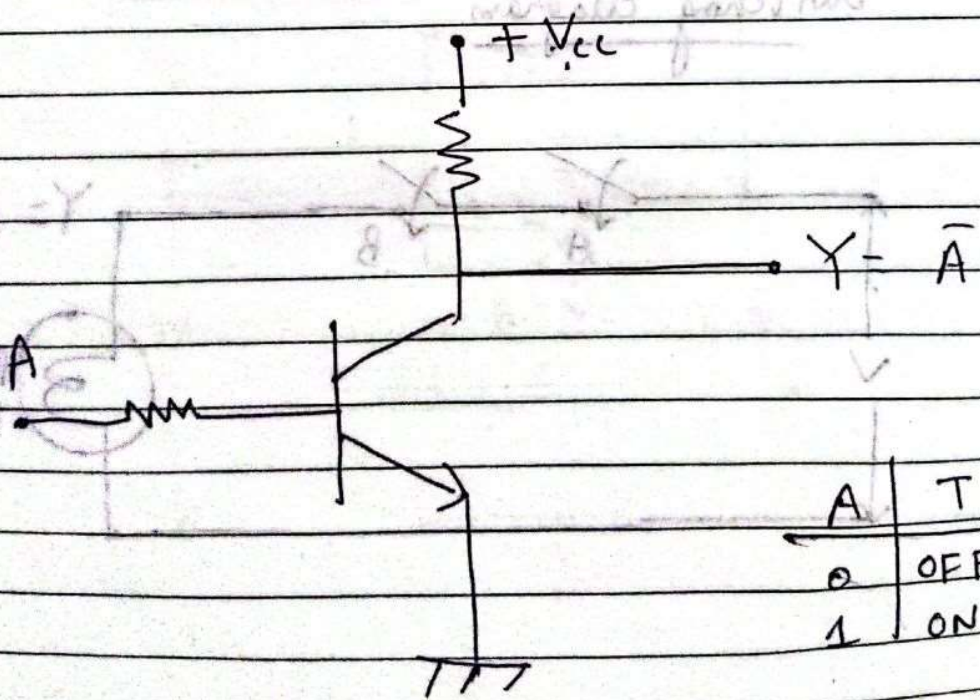
Truth Table

I/P	O/P
0	1
1	0

Switching Circuit diagram (NOT Gate)



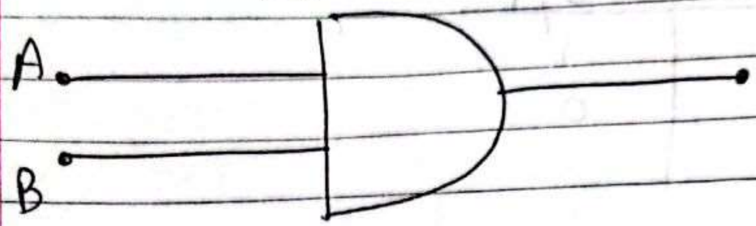
Transistor Circuit



A	T	Y
0	OFF	$V_{cc} \approx 1$
1	ON	Ground = 0

AND Gate

Symbol :-

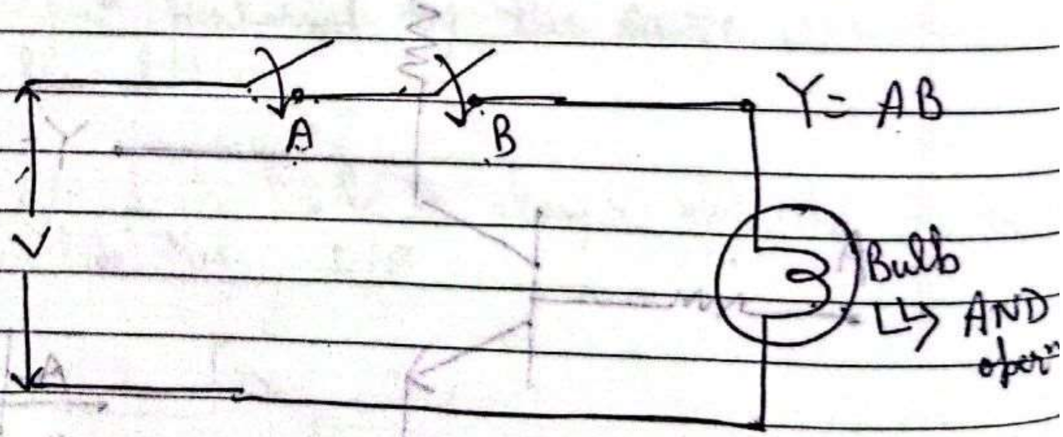


$$Y = A \text{ and } B = A.B = AB$$

Truth Table

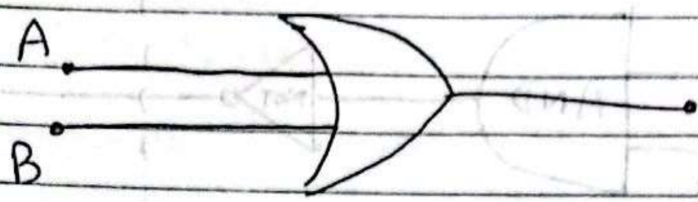
I/P		O/P
A	B	$Y = AB$
0	0	0
0	1	0
1	0	0
1	1	1

Switching diagram



OR Gate

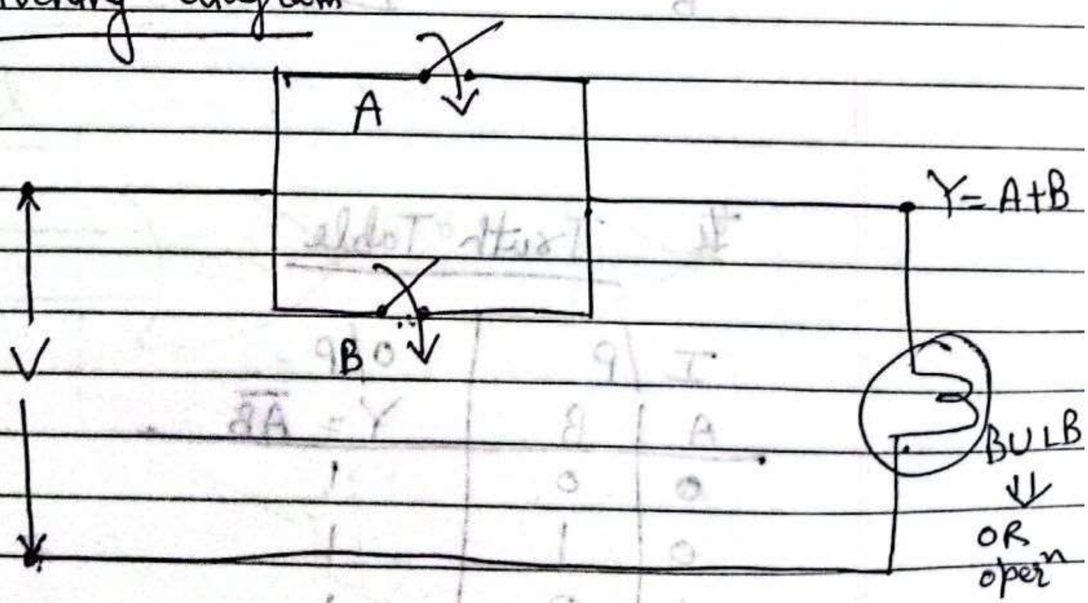
Symbol :-



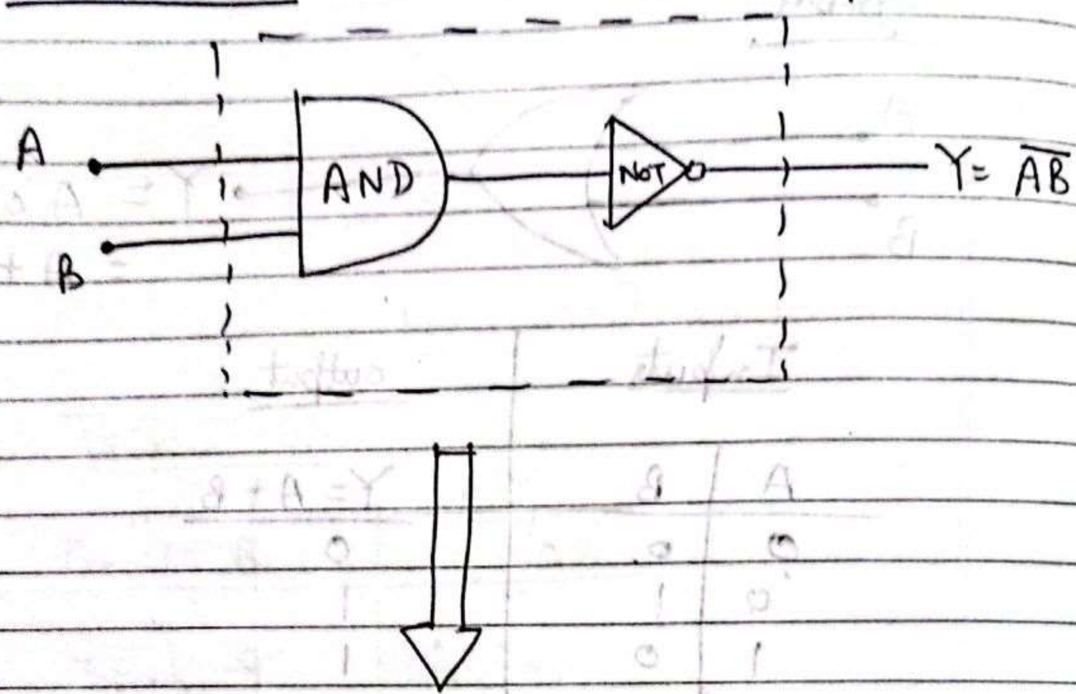
$Y = A \text{ OR } B$
$= A + B$

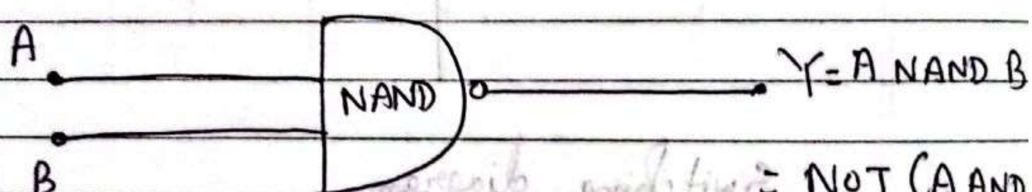
Inputs		output
A	B	$Y = A+B$
0	0	0
0	1	1
1	0	1
1	1	1

Switching diagram



'NAND' GATE ("AND – NOT" operⁿ)





= NOT (A AND B)

$= (\overline{A \cdot B})$

$= \overline{AB}$

Truth Table

I/P		O/P
A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

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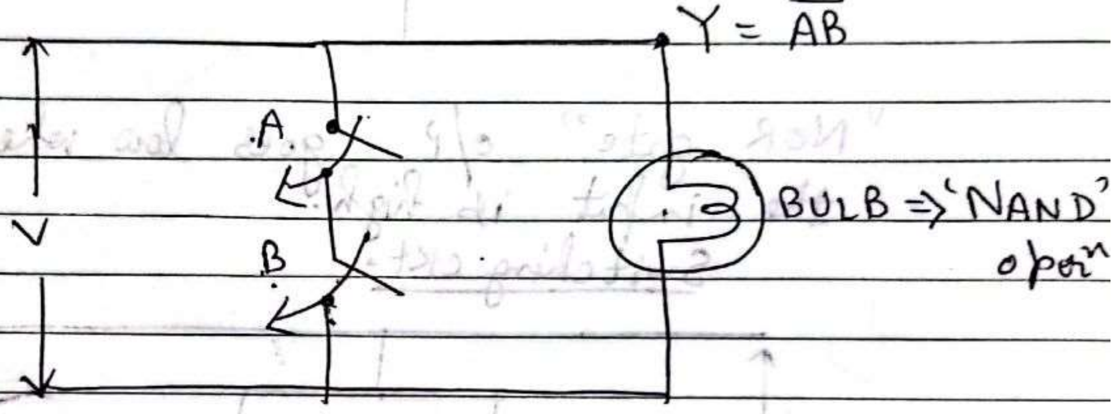
→ "NAND gate" o/p goes low, when all of its inputs are high.

→ "NAND gate" o/p is the exact inverse of the AND gate for all possible i/p Condⁿs.

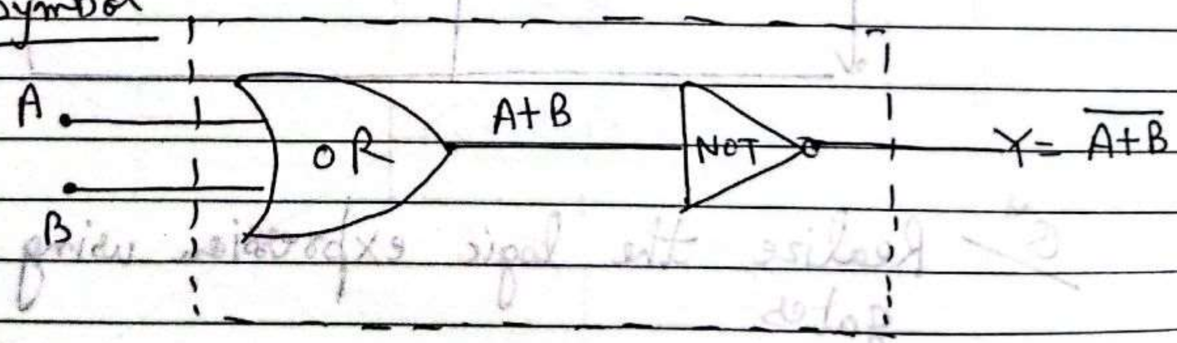
→ A single i/p 'NAND gate' is equvt. to NOT gate.

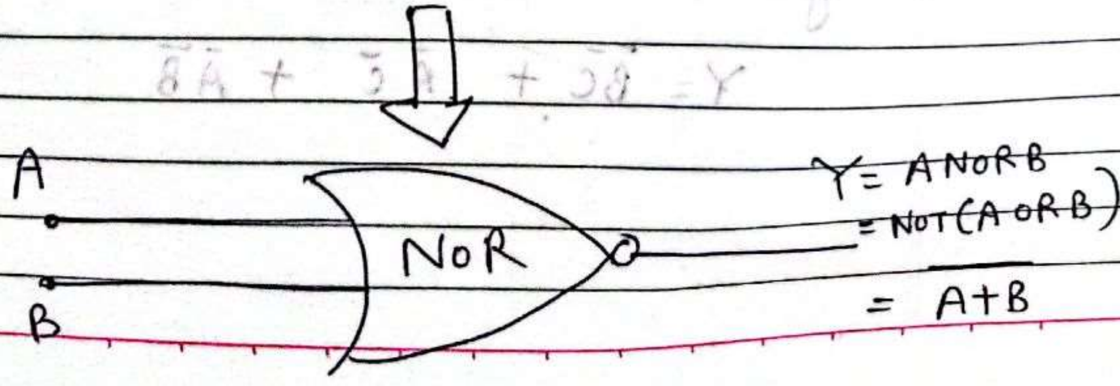
Switching Circuit diagram



'NOR' GATE

Symbol



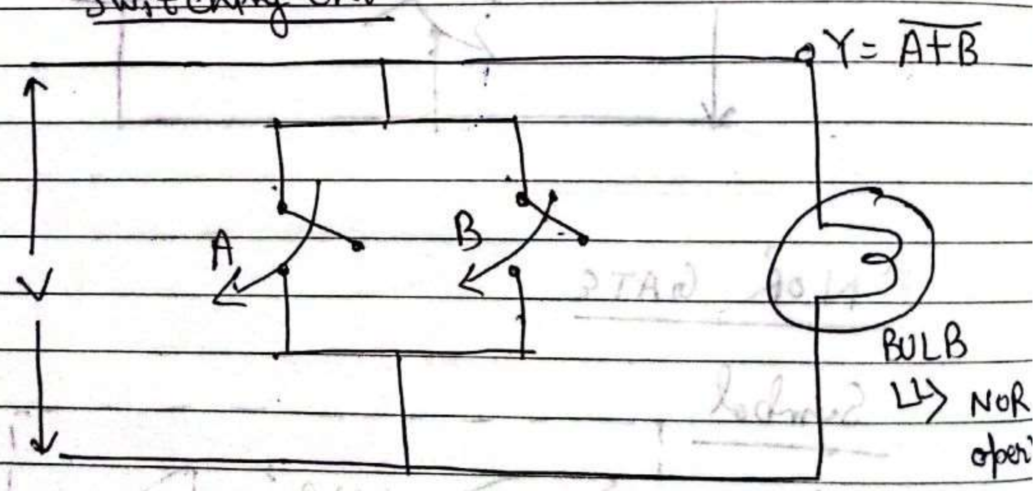


Truth Table

I/P		O/P
A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

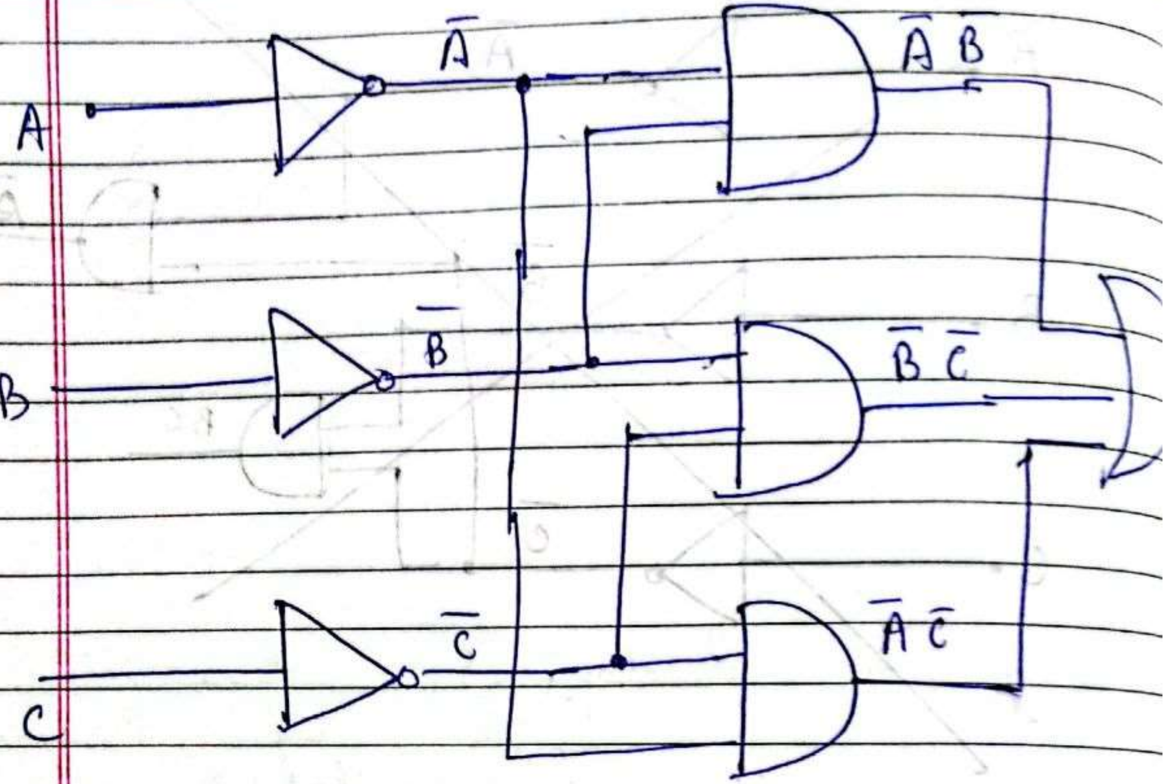
"NOR gate" o/p goes low when any of the input is high.

Switching ckt:



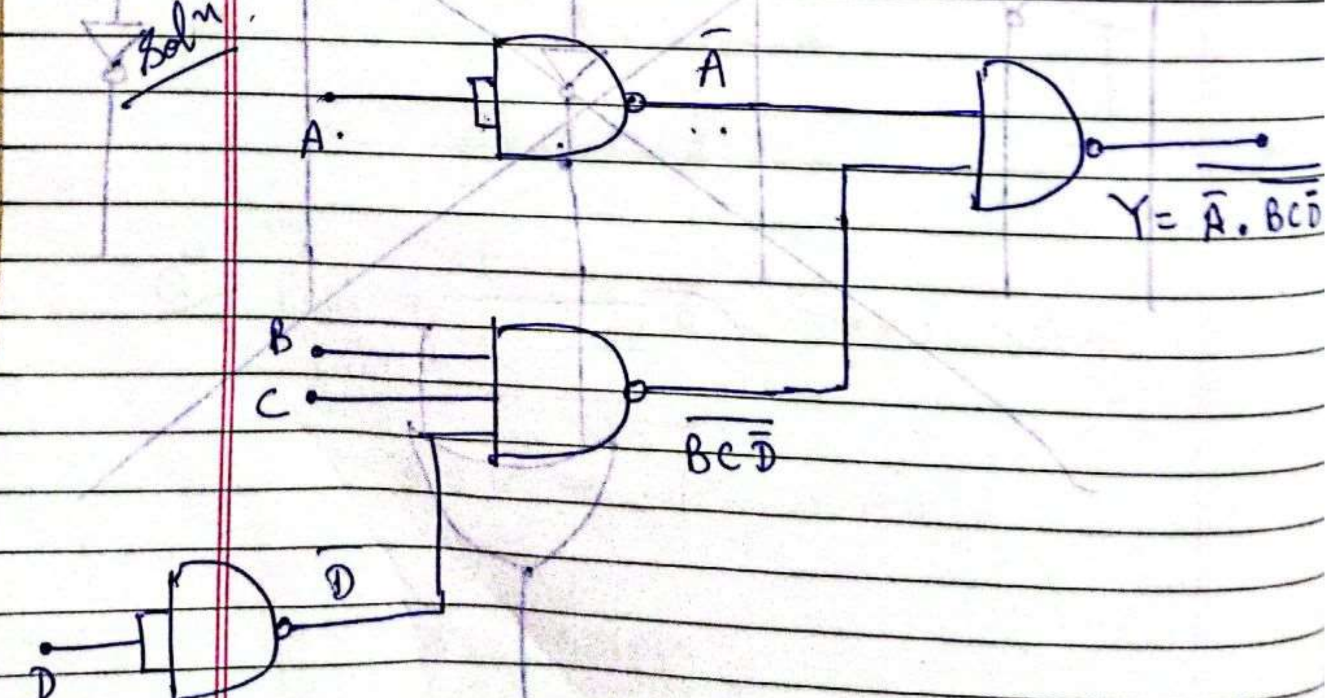
Q. Realize the logic expression using basic gates

$$Y = \bar{B}\bar{C} + \bar{A}\bar{C} + \bar{A}\bar{B}$$



Q^n Realise $Y = A + BC\bar{D}$ using NAND gates only.

Sol^n.



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$$Y = \overline{\bar{A} \cdot \overline{BC\bar{D}}}$$

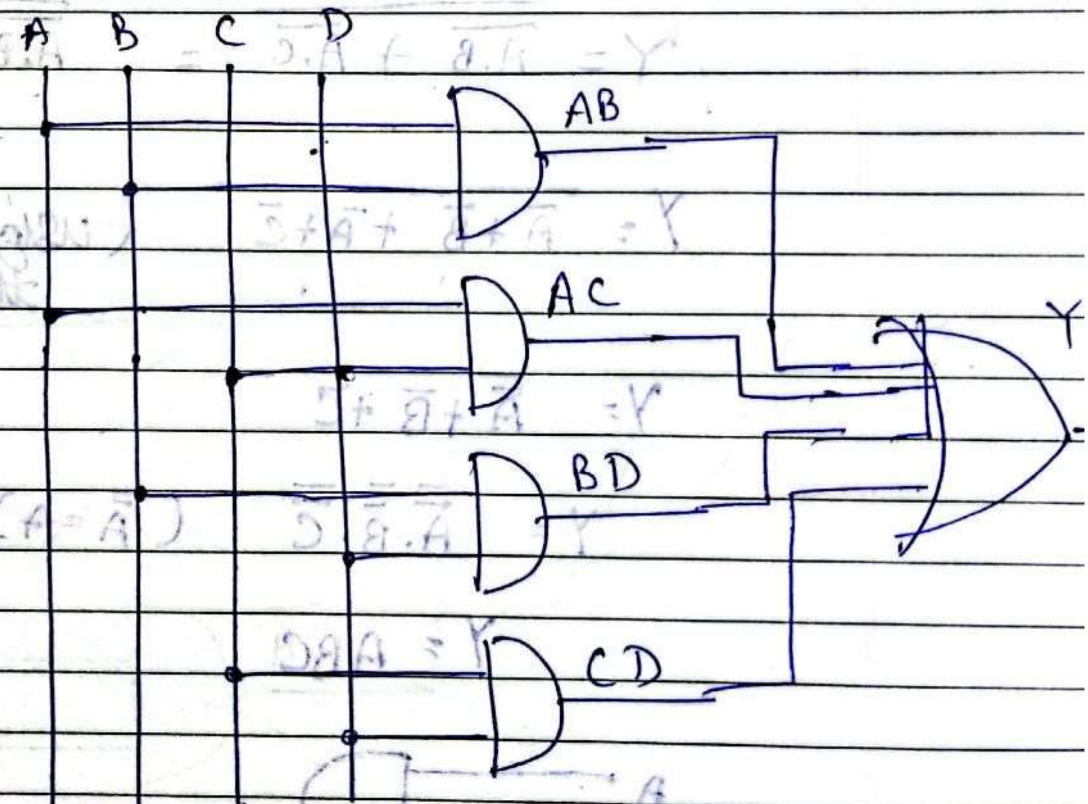
$$= \bar{\bar{A}} + \overline{\overline{BC\bar{D}}}$$

$$Y = A + BC\bar{D}$$

$Y = \overline{\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}}$

Q^n Draw a logic circuit for the Boolean eq^n $Y = AB + AC + BD + CD$

Sol^n



Q^n Minimize the given expression and draw the logic circuit using basic gates.

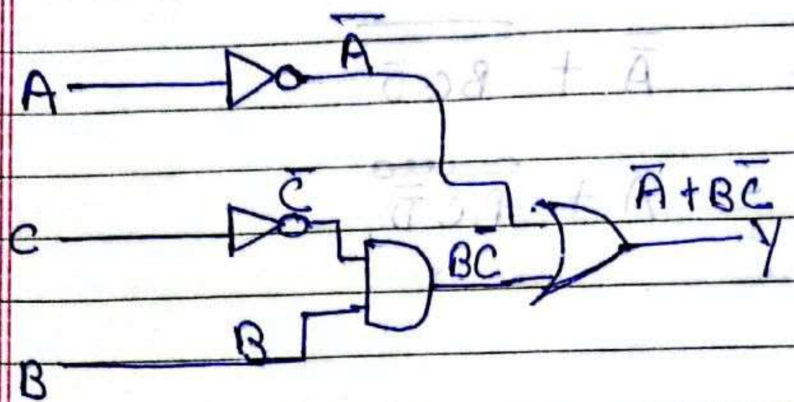
$$Y = B \cdot (\bar{A} + \bar{C}) + \bar{A} \cdot \bar{B}$$

$$Y = \bar{A}B + B\bar{C} + \bar{A} \cdot \bar{B}$$

$$Y = \bar{A}(B + \bar{B}) + B\bar{C}$$

$(A + \bar{A} = 1)$

$Y = \bar{A} + B\bar{C}$



Q. Draw a circuit to realize the func^n —

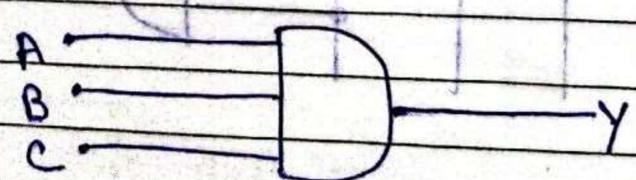
$$Y = \overline{\overline{A.B} + \overline{A.C}} = \overline{\overline{A.B}} \cdot \overline{A.C}$$

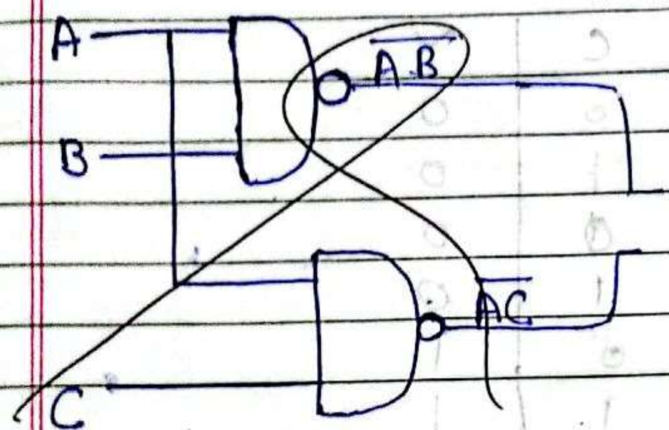
$$Y = \overline{\bar{A} + \bar{B} + \bar{A} + \bar{C}}$$ (using Demorgans theorem)

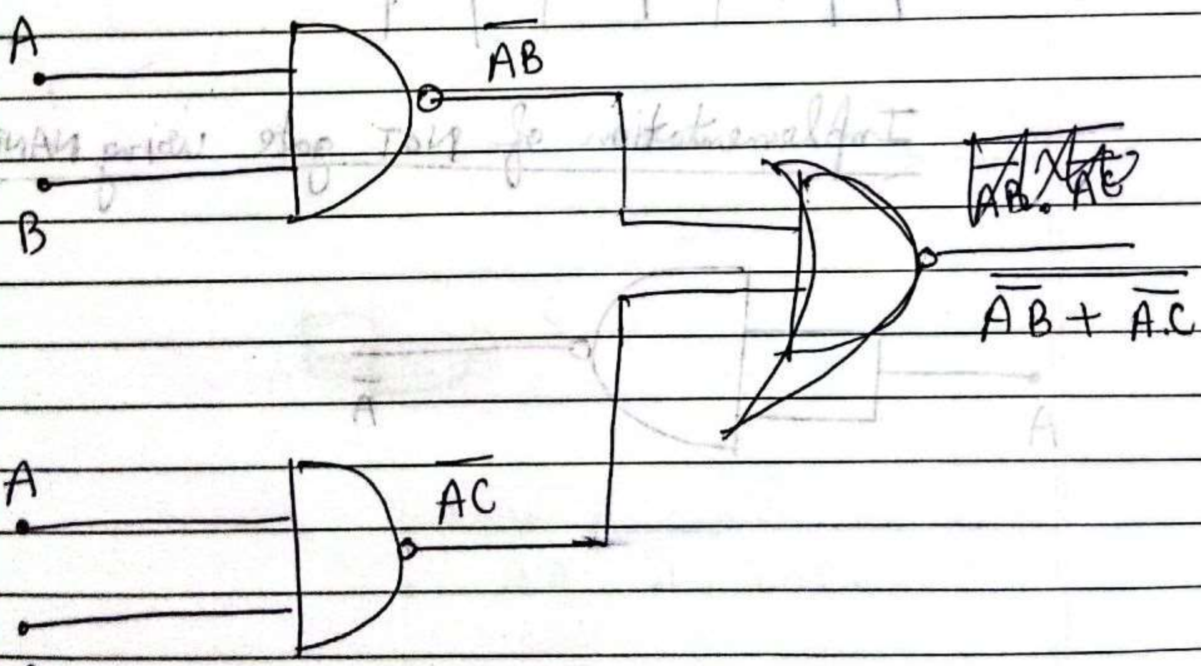
$$Y = \overline{\bar{A} + \bar{B} + \bar{C}}$$

$$Y = \bar{\bar{A}} . \bar{\bar{B}} . \bar{\bar{C}}$$ ($\bar{\bar{A}} = A$)

$$Y = ABC$$







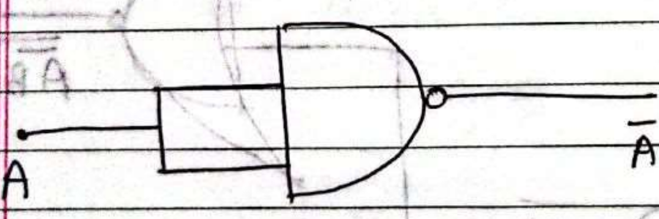
$$\overline{AB\cdot\overline{AC}} = \overline{AB} + \overline{AC}$$

Q. ~~Reduce~~ Draw T.T for the following :—

$$Y = A(B+C) + A$$
$$= A.B + A.C + A$$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Implementation of NOT gate using NAND gate



$$A = B = A$$

$$\overline{A.B} = \overline{A.A}$$
$$= \overline{A}$$

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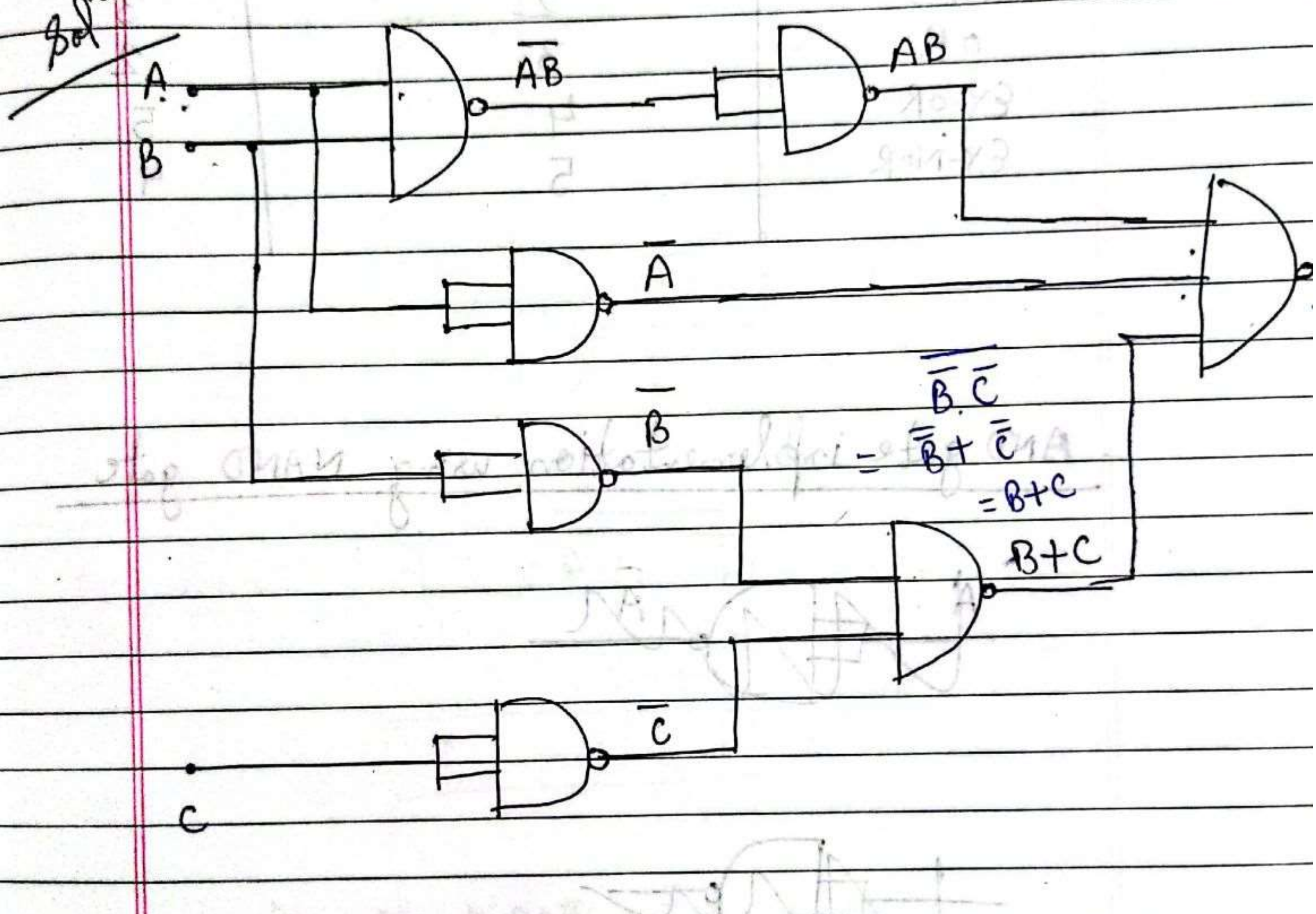
Q. $Y = \overline{AB} + A + (B+C)$

Implement the above Boolean func^n using NAND gates only.

Sol^n



$$Y = \overline{AB \cdot \bar{A} \cdot (B+C)}$$

$$= \overline{AB} + \overline{\bar{A}\,(B+C)}$$

$$= \overline{AB} + \bar{\bar{A}} + \overline{B+C}$$

$$= \overline{AB} + A + (\overline{B+C})$$

* Single i/p NAND or NOR gate cannot be considered as two i/p NAND or NOR gate.

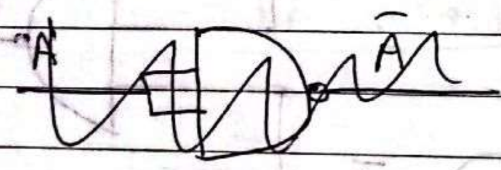
* For inverter, we connect both the inputs.

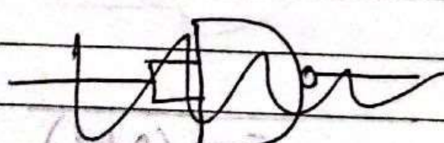
NAND and NOR GATE AS Universal GATE

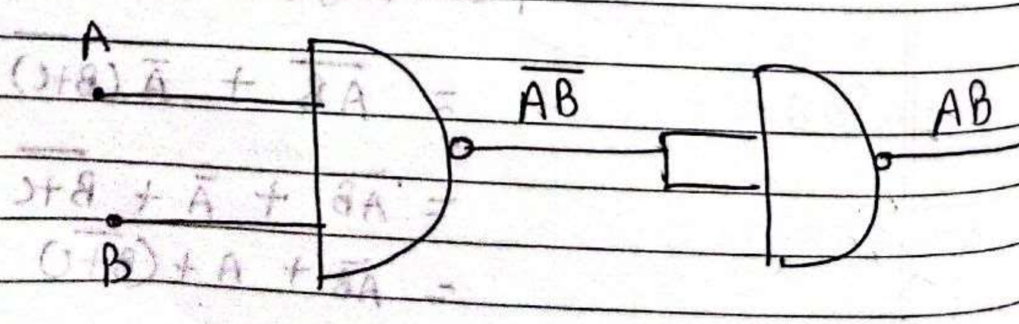
(Min^m No. of two-input NAND and NOR gates reqd.)

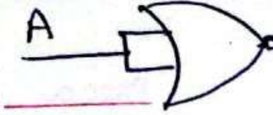
Logic Gates	No. of NAND gates reqd.	No. of NOR gates
NOT	1	1
AND	2	3
OR	3	2
EX-OR	4	5
EX-NOR	5	4

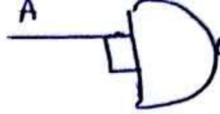
AND gate implementation using NAND gate







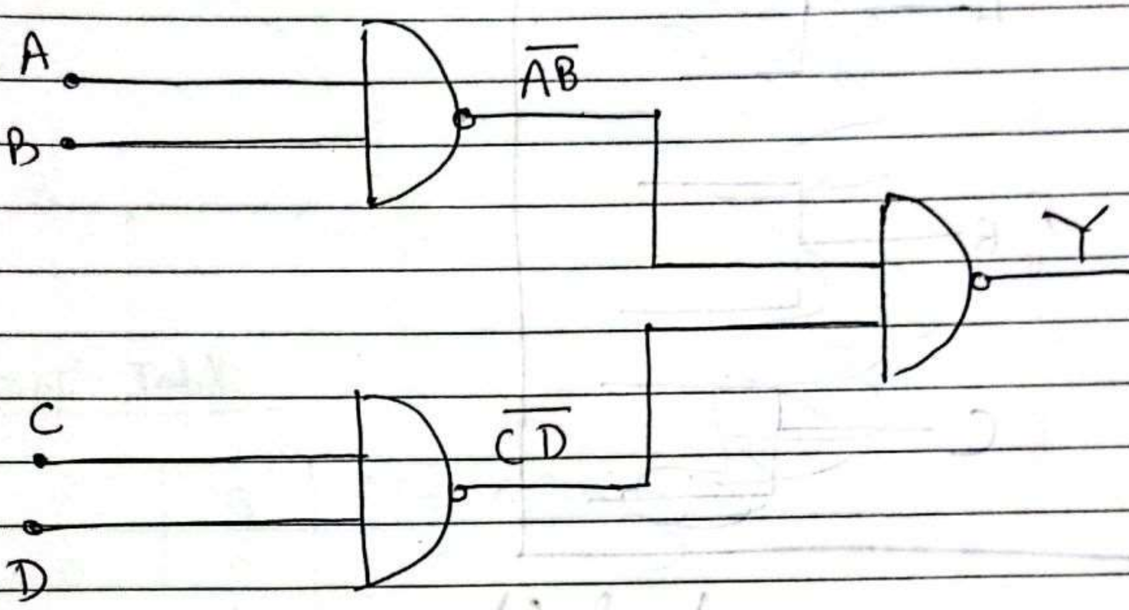
* 



Qn Find the minm no. of two input NAND gates reqd. to be realized for implementing the boolean funcn, Y = AB + CD.

reqd.

Soln

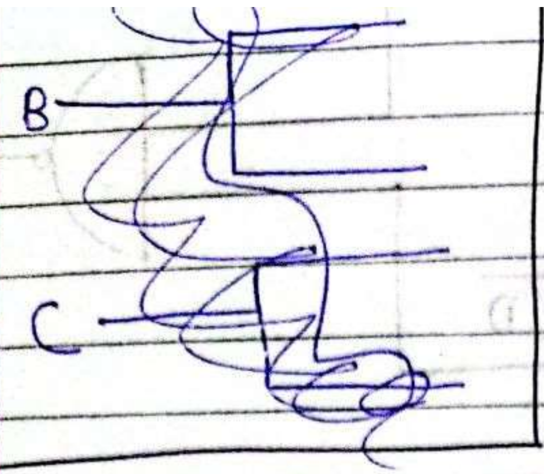


$$Y = \overline{\overline{AB} \cdot \overline{CD}}$$

$$= \overline{\overline{AB}} + \overline{\overline{CD}}$$

$$\boxed{Y = AB + CD}$$

∴ No. of NAND gates reqd. = 3



Special Gates

Special Gates are those Gates which are used for special purpose like arithm and logical operations

It is of two types:-

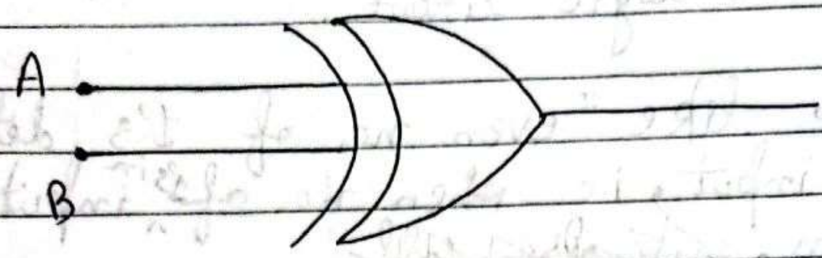
1. EX-OR Gate
2. EX-NOR Gate

EXOR Gate (Exclusive OR)

→ EXOR operⁿ is not a basic operⁿ and can be performed using the basic gates (AND, OR, NOT) or universal gates (NAND, NOR)

Imp → It acts like an "odd no. of 1's detector" in the i/p.

→ It is also called "Stair Case switch".
→ It is mostly used in "parity generation and detection"



$Y = A \text{ EXoR } B$
$= A \oplus B$

Truth Table

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$A = B,\ Y = 0$
$A \neq B,\ Y = 1$

K-Map

A \ B	0	1
0		1
1	1	

$Y = \bar{A}B + A\bar{B}$

$\therefore\ \boxed{Y = A \oplus B = \bar{A}B + A\bar{B}}$

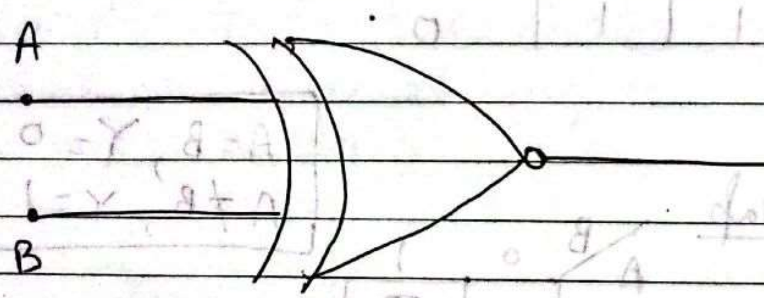
'EXNOR gate'

→ It is also called "Equivalence gate" or "Coincidence logic circuit".

→ It acts like "even no. of 1's detector" in the input, i.e when no. of 1's in input variable are even.

→ EXNOR operⁿ is equal to the EXOR operⁿ followed by NOT gate.

Logic circuit

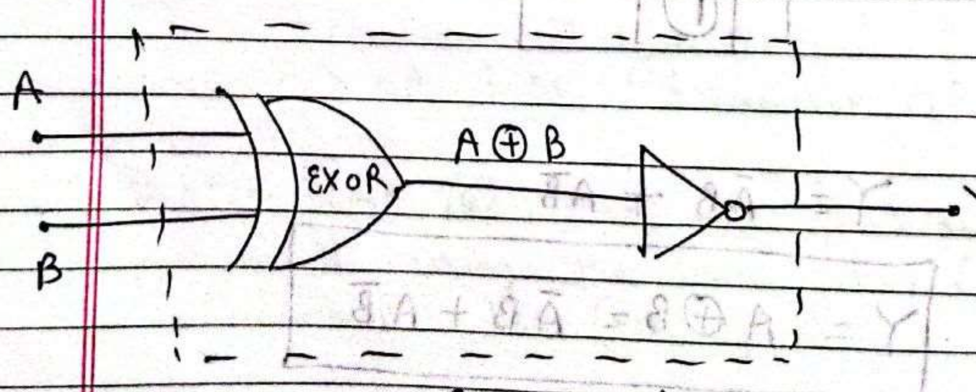


$Y = A \text{ XNOR } B$

$= A\ \overline{\text{XOR}}\ B$

$= \overline{A \oplus B}$

$= A \odot B$



$Y = \overline{A \oplus B}$

$= A \odot B$

EXNOR operⁿ

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Truth Table

I/Ps		O/Ps
A	B	$Y = A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1

K-Map

A \ B	0	1
0	1	
1		1

$$Y = \bar{A}.\bar{B} + AB$$

$$\boxed{A = B \Rightarrow Y = 1}$$

$$\boxed{A \neq B \Rightarrow Y = 0}$$

NOTE :→ The output of "EXNOR" gate is exactly inverse of "EXOR" gate for all input cond^ns.

Combinational Circuits

→ Combinational circuits are those circuits which depends only on present values of inputs.

→ They are memoryless circuits.

→ It consists of logic Gates whose output at any time are determined directly from the present combination of inputs without regard to previous inputs and previous outputs.

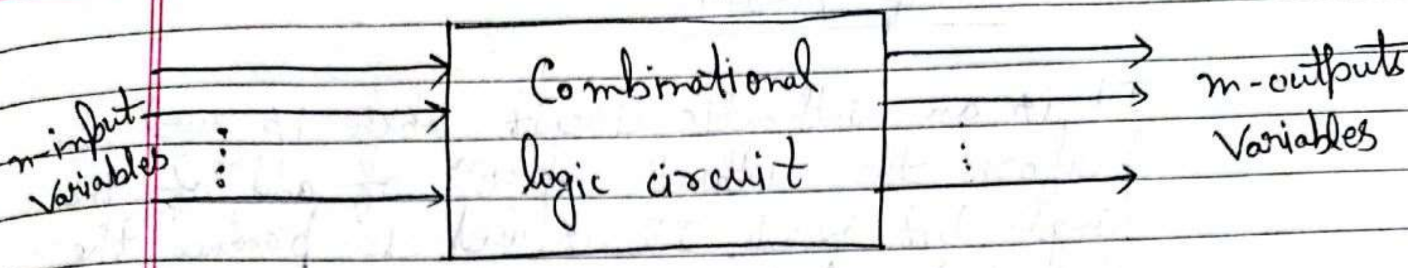

Fig.- Block diagram of combinational logic circuit

Design Procedure

The procedure involves the following steps:-

1. The problem is stated.
2. The no. of available input variables and reqd. output variables is determined
3. The input and output variables are assigned letter symbols.
4. The truth-table is used to define the reqd. relationship between Inputs and outputs.
5. The simplified Boolean func^{n} for each output is obtained.
6. The logic diagram is drawn.

Types of Combinational Circuits

1. Adders (Half adder, Full Adder, Parallel Adder, CLA)
2. Subtractor (Half-subtractor, Full Subtractor)
3. Multiplexer
4. DE-Multiplexer
5. DECODER
6. ENCODER
7. Comparator
8. 7-segment display