CHAPTER 22

Hybrid Parameters

Learning Objectives

- Understand the need of developing hybrid parameters.
- Develop mathematical expressions for the four *h* parameters.
- Explore the *h* parameters of a transistor.
- Learn the approximate hybrid formulas for a transistor amplifier.
- Develop an understanding of experimental determination of transistor *h* parameters.

Introduction

In order to predict the behaviour of a small-signal transistor amplifier, it is important to know its operating characteristics *e.g.*, input impedance, output impedance, voltage gain *etc.* In the text so far, these characteristics were determined by using *β and circuit resistance values. This method of analysis has two principal advantages. Firstly, the values of circuit components are readily available and secondly the procedure followed is easily understood. However, the major drawback of this method is that accurate results cannot be obtained. It is because the input and output circuits of a transistor amplifier are not completely independent. For example, output current is affected by the value of load resistance rather than being constant at the value βI_b. Similarly, output voltage has an effect on the input circuit so that changes in the output cause changes in the input.

One of the methods that takes into account all the effects in a transistor amplifier is the hybrid parameter approach. In this method, four parameters (one measured in ohm, one in siemens, two dimensionless) of a transistor are measured experimentally. These are called hybrid or *h* parameters of the transistor. Once these parameters for a transistor are known, formulas can be developed for input impedance, voltage gain *etc.* in terms of *h* parameters. There are two main reasons for using *h* parameter method in describing the characteristics of a transistor. Firstly, it yields exact results because the inter-effects of input and output circuits are taken into account. Secondly, these parameters can be measured very easily. To begin with, we shall apply *h* parameter approach to general circuits and then extend it to transistor amplifiers.

* Since transistor is generally connected in *CE* arrangement, current amplification factor β is mentioned here.

22.1 Hybrid Parameters of a Linear Circuit

*Every *linear circuit having input and output terminals can be analysed by four parameters (one measured in ohm, one in siemens and two dimensionless) called* **hybrid** *or* **h Parameters.**

Hybrid means "mixed". Since these parameters have mixed dimensions, they are called hybrid parameters. Consider a linear circuit shown in Fig. 22.1. This circuit has input voltage and current labelled v_1 and i_1. This circuit also has output voltage and current labelled v_2 and i_2. Note that both input and output currents (i_1 and i_2) are assumed to flow *into* the box ; input and output voltages (v_1 and v_2) are assumed *positive* from the upper to the lower terminals. These are standard conventions and do not necessarily correspond to the actual directions and polarities. When we analyse circuits in which the voltages are of opposite polarity or where the currents flow out of the box, we simply treat these voltages and currents as negative quantities.

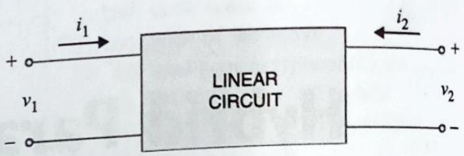


Fig. 22.1

It can be proved by advanced circuit theory that voltages and currents in Fig. 22.1 can be related by the following sets of equations :

$$v_1 = h_{11}\, i_1 + h_{12}\, v_2 \qquad ...(i)$$

$$i_2 = h_{21}\, i_1 + h_{22}\, v_2 \qquad ...(ii)$$

In these equations, the *hs* are fixed constants for a given circuit and are called *h* parameters. Once these parameters are known, we can use equations (*i*) and (*ii*) to find the voltages and currents in the circuit. If we look at eq. (*i*), it is clear that **h_{11} has the dimension of ohm and h_{12} is dimensionless. Similarly, from eq. (*ii*), h_{21} is dimensionless and h_{22} has the dimension of siemens. The following points may be noted about *h* parameters :

(*i*) The *h* parameters of a linear circuit describe the input voltage (v_1) and output current (i_2) in terms of input current (i_1) and output voltage (v_2).

(*ii*) Every linear circuit has four *h* parmeters having different units. Thus h_{11} has units of resistance (Ω), h_{12} unitless, h_{21} is unitless and h_{22} has units of conductance (siemens).

(*iii*) The *h* parameters of a given circuit are constant. If we change the circuit, *h* parameters would also change.

(*iv*) Suppose that in a particular linear circuit, voltages and currents are related as under :

$$v_1 = 10 i_1 + 6 v_2$$

$$i_2 = 4 i_1 + 3 v_2$$

Here we can say that the circuit has *h* parameters given by $h_{11} = 10\ \Omega$; $h_{12} = 6$; $h_{21} = 4$ and $h_{22} = 3\text{S}$.

* A linear circuit is one in which resistances, inductances and capacitances remain fixed when voltage across them changes.

** The two parts on the R.H.S. of eq. (*i*) must have the unit of voltage. Since current (amperes) must be multiplied by resistance (ohms) to get voltage (volts), h_{11} should have the dimension of resistance *i.e.* ohms.

22.2 Determination of *h* Parameters

The major reason for the use of *h* parameters is the relative ease with which they can be measured. The *h* parameters of a circuit shown in Fig. 22.1 can be found out as under :

(*i*) If we short-circuit the output terminals (See Fig. 22.2), we can say that output voltage $v_2 = 0$. Putting $v_2 = 0$ in equations (*i*) and (*ii*), we get,

$$v_1 = h_{11}\, i_1 + h_{12} \times 0$$

$$i_2 = h_{21}\, i_1 + h_{22} \times 0$$

$$\therefore \quad h_{11} = \frac{v_1}{i_1} \quad \text{for } v_2 = 0 \text{ i.e. output shorted}$$

$$\text{and} \quad h_{21} = \frac{i_2}{i_1} \quad \text{for } v_2 = 0 \text{ i.e. output shorted}$$

Let us now turn to the physical meaning of h_{11} and h_{21} . Since h_{11} is a ratio of voltage and current (*i.e.* v_1/i_1), it is an impedance and is called * "*input impedance with output shorted*". Similarly, h_{21} is the ratio of output and input current (*i.e.*, i_2/i_1), it will be dimensionless and is called "*current gain with output shorted*".

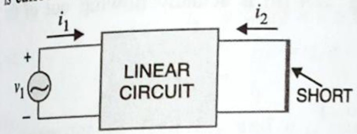


Fig. 22.2

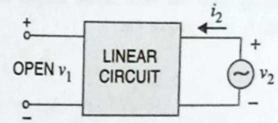


Fig. 22.3

(*ii*) The other two *h* parameters (*viz.* h_{12} and h_{22}) can be found by making $i_1 = 0$. This can be done by the arrangement shown in Fig. 22.3. Here, we drive the output terminals with voltage v_2, keeping the input terminals open. With this set up, $i_1 = 0$ and the equations become :

$$v_1 = h_{11} \times 0 + h_{12}\, v_2$$

$$i_2 = h_{21} \times 0 + h_{22}\, v_2$$

$$\therefore \quad h_{12} = \frac{v_1}{v_2} \quad \text{for } i_1 = 0 \text{ i.e. input open}$$

$$\text{and} \quad h_{22} = \frac{i_2}{v_2} \quad \text{for } i_1 = 0 \text{ i.e. input open}$$

Since h_{12} is a ratio of input and output voltages (*i.e.* v_1/v_2), it is dimensionless and is called "*reverse voltage feedback ratio with input terminals open*". Similarly, h_{22} is a ratio of output current and output voltage (*i.e.* i_2/v_2), it will be admittance and is called "*output conductance with input terminals open*".

The table below gives the meaning of each *h* parameter and the required condition for determining it.

S.No.	Parameter	Meaning	Condition
1.	h_{11}	Input resistance	Output shorted
2.	h_{12}	Reverse voltage feedback ratio	Input open
3.	h_{21}	Current gain	Output shorted
4.	h_{22}	Output conductance	Input open

* Note that v_1 is the input voltage and i_1 is the input current. Hence v_1/i_1 is given the name input impedance.

Example 22.1. *Find the h parameters of the circuit shown in Fig. 22.4 (i).*

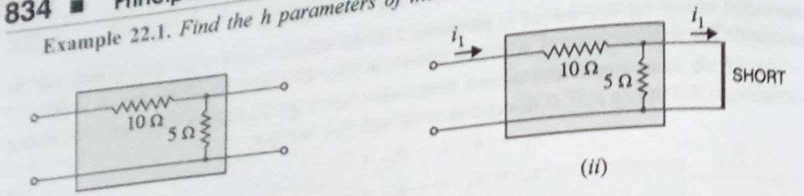


Fig. 22.4

Solution. The h parameters of the circuit shown in Fig. 22.4 (*i*) can be found as under :

To find h_{11} and h_{21}, short - circuit the output terminals as shown in Fig. 22.4 (*ii*). It is clear that input impedance of the circuit is 10 Ω because 5 Ω resistance is shorted out.

$$\therefore \quad h_{11} = 10\ \Omega$$

Now current i_1 flowing into the box will flow through 10 Ω resistor and then through the shorted path as shown. It may be noted that in our discussion, i_2 is the output current flowing into the box. Since output current in Fig. 22.4 (*ii*) is actually flowing out of the box, i_2 is negative *i.e.*,

$$i_2 = -i_1$$

$$\therefore \quad h_{21} = \frac{i_2}{i_1} = \frac{-i_1}{i_1} = -1$$

To find h_{12} and h_{22}, make the arrangement as shown in Fig. 22.4 (*iii*). Here we are driving the output terminals with a voltage v_2. This sets up a current i_2. Note that input terminals are open. Under this condition, there will be no current in 10 Ω resistor and, therefore, there can be no voltage drop across it. Consequently, all the voltage appears across input terminals *i.e.*

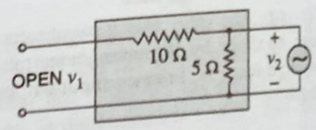


Fig. 22.4 (*iii*)

$$v_1 = v_2$$

$$\therefore \quad h_{12} = \frac{v_1}{v_2} = \frac{v_2}{v_2} = 1$$

Now the output impedance looking into the output terminals with input terminals open is simply 5 Ω. Then h_{22} will be the reciprocal of it because h_{22} is the output conductance with input terminals open.

$$\therefore \quad h_{22} = 1/5 = 0.2\ S$$

The h parameters of the circuit are :

$$h_{11} = 10\ \Omega\ ;\ h_{21} = -1$$

$$h_{12} = 1\ ;\ h_{22} = 0.2\ S$$

It may be mentioned here that in practice, dimensions are not written with h parameters. It is because it is understood that h_{11} is always in ohms, h_{12} and h_{21} are dimensionless and h_{22} is in siemens.

Example 22.2. *Find the h parameters of the circuit shown in Fig. 22.5 (i).*

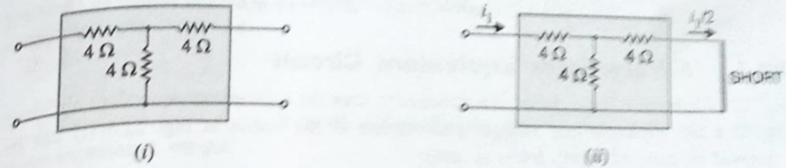


Fig. 22.5

Solution. First of all imagine that output terminals are short-circuited as shown in Fig. 22.5 (*ii*). The input impedance under this condition is the parameter h_{11}.

$$\text{Obviously,} \quad h_{11} = 4 + 4 \parallel 4$$

$$= 4 + \frac{4 \times 4}{4 + 4} = 6\ \Omega$$

Now the input current i_1 in Fig. 22.5 (*ii*) will divide equally at the junction of 4 Ω resistors so that output current is $i_1/2$ *i.e.*

$$i_2 = -i_1/2 = -0.5\ i_1$$

$$\therefore \quad h_{21} = \frac{i_2}{i_1} = \frac{-0.5\ i_1}{i_1} = -0.5$$

In order to find h_{12} and h_{22}, imagine the arrangement as shown in Fig. 22.5 (*iii*). Here we are driving the output terminals with voltage v_2, keeping the input terminals open. Under this condition, any voltage v_2 applied to the output will divide by a factor 2 *i.e.*

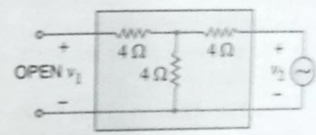


Fig. 22.5 (*iii*)

$$v_1 = \frac{v_2}{2} = 0.5\ v_2$$

$$\therefore \quad h_{12} = \frac{v_1}{v_2} = \frac{0.5\ v_2}{v_2} = 0.5$$

Now the output impedance looking into the output terminals with input terminals open is simply 8 Ω. Then h_{22} will be the reciprocal of this *i.e.*

$$h_{22} = \frac{1}{8} = 0.125\ S$$

Example 22.3. *The h parameters of a linear circuit are measured as under :*

$$h_{11} = 10\ k\Omega\ ;\ h_{12} = 0.5\ ;\ h_{21} = 10\ ;\ h_{22} = 2\ mS$$

Find the input voltage v_1 and output current i_2 if input current i_1 = 1 mA and output voltage v_2 = 2 V.

Solution. The h parameters of the circuit describe the relation of v_1 (input voltage) in terms of i_1 and v_2 as under :

$$v_1 = h_{11}\ i_1 + h_{12}\ v_2$$

Putting the various values, we have,

$$v_1 = 10\ k\Omega \times 1\ mA + 0.5 \times 2\ V = 11\ V$$

Also, $\quad i_2 = h_{21}\, i_1 + h_{22}\, v_2$

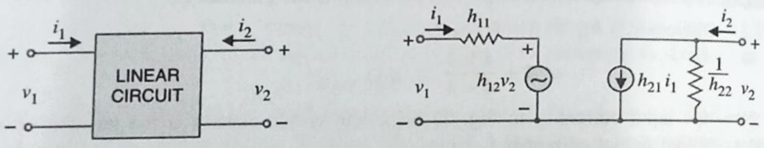
$= 10 \times 1 \text{ mA} + 2 \text{ mS} \times 2 \text{ V} = 14 \text{ mA}$

22.3 *h* Parameter Equivalent Circuit

Fig. 22.6 (*i*) shows a linear circuit. It is required to draw the *h* parameter equivalent circuit of Fig. 22.6 (*i*). We know that voltages and currents of the circuit in Fig. 22.6 (*i*) can be expressed in terms of *h* parameters as under :

$$v_1 = h_{11}\, i_1 + h_{12}\, v_2 \qquad ...(i)$$

$$i_2 = h_{21}\, i_1 + h_{22}\, v_2 \qquad ...(ii)$$



(*i*) (*ii*)

Fig. 22.6

Fig. 22.6 (*ii*) shows *h* parameter equivalent circuit of Fig. 22.6 (*i*) and is derived from equations (*i*) and (*ii*). The *input circuit* appears as a resistance h_{11} in series with a voltage generator $h_{12}\, v_2$. This circuit is derived from equation (*i*). The *output circuit* involves two components ; a current generator $h_{21}\, i_1$ and shunt resistance $1/h_{22}$ and is derived from equation (*ii*). The following points are worth noting about the *h* parameter equivalent circuit [See Fig. 22.6 (*ii*)] :

(*i*) This circuit is called hybrid equivalent because its input portion is a Thevenin equivalent, or voltage generator with series resistance, while output side is Norton equivalent, or current generator with shunt resistance. Thus it is a mixture or a hybrid. The symbol '*h*' is simply the abbreviation of the word hybrid (hybrid means "mixed").

(*ii*) The different hybrid parameters are distinguished by different number subscripts. The notation shown in Fig. 22.6 (*ii*) is used in general circuit analysis. The first number designates the circuit in which the effect takes place and the second number designates the circuit from which the effect comes. For instance, h_{21} is the "short-circuit forward current gain" or the ratio of the current in the output (*circuit* 2) to the current in the input (*circuit* 1).

(*iii*) The equivalent circuit of Fig. 22.6 (*ii*) is extremely useful for two main reasons. First, it isolates the input and output circuits, their interaction being accounted for by the two controlled sources. Thus, the effect of output upon input is represented by the equivalent voltage generator $h_{12}v_2$ and its value depends upon output voltage. Similarly, the effect of input upon output is represented by current generator $h_{21}i_1$ and its value depends upon input current. Secondly, the two parts of the circuit are in a form which makes it simple to take into account source and load circuits.

22.4 Performance of a Linear Circuit in *h* Parameters

We have already seen that any linear circuit with input and output has a set of *h* parameters. We shall now develop formulas for input impedance, current gain, voltage gain *etc.* of a linear circuit in terms of *h* parameters.

(*i*) **Input impedance.** Consider a linear circuit with a load resistance r_L across its terminals as shown in Fig. 22.7. The input impedance Z_{in} of this circuit is the ratio of input voltage to input current *i.e.*

$$Z_{in} = \frac{v_1}{i_1}$$

Now $v_1 = h_{11}\, i_1 + h_{12}\, v_2$ in terms of *h* parameters. Substituting the value of v_1 in the above expression, we get,

$$Z_{in} = \frac{h_{11} i_1 + h_{12} v_2}{i_1} = h_{11} + \frac{h_{12}\, v_2}{i_1} \qquad ...(i)$$

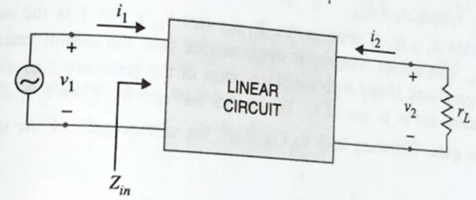


Fig. 22.7

Now, $i_2 = h_{21}\, i_1 + h_{22}\, v_2$ in terms of *h* parameters. Further from Fig. 22.7, it is clear that $i_2 = -v_2/r_L$. The minus sign is used here because the actual load current is opposite to the direction of i_2.

$$\therefore \quad \frac{-v_2}{r_L} = h_{21}\, i_1 + h_{22}\, v_2 \qquad \left[\because\ i_2 = \frac{-v_2}{r_L}\right]$$

or $$-h_{21}\, i_1 = h_{22}\, v_2 + \frac{v_2}{r_L} = v_2\left(h_{22} + \frac{1}{r_L}\right)$$

$$\therefore \quad \frac{v_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}} \qquad ...(ii)$$

Substituting the value of v_2/i_1 from exp. (*ii*) into exp. (*i*), we get,

$$Z_{in} = h_{11} - \frac{h_{12}\, h_{21}}{h_{22} + \frac{1}{r_L}} \qquad ...(iii)$$

This is the expression for input impedance of a linear circuit in terms of *h* parameters and load connected to the output terminals. If either h_{12} or r_L is very small, the second term in exp. (*iii*) can be neglected and input impedance becomes :

$$Z_{in} \simeq h_{11}$$

(*ii*) **Current Gain.** Referring to Fig. 22.7, the current gain A_i of the circuit is given by :

$$A_i = \frac{i_2}{i_1}$$

Now, $$i_2 = h_{21}\, i_1 + h_{22}\, v_2$$

and $$v_2 = -\, i_2\, r_L$$

$$i_2 = h_{21}\, i_1 - h_{22}\, i_2\, r_L$$

$\therefore$
$$i_2 (1 + h_{22}\, r_L) = h_{21}\, i_1$$

or
$$\frac{i_2}{i_1} = \frac{h_{21}}{1 + h_{22}\, r_L}$$

or

But $i_2/i_1 = A_i$, the current gain of the circuit.

$\therefore$
$$A_i = \frac{h_{21}}{1 + h_{22}\, r_L}$$

If $h_{22}\, r_L << 1$, then $A_i \simeq h_{21}$.

The expression $A_i \simeq h_{21}$ is often useful. To say that $h_{22}\, r_L << 1$ is the same as saying that $r_L << 1/h_{22}$. This occurs when r_L is much smaller than the output resistance $(1/h_{22})$. Under such condition, most of the generator current bypasses the shunting $h_{21}\, i_1$ generator. This means that $i_2 \simeq h_{21}\, i_1$ or $i_2/i_1 \simeq h_{21}$. circuit output resistance in favour of r_L.

(iii) Voltage gain. Referring back to Fig. 22.7, the voltage gain of the circuit is given by ;

$$A_v = \frac{v_2}{v_1}$$

$$= \frac{v_2}{i_1\, Z_{in}} \qquad (\because v_1 = i_1 Z_{in}) \qquad ...(iv)$$

While developing expression for input impedance, we found that :

$$\frac{v_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}}$$

Substituting the value of v_2/i_1 in exp. (iv), we get,

$$A_v = \frac{-h_{21}}{Z_{in}\left(h_{22} + \frac{1}{r_L}\right)}$$

(iv) Output impedance. In order to find the output impedance, remove the load r_L, set the signal voltage v_1 to zero and connect a generator of voltage v_2 at the output terminals. Then h parameter equivalent circuit becomes as shown in Fig. 22.8. By definition, the output impedance Z_{out} is

$$Z_{out} = \frac{v_2}{i_2}$$

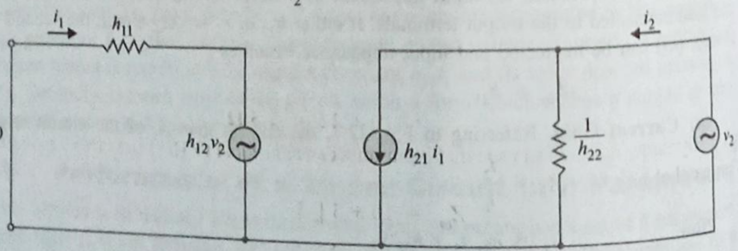


Fig. 22.8

With $v_1 = 0$ and applying Kirchhoff's voltage law to the input circuit, we have,

$$0 = i_1\, h_{11} + h_{12}\, v_2$$

$\therefore$
$$i_1 = -\frac{h_{12}\, v_2}{h_{11}}$$

Now,
$$i_2 = h_{21}\, i_1 + h_{22}\, v_2$$

Putting the value of i_1 $(= - h_{12}\, v_2/h_{11})$ in this equation, we get,

$$i_2 = h_{21}\left(-\frac{h_{12}\, v_2}{h_{11}}\right) + h_{22}\, v_2$$

or
$$i_2 = -\frac{h_{21}\, h_{12}\, v_2}{h_{11}} + h_{22}\, v_2$$

Dividing throughout by v_2, we have,

$$\frac{i_2}{v_2} = \frac{-h_{21}\, h_{12}}{h_{11}} + h_{22}$$

$\therefore$
$$Z_{out} = \frac{v_2}{i_2} = \frac{1}{h_{22} - \frac{h_{21}\, h_{12}}{h_{11}}}$$

Example 22.4. *Find the (i) input impedance and (ii) voltage gain for the circuit shown in Fig. 22.9.*

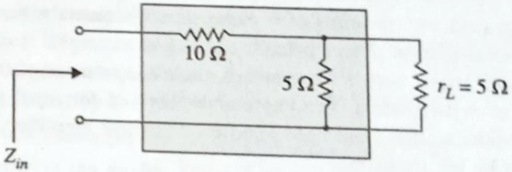


Fig. 22.9

Solution. The h parameters of the circuit inside the box are the same as those calculated in example 22.1 *i.e.*

$$h_{11} = 10\ \Omega\ ; \qquad h_{21} = -1$$
$$h_{12} = 1 \quad \text{and} \quad h_{22} = 0.2\ \text{S}$$

(i) Input impedance is given by;

$$Z_{in} = h_{11} - \frac{h_{12}\, h_{21}}{h_{22} + \frac{1}{r_L}} = 10 - \frac{1 \times -1}{0.2 + \frac{1}{5}}$$

$$= 10 + 2.5 = 12.5\ \Omega$$

By inspection, we can see that input impedance is equal to 10 Ω plus two 5 Ω resistances in parallel *i.e.*

$$Z_{in} = 10 + 5 \parallel 5$$

$$= 10 + \frac{5 \times 5}{5 + 5} = 12.5\ \Omega$$

(*ii*) Voltage gain, $A_v = \dfrac{-h_{21}}{Z_{in}\left(h_{22} + \dfrac{1}{r_L}\right)} = \dfrac{1}{12.5\left(0.2 + \dfrac{1}{5}\right)} = \dfrac{1}{5}$

It means that output voltage is one-fifth of the input voltage. This can be readily established by inspection of Fig. 22.9. The two 5 Ω resistors in parallel give a net resistance of 2.5 Ω. Therefore, we have a voltage divider consisting of 10 Ω resistor in series with 2.5 Ω resistor.

$\therefore$ $$\text{Output voltage} = \frac{2.5}{12.5} \times \text{Input voltage}$$

or $$\frac{\text{Output voltage}}{\text{Input voltage}} = \frac{2.5}{12.5} = \frac{1}{5}$$

or $$A_v = \frac{1}{5}$$

Comments. The reader may note that in a simple circuit like that of Fig. 22.9, it is not advisable to use *h* parameters to find the input impedance and voltage gain. It is because answers of such circuits can be found directly by inspection. However, in complicated circuits, inspection method becomes cumbersome and the use of *h* parameters yields quick results.

22.5 The *h* Parameters of a Transistor

It has been seen in the previous sections that every linear circuit is associated with *h* parameters. When this linear circuit is terminated by load r_L, we can find input impedance, current gain, voltage gain, *etc.* in terms of *h* parameters. Fortunately, for *small a.c. signals*, the transistor behaves as a linear device because the output a.c. signal is directly proportional to the input a.c. signal. Under such circumstances, the *a.c. operation* of the transistor can be described in terms of *h* parameters. The expressions derived for input impedance, voltage gain *etc.* in the previous section shall hold good for transistor amplifier except that here r_L is the a.c. load seen by the transistor.

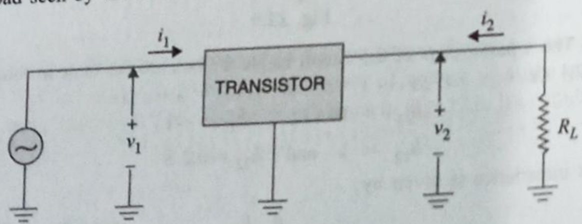


Fig. 22.10

Fig. 22.10 shows the transistor amplifier circuit. There are four quantities required to describe the external behaviour of the transistor amplifier. These are v_1, i_1, v_2 and i_2 shown on the diagram of Fig. 22.10. These voltages and currents can be related by the following sets of equations :

$$v_1 = h_{11}\, i_1 + h_{12}\, v_2$$
$$i_2 = h_{21}\, i_1 + h_{22}\, v_2$$

The following points are worth noting while considering the behaviour of transistor in terms of *h* parameters :

(*i*) For small a.c. signals, a transistor behaves as a linear circuit. Therefore, its a.c. operation can be described in terms of *h* parameters.

(*ii*) The value of *h* parameters of a transistor will depend upon the transistor connection (*i.e. CB, CE* or *CC*) used. For instance, a transistor used in *CB* arrangement may have $h_{11} = 20$ Ω. If we use the same transistor in *CE* arrangement, h_{11} will have a different value. Same is the case with other *h* parameters.

(*iii*) The expressions for input impedance, voltage gain *etc.* derived in Art. 22.4 are also applicable to transistor amplifier except that r_L is the a.c. load seen by the transistor *i.e.*

$$r_L = R_C \parallel R_L$$

(*iv*) The values of *h* parameters depend upon the operating point. If the operating point is changed, parameter values are also changed.

(*v*) The notations v_1, i_1, v_2 and i_2 are used for general circuit analysis. In a transistor amplifier, we use the notation depending upon the configuration in which transistor is used. Thus for *CE* arrangement,

$$v_1 = V_{be} \;;\; i_1 = I_b \;;\; v_2 = V_{ce} \;;\; i_2 = I_c$$

Here V_{be}, I_b, V_{ce} and I_c are the R.M.S. values.

22.6 Nomenclature for Transistor *h* Parameters

The numerical subscript notation for *h* parameters (*viz.* h_{11}, h_{21}, h_{12} and h_{22}) is used in general circuit analysis. However, this nomenclature has been modified for a transistor to indicate the nature of parameter and the transistor configuration used. The *h* parameters of a transistor are represented by the following notation :

(*i*) The numerical subscripts are replaced by letter subscripts.

(*ii*) The first letter in the double subscript notation indicates the nature of parameter.

(*iii*) The second letter in the double subscript notation indicates the circuit arrangement (*i.e. CB, CE* or *CC*) used.

Table below shows the *h* parameter nomenclature of a transistor :

S.No.	*h* parameter	Notation in CB	Notation in CE	Notation in CC
1.	h_{11}	h_{ib}	h_{ie}	h_{ic}
2.	h_{12}	h_{rb}	h_{re}	h_{rc}
3.	h_{21}	h_{fb}	h_{fe}	h_{fc}
4.	h_{22}	h_{ob}	h_{oe}	h_{oc}

Note that first letter *i*, *r*, *f* or *o* indicates the nature of parameter. Thus h_{11} indicates input impedance and this parameter is designated by the subscript *i*. Similarly, letters *r*, *f* and *o* respectively indicate reverse voltage feedback ratio, forward current transfer ratio and output admittance. The second letters *b*, *e* and *c* respectively indicate *CB*, *CE* and *CC* arrangement.

22.7 Transistor *h* Parameter Model

We can visualise a transistor as a two-port or 4-terminal device because one of its three terminals is common to both input and output circuits. Fig. 22.11(*i*) shows the transistor in common emitter (*CE*) arrangement. The *h* parameter *a.c.* equations for this circuit are :

$$V_{be} = h_{ie}\, I_b + h_{re}\, V_{ce} \qquad \text{...}(i)$$

$$I_c = h_{fe}\, I_b + h_{oe}\, V_{ce} \qquad \text{...}(ii)$$

Here V_{be}, V_{ce}, I_b and I_c are the r.m.s. values. It should be noted that h_{ie} has the units of resistance (ohms) while h_{oe} has the units of conductance (siemens). The other two h_{re} and h_{fe} are pure numbers. Further, h_{ie} and h_{fe} are measured with $V_{ce} = 0$ (*i.e.* output short-circuited to a.c.) while h_{re} and h_{oe} are measured with $I_b = 0$ (*i.e.* input open-circuited to a.c.).

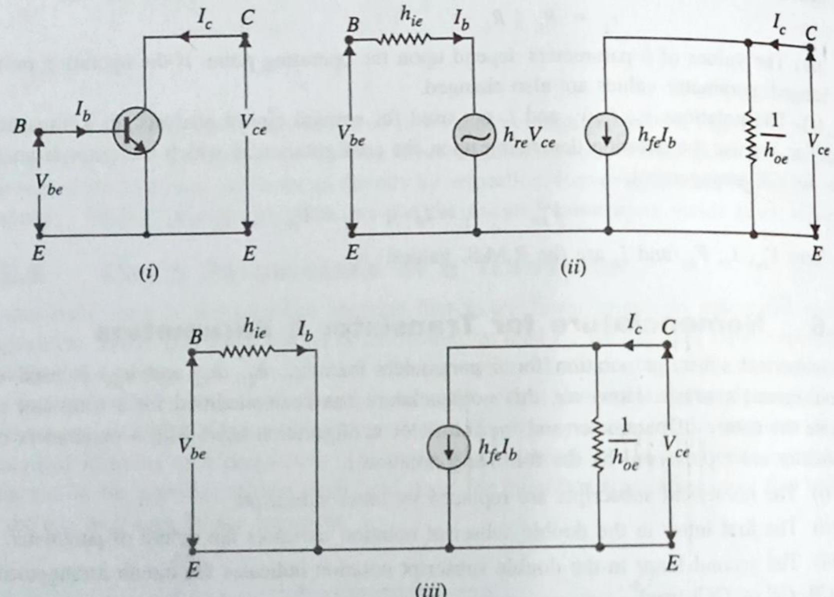


Fig. 22.11

Relations (*i*) and (*ii*) above can be represented by the circuit shown in Fig. 22.11 (*ii*) and is called *transistor h parameter model* or *hybrid equivalent circuit of transistor*. The input resistance h_{ie} appears in series at the input. The reverse voltage feedback ratio h_{re} is multiplied by the output voltage (V_{ce}) to produce an equivalent voltage source ($h_{re}\, V_{ce}$) in series with the input. The current gain h_{fe} is multiplied by the input current (I_b) to produce an equivalent current source ($h_{fe}\, I_b$) in the output. The output conductance h_{oe} appears across the output terminals. Although V_{ce} can be very much greater than V_{be} due to voltage gain obtainable from the transistor, h_{re} is very small (typically 5×10^{-4}) and hence $h_{re}\, V_{ce}$ is small in comparison to V_{be}. Therefore, this voltage source (*i.e.* $h_{re}\, V_{ce}$) can be removed from the input side without any large error. This produces the *simplified hybrid equivalent circuit of transistor* as shown in Fig. 22.11 (*iii*). The following points may be noted about the hybrid equivalent circuit of a transistor :

(*i*) The *h* parameter method is an analytical method. Therefore, it is more accurate than the graphical method for the analysis of a transistor circuit.

(*ii*) The hybrid a.c. equivalent circuit of the input circuit of *CE* circuit is derived using Thevenin's theorem. Thevenin's theorem states that any circuit can be replaced by a single voltage source in series with a single resistance. Here, the input circuit is replaced by a single voltage source $h_{re}\, V_{ce}$ in series with a single resistance h_{ie}.

(*iii*) The hybrid a.c. equivalent circuit of the output circuit of *CE* circuit is derived using Norton's theorem. Norton's theorem states that any circuit can be replaced by a single current source in parallel with a single resistance. Here, the output circuit is *replaced by a single current source $h_{fe}\, I_b$ in parallel with a single resistance $1/h_{oe}$. Note that h_{oe} is output conductance so that $1/h_{oe}$ is the output resistance.

Example 22.5. *The transistor used in the circuit shown in Fig. 22.12 has the following h parameters :*

$$h_{ie} = 2\ \text{k}\Omega\ ;\ h_{fe} = 60\ ;\ h_{oe} = 20\ \mu S\ ;\ h_{re} \text{ is negligible}$$

The input signal is supplied from a source is e.m.f. E in series with a resistance of 10 kΩ. Determine the necessary value of E to give a signal power of 10 mW in a load of 10 kΩ (resistive) connected across the output terminals. The reactances of the coupling capacitors may be neglected.

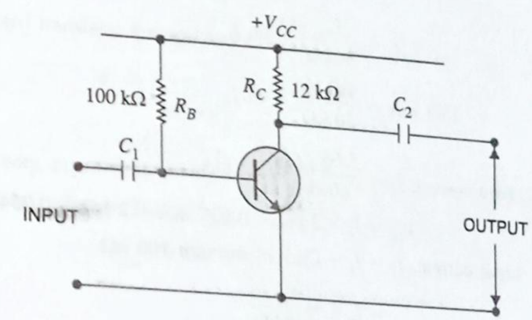


Fig. 22.12

Solution. The hybrid a.c. equivalent circuit of Fig. 22.12 is shown in Fig. 22.13. Note that voltage source $h_{re}\, V_{ce}$ in the input circuit is not shown because h_{re} is negligible.

$$\text{Output resistance} = \frac{1}{h_{oe}} = \frac{1}{20 \times 10^{-6}}\ \Omega = 5 \times 10^4\ \Omega = 50\ \text{k}\Omega$$

$$\text{Load resistance, } R_L = 10\ \text{k}\Omega$$

$$\text{Current source} = h_{fe}\, I_b = 60\ I_b$$

$$\text{Load power, } P = 10\ \text{mW} = 10 \times 10^{-3}\ \text{W}$$

Now,
$$P = \frac{V_{ce}^2}{R_L} \quad \text{or} \quad 10 \times 10^{-3} = \frac{V_{ce}^2}{10 \times 10^3}$$

∴
$$V_{ce} = \sqrt{100} = 10\ \text{V}$$

* The output circuit is replaced by Norton equivalent circuit because I_c depends upon factors outside of the collector-emitter circuit under normal conditions. Thus changing the value of collector resistance will hardly change I_c significantly. However, changing I_b will.

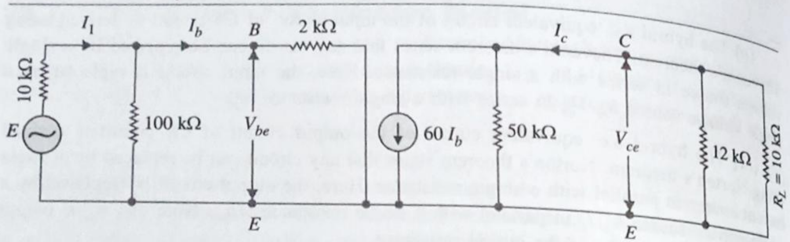


Fig. 22.13

Effective collection load, $r_L = R_C \parallel R_L = \dfrac{R_C\, R_L}{R_C + R_L}$

$$= \frac{12 \times 10}{12 + 10} = \frac{120}{22}\ \text{k}\Omega$$

$\therefore$ A.C. collector current, $I_c = \dfrac{V_{ce}}{r_L} = \dfrac{10\ \text{V}}{(120/22)\text{k}\Omega} = 1.83\ \text{mA}$

Now, $I_c = \dfrac{V_{ce}}{50\,\text{k}\Omega} + 60\ I_b$

or $1.83\ \text{mA} = \dfrac{10\ \text{V}}{50\ \text{k}\Omega} + 60 I_b$

$\therefore$ A.C. base current, $I_b = \dfrac{1.63}{60} = 0.027\ \text{mA}$

$\therefore$ $V_{be} = I_b \times 2\ \text{k}\Omega = 0.027\ \text{mA} \times 2\ \text{k}\Omega = 0.054\ \text{V}$

$\therefore$ Input current, $I_1 = I_b$ + Current through 100 kΩ

$$= I_b + \frac{V_{be}}{100\ \text{k}\Omega}$$

$$= 0.027\ \text{mA} + \frac{0.054\ \text{V}}{100\,\text{k}\Omega} = 0.028\ \text{mA}$$

$\therefore$ $E = V_{be} + I_1 \times 10\ \text{k}\Omega$

$$= 0.054\ \text{V} + 0.028\ \text{mA} \times 10\ \text{k}\Omega = 0.334\ \text{V}$$

22.8 Transistor Circuit Performance in *h* Parameters

The expressions for input impedance, voltage gain *etc.* in terms of *h* parameters derived in Art. 22.4 for general circuit analysis apply equally for transistor analysis. However, it is profitable to rewrite them in standard transistor *h* parameter nomenclature.

(i) Input impedance. The general expression for input impedance is

$$Z_{in} = h_{11} - \frac{h_{12}\ h_{21}}{h_{22} + \dfrac{1}{r_L}}$$

Using standard *h* parameter nomenclature for transistor, its value for *CE* arrangement will be :

$$Z_{in} = h_{ie} - \frac{h_{re}\ h_{fe}}{h_{oe} + \dfrac{1}{r_L}}$$

Similarly, expressions for input impedance in *CB* and *CC* arrangements can be written. It may be noted that r_L is the a.c. load seen by the transistor.

(ii) Current gain. The general expression for current gain is

$$A_i = \frac{h_{21}}{1 + h_{22}\ r_L}$$

Using standard transistor *h* parameter nomenclature, its value for *CE* arrangement is

$$A_i = \frac{h_{fe}}{1 + h_{oe}\ r_L}$$

The reader can readily write down the expressions for *CB* and *CC* arrangements.

(iii) Voltage gain. The general expression for voltage gain is

$$A_v = \frac{-\,h_{21}}{Z_{in}\left(h_{22} + \dfrac{1}{r_L}\right)}$$

Using standard transistor *h* parameter nomenclature, its value for *CE* arrangement is

$$A_v = \frac{-\,h_{fe}}{Z_{in}\left(h_{oe} + \dfrac{1}{r_L}\right)}$$

In the same way, expressions for voltage gain in *CB* and *CC* arrangements can be written.

(iv) Output impedance. The general expression for output impedance is

$$Z_{out} = \frac{1}{h_{22} - \dfrac{h_{21}\ h_{12}}{h_{11}}}$$

Using standard transistors *h* parameter nomenclature, its value for *CE* arrangement is

$$Z_{out} = \frac{1}{h_{oe} - \dfrac{h_{fe}\ h_{re}}{h_{ie}}}$$

In the same way, expression for output impedance in *CB* and *CC* arrangements can be written.

The above expression for Z_{out} is for the transistor. If the transistor is connected in a circuit to form a single stage amplifier, then output impedance of the stage = $Z_{out} \parallel r_L$ where $r_L = R_C \parallel R_L$.

Example 22.6. *A transistor used in CE arrangement has the following set of h parameters when the d.c. operating point is V_{CE} = 10 volts and I_C = 1 mA :*

h_{ie} = 2000 Ω ; $h_{oe} = 10^{-4}$ S ; $h_{re} = 10^{-3}$; h_{fe} = 50

Determine (i) input impedance (ii) current gain and (iii) voltage gain. The a.c. load seen by the transistor is r_L = 600 Ω. What will be approximate values using reasonable approximations ?

Solution. (*i*) Input impedance is given by ;

$$Z_{in} = h_{ie} - \frac{h_{re}\, h_{fe}}{h_{oe} + \dfrac{1}{r_L}} = 2000 - \frac{10^{-3} \times 50}{10^{-4} + \dfrac{1}{600}} \quad \text{.... (i)}$$

$$= 2000 - 28 = \mathbf{1972\ \Omega}$$

The second term in eq. (*i*) is quite small as compared to the first.

$$\therefore \quad Z_{in} \simeq h_{ie} = \mathbf{2000\ \Omega}$$

(*ii*) Current gain, $A_i = \dfrac{h_{fe}}{1 + h_{oe} \times r_L} = \dfrac{50}{1 + (600 \times 10^{-4})} = 47$

If $h_{oe}\, r_L << 1$, then $A_i \simeq h_{fe} = \mathbf{50}$.

(*iii*) Voltage gain, $A_v = \dfrac{-h_{fe}}{Z_{in}\left(h_{oe} + \dfrac{1}{r_L}\right)} = \dfrac{-50}{1972\left(10^{-4} + \dfrac{1}{600}\right)} = -14.4$

The negative sign indicates that there is 180° phase shift between input and output. The magnitude of gain is 14.4. In other words, the output signal is 14.4 times greater than the input and it is 180° out of phase with the input.

Example 22.7. *A transistor used in CE connection has the following set of h parameters when the d.c. operating point is V_{CE} = 5 volts and I_C = 1 mA :*

$h_{ie} = 1700\ \Omega$; $h_{re} = 1.3 \times 10^{-4}$; $h_{fe} = 38$; $h_{oe} = 6 \times 10^{-6}$ S

If the a.c. load r_L seen by the transistor is 2 kΩ, find (i) the input impedance (ii) current gain and (iii) voltage gain.

Solution. (*i*) The input impedance looking into the base of transistor is

$$Z_{in} = h_{ie} - \frac{h_{re}\, h_{fe}}{h_{oe} + \dfrac{1}{r_L}} = 1700 - \frac{1.3 \times 10^{-4} \times 38}{6 \times 10^{-6} + \dfrac{1}{2000}} \simeq 1690\ \Omega$$

(*ii*) Current gain, $A_i = \dfrac{h_{fe}}{1 + h_{oe} r_L} = \dfrac{38}{1 + 6 \times 10^{-6} \times 2000} = \dfrac{38}{1.012} \simeq 37.6$

(*iii*) Voltage gain, $A_v = \dfrac{-h_{fe}}{Z_{in}\left(h_{oe} + \dfrac{1}{r_L}\right)} = \dfrac{-38}{1690\left(6 \times 10^{-6} + \dfrac{1}{2000}\right)} = -44.4$

Example 22.8. *Fig. 22.14 shows the transistor amplifier in CE arrangement. The h parameters of transistor are as under :*

$h_{ie} = 1500\ \Omega$; $h_{fe} = 50$; $h_{re} = 4 \times 10^{-4}$; $h_{oe} = 5 \times 10^{-5}$ S

Find (i) a.c. input impedance of the amplifier (ii) voltage gain and (iii) output impedance.

Solution. The a.c. load r_L seen by the transistor is equivalent of the parallel combination of R_C (= 10 kΩ) and R_L (= 30 kΩ) *i.e.*

$$r_L = \frac{R_C\, R_L}{R_C + R_L} = \frac{10 \times 30}{10 + 30} = 7.5\ \text{k}\Omega = 7500\ \Omega$$

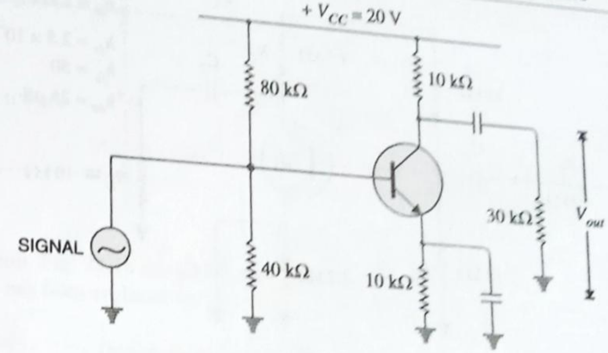


Fig. 22.14

(*i*) The input impedance looking into the base of transistor is given by ;

$$Z_{in} = h_{ie} - \frac{h_{re}\, h_{fe}}{h_{oe} + \dfrac{1}{r_L}} = 1500 - \frac{4 \times 10^{-4} \times 50}{5 \times 10^{-5} + \dfrac{1}{7500}} = 1390\ \Omega$$

This is only the input impedance looking into the base of transistor. The a.c. input impedance of the entire stage will be Z_{in} in parallel with bias resistors *i.e.*

Input impedance of stage = $80 \times 10^3 \parallel 40 \times 10^3 \parallel 1390$ = **1320 Ω**

(*ii*) Voltage gain, $A_v = \dfrac{-h_{fe}}{Z_{in}\left(h_{oe} + \dfrac{1}{r_L}\right)} = \dfrac{-50}{1390\left(5 \times 10^{-5} + \dfrac{1}{7500}\right)} = -196$

The negative sign indicates phase reversal. The magnitude of gain is 196.

(*iii*) Output impedance of transistor is

$$Z_{out} = \frac{1}{h_{oe} - \dfrac{h_{fe}\, h_{re}}{h_{ie}}}$$

$$= \frac{1}{5 \times 10^{-5} - \dfrac{50 \times 4 \times 10^{-4}}{1500}}$$

$$= 27270\ \Omega = 27.27\ \text{k}\Omega$$

∴ Output impedance of the stage

$$= Z_{out} \parallel R_L \parallel R_C$$

$$= 27.27\ \text{k}\Omega \parallel 30\ \text{k}\Omega \parallel 10\ \text{k}\Omega = \mathbf{5.88\ k\Omega}$$

Example 22.9. *Find the value of current gain for the circuit shown in Fig. 22.15. The h parameter values of the transistor are given alongside the figure.*

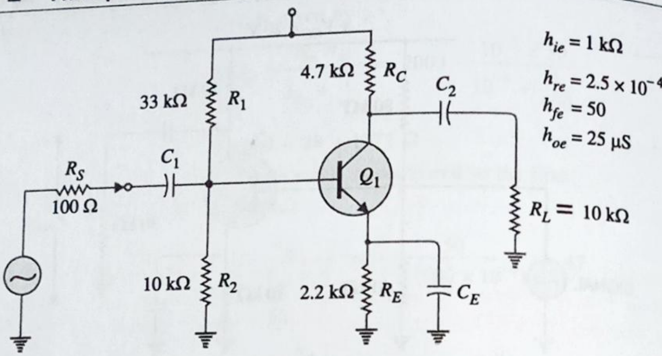


Fig. 22.15

Solution. The current gain A_i for the circuit is given by ;

$$A_i = \frac{h_{fe}}{1 + h_{oe}\, r_L}$$

Here, $r_L = R_C \parallel R_L = 4.7\ \text{k}\Omega \parallel 10\ \text{k}\Omega = 3.2\ \text{k}\Omega$

$$\therefore \quad A_i = \frac{50}{1 + (25 \times 10^{-6})\,(3.2 \times 10^3)} = 46.3$$

Note that current gain of the circuit is very close to the value h_{fe}. The reason for this is that $h_{oe}\, r_L << 1$. Since this is normally the case, $A_i \simeq h_{fe}$.

Example 22.10. *In the above example, determine the output impedance of the transistor.*

Solution. Note that the signal source (See Fig. 22.15) has resistance $R_S = 100\ \Omega$.

$\therefore$ Output impedance Z_{out} of the transistor is

$$Z_{out} = \frac{1}{h_{oe} - \left(\dfrac{h_{fe}\, h_{re}}{h_{ie} + R_S}\right)}$$

$$= \frac{1}{(25 \times 10^{-6}) - \dfrac{(50)\,(2.5 \times 10^{-4})}{(1 \times 10^3) + 100}}$$

$$= 73{\cdot}3 \times 10^3\ \Omega = 73.3\ \text{k}\Omega$$

Example 22.11. *A transistor has the following small-signal hybrid parameters :*

$h_{ie} = 1$ kΩ ; $h_{fe} = 50$; $h_{oe} = 25$ μS ; $h_{re} = 0$

The total effective load connected between the collector and emitter can be considered as a 5 kΩ resistor. The signal is supplied from a source which can be considered as an e.m.f. of 100 mV in series with a resistance of 9 kΩ. Determine (i) current gain (ii) voltage gain (iii) power gain of the transistor and (iv) signal power fed to the load. The effect of the bias components and the reactances of the coupling capacitors may be neglected.

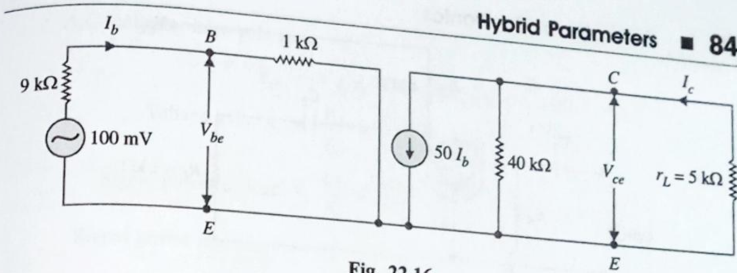


Fig. 22.16

Solution. Fig. 22.16 shows the a.c. equivalent circuit of the transistor amplifier where transistor has been replaced by its *simplified* hybrid equivalent circuit.

$$\text{Output resistance} = \frac{1}{h_{oe}} = \frac{1}{25 \times 10^{-6}}\Omega = 4 \times 10^4\,\Omega = 40\,\text{k}\Omega$$

Effective a.c. load, $r_L = 5$ kΩ

Current source $= h_{fe}\, I_b = 50\, I_b$

By current-divider rule, a.c. collector current I_c is given by ;

$$I_c = 50\, I_b \times \frac{40}{40 + 5} \qquad \text{...}(i)$$

(*i*) Current gain, $A_i = \dfrac{I_c}{I_b} = \dfrac{40 \times 50}{45} = 44.4$ [From eq. (*i*)]

(*ii*) Voltage gain, $A_v = \dfrac{V_{ce}}{V_{be}} = \dfrac{I_c \times 5}{I_b \times 1} = \dfrac{I_c}{I_b} \times 5 = 44.4 \times 5 = 222$

(*iii*) Power gain, $A_p = A_v\, A_i = 222 \times 44.4 = 9860$

(*iv*) A.C. base current, $I_b = \dfrac{E}{(9+1)\ \text{k}\Omega} = \dfrac{100\ \text{mV}}{10\ \text{k}\Omega} = 10\,\mu\text{A}$

$\therefore$ A.C. collector current, $I_c = A_i\, I_b = 44.4 \times 10\ \mu\text{A} = 444\ \mu\text{A}$

$\therefore$ Signal power fed to load $= I_c^2\, r_L = (444 \times 10^{-6})^2 \times 5 \times 10^3$

$= 9.85 \times 10^{-4}$ W $= 0.985$ mW

Example 22.12. *The transistor used in the common base circuit shown in Fig. 22.17 has the following small-signal h parameters :*

$h_{ib} = 50\ \Omega$; $h_{fb} = -0.96$; $h_{ob} = 1$ μS ; h_{rb} is negligible

The signal source can be considered as a constant-current generator of 100 μA and an internal resistance of 450 Ω. Calculate the voltage and power gains of the stage when the load is a 1 kΩ resistor connected across the output terminals. The effect of the bias resistor and the reactances of coupling capacitors may be neglected.

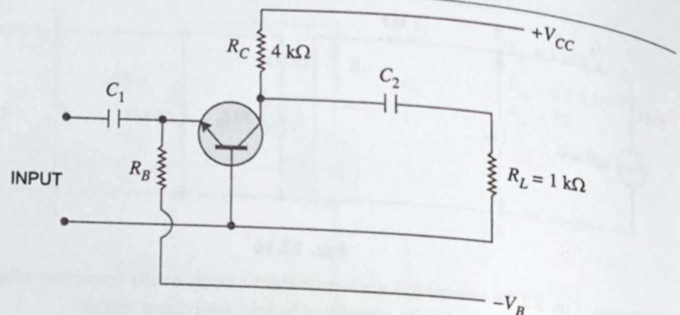


Fig. 22.17

Solution. Fig. 22.18 shows the a.c. equivalent circuit of the transistor amplifier where the transistor has been replaced by its simplified hybrid equivalent circuit.

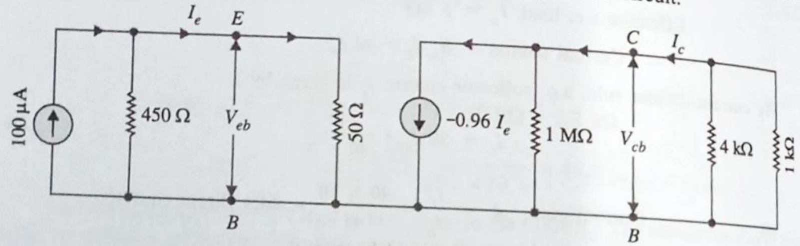


Fig. 22.18

$$\text{Output resistance} = \frac{1}{h_{ob}} = \frac{1}{1 \times 10^{-6}}\ \Omega = 10^6\ \Omega = 1\ \text{M}\Omega$$

$$\text{Load resistance, } R_L = 1\ \text{k}\Omega$$

Referring to the a.c. equivalent circuit in Fig. 22.18, current of 100 μA divides between 450 Ω and 50 Ω resistances. By current-divider rule, the a.c. emitter current I_e is given by ;

$$I_e = 100\ \mu\text{A} \times \frac{450}{450 + 50} = 90\ \mu\text{A}$$

Therefore, a.c. emitter-base voltage V_{eb} is

$$V_{eb} = I_e \times 50\ \Omega = 90\ \mu\text{A} \times 50\ \Omega = 4.5\ \text{mV}$$

$$\text{A.C. collector current, } I_c = -0.96^{*}\ I_e = -0.96 \times 90\ \mu\text{A} = -86\ \mu\text{A}$$

$$\text{Effective a.c. load, } r_L = R_C \parallel R_L = 4\ \text{k}\Omega \parallel 1\ \text{k}\Omega$$

$$= \frac{4\,\text{k}\Omega \times 1\,\text{k}\Omega}{4\,\text{k}\Omega + 1\,\text{k}\Omega} = \frac{4}{5}\ \text{k}\Omega$$

* The output resistance (= 1 MΩ) is very large as compared to 4 kΩ || 1 kΩ. Therefore, $I_c \simeq -0.96\ I_e$.

∴ A.C. collector-base voltage is

$$V_{cb} = -I_c r_L = -(-86\ \mu\text{A}) \times \frac{4}{5}\ \text{k}\Omega = 69\ \text{mV}$$

$$\therefore \quad \text{Voltage gain, } A_v = \frac{V_{cb}}{V_{eb}} = \frac{69\,\text{mV}}{4.5\,\text{mV}} = 15$$

$$\text{Signal power in load} = \frac{V_{cb}^2}{R_L} = \frac{(69 \times 10^{-3})^2}{1 \times 10^3} = 4.8 \times 10^{-6}\ \text{W}$$

$$\text{Signal power into stage} = I_e^2 \times 50\ \Omega = (90 \times 10^{-6})^2 \times 50\ \Omega = 0.41 \times 10^{-6}\ \text{W}$$

$$\therefore \quad \text{Power gain, } A_p = \frac{4.8 \times 10^{-6}}{0.41 \times 10^{-6}} = 12$$

Note. Power gain of transistor = 15 × 0.96 = 14.4

22.9 Approximate Hybrid Formulas for Transistor Amplifier

The *h* parameter formulas (*CE* configuration) covered in Art. 22.8 can be approximated to a form that is easier to handle. While these approximate formulas will not give results that are as accurate as the original formulas, they can be used for many applications.

(*i*) Input impedance

$$\text{Input impedance, } Z_{in} = h_{ie} - \frac{h_{re}\, h_{fe}}{h_{oe} + \dfrac{1}{r_L}}$$

In actual practice, the second term in this expression is very small as compared to the first term.

$$\therefore \quad Z_{in} = h_{ie} \quad \text{... approximate formula}$$

(*ii*) Current gain

$$\text{Current gain, } A_i = \frac{h_{fe}}{1 + h_{oe}\, r_L}$$

In actual practice, $h_{oe}\, r_L$ is very small as compared to 1.

$$\therefore \quad A_i = h_{fe} \quad \text{... approximate formula}$$

(*iii*) Voltage gain

$$\text{Voltage gain, } A_v = \frac{-h_{fe}}{Z_{in}\left(h_{oe} + \dfrac{1}{r_L}\right)}$$

$$= \frac{-h_{fe}\, r_L}{Z_{in}\,(h_{oe}\, r_L + 1)}$$

Now approximate formula for Z_{in} is h_{ie}. Also $h_{oe}\, r_L$ is very small as compared to 1.

$$\therefore \quad A_v = -\frac{h_{fe}\, r_L}{h_{ie}} \quad \text{... approximate formula}$$

(*iv*) Output impedance

$$\text{Output impedance of transistor, } Z_{out} = \frac{1}{h_{oe} - \dfrac{h_{fe}\, h_{re}}{h_{ie}}}$$

The second term in the denominator is very small as compared to h_{oe}.

$$\therefore \quad Z_{out} = \frac{1}{h_{oe}} \quad \text{... approximate formula}$$

The output impedance of transistor amplifier

$$= Z_{out} \parallel r_L \quad \text{where } {}^*r_L = R_C \parallel R_L$$

Example 22.13. *For the circuit shown in Fig. 22.19, use approximate hybrid formulas to determine (i) the input impedance (ii) voltage gain. The h parameters of the transistor are h_{ie} = 1.94 kΩ and h_{fe} = 71.*

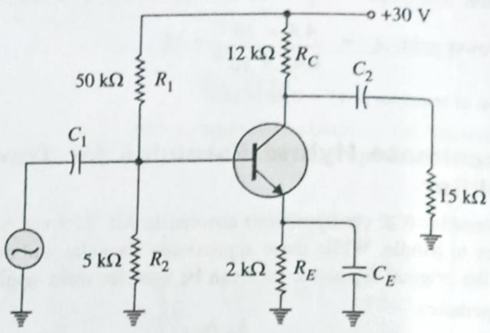


Fig. 22.19

Solution.

A.C. collector load, $r_L = R_C \parallel R_L = 12\ \text{k}\Omega \parallel 15\ \text{k}\Omega = 6.67\ \text{k}\Omega$

(*i*) Transistor input impedance is

$$Z_{in\,(base)} = h_{ie} = 1.94\ \text{k}\Omega$$

$\therefore$ Circuit input impedance $= Z_{in\,(base)} \parallel R_1 \parallel R_2$

$$= 1.94\ \text{k}\Omega \parallel 50\ \text{k}\Omega \parallel 5\ \text{k}\Omega = 1.35\ \text{k}\Omega$$

(*ii*) Voltage gain, $A_v = \dfrac{h_{fe}\, r_L}{h_{ie}} = \dfrac{71 \times 6.67\ \text{k}\Omega}{1.94\ \text{k}\Omega} = 244$

Example 22.14. *A transistor used in an amplifier has h parameter values of h_{ie} = 600 Ω to 800Ω and h_{fe} = 110 to 140. Using approximate hybrid formula, determine the voltage gain for the circuit. The a.c. collector load, r_L = 460 Ω.*

Solution. When *minimum* and *maximum* h parameter values are given, we should determine the *geometric average* of the two values. Thus the values that we would use in the analysis of circuit are found as under :

$$h_{ie} = \sqrt{h_{ie\,(min)} \times h_{ie\,(max)}}$$

$$= \sqrt{(600\ \Omega)\,(800\ \Omega)} = 693\Omega$$

$$h_{fe} = \sqrt{h_{fe\,(min)} \times h_{fe\,(max)}}$$

* If the amplifier is unloaded (*i.e.* $R_L = \infty$), $r_L = R_C$.

$$= \sqrt{(110)\,(140)} = 124$$

$$\text{Voltage gain, } A_v = \frac{h_{fe}\, r_L}{h_{ie}} = \frac{(124)\,(460)}{693} = 82.3$$

22.10 Experimental Determination of Transistor h Parameters

The determination of *h* parameters of a general linear circuit has already been discussed in Art. 22.2. To illustrate how such a procedure is carried out for a *CE* transistor amplifier, consider the circuit of Fig. 22.20. The r.m.s. values will be considered in the discussion. Using standard transistor nomenclature :

$$V_{be} = h_{ie}\, I_b + h_{re}\, V_{ce} \quad \text{...(i)}$$

$$I_c = h_{fe}\, I_b + h_{oe}\, V_{ce} \quad \text{...(ii)}$$

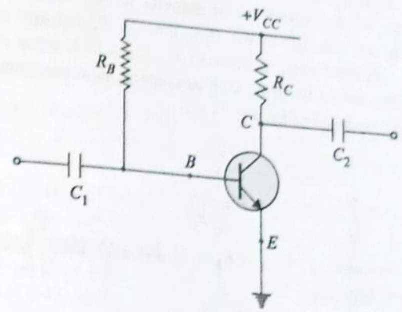


Fig. 22.20

(*i*) **Determination of h_{fe} and h_{ie}.** In order to determine these parameters, the output is a.c. short-circuited as shown in Fig. 22.21 (*i*). This is accomplished by making the capacitance of C_2 deliberately large. The result is that changing component of collector current flows through C_2 instead of R_C and a.c. voltage developed across C_2 is zero *i.e.* $^*V_{ce} = 0$.

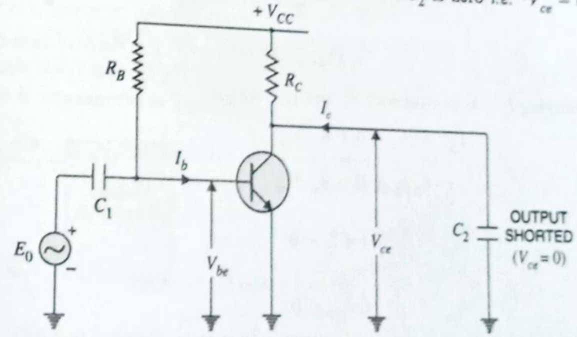


Fig. 22.21 (*i*)

* Note that setting V_{ce} = 0 does not mean that V_{CE} (the d.c. collector-emitter voltage) is zero. Only a.c. output is short-circuited.

Substituting $V_{ce} = 0$ in equations (*i*) and (*ii*) above, we get,

$$V_{be} = h_{ie}\, I_b + h_{re} \times 0$$

$$I_c = h_{fe}\, I_b + h_{oe} \times 0$$

$$\therefore \quad h_{fe} = \frac{I_c}{I_b} \quad \text{for } V_{ce} = 0$$

and $$h_{ie} = \frac{V_{be}}{I_b} \quad \text{for } V_{ce} = 0$$

Note that I_c and I_b are the a.c. r.m.s. collector and base currents respectively. Also V_{be} is the a.c. r.m.s. base-emitter voltage.

(*ii*) **Determination of h_{re} and h_{oe}.** In order to determine these two parameters, the input is a.c. open-circuited, a signal generator is applied across the output and resulting V_{be}, V_{ce} and I_c are measured. This is illustrated in Fig. 22.21 (*ii*). A large inductor L is connected in series with R_B. Since the d.c. resistance of inductor is very small, it does not disturb the operating point. Again, a.c. current cannot flow through R_B because of large reactance of inductor. Further, the voltmeter used to measure V_{be} has a high input impedance and hence there are no paths connected to the base with any appreciable a.c. current. This means that base is *effectively a.c. open-circuited *i.e.* $I_b = 0$.

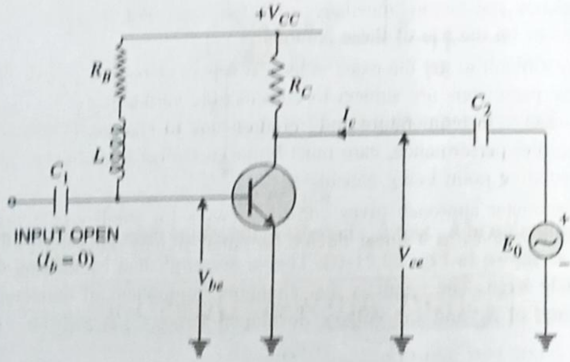


Fig. 22.21 (*ii*)

Substituting $I_b = 0$ in equations (*i*) and (*ii*), we get,

$$V_{be} = h_{ie} \times 0 + h_{re}\, V_{ce}$$

$$I_c = h_{fe} \times 0 + h_{oe}\, V_{ce}$$

$$\therefore \quad h_{re} = \frac{V_{be}}{V_{ce}} \quad \text{for } I_b = 0$$

and $$h_{oe} = \frac{I_c}{V_{ce}} \quad \text{for } I_b = 0$$

* How effectively the base is a.c. open-circuited depends upon the reactance L and the input impedance of the voltmeter used to measure V_{be}.

Example 22.15. The following quantities are measured in a CE amplifier circuit :

(*a*) *With output a.c. short-circuited (i.e. $V_{ce} = 0$)*

$$I_b = 10\ \mu A\ ;\ I_c = 1\ mA\ ;\ V_{be} = 10\ mV$$

(*b*) *With input a.c. open-circuited (i.e. $I_b = 0$)*

$$V_{be} = 0.65\ mV\ ;\ I_c = 60\ \mu A\ ;\ V_{ce} = 1\ V$$

Determine all the four h parameters.

Solution.

$$h_{ie} = \frac{V_{be}}{I_b} = \frac{10 \times 10^{-3}}{10 \times 10^{-6}} = 1000\ \Omega$$

$$h_{fe} = \frac{I_c}{I_b} = \frac{1 \times 10^{-3}}{10 \times 10^{-6}} = 100$$

$$h_{re} = \frac{V_{be}}{V_{ce}} = \frac{0.65 \times 10^{-3}}{1} = 0.65 \times 10^{-3}$$

$$h_{oe} = \frac{I_c}{V_{ce}} = \frac{60 \times 10^{-6}}{1} = 60\ \mu S$$

22.11 Limitations of *h* Parameters

The *h* parameter approach provides accurate information regarding the current gain, voltage gain, input impedance and output impedance of a transistor amplifier. However, there are two major limitations on the use of these parameters.

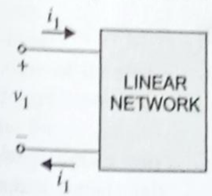
(*i*) It is very difficult to get the exact values of *h* parameters for a particular transistor. It is because these parameters are subject to considerable variation—unit to unit variation, variation due to change in temperature and variation due to change in operating point. In predicting an amplifier performance, care must be taken to use *h* parameter values that are correct for the operating point being considered.

(*ii*) The *h* parameter approach gives correct answers for small a.c. signals only. It is because a transistor behaves as a linear device for small signals only.

Short Answer Questions

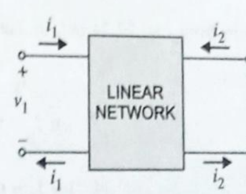
Q. 1. *What is a one-port network ?*

Ans. A one-port network is a two-terminal device or element (such as resistors, capacitors and inductors) through which a current may enter or leave the network. A one-port circuit is represented in Fig. 22.22.



One-port Network

Fig. 22.22



Two-port Network

Fig. 22.23

Q. 2. *What is a two-port network ?*

Ans. A two-port network is an electrical network with two separate ports for input and output. Thus a two-port network has four terminals—two for input and two for output as shown in Fig. 22.23. When voltage v_1 is applied to the input terminals, then input and output currents i_1 and i_2 flow and an output v_2 is produced. Three-terminal devices such as transistors can be arranged as a two-port network by making one of its terminals common to both input and output terminals.

Q. 3. *Why do we study two-port networks ?*

Ans. We study two-port networks for two main reasons. First, such networks are useful in communications, control systems, power systems and electronics. For example, they are used in electronics to model transistors and to facilitate cascaded design. Second, knowing the parameters of a two-port network enables us to treat it as a "black box" when embedded within a larger network.

Q. 4. *Why are h parameters of a two-port linear network called hybrid parameters ?*

Ans. Hybrid means mixed. The four parameters of a two-port linear circuit (*viz* h_{11}, h_{12}, h_{21} and h_{22}) do not have the same dimensions and hence they are called hybrid parameters.

Q. 5. *What are the h parameters h_{11} and h_{21} for the two-port network shown in Fig. 22.24 (i) ?*

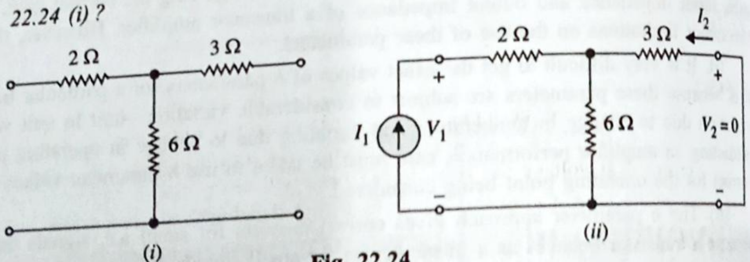


Fig. 22.24

Ans. To find h_{11} and h_{21}, we short-circuit the output and connect a current source I_1 to the input as shown in Fig. 22.24 (*ii*).

From Fig. 22.24 (*ii*), we have,

$$V_1 = I_1 (2 + 3 \parallel 6) = 4I_1$$

$$\therefore \quad h_{11} = \frac{V_1}{I_1} = 4\Omega$$

Again from Fig. 22.24 (*ii*), by current-divider rule

$$-I_2 = \frac{6}{6+3} I_1$$

$$\therefore \quad h_{21} = \frac{I_2}{I_1} = \frac{-2}{3}$$

Q. 6. *Why are hybrid parameters of a transistor named so ?*

Ans. Hybrid means "mixed". A transistor has four *h* parameters. One of them (input resistance) has units of ohms and the other (output conductance) has the units of siemens. The remaining two *h* parameters (reverse voltage feedback ratio and current gain) are pure numbers. Since *h* parameters have mixed units, they are called hybrid parameters.

Q. 7. *What are the two conditions for the measurement of h parameters ?*

Ans. A transistor has four *h* parameters. Two of them (*i.e.* input resistance and current gain) are measured with $V_{ce} = 0$ (*i.e.* output short-circuited to a.c.). The remaining two (*i.e.* reverse voltage feedback ratio and output conductance) are measured with $I_b = 0$ (*i.e.* input open-circuited to a.c.).

Q. 8. *While determining the h parameter input resistance (h_{ie}) of a transistor, why is output short-circuited ?*

Ans. You may recall that any resistance in the emitter circuit is reflected back to the base. This condition is described in the following equation :

$$Z_{in(base)} = h_{fe} (r_e' + R_E) \text{............See Art. 8.18}$$

By shorting the collector and emitter terminals, the measured value of input resistance does not reflect any external resistance in the circuit.

Q. 9. *While determining the h parameter current gain (h_{fe}) of a transistor, why is output short-circuited ?*

Ans. A shorted output represents full-load so current gain (h_{fe}) represents the gain under full-load conditions. If the output was left open, I_c would be zero. Shorting the output gives us a measurable value of I_c that can be reproduced in a particular test.

Q. 10. *While determining the h parameter reverse voltage feedback ratio (h_{re}), why is input open-circuited ?*

Ans. The reverse voltage feedback ratio (h_{re}) is the amount of output voltage that is reflected back to the input. This value is measured with the input open. The value of h_{re} (= V_{be}/V_{ce}) is always less than 1. By measuring h_{re} with input open, you ensure that voltage fed back to the base will always be at its maximum possible value because maximum voltage is always developed across an open-circuit.

Multiple-Choice Questions

1. Hybrid means ....
   (*i*) mixed (*ii*) single
   (*iii*) unique
   (*iv*) none of the above
2. There are .... *h* parameters of a transistor.
   (*i*) two (*ii*) four
   (*iii*) three
   (*iv*) none of the above
3. The *h* parameters approach gives correct results for ....
   (*i*) large signals only
   (*ii*) small signals only
   (*iii*) both small and large signals
   (*iv*) none of the above
4. A transistor behaves as a linear device for ....
   (*i*) small signals only
   (*ii*) large signals only
   (*iii*) both small and large signals
   (*iv*) none of the above
5. The parameter h_{ie} stands for input impedance in ....
   (*i*) *CB* arrangement with output shorted
   (*ii*) *CC* arrangement with output shorted
   (*iii*) *CE* arrangement with output shorted
   (*iv*) none of the above
6. The dimensions of h_{ie} parameter are ....
   (*i*) siemens (*ii*) ohms
   (*iii*) farads
   (*iv*) none of the above