Poisson's Eqn

Laplace Equation $\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Poisson's $\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$

Cauchy Residue theorem

$\int_c f(z)dz = 2\pi i$

Integration around unit Circle

$\sin\theta = e^{i\theta} - e^{-i\theta} / 2i \Rightarrow \frac{1}{2i}\left[z - \frac{1}{z}\right]$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \Rightarrow \frac{1}{2}\left[z + \frac{1}{z}\right]$

Integration round semi Circle

$\int_{-\infty}^{\infty} \frac{f_1(x)}{f_2(x)} = f(x)$

$\boxed{\ln z = \ln r + i(\theta + 2n\pi)}$ $\quad z = 1+i$

$r = \sqrt{2} \quad \theta = \tan^{-1} 1 \Rightarrow \frac{\pi}{4}$

$\ln\sqrt{2} + i\left(\pi/4 + 2\sqrt{2}\pi\right)$

Determine Bilinear Mapping

$z_1, z_2, z_3 \longrightarrow z$ plane

$w_1, w_2, w_3 \longrightarrow w$ plane

$\frac{w - w_1}{w - w_3} \times \frac{w_2 - w_3}{w_2 - w_1} = \frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}$

Unit - 2

Laurent's theorem

$f(z) = a_0 + a_1(z-a) + a_2(z-a)^2 + \dots + \frac{b}{(z-a)} + \frac{b_2}{(z-a)^2}$

$a_n = \frac{1}{2\pi i}\int \frac{f(w)}{(w-a)^{n+1}} dw$

$b_n = \frac{1}{2\pi i}\int \frac{f(w)}{(w-a)^{-n+1}}$

Ex $\Rightarrow$ $f(z) = \frac{1}{(z+1)(z+3)}$

$= \frac{1}{2}\left[\frac{1}{z-1} - \frac{1}{z+3}\right]$ make in form $(1-x)^{-1}$ $(1+x)^{-1}$

$= \frac{1}{2}\left[\frac{1}{z(1-\frac{1}{z})} - \frac{1}{3}\frac{1}{(\frac{z}{3}+1)^{-1}}\right]$ Take Bigger term as common

$= \frac{1}{2z}\left(1 - \frac{1}{z}\right)^{-1} - \frac{1}{6}\left[1 + \frac{z}{3}\right]^{-1}$

Residue of complex func^n

$f(z) = a$ be a pole of order 'm'

$f(z) = \sum_{n=0}^{\infty} a_n (z-a) + \sum_{n=1}^{\infty} b_n (z-a)^{-n}$

(i) Residue at a

$\lim_{z \to a} (z-a) f(z)$

(ii) Residue of pole of order 'n'

$\frac{1}{(n-1)!}\left\{\frac{d^{n-1}}{dz^{n-1}}\left[(z-a)^n f(z)\right]\right\}$

(iii) Residue of pole of order 'm'

$\frac{1}{(m-1)!}\left\{\lim_{z \to z_0} \frac{d^{m-1}}{dz^{m-1}}\left\{(z-z_0)^m f(z)\right\}\right\}$