1

MECHANISMS

Chapter Outline

1.1 INTRODUCTION

A mechanism is a set of machine elements or components or parts arranged in a specific order to produce a specified motion. The machine elements or components are considered rigid or resistant bodies that do not deform under the action of forces. Resistant bodies are bodies that do not suffer appreciable distortion or change in physical form due to forces acting on them, e.g. springs, belts, and fluids. Elastic bodies are also resistant bodies. They are capable of transmitting the required forces with negligible deformation. Rigid bodies are bodies that do not deform under the action of forces. All resistant bodies are considered rigid bodies for the purpose of transmitting motion. In this chapter, we shall study the different ways of connecting rigid (resistant) bodies to obtain various types of mechanisms.

Kinematics is a subject that deals with the study of relative motion of parts constituting a machine, neglecting forces producing the motion. A structure is an assemblage of a number of resistant bodies meant to take up loads or subjected to forces having straining actions, but having no relative motion between its members. Frame is a structure that supports the moving parts of a machine.

1.2 KINEMATIC JOINT

A kinematic joint is the connection between two links by a pin. There is clearance between the pin and the hole in the ends of the links being connected so that there is free motion of the links.

1.2.1 Type of Kinematic Joints

The type of kinematic joints generally used in mechanisms are:

1. *Binary joint*: In a binary joint, two links are connected at the same joint by a pin, as shown in Fig.1.1(a).
2. *Ternary joint*: In a ternary joint, three links are connected at the same joint by a pin. It is equivalent to two binary joints. In Fig.1.1(b), joints B and C are ternary joints and others are binary joints.
3. *Quaternary joint*: When four links are connected at the same joint by a pin, it is called a quaternary joint. One quaternary joint is equivalent to four binary joints. In Fig.1.1(c), joint B is a quaternary joint; A, C, E, F are ternary joints; and D is a binary joint.

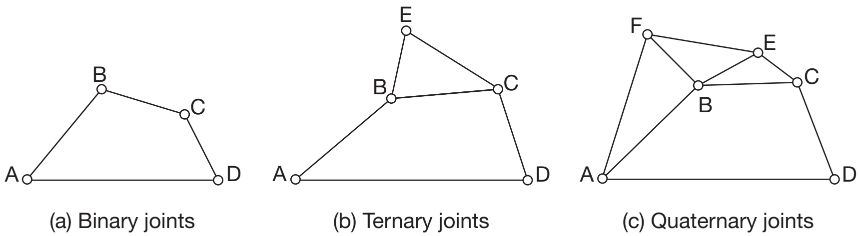


Fig.1.1 Type of kinematic joints

1.3 ELEMENTS OR LINKS

A link (or element or kinematic link) is a resistant body (or assembly of resistant bodies) constituting a part (or parts) of the machine, connecting other parts, which have motion, relative to it. A slider crank mechanism of an internal combustion engine, shown in Fig.1.2, consists of four links, namely, (1) frame, (2) crank, (3) connecting rod and (4) slider.

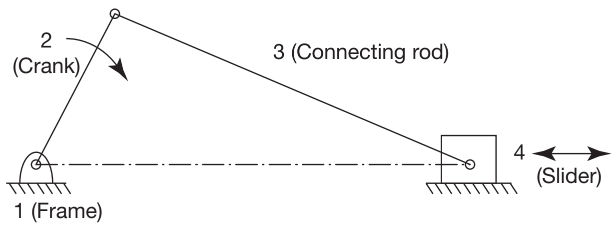
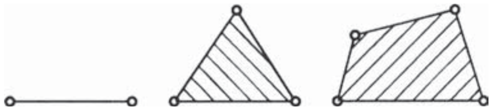


Fig.1.2 Kinematic links of a slider crank mechanism

1.3.1 Classification of Links

Links can be classified as binary, ternary, or quaternary depending upon the ends on which revolute or turning pairs can be placed, as shown in Fig.1.3. A binary link has two vertices, a ternary has three vertices, and a quaternary link has four vertices, and so on.



(a) Binary link (b) Ternary link (c) Quaternary link

Fig.1.3 Types of links

There are four types of links: rigid, flexible, fluid, and floating links.

- *Rigid link*: A rigid link does not undergo any deformation while transmitting motion. Links in general are elastic in nature. They are considered rigid if they do not undergo appreciable deformation while transmitting motion, e.g. connecting rod, crank, tappet rod, etc.
- *Flexible link*: A flexible link is one which while transmitting motion is partly deformed in a manner not to affect the transmission of motion, e.g. belts, ropes, chains, springs, etc.
- *Fluid link*: A fluid link is deformed by having fluid in a closed vessel and the motion is transmitted through the fluid by pressure, as in the case of a hydraulic press, hydraulic jack, and fluid brake.
- *Floating link*: It is a link which is not connected to the frame.

1.4 KINEMATIC PAIR

The two links of a machine, when in contact with each other, are said to form a pair. A kinematic pair consists of two links that have relative motion between them. In Fig.1.2, links 1 and 2, 2 and 3, 3 and 4, and 4 and 1 constitute kinematic pairs.

1.4.1 Classification of Kinematic Pairs

Kinematic pairs may be classified according to the following considerations:

- Type of relative motion
- Type of contact
- Type of mechanical constraint.

1. Kinematic Pairs According to the Relative Motion

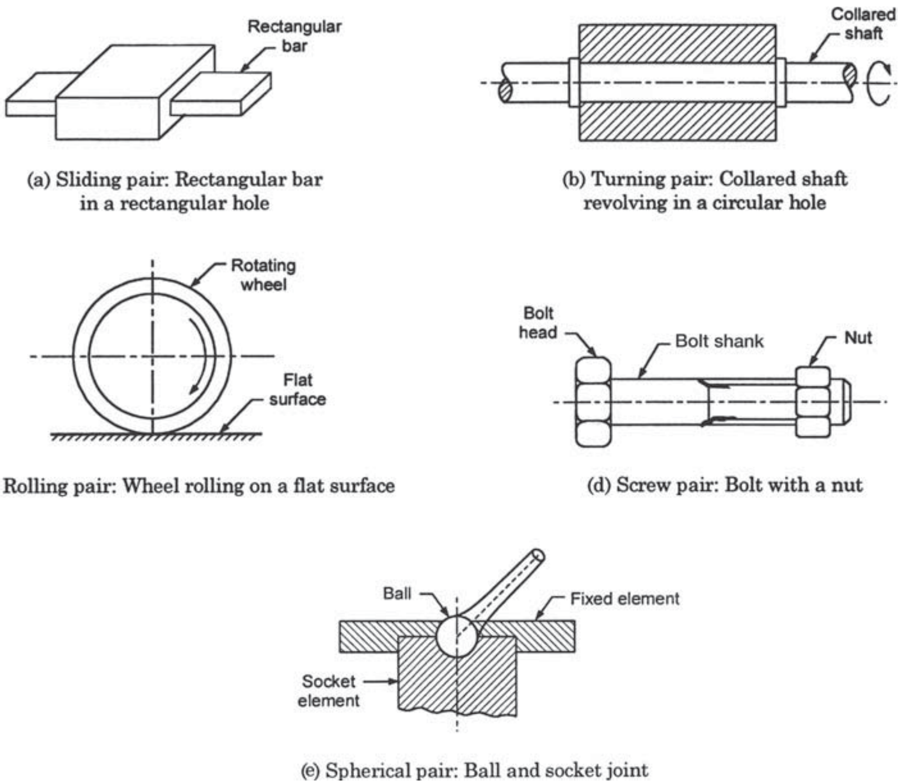
- *Sliding pair*: It consists of two elements connected in such a manner that one is constrained to have sliding motion relative to another. For example, a rectangular bar in a rectangular hole (Fig.1.4(a)), piston and cylinder of an engine, cross-head and guides of a steam engine, ram and its guides in a shaper, tailstock on the lathe bed, etc. all constitute sliding pairs.
- *Turning (revolute) pair*: It consists of two elements connected in such a manner that one is constrained to turn or revolve about a fixed axis of another element. For example, a shaft with

collar at both ends revolving in a circular hole (Fig.1.4(b)) crankshaft turning in a bearing, cycle wheels revolving over their axles, etc. all constitute turning pairs.

- *Rolling pairs*: When two elements are so connected that one is constrained to roll on another element which is fixed, forms a rolling pair. Ball and roller bearings, a wheel rolling on a flat surface (Fig.1.4(c)) are examples of rolling pairs.
- *Screw (or helical) pair*: When one element turns about the other element by means of threads, it forms a screw pair. The motion in this case is a combination of sliding and turning. The lead screw of a lathe with nut, bolt with a nut Fig.1.4(d), screw with nut of a jack, etc. are some examples of screw pairs.
- *Spherical pair*: When one element in the form of a sphere turns about the other fixed element, it forms a spherical pair. The ball and socket joint Fig.1.4(e), pen stand, the mirror attachment of vehicles, etc. are some examples of spherical pair.

2. Kinematic Pairs According to the Type of Contact

   - *Lower pair*: When the two elements have surface (or area) contact while in motion and the relative motion is purely turning or sliding, they are called a lower pair. All sliding pairs, turning pairs, and screw pairs form lower pairs. For example, nut turning on a screw, shaft rotating in a bearing, universal joint, all pairs of a slider crank mechanism, pantograph etc., are lower pairs.



(a) Sliding pair: Rectangular bar in a rectangular hole

(b) Turning pair: Collared shaft revolving in a circular hole

(c) Rolling pair: Wheel rolling on a flat surface

(d) Screw pair: Bolt with a nut

(e) Spherical pair: Ball and socket joint

Fig.1.4 Types of kinematic pairs according to the type of relative motion

- *Higher pairs*: When the two elements have point or line contact while in motion and the relative motion being the combination of sliding and turning, then the pair is known as a higher pair. Belts, ropes, and chains drive, gears, cam and follower, ball and roller bearings, wheel rolling on a surface, etc., all form higher pairs.

3. Kinematic Pairs According to the Type of Mechanical Constraint

- *Closed pair*: When the two elements of a pair are held together mechanically in such a manner that only the required type of relative motion occurs, they are called a closed pair. All lower pairs and some higher pairs (e.g. enclosed cam and follower) are closed pairs (Fig.1.5(a)).
- *Unclosed pair*: When the two elements of a pair are not held mechanically and are held in contact by the action of external forces, are called unclosed pair, e.g. cam and spring loaded follower pair (Fig.1.5(b)).

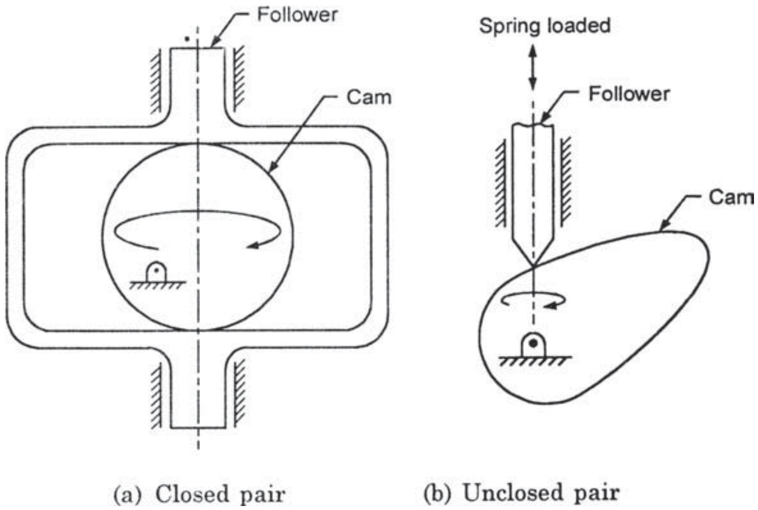


Fig.1.5 Closed and unclosed pairs

1.5 CONSTRAINED MOTION

The three types of constrained motion are as follows:

- *Completely constrained motion*: When the motion between a pair takes place in a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, a square bar in a square hole, a shaft with collars at each end in a circular hole, a piston in the cylinder of an internal combustion engine, have all completely constrained motion.
- *Partially (or successfully) constrained motion*: When the constrained motion between a pair is not completed by itself but by some other means, it is said to be successfully constrained motion. For example, the motion of a shaft in a footstep bearing becomes successfully constrained motion when compressive load is applied to the shaft (Fig.1.6(a)).
- *Incompletely constrained motion*: When the motion between a pair can take place in more than one direction, it is said to be incompletely constrained motion, e.g. a circular shaft in a circular hole. (Fig.1.6(b)).

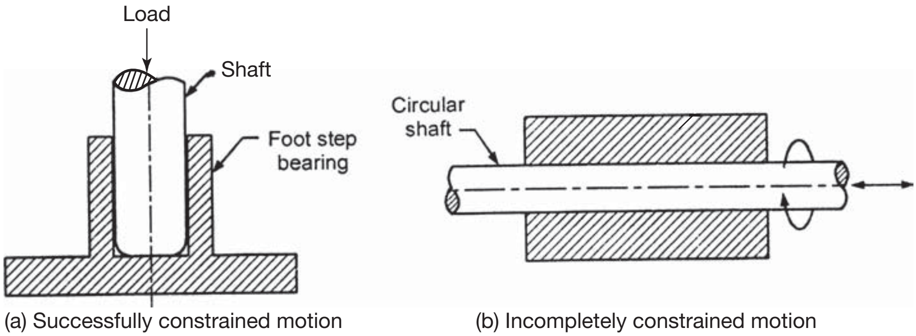


Fig.1.6 Types of constrained motion

1.6 KINEMATIC CHAIN

A kinematic chain may be defined as an assembly of links in which the relative motion of the links is possible and the motion of each relative to the others is definite. The last link of the kinematic chain is attached to the first link. The four-bar mechanism and the slider crank mechanism are some of the examples of a kinematic chain.

The following relationship holds for a kinematic chain having lower pairs only:

$$L = 2P - 4 \tag{1.1a}$$

$$J = 3L/2 - 2 \tag{1.1b}$$

where L = number of binary links
P = number of lower pairs
J = number of binary joints.

If LHS > RHS, then chain is called locked chain or redundant chain.
LHS = RHS, then chain is constrained
LHS < RHS, then chain is unconstrained

For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link. In that case,

$$J + \frac{H}{2} = \frac{3}{2}L - 2 \tag{1.1c}$$

where H = number of higher pairs.

Example 1.1

A chain with three links is shown in Fig.1.7. Prove that the chain is locked.

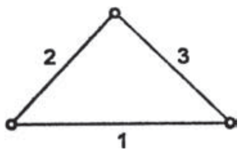


Fig.1.7 Three-bar chain

■ Solution

Number of binary joints, $J = 3$
Number of binary links, $L = 3$
Number of lower pairs, $P = 3$

Now $$L = 2P - 4$$

$$3 = 2 \times 3 - 4 = 2$$

$\therefore$ $$\text{LHS} > \text{RHS}$$

Also $$J = \left(\frac{3}{2}\right) L - 2$$

$$3 = \left(\frac{3}{2}\right) \times 3 - 2 = 2.$$

$\therefore$ $$\text{LHS} > \text{RHS}$$

Therefore, it is a locked chain.

Example 1.2

A four-bar chain is shown in Fig.1.8. Prove that it is a constrained chain.

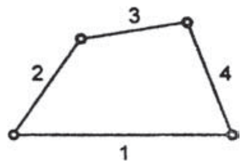


Fig.1.8 Four-bar chain

■ Solution

Number of binary joints, $J = 4$
Number of binary links, $L = 4$
Number of lower pairs, $P = 4$

Now $$L = 2P - 4$$

$$4 = 2 \times 4 - 4 = 4$$

$\therefore$ $$\text{LHS} = \text{RHS}$$

Also $$J = \left(\frac{3}{2}\right) L - 2$$

$$4 = \left(\frac{3}{2}\right) \times 4 - 2 = 4$$

$\therefore$ $$\text{LHS} = \text{RHS}$$

Therefore, it is a constrained chain.

Example 1.3

A five-bar chain is shown in Fig.1.9. Prove that it is an unconstrained chain.

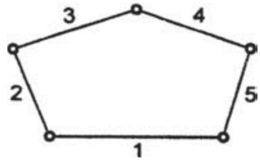


Fig.1.9 Five-bar chain

■ Solution

Number of binary joints, $J = 5$
Number of binary links, $L = 5$
Number of lower pairs, $P = 5$

Now
$$L = 2P - 4$$
$$5 = 2 \times 5 - 4 = 6$$
∴
$$\text{LHS} < \text{RHS}$$

Also
$$J = \left(\frac{3}{2}\right) L - 2$$
$$5 = \left(\frac{3}{2}\right) \times 5 - 2 = 5.5$$
∴
$$\text{LHS} < \text{RHS}$$

Therefore, it is an un-constrained chain.

Example 1.4

Show that the chain shown in Fig.1.10 is an unconstrained kinematic chain.

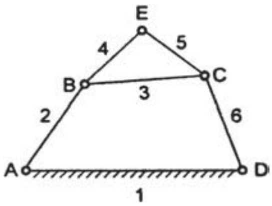


Fig.1.10 Six-bar chain

■ Solution

Number of binary joints, $J = 7$ ($A = 1, B = 2, C = 2, D = 1, E = 1$)
Number of binary links, $L = 6$
Number of lower pairs, $P = 9$ ($1 - 2, 2 - 3, 3 - 4, 2 - 4, 4 - 5, 5 - 3, 5 - 6, 3 - 6, 1 - 6$)

Now $$L = 2P - 4$$
$$6 = 2 \times 9 - 4 = 14$$
$\therefore$ $$\text{LHS} < \text{RHS}$$
Also $$J = \left(\frac{3}{2}\right) L - 2$$
$$7 = \left(\frac{3}{2}\right) \times 6 - 2 = 7$$
$\therefore$ $$\text{LHS} < \text{RHS}$$
Therefore, it is an un-constrained chain.

Example 1.5

Show that the chain shown in Fig.1.11 is not a kinematic chain.

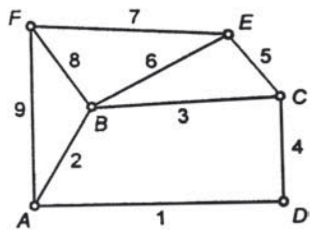


Fig.1.11 Nine-bar chain

■ Solution

Number of binary joints, $J = 13$ ($A = 2$, $B = 4$, $C = 2$, $D = 1$, $E = 2$, $F = 2$)
Number of binary links, $L = 9$
Number of lower pairs, $P = 13$ ($D = 1$, A, C, E, $F = 2$ each, $B = 4$)

Now $$L = 2P - 4$$
$$9 = 2 \times 13 - 4 = 22$$
$$\text{LHS} < \text{RHS}$$
Also $$J = \left(\frac{3}{2}\right) L - 2$$
$$13 = \left(\frac{3}{2}\right) \times 9 - 2 = 11.5$$
$$\text{LHS} > \text{RHS}$$
Therefore, it is not a kinematic chain. It is a locked chain or a frame.

Example 1.6

Determine the type of chain in Fig.1.12(a)–(e).

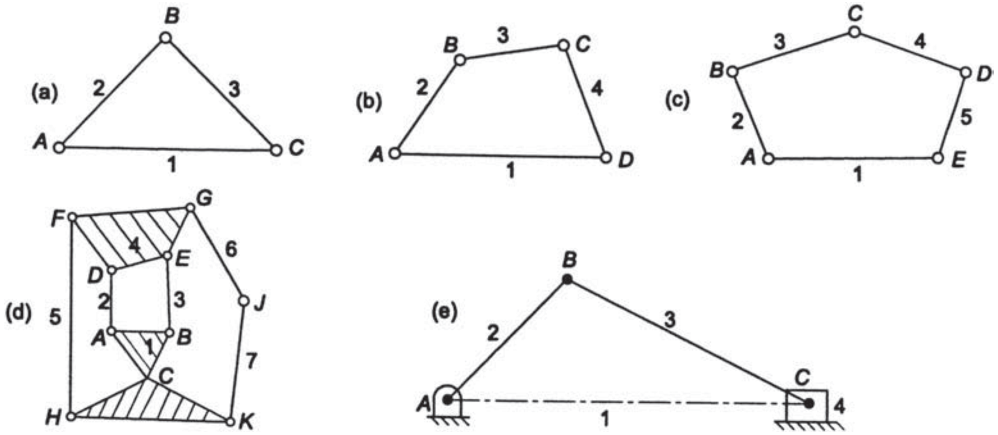


Fig.1.12 Different types of chains

■ **Solution**

(a) (i) $L = 2P - 4$ (ii) $J = 3L/2 - 2$

$L = 3, P = 3, J = 3$

LHS = 3	LHS = 3
RHS = $2 \times 3 - 4 = 2$	RHS = $3 \times 3/2 - 2 = 2.5$
LHS > RHS	LHS > RHS

It is a locked chain and not a kinematic chain.

(b) $L = 4, P = 4, J = 4$

LHS = 4	LHS = 4
RHS = $2 \times 4 - 4 = 4$	RHS = $3 \times 4/2 - 2 = 4$
LHS = RHS	LHS = RHS

It is a constrained kinematic chain.

(c) $L = 5, P = 5, J = 5$

LHS = 5	LHS = 5
RHS = $2 \times 5 - 4 = 6$	RHS = $3 \times 5/2 - 2 = 5.5$
LHS < RHS	LHS < RHS

It is an unconstrained chain and not a kinematic chain.

(d) $L = 6, P = 5, J = 7$

LHS = 6	LHS = 7
RHS = $2 \times 5 - 4 = 6$	RHS = $3 \times 6/2 - 2 = 7$
LHS = RHS	LHS = RHS

It is a constrained kinematic chain.

(e) $L = 4, P = 4, J = 4$

LHS = 4	LHS = 4
RHS = $2 \times 4 - 4 = 4$	RHS = $3 \times 4/2 - 2 = 4$
LHS = RHS	LHS = RHS

It is a constrained kinematic chain.

1.7 MECHANISM

When one of the links of a kinematic chain is fixed, the chain is called a mechanism.

1.7.1 Types of Mechanisms

The mechanisms are of the following types:

- *Simple mechanism*: A mechanism which has four links.
- *Compound mechanism*: A mechanism which has more than four links.
- *Complex mechanism*: It is formed by the inclusion of ternary or higher order floating link to a simple mechanism.
- *Planar mechanism*: When all the links of the mechanism lie in the same plane.
- *Spatial mechanism*: When the links of the mechanism lie in different planes.

1.7.2 Equivalent Mechanisms

Turning pairs of plane mechanisms may be replaced by other types of pairs such as sliding pairs or cam pairs. The new mechanism thus obtained having the same number of degrees of freedom as the original mechanism is called the equivalent mechanism. The equivalent mechanism will have same degrees of freedom and shall be kinematically similar.

The following rules may be used to obtain the equivalent mechanism:

1. A sliding pair is equivalent to a turning pair, as shown in Fig.1.13(a).
2. A spring can be replaced by two binary links, as shown in Fig.1.13(b).
3. A cam pair can be replaced by one binary link together with two turning pairs at each end, as shown in Fig.1.13(c).

1.8 MECHANISM AND MACHINES

A machine is a device that transforms energy available in one form to another to do certain type of desired useful work. The parts of the machine move relative to one another. Its links may transmit both power and motion. On the other hand, a mechanism is a combination of rigid or restraining bodies, which are so shaped and connected that they move upon each other with definite relative motion. A mechanism is obtained when one of the links of the kinematic chain is fixed. A machine is a combination of two or more mechanisms arranged in such a way so as to obtain the required motion and transfer the energy to some desired point by the application of energy at some other convenient point. A machine is not able to move itself and must get the motive power from some source.

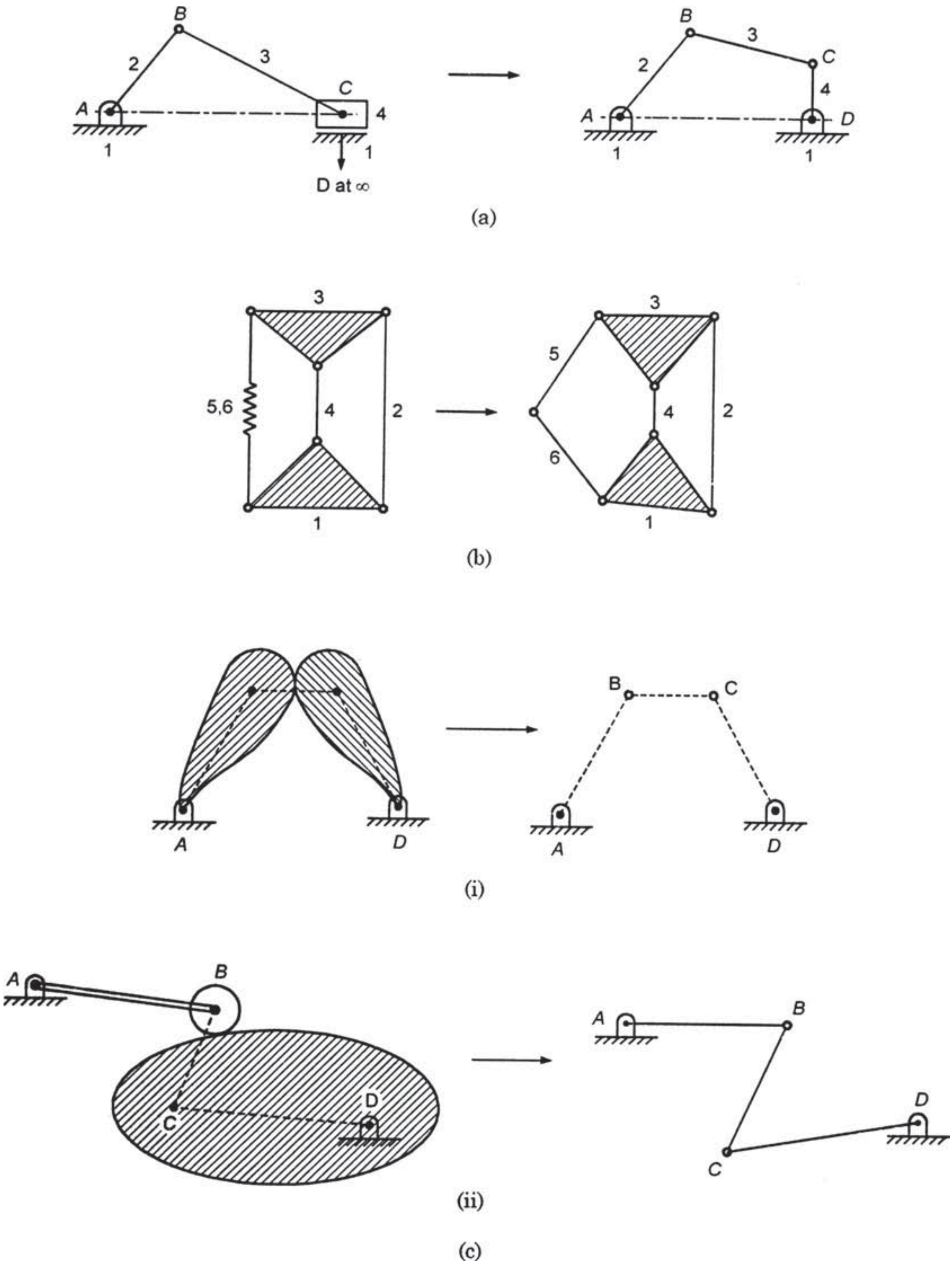


Fig.1.13 Equivalent mechanisms

Some examples of mechanisms are: slider crank, typewriter, clocks, watches, spring toys, etc. Steam engine, internal combustion engine, lathe, milling machine, drilling machine, etc. are some examples of machines.

1.8.1 Classification of Machines

The machines may be classified as the following:

- *Simple machine*: In a simple machine, there is one point of application for the effort and one point for the load to be lifted. Some examples of simple machines are lever, screw jack, inclined plane, bicycle, etc.
- *Compound machine*: In a compound machine, there are more than one point of application for the effort and the load. It may be thought of as a combination of many simple machines. Some examples of compound machines are lathe machine, grinding machine, milling machine, printing machine, etc.

1.9 DEGREES OF FREEDOM

An unconstrained rigid body moving in space can have three translations and three rotational motions (i.e. six motions) about the three mutually perpendicular axes. Degrees of freedom of a kinematic pair is defined as the number of independent relative motions, both translational and rotational, a kinematic pair can have.

$$\text{Degrees of freedom} = 6 - \text{number of restraints} \tag{1.2}$$

The degrees of freedom of some of the systems are as follows:

- A rigid body has 6 degrees of freedom.
- A rectangular bar sliding in a rectangular hole has one degree of freedom as the motion can be expressed by the linear displacement only.
- The position of the crank of a slider crank mechanism can be expressed by the angle turned through and thus has one degree of freedom.
- A circular shaft rotating in a hole and also translating parallel to its axis has two degrees of freedom, i.e. angle turned through and displacement.
- A ball and a socket joint has three degrees of freedom.

1.9.1 Degrees of Freedom of Planar Mechanisms

- *Mobility of a mechanism*: The mobility of a mechanism is defined as the number of degrees of freedom it possesses. An equivalent definition of mobility is the minimum number of independent parameters required to specify the location of every link within a mechanism.
- *Kutzbach criterion*: The Kutzbach criterion for determining the number of degrees of freedom of a planar mechanism is:

$$F = 3(n - 1) - 2p - h \tag{1.3}$$

where F = degrees of freedom
n = total number of links in a mechanism out of which one is a fixed link.

$n - 1$ = number of movable links
p = number of simple joints or lower pairs having one degree of freedom
h = number of higher pairs having two degrees of freedom and so on.

When two links are joined by a hinge, two degrees of freedom are lost. Hence for each joint two degrees of freedom are lost. Therefore, for p number of joints the number of degrees of freedom lost are $2p$. When a kinematic chain is made up of different type of links, then the number of lower pairs p is computed as follows:

$$p = (1/2)\,[2n_2 + 3n_3 + 4n_4 + \cdots\cdots] \tag{1.4}$$

where n_2 = number of binary links
n_3 = number of ternary links, and so on.

To determine the degrees of freedom of a mechanism, the presence of a redundant link or redundant pair may also be considered.

(i) A mechanism may have one or more links which do not introduce any extra constraint. Such links are called redundant links (n_r) and should not be taken into account. Similarly redundant joints (p_r) should also not be taken into account.

In Fig.1.14(a), links 3 and 4 are parallel and are termed as redundant links, as none of them produces extra constraint. By removing one of the two links, the motion remains the same. So one of the two links is considered for calculating the degrees of freedom.

The corresponding kinematic pairs either between links 4 and 2, and 4 and 5; or 3 and 2, and 3 and 5 are considered as redundant pair. Therefore, either of the two links and the corresponding kinematic pair should be considered while calculating the degrees of freedom.

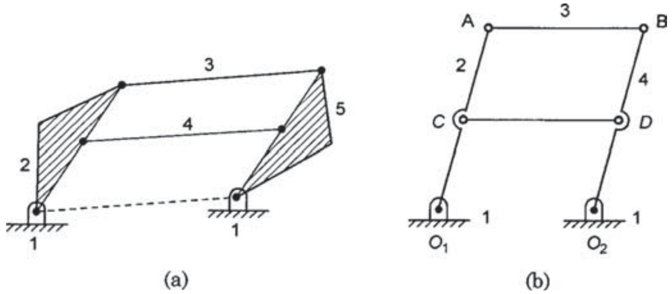


Fig.1.14

In Fig.1.14(b), links AB and CD are identical and each leads to same constraint.

(ii) Sometimes one or more links of a mechanism may have redundant degrees of freedom. If a link can be moved without causing any movement in the rest of the mechanism then the link is said to have redundant degree of freedom (F_r).

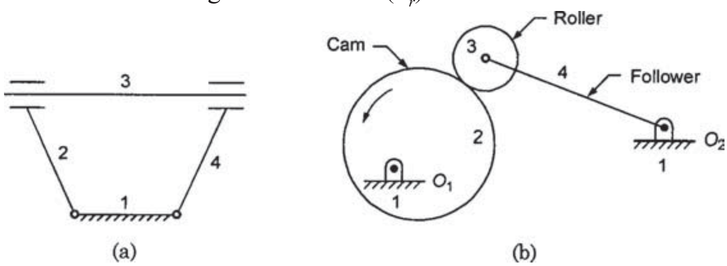


Fig.1.15

In Fig.1.15(a), the link 3 can slide without causing any movement to the mechanism. Thus link 3 represents one redundant degree of freedom. In Fig.1.15(b), roller can rotate without causing any movement in the rest of the mechanism.

Thus Eq. (1) can be modified as:

$$F = 3\,(n - n_r - 1) - 2(p - p_r) - h - F_r \tag{1.5}$$

1.9.2 Planar Mechanisms with Lower Pairs Only

For linkages with lower pairs only, $h = 0$, and

$$F = 3(n - 1) - 2p \tag{1.6}$$

A joint connecting k links at a single joint must be counted as $(k - 1)$ joints. Only four types of joints are commonly found in planar mechanisms. These are the revolute, the prismatic, the rolling contact joints (each having one degree of freedom), and the cam or gear joint (each having two degrees of freedom). These joints are depicted in Fig.1.16. The following definitions apply to the actual degrees of freedom of a device.

$F \geq 1$: the device is a mechanism with F degrees of freedom.
$F = 0$: the device is a statically determinate structure.
$F < -1$: the device is a statically indeterminate structure.

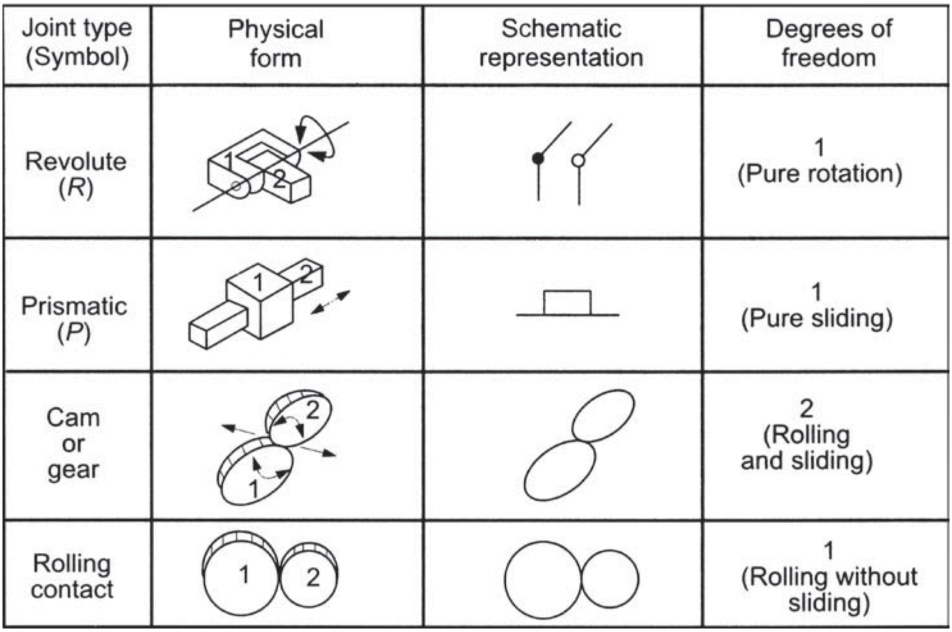


Fig.1.16 Common types of joints found in planar mechanisms

The degrees of freedom of some of the planar mechanisms have been listed in Table 1.1.

Table 1.1 Degrees of Freedom of Planar Mechanisms

Mechanism	n	p	h	$F = 3(n - 1) - 2p - h$
1. Three – bar	3	3	0	0
2. Four – bar	4	4	0	1
3. Five – bar	5	5	0	2
4. Five – bar	5	6	0	0
5. Six – bar	6	8	0	–1
6. Four – bar	4	5	0	–1
7. Three – bar	3	2	1	1
8. Four – bar	4	3	1	2
9. Five – bar	5	11/2	0	1
10. Six – bar	6	7	0	1

Gruebler's criterion: For a constrained motion, $F = 1$, so that

$$1 = 3(n - 1) - 2p - h$$

or

$$2p + h - 3n + 4 = 0 \tag{1.7}$$

Eq. (1.7) represents the Gruebler's criterion.
If $h = 0$, then

$$p = 3n/2 - 2 \tag{1.8}$$

Therefore, a planar mechanism with $F = 1$ and having only lower pairs, cannot have odd number of links.
Eq. (1.8) is similar to Eq. (1.1 b) with $p = J$ and $n = L$. As p and n are to be whole numbers, the relation can be satisfied only if n is even.

For possible linkages made of binary links only,

$n = 4, p = 4$ No excess turning pair
$n = 6, p = 7$ One excess turning pair
$n = 8, p = 10$ Two excess turning pair

and so on.

Thus, we find that the number of excess turning pairs increase as the number of links increase. To get the required number of turning pairs from the same number of binary links is not possible. Therefore, the additional pairs or joints can be obtained only from the links having more than two joining points, i.e., ternary or quaternary links, etc.

1.10 FOUR-BAR CHAIN

A four-bar chain has been shown in Fig.1.17. It consists of four binary links. Link AD is fixed (called frame), AB is the crank (or driver link), BC is the coupler (or connecting rod), and CD the lever (or rocker or follower link). θ is the input angle and ϕ the angle of transmission. The coupler BC may be a ternary link. The number of degrees of freedom of the four-bar chain is one.

A link that makes complete revolutions is the *crank,* the link opposite to the fixed link is the *coupler,* and the fourth link a *lever* or *rocker*, if it oscillates or another *crank*, if it rotates.

The four-bar mechanism with all its pairs as turning pairs is called the "quadric cycle chain." When one of these turning pairs is replaced by a slider pair, the chain becomes "single slider chain." When two turning pairs are replaced by slider pairs, it is called a "double slider chain" or a "crossed double slider chain," depending on whether the two slider pairs are adjacent or crossed.

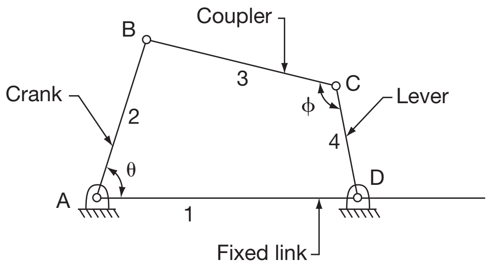


Fig.1.17 Four-bar chain

Example 1.7

Calculate the number of degrees of freedom of the linkages shown in Fig.1.18(a) and (b).

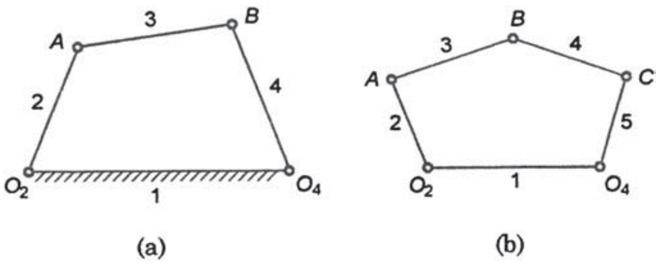


Fig.1.18 Four- and five-bar chains

■ Solution

(a) Number of binary links, $n = 4$

Number of lower pairs, $p = 4$

Degrees of freedom, $F = 3(n - 1) - 2p$

$= 3(4 - 1) - 2 \times 4 = 1$

(b) Number of binary links, $n = 5$

Number of lower pairs, $p = 5$

Degrees of freedom, $F = 3(n - 1) - 2p$

$= 3(5 - 1) - 2 \times 5 = 2$

Example 1.8

Calculate the number of degrees of freedom of the linkages shown in Fig.1.19(a) to (c).

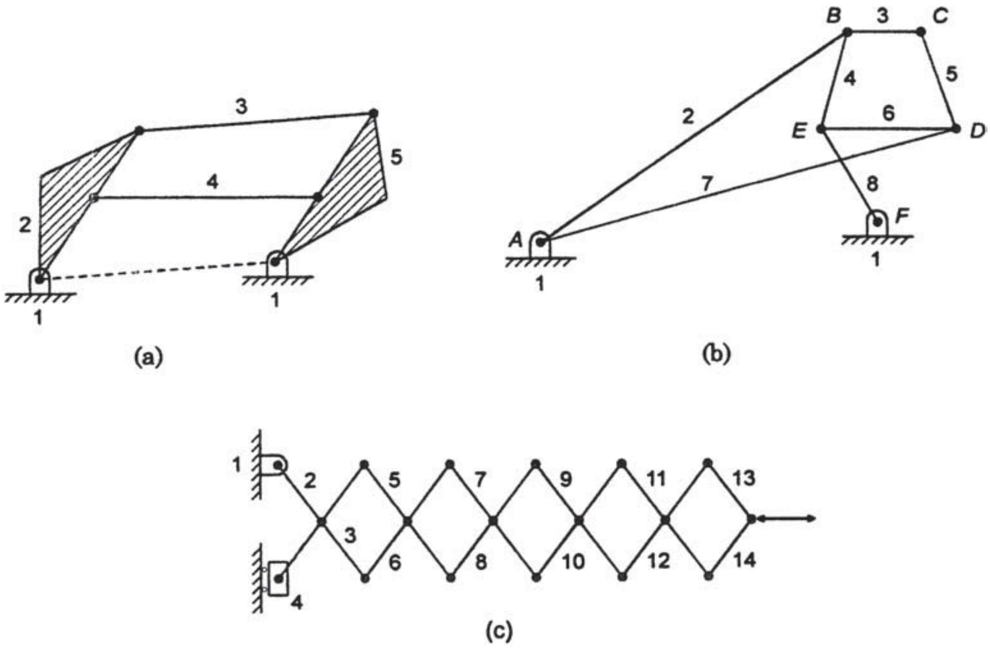


Fig.1.19 Various types of linkages

■ **Solution**

(a) In Fig.1.19(a), links 3 and 4 are parallel and are termed as redundant links, as one of them produces extra constraint. By removing one of these two links, the motion remains same. So one of the two links is considered for calculating the degrees of freedom.

Number of binary links, $n = 4$

Number of lower pairs, $p = 4$

Degrees of freedom, $F = 3\,(n - 1) - 2p$

$= 3\,(4 - 1) - 2 \times 4$

$= 9 - 8 = 1$

(b) Number of binary links, $n = 8$

Number of simple joints:

Binary joints at C and $F = 2$

Ternary joints at A, B, D and $E = 4$

$P = 2 + 2 \times 4 = 10$

Degrees of freedom, $F = 3\,(n - 1) - 2p$

$= 3\,(8 - 1) - 2 \times 10$

$= 21 - 20 = 1$

(c) Number of binary links, $n = 14$

Number of lower pairs, $p = 18$

Number of higher pairs, $h = 1$
(Slider can rotate and slide)

Degrees of freedom, $F = 3\,(n - 1) - 2p - h$

$= 3\,(14 - 1) - 2 \times 18 - 1$

$= 39 - 36 - 1 = 2$

Example 1.9

Determine the number of degrees of freedom of the mechanism shown in Fig.1.20(a) to (f).

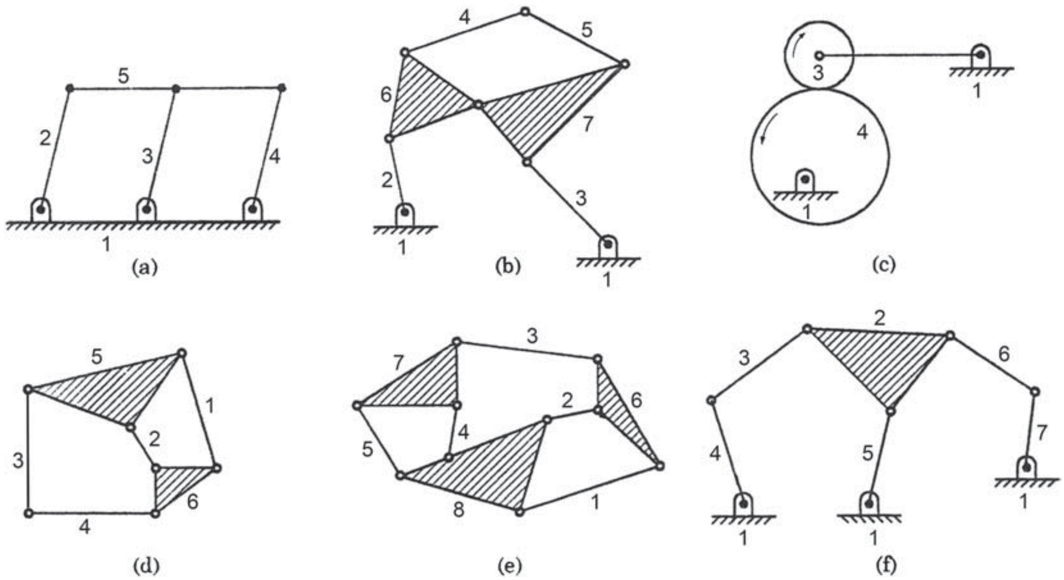


Fig.1.20 Various types of mechanisms

■ Solution

(a) Number of binary links, $n_2 = 3$

Number of ternary links, $n_3 = 2$

Total number of links, $n = n_2 + n_3 = 3 + 2 = 5$

$$2p = 2n_2 + 3n_3 = 2 \times 3 + 3 \times 2 = 12$$

Degrees of freedom, $F = 3(n-1) - 2p$

$$= 3(5-1) - 12$$

$$= 12 - 12 = 0$$

It is a structure.

(b) Number of binary links, $n_2 = 5$

Number of ternary links, $n_3 = 2$

Total number of links, $n = n_2 + n_3 = 5 + 2 = 7$

$$2p = 2n_2 + 3n_3 = 2 \times 5 + 3 \times 2 = 16$$

Degrees of freedom, $F = 3(n-1) - 2p$

$$= 3(7-1) - 16$$

$$= 18 - 16 = 2$$

(c) The roller 3 carried at the end of the output link 2 can be rotated without causing any motion in the rest of the mechanism. Thus roller 3 is a link with redundant degree of freedom. The roller can thus be considered welded to the output link. Hence, there are three binary links 1, 2 and 4 together with two turning pairs 12, 14 and one higher pair 34. Thus

$$n = 3, p = 2, h = 1$$

Degrees of freedom, $F = 3(n-1) - 2p - h$

$$= 3(3-1) - 2\times 2 - 1$$
$$= 6 - 4 - 1 = 1$$

(d) Number of binary links, $n_2 = 4$

Number of ternary links, $n_3 = 2$

Total number of links, $n = n_2 + n_3 = 4 + 2 = 6$

$$2p = 2n_2 + 3n_3 = 2\times 4 + 3\times 2 = 14$$
$$F = 3(n-1) - 2p$$
$$= 3(6-1) - 14$$
$$= 15 - 14 = 1$$

(e) Number of binary links, $n_2 = 5$

Number of ternary links, $n_3 = 2$

Number of quaternary links, $n_4 = 1$

Total number of links, $n = n_2 + n_3 + n_4 = 5 + 2 + 1 = 8$

$$2p = 2n_2 + 3n_3 + 4n_4 = 2\times 5 + 3\times 2 + 4\times 1 = 20$$
$$F = 3(n-1) - 2p$$
$$= 3(8-1) - 20$$
$$= 21 - 20 = 1$$

(f) Number of binary links, $n_2 = 5$

Number of ternary links, $n_3 = 2$

Total number of links, $n = n_2 + n_3 = 5 + 2 = 7$

$$2p = 2n_2 + 3n_3 = 2\times 5 + 3\times 2 = 16$$
$$F = 3(n-1) - 2p$$
$$= 3(7-1) - 16$$
$$= 18 - 16 = 2$$

Example 1.10

Determine the number of degrees of freedom of the mechanism shown in Fig.1.21(a) to (f).

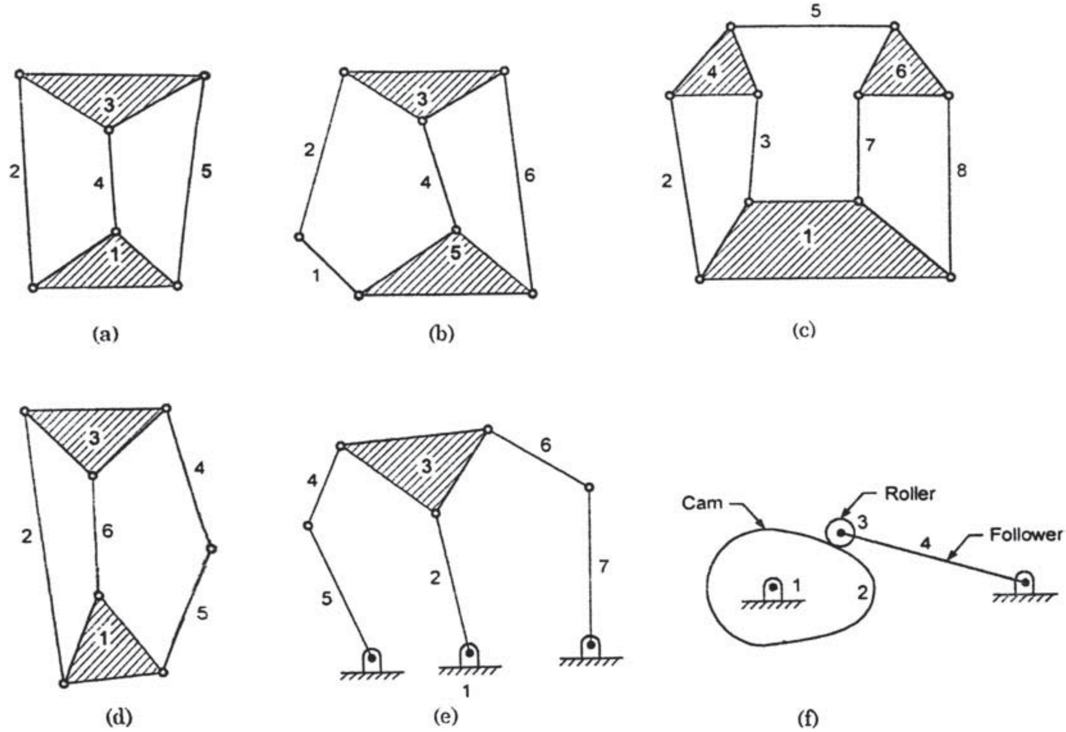


Fig.1.21 Various types of mechanisms

■ Solution

(a) $n_2 = 3, n_3 = 2, n = 5$

$2p = 2 \times 3 + 3 \times 2 = 12$

$F = 3(5 - 1) - 12 = 0$

(b) $n_2 = 4, n_3 = 2, n = 6$

$2p = 2 \times 4 + 3 \times 2 = 14$

$F = 3(6 - 1) - 14 = 1$

(c) $n_2 = 5, n_3 = 2, n_4 = 1, n = 8$

$2p = 2 \times 5 + 3 \times 2 + 4 + 1 = 20$

$F = 3(8 - 1) - 20 = 1$

(d) $n_2 = 4, n_3 = 2, n = 6$

$2p = 2 \times 4 + 3 \times 2 = 14$

$F = 3(6 - 1) - 14 = 1$

(e) $n_2 = 5, n_3 = 2, n = 5 + 2 = 7$

$2p = 2n_2 + 3n_3 = 2\times5+3\times2=16$

$$F = 3(n-1)-2p$$
$$= 3(7-1)-16$$
$$= 18-16 = 2$$

(f) The solution has been given in Example 1.9(c).

Example 1.11

Determine the number of degrees of freedom of the mechanism shown in Fig.1.20 (a)–(h).

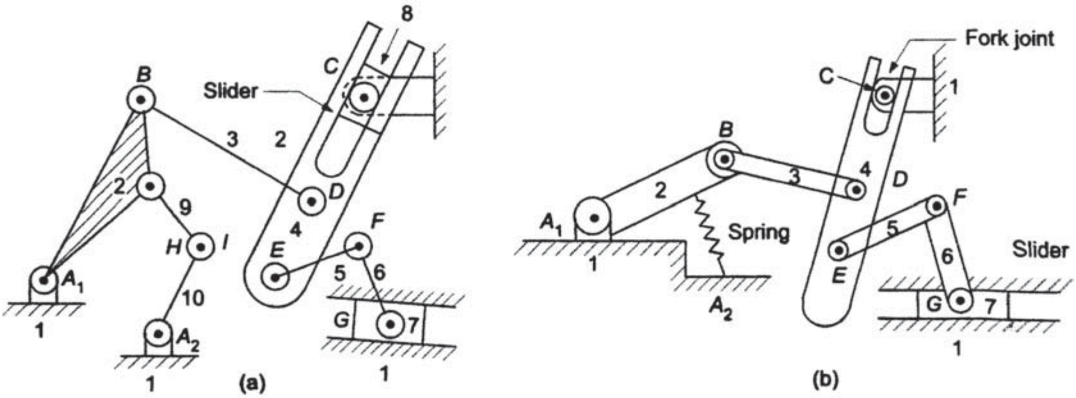


Fig.1.22 Various types of mechanisms

■ **Solution**

(a) Number of binary links, $n_2 = 7$

Number of ternary links, $n_3 = 2$

Number of quaternary links, $n_4 = 1$

Total number of links, $n = n_2 + n_3 + n_4 = 7 + 2 + 1 = 10$

$$2p = 2n_2 + 3n_3 + 4n_4 = 2\times7+3\times2+4\times1 = 24$$
$$F = 3(n-1) - 2p$$
$$= 3(10-1) - 24$$
$$= 27 - 24 = 3$$

(b) $n = 7,\ p = 7,\ h = 1$

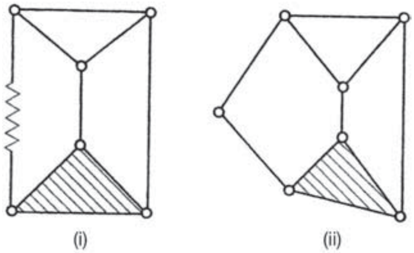
$$F = 3(n-1) - 2p - h$$
$$= 3(7-1) - 2\times7 - 1$$
$$= 18 - 14 - 1 = 3$$

Example 1.12

Find the equivalent mechanisms with turning pairs for the mechanisms shown in Figs.1.23(a) to (d).

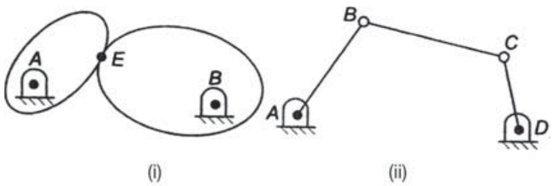
■ **Solution**

(a) A spring can be replaced by two binary links. Therefore, the equivalent mechanism is as shown in Fig.1.23a(ii).



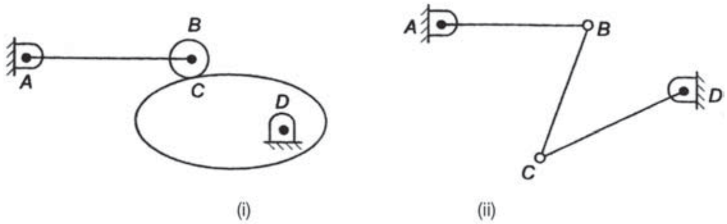
(a) Equivalent mechanism for a spring

(b) A cam pair can be replaced by one binary link with two turning pairs at each end. Therefore, the equivalent mechanism is shown in Fig.1.23b(ii). The centres of curvature at the point of contact *E* lie at *B* and *C*, respectively.



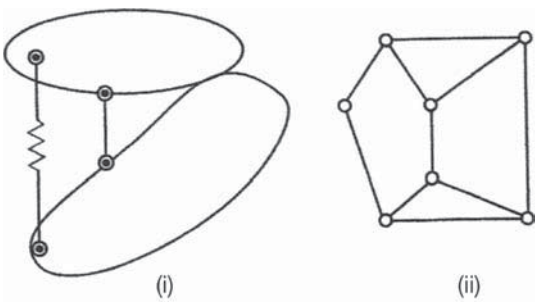
(b) Equivalent mechanism for a cam pair

(c) The equivalent mechanism is shown in Fig.1.23c(ii) as explained in (b) above.



(c) Equivalent mechanism for a cam with roller follower

(d) A spring is equivalent to two binary links connected by a turning pair. A cam follower is equivalent to one binary link with turning pairs at each end. The equivalent chain with turning pairs is shown in Fig.1.23d(ii).



(d) Equivalent mechanism for spring and cam combination

Fig.1.23 Equivalent mechanisms