

Ques In \mathbb{R}^3 , $x = (2, 1 + i, i)$

$$y = (2 - i, 2, 1 + i)$$

i) compute $\langle x, y \rangle$, $\|x\|$, $\|y\|$, $\|x + y\|$
 ii) In $\mathbb{C}[0,1]$, let $f(t) = t$ and $g(t) = e^t$ compute $\langle f, g \rangle$

$$\|f\|, \|g\| \text{ and } \|f + g\|$$

$$\langle f, g \rangle = \int_0^1 f(t)g(t) dt.$$

Also verify Cauchy's Schwartz's inequality and the triangle inequality for both the questions.

DEFINITIONS

- Let V be an inner product space two vectors x and y is V are said to be orthogonal (1) if $\langle x, y \rangle = 0$
- A subset S of V is said to be orthogonal if each of the 2 distinct vectors in S are orthogonal
- A vector x in V is s.t. a unit vector is a unit vector if and only if $\|x\| = 1$
- So finally, a subset S of V is s.t. orthonormal if S is orthogonal and consists of entirely unit vectors.

Ques In \mathbb{R}^3 , for the subset $S = \left\{ \begin{pmatrix} x_1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} x_3 \\ -1 \\ 2 \end{pmatrix} \right\}$ check if S is orthogonal.

Also check if S is orthonormal.

\mathbb{R}^3 over \mathbb{R} is

$$\langle x_1, x_2 \rangle = (1)(1) + (1)(-1) = 1 - 1 = 0$$

$$\langle x_2, x_3 \rangle = -1 + 1 = 0$$

$$\langle x_3, x_1 \rangle = -1 + 1 = 0$$

$\therefore S$ is orthogonal.

$$\|x_1\| = \sqrt{\langle x_1, x_1 \rangle} = \sqrt{(1+1+0)} = \sqrt{2}$$

$$\|x_2\| = \sqrt{\langle x_2, x_2 \rangle} = \sqrt{3}$$

$$\|x_3\| = \sqrt{6}$$

Not orthonormal.

Ques In \mathbb{R}^3 , for the subset $S = \left\{ \frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{3}}(1,-1,1), \frac{1}{\sqrt{6}}(1,1,1) \right\}$ check if S is orthonormal & is orthonormal

PROOF OF CAUCHY SCHWARTZ INEQUALITY

Case 1 If $y=0$, then obviously the inequality holds. Case 2 If $y \neq 0$, then consider $0 \leq \|x - \frac{\langle x, y \rangle}{\|y\|^2} y\|^2 = \langle x - \frac{\langle x, y \rangle}{\|y\|^2} y, x - \frac{\langle x, y \rangle}{\|y\|^2} y \rangle$

$$= \langle x, x \rangle - \frac{\langle x, y \rangle}{\|y\|^2} \langle y, x \rangle - \frac{\langle x, y \rangle}{\|y\|^2} \langle y, x \rangle + \frac{\langle x, y \rangle^2}{\|y\|^4} \langle y, y \rangle$$

Using $\langle x, y \rangle = \overline{\langle y, x \rangle}$ and without loss of generality, consider $\langle y, x \rangle = c$

$$c = \frac{\langle x, y \rangle}{\|y\|^2}$$

$$(*) = \|x\|^2 - \frac{\langle x, y \rangle^2}{\|y\|^2}$$

$$0 \leq \|x\|^2 - \frac{\langle x, y \rangle^2}{\|y\|^2}$$

$$|\langle x, y \rangle| \leq \|x\| \|y\|$$

$$c = \frac{\langle x, y \rangle}{\|y\| \|x\|}$$

$$\begin{aligned} 0 &= \langle x - \frac{\langle x, y \rangle}{\|y\|^2} y, x - \frac{\langle x, y \rangle}{\|y\|^2} y \rangle \\ &= \frac{\langle x, y \rangle^2}{\|y\|^4} \langle y, y \rangle \\ &= \frac{\langle x, y \rangle^2}{\|y\|^2} \end{aligned}$$

TRIANGLE INEQUALITY

$$\|x + y\| \leq \|x\| + \|y\|$$

$$\begin{aligned} \|x + y\|^2 &= \langle x + y, x + y \rangle \\ &= \langle x, x \rangle + \langle y, y \rangle + \langle x, y \rangle + \langle y, x \rangle \\ &= \langle x, x \rangle + \langle y, y \rangle + 2\langle x, y \rangle \end{aligned}$$

$$\begin{aligned} &+ \|x\|^2 \\ &+ 2|\langle x, y \rangle| \\ &+ \|y\|^2 \end{aligned}$$

SUPPLY MEANT FOR EXPORT UNDER BOND OR LETTER OF UNDERTAKING WITHOUT PAYMENT OF INVOICE

Electronics India Private Limited
 while Delta India Electronics Private Limited)
 Trial Building Kharsa no.- 28/18/3 min, 18/3, 18/4, 19/1/2/1 min, 22/2/2, 23/1, 23/2/1, 23/3/1 min, Village Begumpur Khatola, Sector-35, Gurgaon 122001 Haryana India
 PAN NO.: AACCD9731G12P
 N: 06AACCD9731G12P

$$\begin{aligned} & \leq \|x\|^2 + 2\|x\|\|y\| + \|y\|^2 \\ & = (\|x\| + \|y\|)^2 \end{aligned}$$

$$\Rightarrow \|x+y\| \leq \|x\| + \|y\|$$

REMARK

If 'v' is an inner product space then equality holds in Cauchy Schwarz inequality i.e. $|x \cdot y| = \|x\| \|y\|$ if and only if one of the vectors x or y is a multiple of the other.

Proof And out the cond'n under which Δ inequality gets converted to equality.

If the 2 vectors are orthogonal to each other then the Δ inequality gets converted to equality.

