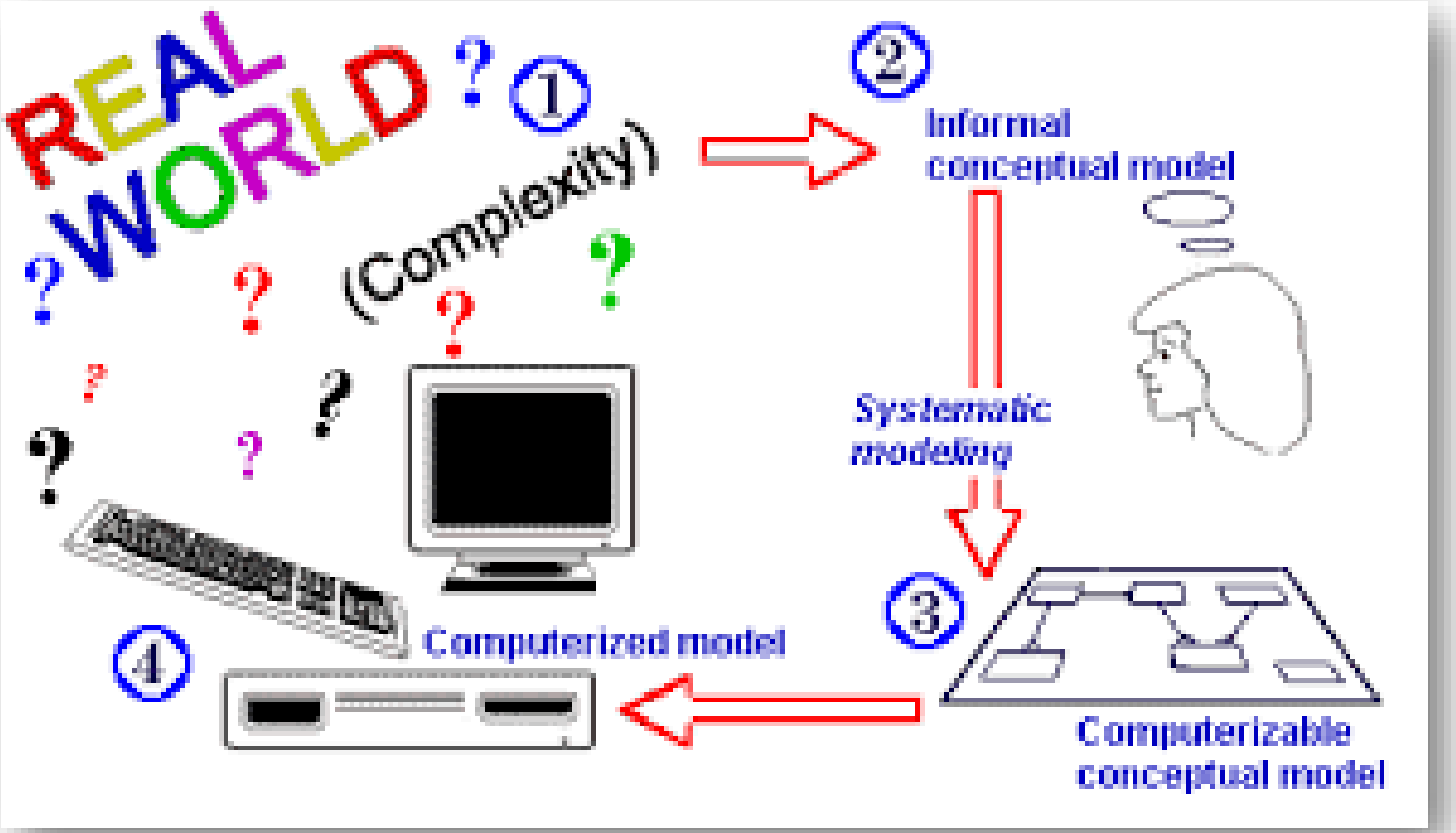
System Modelling & Design of Mechatronics Systems

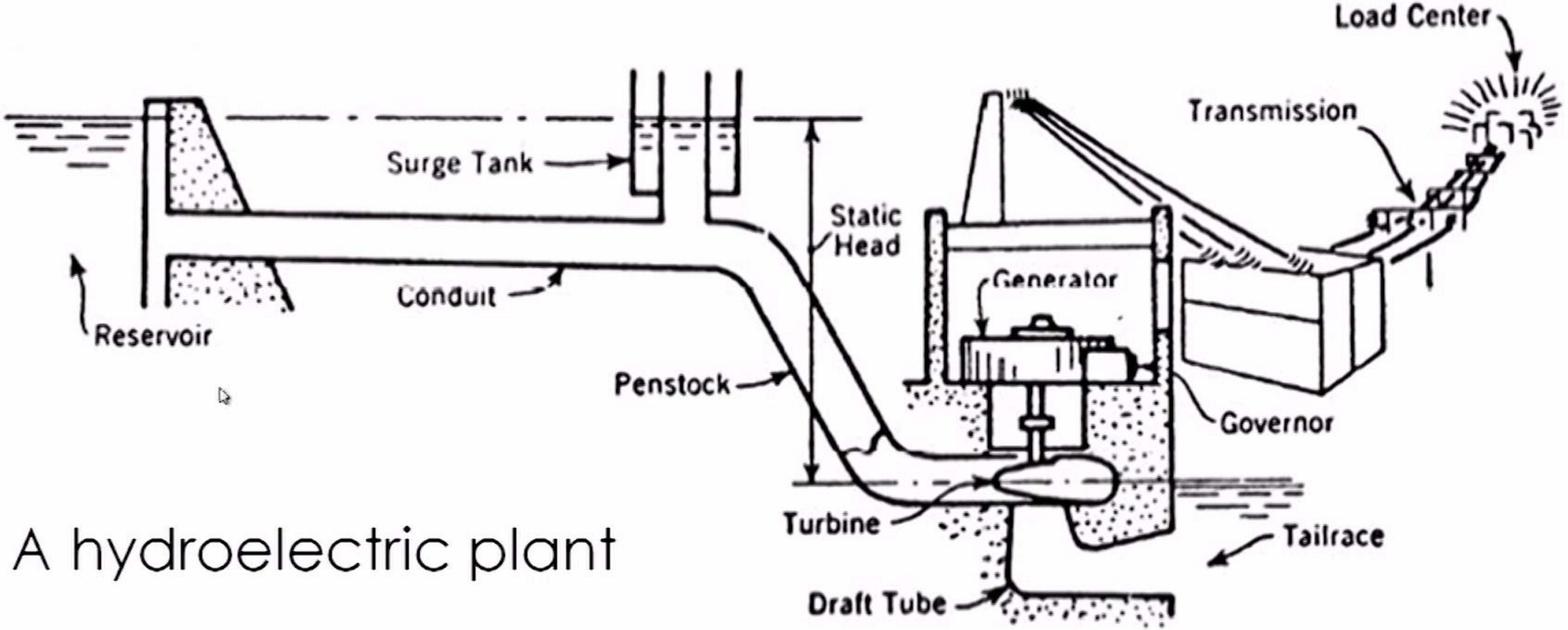
Basic System Models

- Basic System Models-Mechanical Systems
- Basic System Models-Electrical Systems
- Basic System Models-Hydraulic Systems
- Basic System Models-Pneumatic Systems
- Basic System Models-Thermal Systems

- The term **modelling** refers to the development of a mathematical representation of a physical system.

Approach



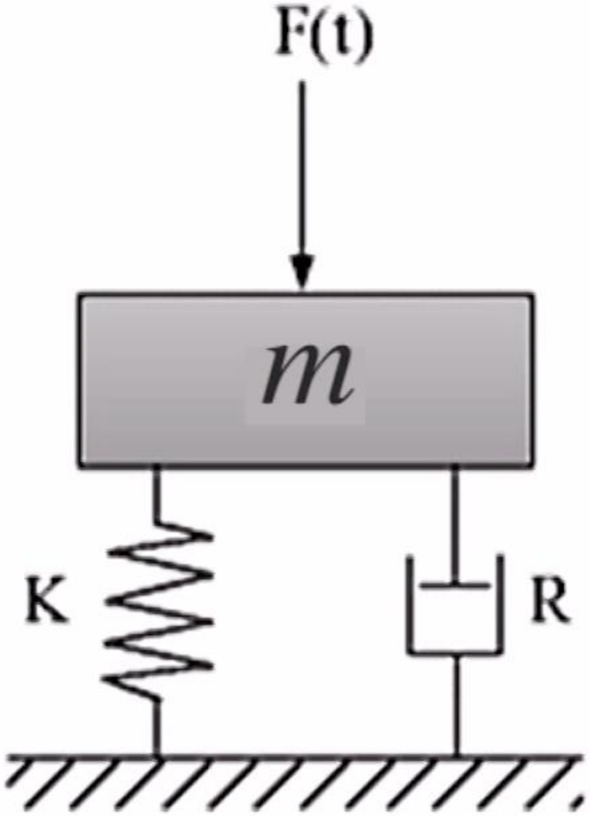


A hydroelectric plant

Various types of systems

- Electrical
- Mechanical
- Electro-Mechanical
- Hydraulic
- Thermal
- Examples
  - Moving car
  - Electric circuits
  - Telescope positioning system.

Physical system



Engineering model

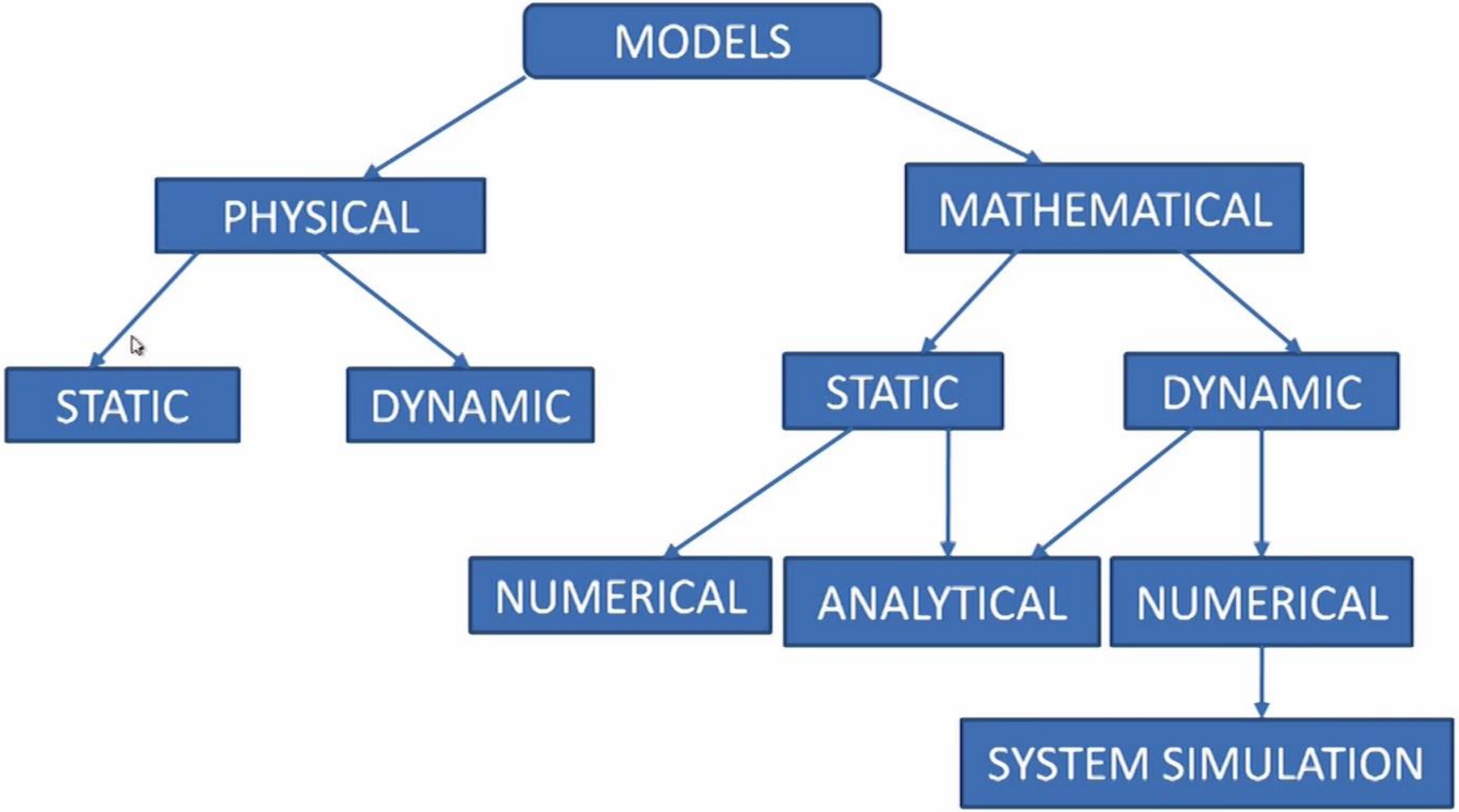
$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + Kx = F(t)$$

Differential equations

Model Definition

- To study the dynamics of a real system, the idea of model of the system is important.
- Models of systems are simplified, abstracted constructs which can be used to predict system behavior.
- A representation of an object, a system, or an idea in some form other than that of the entity itself - (Shannon)

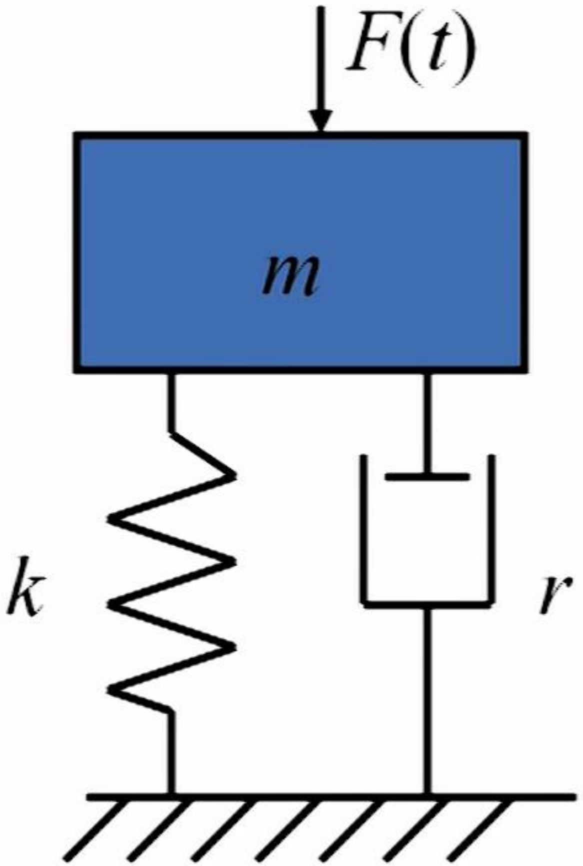
Types of Models

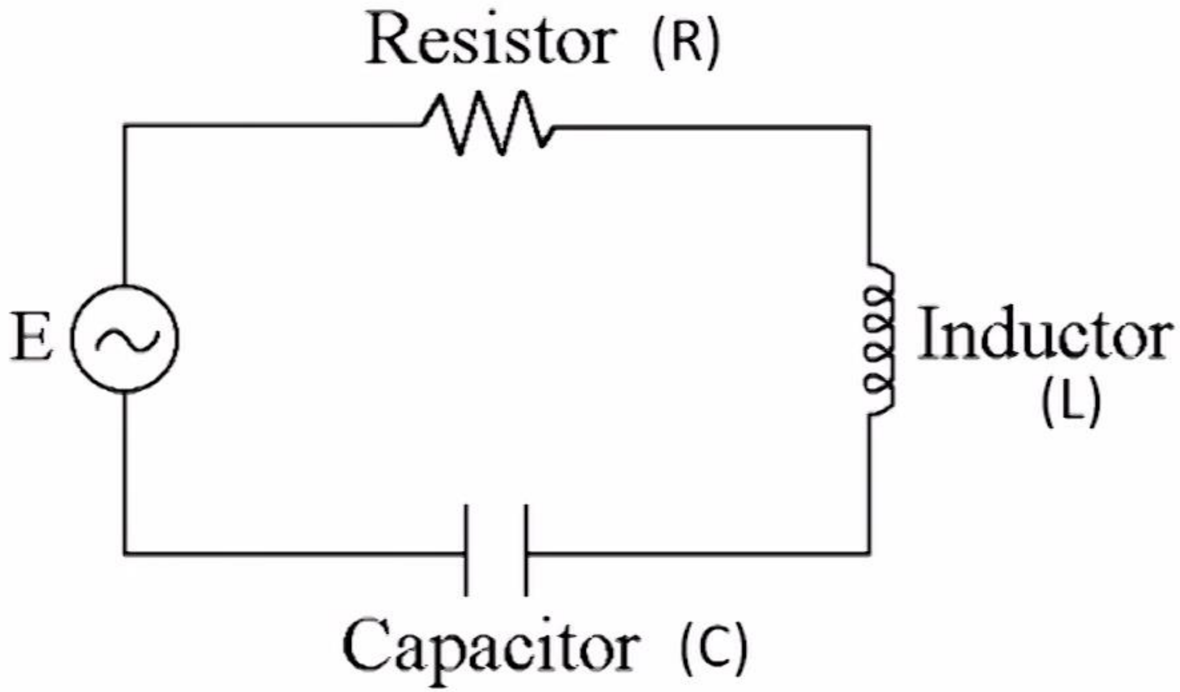


Static v/s Dynamic Models

- **Static Models:**
  - ✓ Static model represents a system at a particular point in time when the system is in balance.
- **Dynamic Models** :
  - ✓ Dynamic model represents systems as they change over time that result from system activities.

Analogy





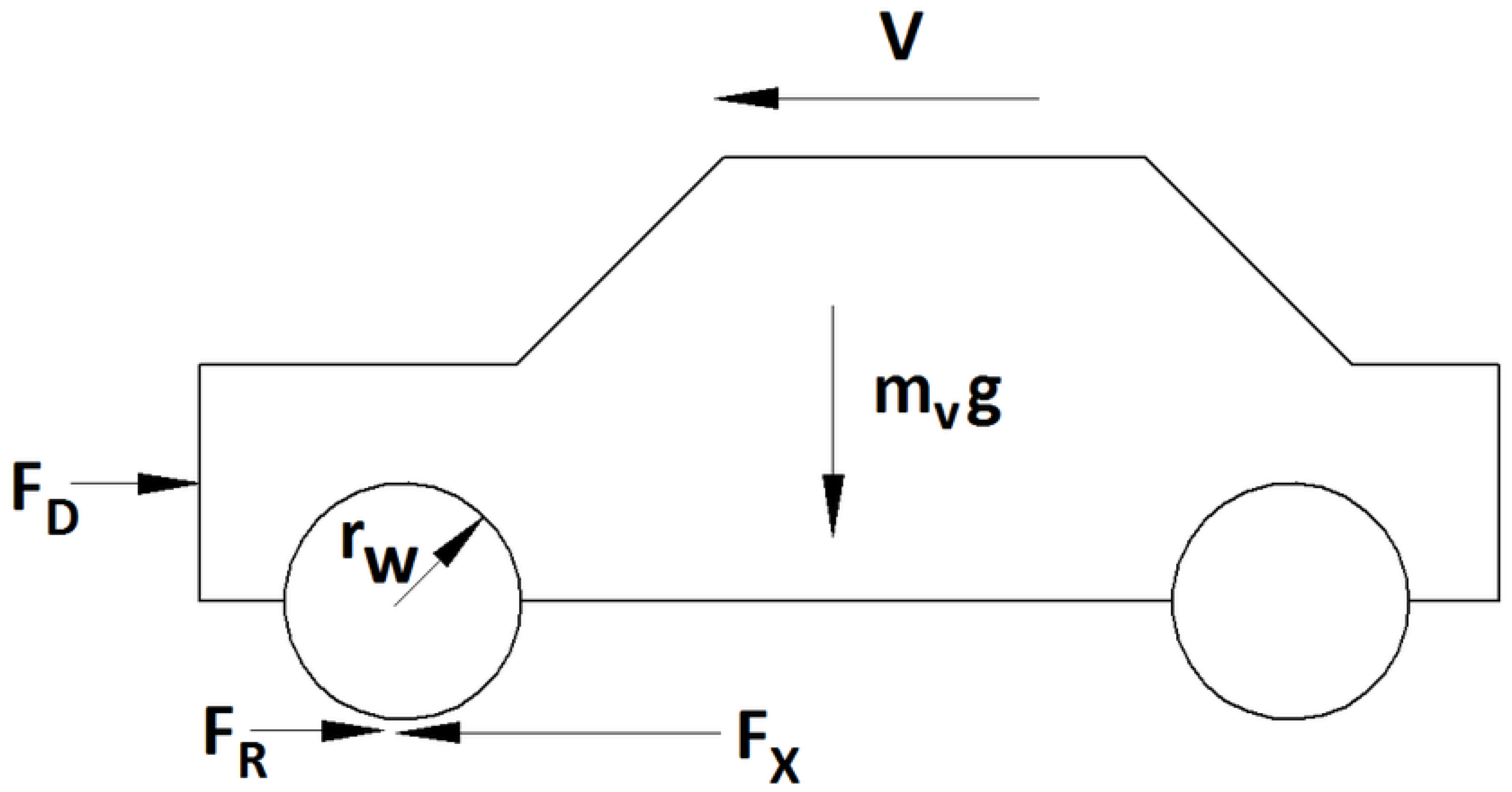
$$m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=F(t)$$

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E(t)$$

Mathematical Models

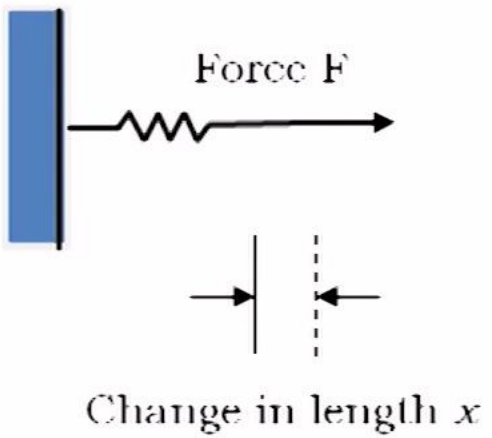
- Think of starting of a motor-motor will not get desired speed immediately
- Filling of a water tank-water will not be filled immediately
- To understand the behaviour of system mathematical models are needed.
- Mathematical models are equations which describe the relationship between the input and output of a system.
- System can be made by using building blocks.

Mechanical Model



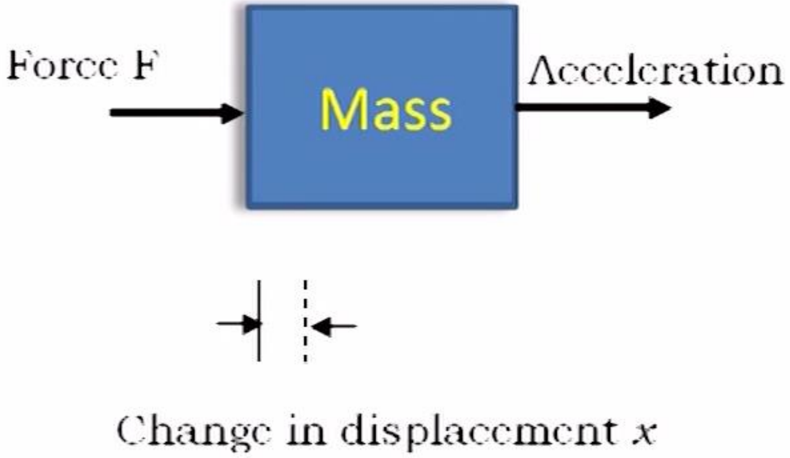
Spring

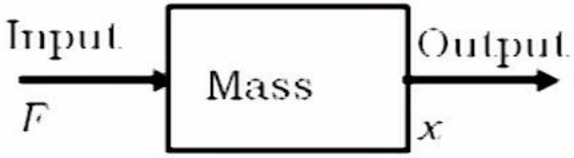
- The stiffness of a spring is defined by the relationship between the force *F* that can extend or compress a spring and the resulting extension or compression x.
- For a linear spring F = kx
- k is here a constant or stiffness
- Higher value of k implies greater force have to be applied to stretch or compress the spring for given displacement.



Mass

- This building block shows the property that bigger is the mass, greater will be the force required to give a specific acceleration.
- The relationship between force and acceleration comes from Newton's second law F=ma, where m is the constant of proportionality.





$$F = ma = m\frac{d^2x}{dt^2}$$

Energy/Power

- Energy is required to stretch a spring, accelerate a mass and move the piston inside a damper.
- In case of spring and mass energy is stored whereas in case of damper it is dissipated.
- The spring when stretched stores energy. This energy is released when spring come back to its original length.
- Energy stored in a spring for an extension x in it is given by $E = \frac{1}{2}kx^2 = \frac{1}{2}\frac{F^2}{k}$ (Since F=kx)

Rotational Systems

- In case of rotational systems the three basic building blocks are torsional spring, a rotary damper and the moment of inertia.
- In these building blocks input are torques and outputs are angle rotated.
- For a torsional spring, the angle rotated (θ)is proportional to the torque (τ) i.e.,

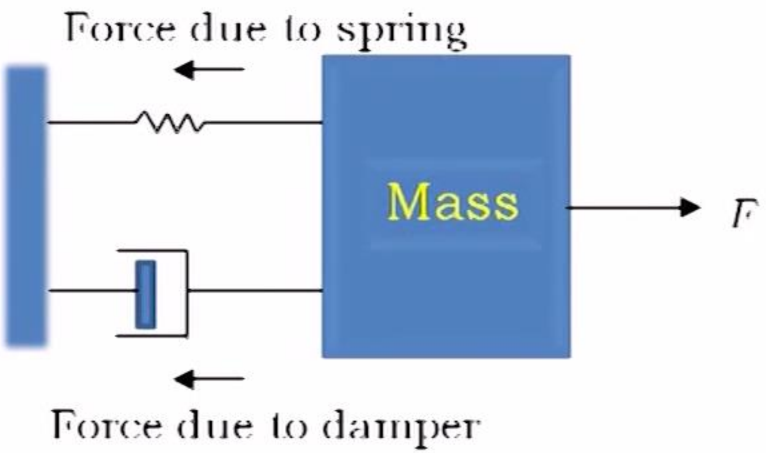
$\tau = k\theta$, where k is torsional stiffness of spring.

Energy/Power

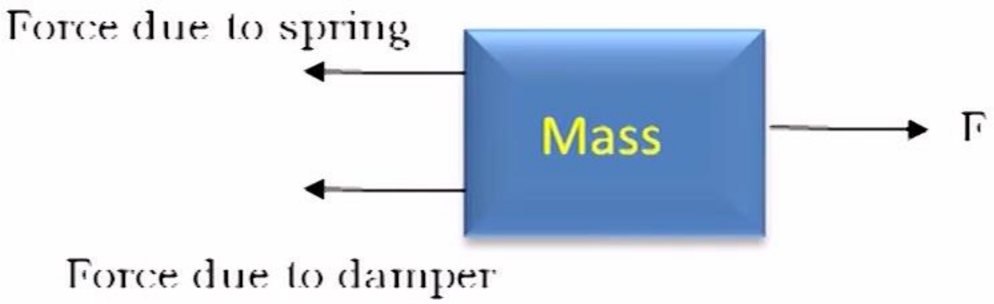
- In case of rotary system torsional spring and rotating mass stores energy whereas rotary damper dissipates energy.
- The energy stored by a torsional spring when it is twisted by an angle θ is given by

$$E = \frac{1}{2}k\theta^2 = \frac{1}{2}\frac{\tau^2}{k}$$

Building up a Mechanical System



Spring–mass-damper system



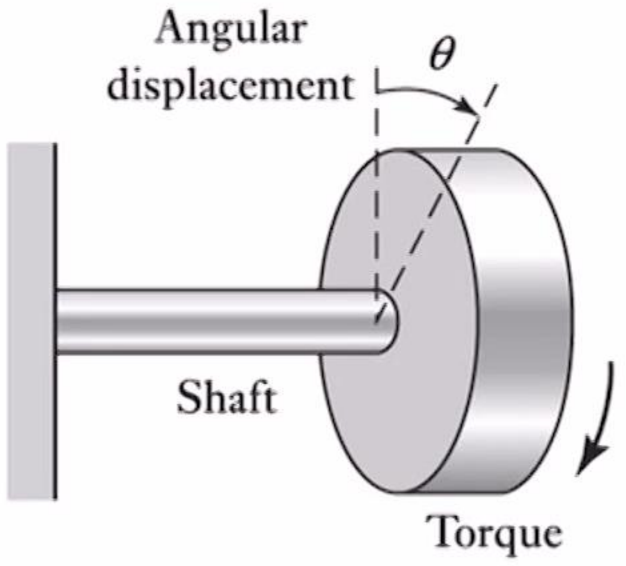
Free-body diagram

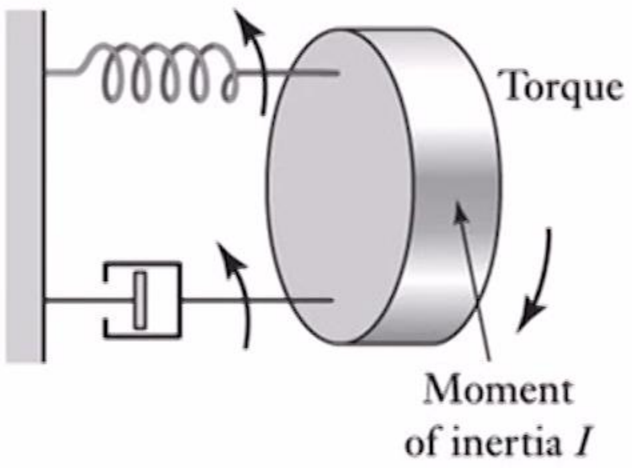
$$F - kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F$$

- This Eq gives relationship between input force F and output displacement x
- It is a 2nd order differential equation

Rotating a mass on the end of a shaft





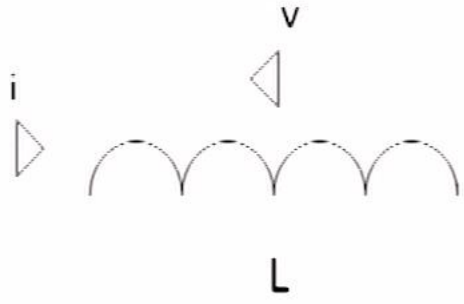
$$T - c\frac{d\theta}{dt} - k\theta = I\frac{d^2\theta}{dt^2}$$

Introduction

- The basic building blocks of electrical systems are inductors, capacitors and resistors.
- Inductor
- Potential difference V across it at any instant depends on the rate of change of current through it

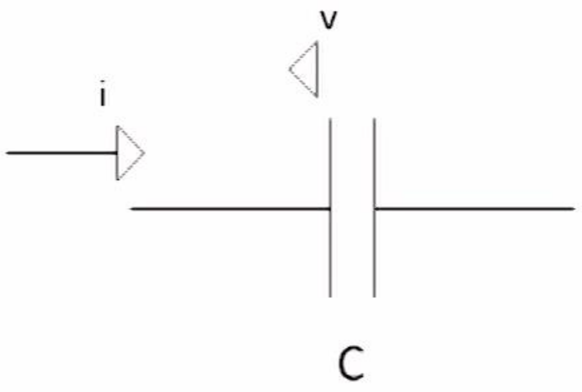
$$V_L = L\frac{di}{dt}$$

- Here L is inductance $i = \frac{1}{L}\int V_L\,dt$



Inductance

- Capacitor
- Potential difference across it depends on the charge q on the capacitor plates at the instant



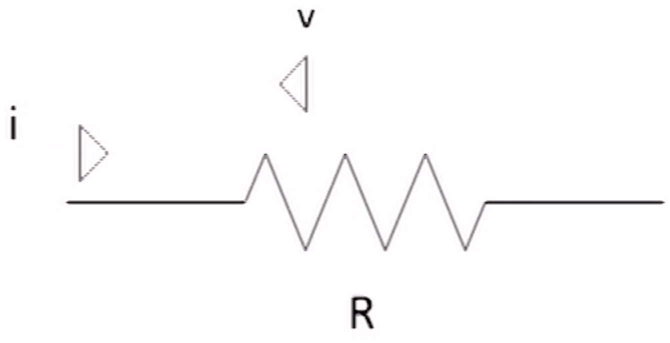
Capacitance

For capacitor $V_c = \frac{q}{c} = \frac{1}{c}\int idt$

$$i = C\frac{dV_c}{dt}$$

- Resistor
- The potential difference across it at any instant depends on the current through it.

$$V_R = iR$$

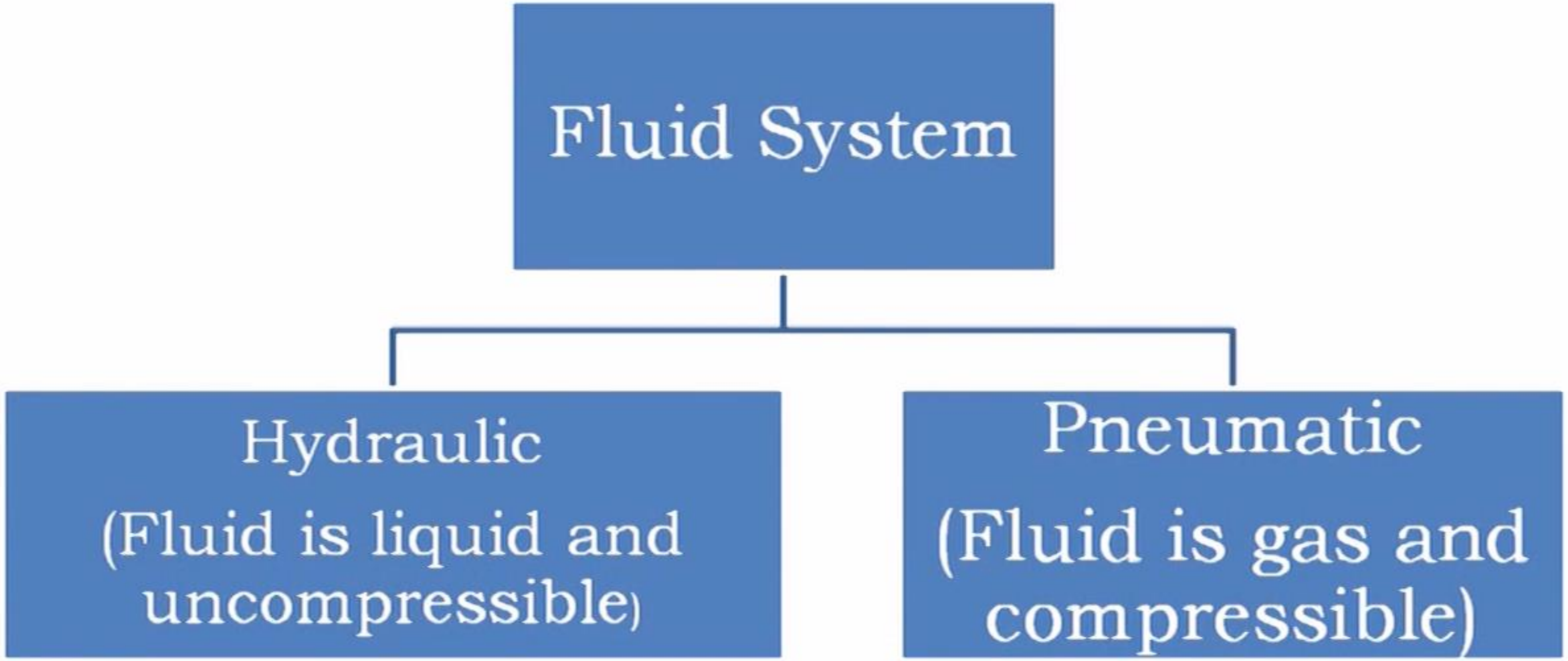


Resistor

Fluid System modelling

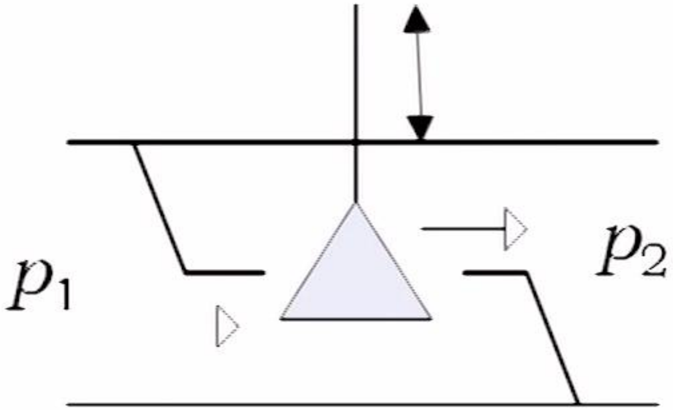
- For hydraulic system the input is volumetric rate of flow (equivalent to electrical current in electrical system) and output is pressure difference (equivalent to potential difference in electrical system)

Classification of Fluid System

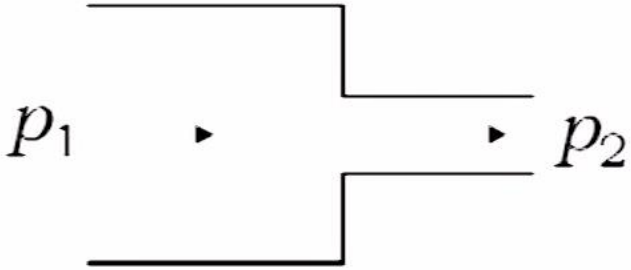


Hydraulic Resistance

- It is the resistance to flow due to liquid flowing through valves or change in pipe diameter.



Valve



Flow through variable pipe diameter

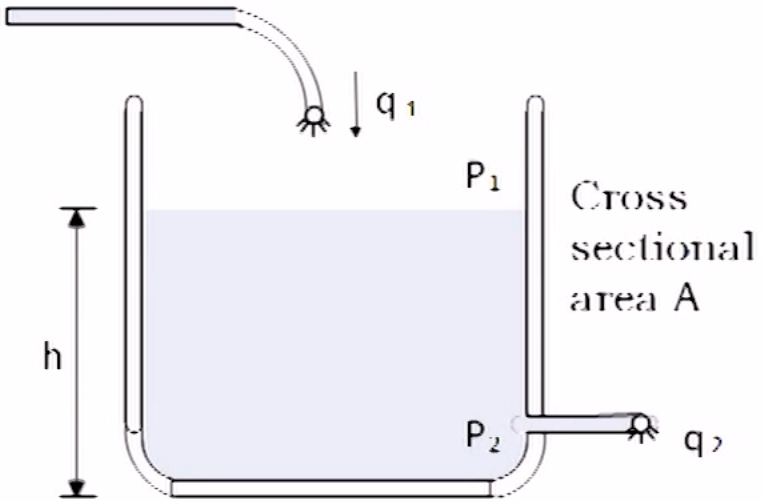
- The relation between volume rate of flow of liquid (q) through the resistive element and resulting pressure difference $(p_1 - p_2)$ is given by

$$(p_1 - p_2) = Rq$$

- Here R is hydraulic resistance.
- This equation is similar to $V = Ri$ in electrical systems

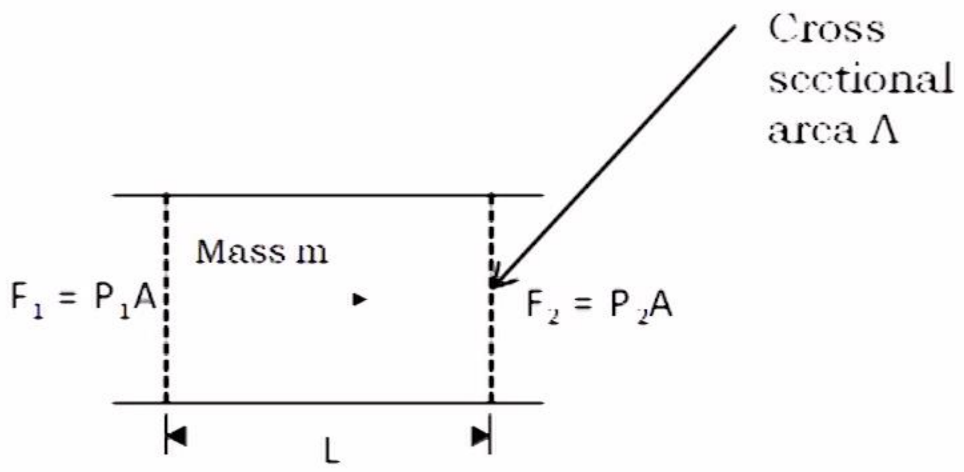
Hydraulic Capacitance

- It describes energy storage with a liquid where it is stored in the form of potential energy.
- Example: height of liquid in a tank.
- For such a capacitance the rate of change of volume (V) in the container is equal to volume rate in (q_1)minus volume rate out(q_2).



$$q_1 - q_2 = \frac{dV}{dt}$$

Hydraulic Inertance



- It is equivalent to inductance in electrical
- A force is required to accelerate a fluid.
- For a block of liquid of mass m
- Net force acting on liquid

$$F_1 - F_2 = (p_1 - p_2)A = ma = m\frac{dv}{dt}$$

$$(p_1 - p_2)A = \rho A L\frac{dv}{dt}$$

$$(p_1 - p_2)A = \frac{\rho A L}{A}\frac{dq}{dt} \text{ (Since } q = Av\text{)}$$

Thermal System Modelling

Thermal Resistance

- There is a net heat flow between two points if there is a temperature difference between them.
- A relationship for thermal resistance can be defined similar to that of an electrical resistance i.e.,
- $q = \frac{T_1 - T_2}{R}$ (Similar to $i = \frac{V}{R}$)
- Here q is rate of heat flow, and $T_1 - T_2$ is the temperature difference

Thermal Capacitance

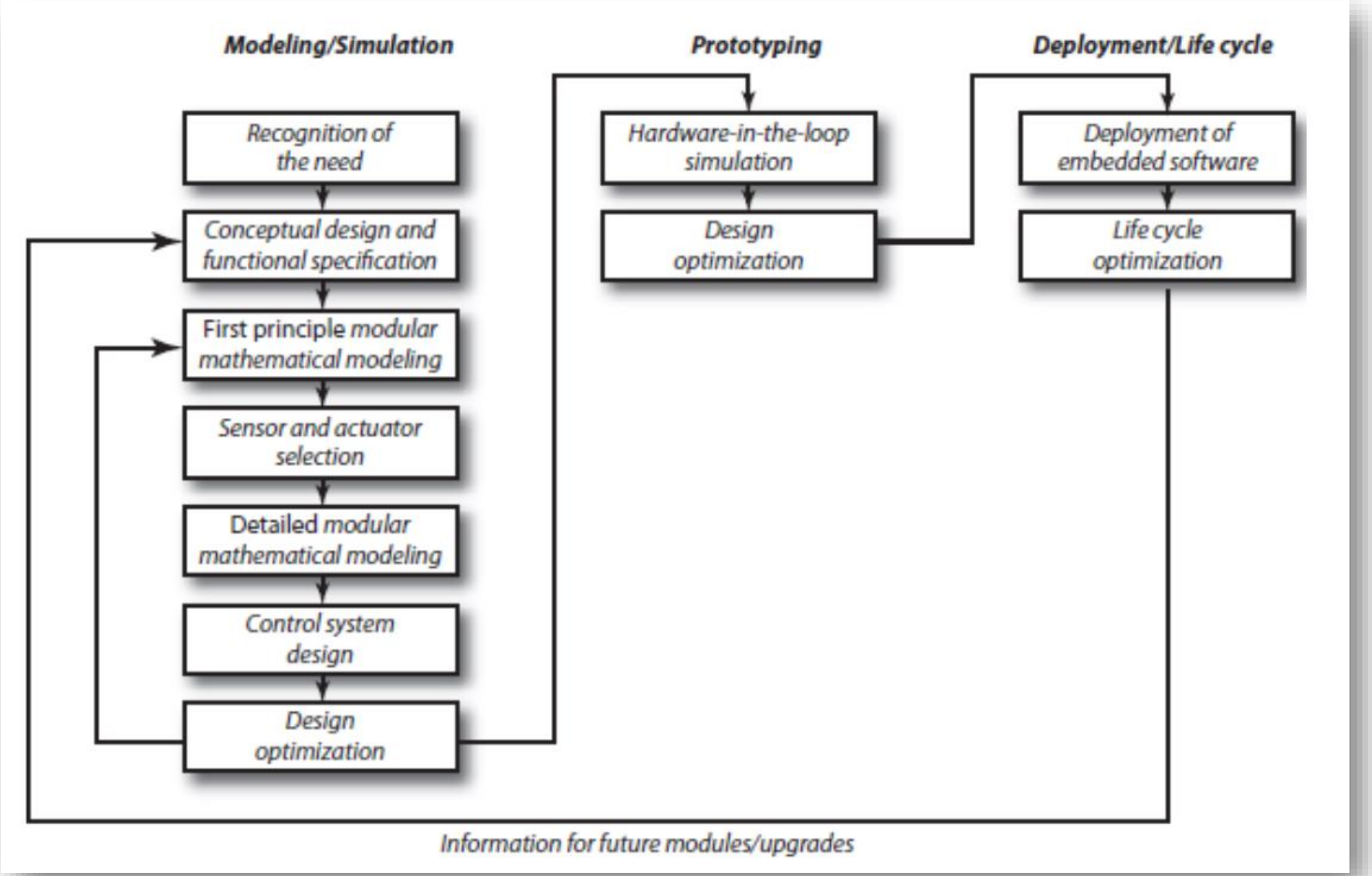
- It is a measure of store of internal energy in a system.
- Thus if input and output flow rate of heat are q_1 and q_2 respectively then
- Rate of change of internal energy

  $= q_1 - q_2 = mc \times$ (rate of change of temperature)

$$q_1 - q_2 = mc\frac{dT}{dt}$$

- Where m is the mass, c is the specific heat capacity, $\frac{dT}{dt}$ is rate of change of temperature

Design of Mechatronics System (Steps)



TRADITIONAL vs MECHATRONICS APPROACH

S.NO	TRADITIONAL APPROACH	MECHATRONICS APPROACH
1	Bulk system	Compact system
2	It is a complex process involving interaction between many skills and disciplines	It is the basic integration of various engineering technology with mechanical engineering
3	Manual control	Control through microprocessor with controller
4	Complex mechanism	Simplified mechanism
5	Non adjustable movement cycles	Programme movements
6	Constant speed drives	Variable speed drives

TRADITIONAL vs MECHATRONICS APPROACH

S.NO	TRADITIONAL APPROACH	MECHATRONICS APPROACH
7	Mechanical synchronisation	Electronic synchronisation
8	Accuracy determined by tolerance of mechanism	Accuracy achieved by feedback
9	It consists of more components	It consists of less components
10	Rigid heavy structures	Light structure
11	Less accuracy	More accurate
12	Less flexibility	More flexibility
13	Less cost	High cost