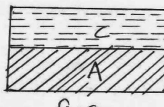
Zeroth Law of Thermodynamics



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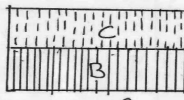


fig②

Mathematically.
A is thermal Equilibrium with C
B also with C
So, A is also thermal Equilibrium with B.

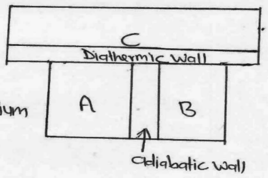


fig ③

It State that if two Systems A and B are Separately in thermal Equilibrium with the third System C then A and B are also in thermal Equilibrium with each Other.

Heat, Internal Energy and Work

Energy in the form of Heat gives to the System → Kinetic Energy → indicated by temperature rise; → potential Energy → indicated by State Change; → Work done. (Kinetic Energy and potential Energy → Internal Energy)

$$\boxed{dQ = dU + dW}$$

Heat (Q) — Heat is the mode of Energy transfer due to temperature difference between System and Surroundings

Work (W) — Work is the mode of Energy transfer by applying force on to a body ~~which~~ or System which produce motion.

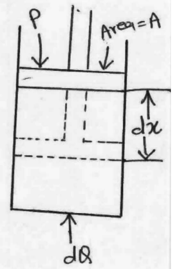
First Law of Thermodynamics

According to the first Law of Thermodynamics, if Some heat is Supplied to a System which is Capable of doing Work, then the quantity of heat absorbed by the System will be Equal to the Sum of the increase in its internal Energy and the External work done by the System on the Surroundings.

Let, dQ is the heat Supplied to the System by the Surroundings, dW is the work done by the System on the Surroundings and dU is the Change in the internal Energy of the System

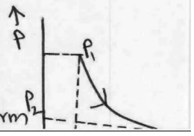
Then
$$dQ = dU + dW$$

Let the System is a gas Contained in a Cylinder provided with a movable piston.



Then the gas does work in moving the piston. The work done by the System against a Constant pressure P is

$$dW = \text{Force} \times \text{distance}$$
$$= \text{Pressure} \times \text{Area} \times \text{distance}$$
$$= PA\,dx$$
$$dW = P\,dV$$



So the first Law of thermodynamics takes the form

Applications of Ist Law of thermodynamics. (19)

I) Isothermal Process

In Isothermal Process as there in no change in temperature, there will be no change in internal Energy.

$$dU = 0$$

According to First law of thermodynamics

$$dQ = dU + dW$$
$$\boxed{dQ = dW}$$

⇒ Heat added or substracted = External work done

II) Isobaric Process:-

In Isobaric Process, as the Pressure remaining constant the amount of heat supplied is used in doing external work and in increasing volume dV.

External work $dW = P\,dV$

According to First law of thermodynamics

~~$dW = PdV$~~

$$dQ = dU + dW$$
$$\boxed{dQ = dU + P\,dV}$$

III) Isochoric Process:-

In Isochoric Process since volume of System remaining constant ~~it known as~~ then $dV = 0$

$$\boxed{dW = P\,dV = 0}$$

According to First law of thermodynamics

(20)

$$dQ = dU + dW$$
$$\boxed{dQ = dU}$$

Hence whole heat supplied to system is used in increasing its internal Energy only.

d) Adiabatic Process:-

In adiabatic Process as no heat enter or leave the System, but when a gas is allowed to expand it does external work and heat energy of gas decrease. Similarly when a gas is compressed work is done and heat energy of gas increase.

According to first law of thermodynamics

$$dQ = dU + dW$$

(since no heat enter or leave the System $dQ = 0$)

$$\Rightarrow \boxed{0 = dU + dW}$$
$$\boxed{dU = \pm dW}$$

Increase in internal Energy = External work done on (or by) the gas.

CONTINUUM MODEL :-

It is a concept in which Molecular gaps or voids are Neglected.

Note:- In case of liquid and solid continuum is always valid, but in case of gas it is not always true.

To check whether continuum is valid or not Knudsen Number (Kn) is Used.

$$Kn = \frac{\text{Mean free Path}}{\text{Characteristic Dimension}}$$

Condition for (Kn) to be satisfy continuum.

$$\boxed{Kn < 0.01 < \frac{1}{100}}$$

Limitations of the first Law of Thermodynamics —

1) It does not say anything about the direction of flow of heat:

Heat always flows from a hot body to a cold body. First Law does not give any reson as to why heat cannot flow from a cold body to a hot body.

2) It does not tell whether a process is possible or not:

First law explains the stopping of a revolving wheels due to conversion of its kinetic energy into heat due to friction. But it fails to explain as to why the heat energy cannot be converted into kinetic energy of rotation of wheel and put it back into rotation.

3) It does not tell that how much work we obtain:

No heat engine can convert all the heat extracted from the source into mechanical work continuously ~~without~~ without rejecting a part of it to the surrounding.

Internal Energy (U) — The internal energy of a system is the sum of molecular kinetic energy and molecular potential energy when the centre of mass of the system is at ~~rest~~ rest.

The molecules of a gas are always in a state of random motion. As the temperature increases, the average kinetic energy of the gas molecules also increase.

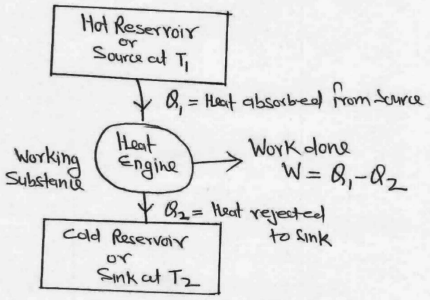
Thus the internal kinetic energy of a gas is a function of its temperature.

The molecules of a real gas exert mutual force of attraction on one another. Hence they possess intermolecular potential energy. If the volume of the gas increases, work is done by the gas against intermolecular attraction. So, its potential energy increases.

Thus intermolecular potential energy of a real gas is a function of its volume.

Heat Engine —

Heat engine is a device that convert heat energy into work. It takes heat from a reservoir then does some work like moving a piston. and finally discharges some heat energy into the sink.



Source — It is a heat reservoir at high temperature T_1.

It is supposed to have infinite thermal capacity so that any amount of heat can be drawn from its without changing its temperature.

Sink — It is a heat reservoir at a lower temperature T_2.

It has also infinite thermal capacity so that any amount of heat can be added to it without changing its temperature.

Working Substance — Working substance is any material (solid, liquid or gas) which perform mechanical work when heat is supplied to it

Working — In every cycle of operation, the working substance absorbs a definite amount of heat Q_1 from the source at higher temperature T_1, converts a part of this heat energy into mechanical work W and rejects the remaining heat Q_2 to the sink at lower temperature T_2. The work done W in a cycle is transferred to the environment by some arrangement.

Efficiency of a heat Engine —

The efficiency of a heat engine is defined as the ratio of the net work done by the engine in one cycle to ~~the~~ the amount of heat absorbed by the working substance from the source.

Net heat absorbed in a cycle = Work done

$$Q_1 - Q_2 = W$$

The efficiency of heat engine

$$\eta = \frac{\text{Work done by engine}}{\text{Heat absorbed from the Source}}$$

$$= \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

Second Law of Thermodynamics —

According to Kelvin-Planck Statement —

It is impossible to construct an engine, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.

This is applicable to a heat engine. It indicates that a working substance, operating in a cycle, cannot convert all the heat extracting from the source into mechanical work. It must reject some heat to the sink at a lower temperature.

According to Clausius Statement —

It is impossible for a self acting machine, to transfer heat from a body to another at higher temperature.

This is applicable to a refrigerator. The working substance can absorb heat from a cold body only if work is done on it. The work is done by an electric compressor. If no external work is done, the refrigerator will not work.

Significance of Second Law —

1) According to Second Law of Thermodynamics, the efficiency of a heat engine can never be unity. i.e. the heat released to the cold reservoir can never be made zero.

2) The coefficient of performance of a refrigerator can never be infinite i.e. External work can never be zero.

Reversible and Irreversible Process —

Reversible Process —

Any process which can be made to proceed in the reverse direction by variation in its condition such that any change occuring in any part of the direct process is exactly reversed in the corresponding part of the reverse process is called reversible process.

For Example —

1) The process of gradual compression and extension of an elastic spring is approximately reversible.
2) A working substance taken along the complete Carnot's cycle.

Irreversible process —

Any process which cannot be retraced in the reverse direction exactly is called an irreversible process.

For Example —

1) Rusting of iron
2) Sudden expansion or contraction of gas
3) Dissolution of salt in water

(13)

Equation of State:

The equation of state relates the pressure P, volume V and temperature T of a physically homogenous system to the ~~state~~ state of thermodynamic equilibrium

$$f(P, V, T)$$

Suppose we have thermodynamic potential as —

Thermodynamic Potential = H, G, A, U

(H – Enthalpy, G – Gibbs Energy
A – Area , U – Internal Energy)

4 Thermodynamic Variables = S, P, V, T

(S – Entropy , P – Pressure
V – Volume , T – Temperature)

So Equation of ~~State~~ State represent the variation in thermodynamic variables or thermodynamic potential.

~~First~~ thermodynamic Equation of State:-

1) we have $dU = TdS - PdV$

$$\Rightarrow \boxed{\left(\frac{dU}{dV}\right)_T = T\left(\frac{dS}{dV}\right)_T - P}$$

~~First~~ Equation of State.

II) For Ideal gas $(PV = RT)$

we have — $\left(\frac{dU}{dV}\right)_T = 0$

III) For 1 mole of Vander wall gas Equation

$$\left(\frac{dU}{dV}\right)_T = \frac{a}{V^2}$$

IV) For n mole of vander wall gas Equation

$$\left(\frac{dU}{dV}\right)_T = \frac{an^2}{V^2}$$

Entropy — Entropy measure the degree of disorderness of the system.

$$\Delta S = \frac{\Delta Q}{T} \quad \text{when } T \text{ is Constant}$$

or

$$dS = \frac{dQ}{T}$$

$$\int dS = \int \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T} \quad \text{When } T \text{ is not Constant}$$

When heat is given to the system, change in entropy is positive.

When heat is taken from the system, change in entropy is negative.

Entropy is a state function. it depends on the state of the system and not the path that is followed.

1) Change in Entropy for Spontaneous process

$$\Delta S = S_f - S_i$$

$$\Delta S = +ve$$

$$\Delta S > 0$$

2) For reversible Process —

$$A \rightleftharpoons B$$

Change in Entropy $\Delta S = +\frac{\Delta Q}{T}$
$A \to B$

for B to A Change in Entropy

$$\Delta S = -\frac{\Delta Q}{T}$$

$$\Delta S_{reversible} = +\frac{\Delta Q}{T} - \frac{\Delta Q}{T} = 0$$

3) Isolated System —

In isolated System $\Delta Q = 0$

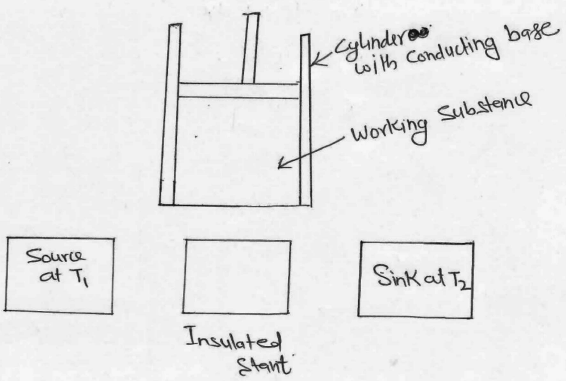
$$\Delta S_{total\ Entropy} = \Delta S_{System} + \Delta S_{Surrounding}$$

~~ΔS_{total}~~

$$\Delta S_{total\ Entropy} = 0$$

** But for Spontaneous process ΔS of System > 0

Carnot Engine — It is an ideal reversible heat engine that operates between two temperatures T_1 (Source) and T_2 (Sink). It operates through a series of two isothermal and two adiabatic processes called Carnot Cycle. It is a theoretical heat engine with which the efficiency of practical engines is compared.



Carnot Engine has the following main parts

1 — Cylinder — This is the main part of Engine has conducting base and insulating walls. It is filled with an insulating and frictionless piston.

Source — It is a heat reservoir at a higher temperature T_1 from which the Engine draw heat. It is supposed that the source has an infinite thermal capacity and so any amount of heat can be drawn from it without changing its temperature.

Sink — It is heat reservoir at a lower temperature T_2 to which any amount of heat can be rejected by the Engine. It has also infinite thermal capacity and so any amout of heat can be added to it without changing its temperature.

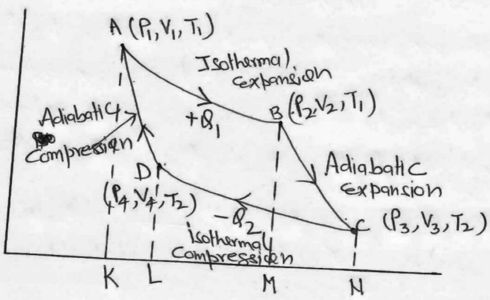
Working Substance — The working substance is an ideal gas contained in the cylinder.

Insulating Stand — When the base of the cylinder is attached to the insulating stand, the working substance gets isolated from the surroundings.

Carnot Cycle — The working substance is carried through a reversible cycle of the following four steps.

Isolthermal Expansion

Step 1 — Place the cylinder on the source so that the gas acquires the temperature T_1 of the source. The gas is allowed to expand by slow outward motion of the piston. The temperature of the gas falls. As the gas absorbs the required amount of heat from the source, it expands isolthermally.



If Q_1 heat is absorbed from the source and W_1 work is done by the gas in isolthermal expansion which takes its state from (P_1, V_1, T_1) to (P_2, V_2, T_2) then

$$W_1 = Q_1 = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = \text{Area of ABMKA}$$

Step 2 — Adiabatic Expansion — The gas is now placed on the insulating stand and allowed to expand slowly till its temperature falls to T_2

If W_2 work is done by the gas in the adiabatic expansion which takes its state from (P_2, V_2, T_1) to (P_3, V_3, T_2) then

$$W_2 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \text{area BCNMB}$$

Step 3 — Isolthermal Compression —

The gas is now placed in thermal contact with the sink at temperature T_2. The gas is slowly compressed so that as heat is produced it easily ~~compressed~~ ~~so that as heat is~~ flows to the sink. The temperature of the gas remains constant at T_2.

If Q_2 heat is released by the gas to the sink and W_3 work is done on the gas by the surroundings in the isothermal compression which takes its state from (P_3, V_3, T_2) to (P_4, V_4, T_2) then

$$W_3 = Q_2 = nRT_2 \ln\left(\frac{V_3}{V_4}\right) = \text{Area CDLNC}$$

Step 4 - Adiabatic Compression — The cylinder is again placed on the insulating stand. The gas is further compressed slowly till its returns to its initial state (P_1, V_1, T_1) then

$$W_4 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \text{area DAKLD}$$

Net Work done by the gas per Cycle —

Total work done by the gas $= W_1 + W_2$

Total work done on the gas $= W_3 + W_4$

$\therefore$ Net work done by the gas in one complete cycle,

$$W = W_1 + W_2 - (W_3 + W_4)$$

But $W_2 = W_4$

$$W = W_1 - W_3 = Q_1 - Q_2$$

Also $W =$ area ABMKA + area BCHMB − area CDLNC − area DAKLD

W = area ABCDA

Hence in a Carnot Engine, the mechanical work done by the gas per cycle is numerically equal to the area of the Carnot cycle.

Efficiency of Carnot Engine —

It is defined as the ratio of the net work done per cycle by the engine to the amount of heat absorbed per cycle by the working substance from the source.

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{nRT_2 \ln (V_3/V_4)}{nRT_1 \ln (V_2/V_1)}$$

Step 2 is an adiabatic expansion, therefore

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \quad \text{—— (A)}$$

Step 4 is an adiabatic compression, therefore

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \quad \text{—— (B)}$$

On dividing Eqn (A) by Eqn (B)

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

UNIT-2 Part-I (waves & oscillations)

Waves:- A wave motion is a disturbance of some kind which moves from one place to another by means of a medium.

Oscillation:- Oscillation is an effect expressible as a quantity that repeatedly & regularly fluctuate above and below some mean position.

I) **Transverse wave:-** Wave motion in which particle of medium vibrate about their mean position at right angle to the direction of propagation.
Eg- Light waves.

II) **Longitudinal waves:-** Wave motion in which wave vibrate about their mean position along the same line as propagation of wave.
Eg. Sound waves

Simple Harmonic Motion:- If the acceleration of a particle in a periodic motion is always directly proportional to its displacement from its equilibrium position.

Types of SHM:-

I) **Linear Harmonic Simple Harmonic Motion:-**
If the displacement of a particle executing SHM is linear is said to be linear simple harmonic motion.
Example- Simple Pendulum.

Angular Simple Harmonic Motion:-
If the displacement of a particle executing SHM is angular. Example- compound Pendulum.

Essential Conditions for SHM:-

1) If f be the linear acceleration and x be the displacement from equilibrium position.
The essential condition is -

$$\boxed{f \propto -x}$$

2) If α be the angular momentum θ be the angular displacement then essential condition is-

$$\boxed{\alpha = -\theta}$$

Time Period:- The smallest time interval during which oscillation repeat itself is called time period, denoted by T. Its unit is seconds.

Frequency:- Number of oscillation that a body complete in one second is called frequency of periodic motion.

It is reciprocal of time period T and is given by-

$$\boxed{n = \frac{1}{T}}$$

Unit - Hertz represented by Hz.

Amplitude:- Maximum displacement of a body from its mean position.

Phase :- It is a physical quantity that express the instaneous Position and direction of Motion of an oscillating system.

Differential Equation of SHM & its solution!-

Let us consider a Particle of Mass m executing SHM along a straight line with x as displacement from the mean position at any time t. Then from the basic condition of SHM restoring force F will be Proportional to displacement and will be directed opposite to it.

Therefore $F \propto -x$

$$\boxed{F = -kx} \quad -(1)$$

k is proportionality constant known as Force constant

If $a = \frac{d^2x}{dt^2}$ be the Acceleration at any instant of time

From (1) $F = -kx$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad -(2)$$

Substituting $\frac{k}{m} = \omega^2$

$$\frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \quad -(3)$$

Equation (1) & (2) are known as differential Equation of SHM

Gauss Divergence Theorm!-

It States that, the volume integral of the divergence of vector field A taken over any volume V bounded by a Closed surface S is equal to surface integral of A taken over the surface S.

Mathematically

$$\iiint_V \mathrm{div} A\, dv = \iint_S A \cdot ds$$

Stokes Theorm!-

It State that the surface integral of a curl of a vector field A taken over any surfaces is equal to the line integral of A around a closed curve.

$$\iint_S (\mathrm{curl} A)\, ds = \oint A \cdot dl$$

OR

$$\iint_S (\nabla \times A)\, ds = \oint A \cdot dl$$

Physical Significance of Maxwell's Equations—

Maxwell's first Eqn $\nabla \cdot \vec{E} = \rho/\epsilon_0$ or $\mathrm{div}\,\vec{E} = \rho/\epsilon_0$ represent the Gauss's Law in Electrostatic for static charges, which state that the electric flux through any closed hypothetical surface is equal to $1/\epsilon_0$ times the total charge enclosed by the surface.

Maxwell Equation (Differential Form)

First

I) $\nabla \cdot D = \rho$ (Gauss law of Electrostatics)

OR

$\nabla \cdot E = \rho/\epsilon_0$ (Using $D = \epsilon_0 E$)

Second

II) $\nabla \cdot B = 0$ (Gauss law of Magnetostatics)

Third

III) $\nabla \times E = -\frac{dB}{dt}$ (Faraday law)

Fourth

IV) $\nabla \times B = \mu_0 \left(J + \frac{dD}{dt}\right)$ (Modified Ampere Circuital law)

OR

$\nabla \times H = \left(J + \frac{dD}{dt}\right)$ (Using $B = \mu_0 H$)

Maxwell Equation (Integral Form)

First

I) $\oint E \cdot ds = \frac{q}{\epsilon_0}$

Second

II) $\oint B \cdot ds = 0$

Third

III) $\oint E \cdot dl = -\frac{d\phi_B}{dt}$

Fourth

IV) $\oint B \cdot dl = \mu_0 \left(J + \frac{dD}{dt}\right) \cdot ds$

First Maxwell Equation!-

$$\nabla \cdot E = \rho/\epsilon_0$$

Physical significance:-

- It is based on gauss law of Electrostatics
- Net Electric flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ the total charge Enclosed by the surface.

$\nabla \cdot E = \frac{q}{\epsilon_0}$ (Differential Form)

$\oint_S E \cdot ds = \frac{1}{\epsilon_0} \int_V q \cdot dv$ (Integral Form)

- It relate Electric flux with charge.
- charge acts a a source or sink for the line of Electric force

Derivation!- Consider a surface bounded by a volume V in a medium having charge density as ρ.

then

$$q = \int_V \rho\, dv$$

Using Gauss law of Electrostatics we have

$$\oint E \cdot ds = \frac{q}{\epsilon_0}$$

Subustituting $q = \int \rho\, dv$

then

$$\oint E \cdot ds = \frac{1}{\epsilon_0} \int \rho\, dv \quad -(2)$$

From Gauss theorm we have

$$\oint_S E \cdot ds = \int_V (\nabla \cdot E)\, dv \quad -(3)$$

(Note Gauss Divergence theorm converts Surface Integral to volume Integral)

Equating (2), (3) we get

$$\int_V (\nabla \cdot E)\, dv = \frac{1}{\epsilon_0} \int_V \rho\, dv$$

$$\int_V \left(\nabla \cdot E - \frac{\rho}{\epsilon_0}\right) dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot E = \rho/\epsilon_0}$$

Maxwell (I) Equation.

Maxwell's Second Equation:-

$$\nabla \cdot B = 0$$

Significance:-

- It is based on Gauss law of Magnetostatics.
- It states that Magnetic flux through any closed Surface is zero

Differential form

$\nabla \cdot B = 0$

Integral form

$\oint B \cdot ds = 0$

- Time independent Equation
- Magnetic flux is zero
- According to this Equation, Isolated Magnetic Poles don't Exist.

Derivation:- we have by Gauss law of Magnetostatics

$$\oint B \cdot ds = 0 \quad -(1)$$

By Gauss divergence theorem we have.

$$\oint_S B \cdot ds = \int_V (\nabla \cdot B) dV \quad -(2)$$

(Note:- By Gauss divergence theorem we have Surface Integral Equal to volume Integral)

Equating (1), (2)

$$\int_V (\nabla \cdot B) dV = 0$$

$$\boxed{\nabla \cdot B = 0} \rightarrow \text{Maxwell Second Equation}$$

Maxwell's Third Equation:-

$$\nabla \times E = -\frac{dB}{dt}$$

Significance:-

- This Equation represent Faraday law of Electro-Magnetic Induction.

Differential form

$\nabla \times E = -\frac{dB}{dt}$

Integral form

$\oint E \cdot dl = -\frac{d\phi}{dt}$

- It is time dependent Equation.
- Relates space variation of E with variation of B
- Time variation of Magnetic field generates Electric field.
- Negative sign Justify Lenz law.

Derivation:- we have by Faraday law.

$$\oint_C E \cdot dl = -\frac{d\phi}{dt}$$

$$= -\frac{d(B \cdot ds)}{dt} \qquad (\phi = B \cdot ds) \quad -(1)$$

By Stokes theorem we have.

$$\oint_C E \cdot dl = \int_S (\nabla \times E) \cdot ds \quad -(2)$$

Equating (1), (2)

$$\oint_S (\nabla \times E) ds = -\frac{d}{dt} \int_S B \cdot ds$$

$$\oint_S (\nabla \times E) ds = -\int \frac{dB}{dt} \cdot ds$$

$$\Rightarrow \boxed{\nabla \times E = -\frac{dB}{dt}} \text{ — Maxwell Third Eqn}$$

Maxwell fourth Equation

$$\text{curl } H = J + \frac{dD}{dt}$$

Significance

- It represent Modified form of Amper's law.
- It is time dependent Equation
- It shows that Magnetic field can be generated by current density vector and time variation

Differential form

$\nabla \times B = \mu_0 \left(J + \frac{dD}{dt} \right)$

Integral form

$\oint_C B \cdot dl = \mu_0 \int_S \left(J + \frac{dD}{dt} \right)$

Derivation:- By Amper's law we have.

$$\oint B \cdot dl = \mu_0 I$$

$$\oint (\mu_0 H) dl = \mu_0 I$$

$$\oint H \cdot dl = I$$

$$I = \int_S J \cdot ds$$

$$\Rightarrow \int_C H \cdot dl = \int_S J \cdot ds \quad -(1)$$

By Stokes theorem we have.

$$\int_C H \cdot dl = \int_S (\nabla \times H) ds \quad -(2)$$

Equating (1), (2)

$$\int_S (\nabla \times H) ds = \int_S J \cdot ds$$

$$\int_S (\nabla \times H - J) ds = 0$$

$$\Rightarrow \nabla \times H - J = 0 \quad \Rightarrow \boxed{\text{Curl } H = J} \quad -(3)$$

Using The equation of time varying field.

$$\text{div } J + \frac{dP}{dt} = 0$$

$$\text{div } J = -\frac{dP}{dt} \quad -(4)$$

Maxwell added a current density J_d to original current density J i.e

$$C = J + J_D$$

Using (3) $\quad \text{Curl } H = J + J_D \quad -(6)$

$$\text{div} (\text{curl } \vec{H}) = \text{div} (J + J_D)$$

$$\text{div} (J + J_D) = 0$$

$$(\text{div} (\text{curl } \vec{H}) = 0 \text{ for time varying field})$$

$$\Rightarrow \text{div } J + \text{div } J_D = 0$$

$$\text{div } J_D = -\text{div } J$$

Using (4) $\quad \text{div } J_D = + dP/dt \quad -(5)$

But $\boxed{\text{div } D = P}$ By maxwell (1) equ

$$\text{div } J_D = \frac{d}{dt} (\text{div } D)$$

$$= \text{div} \left(\frac{dD}{dt} \right)$$

$$\boxed{J_D = \frac{dD}{dt}}$$

Using (6) $\quad \text{Curl } H = J + J_D$

$$\boxed{\text{Curl } H = J + \frac{dD}{dt}}$$

Maxwell's (4) Equation

POYNTING THEOREM:- / Imp

The rate of Energy transport per unit area, is called Poynting vector. It is also termed as instantaneous energy flux density and is represented by S or P.

$$\boxed{\vec{S} = \vec{E}\times\vec{H}}$$

$\vec{S}$ is perpendicular to both $\vec{E}$ and $\vec{H}$.

Unit – W/m^2

Derivation:- We can calculate the energy density carried by Electromagnetic waves with the help of Maxwell's Equation given below-

i) $\nabla\cdot\vec{D} = 0$ — ①

ii) $\nabla\cdot\vec{B} = 0$ — ②

iii) $\nabla\times\vec{E} = -\frac{d\vec{B}}{dt}$ — ③

iv) $\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$ — ④

Now Taking (·) Dot product of ~~③~~ ~~with~~ H with ③ and Dot product of E with ④

$$\vec{H}\cdot(\nabla\times\vec{E}) = -\mu\vec{H}\cdot\frac{d\vec{H}}{dt} \quad — ⑤ \quad (B = \mu\vec{H})$$

$$\vec{E}\cdot(\nabla\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\vec{E}\frac{d\vec{E}}{dt} \quad — ⑥ \quad (D = \epsilon\vec{E})$$

Substracting Equation ⑤ from Equation ⑥

$$\Rightarrow \vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{H}(\vec{\nabla}\times\vec{E}) = \vec{E}\cdot\vec{J} + \epsilon\vec{E}\frac{d\vec{E}}{dt} - \left(-\mu\vec{H}\frac{d\vec{H}}{dt}\right)$$

Since $\vec{A}\cdot(\vec{\nabla}\times\vec{B}) - \vec{B}\cdot(\vec{\nabla}\times\vec{A}) = \vec{\nabla}\cdot(\vec{B}\times\vec{A}) = -\vec{\nabla}(\vec{A}\times\vec{B})$

$$\Rightarrow -\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{J}\cdot\vec{E} + \frac{1}{2}\epsilon\frac{dE^2}{dt} + \frac{1}{2}\mu\frac{dH^2}{dt}$$

using $\left(\because \epsilon\vec{E}\frac{dE}{dt} = \frac{1}{2}\epsilon\frac{dE^2}{dt},\quad \mu\vec{H}\frac{dH}{dt} = \frac{1}{2}\mu\frac{dH^2}{dt}\right)$

$$\Rightarrow -\vec{\nabla}\cdot\vec{S} = \vec{J}\cdot\vec{E} + \frac{1}{2}\frac{d}{dt}(\epsilon E^2 + \mu H^2)$$

or $$\vec{J}\cdot\vec{E} + \frac{d}{dt}\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right) + \vec{\nabla}\cdot\vec{S} = 0$$

Taking volume Integral over the volume V enclosed by the surface S,

$$\int_V \vec{\nabla}\cdot\vec{S}\,dv + \frac{d}{dt}\int_V\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv = -\int_V \vec{J}\cdot\vec{E}\,dv$$

Using Divergence Theorm

$$\boxed{\int_S \vec{S}\cdot d\vec{s} + \frac{d}{dt}\int_V\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv = -\int_V \vec{J}\cdot\vec{E}\,dv}$$

→ $\int_S \vec{S}\cdot d\vec{s}$ → Represent rate of Flow of Energy or Power flux

→ $\frac{d}{dt}\int_V\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv$ = Rate of change of total Energy

→ $\frac{\epsilon E^2}{2}$ & $\frac{\mu H^2}{2}$ represent Energy stored in Electric & Magnetic Field.

$\int_V \vec{J}\cdot\vec{E}\,dv$ = Rate of work done by the Electromagnetic Field in displacing the charge within the volume V

Hence

$$\vec{\nabla}\cdot\vec{S} + \vec{J}\cdot\vec{E} + \frac{d}{dt}\left(\frac{1}{2}\mu H^2\right) + \frac{d}{dt}\left(\frac{1}{2}\epsilon E^2\right) = 0$$

OR

$$\int_V \vec{\nabla}\cdot\vec{S}\,dv + \int_V \vec{J}\cdot\vec{E}\,dv + \frac{d}{dt}\int_V\left(\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right)dv = 0$$

Above Equation is known as Poynting Theorm or Work-Energy Theorm.

According to Poynting theorm the Power transformed into Electromagnetic field is Equal to the Sum of the time rate of change of EM energy within certain volume and the time rate of Energy Flowing out through the boundary surface. This is also known as Energy Conservation law in Electromagnetism.

Electromagnetic Waves

The coupled Oscillating electric and Magnetic fields that moves with the speed of light and exhibit wave behaviour is called Electromagnetic wave. The Electric and magnetic components of plane electromagnetic wave are perpendicular to each other and to the direction of propagation. Such waves are also called plane polarised electromagnetic wave.

Wave Equation

If we consider a linear medium having permittivity ε, permeability μ and Conductivity σ

$D = \epsilon E$, $B = \mu H$, $J = \sigma E$ (current density)

and charge density $\rho = 0$

With these condition maxwell's Eqn reduce to

$$\nabla\cdot D = \rho \Rightarrow \nabla\cdot E = 0 \quad — ①$$

$$\nabla\cdot B = 0 \Rightarrow \nabla\cdot H = 0 \quad — ②$$

$$\nabla\times E = -\frac{\partial B}{\partial t} \Rightarrow \nabla\times E = -\mu\frac{\partial H}{\partial t} \quad — ③$$

$$\nabla\times H = J + \frac{\partial D}{\partial t} \Rightarrow \nabla\times H = \sigma E + \epsilon\frac{\partial E}{\partial t} \quad — ④$$

Now taking the curl of eqn ③

$$\text{Curl Curl } E = -\mu\frac{\partial}{\partial t}(\text{Curl } H)$$

② Electromagnetic Wave Equation in Dielectric:-

Maxwell's Equation in General form is given by-

I) $\nabla \cdot E = \rho/\epsilon_0$

II) $\nabla \cdot B = 0$

III) $\nabla \times E = -dB/dt$

IV) $\nabla \times B = \mu_0 \left(J + \frac{dD}{dt} \right)$

For dielectric conductivity $\boxed{\sigma = 0}$ and $\boxed{J = \sigma E = 0}$

When Dielectric is not charged then $\boxed{\rho = 0}$

Then-

I) $\nabla \cdot E = 0$

II) $\nabla \cdot H = 0$

III) $\nabla \times E = -\mu_0 \frac{dH}{dt}$

IV) $\nabla \times H = \epsilon_0 \frac{dE}{dt}$

Now Replacing μ_0, ϵ_0 by μ & ϵ respectively in Equation ③ & ④ we get.

$$\nabla^2 E = -\mu\epsilon \frac{d^2 E}{dt^2}$$

$$\nabla^2 H = -\mu\epsilon \frac{d^2 H}{dt^2}$$

or
$$\frac{d^2 E_x}{dx^2} - \mu\epsilon \frac{d^2 E_x}{dt^2} = 0$$

$$\frac{d^2 H_x}{dx^2} - \mu\epsilon \frac{d^2 H_x}{dt^2} = 0$$

Comparing with general wave Equation.

$$\frac{d^2 y}{dx^2} - \frac{1}{c^2}\frac{d^2 y}{dt^2} = 0$$

$$\Rightarrow \frac{1}{c^2} = \mu\epsilon \Rightarrow \boxed{c = \frac{1}{\sqrt{\mu\epsilon}}} \text{ - wave velocity}$$

Plane Electromagnetic waves in free space—

Maxwell's Eqns are

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Here $B = \mu H$, $D = \epsilon E$ and $J = \sigma E$

For free space $\rho = 0, \sigma = 0, \mu = \mu_0$ and $\epsilon = \epsilon_0$

therefore

$$\nabla \cdot E = 0 \quad \text{——— ①}$$

$$\nabla \cdot H = 0 \quad \text{——— ②}$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \quad \text{——— ③}$$

$$\nabla \times H = \sigma E + \frac{\partial \epsilon_0 E}{\partial t} \qquad \text{Here } \sigma = 0$$

$$\nabla \times H = \epsilon_0 \frac{\partial E}{\partial t} \quad \text{——— ④}$$

taking the Curl of Eqn ③ and Eqn ④

$$\text{Curl Curl } E = -\mu_0 \frac{\partial}{\partial t} \text{Curl } H$$

$$= -\mu_0 \frac{\partial}{\partial t}\left(\epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\text{Curl Curl } E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\text{Curl Curl } E = \text{grad div } E - \nabla^2 E$$

$$\text{Curl Curl } E = -\nabla^2 E \qquad \{\text{div } E = 0\}$$

So
$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{——— ⑤}$$

Similarly

$$\nabla^2 H - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{——— ⑥}$$

Eqn ⑤ and eqn ⑥ are the wave eqn in free space

or Eqn ⑤ and Eqn ⑥ can be written as also

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \quad \text{——— ⑦}$$

$$\nabla^2 H - \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2} = 0 \quad \text{——— ⑧}$$

Where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/sec.

Let the solⁿ of Eqn ⑦ and Eqn ⑧ are

$$E(r,t) = E_0 e^{ik \cdot r - i\omega t} \quad \text{——— ⑨}$$

$$H(r,t) = H_0 e^{ik \cdot r - i\omega t} \quad \text{——— ⑩}$$

Where E_0 and H_0 are Complex amplitudes which are Constant in space and $\hat{k} = k\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\nu}{c}\hat{n} = \frac{\omega}{c}\hat{n}$

$\hat{n}$ is a unit vector in the direction of wave propagation

In order to apply the condition

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \cdot E = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) E_0 e^{ik \cdot r - i\omega t}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left[(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})\right] e^{i(k_x x + k_y y + k_z z) - i\omega t}$$

$$= i(\hat{i}k_x + \hat{j}k_y + \hat{k}k_z) \cdot (\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z}) e^{ik \cdot r - i\omega t}$$

$$= ik \cdot E_0 e^{ik \cdot r - i\omega t}$$

$$\nabla \cdot E = ik \cdot E$$

Similarly $\nabla \cdot H = ik \cdot H$

Therefore $k \cdot E = 0$

$k \cdot H = 0$

Which means E and H are both perpendicular to the direction of propagation k i.e. electromagnetic waves are transverse in character.

Energy Carried by Electromagnetic Waves

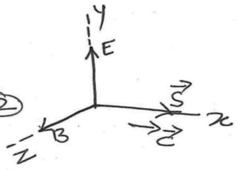
The rate of flow of Energy in an Electromagnetic wave is described by a Vector S, Called pointing Vector, Which is defined by the Expression

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \quad \text{—— (1)}$$

The magnitude of $\vec{S}$ represents power per Unit area. The direction of vector is along the direction of Wave propagation.

$$|\vec{S}| = |\vec{E} \times \vec{B}| = EB$$

$$S = \frac{EB}{\mu_0} \quad \text{—— (2)}$$



But $B = \frac{E}{c}$

So, $S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$ —— (3)

This Eqn represent the instantaneous rate at Which Energy is passing through a unit Area.

The time average of S over one or more cycle of Sinusoidal plane Electromagnetic wave is Called wave intensity I.

**[We obtain an expression involving the time average of $\cos^2(kz-\omega t)$ in the Case of intensity of Sound wave ~~which~~ which is Equals to $\frac{1}{2}$]

Hence the average value of 'S' is

$$I = S_{avg} = \frac{E_{max} B_{max}}{2\mu_0}$$

$$= \frac{E_{max}^2}{2\mu_0 c}$$

$$S_{avg} = \frac{c B_{max}^2}{2\mu_0} \quad \text{—— (4)}$$

The instantaneous energy density u_E associated with an Electric field is

$$u_E = \frac{1}{2}\epsilon_0 E^2 \quad \text{—— (5)}$$

and instantaneous Energy density u_B associated with magnetic field is

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{—— (6)}$$

Because E and B vary with time for an Electromagnetic wave, the Energy densities also vary with time.

Using the relationship $B = E/c$ and $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

The Expression for u_B becomes

$$u_B = \frac{(E/c)^2}{2\mu_0}$$

$$= \frac{\mu_0 \epsilon_0}{2\mu_0} E^2$$

$$u_B = \frac{1}{2}\epsilon_0 E^2 \quad \text{—— (7)}$$

Comparing Eqn (5) and (7)

$$u_B = u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

the instantaneous Energy density associated with the magnetic field of an electromagnetic wave Equals the instantaneous Energy density associated with the electric field.

So, in a given volume, the Energy is Equally Shared by the two fields.

The total instantaneous Energy density 'u' is Equal to the Sum of Energy density associated with the Electric and magnetic field.

$$u = u_E + u_B$$

$$\boxed{u = \epsilon_0 E^2}$$

also $\boxed{u = \frac{B^2}{\mu_0}}$

The total average Energy per unit volume is

$$u_{avg} = \epsilon_0 (E^2)_{avg}$$

$$u_{avg} = \frac{1}{2}\epsilon_0 E_{max}^2$$

also $u_{avg} = \frac{B_{max}^2}{2\mu_0}$

UNIT-2 WAVE OPTICS

INTERFRENCE:- The phenomenon of addition or superposition of two light waves is called interference of light.

At some points the intensity is maximum while at other points the intensity is minimum.

Maximum intensity is called constructive Interference.

Minimum intensity is called destructive Interference.

When two light wave interfere we get Alternate dark and bright ~~fringes~~ bands These are called interference fringes.

Coherent Sources:- Two Sources are said to be coherent if they emit light waves of same frequency, same amplitude and are in same phase with each other.

Temporal coherence: (Coherence in time)

Longitudinal Coherence is known as temporal coherence. It is a ~~relation~~ measure of phase relation of wave reaching at a given points at two different times.

Spatial coherence (Coherence in space).

Transverse coherence or lateral coherence is known as spatial coherence. It is a measure of phase relationship between the waves reaching at two different points in space at same time.

Methods for Producing Interference Pattern:-

It is divided into two Parts-

I) **Division by wave front:-** Interference of light takes place between waves from two sources formed due to ~~two~~ single source.

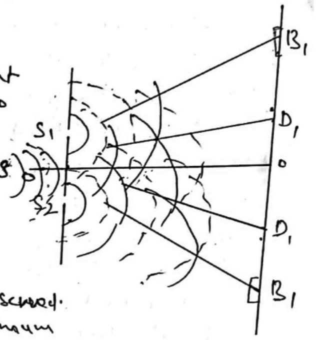
Eg Interference by Young double slit.

II) **Division by Amplitude:-** Interference takes place between the waves from the ~~real~~ real source & virtual source.

Example- Interference by thin film.

Young's Double Slit Experiment:-

In 1801, Thomas Young demonstrated the interference of light experimentally.



A source of monochromatic light S is used for illuminating two narrow slit S_1 & S_2. Two slits lie very close to each other and at equal distance from source S.

The wave from from slit S_1 & S_2 spreadout in all directions and superimpose on the screen.

Alternate bright & dark fringes observed.

At centre O intensity of light is maximum & known as central maxima.

As we move above & below the centre O alternate bright & dark fringes obtained.

From Young's double slits experiment following facts can be verified.

1) Interference pattern disappear, if one of the two slit is closed. It shows that interference pattern is due to superposition of wave from two slits.

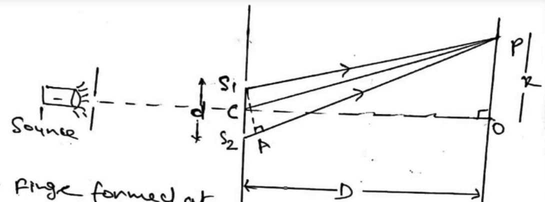
11) Instead of two slits illuminated with a single source, if two independent sources (S_1 & S_2) are used, the position of maximum & minimum intensity dont remain fixed. It shows that for producing interference coherent source (single source) should be used.

Condition for constructive interference:

$\boxed{x = n\lambda}$ $(n = 0, 1, 2, 3 \ldots)$

Condition for Destructive Interference:-

$\boxed{x = (2n+1)\frac{\lambda}{2}}$



Let a bright fringe formed at a point P on the screen.

$OP = x$ $\quad OC = D$ $\quad S_1S_2 = d$

Path Difference $= S_2P - S_1P = S_2A$

We have ΔS_1S_2A & ΔPCO are similar

$$\frac{S_2A}{S_1S_2} = \frac{OP}{CP}$$

& substitute $CP = CO$ as d is very small as compare to OP

Then $\dfrac{S_2A}{S_1S_2} = \dfrac{OP}{CO}$

$\Rightarrow \dfrac{S_2A}{d} = \dfrac{x}{D}$

Path Diff $\quad S_2A = \dfrac{xd}{D}$

For constructive $\dfrac{xd}{D} = n\lambda \quad \Rightarrow x = \dfrac{nD\lambda}{d}$

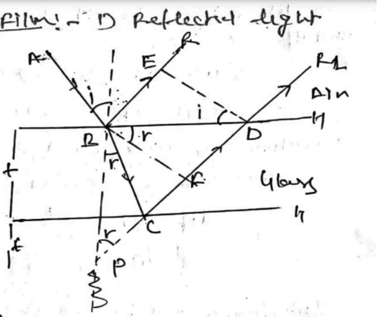
For Destructive $\dfrac{xd}{D} = (2n-1)\dfrac{\lambda}{2}$

$\Rightarrow x = (n - \frac{1}{2})\dfrac{\lambda D}{d}$

Fringe width:- let $x_n = \dfrac{nD\lambda}{d}$ $\quad x_{n+1} = (n+1)\dfrac{D\lambda}{d}$

$(x_{n+1} - x_n) = \dfrac{(n+1)D\lambda}{d} - \dfrac{nD\lambda}{d} = \dfrac{D\lambda}{d} \Rightarrow$ Fringe width $B = D\lambda/d$

Interference due to thin film:- 1) Reflected light



Consider a transparent thin film of thickness t

A ray AB be incident on the upper surface of thin film

AB is partly reflected along BR & partly refracted along BC.

At C again partly reflected along CD.

This process continued throughout the thin film.

Path difference between reflected ray BR & DR_1 can be calculated. Draw ED normal to BR and BF to CD

Angle of incidence is i and angle of refraction is r.

Optical path difference between two reflected rays (BR & DR_1) is given by-

$\Delta =$ Path $(BC+CD)$ in film $-$ Path BE in Air.

$= \mu(BC+CD) - BE$ ——(1)

$\mu = \dfrac{\sin i}{\sin r} = \dfrac{BE/BD}{FD/BD} = \dfrac{BE}{FD}$

$BE = \mu(FD)$ ——(2)

Put (2) in (1)

$\Delta = \mu(BC+CD) - \mu(FD)$

$= \mu(BC + CF + \cancel{FD} - \cancel{FD}) = \mu(BC+CF)$

$\Rightarrow \Delta = \mu(PC+CF) = \mu PF$ ——(3) $\quad \because (PC = BC)$

Now in ΔBPF $\quad \cos r = \dfrac{PF}{BP} \Rightarrow PF = BP\cos r$
$= 2t\cos r$ ——(4)

Equating (3), (4) we get $\Delta = \mu 2t\cos r = 2\mu t\cos r$

Condition for Bright Band

Path Diff. $\Delta = n\lambda$

$2\mu t\cos r \pm \frac{\lambda}{2} = n\lambda$

$2\mu t\cos r = (2n \pm 1)\frac{\lambda}{2}$

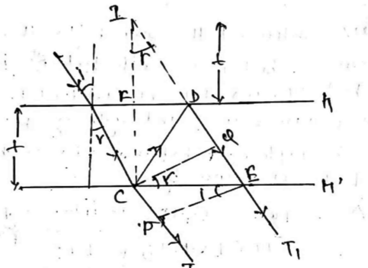
Condition for Dark Band.

Path Diff $\Delta = (2n \pm 1)\lambda/2$

$2\mu t\cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$\boxed{2\mu t\cos r = n\lambda}$

Interference due to Transmitted light:-



Effective Path Difference

$\Delta = \mu(CD+DE) - CP$ ——(1)

$\mu = \dfrac{\sin i}{\sin r} = \dfrac{CP/CE}{QE/CE} = \dfrac{CP}{QE} \Rightarrow CP = \mu(QE)$ ——(2)

From (1), (2)

$\Delta = \mu(CD + DQ + QE) - QE(\mu)$

$= \mu(CD + DQ)$ $\quad \because (CD = ID)$

$= \mu(IQ)$

$\Delta = \mu 2t\cos r \Rightarrow \boxed{\Delta = 2\mu t\cos r}$

Bright- $\quad \Delta = 2\mu t\cos r = n\lambda$

Dark $\quad \Delta = 2\mu t\cos r = (2n \pm 1)\frac{\lambda}{2}$

Interference due to wedge Shape thin Film:-

Thin Film:- Layer of Material deposited on a surface which decide its Properties known as thin Film.

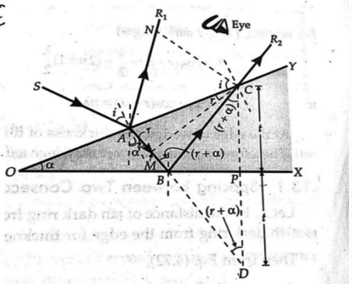
Wedge Shape thin film:- A film of Variable thickness is known as wedge shape thin film.

OR

A thin film of varying thickness having zero thickness at one Point and Progressively increasing to a Particular thickness at other end is known as wedge.

Let us consider OX & OY are two planes inclined at an angle α. YOX is the region inside thin film. μ is the refractive index inside YOX.

Then the Path Difference (Δ) between ray AR_1 & CR_2 is



Interference produced by wedge shaped film.

$\Delta = (AB+BC)_{Med} - (AN)_{Air}$

$= \mu(AM+MB+BC) - (AN)$

$= \mu(AM+MB+BC) - (AN)$ — (1)

from Snell's law

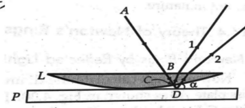
$\mu = \frac{\sin i}{\sin r}$

from ΔANC & AMC,

$\mu = \frac{AN/AC}{AM/AC} \Rightarrow AN = \mu AM$ — (2)

If refractive index of Medium (film) is greater than the refractive index of incident ray (AR_1) then AR_1 suffer Path diff. of $\lambda/2$ then (1) becomes-

$\Delta = \mu(AM+MB+BC) - (AN \pm \frac{\lambda}{2})$



Substituting (2) in (1)

$\Delta = \mu(AM+MB+BC-AM) \pm \frac{\lambda}{2}$

$\Delta = \mu(MB+BC) \pm \frac{\lambda}{2}$ — (3)

Also we have $BC = BD$ & $CP = PD = t$

then (3) becomes

$\Delta = \mu(MB+BD) \pm \frac{\lambda}{2} = \mu MD \pm \frac{\lambda}{2}$

from ΔCMD,

$MD = 2t\cos(r+\alpha)$

$\Rightarrow \boxed{\Delta = 2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2}}$

I) For maxima

$2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2} = n\lambda$

$\Rightarrow \boxed{2\mu t\cos(r+\alpha) = (2n \pm 1)\frac{\lambda}{2}}$

II) For Minima

$2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$\Rightarrow \boxed{2\mu t\cos(r+\alpha) = n\lambda}$

Colours in Thin film

When an oil film on water or a soap film or wedge shaped air film between two glass plate is seen in reflected light it shows brilliant colours.

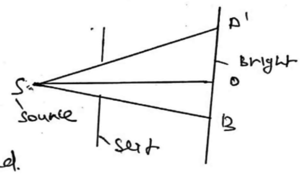
This coloured Phenomenon was offered by Young on the basis of interference of light wave.

Scanned with CamScanner

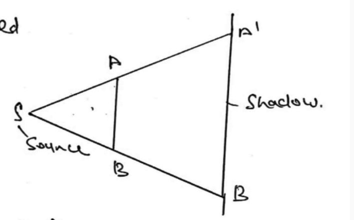
Unit (3). (1)

Diffraction:- The Phenomenon of bending of light round the sharp corner and spreading into the region of geometrical shadow is called Diffraction of light.

I) When a Narrow slit is Placed in the Path of light only the Region A'B' on the screen should get illuminated.

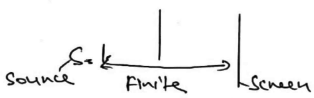
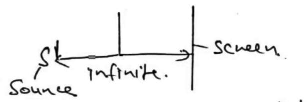


II) When an obstacle AB is Placed in the Path of light, then its distinct geometrical shadow should be obtained on the screen.



Interference	Diffraction
1) It occurs due to superposition of secondary wavelets from ~~two~~ coherent sources of light.	1) It occurs due to superposition of secondary wavelets from exposed Part of single source.
2) All Bright fringes have same Intensity	3) Intensity of successive bright fringes goes on decreasing.
3) fringes due to mono-chromatic light has same width	3) Diffraction fringes are never of Equal width
4) Intensity of all dark fringes are zero	4) Intensity of dark fringes are Not zero.

(2)

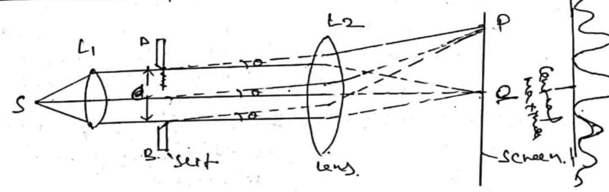
Fresnel Diffraction.	Fraunhoffers Diffraction.
I) In Fresnel Diffraction Source, screen & diffraction device are at finite Distance	I) In Fraunhoffers Diffraction Source, screen and Diffraction device are at Infinite Distance
	
II) No lens & No Mirror Used	II) At infinite distance intensity of light becomes low. Hence lens (convex) is Used.
III) center May be dark or bright	III) Center always bright
IV) wavefront are spherical or cylindrical	IV) Plane wavefront are Used.
V) Diffraction device are zone Plate, ~~the~~ circular Ring etc.	V) Diffraction device are Double slit, n slit, grating etc.

1) dispersive power is directly proportional to the order of the spectrum (n) i.e. higher is the order greater is the dispersive power.

2) dispersive power is inversely proportional to the grating element i.e. the dispersive power of two given lines is greater for a grating having Larger number of lines per cm.

3) dispersive power is inversely proportional to $\cos\theta$ i.e. Larger the value of θ higher is the dispersive power.

Fraunhoffer Diffraction due to single slit:-



* A Monochromatic Source of light S, emitting light wave of wavelength λ is placed at the principle focus of convex lens L1
* Diffraction pattern obtained on screen lying at a distance D from the slit. ~~after convex lens~~

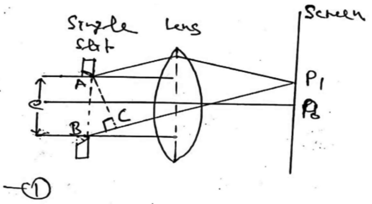
Path Difference = BC

$= e\sin\theta$

Phase Diff = $\frac{2\pi}{\lambda}$ Path diff

$= \frac{2\pi}{\lambda}(e\sin\theta)$

$$\boxed{\delta = \frac{1}{n}\frac{2\pi}{\lambda}(e\sin\theta)} \quad -(1)$$



from the theory of n- harmonic vibrations. we have, Resultant Amplitude R.

$$R = \frac{a\sin\frac{n\delta}{2}}{\sin\delta/2}$$

$$= \frac{a\sin\frac{n}{2}\left(\frac{1}{n}\frac{2\pi}{\lambda}e\sin\theta\right)}{\sin\frac{1}{2}\left(\frac{1}{n}\frac{2\pi}{\lambda}e\sin\theta\right)}$$

$$= \frac{a\sin\frac{\pi}{\lambda}e\sin\theta}{\sin\frac{\pi}{n\lambda}e\sin\theta}$$

let us consider $\frac{\pi}{\lambda}e\sin\theta = \alpha$

$$\Rightarrow \boxed{R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}}$$

if $\frac{\alpha}{n}$ is very small angle

$$\Rightarrow R = \frac{a\sin\alpha}{\frac{\alpha}{n}} = \frac{na\sin\alpha}{\alpha}$$

$$\boxed{R = \frac{A\sin\alpha}{\alpha}} \quad -(2)$$

Scanned with CamScanner

Intensity is Related to Amplitude

Intensity = $(\text{Amplitude})^2$

$$\boxed{I = \frac{A^2\sin^2\alpha}{\alpha^2}}$$

i) Condition for Minima $I = 0 \Rightarrow \frac{A^2\sin^2\alpha}{\alpha^2} = 0$

$\Rightarrow \sin\alpha = 0$

$\alpha = \pm n\pi$

Hence $\pm 1\pi, \pm 2\pi, \pm 3\pi$ -----

ii) Condition for Maxima

I should be maximum.

ie $I = \infty$

$\frac{A^2\sin^2\alpha}{\alpha^2} = \infty$

Means $\alpha = 0$ will give Maximum Intensity.

Fraunhofer's Diffraction due to two slits:-

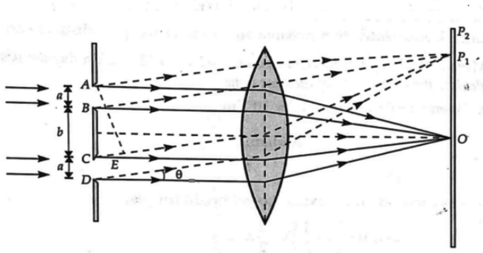
Let us consider a parallel beam of Monochromatic light of wavelength λ incident on double slit and each slit diffract at an angle θ.

Then Path difference ~~(a+b+c)~~ (a+b+c)

$CE = (a+b)\sin\theta$

we have Phase Difference = $\frac{2\pi}{\lambda}$ × Path difference

$$2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta \quad -(1)$$



Fraunhofer's diffraction at double slit.

Here each slit can be treated as an independent small Source of light. Resultant pattern will be same as due to two Sources.

If 2α is the phase difference between the extreme rays from first slit, then

$$2\alpha = \frac{2\pi}{\lambda}a\sin\theta$$

$$\alpha = \frac{\pi}{\lambda}a\sin\theta \quad -(2)$$

The resultant displacement y_1 due to the rays from the first slit is given by -

$$y_1 = A\sin\omega t \quad -(3)$$

Where $A = A_0\frac{\sin\alpha}{\alpha}$ (Amplitude)

A_0 is the Amplitude of the direct ray

A is the Amplitude of diffracted ray at angle θ from first slit.

Then Resultant displacement y_2 due to two rays from the Second slit is given by -

$$y_2 = A\sin(\omega t + 2\beta) \quad -(4)$$

Then Resultant Displacement Y due to rays from two slit diffracted at angle θ is given by -

$Y = y_1 + y_2$

$= A\sin\omega t + A\sin(\omega t + 2\beta)$

$= A(\sin\omega t + \sin(\omega t + 2\beta))$

~~$= 2A\cos\beta\sin(\omega t+\beta)$~~ $= 2A\cos\beta\sin(\omega t + \beta)$ $-(5)$

Substituting (5) in (4)

Then Resultant Amplitude,

$R = 2A\cos\beta$

Substituting $A = A_0\frac{\sin\alpha}{\alpha}$ from (3)

$\Rightarrow R = 2A_0\frac{\sin\alpha}{\alpha}\cos\beta$ But $I \propto A^2$

$$\Rightarrow I \propto 4A_0^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Suppose constant of proportionality is 1 then

$$I = 4A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta$$

Now I) Condition for maxima.

I will be maximum when $\sin\alpha = 0$

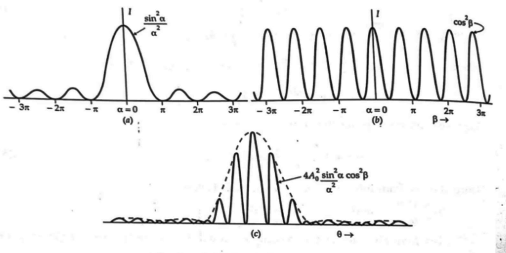
$$\sin\alpha = 0$$
$$\alpha = \pm m\pi$$

As we have $\alpha = \frac{\pi a\sin\theta}{\lambda}$

$$\Rightarrow \frac{\pi a\sin\theta}{\lambda} = \pm m\pi$$
$$\pi a\sin\theta = \pm m\lambda\pi$$
$$\boxed{a\sin\theta = \pm m\lambda}$$

Similarly for maxima

$$\cos^2\beta = 1$$
$$\beta = \pm n\pi$$
$$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$
$$\boxed{(a+b)\sin\theta = \pm n\lambda}$$



Resultant intensity distribution pattern of double slit.

Fraunhoffer Diffraction due to N Slits

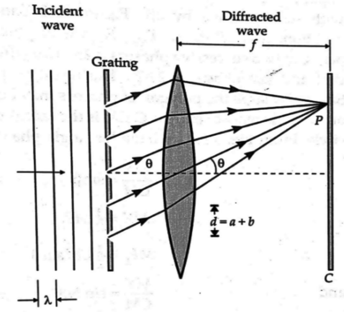
"An arrangement consist of large number of parallel equidistant narrow rectangular slits of same width is known as diffraction grating"

We have considered -

I) Each slit is of width a and has same length

II) All slits are parallel to each other.

III) Space between two slit is same.



Fraunhofer diffraction of a plane wave incident normally on a multiple slit aperture.

Consider point P on the screen where diffracted waves ~~are~~ at angle θ will superimpose.

From theory of diffraction all the points of slit can be summed up into single wavelet of amplitude A.

$$A = A_0\frac{\sin\alpha}{\alpha} \quad -(1)$$

If 2α is the phase difference between two extreme rays then

$$2\alpha = \frac{2\pi}{\lambda}a\sin\theta$$
$$\alpha = \frac{\pi}{\lambda}a\sin\theta \quad -(2)$$

The path difference from two nearby slit is given by -

$$\Delta = (a+b)\sin\theta \quad -(3)$$

The we have phase Difference $= \frac{2\pi}{\lambda}\times$ Path Difference

$$\Rightarrow \quad 2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta \quad -(4)$$

Now to find resultant Intensity (I) we have to superimpose N waves each of amplitude A with phase difference of 2β with nearby wave.

Then we have

$$\frac{MX}{CM} = \sin\beta$$
$$\Rightarrow MX = CM\sin\beta$$

But $MX = \frac{1}{2}MP_1$

$$\Rightarrow \frac{1}{2}MP_1 = CM\sin\beta$$
$$\text{or } MP_1 = 2CM\sin\beta \quad -(5)$$

Also $\frac{MY}{CM} = \sin N\beta$

$$\Rightarrow MY = CM\sin N\beta$$

But $MY = \frac{1}{2}MP_N$

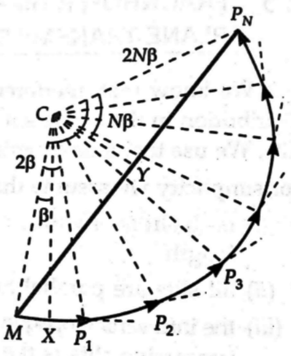
$$\frac{1}{2}MP_N = CM\sin N\beta$$
$$MP_N = 2CM\sin N\beta \quad -(6)$$

Dividing (6) by (5) we have

$$\frac{MP_N}{MP_1} = \frac{\sin N\beta}{\sin\beta}$$

MP_N - Resultant Disturbance of N Slit at θ

MP_1 - Single slit Disturbance



Phasor diagram for N slits.

Hence $R_\theta = A\frac{\sin N\beta}{\sin\beta}$ where $A = A_0\frac{\sin\alpha}{\alpha}$ by (1)

$$\Rightarrow R_\theta = A_0\frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

Resultant Intensity $I = R_\theta^2$

$$\Rightarrow I = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

Here ~~First term~~ $\boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2}$

Here First term $I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$ is intensity due to single slit

+ Second term $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ is due to combined Effect

Now I) Condition for maxima

$$\sin\beta = 0$$
$$\text{or } \boxed{\beta = \pm n\pi}$$

II) Condition for Minima

$$N\beta = \pm m\pi$$
$$N\left(\frac{\pi}{\lambda}(a+b)\sin\theta\right) = \pm m\pi$$
$$\boxed{N(a+b)\sin\theta = \pm m\lambda}$$

Diffraction grating:- An arrangement consist of large no. of close, parallel straight and transparent equidistant slits, each of equal width 'a' seprated by an opaque region b.
The spacing (a+b) between adjacent slit is called diffraction element or grating element.

Grating element = (a+b)

$$I = A^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$$

Resolving Power of an optical Instrument:
The ability or capability of an optical instrument to produce two separate images of two very close object is called resolving power.

OR

Ratio of wavelength of any spectral line to the smallest wavelength difference between very close line for which spectral line can just be resolved.

$$\text{Resolving power} = \frac{1}{\Delta d} = \frac{2a}{\lambda}$$
(d- Distance between two object. a- Numerical Aperture)

Dispersive Power:-
Rate of change of angle of diffraction with wavelength of light used. $(d\theta/d\lambda)$

Rayleigh's criterion:-
To obtain resolving power of an instrument Rayleigh suggested a criterion known as Rayleigh criterion. According to Rayleigh criterion two images can be regarded as seprated if central maxima of one falls on the first minima of other.
Distance between the centre of patterns shall be equal to the radius of central disc. This is called Rayleigh limit of Resolution.

Resolving Power of a Plane transmission grating:-

$$\text{Resolving Power} = \frac{n\ell}{(a+b)} = nN$$
$$\left(N = \frac{\ell}{a+b}\right)$$

Resolving Power of Telescope:-

$$\boxed{r = \frac{1.22 f\lambda}{a}}$$

r- Radius of central bright image
a- Diameter of objective.

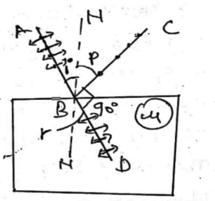
Resolving Power of Microscope:-

$$\boxed{d = \frac{\lambda_0}{2\mu \sin\alpha}}$$

λ_0 - wave length of light in vacuum
μ - Refractive index of Medium
$(\mu \sin\alpha)$ - Numerical Aperture of objective of Microscope.

Brewster's Law:- ③ It state that when light is incident at polarising angle at the interface of ~~reflecting~~ refracting medium, the refractive index of medium is equal to the tangent of the polarising angle.
If P be the polarising angle and μ the refractive index of the refracting medium.
then according to Brewster's law.

$$\boxed{\mu = \tan P} \quad \text{--- ①}$$



When a light is incident at polarising angle P on a refracting medium of refractive index μ
let r be the angle of refraction
then according to Snells law

$$\boxed{\mu = \frac{\sin P}{\sin r}} \quad \text{--- ②}$$

From ① + ②

$$\frac{\sin P}{\sin r} = \tan P$$
$$\frac{\sin P}{\sin r} = \frac{\sin P}{\cos P}$$
$$\Rightarrow \sin r = \cos P \quad \text{or} \quad \sin r = \sin(90 - P)$$
$$\Rightarrow r = 90° - P \quad \Rightarrow \boxed{r + P = 90°}$$

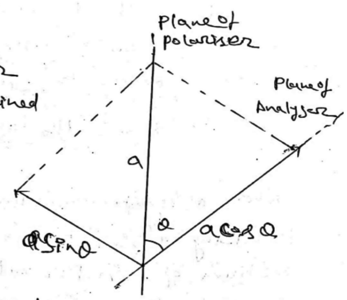
Hence when a ray of light incident at polarising angle, the reflected ray is at right angle to the refracted ray.

Law of Malus:- ④

It state that when a completely plane polarised light beam is incident on an analyser, the intensity of the emergent light varies as the square of cosine of the angle between the plane of transmission of the analyser and polariser.

$$\boxed{I = a^2\cos^2\theta}$$

Consider a plane of polariser and plane of analyser are inclined at an angle θ as shown in figure.
Further suppose that the plane of polarised light of intensity I_0 and amplitude a incident on polariser. Then



i) The component $a\cos\theta$ is along the plane of analyser.
ii) the component $a\sin\theta$ is along the perpendicular to the plane of analyser
then
Intensity of light transmitted from analyser is given by -
$$I = a^2\cos^2\theta$$
$a^2 = I_0$, intensity of incident plane polarised light
therefore
$$\boxed{I = I_0\cos^2\theta} \quad \text{or } I_d\cos^2\theta$$

Condition:- when $\theta = 0$ or $180°$ $\cos\theta = \pm1 \Rightarrow \boxed{I = I_0}$
Hence when polariser and analyser are parallel, the intensity of light transmitted from ~~light~~ analyser is same as from polariser.

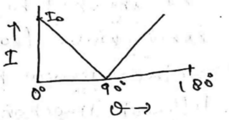
2) When $\theta = 90°$ so that $\cos\theta = 0$

$\boxed{I=0}$

therefore when polariser and analyser are perpendicular the intensity of light transmitted from light is zero ie minimum

3) If we plot a graph between intensity of light and angle between polariser and analyser it will be as -

4) In case light incident on analyser is unpolarised then $\boxed{I = \frac{1}{2} I_0}$



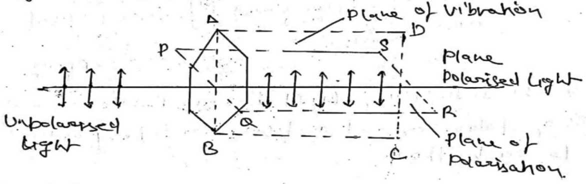
($\cos^2\theta = \frac{1}{2}$ $\cos^2\theta$ ie average value of $\cos^2\theta$)

Hence $I = \frac{1}{2} I_0$

(1)

Polarization:- "The phenomenon due to which vibration of light are restricted in a particular plane is called polarisation of light"

when an ordinary light ie unpolarised light passed through a tourmaline crystal, out of all the vibrations which are symmetrical about the direction of propagation, only those pass through it which are parallel to crystallographic axis AB. Therefore direction of propagation are confined to a single plane.



(6)

Nicol Prism:- "It is an optical device made from Calcite and frequently used for producing and analysis of plane polarised light. It is based on the phenomenon of double refraction."

Phenomenon of Double Refraction or Bire Fringes:- The phenomenon of splitting of unpolarised light into two polarised refracted ray is known as double refraction.

* When a narrow beam of unpolarised light be incident normally on a double refracting crystal such as calcite, it split into two refracted rays- one is 'ORDINARY RAY' or O-ray and other is 'EXTRORDINARY RAY' - E-RAY.

Nicol Prism Principle

When unpolarised beam of light enter the calcite crystal it split into O-ray & E-ray

In Nicol prism O-ray is eliminated by total internal reflection. Hence E-ray only transmitted through the prism.

Construction: A calcite crystal cut into two half. The two half of crystal are properly polished and cemented in their original position with a thin layer of cement named as Canada balsam.

→ Refractive index of Canada Balsam (which is midway for E-ray & O-ray) $\mu = 1.55$

→ Refractive index for calcite of E-ray $\mu_E = 1.49$

→ Refractive index for O-Ray $\mu_o = 1.66$

Nicol Prism

$\mu = \frac{1}{\sin C}$

$C = \sin^{-1}\left(\frac{1.55}{1.66}\right) \Rightarrow \boxed{C = 69°}$

Critical Angle $C = 69°$

Huygen theory of Double Refraction:

I) If two monochromatic ray is incident on a doubly ~~ref~~ refracting crystal it split into two wave front one for ordinary ray and other for extra ordinary ray.

II) Ordinary ray has same velocity in all direction hence wave front is spheroidal.

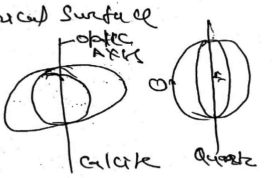
III) Extra ordinary ray has different velocity in different direction hence its wave front is ellipsoidal

IV) The uniaxial crystal has been classified as Negative and positive crystal. In Negative crystal like calcite extraordinary wave velocity (V_e) is greater than ordinary velocity (V_o).

In positive crystal $V_o > V_e$.

V) In Negative crystal like calcite ellipsoidal surface lies outside the spherical surface

In positive crystal ellipsoidal surface lies inside spherical surface.



VI) velocity of ordinary ray & Extra ordinary ray is same along optic Axis.

VII) Ellipsoidal wave surface must be symmetrical about optic Axis.

(7)

Optical Activity:- Certain crystals and solutions posses a natural ability to rotate the plane of polarization about the direction of its propagation. this process known as optical activity

Right handed or dextro-rotatory:- Plane of polarisation or plane of vibrations rotated in clockwise direction.

Eg. Cinnabar, Cane sugar etc.

~~Right~~ Left handed or Laevo rotatory:- Plane of polarisation or plane of vibrations rotated in anti clockwise direction.

Eg - Fruit sugar

Specific Rotation:- Measure of optical activity of a sample

Specific rotation for a given wave length of light at a given temperature is defined conventionally as the rotation produced by one decimeter long column of solution containing 1gm of optically active material per cc of solution.

$$[S]_\lambda^t = \frac{\theta}{l \times C} = \frac{\text{Rotation in Degree}}{\text{length in cc} \times \text{Conc in gm/cc}}$$

Polarimeter:- A polarimeter is an instrument used for determining the optical rotation of solution

→ When used for determining the optical rotation of sugar it is called saccharimeter

Half shade polarimeter:- It consist of two Nicol prism N_1 & N_2, N_1 is polariser & N_2 is analyser. Behind N_1 is a half wave plate of quartz Q which cover one half of the view while other half is covered by Glass G.

Quarter wave Plate:- Plate of double refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of optic Axis.

- If the thickness of the plate is t and the refractive index of ordinary and extraordinary rays are μ_o & μ_e respectively. then the path difference introduced between the two rays is given by $\lambda/4$

To produce a path Difference of $\lambda/4$ in Calcite

$$(\mu_o - \mu_e)t = \lambda/4$$

$$\boxed{t = \frac{\lambda}{4(\mu_o - \mu_e)}}$$

In quartz $\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$

Half wave plate:- This plate is also made from double refracting uniaxial crystal of quartz or calcite with its refracting face cut parallel to the optic axis. The thickness of the plate is such that the ordinary and extraordinary rays have path difference $= \lambda/2$

In Calcite $(\mu_o - \mu_e)t = \lambda/2$

$$\boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}}$$

In Quartz $\boxed{t = \frac{\lambda}{2(\mu_e - \mu_o)}}$

Unit 4 – LASER

LASER:-

LASER:- Light Amplification by Stimulated Emission of Radiations.

LASER action takes place with the interaction of photons with material which results in Stimulated Absorption, Spontaneous Emission & stimulated Emission Process.

Characteristics of LASER beams:-

I) Directionality II) High Intensity

III) High degree of coherence IV) Monochromaticity

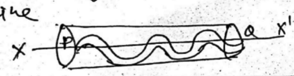
Coherence:- Two light rays which has constant phase with respect to time, are said to be Coherence

Coherence are of two types-

I) **Temporal coherence:-** Temporal coherence is the phase relationship between Radiation fields at different times.

Temporal coherence measure the duration over which this phase relationship is maintained.

- Monochromatic property of laser is due to temporal coherence.
- Consider a light beam travelling along axis XX'. P & Q are two points lying on this line



Two beams are coherent if path difference between P & Q is constant.

Average length of wave train is called coherent length.

If the velocity of light is c then coherent length L_c is given by.

$$L_c = c\,\Delta t$$

$$L_c = \frac{c}{\Delta\nu}$$

$$\Delta\nu = \frac{c}{L_c} \quad -(1)$$

Also we have $\nu = \frac{c}{\lambda} = c\lambda^{-1}$

on Differentiating wrt λ $\frac{\Delta\nu}{\Delta\lambda} = -\frac{c}{\lambda^2}$

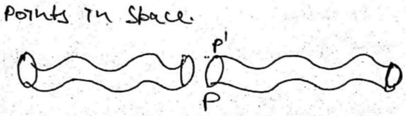
$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$$

using (1) $\frac{\cancel{c}}{L_c} = -\frac{\cancel{c}}{\lambda^2}\Delta\lambda$

$$\Rightarrow \boxed{\Delta\lambda = \frac{-\lambda^2}{L_c}} \quad \text{or} \quad |\Delta\lambda| = \left|\frac{-\lambda^2}{L_c}\right| = \lambda^2/L_c$$

Hence temporal coherence depend on the value of coherent length and coherent time.

Spatial coherence:- Spatial coherence is the phase relationship between the Radiation field at different points in space.



Two beams are said to posses spatial coherence if the phase difference of the wave crossing P & P' at any instant is constant

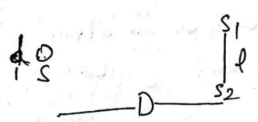
Concept of spatial coherence can be understood by double slit Experiment as shown in figure.

S_1, S_2 are two pinhole with separation l, the source S is at a distance D from S_1 & S_2

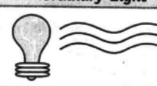
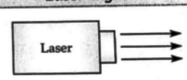
Then the condition for Coherence of S_1, S_2 is.

$$\frac{\lambda}{a} > \frac{l}{D}$$

or $\boxed{l < \frac{D\lambda}{a}}$

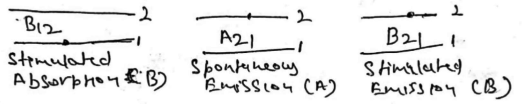


Difference between ordinary light & LASER light

S.No.	Ordinary Light	Laser Light
		
1.	It has many wavelengths or it is not monochromatic.	It is monochromatic.
2.	It is multidirectional.	It is directional.
3.	It is incoherent *i.e.*, the constituent waves are generally not in the same phase.	It is coherent *i.e.*, the constituent waves are exactly in the same phase.
6.	It does not travel as a concentrated and parallel beam.	It travels as a concentrated and parallel beam.
5.	Ordinary light is produced by spontaneous emission.	Laser beam is produced by stimulated emission.

Einstein A & B Coefficients:-

The Probability rate of transition (Absorption, Spontaneous Emission, Stimulated emission) are Expressed in terms of three constants B_{12}, A_{21}, B_{21} known as Einstein Coefficients



Let us consider an assembly of Atom which is in thermal Equilibrium with Electromagnetic radiation of Frequency (ν) and Energy density $u(\nu)$ at temperature T.

Note:- Einstein A coefficient is Related to Spontaneous Emission of light.
Einstein B Coefficient are related to Absorption & Stimulated Emission of light.

Then

I) Number of Absorption transitions Per Unit volume

$= N_1 B_{12} u(\nu)$

(where $u(\nu)$ – Energy Density).

II) Number of Spontaneous transitions Per Unit time Per Unit volume will be.

$= N_2 A_{21}$

III) Number of Stimulated Emission Per Unit time per Unit volume will be.

$= N_2 B_{21} u(\nu)$

Now at thermal Equilibrium the number of Absorption transition and emission transition should be equal.

$$N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$$

$$N_1 B_{12} u(\nu) - N_2 B_{21} u(\nu) = N_2 A_{21}$$

$$(N_1 B_{12} - N_2 B_{21}) u(\nu) = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{(N_1 B_{12} - N_2 B_{21})}$$

Dividing by $N_2 B_{21}$

$$u(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\left(\frac{N_1}{N_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1} \quad \text{— (1)}$$

According to Boltzman law, Number of Atoms N_1 & N_2 in the Energy States E_1 & E_2 at temperature T are given by -

$$N_2 = N_0 e^{-E_2/KT}$$

$$N_1 = N_0 e^{-E_1/KT}$$

$$\frac{N_2}{N_1} = \frac{e^{-E_2/KT}}{e^{-E_1/KT}} = e^{-\frac{(E_2-E_1)}{KT}} = e^{-\frac{h\nu}{KT}}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{h\nu/KT} \quad \text{— (2)}$$

Substituting (2) in (1)

$$\Rightarrow u(\nu) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/KT} - 1} \quad \text{— (3)}$$

According to Plancks Radiation Equn we have-

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/KT} - 1} \quad \text{— (4)}$$

Comparing (3) & (4) we get-

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{— (5)}$$

$$\& \quad \boxed{B_{12} = B_{21}} \quad \text{— (6)}$$

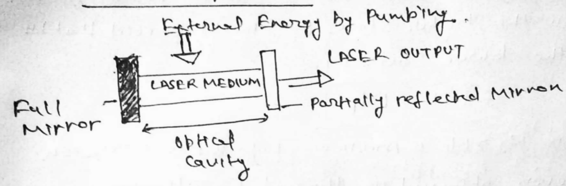
Hence from (6) the Probablity of Stimulated Emission is Equal to Probablity of Stimulated Absorption

From (5) $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$

$$\Rightarrow \frac{A_{21}}{B_{21}} \propto \nu^3 \quad (E_2 - E_1 = h\nu)$$

It means the Probablity of Spontaneous Emission Increase Rapidly when the Energy difference between two State is large.

Components of laser



I) LASER MEDIUM:- It is the Material in which laser action takes place. The active Medium may be Solid Crystal Such as Ruby or gases like CO_2 or Helium or Semiconductor Such as GaAs.

This Medium decide the wavelength of Laser Radiation Laser Medium Contains atom which can Produce more Stimulated emission and Cause Amplification or they are called Active centers.

II) Energy Source (Pumping)

Energy Source Pump the active centes from ground State to excited State to achieve Population inversion Pumping by Energy Source Can be optical Electrical or Chemical depending on laser Medium which we are using.

III) Resonance Cavity: Resonance Cavity Consist of laser Medium (or Active Medium) enclosed between two Mirror one is full Mirror and other is Partially Reflecting Mirror.

How to Achieve Population Inversion: the number of atoms in the higher state E_2 should exceed ~~that~~ than the lower state E_1.

$$\boxed{N_2 > N_1}$$

Condition $N_2 > N_1$ is known as Population Inversion.

* It is never observed in thermal equilibrium.
* It helps in achieving Stimulated Emission.

Pumping Process:- The Process by which population inversion is achieved is known as Pumping process.

1) Optical Pumping
(Excitation by a strong source of light eg flash lamp, arc lamp).

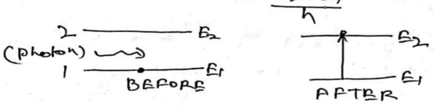
2) Electrical Pumping.
(Pumping is usually provided in the form of light or Electric current).

1) Stimulated Absorption of Radiations:-

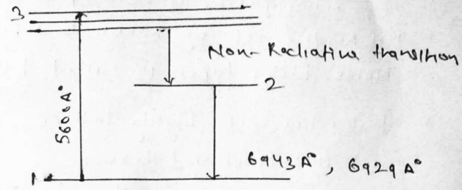
In Stimulated or Induced absorption an atom in a lower level absorb a photon of frequency ν and moves to a upper level.

Consider an atom in a lower state 1 rise to higher state 2 by absorbing Photon of frequency ν given by

$$\nu = \frac{E_2 - E_1}{h}$$



Ruby laser



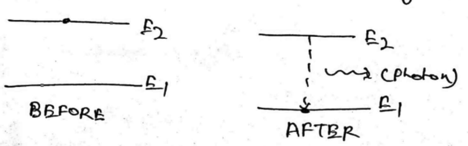
* Lines 1, 2, 3 shows Energy level of Chromium
* Ruby is a crystal of Aluminum (Al_2O_3) doped with Chromium oxide (Cr_2O_3)
* Aluminum atoms in the crystal are replaced by Cr^{+++} ions, it gives Pink color to ruby and give rise to laser action.
* Chromium atom absorb wavelength $\lambda = 5600$Å and get excited from energy level 1 to energy level 3.
* From this level 3 some excited atom returns to ground state, but some other moves to level 2 which is metastable state.
* Life time of atom in the excited state 3 is very less i.e 10^{-8} sec.
* Metastable state has very long life time (3×10^{-3} sec) as compare to energy level 3 (10^{-8} sec).
* The number of Atom in energy level 2 keeps on increasing and ultimately become more populated than 1.
Hence population inversion is established between level 2 & level 1.

* A stage is reached where population inversion caused by xenon tube is used up and laser beam closes till next flash of xenon to repeat the process. Hence Ruby laser is called Pulsed laser.
* Ruby laser is highly intense, coherent, Monochromatic and Unidirectional beam.

Uses:-

I) Precision welding & drilling in metals
II) Drilling of Industrial diamonds.
III) Holography & Photography of moving objects.

2) Spontaneous Emission:- In this Process an electron in the higher energy level decay to lower level and emit a photon of frequency ν.



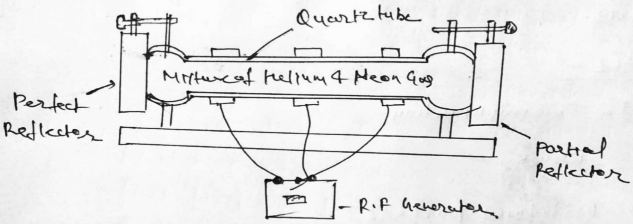
$$h\nu = E_2 - E_1$$

$$\boxed{\nu = \frac{E_2 - E_1}{h}}$$

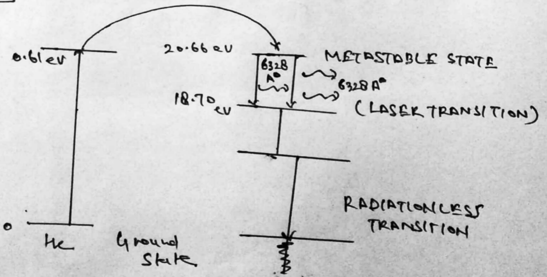
He-Ne laser

Construction:

* The Active Medium is a Mixture of Helium & Neon in the ratio 9:1 at pressure about 1 mm of Hg.
* One of the Mirror is fully silvered & other is Partially silvered



Working



* When a discharge is passed through the gas mixture electron are accelerated down to the tube.
* These accelerated electron collide with the He-Ne atom to Metastable state 20.61 eV and 20.66 eV.

* When an excited Ne atom pass from the Meta Stable State at 20.66eV to 18.70eV it emits a 6328 Å photon. (14)
* This photon travel to gas mixture, reflected back and forth by mirror until it stimulate an excited Ne atom and cause it to emit fresh 6328 Å photon. Process is continued & Laser transition takes place.

Uses –

i) In communication

ii) Surgery

iii) Military purpose

iv) Holography

Conditions for LASER action:-

1) Population Inversion:-

The establishment of situation in which number of atoms in a higher Energy State is greater than the lower energy state is called Population Inversion.

If N_1 is the Number of Atom (Population) in the Energy State E_1 and N_2 is the population in the Energy State E_2, then by using Maxwell Boltzman statistics we have

$N_1 = N_0 e^{-E_1/kT}$ $\quad N_2 = N_0 e^{-E_2/kT}$

Then $\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} \Rightarrow N_2 = N_1 e^{-(E_2-E_1)/kT}$

As $E_2 > E_1$, $N_2 < N_1$ (at thermal Equilibrium)

When stimulated absorption starts population decrease with the increase of Energy States $N_2 > N_1$.

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Michelson Morley Experiment:-

According to Michelson Morley theory, light should travel at different speed through ether.

(Note:- Ether is a theoritical universal substance to act as a medium for transmission of Electromagnetic waves.)

Michelson Morley designed an interferometer to spot minute difference in the arrival of light beams. Experiment compared the relative speed of light to the relative motion of Earth through ether.

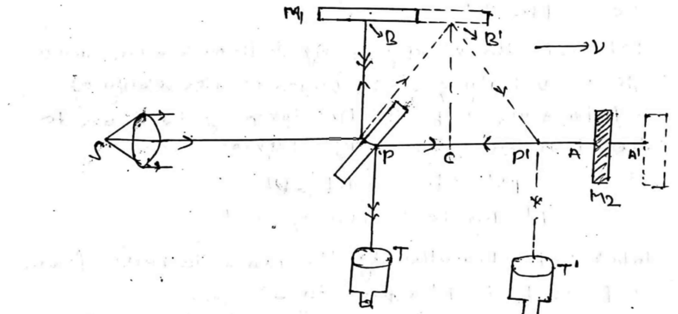

Figure shows that a parallel beam of light from a monochromatic source falls on a semi silvered glass plate at an angle of 45° to the beam. Now reflected beam moves towards M_1 & rest transmitted to M_2. These two beams moves perpendicular to each other and reaches to mirror M_1 & M_2. As the mirrors are highly silvered two beams thus reflected to point P and finally reached towards telescope T.

In this ~~Experiment~~ Apparatus is at rest in ether & distance of M_1 & M_2 from P are equal, then two beam would take same time to return to P.

But in actual Experiment mirror on Apparatus is moving in the direction of motion of Earth round the sun and speed is 3×10^4 m/s.

Therefore, due to the motion of Earth or apparatus, the path of two beams and their reflection from the mirror are shown by dotted lines.

Now optical path from P to mirror M_1 & M_2 be $PA = PB = l$

Let c be the velocity of light through ether and v be the velocity of earth which is also velocity of apparatus, t be the time taken by the beam to travel distance from P to mirror then,

$PB' = ct \quad , BB' = vt$

(B' due to motion of Earth)

Total path travelled by the beam to move from P & reach to P' equal to –

$PB'P' = PB' + B'P'$

$= 2PB'$

Also $B'B = PC$

B'C is ~~far from~~ to PC then,

$(PB')^2 = (PC)^2 + (CB')^2$

$(ct)^2 = (vt)^2 + l^2$

$$\boxed{t = \frac{l}{\sqrt{c^2 - v^2}}}$$

∴ Now time taken by the ray to move distance $PB'P'$ is

$$t_1 = 2t = \frac{2l}{\sqrt{c^2-v^2}}$$

$$= \frac{2l}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\boxed{t_1 = \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)}$$

The transmitting ray through P will move towards M_2 with velocity $(c-v)$ relative to apparatus and with velocity $(c+v)$ it will move back from M_2.

Therefore total time from P to A' & A' to P' will be

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$= \frac{2lc}{c^2-v^2}$$

$$= \frac{2lc}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

$$\boxed{t_2 = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)}$$

Hence the difference between time of travel of two rays is

$$t_2 - t_1 = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right) - \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)$$

$$= \frac{lv^2}{c^3}$$

Thus path difference, δ between two rays is -

$$\boxed{\delta = c(t_2 - t_1) = \frac{lv^2}{c^2}}$$

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In Michelson Morley Experiment

$l = 11$ meter $\quad v = 3\times10^4$ m/s

$\lambda = 5890$ Å $\quad c = 3\times10^8$ m/s

$$\delta n = \frac{l v^2}{\lambda c^2}$$

$$= \frac{2\times 11\times(3\times10^4)^2}{5890\times10^{-10}\times(3\times10^8)^2}$$

$$= \frac{2.2}{5.89} = \underline{0.37}$$

In this Experiment design of Apparatus was such that it could detect a shift ~~but~~ or path difference but no fringe shift was observed in this Experiment. The Experiment was repeated several time along gap of about six month and at several place, but the Result was again Negative.

But this Experiment Results in the Special theory of Relativity.

<u>Refrence frame</u>

A system of coordinate axis which define the position of a particle or event in two or three dimension space is called frame of Refrence.

Refrence frame with four coordinates x, y, z, t is referred to as space time frame.

Lorentz Transformation:- (Used for high speed moving objects)

Lorentz Transformation is the transformation between two inertial frame of refrence when one is moving with constant velocity with respect to other.

According to Lorentz Equation measurement in x-direction made in frame S must be linearly proportional to that made in S' frame. Hence a constant K should be there.

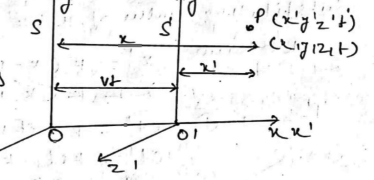
Consider

S – Rest frame

S' – Moving with velocity v along x-Axis

O, O' – Two observer of frame S & S'.

Now pulse of light reaches point P whose coordinates of position & time are – (x, y, z, t) & (x', y', z', t')



Transform Equation of x & x' can be given as-

$$x' = K(x - vt) \quad \text{--- (1)}$$

where K is proportionality constant.

Inverse of above relation can be given as -

$$x = K(x' + vt') \quad \text{--- (2)}$$

Using first postulate of special theory of Relativity

All the laws of physics are valid and same for all inertial frame of refrence (also called principle of Equivalence)

$\therefore$ x should satisfy x'

Substitute value of x' from (1) in (2)

$$x = K(K(x - vt) + vt')$$

$$\frac{x}{K} = (Kx - Kvt + vt')$$

$$\frac{x}{K} - Kx + Kvt = vt'$$

$$t' = \frac{x}{Kv} - \frac{Kx}{v} + Kt$$

$$= Kt - \frac{Kx}{v} + \frac{x}{Kv}$$

$$= Kt - \frac{Kx}{v}\left(1 - \frac{1}{K^2}\right) \quad \text{--- (3)}$$

According to second postulate of special Theory of Relativity speed of light remains constant in both the frame of Refrence, therefore velocity of light at O & O' should be same.

$$\therefore \quad x = ct \quad \& \quad x' = ct' \quad \text{--- (4)}$$

Substituting value of x & x' from (4) in (1) & (2)

we have $\quad x' = K(x - vt)$ --- (1)

$\quad x = K(x' + vt')$ --- (2)

in (1) $\quad ct' = K(ct - vt)$

$\quad ct' = Kct - Kvt$

$\quad ct' = Kt(c - v)$ --- (5)

Similarly $\quad ct = Kt'(c + v)$ --- (6)

Multiplying (5) & (6)

$$c^2 t t' = K^2 t t'(c^2 - v^2)$$

After solving we get $\quad K = \dfrac{1}{\sqrt{1 - v^2/c^2}}$ --- (7)

Now substitute (7) in (1) then Lorentz transformation in position will be

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z}$$

Now <u>Calculation of time</u>:- Substitute (7) in (3)

$$t' = Kt - \frac{Kx}{v}\left(1 - \frac{1}{K^2}\right) \quad (3)$$

$$K = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{1}{K} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{K^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{K^2}$$

Using in (3)

From (3) $\quad t' = Kt - \dfrac{Kx}{v}\left(1 - \dfrac{1}{K^2}\right)$

$$t' = Kt - \frac{Kx}{v}\left(\frac{v^2}{c^2}\right)$$

$$t' = Kt - \frac{Kxv}{c^2}$$

$$= K\left(t - \frac{xv}{c^2}\right)$$

Substitute $\left(K = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right.$ from (7)$\Big)$

$$\Rightarrow \boxed{t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Hence Lorentz transformation Equations becomes-

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z \quad \& \quad t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Inverse of Lorentz

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \quad y = y', \quad z = z' \ \& \ t = t' - \frac{x'v}{c^2} \Big/ \sqrt{1 - v^2/c^2}$$

<u>Special case</u>:- when velocity of moving frame S' is smaller than the velocity of light $v \ll c$ that is $\sqrt{1 - \frac{v^2}{c^2}} = 1$ & $\left(t - \frac{x'v}{c^2}\right) = t$

Then Lorentz transformation Reduce to Galilean Transformation

ie $\quad x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$

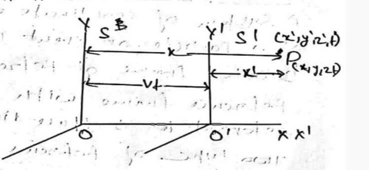
Galilean Transformation!-

Galilean Transformations are those equation which relates the coordinates of a Particle/object in two Different inertial frames

OR.

The Galilean Transformations are used to transform the coordinates of Position and time from one inertial frame to another.

Consider



S-frame of Refrence is stationary (Rest)

S' - Non-Stationary (In Motion v velocity along X-Axis)

V - Constant velocity

O ; O' → Two observer

P - Point P at any particular time.

$(x,y,z,t) \rightarrow$ coordinates wrt S

$(x',y',z',t') \rightarrow$ coordinates wrt S'

Now the Relationship between coordinates of both the frame S & S' will be

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

These Four coordinates are Called Galilean Transformation

1) ~~Inertial Frame of Refrence~~:- A frame of refrence that is not undergoing acceleration. The inertial frame of refrence are non-accelerating frames. In inertial frame else a body is at rest or in uniform motion.

Eg. A car stand still or a bus moving with constant speed are considered to be inertial frame of Refrence

Inertial Frame of Refrence Hold Newton law of motion.

Non Inertial Frame of Refrence:- A frame of refrence that undergoes acceleration wrt an inertial frame.

Non-inertial frame don't hold Newton law of Motion.

Eg. A car just started moving from a standstill.

Time Dilation!- Time slow down at very fast speed. This Phenomenon is known as time Dilation.

Let the clock be situated ~~in the~~ in the frame S giving signals at the interval

$$\Delta t = t_2 - t_1$$

If this interval is recorded ~~by an~~ in the frame S' the

$$\Delta t' = t_2' - t_1'$$

From Lorentz transformation we have-

$$t_1' = \frac{t_1 - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$t_2' = \frac{t_2 - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \boxed{\Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}}$$

Postulates of Special Theory of Relativity —

Postulates 1— the principle of Equivalence (or Relativity)

The principle of Equivalence states that the law of Physics are same in all inertial frame of reference moving with a constant velocity with respect to one another.

Postulates 2— The principle of Consistency of the Speed of light

The Second postulate State that the Speed of light in free Space is always same in all inertial frames of reference and is equal to C, that is it is independent of relative motion of the inertial frame, the Source and observer.

Length contraction!→ (Applications of Lorentz transformation)

According to Special theory of Relativity, length of a Moving object decrease than the length of an object at rest. This Process is called length contraction.

Let a Rod be placed parallel to x-axis in the system S' then the coordinates of the rod be x_1' & x_2' & length l'

$$l' = x_2' - x_1'$$

Also length l of Rod in system S is given by

$$l = x_2 - x_1$$

According to Lorentz transformation Equation.

$$x_2' = \frac{(x_2 - vt)}{\sqrt{1 - v^2/c^2}}$$

$$x_1' = \frac{(x_1 - vt)}{\sqrt{1 - v^2/c^2}}$$

Using $l' = x_2' - x_1'$

$$l' = \frac{(x_2 - vt)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} - \frac{(x_1 - vt)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}} = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \boxed{l = l'\sqrt{1 - \frac{v^2}{c^2}}}$$

Special Case!→ When $v \ll c$ then. $\sqrt{1 - \frac{v^2}{c^2}} \simeq 1$

therefore $\boxed{l = l'}$

Mass energy Equivalence!-

We have amount of Energy E related to Mass m as

$$E = mc^2$$

Rate of change of Momentum as force

$$F = \frac{d(P)}{dt} = \frac{d(mv)}{dt}$$

According to Relativity mass as well as velocity are variables

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

When particle is displaced through a distance dx by force F, then increase in kinetic Energy dk is.

$$dk = F dx$$

$$dk = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$\text{or } \boxed{dk = mv\,dv + v^2 dm} \quad \text{—(1)}$$

We have variation of Mass with velocity as.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}} = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2$$

on Differentiating

$$c^2 2m\,dm - v^2 2m\,dm - m^2 2v\,dv = 0$$

$$\Rightarrow c^2 dm - v^2 dm - mv\,dv = 0$$

$$\Rightarrow c^2 dm = v^2 dm + mv\,dv \quad \text{—(2)}$$

Comparing (1), (2)

$$\Rightarrow \boxed{dK = c^2 dm}$$

If mass of body increase from m_0 to m

$$\int dK = \int_{m_0}^{m} c^2 dm$$

$$K = c^2(m - m_0)$$

This is increase in kinetic Energy due to increase in Mass

Total Energy = Kinetic Energy + Rest Energy

$$= K + m_0c^2$$

$$= c^2(m - m_0) + m_0c^2$$

$$\boxed{E = mc^2}$$

This gives Universal Equivalence between Mass & Energy.