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SUBJECT - Linear Algebra & Numerical Methods

BRANCH - IIOT (B2)

Assignment-1 & 2

A-① Find two different basis of R^3 which contain the vectors $(0,1,-1)$ and $(2,1,-3)$ of R^3.

Solution ⇒ Firstly, let us check for Linear Independency of these two basis -

$$a(0,1,-1) + b(2,1,-3) = (0,0,0)$$

$2b = 0$
$a+b=0$ ⇒ $a=0$ & $b=0$ which means the vectors
$-a-3b=0$ are linearly independent.

Basis 1 :-
Let us a vector $(1,0,0)$

Now, $a(0,1,-1)+b(2,1,-3)+c(1,0,0) = (0,0,0)$

$$\left.\begin{aligned} 2b+c&=0 \\ a+b&=0 \\ -a-3b&=0 \end{aligned}\right\} \Rightarrow a=b=c=0$$

we get $a=b=c=0$ which means the vector $(1,0,0)$ is a basis of R^3 including given two vector.

Basis 2 : Let us a vector $(1,2,0)$

Now, $a(0,1,-1)+b(2,1,-3)+c(1,2,0)=0$

$$\left.\begin{aligned} 2b+c&=0 \Rightarrow c=-2b \\ a+b+2c&=0 \Rightarrow 3b+b+2b=0 \\ -a-3b&=0 \Rightarrow a=-3b \end{aligned}\right\} a=0,\ b=0 \ \& \ c=0$$

Since, we have $a=b=c=0$ which mean these vectors are L.I & $(1,2,0)$ is a basis of R^3 including given two vector.

Hence, $(1,0,0)$ & $(1,2,0)$ are two required basis of R^3.

A-(2) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be linear transformation defined by $T(x,y) = (x+y, x-y, y)$. Find $\ker(T)$ and Range(T).

Solution → Since,

$\{(1,0),(0,1)\}$ is standard basis of $\mathbb{R}^2(\mathbb{R})$

Range(T):

So. by defn-

$\{T(1,0), T(0,1)\}$ is basis of $R(T)$

i.e. $T(1,0) = (1,1,0)$

& $T(0,1) = (1,-1,1)$

$\left\{ \begin{array}{l} \text{Given,} \\ T(x,y) = (x+y, x-y, y) \end{array} \right.$

$\therefore \boxed{\text{Range}(T) = [(1,1,0),(1,-1,1)]}$

In general,

$\boxed{\text{Range}(T) = \{x(1,1,0) + y(1,-1,1) : x,y \in \mathbb{R}\}}$ Answer

Ker(T):

By defn-

$\ker(T) = \{\alpha \in \mathbb{R}^2(\mathbb{R}) : T(\alpha) = (0,0,0)\}$

Consider $\alpha = (x_1, y_1) \in K(T)$

Now,

$T(x_1, y_1) = (0,0,0)$

$(x_1+y_1, x_1-y_1, y_1) = (0,0,0)$

On comparing-

$\left. \begin{array}{l} x_1+y_1 = 0 \\ x_1 - y_1 = 0 \\ y_1 = 0 \end{array} \right\}$ $x_1 = 0$, $y_1 = 0$

$\therefore \boxed{\ker(T) = \{(0,0)\}}$ Answer

ASSIGNMENT - 2

1. Find eigen values & bases of the corresponding eign spaces of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}.$$

Solution ⇒ Let a variable matrix be X such that

$$A\vec{X} = \lambda \vec{X} \Rightarrow (A - \lambda I)\vec{X} = 0$$

Characteristic eqn, $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(1-\lambda)(4-\lambda)+2] = 0$$

$$(2-\lambda)[4-\lambda-4\lambda+\lambda^2+2] = 0$$

$$(2-\lambda)(\lambda^2-5\lambda+6) = 0$$

$\cancel{2\lambda^2 - 10\lambda + 12 - \lambda^3}$

$$(2-\lambda)[\lambda^2-2\lambda+3\lambda+6] = 0$$

$$(2-\lambda)(\lambda-2)(\lambda-3) = 0$$

$$\lambda = 2, 3$$

$$\lambda_1 = 2 \ \& \ \lambda_2 = 3$$

∴ Eigen values: $\lambda_1 = 2$ & $\lambda_2 = 3$

(i) For $\lambda_1 = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}} = \frac{x_2}{\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}} = \frac{x_3}{\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}}$$

$$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{0} = K \text{ (say)}$$

∴ Eigen Vector, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = K \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(ii) For $\lambda_2 = 3$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix}} = \frac{x_2}{\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}} = \frac{x_3}{\begin{pmatrix} 0 & -2 \\ 0 & 2 \end{pmatrix}} = K$$

$$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{0} = K$$

$\therefore$ Eigen bases / vector, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Q(2) Apply the Gram Schmidt process to vectors $(1,0,1)$, $(1,0,-1)$, $(0,3,4)$ to obtain an orthonormal basis for $R^3(R)$ with the standard inner product.

Sol^n: Let $\beta_1 = (1,0,1)$, $\beta_2 = (1,0,-1)$ & $\beta_3 = (0,3,4)$

also let $\alpha_1, \alpha_2 \,\&\, \alpha_3$ be the orthonormal basis for $R^3(R)$.

As

$$\alpha_1 = \frac{\beta_1}{\|\beta_1\|} \Rightarrow \|\beta_1\|^2 = \langle \beta_1, \beta_1 \rangle = (1)^2 + (0)^2 + (1)^2 = 2$$

$$\|\beta_1\| = \sqrt{2}$$

$$\alpha_1 = \frac{(1,0,1)}{\sqrt{2}} \Rightarrow \boxed{\alpha_1 = \left(\tfrac{1}{\sqrt{2}}, 0, \tfrac{1}{\sqrt{2}}\right)}$$

Again, $\alpha_2 = \dfrac{\gamma_2}{\|\gamma_2\|}$ $\quad \|\gamma_2\| = ?$

$$\gamma_2 = \beta_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1$$

$$\gamma_2 = (1,0,-1) - \left[(1,0,-1)\begin{pmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{pmatrix}\right]\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$= (1,0,-1) - \left(\cancel{\frac{1}{\sqrt{2}}} + 0 - \frac{1}{\sqrt{2}}\right)\left(\cancel{\frac{1}{\sqrt{2}}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$\gamma_2 = (1,0,-1)$$

$$\|\gamma_2\|^2 = \langle \gamma_2, \gamma_2 \rangle = 1^2 + 0^2 + (-1)^2 = 2$$

$$\Rightarrow \|\gamma_2\| = \sqrt{2}$$

So, $\alpha_2 = \dfrac{(1,0,-1)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

Now,

$\alpha_3 = \dfrac{\gamma_3}{\|\gamma_3\|}$, $\|\gamma_3\| = ?$

$$\gamma_3 = \beta_3 - \langle \beta_3, \alpha_2 \rangle \alpha_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1$$

$$= (0,3,4) - \left((0,3,4)\begin{pmatrix}1/\sqrt{2}\\0\\-1/\sqrt{2}\end{pmatrix}\right)\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right) - \left((0,3,4)\begin{pmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{pmatrix}\right)(\alpha_1)$$

$$= (0,3,4) - (-2\sqrt{2})\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right) - (0 + 2\sqrt{2})\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$= (0,3,4) + (2,0,-2) - (2,0,2) = (0,3,0)$$

$$\|\gamma_3\|^2 = (0)^2 + (3)^2 + (0)^2 = 9$$

$$\|\gamma_3\| = \sqrt{9} = 3$$

$$\alpha_3 = \frac{(0,3,0)}{\sqrt{9}} = \left(\frac{0}{\sqrt{9}}, \frac{3}{\sqrt{9}}, \frac{0}{\sqrt{9}}\right) = (0,1,0)$$

$\therefore$ $\alpha_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\alpha_2 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ & $\alpha_3 = (0,1,0)$ are the required orthonormal basis for $R^3(R)$ with std. I.P.S.