EMFT

★ Gauss Divergence :- Converts surface integral to volume integral

$$\int_S \vec{A}.ds = \int_V \nabla.\vec{A}\,dV$$

★ Stokes Theorem :- Converts line integral to surface integral

$$\int_c \vec{A}.dl = \int_S \nabla\times\vec{A}\,ds$$

★ Divergence = Flux density = $\frac{\text{Flux}}{\text{Volume}}$

• Flux :- passing through/coming out.
+ve :- outgoing 0 :- no flux -ve : incoming flux

★ Curl :- $\nabla\times\vec{E}$
It gives rotating effect of vector field

D = Displacement vector
$D = E\varepsilon_0$, $B = \mu_0 H$
H = magnetomotive force

★ Maxwell equations.

Differential form	Integral Form
① $\nabla.\vec{E} = \frac{\rho}{\varepsilon_0}$ or $\nabla.\vec{D} = \rho$	① $\oint\vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}\int\rho\,dV$
② $\nabla.\vec{B} = 0$	② $\oint\vec{B}.d\vec{s} = 0$
③ $\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$	③ $\oint\vec{E}.d\vec{l} = -\int\frac{\partial\vec{B}}{\partial t}ds$
④ $\nabla\times\vec{H} = J + \varepsilon_0\frac{\partial E}{\partial t}$	④ $\oint H.dl = \int\left(\vec{J} + \varepsilon_0\frac{\partial E}{\partial t}\right)d\vec{s}$

★ Maxwell's First equation (Gauss Law in electrostatics)
Physical Significance :- It states that electric flux through any closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\oint_S \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0} \qquad \left[\rho = \frac{q}{V}\right]$$

$$\int_S \vec{E}.ds = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

Using Gauss Divergence Theorem

$$\int_V (\vec{\nabla}\cdot\vec{E})\,dV = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

$$\boxed{\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$$

$$\boxed{\nabla\cdot\vec{D} = \rho} \quad (\text{as } \vec{E}\varepsilon_0 = D)$$

* Maxwell 2nd equation (Gauss Law of Magnetostatics)
· P.S:- It states that net magnetic through any closed surface is zero

$$\phi = \oint_S B\cdot ds = 0$$

Using G.D's theorem.

$$\int_V (\nabla\cdot\vec{B})\,dV = 0 \;,\; \boxed{\nabla\cdot\vec{B} = 0}$$

* Maxwell's 3rd equation (Faraday's law of EMI)
· P.S:- It states that induced emf around the closed circuit is equal to negative times the rate of change of magnetic flux linked with the circuit.
Time varying magnetic field always produces electric field or vice-versa

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

$$\oint_l \vec{E}\cdot dl = -\int_S \frac{d\vec{B}}{dt}\cdot ds$$

Using Stokes Theorem.

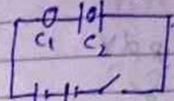
$$\int_S (\nabla\times\vec{E})\,ds = -\int_S \frac{d\vec{B}}{dt}\cdot ds$$

$$(\nabla\times\vec{E}) = -\frac{\partial\vec{B}}{\partial t}$$

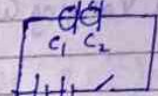
* Maxwell's 4th equation (Modified Ampere's circuital law)
· P.S:- It states that the magnetomotive force around a closed path is equal to conduction current plus displacement current through any surface bounded by the path.

→ Ampere's law

$$\oint B\cdot dl = \mu_0 I.$$



$$\int_{C_1} B\cdot dl = \mu_0 I \;\text{—①} \;;\; \int_{C_2} B\cdot dl = 0 \;\text{—②}$$ (current doesn't flow through dilated material b/w capacitor)



If C_1 and C_2 are very small and close to each other hence

$$\int_{C_1} B\cdot dl = \int_{C_2} B\cdot dl = \mu_0 I \;\text{—③}$$

Hence, eq ① & ② and ③ contradicts each other
So there is inconsistency in Ampere's Law

→ Displacement current:- It is the current which comes into existence in addition to conduction current whenever the electric field or flux changes with time

$$I_d = \varepsilon_0\frac{d\phi_E}{dt}$$

Modified Ampere's Law.

$$\int \vec{B}\cdot dl = \mu_0 I_{total} = \mu_0(I_c + I_d).$$

$$\int \vec{B}\cdot dl = \mu_0 I + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

Conduction Current	Displacement Current
① It obeys Ohm's law	① It doesn't ohm's law
② It can't exist in vacuum	② It can exist even in vacuum
③ Depends on potential diff. and resistance	③ It depends on change in electric field or flux

→ Equation of Continuity.

$$J = \frac{dq}{dt} \;,\; J = \frac{I}{S}$$ (S means Area) (J = current density)

It expresses the quantity of incoming and outgoing charges in a system and follows laws of conservation of charge.

$$\int_s \vec{J}\cdot\vec{ds} = -\frac{dq}{dt} = -\frac{d}{dt}\int_v \rho\, dV$$

Using G.D theorem

$$\int_v (\nabla\cdot\vec{J})\, dV = -\frac{d}{dt}\int_v \rho\, dV$$

$$\boxed{\nabla\cdot\vec{J} + \frac{\partial\rho}{dt} = 0}$$ (Varying Current

Proof of 4th equation

$$\oint_c B\cdot dl = \mu_0 (I_c + I_d)$$

$$\oint_c B\cdot dl = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\oint_c B\cdot dl = \mu_0 \int_s (\vec{J}\cdot ds) + \mu_0 \int_s \varepsilon_0 \frac{d(\vec{E}\cdot ds)}{dt}$$

By Stoke's Theorem.

$$\int_s (\nabla\times\vec{B})\, ds = \int \mu_0 \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right] d\vec{s}$$

$$\boxed{\nabla\times\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right]}$$

$$\boxed{\nabla\times\vec{H} = \vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}}$$

★ ~~Poynting theorem:-~~

★ Poynting Vector:- Rate of flow of energy per unit area in a plane electromagnetic wave can be described by a vector.

SI unit = Watt/m^2

$$\vec{S} = \frac{1}{\mu_0}\cdot(\vec{E}\times\vec{B}), \quad \vec{S} = \vec{E}\times\vec{H}$$

→ ~~The~~ Poynting Theorem:- Rate of decrease of ~~electrons~~ electromagnetic energy in a certain Volume V is equal to work done by the field forces per unit time plus power transferred into the filled

~~The~~ Multiplying $\vec{H}$ with 3rd equation and $\vec{E}$ with eq. ④

$$H(\nabla\times E) = -\frac{dB}{dt} H \quad - ①$$

$$E(\nabla\times H) = E\left(J + \frac{dD}{dt}\right) \quad - ②$$

eq. ① - eq. ②

$$H(\nabla\times\vec{E}) - E(\nabla\times\vec{H}) = -H\frac{dB}{dt} - EJ - E\frac{dD}{dt}$$

Vector Identities :- $\nabla(\vec{A}\times\vec{C}) = \vec{C}(\nabla\times\vec{A}) - \vec{A}(\nabla\times\vec{C})$

~~$\nabla(\vec{H}\times\vec{E})$~~

$$\nabla(\vec{E}\times\vec{H}) = -\left(\vec{E}\times\frac{dD}{dt} + H\frac{dB}{dt}\right) - EJ$$

$$\nabla(\vec{E}\times\vec{H}) = -\left[\frac{d}{dt}\left(\frac{1}{2}\varepsilon_0 E^2\right) + \frac{d}{dt}\left(\frac{1}{2}\mu_0 H^2\right)\right] - EJ$$

$$\nabla(\vec{E}\times\vec{H}) = -\frac{d}{dt}\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) - E^2\sigma \quad [J = \sigma E]$$

Taking Volume integral both sides

$$\int \nabla(\vec{E}\times\vec{H}) = -\frac{d}{dt}\int \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 - \sigma E^2$$

★ Electromagnetic waves in free space

$q = 0, \ \rho = 0$

$I = 0, \ \vec{J} = 0$

★ Wave equation (Amplitude is same at any point in the plane ⊥ to specified direction)

Taking curl of eq. ③

$$\nabla\times(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$$

From vector identity we know

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}$$

~~Eqn~~ $$\vec{\nabla}(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$$

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$\nabla\cdot E \,\&\, \nabla\cdot H = 0$ $\quad \nabla(\nabla\cdot E) - \nabla^2 E = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H})$

$$\cancel{-}\nabla^2 E = \cancel{-}\mu_0 \frac{\partial}{\partial t}\left(\sigma E + \varepsilon_0 \frac{dE}{dt}\right)$$

$$\boxed{\nabla^2 E = \mu_0 \frac{\partial}{\partial t}\left(\sigma E + \varepsilon_0 \frac{dE}{dt}\right)}$$

Plane wave equation.

→ In free space

$\rho = 0, \sigma = 0, \mu_0, \varepsilon_0$.

$\nabla\cdot E = 0, \; \nabla\cdot H = 0$.

$$\boxed{\nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}}.$$

$$\nabla^2 E = v^2 \frac{d^2 E}{dt^2}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \frac{\sqrt{4\pi}}{\sqrt{4\pi}}$$

$$v = \frac{1}{\sqrt{4\pi\varepsilon_0}} \times \frac{\sqrt{4\pi}}{\sqrt{\mu_0}}$$

$$v = \sqrt{9\times10^9} \times \frac{\sqrt{4\pi}}{\sqrt{4\pi\times10^{-7}}}$$

$$v = \sqrt{9\times10^{16}}$$

$$v = 3\times10^8 \text{ m/s} = c$$

$$\vec{E}(r,t) = \varepsilon_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}.$$

★ Energy carried by EM waves.

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}).$$

$$|\vec{S}| = |\vec{E}\times\vec{B}| = EB$$

$$S = \frac{EB}{\mu_0}, \quad c = \frac{E}{B}$$

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}.$$

$$S_{avg} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E^2_{max}}{2\mu_0 c}.$$

$$S_{avg} = \frac{cB^2_{max}}{2\mu_0}.$$

Instantaneous Energy Density.

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$B = \frac{E}{c}, \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}.$$

$$u_B = \frac{(E/c)^2}{2\mu_0.} = \frac{1}{2}\varepsilon_0 E^2 = u_E$$

$$u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$u_{avg} = \frac{1}{2}\varepsilon_0 E^2_{max}$$