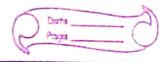
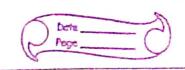


	Moment Generating function (m.g.f)
	The M.g.f. of a random variable X is denoted by
	Mx(t) and is defined as
	$M_x(t) = E(e^{tx}), t \in \mathbb{R}$
	wherever this Expectation Exists.
\rightarrow	9f X is discrete R.V. then
	$M_x(t) = \sum e^{tx} \cdot \rho(x)$
	χ
	where P(x) - Prob man function (P.m.f.)
	1 Augustin ()
\rightarrow	If X is Continuous R.V. then
	$M_x(t) = \int_{0}^{\infty} e^{tx} f(x) dx$
	where f(x) - Poub density function (P.d.f.)
\rightarrow	M.g.f. is used to Compute moments of the distribution.
	> nth moment about 0 is the nth decivative of the
	m.g.f, at 'o'.
3	ie. dn (mx(t)) = nth moments. — (1)
	dtn (x) t=0
	For any positive Integer ex, with moment about o longin)
	$18 \mu_{M} = E(x^{M})$
	So $\mu_i = E(x) = Mean$
	$\mu_{\mathfrak{J}} = E(x^2)$
	$\Rightarrow V(x) = E(x^2) - E(x)$
	$\Rightarrow V(x) = \mu_9 - \mu_0$
	From () My = d4 (MxH) = E(x4) > [mp.
	Her to



Find the Migf of Binomial dist" and hence find its mean and variance. The P.m.f. of binomial dist is $P(x) = n_{x} p^{x} q^{n-x}; x=0,1,...n; p+q=1$ Its M.g.f is n M(t)=E(etx) = Zetx.p(x) $= \sum_{x=0}^{m} e^{tx} n_{cx} p^{x} q^{n-x}$ $= \sum_{n=0}^{\infty} n_{c_{x}} (e^{t_{p}})^{x} q^{n-x}$ = (etp+9)"; provided (etp<9 by using Binomial Expansion of $(a+x)^n = n_G a^0 x^n + n_G a^1 x^{n+1} + - + n_G a^n x^0$ Provided 1x/<a So Mx(t) = (etp+q) provided letp < 9 $M = E(x) = d M_x(t)$ = d (etp+q)" n(etp-19)n-1. etp/t=0 = $n(p+q)^{n+1}$, p = np (::p+q=1) $\mu_{g} = F(x^{2}) = \frac{d^{2}(e^{t} + q)^{n}}{at^{2}}$ = $\frac{d}{dt} \left[n e^{t} b \left(e^{t} b + q \right)^{n+1} \right] = n b \left[(n+1) b + 1 \right]$



Find the M.g.f. of poisson dist and hence find its mean

The p.m.f. of poisson dist is $p(x) = \frac{\overline{e}^{\lambda} \lambda^{\chi}}{\chi!}; \chi = 0,1,2,...$

The M.g.f. is $M_{x}(t) = \mathcal{E}(e^{tx})$ $= \mathcal{E}e^{tx} \cdot \rho(x)$ $\frac{\chi=0}{=\sum_{e} t x \cdot e^{-1} x^{2}}$ $= e^{\lambda} \sum_{\chi=0}^{\infty} (\det)^{\chi}$

$$= e^{-1} \left[1 + \frac{1}{4} e^{t} + \frac{1}{4} e^{t$$

$$= e^{\lambda} e^{\lambda e^{t}}$$

$$= e^{\lambda(e^{t}-1)}$$

$$= e^{\lambda(e$$

$$= \left[e^{\lambda(1-e^{t})} \cdot Ae^{t} \right]_{t=0}$$

$$\mu_{2} = F(x^{2}) = \frac{d^{2} M_{x}(t)}{dt^{2}}$$



$$= \frac{d}{dt} \int_{t=0}^{t} e^{t} e^{t} e^{t} e^{t} e^{t} e^{t} \int_{t=0}^{t} e^{t} e^{t}$$

$$\frac{\mu_{0}}{dt^{2}} = E(x^{2}) = \frac{d^{2}}{dt^{2}} \left(\frac{pe^{t}}{(1-qe^{t})^{2}} \right) |_{t=0}$$

$$= \frac{d^{2}}{dt^{2}} \left(\frac{pe^{t}}{(1-qe^{t})^{2}} \right) |_{t=0}$$

$$= \frac{pe^{t}}{pe^{t}} + \frac{pe^{0}}{pe^{0}} |_{t=0}$$

$$= \frac{p}{p} + \frac{p}{p} = \frac{1+q}{p^{2}}$$

$$V(x) = \frac{\mu_{0}}{p^{2}} - \frac{\mu_{1}^{2}}{p^{2}} = \frac{q}{p^{2}}$$

$$= \frac{p+q}{p^{2}} - \frac{1}{p^{2}} = \frac{q}{p^{2}}$$



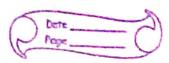
Que	A perfect coin is toxed twice. Find the m.g.f of X
	where X - no of heads.
	Also find mean & variance.
E TO THE PROPERTY OF THE PROPE	S= EHH, HT, TH, TTJ
Sol7	χ $P(x)$.
	0 /4
	$\frac{2}{4} = \frac{1}{2}$
	2 1/4
	$M_{xH} = E(e^{tx}) = \sum_{x=0}^{2} e^{tx} P(x)$
	x=0
	$= P(0) + e^{t} \cdot P(1) + e^{2t} \cdot P(2)$
	$= \frac{1}{4} + \frac{1}{2}e^{t} + \frac{1}{4}e^{2t}$
	4 2 4
	=
	4 4
	$\mu_{l} = E(x) = \frac{d}{dt} \frac{M_{x}(t)}{tz}$
	dt 1 tzs
	$= e^{t} g(t+e^{t}) _{t=0} = 1$
	y 1t=0 T
	$E(x^2) = d^2 M(t) = d e^t (1+e^t)$
	at2 / tes at 2 tes
	\(\frac{1}{8}\)\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
	2 100 2
	$= e^{t} + 3e^{2t} = 3$
	2 t=0 2
	$\delta V(x) = 3 - 1 = 1$
	2 2



Que	Find the might of the dist $f(x) = \int_{0}^{\infty} \frac{2}{3}$; $x = 2$ 0 ; $0 \neq \omega$
Sol	$M_{x(t)} = \sum_{e^{t}} f(x)$ $= e^{t} \cdot f(x) + e^{3t} \cdot f(3)$ $= \frac{3}{3}e^{t} + \frac{1}{3}e^{3t} = \frac{3}{3}e^{t} + e^{3t}$
Si	The might of H.V. X is given by $M_x(t) = e^{3(e^{t-1})}$ Find $P(x=1)$ $M_x(t) = e^{3(e^{t-1})}$ — might of poisson distributes $d = 3$
	So $P(x=x) = e^{-1} A^{x}$; $x=0,1,2$. $P(x=1) = e^{-3} \cdot 3^{1} = 3 = 0.1494$ $ \text{Impositant Psupperties of Migf:} \rightarrow$
(1) (2) (3) (4)	Translation Independence Shifting the oxigin and Scale
(1) So 17	If Mxtt) is the Migf of R.V. X then Mx(ct) is the Mig.f. of the R.V. CX, where C is Ony Constant. Mxtt) = E(etx)
	$\Rightarrow M_{cx}H) = E(e^{tCx})$ $= E(e^{(ct)x}) = M_{x}(ct)$

	and the second s	1
()	Dorre	(5)
6	100000	

	<u>&</u>	$f M_{x}(t) = 1 ; -1 < t < 1$
	<u>Sal</u> n	Then Find the M.g.f of Y=3X My (1) = M3x (1) - Mx (3+1)
		$= \frac{1}{1-(3t)^2} - \frac{1}{1-9t^2}$
		=) My(t) => 1·; -1 < t < 1.
	0)	
	(2)	If XI and X2 are Independent R.V. then
		$M_{X_1 + X_2}(t) = M_{X_1}(t) \cdot M_{X_2}(t)$
	C_{-1}^{η}	$\Gamma(\pm(x,\pm x))$
=	Soln	$M_{x_{1}+x_{2}}(t) = E(e^{t(x_{1}+x_{2})})$ $= E(e^{tx_{1}+tx_{2}}) = E(e^{tx_{1}}.e^{tx_{2}})$
		$= E(e^{tx_1}) \cdot E(e^{tx_2})$
	or programme and district	$= M_{x_1}(t) \cdot M_{x_2}(t)$
		[: E(xy)= E(x). E(y) If x and Y are Independent]
		Ingeneral; If $X_1, X_2 - X_n$ are Independent Random variables then $M_{X_1+X_2+-+X_n} = \frac{n}{i-1} M_{X_1}(t)$
1-		$\frac{1}{x_1 + x_2 + - + x_n} = \frac{1}{i = 1} \frac{1}{x_i} \frac{1}{(t)}$
	Application of the second seco	(ie. M.g.f of Sum = Product of M.g.f of variables)
((3)	If x is transformed into Y by changing both the onigin and scale i.e. $Y = \frac{X-a}{b}$
	And the second s	then $M_{yt} = e^{\alpha t/h} M_{x}(t)$
ALEXAND S.		



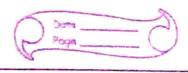
	(C rage
33,	$M_{\gamma}(t) = E(e^{t\gamma})$ $= E[e^{t(x-x)}]$
	$= E\left(e^{tx} - \frac{at}{n}\right)$ $[::E(ax) = aE(x)]$
	$= e^{at/h} E(e^{tx})$ $= e^{at/h} \cdot M_{x}(t)$
Que	
	Than find the M.g.f of $Y = \frac{X-Y}{2}$
Sal	$M_{y}(t) = E(e^{Yt})$
	$= E\left(e^{\pm\left(\frac{x-4}{2}\right)}\right) = E\left(\frac{\pm x}{e^2}, -2t\right)$
	$= e^{2t} E(e^{\frac{t}{2}x})$ $= e^{2t}. M_{x}(\frac{t}{2})$
	$= e^{\partial t} \cdot (1 + e^{t/2})^2$
	$= \frac{1}{4} \frac{(1+e^{t/2})^2}{e^{2t}}$
HW	(i) If M.g.f of X is Mx(t)= 1 (e2t -1)2
	Then find the m.g.f of $Y = X - 6$ An: e^{-3t} (e ^{t-1})
(2)	If M.g.f of X is given by $M_X(t) = \frac{1}{1-t^2}$; $ t < 1$
Committee of the second	



	Page V
	Then find the High of Y = X-4
	Ani- $e^{-t}\left(\frac{16}{16-t^2}\right)$; $ t <4$.
(4)	Uniquenes Theorem: The M. g.f of a dist, if it
	exists, uniquely determines the dist.
	i.e. Coursesponding to a given perob. dist, there is only one
	19.5. and converponding to a given might, there is only
	one prob. dist.
	⇒ If Mxtt) = Mytt) ⇒ x and y are Identically distributed.
	ie. X and Y have some prob. dist.
<u>Moter</u>	M.g.f may not Exist for all Handom variables:
Exi-	Let X have the $P.mf$. $f(x) = \begin{cases} 6 \\ 7 \end{cases}, x = 1,2,$ o , o w
=	$f(x) = \begin{cases} \frac{6}{3} & = 1,2, \end{cases}$
	$\pi^2 x^2$
	0 , 0 W
	Find the M.g.f. of x.
Soln	$M_{x}(t) = E(e^{tx})$
The state of the s	$= \underbrace{\underbrace{\text{getx.}}_{f(x)} = 6}_{\chi=1} \underbrace{\underbrace{\text{getx}}_{\chi=1}}_{\chi=1}$
entral plant of the second	h.
	$= \frac{6}{11^{2}} \left[\frac{e^{t} + e^{2t} + e^{3t}}{1 + 2^{2}} + \frac{1}{3^{2}} \right]$
	17^{2} 1 1 1 1 1 1 1 1 1 1
	This Series is not 9t +420
	=> MxH) does not Exist.



\rightarrow	Find the might of the Exponential dist and hence find
	its mean and variance.
Sol	The P.df of Exponential dist is
	$f(x) = \int de^{dx}; x = 0$
	0 ; 0 w.
	$M_x(t) = E(e^{tx})$
	: X is Continuous R.V.
	=) Mxtt = of etx. 1=1xdx
	$= \int_{-\infty}^{\infty} \left(t^{-\lambda} \right)^{\chi} dx$
	$= d \left[\frac{e^{(t-\lambda)} a}{t-\lambda} \right]^{\infty}$
	[t-1]0
	$= A \left[\frac{e^{(A-t)} x}{e^{(A-t)}} \right]^{\infty}$
	$\left[-\left(\lambda-t\right)\right]_{0}$
	Now = (1-t)x =0 f 1-t>0
	∞ If d-t<0
	$\therefore M_{x}(t) = \frac{1}{t-1} \left[0 - 1 \right]$
	t-1 L J
	= 1 Provided 1-t>0
	d-t on ted
200	
	$\mu_1 = E(x) = d \left[M_x(t) \right] - d \left[d \right]$ $dt \int_{t=0}^{t} dt \left[d - t \right]_{t=0}^{t}$
	dt []t=o dt [1-t]t=o
	$=\lambda + 1$
	$(A-t)^2$ $\int t=0$
	1
	$\mu_{g} = E(x^{2}) = d^{2} \left[M_{x}(t) \right]_{t=0} = d^{2} \left[d^{2} \left(d^{2} - t \right)^{2} \right]_{t=0} = 2d$
	dt2 []t=0 dt [(1-t)]t= (+t)3/t=0



$$\Rightarrow E(x^2) = \frac{2\lambda}{\lambda^3} = \frac{2}{\lambda^2}$$

$$\Rightarrow V(x) = \mu_1 - \mu_1^2$$

$$= \frac{2}{d^2} - \frac{1}{d^2} = \frac{1}{d^2}$$

Find the m.g.f. of the uniform dist over the range [a,b]. Hence find its mean and variance. $f(x) = \int_{b-a}^{b-1} (a < x < b)$

$$M_{x}(t) = E(e^{tx}) = \int_{x}^{x} e^{tx} \int_{y}^{x} dx$$

$$= \frac{1}{b-a} \left[\frac{e^{tx}}{t} \right]_{a}^{b}$$

$$=) M_{x}(t) = e^{tb} - e^{ta}$$

$$+ (b-a)$$