MID TERM EXAMINATION

SECOND SEMESTER [B.TECH.], FEBRUARY 2019

Paper Code: ETMA 102 **Subject: Applied Mathematics-II**

Time: $1\frac{1}{2}$ Hours **Maximum Marks: 30**

Note: Attempt any three questions including Q.No. 1 which is compulsory.

1. (a) If $z = f(x, y)$, what is the geometrical interpretation of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$?

   (b) If $u = \log(V)$, where V is a homogeneous function of degree n in x and y, evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

   (c) If $u^2 + xv^2 - uxy = 0, v^2 - xy^2 + 2uv + u^2 = 0$. Find $\frac{\partial u}{\partial x}$ using Jacobians.

   (d) Find Laplace transform of $f(t) = |t-1| + |t+1|, t \geq 0$.

2. (a) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$ and

   $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = sin4u - sin2u.$$

   (b) Find the point upon the plane $ax + by + cz = p$ at which the function $f(x, y, z) = x^2 + y^2 + z^2$ has a minimum value. Also find minimum value of the Function.

3. (a) Using partial differentiation, evaluate $[(3.82)^2 + 2(2.1)^3]^{\frac{1}{5}}$.

   (b) Using convolution theorem, $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$.

4. (a) Evaluate (i) $L^{-1}\left[log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right]$ (ii) $\int_0^\infty te^{-3t}sint\,dt$.

   (b) Using Laplace transform, find the solution of $\frac{d^2y}{dx^2} + 9y = \cos 2x$, if $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$.

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