Correlation Analysis:→ It is a statistical tool used to measure the strength of the linear relationship b/w two variables and Compute their association.

i.e. Correlation analysis calculas the level of change in one variable due to the change in the other.

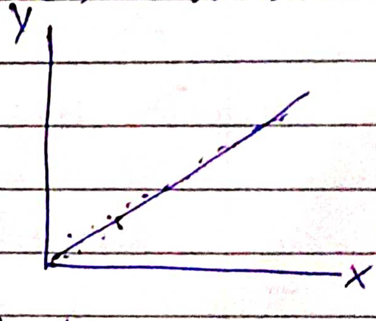
- High Correlation ⇒ Strong relationship] b/w two variables.
  Low Correlation ⇒ Weak relationship]

- If two variables are Correlated then it does not mean that one Causes the other.

Kinds of Correlation:→

(1) **Positive Correlation:→** If the values of the two variables deviate (moves) in the same direction.

X Increases then Y Increases
or X decreases then Y decreases
i.e. X & Y behave alike.



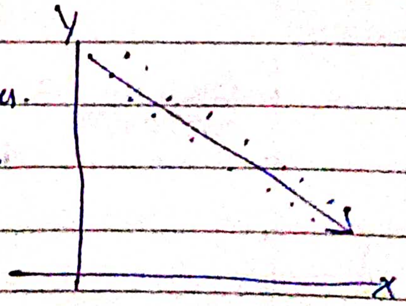
X	2	5	8	11
Y	18	25	36	50

] Positive Correlation.

E.g. family Income and Expenditure on luxury Items are positively Correlated.

(2) **Negative Correlation:→** If the values of two variables deviate (moves) in the opposite direction.

i.e. X Increases then Y decreases.
X decreases then Y Increases.

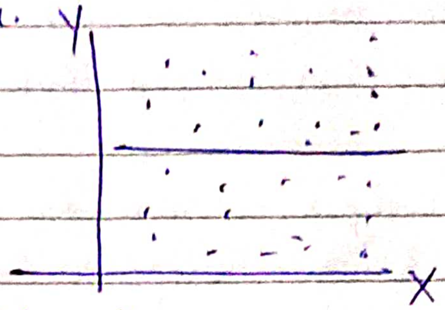


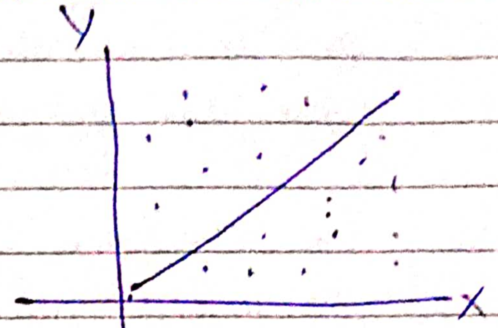
X	8	4	3	1
Y	8	10	15	25

] Negative Correlation

E.g. Price and demand of Commodity.

(3) <u>Zero Correlation:→</u> When one variable does not affect the other.





For Example; there is no Correlation between the age of the student and the marks obtained by him/her.

→ If there is no Correlation b/w two variables then this implies that there is no <u>linear relationship</u> b/w them.

→ However, there may exist some strong Curvilinear (non-linear) relationship between the two variables.

Karl-Pearson Correlation Coefficient:→

- The Correlation Coefficient denoted by r or ρ is a measure that determines the degree to which the movement of two different variables is associated.
- It is used to measure the <u>linear relationship</u> b/w two variables.
- It is also known as <u>Covariance Method</u>.

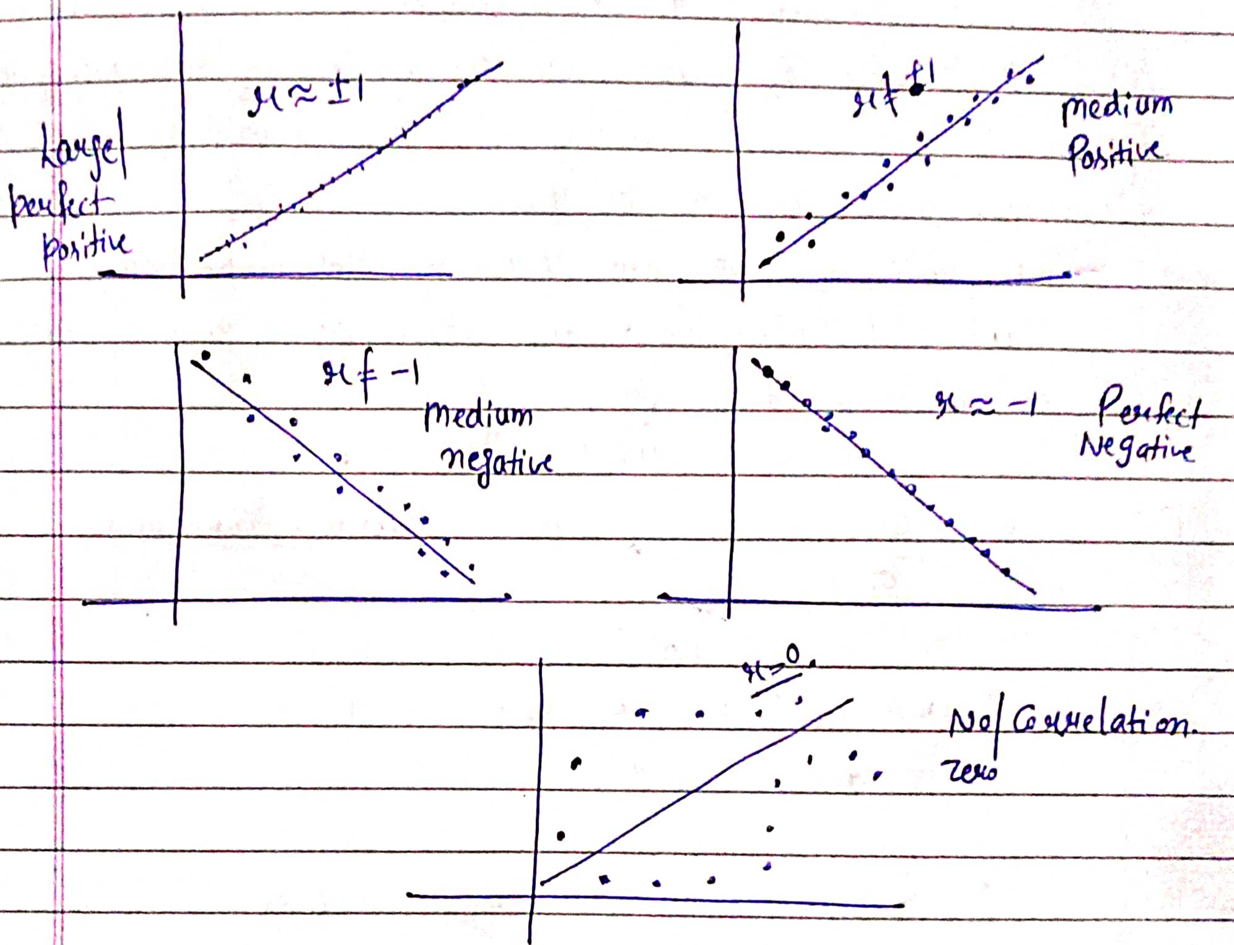
$$r = \frac{Cov(x,y)}{\sqrt{Var(x) \times Var(y)}} = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$$

<u>Covariance:→</u> $Cov(x,y) = E(xy) - E(x) \cdot E(y)$

or

$$Cov(x,y) = \frac{\sum (x-\bar{x}) \cdot (y-\bar{y})}{n}$$

- The value of r or ρ always lies b/w $[-1, 1]$.
- If $r = 1$, then it is perfect positive Correlation.
- If $r = -1$; then it is perfect negative Correlation.
- If $r = 0$; then there is no linear relationship or Zero Correlation.



Ex

X :	-2	-1	0	1	2
Y :	4	1	0	1	4
XY :	-8	-1	0	1	8

$$r = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{E(XY) - E(X) \cdot E(Y)}{\sigma_X \cdot \sigma_Y}$$

$$E(X) = 0, \quad E(Y) = 2, \quad E(XY) = 0$$

$\Rightarrow Cov(XY) = 0 \Rightarrow r = 0 \Rightarrow$ Zero Correlation

$\Rightarrow$ There is no linear relationship b/w X & Y.

- If value of r is very close to zero i.e. let $r \in (-0.1, 0.1)$. then there is no linear relationship b/w the two variables.

- If X and Y are independent, then

$$E(XY) = E(X) \cdot E(Y)$$

$$\Rightarrow Cov(X,Y) = 0 \Rightarrow r = 0$$

So $\Rightarrow$ zero correlation or no linear relationship b/w X and Y.

- Converse need not to be true.

Que The covariance of two perfectly correlated variables X and Y is 0.96. Find the values of σ_x and σ_y when it is given that $\frac{\text{var}X}{\text{var}Y} = \frac{4}{9}$.

Sol^n^

$$r = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = +1 \text{ or } -1 \text{ (Perfect Correlation)}$$

$\sigma_x, \sigma_y > 0$ (always)

and $Cov(X,Y) = 0.96 > 0$

$\Rightarrow$ $r = 1$

$$\Rightarrow 1 = \frac{0.96}{\sigma_x \cdot \sigma_y} \Rightarrow \sigma_x \cdot \sigma_y = 0.96$$

and $\frac{\sigma_x^2}{\sigma_y^2} = \frac{4}{9} \Rightarrow \sigma_x = \frac{2}{3}\sigma_y$

$\therefore \sigma_x \sigma_y = 0.96$

$\Rightarrow \frac{2}{3}\sigma_y^2 = 0.96 \Rightarrow \sigma_y = 1.2$

$\Rightarrow \sigma_x = 0.8$

Que:- Calculate r b/w X and Y from the following data:-

	X	Y
no. of pairs	15	15
Mean	25	18
Sum of Squares of deviations from mean	136	138

And Summation of product deviation of X and Y from their respective mean is 122

ie. $\Sigma(x-\bar{x})(y-\bar{y}) = 122$

$n = 15$

$\bar{X} = 25, \quad \bar{Y} = 18$

$\Sigma(x-\bar{x})^2 = 136 \,;\, \Sigma(y-\bar{y})^2 = 138$

$\Rightarrow \text{var}\, X = \dfrac{\Sigma(x-\bar{x})^2}{n} = \dfrac{136}{15} =$

$\text{var}\, Y = \dfrac{\Sigma(y-\bar{y})^2}{n} = \dfrac{138}{15} =$

$\text{Cov}(x,y) = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = \dfrac{122}{15}$

$\Rightarrow r = 0.8918$