Gamma function: $\Rightarrow$ For $\alpha > 0$, the gamma function $\Gamma\alpha$ is defined as

$$\Gamma\alpha = \int_0^\infty x^{\alpha-1} \cdot e^{-x}\, dx$$

$\rightarrow$ For any $\alpha > 1$, $\alpha \in \mathbb{R}$

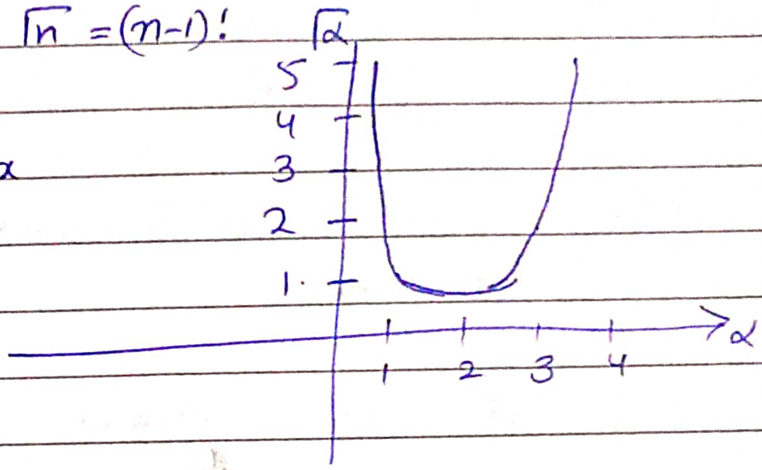
$$\Gamma\alpha = (\alpha-1)\,\Gamma\alpha-1$$

$\rightarrow$ For any Positive Integer n; $\Gamma n = (n-1)!$

So $\quad \Gamma\alpha = \int_0^\infty x^{\alpha-1} \cdot e^{-x}\, dx$

$$\Rightarrow \int_0^\infty \frac{x^{\alpha-1} e^{-x}}{\Gamma\alpha} dx = 1$$

$\Downarrow$ Pdf



$\rightarrow$ A Continuous R.V. X is said to follow Gamma distⁿ if its Pdf is given by

$$f(x) = \begin{cases} \dfrac{x^{\alpha-1} \cdot e^{-x}}{\Gamma\alpha} & ; x > 0 \\ 0 & ; \text{o/w.} \end{cases} \qquad \text{where } \alpha > 0.$$

This is called <u>Gamma distⁿ of 1st kind</u> (Standard Gamma distribution).

$\rightarrow$ <u>Gamma distribution of 2nd kind</u> -

$$f(x) = \begin{cases} \dfrac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma\alpha} & ; x > 0 \\ 0 & ; \text{o/w.} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$. $\quad \alpha$ - Shape parameter

β - Scale parameter.

When $\beta = 1$, then it Converts into Standard Gamma distⁿ.

→ If $\alpha = 1$ and $\beta = \lambda$ Then

Gamma distⁿ $\longrightarrow$ Exponential distribution.

→ If α is positive Integer, then

Gamma distⁿ $\longrightarrow$ Erlang distribution.

⇒ Exponential distⁿ & Erlang distⁿ are special Cases of Gamma distribution.

→ Cdf $= F(x) = P(X \le x)$

$$= \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} \frac{e^{-x} x^{\alpha-1}}{\Gamma\alpha}\,dx$$

<u>Relation b/w Gamma & Normal distⁿ:→</u>

(1) If $X \sim N(\mu, \sigma^2)$ Then $U = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$ i.e. $\frac{Z^2}{2} \sim \Gamma\frac{1}{2}$.

<u>Solⁿ</u> $\because X \sim N(\mu, \sigma^2) \Rightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

$\therefore$ Pdf of Z is

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2)} \quad ; \quad -1 < z < 1$$

The pdf of u is

$$g(u) = f(z) \cdot \left|\frac{dz}{du}\right|$$

$$U = \frac{z^2}{2} \Rightarrow z = \pm\sqrt{2u}$$

$\Rightarrow \frac{dz}{du} = \pm \frac{1}{\sqrt{2}\sqrt{u}}$

So $g(u) = 2 \cdot \frac{1}{\sqrt{2\pi}} e^{-u} \cdot \frac{1}{\sqrt{2}\sqrt{u}}$ ($\because$ f(z) is Even function)

$= \frac{u^{-1/2} e^{-u}}{\sqrt{\pi}}$

$= \frac{u^{-1/2} e^{-u}}{\Gamma\frac{1}{2}}$ $\left[\because \Gamma\frac{1}{2} = \sqrt{\pi}\right]$

$\Rightarrow g(u) = \Gamma\frac{1}{2}$

$\Rightarrow X \sim N(\mu, \sigma^2)$ Then $u \sim \Gamma\frac{1}{2}$.

Mean of Standard Gamma distⁿ:→

$$E(x) = \int_0^\infty x \cdot f(x)\,dx$$

$$= \int_0^\infty x \cdot \frac{x^{\alpha-1} e^{-x}}{\Gamma\alpha}\,dx$$

$$= \frac{1}{\Gamma\alpha} \int_0^\infty x^{\alpha} \cdot e^{-x}\,dx$$

$$= \frac{1}{\Gamma\alpha} \cdot \Gamma\alpha+1 = \frac{\alpha\cancel{\Gamma\alpha}}{\cancel{\Gamma\alpha}} = \alpha$$

Mean of Gamma distⁿ of 2nd kind:→

$$E(x) = \int_0^\infty x \cdot f(x)\,dx$$

$$= \int_0^\infty x \cdot \frac{\beta^\alpha x^{\alpha-1} \cdot e^{-\beta x}}{\Gamma\alpha}\,dx$$

$$= \frac{\beta^\alpha}{\Gamma\alpha} \int_0^\infty x^{\alpha} \cdot e^{-\beta x}\,dx$$

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$$= \frac{1}{\Gamma\alpha} \int_0^\infty (\beta x)^\alpha e^{-\beta x} dx$$

Put $\beta x = y$ $\quad$ $x=0 \Rightarrow y=0$

$\beta dx = dy$ $\quad$ $x=\infty \Rightarrow y=\infty$

So $\frac{1}{\Gamma\alpha} \int_0^\infty y^\alpha \cdot e^{-y} \cancel{\beta} \frac{dy}{\beta}$

$$= \frac{\cancel{\beta}1}{\beta \cancel{\beta}\Gamma\alpha} \Gamma\alpha+1 = \frac{\cancel{\beta}\alpha\Gamma\alpha}{\beta\cancel{\Gamma\alpha}} = \frac{\alpha}{\beta}$$

$\rightarrow$ $V(x) = E(x^2) - (E(x))^2$

For Standard Gamma distⁿ

$$E(x^2) = \int_0^\infty x^2 f(x) dx$$

$$= \int_0^\infty x^2 \cdot \frac{e^{-x} x^{\alpha-1}}{\Gamma\alpha} dx$$

$$= \frac{1}{\Gamma\alpha} \int_0^\infty e^{-x} x^{\alpha+1} dx$$

$$= \frac{\Gamma\alpha+2}{\Gamma\alpha} = \frac{(\alpha+1)(\alpha)\cancel{\Gamma\alpha}}{\cancel{\Gamma\alpha}} = \alpha^2+\alpha$$

So $V(x) = \cancel{\alpha^2} + \alpha - \cancel{\alpha^2} = \alpha$

$\Rightarrow$ Mean = Var

For Gamma distⁿ of Second Kind

$$E(x^2) = \int_0^\infty x^2 \cdot \frac{\beta^\alpha x^{\alpha-1} \cdot e^{-\beta x}}{\Gamma\alpha} dx$$

$$= \frac{\beta^\alpha}{\Gamma\alpha} \int_0^\infty x^{\alpha+1} \cdot e^{-\beta x} dx$$

$$= \frac{1}{\beta\Gamma\alpha} \int_0^\infty (\beta x)^{\alpha+1} \cdot e^{-\beta x} dx$$

Put $\beta x = y$

$$= \frac{1}{\beta\Gamma\alpha} \int y^{\alpha+1} \cdot e^{-y} \cdot \frac{dy}{\beta}$$

$$= \frac{1}{\beta^2 \Gamma\alpha} \Gamma\alpha+2 = \frac{(\alpha+1)(\alpha)\cancel{\Gamma\alpha}}{\beta^2 \cancel{\Gamma\alpha}}$$

$$= \frac{\alpha^2+\alpha}{\beta^2}$$

So $V(x) = \cancel{\frac{\alpha^2}{\beta^2}} + \frac{\alpha}{\beta^2} - \cancel{\frac{\alpha^2}{\beta^2}} = \frac{\alpha}{\beta^2}$

$\Rightarrow$ Var $= \frac{\text{Mean}}{\beta}$

If $\beta < 1$ Then Var $>$ Mean
If $\beta = 1$ Then Var $=$ Mean
If $\beta > 1$ Then Var $<$ Mean

$\rightarrow$ Thus, In Case of Continuous Random variable, only Standard Gamma distn has the prop. that Mean = Variance.
Whereas, in case of Poisson distn, Mean = Var
$\downarrow$
The only distn in Case of discrete R.V.

Ex The daily Consumption of milk in a City in excess of 20000 gallons is appr. distributed as a gamma distn with parameter $\beta = \frac{1}{10000}$ and $\alpha = 2$. The City has daily stock of 30000 gallons. What is the prob. that the stock is Insufficient on a particular day?

So

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Sol^n:- let X – daily Consumption of milk.

$Y = X - 20000 \sim G\left(2, \frac{1}{10000}\right)$

$$f(y) = \frac{1}{(10000)^2 \sqrt{2}} \, y \, e^{-y/10000} \; ; \; y > 0$$

$$P(X > 30000) = P(Y > 10000)$$

$$= \int_{10000}^{\infty} f(y)\, dy$$

$$= \frac{1}{(10000)^2 \sqrt{2}} \int_{10000}^{\infty} y \, e^{-y/10000} \, dy$$

Put $\frac{y}{10000} = z$

$dy = 10000 \, dz$

$$= \frac{1}{\sqrt{2}} \int_{1}^{\infty} z e^{-z} \, dz$$

$$= \frac{1}{\sqrt{2}} \left[z . \frac{e^{-z}}{-1} - \int \frac{e^{-z}}{-1} \right]_1^{\infty}$$

$$= \left[-z e^{-z} - e^{-z} \right]_1^{\infty}$$

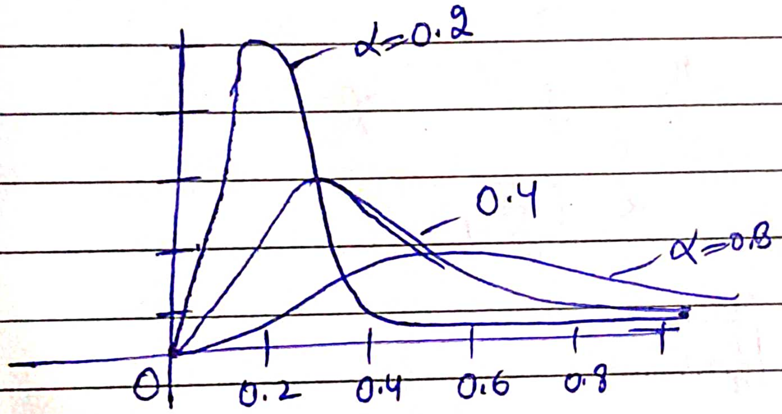
$$= +e^{-1} + e^{-1} = \frac{2}{e}.$$

<u>Weibull dist^n</u>:→ A cts. R.V. X is said to follow weibull dist^n with parameter β, α if its pdf is

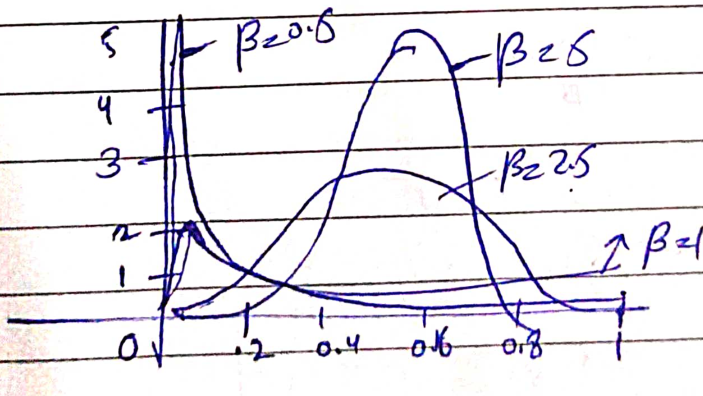
$$f(x) = \begin{cases} \left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^{\beta}} & ; \ \beta, \alpha > 0, \ x > 0 \\ 0 & ; \ o/w. \end{cases}$$

β – Shape parameter, α – Scale parameter.

If $\beta = 1$, then Weibull dist^n ⟶ Exponential dist^n.



α – spread of the curve
$\alpha \uparrow$, spread $\uparrow$.



<u>Mean & Variance</u>

$$E(X^r) = \int_0^\infty x^r f(x)\, dx$$

$$= \int_0^\infty x^r \cdot \left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^{\beta}}\, dx$$

$$= \frac{\beta}{\alpha^{\beta}} \int_0^\infty x^{\beta+r-1}\, e^{-(x/\alpha)^{\beta}}\, dx$$

Put $\left(\frac{x}{\alpha}\right)^{\beta} = y \quad \Rightarrow x = \alpha y^{1/\beta}$

$\Rightarrow dx = \frac{\alpha}{\beta} \cdot y^{\frac{1}{\beta}-1}\, dy$

$\beta \left(\frac{x}{\alpha}\right)^{\beta-1} dx = dy$

So $E(x^r) = \frac{\cancel{\beta}}{\alpha^{\beta}} \int_0^{\infty} e^{-y} \left(\alpha y^{1/\beta}\right)^{r+\beta-1} \cdot \frac{\alpha}{\cancel{\beta}} y^{1/\beta - 1}\, dy$

$= \frac{1}{\alpha^{\beta-1}} \int_0^{\infty} \alpha^{r+\beta-1} \cdot (y)^{(r+\beta-1)\left(\frac{1}{\beta}\right)} y^{(1/\beta)-1} \cdot e^{-y}\, dy$

$= \alpha^{r} \int_0^{\infty} y^{\frac{r}{\beta} + \cancel{1} - \cancel{\frac{1}{\beta}} + \cancel{\frac{1}{\beta}} - \cancel{1}} \cdot e^{-y}\, dy$

$= \alpha^{r} \int_0^{\infty} y^{r/\beta} e^{-y}\, dy$

$= \alpha^{r}\, \Gamma\left(\frac{r}{\beta}+1\right)$

$E(x) = \alpha\, \Gamma\left(\frac{1}{\beta}+1\right)$

$E(x^2) = \alpha^2\, \Gamma\left(\frac{2}{\beta}+1\right)$

$V(x) = E(x^2) - (E(x))^2$

$= \alpha^2\, \Gamma\left(\frac{2}{\beta}+1\right) - \alpha^2 \left(\Gamma\left(\frac{1}{\beta}+1\right)\right)^2$

$= \alpha^2 \left[\Gamma\left(\frac{2}{\beta}+1\right) - \left(\Gamma\left(\frac{1}{\beta}+1\right)\right)^2\right]$

Log-Normal Dist^n $\Rightarrow$ If X is a Continuous R.V. s.t. $\log_e X \sim N(\mu, \sigma^2)$

Then $X \sim \text{lognormal}(\mu, \sigma^2)$

Its Pdf is given by

$$f(x) = \begin{cases} \frac{1}{x\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2} ; & x > 0 \\ 0 ; & x \leq 0. \end{cases}$$

Parameter of dist^n are μ, σ^2

Note – μ, σ^2 are mean & var of the normal dist^n, but not the log-normal distribution.

If we choose $\mu = 0, \sigma = 1$

Then $f(x) = \begin{cases} \frac{1}{x\sqrt{2\pi}} e^{-\frac{1}{2}(\log x)^2} \end{cases}$

So we call this as Standard log-normal dist^n Pdf

i.e. $X \sim SLN(0,1)$.

i.e. $X \sim SLN$ with parameters 0 & 1.

Mean $E(x) = \int_0^\infty x \cdot f(x)\, dx$

$$= \int_0^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2} dx$$

Put $\frac{\log x - \mu}{\sigma} = t \Rightarrow \log x = \sigma t + \mu$

$\Rightarrow x = e^{(\mu + \sigma t)}$

$\frac{1}{x} dx = \sigma dt$

$\Rightarrow dx = \sigma x dt$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} \cancel{\sigma} (e^{\mu + \sigma t})\, dt$$

$$= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\sigma t} e^{-t^2/2} \, dt$$

$$= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(t^2 - 2\sigma t + \sigma^2 - \sigma^2)} \, dt$$

$$= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(t-\sigma)^2 + \frac{\sigma^2}{2}} \, dt$$

$$= \frac{e^{\mu + \frac{\sigma^2}{2}}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(t-\sigma)^2} \, dt$$

If $t - \sigma = z$

$\Rightarrow dt = dz$

$$= e^{\mu + \frac{\sigma^2}{2}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} \, dz = 1$$

=

Pdf of Standard normal variate

$\therefore \; E(x) = e^{\mu + \sigma^2/2}$

Similarly, we can show that

$E(x^2) = e^{2\mu + 2\sigma^2}$

So $V(x) = E(x^2) - (E(x))^2$

$= e^{2\mu + 2\sigma^2} - (e^{\mu + \sigma^2/2})^2$

$= e^{2\mu} \cdot e^{2\sigma^2} - e^{2\mu} \cdot e^{\sigma^2}$

$= e^{2\mu} e^{\sigma^2} (e^{\sigma^2} - 1)$

→ If $X \sim N(\mu, \sigma^2)$ Then what is the distribution of e^x?

let $y = e^x$

$\Rightarrow \log y = x$ is a normal random variable

$\Rightarrow y = e^x$ is a log-normal random variable

i.e. e^x follow log normal distribution.

→ If $X \sim U[0,1]$ Then find the P.d.f of $-2\log X$.

Soln ~~let~~ $X \sim U[0,1]$

$$\Rightarrow f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & ; \text{otherwise} \end{cases}$$

let $y = -2\log X$

$x = 0 \Rightarrow y = \infty$; $x = 1 \Rightarrow y = 0$

The Pdf of y ~~is~~ $g(y) = f(x) \cdot \left|\frac{dx}{dy}\right|$

$$x = e^{-y/2}$$

$$\frac{dx}{dy} = -\frac{1}{2} e^{-y/2} \Rightarrow \left|\frac{dx}{dy}\right| = \frac{1}{2} e^{-y/2}$$

$$\therefore g(y) = 1 \cdot \frac{1}{2} e^{-y/2} \quad ; y > 0 \text{ or } 0 < y < \infty.$$

$$\text{i.e. } g(y) = \begin{cases} \frac{1}{2} e^{-y/2} & ; y > 0 \\ 0 & ; o/w \end{cases}$$

$\Rightarrow y \sim Exp\left(\frac{1}{2}\right)$

Beta distribution:→

This distⁿ Can be used to model the Prob. like – The Conversion rate of Customers actually purchasing on your website.

- The click – through rate of your advertisement.
- ~~The~~ how likely it is that any political party will win.

In Maths, $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx \quad , m,n>0 \quad —(1)$

& $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1-x)^{m+n}}\,dx \quad —(2)$

Based on (1) & (2), we defined Beta distⁿ of first and Second Kind.

$$\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n} \quad , m,n>0.$$

→ A R.V. X is said to follow beta distⁿ of first Kind, denoted by β_1, with parameters m & n $(m,n>0)$ if its pdf is defined as –

$$f(x) = \begin{cases} \frac{1}{\beta(m,n)} x^{m-1}(1-x)^{n-1} & ; 0<x<1; \; m,n>0 \\ 0 & ; \text{otherwise.} \end{cases}$$

$$X \sim \beta_1(m,n)$$

→ If $m=n=1$, then $\beta(1,1) = \frac{\Gamma 1\,\Gamma 1}{\Gamma 2} = 1$

$\Rightarrow f(x) = 1 \quad ; 0<x<1 \quad$ → Pdf of Uniform distⁿ.

$\Rightarrow X \sim U(0,1)$

i.e. beta distⁿ of first Kind $\xRightarrow{m=n=1}$ Uniform distⁿ

→ If $X \sim \beta_1(m,n)$ then $(1-x) \sim \beta_1(n,m)$.

→ A R.V. X is said to follow Beta dist^n of Second kind, denoted by β_2, with parameters m & n $(m,n>0)$ if its pdf is defined as

$$f(x) = \begin{cases} \frac{1}{\beta(m,n)} \frac{x^{m-1}}{(1+x)^{m+n}} & ; 0<x<\infty,\ m,n>0 \\ 0 & ; \text{otherwise.} \end{cases}$$

$X \sim \beta_2(m,n)$.

→ If we choose $y = \frac{1}{1+x}$

then If $X \sim \beta_2(m,n)$ then $Y \sim \beta_1(m,n)$.

$\because y = \frac{1}{1+x} \Rightarrow x = \frac{1}{y} - 1$ and $x \sim \beta_2(m,n)$

$$\therefore f(x) = \frac{1}{\beta(m,n)} \frac{\left(\frac{1}{y}-1\right)^{m-1}}{\left(\frac{1}{y}\right)^{m+n}}$$

$$= \frac{1}{\beta(m,n)} \frac{(1-y)^{m-1}}{y^{m-1}} \cdot y^{m+n}$$

$$= \frac{1}{\beta(m,n)} y^{n-1} (1-y)^{m-1}$$

$$\Rightarrow (1-y) \sim \beta_1(n,m)$$

$$\Rightarrow y \sim \beta_1(m,n)$$

→ In Beta dist^n, $m-1$ — Can be thought of as no. of Successes
$n-1$ — no. of failures.

E.g. — If Prob of Success is 60%. Then set $m=60$
Prob of failure is 40%. $n=40$

→ If m becomes larger then Prob. distⁿ will shift towards the right.

→ If n becomes larger then Prob. distⁿ will shift towards the left.

Eg. If we want to find the Prob. that a person will top the Competitive Exam will be more than 50%, where distⁿ is given to be Beta distⁿ with $m=2$ and $n=8$.

So we want Success rate to be >50 i.e. $P(x>50\%)$ i.e. $P(x>0.5)$

$$f(x) = \frac{1}{B(2,8)} x^{2-1}(1-x)^{8-1}$$

$$= \frac{1}{B(2,8)} x(1-x)^7$$

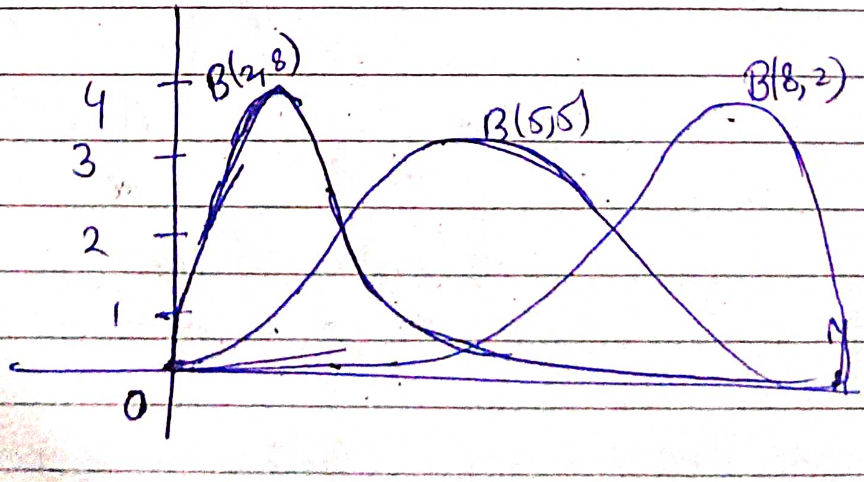
$$B(2,2) = \frac{\Gamma 2 \Gamma 8}{\Gamma 10} = \frac{1}{72}$$

$$P(x>0.5) = \int_{0.5}^{1} f(x)\,dx$$

$$= \frac{1}{72}\int_{0.5}^{1} x(1-x)^7 = \underline{0.01953} \rightarrow \text{very low}$$

(as n was larger)

$m, n > 1$



→ If $m=n=1$ Then ~~B(1,1) =~~

$$f(x) = \frac{1}{B(1,1)} x^0(1-x)^0 = 1.$$

→ If $m=1, n=2$ (say)

then $f(x) = \frac{1}{B(1,2)} x^0 (1-x)^1$

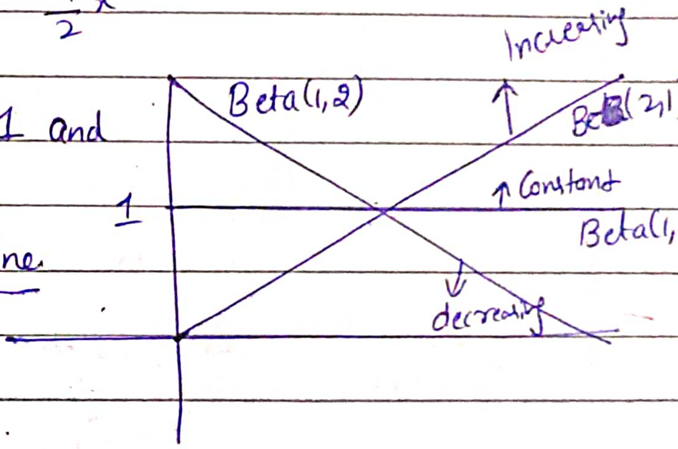
$B(1,2) = \frac{\Gamma 1 \Gamma 2}{\Gamma 3} = \frac{1}{2}$

So $f(x) = \frac{1}{2}(1-x)$

→ If $m=2, n=1$ then $f(x) = \frac{1}{B(2,1)} x (1-x)^0$

$\Rightarrow f(x) = \frac{1}{2} x$

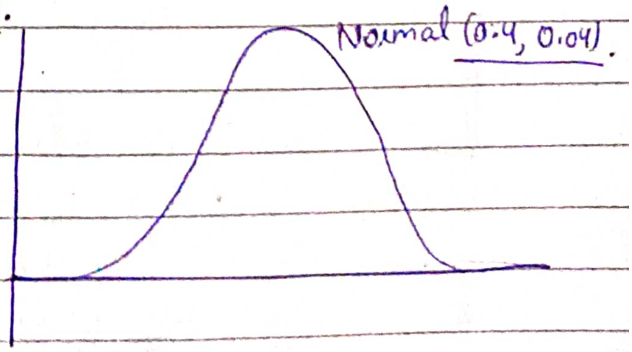
⇒ If any of m or n = 1 and other is 2; then Beta pdf can be a straight line.



→ If $m<1$, $n<1$ i.e. $m \in (0,1)$, $n \in (0,1)$ then PDF of Beta distⁿ is U-shaped.

→ The pdf of Beta distⁿ is normal if m+n is large and m & n are approximately Equal.

Eg. Beta (50,70)



Mean & Variance :→

$$E(x^r) = \int_0^1 x^r f(x)\,dx$$

$$= \frac{1}{\beta(m,n)} \int_0^1 x^r . x^{m-1} (1-x)^{n-1} dx$$

$$= \frac{1}{\beta(m,n)} \int_0^1 x^{m+r-1} (1-x)^{n-1} dx$$

$$= \frac{\beta(r+m, n)}{\beta(m,n)} = \frac{\dfrac{\Gamma(r+m)\,\Gamma(n)}{\Gamma(m+r+n)}}{\dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}}$$

$$= \frac{\Gamma(m+r)\,\Gamma(m+n)}{\Gamma(m)\,\Gamma(m+r+n)}$$

$$\text{So } E(x) = \frac{\Gamma(m+1)\,\Gamma(m+n)}{\Gamma(m)\,\Gamma(m+n+1)} = \frac{m\,\cancel{\Gamma(m)}\,\cancel{\Gamma(m+n)}}{\cancel{\Gamma(m)}\,(m+n)\,\cancel{\Gamma(m+n)}}$$

$$= \frac{m}{m+n}$$

$$E(x^2) = \frac{\Gamma(m+2)\,\Gamma(m+n)}{\Gamma(m)\,\Gamma(m+n+2)} = \frac{(m+1)\,m\,\cancel{\Gamma(m)}\,\cancel{\Gamma(m+n)}}{\cancel{\Gamma(m)}\,(m+n+1)(m+n)\,\cancel{\Gamma(m+n)}}$$

$$= \frac{m(m+1)}{(m+n)(m+n+1)}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{m(m+1)}{(m+n)(m+n+1)} - \left(\frac{m}{m+n}\right)^2$$

$$= \frac{m}{m+n}\left[\frac{(m+1)(m+n) - m(m+n+1)}{(m+n)(m+n+1)}\right]$$

$$= \frac{m\left[\cancel{m^2} + \cancel{nm} + \cancel{m} + n - \cancel{m^2} - \cancel{nm} - \cancel{m}\right]}{(m+n)^2(m+n+1)}$$

$$\Rightarrow V(x) = \frac{mn}{(m+n)^2 (m+n+1)}$$

Que Find the Parameter of the Beta dist^n so that $\mu = 0.260$ and $\sigma = 0.04$. Find $P(X > 0.4)$

Sol^n

$$\text{Mean} = \frac{m}{m+n} = 0.260 \quad -(1)$$

$$\text{S.D.} = \sqrt{\frac{mn}{(m+n)^2 (m+n+1)}} = 0.04 \quad -(2)$$

Solve (1) & (2); we get

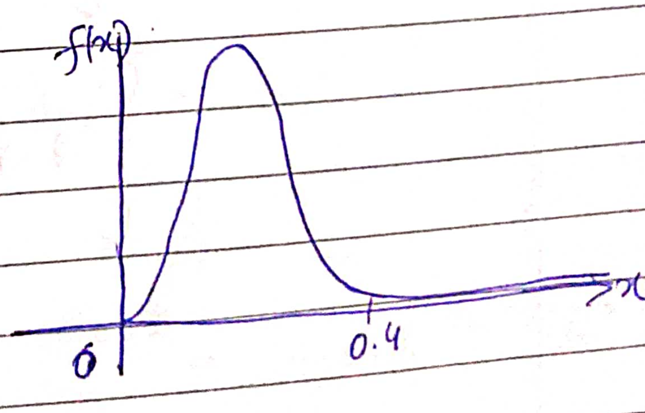
$m = 31.005$; $n = 88.245$.

n — larger

$\Rightarrow$ Prob dist^n will shift towards the left.

$$P(X > 0.4) = 1 - P(X \leq 0.4)$$

$$= 1 - \int_0^{0.4} f(x)\, dx = 0.000\,6077$$



Que The random variable $Y = \log X$ has $N(10,4)$. Find

(1) The Pdf of X

(2) Mean and Variance of X

(3) $P(X \le 1000)$

Sol^n:→ $Y \sim N(10, 2^2)$

Note If $X \sim LN(\mu, \sigma^2)$

Then $\log X \sim N(\mu, \sigma^2)$

Similarly If $Y \sim N(\mu, \sigma^2)$

Then $e^Y \sim LN(\mu, \sigma^2)$

(a) Pdf of X is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2} \quad ; x \geq 0$$

$\sigma = 2,\ \mu = 10$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log x - 10}{2}\right)^2} & ; x \geq 0 \\ 0 & ; o/w. \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{2\sqrt{2}\, x\sqrt{\pi}} e^{-\frac{1}{8}(\log x - 10)^2} & ; x \geq 0 \\ 0 & ; o/w \end{cases}$$

(b) $E(x) = e^{\mu + \frac{\sigma^2}{2}}$; $V(x) = (e^{\sigma^2} - 1)\, e^{(2\mu + \sigma^2)}$

$\Rightarrow E(x) \approx 162.754$; $V(x) = 53.598 \times e^{24}$

(c) $P(X \leq 1000) = P(\log X \leq \log 1000)$

$= P(Y \leq \log 1000)$

$= P\left(\frac{Y-10}{2} \leq \frac{\log 1000 - 10}{2}\right)$

$= P(Z \leq -1.55) = 0.0611$