Que Examine whether the following set of vectors is L.I. or L.D. Also find the rank of the matrix associated.

(1) $\{(2, 3, 6, -3, 4); (4, 2, 12, -3, 6); (4, 10, 12, -9, 10)\}$ Ans:- L.D, Rank 2

(2) $\{(3, 2, 4)\ (1, 0, 2)\ (1, -1, -1)\}$ → Ans L.I.; Rank 3

(3) $\{(1, 2, 3, 1)\ (2, 1, -1, 1), (4, 5, 5, 3), (5, 4, 1, 3)\}$ Ans. L.D., Rank 2

(4) $\{(2, 2, 0, 2); (4, 1, 4, 1); (3, 0, 4, 0)\}$ Ans L.D., Rank 2

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Gauss Elimination method :→

System of Linear Equations :→

Let $a_{11}x_1 + a_{12}x_2 + \_\_\_ + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \_\_\_ + a_{2n}x_n = b_2$ — (1)

$|\qquad |\qquad |\qquad |$

$a_{m1}x_1 + a_{m2}x_2 \_\_\_ + a_{mn}x_n = b_m$

So $AX = b$

where $A = \begin{bmatrix} a_{11} & a_{12} & \_\_ & a_{1n} \\ a_{21} & a_{22} & - & a_{2n} \\ | & | & & | \\ a_{m1} & a_{m2} & - & a_{mn} \end{bmatrix}_{m\times n}$ $b = \begin{bmatrix} b_1 \\ b_2 \\ | \\ b_m \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ | \\ x_n \end{bmatrix}$

(1) is the System of m equations and n unknowns.

System (1) is said to be Consistent if it has atleast one solution and Inconsistent, if it has no solution.

There are three Possibilities :–

(1) The System (1) has unique Solution.

(2) System (1) has no solution.

(3) system (1) has Infinite no. of solutions.

Result 1 The non-hom. System of Equation $Ax = b$ has a solution iff Rank (A) = Rank (A|b) where A|b is the augmented matrix.

⇒ If Rank (A) ≠ Rank (A|b) ⇒ (1) has no solution.

→ If Rank (A) = Rank (A|b) = n (no. of unknowns) then (1) has unique solution.

→ If Rank (A) = Rank (A|b) = $r < n$ then (1) has Infinite no. of solutions.

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Gauss Elimination method for non-homogeneous system:-

(1) Solve $2x + y - z = 4$
$x - y + 2z = -2$
$-x + 2y - z = 2$

Solⁿ:-

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$

$$Ax = b$$

Write the augmented matrix $[A|b]$ and reduce it to row echelon form by applying elementary row operations.

$$[A|b] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ 1 & -1 & 2 & -2 \\ -1 & 2 & -1 & 2 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1/2$
$R_3 \leftarrow R_3 + R_1/2$

$$= \left[\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ 0 & -3/2 & 5/2 & -4 \\ 0 & 5/2 & -3/2 & 4 \end{array}\right]$$

$R_3 \leftarrow R_3 + \frac{5}{3} R_2$

$$= \left[\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ 0 & -3/2 & 5/2 & -4 \\ 0 & 0 & 8/3 & -8/3 \end{array}\right]$$

Hence Rank (A) = Rank $(A|b) = 3$

$\Rightarrow$ System has a unique solution.

And solution is

$$\frac{8}{3} z = -8/3 \Rightarrow z = -1$$

$$-\frac{3}{2} y + \frac{5}{2} z = -4 \Rightarrow y = 1$$

$2x + y - z = 4 \Rightarrow x = 1$

So $(x=1, y=1, z=1)$ is the solution.

(2) $2x + z = 3$

$x - y + z = 1$

$4x - 2y + 3z = 3$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

The augmented matrix is $\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 1 & -1 & 1 & 1 \\ 4 & -2 & 3 & 3 \end{array}\right]$

$R_2 \leftarrow R_2 - R_1/2$

$R_3 \leftarrow R_3 - 2R_1$

$$= \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & -1 & 1/2 & -1/2 \\ 0 & -2 & 1 & -3 \end{array}\right]$$

$R_3 \leftarrow R_3 - 2R_2$

$$= \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & -1 & 1/2 & -1/2 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

$\Rightarrow$ Rank$(A) = 2$ & Rank$(A|b) = 3$

$\Rightarrow$ System has no solution.

(3) $x - y + z = 1$

$2x + y - z = 2$

$5x - 2y + 2z = 5$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 5 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

The augmented form of the matrix is

$$[A|b] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & +1 & -1 & 2 \\ 5 & -2 & 2 & 5 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 5R_1$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -3 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right]$$

$R_3 \leftarrow R_3 - 3R_2$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow$ Rank (A) = Rank (A|b) = 2 < 3

$\Rightarrow$ System has Infinite no. of solutions.

$3y - 3z = 0$

$\Rightarrow y = z$

$x - y + z = 1$

$\Rightarrow x = 1$

So $x = 1$, $y = z$ and z can be chosen arbitrarily.

Que Solve $4x - 3y - 9z + 6w = 0$

$2x + 3y + 3z + 6w = 6$

$4x - 21y - 39z - 6w = -24$

Ans :- Infinite Sol^n

Que Solve $x + 2y - 2z = 1$

$2x - 3y + z = 0$

$5x + y - 5z = 1$

$3x + 14y - 12z = 5$

Ans :- $x = 1, y = 1, z = 1$ Unique Sol^n.

Que $\begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$

Ans $x = 1, y = 2, z = 2$

Que $\begin{bmatrix} 5 & 3 & 14 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 2 \end{bmatrix}$

Ans $x = -1, y = -1/2, z = 3/4$

Que $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$

Ans $[2-\alpha, 1, \alpha, 1]$; α arbitrary.

Que $\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\frac{-1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$