

Poisson distribution \rightarrow It is a limiting case of the binomial distⁿ under the following conditions:

- (1) $n \rightarrow \infty$ i.e. no. of trials are large.
- (2) $p \rightarrow 0$ i.e. Prob of success is very small.
- (3) $np = \lambda$ (say) - finite.

$$\begin{aligned}
 B(x; n, p) &= \binom{n}{x} p^x (1-p)^{n-x} \\
 &= \binom{n}{x} \left(\frac{p}{1-p}\right)^x (1-p)^n \\
 &= \frac{n(n-1)(n-2)\dots(n-x+1)}{x! (n-x)!} \left(\frac{p}{1-p}\right)^x (1-p)^n \\
 &= \frac{n(n-1)(n-2)\dots(n-x+1)}{x!} \left(\frac{\lambda/n}{1-\frac{\lambda}{n}}\right)^x \left(1-\frac{\lambda}{n}\right)^n \\
 &= \frac{x!}{x!} \left(\frac{1}{n}\right) \left(\frac{1}{n}\right) \dots \left(\frac{1-(x-1)}{n}\right) \frac{\lambda^x}{x!} \left(1-\frac{\lambda}{n}\right)^n
 \end{aligned}$$

$$\begin{aligned}
 \lim_{n \rightarrow \infty} B(x; n, p) &= \lim_{n \rightarrow \infty} \frac{x!}{x!} \left(\frac{1}{n}\right) \left(\frac{1}{n}\right) \dots \left(\frac{1-(x-1)}{n}\right) \frac{\lambda^x}{x!} \left(1-\frac{\lambda}{n}\right)^n \\
 &= \frac{\lambda^x}{x!} e^{-\lambda}; x = 0, 1, 2, \dots
 \end{aligned}$$

\rightarrow A r.v. X is said to follow a Poisson distⁿ if its P.m.f. is given by

$$P(X=x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}; & x=0, 1, 2, \dots, \lambda \geq 0. \\ 0; & \text{o/w} \end{cases}$$

λ - Parameter of the distⁿ

$$\rightarrow \sum_{x=0}^{\infty} P(X=x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda} \cdot e^{\lambda} = 1$$

$$\text{Mean} \leftarrow E(x) = \sum_{x=0}^{\infty} x \cdot P(x=x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \cdot \lambda = \lambda.$$

$$E(x^2) = \sum_{x=0}^{\infty} x^2 \cdot P(x=x) = \sum_{x=0}^{\infty} x^2 \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x^2 \lambda^x}{x!}$$

$$= e^{-\lambda} \left[\sum_{x=0}^{\infty} \frac{[x(x-1)+x] \lambda^x}{x!} \right]$$

$$= e^{-\lambda} \left[\lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \right]$$

$$= e^{-\lambda} \left[\lambda^2 \cdot e^{\lambda} + \lambda e^{\lambda} \right]$$

$$= \lambda^2 + \lambda$$

$$V(x) = E(x^2) - (E(x))^2 = \lambda^2 + \lambda - \lambda^2 = \lambda.$$

Ques 3 Coins are tossed 800 times. Find the prob of getting 3 heads x times.

Soln $\Rightarrow n = 800$.

$$p = \text{prob of head} = \frac{1}{2}$$

Prob. of getting 3 heads (when 3 coins are tossed Single time)

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

So n - very large

$$\lambda = np = 800 \times \frac{1}{8} = 100. \quad p - \text{very small}$$

$$\text{So } P(\text{getting 3 heads } x \text{ times}) = P(x=x) = \frac{n!}{x!} p^x (1-p)^{n-x}$$

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-100} (100)^x}{x!}, \quad x=0, 1, 2, \dots$$

Ques In a book of 520 pages, 390 typo errors occur. Find the Prob. that a random sample of 5 pages has no error.

Soln $n = 520$ pages.

The average no. of typo errors per page = $\frac{390}{520} = 0.75$.

$$\text{So } \lambda = 0.75.$$

$$P(\text{x errors per page}) = P(X=x) = \frac{e^{-0.75} (0.75)^x}{x!}$$

$$\Rightarrow P(X=0) = \frac{e^{-0.75} (0.75)^0}{0!} = e^{-0.75}$$

$$\Rightarrow P(\text{no error in 5 pages}) = [P(X=0)]^5 = (e^{-0.75})^5 = e^{-3.75}.$$

Ques $X \sim P(\lambda)$ st. $P(X=2) = 9P(X=4) + 90P(X=6)$,

Find (1) λ (2) $E(X)$.

Soln $X \sim P(\lambda) \Rightarrow P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}; x=0, 1, 2, \dots, \lambda > 0.$

$$\Rightarrow P(X=2) = \frac{e^{-\lambda} \lambda^2}{2!}$$

$$P(X=4) = \frac{e^{-\lambda} \lambda^4}{4!}$$

$$P(X=6) = \frac{e^{-\lambda} \lambda^6}{6!}$$

$$P(X=2) = 9P(X=4) + 90P(X=6)$$

$$\Rightarrow \frac{e^{-\lambda} \lambda^2}{2!} = 9 \left(\frac{e^{-\lambda} \lambda^4}{4!} \right) + 90 \cdot \left(\frac{e^{-\lambda} \lambda^6}{6!} \right)$$

$$\Rightarrow \frac{1}{2} = \frac{9 \lambda^2}{4!} + \frac{90 \lambda^4}{6!}$$

$$\Rightarrow \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\Rightarrow (\lambda^2 - 1)(\lambda^2 + 4) = 0$$

Fitting of Poisson distⁿ:

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}; x=0,1,\dots,\infty$$

$$P(x+1) = \frac{e^{-\lambda} \lambda^{x+1}}{(x+1)!}, x=0,1,2,\dots,\infty$$

$$\Rightarrow \frac{P(x+1)}{P(x)} = \frac{\lambda}{(x+1)} \Rightarrow P(x+1) = \frac{\lambda}{(x+1)} P(x).$$

$$\Rightarrow P(1) = \frac{\lambda}{1}, P(0) = \lambda P(0)$$

$$P(2) = \frac{\lambda}{2} P(1) = \frac{\lambda^2}{2} P(0) \text{ and so on.}$$

Ques Fit a Poisson distⁿ to the following data.

No. of mistakes per page	0	1	2	3	4	Total
No. of pages	109	65	22	3	1	200

x	f	fx
0	109	0
1	65	65
2	22	44
3	3	9
4	1	4
	<u>200</u>	<u>122</u>

$$\bar{x} = \lambda = \frac{\sum fx}{\sum f} = \frac{122}{200} = 0.61.$$

$$P(x \text{ mistakes per page}) = \frac{e^{-0.61} (0.61)^x}{x!}$$

$$P(0) = e^{-0.61} = 0.5432 \Rightarrow N \times P(0) = 109$$

$$P(1) = \lambda \cdot P(0) = 0.61 \times 0.5432 \Rightarrow N \times P(1) = 66$$

$$N \times P(2) \approx 20$$

$$N \times P(3) \approx 4$$

$$N \times P(4) \approx 1$$

Geometric Distribution: \rightarrow When we have to perform an experiment until success occurs.

Eg. (1) What is the prob. of getting 1st head in 8th attempt during tossing a coin repeatedly?

(2) What is the prob. that a person will hit the target on his 10th attempt?

\rightarrow The Geometric distⁿ is a prob. distⁿ of R.V. X - in which independent trials are performed until a success occurs, where the prob. of success ' p ' remains constant in each trial.

Its P.m.f. is given by

$$P(X=x) = \begin{cases} q^{x-1} p & ; x=1, 2, \dots \\ 0 & ; \text{otherwise} \end{cases} \quad p > 0, p \leq 1, p+q=1.$$

E.g. While playing Ludo, you have to throw a die until first time a Six appears.
parameter - p .

Ques 1 Suppose a person shoots a target in an independent manner. If the prob. that the target is hit in any shot is 0.7. Then find the prob. that the target would be hit on the 10th attempt?

Soln

$$p = 0.7 \Rightarrow q = 0.3 \quad (\because p+q=1)$$

$$P(X=x) = q^{x-1} \cdot p$$

$$P(X=10) = q^9 \cdot p = (0.3)^9 \cdot (0.7)$$

(2) Prob. that it takes the person less than 4 shots?

$$\text{Soln} \rightarrow P(X<4) = P(X=1) + P(X=2) + P(X=3)$$

$$= p + qp + q^2p$$

$$= p(1+q+q^2) = 0.7(1+0.3+(0.3)^2) = 0.9730.$$

(3) Prob. that it takes him an even no. of shots?

$$\begin{aligned}
 \text{Soln} \quad P(x=\text{even}) &= P(x=2) + P(x=4) + P(x=6) + \dots \\
 &= q^2 p + q^4 p + q^6 p + \dots \\
 &= q^2 p (1 + q^2 + q^4 + q^6 + \dots) \\
 &= q^2 p \left(\frac{1}{1-q^2} \right) = \frac{q^2 p}{(1-q)(1+q)} = \frac{q}{1+q} \\
 &= \frac{0.3}{0.3+1} = \frac{0.3}{1.3} = 0.2308
 \end{aligned}$$

Ques The prob. for an applicant for a driving license to pass the road test on any given attempt is $\frac{2}{3}$. Find the prob. that the applicant will pass the road test on the 3rd attempt.

$$\text{Soln} \quad p = \frac{2}{3}; \quad q = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(x=3) = q^2 p = \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) = \frac{2}{27}$$

Mean and Variance \rightarrow

$$\begin{aligned}
 E(x) &= \sum_{x=1}^{\infty} x \cdot P(x=x) \\
 &= \sum_{x=1}^{\infty} x \cdot q^{x-1} \cdot p \\
 &= p \sum_{x=1}^{\infty} x \cdot q^{x-1} = p [1 + 2q + 3q^2 + 4q^3 + \dots] \\
 &= p (1-q)^2 \\
 &= \frac{p}{p^2} = \frac{1}{p}
 \end{aligned}$$

$$\begin{aligned}
 E(x^2) &= \sum_{x=1}^{\infty} x^2 \cdot P(x=x) = \sum_{x=1}^{\infty} x^2 \cdot q^{x-1} \cdot p \\
 &= p \sum_{x=1}^{\infty} (x(x-1) + x) q^{x-1} \\
 &= p \left[\sum_{x=1}^{\infty} x(x-1) q^{x-1} + \sum_{x=1}^{\infty} x q^{x-1} \right] \\
 &= p [(2q + 6q^2 + 12q^3 + \dots)] + \frac{p}{p^2}
 \end{aligned}$$

$$= 2pq \left(1 + 3q + 6q^2 + 10q^3 + \dots \right) + \frac{1}{p}$$

$$= 2pq (1-q)^{-3} + \frac{1}{p}$$

$$= \frac{2q}{b^2} + \frac{1}{p}$$

$$\therefore V(X) = E(X^2) - (E(X))^2 = \frac{2q}{b^2} + \frac{1}{p} - \frac{1}{b^2}$$

$$= \frac{2q-1}{b^2} + \frac{1}{p}$$

$$= \frac{2q+b-1}{b^2} = \frac{2q-q}{b^2} = \frac{q}{b^2}$$

In

Ques 1 Average no. of shots required to hit the target?

$$\text{Soln} \quad p=0.7; q=0.3$$

$$\text{Mean} = \frac{1}{p} = \frac{1}{0.7} = 1.4286.$$

Memoryless prop. of Geometric Distⁿ / (Lack of Memory)

If $X \sim \text{geometric dist}^n$, S.T. for any two positive integers s and t , $P(X > s+t | X > s) = P(X > t)$.

Soln

$$P(X=x) = q^{x-1} \cdot p; x=1, 2, 3, \dots$$

For any five integer k , we have

$$\begin{aligned} P(X > k) &= P(X=k+1) + P(X=k+2) + \dots \\ &= q^k \cdot p + q^{k+1} \cdot p + \dots \\ &= q^k \cdot p (1 + q + q^2 + \dots) \\ &= q^k \cdot p \cdot \left(\frac{1}{1-q} \right) = q^k \end{aligned}$$

$$P(X > s+t | X > s) = \frac{P(X > s+t \cap X > s)}{P(X > s)}$$

$$= \frac{P(X > s+t)}{P(X > s)} = \frac{q^{s+t}}{q^s} = q^t = P(X > t)$$

Negative Binomial Dist → When we need to perform an experiment until a total of n successes are obtained.

If $n=1$, Then this a geometric distⁿ.

$$\begin{aligned} P(X=x) &= P(\text{at least } x-1 \text{ successes in } x-1 \text{ trials and a success in } x^{\text{th}} \text{ trial}) \\ &= \binom{x-1}{x-1} p^{x-1} \cdot q^{(x-1)-(x-1)} \cdot p \\ &= \binom{x-1}{x-1} p^{x-1} \cdot q^{x-x} \end{aligned}$$

$$\text{Thus } P(X=x) = \begin{cases} \binom{x-1}{x-1} p^{x-1} q^{x-x}; & x = 1, 2, 3, \dots \\ 0; & \text{o/w} \end{cases}$$

If $n=1$, then $P(X=x) = q^{x-1} \cdot p$ — Geometric Distⁿ.

$$\begin{aligned} \rightarrow \sum_{x=n}^{\infty} P(X=x) &= \sum_{x=n}^{\infty} \binom{x-1}{x-1} p^{x-1} q^{x-n} \\ &= p^n \sum_{x=n}^{\infty} \binom{x-1}{x-1} q^{x-n} \\ &= p^n \frac{(1-q)^n}{\downarrow} = p^n \cdot p^{-n} = 1 \end{aligned}$$

(This is the reason for negative Binomial Distⁿ)

Because of negative Binomial Expansion.

$$\begin{aligned} \text{Mean} = E(X) &= \sum x \cdot P(X=x) \\ &= \sum_{x=n}^{\infty} x \cdot \binom{x-1}{x-1} p^{x-1} q^{x-n} \\ &= p^n \sum_{x=n}^{\infty} \frac{x!}{x!(x+1)(x+2)\dots(x+n)} q^{x-n} \\ &= \frac{np}{p} \quad \text{— (Solve it)} \end{aligned}$$

$$V(X) = \frac{npq}{p^2}$$

Que If the prob. is 0.40 that a child exposed to a certain disease will contract it. What is the prob. that a tenth child exposed to the disease will be the 3rd to catch it?

Soln \rightarrow Rep. Prob. $P(x=10) = {}^9C_2 \cdot p^2 \cdot q^7 \cdot p$; $p = 0.4$, $q = 0.6$.

$$= {}^9C_2 p^3 \cdot q^7 = 0.0645$$

Que Let X be the no. of births in a family until the 2nd girl is born. If the prob. of having a male child is $\frac{1}{2}$. Find the prob. that the sixth child in the family is the 2nd daughter?

Soln

$$P(\text{male}) = \frac{1}{2} = q$$

$$P(\text{girl child}) = \frac{1}{2} = p$$

$$P(x=6) = ({}^5C_5 \cdot p \cdot q^5) = p = {}^5C_5 \cdot p^2 \cdot q^4$$

$$= 5 \times \left(\frac{1}{2}\right)^6 = \frac{5}{64}$$

Que In a company, 5% defective components are produced. What is the prob. that at least 5 components are to be examined in order to get 3 defectives?

Soln

$$P(\text{defective}) = 0.05$$

$$P(x \geq 5) = 1 - P(x < 5)$$

$$= 1 - [P(x=3) + P(x=4)]$$
~~$$= 1 - [{}^3C_3 p^3 q^0 + {}^4C_3 p^3 q^1]$$~~

$$= 1 - [{}^2C_2 p^2 q^0 \times p + {}^3C_2 p^2 q^1 \times p]$$

$$= 1 - [p^3 + 3p^3 q]$$

$$= \boxed{0.9995}$$

Ques If the prob. that a child exposed to certain viral fever will be infected is 0.3. Find the prob. that the eighth child exposed to the disease will be the fourth to be Infected.

$$\begin{aligned} \text{Soln} \quad P(x=8) &= {}^7C_3 p^3 q^4 \times p \quad p = 0.3 \\ &= 0.0681 \text{ Ans} \quad \Rightarrow q = 0.7 \end{aligned}$$

Ques If the prob. that a person will believe a rumour is 0.6. Find the prob. that the tenth person to hear the rumour will be the 3rd person to believe?

$$\begin{aligned} \text{Soln} \quad P(x=10) &= {}^9C_2 p^2 q^7 \times p \quad ; \quad p = 0.6, q = 0.4 \\ &= 0.0127 \text{ Ans} \end{aligned}$$

Ques A person is firing bullets at the target and the prob. of hitting the target at any trial is 0.7. Find the prob. that his Seventh Shot is his fourth hit?

$$\text{Soln} \quad P(x=7) = {}^6C_3 p^3 q^3 \times p = {}^6C_3 p^4 \times q = 0.1297 \text{ Ans}$$

Ques If the prob. of having a male child is 0.5. Find the prob. that in a family, Eighth child is the 3rd boy.

$$\begin{aligned} \text{Soln} \quad P(x=8) &= {}^7C_2 p^2 q^5 \times p \quad ; \quad p = 0.5 \\ &= 0.0820 \text{ Ans} \quad q = 0.5 \end{aligned}$$

Ques In a Company 3% of defective items are produced. Find the prob. that atleast 6 Components are to be examined in order to get 3 defective?

$$\begin{aligned} \text{Soln} \quad P(x \geq 6) &= 1 - P(x < 6) \\ &= 1 - [P(x=3) + P(x=4) + P(x=5)] \end{aligned}$$

Hypergeometric Distribution:

Eg. Let a Box contains 100 balls. 60 Green and 40 Red.

A sample of 7 balls is drawn. What is the prob. that it has 3 Green and 4 Red balls?

\Rightarrow With Replacement

$$P(\text{Green ball}) = \frac{60}{100} = 0.6$$

$$P(\text{Red ball}) = \frac{40}{100} = 0.4$$

Without Replacement

$$P(3G, 4R) = \frac{\binom{60}{3} \binom{40}{4}}{\binom{100}{7}}$$

$$P(3G, 4R) = 7C_3 \cdot (0.6)^3 \cdot (0.4)^4$$

$$= 0.1953$$

$$= 0.1935$$

\rightarrow So when the popⁿ is finite and the sampling is done without replacement. i.e. Prob. of Success changes on each draw. (as each draw decreases the popⁿ) , we obtain hypergeometric distribution.

Notations

The P.m.f of hypergeometric distⁿ is

$$\binom{M}{k} \binom{N-m}{n-k}$$

$$P(x=k) = \frac{\binom{M}{k} \binom{N-m}{n-k}}{\binom{N}{n}}$$

$$k=0, 1, 2, \dots, n.$$

Population

N balls

M green

N-m Red

Sample

n balls

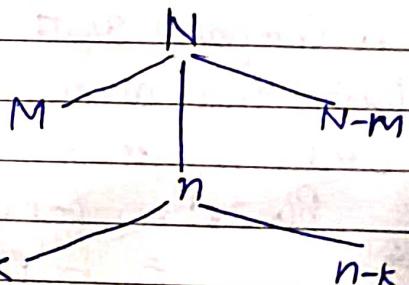
k green

n-k red

Note \rightarrow If the no. of elements in the sample are much

smaller than in the popⁿ,

the hypergeometric pmf \approx binomial pmf.

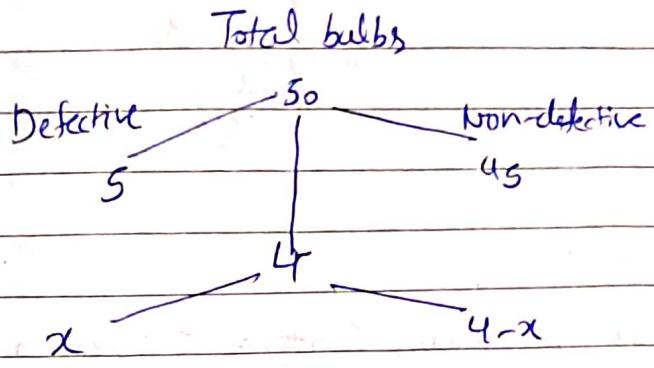


Ex (1) A Create Contains 50 light bulbs of which 5 are defective and 45 are not. A Quality Control Inspector Samples 4 bulbs without Replacement. Let X = no. of defective bulbs selected. Find the P.m.f, $f(x)$, of X .

Solⁿ

$$P(X=x) = \frac{\binom{5}{x} \binom{45}{4-x}}{\binom{50}{4}}$$

$$x=0,1,2,3,4.$$

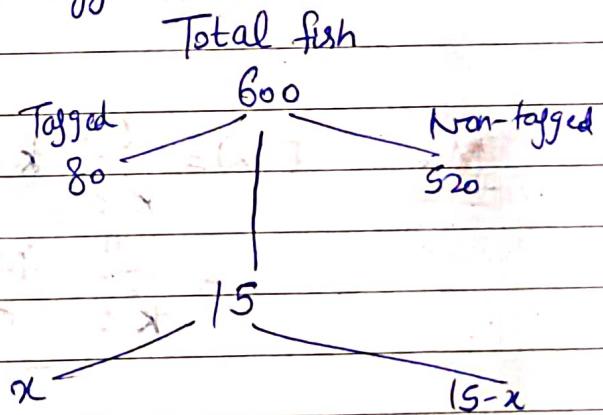


Ex (2) A lake Contains 600 fish, 80 of which have been tagged by Scientist. A person randomly Catches 15 fish from the lake. Find the P.m.f of X - the no of fish in the person's sample which are tagged?

Solⁿ

$$P(X=x) = \frac{\binom{80}{x} \binom{520}{15-x}}{\binom{600}{15}}$$

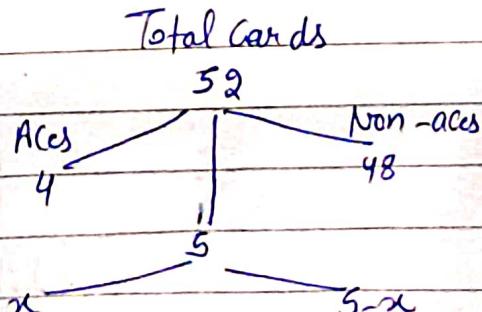
$$x=0,1,\dots,15.$$



Que Let X - no. of aces in a five-Card hand dealt from a 52-Card deck. Find P.m.f of X .

Solⁿ

$$P(X=x) = \frac{\binom{5}{x} \binom{48}{5-x}}{\binom{52}{5}} ; x=0,1,\dots,4.$$

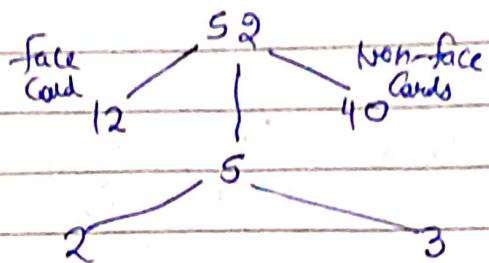


Ex Emma likes to play Cards. She draws 5 cards from 52 Cards deck. What is the prob of that from the 5 cards drawn, She draws only 2 face Cards?

Sol

$$P(X=2) = \frac{\binom{12}{2} \binom{40}{3}}{\binom{52}{5}}$$

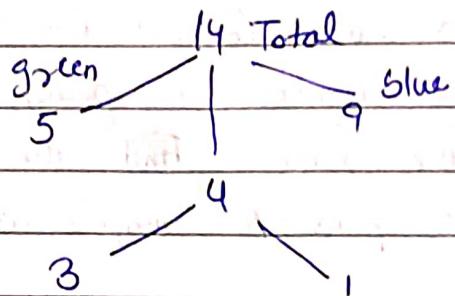
Total Cards



Ques A house Contains 5 Green marbles, And 9 blue marbles. 4 marbles are drawn randomly without replacement. find the prob of getting 3 green marbles.

Sol

$$P(X=3) = \frac{\binom{5}{3} \binom{9}{1}}{\binom{14}{4}}$$



Mean $E(X) = \sum_{k=0}^n k * P(X=k)$

$$= \sum_{k=0}^n k \binom{M}{k} \cdot \binom{N-m}{n-k}$$

$$= \sum_{k=0}^n \frac{k \cdot M(M-1)!}{k(k-1)! (M-k)!} \cdot \frac{(N-m)}{(n-k)}$$

$$\frac{N(N-1)!}{N(n-1)! (N-n)!}$$

$$= \sum_{k=1}^n \frac{M \cdot \binom{M-1}{k-1} \binom{N-m}{n-k}}{\binom{N}{n} \binom{N-1}{n-1}}$$

$$= \frac{nM}{N} \sum_{k=1}^n \left[\frac{\binom{M-1}{k-1} \binom{N-M}{n-k}}{\binom{N-1}{n-1}} \right]$$

$$\text{let } k-1 = x \Rightarrow k = x+1$$

So

$$E(x) = \binom{NM}{N} \left[\sum_{x=0}^n \frac{\binom{M-1}{x} \binom{N-M}{n-x-1}}{\binom{N-1}{n-1}} \right]$$

$$\boxed{E(x) = \frac{nm}{N}}$$

$$\left[: \sum_{x=0}^n P(X=x) = 1 \right]$$

$$\begin{aligned} V(x) &= \frac{nm(N-M)(N-n)}{N^2(N-1)} \\ &= \frac{nm}{N} \left(1 - \frac{M}{N} \right) \left(\frac{N-n}{N-1} \right) \end{aligned}$$

→ Hypergeometric distⁿ tends to Binomial distⁿ as
 $N \rightarrow \infty$ and $\frac{M}{N} \rightarrow p$

$$\therefore E(x) = \frac{nm}{N} = np$$

$$\text{and } V(x) = \frac{nm}{N} \left(1 - \frac{M}{N} \right) \left(\frac{N-n}{N-1} \right)$$

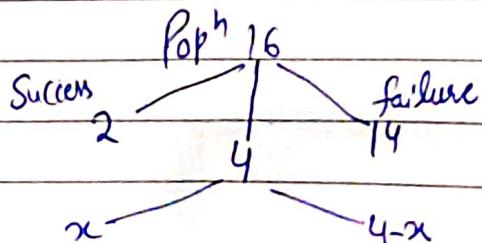
$$= n \cdot p (1-p) \cdot 1 = npq.$$

Ques Find the Expectation of hypergeometric distⁿ s.t. the prob that a 4-trial hypergeometric Experiment results in Exactly 2 Success, when the popⁿ consists of 16 items.

Solⁿ

$$E(x) = \frac{nm}{N}$$

$$n=4, M=2, N=16$$

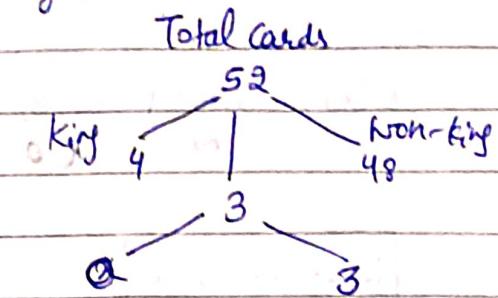


So $E(x) = \frac{2 \times 4}{16} = 0.5$.

Ex Let A draws 3 cards from a pack of 52 cards. what is the prob. of getting no kings?

Soln

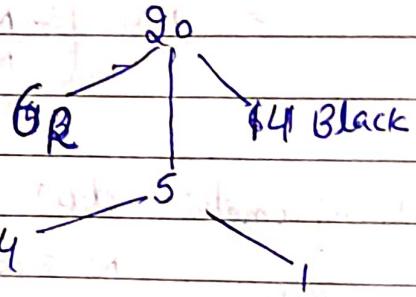
$$P(x=0) = \frac{4 \cdot \binom{48}{3}}{\binom{52}{3}} = 0.7826.$$



Ex A deck of cards contain 20 cards: 6 R, 14 B cards. 5 cards are drawn randomly without replacement. Find the prob. that exactly 4 red cards are drawn.

Soln

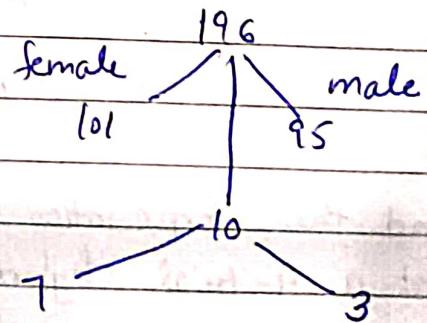
$$P(x=4) = \frac{\binom{6}{4} \binom{14}{1}}{\binom{20}{5}} = 0.0135.$$



Ex A small voting district has 101 female voters and 95 male voters. A sample of 10 voters is drawn. What is the prob that exactly 7 voters are female?

Soln

$$P(x=7) = \frac{\binom{101}{7} \binom{95}{3}}{\binom{196}{10}}$$



Ques A grp. of 10 Individuals are used for a biological calc with the following blood types

Type O - 3 Type A - 4, Type B - 3 people

Find the prob that a random Sample of 5 will contain 1 Type -O, 2 Type -A, 2 Type -B?

Soln

$$\text{Req. Prob} = \frac{(3C_1)(4C_2)(3C_2)}{(10C_5)} \\ = \frac{3}{14}$$

