14

GEARS

Chapter Outline

14.1 INTRODUCTION

A gear may be defined as any toothed member designed to transmit or receive motion from another member by successively engaging tooth. The smaller gear is called the *pinion* and the bigger one the *gear wheel*. They are used in metal cutting machine tools, automobiles, tractors, hoisting and transporting machinery, rolling mills, etc.

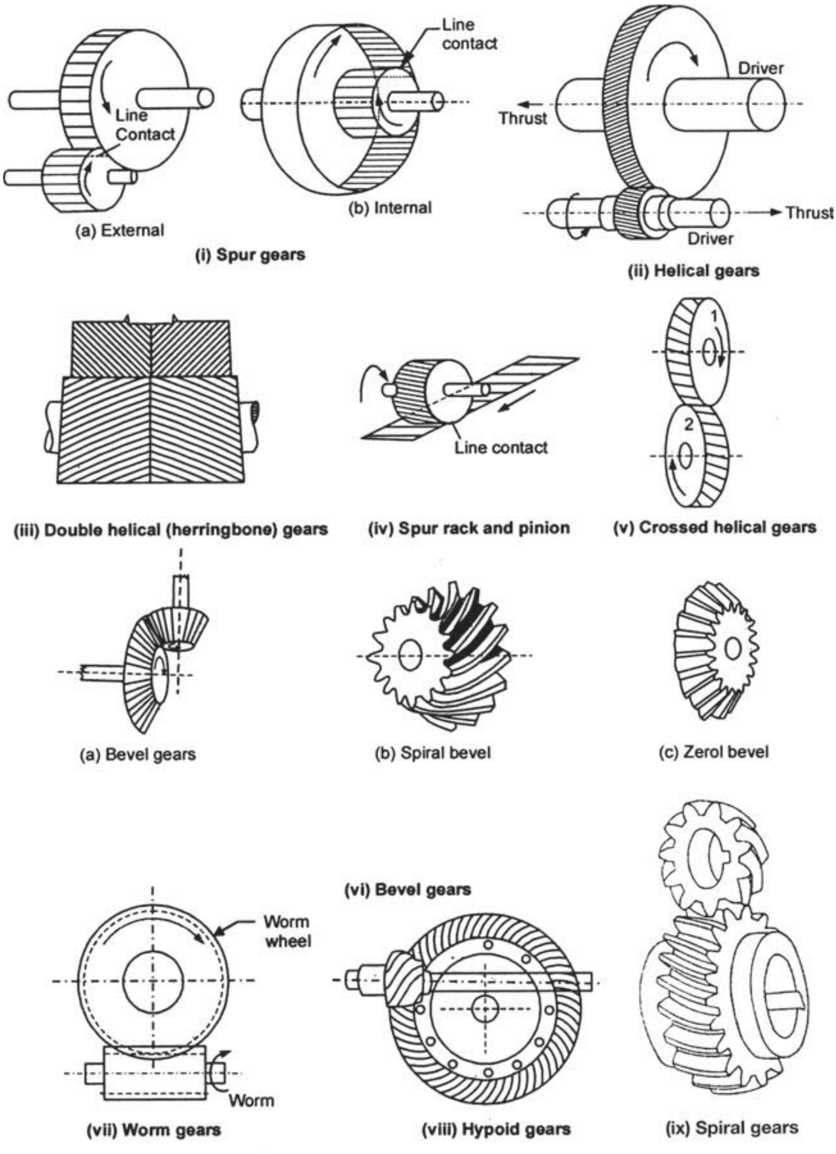

Fig.14.1 Types of gears

The gears provide many advantages over other modes of power transmission likes belts, ropes, and chains etc. Some of their advantages are:

1. They occupy lesser space.
2. There is no slip between the gears in mesh and provide exact speed ratio.
3. They can transmit higher power.
4. Their efficiency is higher.

However, the error in tooth meshing may cause undesirable vibration and noise during operation.

14.2 CLASSIFICATION OF GEARS

The gears may be classified as follows:

1. *Spur gears*: A spur gear is a cylindrical gear whose tooth traces are straight line generators of the reference cylinder [Fig.14.1(i)].
2. *Helical gears*: This is similar to the spur gear in which the tooth traces are helices. [Fig.14.1(ii)].
3. *Double helical (or herringbone) gears*: It is a cylindrical gear in which a part of the face width is right hand and the other left hand, with or without a gap between them [Fig.14.1(iii)].
4. *Spiral gears*: In spiral gears, the tooth traces are curved lines other than helices.[Fig.14.1(ix).
5. *Bevel gears*: The reference surface is a cone in bevel gears. The bevel gears may be straight, spiral, zerol, and face gears. In zerol bevel gears, the teeth are curved in the lengthwise direction and are arranged in such a manner that the effective spiral angle is zero. In face gears, the bevel gear teeth are cut on the flat face of the blank. A crown gear is a bevel gear with a reference cone angle of 90° [Fig 14.1(vi)].
6. *Hypoid gears*: They are similar to the spiral bevel gears with the difference that the axes of the shafts do not intersect. [Fig.14.1(viii)].
7. *Worm gears*: In these gears, there are screw threads on the worm and teeth on the worm wheel. [Fig.14.1(vii)].
8. *Planetary gears*: A gear pair or a gear train one of whose axes, instead of being fixed in position in the mechanism of which the gear pair is a part, moves around the other is called planetary gear train.

Gears may also be classified based on the orientation of the shafts as:

1. Parallel shafts: spur, helical, and double helical gears.
2. Intersecting shafts: straight bevel, spiral bevel, zerol bevel, and face gears.
3. Non-parallel and non-intersecting shafts-spiral, hypoid, and worm gears.

Gears may be of the external, internal, and rack and pinion type. In external gears, the teeth of gears mesh externally, whereas in internal gears the teeth of the two gears mesh internally. A rack is a gear of infinite radius.

14.3 GEAR TERMINOLOGY

A spur gear pair in mesh is shown in Fig.14.2. The various terms releated to gears are defined as follows:

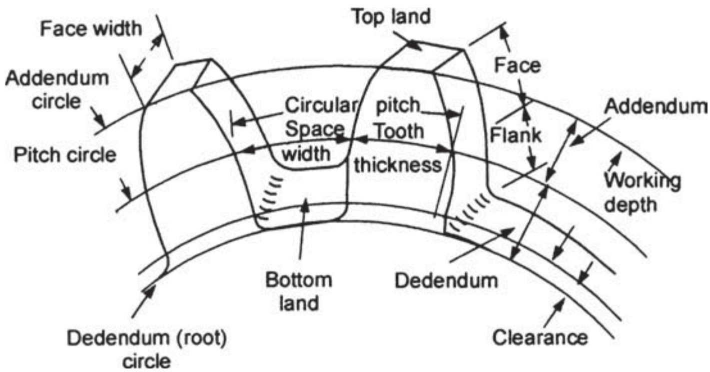

Fig.14.2 Gear terminology

Pitch circle: It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

Pitch circle diameter (*d*): It is the diameter of a circle which by pure rolling action would produce the same motion as the toothed gear wheel.

Base circle: It is the circle from which involute form is generated.

Pitch surface: It is the surface of the disc which the toothed gear has replaced at the pitch circle.

Pitch point: It is the pitch of the tangency or the point of contact of the two pitch circles of the mating gears.

Circular pitch (p): It is the distance measured along the circumference of the pitch circle from a point on one tooth to a corresponding point on the adjacent tooth.

$$p = \pi d/z \tag{14.1}$$

where z = number of teeth.

Base pitch (P_b): It is the distance measured along the circumference of the base circle from a point on one tooth to a corresponding point on the adjacent tooth.

$$\text{Base pitch, } p_b = p \times \cos\alpha \tag{14.2}$$

where α = pressure angle of gear tooth profile.

Diametral pitch (*P*): It is expressed as the number of teeth per unit pitch circle diameter.

$$P = z/d \tag{14.3}$$

$$Pp = \pi \tag{14.4}$$

Module (*m*): It is expressed as the length of the pitch circle diameter per unit number of teeth.

$$m = d/z = 1/P \tag{14.5}$$

Addendum (h_a): The radial height of the tooth above pitch circle.

Addendum circle: A circle bounding the top of the teeth.

Dedendum (h_f): The radial depth of a tooth below the pitch circle.

Dedendum circle: A circle passing through the roots of all the teeth.

Clearance (*c*): The radial height difference between addendum and dedendum of a teeth.

Working depth: It is the radial distance of tooth from addendum circle to clearance circle.

Total depth: It is the sum of addendum and dedendum or the radial distance from dedendum circle to addendum circle.

Face: The part of the tooth surface lying below the pitch surface.

Backlash: The minimum distance between the non driving side of a tooth and adjacent side of the mating tooth at the pitch circle.

Profile: The curve forming face and flank.

Tooth thickness (*t*): This is the arc distance measured along the pitch circle from its intercept with one flank to its intercept with the other flank of the same tooth.

Face width (*b*): The width of the gear tooth measured axially along the pitch surface.

Top land: It is the surface of the top of the tooth

Tooth fillet: The radius that connects the root circle to the profile of the tooth.

Tooth space: It is the width of the space between two teeth measured on the pitch circle.

Pressure angle (*a*): The angle between the common normal at the point of contact and the common tangent at the pitch point. The pressure angle is either 14.5° or 20°.

Path of contact: It is the locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement. It is a straight line.

Path of approach: It is the portion of the path of contact from the beginning of engangement to the pitch point.

Angle of approach: The angle turned by gears during path of approach.

Path of recess: It is the portion of the path of contact from the pitch point to the end of engagement of the two mating teeth.

Angel of recess: The angle turned through during path of recess.

Arc of contact: It is the locus of a point on the pitch circle, from the beginning of engagement to the end of engagement of pair of teeth in mesh.

Involute: The involute of a circle is the curve traced by the end of a thread as it is unwound from a stationary cylinder.

Base circle diameter (d_b): It is the diameter of the base circle.

$$d_b = d \cos \alpha \tag{14.6}$$

Cycloid: It is the locus of a point on the circumference of a circle which rolls without slipping on a fixed straight line.

Centre distance (C): It is the distance between the centres of rotation of the two gears in mesh.

$$\mathrm{C} = (d_1 + d_2)/2 = m(z_1 + z_2)/2 \tag{14.7}$$

14.4 FUNDAMENTAL LAW OF GEARING

Let us consider two curved bodies 1 and 2 rotating about their centers O_1 and O_2 and contacting at point A, as shown in Fig.14.3. A_1 and A_2 are two coincident points, A_1 lying on body 1 and A_2 lyin g on body 2. TT' and NN' represent common tangents and normals, respectively, at point A. Let ω_1 and ω_2 be the angular velocities of body 1 and 2, respectively. Let v_{a1} and v_{a2} be the linear velocities of point A. v_{a1} is perpendicular to O_1A_1 and v_{a2} is perpendicular to O_2A_2. Let the common normal intersect the line of centres at point P. Let O_1G and O_2H be perpendiculars to AP.

If the two bodies are to remain in contact, the component of velocities of A_1 and A_2 along the common normal must be equal.

Therefore $$v_{a1} \cos \alpha = v_{a2} \cos \beta$$

or

$$\omega_1 \cdot O_1A_1 \cos\ \alpha = \omega_2 \cdot O_2A_2 \cos\beta$$

$$\omega_1 \cdot O_1A_1 \cdot \frac{A_1F}{A_1C} = \omega_2 \cdot O_2A_2 \cdot \frac{A_2F}{A_2B}$$

The condition for pure rolling is that the point of contact shall lie on the line of centres.

$$\frac{\omega_1}{\omega_2} = \frac{A_1C}{O_1A_1} \cdot \frac{O_2A_2}{A_2B}$$

Triangles A_1CF and $O_1A_1G_1$ are similar. Also triangles A_2FB and O_2A_2H are similar.

Therefore $$\frac{A_1C}{O_1A_1} = \frac{A_1F}{O_1G}$$

and $$\frac{A_2B}{O_2A_2} = \frac{A_2F}{O_2H}$$

Hence $$\frac{\omega_1}{\omega_2} = \frac{A_1F}{O_1G} \cdot \frac{O_2H}{A_2F}$$

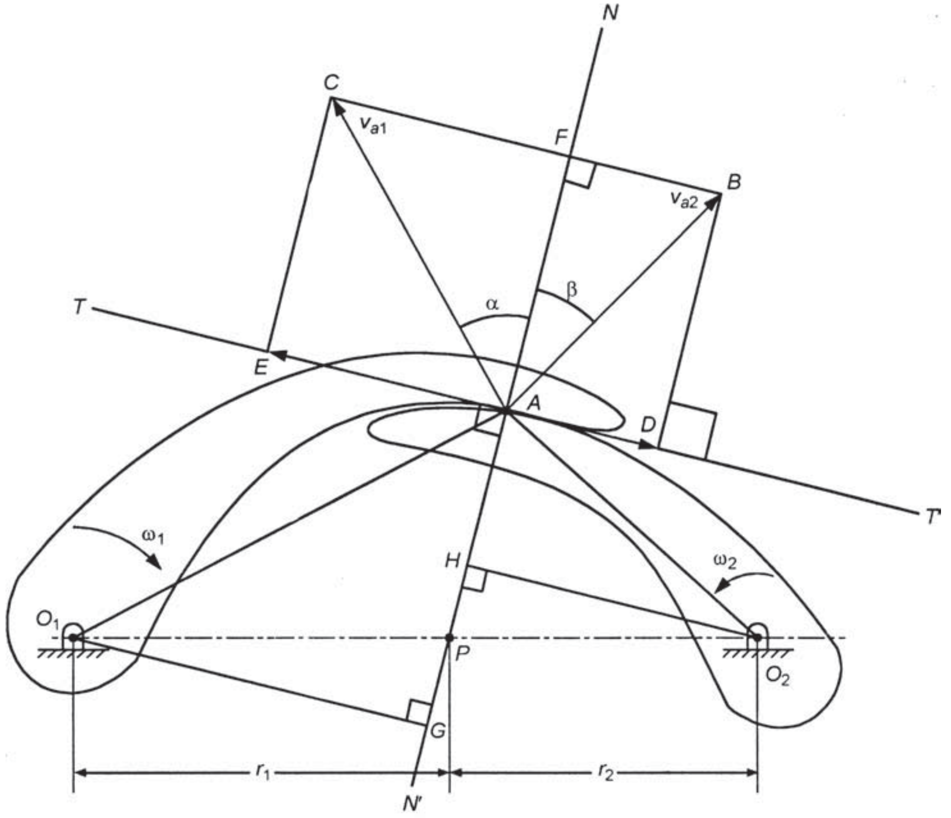

Fig.14.3 Law of gearing

But
$$A_1F = A_2F$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2H}{O_1G}$$

Now triangles O_1PG and O_2PH are similar. Hence

$$\frac{O_2H}{O_1G} = \frac{O_2P}{O_1P}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} \tag{14.8}$$

Therefore, *for constant angular velocity ratio of the two gears in contact the common normal at the point of contact must always intersect the line of centres at a fixed point (pitch point) and divide this line in the inverse ratio of the angular velocities of the two gears.* This is the fundamental law of gearing.

Conjugate action: When the tooth profiles are so shaped so as to produce a constant angular velocity ratio during meshing, then the surfaces are said to be conjugate. Two conjugate surfaces in contact always satisfy the law of gearing.

14.5 SLIDING VELOCITY BETWEEN GEAR TEETH

The relative velocity along the common tangent is called the sliding velocity, v_s. Considering Fig.14.3 again, the sliding velocity v_s along the common tangent,

$$\begin{aligned} v_s &= v_{a1} \sin \alpha + v_{a2} \sin \beta \\ &= \omega_1 \cdot O_1A_1 \cdot \frac{FC}{A_1C} + \omega_2 \cdot O_2A_2 \cdot \frac{FB}{A_2B} \end{aligned}$$

Now $$\frac{A_1C}{O_1A_1} = \frac{A_1F}{O_1G} = \frac{FC}{AG}$$

and $$\frac{A_2B}{O_2A_2} = \frac{A_2F}{O_2H} = \frac{FB}{AH}$$

Hence $$\begin{aligned} v_s &= \omega_1 \cdot AG + \omega_2 \cdot AH \\ &= \omega_1(AP + PG) + \omega_2(AP - PH) \\ &= (\omega_1 + \omega_2)AP + \omega_1 \cdot PG - \omega_2 \cdot PH \end{aligned}$$

Now $$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PH}{PG}$$

Hence $$v_s = \left(\omega_1 + \omega_2\right) AP \tag{14.9}$$

v_s = (sum of the angular velocities) × distance of the point of contact from the pitch point.

Thus, we find that the *velocity of sliding is proportional to the distance of the pitch point from the point of contact.*

14.6 GEAR TOOTH FORMS

There are two types of gear tooth forms: involute and cycloidal. The involute of a circle is the curve traced by the end of a thread as it is unwound from a stationary cylinder. Cycloid is the locus of a point on the circumference of a circle which rolls without slipping on a fixed straight line. If the circle, instead of rolling without slipping on a straight line, rolls on the outside of another circle, the locus of the point on the circumference is called as epicycloids. Conversely, if the circle rolls on the inside of another circle, the corresponding locus of the point on the circumference of the rolling circle is called hypo-cycloid. We shall discuss the involute and cycloidal profiles in brief.

14.6.1 Involute Tooth Profile

Consider two pulleys connected by a crossed wire. The pulleys will rotate in opposite directions with constant angular velocity provided the wire does not slip. Let us assume that one side of the wire is removed and a piece of cardboard is attached to wheel 1, as shown in Fig.14.4(a). Place a pencil at a point Q on the wire and turn wheel 2 counter-clockwise. Point Q will generate an involute on the cardboard relative to wheel 1. If a cardboard is now attached to wheel 2, as shown in Fig.14.4(b), and the process is repeated, an involute is generated on the cardboard of wheel 2. If the cardboards are now cut along the involute, one side of tooth is formed on both the wheels.

The circles that have been used for generating the involutes are known as *base circles*. The angle that is included by a line perpendicular to the line of action through the centre of the base circle and a line from O_1 to O_2 through Q, is known as the *involute pressure angle*, as shown in Fig.14.4(c).

The intersection of the line perpendicular to the base circles and the line of centres has been labelled as P, the pitch point. The circles passing through point P with O_1 and O_2 as centres are called the *pitch circles*, as shown in Fig.14.4(d). At the pitch point, there is pure rolling, and at all other points there is a combination of rolling and sliding.

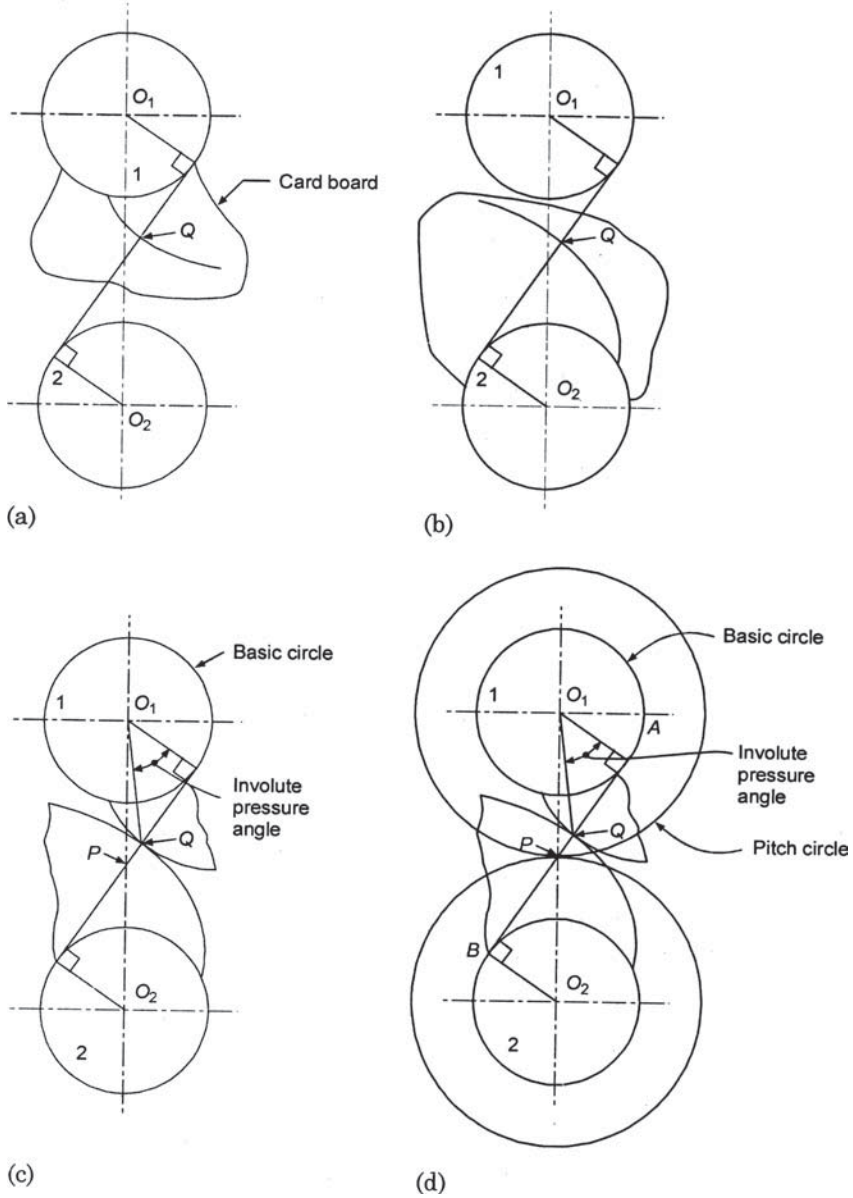

Fig.14.4 Involute tooth profile generation

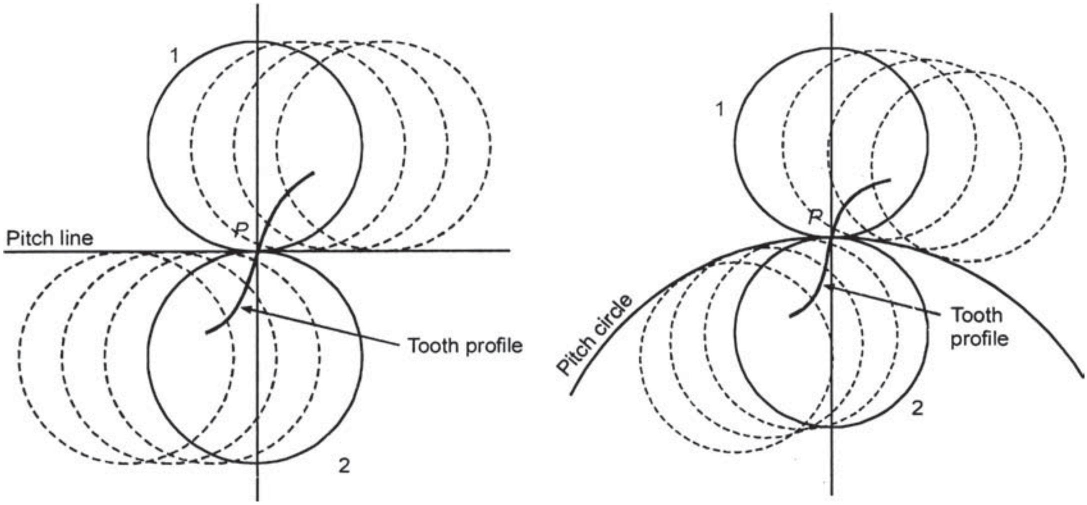

Fig.14.5 Cycloidal tooth profile generation

14.6.2 Cycloidal Tooth Profile

A cycloidal rack tooth profile meshing with a pinion is shown if Fig.14.5(a). The curve generated by a point on a circle 1 by its motion to the right on the straight line, which is the pitch line, is the profile of the face of the cycloidal tooth similarly the curve generated below the pitch line by a point on the rolling circle 2 is the flank of the tooth profile.

Fig.14.5(b) shows the construction of a cycloidal teeth profile of a gear. The circle 1 rolling on the outside of the pitch circle, generates a epicycloid, which is the face portion of the tooth profile. The circle 1 rolls without slipping to the right. The circle 2 rolls without slipping to the left on the inside of the circle generating a hypocycloid, representing the flank profile of the cycloidal tooth.

14.6.3 Comparison between Involute and Cycloidal Tooth Profiles

The comparison of involute and cycloidal tooth profiles is given in Table 14.1. The cycloidal profile is not commonly used for gear tooth, due to the reasons given in Table14.1.

Table 14.1 Comparison between Involute and Cycloidal Tooth Profiles

Characteristic	Involute gears	Cycloidal gears
1. Pressure angle	Constant throughout the engagement	Varies from commencement to end
2. Ease of manufacture	Easy to manufacture	Difficult to manufacture
3. Centre distance	Do not require exact centre distance	Requires exact centre distance
4. Interference	May occur	No interference
5. Strength	Less	More
6. Wear	More wear and tear	Less wear and tear
7. Operation	Smooth	Less smooth

14.7 CONSTRUCTION OF AN INVOLUTE

The construction of an involute tooth profile is shown in Fig.14.6. The following steps may be followed to draw the involute:

1. Draw the base circle.
2. Divide the base circle quadrant into equal number of parts (say 6). Mark the points 0 to 6 on the circumference of the circle.
3. Draw tangents at points 1 to 6.
4. Cut off 1 a = 01 on tangent at 1; 2b = 02 on tangent at 2, and so on.
5. Join o, a, b, c, etc. by a smooth curve to obtain the involute profile.

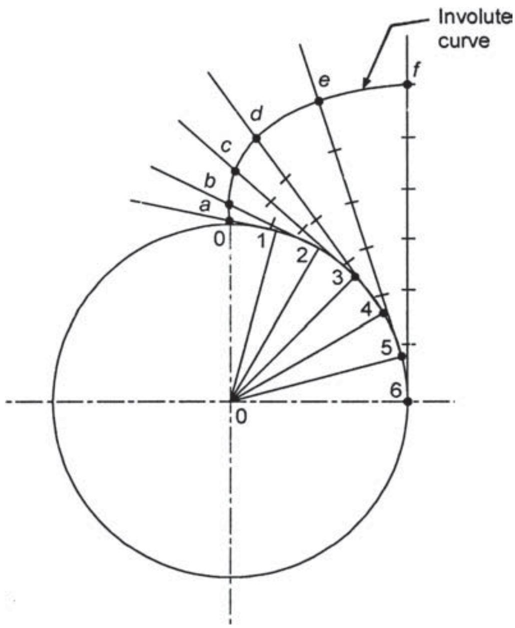

Fig.14.6 Involute profile construction

14.8 INVOLUTE FUNCTION

Consider the involute of a circle shown in Fig.14.7(a).

Let l = length of the thread unwrapped

$$= AB \text{ arc}$$
$$= r_b(\beta + \delta)$$
$$= r_b \tan \alpha$$

Thus $\beta + \delta = \tan \alpha$

where r_b = base radius.

Also from Fig.14.7(b), we have

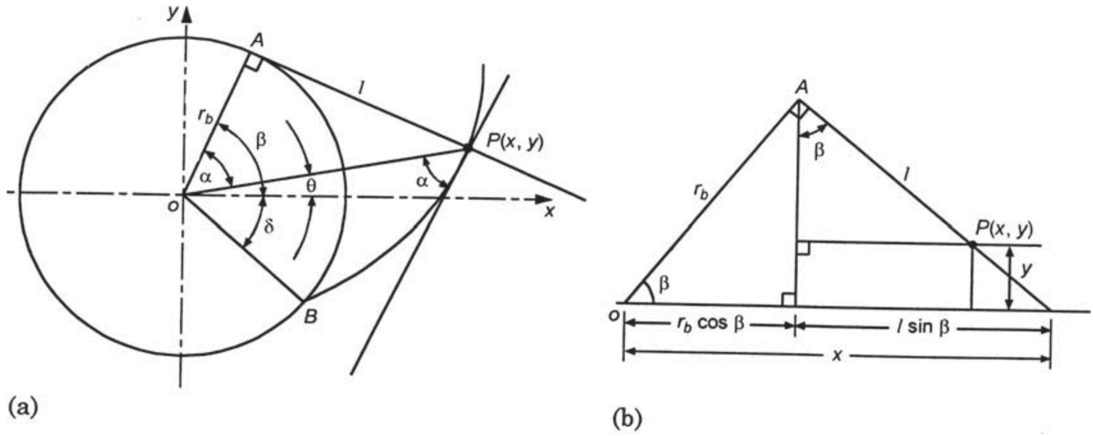

Fig.14.7 Involute function

$$x = r_b \cos\beta + l \sin\beta$$
$$= r_b[\cos\beta + (\beta + \delta)\sin\beta]$$

and $$y = r_b[\sin\beta - (\beta + \delta)\cos\beta]$$

Also $$\theta = \alpha = \beta = \tan\alpha - \delta$$

or $$\theta = \tan\alpha - \alpha - \delta$$
$$= \text{inv}\ (\alpha) - \delta$$

where $$\text{inv}\ (\alpha) = \tan\alpha - \alpha \tag{14.10}$$

Eq. (14.10) represents the involute function. Its values are given in standard tables.

14.9 INVOLUTOMETRY

Fig.14.8 shows an involute which has been generated from a base circle of radius r_b. The involute contains two points A and B with corresponding rodii r_A and r_B and involute pressure angles α_A and α_B.

and $$r_b = r_A \cos\alpha_A$$
$$r_b = r_B \cos\alpha_B$$

Therefore $$\cos\alpha_B = \left(\frac{r_A}{r_B}\right)\cos\alpha_A \tag{14.11}$$

It is possible to evaluate the involute pressure angle at any point on the involute profile from Eq. (14.11).

Now $$\text{arc } DG = \text{length BG}$$

$$\angle DOG = \frac{arc\ DG}{OG} = \frac{BG}{OG}$$

$$\tan\alpha_B = \frac{BG}{OG}$$

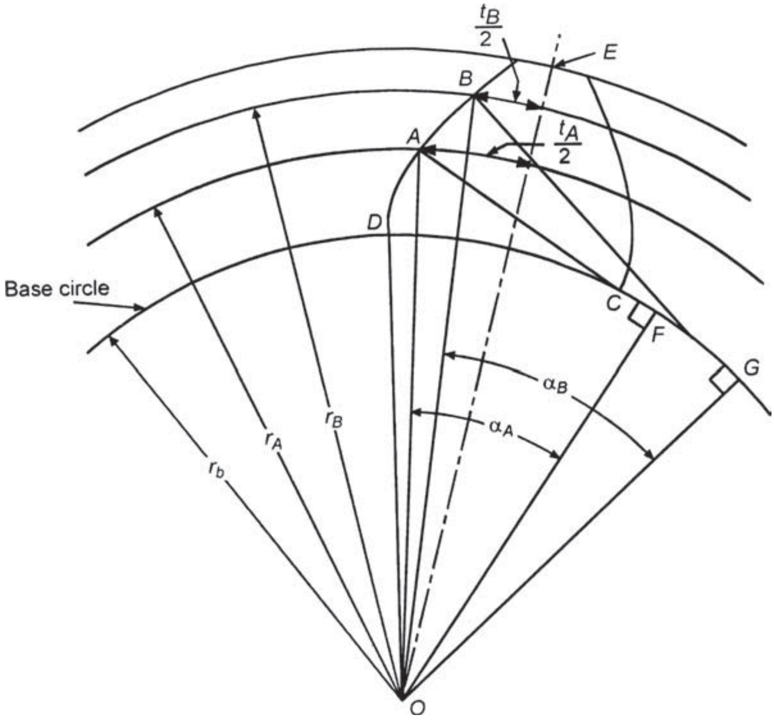

Fig.14.8 Involutometry

Thus $$\angle DOG = \tan \alpha_B$$

Also $$\angle DOB = \angle DOG - \alpha_B$$
$$= \tan \alpha_B - \alpha_B$$

and $$\angle DOA = \tan \alpha_A - \alpha_A$$

Now $$\angle DOE = \angle DOB + \frac{0.5 t_B}{r_B}$$
$$= \text{inv}\,(\alpha_B) + \frac{t_B}{2r_B}$$

Also $$\angle DOE = \angle DOA + \frac{0.5 t_A}{r_A}$$
$$= \text{inv}\,(\alpha_A) + \frac{t_A}{2r_A}$$

Thus $$t_B = 2r_B\left[\frac{t_A}{2r_A} + \text{inv}\,(\alpha_A) - \text{inv}\,(\alpha_B)\right] \qquad (14.12)$$

14.10 INVOLUTE GEAR TOOTH ACTION

The gear tooth action between two gears is shown in Fig.14.9. *P* is the pitch point and line *EF* is tangent to both the base circles, along which all points of contact of two teeth must lie. Line *EF* is called the line of action or the pressure line. Line *XX′* is perpendicular to the line of centres at the pitch point. The angle between *XX′* and *EF* is called the pressure angle. If one gear rotates in clockwise direction then the other gear would rotate in the reverse direction of counter clockwise.

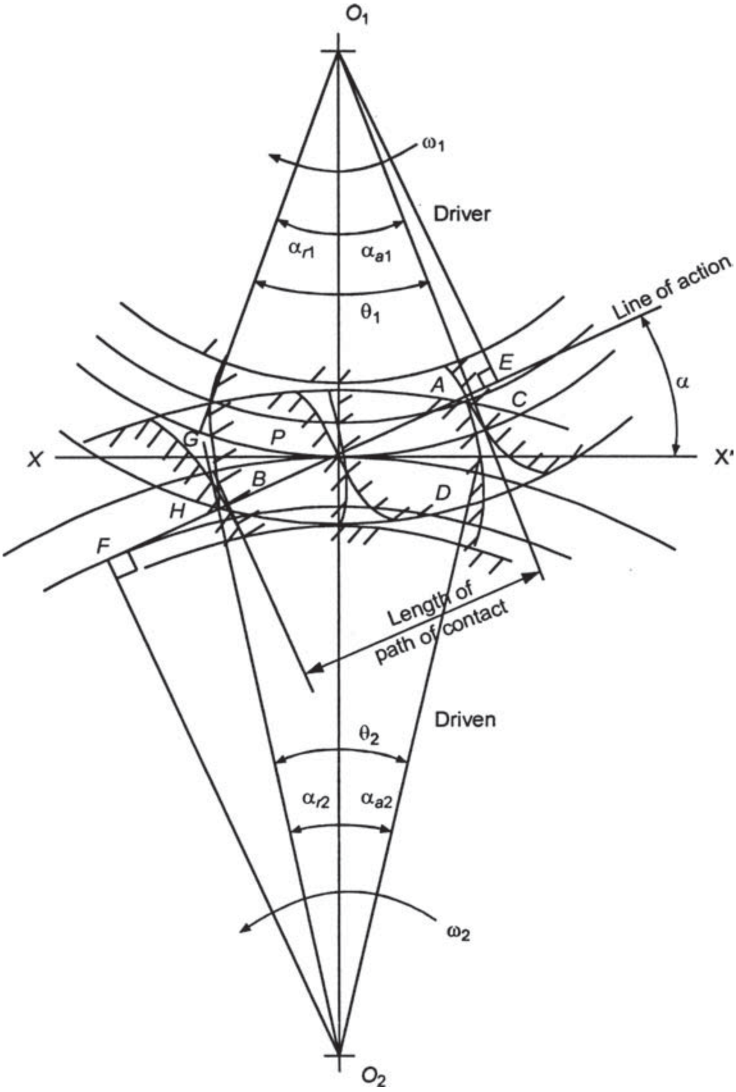

Fig.14.9 Involute gear tooth action

The teeth first come into contact at point A, where the addendum circle of the driven gear cuts the line of action. Contact follows the line of action through point P, and contact ceases at point B, where the addendum circle of the driving gear cuts the line of action. Line AB is called the path of the point of contact, and its length is the length of the path of contact. Point C is the intersection of the tooth profile on gear 2 with its pitch circle when the tooth is at the beginning of contact, and point G is the same point on the profile when the tooth is at the end of contact. Points D and H are the corresponding for gear 1. Arcs CPG and DPH are the arcs on the pitch circles through which the mating tooth profiles move as they pass from the initial to the final point of contact. These arcs action as the *arcs of action*.

Since the pitch circles roll on one another, these are equal. The angles θ_1 and θ_2 which subtend these arcs are called the angles of action. The angles of action are divided into two parts called the *angle of approach* (α_a) and *angle of recess* (α_r). The angle of approach is defined as the angle through which a gear rotates from the instant a pair of teeth comes into contact until the teeth are in contact at the pitch point. The angle of recess is the angle through which a gear rotates from the instant the teeth are in contact at the pitch point contact is broken. In general, the angle of approach is not equal to the angle of recess. Gear tooth action is smoother in recess than in approach.

From Fig.14.10, we have

$$\begin{aligned}
\alpha_{r1} &= \sin^{-1}\left(\frac{PO_1 \sin\alpha_a}{AO_1}\right) \\
\theta_1 &= 180° - (\alpha_a + \alpha_{r1}) \\
AP &= \frac{AO_1 \sin\theta_1}{\sin\alpha_a} \\
\alpha_{r2} &= \sin^{-1}\left(\frac{PO_2 \sin\alpha_a}{BO_2}\right) \\
\theta_2 &= 180° - (\alpha_a + \alpha_{r2}) \\
BP &= \frac{BO_2 \sin\theta_2}{\sin\alpha_a}
\end{aligned}$$

$$\alpha_{a1} = \frac{AP}{r_1} \tag{14.13a}$$

$$\alpha_{r1} = \frac{BP}{r_1} \tag{14.13b}$$

$$\alpha_{a2} = \frac{AP}{r_2} \tag{14.14a}$$

$$\alpha_{r2} = \frac{BP}{r_2} \tag{14.14b}$$

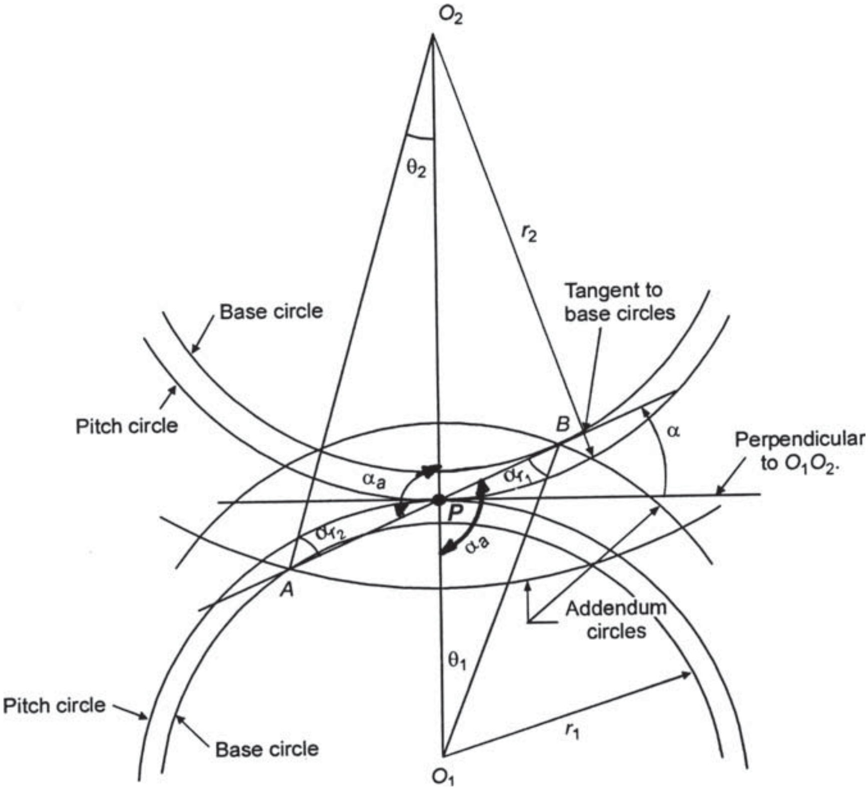

Fig.14.10 Calculating angles of approach and recess

14.11 CHARACTERISTICS OF INVOLUTE ACTION

The characteristics of the involute action are:

1. arc of contact,
2. length of path of contact, and
3. the contact ratio.

As shown in Fig.14.11, the contact of two gear teeth begins at *A* and ends at *B*.

Addendum radius of pinion, $r_{a1} = r_1 + h_{a1}$

Base circle radius of pinion, $r_{b1} = r_1 \cos \alpha$

Addendum radius of gear, $r_{b2} = r_{a2} = r_2 + h_{a2}$

Base circle radius of gear $= r_{b2} \cos \alpha$

where
r_1 = pitch circle radius of pinion
r_2 = pitch circle radius of gear
h_{a1} = addendum of pinion
h_{a2} = addendum of gear
r_{b1} = base circle radius of pinion
r_{b2} = base circle radius of gear

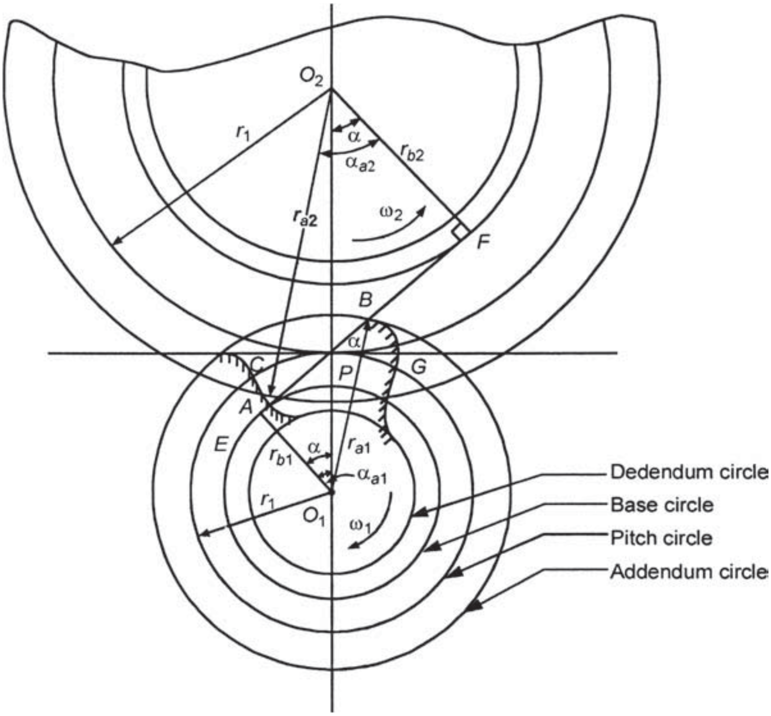

Fig.14.11 Angles of action

Length of path of recess,

$$L_r = PB = EB - EP$$
$$= (r_{a1}^2 - r_{b1}^2)^{0.5} - O_1P \ \sin\alpha$$
$$= (r_{a1}^2 - r_{a1}^2)^{0.5} - r_1 \sin\alpha \qquad (14.15)$$

Length of path of approach,

$$L_a = AP = AF - EF$$
$$= (r_{a2}^2 - r_{b2}^2)^{0.5} - O_2P \ \sin\alpha$$
$$= (r_{a2}^2 - r_{b2}^2)^{0.5} - r_2 \sin\alpha \qquad (14.16)$$

Length of path of contact,

$$AB = L_p = L_r + L_a$$
$$= \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - \left(r_1 + r_2\right)\sin\alpha \qquad (14.17)$$

Length of arc of contact,

$$L_c = \text{arc } CG$$
$$= \frac{AB}{\cos\alpha} \qquad (14.18)$$

Maximum length of path of recess = $r_2 \sin \alpha$ (14.19)
Maximum length of path of approach = $r_1 \sin \alpha$ (14.20)
The *contact ratio* is defined as the average number of pairs of teeth, which are in contact. This can be found by nothing how many times the base pitch fits into the length of the path of contact. The contact ratio (*CR*) can be expressed as:

$$CR = \text{length of path of contact/base pitch}$$

$$= \frac{L_p}{p_b} \tag{14.21}$$

where $p_b = p \cos a = \pi m \cos a$

For a rack and a pinion,

$$L_P = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} - r_1 \sin \alpha + \frac{a}{\sin \alpha} \tag{14.22}$$

where a = addendum.

Example 14.1

A pinion of 24 teeth drives a gear of 60 teeth at a pressure angle of 20°. The pitch radius of the pinion is 38 mm and the outside radius is 41 mm. The pitch radius of the gear is 95 mm and the outside radius is 98.5 mm. Calculate the length of action and contact ratio.

■ Solution

Length of path of contact,

$$L_p = (r_{a1}^2 - r_{b1}^2)^{0.5} + (r_{a2}^2 - r_{b2}^2)^{0.5} - (r_1 + r_2)\sin \alpha$$

Here,

$$r_{a1} = 41 \text{ mm}, r_{a2} = 98.5 \text{ mm}, r_1 = 38 \text{ mm}, r_2 = 95 \text{ mm}, \alpha = 20°.$$

$$r_{b1} = r_1 \cos \alpha = 38 \cos 20° = 35.7 \text{ mm}$$

$$r_{b2} = r_2 \cos \alpha = 95 \cos 20° = 811.27 \text{ mm}$$

$$L_p = (41^2 - 35.7^2)^{0.5} + (98.5^2 - 811.27^2)^{0.5} - (38 + 95)\sin 20°$$

$$= 20.16 + 41.63 - 45.49$$

$$= 16.30 \text{ mm}$$

Contact ratio,

$$m_c = \frac{L_p}{P_b}$$

$$P_b = 2\pi \frac{r_{b1}}{z_1}$$

$$= 2\pi \times \frac{35.7}{24} = 11.37 \text{ mm}$$

$$m_c = \frac{16.30}{11.34} = 1.744$$

Example 14.2

Two equal size spur gears in mesh have 36 number of teeth, 20° pressure angle and 6 mm module. If the arc of contact is 1.8 times the circular pitch, find the addendum.

Solution

Circular pitch, $p = \pi m = \pi \times 6 = 18.85$ mm

Length of arc of contact, $L_a = 1.8p = 1.8 \times 18.85 = 33.93$ mm

Length of path of contact, $L_p = L_a \cos \alpha$

$= 33.93 \cos 20° = 31.88$ mm

Pitch radii,

$$r_1 = r_2 = \frac{mz}{2} = 6 \times \frac{36}{2} = 108 \text{ mm}$$

$$L_p = (r_{a1}^2 - r_{b1}^2)^{0.5} + (r_{a2}^2 - r_{b2}^2)^{0.5} - (r_1 + r_2)\sin\alpha$$

Here $r_{a1} = r_{a2}$ and $r_{b1} = r_{b2} = r\cos\alpha = 108 \cos 20° = 101.49$ mm

$$L_p = 2(r_a^2 - 101.49^2)^{0.5} - 216 \sin 20°$$

$$31.88 = 2(r_a^2 - 101.49^2)^{0.5} - 73.87$$

Addendum, $r_a = 114.44$ mm

$$h_a = r_a - r = 114.44 - 108 = 6.44 \text{ mm}$$

Example 14.3

Two 20° involute gears in mesh have a gear ratio of 2 and 20 teeth on the pinion. The module is 5 mm and the pitch line speed is 1.5 m/s. Assuming addendum to be equal to one module, find (a) angle turned through by pinion when one pair of teeth is in mesh, and (b) maximum velocity of sliding.

Solution

Given: $i = 2$, $z_1 = 20$, $m = 5$ mm, $v = 1.5$ m/s, $h_{a1} = \text{m} = h_{a2}$

(a)

$$r_1 = \frac{mz_1}{2} = 5 \times \frac{20}{2} = 50 \text{ mm}$$

$r_2 = ir_1 = 2r_1 = 100$ mm

$r_{a1} = r_1 + h_{a1} = 50 + 5 = 55$ mm

$r_{a2} = r_2 + h_{a2} = 100 + 5 = 105$ mm

$r_{b1} = r_1 \cos 20° = 50 \cos 20° = 46.98$ mm

$r_{b2} = r_2 \cos 20° = 100 \cos 20° = 93.97$ mm

Length of path of approach, $L_a = [r_{a2}^2 - r_{b2}^2]^{0.5} - r_2 \sin\alpha$

$= [105^2 - 93.97^2]^{0.5} - 100 \sin 20°$

$= 12.65$ mm

Length of path of recess, $L_r = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} - r_1 \sin 20°$

$= [55^2 - 46.98^2]^{0.5} - 50 \sin 20°$

$= 11.50$ mm

Length of path of contact, $L_p = L_a + L_r = 12.65 + 11.50 = 24.15$ mm

Length of arc of contact, $$L_c = \frac{L_p}{\cos\alpha} = \frac{24.15}{\cos 20°} = 25.7 \text{ mm}$$

Angle turned through by the pinion $= L_c \times \dfrac{360}{2\pi r_1}$

$$= 25.7 \times \frac{360}{2\pi \times 50}$$

$$= 211.45°$$

(b) Maximum velocity of sliding, $v_s = (\omega_1 + \omega_2)L_a$

$$\omega_1 = \frac{v}{r_1} = \frac{1.5}{0.05} = 30 \text{ rad/s}$$

$$\omega_2 = \frac{v}{r_2} = \frac{1.5}{0.1} = 15 \text{ rad/s}$$

$$v_s = (30 + 15)12.65 = 5611.25 \text{ mm/s}$$

Example 14.4

The pressure angle of two gears in mesh is 20° and have a module of 10 mm. The number of teeth on pinion are 24 and on gear 60. The addendum of pinion and gear is same and equal to one module. Determine (a) the number of pairs of teeth in contact, (b) the angle of action of pinion and gear, and (c) the ratio of sliding to rolling velocity at the beginning of contact, at pitch point and at the end of contact.

■ Solution

Given: $\alpha = 20°$, $m = 10$ mm, $z_1 = 24$, $z_2 = 60$, $h_{a1} = h_{a2} = 1$ m $= 10$ mm

$$r_1 = \frac{mz_1}{2} = \frac{10 \times 24}{2} = 120 \text{ mm}, \; r_2 = \frac{10 \times 60}{2} = 300 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 120 + 10 = 130 \text{ mm}$$

$$r_{a2} = r_2 + h_{a2} = 300 + 10 = 310 \text{ mm}$$

$$r_{b1} = r_1 \cos\alpha = 120 \times \cos 20° = 112.76 \text{ mm}$$

$$r_{b2} = r_2 \cos\alpha = 300 \times \cos 20° = 281.91 \text{ mm}$$

Length of path of recess, $L_r = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} - r_1 \sin\alpha$

$$= \left[130^2 - (112.76)^2\right]^{0.5} - 120 \sin 20°$$

$$= 23.65 \text{ mm}$$

Length of path of approach, $L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$

$$= \left[310^2 - (281.91)^2\right]^{0.5} - 300 \cos 20°$$

$$= 26.33 \text{ mm}$$

Length of path of contact, $L_p = L_a + L_r = 26.33 + 23.65 = 411.98$ mm

(a) Number of pairs of teeth in contact $= \dfrac{L_p}{\pi m \cos\alpha} = \dfrac{411.98}{\pi \times 10 \times \cos 20^\circ}$

$= 1.69$

(b) Angle of action of pinion, $\alpha_{a1} = \text{Arc of contact} \times \dfrac{360}{2\pi r_1}$

$$= \left(\frac{L_p}{\cos\alpha}\right) \times \frac{360}{2\pi r_1}$$

$$= \left(\frac{411.98}{\cos 20^\circ}\right) \times \frac{360}{2\pi \times 120}$$

$$= 25.4^\circ$$

Angle of action of gear, $\alpha_{a2} = \text{Arc of contact} \times \dfrac{360}{2\pi r_2}$

$$= \left(\frac{L_p}{\cos\alpha}\right) \times \frac{360}{2\pi r_2}$$

$$= \left(\frac{411.98}{\cos 20^\circ}\right) \times \frac{360}{2\pi \times 300}$$

$$= 10.16^\circ$$

(c) Ratio of sliding to rolling velocity $= \dfrac{v_s}{v_r}$

$$v_r = r_1\omega_1 = 120\,\omega_1$$

$$\omega_2 = \frac{24\,\omega_1}{60} = 0.4\,\omega_1$$

At the beginning of contact $= \dfrac{\omega_1 + \omega_2}{\dfrac{L_a}{v_r}}$

$$= \frac{(\omega_1 + 0.4\,\omega_1) \times 26.33}{120\,\omega_1}$$

$$= 0.3072$$

At the pitch point, $v_s = 0.$ Hence $\dfrac{v_s}{v_r} = 0$

At the end of contact $= \dfrac{\omega_1 + \omega_2}{\dfrac{L_r}{v_r}}$

$$= \frac{(\omega_1 + 0.4\omega_1) \times 23.65}{120\omega_1}$$

$$= 0.276$$

Example 14.5

Two 15 mm module, 20° pressure angle spur gears have addendum equal to one module. The pinion has 25 teeth and the gear 50 teeth. Determine whether interference will occur or not. If it occurs, to what value should the pressure angle be changed to eliminate interference.

■ Solution

Given: $\alpha = 20°$, $m = 15$ mm, $z_1 = 25$, $z_2 = 50$, $h_{a1} = h_{a2} = 15$ and $m = 10$ mm
Let the pinion be the driver.

$$r_1 = \frac{mz_1}{2} = \frac{15 \times 25}{2} = 187.5 \text{ mm}, \ r_2 = \frac{15 \times 50}{2} = 375 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 187.5 + 15 = 202.5 \text{ mm}$$
$$r_{a2} = r_2 + h_{a2} = 375 + 15 = 390 \text{ mm}$$
$$r_{b2} = r_2 \cos\alpha = 375 \times \cos 20° = 352.4 \text{ mm}$$

Maximum permissible length of path of approach,

$$(L)_{max} = r_1 \sin\alpha$$
$$= 187.5 \sin 20°$$
$$= 64.13 \text{ mm}$$

Length of path of approach, $$L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$

$$= [390^2 - (352.4)^2]^{0.5} - 375 \sin 20°$$
$$= 38.81 \text{ mm}$$

Since $L_a < (L_{max})$, hence interference will occur.
For $L_a = (L_{max})$, we have

$$r_1 \sin\alpha = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$

$$64.13 = [390^2 - (375\cos\alpha)^2]^{0.5} - 375\sin\alpha$$
$$(64.13 + 375\sin\alpha)^2 = 390^2 - (375\cos\alpha)^2$$
$$4112.65 + 140625 + 48097.5\sin\alpha = 152100$$
$$\sin\alpha = 0.15307$$
$$\alpha = 8.8°$$

Example 14.6

For a pair of involute spur gears, $m = 10$ mm, $\alpha = 20°$, $z_1 = 20$, $z_2 = 40$, $n_1 = 60$ rpm. The addendum on each gear is such that the path of approach and the path of recess on each side is 50% of the maximum possible length. Determine the addendum for the pinion and the gear and the length of arc of contact.

■ **Solution**

$r_1 = mz_1/2 = 10 \times 20/2 = 100$ mm, $r_2 = mz_2/2 = 10 \times 40/2 = 200$ mm

Let the pinion be the driver.

Maximum possible length of approach $= r_1 \sin\alpha = 100 \sin 20° = 34.2$ mm

$$\text{Actual length of approach} = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$
$$= \left[r_{a2}^2 - (200 \cos 20°)^2\right]^{0.5} - 200 \sin 20°$$
$$= \left[r_{a2}^2 - 35321\right]^{0.5} - 68.4 = 0.5 \times 34.2 = 17.1$$

or
$$r_{a2}^2 - 35321 = (85.5)^2$$
$$r_{a2} = 206.5 \text{ mm}$$
$$h_{a2} = 206.5 - 200 = 6.05 \text{ mm}$$

$$\text{Maximum possible length of recess} = r_2 \sin\alpha$$
$$= 200 \sin 20^{\circ} = 68.4 \text{ mm}$$
$$\text{Actual length of recess} = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} - r_1 \sin\alpha$$
$$= \left[r_{a1}^2 - (100 \cos 20°)^2\right]^{0.5} - 100 \sin 20°$$
$$= \left[r_{a1}^2 - 8830\right]^{0.5} - 34.2 = 0.5 \times 68.4 = 34.2$$

or
$$r_{a1}^2 - 8830 = (68.4)^2$$
$$r_{a1} = 116.2 \text{ mm}$$
$$h_{a1} = 116.2 - 100 = 16.2 \text{ mm}$$
$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\alpha} = 0.5\,(r_1 + r_2)\frac{\sin\alpha}{\cos\alpha}$$
$$= 0.5\left(100 + 200\right)\tan 20°$$
$$= 54.6 \text{ mm}$$

Example 14.7

Two involute gear wheels having module 3 mm and pressure angle 20° mesh externally to give a velocity ratio of 3. The pinion rotates at 75 rpm and addendum is equal to one module. Determine (a) the number of teeth on each wheel so that interference is just avoided, (b) the length of path and arc of contact, (c) the number of pairs of teeth in contact, and (d) the maximum velocity of sliding between the teeth.

■ **Solution**

Given: $m = 3$ mm, $\alpha = 20°$, $i = 3$, $n_1 = 75$ rpm

$$r_{a1} = r_1 + h_{a1} = r_1 + 3,\; r_{a2} = r_2 + h_{a2} = 3r_1 + 3 \quad [\therefore \quad r_2 = 3r_1]$$
$$r_{b1} = r_1 \cos 20° = 0.9397 r_1,\; r_{b2} = 3r_1 \cos 20° = 2.819 r_1$$

(a) Let the pinion be the driver.

$$L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$

$$(L_a)_{max} = r_1 \sin\alpha$$

To avoid interference,

$$L_a = (L_a)_{max}$$

$$\left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha = r_1 \sin\alpha$$

$$r_{a2}^2 - r_{b2}^2 = (r_1 - r_2)^2 \sin^2\alpha$$

$$(3r_1 + 3)^2 - (2.819)^2 r_1^2 = (4r_1)^2 \sin^2\alpha$$

or

$$0.8186r_1^2 - 18r_1 - 9 = 0$$

$$r_1 = 22.48 \text{ mm}$$

$$r_2 = 67.44 \text{ mm}$$

$$z_1 = 2r_1/m = 2 \times 22.48/3 = 14.98 \cong 15$$

so that $r_1 = 22.5$ mm, and $z_2 = 45$

(b)

$$r_{a1} = 22.5 + 3 = 25.5 \text{ mm}, r_{a2} = 67.5 + 3 = 70.5 \text{ mm}$$

$$r_{b1} = 22.5 \cos 20° = 21.143 \text{ mm}, r_{b2} = 67.5 \cos 20° = 63.429 \text{ mm}$$

$$L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - (r_1 - r_2)\sin\alpha$$

$$= \left[(25.5)^2 - (21.143)^2\right]^{0.5} + \left[(70.5)^2 - (63.429)^2\right]^{0.5}$$

$$-(22.5 + 67.5)\sin 20°$$

$$= 14.247 \text{ mm}$$

$$p = \pi m = \pi \times 3 = 11.425 \text{ mm}$$

Length of arc of contact,

$$L_c = \frac{L_p}{\cos\alpha} = \frac{14.247}{\cos 20°} = 15.16 \text{ mm}$$

(c) Number of pairs of teeth in contact $= \dfrac{L_c}{p} = \dfrac{15.16}{11.425} = 1.6$

(d) Maximum velocity of sliding $= (\omega_1 + \omega_2)r_1 \sin\alpha$

$$= \left(\frac{2\pi}{60}\right)(75 + 25) \times 22.5 \times \sin 20° = 80.58 \text{ mm/s}$$

14.12 INTERFERENCE AND UNDERCUTTING IN INVOLUTE GEAR TEETH

An involute starts at the base circle and is generated outwards. It is therefore impossible to have an involute inside the base circle. The line of action is tangent to the two base circles of a pair of gears in mesh, and these two points represent the extreme limits of the length of action. These

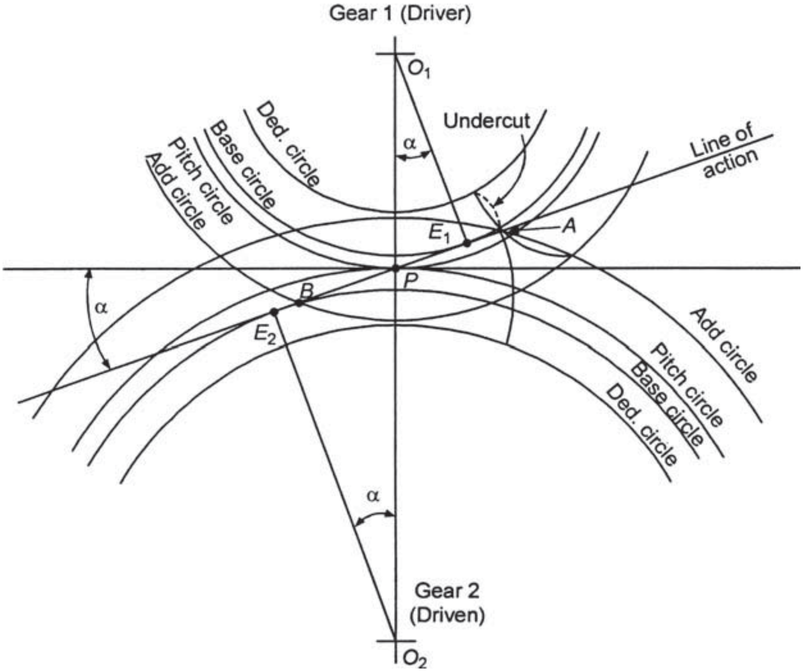

Fig.14.12 Interference in gears

two points are called *interference points*. If the teeth are of such proportion that the beginning of contact occurs before the interference point is met, then the involute portion of the driven gear will mate with a non-involute portion of the driving gear, and *involute interference* is said to occur. This condition is shown in Fig.14.12; E_1 and E_2 show the interference points that should limit the length of action. A shows the beginning of contact, and B shows the end of contact. It can be seen that the beginning of contact occurs before the interference point E_1 is met; therefore, interference is present. The tip of the driven tooth will gauge out or undercut the flank of the driving tooth as shown by the dotted line.

There are several ways of eliminating interference. Interference can be avoided by undercutting, making stub teeth, increasing the pressure angle, and cutting the gears with long and short addendum gear teeth. The method of undercutting is to limit addendum of the driven gear so that it passes through the interference point E_1, thus giving a new beginning of contact. Interference and the resulting undercutting not only weaken the pinion tooth but may also remove and small portion of the involute adjacent to the base circle, which may cause a serious reduction in the length of action.

Fig.14.13 shows a rack and a pinion in mesh. The point of tangency of the line of action and the base circle of the pinion is labelled as the interference point E, which fixes the maximum addendum for the rack. The contact begins as A, and undercutting will occur as shown by the dotted line. If the addendum of the rack extends only to the line that passes through the interference point, E, then the interference point becomes the beginning of contact, and interference is eliminated. If the number of teeth on the pinion is such that it will mesh without interference, it will mesh without interference with any other gear having the same or a larger number of teeth.

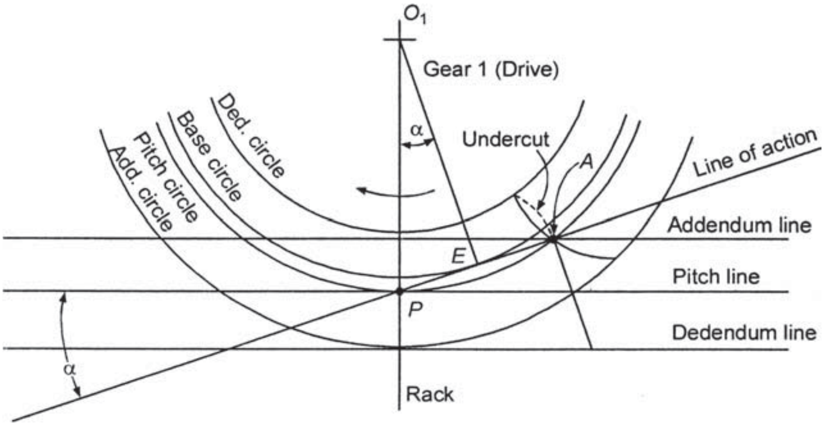

Fig.14.13 Interference in rack and pinion

14.13 MINIMUM NUMBER OF TEETH

14.13.1 Gear Wheel

For a minimum number of teeth to avoid interference, the common tangent to the base circles cuts the addendum circles at A and B, as shown in Fig.14.14.

Let speed ratio, $i = z_2/z_1$

Addendum of pinion, $h_{a1} = a_p m$

Addendum of gear wheel, $h_{a2} = a_\omega m$

where a_p and a_w are the constants by which the module must be multiplied to get the addendum of pinion and gear wheel respectively.

From ΔAO_2P, we have

$$O_2A^2 = O_2P^2 + AP^2 - 2 \cdot O_2P \cdot AP \cdot \cos \angle O_2PA$$

$$= \left(\frac{mz_2}{2}\right)^2 + (O_1P \sin \alpha)^2 - 2 \cdot \frac{mz_2}{2} \cdot O_1P \sin \alpha \cdot \cos\ (90° + \alpha)$$

$$= \left(\frac{mz_2}{2}\right)^2 + \left(\frac{mz_1}{2}\right)^2 \sin^2 \alpha + 2 \cdot \left(\frac{mz_2}{2}\right) \cdot \left(\frac{mz_1}{2}\right) \sin^2 \alpha$$

$$= \left(\frac{mz_2}{2}\right)^2 \left[1 + \left[\frac{1}{i^2}\right] \sin^2 \alpha + \left[\frac{2}{i}\right] \sin^2 \alpha\right]$$

$$O_2A = \left(\frac{mz_2}{2}\right) \left[1 + \left[\frac{1}{i^2}\right] \sin^2 \alpha + \left[\frac{2}{i}\right] \sin^2 \alpha\right]^{0.5} \tag{1}$$

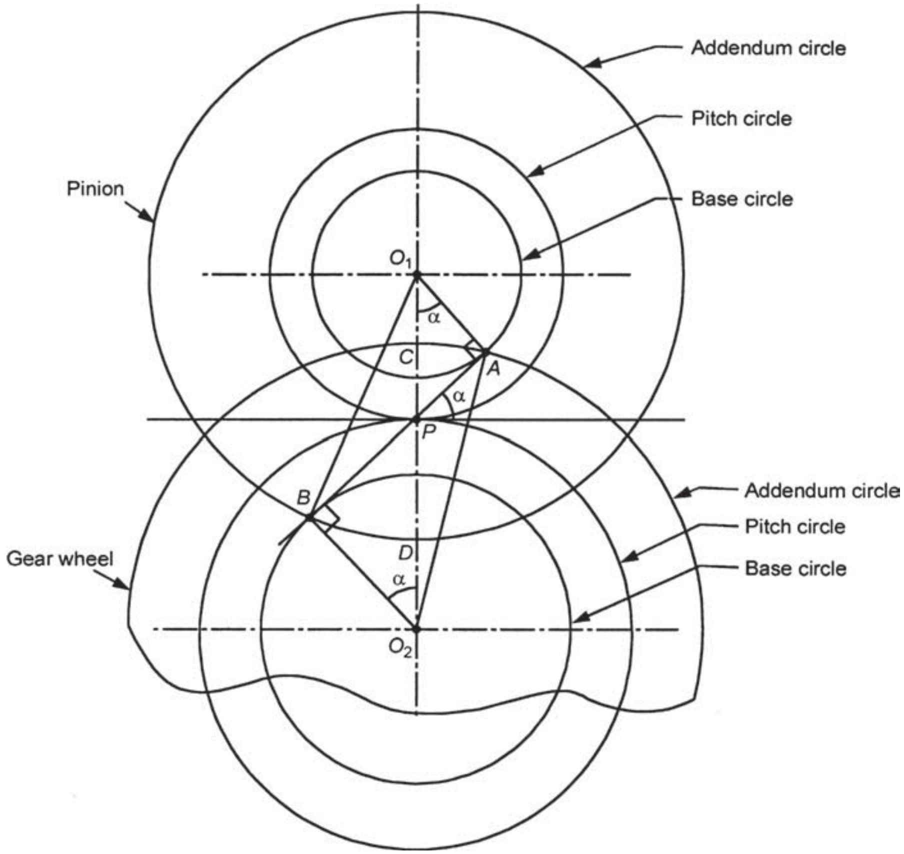

Fig.14.14 Calculating minimum number of teeth on pinion and gear wheel

Also
$$O_2A = O_2P + PC$$
$$= \frac{mz_2}{2} + h_{a2}$$
$$= \frac{mz_2}{2} + a_w m \tag{2}$$

From Eqs. (1) and (2), we get

$$\frac{mz_2}{2} + a_w\ m = \left(\frac{mz_2}{2}\right)\left[1 + \left[\frac{1}{i^2}\right]\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5}$$

or
$$a_w = \left(\frac{z_2}{2}\right)\left[\left\{1 + \left[\frac{1}{i^2}\right]\sin^2\alpha + \frac{2}{i}\sin^2\alpha\right\}^{0.5} - 1\right]$$

or

$$z_2 = \frac{2a_w}{\left[\left\{1+\left(\frac{1}{i^2}\right)\sin^2\alpha+\left(\frac{2}{i}\right)\sin^2\alpha\right\}^{0.5}-1\right]} \tag{14.23}$$

or

$$z_2 = \frac{\left[z_1^2 \sin^2\alpha - 4a^2{}_w\right]}{\left[4a_w - 2z_1 \sin^2\alpha\right]} \tag{14.24}$$

For

$$a_w = 1,\ z_2 = \frac{\left[z^2{}_1 \sin^2\alpha - 4\right]}{\left[4 - 2z_1 \sin^2\alpha\right]} \tag{14.25}$$

For

$$i = 1,\ a_w = a_p \text{ and}$$

$$z_2 = \frac{2a_w}{\left[\left\{1+3\sin^2\alpha\right\}^{0.5}-1\right]} \tag{14.26}$$

14.13.2 Pinion

From ΔBO_1P, we have

$$\begin{aligned}
O_1B^2 &= O_1P^2 + BP^2 - 2\cdot O_1P\cdot BP\cdot\cos\angle O_1PB \\
&= \left(\frac{mz_1}{2}\right)^2 + (O_2P\sin\alpha)^2 - 2\cdot\frac{mz_1}{2}\cdot O_2P\sin\alpha\cdot\cos(90^\circ+\alpha) \\
&= \left(\frac{mz_1}{2}\right)^2 + \left(\frac{mz_2}{2}\right)^2\sin^2\alpha + 2\cdot\left(\frac{mz_1}{2}\right)\cdot\left(\frac{mz_2}{2}\right)\sin^2\alpha \\
&= \left(\frac{mz_1}{2}\right)^2\left[1+i^2\sin^2\alpha+2i\sin^2\alpha\right] \\
O_1B &= \left(\frac{mz_1}{2}\right)\left[1+i^2\sin^2\alpha+2i\sin^2\alpha\right]^{0.5}
\end{aligned} \tag{1}$$

Also

$$\begin{aligned}
O_1B &= O_1P + PD \\
&= \frac{mz_1}{2} + h_{a1} \\
&= \frac{mz_1}{2} + a_pm
\end{aligned} \tag{2}$$

From Eqs. (1) and (2), we get

$$\frac{mz_1}{2} + a_pm = \left(\frac{mz_1}{2}\right)\left[1+i^2\sin^2\alpha+2i\sin^2\alpha\right]^{0.5}$$

or
$$a_p = \left(\frac{z_1}{2}\right)\left[\left\{1 + i^2 \sin^2 \alpha + 2i \sin^2 a\right\}^{0.5} - 1\right]$$

or
$$z_1 = \frac{2a_p}{\left[\left\{1 + i^2 \sin^2 \alpha + 2i \sin^2 \alpha\right\}^{0.5} - 1\right]} \tag{14.27}$$

or
$$z_1 = \frac{\left[z_2^2 \sin^2 \alpha - 4a_p^2\right]}{\left[4a_p - 2z_2 \sin^2\alpha\right]} \tag{14.28}$$

For
$$a_p = 1,\ z_1 = \frac{\left[z_2^2 \sin^2 \alpha - 4\right]}{\left[4 - 2z_2 \sin^2 \alpha\right]} \tag{14.29}$$

For
$$\alpha = 14.5°$$
$$(z_1)_{\min} = \left(\frac{z_2^2 - 63.8}{63.8 - 2z_2}\right) \tag{14.30}$$

For
$$\alpha = 20°$$
$$(z_1)_{\min} = \left(\frac{z^2{}_2 - 34.2}{34.2 - 2z_2}\right) \tag{14.31}$$

14.13.3 Rack and Pinion

A rack is a gear of infinite pitch radius. Thus its pitch circle is a straight line, called the *pitch line*. The line of action is tangent to the base circle at infinity; hence the involute profile of the rack is straight line and is perpendicular to the line of action. For the rack and pinion shown in Fig.14.15, let

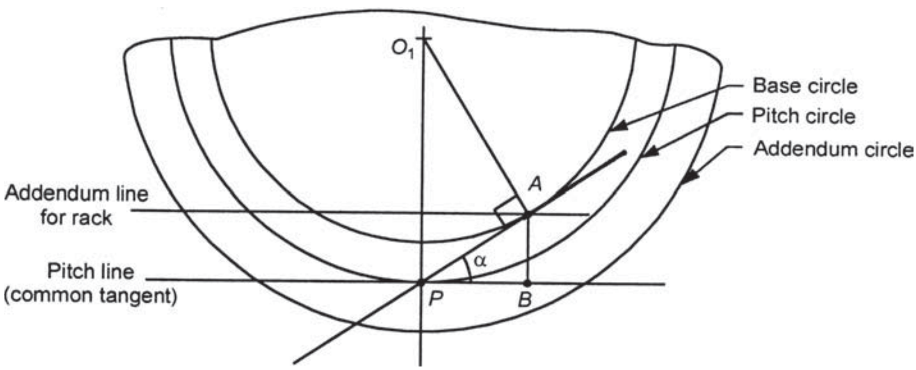

Fig.14.15 Minimum number of teeth on rack and pinion

Addendum of rack, $h_r = a_r m$

$$h_r = AB = AP \sin \alpha$$
$$= O_1P \sin a \sin \alpha$$
$$= O_1P \sin a \sin \alpha$$
$$= r_1 \sin^2 \alpha$$

or
$$a_r m = \left(\frac{mz_1}{2}\right) \sin^2 \alpha$$
$$a_r = \left(\frac{z_1}{2}\right) \sin^2 \alpha$$
$$z_1 = \frac{2a_r}{\sin^2 \alpha} \tag{14.32}$$

For $a_r = 1$, the minimum number of teeth on the pinion are given in Table 14.2.

Table 14.2 Minimum Number of Teeth on the Pinion for a Rack

α	14.5°	20°	20° stub	25°
$(z_1)_{min}$	32	18	14	12

Example 14.8

Determine the minimum number of teeth on the 20° pinion in order to avoid interference with a gear to give a gear ratio of 3:1. The addendum on wheel is equal to one module.

■ Solution

$$z_2 = 2a_w/[\{1 + \left(1/i^2\right)\sin^2 \alpha + \left(2/i\right)\sin^2 \alpha\}^{0.5} - 1]$$
$$z_1 = 2a_w/\text{I} \ \ [\{1 + \left(1/i^2\right)\sin^2 \alpha + \left(2/i\right)\sin^2 \alpha\}^{0.5} - 1]$$
$$= 2 \times 1/3 \left[\{1 + \left(1/9\right)\sin^2 20° + \left(2/3\right)\sin^2 20°\}^{0.5} - 1\right]$$
$$= 14.98 \approx 15$$

Example 14.9

A pinion of 20° involute teeth and 120 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6 mm. Determine the least pressure angle which can be used to avoid interference. With the pressure angle find the contact ratio.

■ Solution

Given: $\alpha = 20°$, $d_1 = 120$ mm, $h_a = 6$ mm

$$h_a = r_1 \sin^2 a$$
$$6 = 60 \sin^2 a$$

$$\alpha = 18.435°$$
$$r_{a1} = r_1 + h_a = 60 + 6 = 66 \text{ mm}$$
$$r_{b1} = r_1 \cos\alpha = 60 \cos 18.435° = 56.92 \text{ mm}$$

Length of path of contact, $L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5}$

$$= \left(66^2 - 56.92^2\right)^{0.5}$$
$$= 33.4 \text{ mm}$$

Base pitch, $p_b = p\cos\alpha = (\pi d_1/z_1)\cos\alpha$

$$= (\pi \times 120/20)\cos 18.435° = 17.88 \text{ mm}$$

Minimum number of teeth in contact $= L_p/p_b$

$$= 33.4/17.88 = 1.87 \approx 2$$

Example 14.10

Two 3 mm module, 20° pressure angle involute spur gears mesh externally to give a velocity ratio of 4. The addendum is 1.2 times the module. The pinion rotates at 150 rpm. Determine (a) the minimum number of teeth on each gear wheel to avoid interference, and (b) the number of pairs of teeth in contact.

■ Solution

Given: $m = 3$ mm, $a_w = 1.2$, $h_a = 1.2 \times 3 = 3.6$ mm, $\alpha = 20°$, $i = 4$, $n_1 = 150$ rpm

(a) $$z_2 = 2a_w\Big/\left[\left\{1 + \left(1/i^2\right)\sin^2\alpha + \left(2/\mathrm{i}\right)\sin^2\alpha\right\}^{0.5} - 1\right]$$

$$= \left(2 \times 1.2\right) / [\{1 + \left(1/6\right)\sin^2 20° + \left(2/4\right)\sin^2 20°^{0.5} - 1]$$

$$= 2.4\Big/\left[\left\{1 + 7.311 \times 10^{-3} + 0.05849\right\}^{0.5} - 1\right]$$

$$= 2.4/\left[1.03237 - 1\right]/$$

$$= 74.13 \cong 76 \text{ so that } z_2 \text{ is divisible by 4.}$$

$$z_1 = 76/4 = 19$$

(b) $$r_1 = mz_1/2 = 3 \times 19/2 = 28.5 \text{ mm}, \; r_2 = mz_2/2 = 3 \times 76/2 = 144 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 28.5 + 3.6 = 32.1 \text{ mm}, \; r_{a2} = r_2 + h_{a2} = 114 + 3.6 = 117.6 \text{ mm}$$

$$r_{b1} = r_1\cos\alpha = 28.5\cos 20° = 26.78 \text{ mm}, \; r_{b2} = r_2\cos\alpha = 114\cos 20° = 107.12 \text{ mm}$$

$$L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} - \left(r_1 + r_2\right)\sin\alpha$$

$$= \left[\left(32.1\right)^2 - \left(26.78\right)^2\right]^{0.5} + \left[\left(117.6\right)^2 - \left(107.12\right)^2\right]^{0.5} - \left(28.5 + 114\right)\sin 20°$$

$$= 17.49 \text{ mm}$$

Number of pairs of teeth in contact $= L_p / (\pi m \cos \alpha)$

$$= 17.49 / (\pi \times 3 \times \cos 20°)$$

$$= 1.975 \simeq 2$$

14.14 GEAR STANDARDIZATION

A set of gears is interchangeable when any two gears selected from the set will mesh with each other and satisfy the fundamental law of gearing. For interchangeability, all gears of the set must have the same circular pitch, module, diametral pitch, pressure angle, addendum, and dedendum; and tooth thickness must be one-half of the circular pitch. Standard tooth forms ensure readily availability of gears.

The standard pressure angles are: 14.5° and 20°. The 20° full-depth system has several advantages when compared with the 25° or 30° full-depth system. The lower pressure angle gives a higher contact ratio which results in quieter operation, reduced wear, reduced tooth load, and reduced bearing loads.

The larger pressure angle tooth forms results in broader teeth at the base and hence are stronger in bending. Also fewer teeth may be used on the pinion without undercutting the teeth. The proportions of standard tooth forms are given in Table 14.3.

Table 14.3 Standard Involute Tooth Forms

	14.5° full-depth	20° full-depth	20° stub
Addendum, h_a	m	m	0.800 m
Dedendum, h_f	1.157 m	1.250 m	m
Clearance, c	0.157 m	0.250 m	0.200 m
Fillet radius, r	0.209 m	0.300 m	0.304 m
Tooth thickness, t	1.5708 m	1.5708 m	1.5708 m

14.15 EFFECT OF CENTRE DISTANCE VARIATION ON VELOCITY RATIO

Consider a pair of teeth in contact at L, as shown in Fig.14.16. The angular velocity ratio is,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

Let the centre distance of rotation of gear 2 be shifted from O_2 to O_2'. As a result of this change, the contact point will shift to L'. Common normal at the point of contact L' is tangent to the base circle, because it is in contact between two involute curves, and they are generated from the base circle. Let the tangent to the base circle $M'N'$ intersect the line joining the centers of rotation O' and O_1 at P'.

Triangles O_1PN and O_2MP are similar. Also triangles $O_1N'P'$ and $O'_2M'P'$ are similar. Therefore,

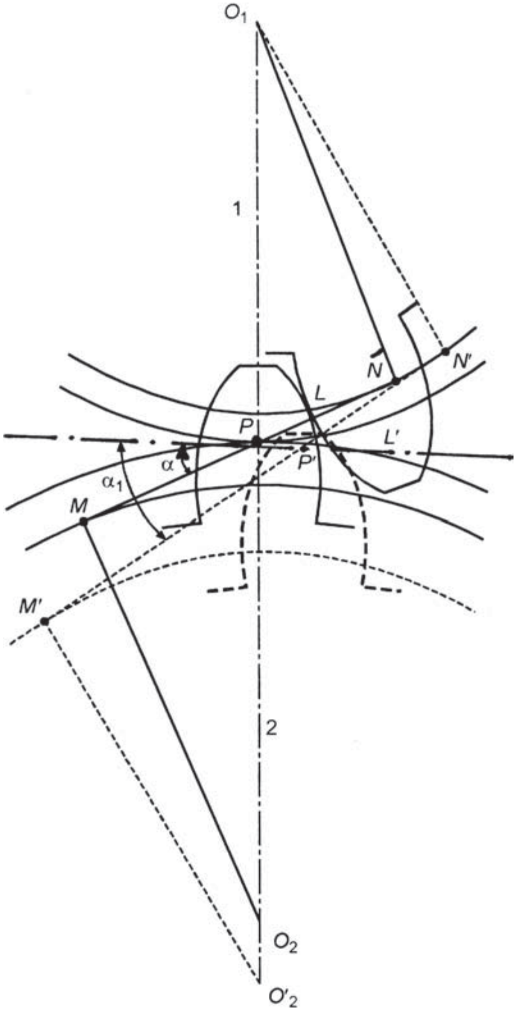

Fig.14.16 Effect of centre distance variation

$$\frac{MO_2'}{N'O_1} = \frac{O_2'P'}{O_1P'}$$

and
$$\frac{MO_2}{NO_1} = \frac{O_2P}{O_1P}$$

But
$$NO_1 = N'O \quad \text{and} \quad O_2M = O_2'M'$$

Therefore,
$$\frac{O_2P}{O_1P} = \frac{O_2'P'}{O_1P'} \tag{14.33}$$

Hence, *the variation in the centre distance, within limits, does not affect the angular velocity ratio, But the length of arc of contact is decreased, and the pressure angle is increased.*

14.16 DETERMINATION OF BACKLASH

Two standard gears in mesh are shown in Fig.14.17(a). The standard centre distance with zero backlash is:

$$c = \frac{m}{2}(z_1 + z_2)$$

The cutting pitch circles are known as standard pitch circles. Fig.14.17(b) shows the condition where the two gears have been pulled apart a distance ΔC to give a new centre distance C'. The line of action now crosses the line of centres at a new pitch point P'. The standard pitch radii r_1 and r_2 are now no longer tangent to each other. The pitch point P' divides the centre distance C' into segments which are inversely proportional to the angular velocity ratio. These segments become the radii r_1' and r_2' of new pitch circles that are tangent to each other at point P'. These circles are known as operating pitch circles.

$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{r'_2}{r'_1}$$

and

$$C' = r_1' + r_2'$$

to give

$$r_1' = \left[\frac{z_1}{z_1 + z_2}\right] C'$$

and

$$r_2' = \left[\frac{z_2}{z_1 + z_2}\right] C'$$

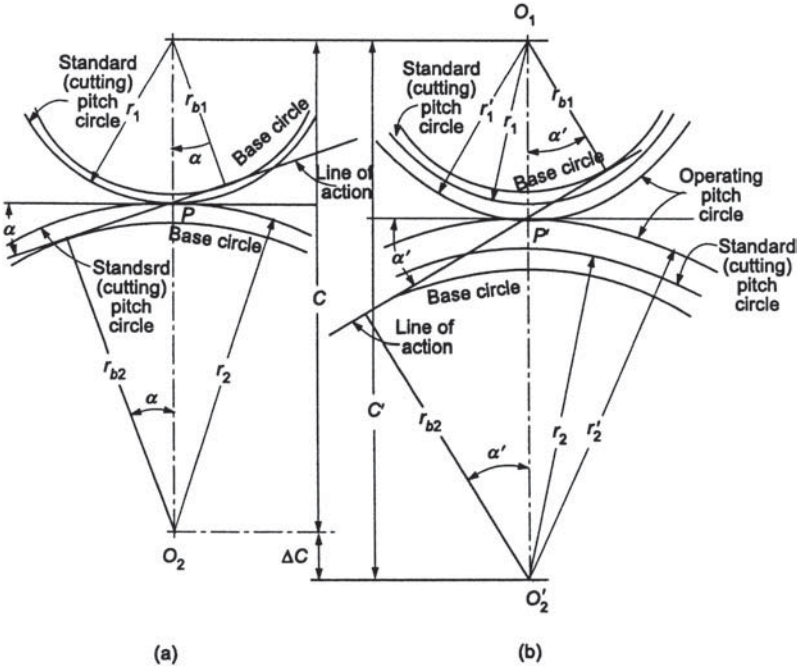

Fig.14.17 Determination of backlash

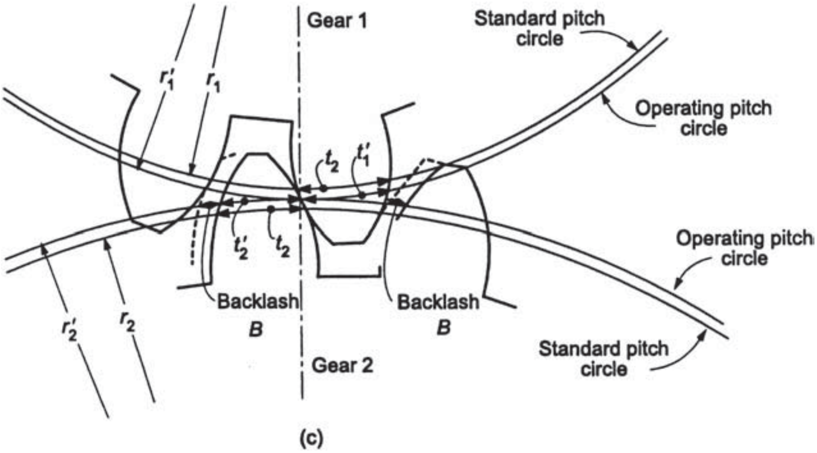

Fig.14.17 Determination of backlash (*Contd.*)

Let α' be the operating pressure angle. Now

$$C' = \frac{r_{b1} + r_{b2}}{\cos\alpha'} = \frac{(r_1 = r_2)\cos\alpha}{\cos\alpha'} = \frac{C\cos\alpha}{\cos\alpha'}$$

or
$$\cos\alpha' = \frac{C\cos\alpha}{C'} \tag{14.34}$$

$$\Delta C = C' - C$$

Also
$$= \frac{C\cos\alpha}{\cos\alpha'} - C$$

$$= C\left[\frac{\cos\alpha}{\cos\alpha'} - 1\right] \tag{14.35}$$

Now from Fig.14.17(c), (14.36)

$$t_1' + t_2' + B = \frac{2\pi r_1'}{z_1} = \frac{2\pi r_2'}{z_2} \tag{14.37}$$

where t' = tooth thickness on operating pitch circle
B = backlash
r' = radius of operating pitch circle
z = number of teeth

Now
$$t_1' = 2r_2'\left[\frac{t_1}{2r_1} + \text{inv}(\alpha) - \text{inv}(\alpha')\right]$$

$$= \frac{r_1' t_1}{r_1} - 2r_1'[\text{inv}(\alpha) - \text{inv}(\alpha')] \tag{14.38}$$

$$t_2' = 2r_2'\left[\frac{t_2}{2r_2} + \text{inv}(\alpha) - \text{inv}(\alpha)'\right] \tag{14.39}$$

$$= \frac{r_2' t_2}{r_2} - 2r_2'[\text{inv}(\alpha) - \text{inv}(\alpha)'] \tag{14.40}$$

where t = tooth thickness on standard pitch circle

$$= \frac{p}{2} = \frac{\pi m}{2}$$

r = radius of standard pitch circle

$$= \frac{mz}{2}$$

Also $$= \frac{r_1}{r_1'} = \frac{r_2}{r_2} = \frac{C}{C'} \tag{14.41}$$

and $$C' = r_1' + r_2' \tag{14.42}$$

Substituting (14.38) to (14.42) in Eq. (14.36), we get

$$B = \left(\frac{C'}{C}\right)\left[\pi m - (t_1 + t_2) + 2C\{\text{inv}(\alpha) - \text{inv}(\alpha')\}\right] \tag{14.43}$$

$$= 2C\left[\text{inv}(\alpha) - \text{inv}(\alpha')\right] \tag{14.44}$$

Example 14.11

A three-module, 20° pinion of 24 teeth drives a gear of 60 teeth. (a) Calculate the length of action and contact ratio, if the gears mesh with zero backlash. (b) If the centre distance is increased 0.5 mm, calculate the radii of the operating pitch circles, the operating pressure angle and the backlash produced.

■ **Solution** Given: $m = 3$ mm, $\alpha = 20°$, $z_1 = 24$, $z_2 = 60$

(a)

$$r_1 = \frac{z_1 m}{2} = 24 \times \frac{3}{2} = 36 \text{ mm}$$

$$r_2 = \frac{z_2 m}{2} = 60 \times \frac{3}{2} = 90 \text{ mm}$$

$$r_{b1} = r_1 \cos\alpha = 36 \times \cos 20° = 33.83 \text{ mm}$$

$$r_{b2} = r_2 \cos\alpha = 90 \times \cos 20° = 84.57 \text{ mm}$$

$$h_{a1} = h_{a2} = m = 3 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 36 = 3 = 39 \text{ mm}$$

$$r_{a2} = r_2 + h_{a2} = 90 + 3 = 93 \text{ mm}$$

$$\text{C} = r_1 + r_2 = 36 + 90 = 126 \text{ mm}$$

Length of path of contact, $$AB = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} + \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - (r_1 + r_2)\sin\alpha$$

$$= \left(39^2 - 33.83^2\right)^{0.5} + \left(93^2 - 84.57^2\right)^{0.5} - 126\sin 20°$$

$$= 14.997 \text{ mm}$$

Contact ratio, $$m_c = \frac{AB}{p_b}$$

Base Pitch, $$p_b = 2\pi \times \frac{33.83}{24} = 8.856 \text{ mm}$$

$$m_c = \frac{14.997}{8.856} = 1.693 \simeq 2$$

(b)
$$C' = C + \Delta C = 126 + 0.5 = 126.5 \text{ mm}$$
$$r_1' = \left[\frac{z_1}{z_1 + z_2}\right] C' = \left[\frac{24}{84}\right] 126.5 = 36.143 \text{ mm}$$
$$r_2' = C' - r_1' = 126.5 - 36.143 = 90.357 \text{ mm}$$
$$\cos\alpha' = \frac{C\cos\alpha'}{C'} = \frac{126\cos 20^\circ}{126.5} = 0.93598$$
$$\alpha' = 20.61^\circ$$

Backlash,
$$\begin{aligned} B &= 2C'\,[\text{inv}(\alpha') - \text{inv}(\alpha)] \\ &= 2 \times 126.5\,[\text{inv}(20.61^\circ) - \text{inv}(20^\circ)] \\ &= 253\,[0.016362 - 0.014904] \\ &= 0.3689 \text{ mm} \end{aligned}$$

14.17 INTERNAL SPUR GEARS

A pinion in mesh with an internal gear is shown in Fig.14.18. Internal gears have some advantages over the external gears. The most important advantage is the compactness of the drive. Other advantages are : greater length of contact, greater tooth strength, and lower relative sliding velocity between meshing teeth.

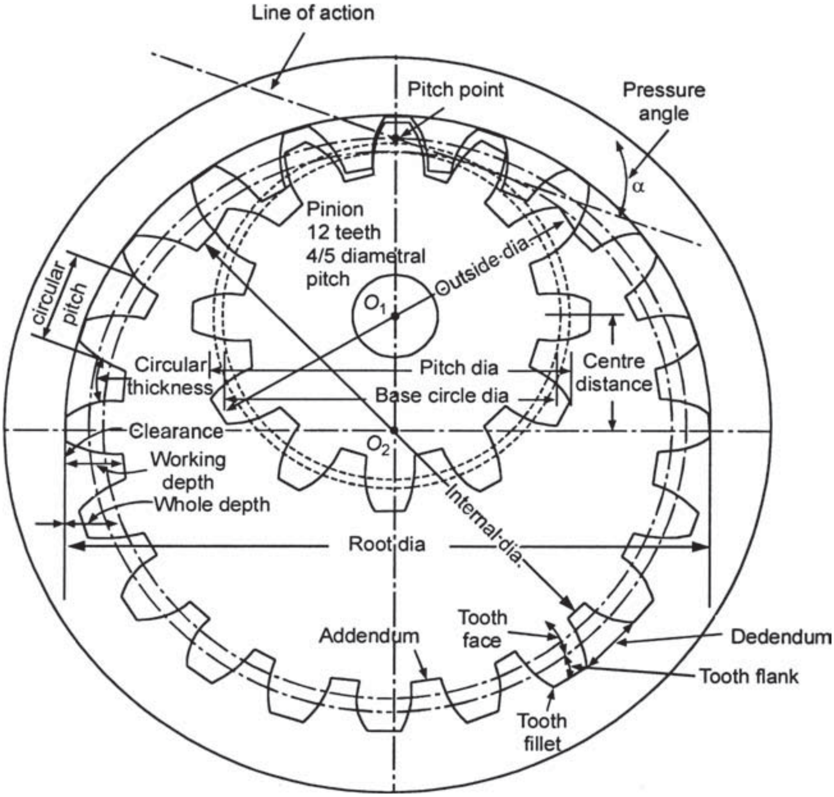

Fig.14.18 Internal spur gears

The tooth profile is concave in internal gears instead of convex as in external gears. Because of this a type of interference called *fouling* may occur in internal gears. Fouling occurs between inactive profiles as the teeth go in and out of the mesh and the there is not sufficient difference between the numbers of teeth on the internal gear and the pinion.

Example 14.12

Two equal spur gears of 48 teeth mesh together with pitch radii of 96 mm and addendum of 4 mm. If the pressure angle is 20°, calculate the length of action and the contact ratio.

Solution

Given: $z_1 = z_2 = 48,\ r = r_1 = r_2 = 96\text{ mm},\ a = a_w = a_p = 4\text{ mm},\ \alpha = 20°$

Length of path contact, $L_p = \left(r_{al}^2 - r_{bl}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - \left(r_1 + r_2\right)\sin\alpha$

$$r_{b1} = r_{b2} = r\cos a = 96\cos 20° = 90.21\text{ mm}$$

$$r_{a1} = r_{a2} = r + a = 96 + 4 = 100\text{ mm}$$

$$L_p = \left[(100)^2 - (90.21)^2\right]^{0.5} + \left[(100)^2 - (90.21)^2\right)^{0.5} - (96 + 96)\ \sin 20°$$

$$= 20.63\text{ mm}$$

Base pitch, $p_b = \dfrac{2\pi\ r_{b1}}{z_1} = 2\pi \times \dfrac{90.21}{48} = 11.81\text{ mm}$

Contact ratio, $CR = \dfrac{L_p}{p_b}$

$$= \frac{20.63}{11.81} = 1.747$$

Example 14.13

A pinion with a pitch radius of 40 mm drives a rack. The pressure angle is 20°. Calculate the maximum addendum possible for the rack without having involute interference on the pinion.

Solution

Given: $r_1 = 40\text{ mm},\ \alpha = 20°$

$$\left(h_r\right)_{\max} = r_i \sin^2\alpha = 40\sin^2 20° = 4.68\text{ mm}$$

Example 14.14

A 0.2-module, 20° pinion of 42 teeth drives a gear of 90 teeth. Calculate the contact ratio. Addendum for pinion and gear is equal to one module.

Solution

Given: $m = 0.2\text{ mm},\ a = 20°,\ z_1 = 42,\ z_2 = 90$

$$d_1 = mz_1 = 0.2 \times 42 = 8.4\text{ mm},\ d_2 = mz_2 = 0.2 \times 90 = 18\text{ mm}$$

$$r_{b1} = r_1\cos a = 4.2\cos 20° = 3.95\text{ mm},\ r_{b2} = r_2\cos a = 9\cos 20° = 8.46\text{ mm}$$

$$r_{a1} = r_1 + m = 4.2 + 0.2 = 4.4\text{ mm},\ r_{a2} = r_2 + m = 9 + 0.2 = 9.2\text{ mm}$$

Length of path of contact, $L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - \left(r_1 + r_2\right)\sin\alpha$

$$= \left[(4.4)^2 - (3.95)^2\right]^{0.5} + \left[(9.2)^2 - (8.46)^2\right]^{0.5} - (4.2 + 9)\sin 20°$$

$$= 1.9384 + 3.6150 - 4.5147 = 1.0387 \text{ mm}$$

Base pitch, $p_b = \dfrac{2\pi\, r_{b1}}{z_1} = 2\pi \times \dfrac{3.95}{42} = 0.5909$ mm

Contact ratio $= \dfrac{Lp}{p_b} = \dfrac{1.0387}{0.5909} = 1.758 \simeq 2$

Example 14.15

Determine the approximate number of teeth in a 20° involute spur gear so that the base circle diameter will be equal to the dedendum circle diameter.

Solution

Given: Dedendum, $h_f = 1.25$ m, $d_d = d - 2h_f = mz - 1.25$ m

Base circle diameter, $d_b = d \cos\alpha = m\,z\cos\alpha$

For $d_b = d_d$, $mz\cos\alpha = mz - 1.25$ m

$$z\cos\alpha = z - 1.25$$

$$z\cos 20° = z - 1.25$$

$$0.9397\,z = z - 1.25$$

$$z = 21$$

Example 14.16

A 4-module, 20° pinion with 30 teeth drives a rack. Calculate the length of action and the contact ratio.

Solution

Given: $m = 4$ mm, $\alpha = 20°$, $z_1 = 30$

For a rack, $L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} - r_1 \sin\alpha + \dfrac{h_a}{\sin\alpha}$

$$h_a = m = 4 \text{ mm}, d_1 = mz = 4 \times 30 = 120 \text{ mm}$$

$$r_{a1} = r_1 + h_a = 60 + 4 = 64 \text{ mm}$$

$$r_{b1} = r_1 \cos\alpha = 60\cos 20° = 56.38 \text{ mm}$$

$$L_p = \left[(64)^2 - (56.38)^2\right]^{0.5} - 60\sin 20° + \frac{4}{\sin 20°}$$

$$= 21.461 \text{ mm}$$

Base pitch $p_b = \dfrac{2\pi r_{b1}}{z_1} = 2\pi \times \dfrac{56.38}{30} = 11.8082$ mm

Contact ratio $= \dfrac{L_p}{p_b} = \dfrac{21.461}{11.8082} = 1.8174 \simeq 2$

Example 14.17

For a 20° pressure angle, calculate the minimum number of teeth in a pinion to mesh with a rack without involute interference. Also calculate the number of teeth in a pinion to mesh with a gear of equal size without involute interference, The addendum equals the module.

■ Solution

Given: $$z_1 = \frac{2a_r}{\sin^2 \alpha}$$

Here $$a_r = 1,\ z_1 = \frac{2}{\sin^2 20°} = 17$$

$$z_1 = \frac{2a_p}{\left[\left\{1 + i^2 \sin^2 \alpha + 2i \sin^2 \alpha\right\}^{0.5} - 1\right]}$$

Here $$i = 1,\ a_p = 1$$

$$z_1 = \frac{2}{\left[\left\{1 + \sin^2 20° + 2 \sin^2 20°\right\}^{0.5} - 1\right]}$$

$$= 12.32 \cong 13$$

Example 14.18

A pair of meshing spur gears has 22 and 38 teeth, a diametral pitch of 0.32, and a pinion running at 1800 rpm. Determine the following: (a) centre distance, (b) pitch diameter, (c) pitch line velocity, and (d) rpm of the gear.

■ Solution

Given: $$z_1 = 22, z_2 = 38, P = 0.32, n_1 = 1800 \text{ rpm}$$

$$m = \frac{1}{P} = \frac{1}{0.32} = 3.125 \text{ mm}$$

(a) Centre distance, $$C = \frac{m(z_1 + z_2)}{2} = \frac{3.125\ (22 + 38)}{2} = 93.75 \text{ mm}$$

(b) Pitch diameters, $$d_1 = m\, z_1 = 3.125 \times 22 = 68.75 \text{ mm},$$

$$d_2 = mz_2 = 3.125 \times 38 = 118.75 \text{ mm}$$

(c) Pitch line velocity, $$v = \frac{\pi d_1 n_1}{60} = \pi \times 0.06875 \times \frac{1800}{60} = 6.48 \text{ m/s}$$

(d) $$n_2 = \frac{n_1 z_1}{z_2} = 1800 \times \frac{22}{38} = 1042.1 \text{ rpm}$$

Example 14.19

A pair of spur gears has 16 and 18 teeth, a module of 13 mm, addendum of 13 mm, and pressure angle of 14.5°. Show that the gears have interference. Determine the amount by which the addendum must be reduced to eliminate the interference.

■ **Solution**

Given: $z_1 = 16, z_2 = 18,$ m = 13 mm, $h_a = 13$mm, $\alpha = 14.5°$

$$r_1 = \frac{mz_1}{2} = 13 \times \frac{16}{2} = 104 \text{ mm}, r_2 = 13 \times \frac{18}{2} = 117 \text{ mm}$$

$$r_{b1} = r_1 \cos \alpha = 104 \cos 14.5° = 100.7 \text{ mm}, r_{b2} = 117 \cos 14.5° = 113.3 \text{ mm}$$

$$r_{a1} = r_1 + h_a = 104 + 13 = 117 \text{ mm}, r_{a2} = 117 + 13 \text{ mm} = 130 \text{ mm}$$

Let pinion be the driver,

Length of approach, $$L_a = \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - r_2 \sin \alpha$$

$$= \left[(130)^2 - (113.3)^2\right]^{0.5} - 117 \sin 14.5°$$

$$= 63.74 - 211.3 = 34.44 \text{ mm}$$

$$(L_a)_{\max} = r_1 \sin \alpha = 104 \sin 14.5° = 26.04 \text{ mm.}$$

Since length of approach is more than the maximum length of approach, therefore interference will occur. To eliminate interference, we make $L_a = (L_a)_{\max}$

$$26.04 = \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - r_2 \sin a$$

$$= \left[r_{a2}^2 - (113.3)^2\right]^{0.5} - 117 \sin 14.5°$$

$$= \left[r_{a2}^2 - (113.3)^2\right]^{0.5} - 211.3$$

$$[r_{a2}^2 - (113.3)^2] = (26.04 + 211.3)^2$$

$$r_{a2} = 126.09 \text{ mm}$$

$$h_a' = r_{a2} - r_2 = 126.09 - 117 = 11.09 \text{ mm}$$

Decrease in addendum = $h_a' - h_a = 13 - 11.09 = 3.91$ mm

Example 14.20

An internal spur gear having 200 teeth and 20° pressure angle meshes with a pinion having 40 teeth and a module of 2.5 mm. Determine (a) the velocity ratio if the pinion is the driver, (b) the centre distance, and (c) If the centre distance is increased by 3 mm, find the resulting pressure angle.

■ **Solution**

Given: $z_1 = 40, z_2 = 200, m = 2.5$ mm

(a) $$d_1 = mz_1 = 2.5 \times 40 = 100 \text{ mm}, d_2 = 2.5 \times 200 = 500 \text{ mm}$$

$$i = \frac{z_2}{z_1} = \frac{200}{40} = 5$$

(b) $$C = \frac{d_2 - d_1}{2} = \frac{500 - 100}{2} = 200 \text{ mm}$$

(c) $C' = 200 + 3 = 203$ mm

$$\cos\alpha' = \frac{C\cos\alpha}{C'} = \frac{200\cos 20°}{203} = 0.9258$$

$$\alpha' = 22.2°$$

Example 14.21

Two spur gears of 24 teeth and 36 teeth of 8 mm module and 20° pressure angle are in mesh. Addendum of each gear is 8 mm. The teeth are of involute form and the pinion rotates at 450 rpm. Determine the velocity of sliding when the pinion is at a radius of 102 mm.

■ Solution

Given: $z_1 = 24$, $z_2 = 36$, $m = 8$ mm, $h_a = 8$ mm, $\alpha = 20°$, $n_1 = 450$ rpm, $r = 102$ mm

$$r_1 = \frac{mz_1}{2} = 8\times\frac{24}{2} = 96 \text{ mm}, \; r_2 = 8\times\frac{36}{2} = 144 \text{ mm}$$

$$r_{b1} = r_1\cos\alpha = 96\cos 20° = 90.21 \text{ mm}, \; r_{b2} = 144\cos 20°$$

$$= 135.31 \text{ mm}$$

$$r_{a1} = r_1 + h_a = 96 + 8 = 104 \text{ mm}, \; r_{a2} = 144 + 8 = 152 \text{ mm}$$

Let pinion be the driver,

Length of recess at 102 mm radius, $L_r = (r_{a1}^2 - r_{b1}^2)^{0.5} - r_1\sin\alpha$

$$= [(102)^2 - (90.21)^2]^{0.5} - 96\sin 20°$$

$$= 47.60 - 32.83 = 14.77 \text{ mm}$$

$$n_2 = \frac{n_1 z_1}{z_2} = 450\times\frac{24}{36} = 300 \text{ rpm}$$

$$\omega_1 = 2\pi\times\frac{450}{60} = 47.124 \text{ rad/s}$$

$$\omega_2 = 2\pi\times\frac{300}{60} = 31.416 \text{ rad/s}$$

Velocity of sliding at 102 mm radius $= (\omega_1 + \omega_2)\, L_r$

$$= (47.124 + 31.416)\times\frac{14.77}{1000} = 1.16 \text{ m/s}$$

Example 14.22

A pair of spur gears with involute teeth is to give a gear ratio of 3:1.The arc of approach is not to be less than the circular pitch and the pinion is the driver. The pressure angle is 20°. What is the least number of teeth than can be used on each gear?

■ **Solution**

Given: For pinion to be the driver, the maximum length of approach = $r_1 \sin \alpha$

Maximum length of arch of approach $= \dfrac{r_1 \sin \alpha}{\cos \alpha} = \pi m$

$$\left(\frac{m z_1}{2}\right)\tan \alpha = \pi m$$

$$z_1 = \frac{2\pi}{\tan 20°} = 17.26 \cong 18$$

$$z_2 = 3 \times 18 = 54$$

Example 14.23

A pinion with 24 involute teeth of 150 mm pitch circle diameter drives a rack. The addendum of the pinion is 6 mm. Find the least pressure angle which can be used if undercutting of the teeth is to be avoided. Using this pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at one time.

■ **Solution**

Given: $z_1 = 24$, $d_1 = 150$ mm, $h = 6$ mm

$$h_{r1} = r_1 \sin^2 \alpha$$

$$6 = 75 \sin^2 \alpha$$

$$\sin^2 \alpha = 0.08$$

$$\sin \alpha = 0.28284$$

$$\alpha = 16.43°$$

$$r_{b1} = r_1 \cos \alpha = 75 \cos 16.43° = 71.94 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 75 + 6 = 81 \text{ mm}, \; r_{a2} = 144 + 8 = 152 \text{ mm}$$

Length of recess, $L_r = L_p = (r_{a1}^2 - r_{b1}^2)^{0.5} - r_1 \sin \alpha$

$$= [(81)^2 - (71.94)^2]^{0.5} - 7.5 \sin\ 16.43°$$

$$= 16 \text{ mm}$$

Number of teeth in contact $= \dfrac{L_p}{p_b}$

$$\frac{16 \times 24}{p \times 150 \times \cos\ 16.43°} = 0.85 \cong 1$$

Example 14.24

A pair of 20° pressure angle gears in mesh has the following data:

Speed of pinion = 400 rpm

Number of teeth on pinion = 24

Number of teeth on gear = 28

Module = 10 mm

Determine the addendum of the gears if the path of approach and recess is half the maximum value. Determine also the arc of contact and the maximum velocity of sliding between the mating surfaces.

■ Solution

Given: $z_1 = 24, z_2 = 28, m = 10 \text{ mm}, h_a = ?, \alpha = 20°, n_1 = 400 \text{ rpm}$

$$r_1 = \frac{mz_1}{2} = 10 \times \frac{24}{2} = 120 \text{ mm}, r_2 = 10 \times \frac{28}{2} = 140 \text{ mm}$$

$$r_{b1} = r_1 \cos\alpha = 120 \cos 20° = 112.76 \text{ mm}, r_{b2} = 140 \cos 20° = 131.56 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 120 + h_{a1}, r_{a2} = 140 + h_{a2}$$

Let pinion be the driver,

Length of approach, $L_a = 0.5\,(L_a)_{\max}$

$$(r_{a2}^2 - r_{b2}^2)^{0.5} - r_2 \sin\alpha = 0.5 r_1 \sin\alpha$$

$$[r_{a2}^2 - (131.56)^2]^{0.5} - 140 \sin 20° = 0.5 \times 120 \times \sin 20°$$

$$r_{a2}^2 - (131.56)^2 = (200)^2 \sin^2 20°$$

$$r_{a2}^2 = 21987, r_{a2} = 148.3 \text{ mm}$$

$$h_{a2} = 148.3 - 140 = 8.3 \text{ mm}$$

Length of recess, $L_r = 0.5 \times (L_r)_{\max}$

$$(r_{a1}^2 - r_{b1}^2)^{0.5} - r_1 \sin\alpha = 0.5 r_2 \sin\alpha$$

$$[r_{a1}^2 - (112.76)^2]^{0.5} - 120 \sin 20° = 0.5 \times 140 \times \sin 20°$$

$$r_{a1}^2 - (112.76)^2 = (190)^2 \sin^2 20°$$

$$r_{a1}^2 = 16938, r_{a1} = 130.14 \text{ mm}$$

$$h_{a1} = 130.14 - 120 = 10.14 \text{ mm}$$

Arc of contact $= \dfrac{L_p}{\cos\alpha} = 0.5(r_1 + r_2)\tan\alpha$

$$= 0.5(120 + 140)\tan 20° = 47.3 \text{ mm}$$

Path of recess,

$$L_r = (r_{a1}^2 - r_{b1}^2)^{0.5} - r_1 \sin\alpha$$

$$= [(130.14)^2 - (112.76)^2]^{0.5} - 120 \sin 20°$$

$$= 23.93 \text{ mm}$$

$$\omega_1 = 2\pi \times \frac{400}{60} = 41.88 \text{ rad/s}$$

$$\omega_2 = 41.88 \times \frac{24}{28} = 35.9 \text{ rad/s}$$

Maximum velocity of sliding $= (\omega_1 + \omega_2) \times L_r$

$$= (41.88 + 35.9) \times \frac{23.91}{1000} = 1.86 \text{ m/s.}$$

Example 14.25

Two gears in mesh have 10 teeth and 40 teeth, respectively. They are full-depth teeth and pressure angle is 20°. The module is 8.5 mm. Determine the (a) reduction in addendum of the gear to avoid interference, and (b) contact ratio.

■ Solution

Given: $z_1 = 10, z_2 = 40, m = 8.5$ mm, $\alpha = 20°$

$$r_1 = mz_1/2 = 8.5 \times \frac{10}{2} = 42.5 \text{ mm}, r_2 = \frac{mz_2}{2} = 8.5 \times \frac{40}{2} = 170 \text{ mm}$$

$$r_{b1} = r_1 \cos a = 42.5 \cos 20° = 311.94 \text{ mm}, r_{b2} = r_2 \cos a = 170 \cos 20° = 1511.75 \text{ mm}$$

$$r_{a2} = r_2 + h_{a2} = 170 + h_{a2}$$

Let pinion be the driver,

Length of approach, $L_a = (L_a)_{\max}$

$$(r_{a2}^2 - r_{b2}^2)^{0.5} - r_2 \sin a = r_1 \sin a$$

$$[r_{a2}^2 - (1511.75)^2]^{0.5} - 170 \sin 20° = 42.5 \times \sin 20°$$

$$r_{a2}^2 (1511.75)^2 = (212.5)^2 \sin^2 20°$$

$$r_{a2}^2 = 30802, r_{a2} = 175.5 \text{ mm}$$

$$h_{a2} = 175.5 - 170 = 5.5 \text{ mm}$$

Reduction in addendum = 8.5 – 5.5 = 3 mm

(a) $r_{a1} = 42.5 + 8.5 = 51 \text{ mm}, r_{a2} = 170 + 8.5 = 178.5 \text{ mm}$

$$L_p = (r_{a1}^2 - r_{b1}^2)^{0.5} + (r_{a2}^2 - r_{b2}^2)^{0.5} - (r_1 + r_2) \sin a$$

$$= [(51)^2 - (1511.75)r^2]^{0.5} + [(51)^2 - (311.94)^2)]^{0.5} - (42.5 + 170) \sin 20$$

$$= 38.67 \text{ mm}$$

$$p = 2\pi \frac{r_1}{z_1} = 2\pi \times \frac{42.5}{10} = 26.7 \text{ mm}$$

$$\text{Contact ratio} = \frac{L_p}{p \cos a} = \frac{38.67}{26.7 \cos 20°} = 1.54 \cong 2$$

Example 14.26

A pinion with 24 involute teeth of 150 mm of pitch circle diameter drives a rack. The addendum of the pinion and rack is 6 mm. Find the least pressure angle which can be used if undercutting of the teeth is to be avoided. Using this pressure angle, find the length of arc of contact and the minimum number of teeth in contact at one time?

■ Solution

Given: $z_p = 24, d_p = 150$ mm, $h_p = h_r = 6$ mm

$$h_r = r_p \sin^2 \alpha$$
$$6 = 75 \sin^2 \alpha$$
$$\sin^2 \alpha = \frac{6}{75} = 0.08$$
$$\sin \alpha = 0.28284$$

Least pressure angle, $\alpha = 16.43°$

Addendum radius of pinion, $r_{ap} = r_p + h_p = 75 + 6 = 81$ mm

Base circle radius of pinion, $r_{bp} = r_p \cos \alpha = 75 \cos 16.43° = 71.937$ mm

Maximum length of path of contact, $L_p = (r_{ap}^2 - r_{bp}^2)^{1/2} = \sqrt{81^2 - (71.937)^2}$

$= 37.23$ mm

Maximum length of arc of contact, $L_c = \dfrac{L_p}{\cos \alpha} = \dfrac{37.23}{\cos 16.43°} = 38.814$ mm

Minimum number of teeth in contact, $\dfrac{\pi d_p}{z_p} = \dfrac{\pi \times 150}{24} = 19.635$

Number of pairs of teeth is contact $= \dfrac{L_c}{19.635} = \dfrac{38.814}{19.635} = 1.976 \simeq 2$

Example 14.27

A pinion of 20 involute teeth and 120 mm pitch circle diameter drives a rack. The addendum of both the pinion and rack is 6.00 mm. What is the least pressure angle which can be used to avoid interference? With this pressure angle, find the length of the arc of contact and minimum number of teeth in contact at a time.

■ Solution

Given: $z_p = 20$, $d_p = 120$ mm, $h_p = h_r = 6$ mm,

$$h_r = r_p \sin^2 \alpha$$
$$\sin^2 \alpha = \frac{6}{60} = 0.1$$
$$\sin \alpha = 0.31623$$
$$\alpha = 18.435°$$
$$r_{ap} = r_p + h_p = 60 + 6 = 66 \text{ mm}$$
$$r_{bp} = r_p \cos \alpha = 60 \cos 18.435° = 56.921 \text{ mm}$$
$$(L_p)_{\max} = \sqrt{r_{ap}^2 - r_{ap}^2} = \sqrt{66^2 - (56.921)^2} = 33.4 \text{ mm}$$

Base pitch, $p_b = \dfrac{\pi d_p}{z_p} \cos \alpha = \dfrac{\pi \times 120}{20} \times \cos 18.435° = 17.822$ mm

Minimum number of teeth in contact $= \dfrac{L_p}{p_b} = \dfrac{33.4}{17.882} = 1.868 \simeq 2$

Example 14.28

A pair of involute spur gears with 16° pressure angle and pitch in module 6 mm is in mesh. The number of teeth on pinion is 16 and its speed is 260 rpm. When the gear ratio is 1.8, find in order that the interference is just avoided, (i) the addenda on pinion and the gear wheel, (ii) the length of path of contact (iii) the maximum velocity sliding of teeth on either side of the pitch point.

[PTU, Dec, 2007

■ Solution

Given:

$$\alpha = 16°, m = 6 \text{ mm}, z = 16, \text{N} = 260 \text{ rpm},$$
$$i = 1.8$$
$$z_2 = 16 \times 1.8 = 28.8 \simeq 29$$
$$d_1 = mz_1 = 6 \times 16 = 96 \text{ mm}, r_1 = 48 \text{ mm}$$
$$d_2 = mz_2 = 6 \times 29 = 174 \text{ mm}, r_2 = 87 \text{ mm}$$

(i)

$$r_{a1} = r_1 + h_{a1} = 48 + h_{a1}$$
$$r_{b1} = r_1 \cos\alpha = 48 \cos 16° = 46.14 \text{ mm}$$
$$r_{a2} = r_2 + h_{a2} = 87 + h_{a2}$$
$$r_{b2} = r_2 \cos\alpha = 87 \cos 16° = 83.63 \text{ mm}$$

Length of path of approach, $L_a = \sqrt{r_{a2}^2 - r_{b2}^2} - r_2 \sin\alpha$

Length of path of recess, $L_r = \sqrt{r_{a1}^2 - r_{b1}^2} - r_1 \sin\alpha$

Maximum length of approach to avoid interference, $(L_a)_{\max} = r_1 \sin\alpha$

Maximum length of recess to avoid interference, $(L_r)_{\max} = r_2 \sin\alpha$

$$\therefore \quad \sqrt{(48 + h_{a1})^2 - (46.14)^2} - 48 \sin 16° = 87 \sin 16°$$
$$(48 + h_{a1})^2 - (46.14)^2 = [(87 + 48) \sin 16°]^2$$
$$= 1384.66$$
$$(48 + h_{a1})^2 = 2513.56$$
$$48 + h_{a1} = 59.27$$
$$h_{a1} = 11.27 \text{ mm}$$
$$\sqrt{(87 + h_{a2})^2 - (83.63)^2} - 87 \sin 16° = 48 \sin 16°$$
$$(87 + h_{a2})^2 - (83.63)^2 = 1384.66$$
$$h_{a2} = 4.53 \text{ mm}$$

(ii) Length of path of contact, $L_p = (r_1 + r_2)\sin\alpha$

$$= (48 + 87)\sin 16° = 37.21 \text{ mm}$$

(iii)

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 260}{60} = 27.227 \text{ rad/s}$$

$$\omega_2 = \frac{\omega_1}{1.8} = 15.126 \text{ rad/s}$$

Path of recess, $L_r = r_2 \sin\alpha = 87 \sin 16° = 23.98$ mm

Path of approach, $L_a = r_1 \sin\alpha = 48 \sin 16° = 13.23$ mm

Maximum velocity of sliding during approach $= (\omega_1 + \omega_2)L_a$

$$= (27.227 + 15.126) \times 13.23$$

$$= 560.33 \text{ mm/s}$$

Maximum velocity of sliding during recess $= (\omega_1 + \omega_2)L_r$

$$= (27.227 + 15.126) \times 23.98$$

$$= 1015.62 \text{ mm/s}$$

14.18 HELICAL GEARS

A helical gear has teeth in the form of a helix around the gear. The helix may be right handed on one gear and left handed on the other gear. The pitch surfaces are cylindrical like spur gears but the teeth wind around the cylinder helically like screw threads. Helical gears are used to transmit power between parallel shafts.

If a plane is rolled on a base cylinder, a line in the plane parallel to the axis of the cylinder will generate the surface of an involute spur gear tooth. If the generating line is inclined to the axis, the surface of a helical gear tooth will be generated. These two conditions are shown in Fig.14.19.

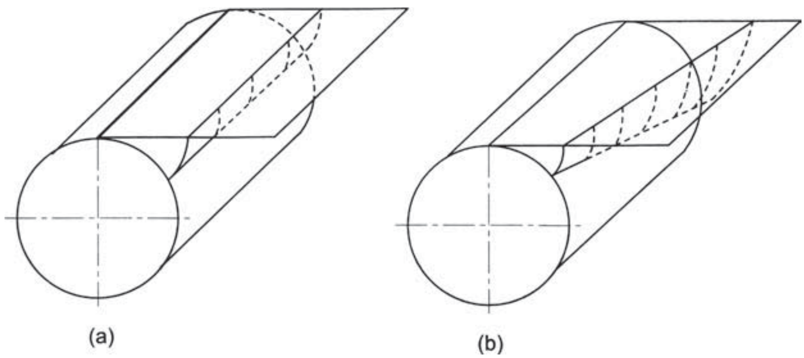

Fig.14.19 Generation of helical gears