

PRODUCTION CONCEPTS AND MATHEMATICAL MODELS

A number of production concepts are quantitative, or they require a quantitative approach to measure them. The purpose of this section is to define some of these concepts. In subsequent chapters, we refer back to these production concepts in our discussion of specific topics in automation and production systems. The models developed in this section are ideal, in the sense that they neglect some of the realities and complications that are present in the factory. For example, our models do not include the effect of scrap rates. In some manufacturing operations, the percentage of scrap produced is high enough to adversely affect production rate, plant capacity, and product costs. Most of these issues are considered in later chapters as we focus on specific types of production systems.

2.4.1 Production Rate

The production rate for an individual processing or assembly operation is usually expressed as an hourly rate, that is, parts or products per hour. Let us consider how this rate is determined for the three types of production: job shop production, batch production, and mass production.

For any production operation, the *operation cycle time*, T_c , is defined as the time that one work unit spends being processed or assembled. It is the time between when one work unit begins processing (or assembly) and when the next unit begins. T_c is the time an individual part spends at the machine, but not all of this time is productive (recall the Merchant study, Section 2.2.2). In a typical processing operation, such as machining, T_c consists of: (1) actual machining operation time, (2) work part handling time, and (3) tool handling time per workpiece. As an equation, this can be expressed:

$$T_c = T_o + T_h + T_{th} \quad (2.8)$$

where T_c = operation cycle time (min/pc), T_o = time of the actual processing or assembly operation (min/pc), T_h = handling time (min/pc), and T_{th} = tool handling time (min/pc). The tool handling time consists of time spent changing tools when they wear out, time changing from one tool to the next, tool indexing time for indexable inserts or for tools on a turret lathe or turret drill, tool repositioning for a next pass, and so on. Some of these tool handling activities do not occur every cycle; therefore, they must be spread over the number of parts between their occurrences to obtain an average time per workpiece.

Each of the terms, T_o , T_h , and T_{th} , has its counterpart in other types of discrete-item production. There is a portion of the cycle when the part is actually being processed (T_o); there is a portion of the cycle when the part is being handled (T_h); and there is, on average, a portion when the tooling is being adjusted or changed (T_{th}). Accordingly, we can generalize Eq. (2.8) to cover most processing operations in manufacturing.

Let us first consider the batch production case and then consider the job shop and mass production. In *batch production*, the time to process one batch consisting of Q work units is the sum of the setup time and processing time; that is,

$$T_b = T_{su} + QT_c \quad (2.9)$$

where T_b = batch processing time (min), T_{su} = setup time to prepare for the batch (min), Q = batch quantity (pc), and T_c = operation cycle time per work unit (min/cycle). We assume that one work unit is completed each cycle and so T_c also has units of min/pc. If more than one part is produced each cycle, then Eq. (2.9) must be adjusted accordingly. Dividing batch time by batch quantity, we have the average production time per work unit T_p for the given machine:

$$T_p = \frac{T_b}{Q} \quad (2.10)$$

The average production rate for the machine is simply the reciprocal of production time. It is usually expressed as an hourly rate:

$$R_p = \frac{60}{T_p} \quad (2.11)$$

where R_p = hourly production rate (pc/hr), T_p = average production time per minute (min/pc), and the constant 60 converts minutes to hours.

For *job shop production* when quantity $Q = 1$, the production time per work unit is the sum of setup and operation cycle times:

$$T_p = T_{su} + T_c \quad (2.12)$$

For job shop production when the quantity is greater than one, then this reverts to the batch production case discussed above.

For *quantity type mass production*, we can say that the production rate equals the cycle rate of the machine (reciprocal of operation cycle time) after production is underway and the effects of setup time become insignificant. That is, as Q becomes very large, $(T_{su}/Q) \rightarrow 0$ and

$$R_p \rightarrow R_c = \frac{60}{T_c} \quad (2.13)$$

where R_c = operation cycle rate of the machine (pc/hr), and T_c = operation cycle time (min/pc).

For *flow line mass production*, the production rate approximates the cycle rate of the production line, again neglecting setup time. However, the operation of production lines is complicated by the interdependence of the workstations on the line. One complication is that it is usually impossible to divide the total work equally among all of the workstations on the line; therefore, one station ends up with the longest operation time, and this station sets the pace for the entire line. The term *bottleneck station* is sometimes used to refer to this station. Also included in the cycle time is the time to move parts from one station to the next at the end of each operation. In many production lines, all work units on the line are moved simultaneously, each to its respective next station. Taking these factors into account, the cycle time of a production line is the sum of the longest processing (or assembly) time plus the time to transfer work units between stations. This can be expressed:

$$T_c = T_r + \text{Max } T_o \quad (2.14)$$

where T_c = cycle time of the production line (min/cycle), T_r = time to transfer work units between stations each cycle (min/pc), and $\text{Max } T_o$ = operation time at the bottleneck station (the maximum of the operation times for all stations on the line, min/cycle). Theoretically, the production rate can be determined by taking the reciprocal of T_c as follows:

$$R_c = \frac{60}{T_c} \quad (2.15)$$

where R_c = theoretical or ideal production rate, but let us call it the cycle rate to be more precise (cycles/hr), and T_c = ideal cycle time from Eq. (2.14) (min/cycle).

Production lines are of two basic types: (1) manual and (2) automated. In the operation of automated production lines, another complicating factor is reliability. Poor reliability reduces the available production time on the line. This results from the interdependence of workstations in an automated line, in which the entire line is forced to stop when one station breaks down. The actual average production rate R_p is reduced to a value that is often substantially below the ideal R_c given by Eq. (2.15). We discuss reliability and some of its terminology in Section 2.4.3. The effect of reliability on automated production lines is examined in Chapters 18 and 19.

It is important to design the manufacturing method to be consistent with the pace at which the customer is demanding the part or product, sometimes referred to as the *takt* time (a German word for cadence or pace). The takt time is the reciprocal of demand rate, but adjusted for the available shift time in the factory. For example, if 100 product units

were demanded from a customer each day, and the factory operated one shift/day, with 400 min of time available per shift, then the takt time would be $400 \text{ min}/100 \text{ units} = 4.0 \text{ min/work unit}$.

2.4.2 Production Capacity

We mentioned production capacity in our discussion of manufacturing capabilities (Section 2.3.3). *Production capacity* is defined as the maximum rate of output that a production facility (or production line, work center, or group of work centers) is able to produce under a given set of assumed operating conditions. The production facility usually refers to a plant or factory, and so the term *plant capacity* is often used for this measure. As mentioned before, the assumed operating conditions refer to the number of shifts per day (one, two, or three), number of days in the week (or month) that the plant operates, employment levels, and so forth.

The number of hours of plant operation per week is a critical issue in defining plant capacity. For continuous chemical production in which the reactions occur at elevated temperatures, the plant is usually operated 24 hr/day, 7 day/wk. For an automobile assembly plant, capacity is typically defined as one or two shifts. In the manufacture of discrete parts and products, a growing trend is to define plant capacity for the full 7-day week, 24 hr/day. This is the maximum time available (168 hr/wk), and if the plant operates fewer hours than the maximum, then its maximum possible capacity is not being fully utilized.

Quantitative measures of plant capacity can be developed based on the production rate models derived earlier. Let PC = the production capacity of a given facility under consideration. Let the measure of capacity = the number of units produced per week. Let n = the number of machines or work centers in the facility. A *work center* is a manufacturing system in the plant typically consisting of one worker and one machine. It might also be one automated machine with no worker, or multiple workers working together on a production line. It is capable of producing at a rate R_p unit/hr, as defined in Section 2.4.1. Each work center operates for H hr/shift. Provision for setup time is included in R_p , according to Eq. (2.11). Let S denote the number of shifts per week. These parameters can be combined to calculate the production capacity of the facility:

$$PC = n S H R_p \quad (2.16)$$

where PC = production capacity of the facility (output units/wk), n = number of work centers producing in the facility, S = number of shifts per period (shift/wk), H = hr/shift (hr), and R_p = hourly production rate of each work center (output units/hr). Although we have used a week as the time period of interest, Eq. (2.16) can easily be revised to adopt other periods (months, years, etc.). As in previous equations, our assumption is that the units processed through the group of work centers are homogeneous, and therefore the value of R_p is the same for all units produced.

EXAMPLE 2.3 Production Capacity

The turret lathe section has six machines, all devoted to the production of the same part. The section operates 10 shift/wk. The number of hours per shift averages 8.0. Average production rate of each machine is 17 unit/hr. Determine the weekly production capacity of the turret lathe section.

Solution: From Eq. (2.10),

$$PC = 6(10)(8.0)(17) = 8160 \text{ output unit/wk}$$

If we include the possibility that each work unit is routed through n_o operations, with each operation requiring a new set up on either the same or a different machine, then the plant capacity equation must be amended as follows:

$$PC = \frac{n_o S H R_p}{n_o} \quad (2.17)$$

where n_o = number of distinct operations through which work units are routed, and the other terms have the same meaning as before.

Equation (2.17) indicates the operating parameters that affect plant capacity. Changes that can be made to increase or decrease plant capacity over the short term are:

1. Change the number of shifts per week (S). For example, Saturday shifts might be authorized to temporarily increase capacity.
2. Change the number of hours worked per shift (H). For example, overtime on each regular shift might be authorized to increase capacity.

Over the intermediate or longer term, the following changes can be made to increase plant capacity:

3. Increase the number of work centers, n , in the shop. This might be done by using equipment that was formerly not in use and hiring new workers. Over the long term, new machines might be acquired. Decreasing capacity is easier, except for the social and economic impact: Workers must be laid off and machines decommissioned.
4. Increase the production rate, R_p by making improvements in methods or process technology.
5. Reduce the number of operations n_o required per work unit by using combined operations, simultaneous operations, or integration of operations (Section 1.5.2: strategies 2, 3, and 4).

This capacity model assumes that all n machines are producing 100% of the time, and there are no bottleneck operations due to variations in process routings to inhibit smooth flow of work through the plant. In real batch production machine shops where each product has a different operation sequence, it is unlikely that the work distribution among the productive resources (machines) can be perfectly balanced. Consequently, there are some operations that are fully utilized while other operations occasionally stand idle waiting for work. Let us examine the effect of utilization.

2.4.3 Utilization and Availability

Utilization refers to the amount of output of a production facility relative to its capacity.

Expressing this as an equation,

$$U = \frac{Q}{PC} \quad (2.18)$$



where U = utilization of the facility, Q = actual quantity produced by the facility during a given time period (i.e., pc/wk), and PC = production capacity for the same period (pc/wk).

Utilization can be assessed for an entire plant, a single machine in the plant, or any other productive resource (i.e., labor). For convenience, it is often defined as the proportion of time that the facility is operating relative to the time available under the definition of capacity. Utilization is usually expressed as a percentage.

EXAMPLE 2.4 Utilization

A production machine operates 80 hr/wk (two shifts, 5 days) at full capacity. Its production rate is 20 unit/hr. During a certain week, the machine produced 1000 parts and was idle the remaining time. (a) Determine the production capacity of the machine. (b) What was the utilization of the machine during the week under consideration?

Solution: (a) The capacity of the machine can be determined using the assumed 80-hr week as follows:

$$PC = 80(20) = 1600 \text{ unit/wk}$$

(b) Utilization can be determined as the ratio of the number of parts made by the machine relative to its capacity.

$$U = 1000/1600 = 0.625 \quad (62.5\%)$$

The alternative way of assessing utilization is by the time during the week that the machine was actually used. To produce 1000 units, the machine was operated

$$H = \frac{1000 \text{ pc}}{20 \text{ pc/hr}} = 50 \text{ hr}$$

Utilization is defined relative to the 80 hr available.

$$U = 50/80 = 0.625 \quad (62.5\%)$$

Availability is a common measure of reliability for equipment. It is especially appropriate for automated production equipment. Availability is defined using two other reliability terms, *mean time between failure* (MTBF) and *mean time to repair* (MTTR). The MTBF indicates the average length of time the piece of equipment runs between breakdowns. The MTTR indicates the average time required to service the equipment and put it back into operation when a breakdown occurs. Availability is defined as follows:

$$A = \frac{\text{MTBF} - \text{MTTR}}{\text{MTBF}} \quad (2.19)$$

where A = availability, MTBF = mean time between failures (hr), and MTTR = mean time to repair (hr). Availability is typically expressed as a percentage. When a piece of equipment is brand new (and being debugged), and later when it begins to age, its availability tends to be lower.

EXAMPLE 2.5 Effect of Utilization and Availability on Plant Capacity

Consider previous Example 2.3. Suppose the same data from that example were applicable, but that the availability of the machines $A = 90\%$, and the utilization of the machines $U = 80\%$. Given this additional data, compute the expected plant output.

Solution: Previous Eq. (2.16) can be altered to include availability and utilization as follows:

$$Q = AU(nSHR_p) \quad (2.20)$$

where A = availability and U = utilization. Combining the previous and new data, we have

$$Q = 0.90(0.80)(6)(10)(8.0)(17) = 5875 \text{ output unit/wk}$$

2.4.4 Manufacturing Lead Time

In the competitive environment of modern business, the ability of a manufacturing firm to deliver a product to the customer in the shortest possible time often wins the order. This time is referred to as the manufacturing lead time. Specifically, we define *manufacturing lead time* (MLT) as the total time required to process a given part or product through the plant. Let us examine the components of MLT.

Production usually consists of a series of individual processing and assembly operations. Between the operations are material handling, storage, inspections, and other non-productive activities. Let us therefore divide the activities of production into two main categories, operations and nonoperation elements. An operation is performed on a work unit when it is in the production machine. The nonoperation elements include handling, temporary storage, inspections, and other sources of delay when the work unit is not in the machine. Let T_c = the operation cycle time at a given machine or workstation, and T_{no} = the nonoperation time associated with the same machine. Further, let us suppose that the number of separate operations (machines) through which the work unit must be routed to be completely processed = n_o . If we assume batch production, then there are Q work units in the batch. A setup is generally required to prepare each production machine for the particular product, which requires a time = T_{su} . Given these terms, we can define manufacturing lead time as:

$$\text{MLT}_j = \sum_{i=1}^{o_j} (T_{suj_i} + Q_j T_{cji} + T_{noji}) \quad (2.21)$$

where MLT_j = manufacturing lead time for part or product j (min), Q_j = quantity of part or product j in the batch being processed (pc), T_{cji} = operation cycle time for operation i (min/ pc), T_{noji} = nonoperation time associated with operation i (min), and i indicates the operation sequence in the processing; $i = 1, 2, \dots, n_{oj}$. The MLT equation does not include the time the raw workpart spends in storage before its turn in the production schedule begins.

To simplify and generalize our model, let us assume that all setup times, operation cycle times, and nonoperation times are equal for the n_{oj} machines. Further, let us suppose that they are all processed through the same number of machines, so that $n_{oj} = n_o$. With these simplifications, Eq. (2.21) becomes:



$$\text{MLT} = n_o(T_{su} + QT_c + T_{no}) \quad (2.22)$$

where MLT = average manufacturing lead time for a part or product (min).

In an actual batch production factory, which this equation is intended to represent, the terms n_o , Q , T_{su} , T_c , and T_{no} would vary by product and by operation. These variations can be accounted for by using properly weighted average values of the various terms. The averaging procedure is explained in the Appendix at the end of this chapter.

EXAMPLE 2.6 Manufacturing Lead Time

A certain part is produced in a batch size of 100 units. The batch must be routed through five operations to complete the processing of the parts. Average setup time is 3 hr/operation, and average operation time is 6 min (0.1 hr). Average nonoperation time due to handling, delays, inspections, etc., is 7 hours for each operation. Determine how many days it will take to complete the batch, assuming the plant runs one 8-hr shift/day.

Solution: The manufacturing lead time is computed from Eq. (2.22)

$$\text{MLT} = 5(3 + 100 \times 0.1 + 7) = 100 \text{ hours}$$

At 8 hr/day, this amounts to $100/8 = 12.5$ days.

Equation (2.22) can be adapted for job shop production and mass production by making adjustments in the parameter values. For a job shop in which the batch size is one ($Q = 1$), Eq. (2.22) becomes

$$\text{MLT} = n_o(T_{su} + T_c + T_{no}) \quad (2.23)$$

For mass production, the Q term in Eq. (2.22) is very large and dominates the other terms. In the case of quantity type mass production in which a large number of units are made on a single machine ($n_o = 1$), the MLT simply becomes the operation cycle time for the machine after the setup has been completed and production begins.

For flow line mass production, the entire production line is set up in advance. Also, the nonoperation time between processing steps is simply the transfer time T_r to move the part or product from one workstation to the next. If the workstations are integrated so that all stations are processing their own respective work units, then the time to accomplish all of the operations is the time it takes each work unit to progress through all of the stations on the line. The station with the longest operation time sets the pace for all stations.

$$\text{MLT} = n_o(T_r + \text{Max } T_o) = n_o T_c \quad (2.24)$$

where MLT = time between start and completion of a given work unit on the line (min), n_o = number of operations on the line; T_r = transfer time (min), $\text{Max } T_o$ = operation time at the bottleneck station (min) and T_c = cycle time of the production line (min/pc). $T_c = T_r + \text{Max } T_o$ from Eq. (2.14). Since the number of stations is equal to the number of operations ($n = n_o$), Eq. (2.24) can also be stated as follows:

$$\text{MLT} = n(T_r + \text{Max } T_o) = nT_c \quad (2.25)$$

where the symbols have the same meaning as above, and we have substituted n (number of workstations or machines) for number of operations n_o .

2.4.5 Work-in-Process

Work-in-process (WIP) is the quantity of parts or products currently located in the factory that are either being processed or are between processing operations. WIP is inventory that is in the state of being transformed from raw material to finished product. An approximate measure of work-in-process can be obtained from the following, using terms previously defined:

$$WIP = \frac{AU(PC)(MLT)}{SH} \quad (2.26)$$

where WIP = work-in-process in the facility (pc), A = availability, U = utilization, PC = production capacity of the facility (pc/wk), MLT = manufacturing lead time, (wk), S = number of shifts per week (shift/wk), and H = hours per shift (hr/shift). Equation (2.26) states that the level of WIP equals the rate at which parts flow through the factory multiplied by the length of time the parts spend in the factory. The units for $(PC)/SH$ (e.g., pc/wk) must be consistent with the units for MLT (e.g., weeks).

Work-in-process represents an investment by the firm, but one that cannot be turned into revenue until all processing has been completed. Many manufacturing companies sustain major costs because work remains in-process in the factory too long.

COSTS OF MANUFACTURING OPERATIONS

Decisions on automation and production systems are usually based on the relative costs of alternatives. In this section we examine how these costs and cost factors are determined.

2.2.1 Fixed and Variable Costs

Manufacturing costs can be classified into two major categories: (1) fixed costs and (2) variable costs. A *fixed cost* is one that remains constant for any level of production output. Examples include the cost of the factory building and production equipment, insurance, and property taxes. All of the fixed costs can be expressed as annual amounts. Expenses such as insurance and property taxes occur naturally as annual costs. Capital investments such as building and equipment can be converted to their equivalent uniform annual costs using interest rate factors.

A *variable cost* is one that varies in proportion to the level of production output. As output increases, variable cost increases. Examples include direct labor, raw materials, and electric power to operate the production equipment. The ideal concept of variable cost is that it is directly proportional to output level. When fixed cost and variable cost are added, we have the following total cost equation:

$$TC = FC + VC(Q) \quad (2.27)$$

where TC = total annual cost (\$/yr), FC = fixed annual cost (\$/yr), VC = variable cost (\$/pc), and Q = annual quantity produced (pc/yr).

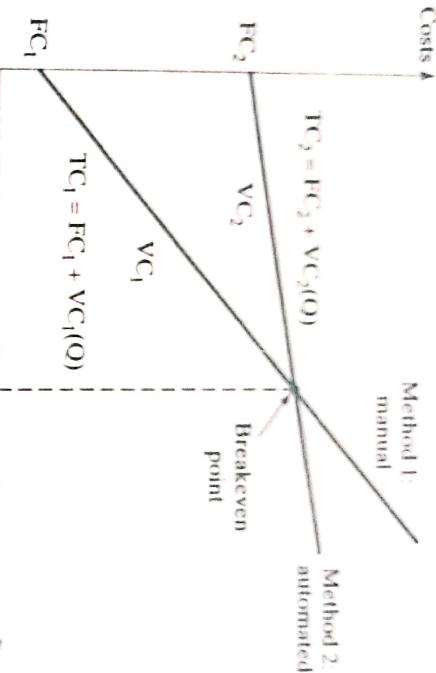


Figure 2.4 Fixed and variable costs as a function of production output for manual and automated production methods.

When comparing automated and manual production methods (Section 1.4), it is typical that the fixed cost of the automated method is high relative to the manual method, and the variable cost of automation is low relative to the manual method, as pictured in Figure 2.4. Consequently, the manual method has a cost advantage in the low quantity range, while automation has an advantage for high quantities. This reinforces the arguments presented in Section 1.4.1 on the appropriateness of manual labor for certain production situations.

2.5.2 Direct Labor, Material, and Overhead

Fixed versus variable are not the only possible classifications of costs in manufacturing. An alternative classification separates costs into: (1) direct labor, (2) material, and (3) overhead. This is often a more convenient way to analyze costs in production. The *direct labor cost* is the sum of the wages and benefits paid to the workers who operate the production equipment and perform the processing and assembly tasks. The *material cost* is the cost of all raw materials used to make the product. In the case of a stamping plant, the raw material consists of the steel sheet stock used to make stampings. For the rolling mill that made the sheet stock, the raw material is the iron ore or scrap iron out of which the sheet is rolled. In the case of an assembled product, materials include component parts manufactured by supplier firms. Thus, the definition of "raw material" depends on the company. The final product of one company can be the raw material for another company. In terms of fixed and variable costs, direct labor and material must be considered as variable costs.

Overhead costs are all of the other expenses associated with running the manufacturing firm. Overhead divides into two categories: (1) factory overhead and (2) corporate overhead. *Factory overhead* consists of the costs of operating the factory other than direct labor and materials. The types of expenses included in this category are listed in Table 2.7. Factory overhead is treated as fixed cost, although some of the items in our list could be correlated with the output level of the plant. *Corporate overhead* is the cost of running the company other than its manufacturing activities. A list of typical corporate overhead expenses is presented in Table 2.8. Many companies operate more than one factory, and this is one of the reasons for dividing overhead into factory and corporate categories. Different factories may have significantly different factory overhead expenses.

TABLE 2.7 Typical Factory Overhead Expenses

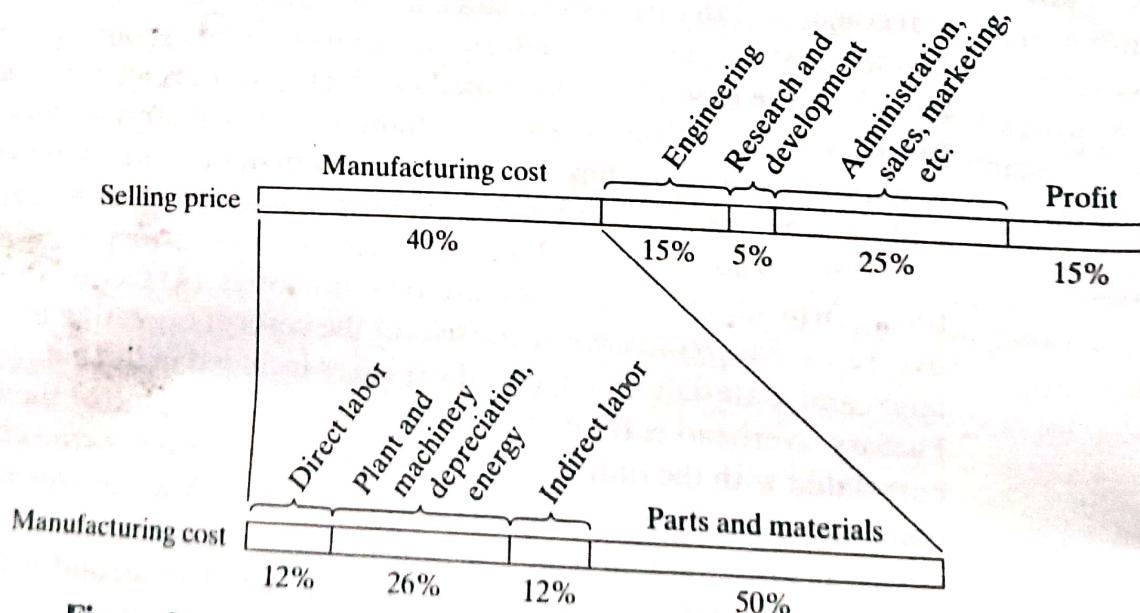
Plant supervision	Applicable taxes
Line foreman	Insurance
Maintenance crew	Heat and air conditioning
Custodial services	Light
Security personnel	Power for machinery
Tool crib attendant	Factory depreciation
Material handling	Equipment depreciation
Shipping and receiving	Fringe benefits

TABLE 2.8 Typical Corporate Overhead Expenses

Corporate executives	Applicable taxes
Sales and marketing	Cost of office space
Accounting department	Security personnel
Finance department	Heat and air conditioning
Legal counsel	Light
Engineering	Insurance
Research and development	Fringe benefits
Other support personnel	Other office costs

J. T. Black [2] provides some typical percentages for the different types of manufacturing and corporate expenses. These are presented in Figure 2.5. We might make several observations about these data. First, total manufacturing cost represents only about 40% of the product's selling price. Corporate overhead expenses and total manufacturing cost are about equal. Second, materials (and parts) make up the largest percentage of total manufacturing cost, at around 50%. And third, direct labor is a relatively small proportion of total manufacturing cost: 12% of manufacturing cost and only about 5% of final selling price.

Overhead costs can be allocated according to a number of different bases, including direct labor cost, material cost, direct labor hours, and space. Most common in industry is

**Figure 2.5** Breakdown of costs for a manufactured product [6].

direct labor cost, which we will use here to illustrate how overheads are allocated and subsequently used to compute factors such as selling price of the product.

The allocation procedure (simplified) is as follows. For the most recent year (or most recent several years), all costs are compiled and classified into four categories: (1) direct labor, (2) material, (3) factory overhead, and (4) corporate overhead. The objective is to determine an *overhead rate* (also called *burden rate*) that could be used in the following year to allocate overhead costs to a process or product as a function of the direct labor costs associated with that process or product. In our treatment, separate overhead rates will be developed for factory and corporate overheads. The *factory overhead rate* is calculated as the ratio of factory overhead expenses (category 3) to direct labor expenses (category 1); that is,

$$\text{FOHR} = \frac{\text{FOHC}}{\text{DLC}} \quad (2.28)$$

where FOHR = factory overhead rate, FOHC = annual factory overhead costs (\$/yr); and DLC = annual direct labor costs (\$/yr).

The *corporate overhead rate* is the ratio of corporate overhead expenses (category 4) to direct labor expenses:

$$\text{COHR} = \frac{\text{COHC}}{\text{DLC}} \quad (2.29)$$

where COHR = corporate overhead rate, COHC = annual corporate overhead costs (\$/yr), and DLC = annual direct labor costs (\$/yr). Both rates are often expressed as percentages. If material cost were used as the allocation basis, then material cost would be used as the denominator in both ratios. Let us present two examples to illustrate (1) how overhead rates are determined and (2) how they are used to estimate manufacturing cost and establish selling price.

EXAMPLE 2.7 Determining Overhead Rates

Suppose that all costs have been compiled for a certain manufacturing firm for last year. The summary is shown in the table below. The company operates two different manufacturing plants plus a corporate headquarters. Determine: (a) the factory overhead rate for each plant and (b) the corporate overhead rate. These rates will be used by the firm in the following year.

Expense Category	Plant 1 (\$)	Plant 2 (\$)	Corporate Headquarters (#)	Totals (\$)
Direct labor	800,000	400,000		1,200,000
Materials	2,500,000	1,500,000		4,000,000
Factory expense	2,000,000	1,100,000		3,100,000
Corporate expense			7,200,000	7,200,000
Totals	5,300,000	3,000,000	3,000,000	15,500,000

Solution: (a) A separate factory overhead rate must be determined for each plant. For plant 1, we have:

$$FOHR_1 = \frac{\$2,000,000}{\$800,000} = 2.5 = 250\%$$

For plant 2,

$$FOHR_2 = \frac{\$1,100,000}{\$400,000} = 2.75 = 275\%$$

(b) The corporate overhead rate is based on the total labor cost at both plants.

$$COHR = \frac{\$7,200,000}{\$1,200,000} = 6.0 = 600\%$$

EXAMPLE 2.8 Estimating Manufacturing Costs and Establishing Selling Price

A customer order of 50 parts is to be processed through plant 1 of the previous example. Raw materials and tooling are supplied by the customer. The total time for processing the parts (including setup and other direct labor) is 100 hr. Direct labor cost is \$10.00/hr. The factory overhead rate is 250% and the corporate overhead rate is 600%. Compute the cost of the job.

- Solution:**
- The direct labor cost for the job is $(100 \text{ hr})(\$10.00/\text{hr}) = \1000 .
 - The allocated factory overhead charge, at 250% of direct labor, is $(\$1000)(2.50) = \2500 .
 - The allocated corporate overhead charge, at 600% of direct labor, is $(\$1000)(6.00) = \6000 .

Interpretation: (a) The direct labor cost of the job, representing actual cash spent on the customer's order = \$1000. (b) The total factory cost of the job, including allocated factory overhead = $\$1000 + \$2500 = \$3500$. (c) The total cost of the job including corporate overhead = $\$3500 + \$6000 = \$9500$. To price the job for the customer and to earn a profit over the long run on jobs like this, the price would have to be greater than \$9500. For example, if the company uses a 10% mark-up, the price quoted to the customer would be $(1.10)(\$9500) = \$10,450$.

2.5.3 Cost of Equipment Usage

The trouble with overhead rates as we have developed them here is that they are based on labor cost alone. A machine operator who runs an old, small engine lathe whose book value is zero will be costed at the same overhead rate as an operator running a new CNC turning center just purchased for \$500,000. Obviously, the time on the machining center is more productive and should be valued at a higher rate. If differences in rates of different production machines are not recognized, manufacturing costs will not be accurately measured by the overhead rate structure.

To deal with this difficulty, it is appropriate to divide the cost of a worker running a machine into two components: (1) direct labor and (2) machine. Associated with each is an applicable overhead rate. These costs apply not to the entire factory operations, but to individual work centers. A *work center* is a production cell consisting of (1) one worker and

one machine, (2) one worker and several machines, (3) several workers operating one machine, or (4) several workers and machines. In any of these cases, it is advantageous to separate the labor expense from the machine expense in estimating total production costs.

The direct labor cost consists of the wages and benefits paid to operate the work center. Applicable factory overhead expenses allocated to direct labor cost might include state taxes, certain fringe benefits, and line supervision. The machine annual cost is the initial cost of the machine apportioned over the life of the asset at the appropriate rate of return used by the firm. This is done using the capital recovery factor, as follows:

$$UAC = IC(A/P,i,n) \quad (2.30)$$

where UAC = equivalent uniform annual cost (\$/yr); IC = initial cost of the machine (\$); and $(A/P,i,n)$ = capital recovery factor that converts initial cost at year 0 into a series of equivalent uniform annual year-end values, where i = annual interest rate and n = number of years in the service life of the equipment. For given values of i and n , $(A/P,i,n)$ can be computed as follows:

$$(A/P, i, n) = \frac{i(1 + i)^n}{(1 + i)^n - 1} \quad (2.31)$$

Value of $(A/P, i, n)$ can also be found in interest tables that are widely available.

The uniform annual cost can be expressed as an hourly rate by dividing the annual cost by the number of annual hours of equipment use. The machine overhead rate is based on those factory expenses that are directly assignable to the machine. These include power to drive the machine, floor space, maintenance and repair expenses, and so on. In separating the factory overhead items in Table 2.7 between labor and machine, judgment must be used; admittedly, the judgment is sometimes arbitrary. Total cost rate for the work center is the sum of labor and machine costs. This can be summarized as follows:

$$C_o = C_L(1 + FOHR_L) + C_m(1 + FOHR_m) \quad (2.32)$$

where C_o = hourly rate to operate the work center (\$/hr), C_L = direct labor wage rate (\$/hr), $FOHR_L$ = factory overhead rate for labor, C_m = machine hourly rate (\$/hr), and $FOHR_m$ = factory overhead rate applicable to machines.

It is the author's opinion that corporate overhead expenses should not be included in the analysis when comparing production methods. Including them serves no purpose other than to dramatically increase the costs of the alternatives. The fact is that these corporate overhead expenses are present whether or not either or none of the alternatives is selected. On the other hand, when estimating costs for pricing decisions, corporate overhead should be included because over the long run, these costs must be recovered through revenues generated from selling products.

EXAMPLE 2.9 Hourly Cost of a Work Center

The following data are given: direct labor rate = \$10.00/hr; applicable factory overhead rate on labor = 60%; capital investment in machine = \$100,000; service life of the machine = 8 yr; rate of return = 20%; salvage value in 8 yr = 0; and applicable factory overhead rate on machine = 50%. The work center will be operated one 8-hr shift, 250 day/yr. Determine the appropriate hourly rate for the work center.