

Engineering Mechanics

By: Dr. Divya Agarwal





UNIT- I

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- Distributed forces: Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

UNIT- II

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- □ **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.





UNIT-III

- Kinematics of Particles: Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates
- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

UNIT-IV

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.
- □ **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.
- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple





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ANALYSIS OF PLANETRUSSES AND FRAMES-ENGINEERING STRUCTURES

• An engineering structure is any connected system of members built to support or transfer forces and to safely

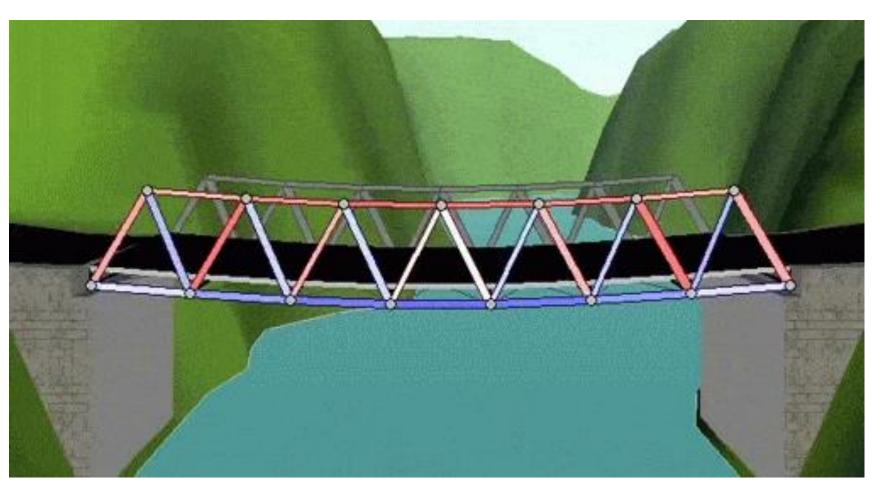
withstand the loads applied to it.

• The term 'structure' refers to anything that is constructed or built from interrelated parts with a fixed location on the ground.

- To determine the internal forces in the structure, dismember the structure and analyze separate FBDs of individual members.
- The engineering structures may be broadly divided into:-
- I. Trusses.
- 2. Frame.
- 3. Machine: Are structures designed to transmit and modify forces and contain some moving members.
- The plane structures are structures whose members lie in one plane.



WHAT ISTRUSS?

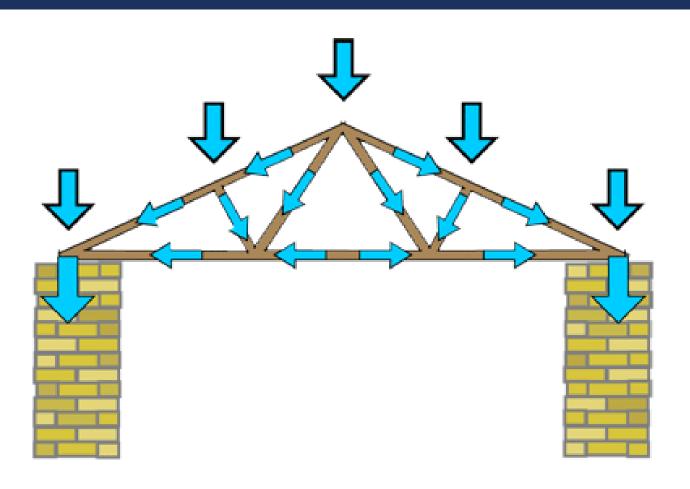


- A truss is a structure that consists of members organized into connected triangles so that the overall assembly behaves as a single object.
- Truss are used in bridges, roofs and towers.

WHAT ISTRUSS?

- It is a system of uniform bars for all members (of the circular section, angle section, channel section etc.) joined together at their ends by riveting or welding and constructed to support loads.
- In other words, A truss is a frictionless pin-connected structure, where an assemblage of slender bars is fastened together at their ends by smooth bolts or ball-and-socket joints acting as hinges.
- The members of a truss are straight members and the loads are applied only at the joints.
- The bar members, therefore, act as two-force members which can either be in tension or in compression; there can be no transverse force in a member of a truss.
- Forces acting at the member ends reduce to a single force and no couple.
- Most structures are made of several trusses joined together to form a space framework.
- Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- A truss consisting of members which lie in a plane and are loaded in the same plane is called plane truss.
- If a truss is made of non-coplanar members, it is referred to as space truss.

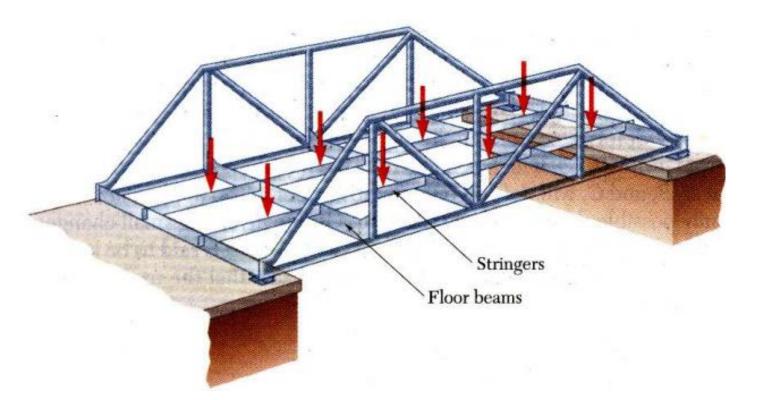
LOAD BALANCING INTRUSS

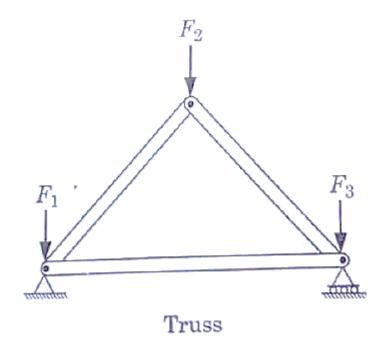


- Truss are used to achieve long span
- Minimize the weight of structure
- Reduced deflection
- Support heavy loads

WHAT ISTRUSS?

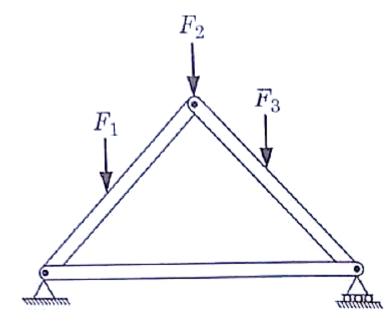
- Members of a truss are slender and not capable of supporting large lateral loads.
- Loads must be applied at the joints. Weights are assumed to be distributed to joints.
- External distributed loads transferred to joints via stringers and floor beams.





WHAT ARE FRAMES?

- A frame structure, on the other hand, consists of members which may be subjected to a transverse load in addition to the axial load.
- In other words, a structure consists of several bars or members pinned together and in which one or more then one of its members is subjected to more than two forces.
- They are designed to support loads and are stationary structures.
- A simple structure is thus a pin-connected frame or truss.
- Similarly, a frame may be a plane frame or a space frame depending upon its structure.

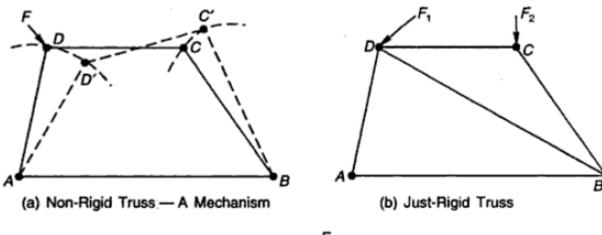


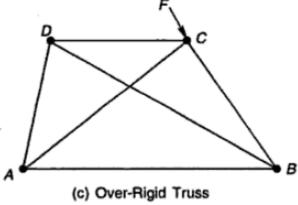
Frame

(Forces may act anywhere on the member)

RIGID OR PERFECT TRUSS

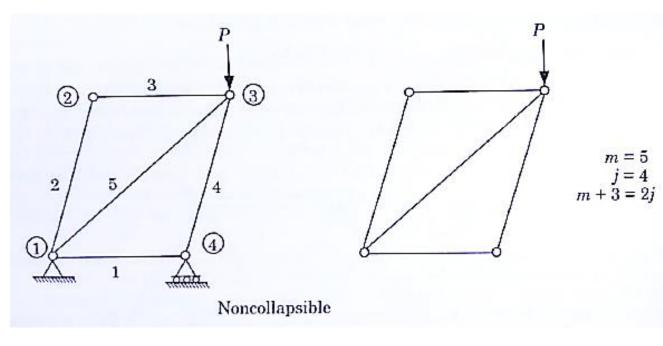
- Trusses are classified as just rigid, over rigid and nonrigid mechanisms.
- If the members are allowed any relative movement, then the assemblage of members is called a non-rigid truss or mechanism
- If the members are not allowed any relative movement, then it is called a rigid truss.
- A just-rigid truss is that which, on the removal of any single member, becomes non-rigid.
- An over-rigid truss is the one that has redundant members which may be removed to render the truss just-rigid.
- Examples of such trusses are shown here.

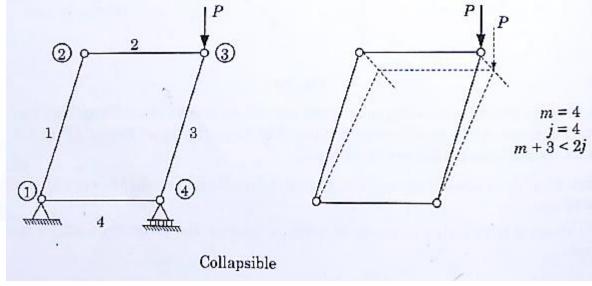




RIGID OR PERFECT TRUSS

The term rigid, with reference to the Truss, is used in the sense that the truss is non collapsible when the external supports are removed.





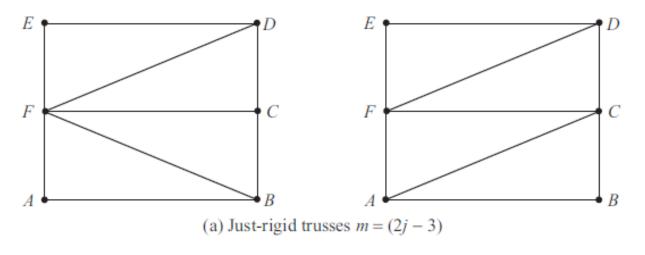
MATHEMATICAL CONDITION FOR RIGID OR PERFECT TRUSS.

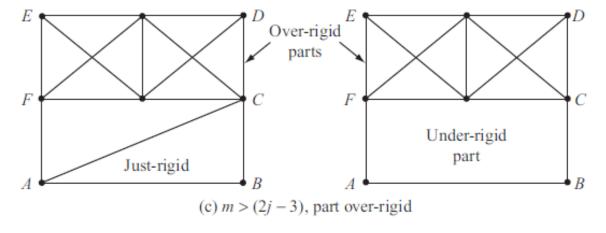
A truss consists of a number of members which are connected together and form a certain number of joints.
 For a truss to be rigid or perfect, the relationship between its number of members and the number of joints is,

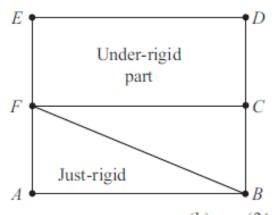
$$m + 3 = 2j$$

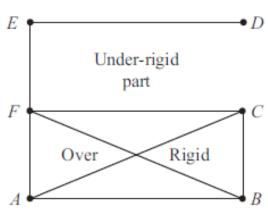
- where, m=number of members in the Truss and
 - j = number of joints in the Truss
- If m+3>2j, it means that the truss contains more members than required to be just rigid and is over rigid and statically indeterminate
- If m+3<2j, it means that the truss contains less members than required to be just rigid and is collapsible or under rigid.</p>
- Statically determinate. A truss is statically determined if the equations of static equilibrium alone are sufficient to determine the axial forces in the members without the need of considering their deformations.

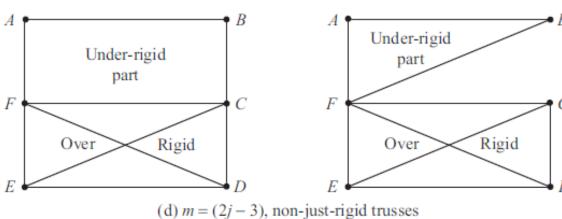
CONDITIONS OF RIGIDITY OF TRUSS











(b) m < (2j - 3), non-rigid trusses

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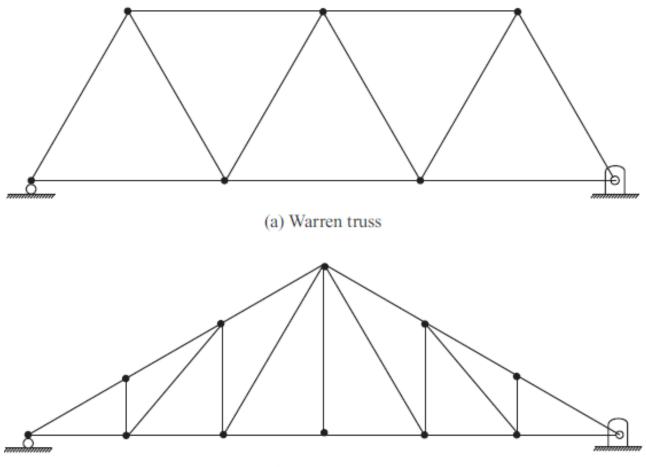
MATHEMATICAL CONDITION FOR RIGID OR PERFECT TRUSS.

- Simple or just-rigid trusses are generated from the basic triangular truss by successively adding a pair of new members to the existing joints and by generating a new joint by connecting the new members.
- For a basic triangular truss which is just-rigid, the number of joints j = 3. For each additional joint, two members must be added to keep it just-rigid.
- If we wish to visualise a truss of j joints, then (j-3) joints must be added to the basic triangular truss. The number of members which will be added are 2(j-3) and the total number of members become

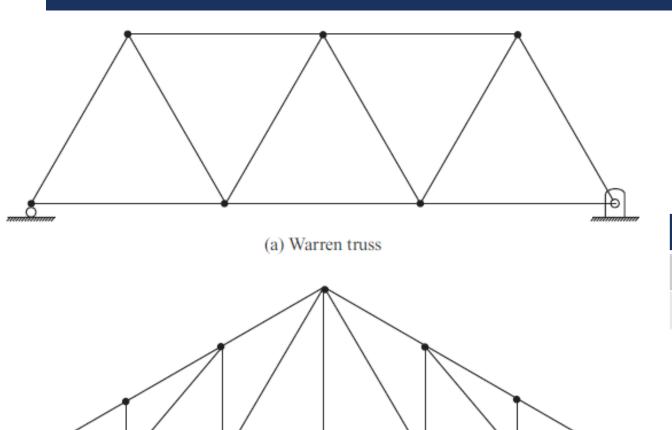
$$m = 2(j-3) + 3$$

Where,

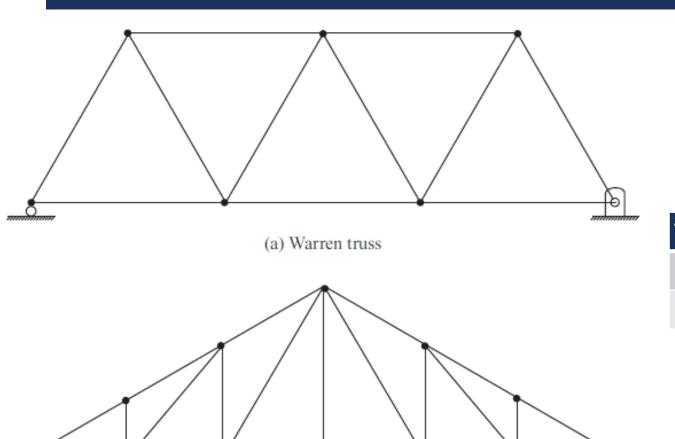
- m = 2 j 3
- Note: Number of members in a simple just-rigid truss must be odd.
- A space truss (or frame) consists of members which do not lie in a single plane. If the non-coplanar members are pin jointed, it is called a simple space truss.
- A necessary relationship between the number of joints j and the number of members m for a just rigid simple space truss is m = 3 j 6



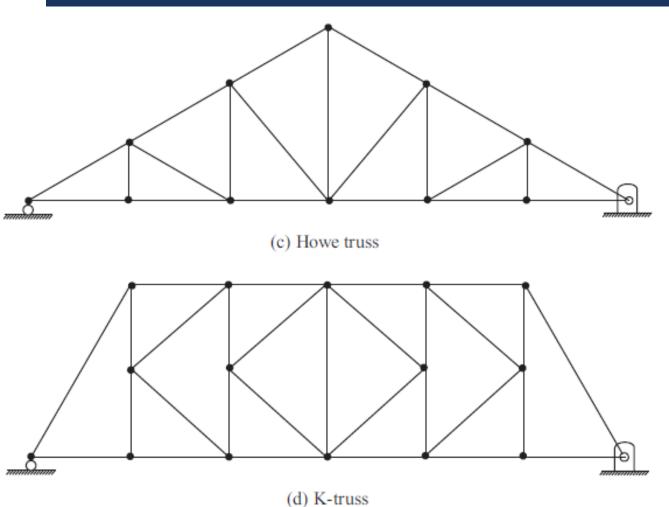
Types of truss	Joints j	Members m	Condition m=2j-3
(a) Warren Truss			
(b) Pratt Truss			



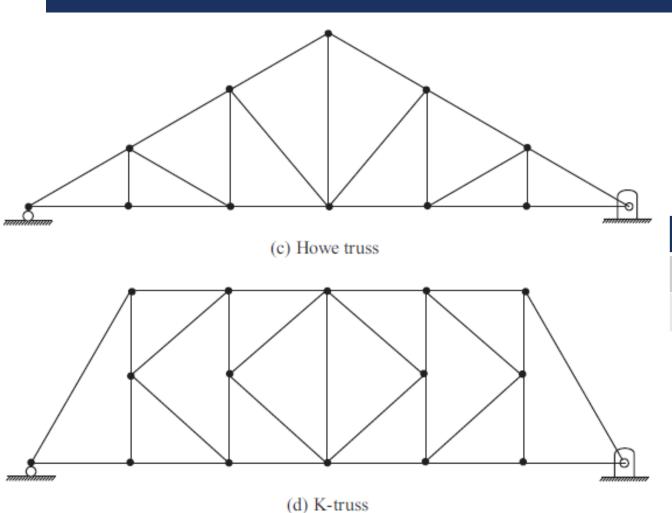
Types of truss	Joints j	Members m	Condition m=2j-3
(a) Warren Truss	7	П	Satisfied
(b) Pratt Truss			



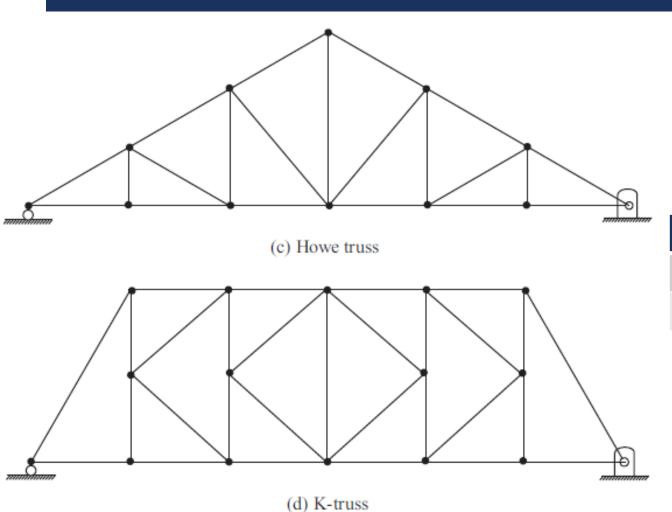
Types of truss	Joints j	Members m	Condition m=2j-3
(a) Warren Truss	7	П	Satisfied
(b) Pratt Truss	12	21	Satisfied



Types of truss	Joints j	Members m	Condition m=2j-3
(c) Howe Truss			
(d) K-Truss			



Types of truss	Joints j	Members m	Condition m=2j-3
(c) Howe Truss	12	21	Satisfied
(d) K-Truss			



Types of truss	Joints j	Members m	Condition m=2j-3
(c) Howe Truss	12	21	Satisfied
(d) K-Truss	16	29	Satisfied

BASIC ASSUMPTIONS FOR THE PERFECT TRUSS ARE

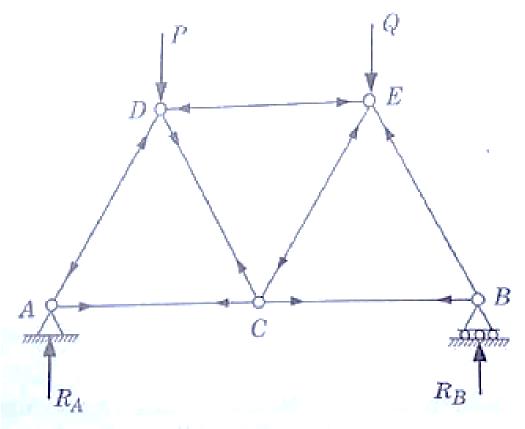
- The joints of a simple Truss are assumed to be pin connections and frictionless therefore, they cannot resist moments.
- The loads and support reactions on the Truss are applied at the joints only.
- All members are two-force members.
- Weight of the members is small compared with the force it supports (weight may be considered at joints), the member weight is often neglected. However, when the member weight is considered, it is applied at the end of each member.
- No effect of bending on members even if weight is considered
- The members of a Truss are straight to force members with the forces acting collinear with the centre line of the members. Or the centroidal axis of each member coincides with the line connecting the centres of the adjacent members and the members only carry axial force.
- The Truss is statically determinate.

TRUSS: DETERMINATION OF AXIAL FORCES IN THE MEMBERS

- The various methods are:
- I. The method of joints
- 2. The method of Sections
- 3. Graphical Method

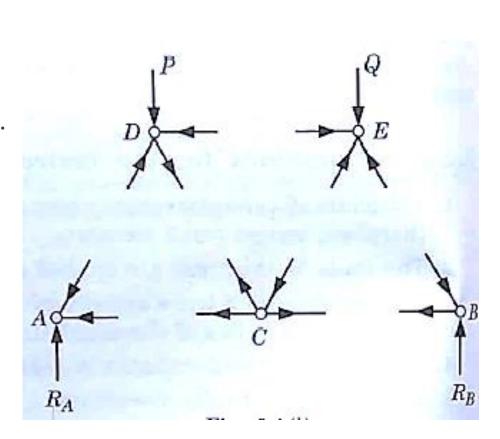
- The method of joints consists of taking up one joint at a time and analysing it for equilibrium.
- At every joint in a truss the forces must be along the members at that joint.
- The forces acting at every joint must satisfy the necessary condition of equilibrium
- The procedure is as follows:
- I. Consider the FBD of the entire Truss
- 2. Compute the support reactions using the equations of equilibrium.
- 3. Assume and mark directions of usual forces in the members on the diagram as shown in figure a.
- 4. If in the solution the magnitude of a force comes out to be negative the assumed direction of the force in the member is simply reversed.

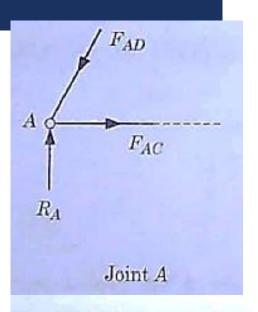
FIGURE A

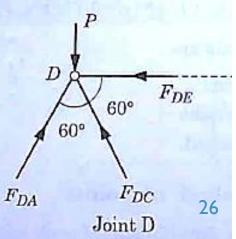


Representation of tension in the member AC. Arrows point away above from the joints A and C (Pull at the joint).

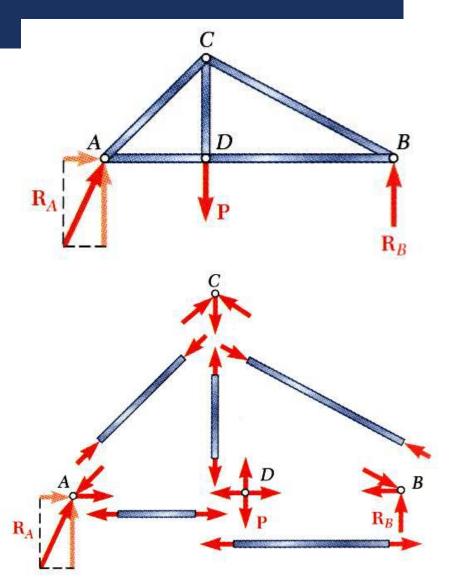
- Choose joint (A and D) as shown in figure and consider its FBD (shown in figure b).
- The forces acting on the joint represent a system of concurrent forces in equilibrium.
- Therefore only two equations of equilibrium can be withdrawn for each joint and can be solved to determine only two unknown forces.
- Therefore, start from a joint where not more than two unknown forces appear.
- For example, we should start from joint A and not from joint D. Consider the equilibrium of the remaining joints.

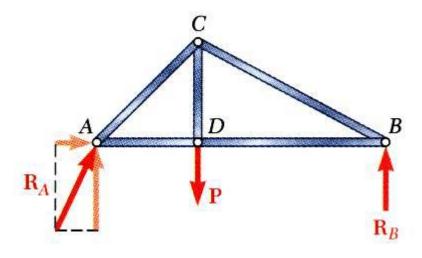


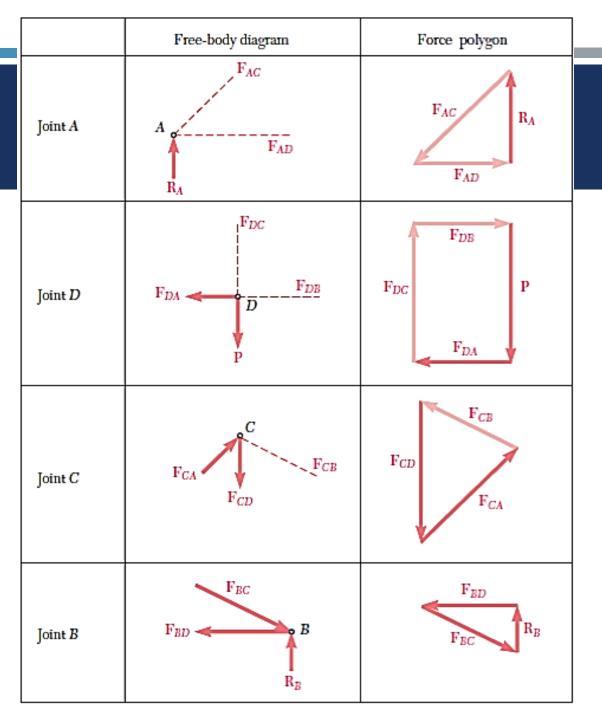


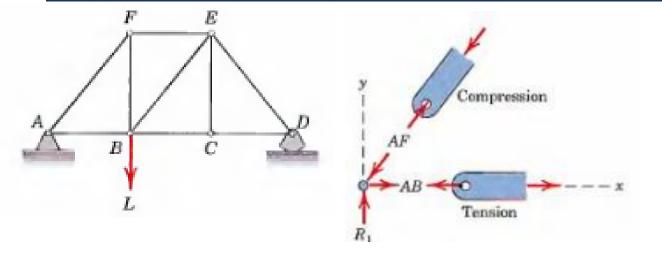


- Dismember the truss and create a FBD for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide 2j equations for 2j unknowns.
- For a simple truss, 2j = m + 3. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.
- Use conditions for equilibrium for the entire truss to solve for the reactions R_A and R_B .

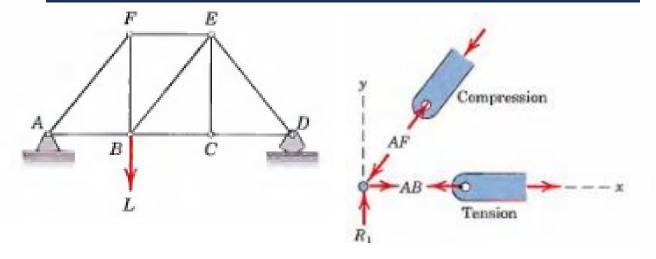




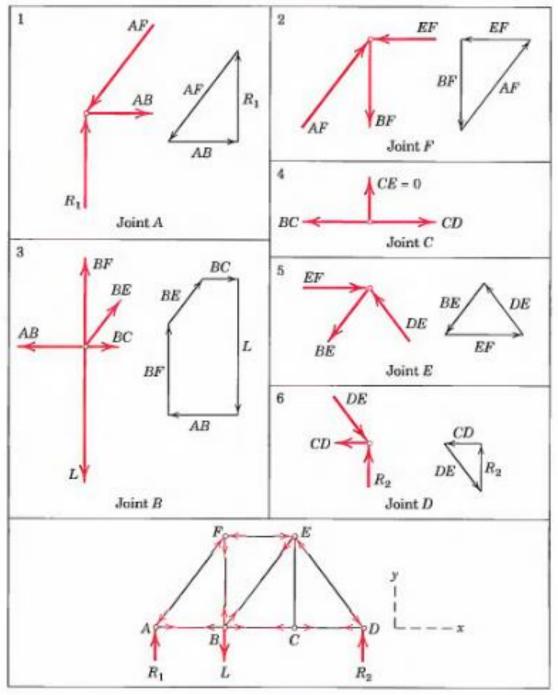




Note the direction of the forces in the members acting on the joints

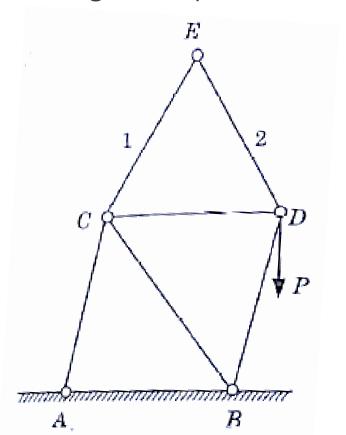


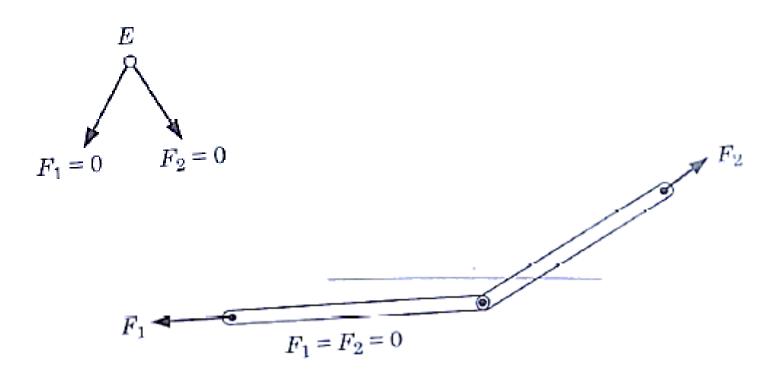
Note the direction of the forces in the members acting on the joints



THE METHOD OF JOINTS: SPECIAL CONDITIONS

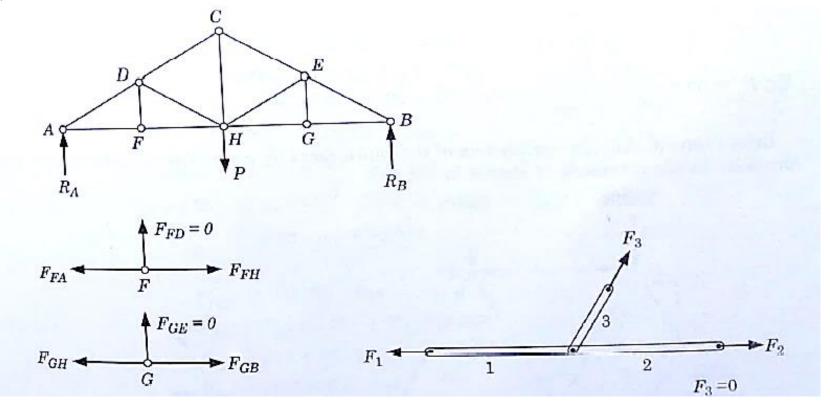
• When two members meeting at a joint are not collinear and there is no external force acting at the joint, then the forces in both the members are zero.





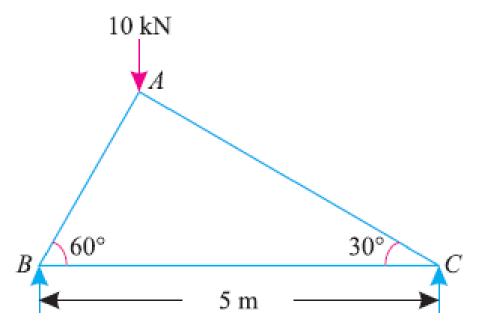
THE METHOD OF JOINTS: SPECIAL CONDITIONS

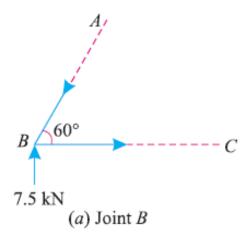
When there are 3 members meeting at a joint, of which two are collinear and third be at an angle and if there is no load at the joint the force in the third member is zero.

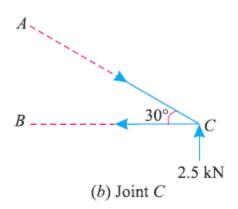


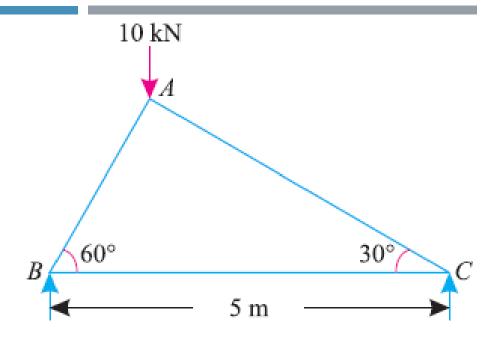
• Consider the joint F, force in the member FD is zero. Consider the joint G, force in the member GE is zero.

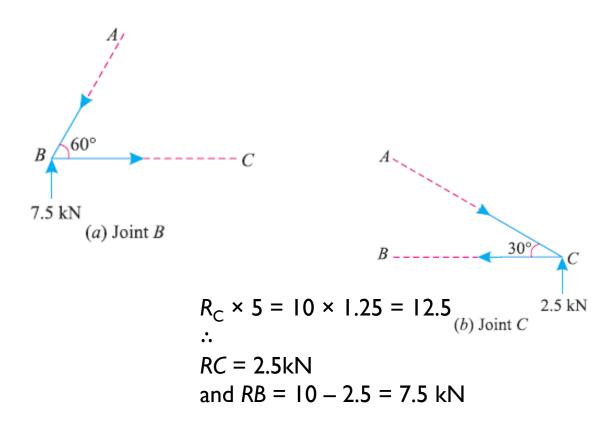
- The truss ABC shown in Fig. 13.5 has a span of 5 metres. It is carrying a load of 10 kN at its apex.
- Find the forces in the members AB, AC and BC.

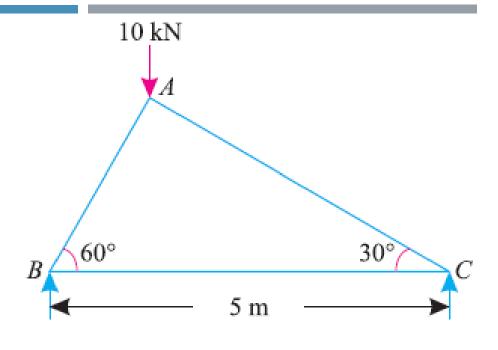




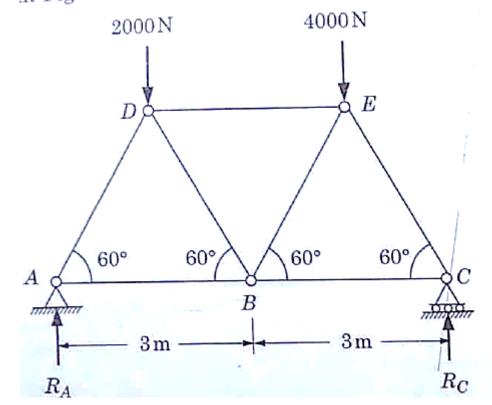








 Using the method of joints, find the axial forces in all the members of a Truss with the loading shown in figure 1.



- To determine support reactions consider equilibrium of entire Truss.
- Reaction at a hinge can have two components acting in horizontal and vertical directions. As there is no horizontal external force acting on the Truss so the horizontal component of reaction at A is zero.
- Taking moments about A,

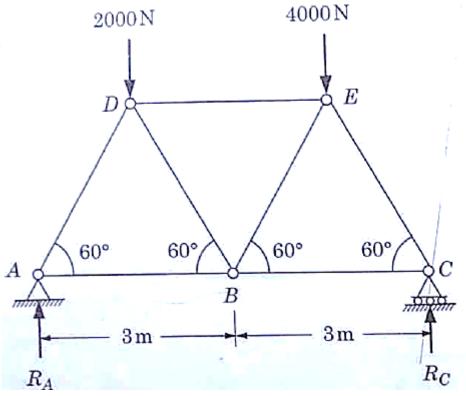
$$\sum M_A = 0: -2000 * (1.5) - 4000 (4.5) + RC * 6 = 0$$

$$R_{C} = 3500N$$

$$\sum_{A} F_{y} = 0: \qquad R_{A} + R_{C} - 2000 - 4000 = 0$$

$$R_A = 2500N$$

Before considering the equilibrium of the joints, mark by inspection the directions of axial forces in all the members as shown in figure.



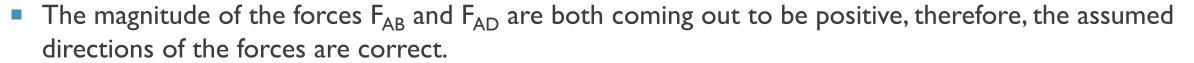
- **Joint A.** Let us begin with the joint A at which there are only two unknown forces. We cannot begin with the joint D because there are three unknown forces acting at the joint D.
- Consider the FBD of the joint A. Equations of equilibrium can be written as:

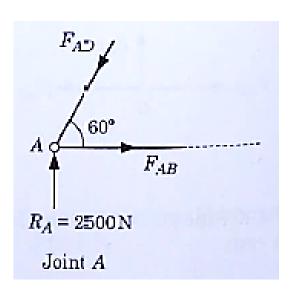
•
$$\sum F_{\chi} = 0$$
: $F_{AB} - F_{AD} \cos 60^{\circ} = 0$

$$F_{AD} = \frac{R_A}{\sin 60^\circ} = \frac{2500}{0.866}$$

•
$$F_{AD} = 2887 N (C)$$

•
$$F_{AB} = F_{AD} \cos 60^{\circ} = 2887 * 0.5 = 1443 N(T)$$





Joint C.

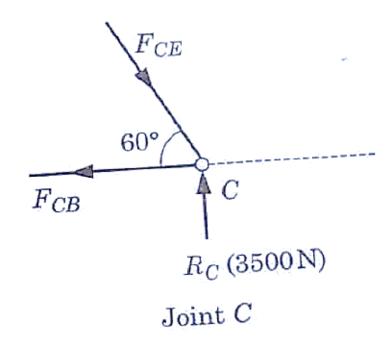
•
$$\sum F_{x} = 0$$
: $F_{CE} \cos 60^{\circ} - F_{CB} = 0$

•
$$\sum F_y = 0$$
: $R_C - F_{CE} \sin 60^\circ = 0$

•
$$F_{CE} = \frac{R_C}{\sin 60^\circ} = \frac{3500}{0.866} = 4041 \text{ N (C)}$$

•
$$F_{CB} = F_{CE} \cos 60^{\circ}$$

= $4041 \text{ N} * 0.5 = 2020.5 \text{ N} \text{ (T)}$



$$F_{AD} = 2887 N (known)$$

$$F_{DB}\cos 60^{\circ} + F_{AD}\cos 60^{\circ} - F_{DE} = 0$$

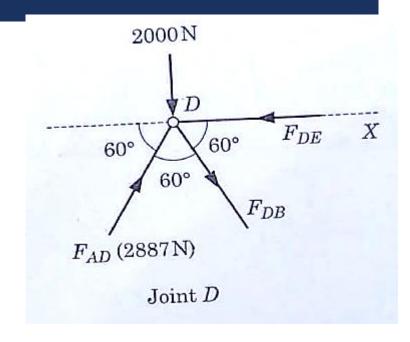
$$F_{AD} \sin 60^{\circ} - F_{DB} \sin 60^{\circ} - 2000 = 0$$

$$F_{DB} = \frac{2887*0.866-2000}{0.866} = 577N \text{ (T)}$$

•
$$F_{DE} = F_{DB} \cos 60^{\circ} + F_{AD} \cos 60^{\circ}$$

$$F_{DE} = 577 * 0.5 + 2887 * 0.5$$

•
$$F_{DE} = 1732 \text{ N (C)}$$



$$F_{DE} = 1732 \text{ N (C)}$$

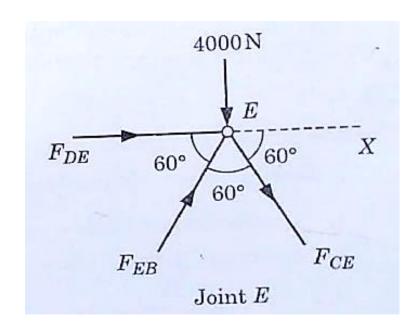
$$F_{CE} = 4041 \text{ N (known)}$$

$$F_{EB}\cos 60^{\circ} - F_{CE}\cos 60^{\circ} + F_{DE} = 0$$

Or

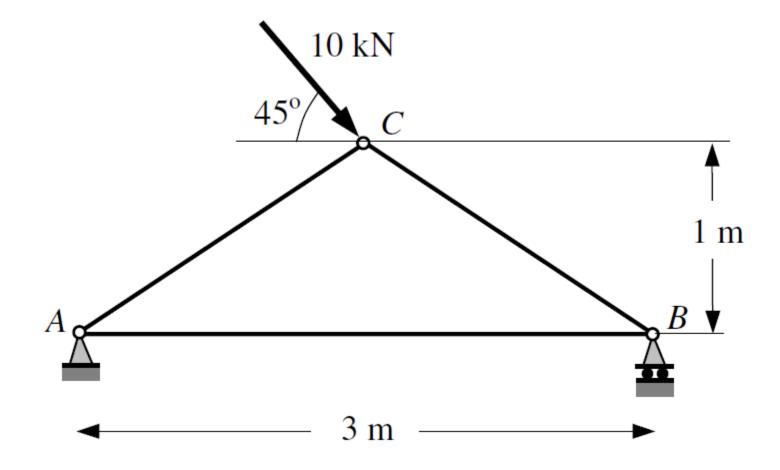
•
$$F_{EB} = \frac{4041*0.5-1732}{0.5} = 577N \text{ (C)}$$

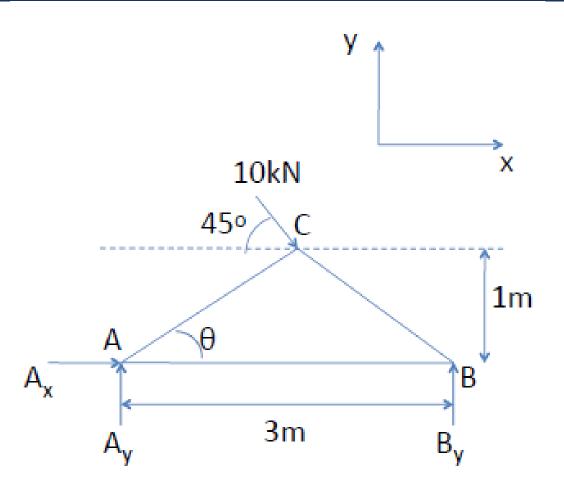
There is no need to consider the equilibrium of the joint B as all the forces have been determined.



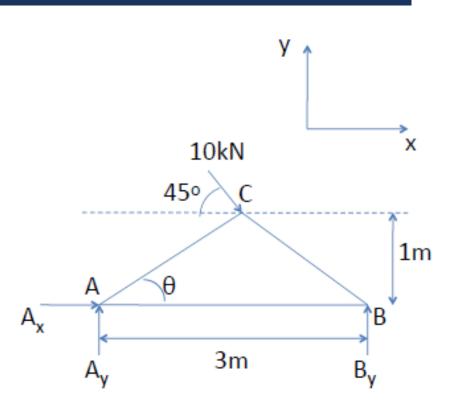
QUESTION

Determine the forces in the members in the following truss





- Consider the FBD of the truss as shown to get the unknown reactions at A and B as shown.
- Consider the equilibrium at each hinge to find the force in the members.
- Assume tensile forces act on all the members.
- $\sum F_{x} = 0 \Rightarrow A_{x} + 10\cos 45^{\circ} = 0 \Rightarrow A_{x} = -7.07kN$
- $\sum M_A = 0$: $\Rightarrow B_y * 3 10\cos 45^\circ * 1 10\sin 45^\circ * 1.5 = 0$ $\Rightarrow B_y = 5.89kN$
- $\sum F_y = 0 : \Rightarrow A_y + B_y = 10 \sin 45^\circ$ $\Rightarrow A_y = 1.18kN$



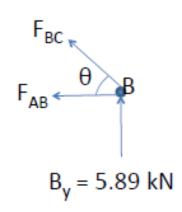
- As we are solving the problem using the method of joints, we take equilibrium at each point.
- As we have assumed the forces in all the members are tensile, the direction of the reaction force they exert on the hinges are as shown.
- $\tan \theta = \frac{1}{1.5} \Rightarrow \sin \theta = \frac{2}{\sqrt{13}} \Rightarrow \cos \theta = \frac{3}{\sqrt{13}}$

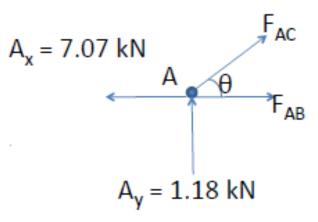
Equilibrium at $B \Rightarrow$

- $\sum F_{x} = 0 \Rightarrow F_{AB} + F_{BC} \cos \theta^{\circ} = 0 \Rightarrow F_{AB} = 8.84kN$

Equilibrium at $A \Rightarrow$

 $\sum F_y = 0 : \Rightarrow F_{AC} \sin\theta + A_y = 0 \Rightarrow F_{AC} = -2.13kN$

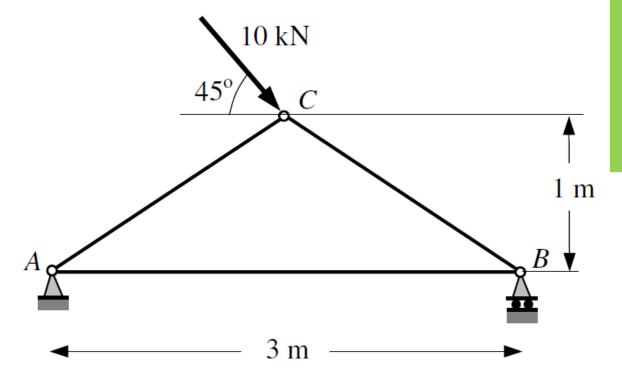




- We assumed that all the forces in the members were tensile.
- But we got some of them negative.

So, the negative sign indicates that the forces in the members are

compressive.



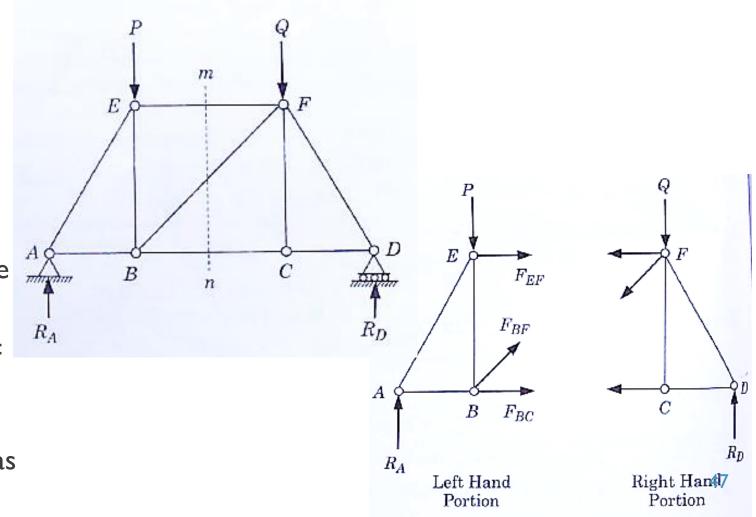
$$F_{AB} = 8.84 \text{kN (T)}$$

$$F_{BC}$$
=10.62kN (C)
 F_{AC} =2.13kN (C)

$$F_{AC} = 2.13 \text{kN (C)}$$

2.THE METHOD OF SECTIONS

- In this method, the equilibrium of a portion of the truss is considered which is obtained by cutting the Truss by some imaginary section.
- Consider a Truss as shown in figure. Cut the Truss into two separate portions by passing an imaginary section through those members in which forces are to be determined.
- The section mn cuts the members EF, BF and BC and the internal forces in these members become the external forces acting on the two portions of the Truss as shown in figure.



2.THE METHOD OF SECTIONS

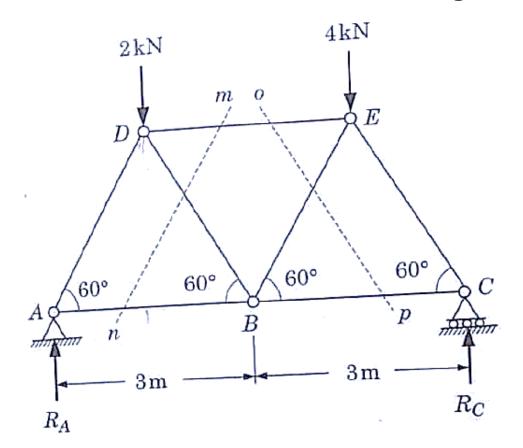
The equilibrium of the entire truss implies that every part of the truss would also be in equilibrium. Therefore, three equations of equilibrium can be written for any one portion of the Truss and can be solved to determine the three unknowns.

$$\Sigma F_x = 0$$
, $\Sigma F_y = 0$ and $\Sigma M = 0$

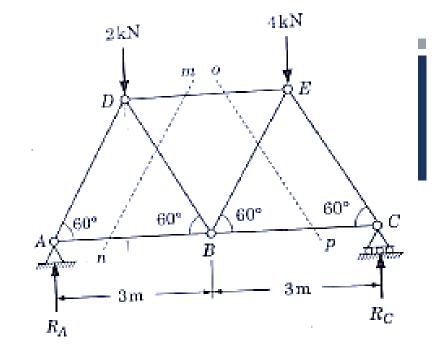
- Following points should be noted while using the method of section:
- The section should be passed through the members and not through the joints.
- A section should divide the truss into two clearly separate and unconnected portions.
- A section should cut only three members since only 3 unknowns can be determined from the three equations of equilibrium. However, in special cases more than three members may be cut.
- When using the moment equation, the moment can be taken about any convenient point which may or may not lie on the section under consideration.

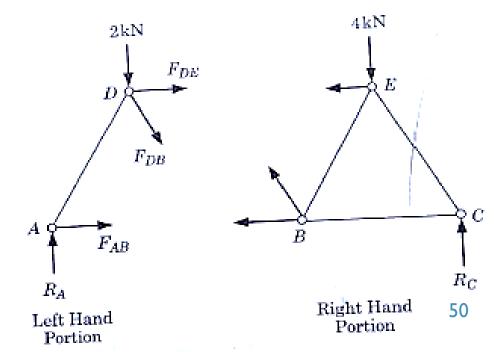
EXAMPLE 2

• Find the axial force in the member DE of the Truss using the method of sections.

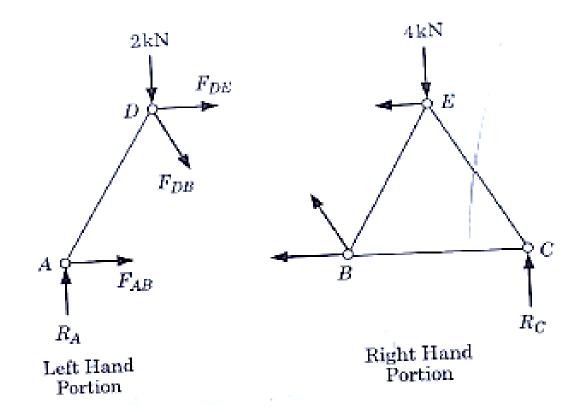


- Determine reactions at the supports, considering entire Truss as a FBD.
- $R_A = 2.5 \text{ kN} \text{ and } R_C = 3.5 \text{ kN}.$
- For determining force in DE, pass a section cutting the member DE and any two other members of the Truss so as to divide the truss into two separate portions. The total number of members cut should not exceed 3.
- There can be more than one way to pass a section (mn or op). Consider the truss as cut by the section mn. The two portions of the Truss are as shown in figure.





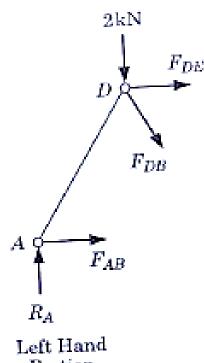
- Assume and mark directions of forces in the cut members.
- Forces in cut members can be assumed to act away from the joints.
- But, the directions of the axial forces assumed in a member in the two portions of the truss must be consistent with the principle of action and reaction.
- For example, if the force in the member DE at the joint D is shown to act from left to right then at the joint E it must be shown to act from right to left (i.e., tension).



- Consider, equilibrium of the left-hand portion of truss.
- The three unknown forces acting on the portion of the truss are F_{DF} , F_{DB} and F_{AB} .
- Write the equations of equilibrium. Taking moments about B,

■
$$\sum M_B = 0$$
: $\Rightarrow 2000 \text{ (3sin30°)} - R_A(3) + F_{DE} \text{ (3 cos 30°)} = 0$
 $\Rightarrow 2000 \text{ (3 * 0.5)} - 3R_A + F_{DE} \text{ (3 * 0.866)} = 0$
 $\Rightarrow F_{DE} = \frac{1500}{0.866}$
 $\Rightarrow F_{DE} = 1732 \text{ N (T)}$

It may be noted that the moment centre B chosen about, does not lie on the section of the Truss under consideration.



Portion.

GRAPHICAL METHOD OF ANALYSIS OF SIMPLE TRUSSES

- The graphical methods of solving the problems of statics are based on the graphical representation of a force by a vector and the law of polygon of forces.
- The implementation of the procedure shall require the concepts of Bows notation.
- The graphical methods have the advantage that a person with the limited knowledge of statics can be trained to use the method mechanically, almost like a tool, and can solve the problem easily and efficiently.

GRAPHICAL METHOD OF ANALYSIS OF SIMPLE TRUSSES: MAXWELL DIAGRAM

- The graphical method for determining the axial forces in the members of a simple truss is the method of joints, implemented graphically. The method consists of following steps:
 - 1. Drawing of the truss to a suitable scale and the representation of the forces acting on it.
 - 2. Representation of all the forces (External forces, reactions and the axial forces in the members of the truss) using Bow's notation. In bow's notation the spaces between the lines of action of the various forces are denoted by letters A, B etc., A force then is represented by the letters denoting the two spaces separated by the line of action of the force.
 - 3. Construction of Maxwell diagram for the truss (also called vector diagram) by considering the equilibrium of the each joint. But at no time the number of unknown forces at a joint should exceed two.
 - 4. Determination of the magnitudes and the nature (tension or compression) or forces in the members using the Maxwell diagram and Bow's notation.
- The method is illustrated with the help of the following examples: (A) A simple supported truss
 - (B) A cantilever truss (C) A truss involving more than two unknown forces at a joint.