(Please write your Exam Roll No.) Exam Roll No. [redacted]

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] APRIL – MAY 2019

Paper Code: ETMA-102 **Subject: Applied Mathematics-II**

(Batch 2013 Onwards)

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt five questions in all including Q no.1 which is compulsory. Select one question from each unit.

Q1 (a) What is geometrical meaning of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$? **(3)**

(b) Verify Euler's theorem for $z = (x^2 + xy + y^2)^{-1}$. **(3)**

(c) Evaluate $L[e^{-t}\{1 - u(t-2)\}]$ **(3)**

(d) Prove that $\int_0^\infty f(t)\delta(t-a)dt = f(a)$, where $\delta(t-a)$ is an impulse function. **(3)**

(e) Show that the limit of the function $f(z) = \frac{\operatorname{Re}(z)}{|z|}, z \neq 0$ and $f(z) = 0,\ z = 0$ as $z \to 0$, does not exist. **(3)**

(f) Prove that the function $e^x(\cos y + i \sin y)$ is analytic and find its derivative. **(3)**

(g) Solve the partial differential equation $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \frac{x-y}{xy}$. **(3)**

(h) Express $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ as a sum of two double integrals and hence evaluate. **(4)**

UNIT-I

Q2 (a) If $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$, determine whether there is a functional relationship between u, v and w and if so, find it. **(6.5)**

(b) Expand $f(x,y) = \frac{y^2}{x^3}$, in powers of $(x-1)$ and $(y-1)$, upto second degree terms. **(6)**

Q3 (a) Find the minimum value of $x^2 + y^2 + z^2$ given that $ax + by + cz = p$. **(6.5)**

(b) Using Charpit's method solve the partial differential equation $2(z + xp + yq) = yp^2$. **(6)**

UNIT-II

Q4 (a) Find the Laplace transform of $\frac{\sin(at)}{t}$. Show that Laplace transform of $\frac{\cos(at)}{t}$ does not exist. **(6.5)**

(b) Using Laplace transform, solve the differential equation $t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \sin t$, when $y(0) = 1$. **(6)**

Q5 (a) Evaluate (i) $L^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\}$ (ii) $L^{-1}\left\{\frac{se^{-\frac{s}{2}} + \pi e^{-s}}{s^2 + \pi^2}\right\}$. **(6.5)**

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(b) Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$. **(6)**

UNIT-III

Q6 (a) Determine the analytic function $w = u + iv$, if $v = \log(x^2 + y^2) + x - 2y$. **(6.5)**

(b) Find the bilinear transformation which maps 1, i, -1 to 2, i, -2 respectively. Also find invariant points of this transformation. **(6)**

Q7 (a) Expand the function $f(z) = \frac{1}{z^2 - 4z + 3}$, for $1 < |z| < 3$. **(6.5)**

(b) Apply calculus of residues to prove that

$$\int_0^{2\pi} \frac{1}{1 - 2a\sin\theta + a^2} d\theta = \frac{2\pi}{1-a^2}, (0 < a < 1).$$ **(6)**

UNIT-IV

Q8 (a) If $\nabla\phi = (y^2 - 2xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k$, find ϕ. **(6.5)**

(b) Using divergence theorem evaluate $\iint_S \vec{F}.d\vec{S}$, where $\vec{F} = x^3 i + y^3 j + z^3 k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(6)**

Q9 (a) Using Green's theorem in xy-plane, evaluate $\int_C [(xy^2 - 2xy)dx + (x^2y + 3)dy]$ around the curve C of the region enclosed by $y^2 = 8x$ and $x = 2$. **(6.5)**

(b) Using Stoke's theorem, evaluate $\int_S curl\, \vec{F}.\hat{n}\, dS$, where $\vec{F} = yi + zj + xk$ and surface S is the part of the sphere $x^2 + y^2 + z^2 = 1$, above the xy-plane. **(6)**

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