

DELHI TECHNICAL CAMPUS

B.TECH I SEM

APPLIED MATHEMATICS-I Paper CodeBS- 111

1. If $u = \log(\tan x + \tan y + \tan z)$, then prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

2. If $u = \log(x^2 + y^2 + z^2)$, then prove that $x \frac{\partial^2 u}{\partial y \partial z} = y \frac{\partial^2 u}{\partial z \partial x}$

3. If $u = (x^2 + y^2 + z^2)$, where $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t$, find $\frac{du}{dt}$.
Ans. $4e^{2t}$

4. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$

5. If $u = \sin(x^2 + y^2)$ and $a^2 x^2 + b^2 y^2 = c^2$, then find $\frac{du}{dx}$.

$$\text{Ans. } 2x \cos(x^2 + y^2) \left(1 - \frac{a^2}{b^2}\right)$$

6. Find the volume of the greatest rectangular parallelepiped that can be inscribed inside the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Ans. $\frac{8abc}{3\sqrt{3}}$

7. Show that the functions $u = x + y + z$, $v = x^3 + y^3 + z^3 - 3xyz$ and $w = x^2 + y^2 + z^2 - xy - yz - zx$ are functionally dependent. find the relation between them.
Ans. $v = uw$

8. Find the extreme values of $\sin x + \sin y + \sin(x + y)$ ans. $\frac{3\sqrt{3}}{2}$

9. If $x = uv$ and $y = \frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$ Ans. $\frac{(u-v)^2}{4uv}$

10. Find the stationary value of $x^3 + y^3 - 3axyz$ Ans. $f_{\min} = -a^3$