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Que $f(x,y) = \cancel{\lim} \dfrac{(x-1)^2 \log x}{(x-1)^2 + y^2}$ Find $\lim\limits_{(x,y)\to(1,0)} f(x,y)$

Sol^n

Put $x-1 = X$

$$\lim_{(X,y)\to(0,0)} f(X,y) = \lim_{(X,y)\to(0,0)} \frac{X^2 \log(X+1)}{X^2+y^2}$$

Put $X = r\cos\theta$, $y = r\sin\theta$

$$\lim_{r\to 0} \frac{\cancel{r^2}\cos^2\theta \log(r\cos\theta+1)}{\cancel{r^2}}$$

$$= \lim_{r\to 0} \cos^2\theta \log(r\cos\theta+1)$$

as $r\to 0$; $r\cos\theta \to 0$ as $\cos\theta$ is bounded

$\therefore$ $\log(r\cos\theta+1) \to \log 1$ as $r\to 0$
$= 0$

$$\therefore \lim_{r\to 0} \cos^2\theta \log(r\cos\theta+1) = 0.$$

Qus $$\lim_{(x,y)\to(2,-2)} \frac{\sqrt{x-y}-2}{x-y-4}$$

Sol^n

Put $x-y = t$

as $x\to 2$, $y\to -2$, $x-y\to 4$

$$\lim_{t\to 4} \frac{\sqrt{t}-2}{t-4} = \lim_{t\to 4} \frac{\sqrt{t}-2}{(\sqrt{t})^2-(2)^2} = \lim_{t\to 4} \frac{\cancel{\sqrt{t}-2}}{\cancel{(\sqrt{t}-2)}(\sqrt{t}+2)}$$

$$= \frac{1}{4} \text{ Ans}$$

Que $f(x,y) = x^2+2y^2$ S.t. $y-x^2+1=0$

Sol^n $F(x,y) = x^2+2y^2+\lambda(y-x^2+1)$

$F_x = 2x - 2\lambda x = 0 \Rightarrow x(1-\lambda) = 0$

$\Rightarrow \lambda = 1$ or $x = 0$

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$F_y = 4y + \lambda = 0$

$y = \frac{-\lambda}{4}$

$\lambda = 1 \Rightarrow y = \frac{-1}{4}$

Now $y - x^2 + 1 = 0$

$\Rightarrow \frac{-1}{4} + 1 = x^2 \Rightarrow x^2 = \frac{3}{4}$

$\Rightarrow x = \pm \frac{\sqrt{3}}{2}$

$\therefore$ at $\left(\pm\frac{\sqrt{3}}{2}, \frac{-1}{4}\right)$ $f\left(\pm\frac{\sqrt{3}}{2}, \frac{-1}{4}\right) = \frac{3}{4} + 2\left(\frac{-1}{4}\right)^2$

$= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$.

$x = 0$; $y = -1$ $(\because y - x^2 + 1 = 0)$

and $\lambda = 4$

at $(0, -1)$, $f(x,y) = 2$

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Que→ $f(x,y) = \begin{cases} \frac{|x|}{|x|+|y|}\sqrt{x^4+y^2} ; & (x,y) \neq (0,0) \\ 0 ; & (x,y) = (0,0) \end{cases}$

Sol^n $f(0,0) = 0$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{r\to 0} \frac{\cancel{|r|}\,|\cos\theta|}{\cancel{|r|}[\,|\cos\theta|+|\sin\theta|\,]} \sqrt{r^4\cos^4\theta + r^2\sin^2\theta}$$

$$= \lim_{r\to 0} r \cdot \frac{|\cos\theta|}{|\cos\theta|+|\sin\theta|} \sqrt{r^2\cos^4\theta + \sin^2\theta}$$

$$= 0$$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{\cancel{|h|}}{\cancel{|h|}}\sqrt{h^4+0}}{h}$$

$$= \lim_{h\to 0} \frac{h^2}{h} = 0$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = 0$$

Que→ $f(x,y) = \begin{cases} \frac{x^2y(x-y)}{x^2+y^2} ; & (x,y) \neq (0,0) \\ 0 ; & (x,y) = (0,0) \end{cases}$ Find $f_{xy} - f_{yx}$?

Sol^n $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = 0$

$$\frac{\partial}{\partial y}(f_x) = f_{yx}(0,0) = \lim_{k\to 0} \frac{f_x(0,k)-f_x(0,0)}{k}$$

$$f_x(0,k) = \lim_{h\to 0} \frac{f(h,k)-f(0,k)}{h} = \lim_{h\to 0} \frac{h^{\cancel{2}}k(h-k)}{(h^2+k^2)\cancel{h}}$$

$$= 0$$

$$\therefore f_{yx}(0,0) = \frac{0-0}{k} = 0$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = 0$$

$$\frac{\partial}{\partial x}(f_y) = f_{xy}(0,0) = \underset{h\to 0}{lt} \frac{f_y(h,0) - f_y(0,0)}{h}$$

$$= \cancel{\underset{h\to 0}{lt}}$$

$$f_y(h,0) = \underset{k\to 0}{lt} \frac{f(h,k) - f(h,0)}{k} = \underset{k\to 0}{lt} \frac{h^2 \cancel{k}(h-k)}{(h^2+k^2)\cancel{k}}$$

$$= h$$

$$\therefore f_{xy}(0,0) = \underset{h\to 0}{lt} \frac{h-0}{h} = 1$$

$$\therefore f_{xy}(0,0) - f_{yx}(0,0) = 1.$$

Que $f(x,y) = \begin{cases} \dfrac{xy}{(x^2+y^2)^\alpha} ; & (x,y) \neq (0,0) \\ 0 ; & (x,y) = (0,0) \end{cases}$ For which value of α, f is cont & diff at $(0,0)$

Solⁿ:→ $f(0,0) = 0$

$\underset{(x,y)\to(0,0)}{lt} f(x,y) = 0$ for f to be Continuous.

$$\Rightarrow \underset{(x,y)\to(0,0)}{lt} \frac{xy}{(x^2+y^2)^\alpha} = \underset{r\to 0}{lt} \frac{r^2 \cdot \cos\theta \sin\theta}{(r^2)^\alpha}$$

$$= \underset{r\to 0}{lt} (r^2)^{1-\alpha} \cos\theta \sin\theta$$

If $1-\alpha = 0$ then lim does not Exist

If $1-\alpha < 0$ then lim does not Exist

If $(1-\alpha) > 0$ Then $\underset{(x,y)\to(0,0)}{\lim} f(x,y) = 0. = f(0,0)$

$\Rightarrow f$ is cts at $(0,0)$ for $\alpha < 1$.

$$f_x(0,0) = \underset{h\to 0}{lt} \frac{f(h,0) - f(0,0)}{h} = 0$$

$$f_y(0,0) = \lim_{k \to 0} \frac{f(0,k) - f(0,0)}{k} = 0$$

$$f(h,k) - f(0,0) = \sqrt{h^2+k^2} \cdot \phi(h,k)$$

$$\Rightarrow \quad \frac{hk}{(h^2+k^2)^\alpha} = \sqrt{h^2+k^2} \cdot \phi(h,k)$$

$$\Rightarrow \quad \phi(h,k) = \frac{hk}{(h^2+k^2)^{\alpha+1/2}}$$

$$\lim_{(h,k)\to(0,0)} \phi(h,k) = \lim_{(h,k)\to(0,0)} \frac{hk}{(h^2+k^2)^{\alpha+1/2}}$$

$$= \lim_{r \to 0} \frac{r^2 \cos\theta \sin\theta}{(r^2)^{\alpha+1/2}}$$

$$= \lim_{r \to 0} (r^2)^{1/2-\alpha} \cos\theta \sin\theta$$

If $\frac{1}{2} - \alpha > 0$ Then $\lim_{(h,k)\to(0,0)} \phi(h,k) = 0$

$$\Rightarrow \alpha < \frac{1}{2}$$

So f is diff at $(0,0)$ for $\alpha < \frac{1}{2}$.

HW Que

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x+y} & \text{If } x+y \neq 0 \\ 0 & \text{If } x+y = 0 \end{cases}$$

Then find the value of $\left(\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y \partial x}\right)$ at $(0,0)$.

Que $f(x,y) = x+y$ and $g(x,y) = xy - 16$.

Sol^n $F(x,y) = x+y + \lambda(xy-16)$

$F_x = 1 + \lambda y = 0 \Rightarrow \lambda = \frac{-1}{y}$

$F_y = 1 + \lambda x = 0 \Rightarrow \lambda = \frac{-1}{x}$

$\frac{-1}{y} = \frac{-1}{x} \Rightarrow x = y$

$xy - 16 = 0$

$x^2 = 16 \Rightarrow x = \pm 4$

$y = \pm 4$

$\cancel{(4,-4)}, \cancel{(-4,4)}$; $(4,4), (-4,-4)$

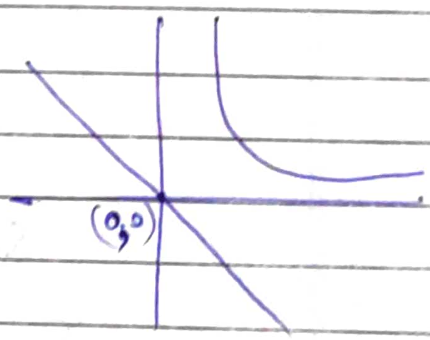
$f(x,y) = \underline{8}, \underline{-8}, \cancel{0, 0}$

$F_{xx} = 0,\ F_{yy} = 0,\ F_{xy} = \lambda$

$$J = \begin{bmatrix} 0 & \lambda \\ \lambda & 0 \end{bmatrix}$$

$|J| = -\lambda^2 < 0 \Rightarrow$ Saddle pt.

Now $F(x,y) = xy - 16 + \lambda(x+y)$



$F_x = y + \lambda = 0 \Rightarrow \lambda = -y$

$F_y = x + \lambda = 0 \Rightarrow \lambda = -x$

$\therefore y = x$

$x + y = 0 \Rightarrow x = 0 = y$

$\therefore (0,0)$ critical point.

$F_{xx} = 0 = F_{yy}$; $F_{xy} = 1$, $F_{yx} = 1$

$$J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow |J| < 0$$

Que $f(x,y) = xy \quad s.t \quad 2x+3y=6$

Sol^n →

$$F = xy + \lambda(2x+3y-6)$$

$F_x = y + 2\lambda = 0 \Rightarrow \lambda = -y/3 \qquad y = -2\lambda$

$F_y = x + 3\lambda = 0 \Rightarrow \lambda = -x/2 \qquad x = -3\lambda$

$\Rightarrow \frac{-x}{2} = \frac{-y}{3} \Rightarrow 3x = 2y$

$\Rightarrow y = \frac{3}{2}x$

$2x + 3\left(\frac{3}{2}x\right) = 6$

$\Rightarrow \left(2 + \frac{9}{2}\right)x = 6 \Rightarrow x = \frac{12}{13}$

$2(-3\lambda) + 3(-2\lambda) = 6$

$\Rightarrow -12\lambda = 6 \Rightarrow \lambda = \frac{-1}{2}$

$x = 3/2, \ y = 1$

$\left(\frac{3}{2}, 1\right)$ – critical point.

$f\left(\frac{3}{2}, 1\right) = \frac{3}{2}$

Que $f(x,y,z) = x + 2y - 2z \quad s.t \quad x^2 + 2y^2 + 4z^2 = 1$

Sol^n

$$F = x + 2y - 2z + \lambda(x^2 + 2y^2 + 4z^2 - 1)$$

$F_x = 1 + 2\lambda x = 0 \qquad x = \frac{-1}{2\lambda}$

$F_y = 2 + 4\lambda y = 0 \qquad y = \frac{-1}{2\lambda}$

$F_z = -2 + 8\lambda z = 0 \qquad z = \frac{1}{4\lambda}$

$\frac{1}{4\lambda^2} + \frac{2}{4\lambda^2} + \frac{4}{16\lambda^2} = 1$

$\frac{1}{4\lambda^2}(4) = 1 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1.$

$\lambda = 1; \quad x = \frac{-1}{2}, \ y = \frac{-1}{2}, \ z = \frac{1}{4}$

$f(x,y,z) = -\frac{1}{2} - \frac{2}{2} - \frac{1}{2} = -\frac{4}{2} = -2$

$\lambda = -1, \quad x = \frac{1}{2}, \ y = \frac{1}{2}, \ z = \frac{-1}{4}$

$f = \frac{3}{2} - \frac{1}{2} = 1$

Qu $f(x,y) = x^2+xy+y^2; \quad x^2+y^2=8$

Sol^n $F = f + \lambda g$

$= (x^2+xy+y^2) + \lambda(x^2+y^2-8)$

$F_x = \cancel{2x+2\lambda x = 0} \quad 2x+y+2\lambda x = 0$

$F_y = x + 2y + 2\lambda y = 0$

$2x(1+\lambda) + y = 0$

$x + 2y(1+\lambda) = 0$

$$\begin{vmatrix} 2(1+\lambda) & 1 \\ 1 & 2(1+\lambda) \end{vmatrix} = 0$$

$4(1+\lambda)^2 = 1 \Rightarrow 1+\lambda^2 = \pm\frac{1}{2}$

$\Rightarrow 1+\lambda = 1/2 \qquad 1+\lambda = -\frac{1}{2}$

$\Rightarrow \lambda = -1/2, \qquad \lambda = -3/2.$

$\cancel{\lambda = -\frac{3}{4}};$

$\lambda = -\frac{1}{2} \qquad 2x\left(\frac{1}{2}\right) + y = 0 \Rightarrow x+y=0$

$y = -x$

$x^2+x^2 = 8$

$2x^2 = 8 \Rightarrow x = \pm 2$

$x=2, y=-2 \Rightarrow (2,-2) \;\&\; (-2,2)$

$x=-2, y=2$

$\lambda = -\frac{3}{2}; \quad 2x\left(-\frac{1}{2}\right) + y = 0$

$\Rightarrow -x+y=0 \Rightarrow x=y$

$(2,2) \; (-2,-2)$

$f(2,2) = 4+4+4 = 12$

$f(-2,-2) = 12$

$f(2,-2) = 4-4+4 = 4$

$f(-2,2) = 4.$

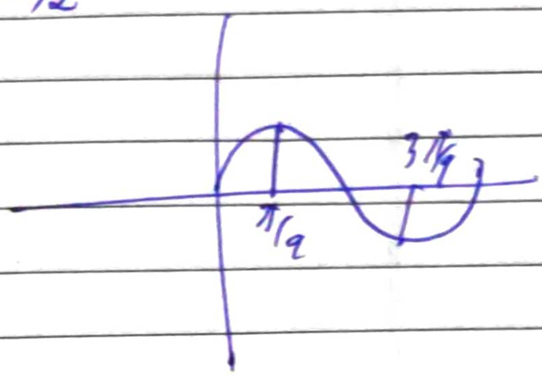
Que $f(x,y) = 5 - 4\sin x + y^2$; $0 < x < 2\pi$, $y \in R$

$f_x = -4\cos x = 0 \Rightarrow \cos x = 0$

$\Rightarrow x = \pi/2, 3\pi/2$

$f_y = 2y = 0 \Rightarrow y = 0$

$\left(\frac{\pi}{2}, 0\right)$ $\left(\frac{3\pi}{2}, 0\right)$



$f_{xx} = 4\sin x$

$f_{yy} = 2$

$$\begin{bmatrix} 4\sin x & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

Positive definite $\Rightarrow$ Minima