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Simple Harmonic Motion

* Periodic Motion : The motion which repeats itself after equal or definite interval of time is called periodic or harmonic motion.

* Oscillatory Motion : A body is set to possess oscillatory or vibratory motion if it moves back and forth repeatedly about the mean position. e.g. bar pendulum

* Simple harmonic motion :
Oscillatory motion in which restoring force is directly proportional to displacement.

* Restoring force :
When a particle is displaced from its equilibrium position a periodic force acts on it in such a direction so as to bring the particle to its equilibrium position.

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* Differential equation of SHM

Consider a block of mass (m) attatched to a spring. When the mass is pulled and left it oscillates. A restoring force (F) acts on its body.

$$F \propto x$$

$$F = -Kx \quad \text{— (1)}$$

where F = restoring force
x = displacement
K = proportionality constant
(elastic constant)

-ve sign indicates that the restoring force is acting opposite to the displacement.

from Newton's Second Law of Motion

$$F = ma$$

$$F = m\frac{d^2x}{dt^2} \quad \text{— (2)}$$

Equating (1) & (2)

$$m\frac{d^2x}{dt^2} = -Kx$$

$$\frac{d^2x}{dt^2} = \frac{-K}{m}x$$

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$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Put $\frac{k}{m} = \omega^2$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$ (differential equation of SHM)

where ω = angular frequency $= \sqrt{\frac{k}{m}}$

f = frequency $= \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

T = Time period $= \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$

* Characteristics of SHM

1) Displacement

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

General solution of this equation is

$$x = A\sin(\omega t + \phi)$$

where A = amplitude

x varies from A to $-A$

2) Velocity

$x = A\sin(\omega t + \phi)$

differentiating w.r.t. t

$$\frac{dx}{dt} = v = A\omega\cos(\omega t + \phi)$$

v varies from $-A\omega$ to $A\omega$

velocity is maximum at mean position

3) Acceleration

$v = A\omega\cos(\omega t + \phi)$

differentiating w.r.t. t

$$\frac{dv}{dt} = a = -A\omega^2\sin(\omega t + \phi)$$

$$a = -\omega^2\left[A\sin(\omega t + \phi)\right]$$

$$a = -\omega^2 x$$

acceleration is directly proportional to displacement.

4) Time Period : It is the time taken for one complete oscillation.

$$x = A\sin(\omega t + \phi)$$

$$t \rightarrow t + \frac{2\pi}{\omega}$$

$$x = A\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right]$$

$$x = A\sin\left[\omega t + 2\pi + \phi\right]$$

$$x = A\sin\left(2\pi + (\omega t + \phi)\right)$$

$$x = A\sin(\omega t + \phi)$$

This shows that after $\frac{2\pi}{\omega}$ time, the particle comes to its equilibrium position again.

Hence, Time Period $= \frac{2\pi}{\omega}$

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5) Frequency - Number of oscillations per second

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

6) Phase - Angle $\omega t + \phi$ is called phase of oscillation.

$$x = A\sin(\underbrace{\omega t + \phi}_{\text{phase}})$$

7) Epoch - Angle when $t = 0$

$$\text{Angle} = \omega t + \phi$$
$$t = 0$$

$$\text{Epoch} = \phi$$

E.g. If two waves are in ~~so~~ the same phase

$$x_1 = a_1 \sin \omega t$$
$$x_2 = a_2 \sin \omega t$$

epoch = 0

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8) Energy of SHM

For SHM, Total Energy = Kinetic Energy + Potential Energy

$$K.E. = \frac{1}{2}mv^2 \quad \cancel{= \frac{1}{2} m(A\sin[\omega t + \theta}$$

$$= \frac{1}{2} m \left[A^2\omega^2 \cos^2(\omega t + \phi)\right]$$

$$= \frac{1}{2} m \left[A^2\omega^2 \{1 - \sin^2(\omega t + \phi)\}\right]$$

$$= \frac{1}{2} m \left[A^2\omega^2 - \omega^2 x^2\right]$$

$$= \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$= \frac{1}{2} k (A^2 - x^2)$$

$$F = -\frac{dU}{dx}$$

$$-kx = \frac{-dU}{dx}$$

$$\int_0^x kx\,dx = \int dU$$

$$\left[\frac{Kx^2}{2}\right]_0^x = U$$

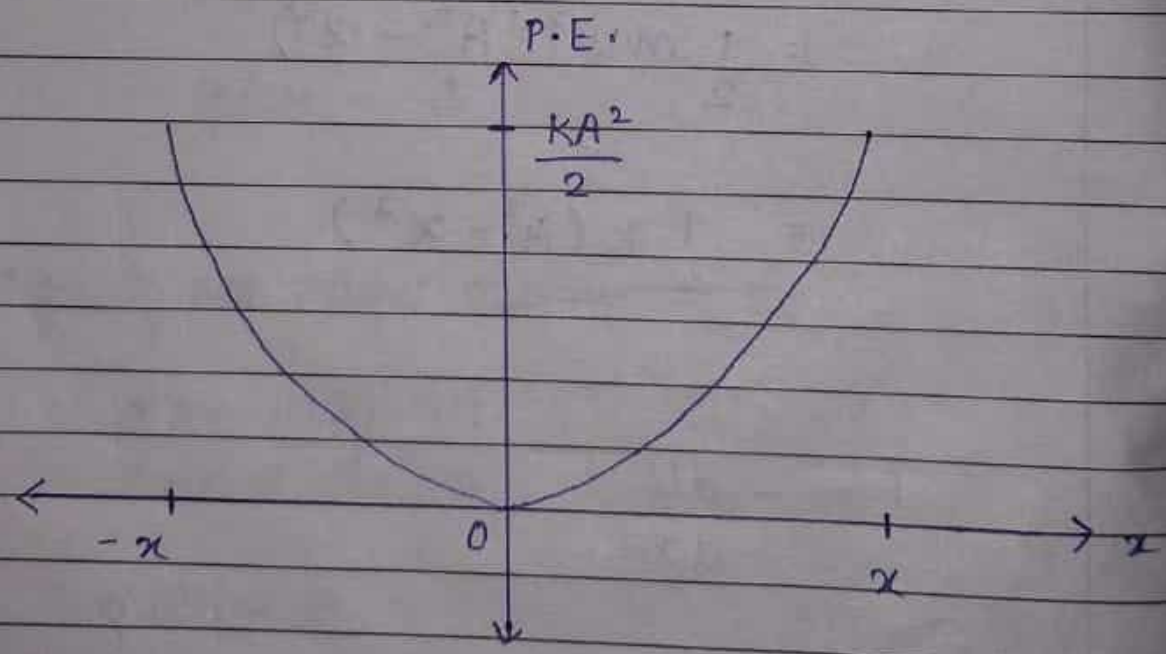
$$U = \frac{Kx^2}{2}$$

Total Energy = K.E. + U

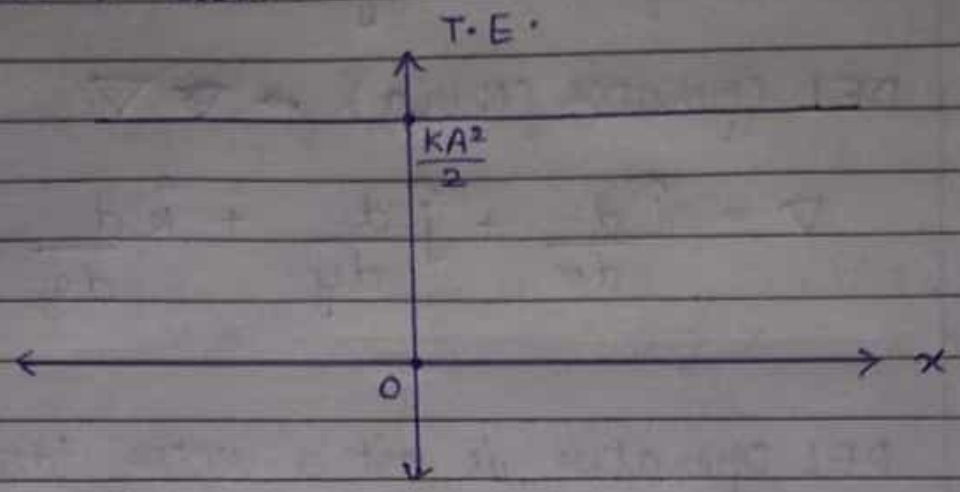
$$= \frac{1}{2}KA^2 - \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2$$

$$= \frac{1}{2}KA^2$$

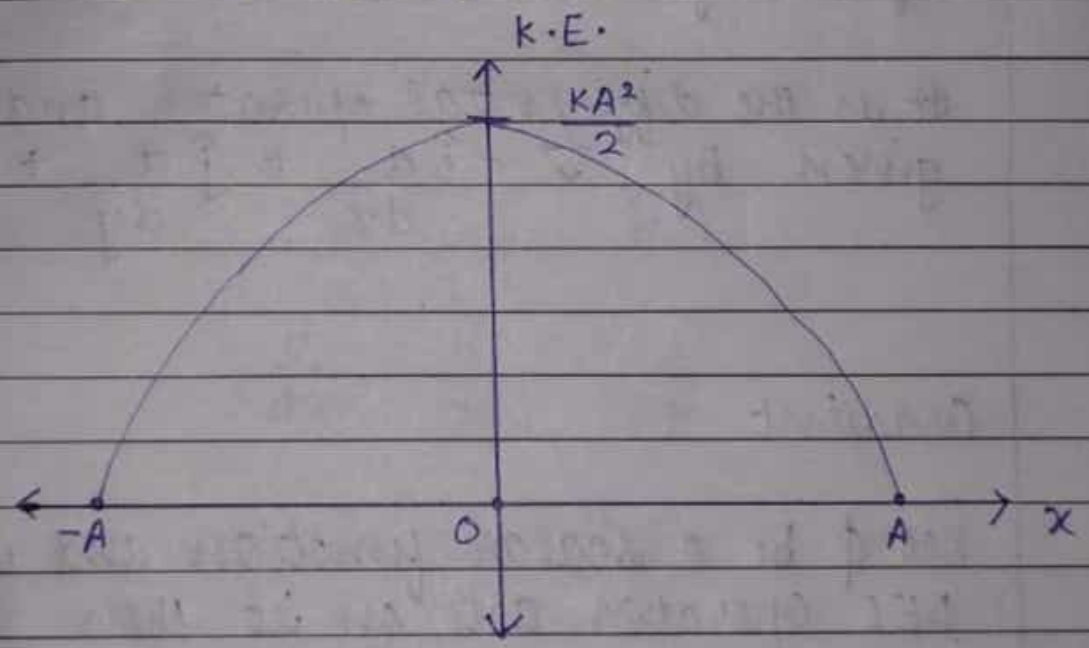
P.E. v/s x curve



T.E. v/s x curve



K.E. v/s x curve



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Electromagnetic Theory

DEL Operator (Nebla) → ∇

$$\nabla = \hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}$$

DEL Operator is not a vector itself but when it acts on a scalar quantity, it ~~cut~~ converts the scalar quantity into vector quantity.

It is the differential operator and is given by $\nabla = \hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}$

Gradient

Let ϕ be a scalar function and when DEL Operator acts on it then

$$\nabla\phi = \hat{i}\frac{d\phi}{dx} + \hat{j}\frac{d\phi}{dy} + \hat{k}\frac{d\phi}{dz}$$

and $\nabla\phi$ is known as gradient.

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Divergence

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\nabla \cdot \vec{A} = \left(\hat{i} \frac{d}{dx} + \hat{j} \frac{d}{dy} + \hat{k} \frac{d}{dz} \right) \cdot (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})$$

$$= \frac{dA_x}{dx} + \frac{dA_y}{dy} + \frac{dA_z}{dz}$$

Curl $\vec{A}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ A_x & A_y & A_z \end{vmatrix}$$

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* $\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$

$$\nabla \cdot \vec{A} = \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) \cdot (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z)$$

$$\text{div } \vec{A} = \frac{dA_x}{dx} + \frac{dA_y}{dy} + \frac{dA_z}{dz}$$

If $\text{div} \cdot \vec{A} = 0$

$\vec{A}$ is solenoidal vector

* $\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$

$$\vec{\nabla} \times \vec{A} = \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) \times (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z)$$

$$= \left(\frac{dA_z}{dy} - \frac{dA_y}{dz}\right)\hat{i} - \left(\frac{dA_z}{dx} - \frac{dA_x}{dz}\right)\hat{j}$$

$$+ \left(\frac{dA_y}{dx} - \frac{dA_x}{dy}\right)\hat{k}$$

If ~~$\vec{A} \times A$~~ $\vec{\nabla} \times \vec{A} = 0$

$\vec{A}$ is irrotational

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Q. If $\vec{A} = 2x\hat{i} + 2y\hat{j} + 3z\hat{k}$. Find curl of $\vec{A}$

$$\vec{\nabla} \times \vec{A} = \left(\frac{d(3z)}{dy} - \frac{d(2y)}{dz}\right)\hat{i} - \left(\frac{d(3z)}{dx} - \frac{d(2x)}{dz}\right)\hat{j}$$

$$+ \left(\frac{d(2y)}{dx} - \frac{d(2x)}{dy}\right)\hat{k}$$

$$= 0$$

$\vec{A}$ is irrotational

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Q. If $\vec{A} = y^2 \hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$

find div $\vec{A}$ at $(1,2,3)$

$$\nabla \cdot \vec{A} = \frac{d(y^2)}{dx} + \frac{d(2xy+z^2)}{dy} + \frac{2yz}{dz}$$

$$\text{div } \vec{A} = 0 + 2x + 2y$$

$$\text{div } \vec{A}\Big|_{(1,2,3)} = 2(1) + 2(2)$$

$$= 6$$

Q. If $\phi = 5xy - 3y^2z^3$ find $\nabla\phi$ at $(-1,2,-1)$

$$\nabla\phi = \frac{d(5xy - 3y^2z^3)}{dx}\hat{i} + \frac{d(5xy - 3y^2z^3)}{dy}\hat{j} + \frac{d(5xy - 3y^2z^3)}{dz}\hat{k}$$

$$= 5y\,\hat{i} + (5x - 6yz^3)\hat{j} + (-9y^2z^2)\hat{k}$$

$$\nabla\phi\Big|_{(-1,2,-1)} = 10\hat{i} + 7\hat{j} - 36\hat{k}$$

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Gauss Divergence Theorem

This theorem is used to convert volume integral into surface integral and vice versa

It states that volume integral of divergence of vector $\vec{A}$ over any volume V is equal to the surface integral of $\vec{A}$ taken over surface S.

$$\int_V \text{div}\, \vec{A}\, dV = \int_S \vec{A}\, dS$$

~~Stope's theorem~~ Stoke's Theorem

It converts line integral into surface integral and vice versa.

It states that surface integral of curl of vector $\vec{A}$ taken over the surface S is equal to line integral of $\vec{A}$ around the closed surface

$$\int_S \vec{\nabla} \times \vec{A}\, dS = \oint A \cdot dl$$

$$\int_S \text{curl}\, \vec{A}\, dS = \oint A \cdot dl$$

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Q. consider a $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Is the field solenoidal ?

$$\text{div } \vec{A} = \frac{d(x^2)}{dx} + \frac{d(y^2)}{dy} + \frac{d(z^2)}{dz}$$

$$= 2x + 2y + 2z$$

$$= 2(x+y+z)$$

No it is not solenoidal

(ii) Is the field irrotational ?

$$\vec{\nabla} \times \vec{A} = \left(\frac{d(z^2)}{dy} - \frac{d(y^2)}{dz}\right)\hat{i} - \left(\frac{d(z^2)}{dx} - \frac{d(x^2)}{dz}\right)\hat{j}$$

$$+ \left(\frac{d(y^2)}{dx} - \frac{d(x^2)}{dy}\right)\hat{k}$$

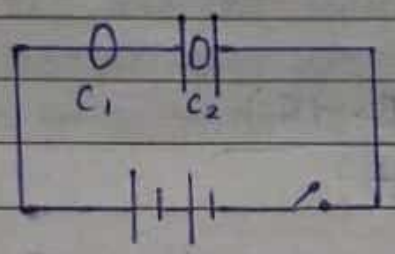
$$= 0$$

Yes it is irrotational

Ampere's Law

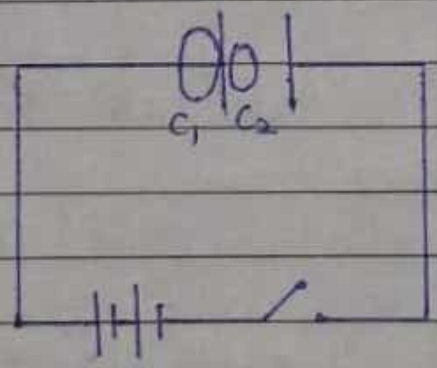
$$\oint B \cdot dl = \mu_0 i$$

line integral of magnetic field



$$\int_{C_1} B \cdot dl = \mu_0 i \quad \text{— (1)}$$

$$\int_{C_2} B \cdot dl = 0 \quad \text{— (2)}$$ (current doesn't flow through dilated material b/w capacitor)



If C_1 and C_2 are very small and close to each other hence

$$\int_{C_1} B \cdot dl = \int_{C_2} B \cdot dl = \mu_0 i \quad \text{— (3)}$$

eq^n ① & ② and ③ contradicts each other
Hence there is inconsistency in Ampere's Law

using Faraday's Law

$$e = -\frac{d\Phi_B}{dt}$$

$$E \cdot dl = -\frac{d(B \cdot dS)}{dt}$$

By symmetry

$$B \cdot dl = -\frac{d(E \cdot dS)}{dt}$$ ← displacement current

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Displacement current

To modify Ampere's Law Maxwell followed a symmetry condition.
By Faraday's Law, a change in magnetic field induces an electric field hence a change in electric field must induce magnetic field
Since current are the usual source of magnetic field it means this change in electric field is equivalent to a current named as displacement current.

It is defined as the current which comes into existence in addition to conduction/conventional current whenever the electric field or flux changes with time

$$i_d = \frac{\varepsilon_0 d\phi_E}{dt}$$

Ampere's Law modified

$$\int \vec{B} \cdot \vec{dl} = \mu_0 i_{total}$$

$$= \mu_0 [i_c + i_d]$$

$$\int B \cdot dl = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

(Modified Ampere's Law)

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Properties of displacement current

(i) It comes into existence when electric flux changes with time.

(ii) It is not a current in itself but it produces magnetic field that is why it is termed as a current.

(iii) It has the same dimension as of current.

Differences b/w conduction current and displacement current

Conduction Current	Displacement Current
It obeys Ohm's law.	It does not obey Ohm's Law.
It ~~d~~ can't exist in vacuum.	It can exist even in vacuum
It depends on potential difference and resistance.	It depends on change in electric field or electric flux.

$$i_c = \frac{dq}{dt}$$

$$i_d = \frac{\varepsilon_0 \, d\phi_\varepsilon}{dt}$$

$$= \frac{\varepsilon_0 \, d(E \cdot dS)}{dt}$$

$$= \frac{\varepsilon_0 \, d\left(\frac{q}{\varepsilon_0}\right)}{dt}$$

$$= \frac{dq}{dt}$$

$$= i_c$$

Equation of continuity

charge can neither be created nor be destroyed.

current through any surface S is the flux of current density vector

$$I = \int J \cdot dS \quad \text{— (1)}$$

$$I = \int J \cdot dS = -\frac{dq}{ds}$$

-ve sign denotes ~~it~~ that charge is flowing away from the surface.

$$I = \frac{-d}{dt}\int_V \rho \, dV$$

$$I = -\int_V \frac{d\rho}{dt} \, dV \quad \text{— (2)}$$

from (1) & (2)

$$\int \vec{J} \cdot dS = -\int_V \frac{d\rho}{dt} \, dV$$

Using Gauss divergence theorem in LHS

$$\int \vec{J} \cdot dS = \int div \cdot \vec{J} \cdot dV$$

$$\int div \, \vec{J} \cdot dV = -\int_V \frac{d\rho}{dt} \, dV$$

$$\int \left(div \, \vec{J} + \frac{d\rho}{dt} \right) dV$$

$$\text{div}\,\vec{J} + \frac{d\rho}{dt} = 0$$

Maxwell's equation in differential form

Maxwell's First equation (Gauss Law in electrostate)

~~It Gauss Law states that electric flux enclos~~
It states that electric flux through any closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface

$$\int \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

$$\left\{ q = \int \rho \, dV \right\}$$

$$\int \vec{E} \cdot d\vec{S} = \frac{\int \rho \, dV}{\varepsilon_0}$$

Using Gauss divergence theorem in LHS

$$\left\{ \int \vec{E} \cdot d\vec{S} = \int \text{div} \cdot \vec{E} \cdot d\vec{V} \right\}$$

$$\int \text{div} \cdot \vec{E} \, d\vec{V} = \frac{1}{\varepsilon_0} \int \rho \, dV$$

$$\int \left(\text{div} \cdot \vec{E} - \frac{\rho}{\varepsilon_0} \right) d\vec{V} = 0$$

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$$div \cdot \vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

$$div\, \vec{E} = \frac{\rho}{\varepsilon_0}$$

Maxwell's Second Equation (Gauss Law in magnetostat

It states that net magnetic flux through any closed surface is always zero.

$$\phi_B = 0$$

$$\int \vec{B} \cdot d\vec{S} = 0$$

Using Gauss divergence theorem in LHS

$$\left\{ \int \vec{B} \cdot d\vec{S} = \int div \cdot \vec{B}\, d\vec{v} \right\}$$

$$\int div \cdot \vec{B}\, d\vec{v} = 0$$

$$div \cdot \vec{B} = 0$$

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Maxwell's third equation (Faraday's Law of EMI)

~~It states that rate of change~~
It states that induces EMF around a closed circuit is equal to negative times of rate of change of magnetic flux linked with the circuit.

$$e = -\frac{d\phi_B}{dt}$$

$$\int \vec{E} \cdot d\vec{l} = -\int \frac{d}{dt} \vec{B} \cdot d\vec{S}$$

Using Stoke's theorem in LHS

$$\left\{ \oint \vec{E} \cdot d\vec{l} = \int \text{curl} \cdot \vec{E}\, d\vec{S} \right\}$$

$$\int \left(\text{curl} \cdot \vec{E} + \frac{d(\vec{B})}{dt} \right) \cdot d\vec{S} = 0$$

$$\text{curl} \cdot \vec{E} = -\frac{d(\vec{B})}{dt}$$

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Maxwell's fourth equation (modified Ampere's Law)

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\phi_\varepsilon}{dt}$$

$$i = \int \vec{J}\cdot\vec{dS} \qquad \phi_\varepsilon = \int \vec{E}\cdot\vec{dS}$$

$$\cancel{\oint \vec{B}\cdot\vec{dl} = \oint \mu_0 \int \vec{J}\cdot\vec{dS} + \mu_0\varepsilon_0 \int E\, d}$$

$$\cancel{\oint \vec{B}\cdot\vec{dl} = \mu_0\left(\int \vec{J}\cdot\vec{dS} + \varepsilon_0 \oint \frac{d}{dt}\int \vec{E}\cdot\vec{dS}\right)}$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\left(\int \vec{J}\cdot\vec{dS} + \int \varepsilon_0 \frac{d\vec{E}}{dt}\cdot\vec{dS}\right)$$

Using stoke's theorem in LHS

$$\left\{\oint \vec{B}\cdot\vec{dl} = \int \text{curl}\cdot\vec{B}\cdot\vec{dS}\right\}$$

$$\int \text{curl}\,\vec{B}\cdot\vec{dS} = \int\left(\mu_0\left[\vec{J} + \varepsilon_0\frac{d\vec{E}}{dt}\right]\right.\cdot\vec{dS}$$

$$\int\left(\text{curl}\,\vec{B} - \mu_0\left[\vec{J} + \varepsilon_0\frac{d\vec{E}}{dt}\right]\right)\cdot\vec{dS} = 0$$

$$\text{curl}\,\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0\frac{d\vec{E}}{dt}\right]$$

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$\rightarrow \text{div } \vec{E} = \dfrac{\rho}{\varepsilon_0}$

$\rightarrow \text{div } \vec{B} = 0$

$\rightarrow \text{curl } \vec{E} = -\dfrac{d(\vec{B})}{dt}$

$\rightarrow \text{curl } \vec{B} = \mu_0 \left[\vec{J} + \varepsilon_0 \dfrac{d\vec{E}}{dt} \right]$

$\vec{D} = \varepsilon_0 \vec{E}$

$\vec{B} = \mu_0 \vec{H}$

eq^n ① $\rightarrow$ $\text{div } \vec{D} = \rho$

eq^n ④ $\rightarrow$ $\text{curl } \vec{H} = \vec{J} + \dfrac{d\vec{D}}{dt}$

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Equation of continuity from Maxwell's eq^n

Maxwell's 4th eq^n

$$\text{curl } \vec{B} = \mu_0 \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \right]$$

Taking divergence on both sides

$$\text{div}(\text{curl } \vec{B}) = \mu_0 (\text{div} \cdot \vec{J}) + \mu_0 \varepsilon_0 \frac{d(\text{div} \cdot \vec{E})}{dt}$$

$$0 = \mu_0 \left[\text{div } \vec{J} + \varepsilon_0 \frac{d(\text{div } \vec{E})}{dt} \right]$$

$$0 = \mu_0 \left[\text{div } \vec{J} + \varepsilon_0 \frac{d}{dt}\left(\frac{\rho}{\varepsilon_0}\right) \right]$$

$$0 = \mu_0 \left[\text{div } \vec{J} + \frac{d\rho}{dt} \right]$$

$$\text{div} \cdot \vec{J} + \frac{d\rho}{dt} = 0$$

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Maxwell's equation in integral form

Maxwell's 1st eq^n * in integral form

$$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$$

Integrating the eq^n over the volume V

$$\int_V \text{div}\cdot\vec{E}\,dv = \int_V \frac{\rho\,dV}{\varepsilon_0}$$

Using Gauss divergence theorem in LHS

$$\left\{ \int_V \text{div}\cdot\vec{E}\,dV = \int_S \vec{E}\cdot d\vec{S} \right\}$$

$$\int_S \vec{E}\cdot d\vec{S} = \int_V \frac{\rho\,dV}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

Physical significance: Net outward electric flux through a surface enclosing a volume V is equal to $\frac{1}{\varepsilon_0}$ times the net charge within that volume.

OR

$$\int_S \vec{D}\cdot d\vec{S} = \int_V \rho\,dV$$

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Maxwell's 2nd eq^n in integral form

$$div \cdot \vec{B} = 0$$

Integrating the eq^n over the volume V

$$\int_V div \cdot \vec{B} \cdot dv = 0$$

Using Gauss divergence theorem in LHS

$$\left\{ \int_V div \cdot \vec{B} \cdot dv = \int_S \vec{B} \cdot d\vec{S} \right\}$$

$$\int_S \vec{B} \cdot d\vec{S} = 0$$

Physical significance : Net magnetic flux through any closed surface is always zero

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Maxwell's 3rd eq^n in integral form :

$$\text{curl } \vec{E} = -\frac{d\vec{B}}{dt}$$

Integrating the eq^n over the surface S -

$$\int_S \text{curl } \vec{E} \cdot d\vec{S} = -\int_S \frac{d\vec{B}}{dt} \cdot d\vec{S}$$

Using Stoke's theorem in LHS

$$\left\{ \int_S \text{curl } \vec{E} \cdot d\vec{S} = \int \vec{E} \cdot dl \right\}$$

$$\int \vec{E} \cdot dl = -\int_S \frac{d\vec{B}}{dt} \cdot d\vec{S}$$

Physical significance : Electromotive force around a closed path is equal to negative times rate of change of magnetic flux linked with the surface.

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Maxwell's 4th eq^n in integral form

$$\text{curl}\,\vec{B} = \mu_0 \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \right]$$

Integrating the eq^n over the surface s

$$\int_S \text{curl}\,\vec{B} \cdot d\vec{S} = \int_S \mu_0 \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \right] \cdot d\vec{S}$$

Using Stoke's theorem in LHS

$$\left\{ \int_S \text{curl}\,\vec{B} \cdot d\vec{S} = \int \vec{B} \cdot dl \right\}$$

$$\int \vec{B} \cdot dl = \mu_0 \left[\int_S \vec{J} \cdot d\vec{S} + \varepsilon_0 \int_S \frac{d\vec{E}}{dt} \cdot d\vec{S} \right]$$

OR

$$\int \vec{H} \cdot dl = \int_S \vec{J} \cdot d\vec{S} + \int_S \frac{d\vec{D}}{dt} \cdot d\vec{S}$$

~~mg~~ magnetomotive force

Physical significance = Magnetomotive force around a closed path is equal to sum of conduction current and displacement current.

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~~Mas~~ Poynting vector ($\vec{S}$)

Whenever the electromagnetic waves propagate in space, they transport electric & magnetic energy from point to point.

Rate of flow of energy per unit area in a plane electromagnetic wave can be described by a vector known as poynting vector.

SI unit = $\frac{\text{Watt}}{m^2}$

Direction → in which wave propagate

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

OR

$$\vec{S} = \vec{E} \times \vec{H}$$

Poynting Theorem

It states that rate of decrease of electromagnetic energy in a ~~cert~~ certain volume V is equal to work done by the field forces per unit time + power transferred into the field.

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Maxwell's eq^n in diff. form

1st eq^n → $\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$ — (a)

2nd eq^n → $\text{div}\,\vec{B} = 0$ — (b)

3rd eq^n → $\text{curl}\cdot\vec{E} = -\frac{d(\vec{B})}{dt}$ — (c)

4th eq^n → $\text{curl}\cdot\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0\frac{d(\vec{E})}{dt}\right]$ — (d)

Taking dot product with $\vec{H}$ in (i) → (c) and with E in (i) (d)

$$\vec{H}\cdot\text{curl}\,\vec{E} = -\vec{H}\cdot\frac{d(\vec{B})}{dt} \quad \text{— (ii)}$$

$$\vec{E}\cdot\text{curl}\,\vec{B} = \mu_0\left[\vec{E}\cdot\vec{J} + \varepsilon_0\vec{E}\cdot\frac{d(\vec{E})}{dt}\right]$$

OR

$$\vec{E}\cdot\text{curl}\,\vec{H} = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{d(\vec{D})}{dt} \quad \text{— (iii)}$$

(ii) − (iii)

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$$\vec{H}\cdot \text{curl}\,\vec{E} - \vec{E}\cdot \text{curl}\,\vec{H} = -\left(\vec{H}\cdot\frac{d(\vec{B})}{dt} + \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{d(\vec{D})}{dt}\right)$$

$$\text{div}\,(\vec{E}\times\vec{H}) = -\left\{\vec{H}\cdot\frac{d(\vec{B})}{dt} + \vec{E}\cdot\frac{d(\vec{D})}{dt}\right\} - \vec{E}\cdot\vec{J}$$

$$\left[\vec{H}\cdot\frac{d(\vec{B})}{dt} = \vec{H}\cdot\frac{d(\mu H)}{dt} = \frac{1}{2}\mu\frac{d(H^2)}{dt}\right]$$

$$\left[\vec{E}\cdot\frac{d(\vec{D})}{dt} = \vec{E}\cdot\frac{d(\varepsilon\vec{E})}{dt} = \frac{1}{2}\varepsilon\frac{d(E^2)}{dt}\right]$$

$$\text{div}\,(\vec{E}\times\vec{H}) = -\left\{\frac{1}{2}\mu\frac{d(H^2)}{dt} + \frac{1}{2}\varepsilon\frac{d(E^2)}{dt}\right\} - \vec{E}\cdot\vec{J}$$

Integrating this eq^n over the vol. 'V'

$$\int \text{div}\,(\vec{E}\times\vec{H})\,dV = -\left[\int\left\{\frac{1}{2}\mu\frac{d(H^2)}{dt} + \frac{1}{2}\varepsilon\frac{d(E^2)}{dt}\right\}dV\right] - \int\vec{E}\cdot\vec{J}\,dV$$

Using gauss divergence theorem in LHS

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$$\left\{\int \operatorname{div}(\vec{E}\times\vec{H})\,dV = \int (\vec{E}\times\vec{H})\cdot d\vec{S}\right\}$$

$$\int (\vec{E}\times\vec{H})\cdot d\vec{S} = -\int\left\{\frac{1}{2}\frac{d(\mu H^2)}{dt} + \frac{1}{2}\frac{d(\varepsilon E^2)}{dt}\right\}$$

$$-\int \vec{E}\cdot\vec{J}\,dV$$

Interpretation of each term

$-\int \vec{E}\cdot\vec{J}\,dV$ = It represents the rate of ~~doing~~ ~~work~~ energy transferred into the ~~elec~~ electromagnetic field through the motion of free charges

$\int -\frac{d}{dt}\left\{\mu H^2 + \varepsilon E^2\right\}$ = It represents the rate of change of electromagnetic field energy

$\int (\vec{E}\times\vec{H})\cdot d\vec{S}$ = It represents the electromagnetic energy crossing the closed surface per second

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Electromagnetic waves in free space

Maxwell's eq^n in differential form

$$\left.\begin{aligned}
&\rightarrow \operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{— (a)} \\
&\rightarrow \operatorname{div} \vec{B} = 0 \quad \text{— (b)} \\
&\rightarrow \operatorname{curl} \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{— (c)} \\
&\rightarrow \operatorname{curl} \vec{B} = \mu_0 \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \right] \quad \text{— (d)}
\end{aligned}\right\} \text{(1)}$$

condition of free space

$q = 0$, $\rho = 0$

$i = 0$, $\vec{J} = 0$

- Therefore Maxwell's eq^n in free space becomes

$$\left.\begin{aligned}
&\rightarrow \operatorname{div} \vec{E} = 0 \quad \text{— (a)} \\
&\rightarrow \operatorname{div} \vec{B} = 0 \quad \text{— (b)} \\
&\rightarrow \operatorname{curl} \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{— (c)} \\
&\rightarrow \operatorname{curl} \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \quad \text{— (d)}
\end{aligned}\right\} \text{(2)}$$

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Taking curl of 2(c)

$$\text{curl}(\text{curl}\,\vec{E}) = -\frac{d(\text{curl}\,\vec{B})}{dt}$$

$$\downarrow$$

$$\underbrace{\text{gradient}\,(\text{div}\,\vec{E})}_{0} - \nabla^2 \vec{E} = -\frac{d(\text{curl}\,\vec{B})}{dt}$$

$$-\nabla^2 \vec{E} = -\frac{d(\text{curl}\,\vec{B})}{dt}$$

$$\nabla^2 \vec{E} = \frac{d(\text{curl}\,\vec{B})}{dt}$$

$$\nabla^2 \vec{E} = \frac{d\left(\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}\right)}{dt}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} \quad \text{— (3)}$$

(Wave eq^n for electric field in free space)

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Taking curl of eq^n 2 (d)

$$\text{curl}(\text{curl}\,\vec{B}) = \mu_0 \varepsilon_0 \frac{d(\text{curl}\,\vec{E})}{dt}$$

$$\downarrow$$

$$\text{gradient}\,(\underbrace{\text{div}\,\vec{B}}_{0}) - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{d(\text{curl}\,\vec{E})}{dt}$$

$$-\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{d(\text{curl}\,\vec{E})}{dt}$$

$$-\nabla^2 \vec{B} = -\mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2}$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} \quad \text{— (4)}$$

(wave eq^n for magnetic field in free space)

Therefore general eq^n for electromagnetic waves in free space

$$\nabla^2 U - \mu_0 \varepsilon_0 \frac{d^2 U}{dt^2} = 0 \quad \text{— (5)}$$

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General wave eq^n in free space

$$\nabla^2 U - \frac{1}{v^2}\frac{d^2U}{dt^2} = 0 \quad \text{—} (6)$$

comparing (5) & (6)

$$\frac{1}{v^2} = \mu_0 \varepsilon_0$$

$$v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{4\pi}{(4\pi\varepsilon_0)\mu_0}}$$

$$v = \sqrt{9 \times 10^9 \times 10^{-7}}$$

$$v = 3 \times 10^8 \text{ m/s}$$

$$v = c$$

Speed of electromagnetic wave in free space is equal to speed of light

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Transverse nature of EMW

Wave eq^n of electric field in free space

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0 \quad — (3)$$

Wave eq^n of magnetic field in free space

$$\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0 \quad — (4)$$

General ~~eq^n~~ sol^n for eq^n (3) & (4)

$$\vec{E} = E_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

E_0 = complex amplitude for Electric field

$\vec{k} = k\hat{n}$ (Propagation vector)

$\vec{k} = \frac{2\pi}{\lambda}\hat{n}$

$\vec{r}$ = position vector $= x\hat{i} + y\hat{j} + z\hat{k}$

ω = angular frequency $= 2\pi\nu$

$$\Rightarrow \vec{B} = B_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

B_0 = complex amplitude for Magnetic field

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$div \cdot \vec{E} = 0$ — 2(a)

$\nabla \cdot \vec{E} = 0$

$i(\vec{k} \cdot \vec{E}) = 0$

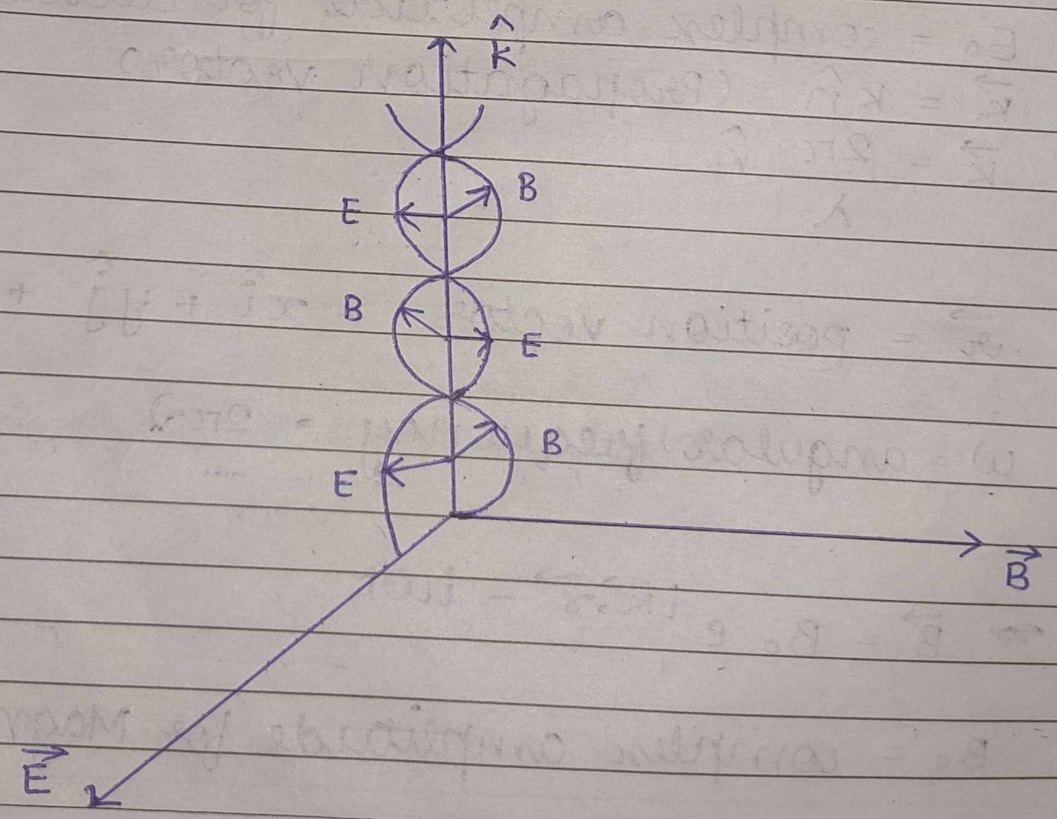
similarly, $\nabla \cdot \vec{B} = 0$ $\quad div \cdot \vec{B} = 0$ — 2(

$i(\vec{k} \cdot \vec{B}) = 0$

$0 = \vec{k} \cdot \vec{E} \Rightarrow \vec{k} \perp \vec{E}$

$0 = \vec{k} \cdot \vec{B} \Rightarrow \vec{k} \perp \vec{B}$

Electric and magnetic field vector is $\perp$r to direction of propogation vector means EMW are transverse in nature.



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Orthogonal nature of EMW

$$\text{curl}\,\vec{E} = \frac{-d\vec{B}}{dt} \quad \text{— } 2(c)$$

$$\text{curl}\,\vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \quad \text{— } 2(d)$$

General sol^n for eq^n 2(c) ~~& 2(d)~~

$$i(\vec{k} \times \vec{E}) = \frac{-d}{dt}\left\{B_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}\right\}$$

$$i(\vec{k} \times \vec{E}) = i\omega \vec{B}$$

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

Similarly general sol^n for eq^n 2(d)

$$i(\vec{k} \times \vec{B}) = -i\mu_0 \varepsilon_0 \omega \vec{E}$$

$$\vec{k} \times \vec{B} = -\mu_0 \varepsilon_0 \omega \vec{E}$$

Electric and magnetic field vector are mutually perpendicular to each other and also perpendical to the direction of propagation vector.

Hence we conclude that $\vec{E}$, $\vec{B}$ and $\vec{k}$ are mutually perpendicular to each other.

EMW is orthogonal in nature.

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Impedence / Wave Impedence / Characteristic Impedence / Intrinsic Impedence

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$\omega \vec{B} = k(\hat{n} \times \vec{E})$$

$$\vec{B} = \frac{k}{\omega}(\hat{n} \times \vec{E})$$

$$\vec{H} = \frac{k}{\omega \mu_0}(\hat{n} \times \vec{E}) \qquad \{ \vec{B} = \mu_0 \vec{H} \}$$

$$\vec{H} = \frac{\frac{2\pi}{\lambda}}{2\pi \nu \mu_0}(\hat{n} \times \vec{E})$$

$$\vec{H} = \frac{1}{\lambda \nu \mu_0}(\hat{n} \times \vec{E})$$

$$\vec{H} = \frac{1}{\lambda \frac{c}{\lambda} \mu_0}(\hat{n} \times \vec{E})$$

$$\vec{H} = \frac{1}{\mu_0 c}(\hat{n} \times \vec{E})$$

$$\vec{H} = \frac{1}{\mu_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}(\hat{n} \times \vec{E})$$

$$\vec{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,(\hat{n} \times \vec{E})$$

$$\frac{\vec{E}}{\vec{H}} = \cancel{\sqrt{\frac{\varepsilon_0}{\mu}}}\,\sqrt{\frac{\mu_0}{\varepsilon_0}}\,\hat{n}$$

$$\left|\frac{\vec{E}}{\vec{H}}\right| = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\frac{\vec{E}}{\vec{H}} = 376.72\ \Omega$$

$$Z_0 \approx 377\ \Omega$$

Poynting vector

Rate of flow of energy in free space.

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{\vec{E} \times (\hat{n} \times \vec{E})}{\mu_0 c}$$

$$= \frac{1}{\mu_0 c}\left\{(\vec{E}\cdot\vec{E})\cdot\hat{n} - (\vec{E}\cdot\hat{n})\cdot\vec{E}\right\}$$

$$= \frac{1}{\mu_0 c}\left\{E^2\cdot\hat{n} - 0\right\}$$

$$= \frac{1}{\mu_0 c} E^2 \cdot \hat{n}$$

$$\vec{S} = E^2 \cdot \hat{n} \frac{1}{\mu_0 \sqrt{\mu_0 \varepsilon_0}} \quad \frac{1}{\mu_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$$= E^2 \cdot \hat{n} \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

$$\vec{S} = \frac{E^2 \hat{n}}{Z_0}$$

Poynting vector propogate along the direction of $\vec{K}$.

Avg. value of Poynting vector

$$\langle S \rangle = \frac{1}{Z_0} \langle E^2 \rangle \hat{n}$$

$$= \frac{1}{Z_0} \langle E_0 e^{i\vec{K}\vec{r} - i\omega t} \rangle^2 \hat{n}$$

$$= \frac{1}{Z_0} E_0^2 \langle e^{i\vec{K}\cdot\vec{r} - i\omega t} \rangle^2 \hat{n}$$

$$= \frac{1}{Z_0} E_0^2 \cdot \frac{1}{2} \hat{n}$$

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$$\langle s \rangle = \frac{1}{Z_0} E^2_{rms} \hat{n}$$

Ratio of electrostatic and magnetostatic energy density.

$$\frac{u_e}{u_m} = \frac{\frac{1}{2} \varepsilon_0 E^2}{\frac{1}{2} \mu_0 H^2}$$

$$= \frac{\varepsilon_0}{\mu_0} \left(\frac{E}{H}\right)^2$$

$$= \frac{\varepsilon_0}{\mu_0} Z_0^2$$

$$= \frac{\varepsilon_0}{\mu_0} \frac{\mu_0}{\varepsilon_0}$$

$$\frac{u_e}{u_m} = 1$$

$$u_e = u_m$$

Electrostatic ~~mag~~ energy density is equal to magnetostatic energy density

Total Energy Density

$$u = u_e + u_m$$

$$= 2u_e$$
$$= \varepsilon_0 E^2$$

Avg value of Total Energy ~~AA~~ over one complete cycle

$$\langle u \rangle = \varepsilon_0 \langle E \rangle^2$$

$$= \varepsilon_0 \langle E_0 e^{i\vec{k}\cdot\vec{r} - i\omega t} \rangle^2$$

$$= \varepsilon_0 E_0^2 \langle e^{i\vec{k}\cdot\vec{r} - i\omega t} \rangle^2$$

$$= \varepsilon_0 E_0^2 \frac{1}{2}$$

$$= \frac{\varepsilon_0 E_0^2}{2}$$

$$= \varepsilon_0 E_{rms}^2$$

$$\frac{\langle S \rangle}{\langle u \rangle} = \frac{1}{Z_0} \frac{\cancel{E^2_{rms}}}{\varepsilon_0 \cancel{E^2_{rms}}} \hat{n}$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}} \times \frac{1}{\varepsilon_0} \hat{n}$$

$$= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \hat{n}$$

$$= c\,\hat{n}$$

~~Ratio~~ Ratio of avg value of poynting vector and avg value of total energy is equal to speed of light.

Q. EMW intensity recieved on the surface of earth from the sun is 1.33 K.Watt/m^2. Find the amplitude of electric field vector associated with sunlight as recieved on earth's surface

$$S = 1330 \frac{\text{Watt}}{m^2}$$

$$E \cdot H = 1330 \quad \text{— (1)}$$

$$\frac{E}{H} = 377 \quad \text{— (2)}$$

eq^n (1) × (2)

$$E^2 = 1330 \times 377$$

$$E = \sqrt{501410}$$

$$E = 708.1 \frac{\text{Volt}}{\text{meter}}$$

$$E_0 = 708.1\sqrt{2} \frac{\text{Volt}}{\text{meter}}$$

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Q. If earth recieves ~~2 cal/min~~ 2 cal min^{-1} cm^{-2}. What are the amplitudes of electric and magnetic field.

$$S = \frac{2 \times 4.18 \times 10^4}{60} \ W/m^2$$

$$S = 1393.3$$

$$E.H = 1393.3 \approx 1400 \ — (1)$$

$$\frac{E}{H} = 377 \ — (2)$$

eq^n (1) x (2)

$$E^2 = 527800$$

$$E = 726.5 \ v/m$$

$$E_0 = 726.5\sqrt{2} \ v/m$$

$$H = 1.93$$

$$B = \mu_0 H$$

$$B = 4\pi \times 10^{-7} \times 1.93$$