Important Questions of UNIT - III

① Find the rank of the following matrices by elementary transformation (Echelon form)

(a) $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$

Ans 3

(b) $\begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

Ans. 3

② Reduce the following matrix into normal form and hence find its rank

(a) $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

Ans 3

(b) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$

Ans 3

③ Find the inverse of following matrices by elementary transformation

(i) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Ans $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

Ans $\frac{1}{5}\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

④ Investigate for what value of d and μ do the system of eqn $x+y+z=6$, $x+2y+3z=10$, $x+2y+dz=\mu$ have (i) no solution (ii) unique solution (iii) infinite solution.

⑤ Find the value of k for which the system of eqn $(3k-8)x+3y+3z=0$, $3k+(3k-8)y+3z=0$, $3x+3y+(3k-8)z=0$ has non-trivial soln. Ans $k=\frac{2}{3}, \frac{11}{3}, \frac{11}{3}$

6. Determine eigen values & eigen vector of

(a) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (b) $-3, -3, 5$ $K_1\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + K_2\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

Ans $K_3\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ Ans $3, 2, 5$ $K_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, K_2\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, K_3\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ Ans $-2, 6, 3$ $K_1\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, K_2\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, K_3\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

7. Verify cayley Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Hence compute A^{-1} also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$

Ans $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 3 \\ -1 & 1 & 3 \end{bmatrix}$, $5A - I = \begin{bmatrix} 9 & -5 & 5 \\ -5 & 9 & -5 \\ 5 & -5 & 9 \end{bmatrix}$

8. Show that the vector $x_1 = (1, -1, 1)$ / $x_2 = (2, 1, 1)$ $x_3 = (3, 0, 2)$ are linearly dependent find the relation b/w them. $x_1 + x_2 = x_3$

9. Examine the vectors $x_1 = (3, 1, 1)$ $x_2 = (2, 0, 1)$ $x_3 = (4, 2, 1)$ are linearly independent.

10. Reduce the matrix $P = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to diagonal form

11. Test the consistency & Hence solve the system of eq^n $3x + 3y + 2z = 1$, $x + 2y = 4$, $10y + 3z = -2$, $2x - 3y - z = 5$.

Ans $x = 2$, $y = 1$, $z = -4$