in later operations and to ensure that only good components are assembled in the product; and some form of final inspection is used on the finished units to ensure that only the highest quality product is delivered to the customer.

QUANTITATIVE ANALYSIS OF INSPECTION

Mathematical models can be developed to analyze certain performance aspects of production and inspection. In this section, we examine three areas: (1) effect of defect rate on production quantities in a series of production operations, (2) final inspection versus distributed inspection, and (3) when to inspect and when not to inspect.

22.5.1 Effect of Defect Rate in Serial Production

Let us define the basic element in the analysis as the unit operation for a manufacturing process, illustrated in Figure 22.7. In the figure, the process is depicted by a node, the input to which is a starting quantity of raw material. Let Q_o = the starting quantity or batch size to be processed. The process has a certain fraction defect rate q (stated another way, q = probability of producing a defective piece each cycle of operation), so the quantity of good pieces produced is diminished in size as follows:

$$Q = Q_o(1 - q) \tag{22.2}$$

where Q = quantity of good products made in the process, Q_o = original or starting quantity, and q = fraction defect rate. The number of defects is given by:

$$D = Q_o q \tag{22.3}$$

where D = number of defects made in the process.

Most manufactured parts require more than one processing operation. The operations are performed in sequence on the parts, as depicted in Figure 22.8. Each process has a fraction-

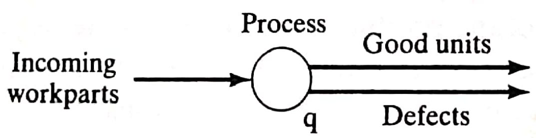

Figure 22.7 The unit operation for a manufacturing process, represented as an input-output model in which the process has a certain fraction defect rate.

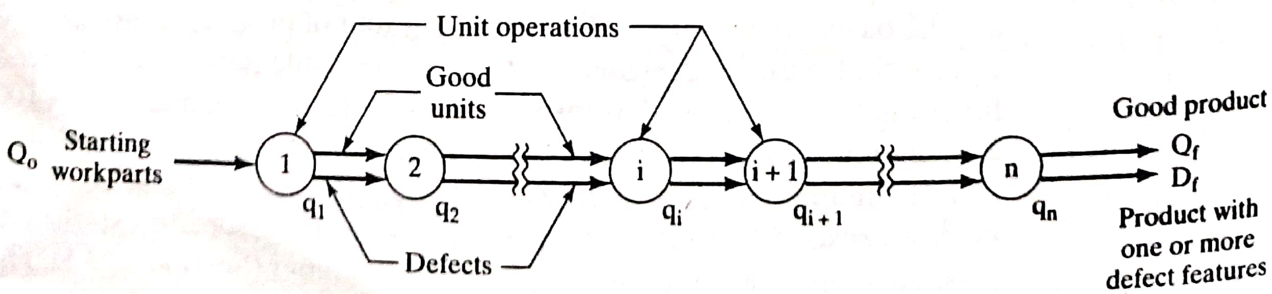

Figure 22.8 A sequence of n unit operations used to produce a part. Each process has a certain fraction defect rate.

tion defect rate q_i, so the final quantity of defect-free parts made by a sequence of n unit operations is given by:

$$Q_f = Q_o \prod_{i=1}^{n} (1 - q_i) \tag{22.4}$$

where Q_f = final quantity of defect-free units produced by the sequence of n processing operations, and Q_o is the starting quantity. If all q_i are equal, which is unlikely but nevertheless convenient for conceptualization and computation, then the preceding equation becomes:

$$Q_f = Q_o(1 - q)^n \tag{22.5}$$

where q = fraction defect rate for all n processing operations. The total number of defects produced by the sequence is most easily computed as:

$$D_f = Q_o - Q_f \tag{22.6}$$

where D_f = total number of defects produced.

EXAMPLE 22.2 Compounding Effect of Defect Rate in a Sequence of Operations

A batch of 1000 raw work units is processed through ten operations, each of which has a fraction defect rate of 0.05. How many defect-free units and how many defects are in the final batch?

Solution: Eq. (22.5) can be used to determine the quantity of defect-free units in the final batch.

$$Q_f = 1000(1 - .05)^{10} = 1000(0.95)^{10} = 1000\,(0.59874) = 599 \text{ good units}$$

The number of defects is given by Eq. (22.6):

$$D_f = 1000 - 599 = 401 \text{ defective units.}$$

The binomial expansion can be used to determine the allocation of defects associated with each processing operation i. Given that q_i = probability of a defect being produced in operation i, let p_i = probability of a good unit being produced in the sequence; thus, $p_i + q_i = 1$. Expanding this for n operations, we have

$$\prod_{i=1}^{n} (p_i + q_i) = 1 \tag{22.7}$$

To illustrate, consider the case of two operations in sequence ($n = 2$). The binomial expansion yields the following expression:

$$(p_1 + q_1)(p_2 + q_2) = p_1 p_2 + p_1 q_2 + p_2 q_1 + q_1 q_2$$

where $p_1 p_2$ = proportion of defect-free parts, $p_1 q_2$ = proportion of parts that have no defects from operation 1 but a defect from operation 2, $p_2 q_1$ = proportion of parts that have no defects from operation 2 but a defect from operation 1, and $q_1 q_2$ = proportion of parts that have both types of defect.

22.5.2 Final Inspection vs. Distributed Inspection

The preceding model portrays a sequence of operations, each with its own fraction defect rate, whose output forms a distribution of parts possessing either (1) no defects or (2) one or more defects, depending on how the defect rates from the different unit operations combine. The model makes no provision for separating the good units from the defects; thus, the final output is a mixture of the two categories. This is a problem. To deal with the problem, let us expand our model to include inspection operations, either one final inspection at the end of the sequence or distributed inspection, in which each production step is followed by an inspection.

Final Inspection. In the first case, one final inspection and sortation operation is located at the end of the production sequence, as represented by the square in Figure 22.9. In this case, the output of the process is 100% inspected to identify and separate defective units. The inspection screen is assumed to be 100% accurate, meaning that there are no Type I or Type II inspection errors.

The probabilities in this new arrangement are pretty much the same as before. Defects are still produced. The difference is that the defective units D_f have been completely and accurately isolated from the good units Q_f by the final inspection procedure. Obviously, there is a cost associated with the inspection and sortation operation that is added to the regular cost of processing. The costs of processing and then sorting a batch of Q_o parts as indicated in Figure 22.9 can be expressed as follows:

$$C_b = Q_o \sum_{i=1}^{n} C_{pri} + Q_o C_{sf} = Q_o\left(\sum_{i=1}^{n} C_{pri} + C_{sf}\right) \tag{22.8}$$

where C_b = cost of processing and sorting the batch, Q_o = number of parts in the starting batch, C_{pri} = cost of processing a part at operation i, and C_{sf} = cost of the final inspection and sortation per part. The processing cost C_{pr} is applicable to every unit for each of the n operations; hence the summation from 1 to n. The final inspection is done once for each unit. We have neglected consideration of material cost. For the special case in which every processing cost is equal $(C_{pri} = C_{pr}$ for all $i)$, we have

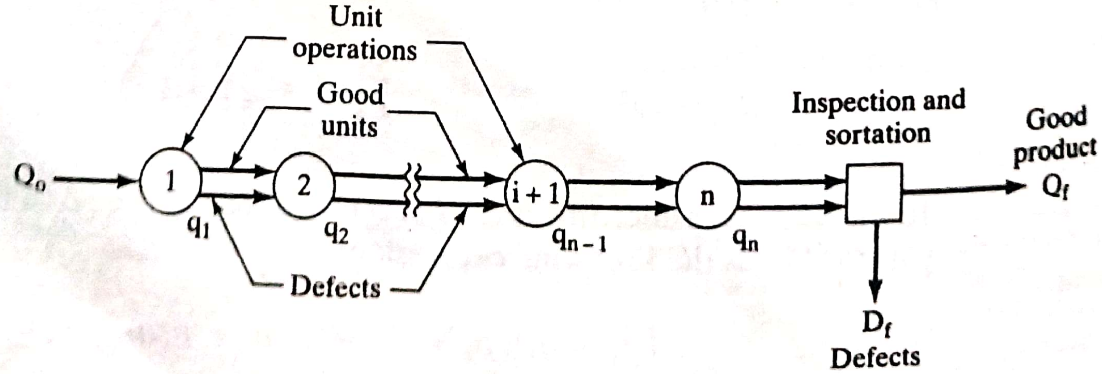

Figure 22.9 A sequence of n unit operations with one final inspection and sortation operation to separate the defects.

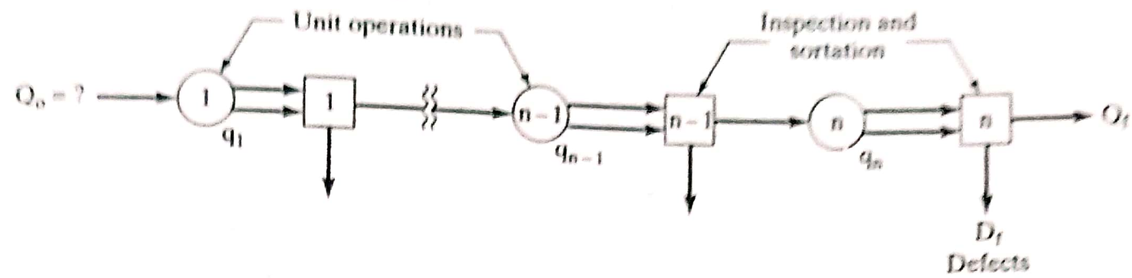

Figure 22.10 Distributed inspection, consisting of a sequence of unit operations with an inspection and sortation after each operation.

$$C_b = Q_o(nC_{pr} + C_{sf}) \tag{22.9}$$

Note that the fraction defect rate does not figure into total cost in either of these equations, since no defective units are sorted from the batch until after the final processing operation. Therefore, every unit in Q_o is processed through all operations, whether it is good or defective, and every unit is inspected and sorted.

Distributed Inspection. Next, let us consider a distributed inspection strategy, in which every operation in the sequence is followed by an inspection and sortation step, as seen in Figure 22.10. In this arrangement, the defects produced in each processing step are sorted from the batch immediately after they are made, so that only good parts are permitted to advance to the next operation. In this way, no defective units are processed in subsequent operations, thereby saving the processing cost of those units. Our model of distributed inspection must take the defect rate at each operation into account as follows:

$$C_b = Q_o(C_{pr1} + C_{s1}) + Q_o(1 - q_1)(C_{pr2} + C_{s2}) + Q_o(1 - q_1)(1 - q_2)(C_{pr3} + C_{s3}) + \ldots$$
$$+ Q_o\prod_{i=1}^{n-1}(1 - q_i)(C_{prn} + C_{sn}) \tag{22.10}$$

where $C_{s1}, C_{s2}, \ldots, C_{si}, \ldots, C_{sn}$ = costs of inspection and sortation at each station, respectively. In the special case where $q_i = q$, $C_{pri} = C_{pr}$, and $C_{si} = C_s$ for all i, the above equation simplifies to:

$$C_b = Q_o(1 + (1 - q) + (1 - q)^2 + \ldots + (1 - q)^{n-1})(C_{pr} + C_s) \tag{22.11}$$

EXAMPLE 22.3 Final Inspection vs. Distributed Inspection

Two inspection alternatives are to be compared for a processing sequence consisting of ten operations: (1) one final inspection and sortation operation following the tenth processing operation and (2) distributed inspection with an inspection and sortation operation after each of the ten processing operations. The batch size Q_o = 1000 pieces. The cost of each processing operation C_{pr} = \$1.00. The fraction defect rate at each operation q = 0.05. The cost of the single final inspection and sortation operation in alternative (1) is C_{sf} = \$2.50. The cost of each inspection and sortation operation in alternative (2) is C_s = \$0.25. Compare total processing and inspection costs for the two cases.

Solution: For the final inspection alternative, we can use Eq. (22.9) to determine the batch cost:

$$C_b = 1000(10 \times 1.00 + 2.50) = 1000(12.50) = \$12{,}500$$

For the distributed inspection alternative, we can use Eq. (22.11) to solve for the batch cost:

$$C_b = 1000(1 + (.95) + (.95)^2 + \ldots+(.95)^9)(1.00 + 0.25)$$
$$= 1000(8.0252)(1.25) = \$10{,}032$$

We see that the cost of distributed inspection is less for the cost data given in Example 22.3. A savings of \$2468 or nearly 20% is achieved by using distributed inspection. The reader might question why the cost of one final inspection (\$2.50) is so much more than the cost of an inspection in distributed inspection (\$0.25). We offer both a logical answer and a practical answer to the question. The logical answer goes like this: Each processing step produces its own unique defect feature (at fraction defect rate q), and the inspection procedure must be designed to inspect for that feature. For ten processing operations with ten different defect features, the cost to inspect for these features is the same whether the inspection is accomplished after each processing step or all at once after the final processing step. If the cost of inspecting for each defect feature is \$0.25, it follows that the cost of inspecting for all ten defect features is simply 10(\$0.25) = \$2.50. In general, this relationship can be expressed:

$$C_{sf} = \sum_{i=1}^{n} C_{si} \tag{22.12}$$

For the special case where all C_{si} are equal ($C_{si} = C_s$ for all i), as in Example 22.3,

$$C_{sf} = n\,C_s \tag{22.13}$$

Given this multiplicative relationship between the single final inspection cost and the unit inspection cost in distributed inspection, it is readily seen that the total cost advantage of distributed inspection in our example problem derives entirely from the fact that the number of parts that are processed and inspected is reduced after each processing step due to the sortation of defective parts from the batch during production rather than afterward.

Notwithstanding the logic of Eqs. (22.12) and (22.13), we are sure that in practice there is some economy in performing one inspection procedure at a single location, even if the procedure includes scrutinizing the product for ten different defect features. Thus, the actual final inspection cost per unit C_{sf} is likely to be less than the sum of the unit costs in distributed inspection. Nevertheless, the fact remains that distributed inspection and sortation reduces the number of units processed, thus avoiding the waste of valuable production resources on the processing of defective units.

Partially Distributed Inspection. A distributed inspection strategy can be followed in which inspections are located between groups of processes rather than after every processing step as in Example 22.3. If there is any economy in performing multiple inspections at a single location, as argued in the preceding paragraph, then this might be a worthwhile way to exploit this economy while preserving at least some of the advantages of distributed inspection. Let us use Example 22.4 to illustrate the grouping of unit operations for inspection purposes. As expected, the total batch cost lies between the two cases of fully distributed inspection and final inspection for the data in our example.

EXAMPLE 22.4 Partially Distributed Inspection

For comparison, let us use the same sequence of ten processing operations as before, where the fraction defect rate of each operation is $q = 0.05$. Instead of inspecting and sorting after every operation, the ten operations will be divided into groups of five, with inspections after operations 5 and 10. Following the logic of Eq. (22.13), the cost of each inspection will be five times the cost of inspecting for one defect feature; that is, $C_{s5} = C_{s10} = 5(\$0.25) = \$1.25$ per unit inspected. Processing cost per unit for each process remains the same as before at $C_{pr} = \$1.00$, and $Q_o = 1000$ units.

Solution: The batch cost is the processing cost for all 1000 pieces for the first five operations, after which the inspection and sortation procedure separates the defects produced in those first five operations from the rest of the batch. This reduced batch quantity then proceeds through operations 6-10, followed by the second inspection and sortation procedure. The equation for this is the following:

$$C_b = Q_o\left(\sum_{i=1}^{5} C_{pri} + C_{s5}\right) + Q_o\prod_{i=1}^{5}(1 - q_i)\left(\sum_{i=6}^{10} C_{pri} + C_{s10}\right) \quad (22.14)$$

Since all C_{pri} are equal ($C_{pri} = C_{pr}$ for all i), and all q are equal ($q_i = q$ for all i), this equation can be simplified to:

$$C_b = Q_o(5C_{pr} + C_{s5}) + Q_o(1 - q)^5(5C_{pr} + C_{s10}) \quad (22.15)$$

Using our values for this example, we have

$$C_b = 1000(5 \times 1.00 + 1.25) + 1000(.95)^5(5 \times 1.00 + 1.25)$$

$$= 1000(6.25) + 1000(0.7738)(6.25) = \$11{,}086$$

This is a savings of \$1414 or 11.3% compared with the \$12,500 cost of one final inspection. Note that we have been able to achieve a significant portion of the total savings from fully distributed inspection by using only two inspection stations rather than ten. Our savings here of \$1414 is about 57% of the \$2468 savings from the previous example, with only 20% of the inspection stations. This suggests that it may not be advantageous to locate an inspection operation after every production step, but instead to place them after groups of operations. The "law of diminishing returns" is applicable in distributed inspection.

22.5.3 Inspection or No Inspection

A relatively simple model for deciding whether to inspect at a certain point in the production sequence is proposed in Juran and Gryna [3]. The model uses the fraction defect rate in the production batch, the inspection cost per unit inspected, and the cost of damage that one defective unit would cause if it were not inspected. The total cost per batch of 100% inspection can be formulated as follows:

$$C_b(\text{100\% inspection}) = Q\,C_s \tag{22.16}$$

where C_b = total cost for the batch under consideration, Q = quantity of parts in the batch, and C_s = inspection and sortation cost per part. The total cost of no inspection, which leads to a damage cost for each defective unit in the batch, would be:

$$C_b(\text{no inspection}) = Q\,q\,C_d \tag{22.17}$$

where C_b = batch cost, as before; Q = number of parts in the batch; q = fraction defect rate; and C_d = damage cost for each defective part that proceeds to subsequent processing or assembly. This damage cost may be high, for example, in the case of an electronics assembly where one defective component might render the entire assembly defective and rework would be expensive.

Finally, if sampling inspection is used on the batch, we must include the sample size and the probability that the batch will be accepted by the inspection sampling plan that is used. This probability can be obtained from the OC curve (Figure 22.1) for a given fraction defect rate q. The resulting expected cost of the batch is the sum of three terms: (1) cost of inspecting the sample of size Q_s, (2) expected damage cost of those parts that are defective if the sample passes inspection, and (3) expected cost of inspecting the remaining parts in the batch if the sample does not pass inspection. In equation form,

$$C_b(\text{sampling}) = C_sQ_s + (Q - Q_s)qC_dP_a + (Q - Q_s)C_s(1 - P_a) \tag{22.18}$$

where C_b = batch cost, C_s = cost of inspecting and sorting one part, Q_s = number of parts in the sample, Q = batch quantity, q = fraction defect rate, C_d = damage cost per defective part, and P_a = probability of accepting the batch based on the sample.

A simple decision rule can be established to decide whether to inspect the batch. The decision is based on whether the expected fraction defect rate in the batch is greater than or less than a critical defect level q_c, which is the ratio of the inspection cost to the damage cost. This critical value represents the break-even point between inspection or no inspection. In equation form, q_c is defined as follows:

$$q_c = \frac{C_s}{C_d} \tag{22.19}$$

where C_s = cost of inspecting and sorting one part, and C_d = damage cost per defective part. If, based on past history with the component, the batch fraction defect rate q is less than this critical level, then no inspection is indicated. On the other hand, if it is expected that the fraction defect rate will be greater than q_c, then the total cost of production and inspection will be less if 100% inspection and sortation is performed prior to subsequent processing.

EXAMPLE 22.5 Inspection or No Inspection

A production run of 10,000 parts has been completed and a decision is needed whether to 100% inspect the batch. Past history with this part suggests that the fraction defect rate is around 0.03. Inspection cost per part is \$0.25. If the batch is passed on for subsequent processing, the damage cost for each defective unit in the batch is \$10.00. Determine: (a) batch cost for 100% inspection and

(b) batch cost if no inspection is performed. (c) What is the critical fraction defect value for deciding whether to inspect?

Solution: (a) Batch cost for 100% inspection is given by Eq. (22.16):

$$C_b(100\%\text{ inspection}) = Q\,C_s = 10{,}000\ (\$0.25) = \$2{,}500$$

(b) Batch cost for no inspection can be calculated by Eq. (22.17):

$$C_b(\text{no inspection}) = Q\,q\,C_d = 10{,}000\ (0.03)(\$10.00) = \$3{,}000$$

(c) The critical fraction defect value for deciding whether to inspect is determined from Eq. (22.19):

$$q_c = \frac{C_s}{C_d} = \frac{0.25}{10.00} = 0.025$$

Since the anticipated defect rate in the batch is $q = 0.03$, the decision should be to inspect. Note that this decision is consistent with the two batch costs calculated for no inspection and 100% inspection. The lowest cost is attained when 100% inspection is used.

EXAMPLE 22.6 Cost of Sampling Inspection

Given the data from the preceding example, suppose that sampling inspection is being considered as an alternative to 100% inspection. The sampling plan calls for a sample of 100 parts to be drawn at random from the batch. Based on the OC curve for this sampling plan, the probability of accepting the batch is 92% at the given defect rate of $q = 0.03$. Determine the batch cost for sampling inspection.

Solution: The batch cost for sampling inspection is given by Eq. (22.18):

$$C_b(\text{sampling}) = C_sQ_s + (Q - Q_s)qC_dP_a + (Q - Q_s)C_s(1 - P_a)$$

$$= \$0.25(100) + (10{,}000 - 100)(0.03)(\$10.00)(0.92) + (10{,}000 - 100)(\$0.25)(1 - 0.92)$$

$$= \$25.00 + 2732.40 + 198.00 = \$2955.40$$

The significance of Example 22.6 must not be overlooked. The total cost of sampling inspection for our data is greater than the cost of 100% inspection and sortation. If only the cost of the inspection procedure is considered, then sampling inspection is much less expensive ($25 versus $2500). But if total costs, which include the damage that results from defects passing through sampling inspection, are considered, then sampling inspection is not the least cost inspection alternative. We might consider the question: what if the ratio $\frac{C_s}{C_d}$ in Eq. (22.19) had been greater than the fraction defect rate of the batch, in other words, the opposite of the case in Examples 22.5 and 22.6? The answer is that if q_c were greater than the batch defect rate q, then the cost of no inspection would be less than the cost of 100% inspection, and the cost of sampling inspection would again lie between the two cost values. The cost of sampling inspection will always lie between the cost of 100% inspection and no inspection, whichever of these two alternatives is greater. If this argument is followed to its logical end, then the conclusion is that either no inspection or 100% inspection is preferred over sampling inspection, and it is just a matter of deciding whether none or all is the better alternative.

22.5.4 What the Equations Tell Us

Several lessons can be inferred from the above mathematical models and examples. These lessons should be useful in designing inspection systems for production.

- Distributed inspection/sortation reduces the total number of parts processed in a sequence of production operations compared with one final inspection at the end of the sequence. This reduces waste of processing resources.
- Partially distributed inspection is less effective than fully distributed inspection at reducing the waste of processing resources. However, if there is an economic advantage in combining several inspection steps at one location, then partially distributed inspection may reduce total batch costs compared with fully distributed inspection.
- The "law of diminishing returns" operates in distributed inspection systems, meaning that each additional inspection station added in distributed inspection yields less savings than the previous station added, other factors being equal.
- As the ratio of unit processing cost to unit inspection cost increases, the advantage of distributed inspection over final inspection increases.
- Inspections should be performed immediately following processes that have a high fraction defect rate.
- Inspections should be performed prior to high cost processes.
- When expected damage cost (of those defects that pass around the inspection plan when the batch is accepted) and expected cost of inspecting the entire batch (when the batch is rejected) are considered, sampling inspection is not the lowest cost inspection alternative. Either no inspection or 100% inspection is a more appropriate alternative, depending on the relative values of inspection/sortation cost and damage cost for a defective unit that proceeds to the next stage of processing.

ERENCES

[1] BARKMAN, W. E., *In-Process Quality Control for Manufacturing*, Marcel Dekker, Inc., New York, 1989.

[2] DRURY, C. G., "Inspection Performance," *Handbook of Industrial Engineering*, Second Edition, G. Salvendy, Editor, John Wiley & Sons, Inc., New York, 1992, pp 2282–2314.

[3] JURAN, J. M., and GRYNA, F. M., *Quality Planning and Analysis*, Third Edition, McGraw-Hill, Inc., New York, 1993.

[4] MONTGOMERY, D. C., *Introduction to Statistical Quality Control*, John Wiley & Sons, Inc., New York, 1985.

[5] MURPHY, S. D., *In-Process M*[illegible] New York, 1990.