12

Rectilinear Motion

The term 'Rectilinear Motion' is defined in the last chapter as the translation in a straight line. In this chapter kinematics (dynamics without referring to the force causing the motion) of rectilinear motion is dealt. Many kinematic problems in linear motion can be solved just by using the definition of speed velocity and acceleration. Motion curves which will be useful in solving kinematic problems are explained and many problems with uniform velocity, uniform acceleration and varying acceleration are dealt with in this chapter.

12.1 MOTION CURVES

Motion curves are the graphical representation of the displacement, velocity and acceleration with time.

Displacement–Time Curve (*s-t* curve). Displacement-Time curve is a curve with time as abscissa and displacement as ordinate (Fig. 12.1). At any instant of time t, velocity v is given by

$$v = \frac{ds}{dt}$$

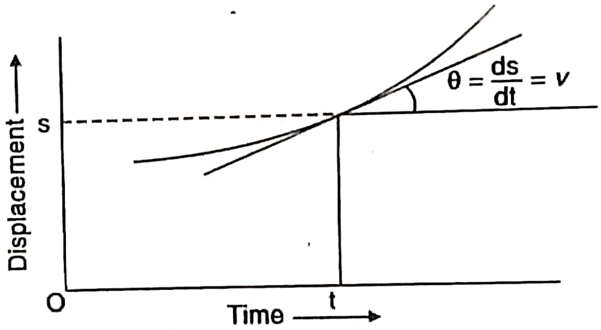

Fig. 12.1

If a body is having non-uniform motion, its displacement at various time intervals may be observed and s-t curve plotted. Velocity at any time may be found from the slope of s-t curve.

Velocity–Time Curve (*v-t* curve). In Velocity-Time curve diagram, the abscissa represents time and the ordinate represents the velocity of the motion. Such a curve is shown in Fig. 12.2. Acceleration 'a' is given by the slope of the v-t curve.

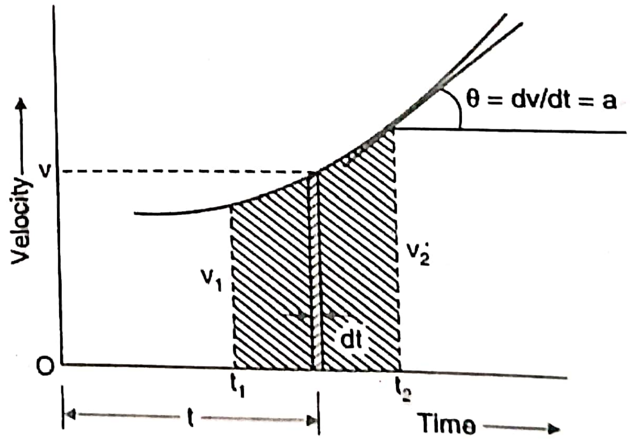

Fig. 12.2

i.e.,
$$a = \frac{dv}{dt} = \theta$$

Thus, acceleration at any time is the slope of v-t curve at the time, as shown in Fig. 12.2.

Now,
$$\frac{ds}{dt} = v$$

$\therefore$
$$ds = v\,dt$$

or

Referring to Fig. 12.2, $v\,dt$ is the elemental area under the curve at time t in th... dt. Hence the shaded area under the curve between t_1 and t_2 shown in Fig. 12.2 rep... displacement of the moving body in the time interval between t_1 and t_2. Thus in v-t cur...

(1) Slope of the curve represents acceleration, and

(2) Area under the curve represents displacement.

Acceleration–Time Curve (a-t curve). If a body is moving with varying acceleration, its motion can be studied more conveniently by drawing a curve with time as abscissa and acceleration as ordinate. Such a curve is called acceleration-time curve. (Fig. 12.3)

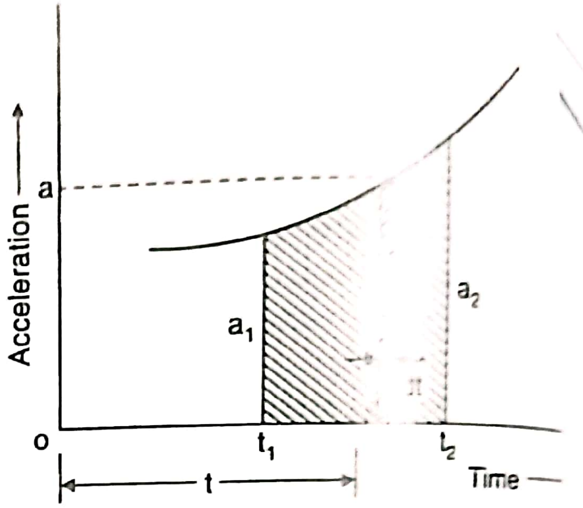

Fig. 12.

Now, $$\frac{dv}{dt} = a \quad \text{or} \quad dv = a\,dt$$

or $$v = \int a\,dt$$

Hence the area under the curve represents velocity.

12.2 MOTION WITH UNIFORM VELOCITY

Consider the motion of a body moving with uniform velocity v. Now,

$$\frac{ds}{dt} = v$$

or $$s = \int v\,dt$$

$$= vt, \text{ since } v \text{ is constant} \qquad ...(12.1)$$

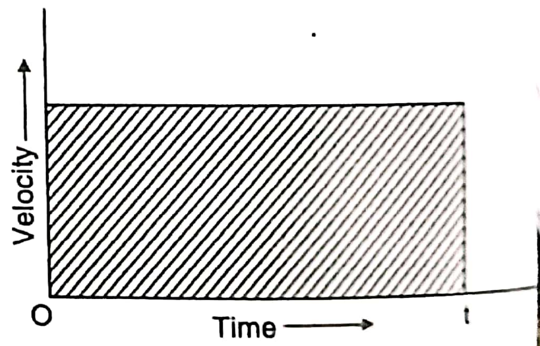

Fig. 12.4

v-t curve for such a motion is shown in Fig. 12.4. It can be easily seen that the distance travelled s, from starting point in time t is given by the shaded area, which is a rectangle

or $$s = vt, \text{ which is same as eqn. (12.1)}$$

12.3 MOTION WITH UNIFORM ACCELERATION

Consider the motion of a body with uniform acceleration a.

Let, u — initial velocity

v — final velocity

and t — time taken for change of velocity from u to v.

Acceleration is defined as the rate of change of velocity. Since it is uniform, we can writ...

$$a = \frac{v - u}{t}$$

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Displa... ment s is given by,

$$s = \text{Average velocity} \times \text{Time}$$

$$= \frac{u+v}{2} t \quad ...(12.3)$$

Substi...ting the value of v from eqn. (12.2) into eqn. (12.3), we get

$$s = \frac{u+u+at}{2} t = ut + \frac{1}{2} at^2 \quad ...(12.4)$$

From, eqn. (12.2), $t = \dfrac{v-u}{a}$

Substituting it into eqn. (12.3)

$$s = \frac{u+v}{2} \frac{v-u}{a} = \frac{v^2-u^2}{2a}$$

i.e.,

$$v^2 - u^2 = 2as \quad ...(12.5)$$

Thus equations of motion of a body moving with constant acceleration are:

$$\left.\begin{aligned} v &= u + at & (a)\\ s &= ut + 1/2\, at^2 & (b)\\ v^2 - u^2 &= 2as & (c) \end{aligned}\right\} \quad ...(12.6)$$

and

Equation (12.6) can be derived by integration technique also as given below:

From definition of acceleration,

$$\frac{dv}{dt} = a$$

$$dv = adt$$

Since 'a' is constant,

$$v = at + C_1 \quad ...(1)$$

where, C_1 is constant of integration.

When $t = 0$, Velocity = Initial velocity, u

Substituting these values in (1), we get,

$$\therefore \quad u = 0 + C_1 \quad \text{or} \quad C_1 = u$$

Thus, $$v = u + at \quad ...(a)$$

From the definition of velocity,

$$\frac{ds}{dt} = v = u + at \quad \text{[from (a)]}$$

$$ds = (u + at)dt$$

$$\therefore \quad s = ut + \frac{1}{2} at^2 + C_2$$

where, C_2 is constant of integration.

When $t = 0$, $s = 0$,

$$\therefore \quad C_2 = 0 \quad \text{and} \quad \text{hence } s = ut + \frac{1}{2} at^2 \quad ...(b)$$

From definition of acceleration,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$= \frac{dv}{ds} v \text{ [since } ds/dt = v] \quad ...(12.7)$$

$$a\, ds = v\, dv$$

$\therefore$

By integrating,

$$a \int ds = \int_u^v v\, dv$$

$$as = [v^2/2]_u^v = \frac{v^2}{2} - \frac{u^2}{2}$$

or

$$v^2 - u^2 = 2as \qquad \text{...}(c)$$

The equations of linear motions can be found conveniently referring to v-t diagram. Since acceleration is uniform, the slope of the curve is constant, *i.e.*, it is a straight line, as shown in Fig. 12.5.

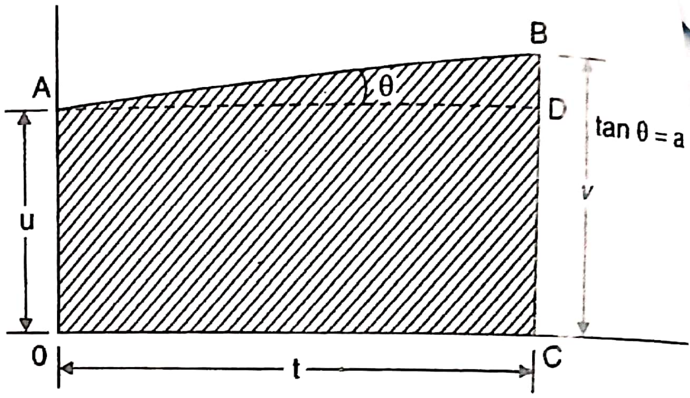

Fig. 12.5

Now, a = Slope of the diagram

$= \tan\theta$

$$= \frac{BD}{AD} = \frac{BC - CD}{OC} = \frac{BC - OA}{OC}$$

$$= \frac{v-u}{t}$$

$\therefore$

$$v = u + at \qquad \text{...}(a)$$

s = Area of $AOCB$

= Area of rectangle $AOCD$ + Area of ΔABD

$$= AO \times OC + \frac{1}{2} \times AD \times BD$$

$$= ut + \frac{1}{2} AD \times AD \tan\theta$$

$$= ut + \frac{1}{2} \times t \times t \times a$$

$$s = ut + \frac{1}{2} at^2 \qquad \text{...}(b)$$

We can also write, s = Area of parallelogram $AOCB$

$$= \frac{1}{2}(AO + BC)\, OC = \frac{1}{2}(u+v)\, t$$

Substituting $t = \dfrac{v-u}{a}$ from Eqn. (a) we get

$$s = \frac{1}{2}(u+v)\frac{(v-u)}{a}$$

i.e.,

$$2as = v^2 - u^2$$

ACCELERATION DUE TO GRAVITY

In chapter 10, it has been shown that the acceleration due to gravity is constant for all practical purposes when we treat the motion of the bodies near earth's surface. Its value is found to be 9.81 m/s² and is always directed towards centre of the earth, *i.e.*, vertically downwards. Hence, if vertically downward motion of a body is considered, the value of acceleration a in eqn. 12.6 is 9.81 m/s² and if vertically upward motion is considered, then,

$$a = -g = -9.81 \text{ m/s}^2.$$

Example 12.1. *A particle is projected vertically upwards from the ground with an initial velocity of u m/s.*

Find:

(*i*) *the time taken to reach the maximum height;*

(*ii*) *the maximum height reached;*

(*iii*) *time required for descending; and*

(*iv*) *velocity when it strikes the ground.*

Solution. Consider the upward motion of the particle.

Initial velocity $= u$

Since vertically upward motion is considered as positive

$$a = -g = -9.81 \text{ m/s}^2$$

When maximum height is reached, final velocity $v = 0$.

Form equation of motion,

$$v = u + at$$

we get,
$$0 = u - gt$$

$$\therefore \quad t = \frac{u}{g} \quad \text{...(1)}$$

Let the maximum height reached (displacement) s be h.

From equation of motion, $v^2 - u^2 = 2as$, we get,

$$0 - u^2 = -2gh$$

or
$$h = \frac{u^2}{2g} \quad \text{...(2)}$$

Now, consider the downward motion of the particle. It starts with zero velocity ($u_2 = 0$) from a height h (= s). Let it strikes the ground with final velocity v_2. Acceleration due to gravity, $g = 9.81$ m/s².

From the equation of motion, $v^2 - u^2 = 2as$, we get

$$v_2^2 - 0 = 2gh$$

$$v_2^2 = 2g\frac{u^2}{2g} \quad \left[\text{from eqn. 2, } h = \frac{u^2}{2g}\right] \quad \text{...(3)}$$

$$= u^2$$

$$v_2 = u$$

When a particle is freely projected, the magnitude of its velocity at any given elevation is the same during both upward and downward motion.

From the relation, $v = u + at$

we get, $v_2 = 0 + gt$

CHAPTER 12

$$t = \frac{v_2}{g} = \frac{u}{g}$$

Thus, the time taken for the upward motion is same as the time taken for the downward motion.

Example 12.2. *A small steel ball is shot vertically upwards from the top of a building 25 m above the ground with an initial velocity of 18 m/s.*

(*a*) *In what time, it will reach the maximum height?*

(*b*) *How high above the building will the ball rise?*

(*c*) *Compute the velocity with which it will strike the ground and the total time it is in motion.*

Solution. For upward motion:

$$u = 18 \text{ m/s}$$
$$v = 0$$
$$a = -9.81 \text{ m/s}^2$$

and $s = h$

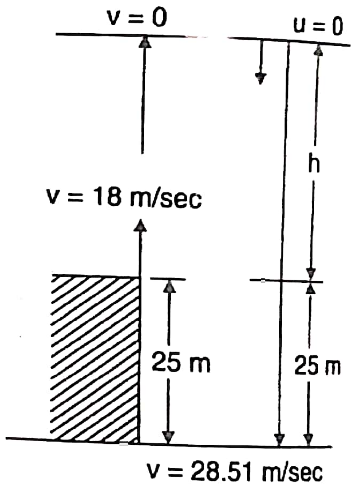

Fig. 12.6

Let t_1 be time taken to reach the maximum height.

From equation of motion,

$$v = u + at$$

we get,

$$0 = 18 - 9.81\, t_1$$

$$t_1 = \mathbf{1.83 \text{ seconds}} \quad \textbf{Ans.}$$

From the relation, $v^2 - u^2 = 2as$ we get,

$$0 - 18^2 = 2(-9.81)h$$

$$\therefore \quad h = \frac{18^2}{2 \times 9.81} = \mathbf{16.51 \text{ m}} \quad \textbf{Ans.}$$

$\therefore$ Total height from the ground

$$= 25 + h = 25 + 16.51 = 41.51 \text{ m}$$

Downward motion:

With usual notations,

$$u = 0,\ v = v_2,\ s = 41.51 \text{ m},\ a = +9.81 \text{ m/s}^2$$
$$t = t_2$$

From the relation $v^2 - u^2 = 2as$, we get

$$v_2^2 - 0 = 2 \times 9.81 \times 41.51$$

$$v_2 = \mathbf{28.54 \text{ m/s.}} \quad \textbf{Ans.}$$

From the relation $v = u + at$ we get,

$$28.54 = 0 + 9.81\, t_2$$

$$\therefore \quad t_2 = 2.91 \text{ m/s}$$

$\therefore$ **Total time during which the body is in motion**

$$= t_1 + t_2 = 1.83 + 2.91 = \mathbf{4.74 \text{ seconds}} \quad \textbf{Ans.}$$

Example 12.3. *If a freely falling stone passes a window of 2.45 m height in half a second, find the height from which the stone fell.*

Solution. Let the stone be dropped from A (Fig. 12.7) at a height h above the window and BC represent the height of window.

Let t be the time to fall from A to B.

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Then for motion from A to B,

$$u = 0,\ s = h \quad \text{and} \quad a = +g = 9.81 \text{ m/s}^2.$$

$$\therefore \quad h = 0 \times t + \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2 \qquad \text{...(1)}$$

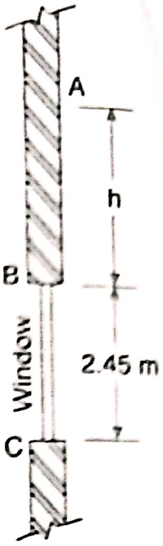

Fig. 12.7

For motion from A to C,

$$u = 0,\ s = h + 2.45,\ a = g = 9.81 \text{ m/s}^2$$

$$\text{Time} = t + 0.5$$

$$\therefore \quad h + 2.45 = 0 \times t + \frac{1}{2} g\,(t + 0.5)^2$$

$$= \frac{1}{2} g\,(t^2 + t + 0.25) \qquad \text{...(2)}$$

Subtracting eqn. (1) from eqn. (2), we get

$$2.45 = \frac{1}{2} g\,(t + 0.25)$$

$$= \frac{1}{2} \times 9.81\,(t + 0.25)$$

$$\therefore \quad t = 0.2495 \text{ second.}$$

$$h = \frac{1}{2} g t^2$$

$$= \frac{1}{2} \times 9.81 \times (0.2495)^2$$

$$\mathbf{h = 0.305\ m} \quad \text{Ans.}$$

Example 12.4. *A ball is dropped from the top of a tower 30 m high. At the same instant a second ball is thrown upward from the ground with an initial velocity of 15 m/s. When and where do they cross and with what relative velocity?*

Solution. Let the two balls cross each other at a height h from the ground (Fig. 12.8) after t seconds.

For the motion of first ball,

$$u = 0,\ s = 30 - h \text{ and } a = 9.81 \text{ m/s}^2.$$

Using the equation, $s = ut + \frac{1}{2} a t^2$, we get

$$30 - h = 0 \times t + \frac{1}{2} \times 9.81 \times t^2 \qquad \text{...(1)}$$

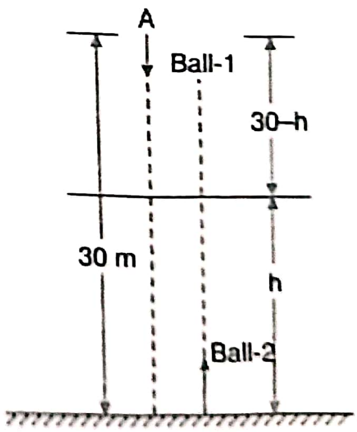

Fig. 12.8

For the motion of second ball,

$$u = 15 \text{ m/s},\ s = h \text{ and } a = -\,9.81 \text{ m/s}^2$$

$$\therefore \quad h = 15\,t - \frac{1}{2} \times 9.81\,t^2 \qquad \text{...(2)}$$

Adding eqns. (1) and (2) we get,

$$30 = 15\,t$$

$$t = 2 \text{ seconds} \quad \text{Ans.}$$

$\therefore \quad h = 15 \times 2 - \frac{1}{2} \times 9.81 \times 2^2$

$h = 10.38$ m Ans.

At $t = 2$ seconds:

(i) Downward velocity of first ball,

$$v_1 = 0 + 9.81 \times 2 = 19.62 \text{ m/s}$$

(ii) Upward velocity of second ball,

$$v_2 = 15 - 9.81 \times 2$$
$$= -4.62 \text{ m/s}$$
$$v_2 = 4.62 \text{ m/s downward}$$

$\therefore$ **Relative velocity** $= 19.62 - 4.62$

$= \mathbf{15 \text{ m/s}}$ **Ans.**

Example 12.5. *A stone dropped into a well is heard to strike the water in 4 seconds. F[ind] the depth of the well, assuming the velocity of sound to be 335 m/s.*

Solution. Let h — depth of the well

t_1 — time taken by stone to strike water

t_2 — time taken by sound to travel the height h.

Then, $\quad t_1 + t_2 = 4$

For the downward motion of the stone,

$$h = 0 \times t_1 + \frac{1}{2} g t_1^2$$

i.e.,
$$h = \frac{1}{2} g t_1^2$$

Since sound moves with uniform velocity, for the upward motion of sound, we have

$$h = 335\, t_2$$

From eqns. (2) and (3), we have

$$\frac{1}{2} g t_1^2 = 335\, t_2$$

But from eqn. (1), $\quad t_2 = 4 - t_1$

Hence
$$\frac{1}{2} g t_1^2 = 335\,(4 - t_1)$$

Substituting the value of $g = 9.81$ m/s², we get

$$\frac{9.81}{2} t_1^2 = 335\,(4 - t_1)$$

$$t_1^2 + 68.30 t_1 - 273.19 = 0$$

$$t_1 = \frac{-68.3 + \sqrt{68.3^2 - 4 \times 1 \times (-273.19)}}{2} = 3.79 \text{ seconds}$$

$\therefore$
$$h = \frac{1}{2} g t_1^2 = \frac{1}{2} \times 9.81 \times (3.79)^2$$

i.e., $\quad h = \mathbf{70.44 \text{ m}}$ **Ans.**

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13 Curvilinear Translation in Vertical Plane (Projectiles)

In the last chapter, we considered the motion of a particle along a straight line. But we observe that a particle moves along a curved path if it is freely projected in the air in the direction other than vertical. These freely projected particles which are having the combined effect of a vertical and a horizontal motion are called projectiles. The motion of a projectile has a vertical component and a horizontal component. The vertical component of the motion is subjected to gravitational acceleration/retardation while horizontal component remains constant, if air resistance is neglected. The motion of a projectile can be analysed independently in vertical and horizontal directions and then combined suitably to get the total effect. In this chapter, after defining important terms to be used, horizontal projection, inclined projection on both horizontal and inclined planes are analysed neglecting air resistance.

13.1 DEFINITIONS

The definitions of the terms used in this chapter are given with reference to Fig. 13.1.

Velocity of projection: The velocity with which the particle is projected is called as velocity of projection (u m/s).

Angle of projection: The angle between the direction of projection and horizontal direction is called as angle of projection (α).

Trajectory: The path traced by the projectile is called as its trajectory.

Horizontal range: The horizontal distance through which the projectile travles in its flight is called the horizontal range or simply range of the projectile.

Time of flight: The time interval during which the projectile is in motion is called the time of flight.

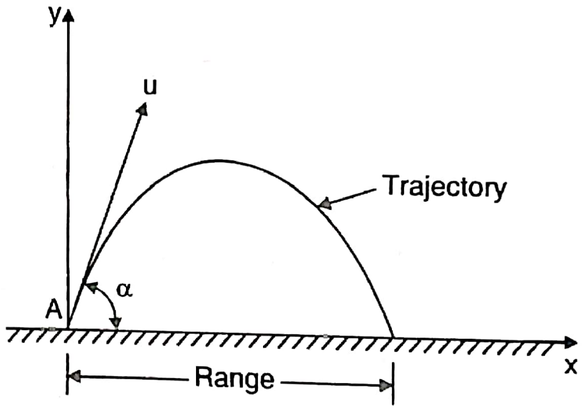


Fig. 13.1

13.2 MOTION OF BODY PROJECTED HORIZONTALLY

Consider a particle thrown horizontally from point A with a velocity u m/s as shown in Fig. 13.2. At any instant the particle is subjected to:

(1) horizontal motion with constant velocity u m/s, and

(2) vertical motion with initial velocity zero and moving with acceleration due to gravity g.

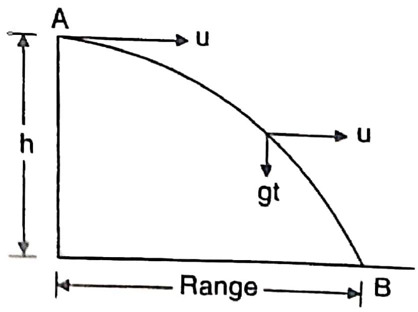


Fig. 13.2

Let h be the height of A from the ground.

Considering vertical motion,

$$h = 0 \times t + \frac{1}{2} gt^2$$

$$= \frac{1}{2} gt^2 \qquad ...(13.1)$$

This expression gives the time of flight. During this period, the particle moves horizontally with uniform velocity, u m/s.

$$\therefore \qquad \text{Range} = ut. \qquad ...(13.2)$$

Example 13.1. *A pilot flying his bomber at a height of 2000 m with a uniform horizontal velocity of 600 kmph wants to strike a target (Ref. Fig. 13.3). At what distance from the target, he should release the bomb?*

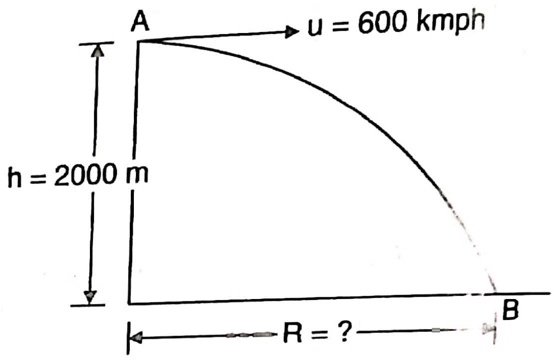


Fig. 13.3

Solution. $h = 2000$ m

$u = 600$ kmph

$$= \frac{600 \times 1000}{60 \times 60}$$

$= 166.67$ m/s

Initial velocity in vertical direction $= 0$

and gravitational acceleration $= 9.81$ m/s^2.

If t is the time of flight, considering vertical motion, we get

$$2000 = 0 \times t + \frac{1}{2} \times 9.81\, t^2$$

$$\therefore \qquad t = 20.19 \text{ seconds}$$

During this period horizontal distance travelled by the bomb

$= ut$

$= 166.67 \times 20.19$

$= 3365.07$ m.

Bomb should be released at 3365.46 m from the target. Ans.

Example 13.2. *A person wants to jump over a ditch as shown in Fig. 13.4. Find the minimum velocity with which he should jump.*

Solution. $h = 2$ m

and Range $= 3$ m

Let t be the time of flight and u the minimum horizontal velocity required. Considering the vertical motion :

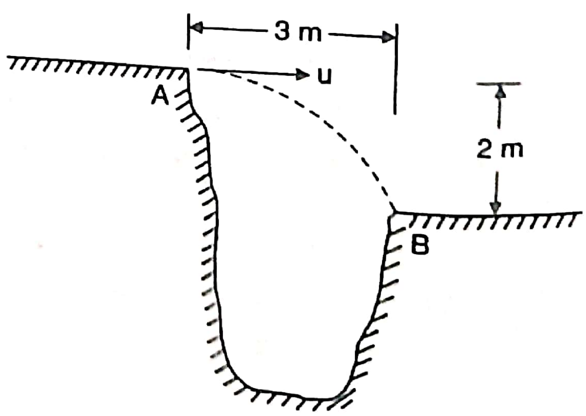


Fig. 13.4

$$h = \frac{1}{2} gt^2$$

$$2 = \frac{1}{2}\, 9.81\, t^2$$

$$\therefore \qquad t = 0.6386 \text{ second}$$

Considering hori... motion of uniform velocity, we get

$$3 = u \times 0.6386$$

$$u = 4.698 \text{ m/s.} \quad \textbf{Ans.}$$

Example 13.3. *A ...ssure tank issues water at A with a horizontal velocity u as shown in ... 13.5. For what rang... values of u, water will enter the opening BC.*

Solution.

Required velocity to enter at ***B*****:**

$$h = 1 \text{ m}$$

If t_1 is the time of flight, considering vertical motion,

$$1 = \frac{1}{2} \times 9.81\, t_1^2$$

$$t_1 = 0.4515 \text{ second}$$

Considering horizontal motion,

$$u_1 t_1 = 3$$

$$\therefore \quad u_1 = \frac{3}{0.4515} = 6.44 \text{ m/s.}$$

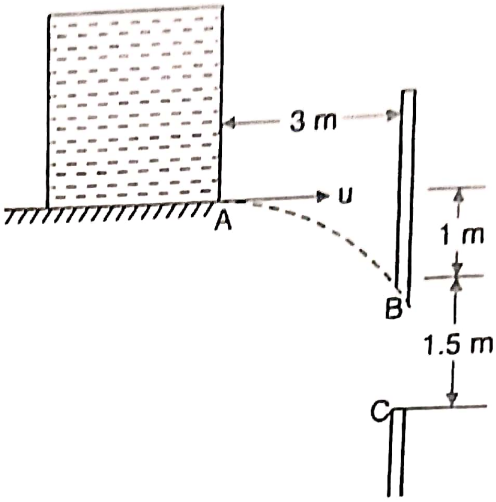


Fig. 13.5

Required velocity to enter at ***C*****:**

Let t_2 be time required for the flight from A to C.

$$h = 2.5 \text{ m} \; ; \text{Range} = 3 \text{ m.}$$

Considering vertical motion,

$$2.5 = \frac{1}{2} \times 9.81\, t_2^2$$

$$t_2 = 0.714 \text{ second}$$

Considering the horizontal motion,

$$u_2\, t_2 = 3$$

$$\therefore \quad u_2 = \frac{3}{0.714} = 4.202 \text{ m/s.}$$

Therefore the range of velocity for which the jet can enter the opening ***BC*** **is 4.202 m/s to 6.44 m/s. Ans.**

Example 13.4. *A rocket is released from a jet fighter flying horizontally at 1200 kmph at an altitude of 3000 m above its target. The rocket thrust gives it a constant horizontal acceleration of 6 m/s². At what angle below the horizontal should pilot see the target at the instant of releasing the rocket in order to score a hit ?*

Solution. Referring to Fig. 13.6, $h = 3000$ m.

In vertical direction, the rocket has initial velocity equal to zero and moves with gravitational acceleration 9.81 m/s².

Hence if t is the time of flight,

$$3000 = 0 + \frac{1}{2} \times 9.81\, t^2$$

i.e.,

$$t = 24.73 \text{ seconds}$$

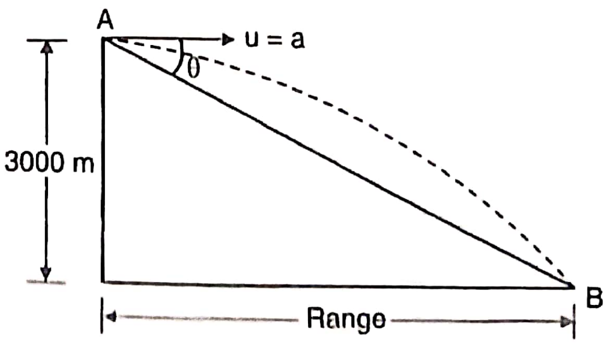


Fig. 13.6

In the horizontal direction, rocket has initial velocity = 1200 kmph and acceleration = 6 m/s²

Now,

$$u = 1200 \text{ kmph}$$

$$= \frac{1200 \times 1000}{60 \times 60} = 333.33 \text{ m/s}$$

∴ Horizontal distance covered during the time of flight = Range

$$= ut + \frac{1}{2}at^2$$

$$= 333.33 \times 24.73 + \frac{1}{2} \times 6 \times 24.73^2$$

$$= 10{,}078.5 \text{ m}$$

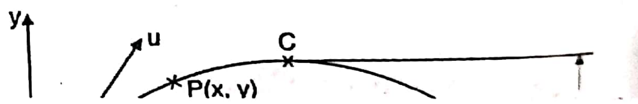
∴ The angle θ below the horizontal at which the pilot must see the target while releasing the rocket, is given by

$$\tan\theta = \frac{3000}{10{,}078.5}$$

∴ $\theta = 16.576°$ **Ans.**

13.3 INCLINED PROJECTION ON LEVEL GROUND

Consider the motion of a projectile, projected from point A with velocity of



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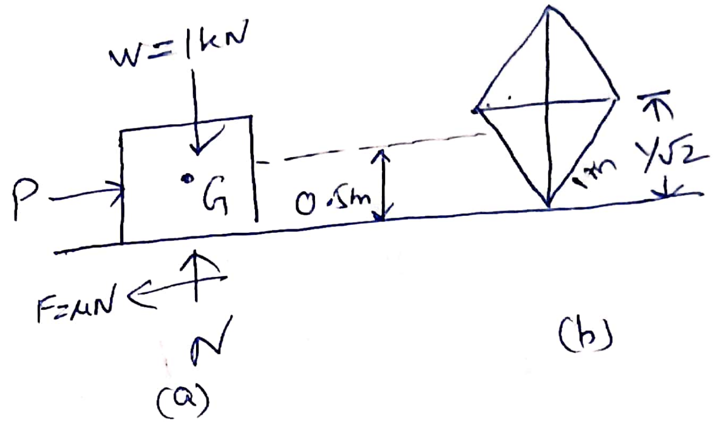
Unit-3

Q) A man wishes to move wooden box of 1 metre cube to a distance of 5 m with the least amount of work. If the block weighs 1 kN and the coefficient of friction is 0.3, find whether he should tip it or slide it.



Sol^n^ workdone in sliding.

Normal reaction $N = W = 1kN$

$\therefore$ Friction force $= F = \mu N = 0.3kN$

Applied force $P = F = 0.3kN$

workdone in sliding to a distance of $5m = P \times 5$ $(W = F \times S)$

$= 0.3 \times 5 = 1.5 \, kNm$

$= 1.5 \, kJ$

work ~~done~~ to be done in tipping.

In one tipping the centre of gravity of box is to be raised to a height $= \frac{1}{\sqrt{2}} - 0.5 = 0.207m$

$\therefore$ workdone in one tipping $= Wh$

$= 1 \times 0.207$

$= 0.207 kJ$

To move a distance of 5m, five tippings are required.

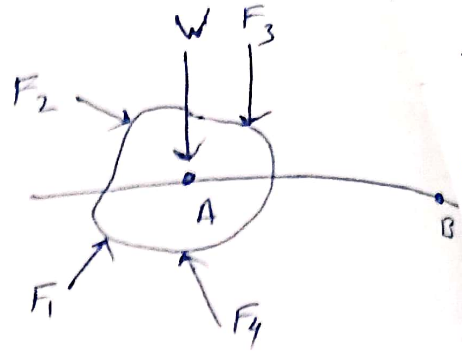
Hence work to be done in moving it by 5 metres by tipping

$= 5 \times 0.207 = 1.035 \, kJ$

Since the man needs to spend only 1.035 kJ while tipping & it is less than 1.5 kJ to be spent in sliding, the man should move the box by tipping.

work energy equation for translation

Consider the body shown in fig. Subjected to a system of forces F_1, F_2 & moving with an acceleration a in x-direction. let its initial velocity at A be u & final velocity when it moves distance $AB = s$ be v. then the resultant of system of forces must be in x direction.



let
$$R = \Sigma F_x$$

From Newton's second law of motion.
$$R = \frac{W}{g} a$$

Multiplying both sides by elementary distance ds, we get
$$R\,ds = \frac{W}{g} a\,ds$$
$$= \frac{W}{g} v \cdot \frac{dv}{ds} \cdot ds. \qquad \text{since } a = v\frac{dv}{ds}$$

Integrating both sides for the motion from A to B, we get
$$\int_0^s R\,ds = \int_u^v \frac{W}{g} v\,dv$$
$$Rs = \frac{W}{g}\left[\frac{v^2}{2}\right]_u^v$$
$$= \frac{W}{2g}(v^2 - u^2)$$

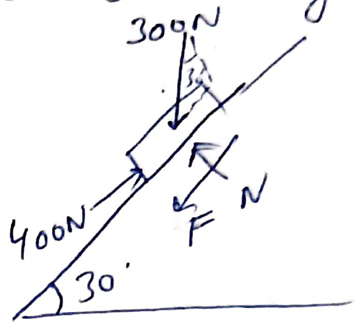
Now Rs is the workdone by the forces acting on a body. $\frac{W}{2g}v^2$ is the final kinetic energy & $\frac{Wu^2}{2g}$ is initial kinetic energy.

Hence we can say, workdone in a motion is equal to change in kinetic energy.

work done = Final kinetic energy − Initial kinetic energy. (3)

& it is called work energy equation.

Q) A body weighing 300N is pushed up a 30° plane by a 400 N force acting parallel to the plane. If the initial velocity of the body is 1.5 m/s & coefficient of kinetic friction is $\mu = 0.2$, what velocity will the body have after moving 6m?



Sol^n Consider the free body diagram of the body shown in fig.

Σ forces normal to plane = 0, gives

$$N = 300 \times \cos 30°$$
$$= 259.81 \text{ newton}$$

$\therefore$ Frictional force $F = \mu N$

$$= 0.2 \times 259.81 = 51.96 N$$

Initial velocity $u = 1.5$ m/s

Displacement $s = 6$ m

Let the final velocity $= V$ m/s

Equating the workdone by forces along the plane to change in kinetic energy, we get.

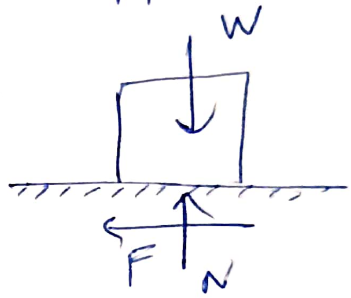
$$(400 - F - W \sin\theta) s = \left(\frac{1}{2}\right) \frac{W}{g} (V^2 - u^2)$$

$$\left(400 - 51.96 - 300 \times \frac{1}{2}\right) 6 = \frac{1}{2} \times \left(\frac{300}{9.81}\right)(V^2 - 1.5^2)$$

$$77.71 = V^2 - 2.25$$
$$V = 8.942 \text{ m/s}$$

Q/ In a police investigation of tyre marks, it was concluded that a car while in motion along straight level road skidded for a total 60 m after the brakes were applied. If the coefficient of friction between the tyres and the pavements is estimated as 0.5, what was the probable speed of the car just before the brakes were applied?

Sol^n Let the probable speed of the car just before brakes were applied be U m/s. Free body diagram of the car is shown in fig.



Now, $\Sigma F_V = 0$

$$N = W$$

$$F = \mu N = \mu W = 0.5W$$

Only force in the direction of motion is F

Now, final velocity $= 0$

displacement, $S = 60\,m$

Applying work energy equation,

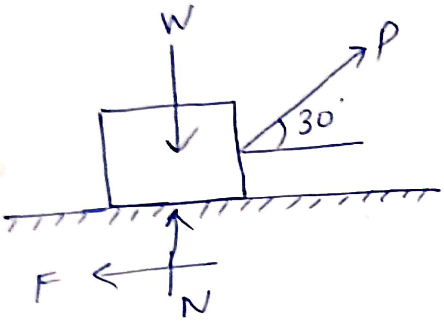
$$-F \times S = \frac{W}{2g}(v^2 - u^2)$$

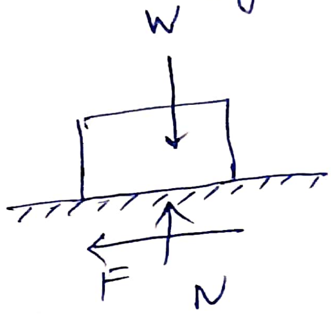
$$-0.5W \times 60 = \frac{W}{2 \times 9.81}(0 - u^2)$$

$$u = 24.261\ m/s$$

$$= \frac{24.261 \times 60 \times 60}{1000} = 87.34\ kmph.$$

A block weighing 2500 N rests on a level horizontal plane for which coefficient of friction is 0.20. This block is pulled by a force of 1000 N acting at an angle of 30° to the horizontal. Find the velocity of the block after it moves 30 m starting from rest. If the force of 1000 N is then removed, how much further will it move? Use work energy method





Sol^n Free body diagrams of the block for the two cases are shown in fig (a) & (b).

when pull P is acting:

$$N = W - P\sin 30°$$
$$= 2500 - 1000\sin 30 = 2000 \text{ newton}$$

$$F = \mu N = 0.2 \times 2000$$
$$= 400 \text{ newton}$$

Initial velocity $= 0$

let final velocity be v.

Displacement $= s = 30$ m

applying work energy equation for horizontal motion

$$(P\cos 30 - F)s = \frac{W}{2g}(v^2 - u^2)$$

$$(0.866 \times 1000 - 400)30 = \frac{2500}{2 \times 9.81}(V^2 - 0)$$

$$V = 10.4745 \text{ m/s}$$

Now, if the force 1000 N is removed, let the distance moved by s before the body comes to rest.

$\therefore$ Initial velocity $= 10.4745$ m/s

Final velocity $= 0$

applying work energy equation for the motion in horizontal direction, we get.

$$-F \times S = \frac{W}{2g}(V^2 - u^2)$$

$$-400 \times S = \frac{2500}{2 \times 9.81}(0 - 10.4745^2)$$

$$S = 34.95 \text{ m.}$$

Impulse Momentum

If R is the resultant force acting on a body of mass m, then from Newton's second law,

$$R = ma$$

But acceleration $a = \frac{dv}{dt}$

$$\therefore \quad R = m\frac{dv}{dt}$$

i.e, $R\,dt = m\,dv$

$$\therefore \int R\,dt = \int m\,dv$$

If initial velocity is u & after time interval t it becomes V, then

$$\int_0^t R\,dt = m\left[v\right]_u^v = mv - mu$$

The term $\int_0^t R.dt$ is called impulse. If the resultant force is in newton and time is in second, the unit of impulse will be Ns.

If R is constant during time interval t, then impulse is equal to $R \times t$.

The term mass x velocity is called momentum.

Now, $mv = \frac{w}{g}v$

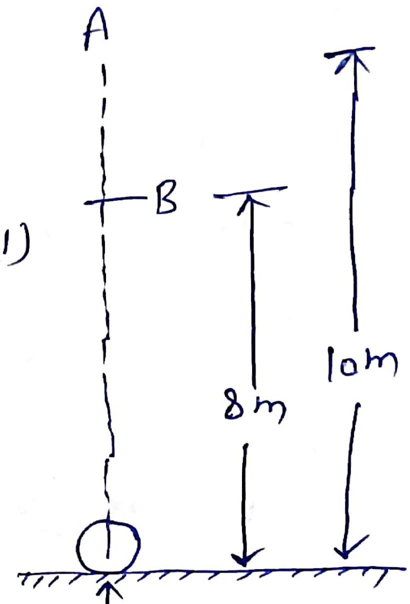
Substituting dimensional equivalence, we get,

$$= \frac{N}{m/s^2}\, m/s = Ns$$

Impulse = Final momentum − Initial momentum.

Q/ A glass marble, whose weight is 0.2 N, falls from a height of 10 m & rebounds to a height of 8 metres. Find the impulse and the average force between the marble and the floor, if the time during which they are in contact is 1/10 of a second.

Soln Applying kinematic equations, for the freely falling body (Ref Fig(i)), the velocity with which marble strikes the floor



fig(i)

$$= \sqrt{2gh}$$

$$= \sqrt{2 \times 9.81 \times 10}$$

$$= 14.007 \text{ m/s (downward)} \quad \text{— (1)}$$

Similarly applying kinematic equations for the marble moving up, we get the velocity of rebound

$$= \sqrt{2g \times 8} = \sqrt{2 \times 9.81 \times 8}$$

$$= 12.528 \text{ m/s} \quad \text{(upward)} \quad \text{— (2)}$$

Taking upward direction as positive and applying impulse momentum equation, we get.

$$\text{Impulse} = \frac{W}{g}(v-u)$$

$$= \frac{0.2}{9.81}[12.52 - (-14.007)]$$

$$= 0.541 \text{ Ns}$$

if F is the average force, then

$$Ft = 0.541$$

$$F \times 1/10 = 0.541 \text{ N}$$

$$F = 5.41 \text{ N}$$

A 1N ball is bowled to a batsman. The velocity of ball was 20 m/s horizontally just before batsman hit it. After hitting, it went away with a velocity of 48 m/s at an inclination of 30° to horizontal as shown in Fig 2(a). Find the average force exerted on the ball by the bat if the impact lasts for 0.02 second. (3)

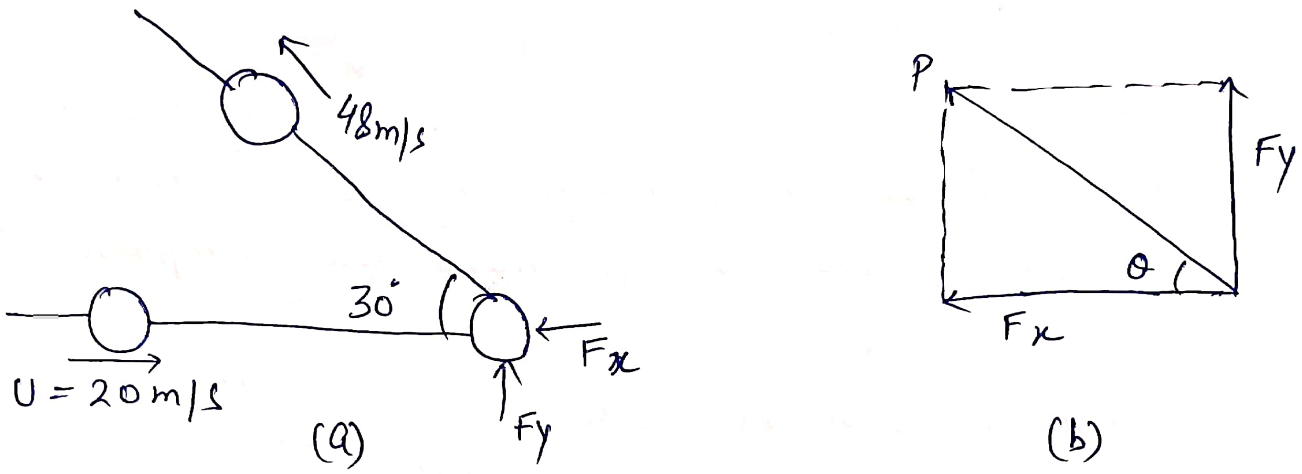


fig 2

Sol: Let F_x be the horizontal component of the force & F_y be the vertical component.

Applying impulse momentum equation in horizontal direction:

$$F_x \times 0.02 = \frac{1}{9.81}\left[48\cos 30^\circ - (-20)\right]$$

$$F_x = 313.81\ N$$

Applying impulse momentum equation in vertical direction

$$F_y \times 0.02 = \frac{1}{9.81}\left(48\sin 30^\circ - 0\right)$$

$$\therefore \quad F_y = 122.32\ N$$

$\therefore$ Resultant force

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{313.81^2 + 122.32^2}$$

$$F = 336.81\ N$$

(c $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{122.32}{313.81}\right)$

$\theta = 21.30°$ to a horizontal as shown in fig (b)

N.Q/ A 1500N block is in contact with a level plane, the coefficient of friction between two contact surfaces being 0.1. If the block is acted upon by a horizontal force of 300N, what time will elapse before the block reaches a velocity of 16m/s starting from rest? If 300N force is then removed, how much longer will the block continue to move? Solve the problem using impulse momentum equation.

Soln Consider the FBD of the block shown in fig (3).

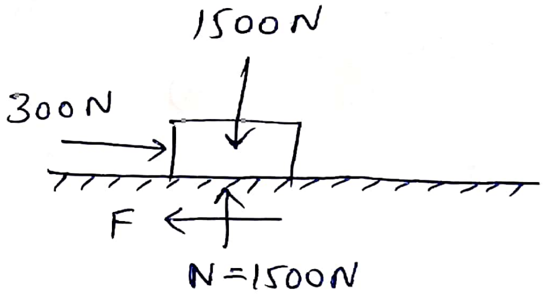


fig (3)

Normal reaction = 1500N

∴ Frictional force $F = \mu N = 0.1 \times 1500$
$= 150N$

Applying impulse momentum equation in the horizontal direction, we have

$$(300-150)t = \left[\frac{1500}{9.81}\right](v-u)$$

$$= \left[\frac{1500}{9.81}\right](16-0)$$

$t = 16.31$ seconds

If force is then removed the only horizontal force is F = 150N. Applying impulse momentum equation for the motion towards right, we have.

$$-150t = \frac{1500}{9.81}(0-16)$$

$$t = 16.31 \text{ Seconds}$$

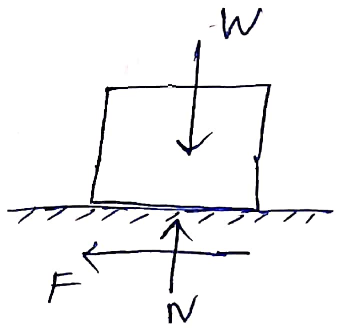
∴ block takes another 16.31 Seconds before it comes to rest.

3/ A 20 kN automobile is moving at a speed of 70 kmph when the brakes are fully applied causing all four wheels to skid. Determine the time required to stop the automobile (a) on concrete road for which $\mu = 0.75$, (b) on ice for which $\mu = 0.08$

Soln Initial velocity of the vehicle.

$$u = 70 \text{ Kmph} = \frac{70 \times 1000}{60 \times 60}$$

$$= 19.44 \text{ m/s}$$



fig(4)

Final velocity $V = 0$

Free body diagram is shown in fig(4).

$$F = \mu N = \mu W = 20\mu$$

Applying impulse momentum equation, we have

$$-Ft = \frac{W}{g}(V-u)$$

$$-20\mu t = \left[\frac{20}{9.81}\right](0-19.44)$$

$$\therefore \quad t = \frac{1.982}{\mu}$$

(a) on concrete road, $\mu = 0.75$

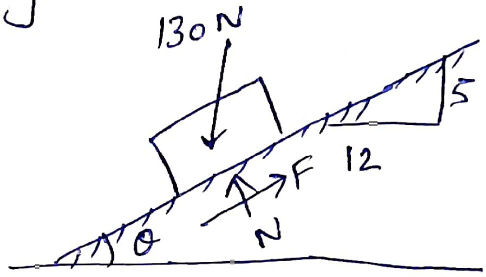
$$\therefore \quad t = \frac{1.982}{0.75} = 2.64 \text{ seconds}$$

(b) on ice, $\mu = 0.08$

$$\therefore \quad t = \frac{1.982}{0.08} = 24.78 \text{ seconds}$$

(6)

Q/ A block weighing 130N is on an incline, whose slope is 5 vertical to 12 horizontal. Its initial velocity down the incline is 2.4 m/s. What will be its velocity in 5 sec later? Take coefficient of friction at contact surface = 0.3.



Sol^n $\tan\theta = \frac{5}{12}$ $\therefore \theta = 22.62'$

$N = W\cos\theta = 130\cos 22.62°$

$= 120$ newton

$F = \mu N = 0.3 \times 120 = 36$ newton

ΣForces down the plane

$= R = W\sin\theta - F$

$= 130 \sin 22.62° - 36$

$= 14.0N$

Initial velocity $u = 2.4$ m/s

Let final velocity be V m/s

Time interval $t = 5$ seconds

Applying impulse momentum equation

$Rt = \frac{W}{g}(V-u)$, we get

$14 \times 5 = \left[\frac{130}{9.81}\right](V-2.4)$

$V = 7.68$ m/s

Conservation of Momentum

When a person jumps off a boat, the action of the person is equal and opposite to the reaction of boat. Hence, the resultant force is zero in the system. If W_1 is the weight of the person and W_2 that of the boat, V is velocity of the person and the boat before the person jumps out of the boat & V_1, V_2 are the velocities of person and the boat after jumping, according to principle of conservation of momentum.

$$\underbrace{\frac{W_1 + W_2}{g} V}_{\text{final momentum}} = \underbrace{\frac{W_1}{g} V_1 + \frac{W_2}{g} V_2}_{\text{initial momentum}}$$

The principle of conservation of momentum may be stated as, the momentum is conserved in a system in which resultant force is zero. In other words, in a system if the resultant force is zero; initial momentum will remain equal to final momentum.

Q, A 800 N man, moving horizontally with a velocity of 3 m/s, jumps off from the end of a pier into a 3200 N boat. Determine the horizontal velocity of the boat (a) if it had no initial velocity & (b) if it was approaching the pier with an initial velocity of 0.9 m/s.

Sol^n

weight of man $W_1 = 800N$

velocity with which man is running $V = 3\ m/s$

weight of the system after man jumps into boat $= 800 + 3200$

$= 4000N$

a) Initial velocity of boat = 0

Since the action of the man is equal to the reaction of the boat, the principle of conservation of momentum can be applied to the system consisting of the man & the boat.

Initial momentum = Final momentum

$$\underbrace{\frac{800}{9.81}}_{\text{person}} \times 3 + \underbrace{\frac{3200}{9.81}}_{\text{boat}} \times \underset{\text{initial velocity}}{0} = \frac{4000}{9.81} V$$

$$V = 0.6 \text{ m/s}$$

b) Initial velocity of boat = 0.9 m/s towards the pier.

$$= -0.9 \text{ m/s}$$

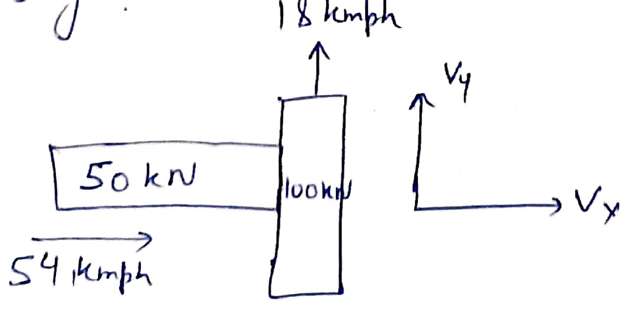
Applying principle of conservation of momentum, we get

$$\frac{800}{9.81} \times 3 + \frac{3200}{9.81} \times (-0.9) = \frac{4000}{9.81} V$$

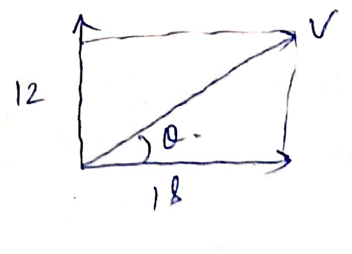
$$V = -0.12 \text{ m/s}$$

i.e velocity of boat & man will be 0.12 m/s towards the pier.

Q/ A car weighing 50 kN & moving at 54 kmph along the main road collides with a lorry of weight 100 kN which emerges at 18 kmph from a cross road at right angles to main road. If the two vehicles lock after collision, what will be the magnitude & direction of the resulting velocity?



(a)



(b)

Let the velocity of the vehicles after collision be V_x in x-direction (along main road) and V_y in y-direction (along cross road) as shown in fig (a). Applying impulse momentum equation along x-direction, we get.

$$\frac{50 \times 54}{9.81} + 0 = \frac{(50+100)}{9.81} V_x$$

$$\therefore \quad V_x = 18 \text{ kmph}$$

Applying impulse momentum equation in y-direction, we get

$$0 + \frac{100 \times 18}{9.81} = \frac{(50+100)}{9.81} V_y$$

$$V_y = 12 \text{ kmph}$$

$\therefore$ Resultant velocity $V = \sqrt{V_x^2 + V_y^2} = \sqrt{18^2 + 12^2}$

$= 21.63$ kmph

Its inclination to main road

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1} 12/18$$

$$= 33.69°$$

as shown in fig (b)

Q/ A gun weighing 300kN fires a 5kN projectile with a velocity of 300 m/s. With what velocity will gun recoil? If the recoil is overcome by an average force of 600kN how far will the gun travel? How long will it take?

Soln Applying principles of conservation of momentum to the system of gun & the projectile, we get

$$0 = 300 \times V + \underbrace{5 \times 300}_{\text{initial velocity}}$$

[gun velocity is stationary)

$$V = -5 \text{ m/s}$$

i.e Gun will have a velocity of 5 m/s in the direction opposite to that of bullet.

Let the gun recoil for a distance s.
using work energy equation, we have

$$-600 \times s = \frac{300}{2\times 9.81}(0-5^2)$$

$$s = 0.637 \text{ m.}$$

Applying impulse momentum equation to gun, we get

$$-600t = (300/9.81)(0-5)$$

$$t = 0.255 \text{ second}$$

Q/ A bullet weighing 0.3N is fired horizontally into a body weighing 100N which is suspended by a string 0.8m long. Due to this impact the body swings through an angle of 30°. Find the velocity of the bullet and the loss in the energy of the system.

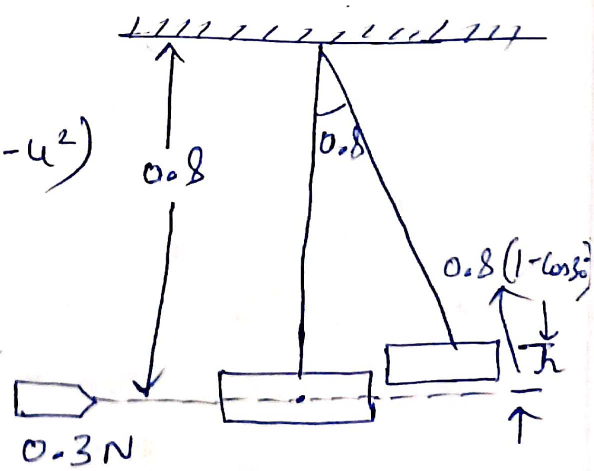
Sol^n Let the velocity of the block be u immediately after bullet strikes it.

Applying work energy equation for the block, we get

$$-Wh = \left(\frac{W}{2g}\right)(V^2 - u^2)$$

$$-100.3 \times 0.8(1-\cos 30°) = \left[\frac{100.3}{2\times 9.81}\right](0-u^2)$$

$$u = 1.45 \text{ m/s}$$



Let v be the velocity of bullet before striking the block. Applying principle of conservation of momentum to the bullet & block system, we get

$\frac{0.3}{9.81}$ V
Bullet O.
Ini

$$\frac{0.3}{9.81} v + \underbrace{0}_{\text{Block velocity}=0} = \frac{100+0.3}{9.81} \mu$$

bullet $0.3 v = 100.3 \times 1.45$

$v = 483.33\ m/s$

Initial energy of bullet $= \frac{0.3}{2 \times 9.81} (483.33)^2 = 3572.04\ J$

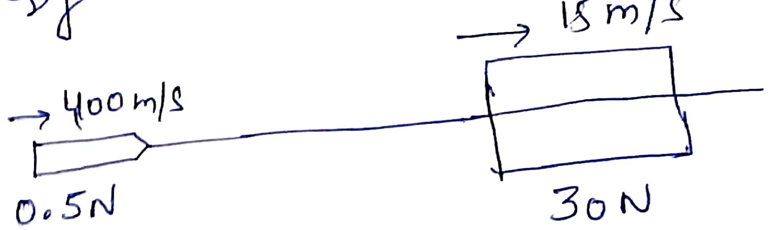
Energy of the block & bullet system

$= \frac{1}{2} \times \frac{100+0.3}{9.81} 1.45^2 = 10.75\ J$

Loss of energy $= 3572.04 - 10.75$
$= 3561.29\ J$

Q) A bullet weighs 0.5N and moving with a velocity of 400 m/s & hits centrally a 30N block of wood moving away at 15 m/s & gets embedded in it. Find the velocity of the bullet after the impact and the amount of kinetic energy lost.

Sol^n



Initial momentum of the system = Final momentum

$$\frac{0.5}{9.81} \times 400 + \frac{30}{9.81} \times 15 = \frac{(30+0.5)}{9.81} v \quad (1)$$

$v = 21.31\ m/s$

kinetic Energy lost = Initial K.E − Final K.E

$$= \left(\frac{1}{2} \times \frac{0.5}{9.81} \times 400^2 + \frac{1}{2} \times \frac{30}{9.81} 15^2\right) - \frac{1}{2} \times \frac{30.5}{9.81} \times 21.3^2$$

$= 3715.57\ J$

Impact of Elastic bodies

(i) line of Impact : Common normal to the colliding surfaces is known as line of impact.

(ii) Direct Impact : If the motion of two colliding bodies is directed along the line of impact, the impact is said to be direct impact.

(iii) oblique impact : If the motion of one or both of the colliding bodies is not directed along the line of impact, the impact is known as oblique impact.

(iv) Central impact : If the mass centres of colliding bodies are on the line of impact, the impact is called central impact.

(V) Eccentric Impact : Even if mass centre of one of the colliding bodies is not on the line of impact, the impact is called eccentric impact.

Coefficient of Restitution

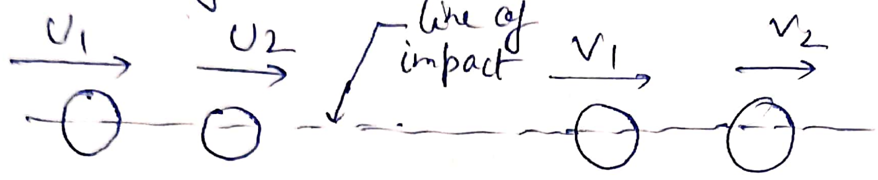
Period of collision (or time of impact) consist of two time intervals. Period of Deformation and period of restitution. Period of Deformation is the time elapse between the instant of the initial contact & the instant of maximum deformation of the bodies. Similarly, Period of Restitution is the time of elapse between the instant of the maximum deformation condition & the instant of separation of the bodies.

Impulse during deformation = $F_\Delta \, dt$

F_Δ is force that acts during period of deformation

Similarly, impulse during restitution = $F_R \, dt$

where F_R refers to the force that acts during period of restitution.



Before impact After impact

m_1 = mass of first body

m_2 = mass of second body

u_1 = velocity of first body before impact

u_2 = velocity of second body before impact

v_1 = velocity of first body after impact

v_2 = velocity of second body after the impact

At the instant of maximum deformation, the colliding bodies will have same velocity. Let the velocity of the bodies at the instant of maximum deformation be $U_{\Delta max}$

Applying impulse Momentum principle for the first body

$$F_\Delta \, dt = m_1 U_{\Delta max} - m_1 u_1 \quad \text{—— (i)}$$

& $$F_R \, dt = m_1 v_1 - m_1 U_{\Delta max} \quad \text{—— (ii)}$$

Now dividing eqn (i) & (ii)

$$\frac{F_R \, dt}{F_\Delta \, dt} = \frac{m_1 v_1 - m_1 U_{\Delta max}}{m_1 U_{\Delta max} - m_1 u_1}$$

ection

$$\frac{\;dt}{{}_\Delta dt} = \frac{V_1 - U_{\Delta max}}{U_{\Delta max} - u_1} \quad \text{—— (3)}$$

Similarly analysis of second body gives,

$$\frac{F_R\,dt}{F_\Delta\,dt} = \frac{U_{\Delta max} - V_2}{u_2 - U_{\Delta max}} \quad \text{—— (4)}$$

from eqn(3) & (4)

$$\frac{F_R\,dt}{F_\Delta\,dt} = \frac{V_1 - U_{\Delta max}}{U_{\Delta max} - u_1} = \frac{U_{\Delta max} - V_2}{u_2 - U_{\Delta max}}$$

$$= \frac{V_1 - U_{\Delta max} + U_{\Delta max} - V_2}{U_{\Delta max} - u_1 + U_2 - U_{\Delta max}} = \frac{V_1 - V_2}{u_2 - u_1} = \frac{V_2 - V_1}{u_1 - u_2}$$

$$= \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$e = \frac{F_R\,dt}{F_\Delta\,dt} = \frac{V_2 - V_1}{u_1 - u_2}$$

$$(V_1 - U_{\Delta max})(U_2 - U_{\Delta max}) = (U_{\Delta max} - u_1)(U_{\Delta max} - V_2)$$

$$V_1 U_2 - U_2 \cancel{U_{\Delta max}} - V_1 U_{\Delta max} + \cancel{U^2_{\Delta max}}$$

$$= \cancel{U^2_{\Delta max}} - u_1 V_{\Delta max} - \cancel{U_{\Delta max} V_2} + u_1 V_2$$

$$V_1 (U_2 - U_{\Delta max}) = u_1 (V_2 - U_{\Delta max})$$

Principle of conservation of momentum as,

$$\frac{W_1}{g}u_1 + \frac{W_2}{g}u_2 = \frac{W_1}{g}V_1 + \frac{W_2}{g}V_2$$

i.e $W_1u_1 + W_2u_2 = W_1V_1 + W_2V_2$ —— (i)

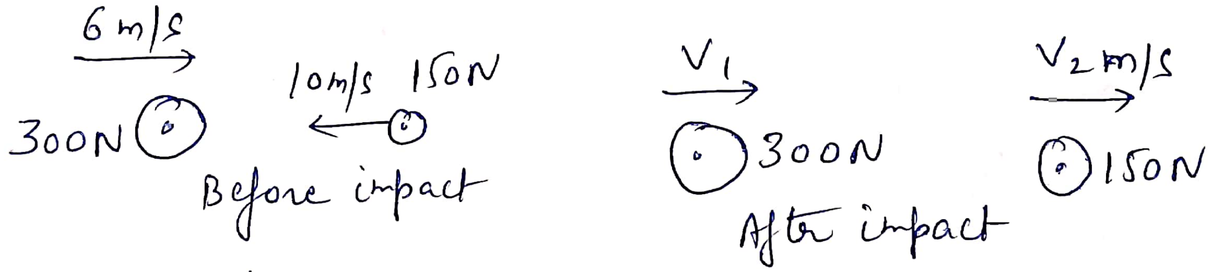
Another equation based on the definition of coefficient of restitution may be as.

$$e(u_1 - u_2) = V_2 - V_1 \quad \text{—— (ii)}$$

from (i) & (ii) unknown velocities V_1 & V_2 may be found

Q/ Direct central impact occurs between a 300 N body moving to the right with velocity of 6 m/s & 150 N body moving to the left with a velocity of 10 m/s. Find the velocity of each body after impact if the coefficient of restitution is 0.8

Sol



initial velocity of 300 N body

$u_1 = 6$ m/s

Initial velocity of 150 N body $u_2 = -10$ m/s

Final velocity of 300 N body $= V_1$ m/s

Final velocity of 150 N body $= V_2$ m/s rom principle of conservation of momentum

$$\frac{300}{g} \times 6 + \frac{150}{g}(-10) = \frac{300}{g} V_1 + \frac{150}{g} V_2$$

i.e $2V_1 + V_2 = 2$ ——— (i)

from the definition of Coefficient of restitution, we have.

$$e(U_1 - U_2) = V_2 - V_1$$

$$0.8(6+10) = V_2 - V_1$$

$$V_2 - V_1 = 12.8 \quad \text{——— (ii)}$$

from eq^n (i) & (ii)

$$3V_1 = -10.8$$

$$V_1 = -3.6 \text{ m/s}$$

& hence, $V_2 = 12.8 + V_1 = 12.8 - 3.6$

$$= 9.2 \text{ m/s}$$

Q/ A 80 N body moving to the right at a speed of 3 m/s strikes a 10 N body that is moving to the left at a speed of 10 m/s. The final velocity of 10 N body is 4 m/s to the right. Calculate the coefficient of restitution and the final velocity of 80 N body.

soln $U_1 = 3$ m/s $\qquad U_2 = -10$ m/s

$V_1 = ?$ $\qquad V_2 = 4$ m/s

applying principles of conservation of momentum to the colliding bodies, we get

6

$$\frac{80}{g}\times 3 + \frac{10}{g}(-10) = \frac{80}{g}v_1 + \frac{10}{g}\times 4$$

$$v_1 = \frac{80\times 3 - 100 - 40}{80} = 1.25\, m/s$$

from the definition of coefficient of restitution, we get

$$e(u_1 - u_2) = v_2 - v_1$$

$$e(3+10) = 4 - 1.25$$

$$e = 0.212$$