

MATHEMATICS

ASSIGNMENT-2

Q11 $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2 e^{2x} \sin 2x$

A $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

$$D^2 - 4D + 4 = 0$$

$$D(D-2) - 2(D-2) = 0$$

$$D = 2, 2$$

$$CF = e^{2x} (C_1 + C_2 x)$$

$$PI = \frac{1}{(D^2 - 4D + 4)} 8x^2 e^{2x} \sin 2x$$
$$= \frac{8e^{2x}}{D^2 + 4D + 4 - 4D - 8 + 4} x^2 \sin 2x$$

$$\Rightarrow 8e^{2x} \frac{1}{D^2} x^2 \sin 2x$$

$$= 8e^{2x} \frac{1}{D} \left[-\frac{1}{2} x^2 \cos(2x) + \frac{1}{2} x \sin(2x) + \frac{1}{4} \cos(2x) + C \right]$$

$$\Rightarrow 8e^{2x} \left[-\frac{1}{4} x^2 \sin(2x) - \frac{1}{4} x \cos(2x) + \frac{1}{8} \sin(2x) + \frac{1}{8} \sin(2x) + \frac{1}{8} \sin(2x) + C_1 \right]$$

$$\Rightarrow e^{2x} [-2x^2 \sin(2x) - 2x \cos(2x) + 3 \sin(2x) + C]$$

$$y = CF + PI$$

$$= e^{2x} (C_1 + C_2 x) + e^{2x} [-2x^2 \sin 2x - 2x \cos 2x + 3 \sin 2x] + C$$

$$\begin{aligned} \text{Q3} \quad D^2x + Dy + 3x &= e^{-t} \quad \text{--- (1)} \\ D^2y - 4Dx + 3y &= \sin 2t \quad \text{--- (2)} \end{aligned}$$

Multiply eq (1) by (D^2+3) & (2) by D

$$(D^2+3)^2x + D(D^2+3)y = (D^2+3)e^{-t} \quad \text{--- (3)}$$

$$-4D^2x + D(D^2+3)y = (\sin 2t)(D) \quad \text{--- (4)}$$

On subtracting

$$(D^2+3)^2x + D(D^2+3)y + 4D^2x - D(D^2+3)y = e^{-t}(D^2+3) - D(\sin 2t)$$

$$\begin{aligned} (D^4 + 10D^2 + 9)x &= D^2e^{-t} + 3e^{-t} - \sin(2t)D \\ &= e^{-t} + 3e^{-t} - 2\cos(2t) \end{aligned}$$

$$(D^4 + 10D^2 + 9)x = 4e^{-t} - 2\cos(2t)$$

$$M = D^2$$

$$(m^2 + 10m + 9) = 0$$

$$m^2 + 9m + m + 9 = 0$$

$$m = -1, -9$$

$$D^2 = -1 \quad ; \quad D^2 = -9$$

$$D = \pm i \quad ; \quad D = \pm 3i$$

$$CF = [(C_1 + iC_2x)\cos x + (C_3 + iC_4x)\sin 3x]$$

$$PI = \frac{1}{(D^4 + 10D^2 + 9)} [4e^{-t} - 2\cos(2t)]$$

$$= \frac{4e^{-t}}{D^4 + 10D^2 + 9} - \frac{2}{D^4 + 10D^2 + 9} \cos 2t$$

$$D = -1 \quad ; \quad D^2 = -4$$

$$= \frac{4e^{-t}}{1+10+9} - \frac{2}{(-4)^2-10 \cdot 4+9} \cos(2t)$$

$$= \frac{e^{-t}}{5} - \frac{2}{15} \cos(2t)$$

$$y = CF + PI$$

$$y = [C_1 + C_2 x] \cos x + [C_3 + C_4 x] \sin 3x + \left[\frac{e^{-t}}{5} - \frac{2 \cos 2t}{15} \right] + c$$

Q3) $\frac{d^2 y}{dx^2} - y = \frac{2}{1+e^x}$

Sol:

$$(D^2 - 1) = 0$$

$$D = \pm 1$$

$$CF = Ae^x + Be^{-x}$$

$$u = e^x$$

$$; v = e^{-x}$$

$$u' = e^x$$

$$; v' = -e^{-x}$$

$$uv' - u'v = e^x \cdot (-e^{-x}) - e^x e^{-x}$$

$$= -1 - 1$$

$$= -2$$

$$R = \frac{2}{1+e^x}$$

$$A = - \int \frac{RV}{uv' - u'v} dx = + \int \frac{x e^{-x}}{(1+e^x)(x)} dx$$

$$A = \int \frac{dx}{e^x + e^{2x}}$$

$$= -\frac{1}{e^x} - \log |e^x| + \log |1+e^x| + C_1$$

$$\begin{aligned} B = \int \frac{R}{U} dx &\Rightarrow \int \frac{x \cdot e^x}{1+e^x} dx \\ &= \int \frac{e^x}{1+e^x} dx \\ &= -\log |1+e^x| + C_2 \end{aligned}$$

$$y = \left[-\frac{1}{e^x} - \log |e^x| + \log |1+e^x| + C_1 \right] e^x - \left[\log |1+e^x| + C_2 \right] e^{-x}$$

Q5) $x^2 y'' + x y' - y = x^2 e^x$

$$y'' + \frac{y'}{x} - \frac{y}{x^2} = e^x$$

$$\left(D^2 + \frac{D}{x} - \frac{1}{x^2} \right) y = e^x$$

$$R = e^x$$

$$x^2 y'' + x y' - y = 0$$

$$[D(D-1) + D-1] y = 0$$

$$D^2 - 1 = 0$$

$$D = \pm 1$$

$$y = A e^x + B e^{-x}$$

$$\Rightarrow x = \log x$$

$$y = A e^{\log x} + B e^{-\log x}$$

$$= Ax + \frac{B}{x}$$

$$u = x, \quad v = \frac{1}{x}$$

$$u' = 1 \quad ; \quad v' = -\frac{1}{x^2}$$

$$uv' - u'v = x \cdot \frac{1}{x^2} - 1 \cdot \frac{1}{x} \Rightarrow -\frac{2}{x}$$

$$A = - \int \frac{B u dx}{\frac{2}{x}} \Rightarrow - \int \frac{e^x \cdot \frac{1}{x}}{\frac{2}{x}} dx$$

$$\Rightarrow \frac{1}{2} e^x + c_1$$

$$B = \int \frac{B v dx}{-\frac{2}{x}} \Rightarrow \int \frac{e^x x}{-\frac{2}{x}} \Rightarrow -\frac{1}{2} \int x^2 e^x dx$$

$$\Rightarrow -\frac{1}{2} [x^2 e^x - 2x e^x + 2e^x] + c_2$$

$$y = \left[\frac{1}{2} e^x \cdot x + c_1 \right] - \frac{1}{2} [x^2 e^x - 2x e^x + 2e^x + c_2] \frac{1}{x} \rightarrow \cancel{2e^x}$$

Q6)

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \Rightarrow \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$$

$$= -e^{-x} \cdot e^x - e^x \cdot e^{-x}$$

$$\Rightarrow -2 \neq 0$$

So, clearly it is linearly independent

Q7) $x^4 + 3x^3 - x^2 + 5x = 2$

Sol: $P_0(x) = 1 \Rightarrow AI = A \cdot P_0(x)$ {where A is const.}

$P_1(x) = x$

$P_2(x) = \frac{1}{2} [3x^2 - 1]$

$2P_2 = 3x^2 - 1$

$x^2 = \frac{2}{3} P_2(x) + \frac{1}{3} P_0(x)$

$P_3(x) = \frac{1}{2} [5x^3 - 3x]$

$5x^3 = 2P_3(x) + 3x$

$x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$

$P_4(x) = \frac{1}{8} (35x^4 - 30x^2 + 3)$

$35x^4 = 8P_4(x) + 30x^2 - 3$

$35x^4 = 8P_4(x) + 30 \left[\frac{2}{3} P_2(x) + \frac{1}{3} P_0(x) \right] - 3P_0(x)$

$= \frac{1}{35} [8P_4(x) + 20P_2(x) + 10P_0(x) - 3P_0(x)]$

$x^4 = \frac{1}{35} [8P_4(x) + 20P_2(x) + 7P_0(x)]$

$x^4 + 3x^3 - x^2 + 5x - 2 = 0$

$\frac{1}{35} [8P_4(x) + 20P_2(x) + 7P_0(x)] + 3 \left[\frac{2}{5} P_3(x) + \frac{3}{5} P_1(x) \right] -$

$\frac{2}{3} P_2(x) + \frac{1}{3} P_0(x) + 5P_1(x) - 2P_0(x) = 0$

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$$P_1(x) = x$$

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$$x^2 = \frac{2}{3} P_2(x) + \frac{1}{3} P_0(x)$$

$$P_3(x) = \frac{1}{2} [5x^3 - 3x]$$

$$5x^3 = 2P_3(x) + 3x$$

$$x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$$

$$P_4(x) = \frac{1}{8} (35x^4 - 30x^2 + 3)$$

$$35x^4 = 8P_4(x) + 30x^2 - 3$$

$$35x^4 = 8P_4(x) + 30 \left[\frac{2}{3} P_2(x) + \frac{1}{3} P_0(x) \right] - 3P_0(x)$$

$$= \frac{1}{35} [8P_4(x) + 20P_2(x) + 10P_0(x) + 3P_0(x)]$$

$$x^4 = \frac{1}{35} [8P_4(x) + 20P_2(x) + 7P_0(x)]$$

$x^4 + 3x^3 - x^2 + 5x - 2 = 0$

$$\frac{1}{35} [8P_4(x) + 20P_2(x) + 7P_0(x)] + 3 \left[\frac{2}{5} P_3(x) + \frac{3}{5} P_1(x) \right] -$$

$$\frac{2}{3} P_2(x) + \frac{1}{3} P_0(x) + 5P_1(x) - 2P_0(x) = 0$$

Q9) $\frac{d}{dx} [x J_n + J_{n+1}] = x [J_n^2 - J_{n+1}^2]$

Sol: $J_n J_{n+1} + x [J'_n J_{n+1} + J_n J'_{n+1}] = 0$ — (1)

from 3rd recurrence relation

$$J'_n = J_{n-1} - \frac{n}{x} J_n$$

Replace by $n+1$

$$J'_{n+1} = J_n - \frac{n+1}{x} J_{n+1} \quad \text{--- (2)}$$

from 4th recurrence relation

$$J'_n = -J_{n+1} + \frac{n}{x} J_n \quad \text{--- (3)}$$

On substituting

$$J_n J_{n+1} - x J_{n+1}^2 + n J_n J_{n+1} + x J_n^2 - (n+1) J_n J_{n+1} = x (J_n^2 - J_{n+1}^2) \rightarrow \text{HP}$$

Q10) $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$

Sol: $P_0(x) = x$; $P_1(x) = 1$; $P_2(x) = x$

Now at $x=0$; $P_0(0) = 0 \rightarrow$ singular point

$$\lim_{x \rightarrow 0} (x-a) \frac{P_1(x)}{P_0(x)}$$

$$\lim_{x \rightarrow 0} x \cdot \frac{1}{x}$$

$$\lim_{x \rightarrow 0} (x-a)^2 \frac{P_2(x)}{P_0(x)}$$

$$\lim_{x \rightarrow 0} x^2 \cdot \frac{x}{x} = 0 \quad \text{finite}$$

By Frobenius method

$$y = \sum_{n=0}^{\infty} a_n x^{m+n}$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} (m+n) a_n x^{m+n-1}$$

$$\frac{d^2y}{dx^2} = \sum_{n=0}^{\infty} (m+n)(m+n-1) a_n x^{m+n-2}$$

Put the value of y , $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ in eq. ①

$$\Rightarrow x \left[\sum_{n=0}^{\infty} (m+n)(m+n-1) a_n x^{m+n-2} \right] + \left[\sum_{n=0}^{\infty} (m+n) a_n x^{m+n-1} \right] + x \left[\sum_{n=0}^{\infty} a_n x^{m+n} \right]$$

$$\Rightarrow \sum_{n=0}^{\infty} (m+n)(m+n-1) a_n x^{m+n-1} + \sum_{n=0}^{\infty} a_n x^{m+n+1}$$

$$\Rightarrow \sum_{n=0}^{\infty} [(m+n)(m+n-1) + (m+n)] a_n x^{m+n-1} + \sum_{n=0}^{\infty} a_n x^{m+n+1} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} [(m+n)^2 a_n x^{m+n-1}] + \sum_{n=0}^{\infty} a_n x^{m+n+1} = 0 \quad \text{--- ②}$$

now equating least power of x i.e. $x^{m-1} = 0$

$$(m^2) a_0 = 0$$

$$m^2 = 0 \Rightarrow m = 0, 0$$

$$m_1 = 0, m_2 = 0$$

from eq. ②

$$\sum_{n=0}^{\infty} (m+n+1)^2 a_{n+1} x^{m+n} + \sum_{n=0}^{\infty} a_{n-1} x^{m+n} = 0$$

$$\sum_{n=0}^{\infty} [(m+n+1)^2 a_{n+1} + a_{n-1}] x^{m+n} = 0 \quad \text{--- ③}$$

Put the coeff of $x^{m+1} = 0$

$$(m+1+1)a_{m+1} + a_{m+1} = 0$$

$$a_{m+1} = -\frac{1}{(m+1)^2} a_m$$

Put $n=1$

$$a_1 = -\frac{1}{(m+1)^2} a_0$$

$n=2$

$$a_2 = -\frac{1}{(m+3)^2} a_1 \quad [a_1 \neq 0]$$

Now by equating coeff of x^m both side

$$(m+1)^2 a_1 x^{m+1} = 0$$

$$a_1 = 0$$

$$a_3 = 0, \text{ put } n=3$$

$$a_4 = -\frac{1}{(m+4)^2} a_3 \quad [a_3 = 0]$$

$$a_4 = 0$$

$n=4$

$$a_5 = -\frac{1}{(m+5)^2} a_4 \quad ; \quad a_5 = 0$$

$$\text{Now } \sum_{n=0}^{\infty} a_n x^{m+n}$$

$$= a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + a_3 x^{m+3} + a_4 x^{m+4} + \dots$$

$$= x^m \left[a_0 + \frac{a_1 x}{(m+1)^2} + (0)x^2 + (0)x^3 + \dots \right]$$

$$\Rightarrow y = x^m \left[a_0 - \frac{a_0 x}{(m+1)^2} \right]$$

$$y = a_0 x^m \left[1 - \frac{x^2}{(m+2)^2} \right] \quad \text{--- (5)}$$

$$y = C_1 (y)_{m=0} + C_2 \left(\frac{dy}{dm} \right)_{m=0}$$

$$(y)_{m=0} = a_0 x^0 \left[1 - \frac{x^2}{2^2} \right]$$

$$\Rightarrow a_0 \left[1 - \frac{x^2}{2^2} \right]$$

Now partially diff. eq (5) w.r.t m

$$\frac{dy}{dm} = a_0 x^m \log x \left[1 - \frac{x^2}{(m+2)^2} \right] + a_0 x^m \left[0 - \left(\frac{-2x^2}{(m+2)^2} \right) \right]$$

$$= a_0 x^m \log x \left[1 - \frac{x^2}{(m+2)^2} \right] + a_0 x^m \left[\frac{2x^2}{(m+2)^2} \right]$$

$$\left(\frac{dy}{dm} \right)_{m=0} \Rightarrow a_0 (1) \log x \left[1 - \frac{x^2}{2^2} \right] + a_0 (1) \left[\frac{2x^2}{2^3} \right]$$

$$y = C_1 \left[a_0 \left(1 - \frac{x^2}{2^2} \right) \right] + C_2 \left[a_0 \left[\log x \left(1 - \frac{x^2}{2^2} \right) + \frac{2x^2}{2^3} \right] \right]$$

$$y = A \left(1 - \frac{x^2}{2^2} \right) + B \left[\log x \left(1 - \frac{x^2}{2^2} \right) + \frac{2x^2}{2^3} \right] \rightarrow \text{---}$$

Q4) $2x(1-x) \frac{d^2 y}{dx^2} + (1-x) \frac{dy}{dx}$

Sol.) $P_0(x) = 2x(1-x)$

$$P_1(x) = 1-x$$

$$P_2(x) = 3$$

Now at $x=0$

$$P_0(0) = 0 \rightarrow \text{singular pt}$$

$$P_1(0) = 1$$

$$P_2(0) = 3$$

(1) Determination of

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$$\lim_{x \rightarrow a} \frac{(x-a) P_1(x)}{P_0(x)}$$

$$\lim_{x \rightarrow a} \frac{(x-a)^2 P_2(x)}{P_0(x)}$$

$$\lim_{x \rightarrow 0} \frac{x(1-x)}{2x(1-x)} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{(x-0)^2 \cdot 3}{2x(1-x)}$$

$$\lim_{x \rightarrow 0} \frac{x^2 \cdot 3}{2x(1-x)}$$

$$\lim_{x \rightarrow 0} \rightarrow 0 \text{ (finite)}$$

Now it is regular singular point

By Frobenius method:-

$$y = \sum_{n=0}^{\infty} a_n x^{m+n}$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} (m+n) a_n x^{m+n-1}$$

$$\frac{d^2y}{dx^2} = \sum_{n=0}^{\infty} (m+n)(m+n-1) a_n x^{m+n-2}$$

Now, put the value of $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$

$$\Rightarrow 2x(1-x) \left[\sum_{n=0}^{\infty} (m+n)(m+n-1) a_n x^{m+n-2} \right] + (1-x) \left[\sum_{n=0}^{\infty} (m+n) a_n x^{m+n-1} \right] + 3 \left[\sum_{n=0}^{\infty} a_n x^{m+n} \right] = 0$$

$$\Rightarrow 2x - 2x^2 \left[\sum_{n=0}^{\infty} (m+n)(m+n-1) a_n x^{m+n-2} \right] + (1-x) \left[\sum_{n=0}^{\infty} (m+n) a_n x^{m+n-1} \right] + 3 \left[\sum_{n=0}^{\infty} a_n x^{m+n} \right] = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(m+n)(m+n-1) a_n x^{m+n-1} - \sum_{n=0}^{\infty} 2(m+n)(m+n-1) a_n x^{m+n} + \sum_{n=0}^{\infty} (m+n) a_n x^{m+n-1} - \sum_{n=0}^{\infty} (m+n) a_n x^{m+n} + \sum_{n=0}^{\infty} 3a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} [2(m+n)(m+n-1)(m+n)] a_n x^{m+n-1} - \sum_{n=0}^{\infty} [2(m+n)(m+n-1)(m+n)-3] a_n x^{m+n}$$

$$\Rightarrow \sum_{n=0}^{\infty} [2(m+n-1+1)(m+n)] a_n x^{m+n-1} - \sum_{n=0}^{\infty} [2(m+n-1+1)(m+n)-3] a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n) a_n x^{m+n-1} - \sum_{n=0}^{\infty} [2m+2n-1)(m+n)-3] a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n) a_n x^{m+n-1} - \sum_{n=0}^{\infty} [2(m+n)^2 - (m+n) - 3] a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n) a_n x^{m+n-1} - \sum_{n=0}^{\infty} [2(m+n)^2 - (m+n) - 3] a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n) a_n x^{m+n-1} - \sum_{n=0}^{\infty} [2(m+n)-3](m+n+1) a_n x^{m+n} = 0$$

Now, put coeff of lowest power of x i.e. x^{m-1} to zero

$$(2m+2n-1)(m+n) a_n = 0$$

$$(2m+0-1)(m+0) a_0 = 0$$

$$(2m-1)(m) a_0 = 0$$

$$(2m-1)(m) = 0$$

$$m = \frac{1}{2} ; m = 0$$

Now $m_1 \neq m_2$

$$m_1 - m_2 = \frac{1}{2} ; \frac{1}{2} \neq \text{integer}$$

from eq ②

$$\Rightarrow \sum_{n=0}^{\infty} [2m+2(n+1)-1](m+n+1) a_{n+1} x^{m+n} - \sum_{n=0}^{\infty} (2m+2n-3)(m+n+1) a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n+1)(m+n+1) a_{n+1} x^{m+n} - (2m+2n-3)(m+n+1) a_n x^{m+n}$$

Now by equating coeff. of x^{m+n} to zero

$$D_n = \ln x R$$

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$$(2m+2n+1)(m+n+1)a_{n+1} = (2m+2n-3)(m+n+1)a_n = 0$$

$$a_{n+1} = \frac{(2m+2n-3)(m+n+1)a_n}{(2m+2n+1)(m+n+1)}$$

put $n=0$ in eq (3)

$$a_1 = \left(\frac{2m-3}{2m+1} \right) a_0$$

put $n=1$ in eq (3)

$$a_2 = \left(\frac{2m-1}{2m+3} \right) a_1 = \frac{2m-1}{2m+3} \left(\frac{2m-3}{2m+1} \right) a_0$$

$$a_2 = \frac{(2m-1)(2m-3)}{(2m+3)(2m+1)} a_0$$

Put $n=2$ in eq 3

$$a_3 = \left(\frac{2m+1}{2m+5} \right) a_2$$

$$\Rightarrow \left(\frac{2m+1}{2m+5} \right) \left[\frac{(2m-1)(2m-3)}{(2m+3)(2m+1)} \right] a_0$$

$$\Rightarrow \frac{(2m-1)(2m-3)}{(2m+5)(2m+3)} a_0$$

$$y = (C_1 y)_{m=m_1} + (C_2 y)_{m=m_2} \quad \text{--- (4)}$$

$$y = \sum_{n=0}^{\infty} a_n x^{m+n} = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + a_3 x^{m+3} + \dots$$

Put the value of a_0, a_1, a_2, a_3 we get

$$y = a_0 x^m + \left[\left(\frac{2m-3}{2m+1} \right) a_0 \right] x^{m+1} + \left[\frac{(2m-1)(2m-3)}{(2m+3)(2m+1)} a_0 \right] x^{m+2} +$$

$$\left[\frac{(2m-1)(2m-3)}{(2m+5)(2m+3)} a_0 \right] x^{m+3} + \dots$$

Examination of wave lengths of No light using
 Newton's Ring
 Diameter for dark ring
 $D_r^2 = 4n\lambda R$

$$|y|_m = \frac{1}{2} = a_0 \sqrt{x} + \frac{(1-3)}{(1!)} a_0 x^{3/2} + \frac{(1-1)(1-3)}{(1!)(2!)} a_0 x^{5/2} + \frac{(1-1)(1-3)}{(1!)(5!)} a_0 x^{7/2}$$

$$\Rightarrow a_0 \sqrt{x} + \frac{(-2)}{2} a_0 x^{3/2} + 0 + 0 = \dots$$

$$\Rightarrow a_0 (x^{1/2} - x^{3/2})$$

Now at $m=0$

$$|y|_{m=0} = a_0 x^0 + \frac{(1-3)}{(1!)} a_0 x^1 + \frac{(1-1)(1-3)}{(1!)(2!)} a_0 x^2 + \frac{(1-1)(1-3)}{(1!)(5!)} a_0 x^3$$

$$|y|_{m=0} = a_0 [1 - 3x + x^2 + \frac{1}{5} x^3 + \dots]$$

Now eq (4)

$$y = C_1 |y|_{m=1} + C_2 |y|_{m=0}$$

$$= C_1 [a_0 (x^{1/2} - x^{3/2})] + C_2 [a_0 (1 - 3x + x^2 + \frac{1}{5} x^3 + \dots)]$$

$$= A (x^{1/2} - x^{3/2}) + B (1 - 3x + x^2 + \frac{1}{5} x^3 + \dots)$$