"Engineering Mechanics"

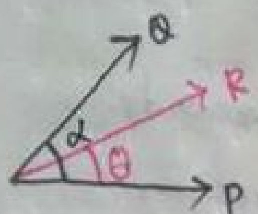
① Newton's law of gravitation

$$F = G \cdot \frac{M \cdot m}{R^2} \qquad G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

Acceleration due to gravity.

$$g = \frac{GM}{R^2} \ (m/sec^2).$$

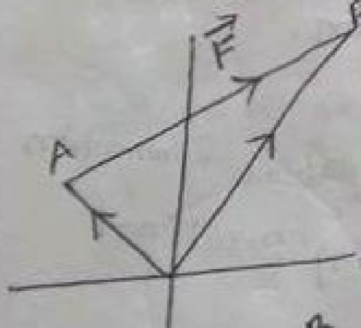
② Parallelogram law of forces.



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

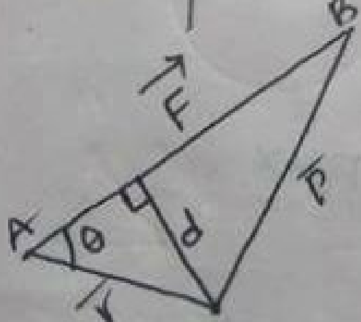
$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}.$$

③ Moment:



$$\vec{F} = F_m \times \vec{AB}$$

$$F_m = \frac{F}{|\vec{AB}|} \rightarrow \text{Force Multiplier.}$$

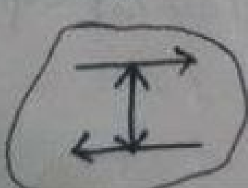


$$M_0^F = F \times d = F \times r \sin\theta$$

$$= \vec{r} \times \vec{F} \text{ (or) } \vec{P} \times \vec{F}.$$

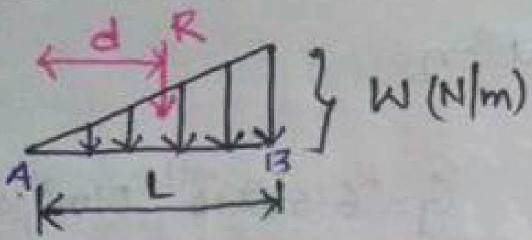
$$= \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

④ Couple:



$$C = F \times d$$

⑤ UVL:

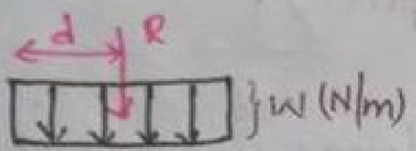


$R = \frac{1}{2} WL$

$d = \frac{2}{3} L$ from A.

$d = \frac{1}{3} L$ from B.

⑥ UDL:

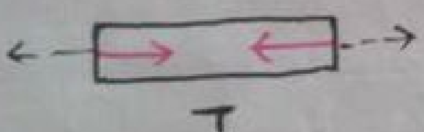


$R = WL$

$d = \frac{L}{2}$

⑧

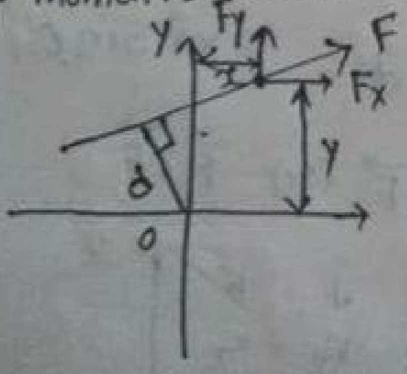
Internal forces towards the joint ⟷ Compression



Internal forces away from the joint ⟷ Tension.

⑨ "Varignon Theorem":-

Momentum due to resultant is equal to Summation of moments due to individual forces forming Resultant



$M_o^F = F \times d$

$= F_x \times y - F_y \times x$

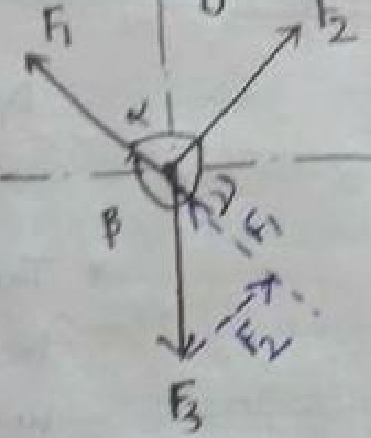
Note: Couple is independent of point of application.

"All Couples are moments, but all moments are not couple".

② "Equilibrium of forces"

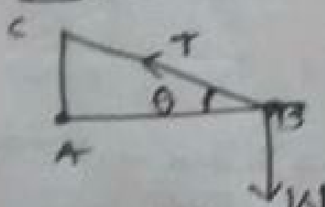
"Lami's Theorem":-

For a coplanar concurrent force system if three forces acting on a body forms the side of a triangle taken in order then they are in eqm condition.



By using lami's theorem we can fix the direction of unknown force.

Note:1



* for $\theta = 45°$: $R_{AX} = W$ ∴ $\left[T = \frac{W}{\sin\theta}\right]$

$R_{Ay} = 0$ only when Tension (T) & Weight (W) are Concurrent.

Since $T_y = T\sin\theta = W$

Note2: Whenever Contact between two takes place we must consider normal support into consideration.

③ FRICTION:-

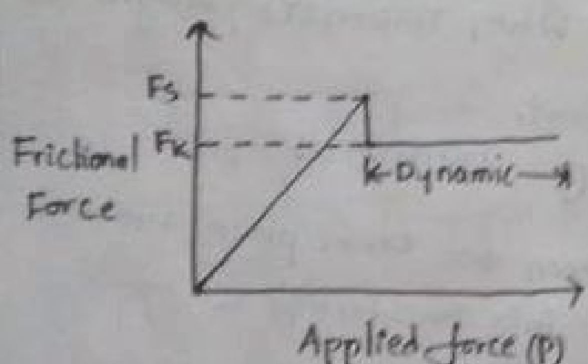
① Frictional Force $\propto$ Normal Reaction $F \propto N$

$F = \mu N$ [μ = coeff. friction]

Friction

- Static / limiting Friction
  $F_S = \mu_S N$
  * Max. Frictional force generated when the body is ready to move.
- Dynamic / Kinetic Friction
  $F_K = \mu_K N$
  * Taken into account when the body is in motion w.r.t. time

* Usually $F_S > F_K \Rightarrow \mu_S > \mu_K$

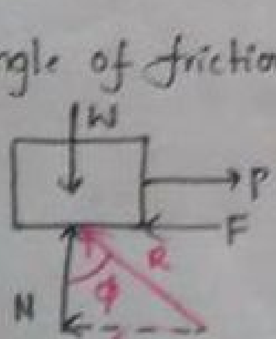


if $P = F_S \rightarrow$ Block is ready to slide

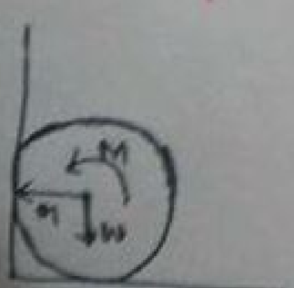
if $P > F_S \Rightarrow P - F_S = ma$ motion

if $P < F_S \rightarrow$ static then frictional force $F = P$.

② Angle of friction:- Angle made by the resultant of frictional force and Normal Reaction with normal reaction.



$\tan\phi = \frac{F}{N} = \frac{\mu N}{N} \rightarrow \boxed{\tan\phi = \mu}$

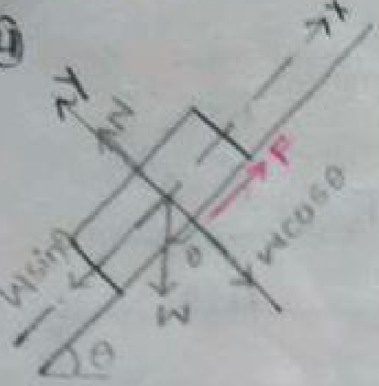


Coeff. friction = f

$N_2 = f N_1$; $M = rf(N_1 + N_2)$

$N_1 = \frac{W}{(1+f^2)}$; $N_2 = \frac{Wf}{(1+f^2)}$

4)



* $F_s = \mu_s N = \mu_s W\cos\theta$

$\mu_s = \tan\theta = \tan\phi$

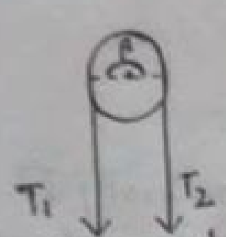
1) $\theta = \phi \Leftrightarrow W_x = F_s$ → Block is ready to slide down

2) $\theta > \phi \Leftrightarrow W_x > F_s$ → Block slides down: motion.

3) $\theta < \phi \Leftrightarrow W_x < F_s$ → Block can't slides down on its own. ∴ Rest.

Note: * Sliding is independent of weight.
* Normal Reaction is independent of applied force.

⑤ Rope & DRUM "BELT FRICTION"



Tightened Side | Slackened Side

$[T_1 > T_2]$

β = Lap angle / Angle of contact. (Rad)

* Smooth pulley $[T_1 = T_2]$.

$$\therefore \frac{T_1}{T_2} = e^{\mu\beta} \quad \text{if } T_1 > T_2$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \quad \text{if } T_2 > T_1$$

→ $\beta = 2\pi n$ n = no of loops over the drum.

"TRUSSES"

- Method of joints
- Method of sections.

Method of joints:

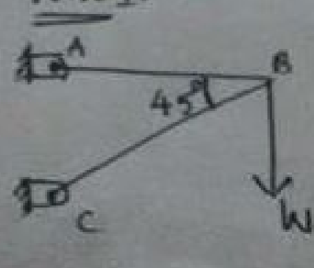
* joint •⟶ Force (away) (Tension)

joint ⟵ Force (Towards) (Compression)

Method of sections:

* Section Cut shouldn't pass through more than '3' members.

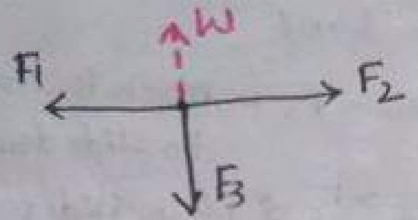
Note 1:



$F_{AB} = W$

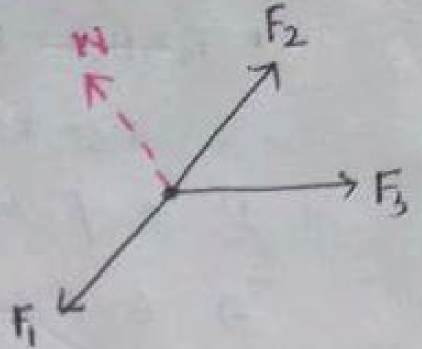
$F_{BC} = \sqrt{2}W$.

* short Cuts:-



$(F_1 = F_2)$ & $F_3 = 0$

$[F_3 = W]$

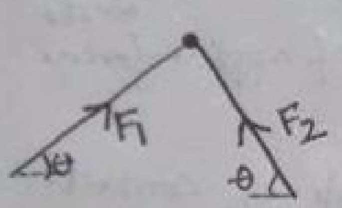


$F_1 = F_2$ &

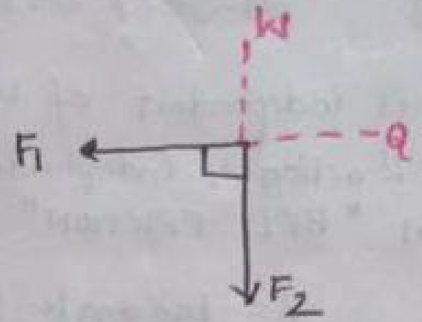
$F_3 = 0$

—x—

$F_3 \neq 0$



$F_1 = F_2 = 0$

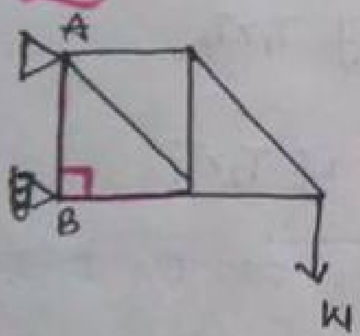


$F_1 = Q$

$F_2 = W$.

$F_1 = F_2 = 0$

Note:



If two members are mutually $\perp^{er}$ and one member is in the direction of Roller, then that carries zero force

"Important points" "plane truss"

- if j = Total no. of joints.
  m = Total no. of members.

① if $m = 2j - 3$ → Perfect | stable | Determinate
(U = E)

② if $m > 2j - 3$ → Redundant / stable | Indeterminate
(U > E)

③ if $m < 2j - 3$ → Deficient & Unstable.
(U < E)

"Dynamics"

Dynamics

- Kinematic (without of consideration of force)
  - Kinematics of particle
    - * Dimensions of body will not involve
    - • Car moving
  - Kinematics of Rigid body
    - * Dimensions will involve
    - * Fan rotation.
- Kinetics (Force would involve)
  - Kinetics of Particle
  - Kinetics of Rigid bodies.

① Kinematics of particle:-

- Rectlilinear motion
  - Constant Acceleration
  - Variable Acceleration
- Curvilinear motion
  - Vertical
    - *Projectile motion
  - Horizontal.
    - * Car moving in curved path

Constant Acceleration

① $a = \frac{dv}{dt}$

$\boxed{V = u + at}$

② $V = \frac{ds}{dt}$

$\boxed{s = ut + \frac{1}{2}at^2}$

③ $\boxed{V^2 - u^2 = 2as}$

Variable Acceleration

$a = f(t, s, v)$

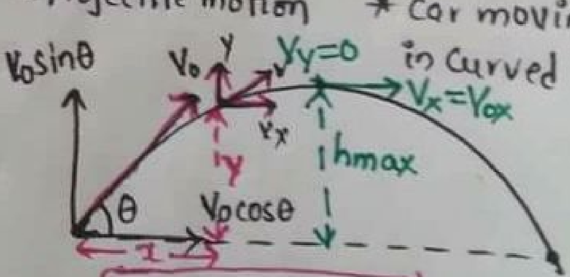
$a = \frac{dv}{dt}$; $V = \frac{ds}{dt}$

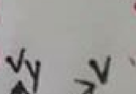
$\boxed{V \cdot dv = a \cdot ds}$

* Integration

a ↓ v ↓ s

∴ chain Rule $a = \frac{dv}{dx} \cdot \frac{dx}{dt}$





$\boxed{V_x = V_{ox} = V_0 \cdot \cos\theta}$

$V_y = V_{oy} - gt$ {∵ $V = u + at$ y-direction}

$\boxed{V_y = V_0 \sin\theta - gt}$

$V = \sqrt{V_x^2 + V_y^2}$: $\tan\alpha = \left[\frac{V_y}{V_x}\right]$

$\boxed{x = V_0 \cos\theta \cdot t}$

$\boxed{y = V_{oy} \cdot t - \frac{1}{2} g t^2}$

at maximum height : $V_y = 0$

∴ $\boxed{t_{hmax} = \frac{V_0 \sin\theta}{g}} = t_{asc}$

$\boxed{h_{max} = \frac{V_0^2 \sin^2\theta}{2g}}$

- from Refernce line

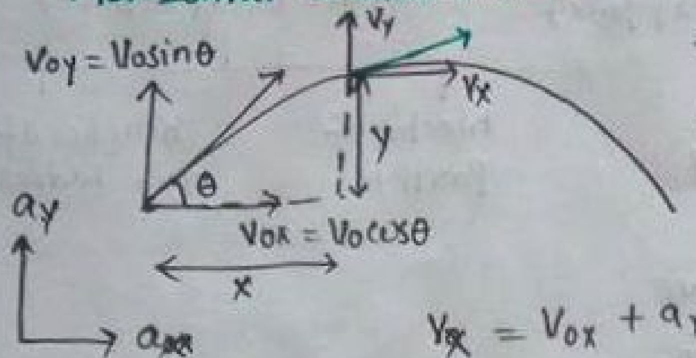
∴ $\boxed{t_R = \frac{2 V_0 \cdot \sin\theta}{g}}$ ⇒ $\boxed{t_R = 2 t_{hmax}}$

$\boxed{\text{Range (R)} = \frac{V_0^2 \sin\theta}{g}}$

* Range : Horizontal distance covered by the body when it reaches it reference line

Note: Velocity is always tangent to the path that the Particle is traced.

"Horizontal Curved Motion":-



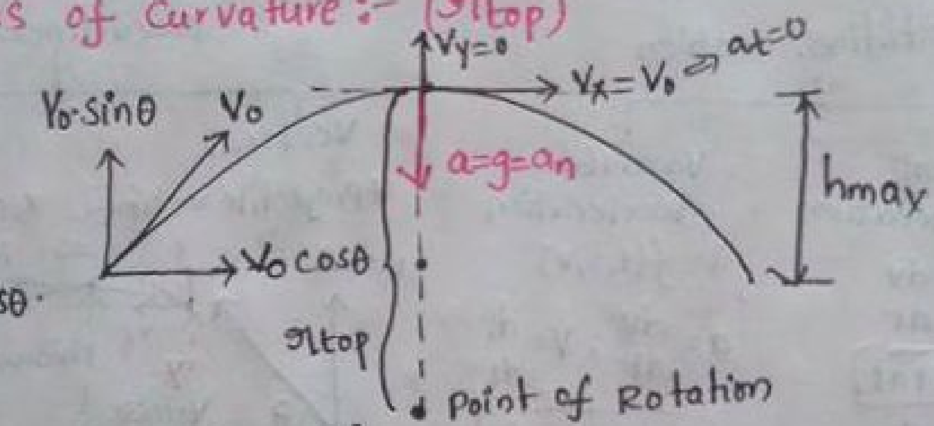
* No influence of gravity
* Body will move with its own acceleration.

$$V_x = V_{ox} + a_x t \Rightarrow x = V_{ox} t + \frac{1}{2} a_x t^2$$

$$V_y = V_{oy} + a_y t \Rightarrow y = V_{oy} t + \frac{1}{2} a_y \cdot t^2$$

$$(V = u + at) \qquad (s = ut + \frac{1}{2} at^2)$$

Radius of curvature:- (r_{top})



at h_{max}

$V = V_x = V_0 \cos\theta$.

$\therefore$

$$a_n = \frac{V^2}{r_{top}}$$

$$\therefore \boxed{r_{top} = \frac{V^2}{a_n} = \frac{V_0^2 \cos^2\theta}{g}}$$

Radius of curvature at any point on curved path.

step 1: find V_x & V_y at that point

$$V_x = V_{ox} \ ; \ V_y = V_{oy} - gt$$

step 2: find $V = \sqrt{V_x^2 + V_y^2}$ & $\alpha = \tan^{-1}(V_y / V_x)$

step 3: find a_t & a_n and $a = \sqrt{a_t^2 + a_n^2}$

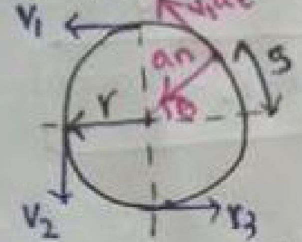
step 4: $r = \frac{V^2}{a_n}$

Note: If particle moving towards $h_{max} \Rightarrow V_y > 0$ (+ve)

② Kinematic of Rigid Bodies:

Fixed Axis Rotation

- Disk, Flywheel, pulley, etc..



$S = r\theta$; $\omega = \frac{d\theta}{dt}$ (rad/sec)

$\boxed{V = r \cdot \omega}$

① Tangential acceleration $\boxed{a_t = \frac{dv}{dt} = r \cdot \alpha}$

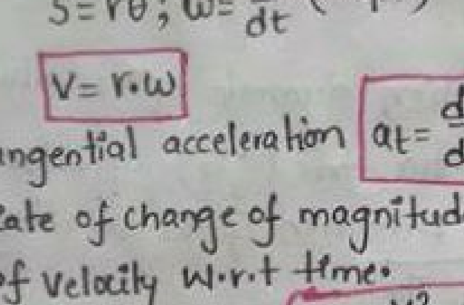
- Rate of change of magnitude only of velocity w.r.t time.

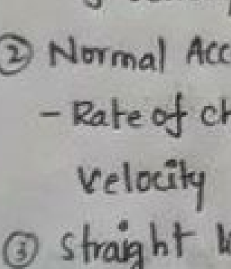
② Normal Acceleration, $\boxed{a_n = \frac{v^2}{r} \text{ (or) } r\omega^2}$

- Rate of change of direction only of velocity w.r.t time.

③ Straight line motion $a_n = 0$.

④ For curvilinear motion $a_n \neq 0$.





$\therefore a_x = a_{tx} + a_{nx}$

$\boxed{a_x = a_t \sin\theta + a_n \cos\theta}$

$a_y = a_{ty} + a_{ny}$

$\boxed{a_y = a_t \cos\theta - a_n \sin\theta}$

$\therefore$ Resultant acceleration $\boxed{a = \sqrt{a_x^2 + a_y^2}}$

$a = \sqrt{a_t^2 + a_n^2}$

General plane Motion.

* Combination of st. line motion & fixed axis Rotation.
* Wheel, Ball bearing, connecting rod, ladder Resting on wall

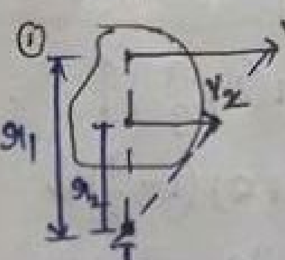
$\vec{V_R} = \vec{V_t} + \vec{V_L}$

(Not constant)

Instantaneous centre:

The point w.r.t which a body undergoing general plane motion is assumed to be undergoing fixed axis Rotation.

* Velocity at instantaneous centre is zero.

① 

if $|v_1| = |v_2|$ — Rectilinear

$|v_1| \neq |v_2|$ — GPM

$\therefore V_1 = r_1 \cdot \omega$

$V_2 = r_2 \cdot \omega$

② 

$V_1 = r_1 \cdot \omega$

$V_2 = r_2 \cdot \omega$

③ 

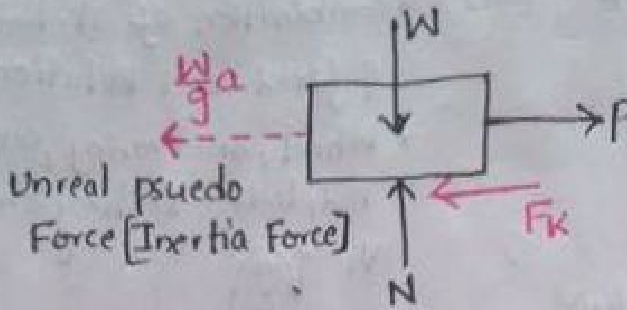
$V_1 = r_1 \cdot \omega$

$V_2 = r_2 \cdot \omega$

"Kinetics of particles & Rigid Bodies"

① Newton's Second Law

$$F = m \cdot a$$



$$P - F_K = \frac{W}{g}a$$

$$\boxed{P - F_K - \frac{W}{g}a = 0}$$

② D'Alembert's principle:

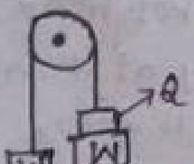
For a body in motion dynamic eq^m is always maintained by representing Inertia force $\left(\frac{W}{g}a\right)$ opposite to the direction of motion [i.e Direction of Acceleration]

③ For fixed axis Rotation: $\boxed{M = I\alpha}$

M = Moment / Torque applied

I = Mass MOI or Rotational Inertia.

Ex:

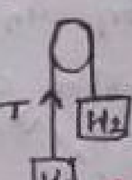


$$Q = \frac{2Wa}{(g-a)}$$

$$T = W\left(1+\frac{a}{g}\right)\uparrow$$

$$T = (W+Q)\left(1-\frac{a}{g}\right)\downarrow$$

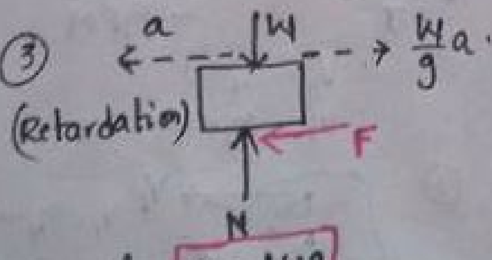
② Elevator



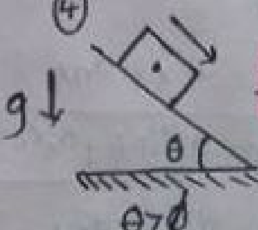
$$a = \left(\frac{W_2 - W_1}{W_2 + W_1}\right)g$$

$$\therefore \boxed{\begin{aligned} T &= W\left(1+\frac{a}{g}\right)\uparrow \\ T &= W\left(1-\frac{a}{g}\right)\downarrow \end{aligned}}$$

W = Weight of Elevator.

③ 

$$\therefore \boxed{a = \mu \cdot g}$$

④ 

$$\boxed{\therefore a = g\sin\theta - \mu g\cos\theta}$$

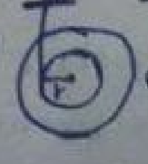
(Gate 2014)

Note: Inertial Force can't contribute any moment for the given loading condition.

① slender Rod

$$I_C = \frac{ML^2}{12}\ ;\ I_b = \frac{ML^2}{3}$$

② $\boxed{I = mk^2}$

③ 

$$I = mk^2$$

$$a = \frac{gr^2}{(r^2+k^2)}$$

Work-Energy principle and Impulse-Momentum Eqn.

① Potential Energy : Due to position $[PE = mgh]$

② Kinetic Energy : Due to motion $KE = \frac{1}{2}mv^2$

for Rotational $KE = \frac{1}{2}I\omega^2$

① Work-Energy principle:-

$F = ma \; : \; v\,dv = a \cdot ds$

$F = m \cdot \frac{v\,dv}{ds}$

$F \cdot ds = m\,v\,dv$

$\int_0^s F \cdot ds = \int_u^v m\,v\,dv$

$$\boxed{F \cdot s = m\left(\frac{v^2 - u^2}{2}\right)}$$

Work = change in KE

→ For Rectilinear motion

For fixed axis Rotation

$$\boxed{M \cdot \theta = \frac{1}{2}I(\omega^2 - \omega_0^2)}$$

② Impulse Momentum Eqn.

(1) For Rectiliner motion

$F = ma = m \cdot \frac{dv}{dt}$

$F = \frac{d}{dt}(mv)$ — Euler's eqn

$$\boxed{F \cdot dt = m \cdot dv}$$

Impulse / Impact ⇓ — Change in Linear momentum ⇓

② For rotational motion

$$\boxed{M \cdot dt = I \cdot d\omega}$$

Statement:

Rate of change of momentum w.r.t time is equal to the Applied force.

③ Law of conservation of momentum

(a) $m_1 + m_2 \cong m_1 + m_2$

$u_1 \rightarrow \quad u_2 \rightarrow \quad v_1 \rightarrow \quad v_2 \rightarrow$

$\because \sum_{i=1}^{n} m_i u_i = \sum_{i=1}^{n} m_i v_i$

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$$ ⇒ Linear momentum

(b) Conservation of Angular momentum.

$$\boxed{I_1 \omega_{01} + I_2 \omega_{02} = I_1 \omega_1 + I_2 \omega_2}$$

Note: If the dimensions of the body has mentioned then it comes under "Conservation of Angular momentum'

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(4). Coefficient of Restitution:— (e)

Ratio of Relative velocity after impact to the relative Velocity before impact.

$$\boxed{e = \frac{|v_2 - v_1|}{|u_2 - u_1|}}$$

(1) $e = 1 \Rightarrow$ Elastic impact (Helical spring)

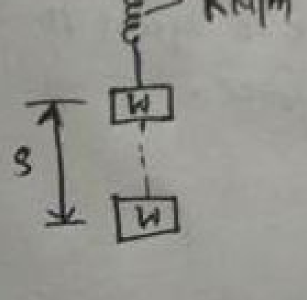
$$\sum_{i=1}^{n} \frac{1}{2} m_i u_i^2 = \sum_{i=1}^{n} \frac{1}{2} m_i v_f^2$$

(2) $e < 1 \Rightarrow$ Inelastic impact [Liquid spring]

(3) $e = 0 \Rightarrow$ Plastic Impact [Bullet & Block]

(4) $e > 1 \Rightarrow$ Super elastic impact [Not prevail].
(Space debris colloid with each other)

Ex:



Energy stored in (the spring) (strain Energy)

$$= \frac{1}{2} P\delta = \frac{1}{2} ks^2$$

$$\left[V = \sqrt{\frac{K \cdot g}{W}} \cdot S \right]$$