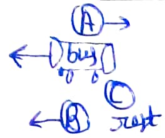
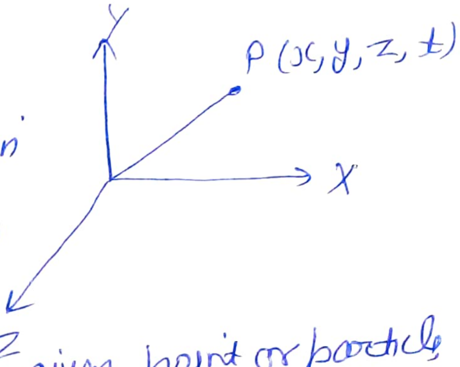
*Exp-



Unit-IV
Special Theory of Relativity

Frame of Reference:- Frame of reference is a coordinate system relative to which the position and motion of a body may be described.

*Exp- The Simple frame of reference is the Cartesian coordinate system in which every point of space may be described by 3 numbers (x, y, z) or three coordinates of that point.



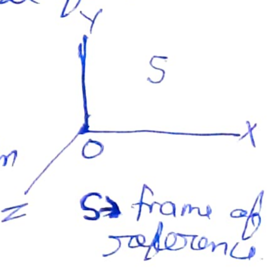
If $\vec{r}$ is the position vector of Point P relative to origin O of a cartesian frame of reference, then $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$

In order to specify the given point or particle exact time of knowing the position of the given point or particle we must assign the more coordinate t, the time coordinate. Such a reference frame is called space-time reference frame. It consists of 4 coordinates (x, y, z, t)

Frame of reference are of two types.

(i) **Inertial Frame of Reference** → A frame of reference in which Newton's Laws of motion holds good is known as inertial frame of reference.

It means a frame of reference which is either at rest or moving with uniform velocity is inertial frame of reference.



From Newton's 2nd law of motion

$$F = ma$$

$$0 = ma \qquad \text{if } F = 0$$

$$a = 0$$

$$\boxed{v = \text{Constant}}$$

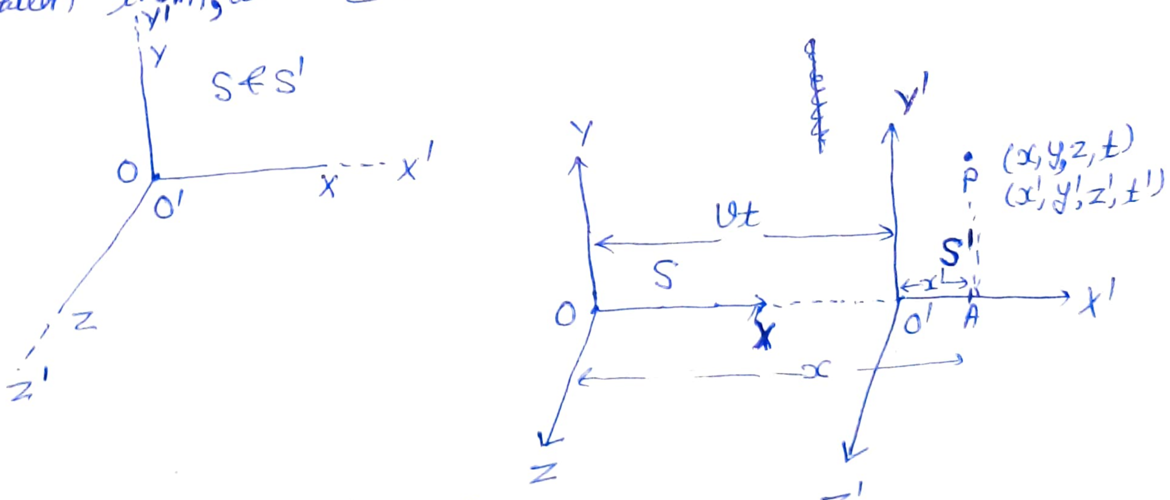
Thus inertial frame of reference is also known as unaccelerated or non-accelerated frame of reference.

(ii) Non-Inertial frame of reference :→ A frame of reference in which Newton's laws of motion are not valid, is known as Non-inertial frame of reference.

A non inertial frame of reference is an accelerated frame of reference.

For Exp:→ When a traveller travels in a bus and bus driver applies force on brake of bus then bus is deaccelerated, but traveller also move forward but no force is applied here on traveller, so without applying force on traveller, traveller move forward, so here Newton' Law is not valid in this frame of reference (bus).

Galilean Transformation :→ The set of equations which relate space-time coordinates of an event w.r.t two inertial frames of references having relative motion between them, are called Galilean transformation equations.



Let us consider 2 inertial frame of references S & S' having cartesian coordinate axes as x, y, z & x', y', z' and origin O and O' respectively. At $t=0$ both the frames are at rest so that their origin O and O' coincides with each other.

Now let frame S' start moving with constant velocity v along +ve direction of X-axis. Let an event occurs at point P at any instant of time. The coordinates of point P wrt to observer O in frame S are (x, y, z, t). The coordinates of point P wrt observer O' in fram S' are (x', y', z', t')

From figure, it is clear that

$$OA = OO' + O'A$$

$$x = vt + x'$$

$$x' = x - vt \quad -① $$

S' is moving in x direction only, so

$$y' = y \quad -②$$

$$z' = z \quad -③$$

its time is considered to be absolute in nature i.e. time becomes same in all inertial frames of references. So $t' = t \quad -④$

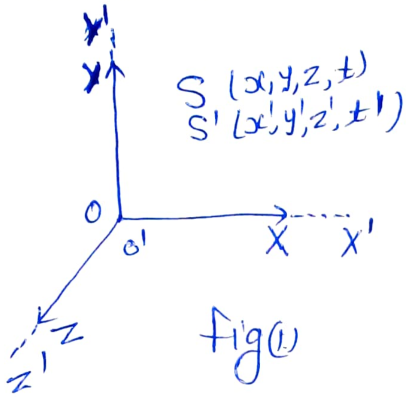
$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$ — Galilean transformation equations

Postulates for Special Theory of Relativity → In 1905, Einstein published his special theory of relativity which is based upon the two postulates –

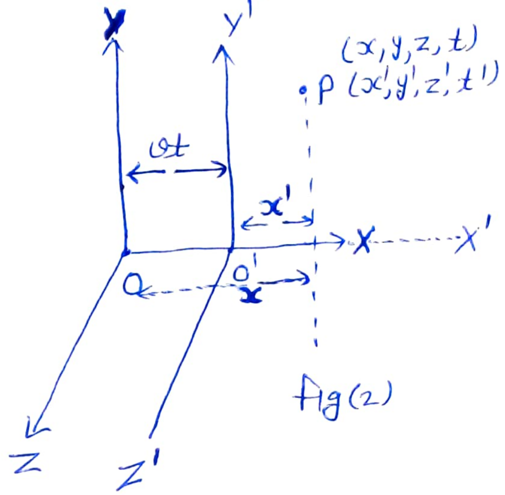
Postulate I :⇒ The laws of physics have the same form in all inertial frames of references moving with constant velocity relative to one another. This is known as principle of relativity.

Postulate II :⇒ The speed of light in free space is the same in all inertial frames of references. This is the principle of constancy of speed of light. This postulate comes from the result of Michelson–Morley Experiment.

Lorentz Transformation Equations → The set of equations which relate the space-time coordinates of two frame of references having relative motion between them.



fig(1)



fig(2)

Let S and S' are two inertial frames of references at $t=0$, they coincide with each other (fig 1). The origin of S and S' are O and O', the coordinate axis of S and S' are (x, y, z, t) and (x', y', z', t') respectively. When S' start to move with constant velocity v relative to S and v is comparable with c, then the relation b/w x & x', y & y' & z & z' may be written as -

$$x' = \gamma(x - vt) \quad \text{①},$$

$$\left.\begin{aligned} y' &= y \\ z' &= z \end{aligned}\right]$$ because S' is moving in +X direction only not in Y and Z directions.

eq ① may be written as by replacing $v \to -v$

∵ Acc to Ist Postulate, The equation of Physics or Laws of Physics must have the same form in both S and S'.

$$x = \gamma(x' + vt') - \text{②}$$

putting the value of x' from equation ① into eq ② -

$$x = \gamma[\{\gamma(x - vt) + vt'\}]$$

$$\frac{x}{\gamma} = \gamma x - \gamma vt + vt'$$

$$vt' = \frac{x}{\gamma} - \gamma x + \gamma vt$$

$$t' = \frac{x}{\gamma v} - \frac{\gamma x}{v} + \gamma t$$

$$t' = \gamma t - \frac{\gamma x}{v}\left(1 - \frac{1}{\gamma^2}\right) \longrightarrow \text{③}$$

Similarly, we can write for t by replacing $v \to -v$

$$t = \gamma t' + \frac{\gamma}{v} x'\left(1 - \frac{1}{\gamma^2}\right) \longrightarrow \text{④}$$

γ can be evaluated from the II postulate.

Suppose, a light signal is given at time $t = 0$, $t' = 0$, that is when O and O' coincides or when S and S' coincides. The signal travels with a speed c which is same for both the frames (II Postulate). Its position as seen from S and S' after some time is given by,

$$x = ct$$
$$x' = ct'$$

putting $x = ct$ and $x' = ct'$ in eq (1), we get

$$ct' = \gamma(ct - vt)$$
$$ct' = \gamma t(c - v) \quad -(5)$$

putting $x = ct$ and $x' = ct'$ in eq (2), we get

$$ct = \gamma t'(c + v) \quad -(6)$$

Solving eq (5) & (6), we get.

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

putting the value of γ in eq (1), and (3), we get

$$x' = \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad -(A)$$

$$t' = \frac{\left(t - \frac{xv}{c^2}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad -(B)$$

These A, B, C, D are Lorentz Transformation equations

S' is not moving in y and z-direction, so

$$y' = y \quad -(C)$$

$$z' = z \quad -D$$

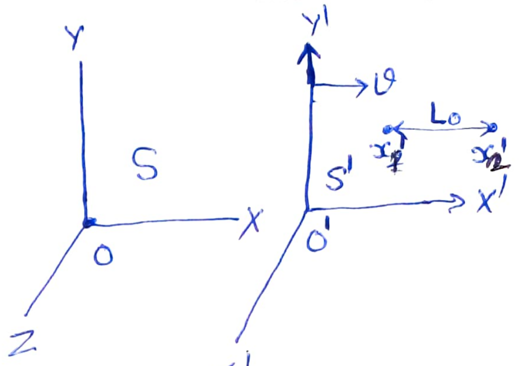
other form of Lorentz transformation equations can be written by replacing $v \to -v$ in eq A, B, C, D - [Inverse Lorentz transformation equation]

$$\boxed{x = \frac{x' + vt'}{\sqrt{(1 - v^2/c^2)}}, \; y' = y, \; z' = z, \; t = \frac{t' + x'v/c^2}{\sqrt{(1 - v^2/c^2)}}}$$

Length Contraction ⇒ Relative motion affects measurement of length. A body moving with velocity v relative to an observer appears to ~~be~~ the observer to be contracted in length in the direction of motion by a factor $\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, whereas its dimensions perpendicular to the direction of motion are unaffected.

$$\boxed{L = L_o\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

Derivation ⇒



Let us consider two frames of reference S and S'. S is in rest position while S' is moving with velocity v. Let a rod having L_o is placed in S'. Suppose x_1' and x_2' be the coordinates of the ends. The length of the rod as observed from S' is,

$$L_o = x_2' - x_1' \quad -(1)$$

Now suppose another observer measures the length of the rod from a stationary frame S, relative to which the rod (or S') is moving with velocity v. If the coordinates of the ends of the rod as x_1 and x_2 at time t, the rod appears to him having length,

$$L = x_2 - x_1 \quad -(2)$$

From the Lorentz transformation, $x_2' = \dfrac{x_2 - vt}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, $x_1' = \dfrac{x_1 - vt}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$

putting these values of x_2' and x_1' in eq (1),

$$L_o = \frac{x_2 - vt}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} - \frac{(x_1 - vt)}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{x_2 - \cancel{vt} - x_1 + \cancel{vt}}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{x_2 - x_1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$L_o = \frac{x_2 - x_1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \qquad \because L = (x_2 - x_1)$$

$$L_0 = \frac{L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$\boxed{L = L_0 \sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

If $v = c$

$L = 0$, Hence If rod moves with speed c (speed of light) then it appears like a point to an stationary observer.

L_0 is known as Proper Length.

→ The Length L_0 measured in the frame of reference in which the rod is at rest.

Time Dialation :- Time intervals are also affected by the relative motion. A clock moving with velocity v with respect to an observer appears him to have slowed down by a factor, than when at rest with respect to him.

$$\boxed{t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}}$$

Derivation :- Suppose a clock is placed at the point x' in the moving frame S'. An observer in S' finds that the clock gives two ticks at times t_1' and t_2'. The time interval b/w the ticks judged from S' is

$$t_0 = t_2' - t_1' \quad -(1)$$

Now ~~s~~ Another observer measures the time interval b/w two ticks from a stationary frame of reference S, relative to which the clock (or S') is moving with velocity v. If he records the ticks at t_1 and t_2, the time interval appears to him as-

$$t = t_2 - t_1 \quad -(2)$$

From Lorentz transformation eqn-

$$t_2 = \frac{t_2' + x'v/c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}, \quad t_1 = \frac{t_1' + x'v/c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

putting the values of t_2 and t_1 in eq (2)

$$t = t_2 - t_1$$

$$t = \frac{t_2' + \frac{x'v}{c^2}}{\sqrt{(1-\frac{v^2}{c^2})}} - \frac{(t_1' + x'v/c^2)}{\sqrt{(1-\frac{v^2}{c^2})}}$$

$$t = \frac{t_2' + \cancel{\frac{x'v}{c^2}} - t_1' - \cancel{x'v/c^2}}{\sqrt{(1-\frac{v^2}{c^2})}}$$

$$t = \frac{t_2' - t_1'}{\sqrt{(1-\frac{v^2}{c^2})}}$$

$$\boxed{t = \frac{t_0}{\sqrt{(1-\frac{v^2}{c^2})}}}$$

It shows that the moving clock appears to be slowed down to a stationary observer. This is known as time dilation.

t_0 is known as Proper time interval → It is the time interval of clock seen from a frame to which the clock is attached.

Velocity Addition Theorem :→ Very high velocities can not be added directly as in classical mechanics. Very high velocities which are comparable with speed of light, should be added using Lorentz transformation equations.

Let two frames of reference S and S'. S' is moving with uniform velocity v relative to S (S is in rest). Let a body move a distance dx in a time interval dt in S frame. then the velocity of the body measured by an observer in S is

$$u = \frac{dx}{dt} \quad -(1)$$

To an observer in S', both the distance and the time interval will be appear different, dx' and dt', for an observer in S'

$$u' = \frac{dx'}{dt'} \quad -(2)$$

From the Lorentz transformation equations -

$$x' = \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} , \quad t' = \frac{t - xv/c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Differentiating, we get

$$dx' = \frac{dx - v\,dt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} , \quad dt' = \frac{dt - dx\,\frac{v}{c^2}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

putting these values in eq (2) -

$$u' = \frac{(dx - v\,dt)/\cancel{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}}{\left(dt - dx\frac{v}{c^2}\right)/\cancel{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}}$$

$$u' = \frac{(dx - v\,dt)}{\left(dt - \frac{v}{c^2}dx\right)} = \frac{\cancel{dt}\left(\frac{dx}{dt} - v\right)}{\cancel{dt}\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

from eq (1), $\frac{dx}{dt} = u$

$$\boxed{u' = \frac{(u - v)}{\left(1 - \frac{uv}{c^2}\right)}}$$

This is relativistic addition of velocities u and v.

If we consider $u = c$, i.e. a ray of light is emitted in the ~~...~~ S frame along the x-axis, then observer in S' frame will measure the velocity ⇒

$$u' = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{\cancel{(c - v)}\,c}{\cancel{(c - v)}} = c$$

$$\boxed{u' = c}$$

So $u = c$, $u' = c$ ⟹ It means that the velocity of light in free space is same in all inertial frames of reference.

Mass-Energy Equivalence :→ According to Einstein's special theory of relativity, The variation of mass m with velocity v is given by-

$$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \quad \text{--- (1)}$$

v - velocity of mass of body
m_0 - rest mass of body.
c - speed of light.

This variation of mass (m) with velocity (v), has modified our ideas about energy. Let us consider a particle having mass m. If a force F is applied on this particle, then the force F may be written as-

$$F = \frac{dP}{dt} , \quad P \to \text{momentum}$$

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Here m and v are variables

This force F will produce K.E in particle, if particle is displaced through distance ds on applying force F, then

$$dK = F\,ds$$

$$= \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)ds$$

$$= m\frac{dv}{dt}ds + v\frac{dm}{dt}ds$$

$$= m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

$$dK = mv\,dv + v^2 dm \quad \text{--- (2)} \quad \because \frac{ds}{dt} = v$$

from eq (1), $m = \dfrac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$

$$dm = m_0\left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v}{c^2}\right)dv$$

$$= \frac{m_0}{c^2}\frac{v\,dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$dm = \frac{m_0 v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$dm = \frac{1}{c^2}\frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}\left(1-\frac{v^2}{c^2}\right)} v\,dv$$

$$= \frac{m\,v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)} \qquad \because \quad m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$= \frac{m\,v\,dv}{\cancel{c^2}\frac{(c^2-v^2)}{\cancel{c^2}}} = \frac{m\,v\,dv}{(c^2-v^2)}$$

$$m\,v\,dv = (c^2-v^2)\,dm$$

putting this value in eqn (2)

$$dK = (c^2 - \cancel{v^2})\,dm + \cancel{v^2}\,dm$$

$$dK = c^2\,dm$$

Suppose the body has mass m_0 when at rest and a mass m when accelerated to a velocity v, The K.E acquired is then,

$$K = \int dK = \int_{m_0}^{m} c^2\,dm$$

$$K = c^2(m-m_0)\,c^2$$

$$\boxed{K.E = c^2(m-m_0)}$$

The total energy = rest mass energy + K.E.

$$= \cancel{m_0 c^2} + c^2(m - \cancel{m_0})$$

$$E = mc^2$$

$$\boxed{E = mc^2}$$

This is the mass-energy relation.

<u>Verification</u> ⇒ The mass and Energy relation has been verified in a number of phenomena. like – Nuclear phenomena,
– Compton effect.

In nuclear phenomena, The explanation of mass defect and release of huge amount of energy in nuclear fission is based on mass-energy relation. It gives the strong support to the relation.

Invariance of Maxwell's Equations under Lorentz Transformations

In special theory of relativity one distinguish between proper charge density ρ_0, measured when the charge under consideration is at rest relative to the observer and the "nonproper" or "relativistic" or "Lorentz contracted" charge density defined as -

$$\rho = \frac{\rho_0}{\sqrt{\left(1-\frac{u^2}{c^2}\right)}}$$

, $u \rightarrow$ speed of charge

then Maxwell' equations can be written as -

$$\boxed{\begin{aligned} \vec{\nabla}\cdot\vec{J} &= \frac{\rho_0}{\sqrt{\left(1-\frac{u^2}{c^2}\right)}}, \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \vec{J}+\frac{\partial \vec{D}}{\partial t} \end{aligned}}$$

Michelson - Morley Experiment :-

Ans. Michelson-Morley Experiment : According to the wave theory of light, a light source sets up a disturbance travelling in all directions through a hypothetical medium called 'ether' which fills all space and penetrates all matter. The assumption of ether, however, created a problem. Does ether remain stationary in space when material bodies (including earth) move in it, or is it dragged along with the moving bodies? Bradley's observation of the aberration of light from stars had indicated that *the ether must be stationary in space*. It means that if a material body, say earth, moves in space*, there is a *relative* motion between the body and the ether. A number of experiments were performed to detect a relative motion between the earth and the ether. The most famous among them is the one performed by Michelson and Morely in 1887 using the Michelson interferometer.

A simplified plan of the experiment is shown in Fig. 3. A beam of light from a source S falls upon a half-silvered glass plate P placed at 45° to the beam and is divided into two beams 1 and 2. The beams 1 and 2 travelling at right angles to each other, fall normally on mirrors M_1 and M_2 which reflect them back to P. The two beams returned to P are directed towards a telescope T in which interference fringes are observed.

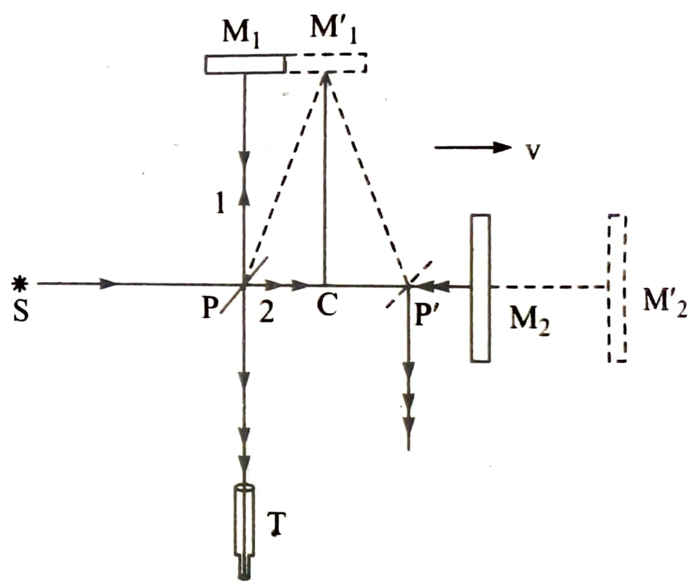


Fig. 3

Let the mirrors M_1 and M_2 be at the same distance l from the plate P. Then, if the apparatus were at rest in ether, the two beams would take the same time to return to P. But, actually the earth, and hence the apparatus, is moving in space through the ether with a velocity v (say). Suppose this motion is in the direction of the initial beam of light. Then,

* The earth is moving round the sun with a velociy of 3×10^4 m/s which is one ten-thousandth of the velocity of light in free space.

if the initial beam strikes the plate P in the position shown, the paths of the two beams and the positions of their reflections from the mirrors will be as shown by the dotted lines. The time taken by the two beams on their journeys are *not* equal.

Let c be the velocity of light through the ether. The beam 2 moving towards M_2 has a velocity $(c - v)$ *relative to the apparatus* on the outgoing trip, and $(c + v)$ on the return trip. If t_2 be the total time taken by this beam to go from P to M_2 and back, then

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2 - v^2} = \frac{2l}{c}\left(\frac{1}{1-\frac{v^2}{c^2}}\right). \qquad \text{...(i)}$$

The beam 1 moving *transversely* with respect to the apparatus retains its velocity c throughout. Let it take a time t' to go from P to strike M_1, travelling a distance ct'. In the same time, the mirror M_1 advances a distance vt'. Thus, in the right-angled triangle PM_1M_1', we have

$$(PM_1')^2 = (PM_1)^2 + (M_1M_1')^2$$

But $PM_1 = l$, $M_1M_1' = vt'$ and $PM_1' = ct'$

$$\therefore \qquad (ct')^2 = l^2 + (vt')^2$$

or

$$t' = \frac{l}{(c^2 - v^2)^{1/2}} = \frac{l}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

If t_1 be the total time taken by the beam to travel the whole path $PM_1'P'$, then

$$t_1 = 2t' = \frac{2l}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{...(ii)}$$

Hence the difference between the times of travel of the two beams is, from eq. (i) and (ii), given by

$$t_2 - t_1 = \frac{2l}{c}\left[\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-1} - \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

Using binomial expansion and dropping terms higher than the second order, we have

$$t_2 - t_1 = \frac{2l}{c}\left[\left(1+\frac{v^2}{c^2}\right) - \left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\right]$$

$$= \frac{2l}{c}\left[\frac{1}{2}\frac{v^2}{c^2}\right] = \frac{lv^2}{c^3}$$

The path difference δ between the beams corresponding to a time difference $(t_2 - t_1)$ is

$$\delta = c\,(t_2 - t_1) = \frac{lv^2}{c^2}$$

If the interferometer is suddenly brought to rest (v is made zero), then the path difference δ would become zero. We know that if the path difference between two interfering waves changes by λ, there is a shift of one fringe across the cross-wires in the field of view. Thus, if ΔN is the number of fringes which shift when the interferometer is stopped, then

$$\Delta N = \frac{\delta}{\lambda} = \frac{lv^2}{c^2\lambda}$$

In the actual experiment, the whole apparatus, which was placed on a block of stone floated on mercury, was rotated through 90°. This introduced a path difference of the same amount in the opposite direction. Hence a shift of $\frac{2lv^2}{c^2\lambda}$ was expected.

To have an observable shift, Michelson and Morley increased the effective value of l upto 11 meters by reflecting the light back and fourth several times. Then, using values : $l = 11$ meter, $v = 3 \times 10^4$ meter/sec, $c = 3 \times 10^8$ meter/sec and $\lambda = 5.5 \times 10^{-7}$ meter (for visible light), the expected shift is

$$\Delta N = \frac{2lv^2}{c^2\lambda} = \frac{2 \times 11 \times (3 \times 10^4)^2}{(3 \times 10^8)^2 \times 5.5 \times 10^{-7}} = 0.4,$$

or a shift of four-tenths a fringe.

Michelson and Morley were extremely surprised to see that there was *no* shift in the fringes when the interferometer was rotated through 90°. They repeated the experiment during various times of the day and various seasons of the year but no shift was observed. Trouton and Noble, in 1902, performed an electromagnetic experiment for the same purpose but with no positive result. Thus, ***the motion of the earth through the ether could not be experimentally detected.***

Explanation of the Negative Result : Three separate explanations were given to the negative result of the Michelson-Morley experiment :

1. Ether-drag Hypothesis : The moving earth *completely drags* the ether with it so that there is not relative motion betwen the two and hence the question of shift does not arise. But this explanation was not accepted for two reasons : (i) It goes against the observed aberration of light from stars. (ii) Fizeau had experimentally shown that a moving body could drag the 'light waves' only *partially*. Furthermore, this partial dragging of light waves was explained by the electromagnetic theory, without introducing the ether-drag hypothesis.

2. Fitzgerald-Lorentz Contraction Hypothesis : Fitzgerald and Lorentz independently put an adhoc hypothesis that all material bodies moving through the ether are contracted in the direction of motion by a factor $\sqrt{1-(v^2/c^2)}$. It is easily seen that such a contraction in the interferometer arm would equalize the times t_1 and t_2 and no fringe-shift would be expected.This explanation also, being purely adhoc could not be accepted. Further, Rayleigh worked out that such a contraction is expected to produce double refraction which was, however, never observed.