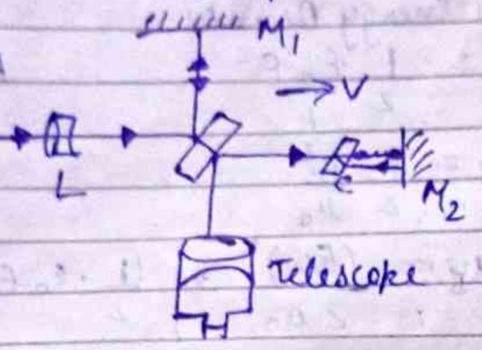
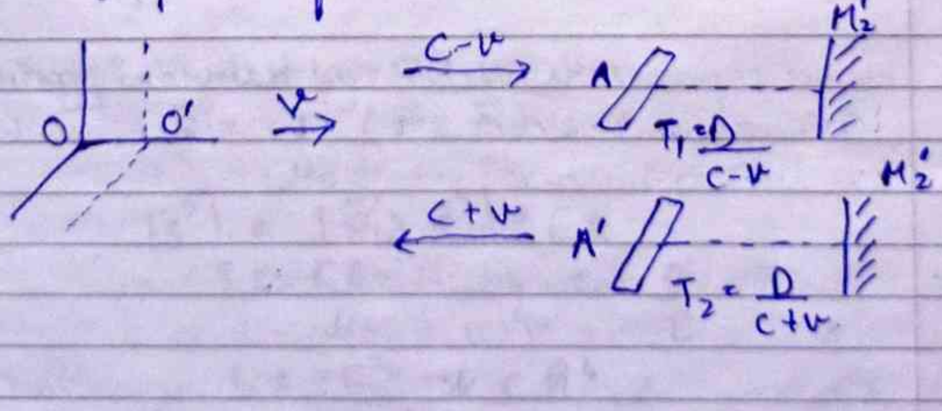
Unit - 4
Theory of Relativity.

* Michelson Morley Experiment



- Light from a monochromatic source S is rendered parallel with the lens L and it is divided into two portions by the half silver plate A.
- One portion of the beam travels to mirror M_1 and is reflected back. The other portion of beam travels towards M_2 and is reflected back. The two reflected beams interfere and the interference fringes are viewed with the help of a telescope.
- The apparatus is arranged to move along the direction of earth's orbit around the sun. The speed of movement of the apparatus is equal to v i.e, speed of the earth in its orbit.



According to Galilean frame of reference moving with a velocity v in the direction of the movement of the apparatus. The velocity of light in the direction of movement of the frame $F' = c - v$ and in the opp. direction it is equals to $c + v$.
Suppose, T_1 is the time taken by light to travel from A to M_1' and T_2 be the time taken from M_1' to A' as shown in fig.

Here, $D = AM_1 = AM_2$

Total time taken by light (to and fro)

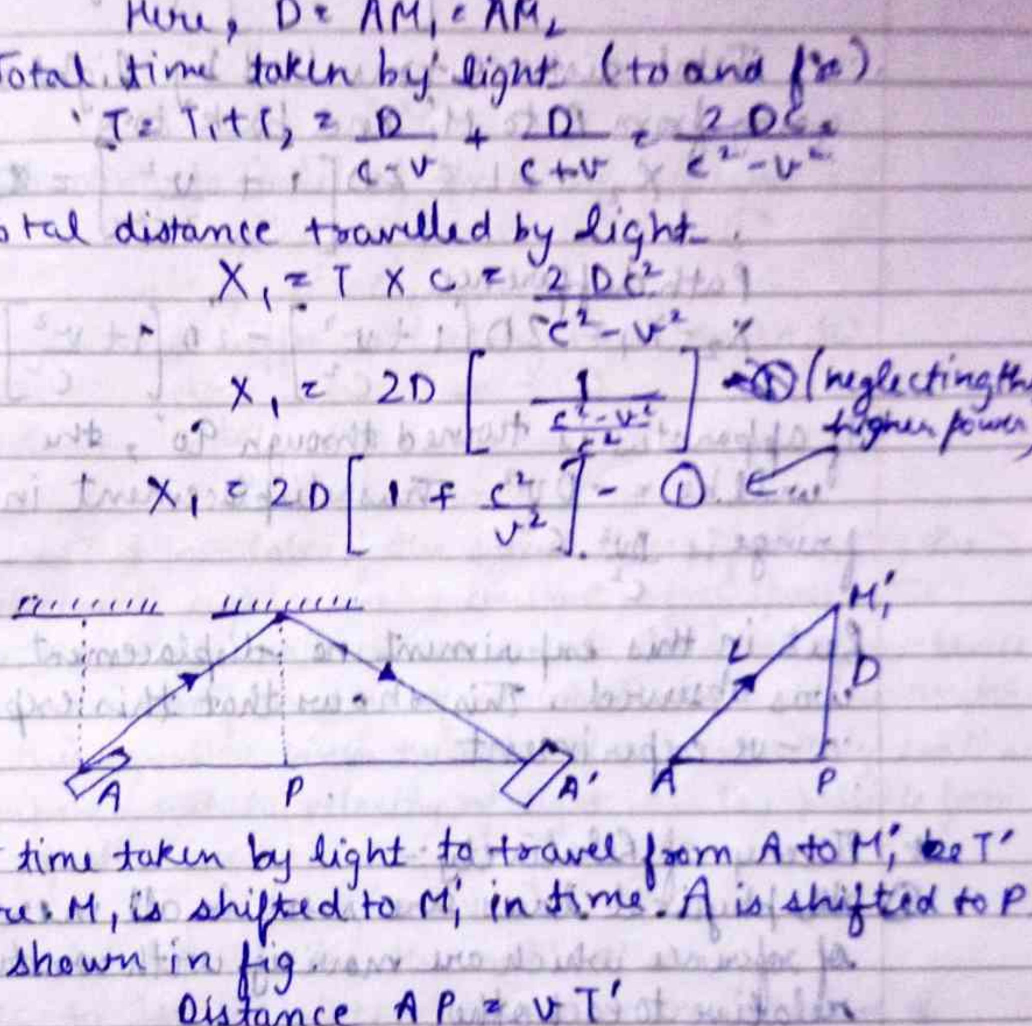
$$T = T_1 + T_2 = \frac{D}{c-v} + \frac{D}{c+v} = \frac{2Dc}{c^2 - v^2}$$

Total distance travelled by light

$$X_1 = T \times c = \frac{2Dc^2}{c^2 - v^2}$$

$$X_1 = 2D\left[\frac{1}{\frac{c^2-v^2}{c^2}}\right]$$ (neglecting the higher power)

$$X_1 = 2D\left[1 + \frac{c^2}{v^2}\right] - (1)$$



Let time taken by light to travel from A to M_1' be T'
Here M_1 is shifted to M_1' in time. A is shifted to P as shown in fig.

Distance $AP = vT'$

$$T' = \frac{L}{c}$$

$$AP = \frac{vL}{c}$$

Applying Pythagoras theorem in $\Delta APM_1'$

$$L^2 = D^2 + \left[\frac{vL}{c}\right]^2$$

$$L^2\left[1-\frac{v^2}{c^2}\right] = D^2$$

$$L = D\left[1-\frac{v^2}{c^2}\right]^{-1/2}$$

$$L = D\left[1+\frac{v^2}{2c^2}\right]$$ (neglecting higher power)

$\therefore$ Total distance travelled by light in time T in going from A to M_1' and back to A'

$$X_2 = 2L = 2D\left[1+\frac{v^2}{2c^2}\right] = X_2$$

Path difference

$$X_2 - X_1 = 2D\left[1+\frac{v^2}{c^2}\right] - 2D\left[1+\frac{v^2}{c^2}\right] = \frac{Dv^2}{c^2}$$

If apparatus is turned through 90°, the difference will be $= -\frac{Dv^2}{c^2}$. Thus displacement in interference fringe is $\frac{Dv^2}{c^2}$.

But in this experiment no displacement of fringes was observed. This shows that this experiment is a -ve experiment.

★ Theory of Relativity:-

① All physical laws are same in all inertial frames of reference which are moving with constant velocity relative to each other.

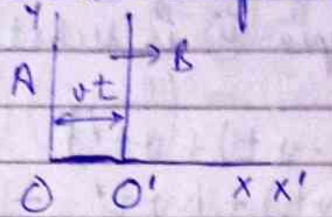
② The speed of light in vacuum is same in every inertial frame

The theory based on these two postulates and applies to all inertial frames is called special theory of relativity.

* Lorentz Transformation of Space and Time

Consider 2 frames of references, A is fixed and B is moving with constant speed v

Initially both frames have same origin of coordinates. After a time t of reference B has moved distance, $OO' = vt$. For point P in space coordinates are (x, y, z) with reference to frame B.



According to Galileon frame of reference;

$$\left.\begin{aligned} x' &= x - vt \\ y' = y,\ z' &= z,\ t' = t, \end{aligned}\right\} - (1)$$

Now differentiating equation in (1)

$$\frac{dx'}{dt} = \frac{dx}{dt} - v \quad - (2)$$

$$c' = c - v \quad - (3)$$

· Acc. to 2nd postulates of the special theory of relativity the velocity of light remains constant in free space. The eq. $x' = x - vt$ is in accordance with the laws of mechanics. The new transformation of x-coordinates must be similar to this equation. When the value of v is extremely small as compared to velocity of light, c. So, possible form of equation can be.

$$x' = k(x - vt) \quad - (4)$$

k only depends on the value of v.

· Acc to 1st postulate, obs. made in the frame of reference B must be identical to those made in A, except for a change in the sign of v and having same value, for the constant of proportionality k

$x = k(x' + vt')$ - (5)

As the relative motion of A and B is confined to only x,

$$y' = y \;(6), \quad z' = z \;(7)$$

As the relative motion of A and B is confined to only x,

Eq. (4) & (5) will not held if $t' = t$

Value of x' from equa (4) is substituted in eq. (5).

This gives

$$x = k[k(x - vt) + vt']$$

$$x = k^2(x - vt) + kvt' \quad - (8)$$

$$kvt' = x - k^2(x - vt)$$

$$t' = \frac{x - k^2x + k^2vt}{kv}$$

$$t' = \frac{k^2vt}{kv} + \frac{x(1-k^2)}{kv}$$

$$t' = kt + \left(\frac{1-k^2}{kv}\right)x \quad - (9)$$

To find the value of k, consider two reference frames A and B. The spaceship in reference from A measures the time t'. The x-coordinates for both the ships will

$$x = ct \quad - (10)$$

$$x' = ct' \quad - (11)$$

Value of c must remain constant.

Substituting value of x' and t' in eq (11)

$$k(x - vt) = c\left[kt + \left(\frac{1-k^2}{kv}\right)x\right]$$

$$kx - kvt = ckt + cx\left(\frac{1-k^2}{kv}\right)$$

$$kx - cx\left(\frac{1-k^2}{kv}\right) = ckt + kvt$$

$$x\left[k - c\left(\frac{1-k^2}{kv}\right)\right] = ckt\left[1+\frac{v}{c}\right]$$

$$x = \cancel{ckt}\ \frac{ckt\left[1+\frac{v}{c}\right]}{k\left[1-c\left(\frac{1-k^2}{k^2 v}\right)\right]} = ct\left[\frac{\left(1+\frac{v}{c}\right)}{1-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right)}\right]$$

This value of x must be equal to value of x in eq (10)

$$ct = ct\left[\frac{\left(1+\frac{v}{c}\right)}{1-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right)}\right]$$

$$1-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right) = 1+\frac{v}{c}, \quad \left(-\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right) = \frac{v}{c}$$

$$1-\frac{1}{k^2} = \frac{v^2}{c^2}, \quad \frac{1}{k^2} = 1-\frac{v^2}{c^2}, \quad k^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

$$\boxed{k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} \quad - (12)$$

When value of v is extremely small in comparison to c, value of k will be 1

Now, putting value of k in eq (4), (6), (7), (9)

$$\boxed{x' = k(x-vt) = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}} \quad - (13)$$

$y' = y$ − (14), $z' = z$ − (15)

$$t' = kt + \left(\frac{1-k^2}{kv}\right)x = kt + \frac{x}{kv} - \frac{kx}{v}$$

$$\boxed{t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} \quad - (16)$$

(13), (14), (15), (16) are Lorentz transformation. These equations gives the conversion for the measurements of time and space made in stationery frame A to their counterparts in moving frame B.

Inverse Lorentz transformation (B to A)

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

* Time dilation :- Word dilation means to lengthen an interval of time. Consider two systems S and S'. Let S' be moving with a velocity v wrt S in the +ve direction ~~of~~ of X-axis.

Let a clock is situated in the system S at position 'x' and give signals at the system S at position x and give signals at interval Δt i.e

$$\Delta t = t_2 - t_1 \quad - (1)$$

If this interval is observed by an observed in system S', then interval $\Delta t'$ is given by :-

$$\Delta t' = t_2' - t_1'$$

Put t_2' and t_1' value from Lorentz.

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\Delta t' > t$ i.e time interval in system S' is greater than time ~~inter~~ interval in system S

$$s = v \times t'$$

* Length Contraction.

Let us consider a rigid rod at rest in a moving frame S', moving along x-axis with a speed v. The length of rod is equal to the difference b/w the coordinates of its 2 end points. Thus an observer in spaceship measures the length of the rod to be

$L_0 = x_2' - x_1'$

L_0 is said to be the proper length of rod.
Now, an observer on the earth ~~surface~~ frame S will have to determine the length of rod by marking the forward end position of rod at the same instant of time. Thus he will mark and measure coordinates of rod x_1 and x_2 in his frame.

Acc. to Lorentz transformation equation.

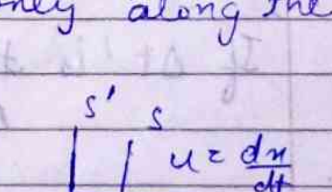
$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}}$$

1 B $\sqrt{1-\frac{v^2}{c^2}}$ is always less than 1,

L is less than L_0. It means if observer at rest wrt to a body measures its length to be l, an observer moving with relative speed v wrt the body will find it shorter than its proper length by a factor $\sqrt{1-\frac{v^2}{c^2}}$. Hence, rod in spaceship is contracted as measured from earth. This effect is symmetric, a rod at rest on earth will undergo length contraction when measured from spaceship. This effect is known as length contraction. It occurs only along the direction of motion

★ Addition of velocities



$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad - (1)$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad - (2)$$

$$dx' = \frac{dx - v\,dt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \frac{v\,dx}{c^2}}$$

Divide num & den by dt

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - v\frac{dt}{dt}}{\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}}$$

$$\boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}}$$

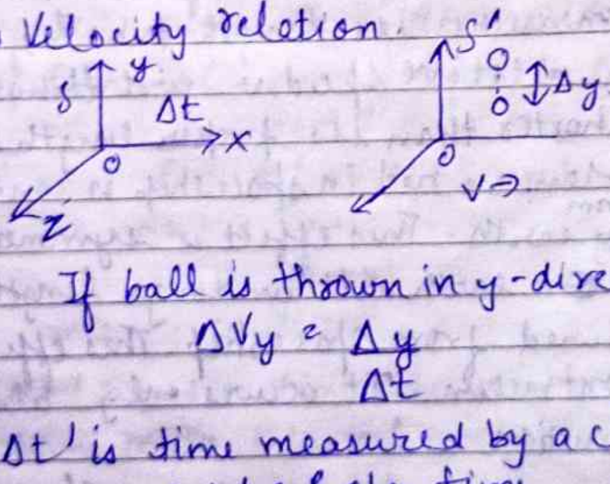
Other frame

From S, $u = \dfrac{u' + v}{1 + \frac{u'v}{c^2}}$

If $u' = c$, $u = \dfrac{c + v}{1 + \frac{cv}{c^2}}$

$$\boxed{u = c}$$

* Mass Velocity relation.



If ball is thrown in y-direction then

$$\Delta V_y = \frac{\Delta y}{\Delta t}$$

If $\Delta t'$ is time measured by a clock moving with particle

$\Delta t'$ = Proper time.

$$\Delta V_y = \frac{\Delta y}{\Delta t'}$$

Momentum is conserved in all frames

$$P_y = m_0 v_{net}$$

$$= m_0 \frac{\Delta y}{\Delta t'}$$

$$= m_0 \frac{\Delta y}{\Delta t} \cdot \frac{\Delta t}{\Delta t'}$$

$$= m_0 v_y \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$P_y = \frac{m_0 v_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

* Energy mass relation :- Frame of reference is moving with velocity comparable to c.

$$E = mc^2$$

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

* Galileon Transformation