Friction

when a body moves or tends to move over another body, a force opposing the motion develops at contact surfaces. This force which opposes the movement or the tendency of movement is called frictional force or simply friction.

Static friction: It is the friction acting on a body when the body is not in motion; but when a force is acting on it. Static friction is the same as the external force being applied (because the body isn't moving).

Limiting friction: It is the friction on a body just before it starts moving. It is the maximum value of static friction that can develop.

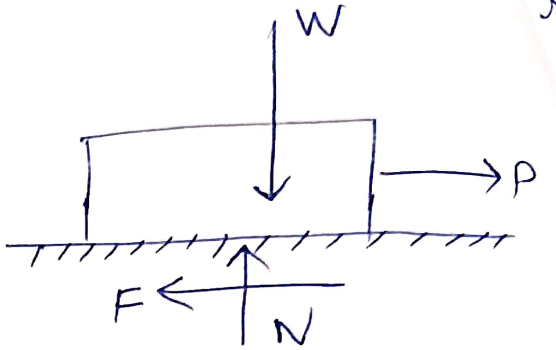
Dynamic friction: If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as Dynamic friction. Dynamic friction is found to be less than limiting friction.

Dynamic friction may be classified into the following two:

a) sliding friction
b) Rolling friction

Sliding friction is the friction experienced by a body when it slides over the other body and the rolling friction is the friction experienced by a body when it rolls over a surface.

It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces and this ratio is called Coefficient of friction.



$$\text{Coefficient of friction} = \frac{F}{N}$$

where, F is limiting friction
& N is normal reaction

Coefficient of friction is denoted by μ,
Thus

$$\mu = \frac{F}{N}$$

Coefficient of friction

Material	Static	Sliding / dynamic
Aluminium – Mild steel	0.61	0.47
Cast Iron – Cast Iron	1.1	0.15
Copper – Cast Iron	1.05	0.29
Copper – Mild steel	0.53	0.36
Glass – Glass	0.9 – 1.0	0.4
Ice – Ice	0.03	
Ice – steel	0.02 – 0.09	0.015
Steel – Brass	0.51	0.44
Steel – Lead	0.95	0.95
Steel (Mild) – Steel (Mild)	0.74	0.57
Steel (Hard) – Steel (Hard)	0.78	0.42
wood – Brick	0.60	—
wood – concrete	0.62	—
wood – Metal	0.2 – 0.6	—

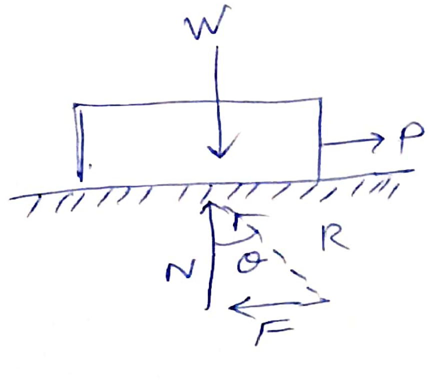
Laws of friction

These principles constitute the laws known as Coulomb's laws of friction / laws of dry friction / laws of solid friction. These laws are listed below:

1. The force of friction always acts in a direction opposite to that in which body tends to move.
2. Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body,
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient of friction.
4. The force of friction depends upon the roughness/ smoothness of the surfaces.
5. The force of friction is independent of the area of contact between the two surfaces.
6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is called coefficient of dynamic friction.

Angle of friction

Consider the block shown in fig resting on a horizontal surface and subjected to horizontal pull P.



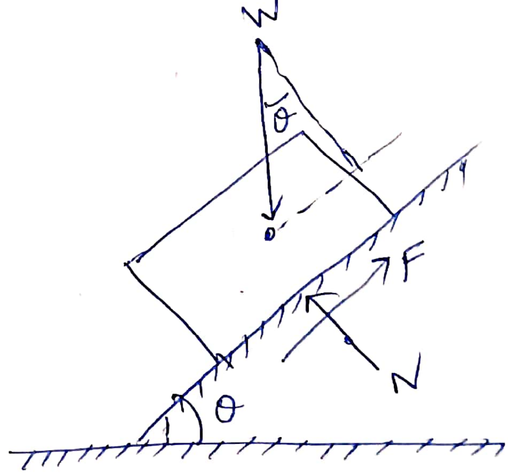
let F be the frictional force developed & N the normal reaction. Thus at contact surface the reactions are F & N. They can combined graphically to get the reaction R which acts at angle θ to normal reaction. This angle θ, called the angle of friction is given by

$$\tan\theta = \frac{F}{N}$$

As P increases, F increases & hence θ also increases. θ can reach the maximum value α when F reaches limiting value. At this stage

$$\tan\alpha = \frac{F}{N} = \mu$$

& this value α is called Angle of limiting friction. Hence angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.



Angle of repose

A maximum inclination of the plane on which a body, free from external forces, can repose (sleep) is called Angle of repose.

Consider the equilibrium of block shown in figure. Since the surface of contact is not smooth, not only normal reaction, but frictional force also develops. Since the body tends to slide downwards, the frictional the reaction ... will be up the plane.

Σ forces normal to the plane $= 0$, gives

$$N = W\cos\theta \qquad —① $$

Σ forces parallel to the plane $= 0$, gives

$$F = W\sin\theta \qquad —② $$

from eq ① & ②

$$\tan\theta = \frac{F}{N}$$

If ϕ is the value of θ when motion is impending, frictional force will be limiting friction and hence

$$\tan\phi = \frac{F}{N}$$

$$= \mu$$

$$= \tan\alpha$$

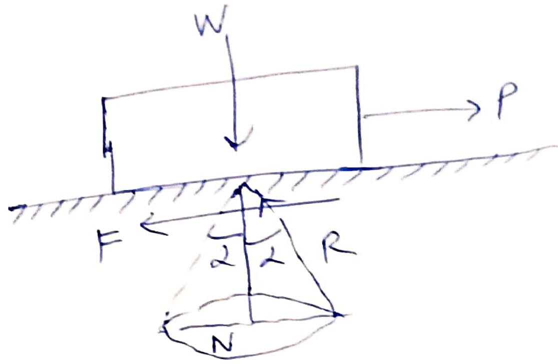
or $\phi = \alpha$

thus the value of angle of repose is same as the value of limiting angle of friction.

Cone of friction

when a body is having impending motion in the direction of force P, the frictional force will be limiting friction & the resultant reaction R will make limiting angle α with the normal as shown in fig. If the body is having impending motion in some other direction, the resultant reaction makes limiting frictional angle α with the normal to that direction. Thus when the direction of force P is gradually changed through 360°, the resultant R generates a right circular cone with semi-central angle equal to α.

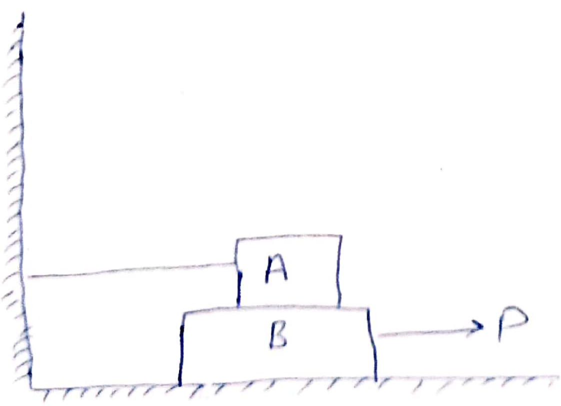
If the resultant R is on the surface of this inverted right circular cone whose semi-central angle is limiting frictional angle (α) the motion of the body is impending. If the resultant is within this cone the body is stationary. This inverted cone with semi-central angle α equal to limiting frictional angle α is called cone of friction.



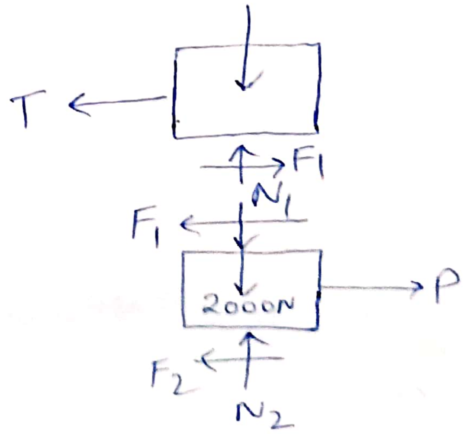
Q) Block A weighing 1000N rests over block B which weighs 2000N as shown in fig 5.5(a) tied to wall with a horizontal string. If the coefficient of friction between blocks A & B is 0.25 and between B & floor is 1/3, what should be the value of P to move the block (B), if.

(a) P is horizontal?

b) P acts at 30° upwards to horizontal?



(a)



(b)

when P is horizontal
considering block A.

$$\Sigma V = 0$$

$$1000 - N_1 = 0 \quad \text{or } N_1 = 1000\,N$$

Since F_1 is limiting friction

$$\frac{F_1}{N_1} = \mu = 0.25$$

$$F_1 = 250\,N$$

$$\Sigma H = 0$$

$$F_1 - T = 0$$

or $T = F_1 = 250N$

Consider equilibrium of block B,

$$\Sigma V = 0$$

$$N_2 - N_1 - 2000 = 0$$

$$N_2 = 2000 + N_1 = 2000 + 1000$$

$$= 3000N$$

Since F_2 is limiting friction

$$F_2 = \mu N_2$$

$$= \frac{1}{3} \times 3000 = 1000N$$

$$\Sigma H = 0$$

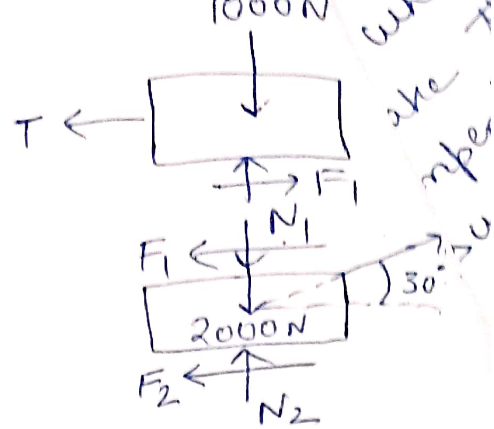
$$F_1 + F_2 - P = 0$$

$$P = F_1 + F_2 = 250 + 1000$$

$$= 1250N$$

(b) when P is inclined

b) when P is inclined



Consider equilibrium of Block A.

$\Sigma V = 0$

$N_1 - 1000 = 0$

$N_1 = 1000\ N$

$F_1 = \mu N_1$

$= 0.25 \times 1000 = 250\ N$

$\Sigma H = 0$

$F_1 - T = 0$

$250 = T$ or $T = 250\ N$

Consider equilibrium of Block B

$\Sigma V = 0$

$N_1 - N_2 + 2000 - P\sin 30 = 0$ or $3000 - N_2 - P\sin 30 = 0$

$3000 - 0.5P = N_2$

~~1000 + 2000 = N2~~

or ~~N2 = 3000~~

from law of friction

$F_2 = \frac{1}{3} N_2 = \frac{1}{3}(3000 - 0.5P)$

$= 1000 - \frac{0.5}{3}P$

$\Sigma H = 0$, gives

$F_1 + F_2 - P\cos 30 = 0$

$-P\cos 30 + 250 + \left(1000 - \frac{0.5}{3}P\right) = 0$

$P\cos 30 - 250 - \left(1000 - \frac{0.5}{3}P\right) = 0$

$P\left(\cos 30 + \frac{0.5}{3}\right) = 1250$

or $P = 1210.43\ N$

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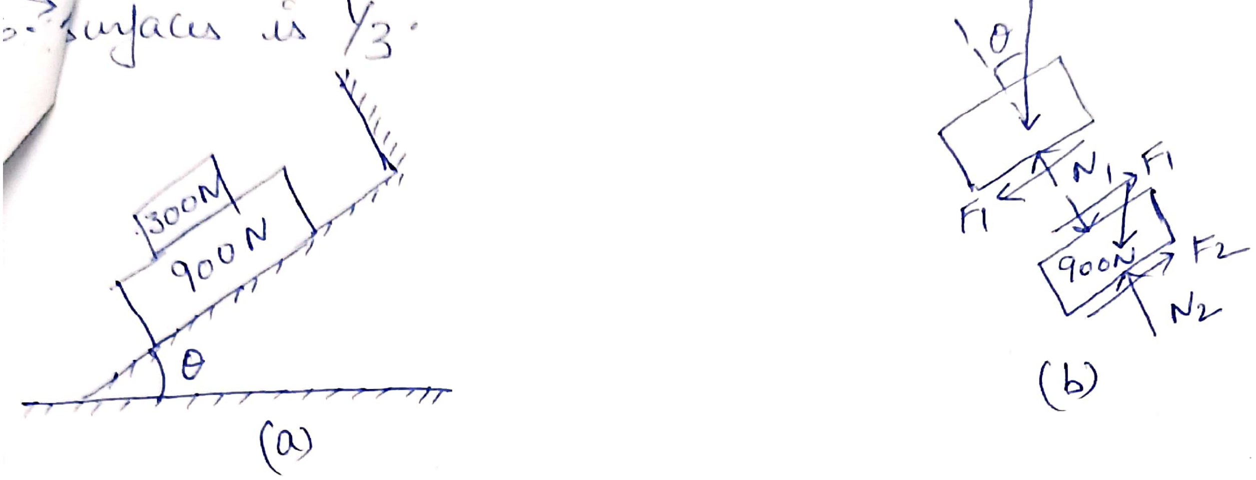
what should be the value of θ in fig that will make the motion of 900 N block down the plane to impend? The coefficient of friction for all contact surfaces is 1/3.



$F_1 + 300 \sin\theta = 0$
$F_1 = -300 \sin\theta$

Consider equilibrium of 300 N block,

Σ Forces normal to plane $= 0$, gives

$$N_1 - 300 \cos\theta = 0$$

or $$N_1 = 300 \cos\theta$$

From law of friction

$$F_1 = \mu N_1 = 300 \cos\theta \times \frac{1}{3} = 100 \cos\theta$$

Consider equilibrium of 900 N block,

Σ Forces normal to the plane $= 0$, gives

$$N_2 - N_1 - 900 \cos\theta = 0$$

$$N_2 = N_1 + 900 \cos\theta$$
$$= 300 \cos\theta + 900 \cos\theta$$
$$= 1200 \cos\theta$$

From law of friction,

$$F_2 = \frac{1}{3} N_2 = \frac{1}{3} \times 1200 \cos\theta$$
$$= 400 \cos\theta$$

Σ Forces parallel to plane $= 0$, gives

$$F_1 + F_2 - 900 \sin\theta = 0$$

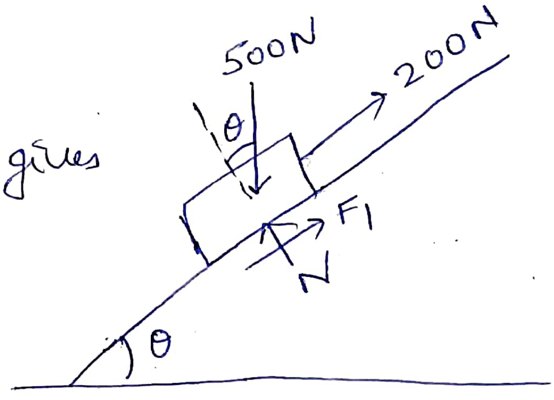
$$100 \cos\theta + 400 \cos\theta = 900 \sin\theta$$

$\therefore \tan\theta = \frac{5}{9}$

$\theta = 29.05°$

Q3 A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. the same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and the coefficient of friction between the inclined plane and the block.

Sol^n



(a)

Σ Forces perpendicular to plane $=0$, gives

$N - 500\cos\theta = 0$

or $N = 500\cos\theta$

from law of friction

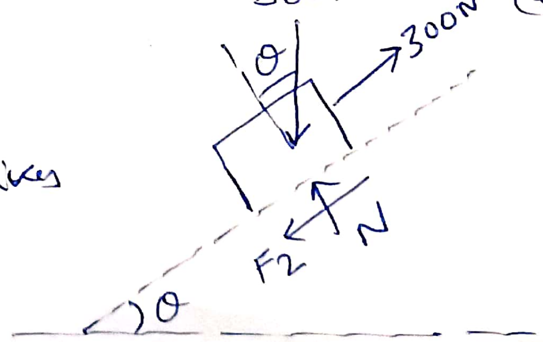
$F_1 = \mu N = 500\mu\cos\theta$

Σ Forces parallel to plane $=0$, gives

$200 + F_1 - 500\sin\theta = 0$

i.e $200 = 500\sin\theta - F_1$ —— (1)

$200 = 500\sin\theta - 500\mu\cos\theta$ —— (1)



(b)

Consider the Equilibrium of (b)

Σ Force normal to plane $=0$, gives

$N - 500\cos\theta = 0$

or $N = 500\cos\theta$

rom the law of friction,

$$F_2 = \mu N = 500 \mu \cos\theta$$

Σ Forces parallel to plane = 0, gives

$$300 - F_2 - 500 \sin\theta = 0$$

$$300 = F_2 + 500 \sin\theta$$

$$= 5 \mu \cos\theta + 500 \sin\theta \quad \text{--- (2)}$$

Adding Eqn (1) & (2), we get,

$$500 = 1000 \sin\theta$$

$$\sin\theta = 0.5$$

$$\theta = 30^\circ$$

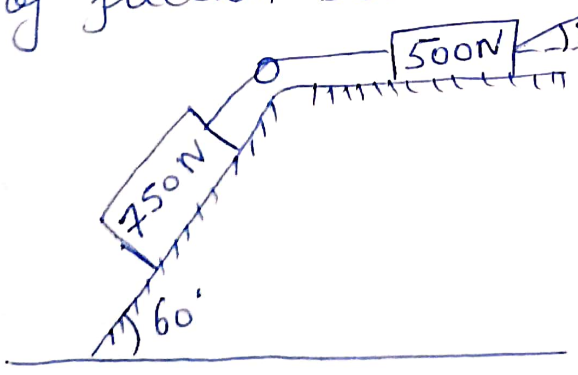
Substituting it in eqn (2), we get

$$300 = 500 \mu \cos 30^\circ + 500 \sin 30^\circ$$

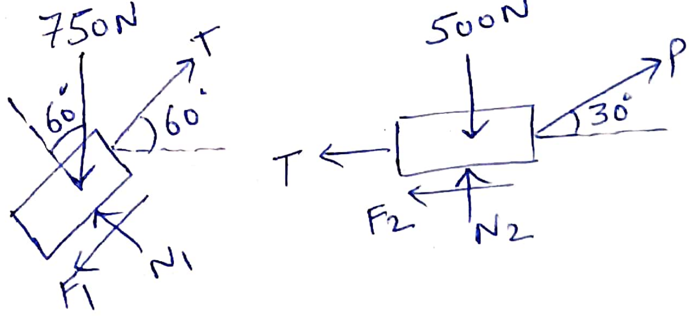
i.e $\quad 500 \mu \cos 30^\circ = 300 - 500 \times 0.5 = 50$

$$\mu = \frac{50}{500 \cos 30^\circ} = 0.11547$$

Q4 What is the value of P in the system as shown in fig (a) to cause the motion of 500 N block to the right side? Assume the pulley is smooth & the coefficient of friction between other contact surfaces is 0.20



(a)



(b)

Sol^n Consider Σ forces normal to the plane $=0$, gives,

$$N_1 - 750\cos 60° = 0 \quad \text{or} \quad N_1 = 375N$$

Since the motion is impending

$$F_1 = \mu N_1 = 0.20 \times 375 = 75N$$

Σ Forces parallel to the plane $=0$, gives

$$T - F_1 - 750\sin 60 = 0$$

$$\therefore \quad T = F_1 + 750\sin 60 = 75 + 750\sin 60°$$
$$= 724.52N$$

Consider the equilibrium of 500 N block,

$$\Sigma V = 0$$

$$500 - N_2 - P\sin 30 = 0$$
$$N_2 + 0.5P = 500$$

i.e. ~~from~~ $N_2 = 500 - 0.5P$

from law of friction,

$$F_2 = 0.2N_2 = 0.2(500 - 0.5P)$$
$$= 100 - 0.1P$$

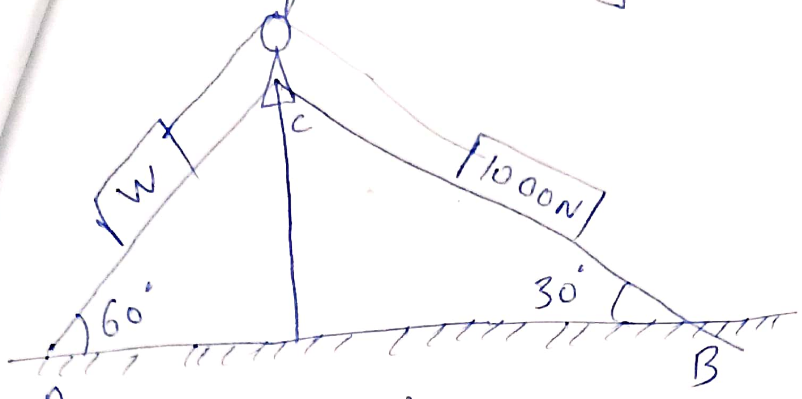
$$\Sigma H = 0$$

$$P\cos 30 - T - F_2 = 0$$

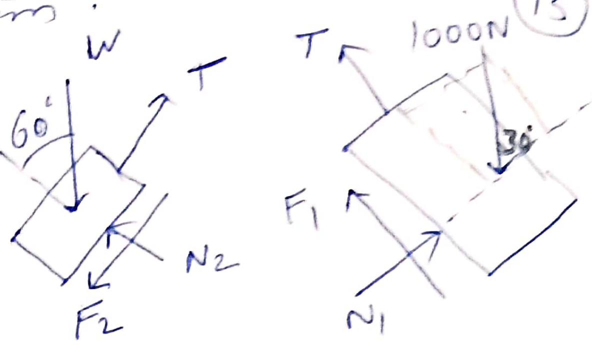
$$P\cos 30 - 724.52 - (100 - 0.1P) = 0$$

$$\therefore \quad P(\cos 30° + 0.1) = 724.52 + 100$$
$$P = 853.52N$$

Q5 Two planes AC & BC inclined at 60° & 30° to the horizontal meet at C as shown in fig (a). A block of ~~weighing~~ weight 1000N rests on the inclined plane BC & is tied by a rope passing over a pulley to a block weighing W newtons and resting on the plane AC. If the coefficient of friction between the block & plane BC is 0.28 & that between the block & the plane AC is 0.20, find the least & greatest value of W n the equilibrium of the system :



(a)



(b)

Sol^n (a) For the least value of W:

Consider the equilibrium of 1000 N block.

Σ Forces normal to the plane $= 0$, gives

$N_1 - 1000\cos 30° = 0 \qquad \therefore N_1 = 866.03$ newton

From the law of friction,

$F_1 = \mu N_1 = 0.28 \times 866.03 = 242.49$ newton

Σ Forces parallel to the plane $= 0$

$T + F_1 - 1000\sin 30 = 0$

$T = -242.49 + 500 = 257.51$ newton

Now consider the equilibrium of block weighing W,

Σ Forces normal to the plane $= 0$, gives

$N_2 - W\cos 60 = 0 \qquad \therefore N_2 = 0.5W$

$\therefore \quad F_2 = \mu N_2 = 0.2 \times 0.5W = 0.1W$

Σ Forces parallel to the plane $= 0$, gives

$T - F_2 - W\sin 60° = 0$

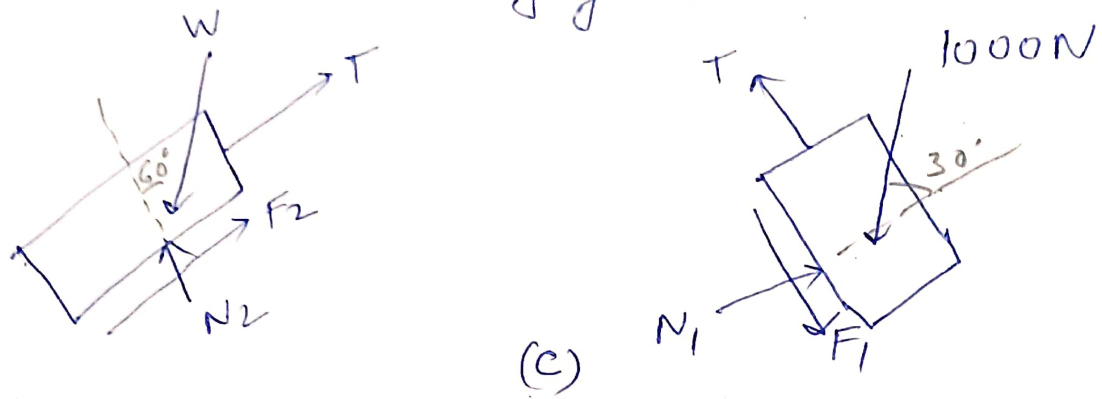
$257.51 - 0.1W - W\sin 60° = 0$

$$W = \frac{257.51}{(0.1 + \sin 60°)} = 266.57$$

(b)

For greatest value of W:

In this case 1000 N block will be on the verge of moving up the plane. the free body diagram for this case is shown in fig (c).



(c)

Considering the equilibrium of block weighing 1000 N

$N_1 = 866.03$ newton

& $F_1 = 242.49$ newton

Σ parallel to plane $=0$, gives

$T - 1000\sin 30' - F_1 = 0$

$T = 1000\sin 30 + 242.49 = 742.49$ newton

Considering the equilibrium of block weighing W,

Σ Forces normal to plane $=0$, gives

$\cancel{N}_2 - W\cos 60° = 0 \quad \therefore N_2 = 0.5W$

$F_2 = \mu N_2 = 0.2 \times 0.5W = 0.1W$

Σ Forces parallel to plane $=0$, gives

$T + F_2 - W\sin 60' = 0$

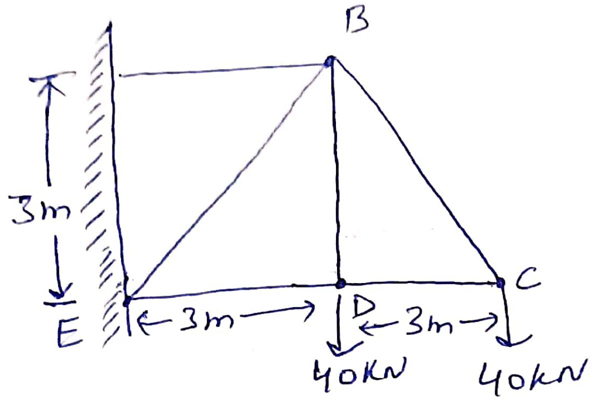
$742.49 - 0.1W - W\sin 60° = 0$

$W = \dfrac{749.49}{\sin 60 - 0.1} = 969.28$ newton

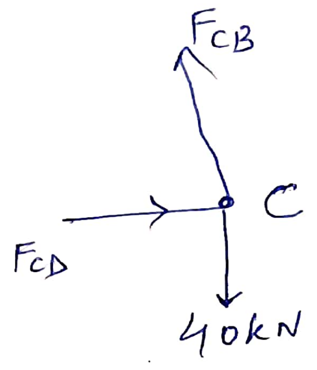
Method of Joint

Q1 Find the forces in all the members of the truss shown in fig. Tabulate the results.

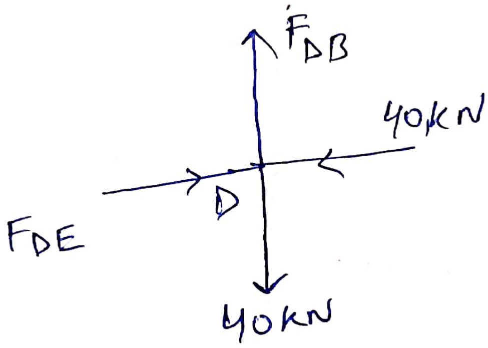
Soln



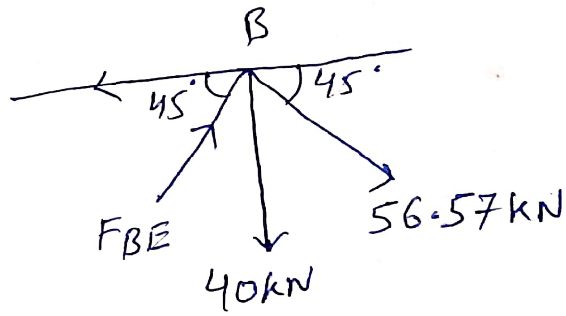
(a)



(b)



(c)



(d)

$\tan\theta = \frac{3}{3} = 1$

$\theta = 45°$

At Joint C,

$F_{CB} \sin 45 - 40 = 0$

$\therefore \quad F_{CB} = 56.57$ kN

Now, $\Sigma F_H = 0$ indicates that F_{CD} should act toward C

$F_{CD} - F_{CB}\cos 45 = 0$

$\therefore \quad F_{CD} = F_{CB}\cos 45 = 56.57\cos 45$

$F_{CD} = 40$ kN

$\Sigma H = 0$

$\cdot$ HA $+ 10 \times 3$

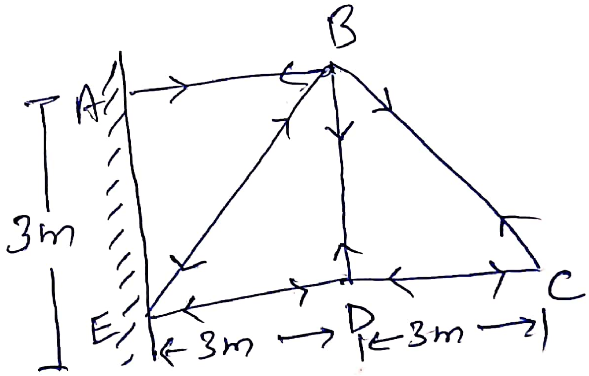
HA $- 10 \times 3$

Trusses

i) Method of Joint

ii) method of section.

At Joint D,



$\Sigma F_V = 0$, gives

$F_{DB} = 40 kN$

$\Sigma F_H = 0$, gives

$F_{DE} = 40 kN$

At Joint B

$\Sigma F_V = 0$,

$F_{BE} \sin 45 - F_{BD} - F_{BC} \sin 45 = 0$

(Note: $F_{BD} = F_{DB}$ & $F_{BC} = F_{CB}$ in magnitude)

$\therefore F_{BE} \sin 45 - 40 - 56.57 \sin 45 = 0$

i.e $F_{BE} = 113.14 kN$

$\Sigma F_H = 0$, gives

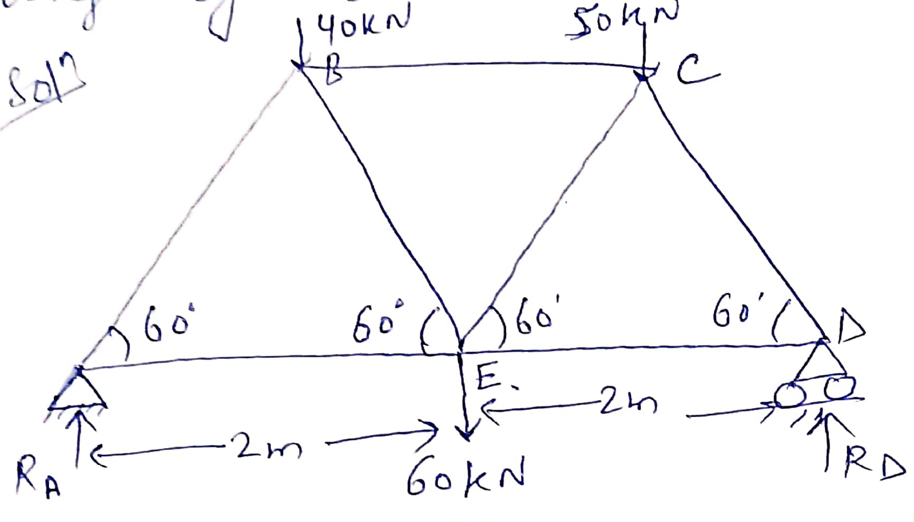
$-F_{BA} + F_{BE} \cos 45 + F_{BC} \cos 45 = 0$

or $F_{BA} = 113.14 \cos 45 + 56.57 \cos 45$

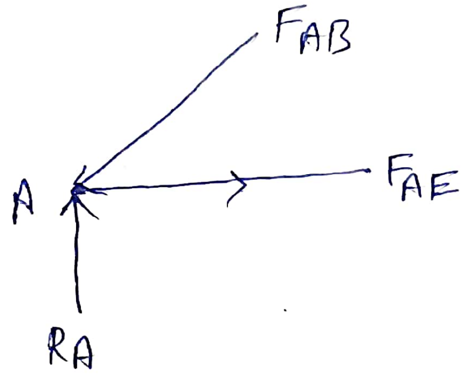
$= 120 kN$

Determine the forces in all the members of truss shown in fig (a) and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal & length of each member is 2m.

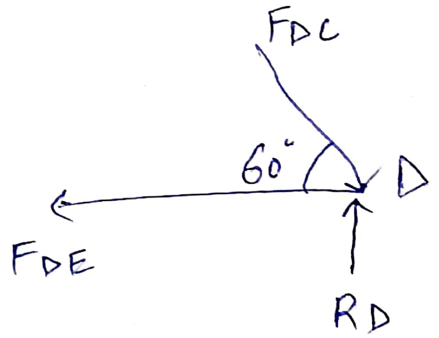
Solⁿ



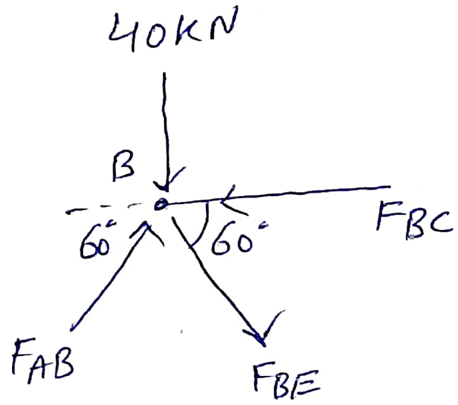
(a)



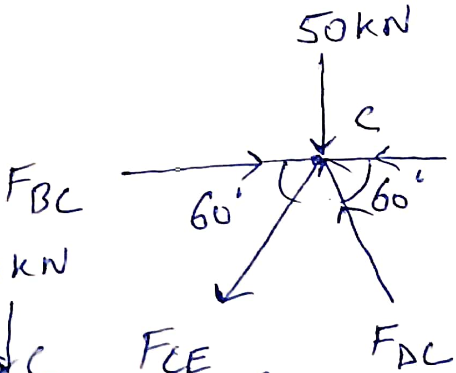
(b)



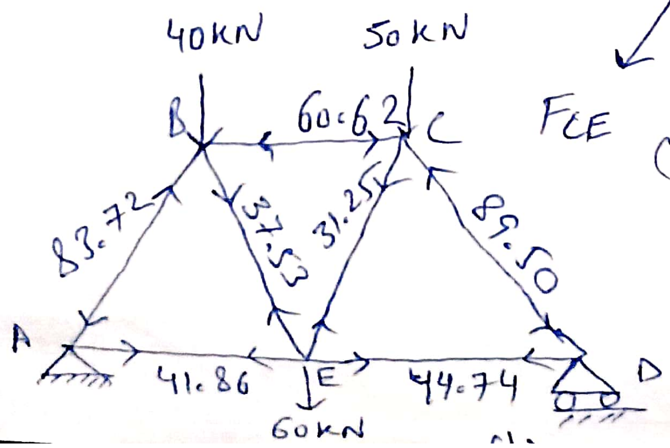
(c)

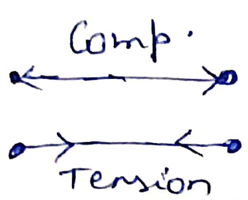


(d)



(e)





Considering the equilibrium of entire frame,

$\Sigma M_A = 0$, gives

$$-R_D \times 4 + 40 \times 1 + 60 \times 2 + 50 \times 3 = 0$$

$$\therefore \quad R_D = 77.5 \text{ kN}$$

$\Sigma F_H = 0$, gives $H_A = 0$

$\therefore$ Reaction at A is vertical

$\Sigma F_V = 0$, gives

$$R_A + R_D - 40 - 60 - 50 = 0$$

since $R_D = 77.5$ kN

$$R_A = 150 - 77.5$$
$$= 72.5 \text{ kN}$$

Now at Joint A,

$$\Sigma F_V = 0$$
$$F_{AB} \sin 60 - R_A = 0$$
$$\therefore \quad F_{AB} = \frac{R_A}{\sin 60} = \frac{72.5}{\sin 60} = 83.72 \text{ kN (Compression)}$$

$\Sigma F_H = 0$, gives

$$F_{AE} - F_{AB} \cos 60 = 0$$
$$\therefore \quad F_{AE} = 83.72 \cos 60^\circ = 41.86 \text{ kN (Tension)}$$

Joint D: $\Sigma F_V = 0$, gives

$$-F_{DC} \sin 60^\circ + R_D = 0$$
$$\therefore \quad F_{DC} = \frac{R_D}{\sin 60} = \frac{77.5}{\sin 60^\circ} = 89.5 \text{ kN (Compression)}$$

$\Sigma F_H = 0$, gives

$$-F_{DE} + F_{DC} \cos 60^\circ = 0$$
$$\therefore \quad F_{DE} = F_{DC} \cos 60^\circ = 89.5 \cos 60^\circ = 44.75 \text{ kN (Tension)}$$

Joint B : Referring to fig (d).

$\Sigma F_V = 0$, gives.

$$F_{AB} \sin 60 - 40 - F_{BE} \sin 60 = 0$$

$$\therefore F_{BE} = \frac{F_{AB} \sin 60 - 40}{\sin 60} = \frac{72.5 - 40}{\sin 60} = 37.53\,kN \text{ (tension)}$$

$\Sigma F_H = 0$, gives

$$F_{AB} \cos 60 + F_{BE} \cos 60° - F_{BC} = 0$$

$$\therefore F_{BC} = 83.72 \cos 60 + 37.53 \cos 60° = 60.62\,kN \text{ (Compression)}$$

Joint C : Referring to fig (e)

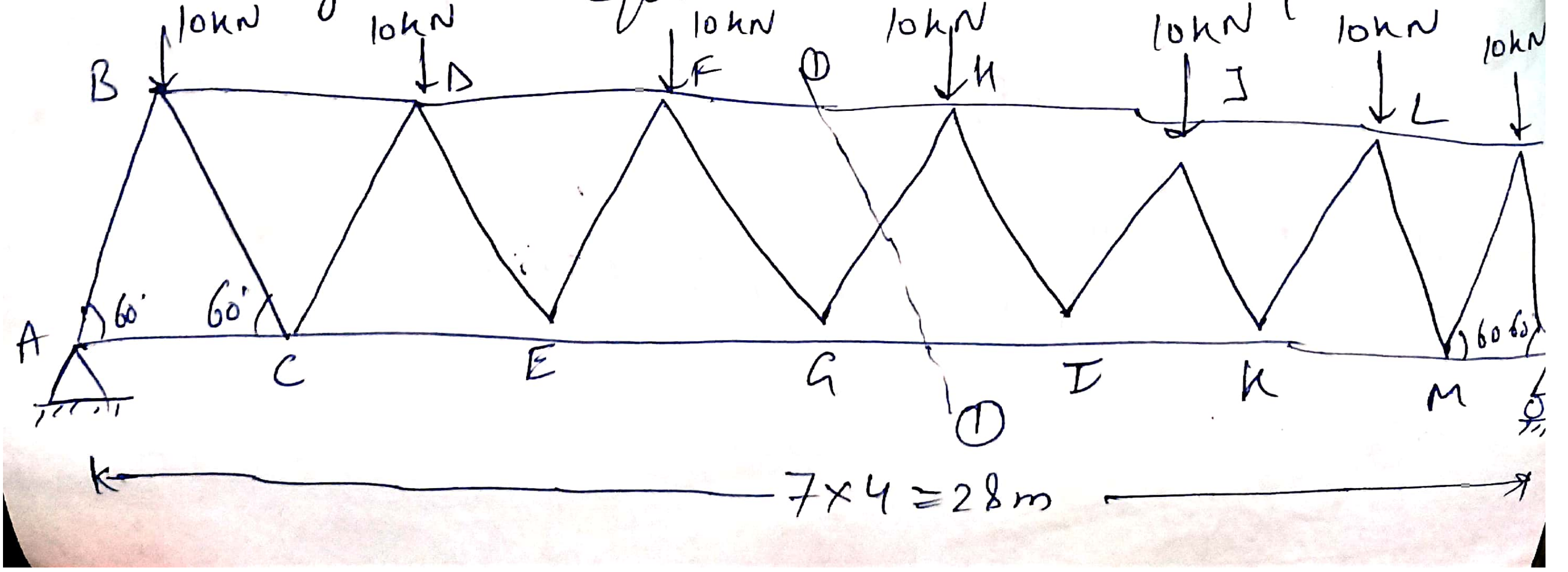
$\Sigma F_V = 0$, gives.

$$F_{DC} \sin 60 - 50 - F_{CE} \sin 60° = 0$$

$$\therefore F_{CE} = \frac{F_{DC} \sin 60 - 50}{\sin 60°} = \frac{89.50 \sin 60 - 50}{\sin 60°} = 31.76\,kN \text{ (tension)}$$

Method of Section

Q/ Determine the forces in the members FH, HG & GI in the truss shown in fig (a). Each load is 10 kN & all traingles are equilateral with sides equal to 4m



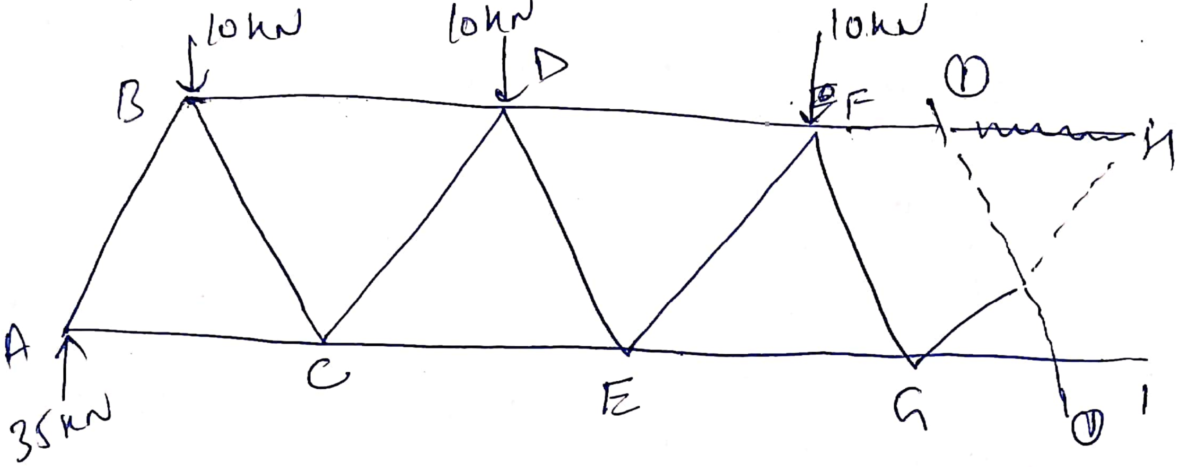
Sol^n ~~to~~ Due to Symmetry

$$R_A = R_B = \frac{1}{2} \times \text{Total load}$$

$$= \frac{1}{2}(10 \times 7) = 35\ kN$$

There is no horizontal component of reaction at A.

Take section (1)-(1) which cuts the members FH, GH, GI and separate the truss into two parts.



$\Sigma M_G = 0$, gives

$$-F_{FH} \times 4 \sin 60^\circ + 35 \times 12 - 10 \times 10 - 10 \times 6 - 10 \times 2 = 0$$

$$F_{FH} = 69.28\ kN\ (\text{Compression})$$

$$\Sigma F_V = 0$$

$$35 - 10 - 10 - 10 - F_{GH} \sin 60' = 0$$

$$F_{GH} = 5.77\ kN\ (\text{compression})$$

$$\Sigma F_H = 0$$

$$F_{GI} - F_{FH} - F_{GH} \cos 60' = 0$$

$$F_{GI} = 69.28 + 5.77 \cos 60 = 72.17\ kN\ (\text{Tension})$$

Ans

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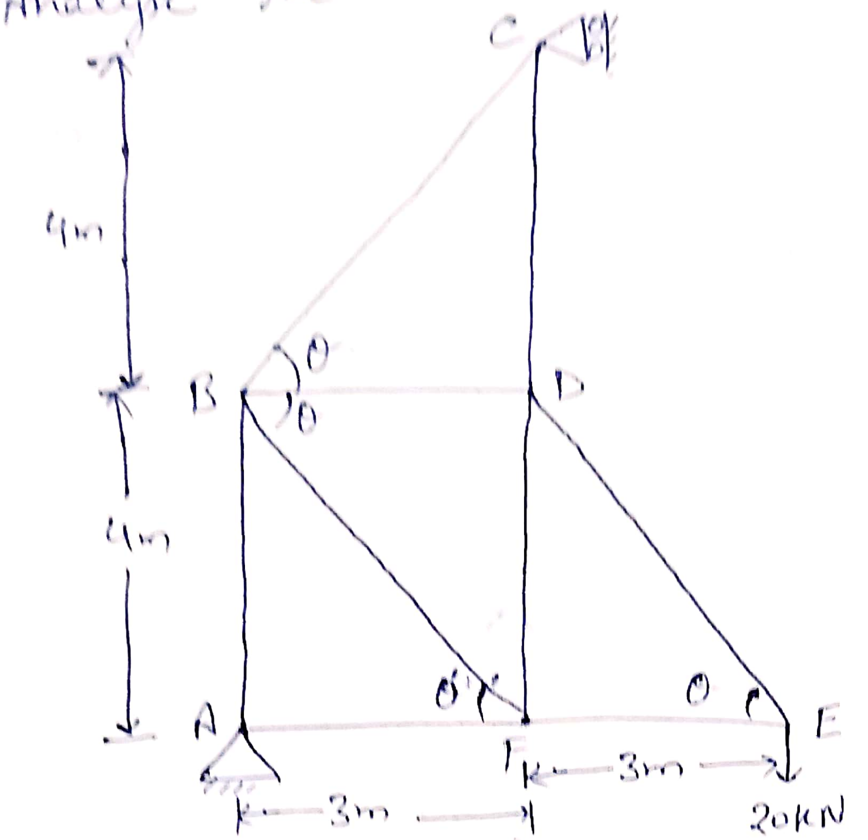
Analyse the truss as shown in figure.

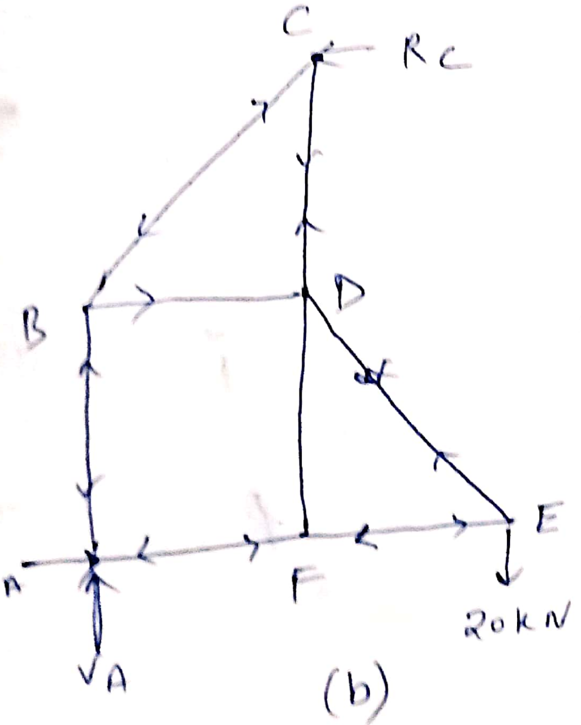


(a)

Sol) inclined members are making angle θ with horizontal,
where,

$$\tan\theta = \frac{4}{3}$$

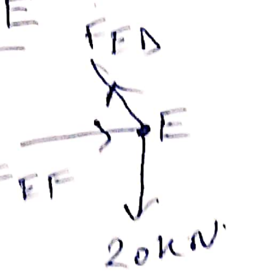
$$\theta = 53.13^\circ$$



(b)

E A C B F

at E



(c)

$\Sigma F_V = 0$

$F_{ED} \sin\theta - 20 = 0$

$$F_{ED} = \frac{20}{\sin 53.13} = 25\,kN\ (\text{Tension})$$

$\Sigma F_H = 0$

$F_{EF} - F_{ED}\cos\theta = 0$

$F_{EF} = 25\cos 53.13 = 15\,kN$ (compression)

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$\Sigma M_A = 0$

$-R_C \times 8 + 20 \times 6 = 0$

$R_C = 15\,kN$

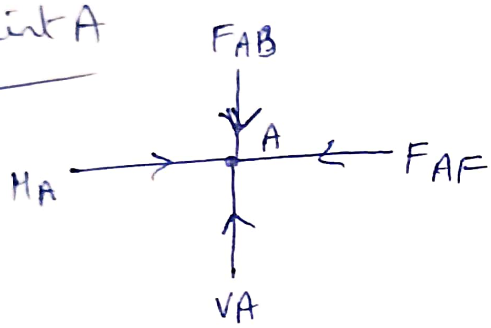
$\Sigma F_V = 0$

$V_A - 20 = 0$ or $V_A = 20$

$\Sigma F_H = 0$

$H_A - R_C = 0$ or $H_A = R_C = 15\,kN$

Joint A



$\Sigma F_V = 0$

$-F_{AB} + V_A = 0$

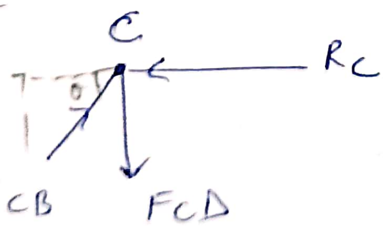
$F_{AB} = V_A = 20\,kN$

$\Sigma F_H = 0$

$H_A - F_{AF} = 0$

$F_{AF} = H_A = 15\,kN$

Joint C



$\Sigma F_H = 0$

$F_{CB} \cos\theta - R_C = 0$

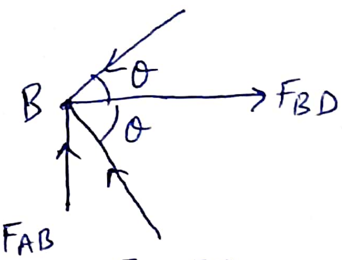
$F_{CB} = \frac{R_C}{\cos\theta} = \frac{15}{\cos 53.13} = 25\,kN$

$\Sigma F_V = 0$

$F_{CB} \sin 53.13 - F_{CD} = 0$

$\therefore F_{CD} = 25 \sin 53.13^\circ = 20\,kN$

Joint B



$F_{BF} = 0$

$\Sigma F_V = 0$

$F_{BF} \sin 53.13^\circ - F_{BC} \sin 53.13 + F_{AB} = 0$

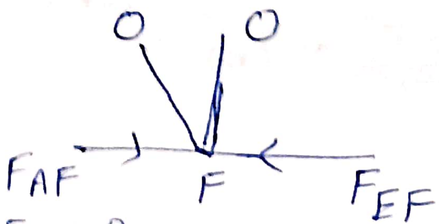
$F_{BF} = \frac{F_{BC} \sin 53.13 - F_{AB}}{\sin 53.13} = \frac{25 \sin 53.13 - 2}{\sin 53.13^\circ} = 0$

$\Sigma F_H = 0$

$F_{BD} - F_{BF} \cos 53.13^\circ - F_{BC} \cos 53.13^\circ = 0$

$F_{BD} = 0 + 25 \cos 53.13^\circ = 15\,kN$

Joint F



$\Sigma F_V = 0$

$F_{FD} = 0$ since $F_{BF} = 0$