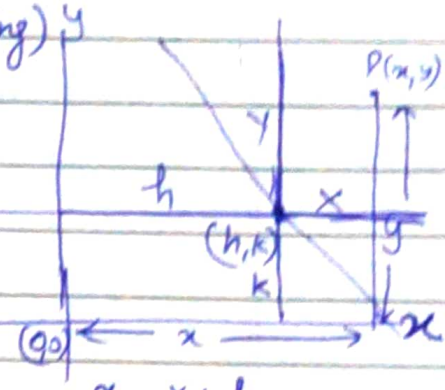
Eqⁿ Reducible to hom. Equation:→

(1) $$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$$

If $\frac{a}{a'} \neq \frac{b}{b'}$ (⇒ lines are Intersecting)



$$\frac{dy}{dx} = \frac{a(x+h)+b(y+k)+c}{a'(x+h)+b'(y+k)+c'}$$

$$= \frac{ax+by+(ah+bk+c)}{a'x+b'y+(a'h+b'k+c')}$$

Find h, k s.t. $ah+bk+c=0$ & $a'h+b'k+c'=0$ ($\because (h,k)$ is the Intersecting points of these two lines)

$$\therefore \frac{dy}{dx} = \frac{ax+by}{a'x+b'y} \rightarrow \text{Reduced into hom. form.}$$

(2) If $\frac{a}{a'} = \frac{b}{b'} = \lambda$ (say) (When lines are Parallel)

So $$\frac{dy}{dx} = \frac{(a'x+b'y)\lambda+c}{a'x+b'y+c'}$$

Put $a'x+b'y = u$

$$a' + b'\frac{dy}{dx} = \frac{du}{dx}$$

$$\therefore \frac{1}{b}\left(\frac{du}{dx} - a'\right) = \frac{u\lambda+c}{u+c'} \rightarrow \text{Reduced Into Separable form.}$$

Que $\frac{dy}{dx} = \frac{x+3y-2}{2x+6y+1}$

$\frac{1}{2} = \frac{3}{6}$

$\therefore$ Put $x+3y = V$

$1+3\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{1}{3}\left(\frac{dv}{dx} - 1\right) = \frac{V-2}{2V+1}$

$\Rightarrow \frac{dv}{dx} = \frac{3V-6}{2V+1} + 1$

$= \frac{3V-6+2V+1}{2V+1} = \frac{5V-5}{2V+1}$

$\Rightarrow \int \frac{2V+1}{5(V-1)}\,dv = \int dx + c$

$\Rightarrow \frac{2}{5}\int \frac{V\,dv}{V-1} + \frac{1}{5}\int \frac{dv}{V-1} = x + c$

$\Rightarrow \frac{2}{5}V + \frac{2}{5}\log|V-1| = x + c$

$\Rightarrow 2(x+3y) + 2\log|x+3y-1| = 5x+c$

$\Rightarrow 6y + 2\log|x+3y-1| = 3x+c$

Que $\frac{dy}{dx} = \frac{y+3}{x+y+2}$

Put $x = X+h$, $y = Y+k$

$\frac{dY}{dX} = \frac{Y+k+3}{X+h+Y+k+2}$

So $k+3=0$, $h+k+2=0$

$k=-3$; $h=1$

$\frac{dy}{dx} = \frac{Y}{X+Y}$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x + vx}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v} - v$$

$$= \frac{v - v - v^2}{1+v}$$

$$= \frac{-v^2}{1+v}$$

$$\Rightarrow \int \frac{1+v}{v^2}\,dv = -\int \frac{dx}{x} + \log c$$

$$\Rightarrow \int \frac{1}{v^2}\,dv + \int \frac{1}{v}\,dv = -\log|x| + \log c$$

$$\Rightarrow \frac{v^{-2+1}}{-2+1} + \log|v| + \log|x| = \log c$$

$$\Rightarrow -\frac{1}{v} + \log|vx| = \log c$$

$$\Rightarrow -\frac{x}{y} + \log|y| = \log c$$

$$\Rightarrow -\frac{x-1}{y+3} + \log(y+3) = \log c$$

H.W. (1) $(2x+y-1)\,dy + (4x+2y-3)\,dx = 0$

(2) $(y-x+1)\,dy - (y-x+2)\,dx = 0$

(3) $x^2 dy - xy\,dx + y^2 e^{x^2/y^2} dy = 0.$