University School of Automation and Robotics
GURU GOBIND SINGH INDRAPRASTHA UNIVERSITY
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USAR
UNIVERSITY SCHOOL OF AUTOMATION AND ROBOTICS

Engineering Mechanics

By: Dr. Divya Agarwal

- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates

- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.

- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.

- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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INTRODUCTION TO DYNAMICS

- Statics - bodies at rest.
- Dynamics is a part of mechanics that deals with the analysis of bodies in motion.
- For convenience, **dynamics** is divided into two branches called **kinematics** and **kinetics**.
- **Kinematics** is the study of the relationship between displacement, velocity, acceleration and time of a given motion without considering the forces that cause the motion.
- **Kinetics** is the study of the relationship between the forces acting on a body, the mass of the body and the motion of the body. It can be used to predict the motion caused by a given force or to determine the forces required to produce a prescribed motion.
- Dynamics is based on natural laws governing motion of a particle.
- Particle is a convenient idealization of the physical objects which need not be small in size, where, mass of the body is assumed to be concentrated at a point and the motion of the body is considered as a motion of an entire unit neglecting any rotation about its own mass centre.

TYPES OF MOTION

- A body is said to be in motion if it changes its position with respect to its surroundings.
- The nature of path of displacement of various particles of a body determines the type of motion.
- When a particle moves in space it describes a curve, called path. This path can be straight or curved.
- The motion may be of the following types :

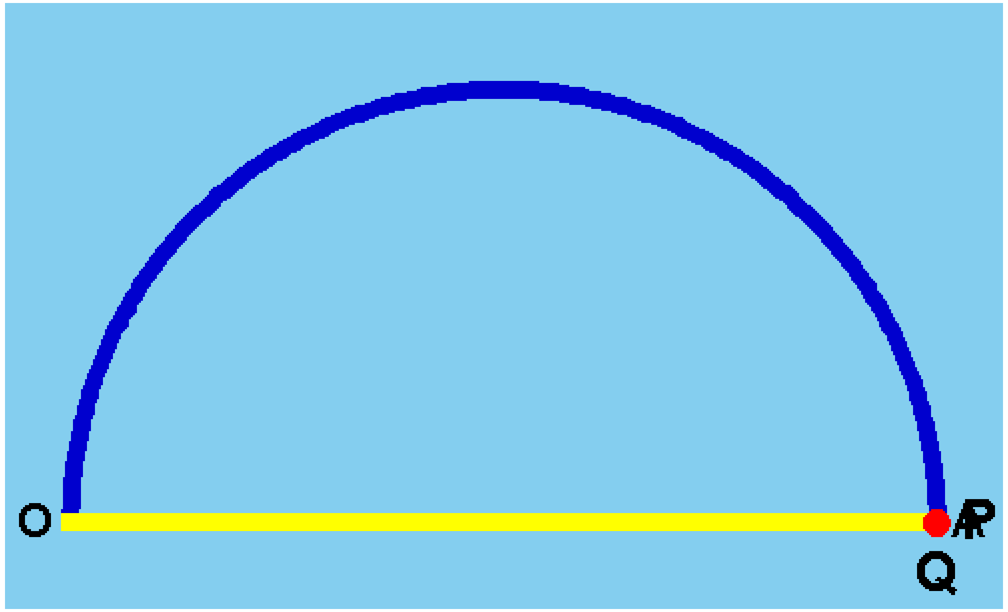
1. **Rectilinear motion**. When a particle moves along a path which is a straight line it is called the rectilinear motion (or, *straight line motion*). *Rectilinear means forming straight lines and translation means behaviour*. Rectilinear translation will mean behaviour by which straight lines are formed. Thus, when a body moves such that its particles form parallel straight paths the body is said to have rectilinear translation.

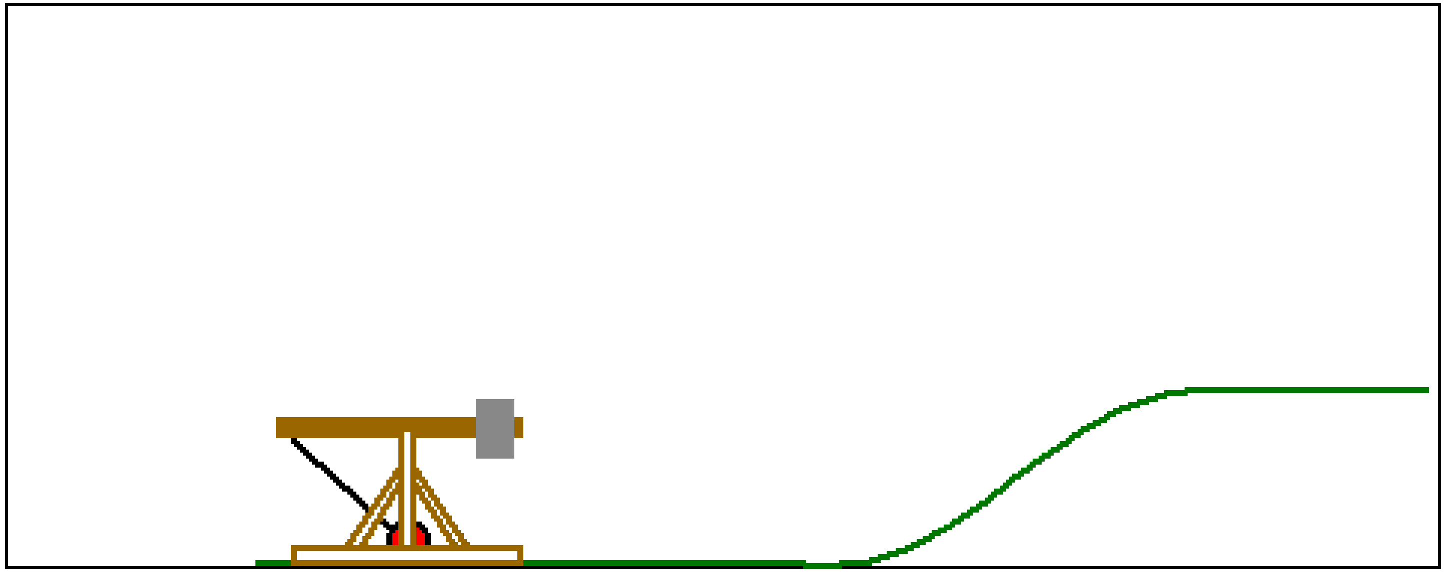
2. **Curvilinear motion.** When a particle moves along *circular arcs or* curved paths it is called curvilinear motion. If the curved path lies in a plane it is called plane curvilinear motion.

3. ***Rotary or circular motion*** is a special case of curvilinear motion where particles of a body move along *concentric circles* and the *displacement* is *measured in terms of angle in radians or revolutions*.

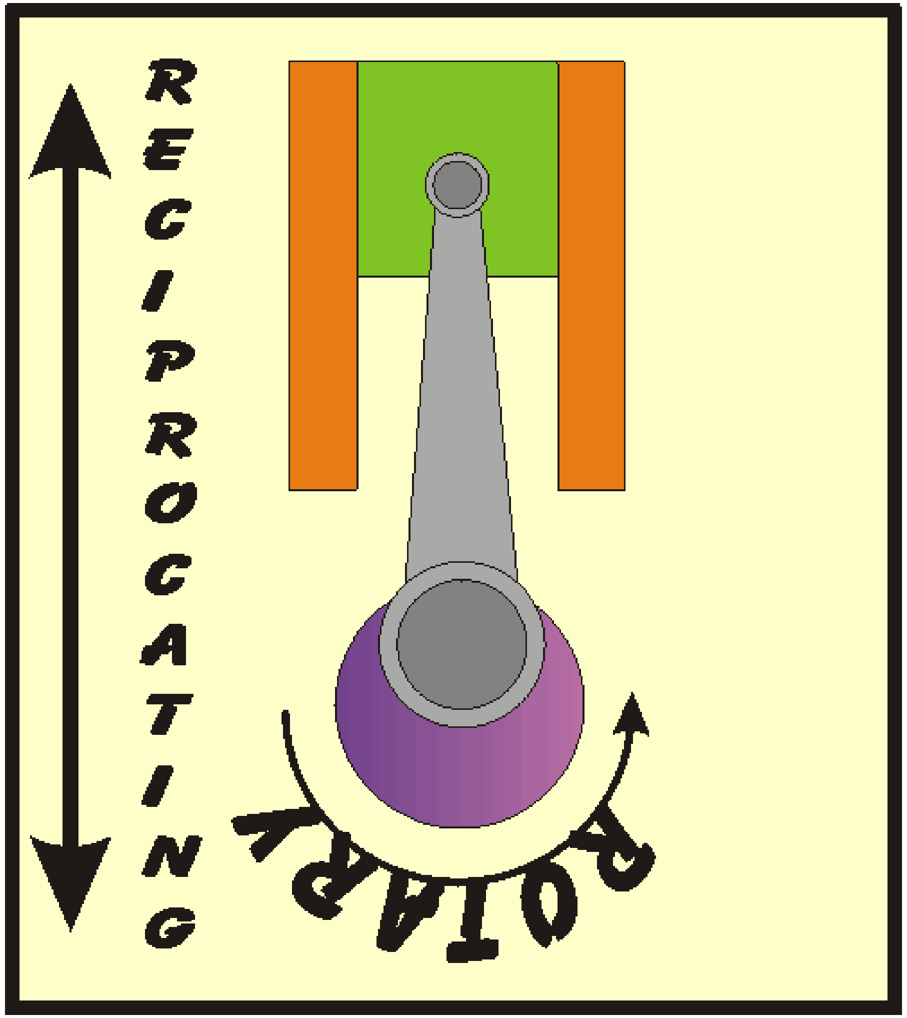
EXAMPLES OF RECTILINEAR MOTION

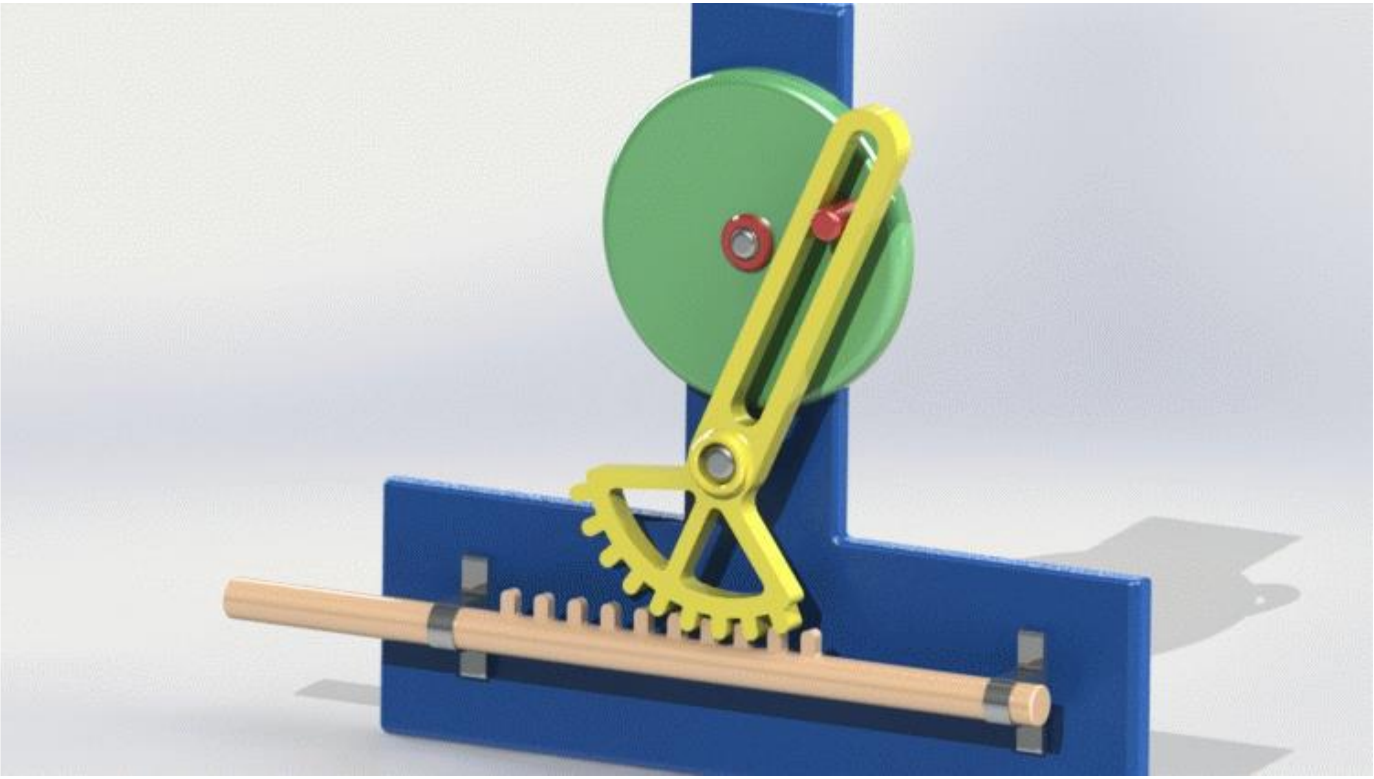
EXAMPLES OF CURVILINEAR MOTION





EXAMPLES OF ROTARY MOTION

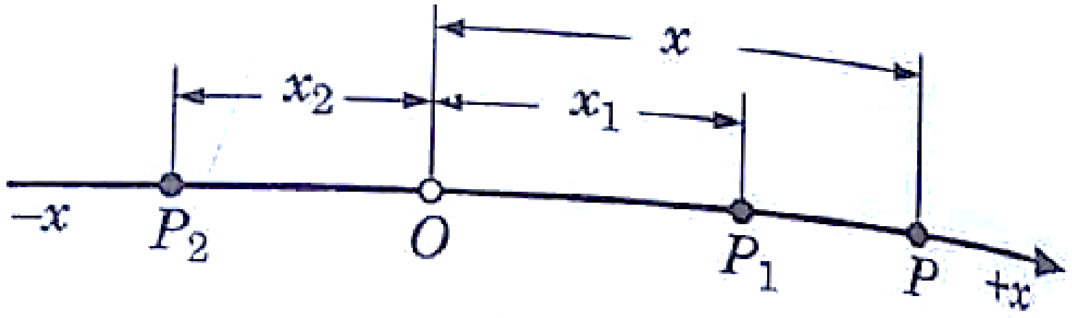




RECTILINEAR MOTION: DEFINITIONS

1. **Rest and motion.** A body is said to be at *rest* at an instant (means a small interval of time) if its position with respect to the surrounding objects remains unchanged during that instant. A body is said to be in *motion* at an instant if it changes its position with respect to its surrounding objects during that instant. Actually, nothing is absolutely at rest or absolutely in motion : *all rest or all motion is relative only*.

2. **Displacement.** If a particle has rectilinear motion with respect to some point which is assumed to be fixed, its displacement is its total change of position during any interval of time. The unit of displacement is same as that of distance or length. In M.K.S. or S.I. system it is *one metre*. The displacement is assumed to be positive to the right of the origin and negative to the left.



3. **Distance travelled**. Distance travelled by a particle, is different than its displacement from the origin. For example if a particle moves from O to position P_1 and then to position P_2, its displacement at the position P_2 is $-x_2$ from the origin but the distance travelled by the particle is $2x_1 + x_2$.

RECTILINEAR MOTION: DEFINITIONS

4. **Speed.** Defined as *its rate of change of its position with respect to its surroundings irrespective of direction*. It is a *scalar quantity*. It is measured by distance covered per unit time. (Speed = Distance covered / Time taken)
5. **Velocity.** Velocity of a body is *its rate of change of its position with respect to its surroundings*. It is a *vector quantity*. It is measured by the distance covered per unit time. *i.e.*, Velocity = Distance covered/Time taken
6. **Uniform velocity.** *If a body travels equal distances in equal intervals of time in the same direction it is said to be moving with a uniform or constant velocity.*
7. **Variable velocity.** *If a body travels unequal distances in equal intervals of time, in the same direction, then it is said to be moving with a variable velocity.*
8. **Average velocity.** *The average or mean velocity of a body is the velocity with which the distance travelled by the body in the same interval of time, is the same as that with the variable velocity.*

   Let the position P be occupied by a particle at any time t be at a distance x from the origin O. Let its position p' at time (t + Δt) be at a distance (x + Δ x) from O. Average velocity of the particle over the time in travel Δ t,

$$v_{average} = \frac{\Delta x}{\Delta t}$$

RECTILINEAR MOTION: DEFINITIONS

9. **Instantaneous velocity.** It is the velocity at a particular instant of time. It can be obtained from the average velocity by choosing the time interval Δt and the displacement Δx very small. The velocity v is positive if the displacement x is increasing and the particle is moving in the positive direction. The unit of velocity is metre per second (m/s). Instantaneous velocity, $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

10. **Acceleration.** The *rate of change of velocity of a body is called its acceleration*. When the velocity is increasing the acceleration is reckoned as *positive*, when decreasing as *negative*. It is represented by *a* or *f*.
If *u* = initial velocity of a body in m/sec, *v* = final velocity of the body in m/sec, *t* = time interval in seconds, during which the change has occurred, Then acceleration, ***a* = (*v* – *u*)/ *t*.**
From above, it is obvious that if *velocity of the body remains constant, its acceleration will be zero.*

11. **Uniform acceleration.** *If the velocity of a body changes by equal amounts in equal intervals of time, the body is said to move with uniform acceleration.*

12. **Variable acceleration.** *If the velocity of a body changes by unequal amount in equal intervals of time, the body is said to move with variable acceleration.*

RECTILINEAR MOTION: DEFINITIONS

13. **Average Acceleration.** Let v be the velocity of the particle at any time t. If the velocity becomes (v + Δv) at a later time (t + Δt) then $Average\ acceleration = \frac{\Delta y}{\Delta t}$

14. **Instantaneous acceleration**. It is the acceleration of a particle at a particular instant of time and can be calculated by using the time interval Δt and the velocity Δv very very small. $\text{Acceleration, a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

- Acceleration is positive if the velocity is increasing. A positive value of acceleration means that the particle is either moving further in the positive direction or slowing down in the negative direction. The unit of acceleration is metre per second square (m/s^2).

$$a = \frac{dv}{dt}, \text{ as } v = \frac{dx}{dt}, \qquad \text{So, } a = \frac{d^2x}{dt^2},$$

$$\text{Also, } a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} \qquad \text{as, } \frac{dx}{dt} = v$$

$$\text{So,} \quad a = v\frac{dv}{dx}$$

RECTILINEAR MOTION: DEFINITIONS

15. **Uniform motion.** *If the velocity of a body changes by equal amounts in equal intervals of time, the body is said to move with uniform acceleration.* Or A particle is said to have a uniform motion when its acceleration is zero and its velocity is constant.
16. **Variable acceleration.** *If the velocity of a body changes by unequal amount in equal intervals of time, the body is said to move with variable acceleration.*
17. **Uniformly accelerated motion.** A particle moving with a constant acceleration is said to be in uniformly accelerated motion.

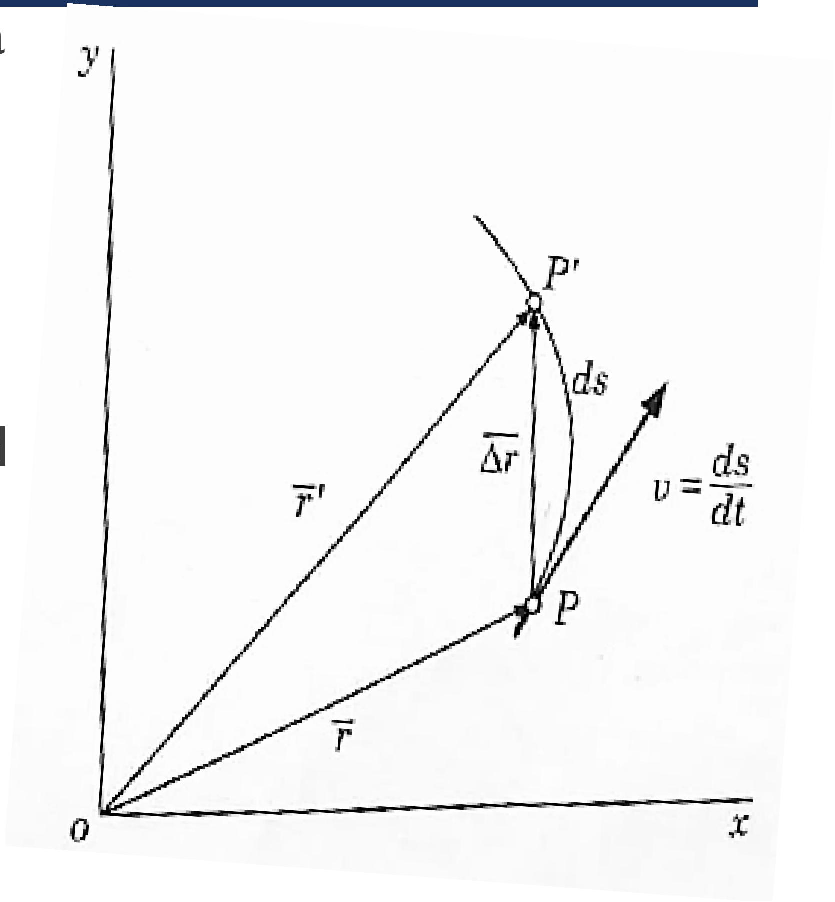
It may be mentioned that velocity and acceleration of a particle are vector quantities processing both the magnitude and direction but as we are presently dealing with the motion along a straight line we can, therefore, specify these quantities by algebraic number with plus or minus signs.

CURVILINEAR MOTION OF A PARTICLE

- When a moving particle describes a curved path it is said to have a curvilinear motion.
- If the curved path lies in a plane it is termed as plane curvilinear motion.
- When the direction of the force acting on a particle varies or when the particle has some initial motion in a direction that does not coincide with the direction of the force acting on the particle, the particle moves in a curved path.
- Curvilinear motion can cause changes in both the magnitude and direction of the position, velocity, and acceleration vectors
- For example, an object when thrown horizontally, with some initial velocity moves in a curved parabolic path, because the force of gravity acting on the object does not coincide with the initial velocity of the object and the object moves in a curved path.

POSITION VECTOR, VELOCITY AND ACCELERATION

- **Position vector.** To define the position of a particle moving along a curved path at any instant, we need to know its coordinates along the x axis as well as the y-axis. (use the concept of the position vector)
- Consider motion of a particle along a curved path as shown.
- To define the motion of the particle P at any instant t choose a fixed reference axis x-y. Join point P to O.
- Line OP is called the position vector r of point P.
- Since vector r is characterized by its magnitude r and direction with reference to the axis, it completely defines position of particle with respect to the axes at any time t.
- Consider now the position P' of the particle, at a later time (t+ Δt) as defined by the position r'.

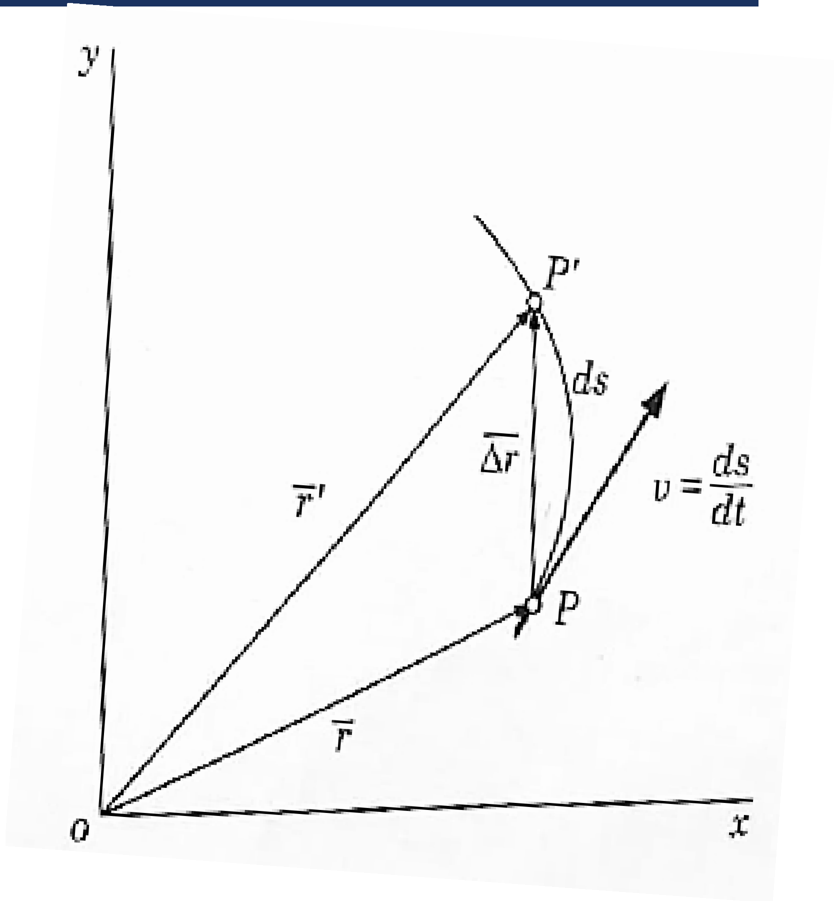


POSITION VECTOR, VELOCITY AND ACCELERATION

- Vector Δr joining P and P' represents change in the position vector during time interval Δt.
- i.e., r+Δr = r' or Δr = r' – r (Applying triangle law)
- Δr thus represents a change in the magnitude as well as the change in the direction of the position vector r.
- **Velocity**. Instantaneous velocity of the particle can be defined as,

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

- As Δt and Δr become smaller, the points P and P' get closer and the vector v obtained at the limit becomes tangent to the part at P.



POSITION VECTOR, VELOCITY AND ACCELERATION

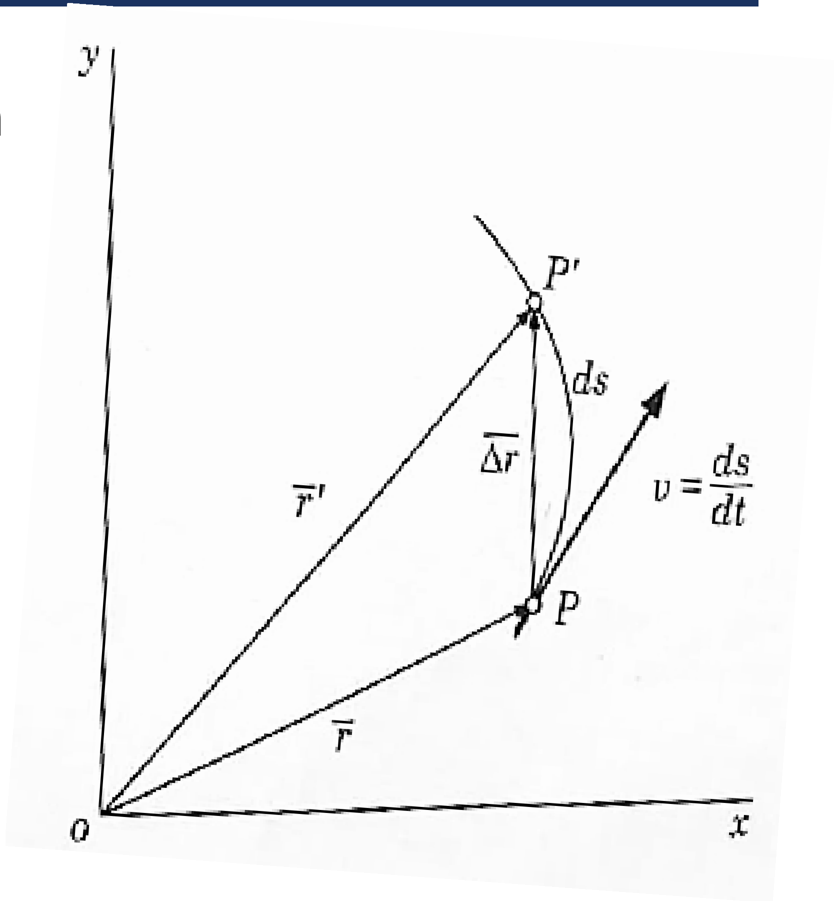
- The magnitude of Δr is given by the length of the line segment PP'. But as Δt approaches zero, length of PP' approaches length Δs of the arc PP'.

- The magnitude of the velocity v, called speed is thus obtained as,

$$v = \lim_{\Delta t \to 0} \frac{PP'}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- The speed of the particle can thus be obtained by differentiating with respect to the time the length of the arc described by the particle.

$$v = \frac{ds}{dt}$$



COMPONENTS OF MOTION: RECTANGULAR COMPONENTS OF VELOCITY

- As the direction of the velocity of a particle in curvilinear motion changes continuously so it is convenient to deal with its velocity v_x and v_y.
- To obtain the values of v_x and v_y let us resolve Δr into components $\overline{PQ}$ and $\overline{QP'}$ parallel to the x and y axis as shown in figure.

$$\Delta r = \overline{PQ} + \overline{QP'}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\overline{PQ}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\overline{QP'}}{\Delta t}$$

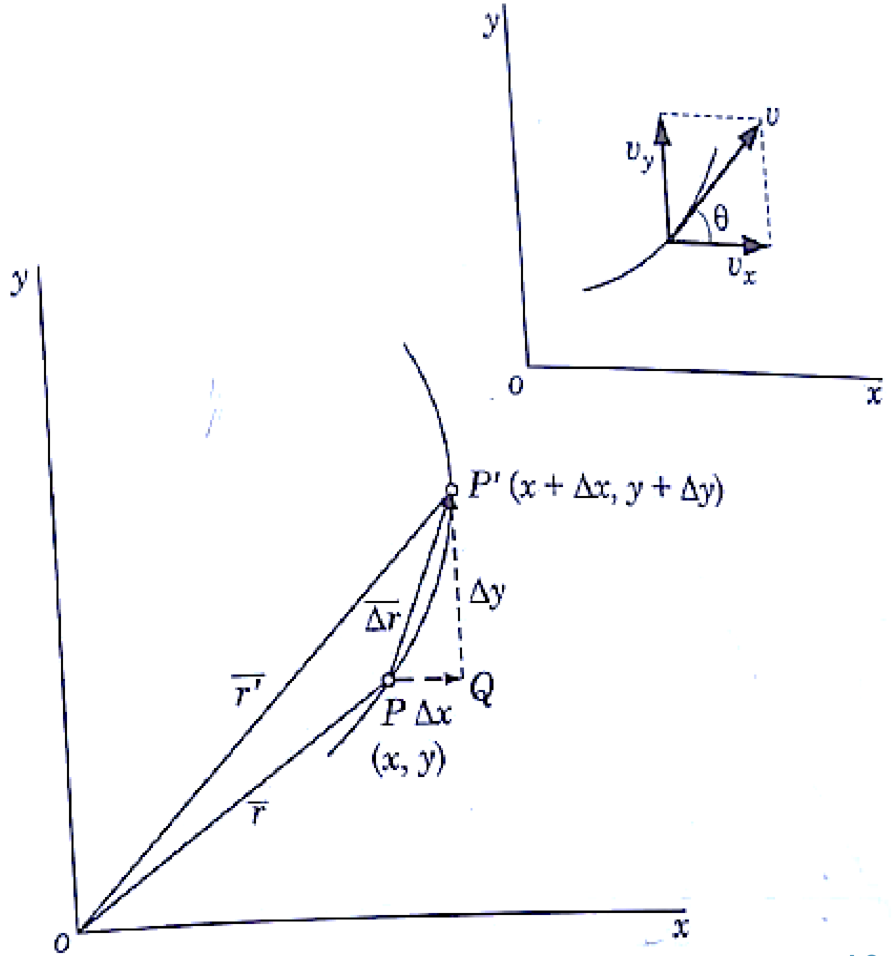


Fig. 15.2

COMPONENTS OF MOTION: RECTANGULAR COMPONENTS OF VELOCITY

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$$v = v_x + v_y \qquad \text{(Vector Sum)}$$

$$\frac{dx}{dt} = v_x = x, \frac{dy}{dt} = v_y = y$$

- Magnitude

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

- Direction

$$\theta = \tan^{-1} \frac{v_y}{v_x}$$

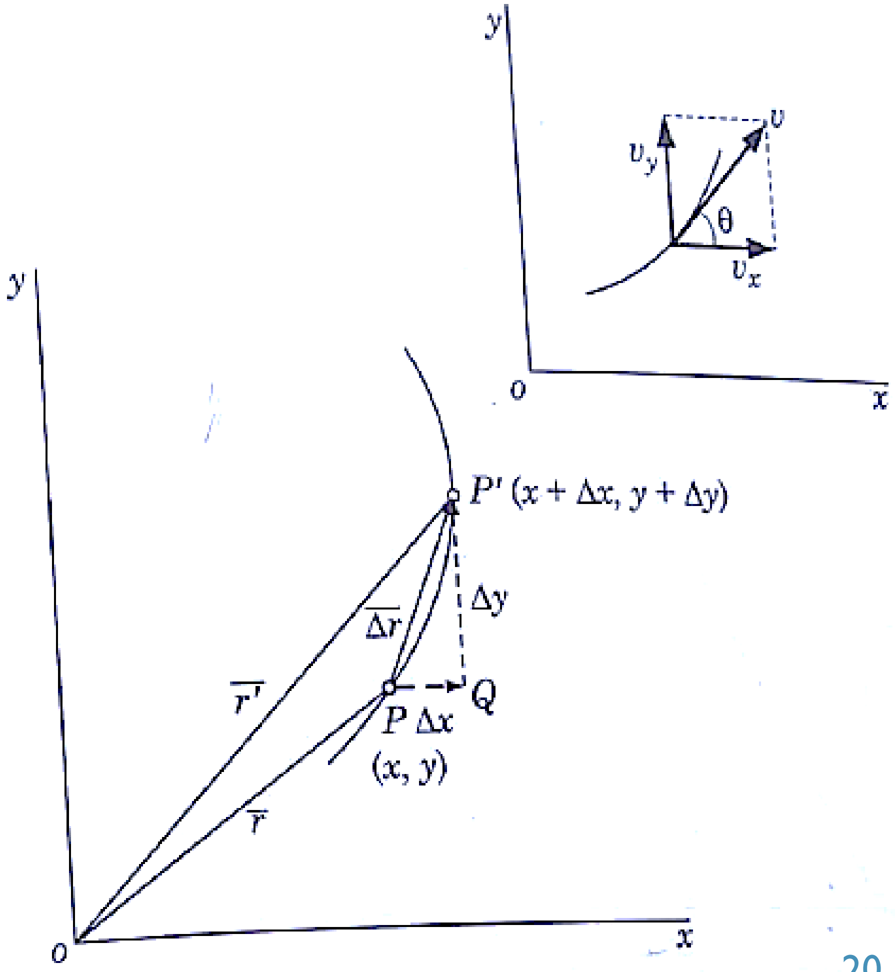


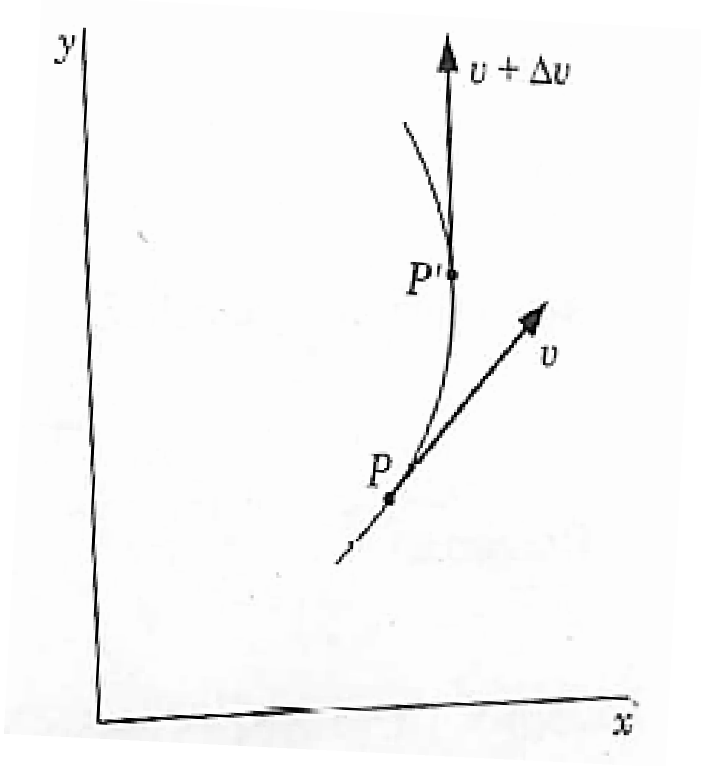
Fig. 15.2

COMPONENTS OF MOTION: RECTANGULAR COMPONENTS OF ACCELERATION

- Consider a particle at position P having a velocity v at any time t.
- After a time Δt its position be defined by P' and its velocity by v + Δv.
- Instantaneous acceleration of the particle is

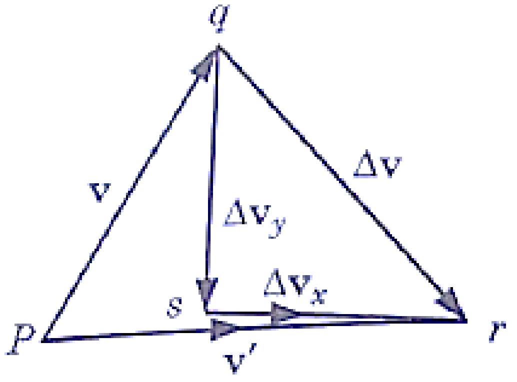
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- In general, the acceleration of the particle at any instant is not tangential to the path of the particle. i.e., direction of acceleration and velocity may not be the same in a curvilinear motion.



COMPONENTS OF MOTION: RECTANGULAR COMPONENTS OF ACCELERATION

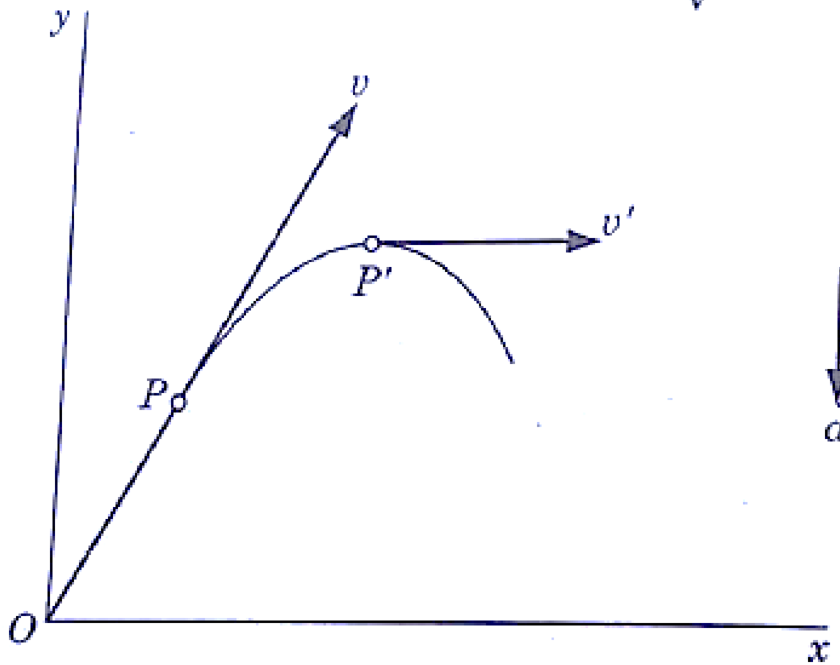
- Let acceleration of the particle be denoted by its rectangular components a_x and a_y parallel to x and y axes respectively.
- Vectors $\overline{pq}$ and $\overline{pr}$ represent velocities v and v' and vector $\overline{qr}$ represents change in velocity Δv of particle.
- Resolving Δv into components Δv_x and Δv_y we can write,

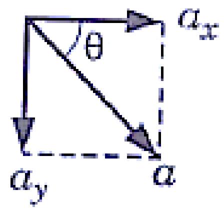


$$\Delta \mathrm{v} = \overline{qr} = \overline{qs} + \overline{sr} \qquad \text{(Vector Sum)}$$

$$a = \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\overline{\mathrm{qs}}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\overline{\mathrm{sr}}}{\Delta t}$$

$$a = \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$





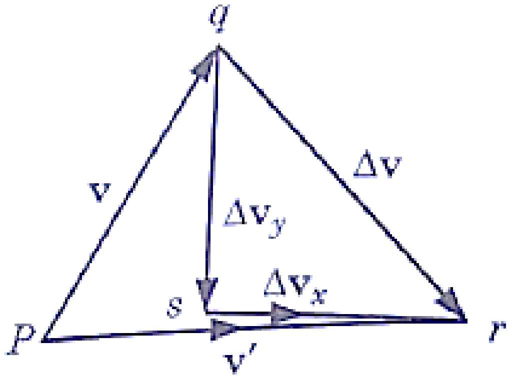
COMPONENTS OF MOTION: RECTANGULAR COMPONENTS OF ACCELERATION

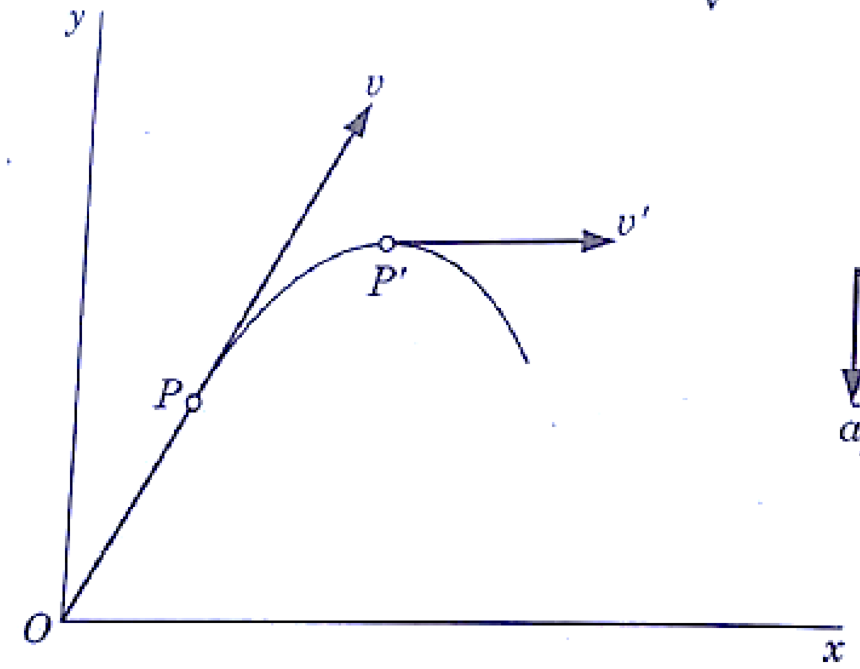
- But, $a_y = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v_y}{\Delta t}$ and $a_x = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v_x}{\Delta t}$
- So, $a = a_x + a_y$ (Vector Sum)
- Magnitude

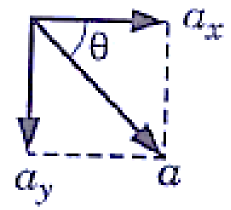
$$a = \sqrt{{a_x}^2 + {a_y}^2}$$

- Direction

$$\theta = \tan^{-1} \frac{a_y}{a_x}$$

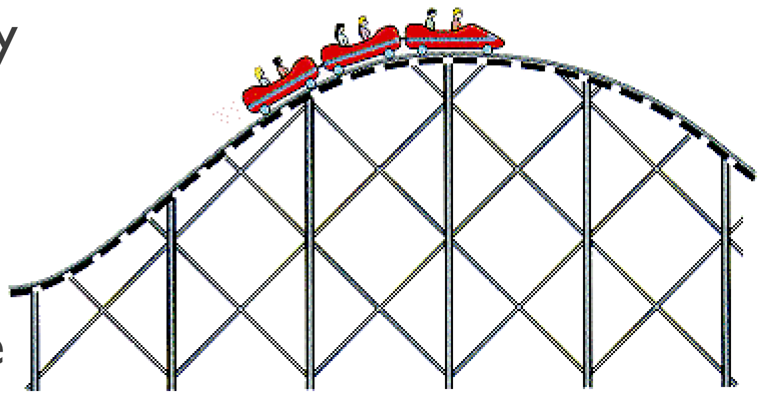


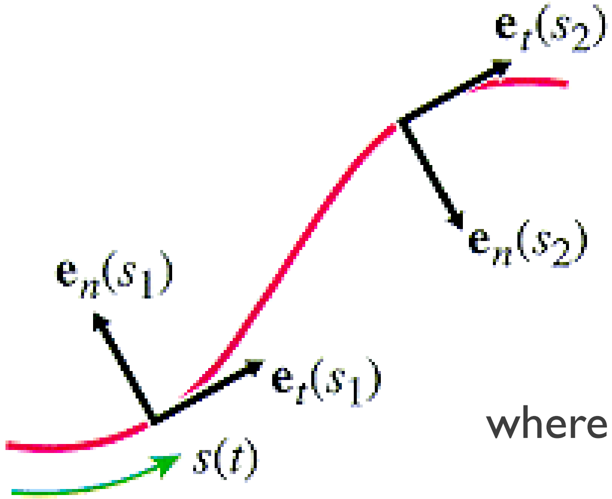




NORMAL/TANGENTIAL COORDINATES

- Sometimes it is useful to describe motion by giving path that a particle is moving along and the speed of the particle at each point along the path.
- This coordinate system is convenient to use when particles move along a surface of known shape.
- Coordinates are chosen at each point along the part such that:
- $\mathbf{e}_t$ is a unit vector tangential to path and pointing in direction of motion, and $\mathbf{e}_n$ is a unit vector normal to path and pointing toward center of curvature.

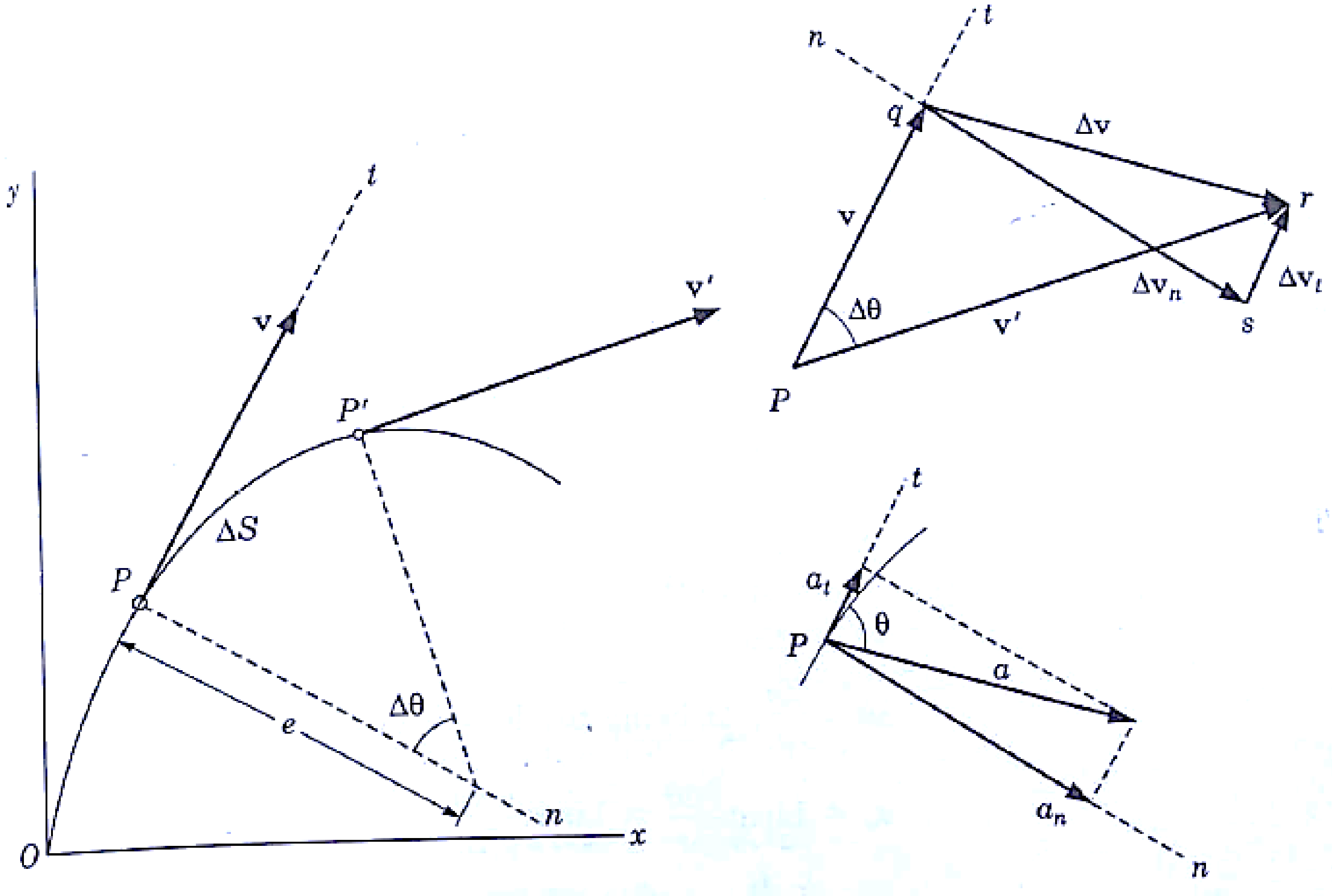




where s is the position along the path.

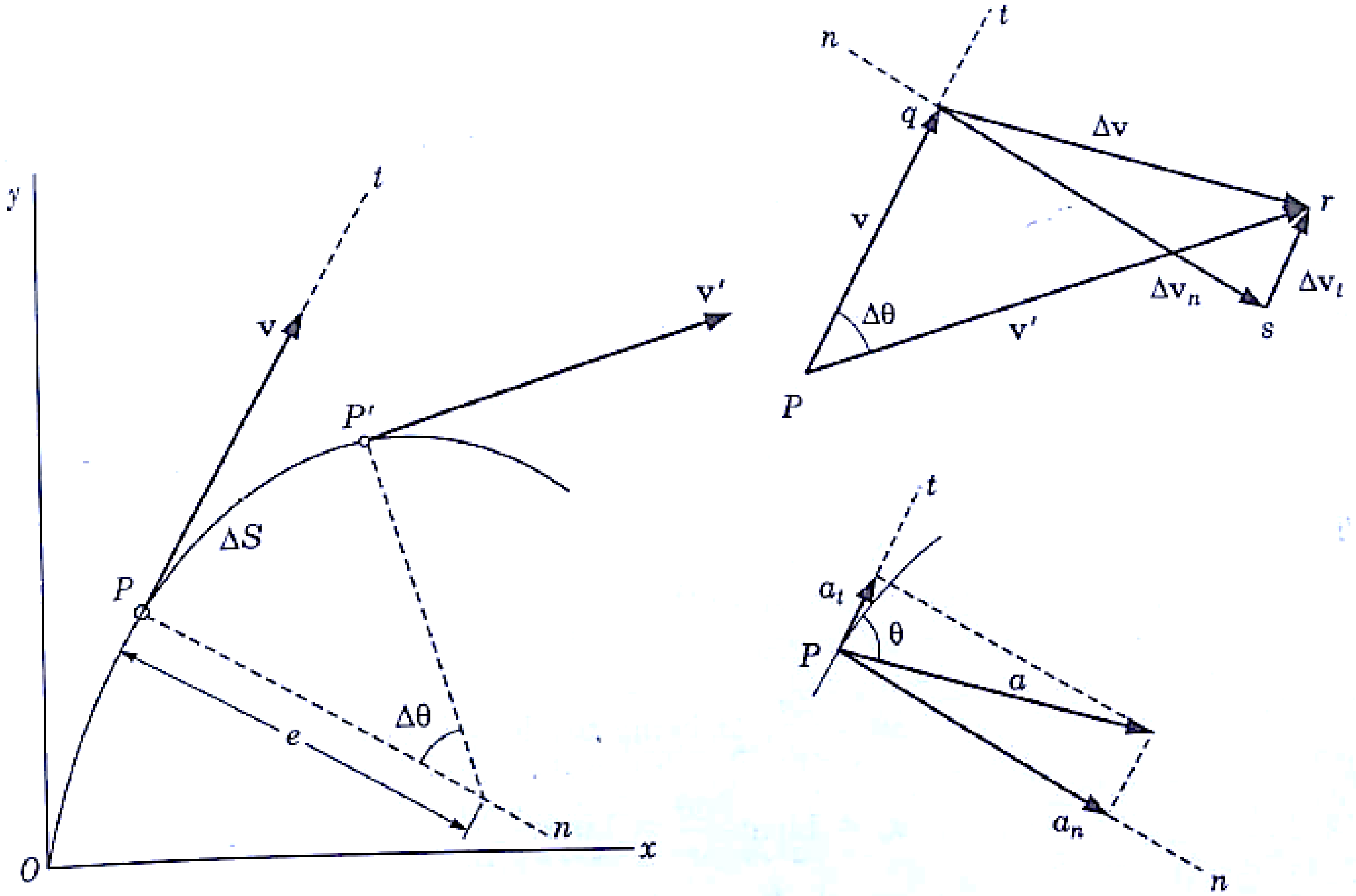
COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

- In case of a particle moving in a curvilinear path we observed that velocity of particle is a vector tangential to the path at any instant.
- But in general, acceleration need not be tangential to the path.
- Thus, sometimes we express acceleration of particle in component form, i.e., one in direction of tangent to the path and other in direction of normal to path.
- These components are called tangential acceleration (a_t) and normal acceleration (a_n) of the particle.



COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

- Consider a particle having velocity v at a time t and a velocity v' at a later time t + Δt.
- To obtain change in velocity of the particle draw $\overline{pq}$ and $\overline{pr}$ representing v and v' respectively.
- Closing side $\overline{qr}$ of triangle represents change in velocity Δv in time Δt.
- Resolve Δv into components Δv_t and Δv_n, along tangent and normal to the path at P.
- The axis in these directions can be denoted by t and n.



COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

$$\Delta \mathrm{v} = \overline{qr} = \overline{qs} + \overline{sr}$$

$$\Delta v = \Delta v_n + \Delta v_t \qquad \text{(Vector Sum)}$$

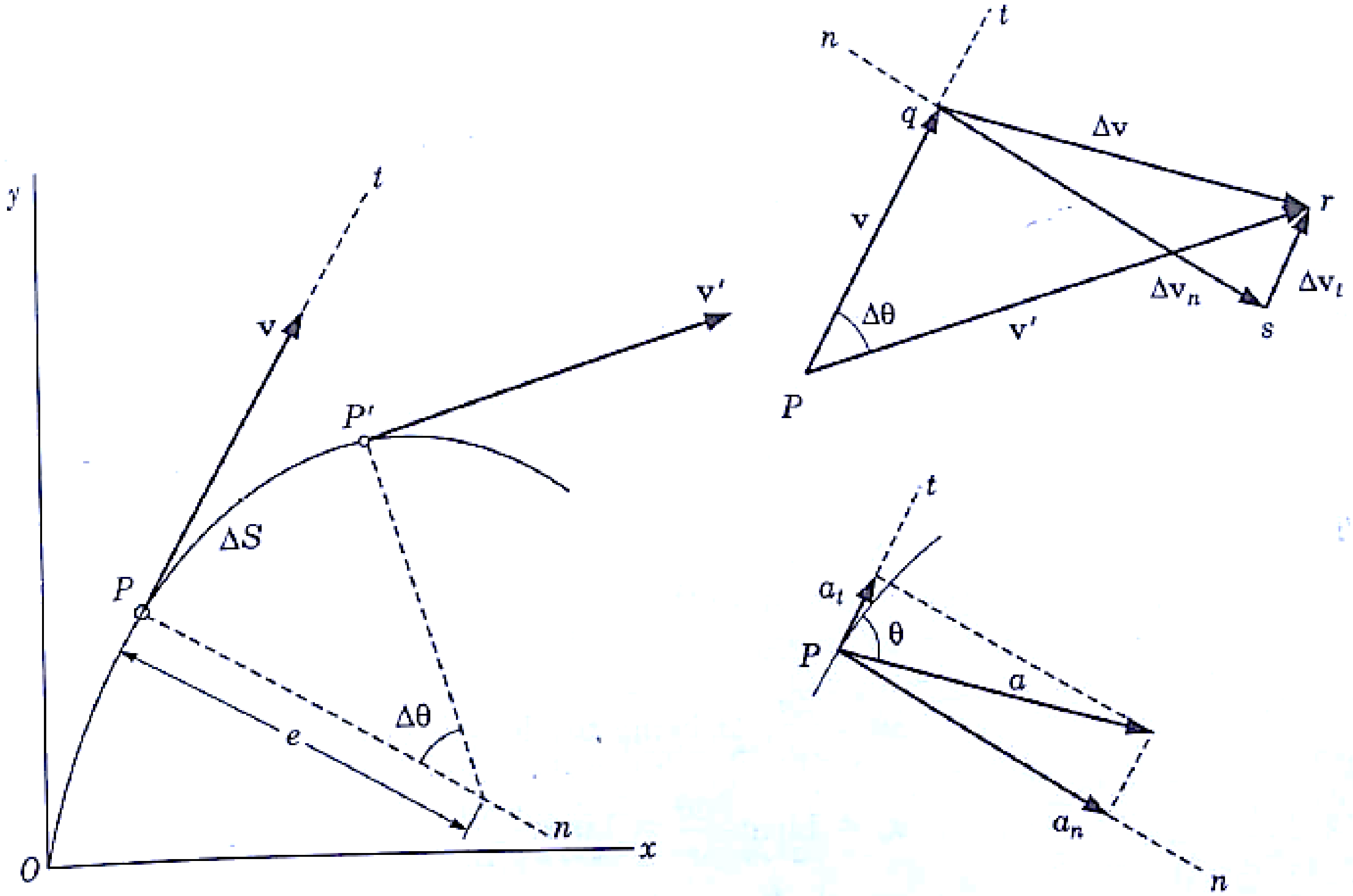
- Acceleration, $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v_t}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta v_n}{\Delta t}$$

- But, $a_t = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v_t}{\Delta t}$ and

$$a_n = \lim_{\Delta t \to 0} \frac{\Delta v_n}{\Delta t}$$

- So, $a = a_t + a_n$ (Vector Sum)



COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

- It may be observed that as Δt approaches zero, point P' coincides with point P and direction of a_n and a_t coincide with direction of tangent and normal to the path at P.
- **Tangential component (a_t).**
- It can be noted that $\bar{sr}$ represents change in magnitude of velocity v, therefore,

$$a_t = \lim_{\Delta t \to 0} \frac{v' - v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- Thus, tangential component of acceleration is equal to rate of change of the speed of the particle.
- Tangential acceleration (a_t) is considered to be positive in the direction of the tangent coinciding with the sense of the motion.

COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

- **Normal acceleration a_n.**
- It may be observed that $\overline{qs}$ represents a change in direction of the velocity.

$$qs = v\Delta\theta$$ (For a small change in angle θ)

$$\Delta v_n = v\Delta\theta$$

- Therefore,

$$a_n = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t}$$

- If ρ is the radius of curvature of the curve then,

$$\Delta s = \rho\Delta\theta$$

$$\Delta\theta = \frac{\Delta s}{\rho},$$ Δs being the length of the arc PP'

$$a_n = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{\rho}\frac{\Delta s}{\Delta t} = \frac{v}{\rho}\frac{ds}{dt}$$ (but, $\frac{ds}{dt} = v$)

COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

- So,

$$a_n = \frac{v^2}{\rho}$$

- Normal acceleration an of a particle at a point is equal to square of its speed divided by radius of curvature of the path at that point.
- The direction of normal acceleration is such that it is always directed towards the centre of the curvature of the path.
- This normal acceleration is also called centripetal acceleration (i.e, centre seeking acceleration)
- It should be noted that total acceleration a of the particle is a vector.
- This acceleration may be caused due to change in magnitude of velocity or change in direction of the velocity or both.

COMPONENTS OF ACCELERATION: NORMAL AND TANGENTIAL

$$\mathrm{a} = a_t + a_n \qquad \text{(Vector Sum)}$$

$$a_t = \frac{dv}{dt}, \qquad and \qquad a_n = \frac{v^2}{\rho}$$

- Magnitude

$$a = \sqrt{{a_t}^2 + {a_n}^2}$$

- Direction

$$\theta = \tan^{-1}\frac{a_n}{a_t}$$

EXAMPLE

- Consider motion of a particle along a circular path of radius r with a constant speed v.

$$a_t = \frac{dv}{dt} = 0 \qquad \text{(as v is constant)}$$

$$a_n = \frac{v^2}{\rho} = \frac{v^2}{r} \qquad (As, \rho = r)$$

- Magnitude

$$a = \sqrt{a_t{}^2 + a_n{}^2} = a_n$$

- Direction

$$\theta = \tan^{-1}\frac{a_n}{a_t} = \tan^{-1}\frac{\frac{v^2}{r}}{0} = 90°$$

- Total acceleration **a** of particle is equal to its normal acceleration a_n and acts in direction as a_n.