Math Assignment-1

Linear Algebra & Numerical Methods

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110T-B1

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Q Find two different basis of R^3 for which contain the vectors $(0, 1, -1)$ and $(2, 1, -3)$ of R^3.

Ans) Firstly checking for linear dependency of these two basis

$$a(0, 1, -1) + b(2, 1, -3) = (0, 0, 0)$$

$2b = 0$

$a + b = 0$ $\Rightarrow a = 0$, $b = 0$ which means the vectors are linearly independent

$-a - 3b = 0$

Basis 1: let us consider a vector $(1, 0, 0)$

Now $a(0, 1, -1) + b(2, 1, -3) + c(1, 0, 0) = (0, 0, 0)$

$$\left.\begin{array}{l} 2b + c = 0 \\ a + b = 0 \\ -a - 3b = 0 \end{array}\right\} \Rightarrow a = b = c = 0$$

We get $a = b = c = 0$ which means the vector $(1, 0, 0)$ is a basis of R^3 including given two vector

Basis 2: let us a vector $(1, 2, 0)$

Now $a(0, 1, -1) + b(2, 1, -3) + c(1, 2, 0) = 0$

$$\left.\begin{array}{l} 2b + c = 0 \Rightarrow c = -2b \\ a + b + 2c = 0 \Rightarrow -3b + b + 2b = 0 \\ -a - 3b = 0 \Rightarrow a = -3b \end{array}\right\} a = 0, b = 0, c = 0$$

Since we have $a = b = c = 0$ which mean these vectors are linear independent & $(1, 2, 0)$ is a basis of $R3$ including given 2 vector

Hence $(1, 0, 0)$ & $(1, 2, 0)$ are 2 reqd basis of $R3$.

Ans 2 Let $T: R^2 \to R^3$ be linear Transformation defined by $T(x,y) = (x+y, x-y, y)$. find ker (T) and Range (T).

Ans) Since $\{(1,0), (0,1)\}$ is a standard basis of $R^2(R)$

So by definition.

$\{T(1,0), T(0,1)\}$ is a basis of $R(T)$

$T(1,0) = (1,1,0)$ { Given

$T(0,1) = (1,-1,1)$ $T(x,y) = (x+y, x-y, y)$)

Range $(T) = [(1,1,0), (1,-1,1)]$

In general

Range $(T) = \{x(1,1,0) + y(1,-1,1) : x, y \in R\}$

Ker(T) by definition

$\ker(T) = \{\alpha \in (R^2(R)) : T(\alpha) = (0,0)\}$

Consider $\alpha = (x,y) \in K(T)$

Now $T(x,y) = (0,0,0)$

$(x_1+y_1, x_1-y_1, y_1) = (0,0,0)$

On comparing

$$\left.\begin{aligned} x_1+y_1 &= 0 \\ x_1-y_1 &= 0 \\ y_1 &= 0 \end{aligned}\right] \quad x_1 = 0, y_1 = 0$$

$\ker(T) = \{(0,0)\}$ Ans

Assignment-2

① Find Eigen values & basis of corresponding eigen spaces of the matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

Soln let a variable matrix be X such that

$$A\vec{X} = \lambda \vec{X} \Rightarrow (A - \lambda I)\vec{X} = 0$$

characteristic eqn $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\,[(1-\lambda)(4-\lambda) + 2] = 0$$

$$(2-\lambda)\,[4-\lambda-4\lambda+\lambda^2+2] = 0$$

$$(2-\lambda)\,[\lambda^2 - 5\lambda + 6] = 0$$

↳ solving this $\boxed{\lambda_1 = 2}$ $\boxed{\lambda_2 = 3}$

(i) For $\lambda_1 = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}} = \frac{x_2}{\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}} = \frac{x_3}{\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}}$$

$$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{0} = k \text{ (Assume)}$$

$\therefore$ Eigen vector: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(ii) For $\lambda_2 = 3$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{pmatrix} 0 & -2 \\ 0 & +2 \end{pmatrix}} = \frac{x_2}{\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}} = \frac{x_3}{\begin{pmatrix} 0 & -2 \\ 0 & 2 \end{pmatrix}} = k \text{ (assume)}$$

$$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{0} = k$$

$$\text{Eigen vector} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Q2 Apply the Graham Schmidt Process to vectors $(1, 0, 1) \cdot (1, 0, -1)$ $(0, 3, 4)$ to obtain orthogonal basis for $R^3(R)$ with standard inner product

Soln Let $\beta_1 = (1, 0, 1) \cdot$ $\beta_2 = (1, 0, -1)$ $\beta_3 = (0, 3, 4)$

$$\alpha_1 = \frac{\beta_1}{\|\beta_1\|} \Rightarrow \|\beta_1\|^2 = \langle \beta_1 \; \beta_2 \rangle = 2$$

$$\alpha_1 = \frac{(1, 0, 1)}{\sqrt{2}} \Rightarrow \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

Again $\cdot$ $\alpha_2 = \dfrac{\gamma_2}{\|\gamma_2\|}$ $\|\gamma_2\| = 2$

$$\gamma_2 = (1, 0, -1) \cdot$$

$$\|\gamma_2\|^2 = 1^2 + 0^2 + 1^2 = 2$$

$$\alpha_2 = \frac{(1, 0, -1)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$$\alpha_3 = \frac{\gamma_3}{\|\gamma_3\|}$$

$$\gamma_3 = \beta_3 - \langle \beta_3, \alpha_2 \rangle \alpha_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1$$

$$\|\gamma_3\|^2 = 0^2 + 3^2 + 0^2 = 9$$

$$\|\gamma_3\| = 3$$

$$\alpha_3 = \frac{(0, 3, 0)}{\sqrt{9}} = \left(\frac{0}{\sqrt{9}}, \frac{3}{\sqrt{9}}, \frac{0}{\sqrt{9}}\right)$$

$$\alpha_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \qquad \alpha_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$\alpha_3 = (0, 1, 0)$ are reqd orthogonal basis for $R^3(R)$ with std IPS

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2. Let $T: R^2 \to R^3$ be linear transformation defined by $T(x,y) = (x+y, x-y, y)$. Find $\ker(T)$ and Range(T).

Assignment 2.

1. Find the eigen values and bases of the corresponding eigen spaces of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

2. Apply the Gram Schmidt process to the vectors $(1,0,1)$, $(1,0,-1)$, $(0,3,4)$ to obtain an orthonormal basis for $R^3(R)$ with the standard inner product.