UNIT 3

INTERFERENCE

- COHERENT SOURCE:

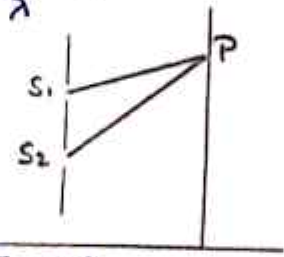
→ Same frequency
→ Same Amplitude
→ Same/constant path phase diff.

→ CONDITION FOR SUSTAINED INTERFERENCE:

- Amplitude should be same
- Same frequency
- Constant phase diff.
- Same wavelength
- Narrow source
- Dist. b/w source should be small

→ PATH DIFF, $d = S_2P \sim S_1P$

→ PHASE DIFF, $\phi = \frac{2\pi}{\lambda} \cdot d$



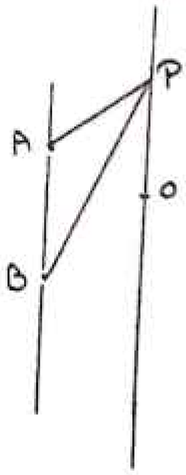
→ ANALYTICAL TREATMENT

- $y_1 = A\sin\omega t$
  $y_2 = A\sin(\omega t + \phi)$

$\boxed{y = y_1 + y_2 = A\sin\omega t(1+\cos\phi) + A\cos\omega t\sin\phi}$ * $y = R\sin(\omega t + \theta)$

- $R^2 = a^2(1+\cos\phi)^2 + a^2\sin^2\phi$
  $= 2a^2(1+\cos\phi)$

$R^2 = 4a^2\cos^2\phi/2 \Rightarrow \boxed{I = 4a^2\cos\phi/2}$



* FOR MAXIMA: $I = 4a^2$

→ PATH DIFF $= 0, \lambda, 2\lambda \ldots \boxed{n\lambda}$
PHASE DIFF $= 0, 2\pi, 4\pi \ldots \boxed{2n\pi}$

* MIN. INTENSITY $\lambda/2, 3\lambda/2 \ldots \boxed{(2n+1)\frac{\lambda}{2}}$

→ PATH DIF = ~~π, 3π ... 2n~~
PHASE DIFF $= \pi, 3\pi \ldots \boxed{(2n+1)\pi}$

→ YOUNGS DOUBLE SLIT EXPERIMENT

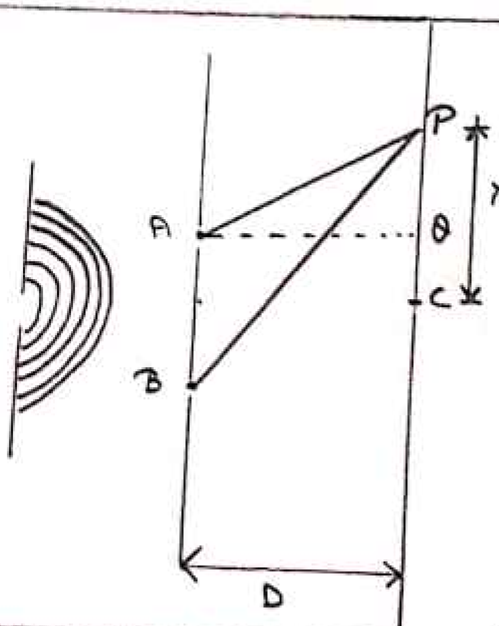
- BRIGHT FRINGE:

$\boxed{x = \frac{n\lambda D}{d}}$

- FRINGE WIDTH, $\boxed{\beta = \frac{\lambda D}{d}}$

- DARK FRINGE:

$\boxed{x = (2n+1)\frac{\lambda D}{2d}}$



→ NEWTON RING

- In $\triangle ABC$, $\boxed{t = \frac{r^2}{2R}}$

BRIGHT
$\Rightarrow \boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$

DARK
$\Rightarrow \boxed{2\mu t\cos r = (n+1)\lambda}$

DARK FRINGE

- $\boxed{r_n = \sqrt{n\lambda R}}$

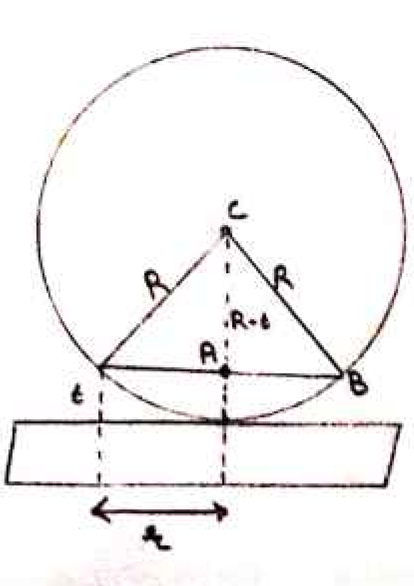
WAVELENGTH

$$\boxed{\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}}$$

BRIGHT FRINGE

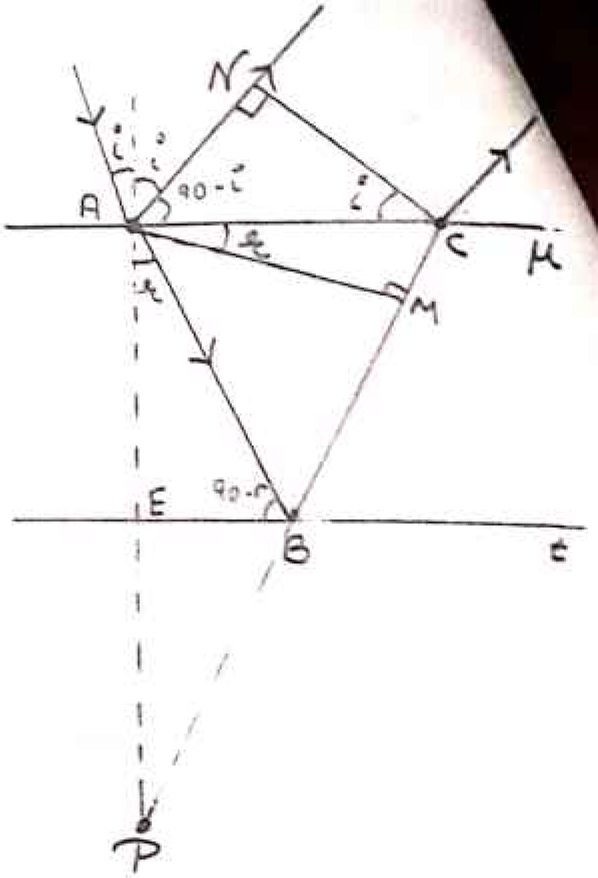
$$\boxed{r_n = \sqrt{(2n-1)\frac{\lambda R}{2}}}$$

$\Rightarrow r_n \propto \sqrt{\lambda} \propto \sqrt{R}$



• DERIVATIONS

1) INTERFERENCE DUE TO REFLECTED LIGHT (THIN FILM):



Let thickness of slab be 't'

Optical Path difference $= \mu(AB+BC) - AN$

Using Snells law, $\mu = \frac{\sin i}{\sin r} = \frac{AN/AC}{CM/AC}$

$\Rightarrow \mu = \frac{AN}{CM}$ — (I)

Now, In $\triangle AEB \cong \triangle PEB$

$\Rightarrow AE = EP$
$AB = BP$

$\Rightarrow x = \{\mu(AB+BC) - AN$
$= \mu(AB+BC-CM)$
$= \mu(PC - CM)$
$= \mu PM$ — (II)

In $\triangle AMP$, $\cos r = \frac{PM}{AP} \Rightarrow PM = AP\cos r$
$= 2AE\cos r$

$PM = 2t\cos r$ — (III)

from (II) & (III),

$$\boxed{x = 2\mu t\cos r}$$

This is Apparent path diff.

According to EMT,
when a ray is reflected from a optically denser medium then there is a path difference of $\lambda/2$.

$\Rightarrow$ Corrected Path diff, $\boxed{x = 2\mu t\cos r \pm \frac{\lambda}{2}}$

• BRIGHT FRINGE

$2\mu t\cos r - \frac{\lambda}{2} = n\lambda$

$\Rightarrow \boxed{2\mu t\cos r = (2n+1)\lambda/2}$

• DARK FRINGE

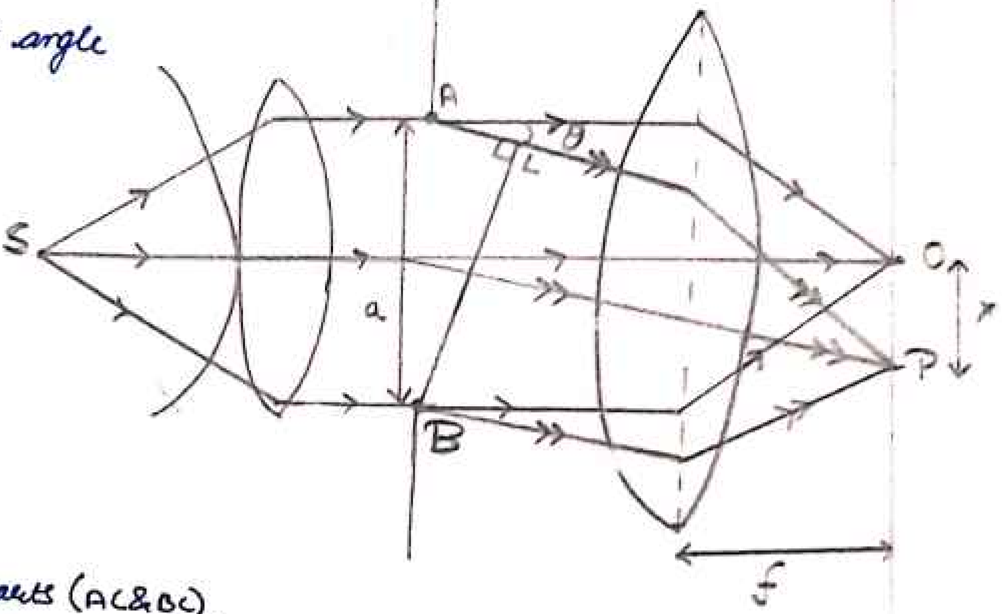
$2\mu t\cos r - \lambda/2 = (2n+1)\lambda/2$

$$\boxed{2\mu t\cos r = (n+1)\frac{\lambda}{2}}$$

FRAUNHOFFER DIFFRACTION FOR SINGLE SLIT:

Central Maxima is concentrated at O,
Since there is no path difference, Maxima is Bright Fringe.

If secondary waves travel at angle θ, they focus at point P.



Path difference, λ = AL

In $\triangle$ABL,

$$\sin\theta = \frac{AL}{AB} \Rightarrow AL = a\sin\theta$$

$$\Rightarrow \boxed{\lambda = a\sin\theta}$$

If slit AB is divided in 2 parts (AC & BC),
AC & BC each generate $\lambda/2$ path difference.
So they form destructive Interference

$$\Rightarrow a\sin\theta = \lambda \rightarrow \text{SECONDARY MINIMA}$$

If $a\sin\theta = 2\lambda$, slit is divided into 4 equal parts of $\lambda/2$.

Similarly $a\sin\theta = n\lambda$, then it forms minima.

FOR FIRST MINIMA,

$$a\sin\theta = \lambda$$
$$\Rightarrow \theta = \frac{\lambda}{a} \quad [\because \theta \approx \text{small}]$$

If lens is very small close to slit or distance of screen from slit is very large.

$$\theta = \frac{x}{f} \Rightarrow \frac{x}{f} = \frac{\lambda}{a} \Rightarrow x = \frac{\lambda f}{a} \quad \therefore \text{WIDTH OF CENTRAL MAX.} = \boxed{\frac{2\lambda f}{a}}$$

MAXIMA,

$$\boxed{a\sin\theta = (2n+1)\frac{\lambda}{2}}$$

• INTENSITY DISTRIBUTION FOR SINGLE SLIT :

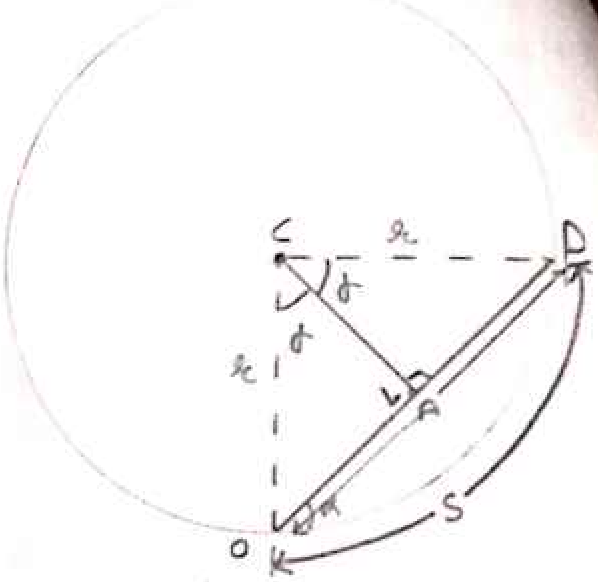
Phase difference = 2α

Path difference = $a\sin\theta$

$\Rightarrow \quad 2\alpha = \frac{2\pi}{\lambda} \cdot a\sin\theta$

$\Rightarrow \quad \boxed{\alpha = \frac{\pi}{\lambda} a\sin\theta}$

Change from $0 \to \frac{\pi}{\lambda} a\sin\theta$



In $\triangle$ OLC, $\frac{OL}{OC} = \sin\alpha = \frac{OL}{r}$

$\Rightarrow$ OL = $r\sin\alpha$

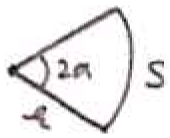
$\Rightarrow$ OP = 2OL

$\Rightarrow \boxed{A = 2r\sin\alpha}$ —(1)

Arc length $\propto$ slit width

$\Rightarrow \quad S \propto a$

$S = Aa = A_0$ (let)



Solid angle, $2\alpha = \frac{S}{r}$

$\Rightarrow 2\alpha = \frac{A_0}{r}$

$\Rightarrow 2r = \frac{A_0}{\alpha}$ —(ii)

from (i) & (ii), $\boxed{A = \frac{A_0\sin\alpha}{\alpha}} \Rightarrow I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \Rightarrow \boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2}}$

1) PRINCIPAL MAXIMA

$\theta = 0, \alpha = 0$

$\Rightarrow I = I_0\left(\frac{\sin 0}{0}\right)^2$

$\boxed{I = I_0}$ $\left[\because \lim \frac{\sin 0}{0} = 1\right]$

2) MINIMA

$\alpha = \frac{\pi}{\lambda} a\sin\theta$

$\Rightarrow \boxed{\alpha = n\pi}$ $[\because a\sin\theta = n\lambda]$ $n = 1, 2, 3 \ldots$

3) SECONDARY MAXIMA

$a\sin\theta = (2n+1)\frac{\lambda}{2}$

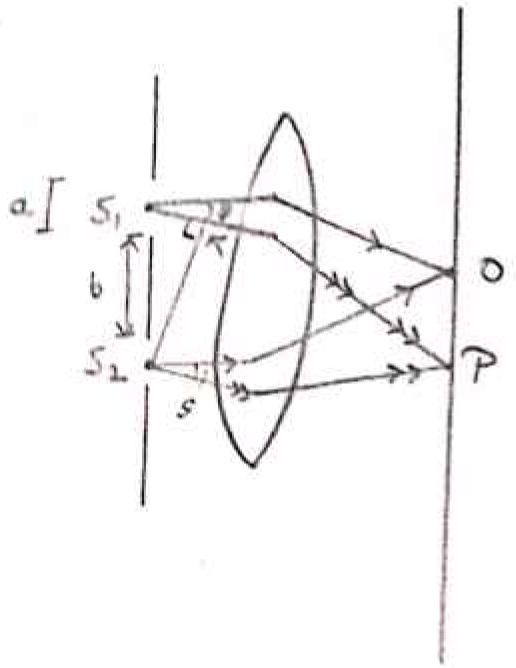
$\Rightarrow \boxed{\alpha = (2n+1)\frac{\pi}{2}}$ $n = 1, 2, 3 \ldots$

$\Rightarrow n = 1$, $\alpha = 3\pi/2$

$I_1 = I_0 \cdot \frac{\sin(3\pi/2)}{(3\pi/2)^2}$

$\boxed{I_1 = \frac{4I_0}{9\pi^2}}$

Fraunhoffer Diffraction at Double Slit:



· At O, central bright maxima is formed.

Dist. b/w slits from center is (a+b),

In ΔS_1KS_2,

$$\sin\theta = \frac{S_1K}{(a+b)}$$

$$\Rightarrow \quad S_1K = (a+b)\sin\theta \quad \text{—(1)}$$

→ Minima

$$\boxed{(a+b)\sin\theta_n = n\lambda}$$

→ Maxima

$$\boxed{(a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2}}$$

* Angular seperation b/w ~~two~~ pair of consecutive minima ~~is~~/maxima is constant.

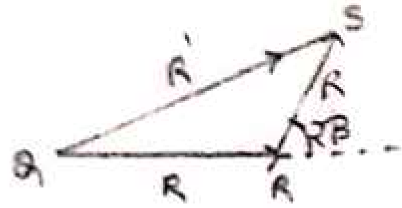
Intensity Distribution

Path Diff = $(a+b)\sin\theta$

Phase Diff = $\frac{2\pi}{\lambda}(a+b)\sin\theta = \beta$

We know that resultant amplitude from each slit is $R = \frac{A_0 \sin\alpha}{\alpha}$ where, $\alpha = \frac{\pi}{\lambda} a\sin\theta$

Resultant R is given by,



$$R'^2 = R^2 + R^2 + 2R^2\cos 2\beta$$

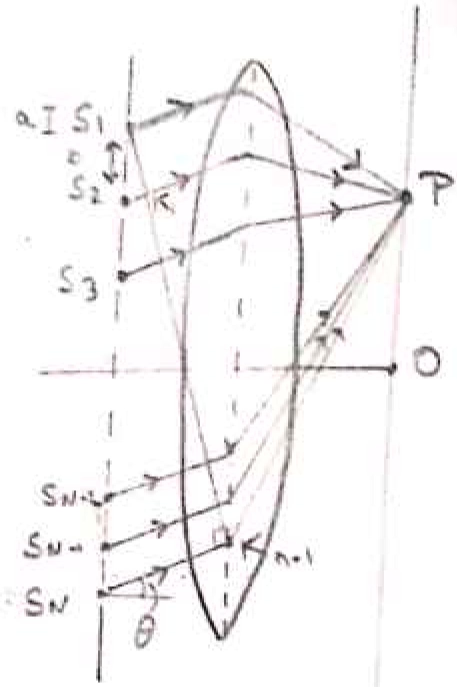
$$R'^2 = 2R^2(1+\cos 2\beta)$$

$$R'^2 = 4R^2\cos^2\beta$$

$$\Rightarrow \quad R'^2 = 4A_0^2\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\beta$$

$$\boxed{I = 4I_0\cos^2\beta\frac{\sin^2\alpha}{\alpha^2}}$$

4) Fraunhoffer Diffraction of 'N' Slit :



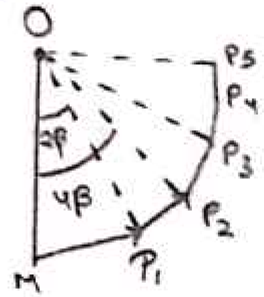
- An arrangement consisting of a large no. of parallel slits, equidistant narrow slits of same width is called "Diffraction Grating".
- The distance b/w any two consecutive slits is known as "Grating Element".
- The parallel rays form central maxima at 'O'.

Path difference b/w S_1 & S_2, $S_2K_1 = (a+b)\sin\theta$

Phase difference, $2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$

- Intensity Distribution :

Using Polygon vector addition rule to find resultant at P, $MP_1 = P_1P_2 = P_2P_3 = \frac{A_0 \sin\alpha}{\alpha}$



$\Rightarrow \sin 2\beta = \frac{MP_1}{OM} \Rightarrow MP_1 = OM \sin 2\beta$

if $\beta \approx$ small
$\cos\beta \approx 1$

$\Rightarrow MP_1 = 2OM \sin\beta$ — (I)

$MP_2 = 2OM \sin 2\beta$

⋮

$MP_N = 2OM \sin N\beta$ — (II)

from (I), $OM = \frac{MP_1}{2\sin\beta}$

$\Rightarrow MP_N = A \frac{\sin N\beta}{\sin\beta} \Rightarrow \boxed{R = \frac{A_0 \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}}$

$\therefore \boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}}$

PRINCIPAL MAXIMA

$\sin\beta = 0 \Rightarrow \beta = \pm n\pi$

$$\lim_{\beta\to n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to n\pi} \frac{N\cos N\beta}{\cos N\beta} \text{ [L' Hospital Rule]}$$

$$= \pm N$$

$$\Rightarrow \boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2} N^2}$$

DIR^n OF PRINCIPAL MAXIMA IN GRATING

$$\Rightarrow \boxed{(a+b)\sin\theta_n = n\lambda}$$

where $m \to$ order of diffraction
$a \to$ Width of slit
$b \to$ dist. b/w slits
$(a+b) \to \frac{1}{N(\text{LINES/cm})}$

- SECONDARY MINIMA

When $\sin N\beta = 0$, $\sin\beta \neq 0$

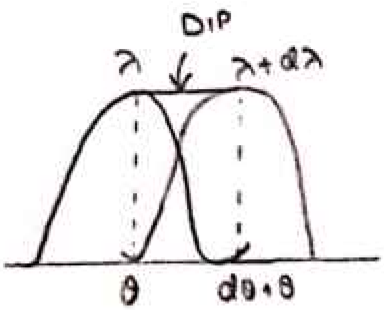
$\Rightarrow I = 0$ (MINIMUM)

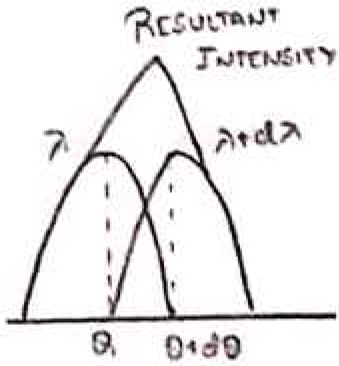
$$\Rightarrow N\beta = \pm m\pi$$

$$\Rightarrow \frac{N\pi}{\lambda}(a+b)\sin\theta = \pm m\pi \Rightarrow \boxed{N(a+b)\sin\theta = \pm m\lambda}$$

→ RAYLEIGH CRITERION:

- The two point sources are equally intense spectral lines are just resolved by an optical instrument when the central maxima of of diff. pattern due to one source fall exactly on first minima of diffraction pattern of other source.





→ RESOLVING POWER OF OPTICAL INSTRUMENT:

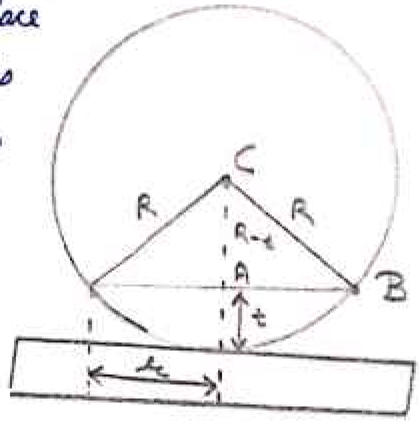
$$\boxed{\theta_{min} = \frac{1.22\lambda}{D}}$$

→ RESOLVING POWER OF A GRATING :

$$\boxed{P = \frac{\lambda}{d\lambda} = nN} \quad \text{OR} \quad \boxed{\frac{\lambda}{d\lambda} = N(a+b)\cos\theta \cdot \frac{d\theta}{d\lambda}}$$

NEWTON RING :

When a plano convex lens is placed on a plain glass plate, then a thin film of air is enclosed b/w upper surface & lower surface of lens. The thickness of air film is very small at pt. of contact & gradually increases as we move outwards. The fringes produced are circular/concentric & uniform in thickness.



In $\triangle ABC$,

$$R^2 = (R-t)^2 + r^2$$
$$R^2 = R^2 + t^2 - 2Rt + r^2$$
$$\Rightarrow \boxed{t = \frac{r^2}{2R}}$$

* INTERFERENCE IS DUE TO REFLECTED LIGHT,

$$\Rightarrow 2\mu t\cos r = (2n+1)\lambda/2 \; [\text{BRIGHT}]$$
$$2\mu t\cos r = (n+1)\lambda/2 \; [\text{DARK}]$$

• BRIGHT RING

$2\mu t\cos r = (2n+1)\lambda/2$

$n \to n-1$

$2\mu t\cos r = (2n-1)\lambda/2$

$\mu = 1$, $\cos\theta \simeq 1$ since θ is v.small

$\therefore \; 2t = (2n-1)\lambda/2$

$\Rightarrow \boxed{r_n = \sqrt{(2n-1)\lambda/2}}$

• DARK RING

$2\mu t\cos r = (n+1)\lambda$

$n \to n-1$

$\Rightarrow 2\mu t\cos r = n\lambda$

$\Rightarrow 2t = n\lambda \; [\because \mu = 1, \cos r \simeq 1]$

$\therefore \; \boxed{r_n = \sqrt{n\lambda R}}$

FOR n^{th} DARK RING

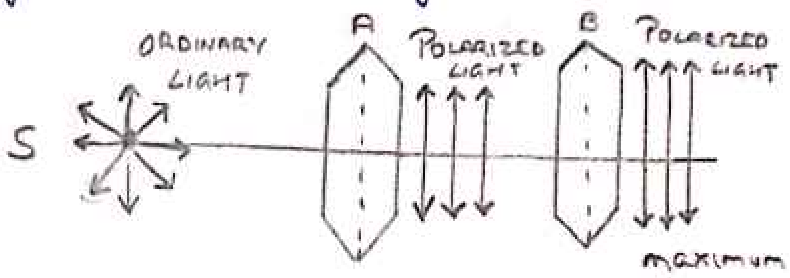
$$\boxed{D_n^2 = 4n\lambda R}$$

Polarization

- **Polarization**

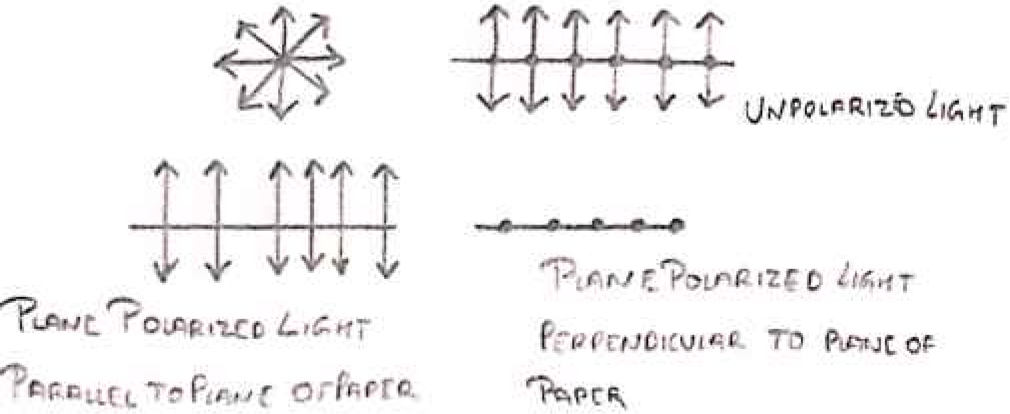
- The process of transforming unpolarized light wave to polarized light wave.

- When ordinary light is incident on a pair of parallel Tourmaline crystal.



1) When axis of B is parallel to A, $I = max$
2) When axis of B is perpendicular to A, $I = min$

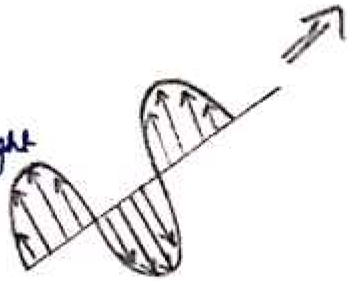
- Pictorial Representation



- Types of Polarization

1) Plane Polarized Light

Electric vector vibrates in a fixed straight line $\perp$ to dirn of propagation of light



2) Circularly Polarized Light

When two plane polarised lights are superimposed under certain conditions, the resultant vector rotates with const. magnitude in a plane $\perp$ to dirn of propagation of light. The tip of a vector space traces a circle & the light is said to be circularly polarised.

It contains two EM waves of equal amplitude &

3) Elliptically Polarised

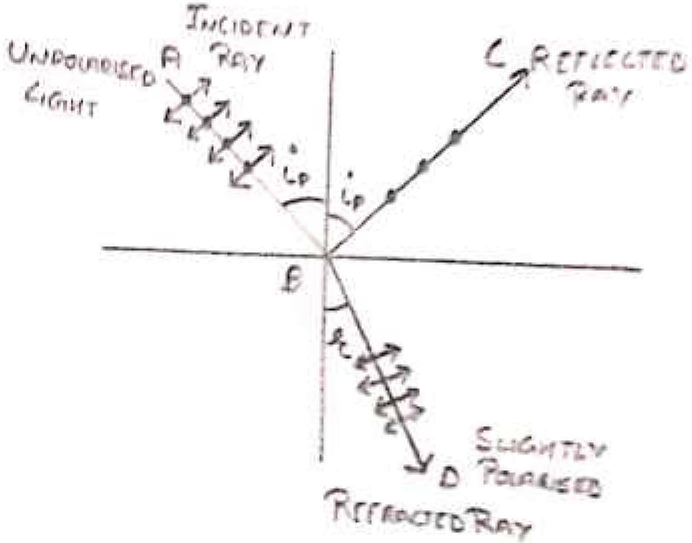
When two plane polarised light waves are superimposed, the resultant vector rotates in perpendicular to dir of propagation of light, the tip of vector traces an Ellipse.

Elliptically Polarised light consists of two perpendicular EM waves of unequal amplitude having phase difference of 90°

→ Polarisation by Reflection

• Brewsters Law

Suppose AB is incident on glass surface at angle i_p. It is reflected along BC & refracted along BD,



$$\boxed{\mu = \tan i_p = \frac{\sin i_p}{\cos i_p}}$$

From Snells Law,

$$\mu = \frac{\sin i_p}{\sin r} \Rightarrow \frac{\sin i_p}{\sin(\pi/2 - i_p)} = \frac{\sin i_p}{\cos r \sin r}$$

$$\therefore \pi/2 - i_p = r$$

$$\Rightarrow \boxed{i_p + r = \pi/2}$$

$$i_p + \angle CBD + r = 180°$$

$$\Rightarrow \angle CBD = 90°$$

∴ The reflected ray & refracted ray are at right angle with each other.

Wave plates

- We make a crystal plate of such a thickness, it can introduce a path diff of $\lambda/4$ or $\lambda/2$ b/w Ordinary & Extraordinary ray.

→ Quarter Waveplate (Calcite)

$$\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$$

$\mu_e \rightarrow$ R.I of Calcite for E-Ray

$\mu_o \rightarrow$ R.I of Calcite for O-Ray

→ Half Wave Plate

$$\boxed{t = \frac{\lambda}{2(\mu_e - \mu_o)}}$$

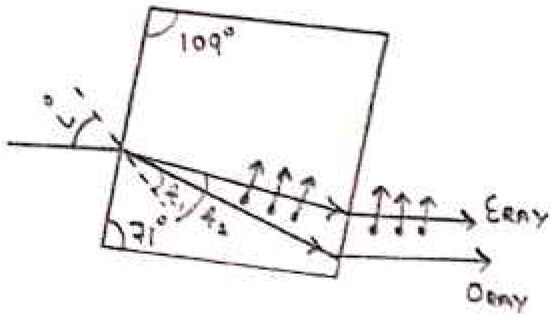
* For Quartz Crystal $(\mu_o - \mu_e)t = P.D$

Double Refraction or Birefringes:

- Structure of Calcite Crystal is "Rhombohedral".

O ray → Follows Snells Law.

E ray → Doesn't follow Snell's Law



For O ray, $\mu_o = \frac{\sin i}{\sin r_1}$ [Fixed]

E ray, $\mu_e = \frac{\sin i}{\sin r_2}$ [Varying]

Since $r_2 > r_1$

$\Rightarrow \mu_o > \mu_e \quad \therefore \boxed{v_e > v_o}$

$\Rightarrow$ Velocity of light for O ray ~~is~~ inside the crystal is less than E ray.