

UNIT-I (Important Questions)

① If $u = (1 - 2xy + y^2)^{-1/2}$ then Prove that

$$\frac{\partial}{\partial x} \left\{ (1 - x^2) \frac{\partial u}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ y^2 \frac{\partial u}{\partial y} \right\} = 0$$

② If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$$

③ If $u = (x^2 + y^2 + z^2)^{-1/2}$ show that

$$(i) x u_x + y u_y + z u_z = -u$$

$$(ii) u_{xx} + u_{yy} + u_{zz} = 0$$

④ verify that v satisfies $v_{xx} + v_{yy} + v_{zz} = 0$

$$\text{if (a) } v = x^2 + y^2 - 2z^2$$

$$(b) v = e^{3x+4y} \cos 5x$$

⑤ If $u = f(r)$ where $r^2 = x^2 + y^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

$$\text{⑥ If } u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$$

then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + 2u(x, y) = 0$

⑦ If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ evaluate $\frac{\partial^2 u}{\partial x \partial y}$

$$\text{Ans. } \frac{x^2 - y^2}{x^2 + y^2}$$

⑧ If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ then show that

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$$

⑨ If $u = r^n (3 \cos^2 \theta - 1)$ satisfies the differential

$$\text{eq}^n \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) = 0 \text{ then prove that } n=2 \text{ or } -3$$

Q. 10 If $u = x+y+z$, $uv = y+z$, $uvw = z$ (2)
 find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ Ans $\frac{1}{u^2v}$

Q. 11 If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ then
 find $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$ Ans . 4

Q. 12 If $x+y+z = u^3 + v^3 + w^3$, $x^3 + y^3 + z^3 = u^2 + v^2 + w^2$,
 $x^2 + y^2 + z^2 = u + v + w$ then show that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

Q. 13 Are the ~~possible~~ function $u = \frac{x-y}{x+z}$, $v = \frac{x+z}{y+z}$
 functionally dependent? If so find the relation
 between them.

Q. 14 If $x+y+z = u$, $y+z = u^2 v$, $z = u^2 w$ then
 find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ Ans u^{-5}

Q. 15 If $u = x+y+z$, $v = x^2 + y^2 + z^2$, $w = x^3 + y^3 + z^3 - 3xyz$
 Prove that u, v, w are not independent and hence
 find the relation between them. Ans $w = \frac{u(u^2 + 3v)}{4}$

Q. 16 The temperature T at any point (x, y, z) in
 space is $T = 400xyz^2$. find the highest temperature
 at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$

Q. 17 If $u = ax^2 + by^2 + cz^2$, where $x^2 + y^2 + z^2 = 1$ and
 $lx + my + nz = 0$ prove that stationary values of u
 satisfy the equation $\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$

Q. 18 find the shortest and longest distance from the
 point $(1, 2, -1)$ to the sphere. Ans $\sqrt{6}, 3\sqrt{6}$

Q₁₉ Divide a number into three parts such that the product of first, square of second and cube of third is maximum. ③

Ans 4, 8, 12

20 A rectangular box which is open at the top has a capacity of 256 cubic feet. Determine the dimension of the box such that the least material is required for the construction of the box.

Ans 8, 8, 4

21 Find the volume of the largest parallelepiped with edges parallel to the axes that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Ans $\frac{8abc}{3\sqrt{3}}$