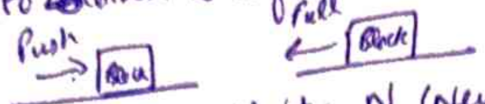
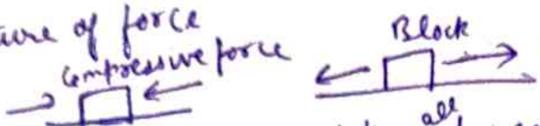
Force :- External energy required to move the body from one place to another is a force



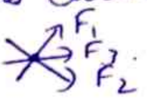
Vector quantity, Unit :- N (Newton), $KN = 10^3 N$.

Nature of force

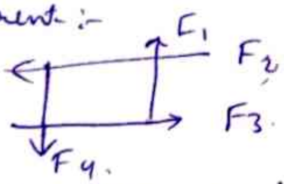


Tensile forces.

* Coplanar forces :- When all forces lie on same plane.
  Non coplanar :- When all forces lie on diff. plane
* Types of coplanar
(1) Collinear :- When all the forces are in a straight line then that force system is called collinear force system
(2) Concurrent :- All forces passing through a single point then that force system is called a concurrent / When line of action of forces intersect with each other.



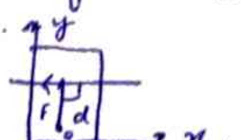
(3) Non concurrent :-



(4) Parallel force :- When all forces are parallel to each other.
(1) Like parallel :- parallel & same direction.
(2) Unlike :- Parallel but opp. direction.

* Principle of transmissibility of force
Force acting on rigid body at a point, it can be transferred to the other point on same body, keeping same magnitude, same direction, same line of action. So, that effect will remain same

* Moment of force :- To define moment of force we need.
(1) a point which we have to define "M"
(2) ⊥ Distance from line of action of "F" to that point
(3) Sense of Moment of Force (M) (clockwise or Anticlockwise)
(4) Direction of M.



$M_0 = F \times d$.

Anticlockwise +ve

$M_0 = +F \times d$

Clockwise -ve

$M_0 = -F \times d$

By right hand thumb rule.

XY plane → M_z, ZX → M_y

YZ plane → M_x

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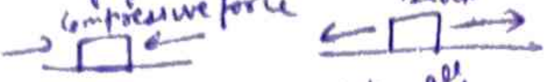
Vector Quantity, Unit :- N (Newton), $kN = 10^3 N$

Nature of force



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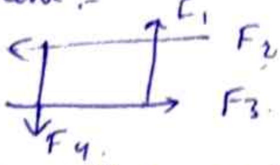

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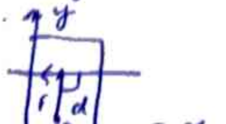


$M_0 = F \times d$

Anticlockwise +ve

$M_0 = +F \times d$

By right hand thumb rule.

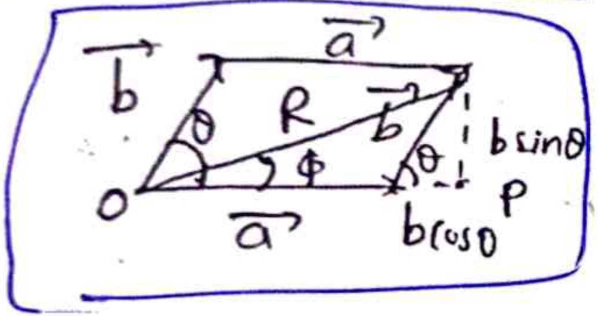
XY plane → M_z , ZX → M_y

YZ plane → M_x

Clockwise -ve

$M_0 = -F \times d$

RESULTANT OF VECTORS



$$R^2 = (b\sin\theta)^2 + (a+b\cos\theta)^2$$

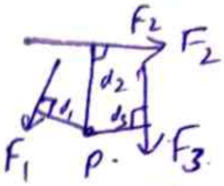
$$R^2 = b^2\sin^2\theta + a^2 + b^2\cos^2\theta + 2ab\cos\theta$$

$$R^2 = a^2+b^2+2ab\cos\theta$$

$$\boxed{R = \sqrt{a^2+b^2+2ab\cos\theta}}$$

$$\boxed{\tan\phi = \frac{b\sin\theta}{a+b\cos\theta}}$$

Eg:-



$M_P = -F_3 \times d_3 + F_1 d_1 - F_2 \times d_2$.

$\boxed{\Sigma M}$ → −ve (clockwise)
→ +ve (Anticlockwise)

* Equilibrium :- State of balance due to equal action of opposing forces.

→ If body is having zero effect under action of forces, then body will be in stable equilibrium

→ Net force (ΣF)
→ Net Moment (ΣM)

→ Static equilibrium.

Coplanar Concurrent → $\Sigma f_x = 0, \Sigma f_y = 0$

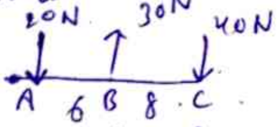
$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

$\theta = \tan^{-1} \left| \frac{\Sigma f_y}{\Sigma f_x} \right|$

* Varignon's theorem :-

If many coplanar forces are acting on a body, then algebraic sum of moments of all forces about a point in the plane of forces is equal to the moment of their resultant about same point

Eg:-



$\Sigma M_A = -30 \times 6 + \cancel{800}\ 40 \times 14$.

$R \times x = 380\ N/m$

$30 \times x = 38$

Find resultant.

$\Sigma f_x = 0$.

$\Sigma f_y = 30 - 40 - 20$.

$= -30\ N$.

$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2} = \sqrt{0 + 30^2}$

$R = 30 N$.

Resultant is acting downward.

$\Sigma M_A = M_A^R$.

$\Sigma M_A = R \times x$.

~~$\Sigma M = 380$~~ $-30 \times 6 + 40 \times 14$

* Couple :- Couple consists of 2 parallel forces that are equal in magnitude, opp. in direction & do not share a line of action

Props of couple :-

1. Algebric sum of forces consisting the couple is zero
2. A couple cannot be balanced by a single force but can be balanced only by a couple but of opp. direction
3. The algebric sum of moment of forces ~~consulting~~ constituting the couple about any point is same and equal to moment of couple itself.
4.

* Lamis Theorem :- When 3 forces acting at a point are in equillibrium, then each force is proportional to sine of the angle b/w the other 2 forces

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{T_3}{\sin\gamma}.$$

* Centre of gravity :- An imaginary point in a body of matter where the total weight of body is thought to be concentrated.
* Centre of mass :- A point where the whole of the mass of body ~~or~~ appeared to be concentrated.

~~Centre~~

$$x = \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots + m_n}$$

$$y = \frac{m_1y_1 + m_2y_2 + \dots + m_ny_n}{m_1 + m_2 + \dots + m_n}$$

Centre of gravity :- $x = \dfrac{W_1x_1 + W_2x_2 + \dots + W_nx_n}{m_1 + m_2 + \dots + m_n}$

* Centroid :- Centre point of the object

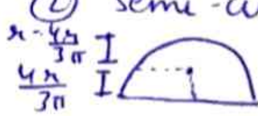
$$x = \frac{A_1x_1 + \dots + A_nx_n}{x_1 + x_2 + \dots + x_n}.$$

$$y = \frac{A_1y_1 + A_2y_2 + \dots + A_ny_n}{y_1 + y_2 + \dots + y_n}$$

① Rectangle

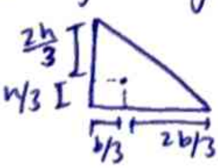
$A = b \times h$

② Semi-circle



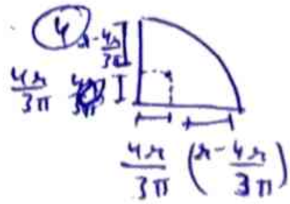
$A = \dfrac{\pi r^2}{2}$

③ Right Angled Triangle



$A = \frac{1}{2} \times b \times h$.

④

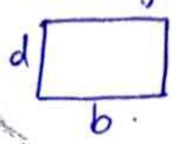


$A = \dfrac{\pi r^2}{4}$

* Moment of inertia :- Resistance offered by physical object against rotational motion

① Rectangle

②



$MI_{xx} = \dfrac{bd^3}{12}$.

$MI_{yy} = \dfrac{db^3}{12}$

Then $MI_{x_1} = MI_{xx} + Al_y^2$

$MI_{y_1} = MI_{yy} + Al_x^2$

Then $MI_{xxT} = MI_{x_1} + MI_{x_2} + MI_{x_3}$

$MI_{yyT} = MI_{y_1} + MI_{y_2} + MI_{y_3}$

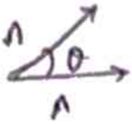
(i) DOT PRODUCT (SCALAR PRODUCT) — ii) CROSS PRODUCT (VECTOR PRODUCT)

$$\boxed{\vec{a}\cdot\vec{b} = |a|\,|b|\cos\theta}$$

$$\boxed{\vec{a}\times\vec{b} = |a|\,|b|\sin\theta\;\hat{n}}$$

Q1) Two same vectors of magnitude A with an angle θ. Find the magnitude and direction of the resultant.

Ans)



$$R = \sqrt{A^2 + A^2 + 2AA\cos\theta}$$
$$= \sqrt{2A^2 + 2A^2\cos\theta}$$
$$= A\sqrt{2(\cos\theta+1)}$$
$$= \sqrt{2}A\sqrt{\cos\theta+1}$$

$[\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1-\cos^2\theta)$

$= 2\cos^2\theta - 1]$

$\left[\frac{\cos 2\theta + 1}{2} = \cos^2\theta\right]$

$[\cos 2\theta + 1 = 2\cos^2\theta]$

$$\boxed{|R| = 2A\cos\theta/2}$$

Rectangle (b, d): $MI_{xx} = \frac{bd^3}{12}$, $MI_{yy} = \frac{db^3}{12}$

Triangle (b, d): $MI_{xx} = \frac{bh^3}{36}$, $MI_{yy} = \frac{hb^3}{36}$

Circle: $MI_{xx} = MI_{yy} = \frac{\pi}{64}d^4$

Semicircle: $MI_{xx} = \cancel{...}\, MI_{yy} = 0.055\,r^4$

Quarter circle: $MI_{xx} = 0.392\,r^4$, $MI_{yy} = 0.11\,r^4$

Parallel axis: $\cancel{MI_{xx} = I + Al^2}$

$MI_{x_1} = MI_{xx} + Al_y^2$

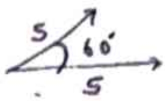
$MI_{y_1} = MI_{yy} + Al_x^2$

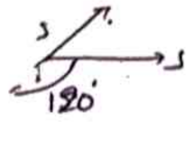
$MI_{YYT} = MI_{y_1} + MI_{y_2} + MI_{y_3}$

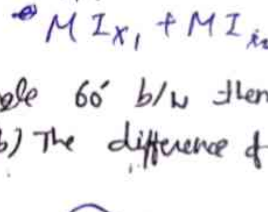
$MI_{XXT} = MI_{x_1} + MI_{x_2} + MI_{x_3}$

2) Two vectors of equal magnitude 5 units have an angle 60° b/w them. Find the magnitude of (a) The sum of the vectors and (b) The difference of the vectors.

1)







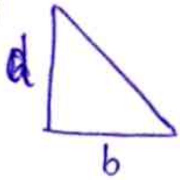
(a)

$$|\text{sum}| = \sqrt{25+25+2(5)(5)\cos 60°}$$
$$= \sqrt{75}$$
$$= 5\sqrt{3}$$

$$|\text{Diff.}| = \sqrt{25+25+2(5)(5)\cos 120°}$$
$$= \sqrt{25+25-25}$$
$$= 5$$

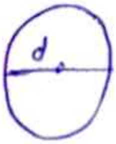
Mass MOI: $MI_{xx} = \int y^2\,dm$, $MI_{yy} = \int x^2\,dm$

$I = mk^2$, $k = \sqrt{\frac{I_G}{M}}$ (Radius of gyration)

② 

$MI_{xx} = \frac{bd^3}{36}$, $MI_{yy} = \frac{bd^3}{36}$ $\frac{db^3}{36}$

MOI me figures alag alag karke solve karte hai

③ 

$MI_{xx} = MI_{yy} = \frac{\pi}{64} d^4$

④ 

$MI_{xx} = MI_{yy} = 0.055r^4$

⑤ 

$MI_{xx} = 0.392r^4$, $M_{yy} = 0.11r^4$

→ Polar MOI :- Resistance to twisting is polar momentum

Polar MOI = $\frac{1}{2}$ MOI

Area MOI :- resistance to bending.

Polar MOI :- resistance to twisting.

→ Mass MOI - $MI_{x_1} = MI_{xx} + m l_y^2$

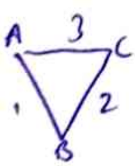
★ Radius of gyration :- Imaginary distance from centroid at which area of cross section is imagined to be focused at a point in order to obtain same MOI

$k = \sqrt{\frac{I_{xx}}{\Sigma A}}$ $MI = mk^2$ $R = \sqrt{\frac{MI}{m}}$

★ Truss :- We have 3 lines joined by lower pair or pin joint.

$DOF = 3(n-1) - 2l - h$ → higher pair.

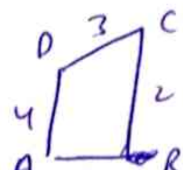
n ↓ Total lines l ↓ Lower pair



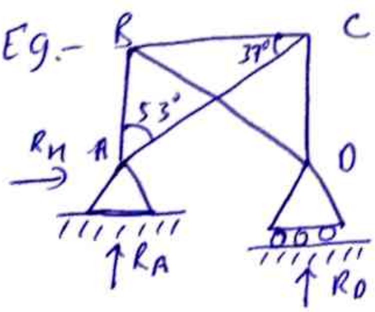
$n = 3$.
$l = 3$
$h = 0$.

DOF = 0

Stable



$n = 4$
$l = 4$
$h = 0$
DOF = 1

Eg.- 

* Perfect truss :- A truss has got enough members to resist the loads without undergoing deformation in its shape.

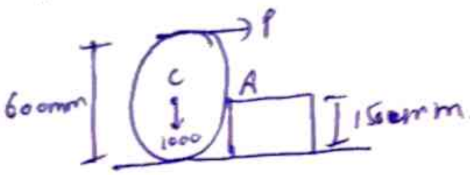
* Imperfect truss :- One which doesn't satisfy relation given by equation $m = 2j - 3$

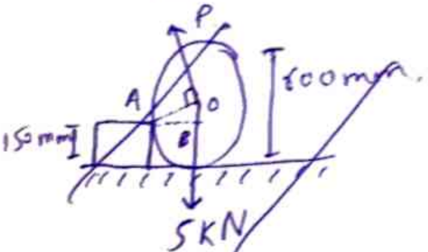
j = joints
m = no of members
$m > 2j - 3$. Redundant

Q) Imp
Wheel :- 6 cm diameter.
Weight :- 1000 N.
rectangular block :- 15 m high

Force P.
Find Force P, when wheel is just about to roll over block

Ans





In Δ OAB.

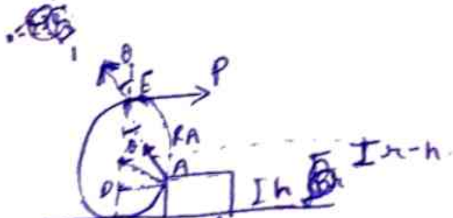
$AB = \sqrt{(300)^2 - (150)^2} = 260mm$

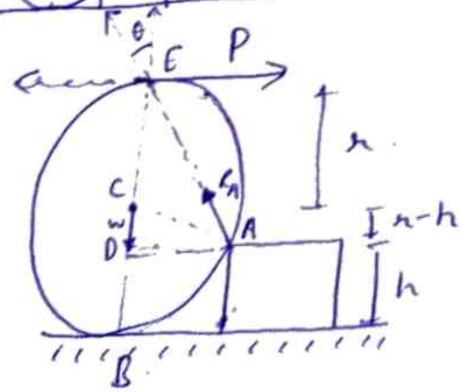
$M_A = P \times 300 - 5 \times 260 = 0$

$P = 4.33 KN$

Reaction on block → $R \cos 30° = P \sin 30°$

$R = \frac{4.33 \times 0.5}{0.866} = 2.5 KN$





$\Sigma F_x = 0.$

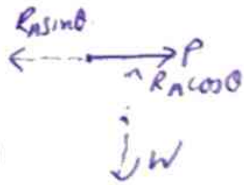
$P - R_A \sin\theta = 0$

$P = R_A \sin\theta$

$\Sigma F_y = 0$

$R_A \cos\theta = W$

$\tan\theta = \frac{P}{W}.$



In ΔADE

$$\tan\theta = \frac{AD}{DE}$$

$DE = 2r - h.$

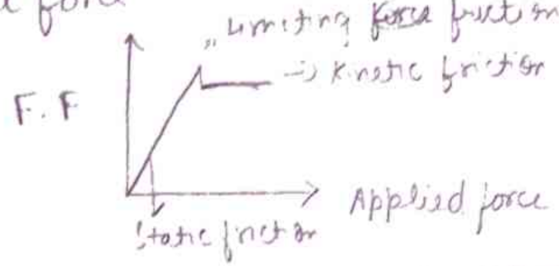
$AD = \sqrt{r^2 - (r-h)^2}$

$$= \frac{\sqrt{r^2 - (r-h)^2}}{2r-h}$$

$$\tan\theta = 0.577$$

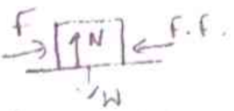
$$P = W\tan\theta = 0.577 \times 1000$$
$$= 577N$$

(1) Friction :- When 2 rigid bodies have relative motion, an opposing force is developed in the direction opp. to the motion known as frictional force



$F.F = \mu.N$

→ Static friction :- F.F. observed b/w 2 bodies before relative motion
→ Kinetic :- F.F observed when 2 bodies are having steady state of motion
→ Limiting - F.F observed when relative motion b/w 2 bodies just start



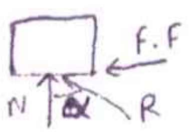
$F = F.F$
$N = W$

* Laws of Dry friction
(1) Friction b/w 2 surfaces depends upon upon type of surface in contact
(2) It always acts tangential to area of contact and opposes motion
(3) $F.F \propto N$ $\quad F.F \perp N$
(4) It is independent of area of contact.

μ - independent ... velocity.

$$\mu_S > \mu_K, \quad \mu_S \le \mu_L$$

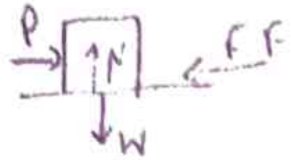
* Angle of friction :- It is angle made by resultant of F.F and normal reaction with the normal rxn.



$\tan\alpha = \mu$

$$\tan\alpha = \frac{F.F}{N}$$

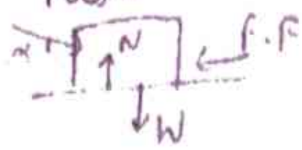
★

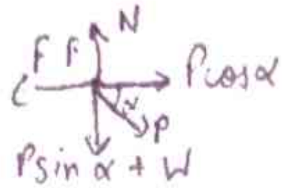


$P = F.F$

$N = W$

Push

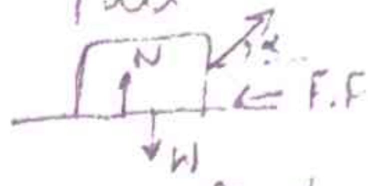


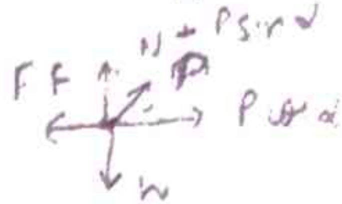


$N = P\sin\alpha + W$

$F.F = P\cos\alpha$

Pull

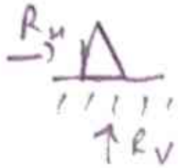




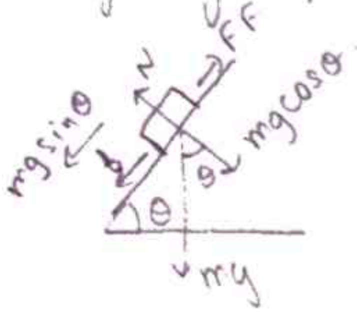
$F.F = P\cos\alpha$

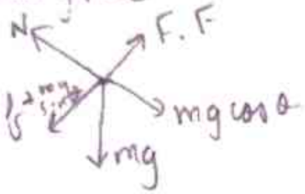
$N + P\sin\alpha = W$

→ Hinge



★ Angle of Repose :- Min. angle for which body slips due to its weight





$N = mg\cos\theta$

$F.F = f + mg\sin\theta$

When $f_{max} = mg\sin\theta = F.F$

$mg\sin\theta = \mu N$

$\cancel{mg}\sin\theta = \mu \cancel{mg}\cos\theta$

$\boxed{\mu = \tan\theta}$

If angle $< \tan^{-1}\mu \Rightarrow$ No slipping

If angle $> \tan^{-1}\mu \Rightarrow$ Slipping

$\boxed{\alpha = \tan^{-1}\frac{1}{3}}$ Max possible α without slipping

Varigon's theorem :- According to this theorem, moment of all forces from point A is equal to the moment of resultant from point A.

$$\Sigma M_A^F = M_A^R \quad \circlearrowleft +ve$$

$$(500 \times 3) + (-350 \times 7.5) + (150 \times 10) + (200 \times 12) = 300 \times d$$

$$3000 - 225 = 300d$$

$$d = \frac{2775}{300} = 9.25 \text{ m}$$

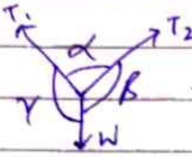
Hence, Resultant force $R = 300$ N↑ lies at ⊥ distance d

$d = 2.5$ m to the right of A.

★ Lami's Theorem (3 forces in equilibrium

It states that "If three concurrent forces act on a body keeping it in Equillibrium, then each force is proportional to the sine of angle b/w 2 forces

Eg:-

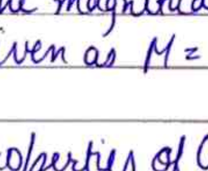


$$\frac{T_1}{\sin\beta} = \frac{T_2}{\sin\gamma} = \frac{W}{\sin\alpha}$$

- It is not applicable for parallel and general force system
- It is applicable only when three forces acting at a point are in equilibrium

★ Couple :- Couple is a special case of parallel forces.

- Two parallel forces of equal magnitude and opposite sense a couple.
- The effect of a couple is to rotate the body on it acts.



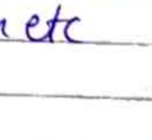
d = arm of couple

$M = F \times d$

Clockwise couple

+ve

-ve



$= \circlearrowright$

$M = F \times d$

Anticlockwise couple

- The magnitude of rotation is known as the moment of a couple given as $M = F \times d$ are N.m, kN.m etc

★ Properties of Couple

1. Couple tend to cause rotation of the body about an axis ⊥ to the plane containing 2 parallel forces.
2. Moment of a couple is equal to the product of one force and the arm of the couple.
3. The resultant force of a couple system is 0.
4. To balance a system whose resultant is couple, "another couple" of opposite direction is required.
5. To shift a force to a new parallel position, a couple is to be added to the system.

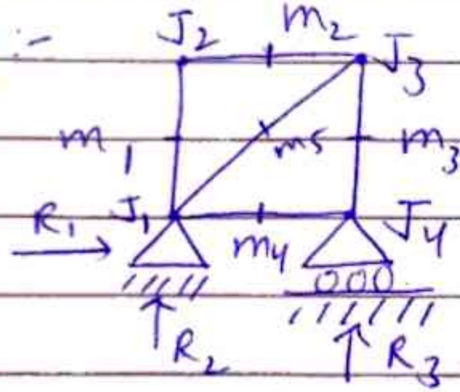
★ Trusses

1) Perfect $m = 2J - r$

2) deficient J = Joints , m = members

3) Redundent R = reactions

Eg:-



$4 = 2 \times 4 - 3 = 5$

$4 \neq 5$

$5 = 5$ after adding m_5

Perfect :- means $m = 2J - r$

Deficient :- means $m < 2J - r$

Redundent :- means $m > 2J - r$

1) Rigid :- Truss will not bend and sustain load
2) Axial forces only
3) Self weight is zero
4) Forces will act only on joints
5) Joints are frictionless
6) Members are straight