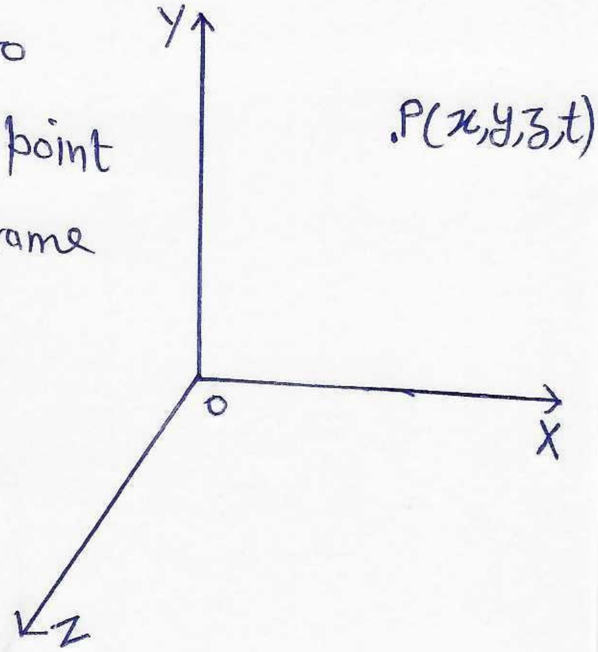
UNIT-IV Theory of Relativity

Frame of Reference —

A Co-ordinate System with respect to Which we measure the position of a point object of an event is Called a frame of reference



For Example— the motion of the body has no meaning unless it is described with respect to Some well defined Co-ordinate System

or

The position of a train Can be found w.r.t. a plateform On which we Stand. The plate form is Said to Constitute a frame of reference.

The frame of reference are of two types:

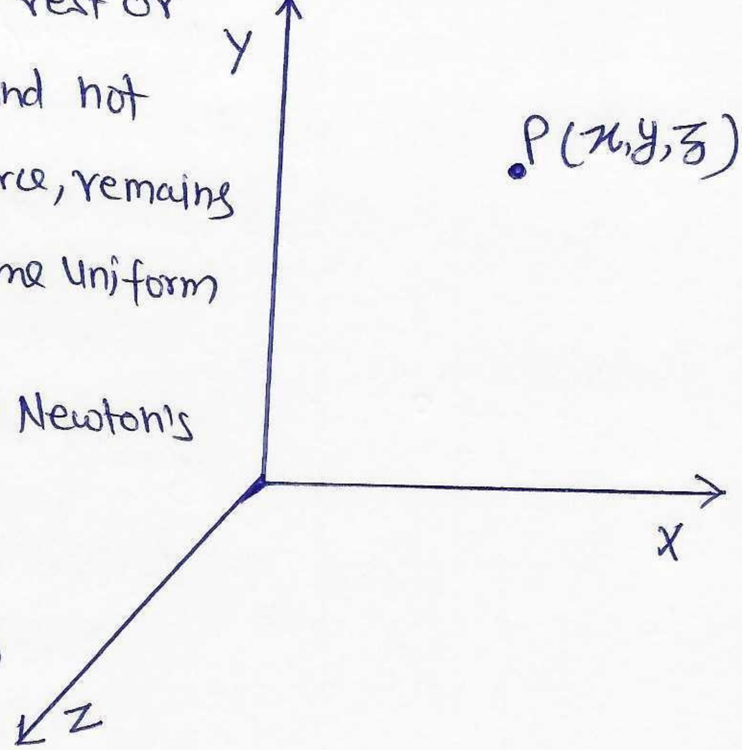
1) Inertial or Unaccelerated frame of reference

2) Non-inertial or accelerated frame of reference

Inertial frame of reference —

A frame of reference is said to be inertial in which a body at rest or moving with uniform velocity and not under the influence of any force, remains at rest or moving with the same uniform velocity.

i.e. A body in this frame obey Newton's Law of inertia



So, the body is not acted upon by any external force, then

$$m\frac{d^2x}{dt^2} = 0, \quad m\frac{d^2y}{dt^2} = 0 \text{ and } m\frac{d^2z}{dt^2} = 0$$

or $$\frac{d^2x}{dt^2} = 0 \quad \frac{d^2y}{dt^2} = 0 \text{ and } \frac{d^2z}{dt^2} = 0$$

Which gives

$$\frac{dx}{dt} = u_x \text{ (Constant)}$$

$$\frac{dy}{dt} = u_y \text{ (Constant)}$$

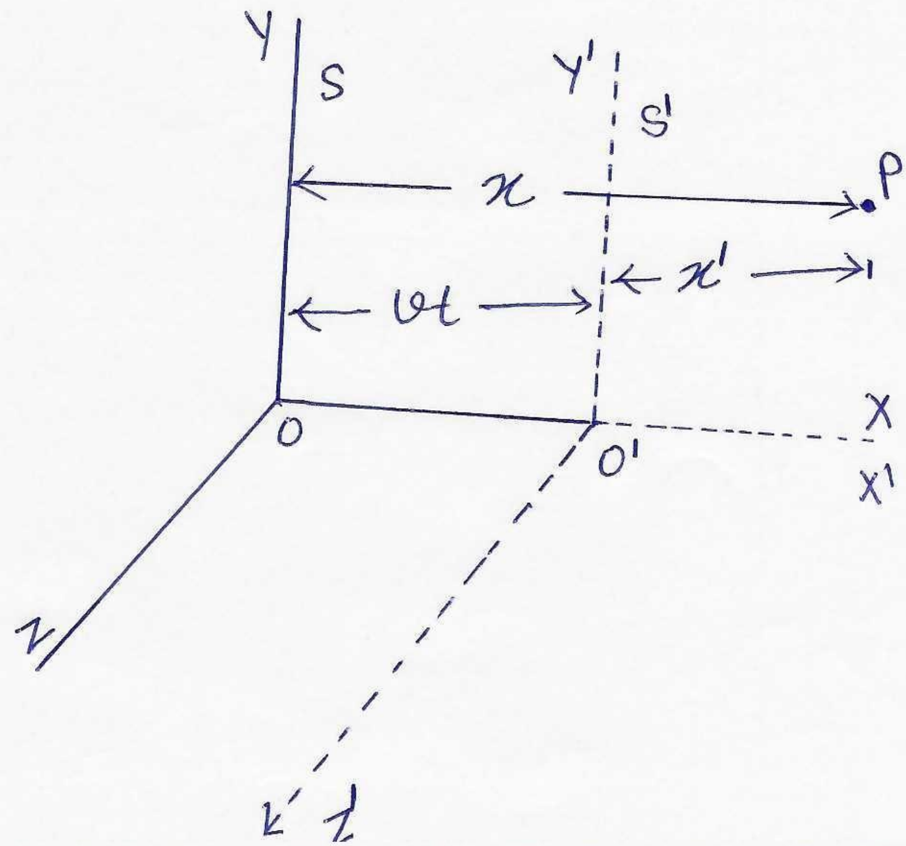
$$\frac{dz}{dt} = u_z \text{ (Constant)}$$

Non-inertial frame of reference —

A frame of reference with respect to which an unaccelerated body appears accelerated are called non-inertial frame of reference i.e. accelerated frame are called non-inertial. In non-inertial frame Newton's law does not hold.

Galilean Transformations —

The equations relating the co-ordinates of a particle in two inertial frame (whose relative velocity is negligible in comparision of speed of light) are called as Galilean transformations



Suppose we have two frame of reference S and S'

the velocity of S' relative to S be v. Consider an event happening at P at any particular time.

Let the coordinates of P with respect to S be x, y, z, t and with respect to S' be x', y', z', t'

The axis X and X' are parallel to v and Y', Z' be parallel to Y and Z respectively.

The time from the instant at which the origins O and O' Coincide.

We have

$$X = X' + vt$$
$$X' = X - vt$$

As there is no relative motion along Y and Z axes.

So $Y' = Y$ and $Z' = Z$

time $t' = t$ {In Classical Physics, time is independent of space Coordinate System}

Therefore, the four eqn are

$$X' = X - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

These Equations are Called as Galilean

The inverse Galilean transformation can be expressed as

$$X = X' + vt$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

For Velocity transformation

$$u'_x = \frac{dx'}{dt} = \frac{d}{dt}(x - vt)$$

$$u'_x = \frac{dx}{dt} - \frac{v\,dt}{dt}$$

$$u'_x = u_x - v$$

$$u'_y = \frac{dy'}{dt} = \frac{dy}{dt} = u_y$$

and $$u'_z = \frac{dz'}{dt} = \frac{dz}{dt} = u_z$$

In Vector form,

$$\boxed{u' = u - v}$$

For acceleration transformation eqns

$$a'_x = \frac{du'_x}{dt} = \frac{d}{dt}(u_x - v)$$

$$a'_x = \frac{du_x}{dt} - \frac{dv}{dt}$$

$$a'_x = a_x \qquad \{v \text{ is Constant}\}$$

Similarly $a'_y = a_y$

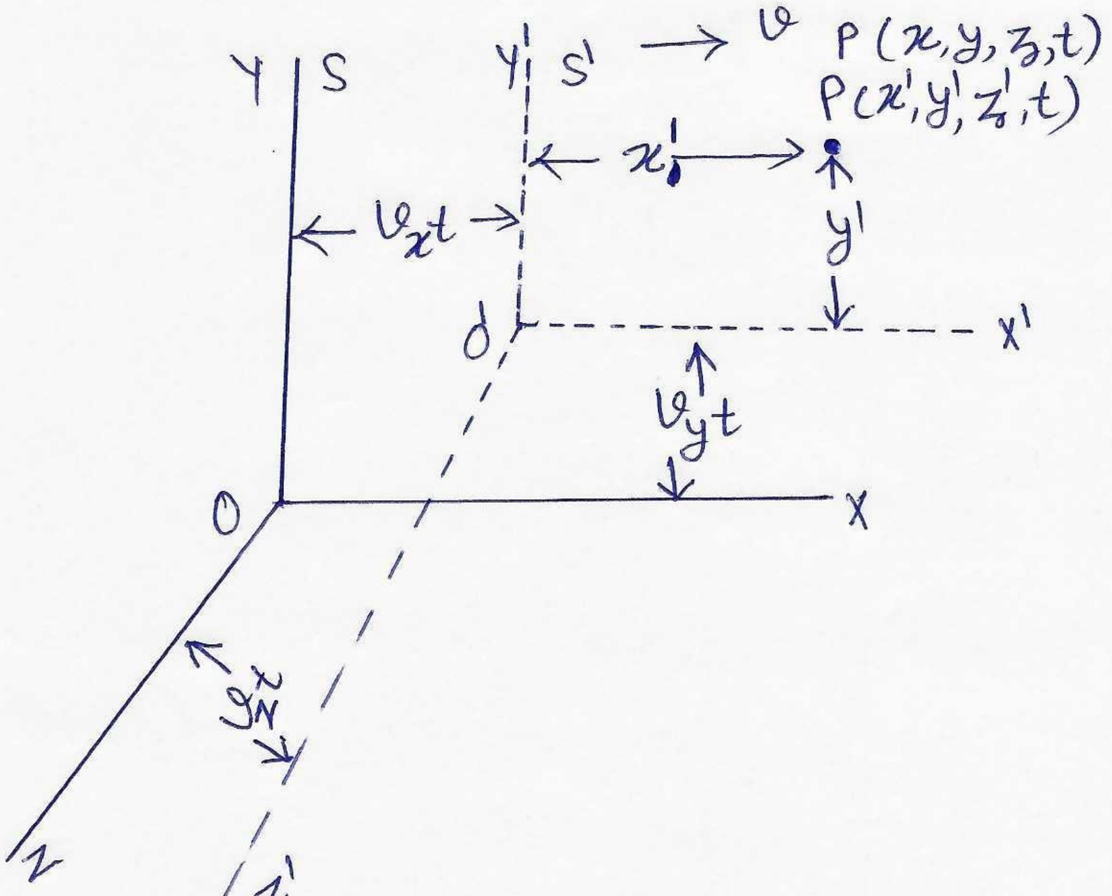
$$a'_z = a_z$$

In vector form $\boxed{a' = a}$

This shows that in all inertial frame a body will be observed to have the same acceleration.

General Case—

Frame S' is moving along a Straight line relative to Frame S along any direction —



Let us Consider another Situation in which frame S' is moving with respect to frame S with Constant Velocity $\vec{v}$ along any direction Such that

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

Let the origin O and O' of two Systems Coincide initially the Co-ordinates of P with respect to S and S' are

When frame S' is Seprated from S by a distance $v_x t$, $v_y t$ and $v_z t$ along the three axis.

So
$$x' = x - v_x t$$
$$y' = y - v_y t$$
$$z' = z - v_z t$$
and
$$t' = t$$

these are the Galilean transformation Equations relating the observations of two observers when one of the two frame is moving along any direction.

The Galilean transformations of the velocity of particle

$$\frac{dx'}{dt} = \frac{dx}{dt} - v_x$$

$$u'_x = u_x - v_x$$

$\left\{ v_x, v_y \text{ and } v_z \text{ are Constant} \right\}$

$$\frac{dy'}{dt} = \frac{dy}{dt} - v_y$$

$$u'_y = u_y - v_y$$

In vector form

$$\boxed{\vec{u}' = \vec{u} - \vec{v}}$$

and
$$\frac{dz'}{dt} = \frac{dz}{dt} - v_z$$

$$u'_z = u_z - v_z$$

Where u_x, u_y and u_z are the velocity Component of particle inthe frame S and u'_x, ~~[illegible]~~ u'_y and u'_z that are in frame S'

$u' = u - v$ represents the Galilean transformation of velocity of the particle.

Galilean tranformation equations for the acceleration of particle may be obtain by differentiating above

$$\frac{du'}{dt} = \frac{du}{dt} - 0$$

$$\boxed{a' = a}$$

Thus the acceleration observed by the observers in different inertial frame of reference is same.
It means that Newton's Second law is valid in every inertial frame of reference
So it proved that the basic laws of Physics do not change under Galilean transformations.

Postulates of Special Theory of Relativity —

Postulates 1 — the principle of Equivalence (or Relativity)

The principle of Equivalence States that the law of Physics are Same in all inertial frame of reference moving with a Constant Velocity with respect to one another.

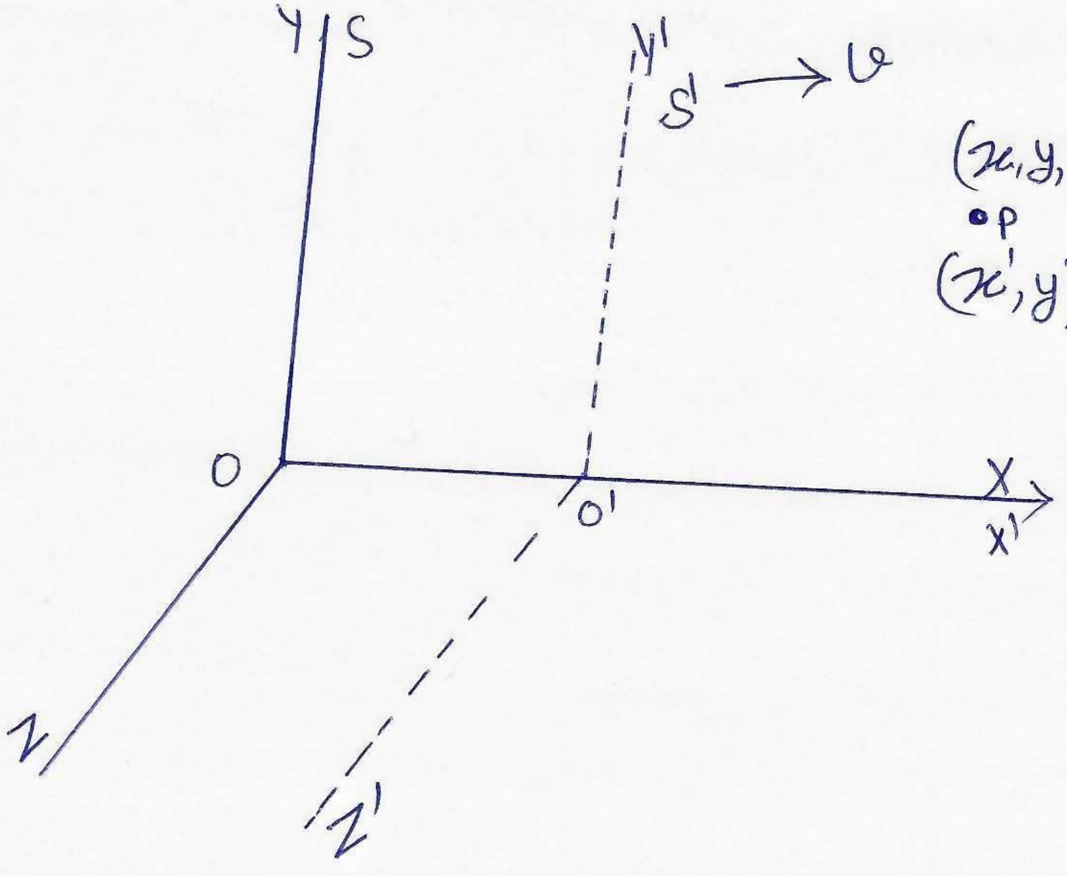
Postulates 2 — The principle of Consistency of the Speed of light

The Second postulate State that the Speed of light in free Space is always Same in all inertial frames of reference and is equal to c, that is it is independent of relative motion of the inertial frame, the Source and observer.

Lorentz Transformation Equations—

The Galilean transformation equations are not Suitable under the new Concept of Special theory of relativity. Where the Speed of the object or observer is Comparable with the velocity of light, therefore, Galilean transformation Equations must be replaced by new ones Consistent with Experiment.

The Equations relating the coordinates of a particle in two inertial frame are needed when one goes from one inertial frame to another in uniform motion. New transformation Equations ~~are~~ were discovered by Lorentz and are Know as Lorentz transformation Equations of Space and time.



Let us suppose a system of two inertial frame of reference S and S'. Let S' is moving with uniform velocity $\vec{v}$ relative to S and S is at rest. Let two observers situated at O and O' are observing any event P.

The event P is determined by the coordinates x, y, z, t for an observer O on the stationary frame S, while the same event is determined by the coordinates x', y', z', t' for an observer O' on the moving frame S'.

In new transformation, the measurement in X-direction made in frame S must be linear proportional to that made in S'.
(not equal same as Galilean)

$$x' = \gamma(x - vt) \quad \text{——} \quad (1)$$

γ is proportionality constt.

and
$$x = \gamma(x' + vt') \quad \text{——} \quad (2)$$

Substituting the value of x' from eqn (1) in eqn (2)

$$x = \gamma\left[\gamma(x - vt) + vt'\right]$$

$$\frac{x}{\gamma} = \gamma x - \gamma vt + vt'$$

or
$$t' = \frac{x}{\gamma v} - \frac{\gamma x}{v} + \gamma t$$

Similarly, we can achieve

$$t = \gamma t' + \frac{\gamma x'}{v}\left(1 - \frac{1}{\gamma^2}\right) \qquad (4)$$

The value of γ can be evaluated with the help of second postulate.

Let a flash of light is emitted from the common origin of S and S' at $t = t' = 0$. The flash travels with the velocity of light c which is same in both frame.

After some time ~~the position~~ the position of flash as seen from the observers in S and S' is given by

$$x = ct \quad \text{and} \quad x' = ct'$$

Put the value of x and x' in Eqn (1) and (2)

$$ct' = \gamma(ct - vt) \quad \text{and} \quad ct = \gamma(ct' + vt')$$

$$ct' = \gamma t(c - v) \quad \text{and} \quad ct = \gamma t'(c + v)$$

Multiplying both these with each other.

$$c^2 tt' = \gamma^2 tt'(c^2 - v^2)$$

$$\gamma^2 = \frac{c^2}{c^2 - v^2} = \frac{c^2}{c^2(1 - v^2/c^2)} = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad (5)$$

Substituting the value of γ in Eqn ① we get

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{——— ⑥}$$

But $\gamma^2 = \dfrac{1}{1 - \frac{v^2}{c^2}}$

or $\quad 1 - \dfrac{v^2}{c^2} = \dfrac{1}{\gamma^2}$

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}$$

Substituting the value of $1 - \frac{1}{\gamma^2}$ in Eqn ③ and ④

$$t' = \gamma t - \frac{\gamma x}{v}\left(\frac{v^2}{c^2}\right)$$

$$= \gamma t - \gamma \frac{xv}{c^2}$$

$$t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

$$t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{——— ⑦}$$

In case where flash of light is not restricted to travel along X-axis, we have

$$y' = y \text{ and } z' = z \quad \text{——— ⑧}$$

Eqn ⑥, Eqn ⑦ and Eqn ⑧ are known as Lorentz transformation equations for space and time.

Inverse Lorentz Transformation Equations —

The Space and time Coordinates (x, y, z, t) of an event in the Stationary System S Can be obtained from the Coordinates (x', y', z', t') in the moving System S' by replacing v by $-v$ and by interchanging the primed and unprimed Co-ordinates in Equations ⑥, ⑦ and ⑧. The resultant transformation Equations are

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t = \frac{t' + x'v/c^2}{\sqrt{1 - v^2/c^2}}, \quad y = y'$$

$$\text{and } z = z'$$

These Equations are identical with Lorentz transformation Equations and are Called Inverse Lorentz transformation Equations.

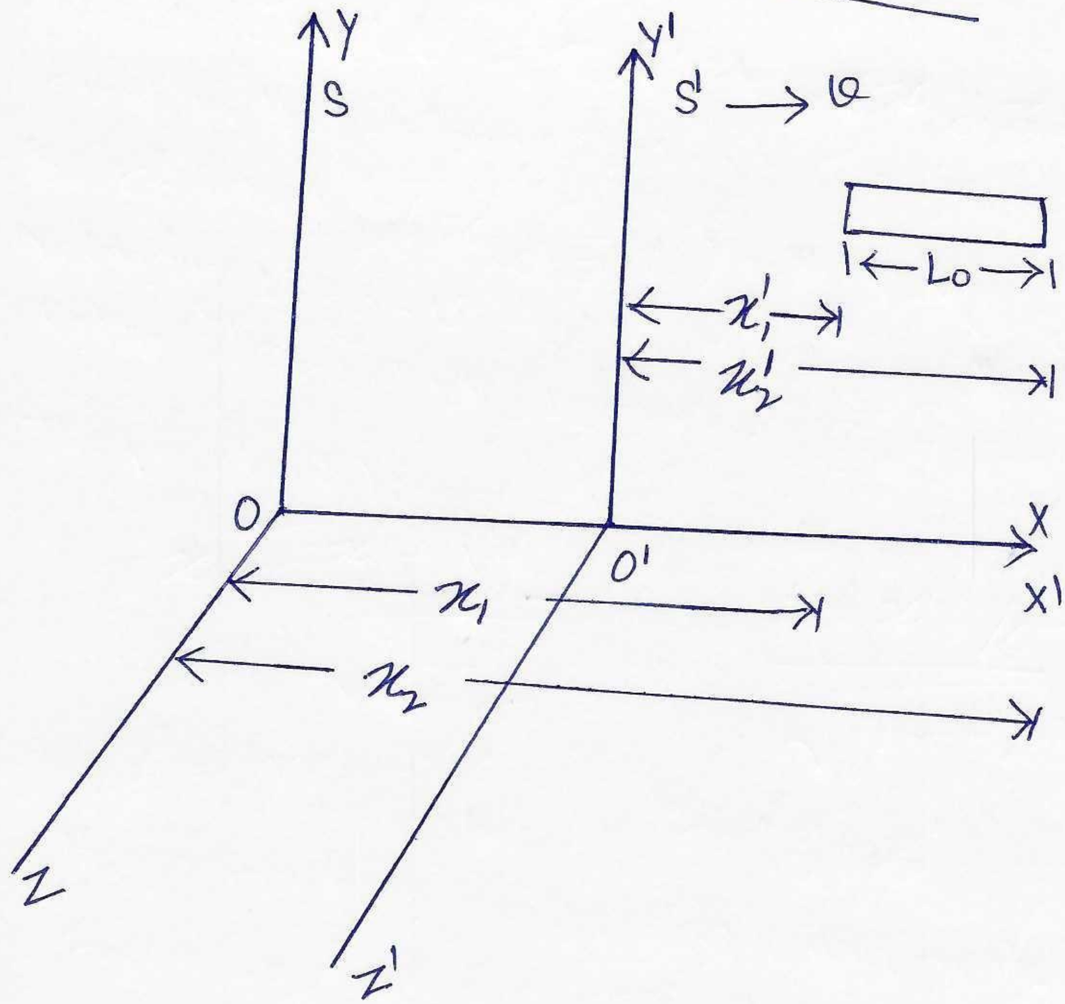
** The most Significant Conclusion of Lorentz transformations is that it limits the maximum Velocity of the material bodies. According to the Conclusion, ~~noting~~ nothing can move with a velocity greater than the velocity of light, c or v Should always be less than c.

The Lorentz transformation Equations reduce to the Classical Galilean transformation Equations when $v \ll c$

i.e $\sqrt{1 - v^2/c^2} = 1$ and $t - \frac{xv}{c^2} = t$

So

Length Contraction (Lorentz – Fitzgerald) —



According to Lorentz-Fitzgerald when an object moves with a velocity v (comparable with the velocity of light) relative to a stationary observer, its measured length appears to be contracted in the direction of its motion by a factor $\sqrt{1-v^2/c^2}$, where as its other dimensions perpendicular to the direction of motion remain unaffected.

<u>To drive</u> Expression for length Contraction —

Let us consider a frame of reference ...
a Uniform ...

in a positive direction of X-axis. A rigid rod of proper length L_o be placed with its length parallel to the X-axis in moving frame s'

[proper length is the length of the rod as measured by an observer at rest with respect to the rod.]

Suppose x_1' and x_2' be the coordinates of two end points of the rod with respect to an observer in frame s'.

So
$$L_o = x_2' - x_1' \quad \text{——— (1)}$$

The length of the rod L measured by an observer in stationary frame s at the same instant will be

$$L = x_2 - x_1 \quad \text{——— (2)}$$

From Lorentz equations for a given value of t $(t=t')$

$$x_2' = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}} \quad \text{and} \quad x_1' = \frac{x_1 - vt}{\sqrt{1-v^2/c^2}}$$

Substituting these values of x_1' and x_2' in eqn (1)

So
$$L_o = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}} - \frac{x_1 - vt}{\sqrt{1-v^2/c^2}}$$

$$L_o = \frac{1\,(x_2 - x_1)}{\sqrt{1-v^2/c^2}}$$

$$L_o = \frac{L}{\sqrt{1-v^2/c^2}}$$

or $L = L_0 \sqrt{1 - v^2/c^2}$ ——— (3)

Since $\sqrt{1 - v^2/c^2} < 1$

So $\boxed{L < L_0}$

Thus the measured length L of the moving rod along the direction of motion is Contracted by a factor $\sqrt{1 - v^2/c^2}$ from its proper length L_0

As the rod more faster, its appears Shorter. If $v = c$ then $L = 0$ i.e. a rod moving with velocity of light will appears as a point to a Stationary observer.

Time Dilation (or Apparent Retardation of Clocks)

A Clock in a moving frame of reference S' measure a Longer time interval between two events that occur in a Stationary frame of reference than the same time interval measured by the Clock in Stationary frame S. This is known as time dilation.

Let us consider initially that the observer O in frame S and O' in frame S' are at rest with respect to each other. They Synchronise their respective clocks and observe that the time interval between any two events measured by their own clocks is the same (when S and S' are at rest).

Let the clock in the stationary frame S gives signals at regular intervals and suppose the frame S' moves to right along the X-axis with a uniform velocity v with respect to S.

The clock in S is situated at a position x and give out signals at two instants of time t_1 and t_2 as measured by an observer in S.

$$t_0 = t_2 - t_1 \qquad \text{——— (1)}$$

Let the time measured by an observer O' in moving frame S' between two events be t_1' and t_2'.

Thus $$t = t_2' - t_1' \qquad \text{——— (2)}$$

From Lorentz transformation equation for time

$$t_1' = \frac{t_1 - xv/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t_2' = \frac{t_2 - xv/c^2}{\sqrt{1 - v^2/c^2}} \qquad \text{—— (3)}$$

So $t = t_2' - t_1'$

$$= \frac{t_2 - xv/c^2}{\sqrt{1 - v^2/c^2}} - \frac{t_1 - xv/c^2}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{t_2 - t_1}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$t_0 = t\sqrt{1 - v^2/c^2} \qquad \text{—— (4)}$$

Since $v < c$, $\sqrt{1 - v^2/c^2} < 1$

Hence $t > t_0$

Thus, the observer O' in S' measure a Longer time interval between two events with his clock at rest with respect to him.

If $v << c$ then v^2/c^2 Can be neglected,

Therefore from eqn (4) $t = t_0$

Hence, the time interval measured by observer in moving clock is Same as when the clock is at rest

if $v = c$

Velocity addition

Suppose a frame of reference s' is moving with uniform velocity v relative to a stationary frame S along the positive direction of X-axis. Suppose a particle is also moving along the positive direction of X-axis.

If particle moves through a distance dx in a time interval dt in the frame S, then the velocity of particle as measured by an observer in stationary frame S is given by.

$$u = \frac{dx}{dt} \quad \text{——— (1)}$$

~~To~~ An observer in moving frame S', the same distance covered by the particle and the time interval taken ~~with~~ will appears different. If the distance covered and time interval taken in S' frame are as dx' and dt'. Then the velocity of the particle in moving frame S' will be

$$u' = \frac{dx'}{dt'} \quad \text{——— (2)}$$

From the Lorentz transformation equations, we have

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{——— (3)}$$

differentiating these

$$dx' = \frac{dx - v\,dt}{\sqrt{1-v^2/c^2}} \quad \text{and} \quad dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1-v^2/c^2}}$$

Substituting these values of dx' and dt' in eqn ②, we have

$$u' = \frac{dx - v\,dt}{dt - \frac{v\,dx}{c^2}}$$

$$= \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

$$\boxed{u' = \frac{u-v}{1-\frac{uv}{c^2}}} \quad \text{——— ④}$$

Eqn ④ represents the relativistic velocity addition formula or relativistic velocity addition theorem.

In classical mechanics it is simply expressed as

$$u' = u - v$$

If a particle has velocity u' in a frame S' which is moving with velocity v relative to an another frame S. The velocity of particle in stationary frame S is obtained by inverse Lorentz transformation obtained by v by $-v$ in eqn ④

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} \qquad \text{——} \; (5)$$

Consistency of Einstein Second postulate ——

If $u' = c$ i.e. moving particle be a photon moving with the velocity of light in positive direction of X-axis. Then the velocity observed by an observer in frame S is

$$u = \frac{c + v}{1 + \frac{cv}{c^2}}$$

$$u = \frac{c(c+v)}{(c+v)}$$

$$u = c$$

Thus observers in frame S' and S record the same value for the velocity of Photon.

Thus the velocity of light c is the same in all inertial frames of reference. This is the second postulate of Einstein special Theory of relativity. Thus we conclude that relativistic velocity addition theorem is consistent with second postulate of special theory of relativity. if $u' = c$ and $v = c$ then

$$u = c+c$$

Mass Energy Equivalence (Einstein's Mass-Energy Relation)—

The mass-Energy equation ($E=mc^2$) is the most famous and most significant relationship obtained by Einstein from the postulates of their special Theory of relativity.

Consider a particle of mass 'm' acted upon by a force 'F' in the same direction as its velocity v.

If the force F displaces the particle through a small distance ds, then work done, dW is stored by the particle as its kinetic energy dK.

So, $$dW = dK = F.ds \quad \text{———} \; (1)$$

But, according to Newton's Law of motion, the force is the rate of change of momentum p, i.e.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} \quad \text{———} \; (2)$$

According to the theory of relativity, mass of the particle varies with velocity.. Hence m and v both are variable in Equation (2).

Therefore,

$$F = \frac{m\,dv}{dt} + \frac{v\,dm}{dt} \quad \text{———} \; (3)$$

Substituting the value of F from eqn (3) in eqn (1)

$$dK = m\frac{dv}{dt}\,ds + v\frac{dm}{dt}\,ds$$

$$= m\frac{ds}{dt}\,dv + v\frac{ds}{dt}\,dm$$

$$dK = m v\,dv + v^2\,dm \qquad \text{———} \; (4)$$

But mass 'm' of the particle moving with velocity v varies in accordance with the relation

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2} \; \text{—} \; (5)$$

Where, m_0 is the rest mass of the particle

differentiating Eqn (5), we get

$$dm = m_0\left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v}{c^2}\,dv\right)$$

$$= \frac{m_0 v\,dv}{c^2\left(1-v^2/c^2\right)^{3/2}}$$

But from Eqn (5) $\quad m_0 = m\left(1-v^2/c^2\right)^{1/2}$

So,

$$dm = \frac{m\left(1-v^2/c^2\right)^{1/2} v\,dv}{c^2\left(1-v^2/c^2\right)^{3/2}}$$

$$dm = \frac{m v dv}{(c^2 - v^2)}$$

or $$m v dv = (c^2 - v^2) dm \quad \text{—— (6)}$$

Put the value of $m v dv$ in eqn (4), we get

$$dK = (c^2 - v^2) dm + v^2 dm$$

$$dK = c^2 dm$$

If the change in KE of particle be K, when its mass changes from rest mass m_0 to effective mass m, then

$$K = \int dK = \int_{m_0}^{m} c^2 dm$$

$$K = (m - m_0) c^2$$

$$K = c^2 \left(\frac{m_0}{\sqrt{1 - v^2/c^2}} - m_0 \right) \quad \text{—— (7)}$$

This is the relativistic expression for kinetic energy of particle. It is clear that increase in KE is due to the increase in the mass of the particle on account of its relative motion and equal to the product of gain in mass and square of the velocity of light.

So, Total Energy of moving particle is

$$E = \text{rest energy} + \text{relativistic K.E.}$$

$$E = m_0 c^2 + (m - m_0) c^2$$

Numerical Problems

Q1 Using Galilean transformation to prove that the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is invariant in two inertial frames.

<u>Sol^n</u>

Suppose a frame of reference S' is moving with velocity v relative to frame S at rest such that $\vec{v} = v_x i + v_y j + v_z k$.

The coordinates of two points in frame S be (x_1, y_1, z_1) and (x_2, y_2, z_2) while in frame S' be (x_1', y_1', z_1') and (x_2', y_2', z_2')

From Galilean transformation

$$x_1' = x_1 - v_x t, \quad y_1' = y_1 - v_y t, \quad z_1' = z_1 - v_z t$$

and $$x_2' = x_2 - v_x t \quad y_2' = y_2 - v_y t \quad z_2' = z_2 - v_z t$$

So, The distance between two points in frame S'

$$d = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}$$

$$d = \sqrt{[(x_2 - v_x t) - (x_1 - v_x t)]^2 + [(y_2 - v_y t) - (y_1 - v_y t)]^2 + [(z_2 - v_z t) - (z_1 - v_z t)]^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

= The distance between the two points in stationary frame S

Hence distance between any two points is invariant under Galilean transformation.

Q2. Show that the space-time interval, $x^2+y^2+z^2-c^2t^2$ is invariant under Lorentz transformation. or

Prove $x^2+y^2+z^2-c^2t^2 = x'^2+y'^2+z'^2-c^2t'^2$

Sol^n. Let (x, y, z, t) and (x', y', z', t') are the coordinates of the same event observed by two observers in stationary frame S and moving fram S'.

The Lorentz transformation equations are

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z \text{ and } t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}}$$

Put the value x', y', z' and t' in $x^2+y^2+z^2-c^2t^2 = x'^2+y'^2+z'^2-c^2t'^2$

$$= \frac{(x - vt)^2}{1 - v^2/c^2} + y^2 + z^2 - \frac{c^2(t - xv/c^2)^2}{1 - v^2/c^2}$$

$$= \frac{1}{1 - v^2/c^2}\left[x^2 + v^2t^2 - 2xvt - c^2\left(t^2 + \frac{x^2v^2}{c^4} - \frac{2xvt}{c^2}\right)\right] + y^2 + z^2$$

$$= \frac{1}{1 - v^2/c^2}\left[x^2 + v^2t^2 - 2xvt - c^2t^2 - \frac{c^2x^2v^2}{c^4} + \frac{2c^2xvt}{c^2}\right] + y^2 + z^2$$

$$= \frac{1}{1 - v^2/c^2}\left[x^2\left(1 - \frac{v^2}{c^2}\right) - c^2t^2\left(1 - \frac{v^2}{c^2}\right)\right] + y^2 + z^2$$

$$= \frac{1}{(1 - v^2/c^2)} \times (1 - v^2/c^2)\left[x^2 - c^2t^2\right] + y^2 + z^2$$

$$= x^2 + y^2 + z^2 - c^2t^2$$

Hence $x^2+y^2+z^2-c^2t^2$ is invariant under Lorentz

Q3 A rod has length 100cm. When the rod is in Satellite with a velocity that is one half of the velocity of light relative to Laboratory. What is the length of rod as determined by an observer

1) in the Satellite 2) in a Laboratory

Soln

1) As the rod is at rest relative to an ~~observer~~ observer in the Satellite. Hence, the length of the rod as determined by an observer in the Satellite is 100 Cm.

2) For an observer in the Laboratory, that is in the Stationary frame, the rod is in the motion. Thus according to Length Contraction

$$L_0 = \frac{L}{\sqrt{1-v^2/c^2}}$$

$$L = L_0 \sqrt{1-v^2/c^2}$$

$$= 100 \sqrt{1-\left(\frac{0.5c}{c}\right)^2}$$

$$= 100 \sqrt{1-0.25}$$

$$= 100 \times .866 \text{ Cm}$$

$$L = 86.6 \text{ Cm}$$

Ans

Q4 What will be the length of a meter rod appear to be for a person travelling parallel to the length of the rod at a speed of 0.8C relative to the rod.

Soln

According to the length contraction formula

$$L = L_0 \sqrt{1 - v^2/c^2}$$

Here $v = 0.8c$ and $L_0 = 1\,m$

So

$$L = 1 \sqrt{1 - \left(\frac{.8c}{c}\right)^2}$$

$$= 1 \sqrt{0.36}$$

$$= 1 \times 0.6\,m$$

$$L = 60\,cm$$ Ans.

Q5 At what Speed Should a Clock be moved so that it may appear to Lose 1 minute in each hour.

Soln let v be the Speed with which a clock moves relative to an observer. if it Loses 1 min in every hour then it must ~~record~~ record 59 min, for 60 minutes recorded by stationary clock with respect to the observer.

According to the time dilation formula

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}}$$

here $t_0 = 59$ min (proper time)
$t = 60$ min (time dilation)

$$\sqrt{1-v^2/c^2} = t_0/t$$

$$1-\frac{v^2}{c^2} = t_0^2/t^2$$

$$v^2/c^2 = 1-t_0^2/t^2$$

$$v = c\sqrt{1-t_0^2/t^2}$$

$$v = 3\times10^8\sqrt{1-\left(\frac{59}{60}\right)^2} = 3\times10^8\sqrt{1-0.967}$$

$$= 3\times10^8\sqrt{0.033} = 3\times10^8\times0.1816$$

$$v = 5.45\times10^7 \text{ m/sec}$$

Q6 The mean life of a meson is 2×10^{-8} sec. Calculate the mean life of meson moving with velocity 0.8C.

Solⁿ If t_0 be the mean life of meson in its own frame and 't' that measured by an observer in stationary frame, then according to the time dilation

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}}$$

$$= \frac{2 \times 10^{-8}}{\sqrt{1-\left(\frac{.8c}{c}\right)^2}}$$

$$= \frac{2 \times 10^{-8}}{\sqrt{1-.64}}$$

$$= \frac{2 \times 10^{-8}}{0.6}$$

$$t = 3.33 \times 10^{-8} \text{ sec.}$$

Ans

Q7 At what speed will the mass of a body be 2.25 times its rest mass.

Soln According to the variation of mass with velocity formula

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Given $m = 2.25 m_0$

therefore

$$2.25 m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\sqrt{1 - v^2/c^2} = \frac{1}{2.25}$$

$$1 - v^2/c^2 = \frac{1}{5.06}$$

$$v^2/c^2 = 1 - \frac{1}{5.06}$$

$$v^2 = c^2 \left(1 - \frac{1}{5.06}\right)$$

$$v = 3 \times 10^8 \sqrt{\frac{4.06}{5.06}}$$

$$v = 2.68 \times 10^8 \text{ m/sec}$$

Ans

Q8 Find the mass and Speed of 2 Mev electron.

Soln The Energy of particle is

$$E = mc^2$$

$$m = E/c^2$$

Given $E = 2\,\text{MeV}$

$$= 2 \times 10^6 \times 1.6 \times 10^{-19} \text{ Joule}$$

$$c = 3 \times 10^8 \text{ m/sec}$$

$$m = \frac{2 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2}$$

$$m = 3.55 \times 10^{-30} \text{ kg}$$ Ans

According to the Variation of mass with Velocity

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

or $$1 - v^2/c^2 = \left(\frac{m_0}{m}\right)^2$$

$$v^2/c^2 = 1 - \left(\frac{m_0}{m}\right)^2$$

$$v = c\sqrt{1 - (m_0/m)^2}$$

$$= 3 \times 10^8 \sqrt{1 - \left(\frac{9.1 \times 10^{-31}}{3.55 \times 10^{-30}}\right)^2}$$

$$= 3 \times 10^8 \times 0.967$$