Exam Roll No. [redacted]

(Please write your Exam Roll No.)

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] MAY- JUNE 2018

Paper Code: ETMA-102	Subject: Applied Mathematics-II
	(Batch 2013 onwards)

Time: 3 Hours | Maximum Marks: 75

Note: Attempt five questions in all including Q.No1 which is compulsory. Select one question from each unit. Use of scientific calculator is allowed.

Q1 (a) If $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$ **(4)**

(b) Form a partial differential equation by eliminating the function f from the relation $f(x+y+z, x^2+y^2+z^2) = 0$. **(4)**

(c) Find the inverse Laplace transform of $\frac{se^{-\frac{s}{2}}+\pi e^{-s}}{s^2+\pi^2}$. **(4)**

(d) Evaluate the following integral by changing the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$. **(4)**

(e) Evaluate $L(t^2 e^t \sin 4t)$ **(4)**

(f) if $\nabla\varphi = (y^2 - 2xyz^3)\,\hat{\imath} + (3 + 2xy - x^2z^3)\,\hat{\jmath} + (6z^3 - 3x^2yz^2)\,\hat{k}$, find φ. **(5)**

UNIT-I

Q2 (a) If $Z = f(x,y)$, where $x = e^u\cos v, y = e^u\sin v$, show that $x\frac{\partial f}{\partial v} + y\frac{\partial f}{\partial u} = e^{2u}\frac{\partial f}{\partial y}$ **(6.5)**

(b) Expand sin (xy) in powers of (x-1) and $(y-\frac{\pi}{2})$, up to and including second degree terms. **(6)**

Q3 (a) Show that the volume of the greatest rectangular parallel piped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ is $\frac{8}{3\sqrt{3}}abc$. **(6.5)**

(b) Solve the partial differential equation $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. **(6)**

UNIT-II

Q4 (a) If $L[f(t)] = f(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty f(s)ds$, provided integral exist. **(6.5)**
Hence evaluate $L\left[\frac{\sin at}{t}\right]$.

(b) Evaluate $L^{-1}\left(\frac{s}{s^4+s^2+1}\right)$. **(6)**

Q5 (a) Using Laplace transform, solve $\frac{d^3x}{dt^3} - 2\frac{d^2x}{dt^2} - 5\frac{dx}{dt} = 0$, given that $x = 0, \frac{dx}{dt} = 1$ at $t = 0$ and $x = 1$ at $t = \frac{\pi}{8}$. **(6.5)**

(b) Using convolution theorem, evaluate $L^{-1}\left(\frac{s^2}{s^4-a^4}\right)$. **(6)**

UNIT-III

Q6 (a) Determine analytic function $f(z) = u + iv$ in terms of z, if $v = \log(x^2+y^2) + x - 2y$. **(6.5)**

(b) Under the transformation $w = \frac{1}{z}, z \neq 0$, find the image of $|z - 2i| = 2$. **(6)**

Q7 (a) If $f(\alpha) = \int_c \frac{3z^2+7z+1}{z-\alpha}$ dz, where C is the circle $x^2+y^2=4$, find the value of $f(3), f^{/}(1-i)$ and $f^{//}(1-i)$. **(6.5)**

(b) Prove that if $a > 0$, then $\int_0^{\infty} \frac{1}{x^4+a^4}\, dx = \frac{\pi\sqrt{2}}{4a^3}$ **(6)**

UNIT-IV

Q8 (a) A fluid motion is given by $\vec{v} = (y+z)\hat{\jmath} + (x+y)\hat{k}$. Is this motion is irrotational? If so find the velocity potential. Is the motion possible for incompressible fluid? **(6.5)**

(b) Apply Stoke's theorem to evaluate $\int_c [(x+y)dx + (2x-z)dy + (y+z)dz]$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0.6). **(6)**

Q9 (a) Use Divergence theorem to evaluate $\iint_s \vec{F}\,\hat{n}ds$, where $\vec{F} = x^3\,\hat{\imath} + y^3\hat{\jmath} + z^3\,\hat{k}$ and s is the surface of the sphere $x^2+y^2+z^2=a^2$. **(6.5)**

(b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dzdydx$. (6)

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