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Q) Find the Fourier Transform of

$$x(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}$$

(t) time  $\rightarrow$  continuous  
(a, c, d)

(n) number  $\rightarrow$  discrete  
(b)

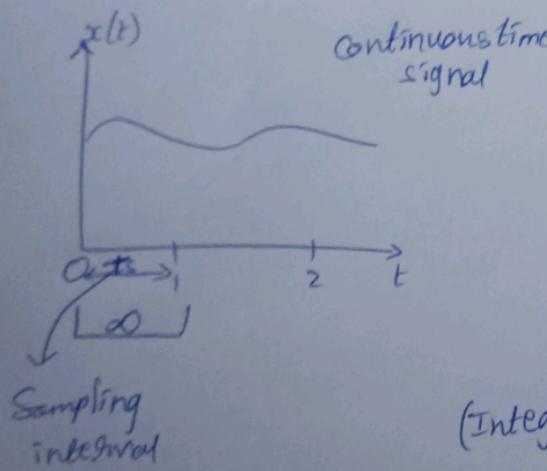
a)  $x(n) = \delta(n)$

b)  $x(t) = 2\pi \operatorname{rect}(t)$

c)  $x(t) = \cos 2\pi(60t) + \sin 2\pi(25t)$

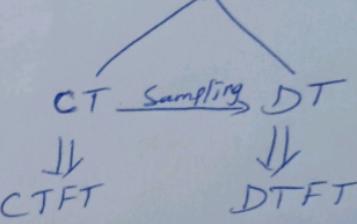
Time  $\rightarrow$  independent

$x(t) \rightarrow$  dependent



Continuous  $\rightarrow$  discrete  
sampling

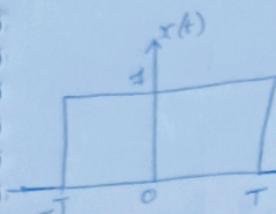
(similarity of the signal with  
Complex Exponential)  
Fourier Transform



$$\begin{aligned}
 & \langle x(t), e^{j\omega t} \rangle \quad \langle x(n), e^{j\omega n} \rangle \\
 & \text{(Integral)} \quad \text{(Sum)} \\
 & \text{inner product} \\
 & \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad \sum_{n=-\infty}^{+\infty} x(n) e^{-j\omega n}
 \end{aligned}$$

$$Q) x(t) = \int_0^1 u(t) dt \quad -T \leq t \leq T$$

otherwise

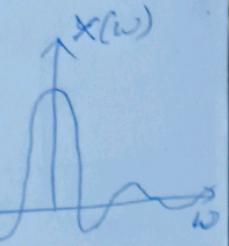


rectangle ABCD

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos \theta$$

$$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin \theta$$

FT



CTFT:-

$$\langle x(t), e^{j\omega t} \rangle = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-T}^{+T} 1 \cdot e^{-j\omega t} dt$$

$$= \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_T^{-T}$$

$$= \frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega}$$

$$= -\left( \frac{e^{-j\omega T} - e^{j\omega T}}{j\omega} \right)$$

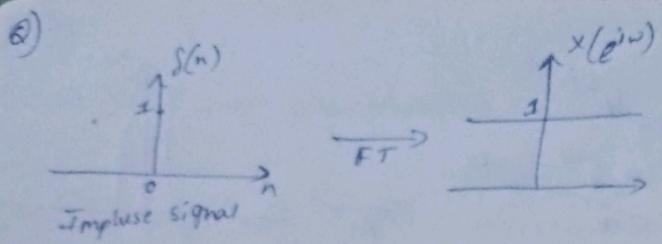
$$= \frac{e^{j\omega T} - e^{-j\omega T}}{j\omega}$$

$$= \frac{2j \sin \omega T}{j\omega}$$

$$= \frac{2 \sin \omega T}{\omega}$$

Multiplying & dividing T

$$= \frac{2T \sin \omega T}{\omega T} = (2T) \operatorname{sinc}(WT)$$



$$x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega} = \sum_{n=0}^{\infty} x(n) e^{-jn\omega} = \sum_{n=0}^{\infty} 1 \cdot 1 = 1.$$

(b)  $x(t) = \cos \omega_0 t$

$$\int x(t) e^{-j\omega t} dt$$

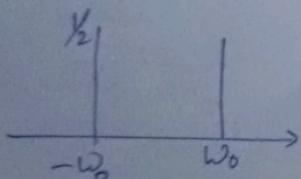
Similarity  $\rightarrow$  Inner product

$$\langle x(t) e^{-j\omega t} \rangle$$

$$\langle \cos \omega_0 t e^{-j\omega t} \rangle$$

$$\left\langle \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} e^{-j\omega t} \right\rangle$$

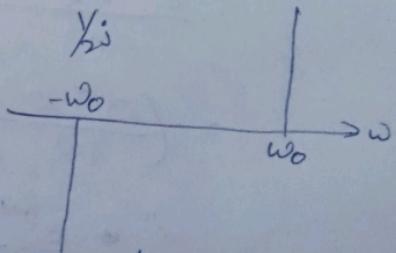
$$\frac{1}{2} \left[ \langle e^{j\omega_0 t} e^{-j\omega t} \rangle + \langle e^{-j\omega_0 t} e^{-j\omega t} \rangle \right]$$



$e^{j2\pi f t}$  angular frequency

$$\langle \left( \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \right) e^{-j\omega t} \rangle$$

$$\frac{1}{2j} \left[ \underbrace{\langle e^{j\omega_0 t} e^{-j\omega t} \rangle}_{\omega_0} - \underbrace{\langle e^{-j\omega_0 t} e^{-j\omega t} \rangle}_{-\omega_0} \right]$$



(c)  $x(t) = \cos 50\pi t$

$$\langle \cos 50\pi t \cdot e^{j2\pi f t} \rangle$$

$$\left\langle \frac{e^{-j50\pi t} + e^{j50\pi t}}{2} \cdot e^{j2\pi f t} \right\rangle$$

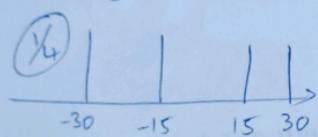
$$\frac{1}{2} \left[ \langle e^{-j50\pi t} e^{j2\pi f t} \rangle + \langle e^{j50\pi t} e^{j2\pi f t} \rangle \right] \Rightarrow$$

$$Q) x(t) = \cos 30\pi t + \cos 60\pi t$$

$$\langle (\cos 30\pi t + \cos 60\pi t) e^{j2\pi ft} \rangle$$

$$\left\langle \left[ \frac{e^{j30\pi t} + e^{-j30\pi t}}{2} \right] + \left[ \frac{e^{j60\pi t} + e^{-j60\pi t}}{2} \right] e^{j2\pi ft} \right\rangle$$

$$\frac{1}{4} \left[ \langle e^{j30\pi t} + e^{-j30\pi t} \rangle e^{j2\pi ft} + \langle e^{j60\pi t} + e^{-j60\pi t} \rangle e^{j2\pi ft} \right]$$



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Q. Quasi Periodic:

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Short Time Fourier Transform (STFT)

$$STFT_p^u(t', u) = \int_{-t}^t [f(t) \cdot w(t-t')] \cdot e^{-j2\pi ut} dt$$

Signal to be analyzed

Time Parameter Frequency Parameter

Windowing function.

centered at  $t=t'$

magnitude  
frequency  
time

Specrogram  
Energy (time, frequency)

\* window is too long  $\rightarrow$  FT

\* window is too short  $\rightarrow$  time resolution is best, frequency resolution is not better.

If window signal increases then in the Fourier transform of a signal we can say the particular frequency.

Overlap can be there in the signal.

Heisenberg (or uncertainty) principle:-

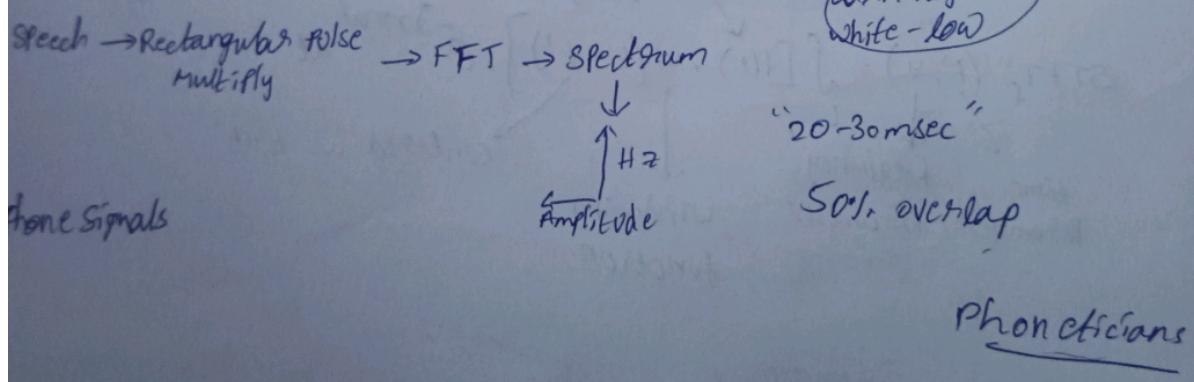
$$\Delta t \cdot \Delta f \geq \frac{1}{4\pi}$$

Time resolution                      Frequency resolution.

$\frac{1}{4\pi}$   
The two dimensional function  $|x(n, \omega)|^2$  is called the spectrogram.

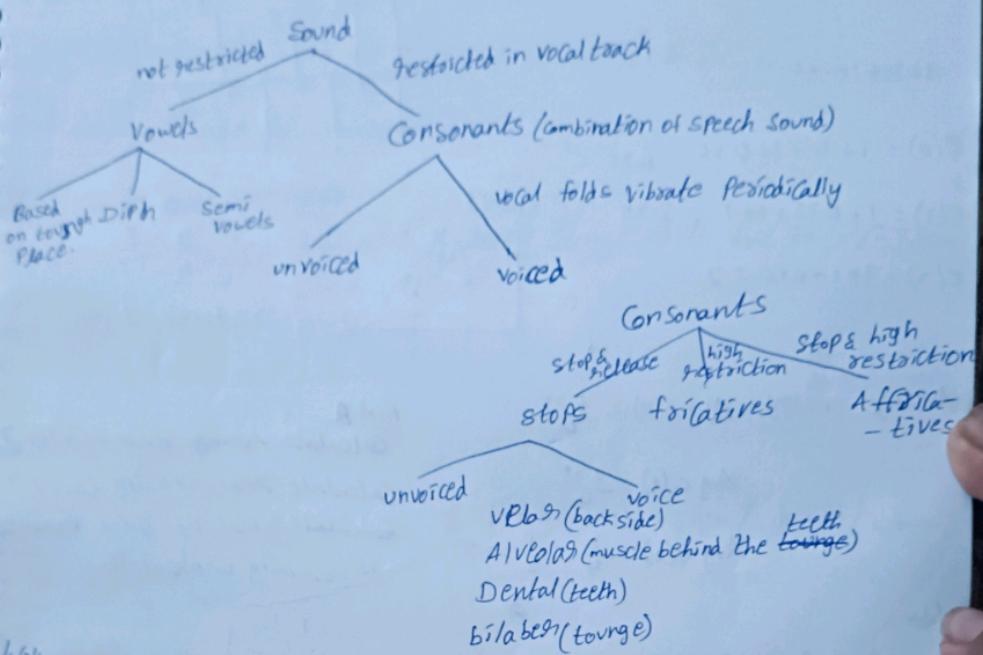
Spectrogram  
Narrow                      wideband  
 $\equiv$                        $\equiv$   
 $\vdots \vdots \vdots \vdots \vdots \vdots$

Formants



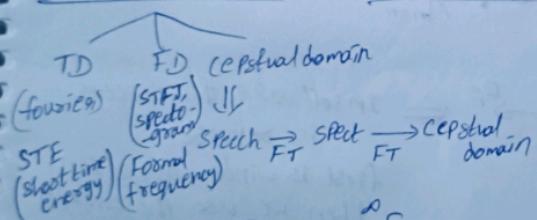
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### Classification of speech, speech Signal characteristics.



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Time domain features into a short time energy :-



$$\text{Energy} = (\text{Signal})^2 \text{ sum of it.}$$

$$E_n = \sum_{m=0}^n [x[m]w[n-m]]^2 \quad \text{It is variance over the time}$$

- Q) For a speech signal sample at 16KHz how many samples will be there inside the analysis window if the window size is taken for 20ms. Overlapping 50%.

A) 320 Samples.  $\frac{320}{2} = 160$

Q) Find the short time energy for the signal given in the figure  
 windowsize  $\rightarrow$  4, overlap  $\rightarrow$  50%.

$$1+4+1+1+0.25$$

$$e(0) = 1+4+1+1+0.25 = 6.25$$

$$e(1) = 1+0.25+1+1 = 3.25$$

$$e(2) = 1+1+0+0 = 2$$

Average energy =  $\text{Avg } e(0) = \frac{6.25}{4}$

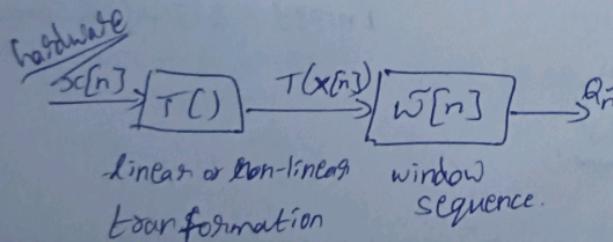
$$\text{Avg } e(1) = \frac{3.25}{4}$$

$$\text{Avg } e(2) = \frac{2}{4} = \frac{1}{2} = 0.5$$

isolate

Short time Processing:-

$$Q_n = \left( \sum_{m=-\infty}^{\infty} T(x[m]) \hat{w}[n-m] \right)_{n=\hat{n}}$$



Part A:-

Calculate Average magnitude?

Calculate zero crossing count?

Calculate repeat all these parameters if hamming window is

In software, this process is reverse.

first is window function then we use the energy function.

Short time energy - (voice & no-voice signal)  
 average Magnitude

Zero Crossing

AutoCorrelation.

Pitch/Fundamental frequency

magnitude = 1 through the signal then it is known as rectangular window.

Hamming window (Raised Cosine window) :-

$$h[n] = 0.54 - 0.46 \cos\left(2\pi n/(N-1)\right) \quad 0 \leq n \leq N-1$$

Note:-

Each frame should be 20-30ms

Short time Average Magnitude:-

\* Short time energy is very sensitive to large signal levels due to  $x^2[n]$  term

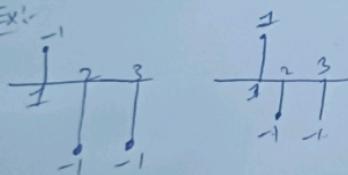
$$M_n = \sum_{m=0}^{N-1} |x[m]| \bar{w}[n-m]$$

Zero Crossing:-

$$\text{Sign}(n) = \begin{cases} 1 & x(n) > 0 \\ -1 & x(n) < 0 \end{cases}$$

$$Z = \frac{1}{2N} \sum_{n=0}^{N-1} |\text{sign}(s(n)) - \text{sign}(s(n-1))| / w(n-m)$$

Ex:-



$[1 - (-1)]$  for 1-2 sec = 2 (where there is an zero crossing)  
 $[1 - (-1)]$  for 2-3 sec = 0 (No zero crossing)

\* zero crossing are more in unvoiced regions.

