UNIT-II The modelling process, proportionality and geometric Similarity

To gain an understanding of the process involved in mathematical modeling, Consider the two words i.e. real world and mathematical world.

① Explain real world and mathematical world involved in the process of M-M

The real & mathematical worlds

Real world system

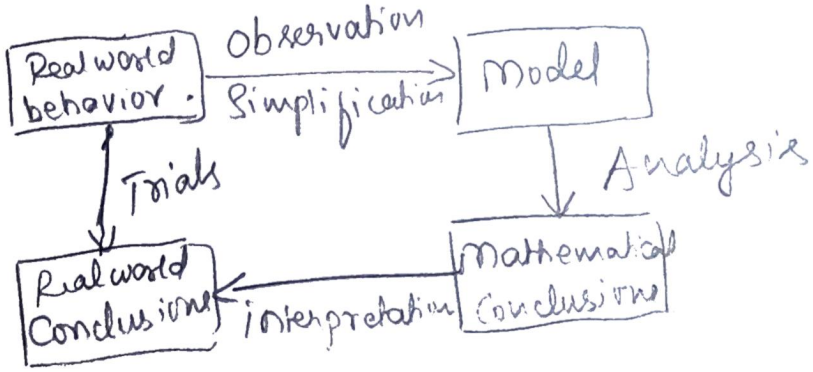
observed behavior or phenomenon

Mathematical world

Models
Mathematical operations & rules
Mathematical Conclusions

A system is an assemblage of objects joined in some regular interaction or interdependence. The modeler is interested in understanding how a particular system works, what causes changes in the system and how sensitive system is to certain changes. He or She is also interested in predicting what changes might occur and when they occur. How might such information be obtained?

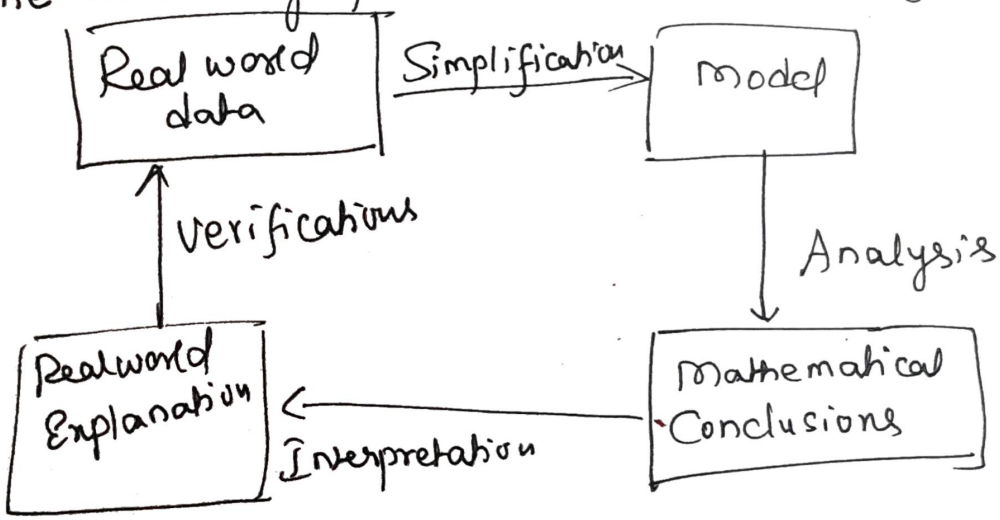
our goal is to draw conclusions about an observed phenomenon in the real world. one procedure would be to conduct experiment and observe their effect on the real world behavior. This is depicted as follows



Because we made some simplications in constructing the model and because the observations on which the model is based invariably contain errors and limitations, we must carefully account for these anomalies before drawing any inferences about the real world behavior ∴ we have the following rough modeling procedure.

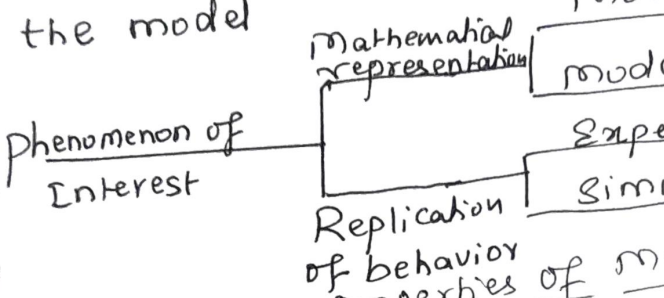
1. Through observations, identify the primary factors involved in the real world behavior, possibly making simplifications
2. Conjecture tentative relationship among the factors
3. Apply mathematical analysis to the resultant model.
4. Interpret mathematical conclusions interms of real world problem.

The modelling process as a closed system



Mathematical models

Mathematical model is defined as mathematical construct designed to study a particular real world system or phenomenon. The nature of the model

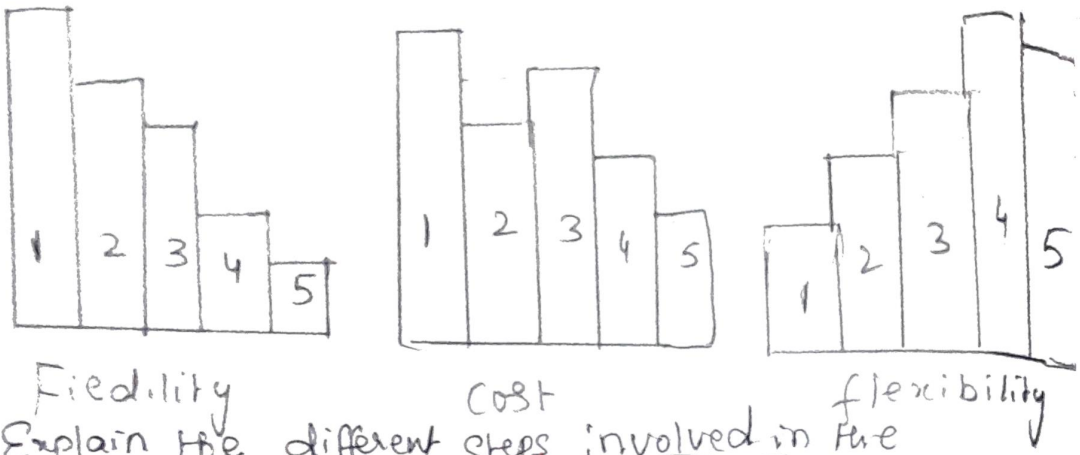


(2) Explain the properties of mathematical model we consider the following properties of a model

1. Fidelity : The preciseness of a model's representation of reality
2. Costs : The total cost of the modelling process
3. Flexibility : The ability to change and control conditions affecting the model as required data are gathered.

It is useful to know the degree to which a given model possesses each of these characteristics. The comparisons are depicted below where the ordinate axis denotes the degree of effectiveness of each class.

1. Real world observations
2. Experiments
3. Simulations
4. Constructed models
5. Selected models



Explain the different steps involved in the Construction of models

Construction of M.M

The following steps are involved in the Construction of models.

Step 1: Identify the model

Step 2: Make assumptions

a) identify and classify the variables

b) Determine interrelationships between the variables and submodels.

Step 3: Solve the model

Step 4: Verify the model

a) Does it address the problem?

b) Does it make common sense?

c) Test it will real world data

Step 5: Implement the model

Step 6: Maintain the model.

Maintain the model: model is derived from a specific problem identified in step 1 and from the assumptions made in step 2. Has the original problem changed in anyway or have some previously

neglected factors become important?
Does one of the submodels need to be adjusted?

(3) Modeling Using proportionality

Two variables x and y are said to be proportional if one is constant multiple of the other.

$x \propto y$ $\quad y = kx$.

if $x \propto y$, $y \propto z \Rightarrow x \propto z$

(4) Name any

we now consider the famous physical laws which follow the rule of proportionality

Hooke's law: $F = ks$, where F is the restoring force in a spring stretched or compressed a distance s.

Newton's law: $F = ma$, where a is the acceleration of a mass m subjected to a net External force F.

Ohm's law: $V = IR$, where I is the current induced by a voltage V across a resistance R.

Boyle's law: $V = k/p$ where under a constant temperature k the volume V is inversly proportional to the pressure p.

Einstein's theory of relativity:

$E = Mc^2$, where under the constant speed of light squared c^2 the energy E is

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proportional to the mass M of the object.

Kepler's third law:

$T = CR^{3/2}$ where T is the period (days) and R is the mean distance to the Sun.

If a spring is stretched 0.37 inch by a 14 lb force, what stretch will be produced by a 9-lb force? By a 22-lb force?
Assume Hooke's law which asserts the distance stretched is proportional to the force applied.

Hooke's law $F = kS$

$$14 = k(0.37) \Rightarrow k = \frac{14}{0.37} = 37.83$$

the stretch produced by a 9-lb force is

$$F = kS = (37.83)(9) = 340.47$$

$$F = kS = (37.83)(22) = 832.26$$

$$S = \frac{F}{k} = \frac{9}{37.83} = 0.2379 \quad S = \frac{22}{37.83} = 0.5815 //$$

A new planet is discovered beyond pluto at a mean distance to the Sun of 4004 million miles. Using kepler's third law, determine an estimate for the time T to travel around the Sun in an orbit.

$$T = CR^{3/2}$$

$$365 = C(93)^{3/2} \Rightarrow C = \frac{365}{(93)^{3/2}} = 0.406$$

$$T = CR^{3/2}$$

$$T = 0.406\,(4004)^{3/2} = 102864.88$$