Burglary-Alarm example

Finding P (Burglary|John Calls)?

$$\mathrm{P(B \mid J)} = \frac{P(JB)}{P(J)}$$

Now we need to find P(JB)

$$P(JB) = P(JBA) + P(JBA')$$

$$= P(J|BA).P(AB) + P(J|BA').P(A'B)$$

$$= P(J|A).P(AB) + P(J|A').P(A'B)$$

$$= 0.9 \times P(AB) + 0.05 \times P(A'B) \quad \text{---------- (1)}$$

Computing P(Burglary|John Calls)

P(Burglary|John Calls) = ???

$$P(AB) = P(ABE) + P(ABE')$$

$$= P(A|BE)P(BE) + P(A|BE')P(BE')$$

$$= 0.95 \times 0.001 \times 0.002 + 0.95 \times 0.001 \times 0.998$$

$$P(AB) = 0.00095 \quad \text{---------- (2)}$$

Similarly P (A'B) = 0.00005 ---------- (3)

P(JB) = 0.9×0.00095 + 0.05×0.00005 = 0.00086

$$P(B|J) = \frac{P(JB)}{P(J)}$$

$$= \frac{0.00086}{P(J)}$$

Now let's compute P(J)

$$P(J) = P(JA) + P(JA')$$

$$= P(J|A)P(A) + P(J|A')P(A')$$

$$= 0.90 \times 0.002526422 + 0.05 \times P(A') \quad \text{---------- (4)}$$

Now we compute P(A)

$$P(A) = P(ABE) + P(ABE') + P(AB'E) + P(AB'E')$$

$$= P(A|BE)P(BE) + P(A|BE')P(BE') + P(A|B'E)P(B'E) + P(A|B'E')P(B'E')$$

$$= 0.95 \times 0.001 \times 0.002 + 0.95 \times 0.001 \times 0.998 + 0.29 \times 0.999 \times 0.002 + 0.001 \times 0.999 \times 0.998$$

$$= 0.002526422$$

Similarly P (A')

$$P(A') = P(A'BE) + P(A'BE') + P(A'B'E) + P(A'B'E')$$

$$= P(A'|BE)P(BE) + P(A'|BE')P(BE') + P(A'|B'E)P(B'E) + P(A'|B'E')P(B'E')$$

$$= 0.05 * 0.001 * 0.002 + 0.05 * 0.001 * 0.998 + 0.71 * 0.999 * 0.002 + 0.999 * 0.999 * 0.998$$

$$= 0.997473$$

Putting the above value in eqation (4), we get

$$P(J) = 0.90 \times 0.002526422 + 0.05 \times 0.997473$$

$$\boldsymbol{P(J) = 0.0521474}$$

Putting P(J) for computing P(B|J)

$$P(B|J) = \frac{P(JB)}{P(J)} = \frac{0.00086}{0.0521474} = 0.01649$$