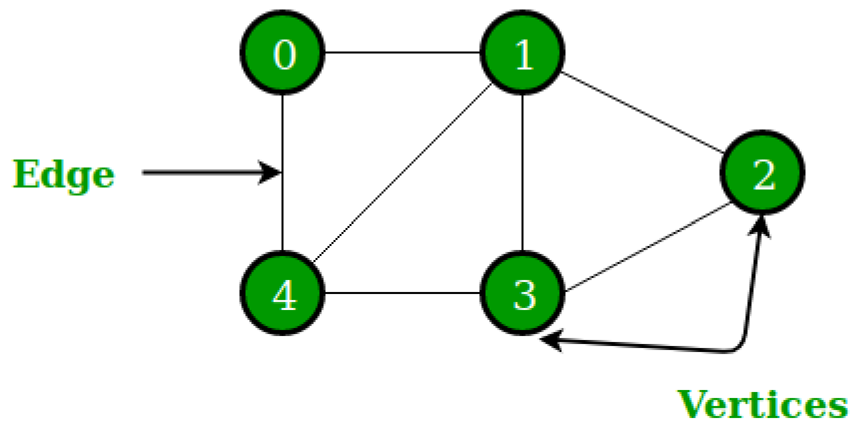
Graphs

Graph- A Graph is a non-linear data structure consisting of nodes and edges. The nodes are sometimes also referred to as vertices and the edges are lines or arcs that connect any two nodes in the graph. More formally a Graph can be defined as,

“A Graph consists of a finite set of vertices (or nodes) and set of Edges which connect a pair of nodes.”



Types of Graphs

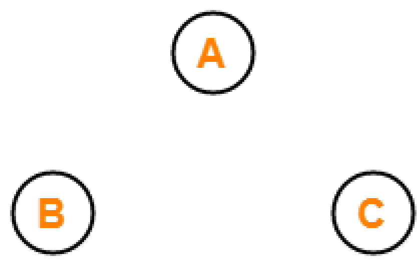
1. Null Graph
2. Trivial Graph
3. Non-directed Graph
4. Directed Graph
5. Connected Graph
6. Disconnected Graph
7. Regular Graph
8. Complete Graph
9. Cycle Graph
10. Cyclic Graph
11. Acyclic Graph
12. Finite Graph
13. Infinite Graph
14. Bipartite Graph
15. Planar Graph
16. Simple Graph

17. Multi Graph
18. Pseudo Graph
19. Euler Graph
20. Hamiltonian Graph

1. Null Graph-

- A graph whose edge set is empty is called as a null graph.
- In other words, a null graph does not contain any edges in it.

Example-



Example of Null Graph

Here,

- This graph consists only of the vertices and there are no edges in it.
- Since the edge set is empty, therefore it is a null graph.

2. Trivial Graph-

- A graph having only one vertex in it is called as a trivial graph.
- It is the smallest possible graph.

Example-

Example of Trivial Graph

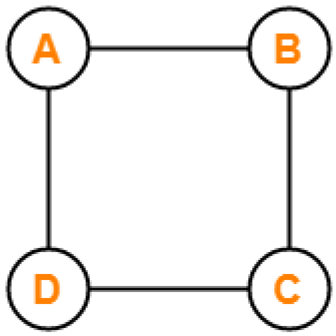
Here,

- This graph consists of only one vertex and there are no edges in it.
- Since only one vertex is present, therefore it is a trivial graph.

3. Non-Directed Graph-

- A graph in which all the edges are undirected is called as a non-directed graph.
- In other words, edges of an undirected graph do not contain any direction.

Example-



Example of Non-Directed Graph

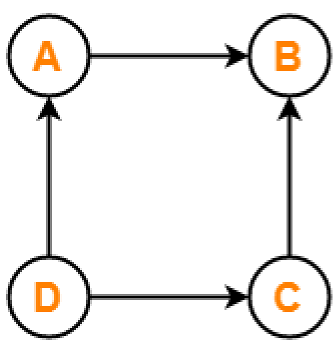
Here,

- This graph consists of four vertices and four undirected edges.
- Since all the edges are undirected, therefore it is a non-directed graph.

4. Directed Graph-

- A graph in which all the edges are directed is called as a directed graph.
- In other words, all the edges of a directed graph contain some direction.
- Directed graphs are also called as **digraphs**.

Example-



Example of Directed Graph

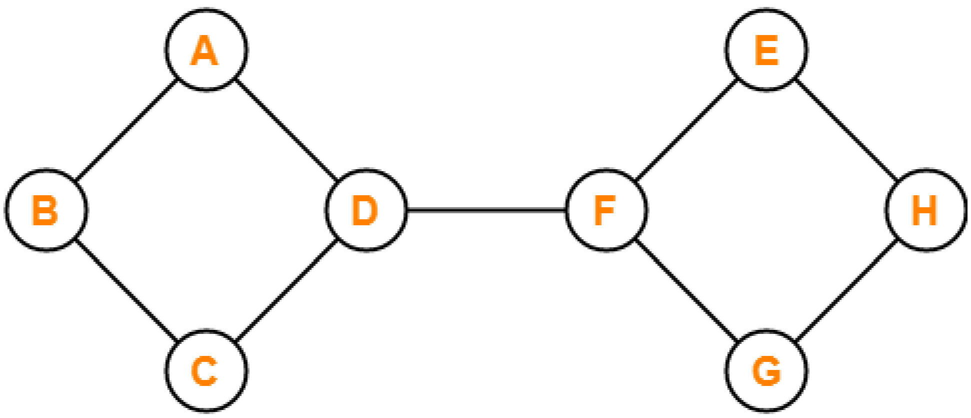
Here,

- This graph consists of four vertices and four directed edges.
- Since all the edges are directed, therefore it is a directed graph.

5. Connected Graph-

- A graph in which we can visit from any one vertex to any other vertex is called as a connected graph.
- In connected graph, at least one path exists between every pair of vertices.

Example-



Example of Connected Graph

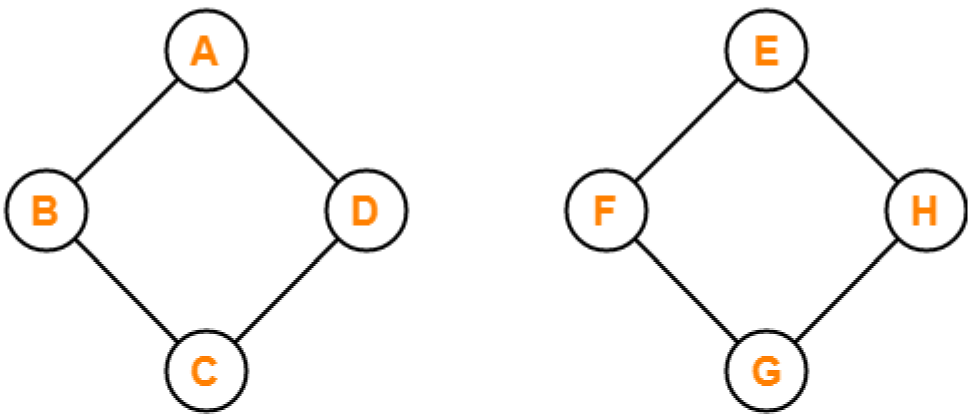
Here,

- In this graph, we can visit from any one vertex to any other vertex.
- There exists at least one path between every pair of vertices.
- Therefore, it is a connected graph.

6. Disconnected Graph-

- A graph in which there does not exist any path between at least one pair of vertices is called as a disconnected graph.

Example-



Example of Disconnected Graph

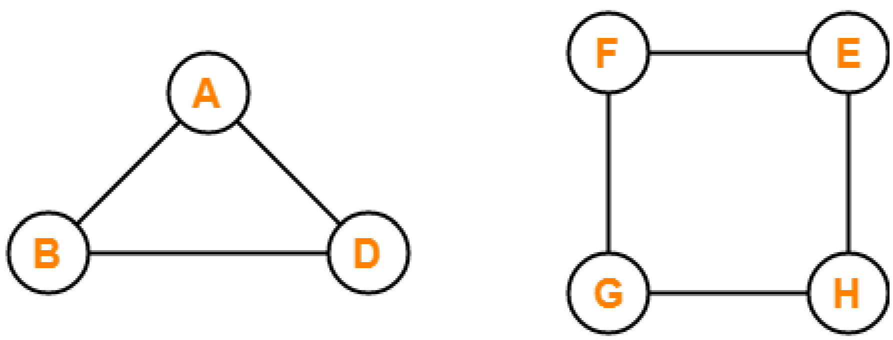
Here,

- This graph consists of two independent components which are disconnected.
- It is not possible to visit from the vertices of one component to the vertices of other component.
- Therefore, it is a disconnected graph.

7. Regular Graph-

- A graph in which degree of all the vertices is same is called as a regular graph.
- If all the vertices in a graph are of degree ‘k’, then it is called as a “**k-regular graph**“.

Examples-



Examples of Regular Graph

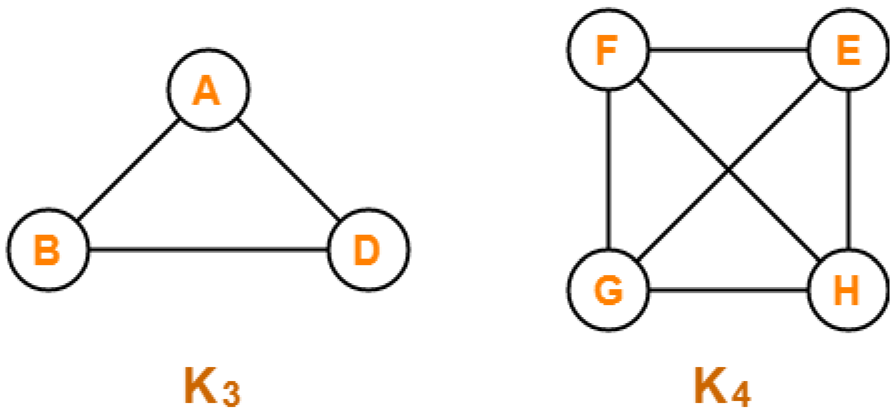
In these graphs,

- All the vertices have degree-2.
- Therefore, they are 2-Regular graphs.

8. Complete Graph-

- A graph in which exactly one edge is present between every pair of vertices is called as a complete graph.
- A complete graph of 'n' vertices contains exactly ${}^{n}C_2$ edges.
- A complete graph of 'n' vertices is represented as $\mathbf{K_n}$.

Examples-



Examples of Complete Graph

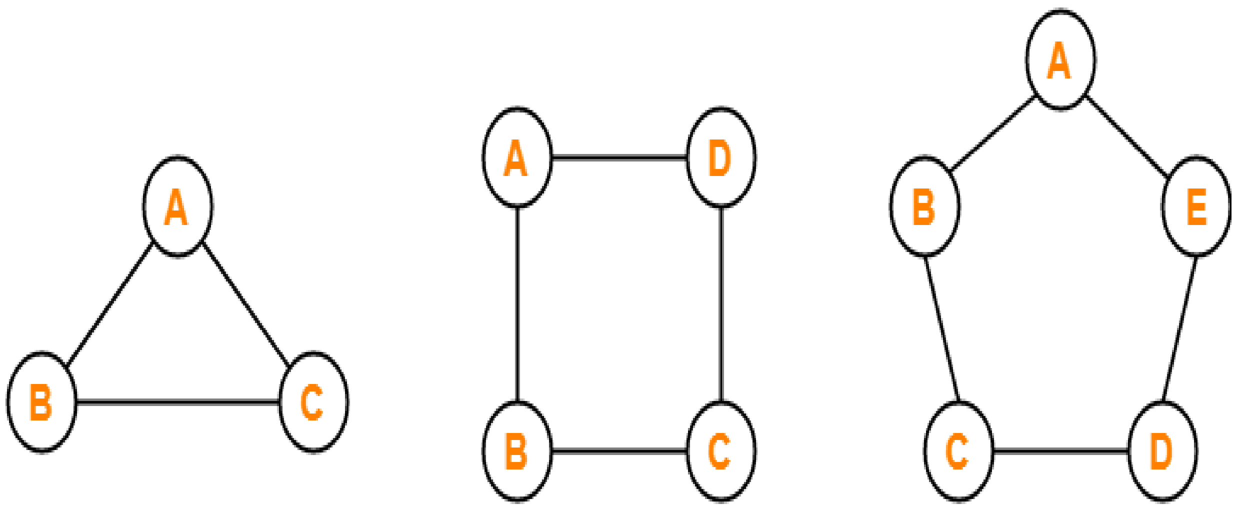
In these graphs,

- Each vertex is connected with all the remaining vertices through exactly one edge.
- Therefore, they are complete graphs.

9. Cycle Graph-

- A simple graph of ‘n’ vertices (n>=3) and n edges forming a cycle of length ‘n’ is called as a cycle graph.
- In a cycle graph, all the vertices are of degree 2.

Examples-



Examples of Cycle Graph

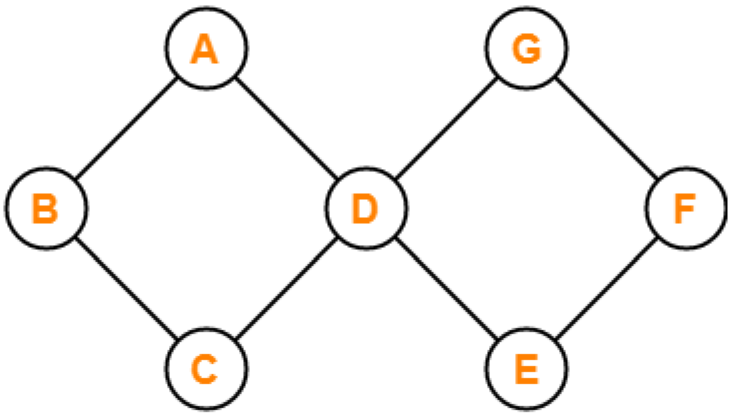
In these graphs,

- Each vertex is having degree 2.
- Therefore, they are cycle graphs.

10. Cyclic Graph-

- A graph containing at least one cycle in it is called as a cyclic graph.

Example-



Example of Cyclic Graph

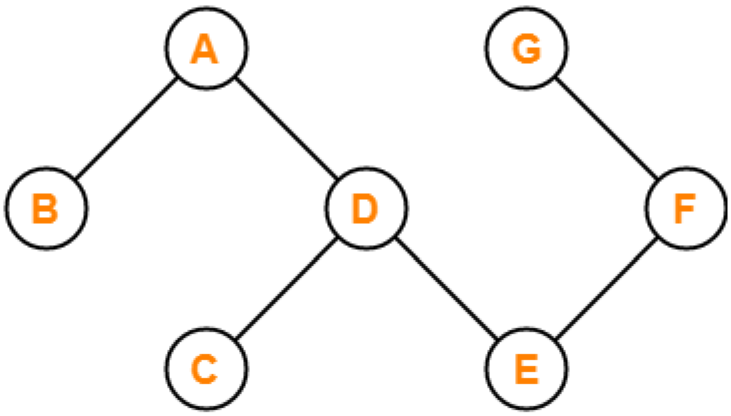
Here,

- This graph contains two cycles in it.
- Therefore, it is a cyclic graph.

11. Acyclic Graph-

- A graph not containing any cycle in it is called as an acyclic graph.

Example-



Example of Acyclic Graph

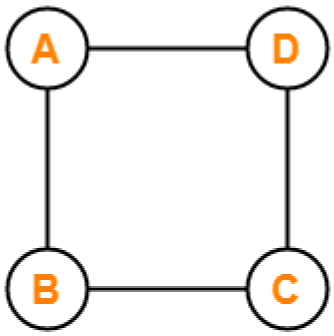
Here,

- This graph do not contain any cycle in it.
- Therefore, it is an acyclic graph.

12. Finite Graph-

- A graph consisting of finite number of vertices and edges is called as a finite graph.

Example-



Example of Finite Graph

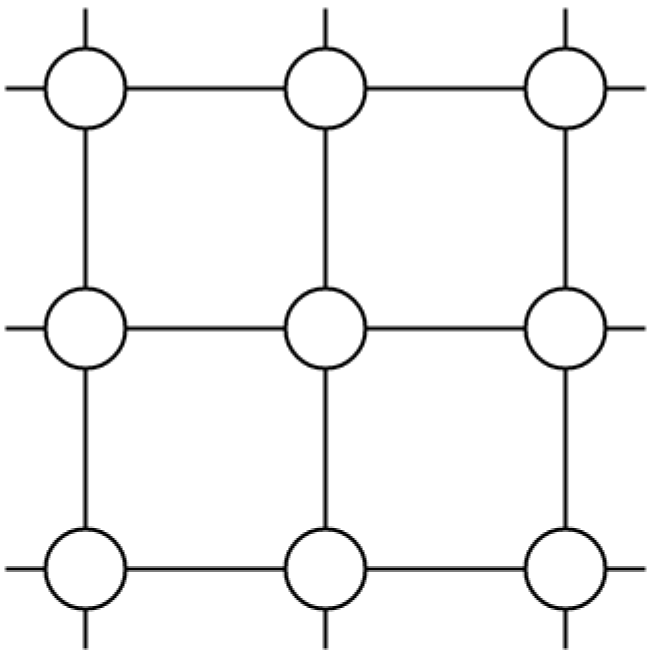
Here,

- This graph consists of finite number of vertices and edges.
- Therefore, it is a finite graph.

13. Infinite Graph-

- A graph consisting of infinite number of vertices and edges is called as an infinite graph.

Example-



Example of Infinite Graph

Here,

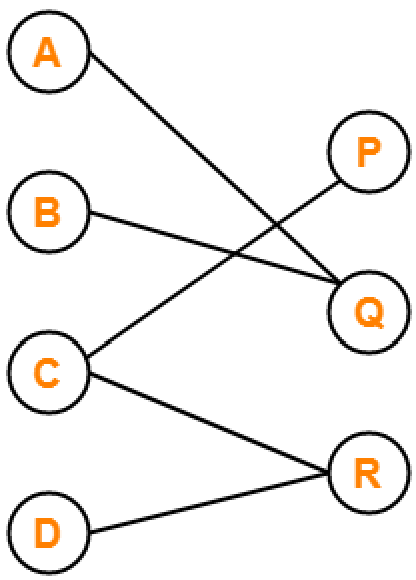
- This graph consists of infinite number of vertices and edges.
- Therefore, it is an infinite graph.

14. Bipartite Graph-

A bipartite graph is a graph where-

- Vertices can be divided into two sets X and Y.
- The vertices of set X only join with the vertices of set Y.
- None of the vertices belonging to the same set join each other.

Example-



Example of Bipartite Graph

Complete Bipartite Graph-

A complete bipartite graph may be defined as follows-

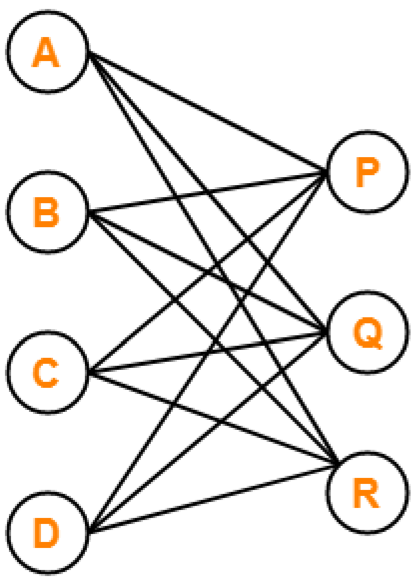
A bipartite graph where every vertex of set X is joined to every vertex of set Y is called as complete bipartite graph. **OR** Complete bipartite graph is a bipartite graph which is complete.

OR

Complete bipartite graph is a graph which is bipartite as well as complete.

Complete Bipartite Graph Example-

The following graph is an example of a complete bipartite graph-



Example of Complete Bipartite Graph

Here,

- This graph is a bipartite graph as well as a complete graph.
- Therefore, it is a complete bipartite graph.
- This graph is called as $\mathbf{K_{4,3}}$.

Bipartite Graph Chromatic Number-

To properly color any bipartite graph,

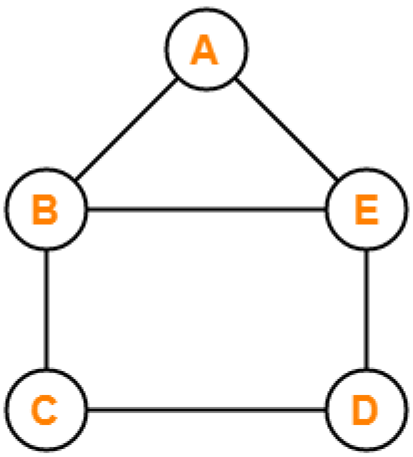
- Minimum 2 colors are required.

- This ensures that the end vertices of every edge are colored with different colors.
- Thus, bipartite graphs are 2-colorable.

15. Planar Graph-

- A planar graph is a graph that we can draw in a plane such that no two edges of it cross each other.

Example-



Example of Planar Graph

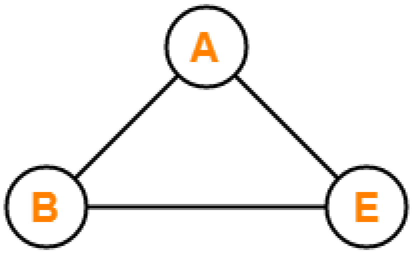
Here,

- This graph can be drawn in a plane without crossing any edges.
- Therefore, it is a planar graph.

16. Simple Graph-

- A graph having no self loops and no parallel edges in it is called as a simple graph.

Example-



Example of Simple Graph

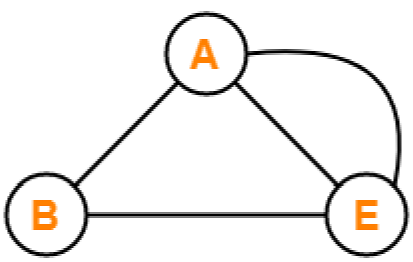
Here,

- This graph consists of three vertices and three edges.
- There are neither self-loops nor parallel edges.
- Therefore, it is a simple graph.

17. Multi Graph-

- A graph having no self-loops but having parallel edge(s) in it is called as a multi graph.

Example-



Example of Multi Graph

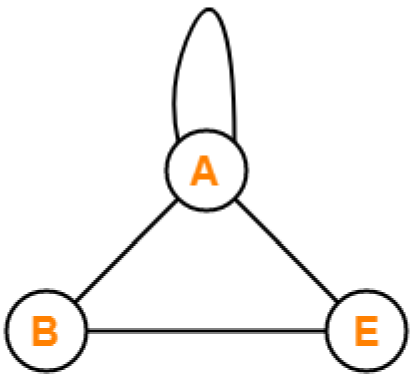
Here,

- This graph consists of three vertices and four edges out of which one edge is a parallel edge.
- There are no self-loops but a parallel edge is present.
- Therefore, it is a multi-graph.

18. Pseudo Graph-

- A graph having no parallel edges but having self-loop(s) in it is called as a pseudo graph.

Example-



Example of Pseudo Graph

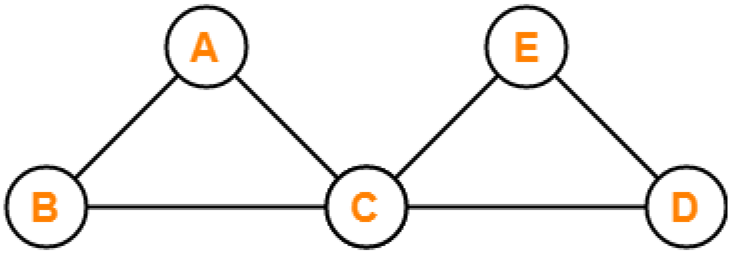
Here,

- This graph consists of three vertices and four edges out of which one edge is a self-loop.
- There are no parallel edges but a self-loop is present.
- Therefore, it is a pseudo graph.

19. Euler Graph-

- Euler Graph is a connected graph in which all the vertices are even degree.

Example-



Example of Euler Graph

Here,

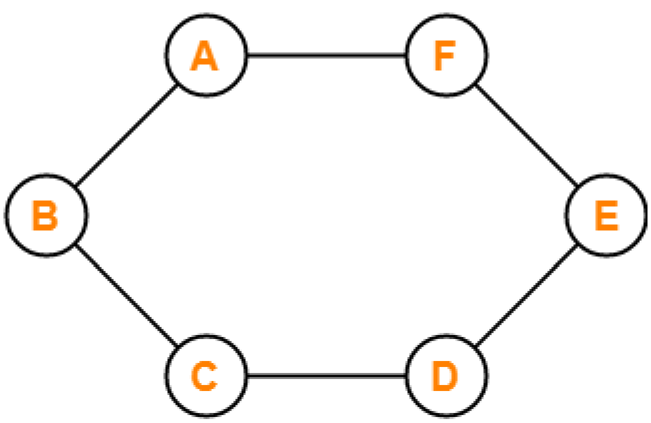
- This graph is a connected graph.
- The degree of all the vertices is even.
- Therefore, it is an Euler graph.

Read More- **Euler Graphs**

20. Hamiltonian Graph-

- If there exists a closed walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges, then such a graph is called as a Hamiltonian graph.

Example-



Example of Hamiltonian Graph

Here,

- This graph contains a closed walk ABCDEFG that visits all the vertices (except starting vertex) exactly once.
- All the vertices are visited without repeating the edges.
- Therefore, it is a Hamiltonian Graph.

Important Points-

- Edge set of a graph can be empty but vertex set of a graph cannot be empty.
- Every polygon is a 2-Regular Graph.
- Every complete graph of ‘n’ vertices is a (n-1)-regular graph.
- Every regular graph need not be a complete graph.

Remember-

The following table is useful to remember different types of graphs-

	Self-Loop(s)	**Parallel Edge(s)**
Graph	Yes	Yes
Simple Graph	No	No
Multi Graph	No	Yes
Pseudo Graph	Yes	No

Applications of Graph Theory-

Graph theory has its applications in diverse fields of engineering-

1. Electrical Engineering-

- The concepts of graph theory are used extensively in designing circuit connections.
- The types or organization of connections are named as topologies.
- Some examples for topologies are star, bridge, series and parallel topologies.

2. Computer Science-

Graph theory is used for the study of algorithms such as-

- **Kruskal's Algorithm**
- **Prim's Algorithm**
- **Dijkstra's Algorithm**

3. Computer Network-

The relationships among interconnected computers in the network follow the principles of graph theory.

4. Science-

Following structures are represented by graphs-

- Molecular structure of a substance
- Chemical structure of a substance
- DNA structure of an organism etc

5. Other Applications-

- Routes between the cities are represented using graphs.
- Hierarchical ordered information such as family tree are represented using special types of graphs called trees.

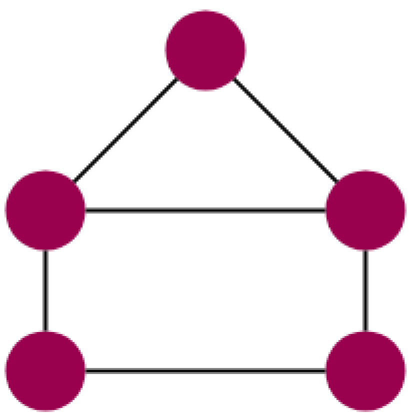
Complement Of Graph-

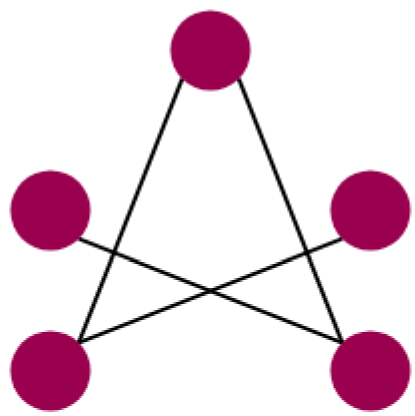
Complement of a simple graph G is a simple graph G’ having-

- All the vertices of G.
- An edge between two vertices v and w iff there exists no edge between v and w in the original graph G.

Complement Of Graph Example-

The following example shows a graph G and its complement graph G’-





Graph G

Complement Graph $\overline{G}$

Relationship Between G & G’-

The following relationship exists between a graph G and its complement graph G’-

1. Number of vertices in G = Number of vertices in G’.

|V(G)| = |V(G’)|

2. The sum of total number of edges in G and G’ is equal to the total number of edges in a complete graph.

$$|E(G)| + |E(G')|$$

$$= C(n,2)$$

$$= n(n-1) / 2$$

where n = total number of vertices in the graph

Important Terms-

It is important to note the following terms-

- **Order of graph** = Total number of vertices in the graph
- **Size of graph** = Total number of edges in the graph

Problem-01:

A simple graph G has 10 vertices and 21 edges. Find total number of edges in its complement graph G’.

Solution-

Given-

- Number of edges in graph G, $|E(G)| = 21$
- Number of vertices in graph G, $n = 10$

We know $|E(G)| + |E(G')| = n(n-1) / 2$.

Substituting the values, we get-

$21 + |E(G')| = 10 \times (10-1) / 2$

$|E(G')| = 45 – 21$

$\therefore |E(G')| = 24$

Thus, Number of edges in complement graph G’ = 24.

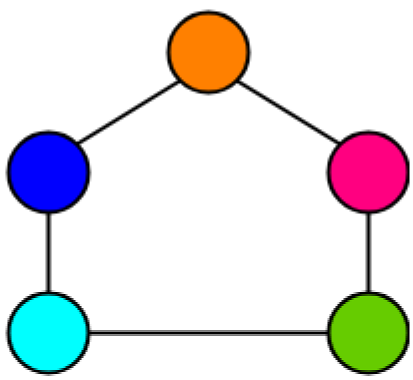
Graph Coloring-

> Graph Coloring is a process of assigning colors to the vertices of a graph
>
> such that no two adjacent vertices of it are assigned the same color.

- Graph Coloring is also called as **Vertex Coloring**.
- It ensures that there exists no edge in the graph whose end vertices are colored with the same color.
- Such a graph is called as a **Properly colored graph**.

Graph Coloring Example-

The following graph is an example of a properly colored graph-



Graph Coloring

In this graph,

- No two adjacent vertices are colored with the same color.
- Therefore, it is a properly colored graph.

Graph Coloring Applications-

Some important applications of graph coloring are as follows-

- Map Coloring
- Scheduling the tasks
- Preparing Time Table
- Assignment
- Conflict Resolution
- Sudoku

Chromatic Number-

Chromatic Number is the minimum number of colors required to properly color any graph.

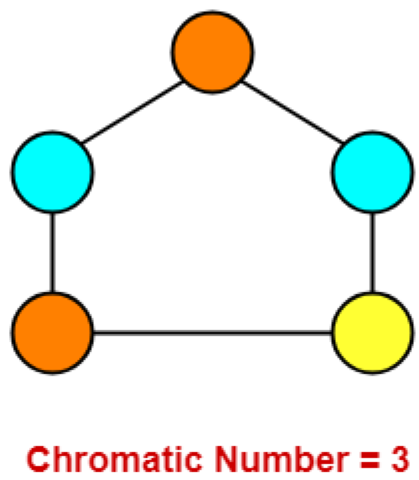
OR

Chromatic Number is the minimum number of colors required to color any graph

such that no two adjacent vertices of it are assigned the same color.

Chromatic Number Example-

Consider the following graph-



In this graph,

- No two adjacent vertices are colored with the same color.
- Minimum number of colors required to properly color the vertices = 3.
- Therefore, Chromatic number of this graph = 3.
- We cannot properly color this graph with less than 3 colors.

\

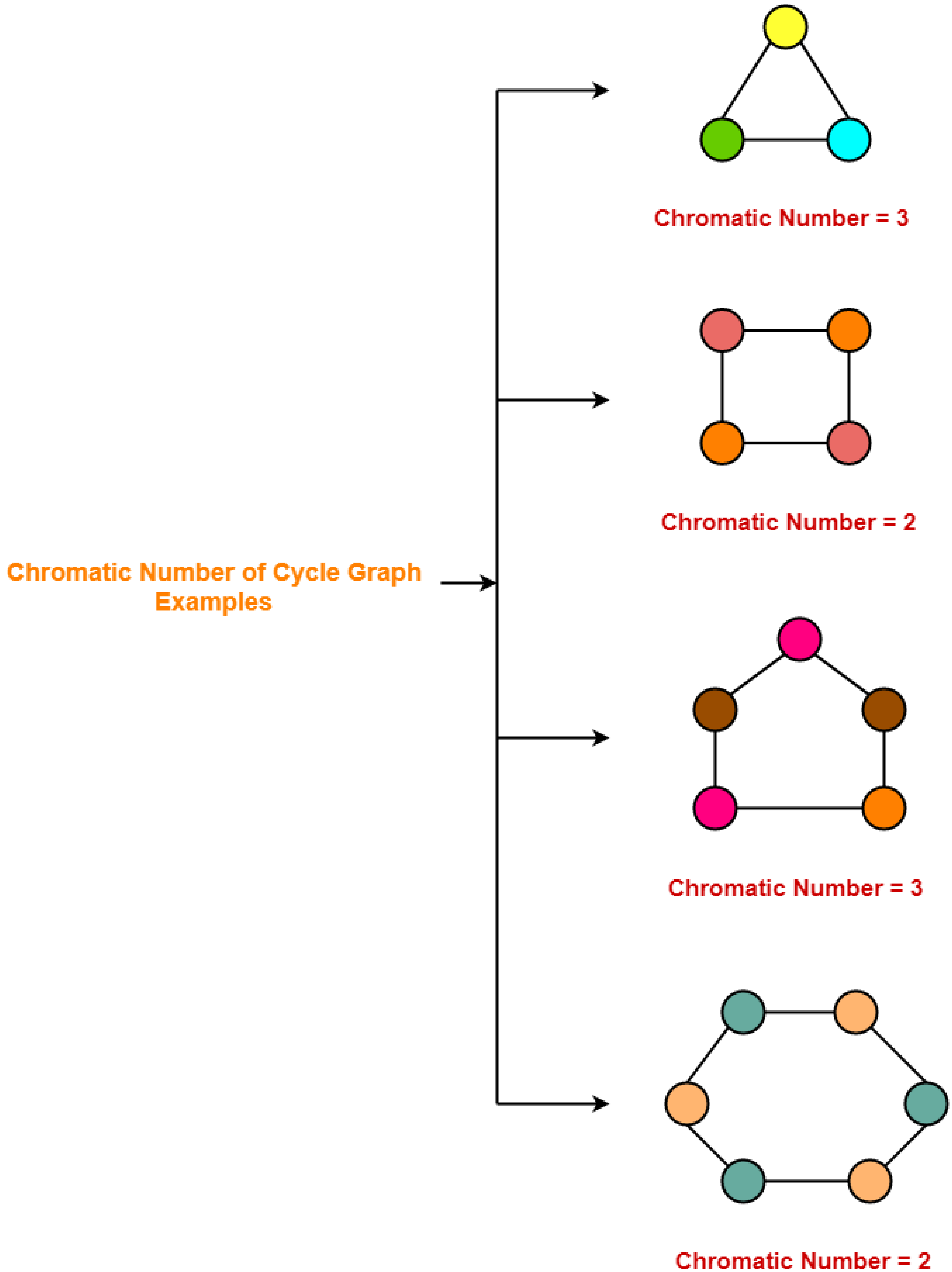
Chromatic Number Of Graphs-

Chromatic Number of some common types of graphs are as follows-

1. Cycle Graph-

- A simple graph of ‘n’ vertices (n>=3) and ‘n’ edges forming a cycle of length ‘n’ is called as a cycle graph.
- In a cycle graph, all the vertices are of degree 2.

Chromatic Number
• If number of vertices in cycle graph is even, then its chromatic number = 2. • If number of vertices in cycle graph is odd, then its chromatic number = 3.


Chromatic Number of Cycle Graph Examples
Chromatic Number = 3
Chromatic Number = 2
Chromatic Number = 3
Chromatic Number = 2

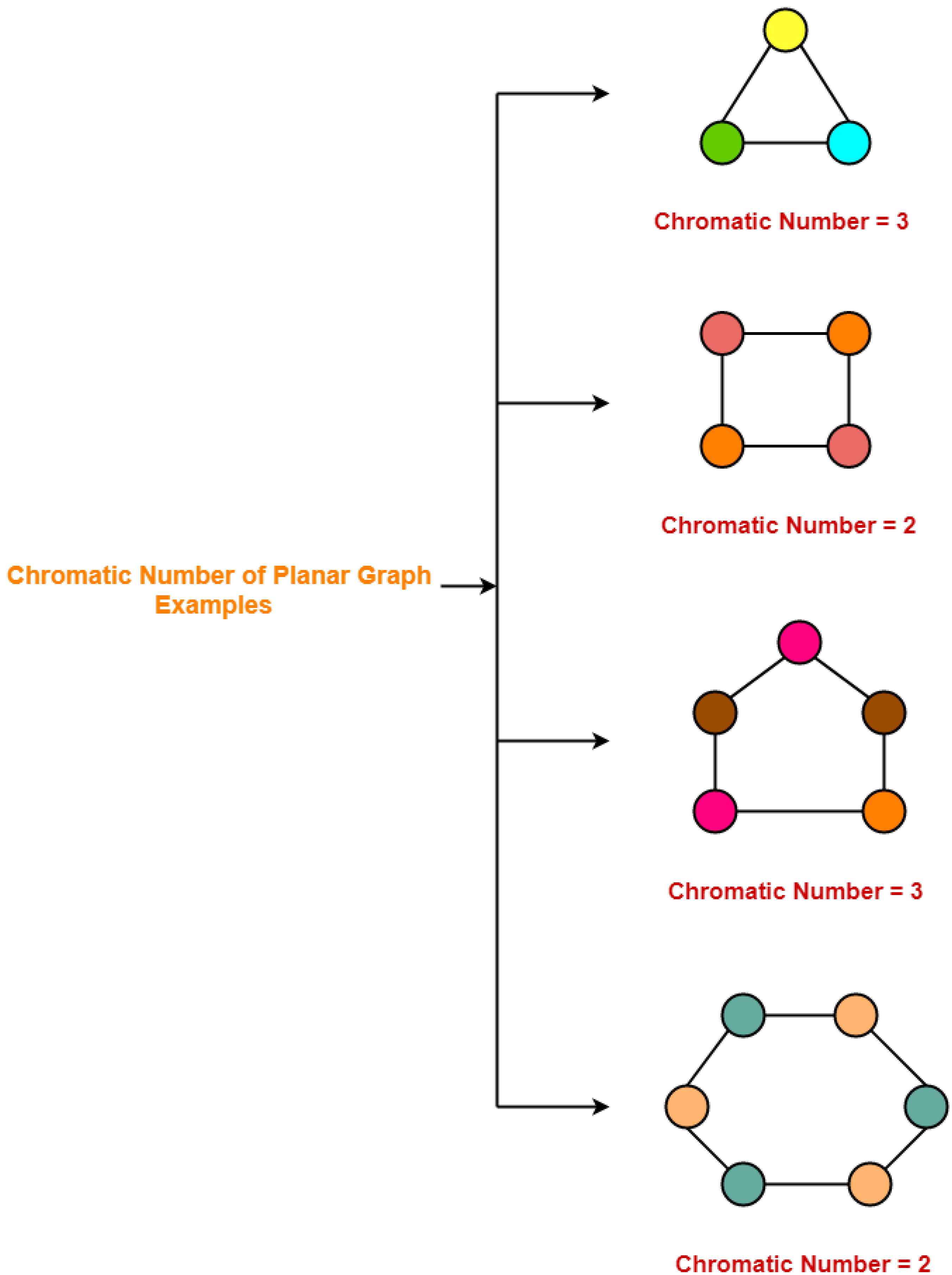
2. Planar Graphs-

A **Planar Graph** is a graph that can be drawn in a plane such that none of its edges cross each other.

Chromatic Number
Chromatic Number of any Planar Graph
= Less than or equal to 4

Examples-

- All the above cycle graphs are also planar graphs.
- Chromatic number of each graph is less than or equal to 4.


Chromatic Number of Planar Graph Examples
Chromatic Number = 3
Chromatic Number = 2
Chromatic Number = 3
Chromatic Number = 2

3. Complete Graphs-

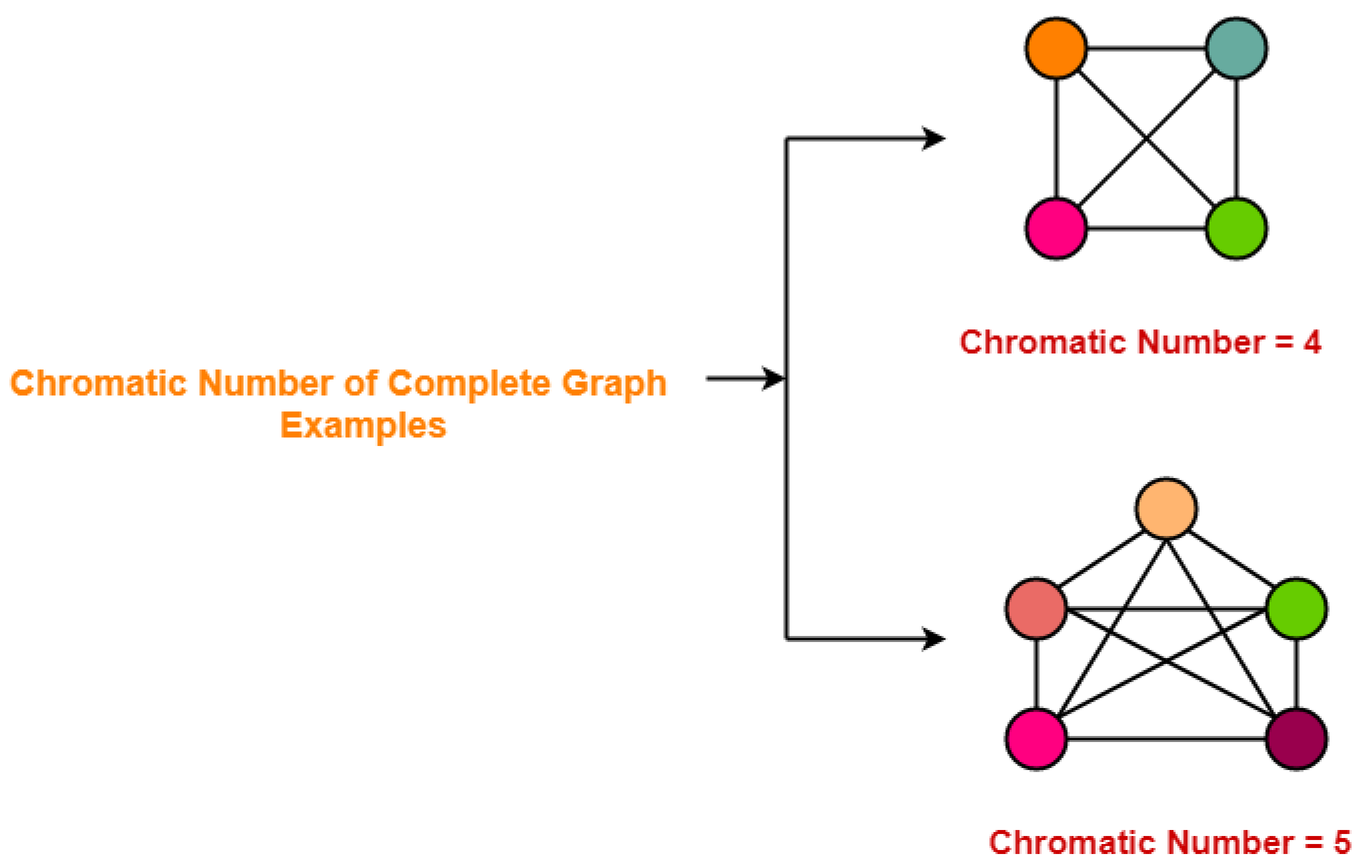
- A complete graph is a graph in which every two distinct vertices are joined by exactly one edge.
- In a complete graph, each vertex is connected with every other vertex.
- So to properly it, as many different colors are needed as there are number of vertices in the given graph.

Chromatic Number

Chromatic Number of any Complete Graph

= Number of vertices in that Complete Graph

Examples-

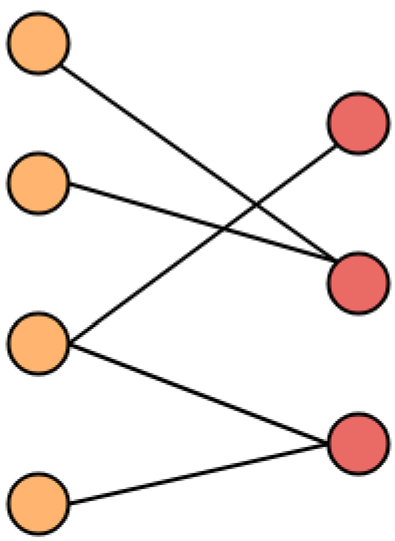


4. Bipartite Graphs-

- A **Bipartite Graph** consists of two sets of vertices X and Y.
- The edges only join vertices in X to vertices in Y, not vertices within a set.

Chromatic Number
Chromatic Number of any Bipartite Graph
= 2

Example-



Chromatic Number = 2

5. Trees-

- A **Tree** is a special type of connected graph in which there are no circuits.
- Every tree is a bipartite graph.
- So, chromatic number of a tree with any number of vertices = 2.

Chromatic Number

Chromatic Number of any tree

= 2

Examples-