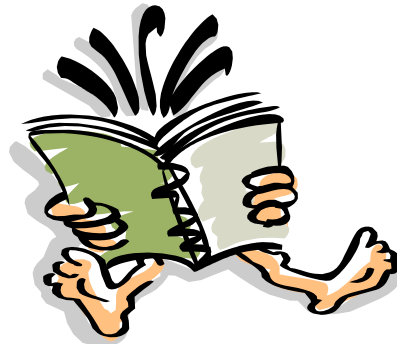


# BINARY SEARCH ALGORITHM

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Ref Book: Computer Algorithms , By Sartaj Sahni



# Binary Search Algorithm

- Can only be performed on a sorted (non decreasing order) list !!!
- Uses *divide and conquer* technique to search list

# Binary Search Algorithm

- Search item is compared with middle element of list
- If search item  $<$  middle element of list, search is restricted to first half of the list
- If search item  $>$  middle element of list, search second half of the list
- If search item  $=$  middle element, search is complete

# Binary Search Algorithm

- BinSearch(a,i,l,x)  
//a[i:l] –sorted array - Whether x is present or not  
{  
    if (l==i) {  
        if (x==a[i])  
            return i;  
        else  
            return 0;  
    }  
    else{  
        mid=(i+l)/2;  
        if (x==a[mid])  
            return mid;  
        else if (x<a[mid])  
            return BinSearch(a,l,mid-1,x);  
        else  
            rerun BinSearch(a,mid+1,l,x);  
    }  
}

# Binary Search Algorithm

- Problem  $P$  is divided into one sub problem.
- Division takes only  $O(1)$  time.
- Answer to subproblem is answer to original problem  $P$  ; there is no need for any combining.

# Binary Search

Example: sorted array of integer keys. Target=7.

[ 0 ]	[ 1 ]	[ 2 ]	[ 3 ]	[ 4 ]	[ 5 ]	[ 6 ]
3	6	7	11	32	33	53

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Find approximate midpoint

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Is 7 = midpoint key? NO.



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3	6	7	11	32	33	53




Is  $7 < \text{midpoint key}$ ? YES.

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[ 0 ]	[ 1 ]	[ 2 ]	[ 3 ]	[ 4 ]	[ 5 ]	[ 6 ]
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Search for the target in the area before midpoint.

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Target < key of midpoint? NO.

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Find approximate midpoint.

Is target = midpoint key? YES.



# Binary Search

- Element at middle element requires minimum comparison

100	110	125	150	170	190	200	210
-----	-----	-----	-----	-----	-----	-----	-----

- [0]      [1]      [2]      [3]      [4]      [5]      [6]      [7]

- Target =210

Low	high	middle	
0	7	3	150
4	7	5	190
6	7	6	200
7	7	7	210 <-Target

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- Element at middle element requires minimum comparison

100	110	125	150	170	190	200	210
-----	-----	-----	-----	-----	-----	-----	-----

- [0]      [1]      [2]      [3]      [4]      [5]      [6]      [7]

- Target =205

Low	high	middle	
0	7	3	150
4	7	5	190
6	7	6	200
7	7	7	210 <-Target

# Binary Search

- Successful Search
    - Best case:  $\Theta(1)$
    - Average and worst case =  $\Theta(\log_2 n)$
  - Unsuccessful Search
    - $\Theta(\log_2 n)$
- If  $n$  (no of elements) is in the range  $[2^{k-1}, 2^k)$  then binarysearch makes at most  $K$  element comparisons for a successfully search and either  $k-1$  or  $k$  comparisons for unsuccessfully search. ( $k=\log_2 n$ )*

# Binary Search

Recurrence Relation for worst and average:

$$T(N) = \begin{cases} \Theta(1) & N == 1 \\ T\left(\frac{N}{2}\right) + 1 & \text{otherwise} \end{cases}$$

- Solve this:

- Ans:  $\Theta(\log_2 N)$

# Binary Search

*It means !!!*

*Let's say Tycho-2 star catalog contains information about the brightest **2,539,913** stars in our galaxy. Suppose that you want to search the catalog for a particular star, based on the star's name.*

*If the program examined every star in the star catalog in order starting with the first (an algorithm called **linear search**), the computer might have to examine all **2,539,913** stars to find the star you were looking for, in the worst case.*

*If the catalog were sorted alphabetically by star names, **binary search** would not have to examine more than **22 stars**, even in the worst case.*