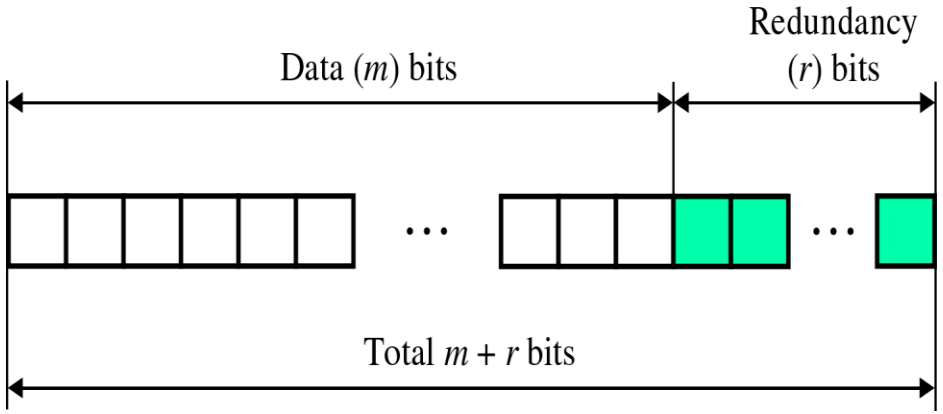


Data link Layer
Error Detection and Correction

Error Detection and Correction

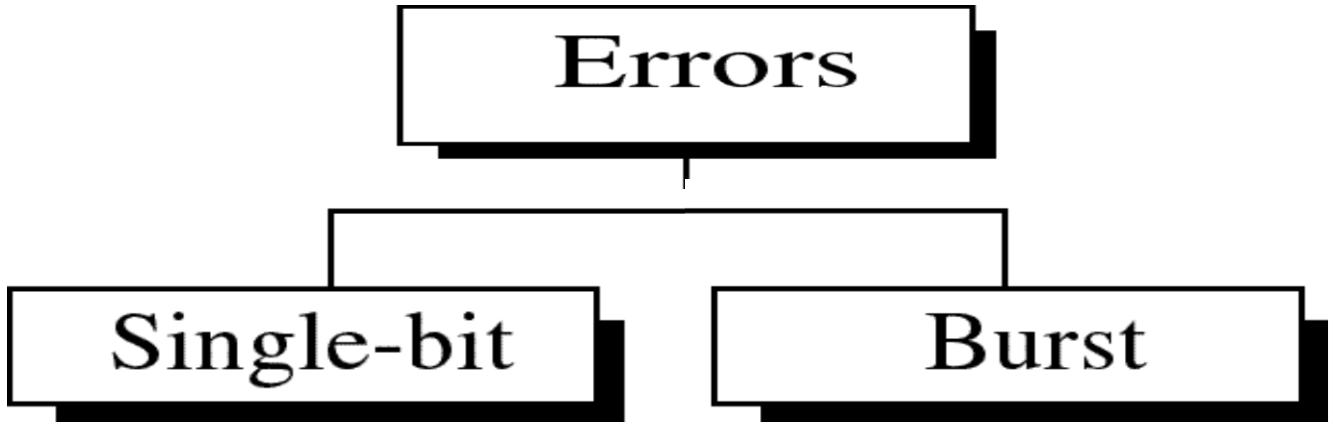
- Networks must be able to transfer data from one device to another with complete accuracy.
- Data can be corrupted during transmission.
- For reliable communication, errors must be detected and corrected.
- Error detection and correction are implemented either at the data link layer or the transport layer of the OSI model.

Error detection and correction

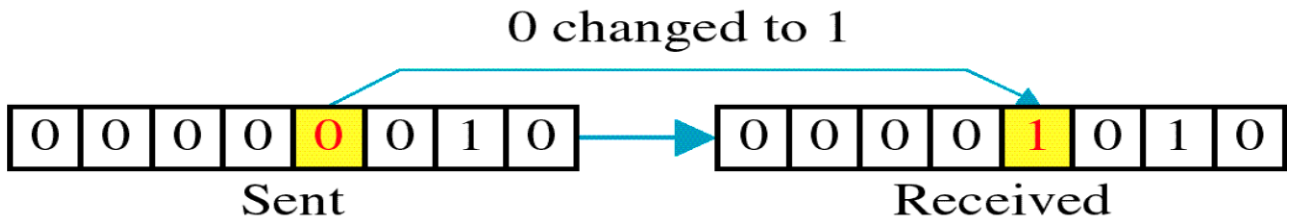
- Error detection:
  - errors caused by signal attenuation, noise.
  - receiver detects presence of errors:
  - signals sender for retransmission or drops frame

- Error correction:
  - receiver identifies and corrects bit error(s) without resorting to retransmission

Types of Error



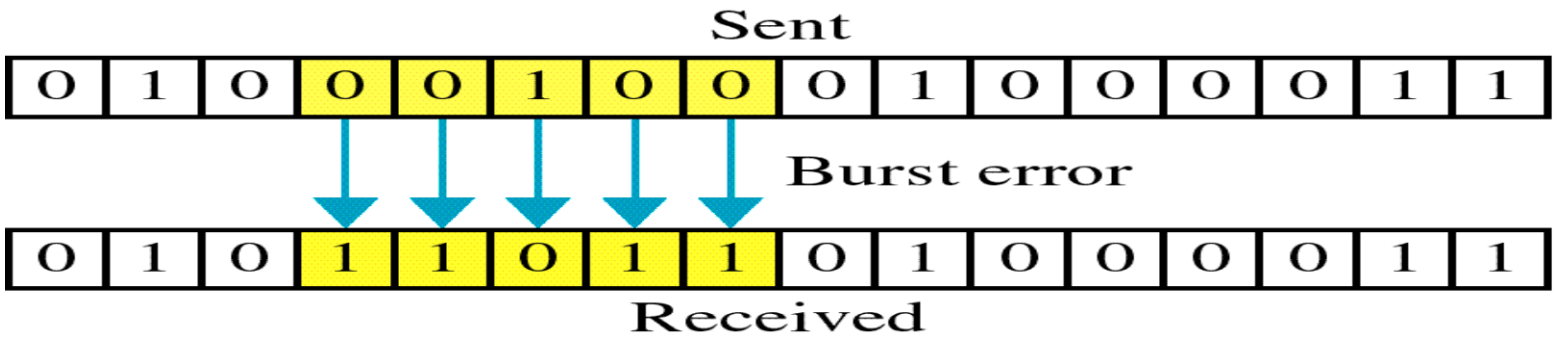
Single-bit error



- Single bit errors are the least likely type of errors in serial data transmission because the noise must have a very short duration which is very rare.
- However this kind of errors can happen in parallel transmission.

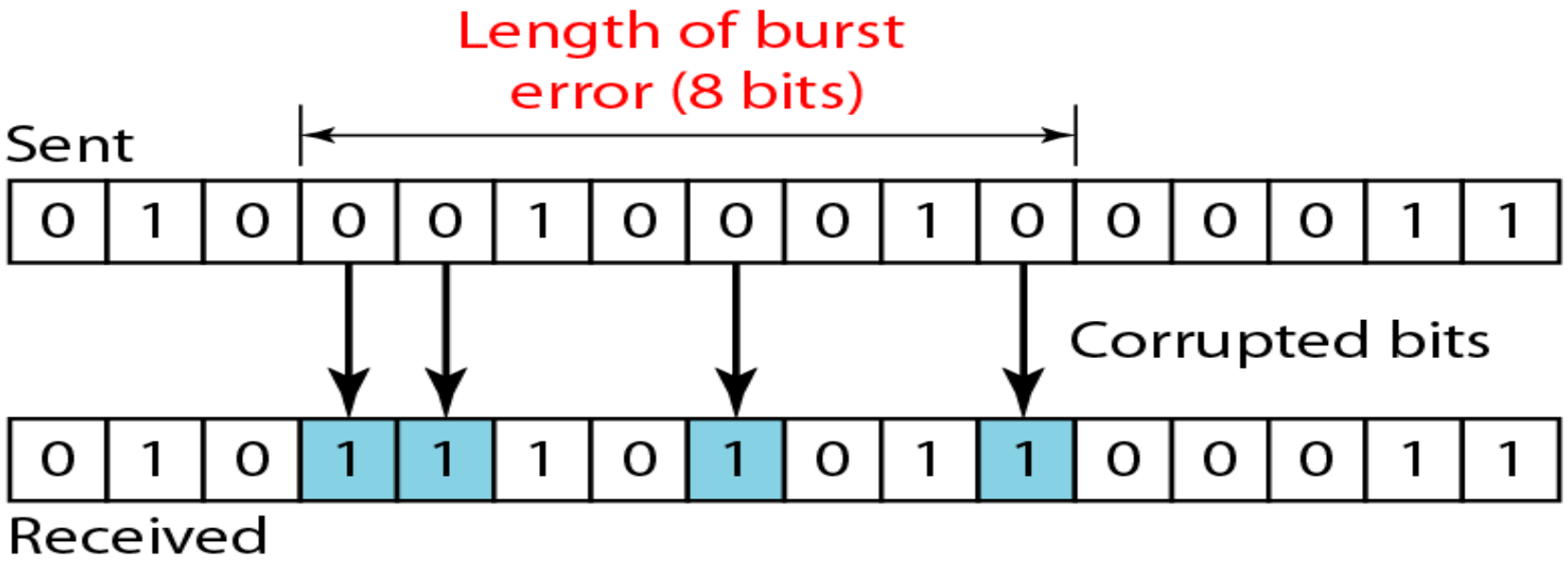
- If data is sent at rate = 1Kbps then a noise of 1/100 sec can affect 10 bits.(1/100*1000)
- If same data is sent at rate = 1Mbps then a noise of 1/100 sec can affect 10,000 bits.(1/100*10^6)

Burst error



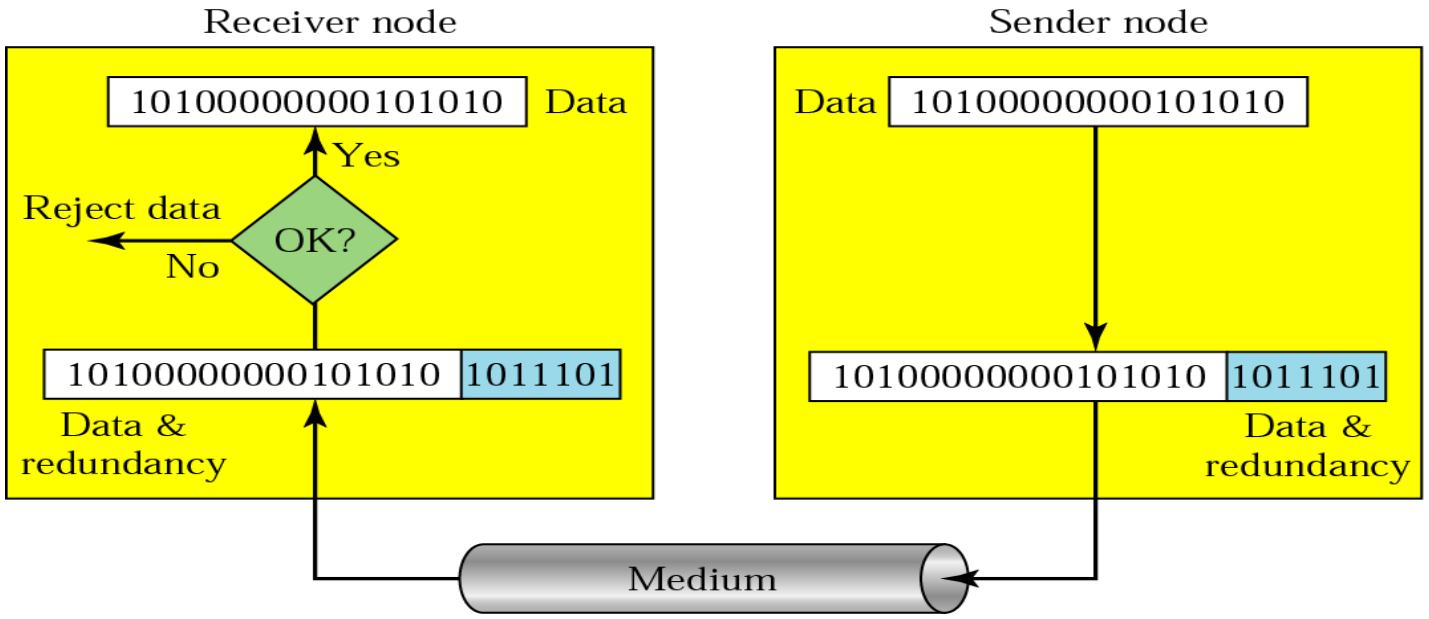
- The term burst error means that two or more bits in the data unit have changed from 1 to 0 or from 0 to 1.
- Burst errors does not necessarily mean that the errors occur in consecutive bits, the length of the burst is measured from the first corrupted bit to the last corrupted bit. Some bits in between may not have been corrupted.

Burst error with 8 bit length



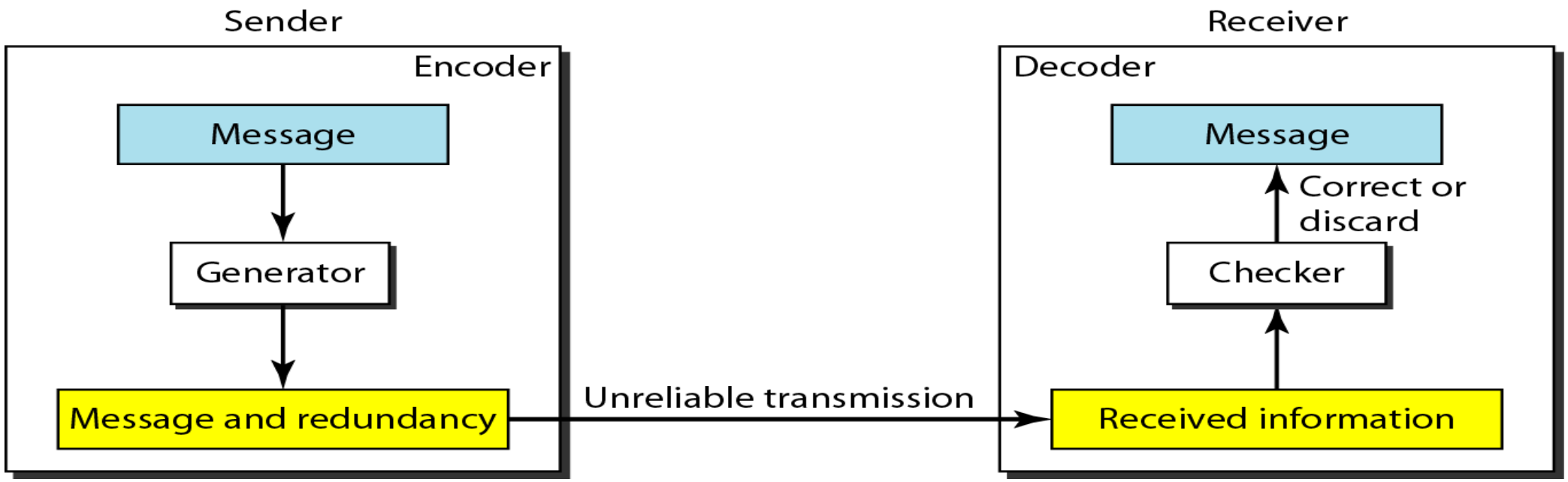
Redundancy

- Error detection means to decide whether the received data is correct or not without having a copy of the original message.



- Error detection uses the concept of redundancy, which means adding extra bits for detecting errors at the destination.

Redundancy


Sender
Encoder
Message
Generator
Message and redundancy
Unreliable transmission
Receiver
Decoder
Message
Correct or discard
Checker
Received information

Forward error correction vs Retransmission

- Forward error correction is process in which the receiver tries to guess the message by using redundant bits
- Used when number of errors is small
- Retransmission is technique in which the receiver detects the occurrence of an error and asks the sender to resend the message

Coding

- Redundancy is achieved through various coding schemes.
- The sender adds redundant bits through a process that creates a relationship between the redundant bits and the actual data bits.
- The receiver checks the relationships between the two sets of bits to detect or correct the errors. The ratio of redundant bits to the data bits and the robust ness of the process are important factors in any coding scheme.

  (1) Block Coding

  (2) Linear Block Coding

Modular Arithmetic

Modulo N Arithmetic

- Integers from 0 to N-1

Modulo 2 Arithmetic

$0 \oplus 0 = 0$ $\quad$ $1 \oplus 1 = 0$

a. Two bits are the same, the result is 0.

$0 \oplus 1 = 1$ $\quad$ $1 \oplus 0 = 1$

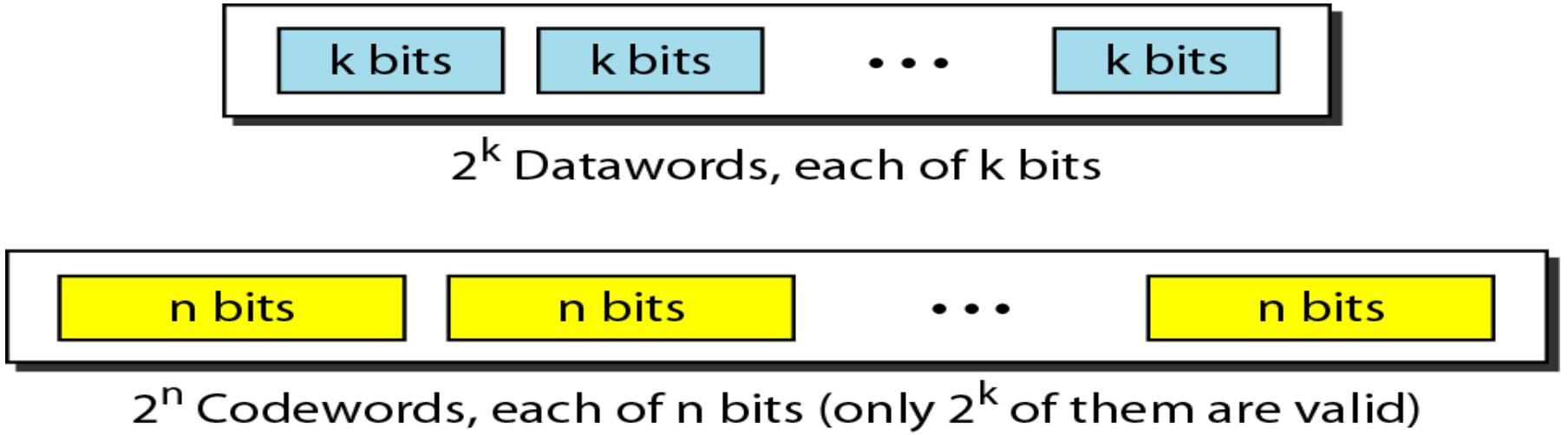
b. Two bits are different, the result is 1.

	1	0	1	1	0
$\oplus$	1	1	1	0	0
	0	1	0	1	0

c. Result of XORing two patterns

Block Coding

In block coding, we divide our message into blocks, each of k bits, called datawords. We add r redundant bits to each block to make the length n = k + r. The resulting n-bit blocks are called codewords.



Data Words and Codewords

Let us assume that k = 2 and n = 3. Table shows the list of datawords and codewords.

Datawords	*Codewords*
00	000
01	011
10	101
11	110

An error-detecting code can detect only the types of errors for which it is designed; other types of errors may remain undetected.

Data Words and Codewords

- The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.

- The codeword is corrupted during transmission, and 111 is received. This is not a valid codeword and is discarded.

- The codeword is corrupted during transmission, and 000 is received. This is a valid codeword. The receiver incorrectly extracts the dataword 00. Two corrupted bits have made the error undetectable.

Data Words and Codewords

In this coding scheme,
$k = 4$ and $n = 5$.
we have $2^k = 16$ datawords
and
$2^n = 32$ codewords.
We saw that 16 out of 32 codewords are used for message transfer
rest are either used for other purposes or unused.

Hamming Distance

The Hamming distance between two words is the number of differences between corresponding bits.

Hamming distance between two words x and y as d(x,y)

Example 10.4

Let us find the Hamming distance between two pairs of words.

1. *The Hamming distance d(000, 011) is 2 because*

$$000 \oplus 011 \text{ is } 011 \text{ (two 1s)}$$

2. *The Hamming distance d(10101, 11110) is 3 because*

$$10101 \oplus 11110 \text{ is } 01011 \text{ (three 1s)}$$

Note

The minimum Hamming distance is the smallest Hamming distance between all possible pairs in a set of words.

Minimum hamming distance

Find the minimum Hamming distance of the coding scheme in Table 10.1.

Solution

We first find all Hamming distances.

$d(000, 011) = 2$ $d(000, 101) = 2$ $d(000, 110) = 2$ $d(011, 101) = 2$
$d(011, 110) = 2$ $d(101, 110) = 2$

The d_{min} in this case is 2.

Datawords	*Codewords*
00	000
01	011
10	101
11	110

Example 10.6

Find the minimum Hamming distance of the coding scheme in Table 10.2.

Solution

We first find all the Hamming distances.

$d(00000, 01011) = 3$	$d(00000, 10101) = 3$	$d(00000, 11110) = 4$
$d(01011, 10101) = 4$	$d(01011, 11110) = 3$	$d(10101, 11110) = 3$

The d_{min} in this case is 3.

Dataword	*Codeword*
00	00000
01	01011
10	10101
11	11110

Parameters for coding scheme

codeword size n
dataword size k

Minimum hamming distance d_{min}

Coding scheme C *is written as* $C(n,k)$

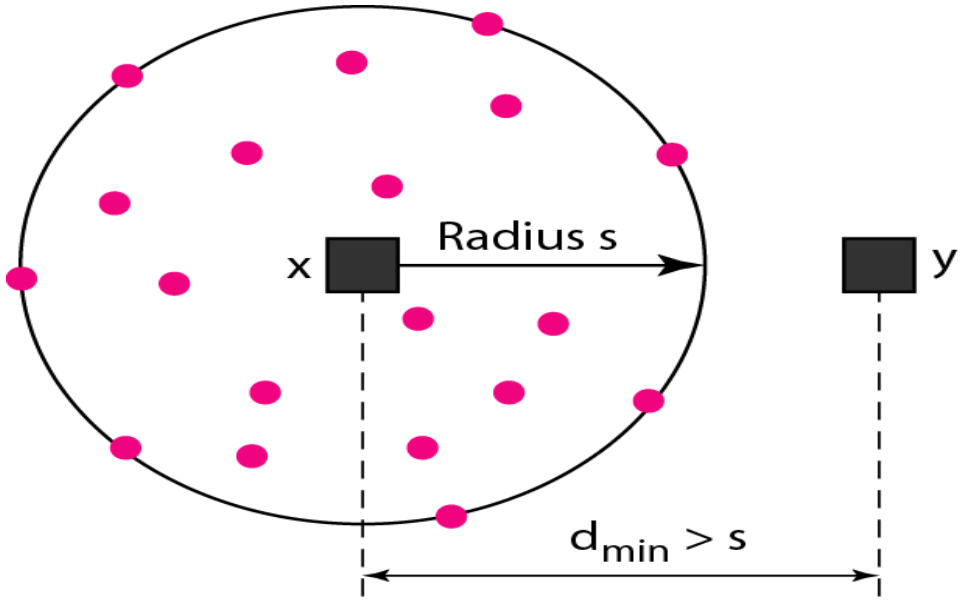
Hamming distance and Error

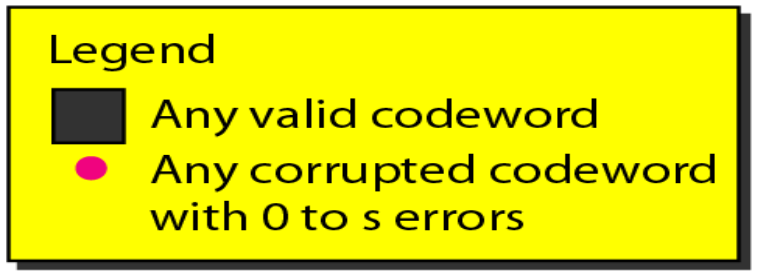
- Find minimum distance in code if we want to be able to detect up to ***S*** errors
- If our code is to detect up to ***S*** errors, the minimum distance between the valid codes must be ***S + 1***

To guarantee the detection of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = s + 1$.

Hamming distance and Error

- If sent codewords X is at the center of a circle with redius *S*.
- All received codewords that are created by 1 to *S* errors are points inside the circle or on the perimeter of the circle.
- All valid code words must be outside the circle





Error Detection Techniques (Linear Block Codes)

- Basic approach used for error detection is the use of redundancy bits, where additional bits are added to facilitate detection of errors.
- Some popular techniques for error detection are:

  1. Simple Parity check
  2. Two-dimensional Parity check
  3. Checksum
  4. Cyclic redundancy check

1. Simple Parity Check

- Blocks of data from the source are subjected to a check bit or parity bit generator form, where a parity of :

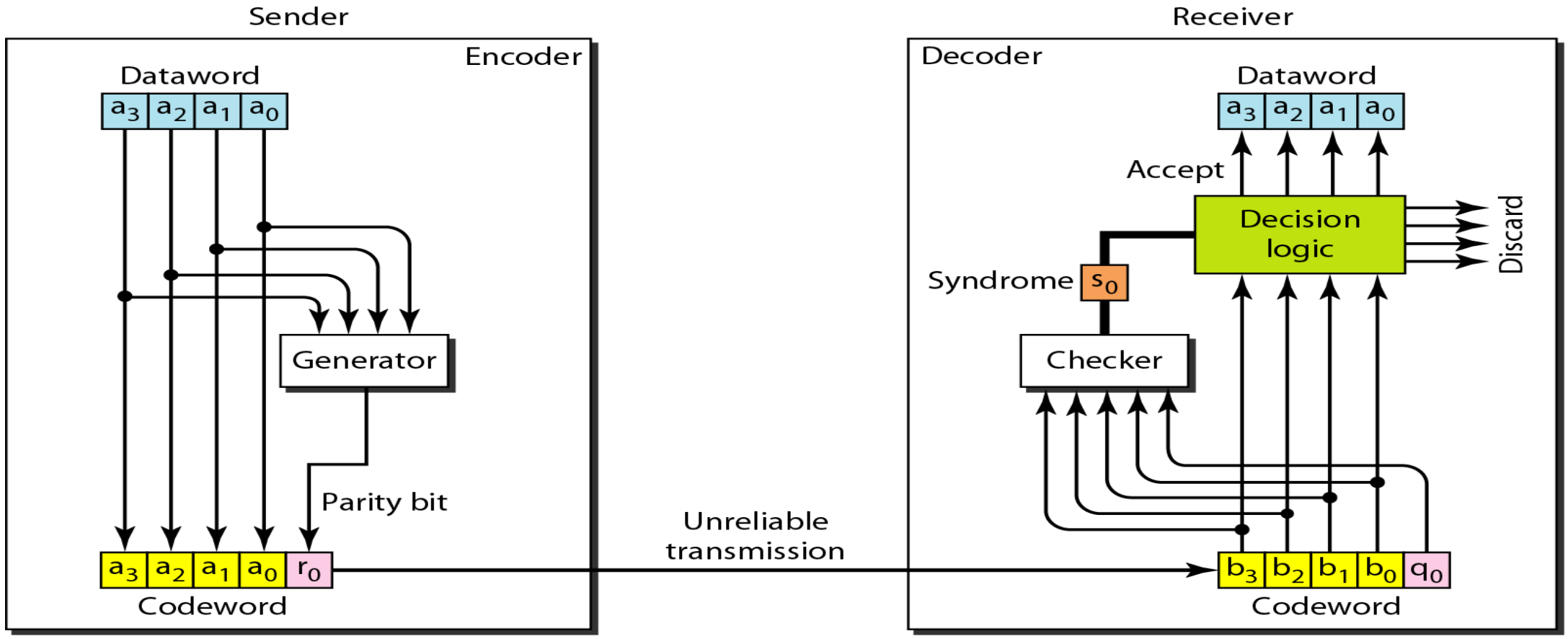
  1 is added to the block if it contains odd number of 1's, and

  0 is added if it contains even number of 1's

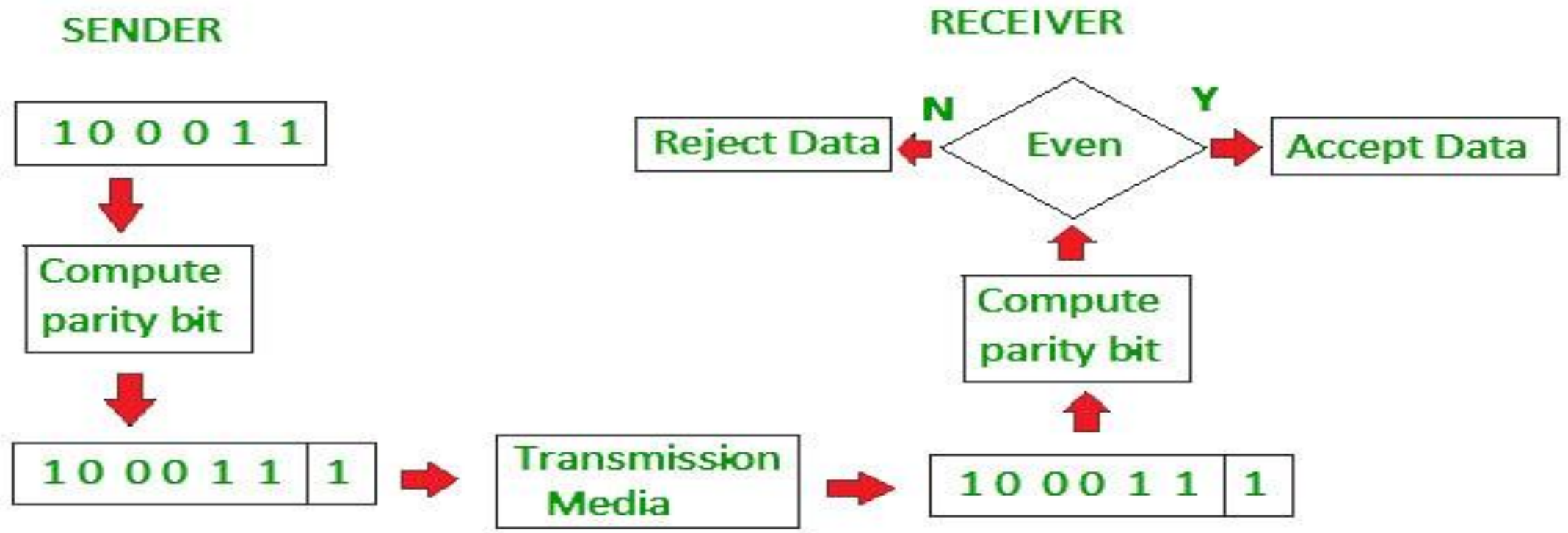
This scheme makes the total number of 1's even, that is why it is called even parity checking.

A simple parity-check code is a single-bit error-detecting code in which $n = k + 1$ with $d_{min} = 2$.

1. Simple Parity Check



1. Simple Parity Check



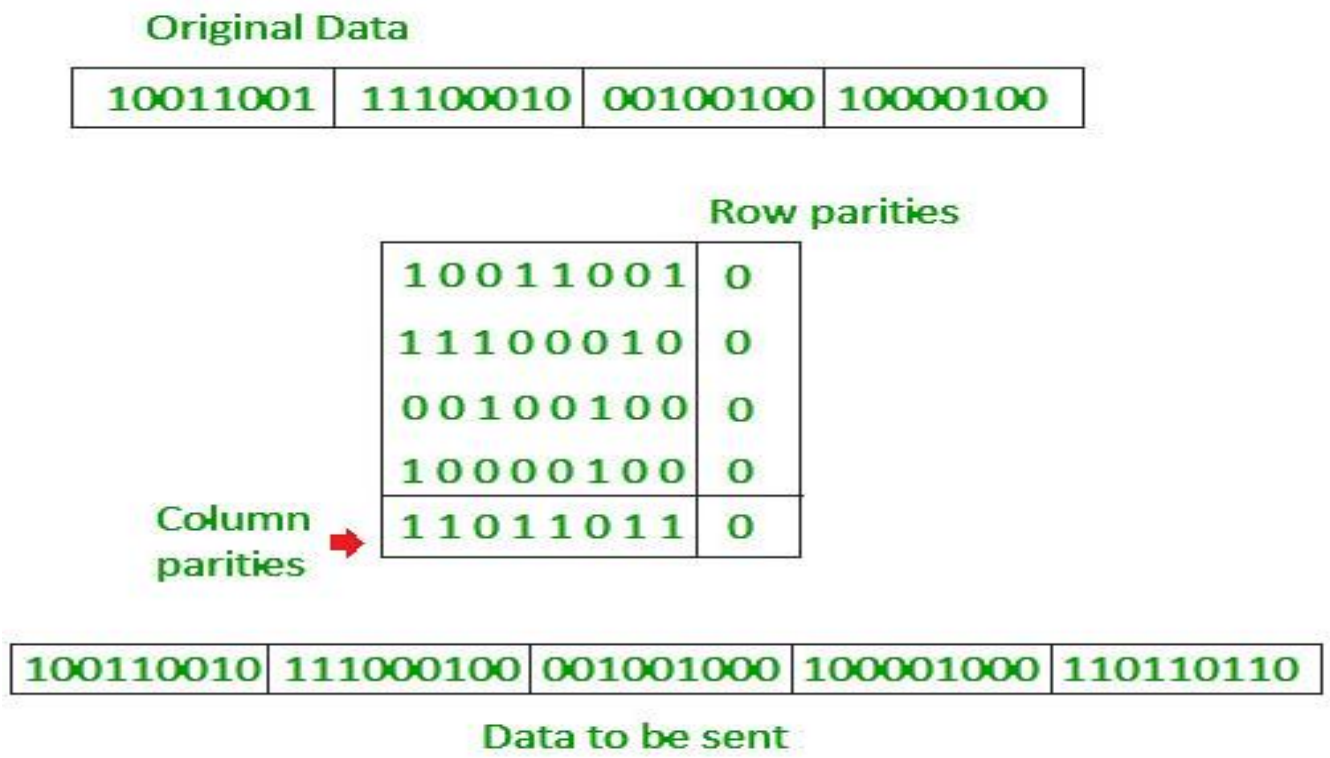
1. Simple Parity Check

Datawords	*Codewords*	*Datawords*	*Codewords*
0000	00000	1000	10001
0001	00011	1001	10010
0010	00101	1010	10100
0011	00110	1011	10111
0100	01001	1100	11000
0101	01010	1101	11011
0110	01100	1110	11101
0111	01111	1111	11110

Example 10.12

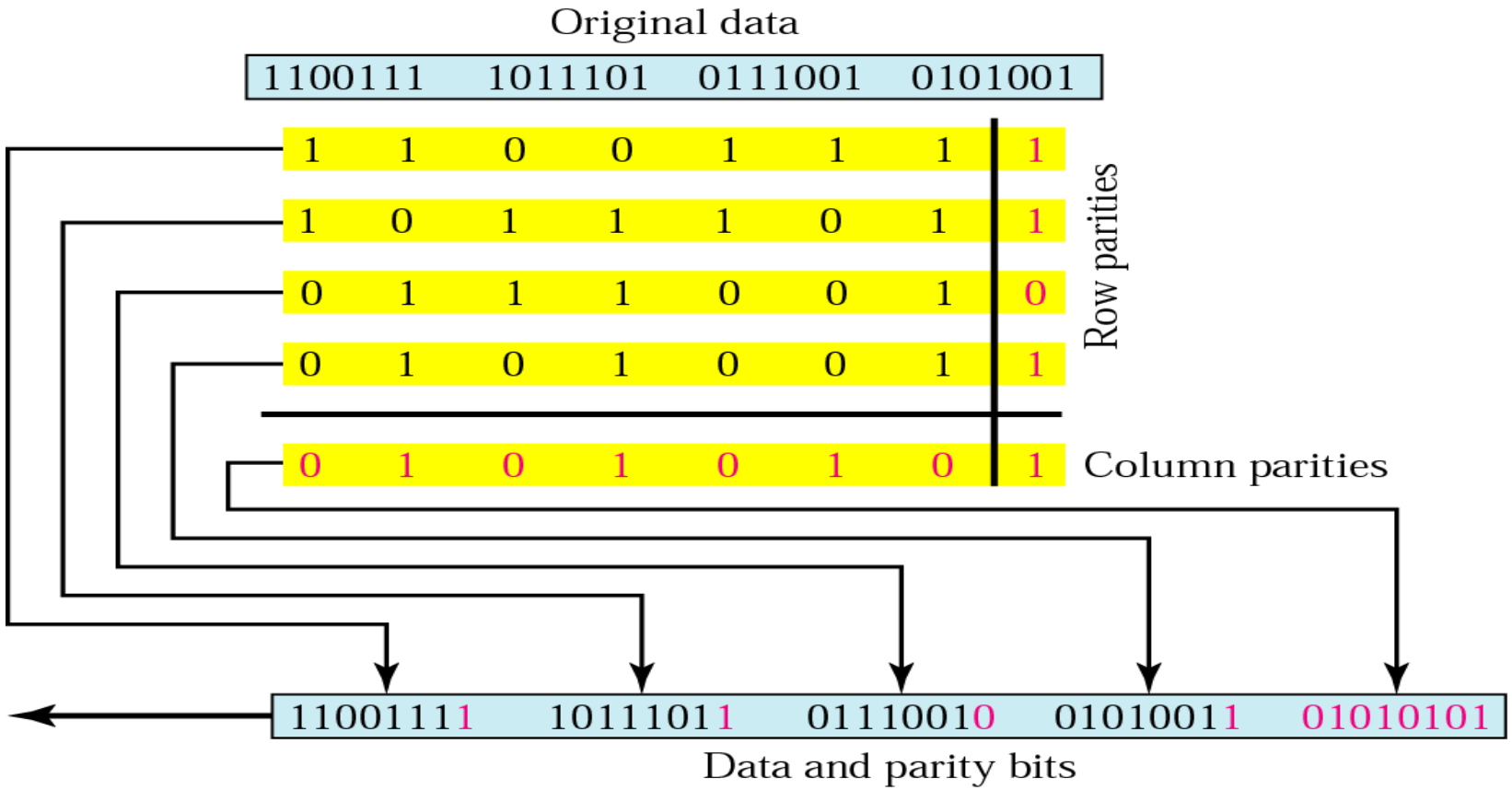
- Assume the sender sends the dataword **1011**. The codeword created from this dataword is **10111**, which is sent to the receiver. We examine five cases:
- No error occurs; the received codeword is **10111**. The syndrome is 0. The dataword **1011** is created.
- One single-bit error changes a1 . The received codeword is **10011.** The syndrome is 1. No dataword is created.
- One single-bit error changes r0 . The received codeword is **10110**. The syndrome is 1. No dataword is created.
- An error changes r0 and a second error changes a3 . The received codeword is **00110**. The syndrome is 0. The dataword **0011** is created at the receiver. Note that here the dataword is wrongly created due to the syndrome value.
- Three bits—a3, a2, and a1—are changed by errors. The received codeword is **01011**. The syndrome is 1. The dataword is not created. This shows that the simple parity check, guaranteed to detect one single error, can also find any odd number of errors.

2. Two-dimensional Parity check

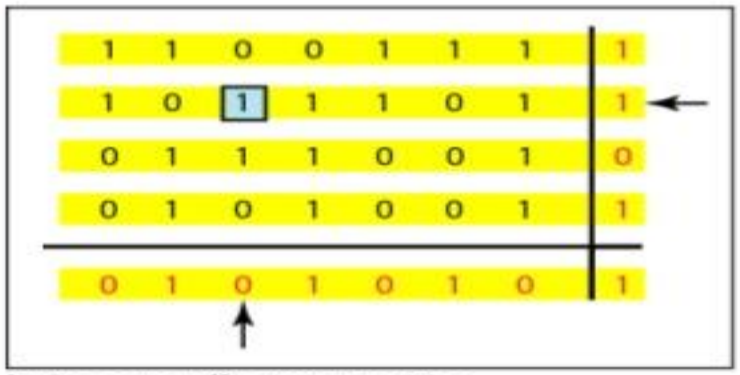


- Parity check bits are calculated for each row, which is equivalent to a simple parity check bit.
- Parity check bits are also calculated for all columns, then both are sent along with the data. At the receiving end these are compared with the parity bits calculated on the received data.

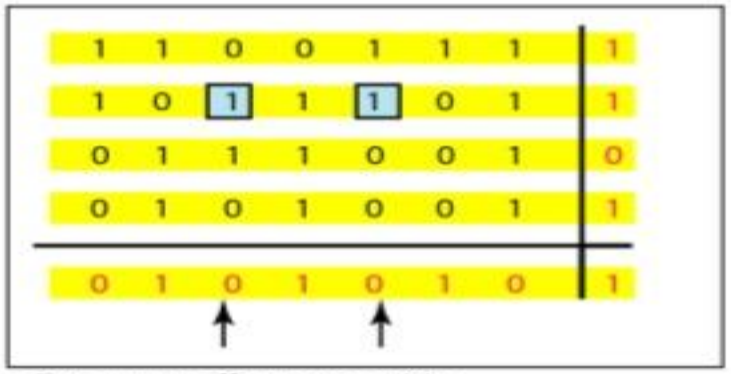
2. Two-dimensional Parity check



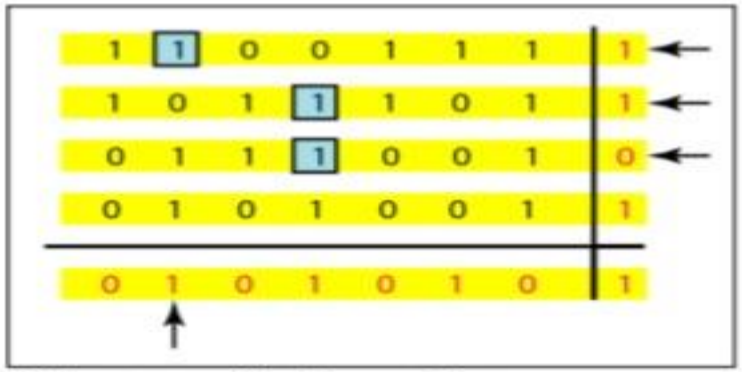
2. Two-dimensional Parity check



b. One error affects two parities



c. Two errors affect two parities

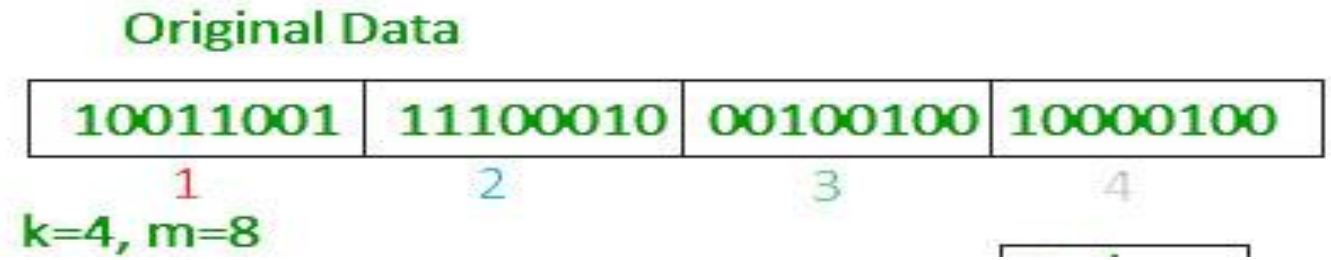


d. Three errors affect four parities

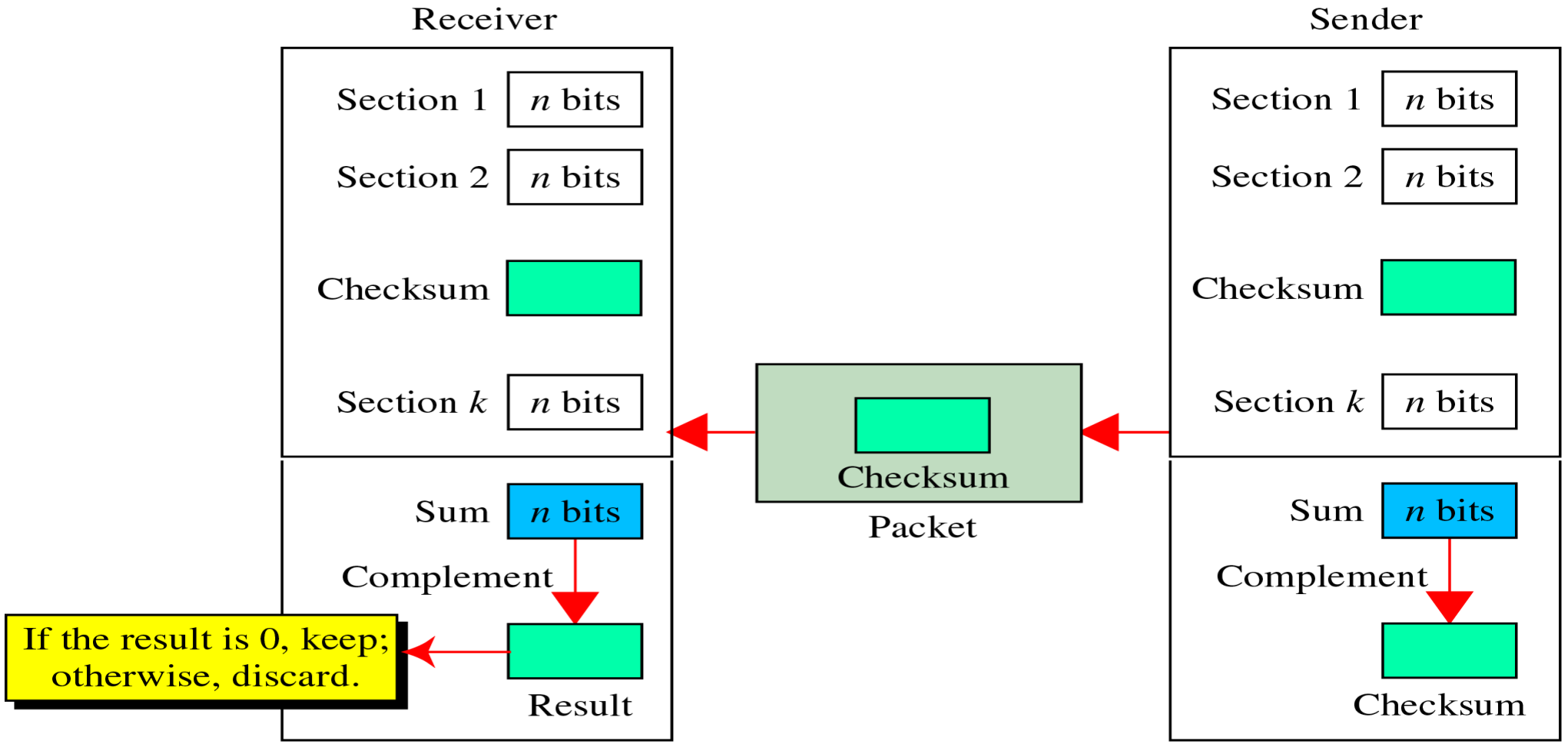
e. Four errors cannot be detected

3. Checksum

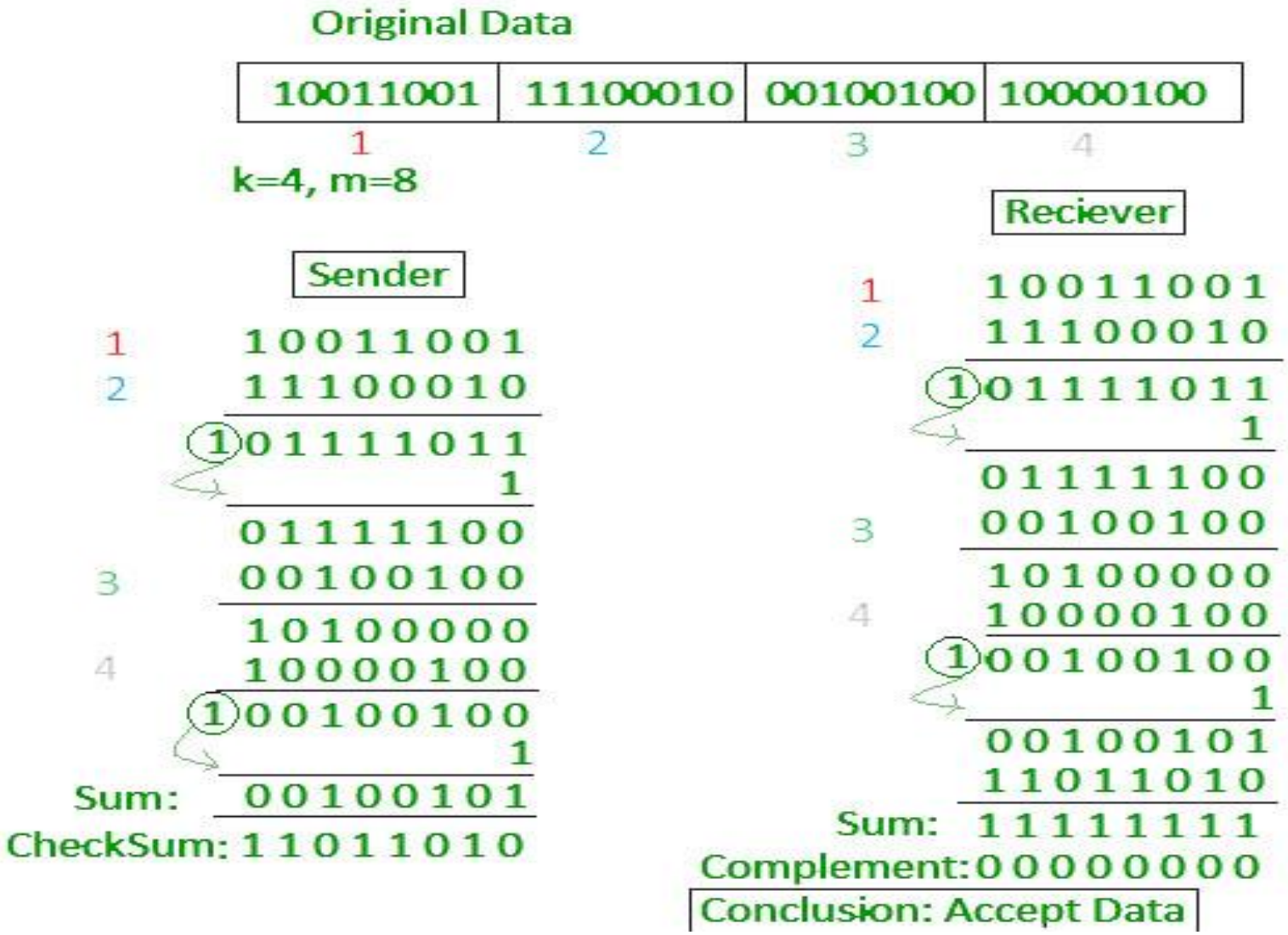
- In checksum error detection scheme, the data is divided into *k* segments each of m bits.
- In the sender's end the segments are added using 1's complement arithmetic to get the sum. The sum is complemented to get the checksum.
- The checksum segment is sent along with the data segments.
- At the receiver's end, all received segments are added using 1's complement arithmetic to get the sum. The sum is complemented.
- If the result is zero, the received data is accepted; otherwise discarded.



3. Checksum


Receiver
Sender
Section 1
n bits
Section 2
n bits
Checksum
Section k
n bits
Sum
Complement
Result
If the result is 0, keep; otherwise, discard.
Checksum
Packet

3. Checksum


Original Data
10011001
11100010
00100100
10000100
1
2
3
4
k=4, m=8
Sender
1 10011001
2 11100010
1 01111011
1
01111100
3 00100100
10100000
4 10000100
1 00100100
1
Sum: 00100101
CheckSum: 11011010
Reciever
1 10011001
2 11100010
1 01111011
1
01111100
3 00100100
10100000
4 10000100
1 00100100
1
00100101
11011010
Sum: 11111111
Complement: 00000000
Conclusion: Accept Data

Example 10.18

- *Suppose our data is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers.*

- *For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, 36), where 36 is the sum of the original numbers.*

- *The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum.*

- *Otherwise, there is an error somewhere and the data are not accepted.*

Example 10.19

We can make the job of the receiver easier if we send the negative (complement) of the sum, called the checksum. In this case, we send (7, 11, 12, 0, 6, −36). The receiver can add all the numbers received (including the checksum). If the result is 0, it assumes no error; otherwise, there is an error.

Example 10.20

How can we represent the number 21 in one's complement arithmetic using only four bits?

Solution

The number 21 in binary is 10101 (it needs five bits). We can wrap the leftmost bit and add it to the four rightmost bits. We have (0101 + 1) = 0110 or 6.

Example 10.21

How can we represent the number −6 in one's complement arithmetic using only four bits?

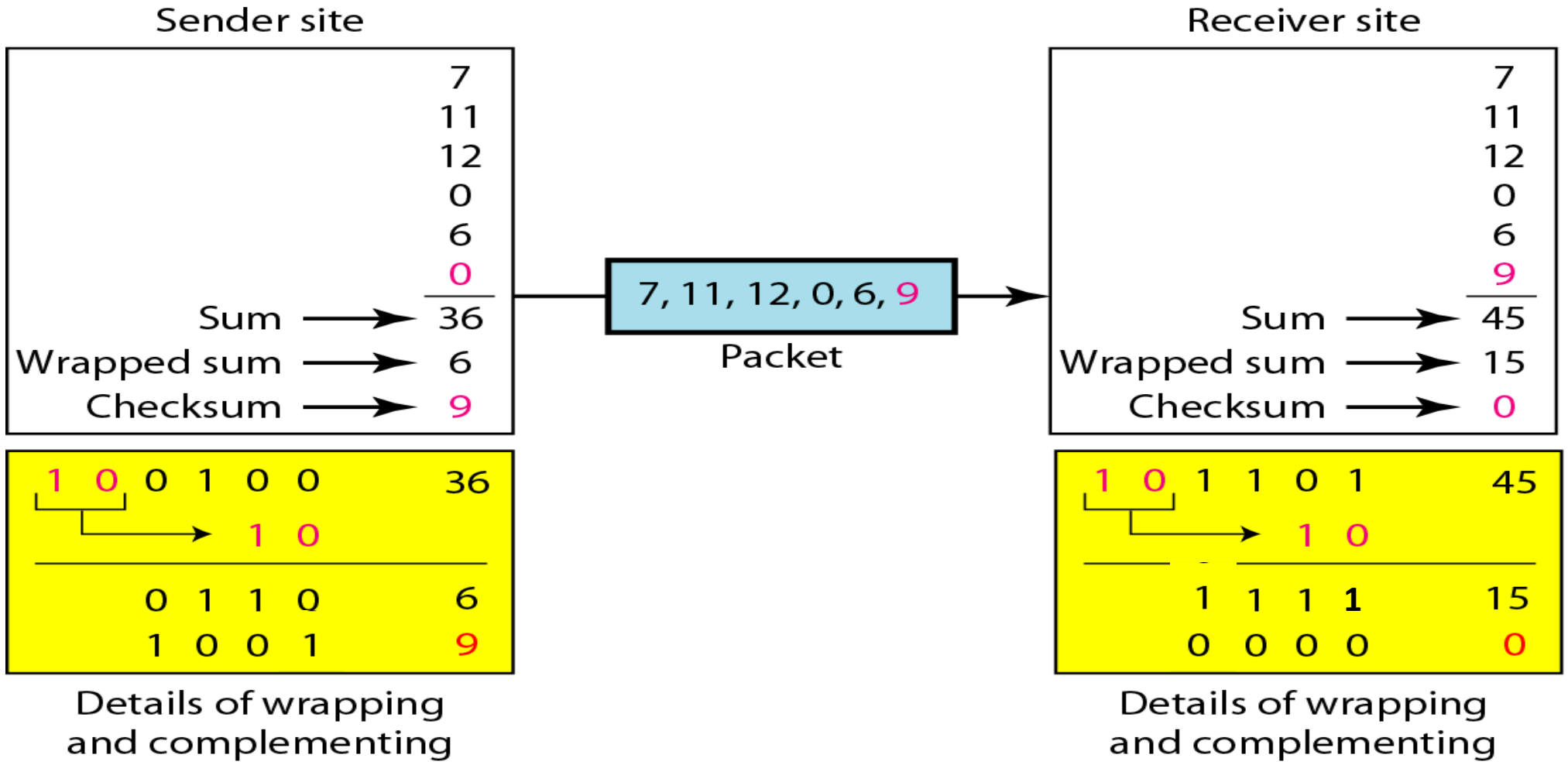
Solution

In one's complement arithmetic, the negative or complement of a number is found by inverting all bits. Positive 6 is 0110; negative 6 is 1001. If we consider only unsigned numbers, this is 9. In other words, the complement of 6 is 9. Another way to find the complement of a number in one's complement arithmetic is to subtract the number from $2^n - 1$ (16 − 1 in this case).

Example 10.22 (continued)

The receiver follows the same procedure as the sender. It adds all data items (including the checksum); the result is 45. The sum is wrapped and becomes 15. The wrapped sum is complemented and becomes 0. Since the value of the checksum is 0, this means that the data is not corrupted. The receiver drops the checksum and keeps the other data items. If the checksum is not zero, the entire packet is dropped.

Example 10.22



Note

Sender site:

1. The message is divided into 16-bit words.
2. The value of the checksum word is set to 0.
3. All words including the checksum are added using one's complement addition.
4. The sum is complemented and becomes the checksum.
5. The checksum is sent with the data.

Note

Receiver site:

1. The message (including checksum) is divided into 16-bit words.
2. All words are added using one's complement addition.
3. The sum is complemented and becomes the new checksum.
4. If the value of checksum is 0, the message is accepted; otherwise, it is rejected.

Example 10.23

1	0	1	3	Carries
4	6	6	F	(Fo)
7	2	6	7	(ro)
7	5	7	A	(uz)
6	1	6	E	(an)
0	0	0	0	Checksum (initial)
8	F	C	6	Sum (partial)
			1	
8	F	C	7	Sum
7	0	3	8	Checksum (to send)

a. Checksum at the sender site

1	0	1	3	Carries
4	6	6	F	(Fo)
7	2	6	7	(ro)
7	5	7	A	(uz)
6	1	6	E	(an)
7	0	3	8	Checksum (received)
F	F	F	E	Sum (partial)
			1	
8	F	C	7	Sum
0	0	0	0	Checksum (new)

a. Checksum at the receiver site

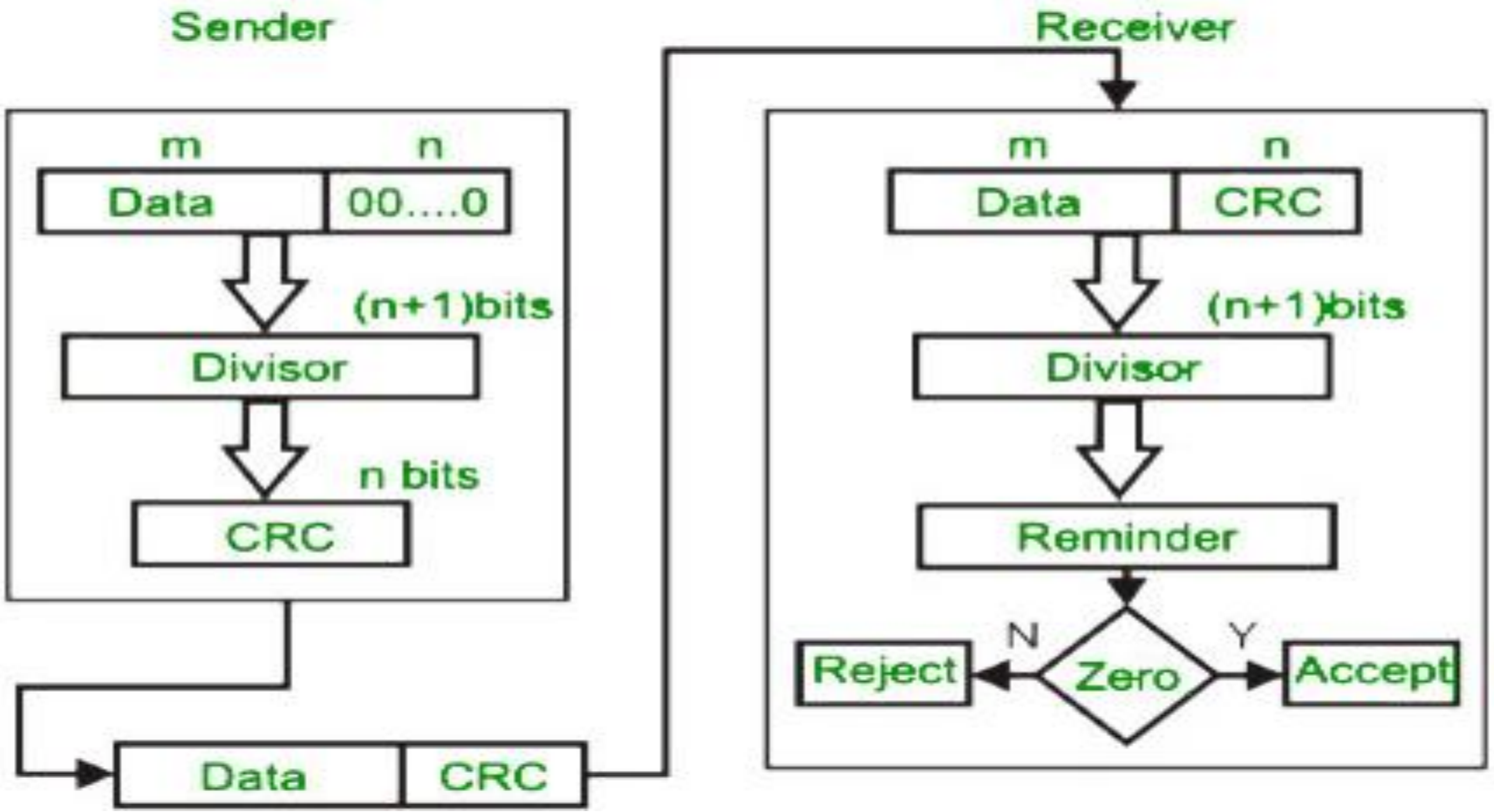
Performance of Checksum

- The checksum detects all errors involving an odd number of bits.
- It detects most errors involving an even number of bits.
- If one or more bits of a segment are damaged and the corresponding bit or bits of opposite value in a second segment are also damaged, the sums of those columns will not change and the receiver will not detect a problem.

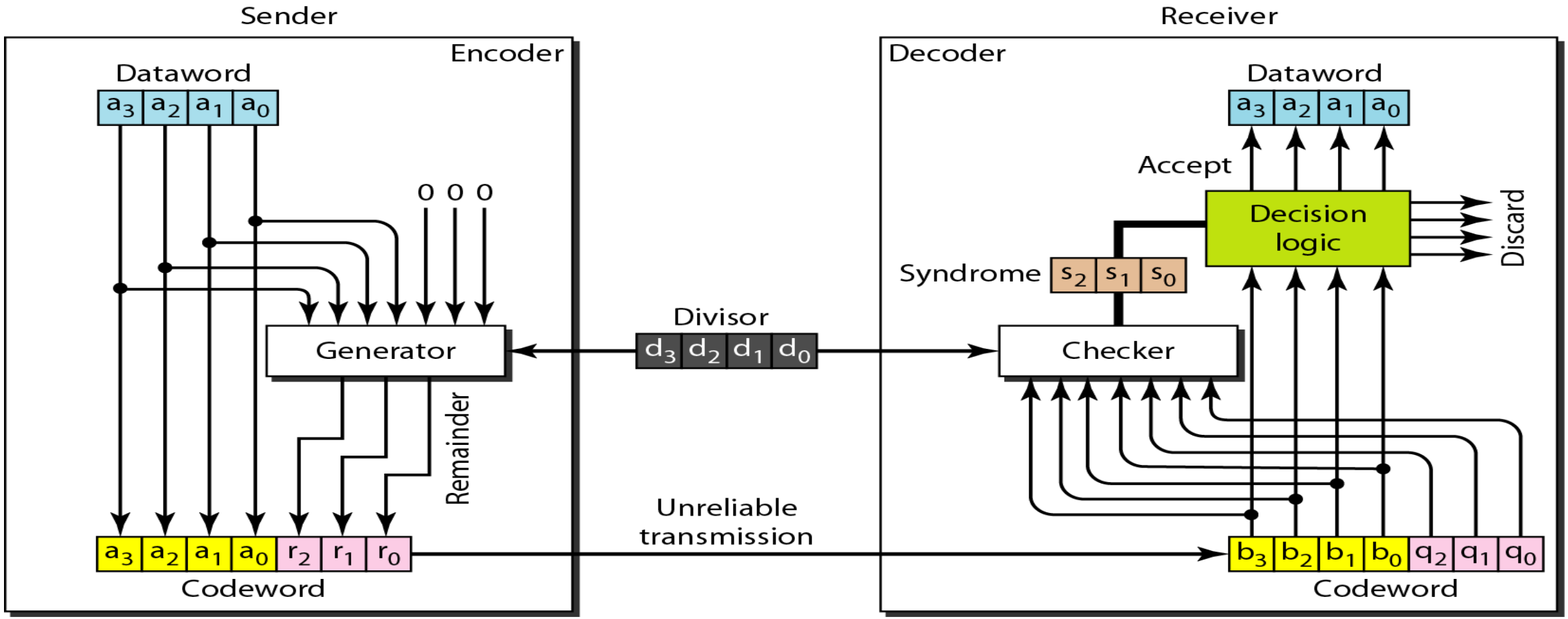
3. Cyclic redundancy check (CRC)

- Unlike checksum scheme, which is based on addition, CRC is based on binary division.
- In CRC, a sequence of redundant bits, called cyclic redundancy check bits, are appended to the end of data unit so that the resulting data unit becomes exactly divisible by a second, predetermined binary number.
- At the destination, the incoming data unit is divided by the same number. If at this step there is no remainder, the data unit is assumed to be correct and is therefore accepted.
- A remainder indicates that the data unit has been damaged in transit and therefore must be rejected.

3. Cyclic redundancy check (CRC)

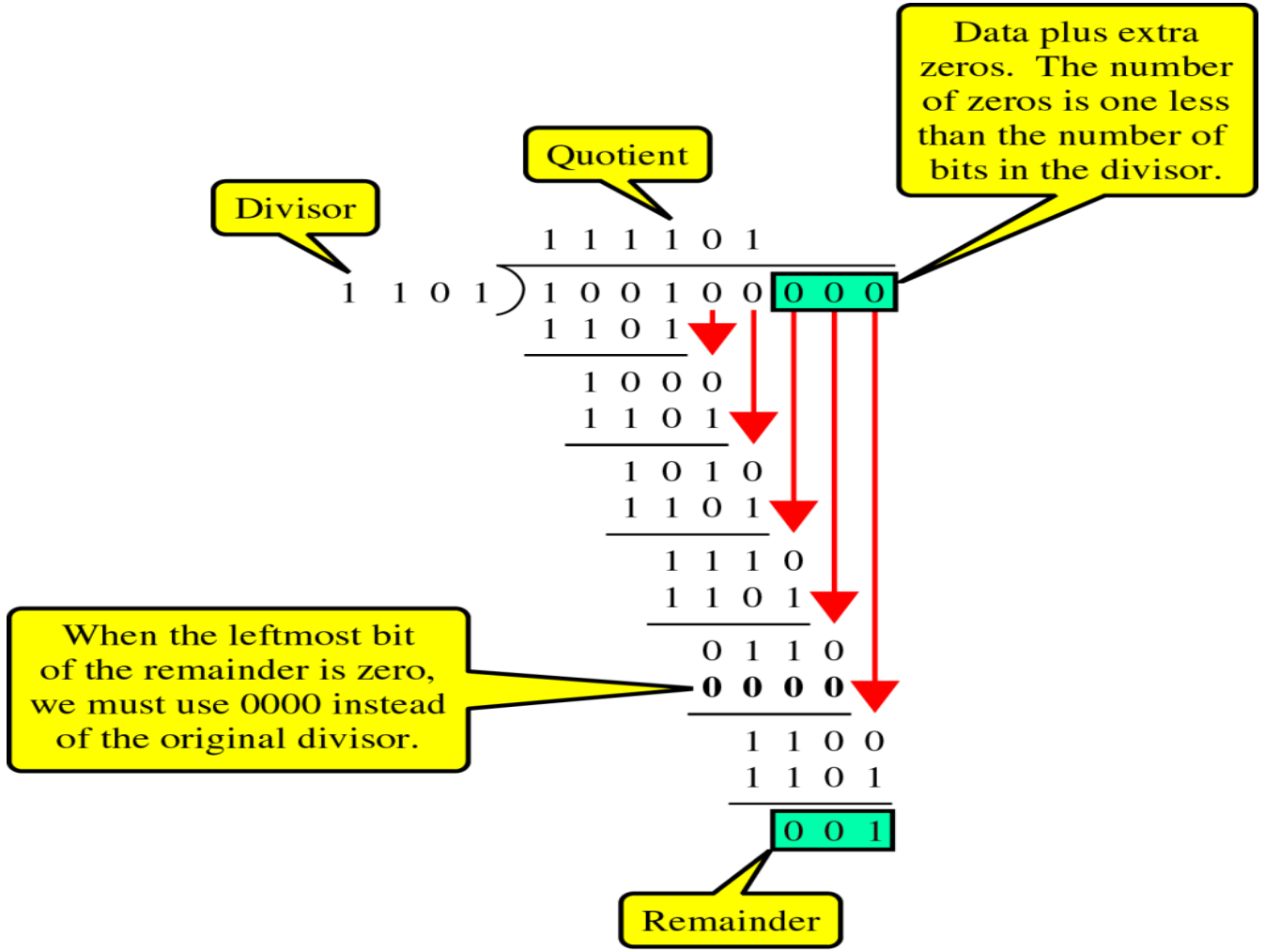


CRC Encoder and Decoder



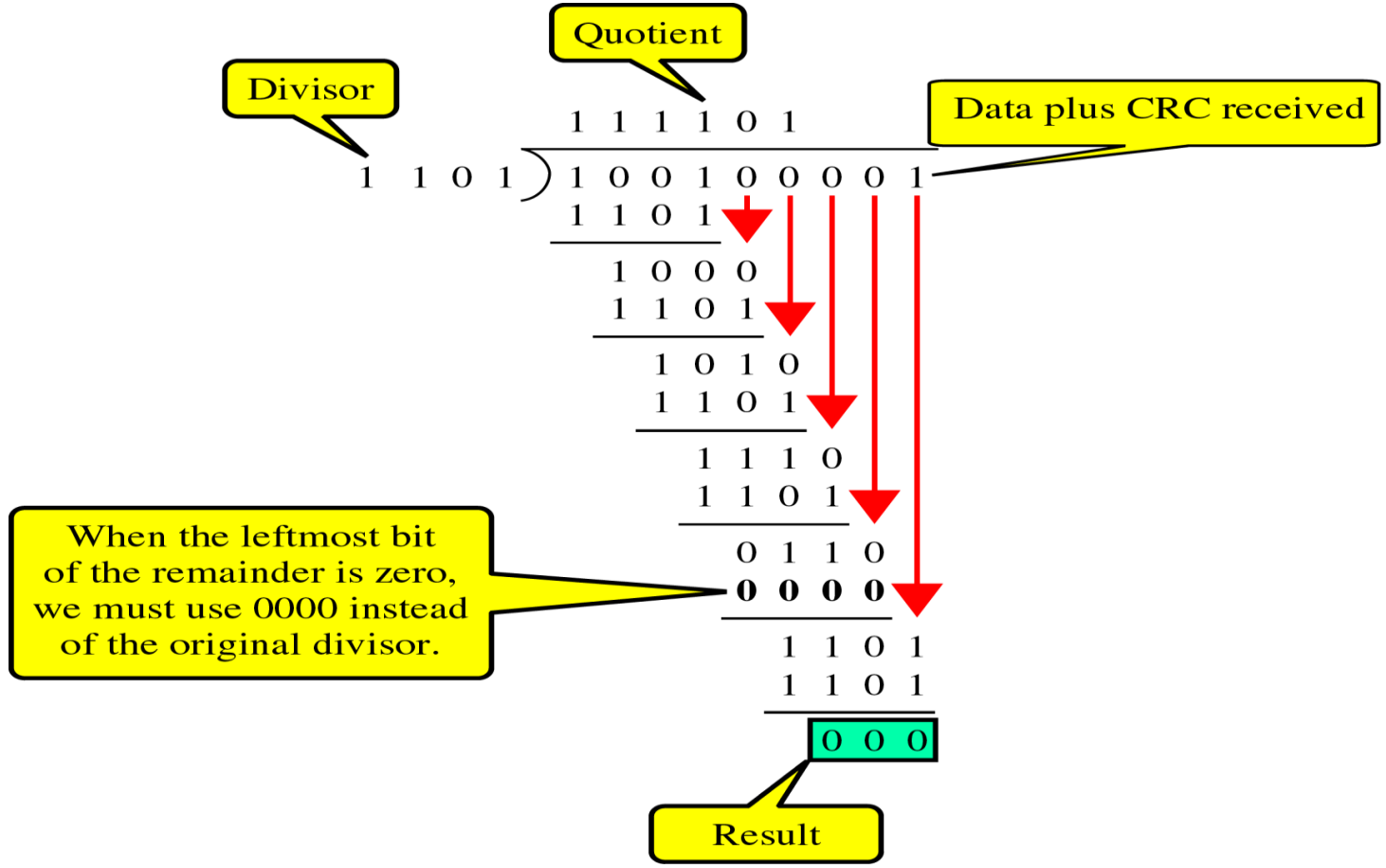
Cyclic redundancy check (CRC)

CRC Generator

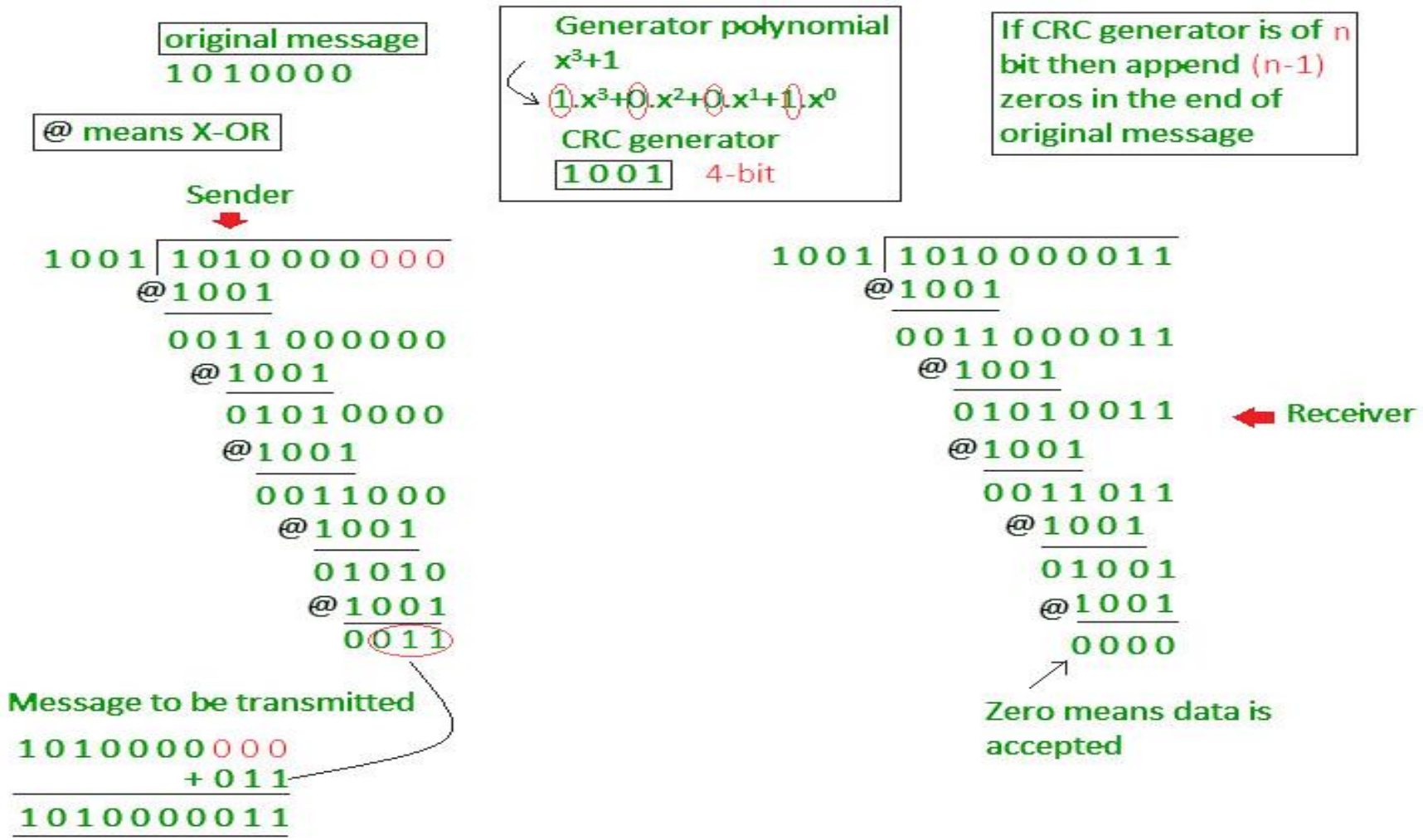


Cyclic redundancy check (CRC)

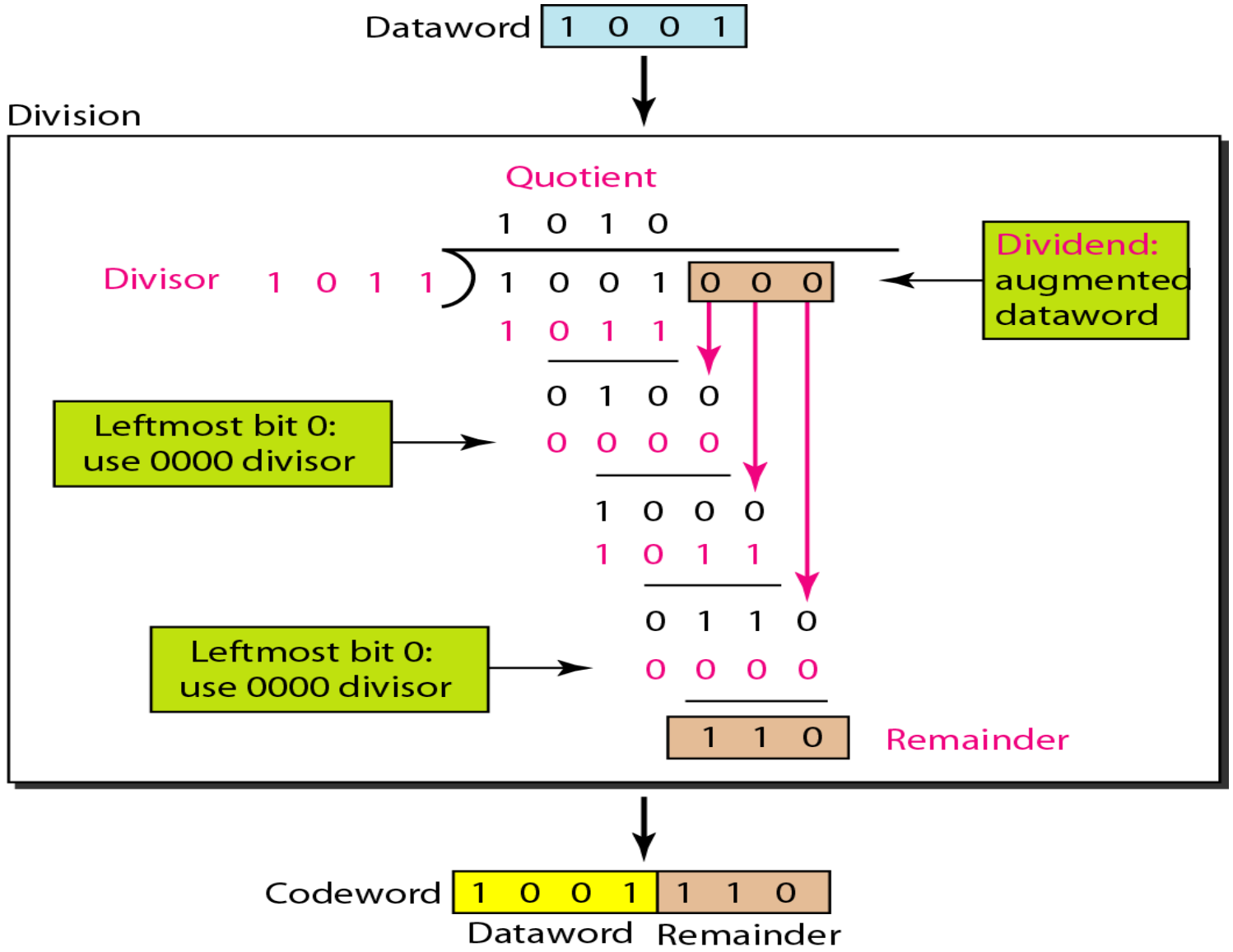
CRC Checker



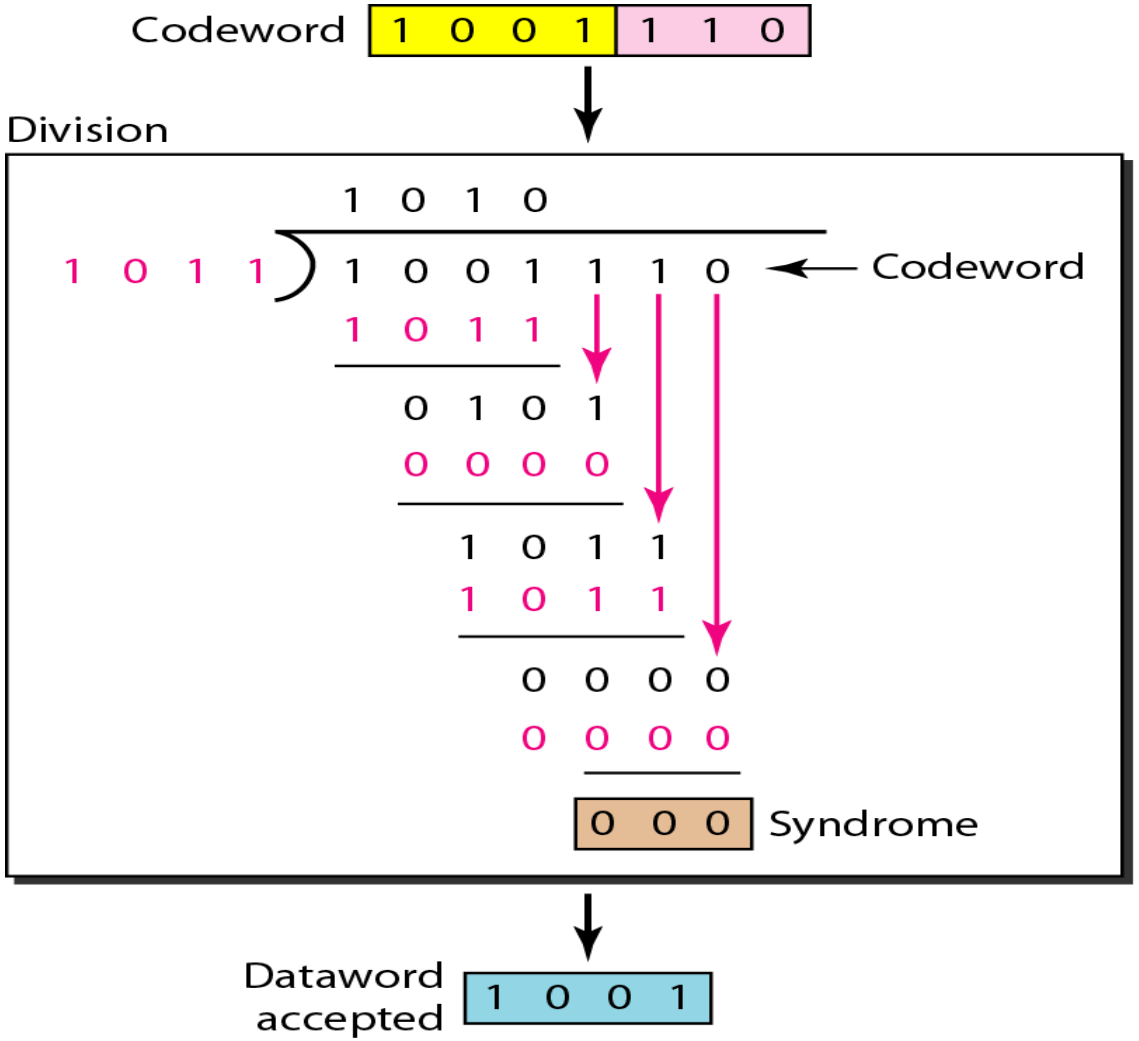
Cyclic redundancy check (CRC)

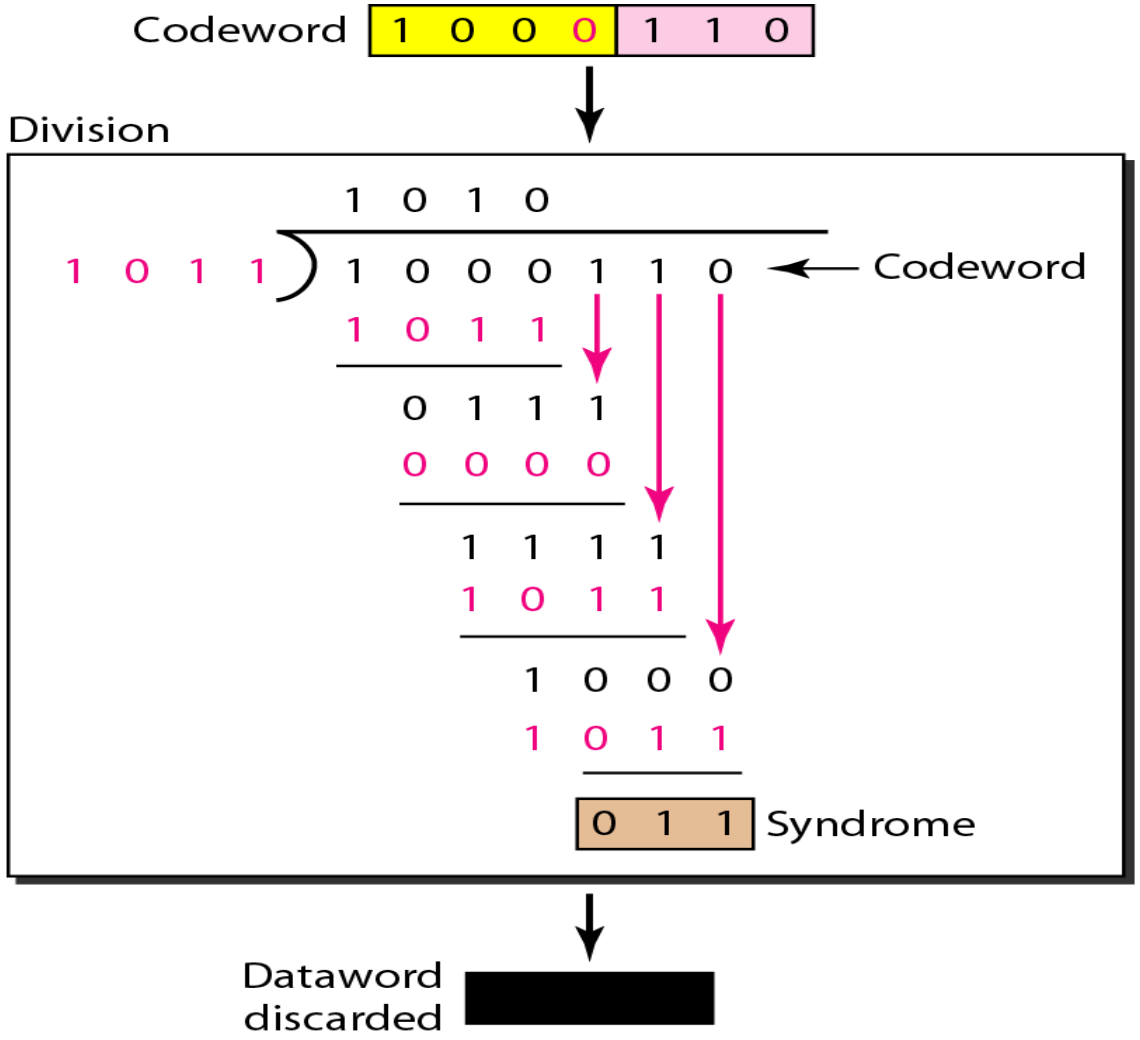


Division in CRC encoder



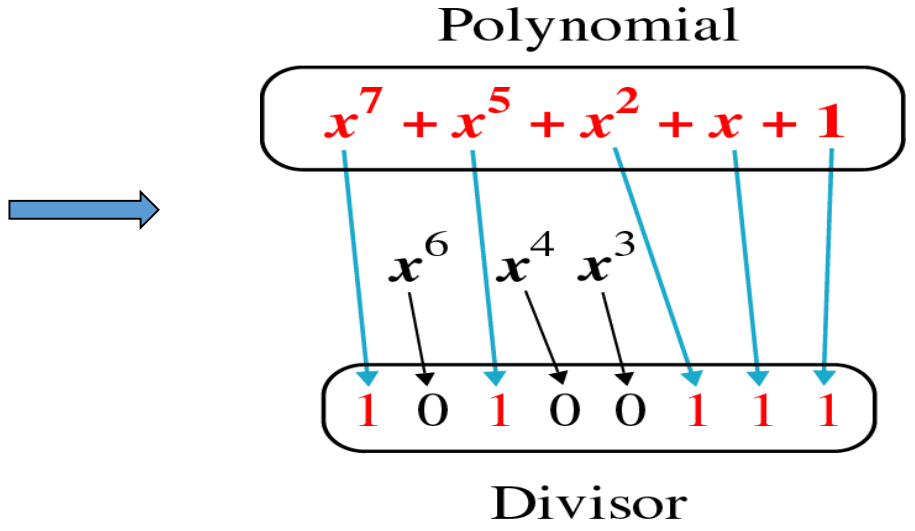
Division in the CRC decoder for two cases





Cyclic redundancy check (CRC)

- CRC generator(divisor) is most often represented not as a string of 1s and 0s, but as an algebraic polynomial.



- Standard polynomials

CRC-12

$x^{12} + x^{11} + x^3 + x + 1$

CRC-16

$x^{16} + x^{15} + x^2 + 1$

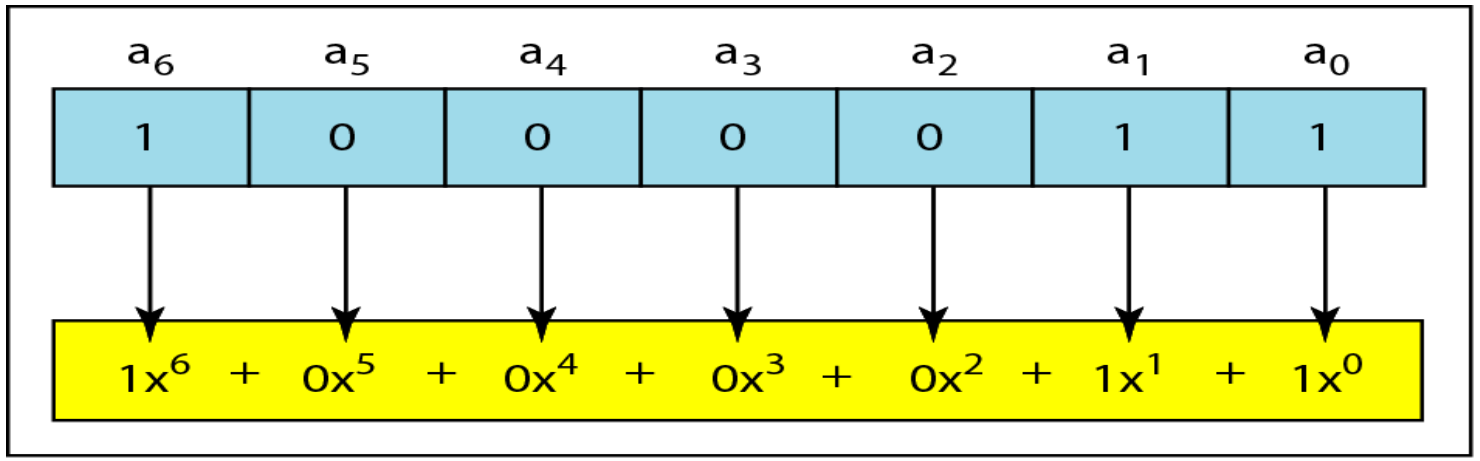
CRC-ITU-T

$x^{16} + x^{12} + x^5 + 1$

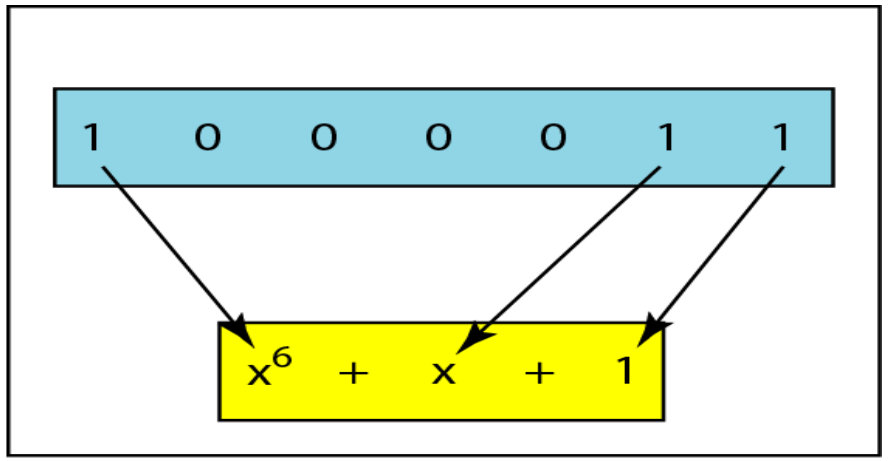
CRC-32

$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$

A polynomial to represent a binary word



a. Binary pattern and polynomial



b. Short form

Cyclic redundancy check (CRC)

Dataword $d(x)$: 1001 $= x^3 + 1$
Appending three 0's. Multiplying by x^3.
We get $x^6 + x^3 = 1001000$
Generator $g(x) = x + 1$

$$
\begin{array}{r|l}
x + 1 & x^5 + x^4 + x^3 \\
\hline
 & x^6 + x^3 \\
 & x^6 + \quad + x^5 \\
\hline
 & x^5 + x^3 \\
 & x^5 + \quad + x^4 \\
\hline
 & x^4 + x^3 \\
 & x^4 + x^3 \\
\hline
 & 0 \quad \text{Remainder}
\end{array}
$$

Codeword $c(x)$ = Dataword + Remainder
= 1001000 $= x^6 + x^3$

Lets say, **single bit error** $e(x) = x^5$ i.e. 6th LSB is inverted: 0100000. It can be seen that $e(x)$ is not divisible by $g(x)$ i.e. the error is caught.

$$
\begin{array}{r|l}
x + 1 & x^4 + x^3 + x^2 + x^1 + 1 \\
\hline
 & x^5 \\
 & x^5 + x^4 \\
\hline
 & x^4 \\
 & x^4 + x^3 \\
\hline
 & x^3 \\
 & x^3 + x^2 \\
\hline
 & x^2 \\
 & x^2 + x^1 \\
\hline
 & x \\
 & x + 1 \\
\hline
 & 0
\end{array}
$$

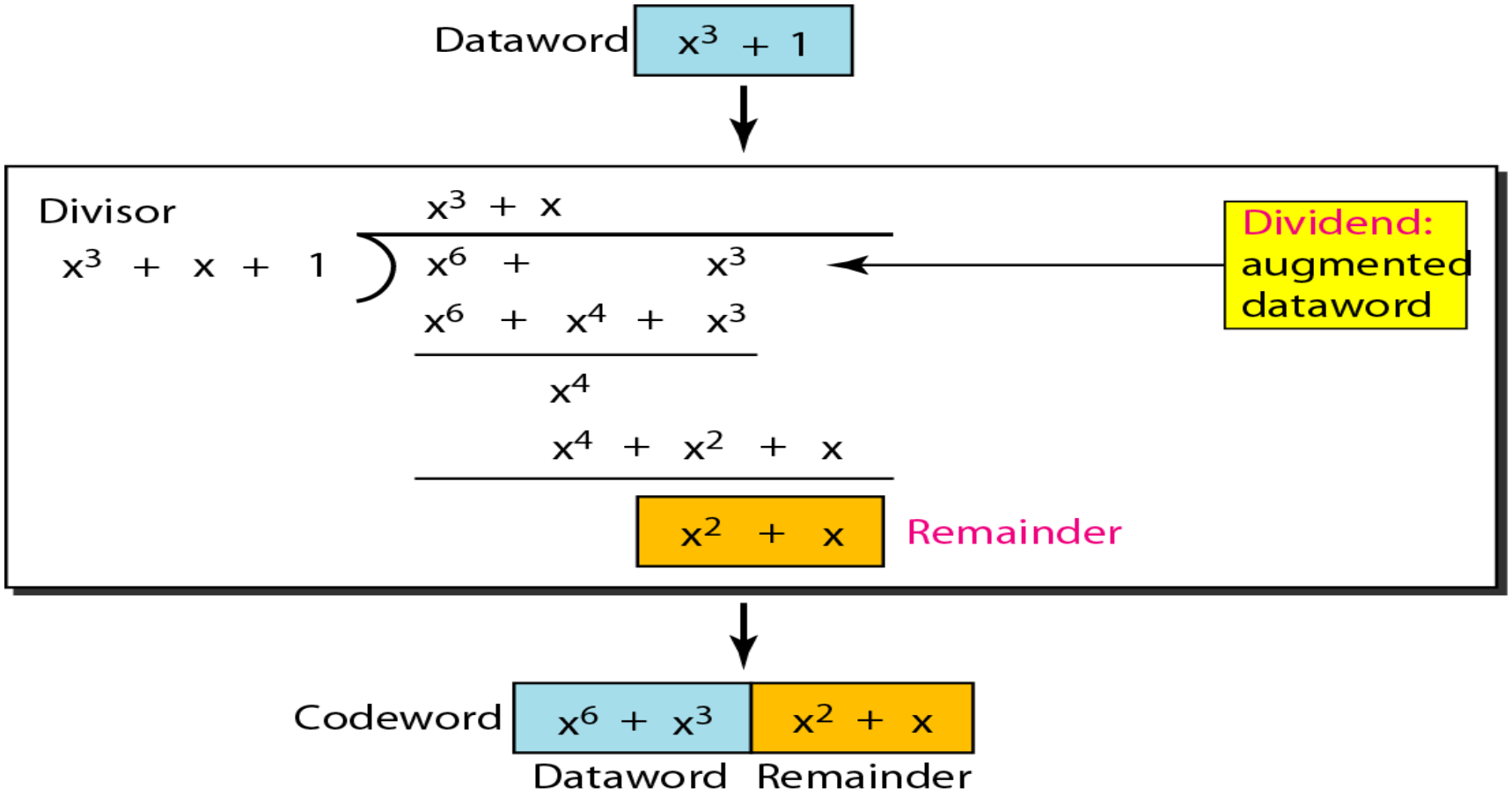
Remainder not zero, thus error occurred and is caught.

It can also be seen that distorted codeword $c(x) + e(x) = x^6 + x^5 + x^3$ is also not divisible by $g(x)$ i.e. the error is caught

$$
\begin{array}{r|l}
x + 1 & x^5 + x^2 + x^1 + 1 \\
\hline
 & x^6 + x^5 + x^3 \\
 & x^6 + x^5 \\
\hline
 & x^3 \\
 & x^3 + x^2 \\
\hline
 & x^2 \\
 & x^2 + x^1 \\
\hline
 & x \\
 & x + 1 \\
\hline
 & 0
\end{array}
$$

Remainder not zero, thus error occurred and is caught.

CRC division using polynomials



Note

The divisor in a cyclic code is normally called the generator polynomial or simply the generator.

Note

In a cyclic code,

If $s(x) \neq 0$, one or more bits is corrupted.

If $s(x) = 0$, either

a. No bit is corrupted. or
b. Some bits are corrupted, but the decoder failed to detect them.

Note

In a cyclic code, those $e(x)$ errors that are divisible by $g(x)$ are not caught.

Note

If the generator has more than one term and the coefficient of x^0 is 1, all single errors can be caught.

The receiver received 10011101101 from sender. The original message is transmitted using CRC polynomial $X^3 + 1$ to protect from errors. Check whether message bits are corrupted during transmission or not?

Example 10.17

Find the suitability of the following generators in relation to burst errors of different lengths.

a. $x^6 + 1$ *b.* $x^{18} + x^7 + x + 1$ *c.* $x^{32} + x^{23} + x^7 + 1$

Solution

a. This generator can detect all burst errors with a length less than or equal to 6 bits; 3 out of 100 burst errors with length 7 will slip by; 16 out of 1000 burst errors of length 8 or more will slip by.

Example 10.17 (continued)

b. This generator can detect all burst errors with a length less than or equal to 18 bits; 8 out of 1 million burst errors with length 19 will slip by; 4 out of 1 million burst errors of length 20 or more will slip by.

c. This generator can detect all burst errors with a length less than or equal to 32 bits; 5 out of 10 billion burst errors with length 33 will slip by; 3 out of 10 billion burst errors of length 34 or more will slip by.

Note

A good polynomial generator needs to have the following characteristics:

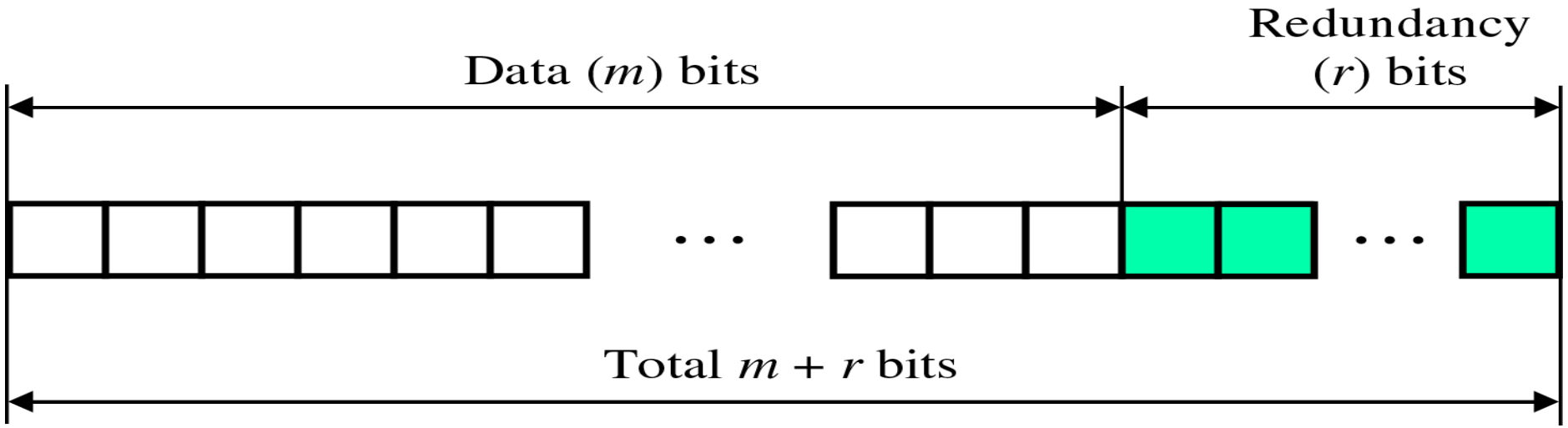
1. It should have at least two terms.
2. The coefficient of the term x^0 should be 1.
3. It should not divide $x^t + 1$, for t between 2 and $n - 1$.
4. It should have the factor $x + 1$.

Error Correction

- To correct an error, the receiver reverses the value of the altered bit. To do so, it must know which bit is in error.
- Single – bit Error correction used parity bit
- The secret of error correction is to locate the invalid bit or bits

Error Correction - Calculation of Redundancy Bits

- To calculate the number of redundancy bits (R) required to correct a given number of data bit (M)



Error Correction - Calculation of Redundancy Bits

- To calculate the number of redundancy bits (R) required to correct a given number of data bit (M)
- If the total number of bits in a transmittable unit is m+r, then r must be able to indicate at least m+r+1 different states

$$2^r \geq m + r + 1$$

- For value of m is 7(ASCII), the smallest r value that can satisfy this equation is 4

$$2^4 \geq 7 + 4 + 1$$

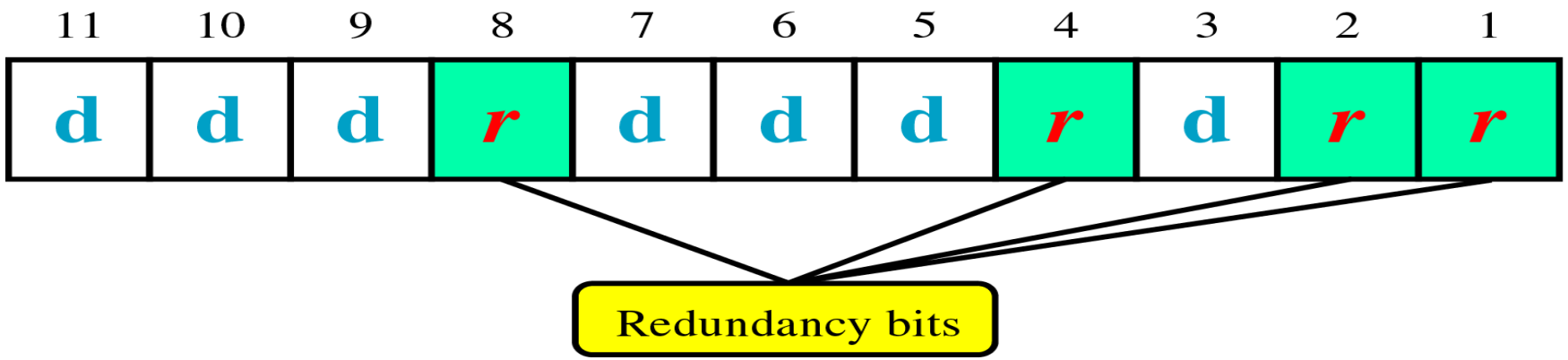
Relationship between data bits and Redundancy bits

Number of Data Bits (m)	Number of Redundancy Bits (r)	Total Bits (m+r)
1	2	3
2	3	5
3	3	6
4	3	7
5	4	9
6	4	10
7	4	11

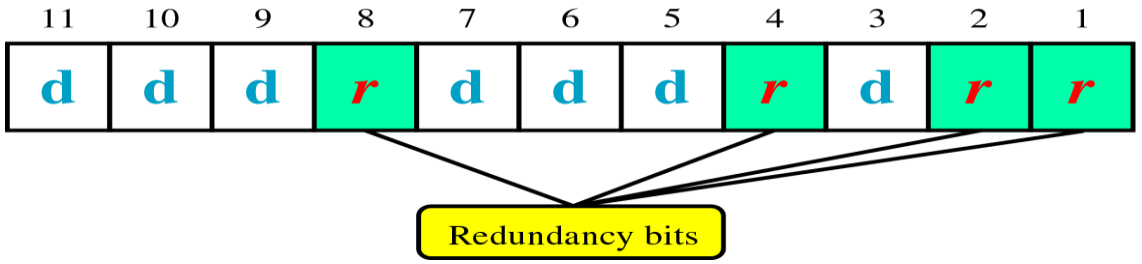
Error Correction - Hamming Code Method

- Develop by R.W. Hamming

positions of redundancy bits in Hamming code



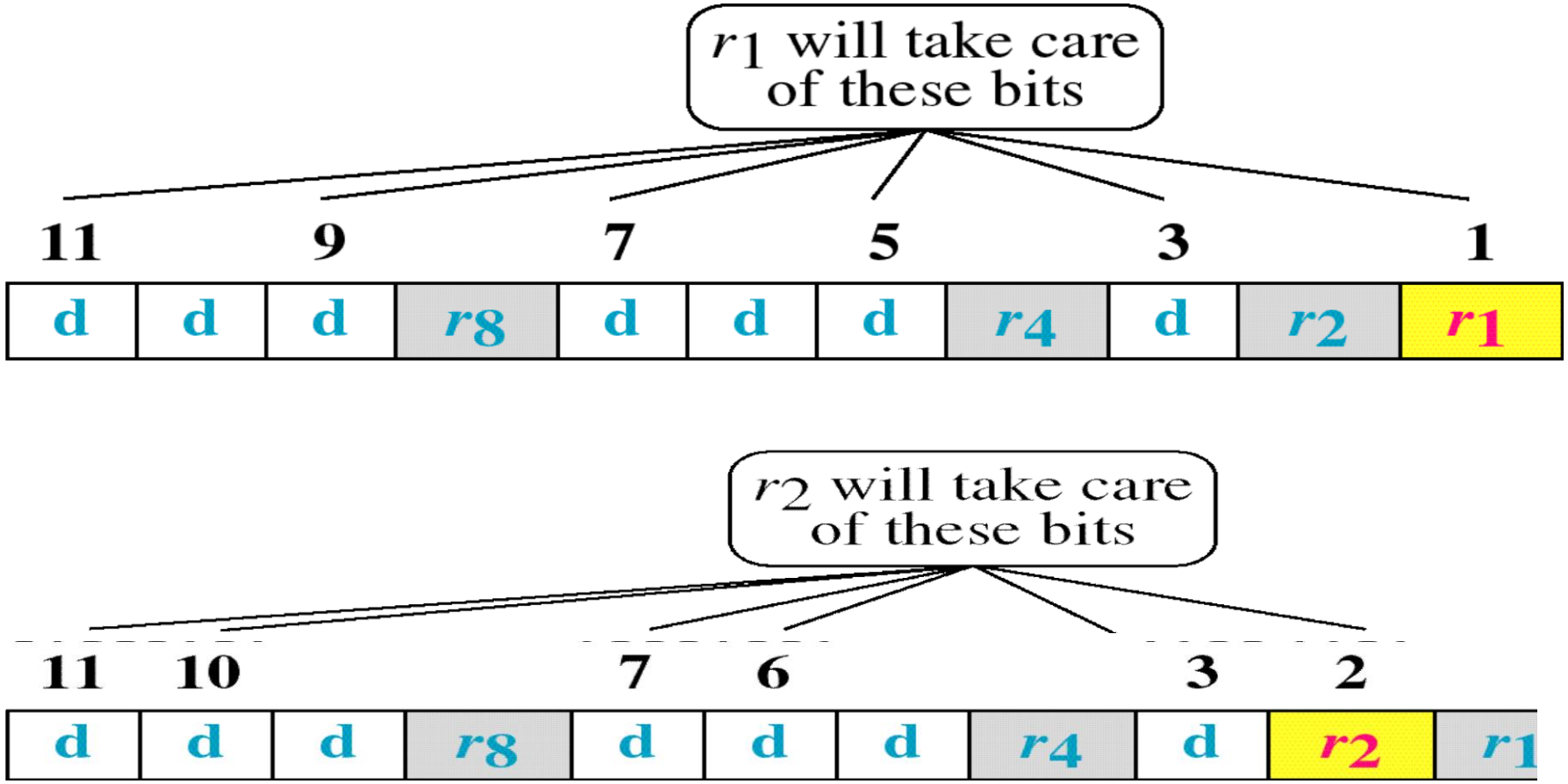
Error Correction - Hamming Code Method



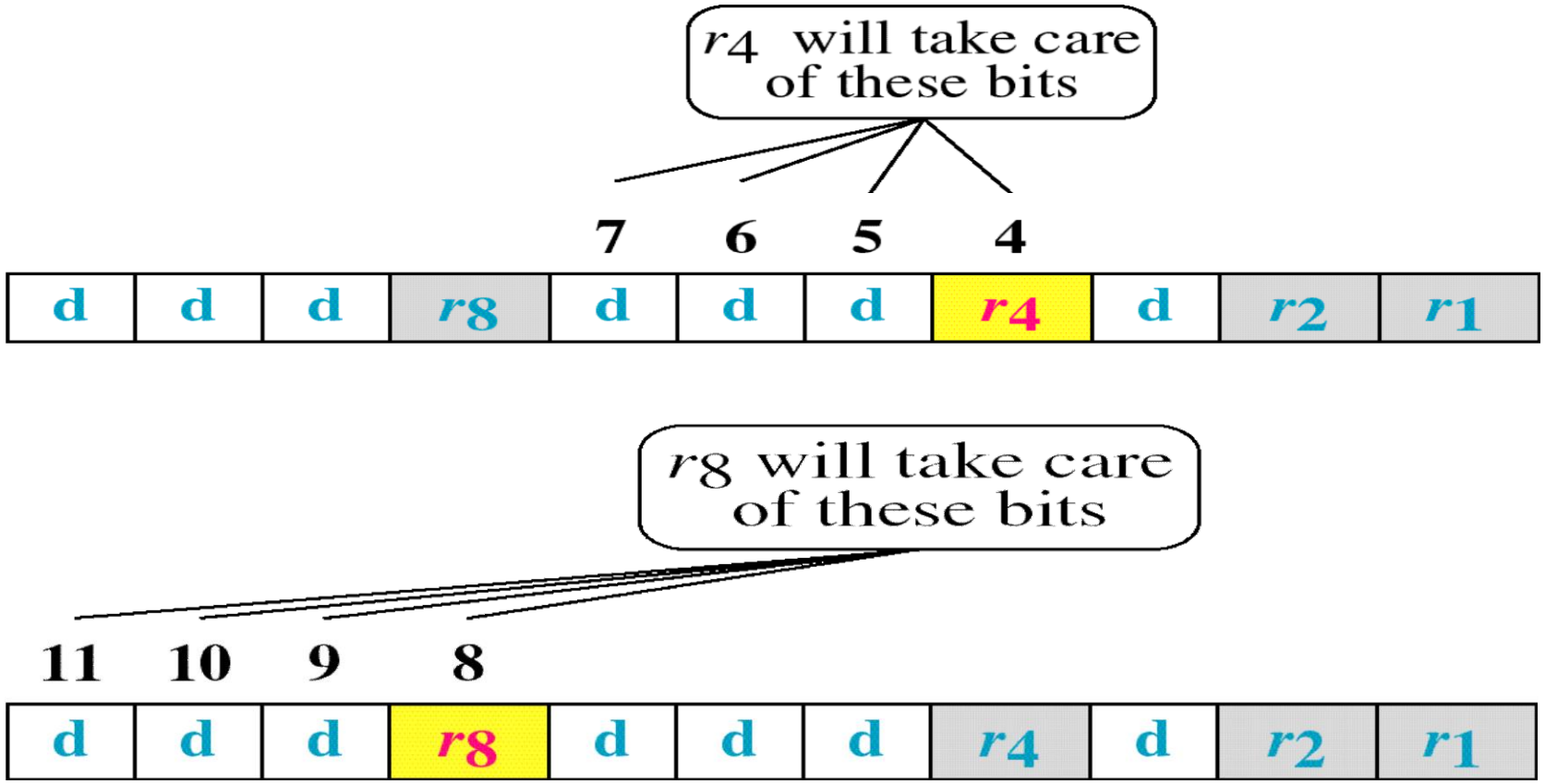
each r bit is calculating by using combination of data bits

r_1 = bits 1, 3, 5, 7, 9, 11
r_2 = bits 2, 3, 6, 7, 10, 11
r_4 = bits 4, 5, 6, 7
r_8 = bits 8, 9, 10, 11

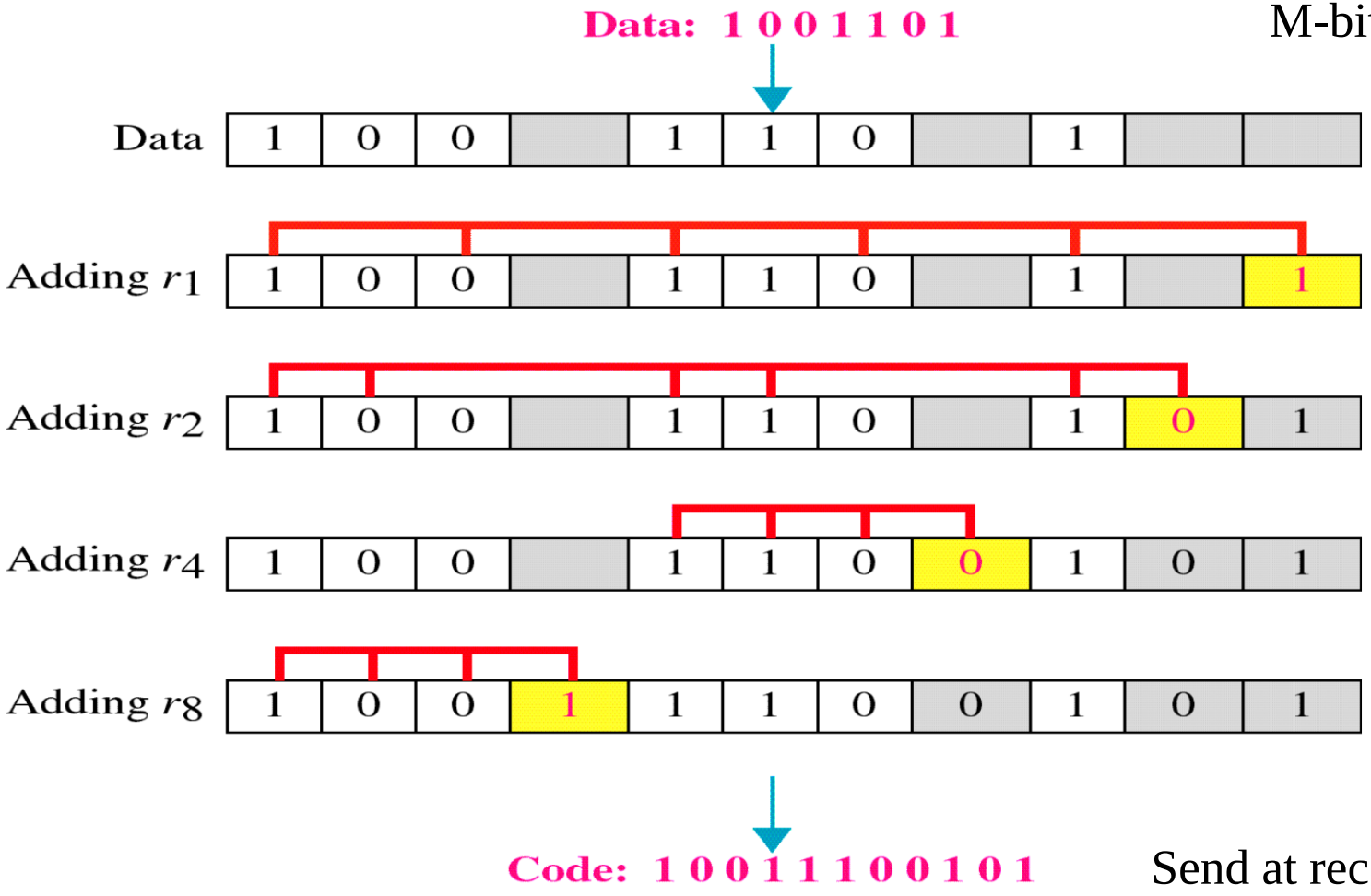
Hamming Code Method – r bit calculation



Hamming Code Method – r bit calculation



Hamming Code - Example



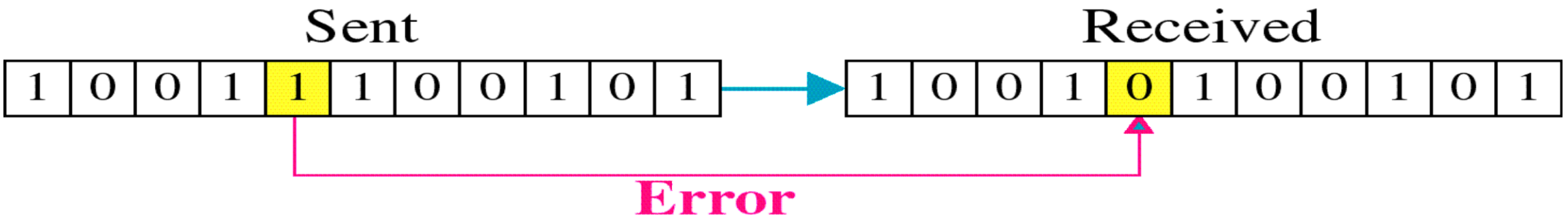
M-bits are 7,so r-bits are 4

Send at receiver

Hamming Code - Example

- At Receiver End recalculate all r bit value and if all having value is 0 then no error.
- If Error then?

Hamming Code - Example



Hamming Code – Error Detection and Correction