

Q1 Given

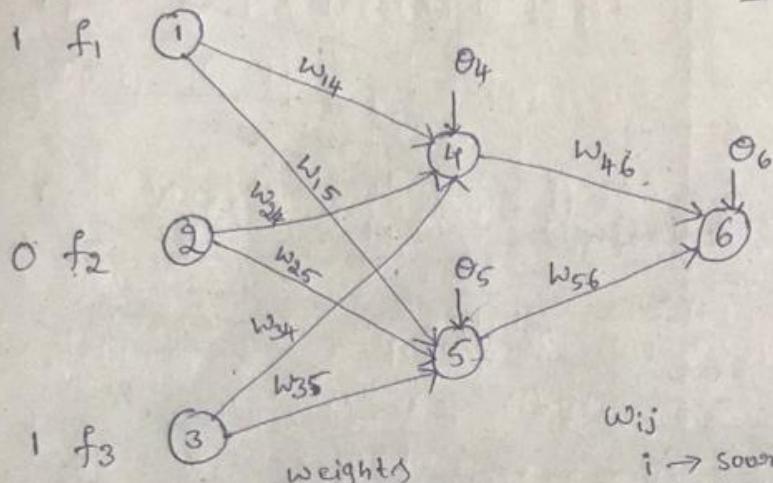
Dataset
 $D[1 \ 0 \ 1]$

$f_1 \ f_2 \ f_3$

Class label [1]

Assume $\eta = 0.9$

Activation = Sigmoid



$$w_{14} = 0.2$$

$$w_{15} = -0.3$$

$$w_{24} = 0.4$$

$$w_{25} = 0.1$$

$$w_{34} = -0.5$$

$$w_{35} = 0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

w_{ij}

$i \rightarrow \text{source}$
 $j \rightarrow \text{Target}$

A

Bias

$$\theta_4 = -0.4$$

$$\theta_5 = 0.2$$

$$\theta_6 = 0.1$$

SOLN

EPOCH 1

Step 1: Forward Propagation.

$$Z_4 = w_{14}f_1 + w_{24}f_2 + w_{34}f_3 + \theta_4$$
$$Z_4 = (0.2)(1) + (0.4)(0) + (-0.5)(1) + (-0.4)$$

$$Z_4 = 0.2 + 0 - 0.5 - 0.4$$

$$Z_4 = 0.2 - 0.9 = -0.7$$

$$\boxed{Z_4 = -0.7} //$$

Activation is Sigmoid

$$\therefore O_4 = A(Z_4) = \frac{1}{1+e^{-Z_4}} = \frac{1}{1+e^{(-0.7)}}$$

$$O_4 = \frac{1}{1+e^{0.7}} = \frac{1}{1+2.0137}$$

$$O_4 = \frac{1}{2.0137} = \frac{1}{3.0137}$$

$$\boxed{O_4 = 0.3318} //$$

$$Z_5 = w_{15}f_1 + w_{25}f_2 + w_{35}f_3 + \theta_5$$

$$Z_5 = (-0.3)(1) + (0.1)(0) + (0.2)(1) + 0.2$$

$$Z_5 = -0.3 + 0 + 0.2 + 0.2$$

$$Z_5 = -0.3 + 0.4 = 0.1$$

$$\boxed{Z_5 = 0.1} //$$

$$O_5 = A(Z_5) = \frac{1}{1+e^{-0.1}} = \frac{1}{1+0.9048}$$

$$O_5 = \frac{1}{1.9048}$$

$$\boxed{O_5 = 0.525} //$$

$$Z_6 = w_{46} \times Z_4 + w_{56} \times Z_5 + \theta_6$$

$$Z_6 = (-0.3) \times (-0.7) + (-0.2)(0.1) + 0.1 \quad \times$$

$$Z_6 = 0.21 + (-0.02) + 0.1$$

$Z_6 = 0.29$

$$O_6 = A(Z_6) = \frac{1}{1+e^{-0.29}} = \frac{1}{1+0.7483}$$

$$= \frac{1}{1.7483} = 0.5719 \quad \times$$

OR

$Z_6 = w_{46} \times A(Z_4) + w_{56} \times A(Z_5) + \theta_6$

$$Z_6 = (-0.3)(0.3318) + (-0.2)(0.525) + 0.1$$

$$Z_6 = -0.09954 - 0.105 + 0.1$$

$Z_6 = -0.1045$

$$A_6 = O_6 = A(Z_6) = \frac{1}{1+e^{-(0.1045)}}$$

$$= \frac{1}{1+e^{0.1045}}$$

$$= \frac{1}{2.1101}$$

$O_6 = A_6 = 0.4739$

Backward propagation

$$\frac{\text{Error}}{6} \\ \frac{5}{4}$$

T₁ = 1

$$E_{\text{out}} = O_6(1-O_6)(T_1 - O_6)$$

$$E_6 = (0.4739)(1-0.4739)(1-0.4739)$$

$$E_6 = 0.1311$$

Hidden layer Calc

$$E_5 = O_5(1-O_5)(\sum_k E_k w_{jk})$$

K → next
j → current
i → previous

$$E_5 = O_j(1-O_j)(\sum_k E_k w_{jk})$$

$$E_5 = O_5(1-O_5)(E_6 w_{56})$$

$$E_5 = 0.5(1-0.5)(0.1311)(-0.2)$$

$$E_5 = -0.00654$$

$$E_4 = O_4(1-O_4)(E_6 w_{46})$$

$$E_4 = (0.3318)(1-0.3318)(0.1311)(-0.3)$$

$$E_4 = -0.00872$$

Calc the updating weights

$$\Delta w_{ij} = \eta E_j O_i$$

$$w_{46\text{new}} = w_{46} + \Delta w_{46}$$

$$= w_{46} + \eta E_6 \cdot O_4$$

$$= -0.3 + 0.9(0.1311)(0.3318)$$

$$= -0.3 + 0.03926$$

$$w_{46\text{new}} = -0.26074$$

$$\begin{aligned}
 w_{56\text{new},1} &= w_{56} + \Delta w_{56} \\
 &= w_{56} + \eta \cdot 0.5 E_6 \\
 &= -0.2 + 0.9 \times 0.525 (0.1311) \\
 &= -0.2 + 0.0619
 \end{aligned}$$

$$w_{56\text{new},1} = -0.1381$$

$$\begin{aligned}
 w_{14\text{new},1} &= w_{14} + \Delta w_{14} \\
 &= w_{14} + \eta \cdot 0.1 \cdot E_4 \\
 &= 0.2 + 0.9 (1) (-0.0087) \\
 &= 0.2 - 0.00783
 \end{aligned}$$

$$w_{14\text{new},1} = 0.1921$$

$$\begin{aligned}
 w_{15\text{new},1} &= w_{15} + \Delta w_{15} \\
 &= w_{15} + \eta \cdot 0.1 \cdot E_5 \\
 &= -0.3 + (0.9) (f_1) (-0.00654) \\
 &= -0.3 + (0.9) (-0.00654) \\
 &= -0.3 - 0.005886 \\
 &= -0.305886
 \end{aligned}$$

$$w_{15\text{new},1} = -0.306$$

$$\begin{aligned}
 w_{24\text{new}} &= w_{24} + \Delta w_{24r} \\
 &= w_{24} + \eta \cdot o_2 \cdot E_4 \\
 &= 0.4 + (0.9)(0)(-0.00872) \\
 &= 0.4 + 0 \\
 w_{24\text{new}} &= 0.4 \quad //.
 \end{aligned}$$

$$\begin{aligned}
 w_{25\text{new}} &= w_{25} + \Delta w_{25} \\
 &= w_{25} + \eta \cdot o_2 \cdot E_5 \\
 &= 0.1 + (0.9)(0)(-0.00654) \\
 &= 0.1 + 0 \\
 w_{25\text{new}} &= 0.1 \quad //.
 \end{aligned}$$

$$\begin{aligned}
 w_{34\text{new}} &= w_{34} + \Delta w_{34\text{new}} \\
 &= w_{34} + \eta \cdot o_3 \cdot E_4 \\
 &= (-0.5) + (0.9)(1)(-0.00872) \\
 &= -0.5 - 0.007848 \\
 w_{34\text{new}} &= -0.508 \quad //.
 \end{aligned}$$

$$\begin{aligned}
 w_{35\text{new}} &= w_{35} + \Delta w_{35} \\
 &= 0.2 + \eta \cdot o_3 \cdot E_5 \\
 &= 0.2 + (0.9)(1)(-0.00654) \\
 &= 0.2 - 0.005886 \\
 w_{35\text{new}} &= 0.194 \quad //.
 \end{aligned}$$

Calc bias

$$\Delta \theta_j = \eta \epsilon_j$$

$\eta \rightarrow$ learning rate

$\eta \rightarrow$

$$\theta_{6\text{new}} = \theta_6 + \Delta \theta_6 \quad \boxed{\underline{j=1}}$$

$$= \theta_6 + \eta(E_6)$$

$$= \theta_6 + E_6$$

$$= 0.1 + (0.1311)(0.9)$$

$$= 0.21799$$

$$\boxed{\theta_{6\text{new}} = 0.218}$$

$$\theta_{5\text{new}} = \theta_5 + \Delta \theta_5$$

$$= \theta_5 + \eta \cdot E_5$$

$$= 0.2 + (0.9)(-0.00654)$$

$$= 0.2 - 0.005886$$

$$\boxed{\theta_{5\text{new}} = 0.194} //$$

$$\theta_{4\text{new}} = \theta_4 + \Delta \theta_4$$

$$= \theta_4 + \eta E_4$$

$$= -0.4 + (0.9)(-0.00872)$$

$$= -0.4 - 0.007848$$

$$\boxed{\theta_{4\text{new}} = -0.408} //$$

EPOCH 2

New weights

$$w_{14\text{new}} = w_{14n_1} = 0.192$$

$$w_{15\text{new}} = w_{15n_1} = -0.306$$

$$w_{24\text{new}} = w_{24n_1} = 0.4$$

$$w_{25\text{new}} = w_{25n_1} = 0.1$$

$$w_{34\text{new}} = w_{34n_1} = -0.508$$

$$w_{35\text{new}} = w_{35n_1} = 0.194$$

$$w_{46\text{new}} = w_{46n_1} = -0.260$$

$$w_{56\text{new}} = w_{56n_1} = -0.138$$

Learning Rate

$$\eta = 0.9$$

Biases

$$\theta_{6\text{new}} = \theta_{6n_1} = 0.218$$

$$\theta_{5\text{new}} = \theta_{5n_1} = 0.194$$

$$\theta_{4\text{new}} = \theta_{4n_1} = -0.408$$

Step 1: Forward Propagation

$$Z_4 = O_{4n_1} + w_{14}f_1 + w_{24}f_2 + w_{34}f_3 \quad f_1 = 1$$

$$f_2 = 0$$

$$Z_4 = -0.408 + (0.192)(1) + 0(0.4) + 1(-0.508) \quad f_3 = 1$$

$$Z_4 = -0.408 + 0.192 + 0 - 0.508$$

$$\boxed{Z_4 = -0.724} //$$

$$O_4 = \frac{1}{1 + e^{(-0.724)}} = \frac{1}{1 + e^{0.724}}$$

$$\boxed{O_4 = 0.3265} //$$

$$Z_5 = O_{5n_1} + w_{15}f_1 + w_{25}f_2 + w_{35}f_3$$

$$Z_5 = 0.194 + (-0.306)(1) + 0(0.4) + 1(0.194)$$

$$Z_5 = 0.194 - 0.306 + 0.194$$

$$\boxed{Z_5 = 0.082} //$$

$$O_5 = \frac{1}{1 + e^{-0.082}} = \frac{1}{1 + 0.9212}$$

$$O_5 = \frac{1}{1.9212}$$

$$\boxed{O_5 = 0.5205} //$$

$$Z_6 = \theta_{6n_1} + w_{46} \times o_4 + w_{56} \times o_5$$

$$Z_6 = 0.218 + (-0.26)(0.3265) + (-0.138)(0.5205)$$

$$Z_6 = 0.218 - 0.08489 - 0.07182$$

$$Z_6 = 0.0613 \quad //..$$

$$O_6 = \frac{1}{1 + e^{-0.0613}} = 0.515 \quad //..$$

Backward propagation

$$E_6 = O_6(1-O_6)(T_1 - O_6)$$

$$E_6 = (0.515)(1-0.515)(1-0.515)$$

$$E_6 = 0.121 \quad //..$$

Hidden layer calc

$$E_5 = O_5(1-O_5) \sum_k E_k \cdot w_{jk}$$

j → current
k → future
i → previous

$$E_5 = O_5(1-O_5) E_6 \cdot w_{56}$$

$$E_5 = (0.5205)(1-0.5205)(0.121)(-0.138)$$

$$E_5 = -0.0042 \quad //..$$

$$E_4 = O_4(1-O_4) \sum_k E_k \cdot w_{jk}$$

$$E_4 = O_4(1-O_4) E_6 \cdot w_{46}$$

$$E_4 = (0.3265)(1-0.3265)(0.121)(-0.260)$$

$$E_4 = -0.0069 \quad //..$$

Updated weights

$$\begin{aligned} w_{46\text{new}_2} &= w_{46\text{new}_1} + \Delta w_{46} & \Delta w_{ij} = \eta D_i E_j \\ &= w_{46\text{new}_1} + \eta \cdot 0.4 \cdot E_6 \\ &= -0.26 + 0.9(0.3265)(0.121) \\ &= -0.26 + 0.036 \\ w_{46\text{new}_2} &= -0.224 \end{aligned}$$

$$\begin{aligned} w_{58\text{new}_2} &= w_{58\text{new}_1} + \Delta w_{58} \\ &= -0.138 + \eta \cdot 0.5 \cdot E_6 \\ &= -0.138 + 0.9 \times 0.5205 \times 0.121 \\ w_{58\text{new}_2} &= -0.081 \end{aligned}$$

$$\begin{aligned} w_{14\text{new}_2} &= w_{14\text{new}_1} + \Delta w_{14} \\ &= w_{14\text{new}_1} + \eta \cdot 0.1 \cdot E_4 \\ &= 0.1921 + 0.9 \times 1 \times (-0.0069) \\ &= 0.1921 - (0.9)(0.0069) \\ w_{14\text{new}_2} &= 0.186 \end{aligned}$$

$$\begin{aligned} w_{15\text{new}_2} &= w_{15\text{new}_1} + \Delta w_{15} \\ &= -0.306 + \eta \cdot 0.1 \cdot E_5 \\ &= -0.306 + (0.9)(0.1)(-0.0042) \\ &= -0.306 - (0.9)(0.0042) \\ w_{15\text{new}_2} &= -0.309 \end{aligned}$$

$$w_{24\text{new2}} = w_{24\text{new1}} + \Delta w_{24}$$
$$= 0.4 + 0.02 \cdot E_4$$

$O_2 = 0.$

$$\boxed{w_{24\text{new2}} = 0.4} //.$$

$$w_{25\text{new2}} = w_{25\text{new1}} + \Delta w_{25}$$
$$= 0.1 + 0.02 \cdot E_5$$
$$= 0.1 + (0.9)(0)(E_5)$$

$$\boxed{w_{25\text{new2}} = 0.1} //.$$

$$w_{34\text{new2}} = w_{34\text{new1}} + \Delta w_{34}$$
$$= -0.508 + (0.9)(0_3)(E_4)$$
$$= -0.508 + (0.9)(1)(-0.007)$$
$$= -0.508 - 0.0063$$
$$\boxed{w_{34\text{new2}} = -0.5143} //.$$

$$w_{35\text{new2}} = w_{35\text{new1}} + \Delta w_{35}$$
$$= 0.194 + (0.9)(0_3)(E_5)$$
$$= 0.194 - (0.9)(1)(0.0042)$$
$$= 0.194 - 0.00378$$

$$\boxed{w_{35\text{new2}} = 0.19} //.$$

Calc bias

$$\Delta\theta_j = \eta \cdot \varepsilon_j$$

$$\begin{aligned}\theta_{6\text{new}2} &= \theta_{6\text{new}1} + \Delta\theta_6 = \theta_{6\text{new}1} + \eta \cdot \varepsilon_6 \\ &= 0.218 + (0.9)(0.121)\end{aligned}$$

$$\boxed{\theta_{6\text{new}2} = 0.327} //..$$

$$\begin{aligned}\theta_{5\text{new}2} &= \theta_{5\text{new}1} + \eta \cdot \varepsilon_5 \\ &= 0.194 + (0.9)(-0.00372)\end{aligned}$$

$$= 0.194 - 0.00378$$

$$\boxed{\theta_{5\text{new}2} = 0.190} //..$$

$$\begin{aligned}\theta_{4\text{new}2} &= \theta_{4\text{new}1} + \eta \cdot \varepsilon_4 \\ &= -0.408 + 0.9 \times (-0.0069)\end{aligned}$$

$$= -0.408 - 0.00621$$

$$\boxed{\theta_{4\text{new}2} = -0.414} //..$$

EPOCH 3

Weights from Previous Step

$$w_{14\text{new2}} = 0.186$$

$$w_{24\text{new2}} = 0.4$$

$$w_{15\text{new2}} = -0.309$$

$$w_{25\text{new2}} = 0.1$$

$$w_{34\text{new2}} = -0.5143$$

$$w_{35\text{new2}} = 0.19$$

$$w_{46\text{new2}} = -0.224$$

$$w_{56\text{new2}} = -0.081$$

Bias

$$\theta_{6\text{new2}} = 0.327$$

$$\theta_{5\text{new2}} = 0.190$$

$$\theta_{4\text{new2}} = -0.414$$

Learning rate

$$\eta = 0.9$$

Step 1: Forward propagation

$$Z_4 = \Theta_{4n_2} + w_{14n_2}f_1 + w_{24n_2}f_2 + w_{34n_2}f_3$$

$$Z_4 = -0.414 + (0.186)(1) + (0.4)(0) + (-0.5143)(1)$$

$$Z_4 = -0.414 + 0.186 + 0 - 0.5143$$

$$\boxed{Z_4 = -0.7423}$$

$$O_4 = A(Z_4) = \frac{1}{1 + e^{(-0.7423)}} = \frac{1}{1 + e^{0.7423}}$$

$$O_4 = \frac{1}{1 + 2.1}$$

$$\boxed{O_4 = 0.32} //$$

$$Z_5 = \Theta_{5n_2} + w_{15n_2}f_1 + w_{25n_2}f_2 + w_{35n_2}f_3$$

$$Z_5 = 0.19 + (-0.309)(1) + (0.1)(0) + (0.19)(1)$$

$$Z_5 = 0.19 - 0.309 + 0.19$$

$$\boxed{Z_5 = 0.071} //$$

$$O_5 = A(Z_5) = \frac{1}{1 + e^{-0.071}}$$

$$= \frac{1}{1 + 0.93}$$

$$= \frac{1}{1.93}$$

$$\boxed{O_5 = 0.518} //$$

$$Z_6 = \theta_{6n_2} + w_{46n_2} \cdot O_4 + w_{56n_2} \cdot O_5$$

$$Z_6 = 0.327 + (-0.224)(0.32) + (-0.081)(0.518)$$

$$Z_6 = 0.327 - (0.224)(0.32) - (0.081)(0.518)$$

$$\boxed{Z_6 = 0.213}$$

$$O_6 = A(Z_6) = \frac{1}{1 + e^{-0.213}}$$

$$= \frac{1}{1 + 0.808}$$

$$\boxed{O_6 = 0.553} //$$

Backward propagation

$$E_6 = O_6(1-O_6)(T_1 - O_6)$$

$$= (0.553)(1-0.553)(1-0.553)$$

$$\boxed{E_6 = 0.11} //$$

Hidden layer calc

$$E_4 = O_4(1-O_4) \sum_K E_K w_{JK}$$

$$= (0.32)(1-0.32) E_6 \cdot w_{46}$$

$$= 0.32(1-0.32)(0.11)(-0.224)$$

$$\boxed{E_4 = -0.00534} //$$

$$E_5 = O_5(1-O_5) \sum_K E_K w_{JK}$$

$$= O_5(1-O_5) E_6 \cdot w_{56}$$

$$= (0.518)(1-0.518)(0.11)(-0.081)$$

$$\boxed{E_5 = -0.0022} //$$

Updated weights

$$\begin{aligned} w_{46\text{new}_3} &= w_{46\text{new}_2} + \Delta w_{46} \\ &= w_{46\text{new}_2} + \eta \cdot o_4 \cdot E_6 \\ &= -0.224 + (0.9)(0.32)(0.11) \end{aligned}$$

$$w_{46\text{new}_3} = -0.19$$

$$w_{56\text{new}_3} = w_{56\text{new}_2} + \Delta w_{56}$$

$$= w_{56\text{new}_2} + \eta \cdot o_5 \cdot E_6$$

$$= -0.081 + (0.9)(0.518)(0.11)$$

$$w_{56\text{new}_3} = -0.03$$

$$w_{14\text{new}_3} = w_{14\text{new}_2} + \Delta w_{14}$$

$$= w_{14\text{new}_2} + \eta \cdot o_1 \cdot E_4$$

$$= 0.186 + (0.9)(1)(-0.0054)$$

$$w_{14\text{new}_3} = 0.181$$

$$w_{24\text{new}_3} = w_{24\text{new}_2} + \Delta w_{24}$$

$$= w_{24\text{new}_2} + \eta \cdot o_2 \cdot E_4$$

$$= 0.4 + (0.9)(0)(-0.0054)$$

$$w_{24\text{new}_3} = 0.4$$

$$\begin{aligned}
 w_{15\text{new}3} &= w_{15\text{new}2} + \Delta w_{15} \\
 &= -0.309 + \eta \cdot 0_1 \cdot E_5 \\
 &= -0.309 + (0.9)(1)(-0.0022)
 \end{aligned}$$

$$w_{15\text{new}3} = -0.31 //..$$

$$\begin{aligned}
 w_{25\text{new}3} &= w_{25\text{new}2} + \Delta w_{25} \\
 &= 0.1 + \eta \cdot 0_2 \cdot E_5
 \end{aligned}$$

$$w_{25\text{new}3} = 0.1 //..$$

$$\begin{aligned}
 w_{34\text{new}3} &= w_{34\text{new}2} + \Delta w_{34} \\
 &= -0.5143 + (0.9)(1)(-0.0054) \\
 &= -0.5143 - (0.9)(0.0054)
 \end{aligned}$$

$$w_{34\text{new}3} = -0.52 //..$$

$$\begin{aligned}
 w_{35\text{new}3} &= w_{35\text{new}2} + \Delta w_{35} \\
 &= w_{35\text{new}2} + \eta \cdot 0_3 \cdot E_5 \\
 &= 0.19 + (0.9)(1)(-0.0022)
 \end{aligned}$$

$$w_{35\text{new}3} = 0.19 //..$$

Calc Bay

$$\Delta \theta_j = \eta \cdot \varepsilon_j$$

$$\Delta \theta_6 \cdot \theta_{6\text{new3}} = \theta_{6\text{new2}} + \eta \cdot \varepsilon_6 \\ = 0.327 + 0.9 \times 0.11$$

$$\boxed{\theta_{6\text{new3}} = 0.426} //..$$

$$\theta_{5\text{new3}} = \theta_{5\text{new2}} + \eta \cdot \varepsilon_5 \\ = 0.190 + (0.9)(-0.0022)$$

$$= 0.188$$

$$\boxed{\theta_{5\text{new3}} \approx 0.19} //..$$

$$\theta_{4\text{new3}} = \theta_{4\text{new2}} + \eta \cdot \varepsilon_4 \\ = -0.414 + 0.9(-0.0054) \\ = -0.418$$

$$\boxed{\theta_{4\text{new3}} \approx 0.42} //..$$