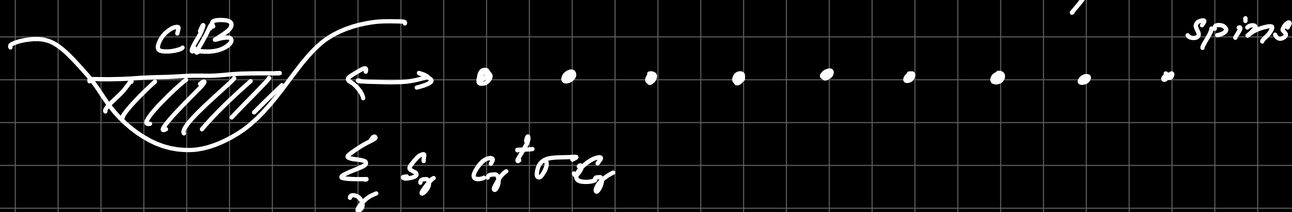
Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover   Lecture 9

Topic: Overview of Kondo Problem and heavy fermions, Kondo variational (continued), renormalization group analysis

Why this topic?

@ 211B :- Couple a C.B. to a spin lattice like (Kondo lattice problem)



size of fermi surface = $n_{CB} + n_{spins}$ (using Oshikawa's flux threading argument)

$\Downarrow$

a supposedly striking result

Introduce parton construction:

$$\vec{S} = f^{\dagger}\frac{\vec{\sigma}}{2}f \qquad f = \begin{bmatrix} f_{\uparrow} \\ f_{\downarrow} \end{bmatrix}$$

$$f^{\dagger}f = 1$$

f's are called partons (like protons formed of quarks)

↳ such a const. helps explains kondo lattice problem easily.

$f \to f e^{i\theta(r)}$ does not changes $\vec{S}$ i.e. parton constr. calls for a gauge field

Heavy fermions = Higgs phase of the gauge field

* When there's a gauge field
$\Downarrow$
Phases of a Gauge field

deconfined phase
everything is freely propagating
(e.g. Griffiths E&M)
= e.g. QSL = deconfined phase

Superconductor
Condense an object that carries gauge charge
e.g. in a SC
$\langle c^+ c^+ \rangle$ is condensed
$\Downarrow$
gapping out gauge field, acquires mass, meissner effect
= "Heavy fermions" in a higgs phase
$\langle c^+ f \rangle \to$ condenses

Confined phase

* "dynamical" vs "non-dynamical" gauge field

Variational wavefunction for Anderson model

Anderson model:

$$\epsilon_k c^+_{k\sigma} c_{k\sigma} + \left[c^+_r (r=0)\, d_\sigma + h.c. \right] + \epsilon_d d_\sigma^+ d_\sigma + U n_{d\uparrow} n_{d\downarrow}$$

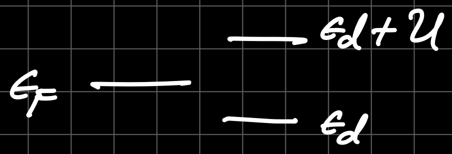
or

$$\left[V_k\, c_k^+ d_\sigma + h.c. \right]$$

Kondo impurity:

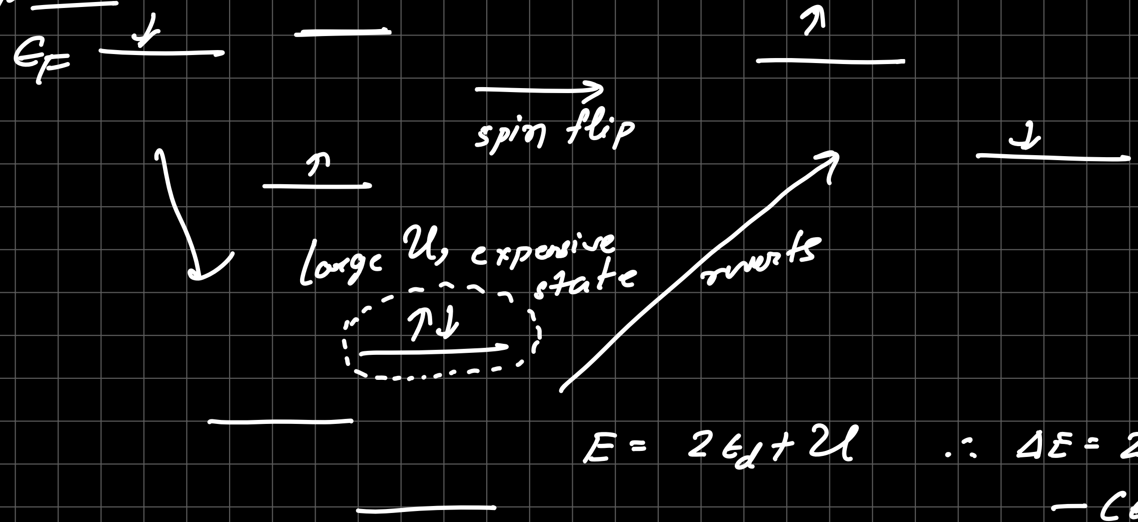
$$\sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \vec{S}\cdot c^\dagger \vec{\sigma} c$$

$$\text{And. model} \xrightarrow[\&]{\text{for large } U} \text{Kondo impurity}$$



U large

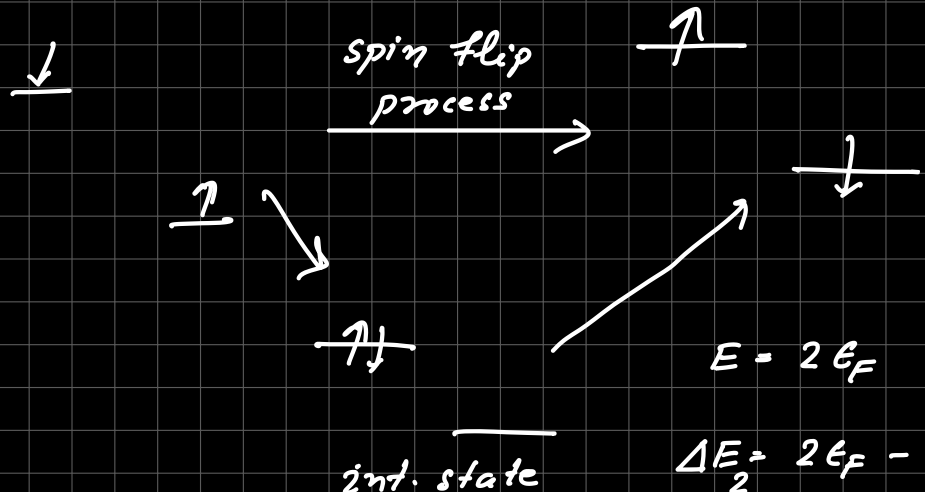
1st process



$E = 2\epsilon_d + U$

$$\therefore \Delta E = 2\epsilon_d + U - (\epsilon_d + \epsilon_F) = \epsilon_d + U - \epsilon_F = \tilde{\epsilon}_d + U$$

2nd process



$E = 2\epsilon_F$

$$\Delta E_2 = 2\epsilon_F - (\epsilon_F + \epsilon_d)$$

$$H_{eff} = |V|^2 \left[\frac{1}{\Delta E_1} + \frac{1}{\Delta E_2}\right] \sim \vec{S}\cdot c^\dagger \vec{\sigma} c$$

$$\simeq J\, \vec{S}\cdot c^\dagger \vec{\sigma} c$$

↳ AFM (∵ we form singlet

in Int. states)

$$J \sim \frac{|V|^2}{|\tilde{\epsilon}_d|}$$ Antiferromagnetic coupling

Variational wfn: (for Anderson Model)

$$|\psi\rangle = [\alpha_0 + \sum_{k<k_F} \alpha_k^\sigma c_{k\sigma} d_\sigma^\dagger]|0\rangle$$

↳ |filled FS⟩ ⊗ |empty impurity⟩

- spin-singlet
- No double-occupancy

$$P\, e^{\sum_k \frac{\alpha_k}{\alpha} c_{k\sigma} d_\sigma^\dagger} |0\rangle$$

$$\xi_x \to \text{F.T. of } \left(\frac{\alpha_k}{\alpha_0}\right)$$

doesn't allow for double occupancy

Define Δ_K as

$$\Delta_K = E_{var} - \epsilon_d$$ (assume filled FS has zero energy)

$\underset{\sim}{\Delta_K < 0}$

↳ entanglement is preferred

$$\Delta_K = -\epsilon_F\, e^{-\frac{1}{2N(0)\, V^2/\tilde{\epsilon}_d}}$$

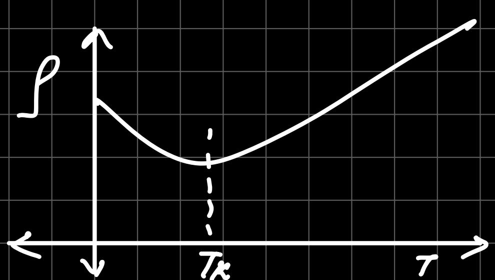
$$\frac{V^2}{|\tilde{\epsilon}_d|} \to \text{kondo en. scale}$$

shows up in 2nd order pert. theory

shows up here too

$T_K \sim \Delta_K$

[not a phase transition (0-D SSB) yet a relevant energy scale]



Kondo screening length:

$$\alpha(\vec{r}) = \int_k \alpha_k \, e^{ikr} \qquad \alpha_k = \frac{\alpha_0 V_k^{\sigma}}{\Delta_k + \tilde{\xi}_k}$$

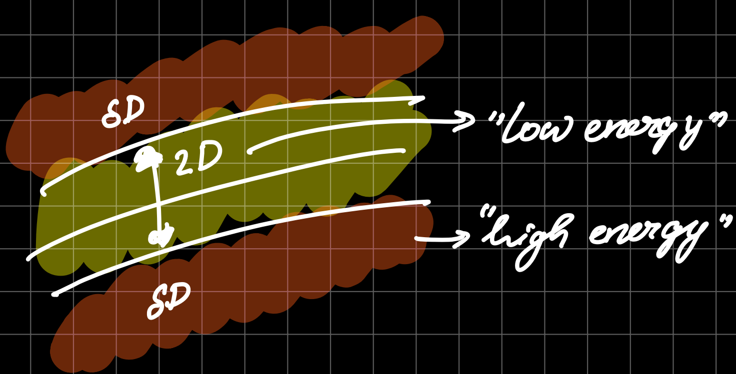
$$= \int_k \frac{\alpha_0 V_k^{\sigma}}{\underbrace{\Delta_k}_{<0} + \underbrace{\tilde{\xi}_k}_{<0}} e^{ikr} \sim e^{-r/\xi_k}$$

→ no pole on real axis

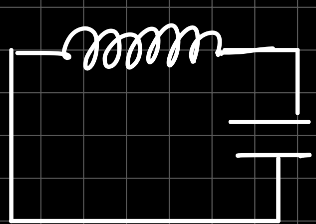
$$|\tilde{\xi}_k| \sim |k - k_F| v_F \qquad \xi_k = \frac{v_F}{\Delta_k}$$

Anderson's Poor man RG for Kondo problem

* integrate high energy modes sequentially to obtain low energy eff. hamiltonian



e.g. RG



$$S_{action} \sim \frac{\dot{x}^2}{2} - \frac{\omega_0^2 x^2}{2} + \frac{\dot{\varepsilon}^2}{2} - \frac{\varepsilon^2 \omega_{cL}^2}{2}$$

$$\omega_0 >>> \omega_{cL}$$

Ans:- $L_{eff} \sim \Big(\qquad \Big)$

From Xiao Gang's book

$$\mathcal{H} = \sum_k \varepsilon_k c^+_{k\sigma} c_{k\sigma} + \frac{J_K}{V} \sum_{kk'} \left(c^+_k \vec{\sigma}\, c_{k'} \right) \cdot \vec{S}$$

$$\parallel$$

$$c^+(r=0)\, \vec{\sigma}\, c(r=0) \cdot \vec{S}$$

$\downarrow$

separate k, k' into bands & integrate out high energy mode

leads to a term of the kind

$$\sum_{k'} J_K^2 \left\langle c^+_{k_1} \vec{\sigma}\, c_{k'} \cdot \vec{S}\, c^+_{k'} \vec{\sigma}\, c_{k_2} \cdot \vec{S} \right\rangle$$

$k_1, k_2 \in$ Low energy modes

$k' \in$ high " "

$\langle\,\rangle$ = avg wrt Hamiltonian of high energy modes

$$e^{-(\mathcal{H}_0 + \mathcal{H}_I)} \qquad \langle e^{-\mathcal{H}_I} \rangle = e^{-\left[\langle \mathcal{H}_I \rangle - \frac{1}{2}\langle \mathcal{H}_I^2 \rangle + \dots\right]}$$

$$J_K^2 \left\langle \bar{c}_\alpha(k_1, \omega_1)\, \sigma^a_{\alpha\beta}\, c_\beta(k', \omega_1)\, S^a\, \bar{c}_\gamma(k', \omega_2)\, \sigma^b_{\gamma\delta}\, c_\delta(k_2, \omega_2)\, S^b \right\rangle$$

$$\underset{(k', \omega_1)\ (k', \omega_2)}{\langle c_\beta\, \bar{c}_\gamma \rangle} = \frac{1 - n_F(\varepsilon_{k'})}{i\omega_1 - \tilde{\xi}_{k'}}\, \delta_{\omega_1, \omega_2}\, \delta_{\beta\gamma}$$

$$\omega_1 = \xi_{k_1} = \xi_{k_2} \quad \text{(elastic scattering)}$$

$$\frac{\sum_{k'} c^+_\alpha(k_1)\, c_\delta(k_2)\, \sigma^a_{\alpha\beta}\, \sigma^b_{\gamma\beta}\, \overbrace{S^a S^b}^{= i\epsilon_{abc} S^c + \delta_{ab} \mathbb{1}}}{(\tilde{\xi}_{k_2} - \tilde{\xi}_{k'})} \cdot \left[1 - n_F(\xi_{k'})\right]$$

$$\approx \left(c^+_{k_1} \vec{\sigma}\, c_{k_2} \cdot \vec{S} \right)$$

$$\delta J_K \simeq J_K^2 \sum_{k'} \frac{1}{\xi_{k'} - \tilde{\xi}_{k_1}} \approx J_K^2 N(0) \frac{\delta D}{D}$$

this is the eff RG flow

$$\boxed{\frac{d J_K}{d \log D} \simeq 2 J_K^2 N(0)}$$

(very much 1d ising model
↓
"asymptotic freedom")

$$\frac{d J_K}{d l} = 2 J_K^2 N(0)$$

$l=0 \longrightarrow l \ggg 1$ (Temp reduces)

$$\frac{T_K}{\epsilon_F} = e^{-l} \qquad e^{l} = b$$

integrating

$$-\frac{1}{J_K(l \gg 1)} + \frac{1}{J_K(l=0)} = 2N(0)\, l$$

$$\therefore T_K = \epsilon_F \, e^{-\frac{1}{2N(0) J_K}}$$

$e^{l} = \dfrac{\xi_K}{a}$ $\quad a \to$ lattice spacing $\Big\} \to$ gives a kondo scale

<u>Ref</u>:- Philip Philips book, Coleman's book

$$\xi_K = a\, e^{\frac{1}{N(0) J_K}}$$

crossover, not a phase transition.

↓

things progressively decrease from some high temp. behaviour into low temp. phenomena