Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover Lecture 13

Topic: Mean-field for AFM (large S), Schwinger boson approach, Intro to $\mathbb{Z}_2$ gauge theory

Large-S for AFM

$$\mathcal{H} = |J| \sum \vec{S}_i \cdot \vec{S}_j$$

(classically)

$$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$$

Neel state (not an eigenstate of $\mathcal{H}_{AFM}$)

$$\mathcal{H}_{FM} |\uparrow\uparrow \cdots \uparrow\rangle = E_{gs} |\uparrow \cdots \uparrow\rangle$$

$$S^+ = S_x + i S_y = \left(\sqrt{2S - b^\dagger b}\right) b$$

$$S^z = S - b^\dagger b \qquad (b^\dagger b) \text{ small}$$

define $\tilde{S}^z = -S^z$ on B sublattice; $\tilde{S}^x = S^x$, $\tilde{S}^y = -S^y$ $\Big\}$ for comm. relations to stay the same

$$S^z \text{ or } \tilde{S}^z = S - n_b$$

$$S^+ \text{ or } \tilde{S}^+ = \left(\sqrt{2S - n_b}\right) b$$

$$\mathcal{H} = +|J| \sum_{\langle ij\rangle} \left[S_i^z S_j^z + \frac{1}{2}(S_i^+ S_j^- + h.c.)\right]$$

$$= |J| \sum_{\langle ij\rangle} \left[-S_i^z \tilde{S}_j^z + \frac{1}{2}(S_i^+ \tilde{S}_j^+ + h.c.)\right]$$

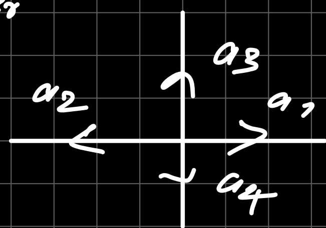
$$\sqrt{2S - n_b} \approx \left(\sqrt{2S}\right)\left[1 - \frac{n_b}{4S} + \cdots\right]$$

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 + \cdots\cdots$$

$$\mathcal{H}_0 = -\frac{S^2 |J| V Z}{2}$$

$$\mathcal{H}_1 = |J| S Z \sum_k \left[b_k^\dagger b_k + \frac{\gamma_k}{2}\left(b_k^\dagger b_{-k}^\dagger + h.c.\right)\right]$$

$$\gamma_k = \frac{1}{Z} \sum_{\vec{a}_r} e^{i \vec{k}\cdot\vec{a}_r}$$



$$= \frac{1}{4}\left[2\cos k_x a + 2\cos k_y a\right]$$

$$\mathcal{H}_2 = -\frac{|J|}{4} \sum_{\langle ij \rangle} \left(\cdots\cdots\cdots \right)$$

$$b_k = \cosh(\theta_k)\,\alpha_k + \sinh(\theta_k)\left(\alpha_{-k}^\dagger\right)$$

$$\left[\alpha_k, \alpha_{k'}^\dagger\right] = \delta_{kk'}$$

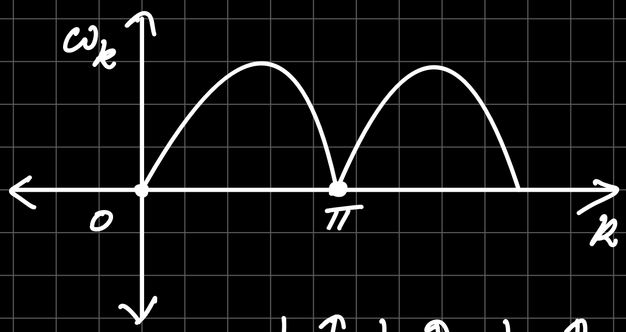
$$\sum \alpha_k^\dagger \alpha_k E_k$$

$$\mathcal{H}_1 = |J| S Z \sum_k \left[\cosh(2\theta_k) + \gamma_k \sinh(\theta_k)\right] \alpha_k^\dagger \alpha_k + \frac{1}{2}\left[\sinh(2\theta_k) + \gamma_k \cosh(2\theta_k)\right]\left[\alpha_k^\dagger \alpha_{-k}^\dagger + h.c.\right]$$

$$\mathcal{H}_1 = \sum_k \omega_k \left[\alpha_k^\dagger \alpha_k + \frac{1}{2}\right] - \frac{JZNS}{2}$$

$$\omega_k = |J| S Z \sqrt{1 - \gamma_k^2} \longrightarrow$$ Vanishes at $\vec{k} = 0$ & $\vec{k} = (\pi, \pi \cdots \pi) \equiv \vec{Q}$

$$\gamma_k = \frac{\cos k_x + \cos k_y}{2} \begin{array}{l} \nearrow k=0 \\ \searrow k=-1 \end{array} \Big] \ \gamma_k^2 = 1$$



$\downarrow \uparrow \downarrow \uparrow \downarrow \uparrow$

$S_i^z S_j^z \qquad |s| = 1$

$\approx |S(\vec{r})|^2 (\cos k_x + \cos k_y)$

$k \approx 0$

$\omega_k \approx JS\sqrt{z}\,|k| = c_s |k|$

$k \approx \vec{\varphi}$

$\omega_k \sim JS\sqrt{z}\,|\vec{k} - \vec{\varphi}|$

energy diff

$$\Delta E = E_{quantum} - E_{classical}$$

$$= \frac{1}{2} \sum_k |J|\, sz \left(\sqrt{1-\gamma_k^2} - 1\right) \leq 0$$

<u>Fluctuations</u> / <u>Stability</u> :-

$$\Delta m = \frac{1}{V} \sum_{\vec{r}} e^{i\vec{\varphi}\cdot\vec{r}} \langle S^z(\vec{r}) \rangle \qquad - S$$

$(\equiv m_\varphi - m_{cl})$

$$= \frac{1}{V} \sum_{\vec{r}} \left[S - n_b(\vec{r}) \right] - S$$

$$= - \int \frac{d^d k}{(2\pi)^d} \langle b_k^\dagger b_k \rangle$$

check $\to \; = \frac{1}{2} - \int \frac{d^dk}{(2\pi)^d} \frac{\left(n_B(k)+\frac{1}{2}\right)}{\sqrt{1-\gamma_k^2}}$

$$\approx -\int \frac{d^dk}{k\left(e^{\beta k}-1\right)} + \underbrace{\int \frac{d^dk}{k}}_{\text{diverges in } d=1, \text{ contributes at } T=0}$$

$\therefore$ At $T=0$, no order in $d=1$

At $T>0$, $T\int \frac{d^dk}{k^2}$ diverges in $d \leq 2$.

Some discussions on

mermin-Wagner

theorem

Large $\mathbb{N}$ approximation

$SU(2) \rightarrow SU(N)$ using Schwinger bosons

$$\vec{S} = b^{+}\frac{\vec{\sigma}}{2} b \qquad b = [b_{\uparrow}\ b_{\downarrow}]$$

$$S^{+} = b_{\uparrow}^{+} b_{\downarrow} \qquad S^{z} = \frac{(b_{\uparrow}^{+}b_{\uparrow} - b_{\downarrow}^{+}b_{\downarrow})}{2}$$

$$[b_{\uparrow}, b_{\uparrow}^{+}] = \mathbb{1}$$

$$[b_{\uparrow}, b_{\downarrow}^{+}] = 0$$

$$\vec{S}^2 = S^{x2} + S^{y2} + S^{z2} = S(S+1)$$

$$b_{\uparrow}^{+}b_{\uparrow} + b_{\downarrow}^{+}b_{\downarrow} = 2S$$

$$|s,m\rangle = \frac{(b_{\uparrow}^{+})^{s+m}(b_{\downarrow}^{+})^{s-m}|0\rangle}{\sqrt{(s+m)!\,(s-m)!}}$$

$z^{+}z$ is invariant $SU(N)$ transformation.

$$z \rightarrow Uz \qquad U \in SU(N)$$

$$S^{x}, S^{y}, S^{z} \longrightarrow [S_{\alpha\beta}, S_{\gamma\delta}] = \delta_{\beta\gamma}\delta_{\alpha\delta} - \delta_{\alpha\delta}\delta_{\gamma\beta}$$

$$S_{\alpha\beta}^{+} = b_{\alpha}^{+} b_{\beta}$$

check that it works out