Keldysh field theory 1.1

$$= \frac{Tr(U^{+} O U \cdot \rho)}{Tr(\rho)} = \frac{Tr(U^{+} O U \rho)}{Tr(\rho(-\infty))}$$

Chapter-2 Komenev

$$\# \; Z_{J,\bar{J}} = \int d(\bar{\phi},\phi)\, e^{-\bar{\phi}\phi + \bar{J}\phi + \phi\bar{J}} = \exp(J\bar{J})$$

J & $\bar{J}$ are complex nos

$$d(\bar{\phi},\phi) = \frac{d(Re\,\phi)\, d(Im\,\phi)}{\pi}$$

$$\# \int d(\bar{\phi},\phi)\, e^{-|\phi|^2} (\bar{\phi})^{n} (\phi)^{n'} = \frac{\partial^{n+n'} Z_{J,\bar{J}}}{\partial J^{n} \partial (\bar{J})^{n'}}\Big|_{J=0}$$

$$= n!\, \delta_{n,n'}$$

2 quantum level, occupied by different no. of bosons.

$$\# \# \quad f(\rho) = \langle \phi | \rho^{b^{+}b} | \phi' \rangle = e^{\bar{\phi}\phi'\rho} = \exp[\bar{\phi}\phi'\rho]$$

pf:-

$$\partial_{\rho} f(\rho) = \langle \phi | b^{+}b\, \rho^{b^{+}b-1} | \phi' \rangle$$

operator

$$\rho^{b^{+}b}\, b|n\rangle = \sqrt{n}\, \rho^{n-1} |n-1\rangle = b\, \rho^{b^{+}b-1} |n\rangle$$

$$\rho^{b^{+}b} = \exp[b^{+}b \ln \rho] \approx \rho^{n}$$

$$b\, \rho^{b^{+}b-1} = b \exp[(b^{+}b-1)\ln\rho]$$

$$= \rho^{n-1}\, b|n\rangle = (\sqrt{n-1})\, \rho^{n-1} |n-1\rangle$$

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$\therefore\ b\,\rho^{b+b-1} = \rho^{b+b}\,b \qquad \therefore\ \partial_\rho f(\rho) = \langle\phi|\, b^\dagger \rho^{b+b}\, b\,|\phi\rangle$ (1.5)

$\Rightarrow \partial_\rho f(\rho) = (\bar{\phi}\phi)\, f(\rho) \rightarrow \boxed{f(\rho) = e^{\bar{\phi}\phi\rho}}$

* PARTITION function Calculation

$Z := \dfrac{tr(\hat{U}_c \hat{\rho})}{tr(\hat{\rho})} \qquad \hat{U}_c = \hat{U}_{-\infty,0}\,\hat{U}_{0,-\infty}$

$\hat{\rho}(-\infty)$ $\quad$ $E(t)$ same on $\longleftarrow t \longrightarrow$, then $\hat{U}_c = 1$.

idea $\longrightarrow$ can choose eqm $\hat{\rho}(H)$ at $t \to -\infty$ & switch on & off part to study non-eqm processes.

$Tr(\rho^0) = Z_{non\text{-}int} = \sum_{n=0}^{\infty} e^{-\beta \xi n}, \quad \xi = \omega_0 - \mu$

Note the difference from normal eqm def^n of $\rho = e^{-\beta H}/Z$

$\hat{\rho}(H) = \exp(-\beta(\hat{H} - \mu\hat{N})) = \dfrac{\exp[-\beta(\omega_0-\mu)\hat{b}^\dagger\hat{b}]}{Z}$

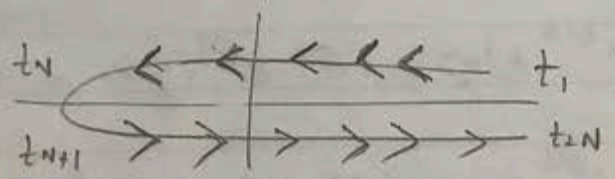
$Tr(\hat{\rho}) = \dfrac{1}{1 - e^{-\beta(\omega_0-\mu)}} = [1 - \rho(\omega_0)]^{-1}$

$\boxed{(\text{i.e. } Tr(\hat{\rho}) \neq 1)}$

disorder $\Rightarrow$ breaking of certain symmetry. $\hookrightarrow$ *4

interaction + disorder indpt $\boxed{\text{current}}$.

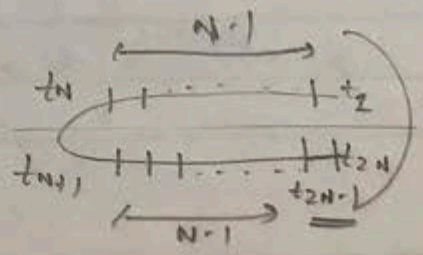
#



$Z = \dfrac{tr[U_c \hat{\rho}]}{tr[\hat{\rho}]}$

$U_c = ?$:-

Discretize time



$\Rightarrow$ 2N-2 parts

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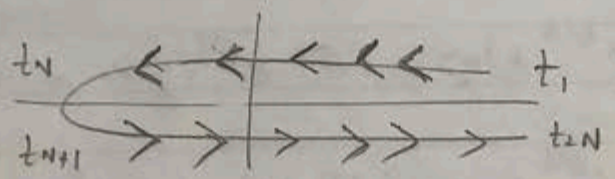

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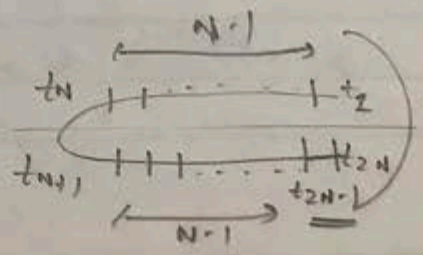

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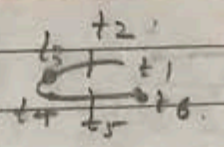
Exp. No.

identification:- $t_3 \equiv t_4$

$t_1 = t_{2N} \longrightarrow$ -a

NT:- $tr(\hat{U}_c \hat{\rho})$ ~

$N=3$



$tr(\hat{U}_c \rho) =$

$\langle\psi|\hat{U}_{\delta t}|\phi_5\rangle\langle\phi_5|\hat{U}_{\delta t}|\phi_4\rangle\langle\phi_4|\mathbb{1}|\phi_3\rangle\langle\phi_3|\hat{U}_{\delta t}|\phi_2\rangle\langle\phi_2|\hat{U}_{\delta t}|\phi_1\rangle\langle\phi_1|\rho|\psi\rangle$

set $t = t_6$ & we're done

assumed cont.

$\langle\phi_{j+1}|\hat{U}_{\pm\delta t}|\phi_j\rangle = \langle\phi_{j+1}|\exp(\mp i H(t_j)\delta t)|\phi_j\rangle$

linear order in δt.

$= \langle\phi_{j+1}|\mathbb{1} \mp i H_{t_j}\cdot\delta t|\phi_j\rangle$

normal ordered H.

$= \langle\phi_{j+1}|\mathbb{1} \mp i H_{t_j}(b^\dagger, b)\,\delta t|\phi_j\rangle$

$= \left(e^{\bar{\phi}_{j+1}\phi_j}\right)\left(\mathbb{1} \mp i H(\bar{\phi}_{j+1}, \phi_j)\,\delta t\right)$

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collect terms

collect t terms

$$Z = \left(\frac{1}{tr\,\rho}\right)\int\prod_{j=1}^{2N} d[\bar{\phi}_j, \phi_j]\ \prod_{j=1}^{2N}\exp[-\bar{\phi}_j\phi_j]\ \times \prod_{j=1}^{2N-1} e^{\bar{\phi}_{j+1}\phi_j}\, e^{-iH(\bar{\phi}_{j+1}, \phi_j)\delta t}$$

$$\times\ e^{\bar{\phi}_{N+1}\phi_N} \times \prod_{j=N+1}^{2N-1} e^{\bar{\phi}_{j+1}\phi_j}\, e^{+iH(\bar{\phi}_{j+1}, \phi_j)\delta t}.$$

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Teacher's Signature .....

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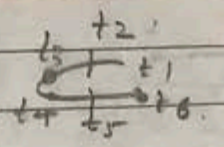

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$$\langle \phi_1 | \exp[-\beta(\omega_0-\mu) b^\dagger b] | \phi_{2N} \rangle = \exp[\bar{\phi}_1 \phi_{2N} \rho(\omega_0)]$$

where $\rho(\omega_0) = \exp[-\beta(\omega_0-\mu)]$ {ASSUME thermal ρ_0 at $t \to -\infty$ then adiabatically turn on interactions}.

$$Z = \frac{1}{Tr(\hat{\rho})} \left[\int \prod_{j=1}^{2N} d[\bar{\phi}_j, \phi_j] \exp\left[i \sum_{j,j'=1}^{2N} \bar{\phi}_j \, G^{-1}_{jj'} \, \phi_{j'} \right] \right.$$

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$\rho(\omega_0)$ → $\langle \phi_1 | \rho | \phi_{2N} \rangle$

from unity req.

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* still real time

matrix elements

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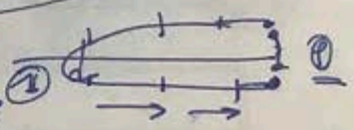
real time is hidden in the matrix elements



(Still, why do I need field correlator?).

Bosonic Integrals:-

$$\# \; I_{j,j} = \int \prod_{j=1}^{N} d[\bar{z}_j, z_j] \exp\left[-\sum_{i,j}^{N} \bar{z}_i A_{ij} z_j + \sum_j^{N} (\bar{z}_j J_j + \bar{J}_j z_j) \right]$$

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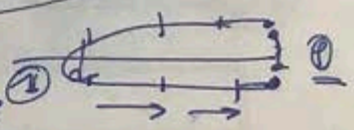

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require A is hermitian

$$Z_{J\bar{J}} = \frac{1}{\det A} \times \exp\left[+\sum_{ij}^{N} \bar{J}_i (A_{ij})^{-1} J_j\right]$$

$$Z_{J\bar{J}} = \frac{e^{\sum \bar{J}_i (A_{ij})^{-1} J_j}}{\det A}$$

→ valid as long as eigenvalues of A have a non-negative real part

observe that in complex case, $\sqrt{\ }$ as square root.

$$Z_{J\bar{J}} = \frac{\exp\left[\sum \bar{J}_i (A_{ij})^{-1} J_j\right]}{\det A}$$

Wick's theorem :-

$$\langle z_a \bar{z}_b \rangle = \int \prod_{j=1}^{N} d[\bar{z}_j, z_j] \, (\bar{z}_b z_a) \exp\left[-\sum_i \bar{z}_i (A_{ij}) z_j\right]$$

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Real case →

$$Z(J) = \int \prod_{i=1}^{N} \left(\frac{dx_i}{\sqrt{2\pi}}\right) \exp\left[-\frac{1}{2} \sum x_i A_{ij} x_j + \sum_i J_i x_i\right]$$

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$$\langle x_a x_b \rangle = A^{-1}_{ab} = \langle x_b x_a \rangle$$ [A is a Symmetric matrix] ⇓ can be complex too

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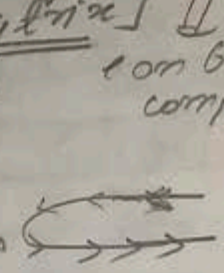
★ Calculation of Z_c: ($C \equiv$ )

$$Z_c = \left(\frac{1}{Tr(\hat{\rho})}\right) \times \int \prod_{j=1}^{2N} d[\bar{\phi}_j, \phi_j] \exp\left[\, \xi_{ij} \bar{\phi}_i (\mathcal{G}^{-1})_{ij} \phi_j \right]$$

$\boxed{Tr\, \hat{\rho}_D \neq 1}$ by def^n see (4A)

$$= \left\{\frac{1}{Tr(\hat{\rho})}\right\} \times \frac{1}{\det(-i\mathcal{G}^{-1})}.$$

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$$= 1 - \rho(\omega_0)\,(h_+ h_-)^{N-1}$$ [no trick, requires brute checking]

$$\det(-i\mathcal{G}^{-1}) = 1 - \rho(\omega_0)\,(h_+ h_-)^{N-1}$$

$$\approx 1 - \rho(\omega_0)\,(1 + \omega_0^2 \delta t^2)^N$$ for large N.

$$\approx 1 - \rho(\omega_0)\, e^{\omega_0^2 \delta_t^2 N} \simeq 1 - \rho(\omega_0)$$ as it should hold for a non-interacting system theory.

divide C s.t. $N\delta_t = \text{const}$.

$$\therefore Z_c = \frac{1}{tr(\hat{\rho}_0)} \times \frac{1}{\det[-i\mathcal{G}^{-1}]} = \frac{1}{[1-\rho(\omega_0)]^{-1}} \frac{1}{[1-\rho(\omega_0)]} = 1$$

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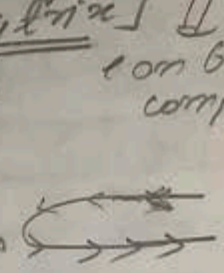
★ Calculation of Z_c: ($C \equiv$ )

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$$\approx 1 - \rho(\omega_0)\,(1 + \omega_0^2 \delta t^2)^N$$ for large N.

$$\approx 1 - \rho(\omega_0)\, e^{\omega_0^2 \delta_t^2 N} \simeq 1 - \rho(\omega_0)$$ as it should hold for a non-interacting system theory.

divide C s.t. $N\delta_t = \text{const}$.

$$\therefore Z_c = \frac{1}{tr(\hat{\rho}_0)} \times \frac{1}{\det[-i\mathcal{G}^{-1}]} = \frac{1}{[1-\rho(\omega_0)]^{-1}} \frac{1}{[1-\rho(\omega_0)]} = 1$$

Take cont limit, $N \to \infty$ with $N\delta_t = \text{const}$.

$\phi_j \equiv \phi(t_j)$ or $\phi(t)$

$$Z_0 = \int \underbrace{D[\bar{\phi}(t), \phi(t)]}_{\prod \frac{d[\bar{\phi}_j, \phi_j]}{\pi(i\delta_0)}} \exp[i S[\phi(t), \bar{\phi}(t)]]$$

$$S[\bar{\phi}, \phi] = \sum_{j=2}^{2N} \delta t_j \left[i\bar{\phi}_j \left(\frac{\phi_j - \phi_{j-1}}{\delta t_j} \right) - \omega_0 \bar{\phi}_j \phi_{j-1} \right] + \bar{\phi}_1 [\phi_1 - i\delta_0(\omega_0)\, \phi_{2N}]$$

where $\delta t_j = t_{j+1} - t_j = \delta t$ for $j \geq N+1$, $\delta t_j = -\delta t$

Boundary term

take continuum limit

$$S(\bar{\phi}, \phi) = \int_{\boxed{C}} dt\, \bar{\phi}(t)\, \mathcal{G}^{-1}\, \phi(t)$$

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→ Continuum limit → "Only an abbreviation"

Q. Where does the boundary term go?
↳ into the off field correlators.

Observe :- $\mathcal{G}^{-1} = i\partial_t - \omega_0 \longrightarrow \mathcal{G}^{-1}(e^{i\omega_0 t}) = 0$

not a unique inverse due to non-invertibility

include bdry term to obtain a unique $\hat{\mathcal{G}}$.

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$\underline{\phi_+ \,\&\, \phi_-}$ fields :- $\phi_+^+ \to$ fwd contr $\phi_-^- \to$ bwd contour

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Caution:- $\phi^+ \,\&\, \phi^-$ aren't uncorrelated.

$\phi^+(+\infty) = \phi^-(-\infty)$. & at $t \to -\infty$, there is a correlation term for $\phi^+ \,\&\, \phi^-$ [the off diagonal element]

Basically the $i\bar\phi^+(-\infty)[\phi^+(-\infty) - \rho(\omega_0)\phi^-(-\infty)]$ boundary term. $\boxed{\text{Imp.}}$

$$\therefore\ Z = \int \mathcal{D}[\bar\phi,\phi]\, e^{i\int dt\, (\bar\phi^+ g^{-1}\phi^+ - \bar\phi^- g^{-1}\phi^-) + \text{bdry terms}}$$

Important:- So for eqm theory,

$$\langle \phi_j \bar\phi_{j'} \rangle = \frac{Tr(\phi_j \bar\phi_{j'} \rho)}{Z} = \frac{Tr(\phi_j \bar\phi_{j'} e^{-\beta H})}{Z}$$

$$\Rightarrow \langle \phi_j \bar\phi_j \rangle = i\, g_{jj'}$$

But here,

$$\langle \phi_j \bar\phi_{j'} \rangle = \frac{\delta^2 Z(0,0)}{\delta J_j \delta \bar J_{j'}}$$

What about interacting partners?

$$Z = \frac{\int \prod d\phi\, e^{iS}}{Tr(\rho(-\infty))} = \boxed{Z(0,0)} = \frac{1}{Tr\rho(0)} \neq 1$$

at $t \to -\infty$, particles aren't interacting.

now suppose we switch on adiabatic interactions,

$$Z = \frac{Tr(\mathcal{U}_c \rho)}{Tr(\rho)} = 1 \text{ if hamiltonian is same for both fwd \& bwd sites.}$$

non-interacting d. matrix

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absent at $t \to -\infty$ but present at $t \to \infty$

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would be true (i.e. the $Z=1$ fact) if the hamiltonian in the fwd & bwd directions are same, though the system evolves in opposite direction.

$\Rightarrow$ for the current case, $Z = 1$ (non-interacting)

GREEN'S FUNCTIONS

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note

$$\langle \phi_j \bar{\phi}_{j'} \rangle = \frac{\frac{\int (\,)(\,)}{Tr(\rho)}}{Z(0,0)}$$

$\to Z(J, \bar{J})$

motion equations

where $Z(0,0) = \frac{Tr(\hat{U}_c \rho_{-\infty})}{Tr(\rho_{-\infty})}$, here $Z(0,0) = 1$

Q. If at $t \to -\infty$, we have a non-interacting system, then won't we have a linear term while evaluating G.F.?

Teacher's Signature

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$\bar{\phi}_j \, \phi_j \qquad \langle \phi_j \, \bar{\phi}_{j'} \rangle = G \qquad \phi_j \quad i(G_{jj'})$

$\frac{\delta}{\delta \bar{J}_j} \frac{\delta}{\delta \bar{J}_{j'}}$

$J\phi + J\bar{\phi}$

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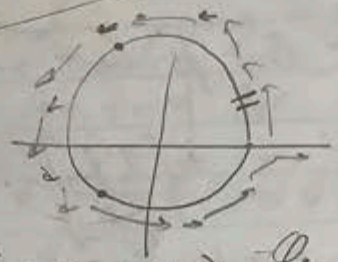
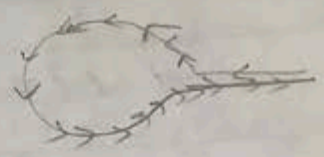
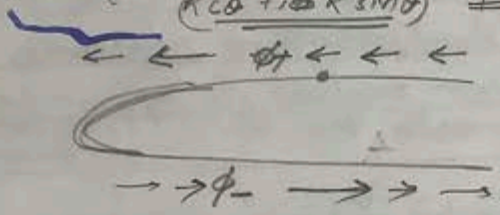




$S(t) \to U + U^{\dagger}$

$t = (R\cos\theta + i R\sin\theta) \qquad \dot{t} = -R\sin\theta\, d\theta + i R\cos\theta\, d\theta$



Various G.F or correlations can be calculated :-

Let $\boxed{j;\ j = 1, 2, 3 \ldots N}$

i.e. j arrives before j' on the Keldysh contour

$$\langle \phi_j^+ \bar{\phi}_{j'}^- \rangle \equiv i G^{<}_{jj'} = \frac{\rho\, h_+^{j'-1} h_-^{j'-1}}{\det[-iG^{-1}]}$$

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$$\langle \phi_j^+ \bar{\phi}_{j'}^+ \rangle = i G^{T}_{jj'} = \frac{(h_-)^{j-j'}}{\det(-iG^{-1})} \times \begin{cases} 1, & j \geq j' \\ \rho (h_+ h_-)^{N-1}, & j < j' \end{cases}$$

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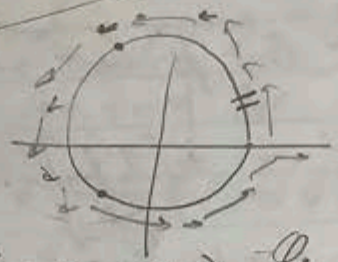

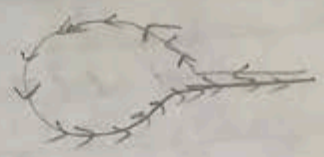

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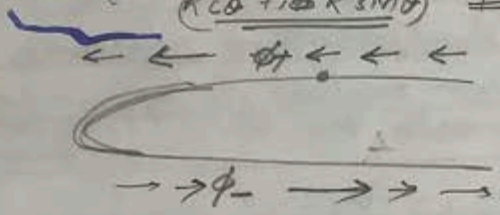

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Continuum limit → $(h_+ h_-)^{N-1}$ → (2)

$$(h_\pm)^j \xrightarrow{N\to\infty} e^{\pm i\omega_0 \delta t \times j} = e^{\pm i\omega_0 t_j}, \quad t_j = j\,\delta t$$

$$S[\phi^+,\phi^-] = \int_{-\infty}^{\infty} dt\left[\bar\phi^+(t)\left[i\partial_t-\omega_0\right]\phi^+(t) - \bar\phi^-(t)\left[i\partial_t-\omega_0\right]\phi^-(t)\right]$$

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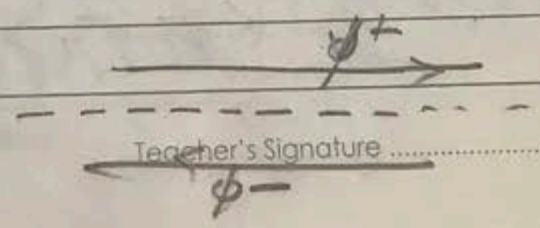

Naba Kishore

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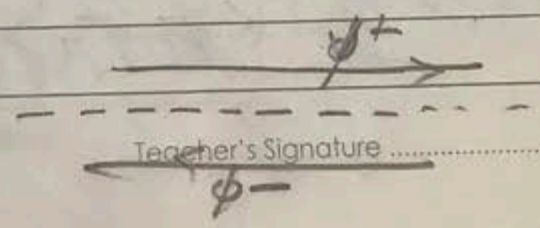

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complement each other

$$\langle \phi^-(t)\, \bar{\phi}^-(t') \rangle = i\,\underbrace{\tilde{\mathcal{G}}^{\bar{T}}}_{\text{anti-time ordering}}(t,t') = \theta(t-t')\, i\mathcal{G}^<_{t,t'} + \theta(t'-t)\, i\mathcal{G}^>_{t,t'}$$

just upper case on fwd/bwd contour to negate the time ordering.

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[this def^n is general & isn't confined to the case of eqm. stat mech]

$$[\mathcal{G}^<(t,t')]^+ = -\mathcal{G}^<(t,t')$$

exchange labels & perform h.c.

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issue about equal time → discrete $\xrightarrow{\text{ambiguity}}$ cont.

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then still $\langle \phi_j^- \overline{\phi_{j'}^+} \rangle \approx e^{i\omega_0 (j'-j)\delta t}$ (1)

Still correlated(??)

is $\overline{\phi^+}$ living on the Backward contour in some sense? (similar to ϕ^-).

not all + G.F. are independent

$$G^{11}(t,t') + G^{\tilde{11}}(t,t') - G^{>}(t,t') - G^{<}(t,t') = 0$$

(not true at $t=t'$)

need a linear trans to get rid of such a degeneracy.

Keldysh Rotation :-

$$\phi^{cl}(t) = \frac{1}{\sqrt{2}}(\phi^+(t) + \phi^-(t)) \qquad \phi^q(t) = \frac{1}{\sqrt{2}}(\phi^+ - \phi^-)$$

$$\langle \phi^\alpha(t)\, \phi^\beta(t') \rangle = i G^{\alpha\beta} = \begin{pmatrix} iG^K & iG^R \\ iG^A & 0 \end{pmatrix} \qquad (\alpha,\beta) = (cl, q)$$

$G^{11} + G^{\tilde{11}} - G^{<} - G^{>} = 0$ consequence.

Green's functions are to be seen as matrices in time domain

$$\left[\begin{aligned} G^A_{t,t'} &= \theta(t'-t)\,[G^< - G^>] \\ G^R_{t,t'} &= \theta(t-t')\,[G^> - G^<] \\ G^K_{t,t'} &= G^> + G^< \end{aligned}\right\} \quad \begin{aligned} (G^R)^\dagger &= G^A \\ (G^K)^\dagger &= -G^K \end{aligned}$$

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$$\phi^{cl}(t) = \frac{1}{\sqrt{2}}(\phi^+(t) + \phi^-(t)) \qquad \phi^q(t) = \frac{1}{\sqrt{2}}(\phi^+ - \phi^-)$$

$$\langle \phi^\alpha(t)\, \phi^\beta(t') \rangle = i G^{\alpha\beta} = \begin{pmatrix} iG^K & iG^R \\ iG^A & 0 \end{pmatrix} \qquad (\alpha,\beta) = (cl, q)$$

$G^{11} + G^{\tilde{11}} - G^{<} - G^{>} = 0$ consequence.

Green's functions are to be seen as matrices in time domain

$$\left[\begin{aligned} G^A_{t,t'} &= \theta(t'-t)\,[G^< - G^>] \\ G^R_{t,t'} &= \theta(t-t')\,[G^> - G^<] \\ G^K_{t,t'} &= G^> + G^< \end{aligned}\right\} \quad \begin{aligned} (G^R)^\dagger &= G^A \\ (G^K)^\dagger &= -G^K \end{aligned}$$

* Some relations :-

Q. Suppose I need to know the avg no. of bosons at any time. what do I do?

Soln :- $\langle b^+ b \rangle$ → either in fwd. / " " bwd.

fwd :- $\langle b^+ b \rangle = \dfrac{Tr((b^+b)_t \rho)}{Tr(\rho)} = \dfrac{Tr((b^+b)_t\, U \rho U^+)}{Tr(\rho)}$

$\Rightarrow (\langle b^+ b \rangle)_t = \dfrac{Tr(U^+ (b^+b)_t \hat{U} \rho)}{Tr(\rho)}$

$U^+ \quad U \quad b^+b$

$U^+ b^+ b\, U = \dfrac{\delta Z_v}{\delta v}$

$\dfrac{\delta^2 Z(J,\bar{J})}{\delta J(t)\,\delta \bar{J}(t)}$ $Z(v) = \dfrac{tr(\hat{U}_c(v)\, \rho(-\infty))}{tr(\rho(-\infty))}$ $\langle O \rangle$ or $\bar{O}$ $= 1$

Q. Why is ~~tr of disorder~~ tr of disorder/int. indpt?

$tr(\rho_t) = tr(U_c \rho)$ → $U_c \neq 1$ always

$\langle \bar{O} \rangle_t = \left(\dfrac{2}{2}\right) \dfrac{\delta Z[v]}{\delta v(t)}\Big|_{v=0}$

or

$\langle b^+ b \rangle_t = \dfrac{\delta^2 Z(J,\bar{J})}{\delta J(t)\,\delta \bar{J}(t)}\Big|_{J=0}$ b^+b

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Don't get convinced if you aren't.

Q. What is the # of bosons at any time?

$$\langle b^\dagger b\rangle = \frac{1}{Tr(\rho)} \times Tr(b^\dagger b\, \rho(t)) = \frac{1}{Tr(\rho)} Tr(b^\dagger b\, \mathcal{U}\, \rho\, \mathcal{U}^\dagger)$$

$t \leftarrow -\infty$.

$$\therefore \frac{1}{Tr(\rho)} \times Tr(\mathcal{U}^\dagger (b^\dagger b)\, \mathcal{U}(t,-\infty)\, \rho(-\infty))$$

$$= \frac{1}{Tr(\rho)} Tr[\mathcal{U}_{-\infty,t}\, (b^\dagger b)\, \mathcal{U}(t,-\infty)\, \rho(-\infty)].$$

$$= \left(\frac{1}{Tr\rho}\right) Tr[\mathcal{U}_{-\infty,\infty}\, \mathcal{U}_{\infty,t}\, (b^\dagger b)\, \mathcal{U}(t,-\infty)\, \rho(-\infty)]. \quad (1)$$

$\mathcal{U}_{\infty,t} \to \hat{\mathcal{U}}_c(J)$.

Now, $Z_c[J(t), \bar{J}(t)] = \frac{1}{Tr(\rho)} Tr[\hat{\mathcal{U}}_c\, \rho(-\infty)]$.

$$\frac{1}{Tr(\rho)} \times \int \mathcal{D}[\phi(t), \bar{\phi}]. \times \exp[-S]$$

$$S = \int_{-\infty}^{\infty} dt\, \left(\bar{\phi}^+(i\partial_t - \omega_0)\phi^+ + \bar{J}\phi^+ + J\bar{\phi}^+\right) - \left[\bar{\phi}^-(i\partial_t - \omega_0)\phi^- + \bar{J}\phi^- + J\bar{\phi}^-\right]$$

fwd C

Now, $\langle b^\dagger b\rangle$ when in continuous language $\langle \phi_j^+ \bar{\phi}_{j+1}^+ \rangle$ or $\langle \phi_{j+1}^- \bar{\phi}_j^- \rangle$ bwd C

$t' = t + \delta t$

continuum limit.

$$i\, G^<(t,t') = \left(\frac{\rho}{1-\rho}\right) e^{-i\omega_0(t-t')}$$

$b_t^\dagger b_t$ could occur (or (can be made to occur) either in fwd or backward contour (can simply reorder $\mathcal{U}_{t,-\infty}$ & other $\hat{\mathcal{U}}$ operators to change fwd to bwd).

$\therefore$ irrespective of where we compute $b^\dagger b$, the answer should be same.

Teacher's Signature

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Teacher's Signature

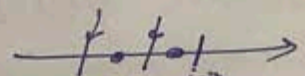
$\boxed{\langle \phi^-_{j+1} \bar{\phi}^-_j \rangle}$

$$= i\, G^T(t_{j+1}, t_j) \quad \text{cont. limit}$$

$$= i\, G^<(t_n, t_j)$$

$$= \left(\frac{\rho}{1-\rho}\right) e^{-i\omega_k \delta t} \xrightarrow{\delta t \to 0} \boxed{n_B}$$

So in both cases, the no. of bosons is governed by $\bar{\phi}^+_{j+1} \phi_j$ B.E.f^n.



$\mathcal{U}_{j-1,j}$

$\mathcal{U}_{j-1,j} \langle \phi_j | b^+ b | \phi_j \rangle \mathcal{U}_{j,j+1}$

$\langle \phi_{j-1} | \mathcal{U}_{j-1,j} \rangle \langle \phi_j \mathcal{U}_{j,j+1}$

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the ordering of arguments is imp here.

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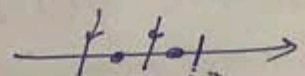

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$$G^A = G^{ret} = \left(\frac{1}{2}\right)\left[G^T + G^< - G^> - G^{\tilde{T}}\right]$$

Now, from the direct case, we know

$$G^T + G^{\tilde{T}} - G^< - G^> = 0$$

$$\text{or } G^T + G^{\tilde{T}} = G^< + G^>$$

also

$$\therefore \quad G^T(t,t') = \theta(t-t')\,G^> + \theta(t'-t)\,G^<$$

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$$\therefore \quad G^K = G^< + G^> \quad \text{or} \quad \boxed{(G^K)^\dagger = -G^K}$$

$$G^R = \left(\frac{1}{2}\right)\left[\left(G^T - G^{\tilde{T}}\right) + G^> - G^<\right].$$

now, $G^T - G^{\tilde{T}} = \theta(t-t')\left[G^> - G^<\right] + \theta(t'-t) \times \left[G^< - G^>\right]$

$$\Rightarrow G^R(t,t') = \left[1 - \theta(t'-t)\right] \times \left[G^> - G^<\right]$$

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$$\begin{cases} G^<(t,t') = -i\, n_B\, e^{-i\omega_0(t-t')} \\ G^>(t,t') = -i\,(n_B+1)\, e^{-i\omega_0(t-t')} \\ G^T(t,t') = [\theta(t-t')[n_B+1] + \theta(t'-t)\, n_B]\, e^{-i\omega_0(t-t')} \\ G^{\tilde{T}}(t,t') = [\theta(t'-t)(n_B+1) + \theta(t-t')\, n_B]\, e^{-i\omega_0(t-t')} \end{cases}$$

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only Spectrum info

$$\Rightarrow \boxed{G^A = i\,\theta(t'-t)\, e^{-i\omega_0(t-t')}}$$

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(Spectrum + distribution info)

(General & not the eqm. distribution function).

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Keldysh action:- Construct a Z so as to reproduce the the correlators.

$$\langle \phi^\alpha \bar{\phi}^\beta \rangle = \int D[\phi^\alpha, \bar{\phi}^\beta] \, \phi^\alpha \bar{\phi}^\beta \, \exp[iS]$$
$$= \frac{1}{Z[0,0]} \times \frac{\delta^2 Z_{J,\bar{J}}}{\delta J_\beta \delta \bar{J}_\alpha}$$

where $Z(0,0) = \int D[\phi, \bar{\phi}] \exp[iS]$

In case of f.t done till now,

$$\langle \phi^\alpha \bar{\phi}^\beta \rangle = \frac{1}{Tr[\rho(t)]} \times Tr(\text{(pseudo data ..)} \, \rho(-\infty))$$
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[this is in contrast to Z_0. $\frac{Tr(\rho(t))}{Tr(1)} = \frac{Tr(\hat{U}_c \rho)}{Tr\rho}$.

Let denote this Z by Z_0.]

in this case $Z[0,0] = 1$.

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$$\langle \phi^\alpha \bar{\phi}^\beta \rangle = \frac{Tr(a_{\uparrow\alpha} a^\dagger_{\uparrow\beta} \rho(t))}{Tr(\rho(t))} = \frac{\frac{Tr(a_{\uparrow\alpha} a^\dagger_{\uparrow\beta} \rho(t))}{Tr(\rho_0)}}{\frac{Tr(\rho(t))}{Tr(\rho_0)}} \Big|_{Z_0}$$

$$\frac{\delta^2 Z[J,J']}{\delta \bar{J}_\alpha \delta J_\beta}$$

$\therefore$ define:-

$$Z(J,\bar{J}) = \frac{Tr(\hat{U}_c(J,\bar{J}) \rho)}{Tr(\rho_0)}$$

So it looks like there is still some hope for Wick's theorem.

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absorb

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$$\langle \phi^{+}(t)\,\bar{\phi}^{-}\rangle = \frac{\delta^2 Z(J,\bar{J})}{\delta J_t\,\delta J_p\; Z(0,0)} \longrightarrow \frac{Tr(\hat{O}\,\rho(t))}{Tr(\rho(t))}$$

or something like that

keldysh $\longrightarrow$ make action so as to reproduce the Green's functions or field correlators.

$$S[\phi^{u},\phi^{q}] = \int dt\,d\vec{x}\;(\bar{\phi}^{u},\bar{\phi}^{q}) \begin{bmatrix} 0 & (G^{A})^{-1} \\ (G^{R})^{-1} & (G^{K})^{-1} \end{bmatrix} \begin{pmatrix} \phi^{u} \\ \phi^{q} \end{pmatrix}$$

$$\because \quad G = \begin{pmatrix} G^{K} & G^{R} \\ G^{A} & 0 \end{pmatrix} \Rightarrow G^{-1} \xrightarrow{\text{formally}} \begin{pmatrix} 0 & (G^{A})^{-1} \\ (G^{R})^{-1} & (G^{K}) \end{pmatrix}$$

Now, $(G^{A})^{-1} \cdot G^{R} = 1$ $\longrightarrow$ matrix

how? 2x2 matrix inversion

$$\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \xrightarrow{\text{invert}} \begin{pmatrix} 0 & \frac{1}{c} \\ \frac{1}{b} & \frac{a}{c} \end{pmatrix}$$

$$-\frac{1}{bc}\begin{bmatrix} 0 & -b \\ -c & a \end{bmatrix} \quad \begin{pmatrix} (b^{-1})^{A} \\ -ab^{-1} \end{pmatrix}$$

$$-\frac{a}{bc}$$

$$\begin{pmatrix} (G^{A})^{-1} & \\ 0 & \\ (G^{R})^{-1} & \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ G^{R-1} \end{pmatrix}$$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$(G^{K})(G^{A})^{-1} + (G^{R})\,G^{M}$$

$$M^{+} = -M$$

$$G^{K} = G^{R}\,M\,G^{A}$$

$$G^{K+} = G^{R}\,M^{+}\,G^{A}$$

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$$\because \quad G = \begin{pmatrix} G^{K} & G^{R} \\ G^{A} & 0 \end{pmatrix} \Rightarrow G^{-1} \xrightarrow{\text{formally}} \begin{pmatrix} 0 & (G^{A})^{-1} \\ (G^{R})^{-1} & (G^{K}) \end{pmatrix}$$

Now, $(G^{A})^{-1} \cdot G^{R} = 1$ $\longrightarrow$ matrix

how? 2x2 matrix inversion

$$\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \xrightarrow{\text{invert}} \begin{pmatrix} 0 & \frac{1}{c} \\ \frac{1}{b} & \frac{a}{c} \end{pmatrix}$$

$$-\frac{1}{bc}\begin{bmatrix} 0 & -b \\ -c & a \end{bmatrix} \quad \begin{pmatrix} (b^{-1})^{A} \\ -ab^{-1} \end{pmatrix}$$

$$-\frac{a}{bc}$$

$$\begin{pmatrix} (G^{A})^{-1} & \\ 0 & \\ (G^{R})^{-1} & \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ G^{R-1} \end{pmatrix}$$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$(G^{K})(G^{A})^{-1} + (G^{R})\,G^{M}$$

$$M^{+} = -M$$

$$G^{K} = G^{R}\,M\,G^{A}$$

$$G^{K+} = G^{R}\,M^{+}\,G^{A}$$

$\bar{t} = t_{j+1}$ $\quad t = t_j$

$\bar{\phi}\,(i\partial_t - \omega_0)\,\phi$ → although G^{-1} in discrete case had a non-vanishing det, in cont. case it doesn't seem to get that behaviour.

★★★ (see explanatn of text).

$\Rightarrow$ Off diagonal G.F. $\langle \phi^+ \bar{\phi}^- \rangle$ or $\langle \phi^- \bar{\phi}^+ \rangle$ does not CONTRADICT the continuous notation.

$(i\partial_t - \omega_0)^{-1} \begin{pmatrix} G^{\uparrow} & G^{>} \\ G^{<} & G^{\downarrow} \end{pmatrix}$

$$\left[(i\partial_t - \omega_0)\, G^{<}\right]_{t,t'} = (i\partial_t - \omega_0)\, n_B\, e^{-i\omega_0(t-t')} = 0$$

Same for $(i\partial_t - \omega_0)\, G^{>} = 0$

$$\& \boxed{(i\partial_t - \omega_0)\, G^{\uparrow/\downarrow} = \pm\delta(t-t')}$$

So broadly, $G^{-1} \circ G = \begin{pmatrix} G^{\uparrow} & G^{>} \\ G^{<} & G^{\downarrow} \end{pmatrix}$ $\quad \bar{\phi}^{+}\ \phi$

$$G^{-1} \circ \begin{pmatrix} G^{\uparrow} & G^{>} \\ G^{<} & G^{\downarrow} \end{pmatrix} = \begin{pmatrix} G^{\uparrow}/G^{<} & G^{>}/G^{\downarrow} \\ G^{<}/G^{\uparrow} & G^{\downarrow}/G^{>} \end{pmatrix}$$

Mus simplifyme.

Now, $G^{-1}\begin{pmatrix} G^{\uparrow} \\ G^{<} \end{pmatrix} = \begin{pmatrix} \pm\delta(t-t') \\ 0 \end{pmatrix}$ [acm $(i\partial_t - \omega_0)$]

upper ½ is identity i.e. it makes

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ → this component of the identity matrix.

Similarly, $G^{-1} \circ \begin{pmatrix} G^{>} \\ G^{\downarrow} \end{pmatrix} = \begin{pmatrix} 0 \\ \pm\delta(t-t') \end{pmatrix}$ → here we use $-(i\partial_t - \omega_0)$

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$\triangleq$ $G^R = -i\,\theta(t-t')\,e^{-i\omega_0(t-t')}$ $\quad G^A = +i\,(\theta)(t'-t)$

$$\theta(t)\,e^{-i\omega_0 t} = \int_{-\infty}^{\infty} \theta(t)\,e^{i\omega_0 t} = \int e^{i\omega_0 t} = \frac{1}{i\omega_0} = \frac{1}{(-i\omega_0)}$$

$$\propto \left(\frac{1}{i\omega_0}\right) \propto \left(\frac{1}{-i\omega'}\right) e^{-i(\omega_0+\omega')t} \quad t \qquad \left[\frac{1}{-i(t-t')}\right] \qquad \frac{1}{(-i\omega_0)}$$

$$\triangleq \langle \phi^+(t)\,\phi^-(t') \rangle = i G^< = \frac{Tr(\,a^+(t')\;a(t)\;\rho(-\infty))}{Tr(\rho)}$$

$$\boxed{\left(U_{-\infty,t'}\, a^+\, U_{t',-\infty}\, U_{-\infty,t}\, \hat{a} \quad U_{t,-\infty}\, \rho(-\infty) \right)}$$

$=$

$U_{0,-\infty}$ $\quad U_{0,t'}$

$$\langle \phi^-(t)\,\phi^+(t') \rangle = i G^> \approx \frac{Tr(\,a\;a^+\;\rho)}{Tr(\rho_0)} \qquad e^{iHt'}$$

$a \quad a^+$

$$\downarrow$$

$$Tr(\,e^{iHt}$$

Now # $G^R - G^A = (-i)\,\langle (bb^+ - bb^+) \rangle$

$$\left[[G^{-1}]^R \circ (G^K) + (G^{-1})^K \circ (G^A) \right] = A \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$(G^R)^{-1}\, G^K \qquad G^R G^K + G \qquad (G^{-1})^R = -\frac{1}{bc}\begin{bmatrix} 0 & -b \\ -b & a \end{bmatrix}.$$

$$+ (G^K)^{-1}\, G^R \qquad (G^K)^2 + (G^R G^A) \qquad = \begin{bmatrix} 0 & (G^A)^{-1} \\ (G^R)^{-2} & (G^K) \end{bmatrix}$$

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$$G^R = -i\theta(t-t')\, e^{-i\omega_0(t-t')} \qquad G^K = -i(2n_B+1)\, e^{-i\omega_0(t-t')}$$

$$G^A = i\theta(t'-t)\, e^{-i\omega_0(t-t')}$$

$n_B(\omega_0)$

F.T. or the so called energy rep

w.r.t $\tau = t-t'$

$$G^R(\epsilon) = \frac{1}{(\epsilon-\omega_0+i0)}$$

$$G^A(\epsilon) = \frac{1}{(\epsilon-\omega_0-i0)}$$

$$G^K(\epsilon) = (2\pi)\, i\left[2n_B(\epsilon)+1\right]\delta(\epsilon-\omega_0)$$

(***) So even though

$$S = \int dt\left[\bar{\phi}^+(i\partial_t-\omega_0)\phi^+ - \bar{\phi}^-(i\partial_t-\omega_0)\phi^-\right]$$

& resembles two free fields, we get correlations like $G^< = -i\langle\phi^+\bar{\phi}^-\rangle$ & $G^> = -i\langle\phi^-\bar{\phi}^+\rangle$

this is due to presence of $\begin{pmatrix} & (S) \\ (S) & \end{pmatrix}$ in the off diagonal blocks (i.e. in real time they correspond to agreement at boundaries).

So is the cont. rep of S. wrong?

Ans:- No.

So, $(i\partial_t-\omega_0)\, G^<(\tau) = 0$ $\left[\because G^< = n_B e^{-i\omega_0|t-t'|},\ G^> = -(n_B+1)\, e^{-i\omega_0|t-t'|}\right]$

Now if $G^{-1} = \begin{pmatrix} (i\partial_t-\omega_0) & 0 \\ 0 & -(i\partial_t-\omega_0) \end{pmatrix}$ then $G^{-1}\circ\begin{pmatrix} G^T & G^> \\ G^< & G^{\tilde{T}} \end{pmatrix}$

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$$G^R = -i\theta(t-t')\, e^{-i\omega_0(t-t')} \qquad G^K = -i(2n_B+1)\, e^{-i\omega_0(t-t')}$$

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So, $$\mathcal{G}^{-1} \circ \mathcal{G} = \begin{pmatrix} i\eta_t \cdot \omega_0 & 0 \\ 0 & -(i\eta_t \cdot \omega_0) \end{pmatrix} \circ \begin{bmatrix} \mathcal{G}^{\lceil} & \mathcal{G}^{>} \\ \mathcal{G}^{<} & \mathcal{G}^{\rceil} \end{bmatrix}$$

$$= \begin{pmatrix} \delta(t-t') & 0 \\ 0 & \delta(t-t') \end{pmatrix} = \begin{pmatrix} \delta(t-t') & 0 \\ 0 & \delta(t-t') \end{pmatrix} = \mathbb{1}$$

$\therefore$ $\mathcal{G}^{-1}$ is a valid inverse of the $\mathcal{G}$ matrix.

$$\therefore \mathcal{G}^{-1} = \begin{pmatrix} 0 & (\mathcal{G}^A)^{-1} \\ (\mathcal{G}^A)^{-1} & [\mathcal{G}^K]^{-1} \end{pmatrix} = \left\{ \begin{pmatrix} \mathcal{G}^K & \mathcal{G}^R \\ \mathcal{G}^A & 0 \end{pmatrix} \right\}^{-1}$$

so $[\mathcal{G}^A]^{-1} = [(\mathcal{G}^A)^{-1}]$ (just the matrix inverse in time domain).

$[\mathcal{G}^R]^{-1} = [(\mathcal{G}^R)^{-1}]$ (" " " ")

Now, $[\mathcal{G}^K]^{-1} \times \mathcal{G}^A = -[\mathcal{G}^R]^{-1} (\mathcal{G}^K)$

$$\Rightarrow [\mathcal{G}^K]^{-1} = -[(\mathcal{G}^R)^{-1}] (\mathcal{G}^K) [(\mathcal{G}^A)^{-1}]$$

$\mathcal{G}^K = \mathcal{G}^{>} + \mathcal{G}^{<} = \mathcal{G}^{>} + \mathcal{G}^{<}$

[note:- the bd terms appear as correlations in cross field $\mathcal{G}$ i.e. $\mathcal{G}^{>}$ captures the presence of ~~the~~ $\delta(\omega_0)$ term while $\mathcal{G}^{<}$ captures presence of $\mathbb{1}$ in the off-diagonal block].

So, $$\mathcal{G}^{-1} \circ \mathcal{G} = \begin{pmatrix} i\eta_t \cdot \omega_0 & 0 \\ 0 & -(i\eta_t \cdot \omega_0) \end{pmatrix} \circ \begin{bmatrix} \mathcal{G}^{\lceil} & \mathcal{G}^{>} \\ \mathcal{G}^{<} & \mathcal{G}^{\rceil} \end{bmatrix}$$

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Now we look for the exact forms of the correlators.

$$G_R \longrightarrow -i\,\theta(t-t')\, e^{-i\omega_0(t-t')} \xrightarrow{F.T.}$$

$\int f.T.$

$$\epsilon \text{ mode} \Rightarrow (-i) \int_0^\infty e^{i[\epsilon-\omega_0]\tau}\, d\tau.$$

insert a $i\eta$ & $\eta \to 0$

here the fourier sum is of the form $f(t)\, e^{-i\epsilon t}$ note $+i\epsilon t$

$$= (-i) \times \frac{-1}{i(\epsilon-\omega_0+i\eta)}$$

$$= \left(\frac{1}{\epsilon-\omega_0+i\eta}\right) \quad \eta \to 0^+ \quad \text{Regularisation.}$$

Similarly, $G_A \to \dfrac{1}{\epsilon-\omega_0-i\eta}$, $\eta \to 0^+$

Now,

$$\begin{cases} (G_R)^{-1}_{\epsilon\,\text{mode}} = (\epsilon-\omega_0+i\eta), & \eta \to 0 \\ (G_A)^{-1}_{\epsilon\,\text{mode}} = (\epsilon-\omega_0-i\eta), & \eta \to 0 \end{cases}$$

Using the property that Green functions & its inverse have their modes in reciprocal

$\downarrow$ G.F.T. (inverse fourier transf)

$$(+i\partial_t - \omega_0)\, \delta(t-t') \longrightarrow \delta(t-t')\left[i\partial_{t'} - \omega_0\right]$$

flow?

$$\int dt\,dt' \left\{\partial_t\, \delta(t-t')\right\} f(t') = \int dt\,dt'\, g(t)\, \delta(t-t') - \partial_t f(t)$$

$$\partial_t \delta(t-t') \to \delta(t-t')\,\partial_{t'}\, g(t)$$

$$\text{i.e.} \int g(t)\,\partial_t \delta(t-t')\, f(t') \xrightarrow{\int_{t'}} -\int (\partial_t g)\, f(t)\, dt$$

$$\therefore \int g(t)\left[\delta(t-t')\,\partial_{t'}\right] f(t')\, dt\,dt' \longleftarrow \int g(t)\, \partial_t f(t)\, dt$$

$\xrightarrow{\text{int over } t}$

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P.S. $\longrightarrow$ also $\partial_t^2\, \delta(t-t') = \left[\delta(t-t')\, \partial_{t'}^2\right]$ ...

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looks diagonal.

But G^R was lower Δ^r. ($G^R \sim \theta(t-t')\, e^{-i\omega_0(t-t')}$)

even in continuum limit.

Q. Questions on the adiabatic theorem:- ~~What time~~

where does this time-scale idea come from?

it's the lifetime of the state, then how is it the time scale for ~~evol~~ evolution of Ψ.

$G(t,t')$

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$\begin{pmatrix} 1 \\ r \\ 1 \end{pmatrix}^t$

Q. $[G^{-1}]^K = ?$ (See parts on the fermionic case)

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$$[G^K]^{-1} = -[G^R]^{-1} \circ G^K \circ [G^A]^{-1}$$

parametric

$$G^K = G^R \circ F - F \circ G^A \quad , \text{s.t. } F^\dagger = F$$

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① $S[\phi^{(1)}, 0] = 0$ $\quad \phi^{(1)} = 0 \Rightarrow \phi^+ = \phi^-$

$S[\phi^{(1)}] = 0$

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→ Responsible for "Causality" of response functions" (???)

③ q-q component → ⓐ Anti-hermition.

ⓑ $[G^K]^{-1} = 2i\eta F(t)$

s.t. $F = F^\dagger$

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Specific model, generally large in interacting systems

→ Q. flow???

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$2i\eta F(t)$

Keldysh with ext source

Put $V(t) = V^+(t) \neq V^-(t)$ b/b Z for $t_{a,c}$ time being let

(which is what's expected of a common source)

set $V^q = 0$ at the end of calculation

→. Continuum one

$$Z_c[V] = \int D[\bar{\varphi}^{cl}, \varphi^q]\, e^{iS - iS_V}$$

$$S_V = \int \bar{\phi}^+ V^+ \phi^+ - \int \bar{\phi}^- V^- \phi^-$$

$$\downarrow$$

$$= \int \phi(\bar{\varphi}^{cl}\ \bar{\varphi}^q) \begin{pmatrix} V^q & V^{cl} \\ V^{cl} & V^q \end{pmatrix} \begin{pmatrix} \varphi^{cl} \\ \varphi^q \end{pmatrix}$$

$\therefore\ Z_c[V^{cl}, V^q]$ where $V^{cl} = \dfrac{V^+ + V^-}{2}$ not $\sqrt{2}$

& $V^q = \dfrac{V^+ - V^-}{2}$

$$= \int D[\bar{\varphi}, \varphi]\, e^{i\int dt\, \bar{\varphi}(G^{-1} - V)\varphi}$$

$$= \frac{1}{\mathrm{Tr}(G_0)} \times \frac{1}{\det[-iG^{-1} + iV]} = \frac{\det[-iG^{-1}]}{\det[-iG^{-1} + iV]}.$$

using $\frac{1}{\mathrm{Tr}(G_0)} = \det[-iG^{-1}]$

Keldysh with ext source

Put $V(t) = V^+(t) \neq V^-(t)$ b/b Z for $t_{a,c}$ time being let

(which is what's expected of a common source)

set $V^q = 0$ at the end of calculation

→. Continuum one

$$Z_c[V] = \int D[\bar{\varphi}^{cl}, \varphi^q]\, e^{iS - iS_V}$$

$$S_V = \int \bar{\phi}^+ V^+ \phi^+ - \int \bar{\phi}^- V^- \phi^-$$

$$\downarrow$$

$$= \int \phi(\bar{\varphi}^{cl}\ \bar{\varphi}^q) \begin{pmatrix} V^q & V^{cl} \\ V^{cl} & V^q \end{pmatrix} \begin{pmatrix} \varphi^{cl} \\ \varphi^q \end{pmatrix}$$

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~~Quantum~~ Claim:- $Z[V^{cl}, V_q = 0] = 01$

1st order

$$Z = \exp[-\mathrm{tr}\ln(1 - \hat{G}\hat{V})]$$

1st order $\hat{V} = \begin{pmatrix} 0 & V^q \\ V^{cl} & 0 \end{pmatrix}_{t'}$ $\hat{G} = \begin{pmatrix} G^R & G^R \\ G^A & 0 \end{pmatrix}_{t,t'}$

$$= \exp\left[(-)\left[\hat{G}\hat{V} - \frac{(GV)^2}{2} + \frac{(GV)^3}{3} - \cdots\right]\right]$$

$$\mathrm{tr}\, GV = \mathrm{tr}(G^R V^{cl}) + \mathrm{tr}(G^A V^{cl})$$

$$= \int dt\, [\underbrace{G^R(t,t) + G^A(t,t)}_{=0}] V^{cl}(t)$$

$$= 0 \qquad G_{t,t'}\, V_{t'}$$

2nd order

$$\mathrm{tr}[(GV)(GV)] = \cancel{\mathrm{tr}(\ )}$$

$$= \mathrm{tr}\begin{pmatrix} G^R V^{cl} & \sim \\ \sim & G^A V^{cl} \end{pmatrix} \cdot \begin{pmatrix} G^R V^{cl} & \sim \\ \sim & G^R V^A \end{pmatrix}$$

$$= \sum_{R,A} G^R V^{cl}\, G^R V^{cl}$$

$G(t,t')$ t'

$$V(t) = \begin{pmatrix} V^q & V^{cl} \\ V^{cl} & V^q \end{pmatrix}$$

$\int dt\, G(t,t') V(t)$

~~$G^R(t,t')$~~

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Now,

$$\therefore Z_c = \frac{1}{\det[i\hat{g}]\,\det[-i\hat{g}^{-1}\hat{\theta} + iV]}$$

$$= \frac{1}{\det[1-\hat{g}V]} = \exp[-\mathrm{Tr}\ln\det(1-gV)]$$

$$\Rightarrow Z_c = \exp[-\mathrm{Tr}\ln(1-\hat{g}V)]$$

where $\hat{g}\theta(t,t') = -i\,e^{-i\omega_0(t-t')}\begin{bmatrix} f(\omega_0) & \theta(t-t') \\ -\theta(t'-t) & 0 \end{bmatrix}$

where $f(\omega_0) = (2n_B(\omega_0)+1)$

$$(gV)_{V_q=0} = \begin{pmatrix} g^{AK} & g^{R} \\ 0 & 0 \\ g^{K} & g^{A} \end{pmatrix}\begin{pmatrix} 0 & V^{cl} \\ V^{cl} & 0 \end{pmatrix} = \begin{matrix} g^{R}V^{cl} \\ + g \end{matrix}$$

$$= \begin{pmatrix} g^{R}V^{cl} & g^{R}V^{cl} \\ 0 & g^{A}V^{cl} \end{pmatrix}\begin{pmatrix} g^{R}V^{cl} & g^{R}V^{cl} \\ 0 & g^{A}V^{cl} \end{pmatrix}$$

$$= \begin{pmatrix} g^{R}V^{cl}g^{R}V^{cl} & \\ & g^{A}V^{cl}g^{A}V^{cl} \end{pmatrix}$$ Second

$$\mathrm{Tr}(g^{R}(t,t')\,V^{cl}(t')\,g^{R}(t',$$

(t,t') → $\mathrm{Tr}(\ )$ $\quad \mathrm{Tr}(g^{R}Vg^{R}V)$

$$= \int dt\, g^{R}(t,t')V(t')\,g^{R}(t')V(t)\,dt'$$

(t,t')

$(t,t')\times(t',t)$

$$= \int dt\,dt'\; g^{R}(t,t')\,V(t')\,g^{R}(t',t)\,V(t)$$

(t,t) →

non-zero for $t=t'$ → Integral is 0 because its area is 0.

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Q. $\because [H_0, \hat{n}] = 0$, Classical sources can't factor in on the action. [? ? ?]

$\Rightarrow$ Discrete Case :-

Put in $H_v = V(t)$ btb

$$Z_d[V] = \frac{1}{Tr(\rho_0)} \int \prod_{j=1}^{2N} d[\bar{\phi}_j, \phi_j] \exp\left[i \sum \bar{\phi}_j \, G_{jj'}^{-1} \phi_{j'}\right]$$

$$\Rightarrow \cancel{Z_d[V] = \frac{1}{Tr(\rho_0)} \int \prod_{j=1}^{2N}} \quad i G_{jj}^{-1} = \begin{pmatrix} \begin{matrix} -1 & & \\ h_- & 1 & \\ & h_- & \ddots \end{matrix} & \begin{matrix} & & \\ & & \\ & & \rho(\omega_0) \end{matrix} \\ 0 & \begin{matrix} -1 & & \\ h_+ & 1 & \\ & h_+ & \ddots \end{matrix} \end{pmatrix}$$

$$h_- = 1 - i(\omega_0 + V(t_j))\,\delta t_j$$
$$= 1 - i(\omega_0 + V_j)\,\delta t$$

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$$Z_d = \frac{1}{Tr(\rho_0)} \times \frac{1}{\det[-iG^{-1}]} = \frac{1}{Tr(\rho_0)} \times \frac{1}{1 - \rho(\omega_0)(h_-)^{N-1} h_+^{N-1}}$$

now, $h_- = 1 - i(\omega_0 + V_j)\delta t \approx e^{-i\omega_0 \delta t} e^{-iV_j \delta t}$

$$\underset{1}{\downarrow} \qquad e^{-i\int_c V dt}$$

$$\therefore Z_d = \frac{1}{Tr(\rho_0)} \times \frac{1}{1 - \rho(\omega_0) e^{-i\int_c V dt}} = \frac{1}{Tr\rho_0} \times \frac{1}{1 - \rho(\omega_0) e^{-i 2\int_c V_q dt}}$$

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$$\Rightarrow Z_d = \frac{1-f(\omega_0)}{1-f(\omega_0)\, e^{-2i\int V_q dt}} = \frac{1}{1 \pm \frac{f(\omega_0)}{1-f(\omega_0)}\left[e^{-i2\int V_q dt}\right]}$$

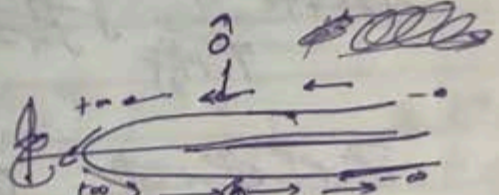
$$= \left[1 - n_B\left(e^{-2i\int_{-\infty}^{\infty} V_q\, dt} - 1\right)\right]^{-1}$$

$$\boxed{Z_d = \left[1 - n_B\left(e^{-2i\int_{-\infty}^{\infty} V_q\, dt} - 1\right)\right]^{-1}}$$

clearly, $Z_d[V'', V_q = 0] = 1$

↳ need of fictitious potential to generate Observables.

$\langle \hat{n}(t) \rangle$:-



$$= \langle \hat{n}(t) \rangle_{\hat{0}}$$

$$i\,\frac{1}{Z[0]}\,\frac{\delta}{\delta V_q(t)}\, Z_d[V_q]\Big|_{V_q=0}$$

fwd ⇒ $+ V(t)\hat{0}$

bwd → $-V^{-}(t)\,\hat{0}$

$-\hat{V}^{-}(t)\,\hat{0}$

$\leftarrow \hat{0} \leftarrow$

$\rightarrow -\hat{0} \rightarrow$

(1/2) (sum of fwd + sum of bwd)

in Bwd we reverse sign.

i.e. $H_V = H_0 \pm \hat{0}\, V(t)$ + → fwd, − → bwd

$$\therefore \langle \hat{n}(t) \rangle = \left(\frac{i}{2}\right) \frac{1}{Z(0)}\, \frac{\delta}{\delta V_q(t)}\, Z[V]\Big|_{V_q=0}$$

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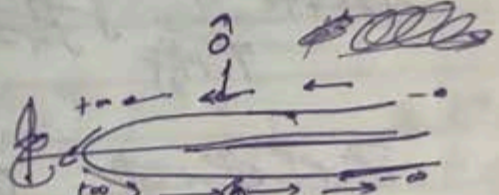

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Rough

$$\left([G_R]^{-1} \cdot F - F[G_A]^{-2}\right)_{t,t'} \phi(t')$$

$$\int dt'' \, \delta(t-t'') \left[i\partial_{t''} - \omega_0 + i\eta\right] f(t'',t') - f(t,t'') \, \delta(t''-t') \left[i\partial_{t'} - \omega_0 - i\eta\right]$$

$$= \left((i\partial_t - \omega_0 + i\eta) f(t,t') - f(t,t') \left[i\partial_{t'} - \omega_0 - i\eta\right] \right)$$

$$2i\eta \, f(t,t') \qquad (\partial_t)\,(f(t,t'))$$

$$\bar{\phi}(t) \left[\partial_t f(t,t')\right] \phi(t') - \bar{\phi}(t) \, f(t,t') \, \partial_{t'} \phi(t')$$

$$f^*(t,t') = f(t',t) \qquad + \bar{\phi}(t) \, \partial_{t'} f(t,t') \, \phi(t')$$

$$\left[\partial_t f(t,t') + \partial_{t'} f(t,t')\right]$$

$$\int dt'' \, (i\partial_t - \omega_0 + i\eta) \, \delta(t-t'') \, f(t'',t')$$

$$(i\partial_t - \omega_0 + i\eta) f(t,t') - f(t,t') \, (i\partial_{t'} - \omega_0 - i\eta)$$

$$\int \bar{\phi}(t) \left[\partial_t f(t,t')\right] \phi(t') - \int \bar{\phi}(t) \, f(t,t') \, \partial_{t'} \phi(t')$$

$$i\theta(a + \vec{a}) \in R \qquad f^*(t,t')$$

$$+ \int \bar{\phi}(t) \, \partial_{t'} (f(t,t')) \, \phi(t')$$

$$\int \bar{\phi}(t') \, \partial_t (f(t',t)) \, \phi(t)$$

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$$\frac{\alpha - \rho(\omega_0)}{1-\alpha\rho(\omega_0)} \times \alpha = \left[\frac{(\alpha-1)}{1-\alpha\rho(\omega_0)} + 1\right] =$$

$$\frac{1}{\frac{1-\rho(\omega_0)\alpha}{1-\rho(\omega_0)}} = \frac{1}{1-\rho(\omega_0)+\rho(\omega_0)(1-\alpha)} = \frac{1}{1+n_B(1-\alpha)}$$

$$[1+n_B(1-\alpha)]^{-1} = [1-n_B(\alpha-1)]^{-1} = \mathcal{G}_d$$

* # Particle no. is conserved, then what is

$$\Longleftrightarrow \quad \Psi \begin{pmatrix} 0 & [G^A] \\ [G^A]^{-1} & [G^K]^{-1} \end{pmatrix} \psi$$

$$\mathcal{H}_0 = \delta(t-t')(i\partial_{t'} - \omega_0 + i\eta)$$

$$[G^K]^{-1} = [G^A]^{-1} \circ F - F \circ [G^A]^{-1}$$

$$\phi_{t''} = \delta(t-t'')(i\partial_{t''} - \omega_0) F(t'',t'') - F(t,t'')\,\delta(t''-t'')(i\partial_{t''} - \omega_0 - i\eta)$$

$$\bar{\phi}_t f_{t,t''} \partial_{t''} \phi_{t''} - \bar{\phi}_t (\partial_t f)_{t''} \phi_{t''}$$

$$= 2i\eta F(t,t'') + i\left[\partial_t F(t,t'') - \partial F(t,t'')\partial_t\right]$$

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$$\bar{\phi}_t (\partial_t f)\phi_{t'} \qquad \bar{\phi}_t f(t,t'')\partial_{t'}\phi_{t'}$$

$$\frac{2\eta}{\phi_t}$$

Teacher's Signature .....

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$$\frac{2\eta}{\phi_t}$$

Teacher's Signature .....

$V_q \phi^+ - V_q \phi^-$ $\quad$ V_q $\quad$ φ^2

$$\frac{1}{Tr\,\rho}\int_1^{\infty}\prod d[\phi_j,\phi_j]\ \exp\Big[i$$

★ because when performing $\frac{\delta}{\delta V(t)}$, we perform diff. wrt both fwd & bwd parts

b^+b for each part

bwd part

$H_V = H - V(t)\, b^+ b$

$\phi^+(\varphi^{-1})\phi^+ - \phi^-\varphi^{-1}\phi^-$

$(i\partial_t - \omega_0)$

We go in reverse

$\langle j|\, b^+ b\, |j+1\rangle \longrightarrow \langle \phi_{j+1}, \phi_j\rangle \to n_B(\omega_0)$ as $N\to\infty$

Continuum Case:-

$$\frac{\delta}{\delta V_q} Z_c(V_q)\Big|_{V_q=0} = \langle \bar\phi^+\phi^+ + \bar\phi^-\phi^-\rangle$$

$$\frac{\overline{\varphi^{c1}}\,\varphi^{c1} + \overline{\varphi^{q}}\,\varphi^{q}}{2}$$

$\langle \bar\phi^+(t)\phi^+ + \bar\phi^-(t)\phi^-(t)\rangle$

might look like b^+b inserted at fwd & bwd branches.

$b^+(t)\, b(t)$

$V_q \phi^+ - V_q \phi^-$ $\quad$ V_q $\quad$ φ^2

$$\frac{1}{Tr\,\rho}\int_1^{\infty}\prod d[\phi_j,\phi_j]\ \exp\Big[i$$

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$H_V = H - V(t)\, b^+ b$

$\phi^+(\varphi^{-1})\phi^+ - \phi^-\varphi^{-1}\phi^-$

$(i\partial_t - \omega_0)$

We go in reverse

$\langle j|\, b^+ b\, |j+1\rangle \longrightarrow \langle \phi_{j+1}, \phi_j\rangle \to n_B(\omega_0)$ as $N\to\infty$

Continuum Case:-

$$\frac{\delta}{\delta V_q} Z_c(V_q)\Big|_{V_q=0} = \langle \bar\phi^+\phi^+ + \bar\phi^-\phi^-\rangle$$

$$\frac{\overline{\varphi^{c1}}\,\varphi^{c1} + \overline{\varphi^{q}}\,\varphi^{q}}{2}$$

$\langle \bar\phi^+(t)\phi^+ + \bar\phi^-(t)\phi^-(t)\rangle$

might look like b^+b inserted at fwd & bwd branches.

$b^+(t)\, b(t)$

Q Quantum CLT → How does non-qm situations violate CLT?
Central limit theorem

$$Z[\bar{x}, x] = \int \prod_{j=1}^{N} d[\bar{\psi}_j, \psi_j] \; e^{-\sum_{ij}^{N} \bar{\psi}_i (A)_{ij} \psi_j + \sum_j \bar{\psi}_j x_j + \sum \bar{x}_i \psi_i}$$

$$= \det A \; \exp\left[\sum_{ij}^{N} \bar{x}_i (A^{-1})_{ij} x_j \right]$$

→ true for any invertible matrix

not necessarily → true for convergente $\to N \to 1 \quad A = 0$

$$Z = \int d\bar{\psi} d\psi \, e^{\bar{\psi} x + \bar{x} \psi}$$

$$= \int d\bar{\psi} \, d\psi \, \bar{x} \psi \, e^{\bar{\psi} x}$$

$$= \int d\bar{\psi} \, (-\bar{x}) \, [1 + \bar{\psi} x]$$

$$Z = \bar{x} x$$

Self energy

$$\langle \psi_1 | \hat{\rho}(t_0) | -\psi_{2N} \rangle = \langle \psi_1 | e^{-\beta t_0 c^\dagger c} | -\psi_{2N} \rangle$$

Imp:- $\langle \psi_1 | e^{-\beta t_0 c^\dagger c} | -\psi_{2N} \rangle = e^{\bar{\psi}_1 - \psi_{2N} (e^{-\beta t_0})}$

$$= e^{\bar{\psi}_1 \psi_{2N} - \rho(t_0)}$$

∃ a proof of coherent states

$$f(\rho) = \langle \phi | \rho^{b^\dagger b} | \phi' \rangle = e^{(\bar{\phi} \phi' \rho)}$$

$$f(1) = e^{\bar{\phi} \phi'}$$

$$\partial_\rho f(\rho) = \langle \phi | b^\dagger b \, \rho^{b^\dagger b - 1} | \phi' \rangle$$

$$= \langle \phi | b^\dagger \rho^{b^\dagger b} b | \phi' \rangle = \bar{\phi} \phi' f(\rho)$$

Teacher's Signature

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Teacher's Signature

★ fermionic Case :- $\psi \to$ Grassmann

★ $|\psi\rangle = e^{-\psi c^\dagger}|0\rangle$ ★ $\langle\bar\psi|\bar\psi\rangle = e^{\bar\psi\psi}$

$\Rightarrow \boxed{|\psi\rangle = |0\rangle - \psi|1\rangle}$

★ $Tr\{\hat O\} = \int (d\bar\psi d\psi)\, e^{-\bar\psi\psi} \langle\psi|\hat O|\psi\rangle$ → important.

why?

$\langle n|\psi\rangle\langle\psi|\hat O|n\rangle \to$ $\langle\psi| = \langle 0| - \langle 1|\bar\psi$

$\langle n|\, e^{\psi c^\dagger}|0\rangle\langle 0|\, e^{-c\bar\psi}\, |\hat O|n\rangle$ $= \langle 0|\, e^{-c\bar\psi}$

$\to \langle n|(|0\rangle - \psi|1\rangle)$

some overlap) $\psi \times$ ($\bar\psi$ (some overlap).

$-\bar\psi\,\psi = -\bar\psi$ (and overlap) $\times\, \psi$ (1st overlap)

★ $Tr\{\hat O\} = \int d\bar\psi\, d\psi\; e^{-\bar\psi\psi} \langle\psi|\hat O|-\psi\rangle$

[Entropic inequality violations in non-eqm situations ??]

★ ~~$Z[\bar\psi] = \int \prod_{j=1}^{N} [d\psi_j, d\bar\psi_j]$~~

contrast $Z_{eqm} = \int d(\phi^*, \phi)\, e^{-\phi^*\phi}$

$e^{-\beta H}$

$\langle \phi|\hat O|\phi\rangle$

(±1 for bosons, −1 for fermions

So, correct

$\phi_\beta(x,\beta) = \phi(x,0)$

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$\mathcal{U}^{+}\mathcal{U}.\rho$

Now,

$$Z_{non\text{-}eqm} = \int d(\phi,\phi)\, e^{-\beta\phi} \langle \phi | \hat{\rho}(t) | \phi \rangle$$

$\mathrm{Tr}(\hat{\rho})$

But the way we carry out the trace is a very time indpt procedure.

$$Z_{non\text{-}eqm} = \mathrm{Tr}(\hat{\rho}(t)) = \mathrm{Tr}[\mathcal{U}\rho\mathcal{U}^{+}] \quad \text{, where } \mathcal{U} = \mathcal{U}_{t,-\infty}$$

$$= \mathrm{Tr}[\mathcal{U}_{-\infty,t}\, \mathcal{U}_{t,-\infty}\, \rho_{-\infty}]$$

remove t completely

a testament to the fact that Trace is time independent so is partition fn in non-eqm case.

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$\langle \phi | \hat{\rho}(t) | \phi \rangle$ can't be thought as a transition amplitude in imaginary time, simply because we don't have a form for $\hat{\rho}$ (to invoke the comparison).

Generating function.

$$Z[\bar{x}, x] = \int \prod_{i=1}^{N} d[\bar{\psi}_i, \psi_i]\, e^{-\sum \bar{\psi}_i (A^{-1})_{ij} \psi_j + \sum \bar{x}_i \psi_i + \sum \bar{\psi}_j x_j}$$

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condition

A needs to be invertible for this result to work

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difference from normal Wick's theorem

$$\langle \psi_a \psi_b \bar{\psi}_c \bar{\psi}_d \rangle = A_{bc} A_{ad} - A_{ac} A_{bd}$$

Partition function :-

same as Bosonic case

$$Z_{for\ B^*} = \int \frac{\mathcal{D}[\bar{\psi},\psi]}{Tr(\hat{\rho}_0)} \exp\left[i \sum \bar{\psi}_i (G^{-1})_{ij} \psi_j \right]$$

where iG^{-1} is same as Boson except it features

$-\rho(t_0)$ instead of $\rho(\infty_0)$ at $(1, 2N)$ location

$$-\rho(t_0) \longrightarrow \langle \psi_1 | \hat{\rho}_0 | \psi_{2N} \rangle \xrightarrow[\text{d.m.}]{\text{eqm}} \langle \psi_1 | \cancel{e^{-\beta(t_0-\mu)}} |$$

$$\hookrightarrow \langle \psi_1 | e^{-\beta_1 e^{(t_0-\mu)}} | \psi_{2N} \rangle$$

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$$-\rho(t_0)$$

$$Tr(\hat{\rho}_0) = 1 + \rho(t_0)$$

$$\det(-iG^{-1}) = 1 + \rho(t_0) \qquad \therefore \underline{Z = 1}$$

$$Z_{cont} = \int \mathcal{D}[\bar{\psi},\psi]\, e^{iS}$$

$$S_N = \int dt \left[\bar{\psi}^+ (i\partial_t - \epsilon_0) \psi^+ - \bar{\psi}^- (i\partial_t - \epsilon_0) \psi^- \right] + \text{bd terms}$$

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$G_{fermion}$ & $S_{K_{fermion}}$:-

$$\langle \psi^+(t)\bar{\psi}^-(t')\rangle = iG^< = -n_F\, e^{-i\epsilon_0(t-t')}$$

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Similarly $iG^{\bar{T}}$

$$G^T + G^{\bar{T}} = G^> + G^<$$ still holds (Keldysh rotation)

Keldysh rotat^n :-

$\longrightarrow$ no cl & q fields, because Grassmann variables have no classical meaning. (always integrated out at some time in the calculation).

Define

① $\psi_1 = \left(\frac{\psi^+ + \psi^-}{\sqrt{2}}\right)$ $\quad \psi_2 = \left(\frac{\psi^+ - \psi^-}{\sqrt{2}}\right)$.

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Now green's functions look like

$$-i\langle \psi_\alpha(t)\bar{\psi}_\beta(t')\rangle = G_{\alpha\beta} = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}$$ fermion

[bosonic :- $\begin{pmatrix} G^K & G^R \\ G^A & 0 \end{pmatrix}$]

Imp property :- $\boxed{T_c\left[\hat{G}^{(1)}\circ\hat{G}^{(2)}\circ\hat{G}^{(3)}\cdots\circ\hat{G}^{(n)}\right] = 0}$

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Why such a structure?

→ ~~Because~~ Relic of the different choice for transformation of bar fields.

→ One can choose the bosonic convention here as well to recreate the same structure. ~~but this shall not~~ LSO (of FFLO states) decided for this choice as

as $\hat{G}$, G^{-1} & fermionic self energy $\hat{\Sigma}_F$ all have the same appearance as $\hat{G}$

i.e. $G^{-1} = \begin{pmatrix} (G^R)^{-1} & (G^{-1})^K \\ & (G^A)^{-1} \end{pmatrix}$

$\Sigma_F = \begin{pmatrix} \Sigma^R & \Sigma^K \\ 0 & \Sigma^A \end{pmatrix}$ [in case of bosons however, the self energy looks different so does G^{-1}.

$\therefore G = \begin{pmatrix} G^K & G^R \\ G^A & 0 \end{pmatrix}$

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Fluctuation Dissipation theorem:- (what?)

$G^K(t) = \tanh\left(\frac{t-\mu}{2T}\right)\left[G^R(t) - G^A(t)\right]$

Be stupid! Practically ignorant

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$= Tr(\partial_t \partial_t P)$

Many particles :- Degenerate fermi gas

$$S = \int dx\, dx'\, dt \;\; F(x,t) \left[G^{-1} \right]_{x,x',t,t'} \psi(x,t)$$

$$\downarrow$$

$$\int dx\, dx'\, dt\, dt' \;\; \underbrace{F(x,t) \left[G^{-1} \right]_{x,x',t,t'} \psi(x,t)}$$

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$$\downarrow \text{ q.F.T.} \qquad \rightarrow e^{-i\epsilon t}$$

$$\left[G_R \right]^{-1} \text{ in } t\text{-domain} = (i\partial_t - \omega_0 + i\eta)\, \delta(t-t')$$

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So in k, ω domain

$$\left[i\partial_t - \frac{k^2}{2m} - \mu_2 \right] \qquad e^{ikx} \qquad \text{ignore}$$

$$\downarrow x \qquad \sum_{k} e^{ik(x-x')}$$

$$\left[i\partial_t + \frac{\nabla_{\vec{r}}^2}{2m} \right] \qquad e^{i\vec{k}(\vec{r}-\vec{r}')} \quad \delta(\vec{r}-\vec{r}')\, \delta(t-t')$$

$$\left(i\partial_t + \frac{\nabla_{\vec{r}}^2}{2m} \right) \delta(\vec{r}-\vec{r}')\, \delta(t-t')$$

$$\downarrow$$

full $\left(i\partial_t + \frac{\nabla_{\vec{r}}^2}{2m} \right) - \mu$

Teacher's Signature .....

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$$\downarrow$$

full $\left(i\partial_t + \frac{\nabla_{\vec{r}}^2}{2m} \right) - \mu$

Teacher's Signature .....

Keldysh

$= \frac{Tr(\rho^{+} O \rho)}{Tr(\rho(-\infty))}$

$G_0 \rightarrow G_{k,\sigma} = \frac{k^2}{2m} + h_z \sigma$

$\psi_\sigma(\vec{r},t) = \sum_{\vec{k}} \psi_{k,\sigma}(t) e^{i\vec{k}\cdot\vec{r}}$

$\bar{\psi}_\sigma(\vec{r},t) = \sum_{\vec{k}} \bar{\psi}(\vec{k},t)\, e^{-i\vec{k}\cdot\vec{r}}$

How to write the Keldysh action?

$\bar{\psi}\, (G_R)^{-1}\, \psi$ Now, $G_R^{-1} \xrightarrow{\text{in } k,t \text{ domain}} \left(i\partial_t - \frac{k^2}{2m} - h_z\sigma\right) \delta_{k,k'}\, \delta(t-t')$

Zeeman magnet

real space

$\left(i\partial_t + \frac{\nabla_x^2}{2m} - h_z\sigma\right) \delta(x-x')\, \delta(t-t')$

$\begin{pmatrix} (G_R)^{-1} & (G_K)^{-1} \\ 0 & (G_A)^{-1} \end{pmatrix}$

$\eta \rightarrow 0$

Now G_R^{-1} could look like

$\underbrace{\left(i\partial_t + \frac{\nabla_x^2}{2m} - h_z\sigma + i\eta\right)}_{G_R^{-1}}$

G_A^{-1}

$\left(i\partial_t + \frac{\nabla_x^2}{2m} - h_z \delta_{\sigma,\sigma'} - i\eta\right)$

$h_z\sigma$ $\sigma = \pm 1$ for $\sigma = \uparrow\downarrow$

$\vec{h}\cdot\vec{\sigma}$

$h_z\, c_\sigma^{+} c_\sigma$

σ_z $c_\uparrow^{+} c_\uparrow - c_\downarrow^{+} c_\downarrow$

$\sigma_x \Rightarrow$ $c_\downarrow^{+} c_\uparrow + c_\uparrow^{+} c_\downarrow$

$\vec{h}\cdot\vec{S}_{\sigma,\sigma'} = \vec{h}\cdot\vec{S}_{\sigma,\sigma'}$

Keldysh

$= \frac{Tr(\rho^{+} O \rho)}{Tr(\rho(-\infty))}$

$G_0 \rightarrow G_{k,\sigma} = \frac{k^2}{2m} + h_z \sigma$

$\psi_\sigma(\vec{r},t) = \sum_{\vec{k}} \psi_{k,\sigma}(t) e^{i\vec{k}\cdot\vec{r}}$

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$$\delta(t-t')$$

$$\therefore \left[G^R_{\sigma,\sigma'}(x,x')\right]^{-1} = \delta(x-x')\left[i\partial_t + \frac{\nabla_x^2}{2m} - \vec{h}\cdot\vec{\sigma}_{\sigma,\sigma'} + i\eta\right]$$

$$S = \int_{dt'} dt\, d\vec{x}\, d\vec{x}' \sum_{\substack{\sigma,\sigma'=\pm1 \\ a,b=1,2}} \bar{\psi}_{a,\sigma}(\vec{x},t)\, [G^{-1}]_{\substack{\sigma,\sigma' \\ x,x' \\ b,t'}}\, \psi_{b,\sigma'}(\vec{x}',t')$$

Let $x = \{\vec{x}, t\}$

Then,

$$S = \int \underbrace{dx\, dx'}_{(dt\,dx) \to (dx'\,dt')} \sum_{\substack{\sigma,\sigma'=\pm1 \\ a,b=1,2}} \bar{\psi}_{a,\sigma}(x)\, [G^{-1}]_{\sigma,\sigma'}\, \psi_{b,\sigma'}(x)$$

Now,

$$G^{-1} = \begin{pmatrix} G^A & [G^{-1}]^K \\ 0 & (G^A)^\dagger \end{pmatrix}$$

?

What about G^K or $[G^{-1}]^K$?

Statement ①:- Keldysh is a pure regularization.
"Does not have a continuum limit"

Eqn:- 2.52 (BOSONIC CASE)

$$[G^{-1}]^K = [G^R]^{-1} \circ F - F \circ [G^A]^{-1}$$

$$[G^R]^{-1}_{t,t'} = \delta(t-t')\left[i\partial_t - \omega_0 \sigma_z + i\eta\right]$$

$$\left(\delta(t-t')\left[i\partial_{t'} - \omega_0\right]\right) F(t',t'') - F(t,t'')$$

$$(i\partial_t)\, F(t',t'') - F(t,t'')\left[i\partial_{t''} - \omega_0\right] \delta(t'-t'')\left[i\partial_{t''} - \omega_0\right].$$

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Keldysh field theory

Claimed $[G^{-1}]^R \cdot 2i\eta F(t,t') \,,\ \eta \to 0$

hides info of the distribution of particles.

$$\boxed{G^K = G^R \circ F - F \circ G^A}$$

how?

Claim: Wigner transform of F is the Distribution f^n

$2i\eta F \to$ ensures convergence of the Integral.

So what? Is it just for that cause only?

$$\boxed{\partial_{t'} f(t;t') + \partial_t f(t,t') = 0}$$

GK:- fermion case :-

$$[G^{-1}]^K = [G^{-1}]^R \circ F - F \circ [G^{-1}]^A$$

$$= [G^R]^{-1} \circ F - F \circ [G^A]^{-1}$$

① $F = F^\dagger$

② w.t. of $F = 1 - 2n_F$ is the fermionic Distribution f^n (?) $\longrightarrow$ isn't n_F the distribut^n f^n (?)

Result

$$G^R G_0^K{}_F = (1 - 2n_F)\, e^{-i\omega_0(t-t')}$$

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§

§ Some remarks from Ch-2 :-

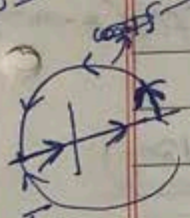
$\star$ $G_R(t,t) + G_A(t,t) = 0 \xrightarrow{f.t.} \int \left(\frac{dk}{2\pi}\right) \left[G_R(k) + G_A(k)\right] = 0$

$\Rightarrow \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \left[G_R(\epsilon) + G_A(\epsilon)\right] = 0$ (in Romenev's lingo, f.T. is en.rep) — (1)

$\int d\epsilon$

$0^{(?)}$

" $\int$ integral of a function analytic in Upper/lower $\frac{1}{2}$ plane is 0".



$\phi_\alpha \phi_\beta$

$b^\dagger b - b b^\dagger$

Now, $G_R(t,t) - G_A(t,t) = (G^> - G^<)_{t=t'=t} = \langle (b b^\dagger - b^\dagger b) \rangle - i$

$= -i \langle \underbrace{b b^\dagger - b^\dagger b}_{\text{a+c q. times}} \rangle = -i$

$\int d\epsilon \, \boxed{G_R(\epsilon)} e^{-i\epsilon(t-t')}$

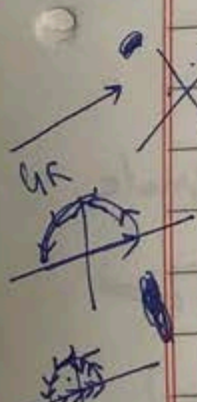
$t > t'$

$\Rightarrow \int \frac{d\epsilon}{2\pi} \left[G_R(\epsilon) - G_A(\epsilon)\right] = -i$

$\Rightarrow \int d\epsilon \left[G_R(\epsilon) - G_A(\epsilon)\right] = -2\pi i$ — (2)

(1) & (2), gives

$\boxed{\begin{aligned} \int G_R \, d\epsilon &= -\pi i \\ \int G_A \, d\epsilon &= +\pi i \end{aligned}}$



G_R

$\int_{-\infty}^{\infty} G_R(\epsilon) \, d\epsilon = -\pi i \qquad \int_{-\infty}^{\infty} G_A(\epsilon) \, d\epsilon = +\pi i$

$\star$ Claim:- $G_R - G_A$ always come with distribution function.

"to be thought as int over the complex upper (lower) semi-circle"

$= \frac{Tr(U^\dagger O U \cdot \rho)}{Tr(\rho)} = \frac{Tr(U^\dagger O U \rho)}{Tr(\rho(-\infty))}$

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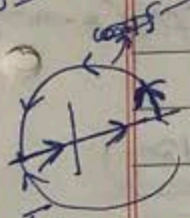

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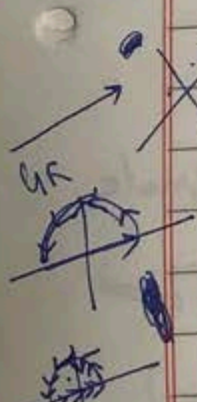

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$= Tr(U^\dagger O \rho O \lambda$ $\quad = \frac{Tr(U^\dagger O U \rho)}{Tr(\rho(\infty))}$

§ fluctuation-Dissipation theorem

$$G^K(\epsilon) = \coth\left(\frac{\epsilon-\mu}{2T}\right)\left[G^R(\epsilon) - G^A(\epsilon)\right]$$

$\longrightarrow$ property of thermal eq^m.

$\Rightarrow$ Read linear Response theory

$\#$ $G^K = G^R \circ F - F \circ G^A \xrightarrow{F.T.} \left[\epsilon - \omega_0 + i\eta\right] f(\epsilon) - (f(\epsilon))\left[\epsilon - \omega_0 - i\eta\right]$

$\Rightarrow G^K(\epsilon) = 2i\eta f(\epsilon)$

Claim:- In thermal eqm, $F(\epsilon) = \coth\left(\frac{\epsilon-\mu}{2T}\right) = (1+2n_B)$

claim:- Wigner transform of $F(t,\epsilon)$ of the matrix F is referred to as the distribution function ($\neq n_B / n_F$).

§ Wigner transform:-

$A(x_1, x_2)$. , $x_i = (\xi, t)$ & $p = (k, \epsilon)$.

$$A(x_1,x_2) \longrightarrow A\Big(\underbrace{\frac{x_1+x_2}{2}}_{\text{COM coor}}, \underbrace{x_1-x_2}_{\text{rel. coordinate}}\Big)$$

$\downarrow$ f.t. with rel coord.

$$A(x,p) = A\left(\frac{x_1+x_2}{2}, p_{rel}\right)$$

$$A(x,p) = \int dx' \, e^{-ipx'} A\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right)$$

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$\longrightarrow$ A = matrix with indices (t_1, t_2)

§§: Prop^n:- ① $A^+(x_1, x_2) = [A(x_2, x_1)]^*$

$\therefore$ W.T. $A^+(x_1, x_2) = [A(x, p)]^*$.

~~A(x,p)~~ Pf:-

$$[A(x,p)]^* = \int_{x \to x'} dx' \, e^{+ipx'} A^*\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right)$$

$$= \int dx' \, e^{-ipx'} \underbrace{A^*\left(x-\frac{x'}{2}, x+\frac{x'}{2}\right)}_{A(x_2, x_1)}$$

② 2pnt f^n:- to extract W.T. of C when A & B are known

$C = A \circ B$ $\quad C(x,p) = \int dx' \, e^{-ip(x')} (A \circ B)\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right)$.

$$C(x_1, x_2) = \int dx_3 \, A(x_1, x_3) \, B(x_3, x_2)$$

$$C(x,p) = \int dx' \, e^{-ipx'} \int dx_3 \, A\left(x+\frac{x'}{2}, x_3\right) B\left(x_3, x-\frac{x'}{2}\right)$$

$$= \cancel{\int dx' e^{ipx'} \int dx_3 \, \sum_p e}$$

use the identity

$$A(x_1, x_2) = \sum_p e^{ip(x_1 - x_2)} A\left(\frac{x_1+x_2}{2}, p\right).$$

$$\sum_p \cdot \sum_{\substack{\cdot \\ C(x,p)}} \int \frac{d^3k}{(2\pi)^3} = \sum_p e^{ip(x_1-x_2)} A\left(\frac{x_1+x_2}{2}, p\right)$$

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$$\therefore C(x,p) = \int dx' e^{-ipx'} \int dx_3 \sum_{p_a,p_b} e^{+ip_a\left(x+\frac{x'}{2}-x_3\right)} e^{ip_b(\ldots}$$

$$A\left(\frac{x+\frac{x'}{2}+x_3}{2}, p_a\right) B\left[\frac{x_3+\ldots}{2}, p_b\right]$$

$$= \int dx' e^{-ipx'} \int dx_3 \sum_{p_a,p_b} e^{ip_a\left(x+\frac{x'}{2}-x_3\right)} e^{ip_b\left(x_3+\frac{x'}{2}-x\right)} \cdot \times$$

$$A\left(\frac{x+\frac{x'}{2}+x_3}{2}, p_a\right) B\left(\frac{x_3+x-\frac{x'}{2}}{2}, p_b\right)$$

$$= \cancel{\int dx' e^{-ipx'} \int dx_3 \sum_{p_a,p_b} e^{ip_a}}$$

$$e^{ikL} \qquad \psi(x) = \psi(x+L)$$

$$kL = 2\pi n \qquad e^{ikL}$$

$$k = \frac{2\pi n}{L}$$

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§ Clarification of $[G^{-1}]^K$ part :-

$$[G^K]^{-1} = (G^R)^{-1} \circ F - F \circ (G^A)^{-1}$$

$$\cancel{[G^K]^{-1}(t_1,t_2) = \oint_K \sum [\epsilon - \omega_0 \; e^{-i\epsilon(t_1-t_2)}}$$

$$[G^K]^{-1}(t_1,t_2) = [G^R]^{-1} \circ F - F \circ [G^A]^{-1}$$

$$= \int dt_3 \int \frac{d\epsilon}{d\epsilon'} (\epsilon - \omega_0 + i\eta)\, e^{-i\epsilon(t_1-t_3)} f(\epsilon')\, e^{-i\epsilon'(t_3-t_2)} - (\;)$$

$$= \int d\epsilon \; (\epsilon - \omega_0 + i\eta)\, e^{-i\epsilon(t_1-t_2)} f(\epsilon)$$

$$- \int d\epsilon \; f(\epsilon) [\epsilon - \omega_0 - i\eta]\, e^{-i\epsilon(t_1-t_2)} = [G^K]^{-1}(t_1,t_2)$$

$$\Rightarrow [G^{-1}]^K(t_1,t_2) = \int \{2i\eta F(\epsilon)\}\, e^{-i\epsilon(t_1-t_2)}$$

$$\boxed{[G^{-1}]^K(\epsilon) = 2i\eta F(\epsilon)}$$

at eqm, $n_B = \dfrac{1}{e^{\beta(\epsilon-\mu)}-1} = \dfrac{1}{e^{(\beta)(\epsilon-\mu)}-1}$

however it could be different at non-eqm. conditions.

$f(t,t') \xrightarrow{W.T.} F(t,\epsilon)$ is called as the Distribution function (?).
(Time dependent Distribution function)

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$$= \int dt_3 \int \frac{d\epsilon}{d\epsilon'} (\epsilon - \omega_0 + i\eta)\, e^{-i\epsilon(t_1-t_3)} f(\epsilon')\, e^{-i\epsilon'(t_3-t_2)} - (\;)$$

$$= \int d\epsilon \; (\epsilon - \omega_0 + i\eta)\, e^{-i\epsilon(t_1-t_2)} f(\epsilon)$$

$$- \int d\epsilon \; f(\epsilon) [\epsilon - \omega_0 - i\eta]\, e^{-i\epsilon(t_1-t_2)} = [G^K]^{-1}(t_1,t_2)$$

$$\Rightarrow [G^{-1}]^K(t_1,t_2) = \int \{2i\eta F(\epsilon)\}\, e^{-i\epsilon(t_1-t_2)}$$

$$\boxed{[G^{-1}]^K(\epsilon) = 2i\eta F(\epsilon)}$$

at eqm, $n_B = \dfrac{1}{e^{\beta(\epsilon-\mu)}-1} = \dfrac{1}{e^{(\beta)(\epsilon-\mu)}-1}$

however it could be different at non-eqm. conditions.

$f(t,t') \xrightarrow{W.T.} F(t,\epsilon)$ is called as the Distribution function (?).
(Time dependent Distribution function)

Signature .....

$F(t_1,t_2) \xrightarrow{W.T} F\left(\frac{t_1+t_2}{2}, \epsilon\right) = F(t,\epsilon)$ = Distrib. fn.

$$F(t,\epsilon) = \int dt' \, e^{+i\epsilon t'} \, F\left(t+\frac{t'}{2}, t-\frac{t'}{2}\right).$$

$$F(t,\epsilon) = \int dt' \, e^{+i\epsilon t'} \, f(t') \quad (\text{for } f(t_1,t_2) \text{ as } f(t_1-t_2)).$$

$$\Rightarrow F(t,\epsilon) = F(\epsilon) \quad (= F(p))$$

$$[\rho^{-1}K](\eta) = 2i\epsilon F(\eta), \quad \epsilon \to 0$$

$$[\rho K] = \rho^R \circ F - \rho^A \circ F$$

$$\Rightarrow [\rho K]_\epsilon \circ = \frac{1}{\epsilon - \omega_0 + i\eta} f(\epsilon) - \frac{f(\epsilon)}{\epsilon - \omega_0 - i\eta} \xrightarrow{\eta \to 0} -2\pi i \, \delta(\epsilon - \omega_0) f(\epsilon)$$

$$\therefore f(\epsilon) = [2n_B(\epsilon)+1] = \coth\left(\frac{\beta\epsilon}{2}\right)$$

$$\boxed{\begin{aligned} [\rho K]_\epsilon &= 2i\eta f(\epsilon) \\ [\rho K]_\epsilon &= -2\pi i \delta(\epsilon-\omega_0) f(\epsilon) \end{aligned}}$$

if T.it $\rho = \rho_0 = e^{-\beta(\epsilon)}$

$\Downarrow$

in thermal eqm.

Q. Why do we need Wigner transform & not just simply f.t.?

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$\# \; Z_{J\bar{J}} = \int d(\bar{\phi},\phi) \; e^{-\bar{\phi}\phi + \bar{J}\phi + \phi\bar{J}} = \exp(J\bar{J})$

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Date.

Keldysh 1.3
Ch-5

§ Many particle φ. system (BOSONIC)

$$H = \sum_{k} \omega_k b_k^{+} b_k = \sum_{k} \omega_k b_k^{+} b_k$$

$$Z = \frac{tr(\hat{U}_c \hat{\rho}_{in})}{tr(\hat{\rho}_{in})} = 1$$

$$Z = \int D[\varphi^{+}, \varphi^{-}] \exp[iS_0]$$

$$S_0 = \int dt \left[\sum_{k} \left(\bar{\varphi}_k^{+} (i\partial_t - \omega_k) \varphi_k - \bar{\varphi}_k^{-} (i\partial_t - \omega_k) \varphi_k^{-} \right) \right]$$

$$= \int dt \sum_{k} \left(\bar{\varphi}_k^{+} (i\partial_t - \omega_k) \varphi_k^{-} - \bar{\varphi}_k^{-} (i\partial_t - \omega_k) \varphi_k^{-} \right).$$

Keldysh rotation.

$$Z = \int D[\varphi^{cl}, \varphi^{q}] \; e^{iS[\varphi^{cl}, \varphi^{q}]}$$

where $D[\varphi^{cl}, \varphi^{q}] = \prod_{k} \prod_{j=1}^{N} \frac{d(\mathrm{Re}\, \varphi_j^{cl}(k)) \, d(\mathrm{Im}\, \varphi_j^{cl}(k))}{\pi} \frac{d(\mathrm{Im}\, \varphi_j^{q}(k)) \, d(\mathrm{Re}\, \varphi_j^{q}(k))}{\pi}$

& $N \to \infty$.

$$S_0[\varphi^{cl}, \varphi^{q}] = \int dt \sum_{k} \left(\bar{\varphi}^{cl}(k) \;\; \bar{\varphi}^{q}(k) \right) \begin{pmatrix} 0 & (i\partial_t - \omega_k + i\eta) \\ (i\partial_t - \omega_k - i\eta) & 2i\eta F(\omega_k) \end{pmatrix} \begin{pmatrix} \varphi^{cl}(k) \\ \varphi^{q}(k) \end{pmatrix}$$

$$= \sum_{k} \int dt \left[\bar{\varphi}^{cl} \;\; \bar{\varphi}^{q} \right] \begin{pmatrix} 0 & \overbrace{i\partial_t - \omega_k + i\eta}^{\delta_{t,t'}} \\ \underbrace{i\partial_t - \omega_k - i\eta}_{\delta_{t,t'}} & \underbrace{2i\eta F(\omega_k)}_{\delta(t,t')} \end{pmatrix} \begin{bmatrix} \varphi^{cl} \\ \varphi^{q} \end{bmatrix} \quad (k, t')$$

(k, t)

[i.e. not a $\delta(t-t')$]

Teacher's Signature .....

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Keldysh 1.3
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Teacher's Signature .....

Chapter-2 Komenev

q-q component of g^{-1} i.e. $\left([g^{-1}]^K = 2i\eta F(\omega_k)\right)$ here

a pure regularization for the "free field"

becomes finite (& non-local cort state & time indices) once interactions are included.

$$g^{-1} = \begin{pmatrix} 0 & i\partial_t - \omega_k + i\eta \\ i\partial_t - \omega_k + i\eta\, F(\omega_k) & \end{pmatrix}$$

$$g_0 = \begin{pmatrix} g^A & g^R \\ g^A & 0 \end{pmatrix}$$

$$\therefore \quad g^{\alpha\beta}(k,k',t,t') = (\delta_{k,k'}) \begin{bmatrix} g^K(\vec{k},t,t') & g^R(\vec{k},t,t') \\ g^A(\vec{k},t,t') & 0 \end{bmatrix}$$

& by analogy to the single quantum state, we have

$$g^R(\vec{k},t,t') = -i\theta(t-t')\, e^{-i\omega_k(t-t')} \longrightarrow (\epsilon - \omega_k + i\eta)^{-1}$$

$$g^A(\vec{k},t,t') = i\theta(t-t')\, e^{+i\omega_k(t-t')} \longrightarrow (\epsilon - \omega_k - i\eta)^{-1}$$

$$g^K(\vec{k},t,t') = -i\underbrace{\left(2n_B(\omega_k)+1\right)}_{F(\omega_k)}\, e^{-i\omega_k(t-t')}$$

$$\overset{\text{def.}}{=} -2\pi i\, f(\epsilon)\, \delta(\epsilon - \omega_k)$$

Changing to Real Space

Define $\varphi^\alpha(\vec{r},t) = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}\, \varphi^\alpha(\vec{k},t)$ $\quad \alpha = (c,q)$

Chapter-2 Komenev

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$$S_0 = \int dt \sum_{k^m} \left[\bar{\varphi}^{a1}, \bar{\varphi}^{a2}\right] \; \mathcal{G}^{-1}_{x,x',t,t'} \begin{bmatrix} \varphi^{a1} \\ \varphi^{a2} \end{bmatrix}.$$

q.f.t.

$$S_0 = \int_{t=-\infty}^{t=+\infty} dt \, d^3x^m \; \left[\bar{\varphi}^{a1} \; \bar{\varphi}^{a2}\right] \; \mathcal{G}^{-1}_{x,x',t,t'} \begin{bmatrix} \varphi^{a1} \\ \varphi^{a2} \end{bmatrix}$$

$$\varphi^{\alpha} \equiv \varphi^{\alpha}(\vec{x}, t)$$

But $\mathcal{G}^{-1}$ here looks

$$\mathcal{G}^{-1}_{x,x',t,t'} = \delta(\vec{x}-\vec{x}')\,\delta(t-t') \begin{bmatrix} 0 & i\partial_{t'} + \frac{\nabla_{x'}^2}{2m} - i\eta \\ i\partial_{t'} + \frac{\nabla_{x'}^2}{2m} + i\eta & 2i\eta F(t) \end{bmatrix}$$

$$2i\eta F(\omega)$$

~~term of the form.~~

$$\cancel{\bar{\varphi}^{a}(\vec{x}, t) \; 2i\eta F}$$

Q. Shouldn't the term be of the form

$$\bar{\varphi}^{a}(\vec{x}, t') \; 2i\eta F(t, t') \; \varphi^{a}(\vec{x}, t)$$

Ans:- Yes, $[\mathcal{G}^{-1}]^K$ is in general non-local but being a pure boundary term, it is frequently omitted.

Now, correlators look like.

$$\langle \varphi^{\alpha}(\vec{x}, t) \, \varphi^{\beta}(\vec{x}', t') \rangle = i\mathcal{G}^{\alpha\beta}_{0}(\vec{x}, \vec{x}', t, t') = i \sum_{\vec{k}} \mathcal{G}^{\alpha\beta}_{0\,\vec{k}}(t, t') \; e^{i\vec{k}(\vec{x}-\vec{x}')}$$

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Single Particle QM

present some difference from the usual Complex bosonic Keldysh theory.

One can write a Keldysh path integral for one-particle QM over the closed time contour.

$$S_K = \int_C dt \left[\frac{\dot{x}^2}{2} - V(x) \right]$$

decouple as $x^+(t)$ & $x^-(t)$ & perform Keldysh rotation.

without boundary terms (i.e. $+\infty$ & $-\infty$ ones), we'll lose the cross field correlations.

$$\therefore \quad S_K = \int_C dt \left[\frac{\dot{x}^2}{2} - V(x) \right] + \text{Boundary terms}$$

$\rightarrow$ Define $x^{cl} = \frac{1}{2}(x^+(t) + x^-(t))$ & $x^q = \left(\frac{x^+ - x^-}{2}\right)$

Then, $S_K = \int_C dt \left[\frac{\dot{x}^2}{2} - V(x) \right]$

$$= \int dt \left[2\dot{x}^{cl}\dot{x}^q - \{V(x^{cl}+x^q) - V(x^{cl}-x^q)\} \right]$$

$$\Rightarrow S_K = \int dt \left[2\dot{x}^{cl}\dot{x}^q - \{V(x^{cl}+x^q) - V(x^{cl}-x^q)\} \right]$$

$$\Rightarrow \boxed{S_K = \int dt \left[-2\ddot{x}^{cl} x^q + \{V(x^{cl}+x^q) - V(x^{cl}-x^q)\} \right]}$$

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where we left the boundary term $2x^q \dot{x}^{cl}\Big|_{-\infty}^{\infty}$

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$$S_K = -\int dt \left[2x^q \ddot{x}^{cl} + \{V(x^{cl}+x^q) - V(x^{cl}-x^q)\} \right]$$

If we assume x^q fluctuations to be small (???),

we can have

$$S_K = -2\int dt \left[x^q(\ddot{x}^{cl} + V'(x^{cl})) + O((x^q)^3) \right]$$

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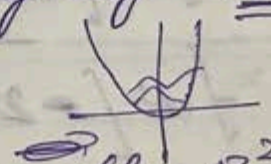
$\Big\downarrow$ integrate out x^q

$\delta(\ddot{x}^{cl} + V'(x^{cl}))$ which generates the Classical dynamical equation

$$\boxed{\ddot{x}^{cl}(t) = -V'(x^{cl})}$$

Reason why symmetric combination of fwd & bwd field is called the classical component. (nomenclature true only till 1st order in x^q, take higher orders & we shall get quantum corrections on both x^q & x^{cl}).

Equivalence of a ~~many body~~ single bosonic level with SHO



ω_0 ..... $\equiv$

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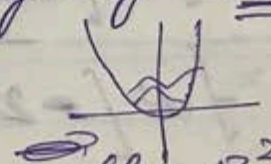

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$$S_K = \int_C dt\, \bar{\varphi}\,(i\partial_t - \omega_0)\,\varphi = \int_C dt\, \bar{\varphi}\,(i\partial_t - \omega_0)\,\varphi$$

define $\varphi = \frac{1}{\sqrt{2\omega_0}}\left(P(t) - i\omega_0 X(t)\right)$

$\bar{\varphi} = \frac{1}{\sqrt{2\omega_0}}\left(P(t) + i\omega_0 X(t)\right)$

$$S_K = \left(\frac{1}{2\omega_0}\right)\left[\int i\left(P\partial_t P + \omega_0^2 x\partial_t x\right) - \omega_0\left(P^2 + \omega_0^2 x^2\right)\right]$$

$$= \left(\frac{1}{2\omega_0}\right)\left[\int_C dt\left(i\left(P\partial_t P + \omega_0^2 x\partial_t x - i\omega_0 P\dot{x} + i\omega_0 x\dot{P}\right) - \omega_0\left(P^2 + \omega_0 x^2\right)\right.\right.$$

$$= \left(\frac{1}{2\omega_0}\right)\int_C dt\left(iP\dot{P} + i\omega_0^2 x\dot{x} + \omega_0 P\dot{x} - \omega_0 x\dot{P} - \omega_0\left(P^2 + \omega_0^2 x^2\right)\right)$$

remove $(P^2)^{\cdot}$, $(X^2)^{\cdot}$ & xP at boundary to get

$$= \int_C dt\, P\dot{x} - \frac{P^2}{2} - \frac{\omega_0^2 x^2}{2}$$

integrate out P

$$S_K = \int_C \frac{1}{2}\dot{x}^2 - \frac{\omega_0^2 x^2}{2} = S_K(\text{SHO})$$

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$$= \left(\frac{1}{2\omega_0}\right)\int_C dt\left(iP\dot{P} + i\omega_0^2 x\dot{x} + \omega_0 P\dot{x} - \omega_0 x\dot{P} - \omega_0\left(P^2 + \omega_0^2 x^2\right)\right)$$

remove $(P^2)^{\cdot}$, $(X^2)^{\cdot}$ & xP at boundary to get

$$= \int_C dt\, P\dot{x} - \frac{P^2}{2} - \frac{\omega_0^2 x^2}{2}$$

integrate out P

$$S_K = \int_C \frac{1}{2}\dot{x}^2 - \frac{\omega_0^2 x^2}{2} = S_K(\text{SHO})$$

$$S_K = (2HO) = \left(\frac{1}{2}\right)\int_{-\infty}^{\infty} \dot{x}^2 - \omega_0^2 x^2 \xrightarrow{K\text{-rot}}$$

$$\left(\frac{1}{2}\right)\int_{-\infty}^{\infty} \left(2\dot{x}^{a}\dot{x}^{a} - \omega_0^2\left(2x^{a}2x^{a}\right)\right).$$

$$S_K(HO) = \int_{-\infty}^{\infty} \left(2\dot{x}^{a}\dot{x}^{a} - 2\omega_0^2 x^{a}x^{a}\right).$$

$$= \int_{-\infty}^{\infty} \left(2\dot{x}^{a}\dot{x}^{a} - 2\omega_0^2 x^{a}x^{a}\right).$$

$$= \left(\frac{1}{2}\right)\int x^T D^{-1} x \; dt$$

ensures causality structure

$$D^{-1} = \begin{pmatrix} 0 & (D^{-1})^A \\ (D^{-1})^K & (D^{-1})^K \end{pmatrix}$$

where $\frac{1}{2}D^{-1} = \begin{pmatrix} 0 & -\partial_t^2 - \omega_0^2 \\ -\partial_t^2 - \omega_0^2 & (D^{-1})^K \end{pmatrix}$ $\quad$ s.t. $S = \left(\frac{1}{2}\right)\int x^T D^{-1} x \, dt$.

why is $(D^{-1})^K$ added here?

$\mathrm{Det}\left(\frac{D^{-1}}{2}\right) \longrightarrow 0$ with/without $(D^{-1})^K$.

& why is it so?

(1) # Also, isn't $(-\partial_t^2 - \omega_0^2)$ non-invertible, courtesy of a zero mode $e^{-i\omega_0 t}$?

Ans:- $(-\partial_t^2 - \omega_0^2)$ is a continuum discretized abbreviation of the underlying time matrix.

For e.g., in $\int dt\left\{\bar{\phi}^+ (i\partial_t - \omega_0)\phi^+ - \bar{\phi}^-(i\partial_t - \omega_0)\phi^-\right\}$,

$$G^{-1} := \begin{pmatrix} i\partial_t - \omega_0 & 0 \\ 0 & -(i\partial_t - \omega_0) \end{pmatrix}$$

& hence naively it looks as if

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But looking at the discrete matrix,

$$i\mathcal{G}^{-1} = \begin{pmatrix} \begin{matrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{matrix} & \rho \\ \rho & \begin{matrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{matrix} \end{pmatrix}_{4\times N}$$

, clearly, we see that

$\det = 1 - \rho(\omega_0)$ [as $N \to \infty$]

i.e. non-vanishing.

This also appears in,

$$\mathcal{G}^{-1}_K = \begin{pmatrix} 0 & (\mathcal{G}^A)^{-1} \\ (\mathcal{G}^R)^{-1} & [\mathcal{G}^{-1}]^K \end{pmatrix}.$$

& $(\mathcal{G}^A)^{-1}(t) = (t - \omega_0 - i\eta)$.

$(\mathcal{G}^R)^{-1}(t) = (-\omega_0 + i\eta)$

or

$$(\mathcal{G}^A)^{-1} = (i\partial_t - \omega_0 - i\eta)\,\delta(t-t').$$

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moreover, here $(\mathcal{G}^A)^{-1}$ & $(\mathcal{G}^R)^{-1}$ looks as if they are diagonal but in reality they are upper (lower) Δ^r matrices.

Again, it seems as if $\det(\mathcal{G}^{-1}_K) \to 0$

But in discrete case,

(Check the following for accuracy),

$[\mathcal{G}^A]^{-1}$ is a matrix with $+i$ along main diagonal & $-i + -\omega_0 \delta t$ along lower subdiagonal.

for $[\mathcal{G}^R]^{-1}$, $-i$ along M.D., & $+i - \omega_0 \delta t$ along upper sub diagonal.

$\therefore$ From discrete case

$$\det[(\mathcal{G}^R)^{-1}]\det[(\mathcal{G}^A)^{-1}] = \prod_N (+i)(-i) = 1$$

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$= \frac{Tr(U^\dagger O U \cdot \rho)}{Tr A} = Tr(U^\dagger O U \rho)$

In fact, ~~the~~ due to the above condition

$$Z = \int D[\bar{\psi}^a, \psi^a] \, e^{i \int \bar{\psi}^a \psi^a \, G^{-1} \begin{pmatrix} \psi^a \\ \bar{\psi}^a \end{pmatrix}}$$

$$= \frac{1}{(-i)^n \det G^{-1}}, \text{ where } n = \text{no. of fields}$$

$n=2$

$$= \frac{1}{(-)\,(-)\,\det G^{R\,-1} \times \det G^{A\,-1}} = \underline{1}$$

i.e. normalization is maintained

But the question remaining how does one evaluate $\det[G^R]^{-1} \det[G^A]^{-1}$ in the continuous picture?

$\rightarrow$ $\det([G^R]^{-1}) \det([G^A]^{-1})$

$= \prod$ eigenvalues $= \prod_{E} [G^R(E)\, G^A(E)]^{-1}$ (fishy?)

$$= \exp\left[-\int \frac{dE}{2\pi} \left\{ \ln(G^R(E)) + \ln G^A(E) \right\}\right]$$

Now, we know, $G^A(E,E) + G^R(E,E) = 0$

$$\Rightarrow \int \frac{dE}{2\pi} \left[G^A(E) + G^R(E) \right] = 0$$

Also, $G_{R_1} \circ G_{R_2} - - \circ G_{R_N} = G_R$ (i.e. a retarded propagator)

$$\Rightarrow G_R(E) = G_{R_1}(E)\, G_{R_2}(E) - \cdot G_{R_N}(E)$$

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we get $\int (G_R \circ G_R)(t) + (G_A \circ G_A)(t) \times \frac{dt}{2\pi} = 0$ — ①

(i.e. set $R_1 = R_2 = R$ & ~~rest~~ as ~~matrix~~ identity)

Similarly for higher powers. — ②

$$\therefore \int \frac{dt}{2\pi} \left[\ln G_R(t) + \ln G_A(t) \right] = 0$$

(series expand & use ① & ②).

$$\therefore \Pi_t \left[G_A^{-1}(t)\, G_R^{-1}(t) \right] = \Pi_t \left[\underbrace{G_A(t)\, G_R(t)}_{} \right]^{-1}$$

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By similar arguments, $\det \begin{pmatrix} 0 & (-\partial_t^2 - \omega^2)^A \\ (-\partial_t^2 - \omega^2)^R & [D^{-1}]^K \end{pmatrix} = (-1) \times [1]$

$= -1$

i.e. has a non-vanishing determinant.

(Answer of Qn-①)

Teacher's Signature .....

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Keldysh F.T. 1.3 continued

Komenev Ch-5 :-

Recap

many particle b. sys.,

$$Z_{SK} = \int \mathcal{D}[\varphi^{cl}, \varphi^{q}] \exp\left\{ i S_0[\varphi_{cl}, \varphi_q] \right\}$$

$$S_0 = \int dt\, dx \, (\overline{\varphi^{cl}} \;\; \overline{\varphi^{q}})_{x,t} \; \mathcal{G}^{-1}_{xx',tt'} \begin{pmatrix} \varphi^{cl} \\ \varphi^{q} \end{pmatrix}_{x',t'}$$

$$\mathcal{G}^{-1}_{xx',tt'} = \delta(x-x')\,\delta(t-t') \begin{bmatrix} 0 & i\partial_{t'} + \frac{\nabla_{x'}^2}{2m} - i\eta \\ i\partial_{t'} + \frac{\nabla_{x'}^2}{2m} + i\eta & 2i\eta F \end{bmatrix}$$

also, the position space correlator is given by

$$\langle \varphi^{\alpha}(\vec{r},t)\, \varphi^{\beta}(\vec{r}',t') \rangle = i\, \mathcal{G}_0^{\alpha\beta}(\vec{r}-\vec{r}', t-t')$$

$$= \sum_{\vec{k}} i\, \mathcal{G}_0^{\alpha\beta}(\vec{k}, t-t')\, e^{i\vec{k}(\vec{r}-\vec{r}')}$$

$$\begin{pmatrix} 0 & \epsilon - \omega + i\eta \\ \epsilon - \omega - i\eta & 2i\eta F(\omega) \end{pmatrix}$$

only for time-independent situations. (i.e. $\hat{H} \neq \hat{H}(t)$)

5.2 Interactions (2 Body) [Valid only for translation invariant physical situation

$$H_{int} = \left(\frac{1}{2}\right) \sum_{q, k, k'} U(q)\, a^{\dagger}_{k} a^{\dagger}_{k'} a_{k'+q} a_{k-q}$$

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$\Rightarrow Z_{J,\bar{J}} = \int d(\bar{\varphi},\varphi)\, e^{-\bar{\varphi}\varphi + \bar{J}\varphi + \varphi\bar{J}} = \exp(J\bar{J})$

$J, \bar{J}$ are complex

where $\mathcal{U}(\vec{q})$ is the f.t. of $\mathcal{U}(\vec{r})$.

$Z_{J,\bar{J}}\Big|_{J=0}$

$\Rightarrow$ For dilute atomic gas, assume a short contact potential i.e. $\mathcal{U}(\vec{r}) \sim \delta(\vec{r})$.

$\Rightarrow \mathcal{U}(\vec{q}) = g = \text{const.}$ $\left[g \rightarrow \text{measure it from s-wave scattering length. } g = \frac{4\pi a_s}{m} \right]$

$$S_{int} = \left(\frac{-g}{2}\right) \sum_{\vec{q},\vec{k},\vec{k}'} \int_C dt\, \bar{\varphi}_{\vec{k}}(t)\, \bar{\varphi}_{\vec{k}'}(t)\, \varphi_{\vec{k}'-\vec{q}}(t)\, \varphi_{\vec{k}+\vec{q}}(t).$$

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Keldysh rotation.

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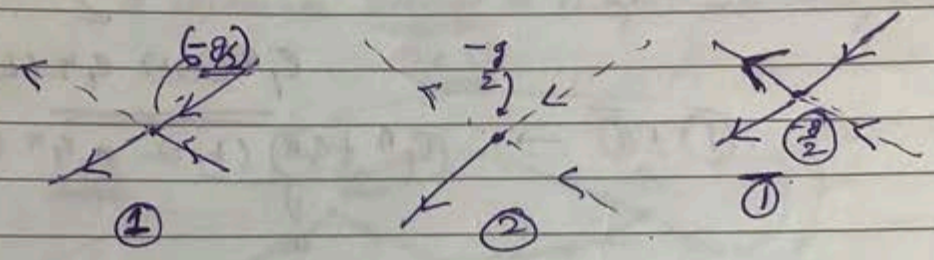
~~Check~~ Causality check :- $S[\varphi^{cl}, \varphi^{q}=0] = 0$

$S_{int}[\varphi^{cl}, \varphi^{q}=0] = 0$

2 types of vertices :-



→ interactions switched. Switched on adiabatically in fwd & ~~off~~ switched off correspondingly in bwd direction.

Claim :- $Z = 1$ even after interactions are added.

Pf :- ~~follow~~ → idea :- Show that corrections to Z are 0, when viewed perturbatively.

$$Z = \int \mathcal{D}[\varphi^{cl}, \varphi^{q}] \exp\{iS[\varphi^{cl}, \varphi^{q}] + iS_{int}\}$$

1st order :-

$$\langle S_{int} \rangle = \left(-\frac{g}{2}\right)\left[① + ② + \bar{①} + \bar{②}\right]$$

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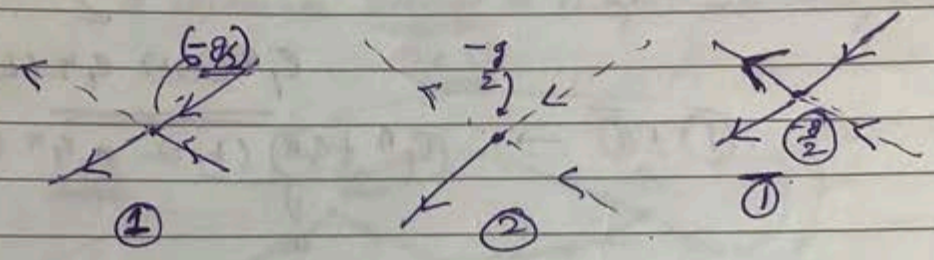

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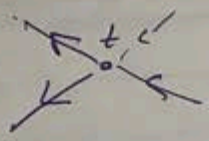
① →

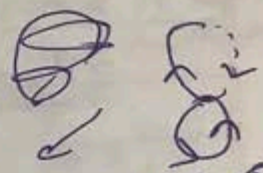
$\xrightarrow[\text{Thm.}]{\text{coic.}}$

$G_A(t,t)\, G_K(t,t) \times$ ② → diag

Ī →



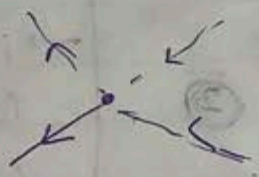
$\xrightarrow{\text{w. Thm.}}$



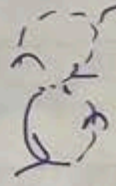
$G_R(t,t)\, G_K(t,t) \times$ ②

$\therefore$ ① + Ī → $\underbrace{(G_A + G_R)(t,t)}_{\overset{!!}{0}} \times G_K(t,t) \times 2$

② →



→



$G_{R,A} = 0$

Ī →

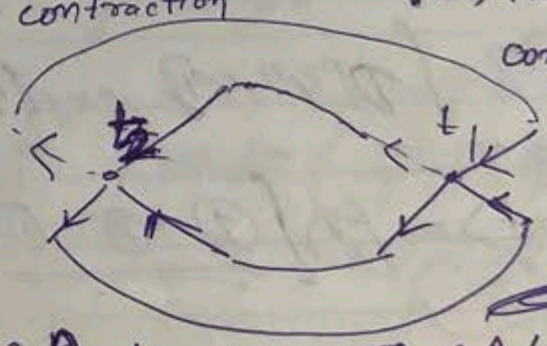
→ 0 $\quad (G_{q,q} = 0)$

2nd order

$$\langle S_{int} \times S_{int} \rangle = \langle (1+2+\bar{1}+\bar{2})\,(1+2+\bar{1}+\bar{2}) \rangle$$

X → cross contraction $\qquad$ • → independent contraction.

1×1 →



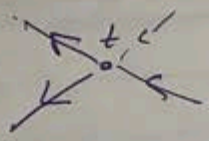
$G_A(t_1,t_2) \times G_A(t_2,t_1) \times G_K(t_1,t_2)$
$G_K(t_2,t_1)$

① →

$\xrightarrow[\text{Thm.}]{\text{coic.}}$

$G_A(t,t)\, G_K(t,t) \times$ ② → diag

Ī →



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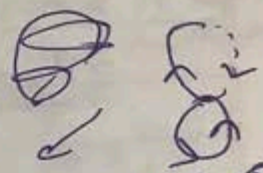

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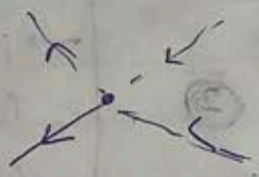

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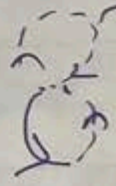

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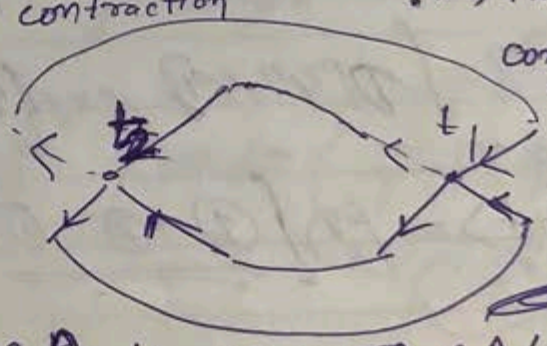

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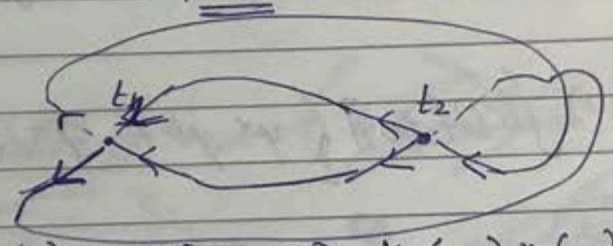
again,

$1\times 1 \rightarrow \underbrace{G^A(t_1,t_2)\ G^A(t_2,t_1)}_{\parallel} \times (\ ) \times (\ )$

non-zero only for $t_1 = t_2$

a measure zero set on t_1, t_2 plane & integration is 0.

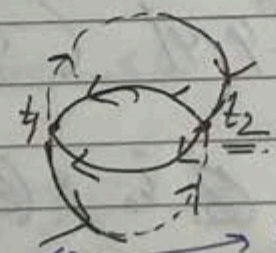
$1\times\bar{1} \rightarrow$



$\underbrace{G^A(t_1,t_2) \times G^R(t_1,t_2)}_{\parallel} \times (\ ) \times (\ ) \longrightarrow 0$

$\int \longrightarrow 0$

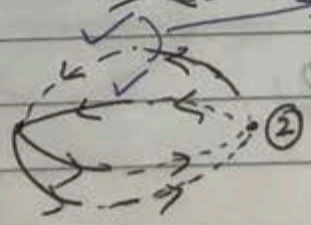
Similarly for all other higher order terms in g. (i.e. the vanishing behaviour).



$\rightarrow G^A G^R \rightarrow 0$

other diagrams

$\bar{1}$



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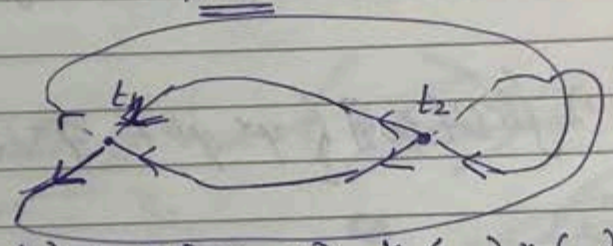

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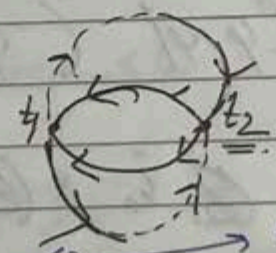

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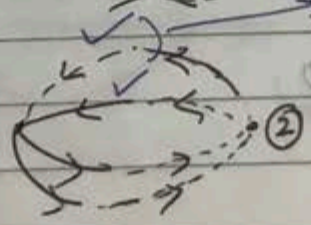

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$$= \frac{\partial^{n+n'} Z_{int}}{\partial J^n \partial \bar{J}^{n'}}\Big|_{J=0}$$

Dyson Equations:-

→ describes how the Green's function evolve in an interacting environment.

$$\Rightarrow \varphi^{\alpha\beta}(\vec{x},t,\vec{x}',t') = (-i)\int D[\bar{\varphi}\varphi]\, \varphi^{\alpha}(\vec{x},t)\, \bar{\varphi}^{\beta}(\vec{x}',t')\, e^{i(S+S_{int})}$$

we then expand perturbatively, ~~to get~~

$$(-i)\int D[\bar{\varphi},\varphi]\left\{ \varphi^{\alpha}\bar{\varphi}^{\beta} + \varphi^{\alpha}\, iS_{int}\, \bar{\varphi}^{\beta} + \varphi^{\alpha}\frac{(-S_{int})^2}{2!}\bar{\varphi}^{\beta} \right\} e^{iS_0}$$

Now,

$$(-i)\int D(\bar{\varphi},\varphi)\left[\varphi^{\alpha}\bar{\varphi}^{\beta}\right] e^{iS_0} = \hat{\xi}_0$$

1st order term

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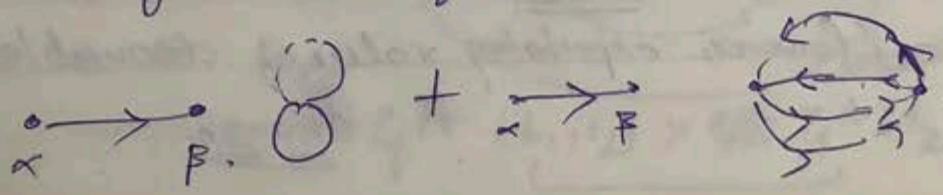
$$= \hat{\xi}_0 \times \underbrace{(\text{individual/disconnected diagrams})}_{=0}$$

$= 0$ (from $Z_{int} = Z_0$, we know that disconnected corrections are exactly 0).

diagrams are here of the form



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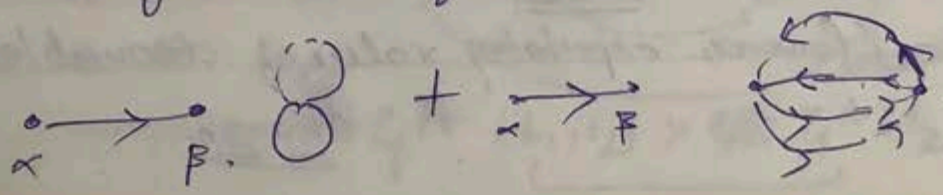

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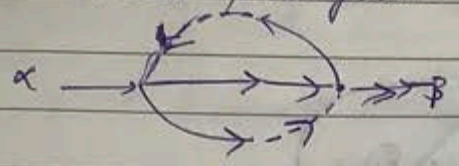
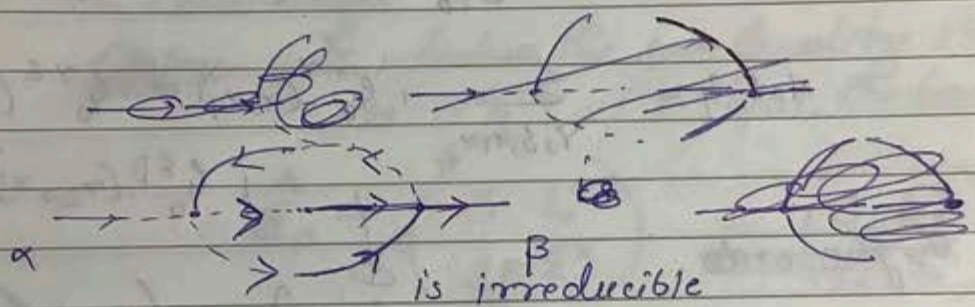
∴ We focus on diagrams of the form



i.e. of the form $\psi^{\alpha} \times \sin t \times \bar{\psi}^{\beta}$

Such a diagram that can't be cut into 2 diagrams by a single line, is termed as an Irreducible diagram.

For e.g.:



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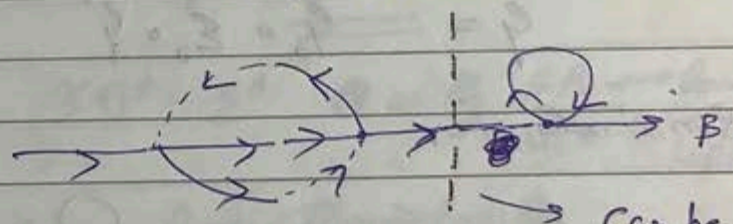
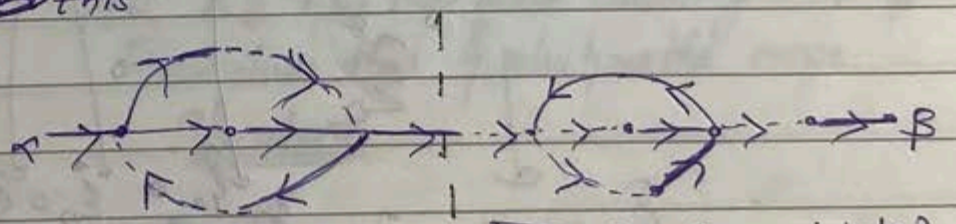


Can be cut into 2 by a line.

isn't irreducible.

So isn't this



can be cut into 2 parts by a line.

Therefore we sum all such irreducible diagrams & call it self energy Σ.

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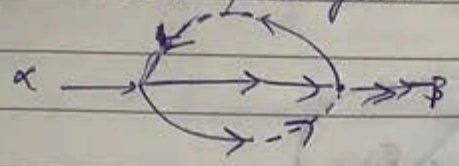

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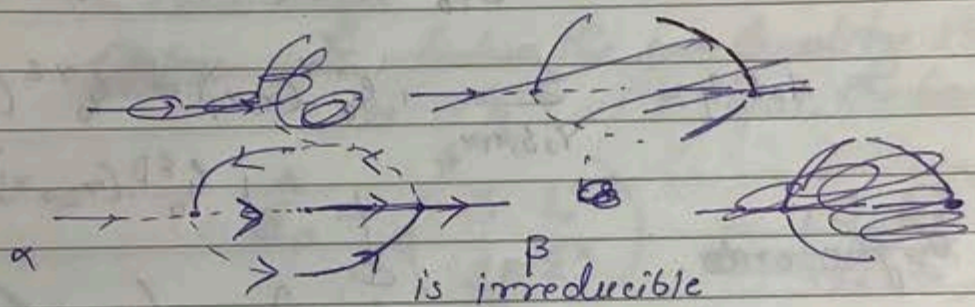

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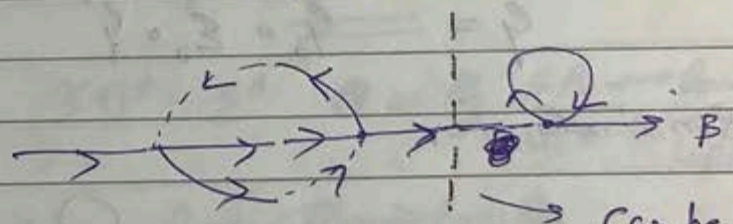

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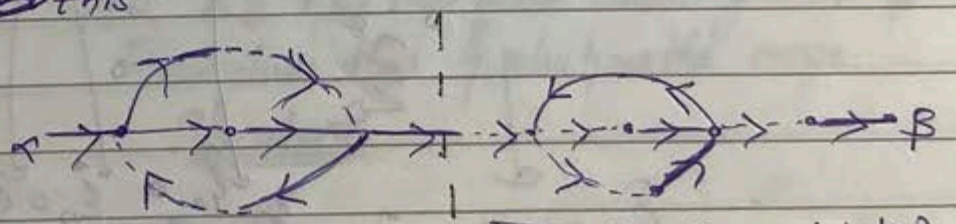

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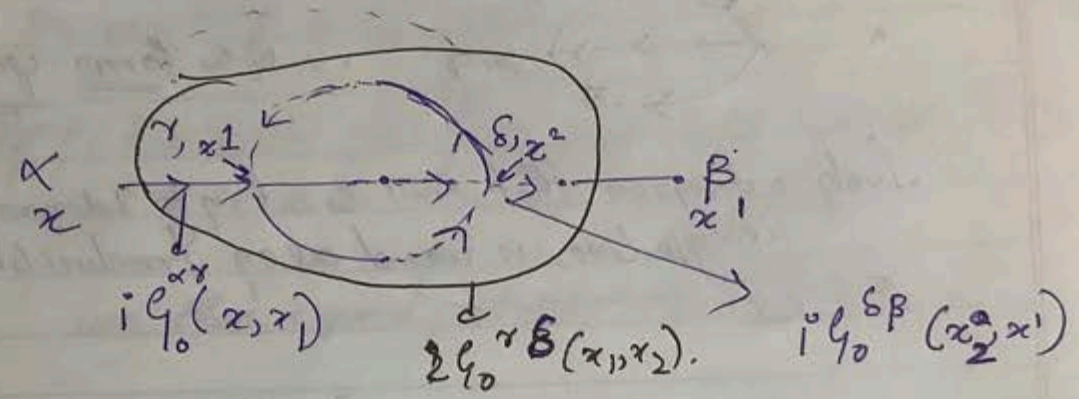

$$\therefore \text{Net } \mathcal{G} = \sum_{\gamma, \delta, x_1, x_2} i\,\mathcal{G}_0^{\alpha\gamma}(x, x_1)\; i\,\mathcal{G}_0^{\gamma\delta}(x_1, x_2)\; i\,\mathcal{G}_0^{\delta\beta}(x_2, x').$$

$\therefore$ ~~Higher order~~

$$\mathcal{G} = \mathcal{G}_0 \circ \hat{\xi}_0 \circ \hat{\mathcal{G}} \qquad \left(\circ \to \sum_{\alpha, \delta, x_1, x_2}\right).$$

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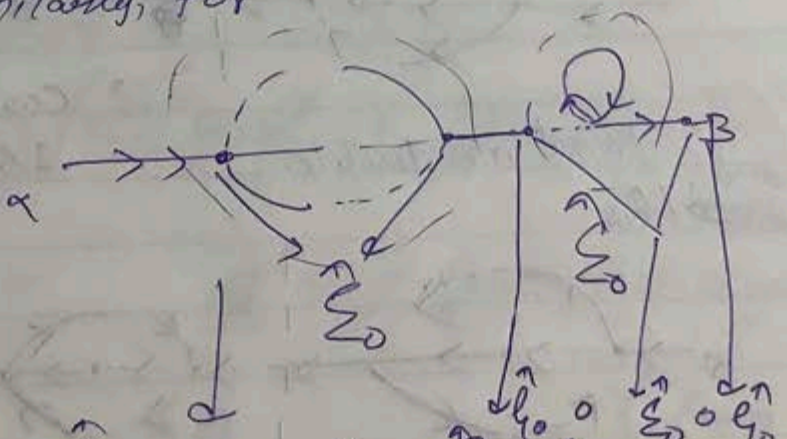

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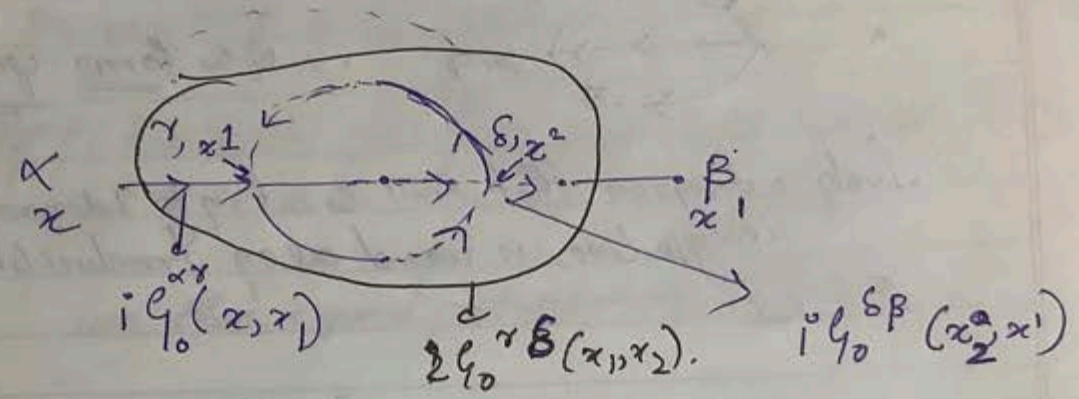

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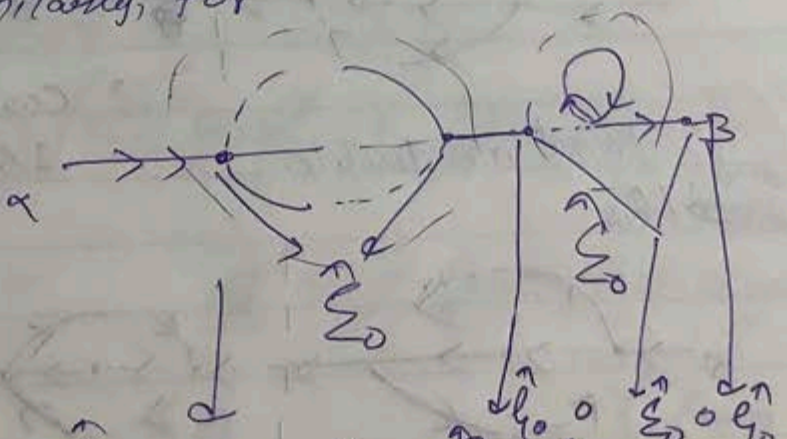

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$$\boxed{\Rightarrow \hat{G} = G_0 + G_0 \circ \Sigma_0 \circ G}$$

$\circ \rightarrow$ sum over Keldysh indices + other Lorentz spin indices (i.e. p, $\vec{r}$, σ etc.) + t

$$\Rightarrow \boxed{(G_0^{-1} - \Sigma_0) \circ G = 1} \quad \text{(Dyson's eqn)}$$

~~Non trivial feature~~

Claim:- $\hat{\Sigma}_0$ features the same Causality structure as G_0^{-1}. i.e. has the form.

$$\hat{\Sigma}_0 = \begin{pmatrix} 0 & \Sigma_0^A \\ \Sigma_0^R & \Sigma_0^K \end{pmatrix} = \begin{pmatrix} 0 & \Sigma_0^A \\ \Sigma_0^R & \Sigma_0^K \end{pmatrix}$$

with $(\Sigma^R)^\dagger = \Sigma^A$ with $\Sigma^R \rightarrow$ lower Δ^r
$\Sigma^A \rightarrow$ upper Δ^r

& $\Sigma^K \rightarrow$ Anti-Hermitian

$\because$ G^{-1} & Σ has the same Causality str, we conclude that $\hat{G}$ also has the same Structure.

Dyson eqn now becomes.

$$\begin{pmatrix} 0 & G_0^{A^{-1}} - \Sigma^A \\ (G_0^R)^{-1} - \Sigma^R & -\Sigma^K \end{pmatrix} \circ \begin{pmatrix} G^K & G^R \\ G^A & 0 \end{pmatrix} = \mathbb{1}.$$

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$\underline{\underline{A}}$ where we neglected $2\eta F$ term from $= \frac{\partial^{n+1} \Sigma_{ij}}{\partial (J)^{n+1}}\Big|_{j=0}$

we have equations as follows:-

$$\cancel{(G_0^R)^{-1} - \Sigma_0^R}$$

$$\left\{ \left[G_0^{R(A)} \right]^{-1} - \left(\Sigma_0^{R(A)} \right) \right\} \circ G^{R(A)}(x,x') = \underline{\underline{\delta(t-t')\,\delta(x-x')}}$$

$$\Rightarrow \left(i\partial_t + \frac{\nabla_r^2}{2m} - \Sigma_0^{R(A)} \right) \circ G^{R(A)}(x,x') = \delta(t-t')\,\delta(x-x')$$

$$\Big\downarrow f.t.$$

$$G^{R(A)}[\vec{k},\epsilon] = \frac{1}{\epsilon - \frac{k^2}{2m} - \Sigma_0^{R(A)}(\vec{k},\epsilon)}$$

Now, $\left[\Sigma_0^R(x,x')\right]^{\dagger} = \left[\Sigma^A\right](x,x')$

$$\Rightarrow \boxed{\mathrm{Re}\left[\Sigma_0^R\right](k,\epsilon) = \mathrm{Re}\left[\Sigma^A\right](\vec{k},\epsilon)}$$

$$\& \boxed{\mathrm{Im}\left[\Sigma_0^R\right](k,\epsilon) = -\mathrm{Im}\left[\Sigma_0^A\right](\vec{k},\epsilon)}$$

$\underline{\underline{Q.}}$ Physical significance of $\mathrm{Re}\left[\Sigma^{R(A)}\right](\vec{k},\epsilon)$ & $\mathrm{Im}\left(\Sigma^{R(A)}\right](\vec{k},\epsilon)$

① $\underline{\underline{\text{Real part}}}$:- Gives a renormalized/dressed version of the ~~energy~~ dispersion relation.

i.e. $\epsilon = \frac{k^2}{2m} \xrightarrow[\text{to}]{\text{modifies}} \underline{\epsilon - \frac{k^2}{2m} - \Sigma^{R(A)}(\vec{k},\epsilon) = 0}$

$\underline{\underline{(\text{new disp rel.})}}$

$\underline{\underline{A}}$ where we neglected $2\eta F$ term from $= \frac{\partial^{n+1} \Sigma_{ij}}{\partial (J)^{n+1}}\Big|_{j=0}$

we have equations as follows:-

$$\cancel{(G_0^R)^{-1} - \Sigma_0^R}$$

$$\left\{ \left[G_0^{R(A)} \right]^{-1} - \left(\Sigma_0^{R(A)} \right) \right\} \circ G^{R(A)}(x,x') = \underline{\underline{\delta(t-t')\,\delta(x-x')}}$$

$$\Rightarrow \left(i\partial_t + \frac{\nabla_r^2}{2m} - \Sigma_0^{R(A)} \right) \circ G^{R(A)}(x,x') = \delta(t-t')\,\delta(x-x')$$

$$\Big\downarrow f.t.$$

$$G^{R(A)}[\vec{k},\epsilon] = \frac{1}{\epsilon - \frac{k^2}{2m} - \Sigma_0^{R(A)}(\vec{k},\epsilon)}$$

Now, $\left[\Sigma_0^R(x,x')\right]^{\dagger} = \left[\Sigma^A\right](x,x')$

$$\Rightarrow \boxed{\mathrm{Re}\left[\Sigma_0^R\right](k,\epsilon) = \mathrm{Re}\left[\Sigma^A\right](\vec{k},\epsilon)}$$

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$\int d(\bar{\phi},\phi)\, e^{-|\phi|^2} (\bar{\phi})^n (\phi)^{n'} = \delta^{n+n'}\, 2_{n}\, n!$

② Imaginary part :-

$\mathrm{Im}(\Sigma_0)$ has meaning of INVERSE LIFETIME (τ^{-1}) a particle spends in a given (renormalized) eigenstate $\vec{k}$ of the non-interacting system. (???) (flow?)

G^K in the dyson expansion :-

$$G^K = G^R \circ F - F \circ G^A \quad [\text{Causal Structure}]$$

~~we have~~ From dyson eqn.

$$\left([G_0^R]^{-1} - \Sigma^R\right)\left[G^R \circ F - F \circ G^A\right] = \Sigma^K \circ G^A$$

$$\Rightarrow \Sigma^K = \left([G_0^R]^{-1} - \Sigma^R\right)\left[G^R \circ F - F \circ G^A\right]\left[(G_0^A)^{-1} - \Sigma^A\right]$$

$$\Rightarrow \Sigma^K = F \circ \left((G_0^A)^{-1} - \Sigma^A\right) - \left((G_0^R)^{-1} - \Sigma^R\right) \circ F$$

$$\Rightarrow \Sigma^K - \left[\Sigma^R \circ F - F \circ \Sigma^A\right] = -\left[(G_0^R)^{-1} \circ F - F \circ (G_0^A)^{-1}\right]$$

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we put in the explicit form of $(G_0^R)^{-1}$ & $(G_0^A)^{-1}$, ignoring in it any.

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$$\Rightarrow -\left[\left(i\partial_t + \frac{\nabla_x^2}{2m} - V^{(1)}(x)\right); F\right] = \Sigma_c^K - (\Sigma^R \circ F - F \circ \Sigma^A)$$

Quantum Kinetic equation for distribution matrix $F(x,x')$

Kinetic term.

Collision term

or ~~Collision~~ collision integral

~~LHS vani~~

At eqm, LHS vanishes

$$\Rightarrow \boxed{\Sigma^K = \Sigma^R \circ F - F \circ \Sigma^A} \quad (\text{at eqm}).$$

Real Bosonic field :-

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Real Bosonic field :-

$\# \int d(\bar{\phi},\phi)\, e^{-|\phi|^2} (\bar{\phi})^n (\phi)^{n'} = \partial^{n+n'} \Sigma_{n,n'}$

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Wigner transform

→ To avoid the separation b/w intrinsic microscopic scale (i.e. time, length of the system's inner dynamics) & extrinsic scales (i.e. scale/time associated with external perturbation).

⇒ In the latter is macroscopic/mesoscopic, one can simplify Kinetic theory. i.e. much larger than the internal/relative time scale.

$p = (k^m, \omega)$ $\quad x = (\vec{r}^m, t)$

$px := k^m r^m - \omega t$

Let $A = A(x_1, x_2)$. let $x = \left(\frac{x_1+x_2}{2}\right)$ $\quad x' = x_1 - x_2$.

∴ W.T. of $A(x_1, x_2)$'s defined as,

$$A(x,p) = \int dx'\, e^{-ipx'} A\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right).$$

~~$A^+(x_1,x_2) = \int dx'$~~ W.T. →

W.T. of $A^+(x_1, x_2)$:-

$$A^+(x_1,x_2) = [A(x_2,x_1)]^*$$

W.T.

$[A(x,p)]^*$

$$\int dx'\, e^{+ipx'} A^*\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right) \xleftarrow{x' \to -x'} \int dx'\, e^{-ipx'} A^*\left(x-\frac{x'}{2}, x+\frac{x'}{2}\right)$$

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$$A^{\dagger}(x_1,x_2) \xrightarrow{W.T.} A'(x,p) = \int_{x'} e^{-ipx'} \hat{A}\left(x-\frac{x'}{2}, x+\frac{x'}{2}\right)$$

$$\Big\downarrow x' \to -x'$$

$$\underline{\underline{A(x,p)^{*}}} \longleftarrow \int_{x'} e^{-ipx'} A^{*}\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right)$$

<u>2pnt</u> <u>expansion</u>

$$\cancel{C} = A \circ B \Rightarrow \underline{\underline{C = A \circ B}}$$

$$C(x_1,x_2) = \int dx_3\, A(x_1,x_3)\, B(x_3,x_2)$$

$$C(x,p) = \cancel{\int dx\, e^{-ipx}} \int_{x^3,x'} e^{-ipx'} \underbrace{A\left(x+\frac{x'}{2}, x_3\right) B\left(x_3, x-\frac{x'}{2}\right)}_{\text{Fn. do conv. W.T. of } A \,\&\, B \to 0}.$$

Fn. W.T.

$$A(x_1,x_2) \underset{FWT}{\overset{WT.}{\rightleftarrows}} A(x,p)$$

$$A(x_1,x_2) = \sum_{p} e^{ip(x_1-x_2)} A\left(\frac{x_1+x_2}{2}, p\right)$$

$$C(x,p) = \int_{x^3,x'} e^{-ipx'} \sum_{p_1,p_2} e^{ip_1\left(x+\frac{x'}{2}-x_3\right)+ip_2\left(x_3-x-\frac{x'}{2}\right)} \times$$

$$A\left(\frac{x+\frac{x'}{2}+x_3}{2}, p_1\right) B\left(\frac{x_3+x-\frac{x'}{2}}{2}, p_2\right)$$

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$$C(x,p) = \int_{x^3,x'} \sum_{p_1,p_2} \exp\left[i p_1 \left(\frac{x+x'}{2} - x_3\right) + i p_2 \left(x_3 - \frac{x+x'}{2}\right)\right] \times e^{-ipx'}$$

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$x_{a,b} = \cancel{x+x_3+x'}$ $\quad p_{a,b} = p_{1,2}$

$x' = x_1 - x_2$

~~$C(x,p)$~~

$$C(x,p) = \int_{x^3,x'} \sum_{p_1,p_2} \exp\left[i p_1 \left(\frac{x+x'}{2} - x_3\right) + i p_2 \left(x_3 - x + \frac{x'}{2}\right)\right].$$

$$\exp[-i p x'] A\left(\frac{x+\frac{x'}{2}+x_3}{2}, p_1\right) \times$$

$$B\left(\frac{x_3+x-\frac{x'}{2}}{2}, p_2\right).$$

Change of coordinates

$x_{a,b} = \left(x_3 - x \pm \frac{x'}{2}\right)$

$p_a, p_b = p_{1,2} - p$

Check $|J| = 2$

$$C(x,p) = \int_{x_a,x_b} \sum_{p_a,p_b} \exp[i(p_b x_a - p_a x_b)]$$

$$A\left(x+\frac{x_a}{2}, p+p_a\right) B\left(x+\frac{x_b}{2}, p+p_b\right).$$

expand in taylor series of momenta p_a, p_b

$$C(x,p) = A(x,p) \exp\left[\frac{i}{2}\left(\overleftarrow{\partial_x}\overrightarrow{\partial_p} - \overleftarrow{\partial_p}\overrightarrow{\partial_x}\right)\right] B(x,p)$$

Teacher's Signature .....

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Teacher's Signature .....

where $\partial_x \partial_p = \nabla_{x'} \nabla_x - \partial_t \partial_\epsilon$

$$A\left(x+\frac{x_a}{2}, p+p_a\right) = e^{p_a \nabla} A\left(x+\frac{x_a}{2}, p\right).$$

$$\int_{x_a, x_b} \sum_{p_a, p_b} e^{-ip_a(x_b)} e^{p_a \nabla_p} A\left(x+\frac{x_a}{2}, p\right).$$

$$= \int_{x_a, x_b} \sum_{p_a, p_b} e^{-ip_a(x_b + i\nabla_p)} A(x+x_a, p).$$

$$\int_{x_a, x_b} \delta(x_b + i\nabla_p) \; A(x+x_a, p)$$

$$\sum_{p_b} \frac{\delta^{(n)}(x_b)}{n!} \left\{ (i\nabla_p)^n \; A(x+x_a, p) \right\}$$

$$\int dx_b \, dx_a \sum_{n=0, p_b}^{\infty} \frac{\delta^{(n)}(x_b)}{n!} (i)^n \left\{ \nabla_p^n A(x+x_a, p) \right\}$$

$$e^{ip_b x_a} B\left(x+\frac{x_b}{2}, p+p_b\right)$$

$$\int B\left(x+\frac{x_b}{2}\right) \delta(x_b) \, dx_b = B(x).$$

Goal:- $\int \delta^{(n)}(x_b) \; B\left(x+\frac{x_b}{2}, p+p_b\right) dx_b = ?$

$$\int \delta(x_b - \epsilon) \; B\left(x+\frac{x_b}{2}, p+p_b\right) dx_b$$

$$= B\left(x+\frac{\epsilon}{2}, p+p_b\right).$$

$$\Rightarrow \int_{x_b} (-1)^n \, \delta^{(n)}(x_b - \epsilon) \; B\left(x+\frac{x_b}{2}, p+p_b\right) dx_b$$

$$= (\partial_\epsilon^n) \; B\left(x+\frac{\epsilon}{2}, p+p_b\right)$$

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1.3 – 3

$$\int_{x_b} \delta^n(x_b)\, B\left(x+\frac{x_b}{2}, p+p_b\right) = \frac{(-1)^n}{2^n\, n!} \nabla_x^n B(x,p+p_b)$$

$$\int_{x_a} \sum_{p_b} e^{ip_b x_a} \frac{(i/2)^n}{n!} \left[\nabla_p^n A(x+x_a, p)\right] (-1)^n \frac{n!}{2^n} \nabla_x^n B(x, p+p_b)$$

$$= \int_{x_a} \sum_{p_b, n} (-1)^n \left[\nabla_p^n A(x+x_a), p\right] \underbrace{e^{ip_b x_a}\, e^{p_b \nabla_p}}_{\delta(x_a - i\nabla_p)} \nabla_x^n B(x,p)$$

$$\delta^m(x_a)\,(-1)^m \nabla_p^m \nabla_x^n B(x,p)$$

$$= \sum_{n,m} \nabla_x^m \nabla_p^n A(x,p) \quad \nabla_p^m \nabla_x^n B(x,p).$$

$$C(x,p) = A(x,p) \exp\left[\left(\frac{i}{2}\right)\left(\overleftarrow{\partial_x}\, \overrightarrow{\partial_p} - \overleftarrow{\partial_p}\, \overrightarrow{\partial_x}\right)\right] B(x,p).$$

lowest order.

$$A(x,p)\, B(x,p) + \left(\frac{i}{2}\right)\left\{\partial_x A(x,p)\, \partial_p B(x,p) - \partial_p A\, \partial_x B\right\} - \dots$$

$$\left(\frac{i}{2}\right)\left\{\left\{\left(\nabla_x^n A\, \nabla_p^n B(x,p)\right) - \partial_x A\, \partial_x B\right\} - \left\{\left(\nabla_x^n A\, \nabla_p^n B\right) - (\partial_x A)\, \partial_p B\right\}\right\}$$

→ Why is this useful?

Although the expression

$$C(x,p) = A(x,p) \exp\left[\frac{i}{2}\left(\overleftarrow{\partial_x}\, \overrightarrow{\partial_p} - \overleftarrow{\partial_p}\, \overrightarrow{\partial_x}\right)\right] B(x,p).$$

is formally exact; one central coordinate ~~should~~ gains vital info when lower order terms can be kept

1.3 – 3

$$\int_{x_b} \delta^n(x_b)\, B\left(x+\frac{x_b}{2}, p+p_b\right) = \frac{(-1)^n}{2^n\, n!} \nabla_x^n B(x,p+p_b)$$

$$\int_{x_a} \sum_{p_b} e^{ip_b x_a} \frac{(i/2)^n}{n!} \left[\nabla_p^n A(x+x_a, p)\right] (-1)^n \frac{n!}{2^n} \nabla_x^n B(x, p+p_b)$$

$$= \int_{x_a} \sum_{p_b, n} (-1)^n \left[\nabla_p^n A(x+x_a), p\right] \underbrace{e^{ip_b x_a}\, e^{p_b \nabla_p}}_{\delta(x_a - i\nabla_p)} \nabla_x^n B(x,p)$$

$$\delta^m(x_a)\,(-1)^m \nabla_p^m \nabla_x^n B(x,p)$$

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$$\left(\frac{i}{2}\right)\left\{\left\{\left(\nabla_x^n A\, \nabla_p^n B(x,p)\right) - \partial_x A\, \partial_x B\right\} - \left\{\left(\nabla_x^n A\, \nabla_p^n B\right) - (\partial_x A)\, \partial_p B\right\}\right\}$$

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many particle system

$\Rightarrow$ [$\pi r(x_1, x_2)$] exp[$i S_0[x_1, x_2]$]

→ flow: Suppose we explore the limit where ∂_x & ∂_p maybe regarded as small. i.e. $\delta x \, \delta p \gg 1$.

where δx & δp are scales over which SIGNIFICANT VARIATIONS TAKE PLACE, then

Here, we can work with only the lower order terms which is still a legitimate procedure.

→ physically this would mean having a function that relatively slow in the central coordinate & ~~relatively~~ faster in relative coordinate (i.e. $x_1 - x_2$)

extreme example:-

Consider a function ~~of the dep~~ $A(x_1, x_2) = A(x_1 - x_2)$.

Here $\partial_x = 0 \Rightarrow$ only 0th order survives.

$$\Rightarrow C(p) = A(p)\, B(p).$$

→ ~~Commutator~~ Commutator modification.

convolutⁿ

$$[A \overset{\circ}{,} B] = i\Big[\underbrace{\partial_x A \, \partial_p B - \partial_p A \, \partial_x B}_{\text{Poisson bracket}}\Big]. \quad (1)$$

[in fact curious to see the principle "replace ~~commutator~~ P. bracket by commutator" becoming an actual reality in eqⁿ (1)].

many particle system

$\Rightarrow$ [$\pi r(x_1, x_2)$] exp[$i S_0[x_1, x_2]$]

→ flow: Suppose we explore the limit where ∂_x & ∂_p maybe regarded as small. i.e. $\delta x \, \delta p \gg 1$.

where δx & δp are scales over which SIGNIFICANT VARIATIONS TAKE PLACE, then

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Expt. No.

Kinetic Term :-

The Kinetic equation for $\mathbb{F}$ is given by

$$[(2\partial_t^2 + 2\kappa \nabla_k^2 + 2\omega_0^2) ; \mathbb{F}] = \Sigma^{K-} (\Sigma^R \circ \mathbb{F} - \mathbb{F} \circ \Sigma^A) \quad \text{(roll b.F.)}$$

& for complex free field with $(\bar{\varphi}\varphi)^2$ interaction

$$-\left[i\partial_t + \frac{\nabla_k^2}{2m} - V^{cl}(x) ; \mathbb{F}\right] = \underbrace{\Sigma^{K-} (\Sigma^R \circ \mathbb{F} - \mathbb{F} \circ \Sigma^A)}_{\text{captures the interact}^n}$$

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① we perform a W.T. expansion on LHS

(a) $V^{cl}(x) \xrightarrow{W.T.} V^{cl}(x)$

$$\Rightarrow [V^{cl} ; \mathbb{F}] = (2i)\left[\partial_x V^{cl} \partial_p \mathbb{F}\right] \{p \text{ indpt } V^{cl}\}$$

$$= i\left[\nabla_{x^m} V^{cl} \nabla_{k^m} \mathbb{F} - \partial_t V^{cl} \partial_\epsilon \mathbb{F}\right]$$

(b) $i\partial_t \rightarrow$ translat^n invariant operator

$$i\partial_t \xrightarrow{W.T.} \epsilon$$

$$[i\partial_t ; \mathbb{F}] \xrightarrow{W.T.} -2i\left[\partial_p(\epsilon)\, \partial_x \mathbb{F}\right]$$

$$= (-2i) \times 0 \quad \partial_t \mathbb{F} = i\partial_t \mathbb{F}$$

Similarly,

$$-\partial_t^2 \xrightarrow{W.T.} \epsilon^2$$

$$[-\partial_t^2 ; \mathbb{F}] \xrightarrow{W.T.} -i\left(\partial_p \epsilon^2 \partial_x \mathbb{F}\right)$$

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Teacher's Signature .....

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Teacher's Signature .....

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$$\Sigma - (\Sigma^R F - F \cdot \Sigma^A) \xrightarrow{W.T.} \cancel{\left(\Sigma^R F - F\Sigma^A\right)}$$

$$(\Sigma^A)^+ = \Sigma^R \quad \& \quad A^+ \xrightarrow{W.T.} A(x,p)^* \text{ if } A \xrightarrow{W.T.} A(x,p)$$

$$\Sigma_c^R \cdot F - F \cdot \Sigma_c^A \xrightarrow{W.T.} \left(\Sigma_c^R F - F\Sigma_c^A\right) +$$

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$$F\left[\Sigma^R - \Sigma^A\right] +$$

assume lower order expansion.

$$\left(\Sigma^R F - F\Sigma^A\right) + \frac{i}{2}\left[\partial_x \Sigma^R \partial_p F - \partial_p \Sigma^R \partial_x F\right]$$

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$$\Sigma_c^A = (\Sigma^R)^*$$

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Kinetic Term :-

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~~Simplifying~~, Simplifying,

$$\cancel{-\left[\hbar\partial_t - i\nabla_{\vec{p}}V^{cl}\nabla_{\vec{x}} + i\partial_t V^{cl}\partial_{\vec{p}} + 2i\vec{p}\,\nabla_{\vec{p}}\right]} \quad \mathbb{F} = \underbrace{I_{coll}[\mathbb{F}]}_{\cancel{\text{Collision integral}}}$$

Shifting all the $\mathbb{F}$ derivatives to lhs

$$(1 - \partial_t \mathrm{Re}\,\Sigma^R)\partial_t + (\partial_t \tilde{V})\partial_t + \tilde{v}_{\vec{p}}$$

$$\left[(1-\partial_t \mathrm{Re}\,\Sigma^R)\partial_t + (\partial_t\tilde{V})\partial_t + \tilde{v}_k\nabla_{\vec{p}} - (\nabla_{\vec{p}}\tilde{V})\nabla_{\vec{x}}\right]\mathbb{F}$$

$$= I^{coll}(\mathbb{F}).$$

where $\tilde{V} = V^{cl}(x) + \mathrm{Re}[\Sigma^R(x,p)]$

$\tilde{v}_k = \nabla_{\vec{p}}[\omega_{\vec{p}} + \mathrm{Re}\,\Sigma^R]$

$$I^{coll}[\mathbb{F}] = i\Sigma^K(x,p) + 2\mathbb{F}(x,p)\,\mathrm{Im}[\Sigma^R(x,p)]$$

→ At eqm, $\partial_t = 0$, & $\mathbb{F} = \mathbb{F}(\epsilon)$ only (i.e. of f^n of energies only), LHS vanishes.

RHS ~~vanishes~~ can vanish for

$$F(\epsilon) = \coth\left(\frac{\epsilon-\mu}{2k_BT}\right).$$

* This is called as equilibrium sol^n.

For a general dispersion relation, $\vec{p}^2 \to k^2$ is replaced with

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Signature .....

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Kinetic Term :-

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~~Simplifying~~, Simplifying,

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Signature .....

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Kinetic Term :-

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Keldysh 1.4

Some properties of Quantum Green's functions

① GF in QMP:-

$$G(r^0, t^0, r^1, t^1) = \langle r^0, t^0 | r^1, t^1 \rangle = \langle r^0 | e^{-iH(t-t^1)} | r^1 \rangle$$

relation to poles (looking at single particle $\mathcal{H}$)

$$G(r^0, t^0, r^1, t^1) = \sum_n \langle r^0 | n \rangle \langle n | r^1 \rangle e^{-iH(t-t^1)}$$

$$G = \sum_n \varphi_n(r^0) \varphi_n^*(r^1) e^{-iE(t-t^1)}$$

$$= \sum_n \varphi_n(r^0) \varphi_n^*(r^1) e^{-iE_n(t-t^1)}$$

f.t.

$$G(r^0, r^1, \omega) = \sum_n i \frac{\varphi_n(r^0) \varphi_n^*(r^1)}{(E - E_n)}$$

Poles at Eigen energies when looked in frequency domain.

② GF in QFT:-

Broadly looks at the propagation amplitude by creating a 1-particle excitation, allowing it to propagate (aha!) & then annihilating at a certain site.

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Keldysh 1.4

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Teacher's Signature

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avg is either over G.S. (interacting & $T\to 0$ limit)
or over an ensemble (hence Temp dependent)

$T \to$ Causal

$$G(\vec{r},t,\vec{r}',t') = \theta(t-t')\langle\psi(\vec{r},t)\,\psi^{+}(\vec{r}',t')\rangle$$
$$- \theta(t'-t)\langle\psi^{+}(\vec{r}',t')\,\psi(\vec{r},t)\rangle$$

create a particle at $\vec{r}',t'$, allow propagation till $\vec{r},t$ & then annihilate

same but now with anti-particle

GF for electrons

$$G^{c}_{ij}(t,t') = -i\langle T[c_i(t)\,c_j^{+}(t')]\rangle$$, i & j are sites

now define Two more G.F.

$$G^{R}_{ij}(t,t') = -i\,\theta(t-t')\langle\{c_i(t),c_j^{+}(t')\}\rangle$$

commutator

$t' \longrightarrow t$ future propagation (hence retarded)

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Simplifying

Spectral Representation

→ representation of G_F in energy domain. (e.

(useful for time indpt $\mathcal{H}$ situations).

→ non-interacting example

$$\mathcal{H} = \sum \epsilon_m c_m^\dagger c_m .$$

$$G^R_{mm'}(t-t') = (\delta_{mm'}) \; -i\,\theta(t-t')\, e^{-i\epsilon_m(t-t')}$$

$$\theta(t) = \int \frac{d\omega}{2\pi} \left(\frac{e^{-i\omega t}}{\omega + i\eta} \right) \times (i)$$

$$G^R_{mm'}(t-t') = \delta_{mm'} \int \frac{d\omega}{2\pi} \times \frac{e^{-i(\omega+\epsilon_m)(t-t')}}{\omega + i\eta}$$

$$G^R_{mm'}(t-t') = \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega - \epsilon_m + i\eta} \times \delta_{mm'}$$

dagger

$$G^A_{m,m'}(t-t') = \delta_{mm'} \times \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega - \epsilon_m - i\eta} .$$

poles at eigen energies.

$$G^R_{m,m'}(\omega) = \frac{1}{\omega - \epsilon_m + i\eta}$$

$$= \frac{1}{\omega - \epsilon_m} - \frac{i\eta}{(\omega-\epsilon_m)^2 + \eta^2} \xrightarrow{\eta \to 0} -\frac{1}{\pi}\delta(\omega - \epsilon_m)$$

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$\therefore \mp \mathrm{Im}(G^R_{ii}(\omega)) = \rho_i(\omega)$ = Local density of states

how?

$G^{r,a}_{ij} = \sum \langle i|n\rangle\, g_{nn}(\omega) \langle n|j\rangle$ non-interacting, diagonal.

$$G^{r,a}_{ij}(\omega) = \sum \frac{\langle i|n\rangle \langle n|j\rangle}{\omega - \epsilon_n \pm i\eta}$$

$$G^{r,a}_{ii}(\omega) = \sum_n \frac{|\langle n|i\rangle|^2}{\omega - \epsilon_n \pm i\eta} =$$

$$= \frac{1}{\omega - \epsilon_i \pm i\eta} \quad \text{part}$$

$\mp \mathrm{Im}$

$\mp \rho_i(\omega) \longrightarrow$ LOCAL density of states with energy ω at site "i".

$\delta(\epsilon - \omega_i)$ broader

In interacting case, this broadens out with a finite gap ($= 2\eta$).

Imp:- D.O.S. is for single particle excitations only.

$$G^{r,a}_{ii}(\omega) = \sum_n |\langle n|i\rangle|^2\, \delta(\omega - \epsilon_n)$$

Teacher's Signature .....

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$\mp \mathrm{Im}$

$\mp \rho_i(\omega) \longrightarrow$ LOCAL density of states with energy ω at site "i".

$\delta(\epsilon - \omega_i)$ broader

In interacting case, this broadens out with a finite gap ($= 2\eta$).

Imp:- D.O.S. is for single particle excitations only.

$$G^{r,a}_{ii}(\omega) = \sum_n |\langle n|i\rangle|^2\, \delta(\omega - \epsilon_n)$$

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$$G^R(r,r',t,t') = -i\,\theta(t-t')\,\langle\{c_r(t), c^\dagger_{r'}(t')\}\rangle$$

$$\{c_r, c^\dagger_{r'}\} = \delta(\vec{r}-\vec{r}')$$

$$G^R(r,r',t,t') = -i\,\theta(t-t')\,\langle\{\psi(\vec{r},t), \psi^\dagger(\vec{r}',t')\}\rangle$$

$$H = \sum \epsilon_k\, \psi_k^\dagger \psi_k$$

$$[\psi(\vec{r},t), H] = \sum_k [\psi_q(t), \psi_k^\dagger \psi_k]\, e^{i\vec{q}\cdot\vec{r}}$$

$$= \sum_k \exp[i\vec{q}\cdot\vec{r}]$$

$$\{c_r(t), c^\dagger_{r'}(t')\} = \sum_{k,q} \{\psi_k e^{i\vec{k}\cdot\vec{r}} e^{-i\epsilon_k t}, \psi_q^\dagger e^{-i\vec{q}\cdot\vec{r}'} e^{i\epsilon_q t'}\}$$

$$= \sum_{k,q} e^{i(k\vec{r}-q\vec{r}')}\, e^{-i\epsilon_k t + i\epsilon_q t'}$$

$$\boxed{\delta(\vec{r}-\vec{r}')} = \sum_k e^{ik(\vec{r}-\vec{r}')} - \epsilon_k(t-t')$$

$$= \delta(\omega-\epsilon_k)\, e^{iq(x-x')}$$

$$\{\psi(t), c^\dagger_{r'}(t)\} = \delta(\vec{r}-\vec{r}')$$

$$G^R(\vec{r},\vec{r}',t,t') = \int \frac{d\omega}{2\pi} \frac{e^{-i\omega T}}{\omega-\epsilon}$$

$$T = t-t'$$

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$$G^R(\vec{r},\vec{r}',T) = \int_{\substack{\omega \\ \eta \to 0}} \frac{d\omega}{2\pi} \times \frac{e^{-i\omega T}}{\omega + i\eta} \times \sum_k e$$

$$G^R(\vec{r},\vec{r}',\omega) = \sum_k \frac{e^{i\vec{k}(\vec{r}-\vec{r}')}}{\omega - \varepsilon_k + i\eta}$$

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set $\vec{r} = \vec{r}'$

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$$\eta(\varepsilon, x) = \sum_n |\varphi_n(x)|^2 \, \delta(\varepsilon - \varepsilon_n)$$

weighted by density

i.e. a state contributes more to the DOS at a place where P. density is high.

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$$\left[(1-\partial_t \mathrm{Re}\,\Sigma^R)\partial_t + (\partial_t \tilde{v})\partial_t + \tilde{v}_k \nabla_k - (\nabla_k \tilde{v})\nabla_k\right] F$$

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Generalization of properties to Interacting Time indpt case

To simplify let i, j be sites in space & we'll work from there.

$$G_{ij}(t-t') = -i\,\theta(t-t')\langle \{ c_i(t), c_j^\dagger(t') \} \rangle$$

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$$= -i\,\theta(t-t')\langle e^{iH(t-t')} c_i(0) e^{-iH(t-t')} c_j^\dagger(0) + c_j^\dagger(0) e^{iH(t-t')} c_i(0) e^{-iH(t-t')} \rangle$$

$$= (-i)\,\theta(t-t')\langle e^{iH(t-t')} c_i(0) e^{-iH(t-t')} c_j^\dagger(0) \rangle + \langle c_j^\dagger(0) e^{2iH(t-t')} c_i(0) e^{-iH(t-t')} \rangle$$

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$$\cancel{= -i\,\theta(t-t') \sum_m \langle e^{iH(t-t')} c_i(0) |m\rangle\langle m| c_j^\dagger(0)}$$

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$(t,t') = -i\theta(t-t') \sum_m$.

some algebra

$$g^r_{ii'}(\omega) = \frac{1}{Z} \sum_{nm} \frac{|\langle n|c_i(0)|m\rangle|^2 \left[e^{-\beta\epsilon_n} + e^{-\beta\epsilon_m}\right]}{\omega - (\epsilon_m - \epsilon_n) + i\eta}$$

① Called the "Lehmann" or "Spectral Rep of G.F."

② $|m\rangle$ has one more particle than $|n\rangle$ at site i & hence $\epsilon_\eta = E_m - E_n$ ~~i facing~~ are all such single particle excitations

③ ~~LDOS~~ LDOS

$$\rho_i(\omega) = \frac{1}{Z} \sum_{nm} |\langle n|c_i(0)|m\rangle|^2 \left(e^{-\beta\epsilon_n} + e^{-\beta\epsilon_m}\right) \times \delta\left[\omega - (E_m - E_n)\right]$$

④ Both. GF & LDOS are functions of Temperature.

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E.O.M for G.F

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G's coupling

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