

Recap :- ① Haldane

Bond structure      finite

+

Chern  $\neq$

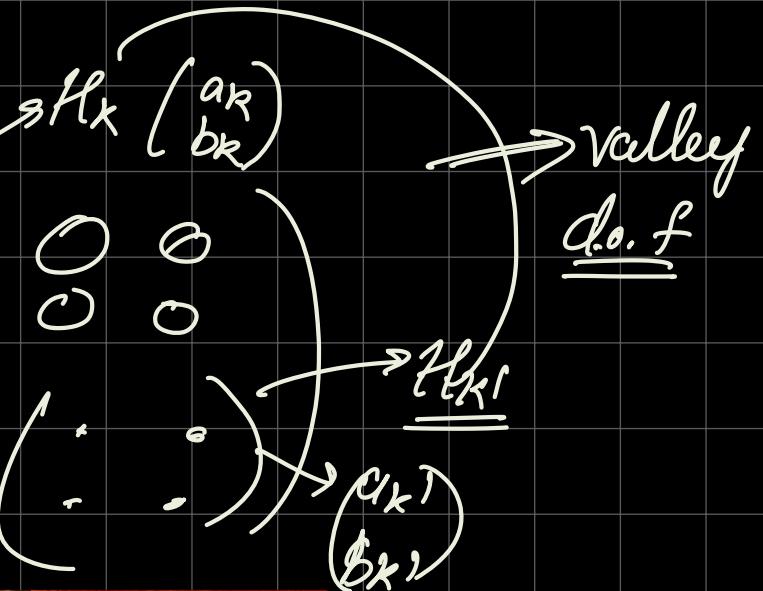
size edge modes

Bulk       $\longleftrightarrow$       Boundary correspondence

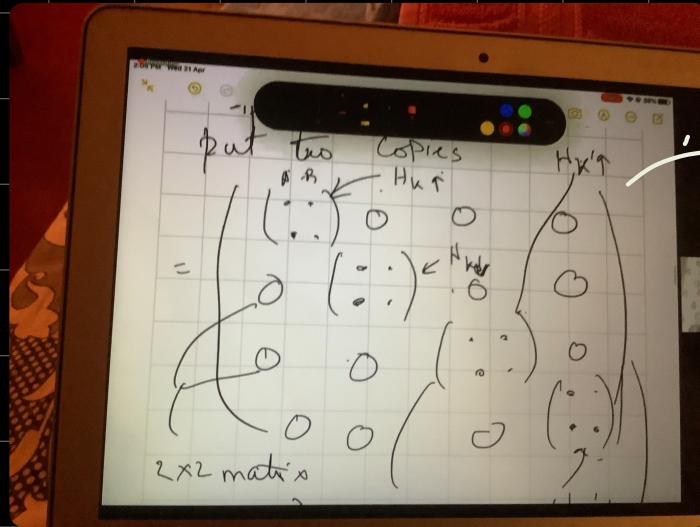
- # Today  $\Rightarrow$
- Kane-Mele model
  - Haldane model's extension

Basis of Hamiltonian

$$\mathcal{H} = \begin{pmatrix} \ddots & & \\ & \ddots & \\ & & \ddots \end{pmatrix}$$



Spin dof



spin

8x8 matrix

• Kane & Mele  $\rightarrow$  spin & orbit interaction was explicitly added.

$\xrightarrow{\text{Haldane model}}$  Chern band without  $\vec{B}$

$$1 \rightarrow \text{Chern } \# +1 \quad \downarrow \rightarrow \text{Chern } \# (-1)$$

Kane & Mele  $\rightarrow$  preserve TRS, no ext  $\vec{B}$  field.

$\downarrow$   
invoke spin



Overall Chern # was 0  $\rightarrow$  but they add up for  
+1 -1

$C=0$   $\rightarrow$  but still top. non trivial

$\hookrightarrow$  characterizes  $\mathbb{Z}_2$  index

$\rightarrow \mathbb{Z}_2$

$$H = \sum_i C_j^\dagger C_j + 2\lambda_{so} \sum_{i,j} V_{ij} C_j^\dagger S^z C_j$$

$\delta OC$

honeycomb

similar to halolane with

$$\phi = \frac{\pi}{2}$$

SOC preserves ZRS

$$\vec{L} \cdot \vec{s} \rightarrow (-\vec{L}) \cdot (-\vec{s}) = \underline{\underline{\vec{L} \cdot \vec{s}}}$$

additional terms

$$+ 2 \lambda_{Rashba} \sum_{ij} C_j^+ (\vec{s} \times \vec{d}_{ij}) C_j$$



broken  
mirror  
symmetry

$$+ \lambda_V \sum_i \sum_j C_j^+ C_j$$

can break  
inversion  
symmetry

• QS  $\Rightarrow$  Topo insulator of 2 Dim.

$\hookrightarrow$  also called Quantum spin hall syst

$\hookrightarrow$  breaks sublattice symmetry

#

$$H = a_L^\dagger b_R^\dagger ( ) (a_R^\dagger b_L^\dagger)$$

$$(a_{kp}^\dagger a_{k'p'}^\dagger b_{kp}^\dagger b_{k'p'}) ( ) X ( ) X$$

add spins

$$(a_{k\uparrow}^\dagger a_{k\downarrow}^\dagger b_{k\uparrow}^\dagger b_{k\downarrow}^\dagger)$$

$b_{k1}$ )

So interaction gives a non-zero int here

# High symmetry points:  $\underline{k}, \underline{x}^1$

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kane and mele model updated.nb - Wolfram Mathematica 12.1
File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
FullSimplify[MatrixForm[hamapproxk]]

$$\begin{pmatrix} -3\sqrt{3}\lambda so + \lambda v & 0 & -\frac{3}{4}(1+\sqrt{3})(qx-iqy) & \frac{3}{4}(1+\sqrt{3})(qx+iqy) \\ 0 & 3\sqrt{3}\lambda so + \lambda v & \frac{3}{2}i(\frac{1}{2}+\sqrt{3})\lambda R & -\frac{3}{4}(1+\sqrt{3})(qx-iqy) \\ -\frac{3}{4}(-i+\sqrt{3})(qx+iqy) & -3(-1)^{1/3}\lambda R & 3\sqrt{3}\lambda so - \lambda v & 0 \\ -\frac{3}{2}(-1)^{1/3}(i qx + qy) \lambda R & -\frac{3}{4}(-i+\sqrt{3})(qx+iqy) & 0 & -3\sqrt{3}\lambda so - \lambda v \end{pmatrix}$$

FullSimplify[Eigenvalues[hamapproxk]]

$$\left\{ -\frac{1}{4}(-1)^{1/3} \text{Root}\left[-648 qx^4 + 648 i \sqrt{3} qx^4 - 1296 qx^2 qy^2 + 1296 i \sqrt{3} qx^2 qy^2 - 648 qy^4 + 648 i \sqrt{3} qy^4 - 7776 qx^2 qy^2 - 7776 i \sqrt{3} qx^2 qy^2 \lambda R^2 + 2592 qy^3 \lambda R^2 - 2592 i \sqrt{3} qy^3 \lambda R^2 - 2592 qx^2 \lambda R^2 + 2592 i \sqrt{3} qx^2 \lambda R^2 - 2592 qy^2 \lambda R^2 + 2592 i \sqrt{3} qy^2 \lambda R^2 - 15552 qx^2 \lambda so^2 + 15552 i \sqrt{3} qx^2 \lambda so^2 - 15552 qy^2 \lambda so^2 + 15552 i \sqrt{3} qy^2 \lambda so^2 + 31184 i \lambda R^2 \lambda so^2 - 31104 i \sqrt{3} \lambda R^2 \lambda so^2 + 7776 qx^2 \lambda R^2 \lambda so^2 - 7776 i \sqrt{3} qx^2 \lambda R^2 \lambda so^2 - 93312 \lambda so^4 + 7776 i \sqrt{3} qx^2 \lambda R^2 \lambda so^2 - 7776 qy^2 \lambda R^2 \lambda so^2 - 93312 \lambda so^4, -576 qx^2 \lambda v^2 + 576 i \sqrt{3} qx^2 \lambda v^2 - 576 qy^2 \lambda v^2 + 576 i \sqrt{3} qy^2 \lambda v^2, 1152 \lambda R^2 \lambda v^2 + 1152 i \sqrt{3} \lambda R^2 \lambda v^2 - 288 qx^2 \lambda R^2 \lambda v^2 + 288 i \sqrt{3} qx^2 \lambda R^2 \lambda v^2, 1152 qy^2 \lambda R^2 \lambda v^2 + 1152 i \sqrt{3} qy^2 \lambda R^2 \lambda v^2 - 288 qx^2 \lambda R^2 \lambda v^2 + 288 i \sqrt{3} qy^2 \lambda R^2 \lambda v^2] \right\}$$


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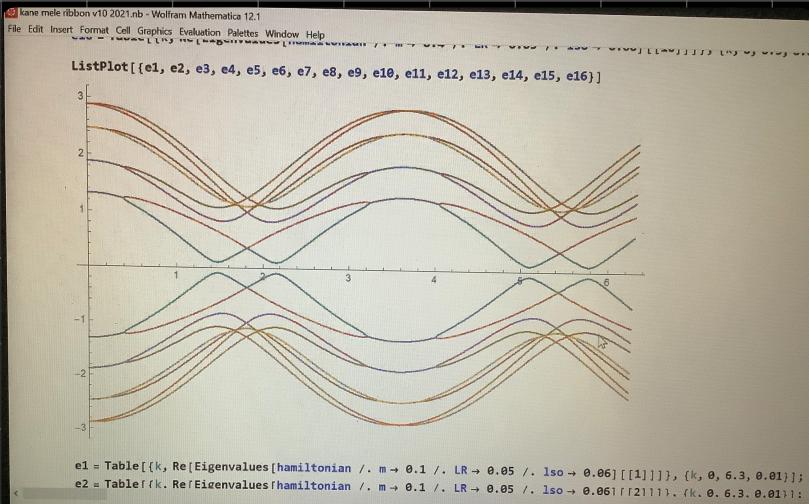
Rashba term

$\Rightarrow$  couples  $\uparrow\downarrow$

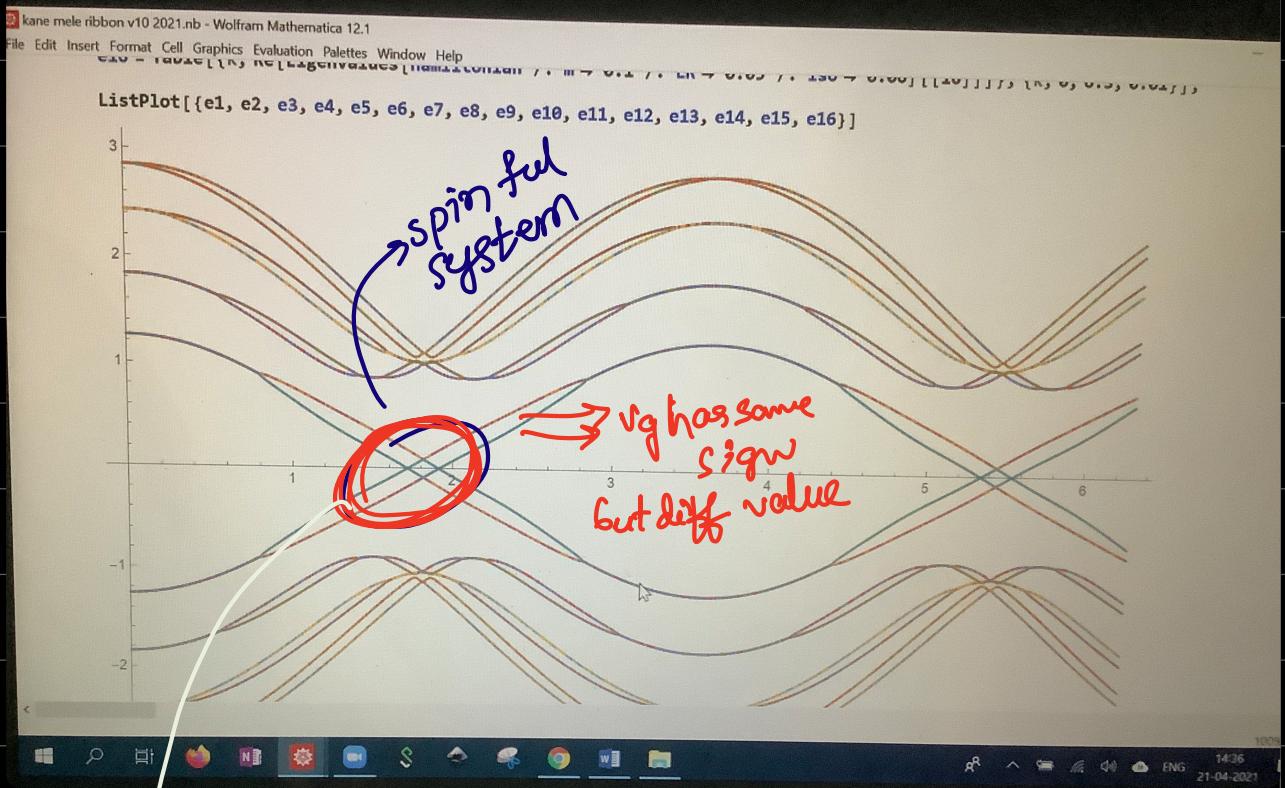
spin

net Chern number  $\Rightarrow \underline{0}$   $\rightarrow$  Spin projected Chern # will be 0.

# Finite ribbon calculation

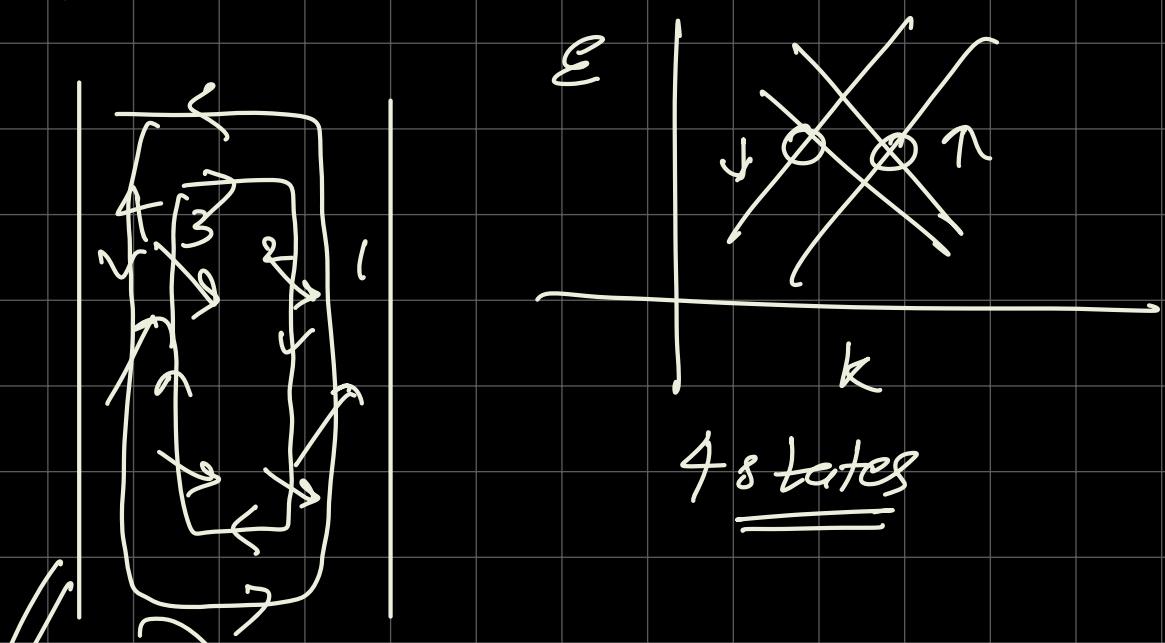


Gapped trivial



edge modes → have a spin flavour attached to it  
 of the system (and I'm totally failing  
lost here)

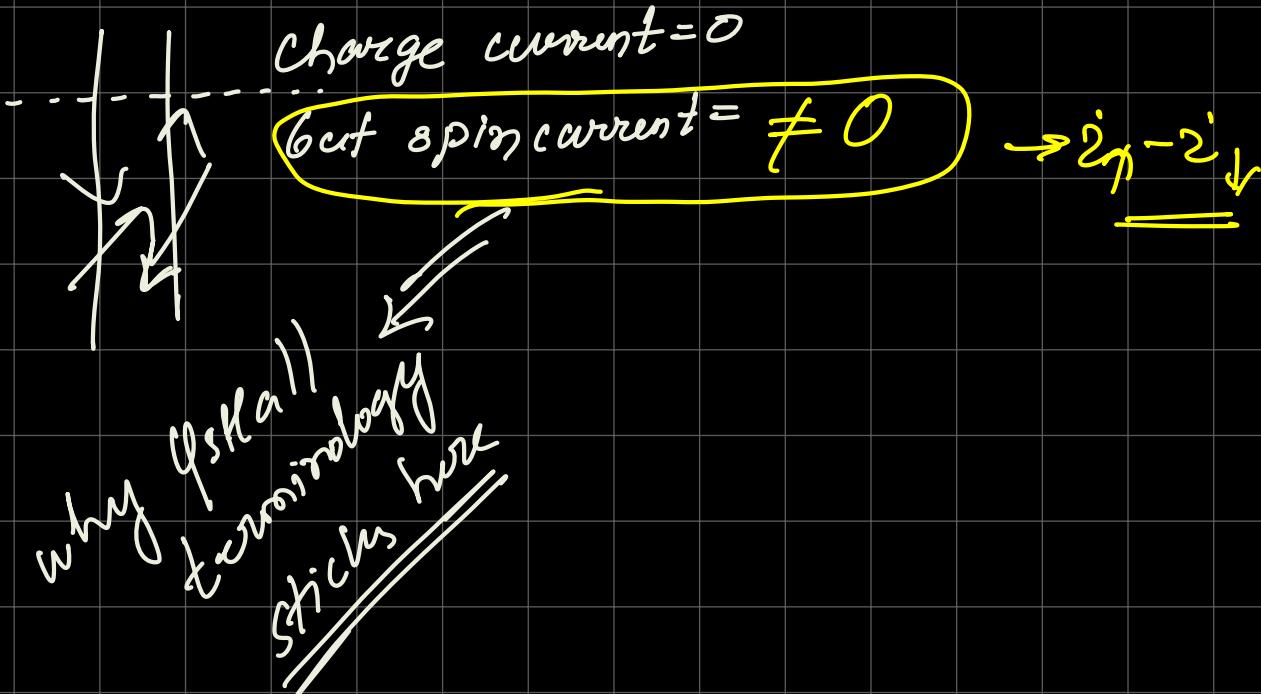
→ haldane model



~~Quantum  
spin Hall  
effect~~

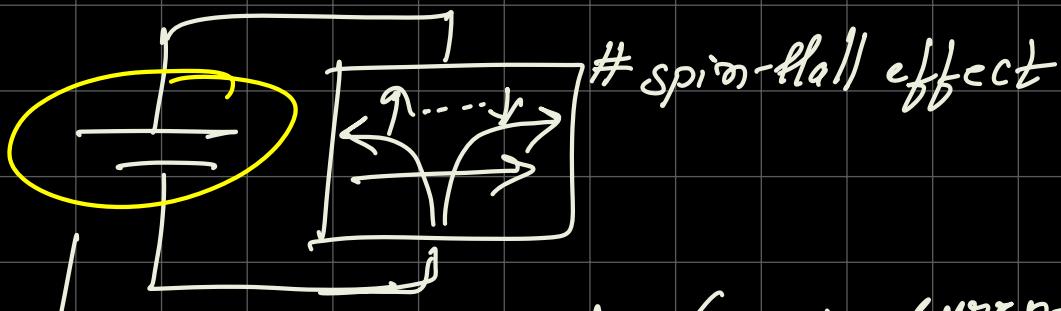
→ no net charge at the edge

Charge current  $\rightarrow 0$



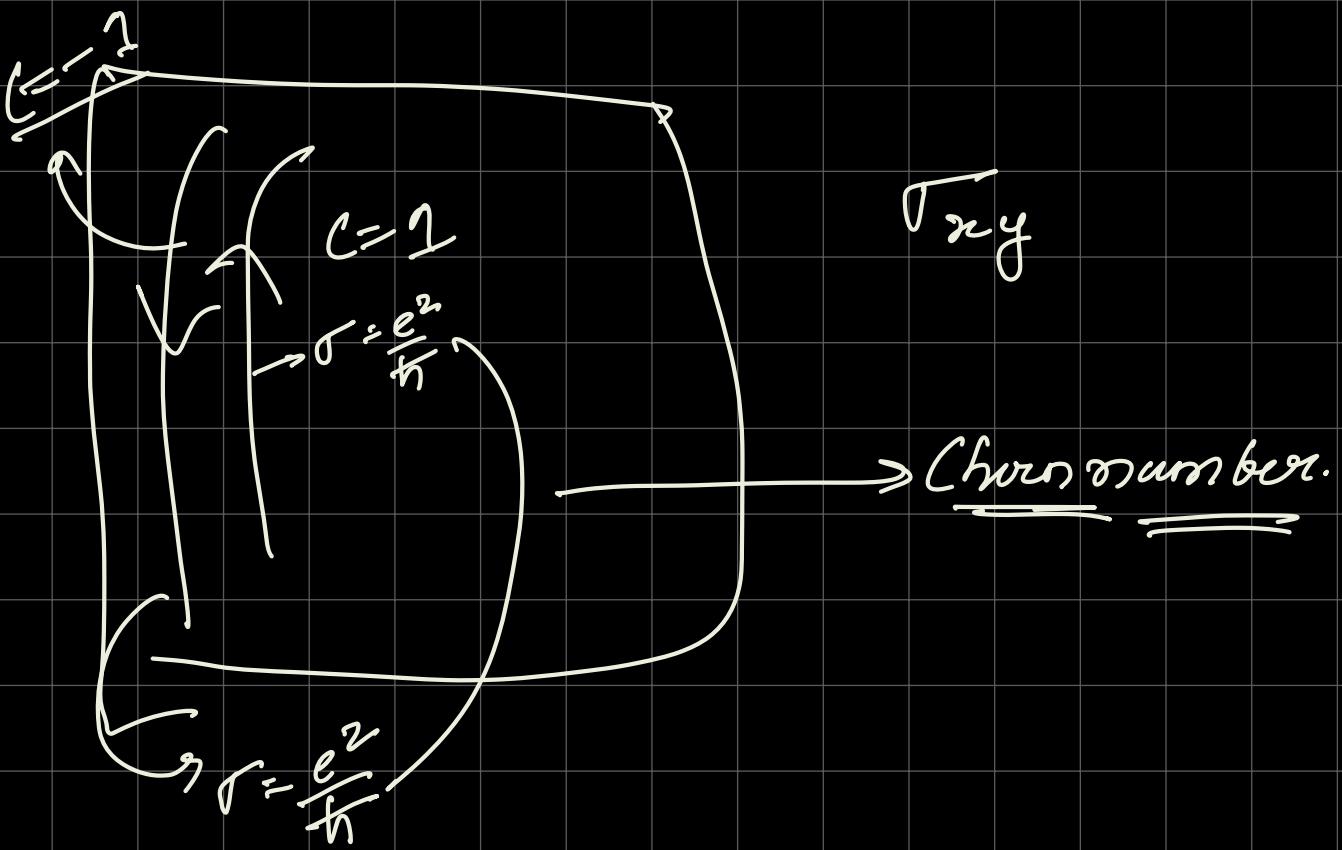
# But Q. Spin Hall effect

"classical" Spin Hall  $\rightarrow$  material with large atomic #

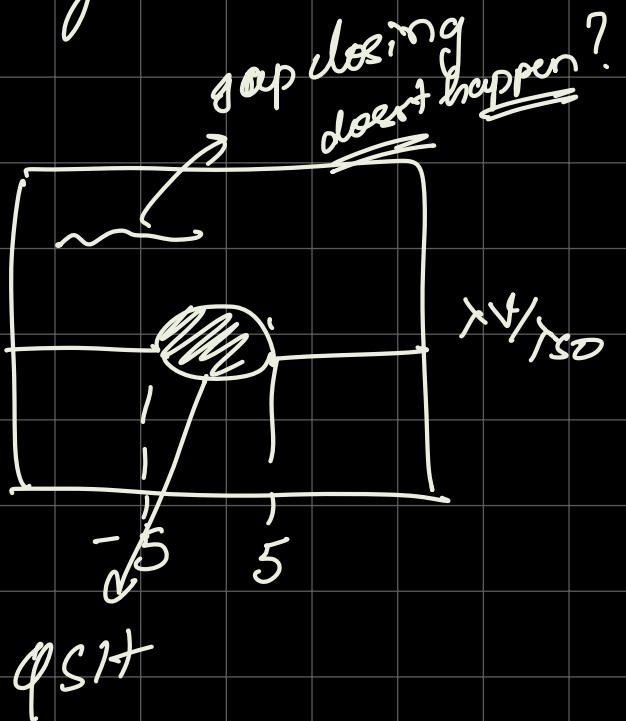
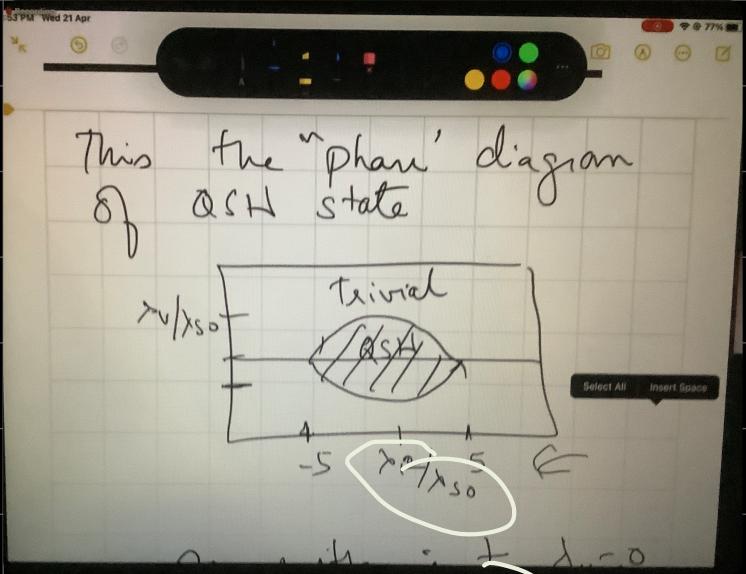


Charge current ✓ spin current  $\neq 0$

non-eqm of spin ↑ more on one side & less on the other side



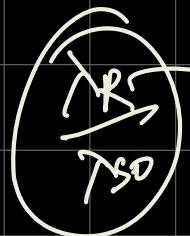
next class → Landauer-Büttiker picture



$\eta\eta\eta \rightarrow$  does the job of gap closing

if  $\lambda_v$  is large, then  $\lambda_{SO}$  can't

close the gap anymore



→ causes scattering b/w rashba term.

=> Shen + Vanderbilt

=> Bernard