Problem 1: assuming 2BP, $m_{s/c} \ll m_{\odot}$, $m_{s/c} \ll m_{\sigma^7}$

given: $GM_{\odot} = 1.32712428 \times 10^{11}\ km^3/s^2$,

$V = 22.4346\ km/s$, $V_r = 4.1219\ km/s$, $h = 4.9775 \times 10^9\ km^2/s$

a) $\vec{V} = V_r \hat{r} + V_\theta \hat{\theta}$

$V = |\vec{V}| = \sqrt{V_r^2 + V_\theta^2} \Rightarrow V^2 = V_r^2 + V_\theta^2 \Rightarrow V_\theta = \sqrt{V^2 - V_r^2}$

Plug in! → make sure units match!

$V_\theta = (22.4346^2 - 4.1219^2)^{1/2}$

$V_\theta = 22.0527\ km/s$ select positive value since V_θ is positive by definition

$\therefore \vec{V} = 4.1219 \hat{r} + 22.0527 \hat{\theta}\ km/s$ ← both vectors have a zero component in the $\hat{h}$ direction in the relative 2BP in the rotating frame.

$h = r^2 \dot{\theta} = r V_\theta \Rightarrow r = h / V_\theta$

$r = 4.9775 \times 10^9 / 22.0527$

$r = 2.2571 \times 10^8\ \hat{r}\ km$

b) $\mathcal{E} = \frac{1}{2} V^2 - \mu_\odot / r$

$= -336.324\ km^2/s^2$

$\vec{e} = \frac{\vec{V} \times \vec{h}}{\mu_\odot} - \frac{\vec{r}}{r}$

$= \frac{1}{\mu_\odot} (4.1219 \hat{r} + 22.0527 \hat{\theta}) \times (4.9775 \times 10^9 \hat{h}) - \frac{2.2571 \times 10^8 \hat{r}}{2.2571 \times 10^8}$

$\vec{e} = -0.17289 \hat{r} - 0.15460 \hat{\theta} + 0 \hat{h}$

$e = |\vec{e}| = 0.2319$

alternative: $e = \sqrt{1 + \frac{2h^2 \mathcal{E}}{\mu^2}}$

$p = h^2 / \mu_\odot = 1.86686 \times 10^8\ km$

$a = p/(1 - e^2) = 1.97299 \times 10^8\ km$

$\mathbb{P} = 2\pi \sqrt{a^3/\mu} = 4.77981 \times 10^7\ s = 553.22\ days$

type of conic: $\mathcal{E} < 0$, $0 \le e \le 1$, so it's an ellipse!

c) start w/ the glorious conic equation

$r = \frac{h^2/\mu}{1 + e\cos\theta^*} \Rightarrow \cos\theta^* = \frac{p - r}{re}$

$\cos\theta^* = -0.7454$

$\theta^* = \pm \cos^{-1}(-0.7454)$ but which sign??

$V_r > 0$ so the body is moving away from periapsis, $\therefore \theta^* > 0°, \phi_{fpa} > 0°$

$\theta^* = 138.19°$

ϕ_{fpa} values $[-90°, 90°]$, here ϕ_{fpa} is $> 0°$

$\tan(\phi_{fpa}) = V_r / V_\theta$

$\phi_{fpa} = 10.587°$

d) given: $\vec{R} = 5.3243\times10^{7}\hat{x} + 2.1925\times10^{8}\hat{y} + 6.2724\times10^{6}\hat{z}$ km; $\vec{V} = -2.0449\times10\hat{x} + 9.2202\hat{y} - 3.8811\times10^{-1}\hat{z}$ km/s

inertial frame = $[\hat{x}\ \hat{y}\ \hat{z}]$

$r = |\vec{r}| = \sqrt{r_1^2 + r_2^2 + r_3^2} = ((5.3243\times10^{7})^2 + (2.1925\times10^{8})^2 + (6.2724\times10^{6})^2)^{1/2} = 2.25709\times10^{8}$ km

$v = |\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = 22.4349$ km/s

$\vec{h} = \vec{r}\times\vec{v} = [r_2v_3 - r_3v_2,\ r_3v_1 - r_1v_3,\ r_1v_2 - r_2v_1] = [-1.429\times10^{8},\ -1.076\times10^{8},\ 4.9743\times10^{9}]$ in $\hat{x}\hat{y}\hat{z}$

$h = |\vec{h}| = 4.9775\times10^{9}$ km²/s

$\vec{e} = \dfrac{\vec{v}\times\vec{h}}{\mu_\odot} - \dfrac{\vec{r}}{r}$

$= [0.10938,\ -0.20449,\ -0.00128]$ in $\hat{x}\hat{y}\hat{z}$

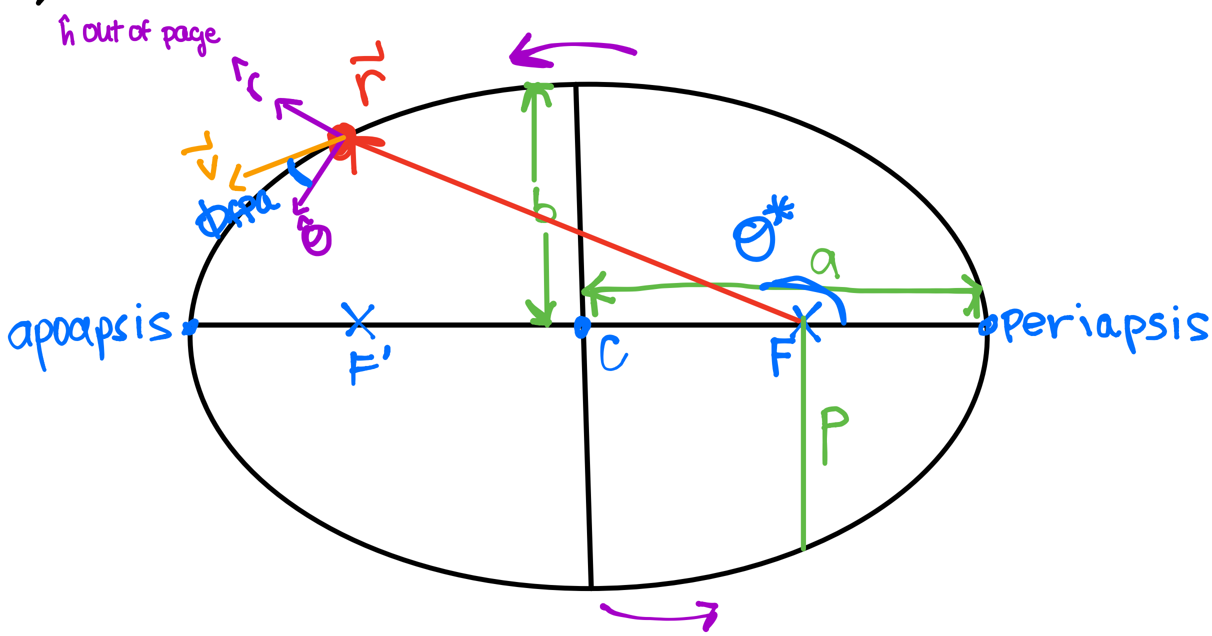
$e = |\vec{e}| = 0.2319$

$\varepsilon = \dfrac{v^2}{2} - \dfrac{\mu}{r} = \dfrac{-\mu}{2a} \Rightarrow a = -\mu/2\varepsilon$

$a = 1.9730\times10^{8}$ km

all of the magnitudes match up with parts a-c above! yay! ☺

e)



a: semi major axis

b: semi minor axis

p: semi latus rectum

position of s/c

direction of motion →

C: center

F: focus

F': vacant focus

f) if the s/c is at the top of the minor axis, $\theta_b^* = \cos^{-1}(-e)$

$\theta_b^* = \cos^{-1}(-0.2319) \approx 103.4°$

since $\theta^* > \theta_b^*$ moving away from periapsis, then s/c must be between the top minor axis and apoapsis.

also, $r > a$, so it must be past the top of the minor axis where $r_b = a$

Problem 2 $GM_{♂} = 4.305\times10^4\ km^3/s^2$, $R_{♂} = 3397.2$ km

given $r_p = 2342.8km + R_{♂} = 5740km$ $\delta = 66.7°$

assuming 2BP, $m_{s/c} \ll m_{♂}$

a) Calculate V_∞, θ^*_∞, hyperbolic orbit $a<0$

$$\frac{\delta}{2} + 90° = \theta^*_\infty$$

$$\theta^*_\infty = 123.35°$$

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right) \Rightarrow 1/\sin(\delta/2) = e = 1.819$$

$$r_p = a(1-e) \Rightarrow a = r_p/(1-e) = -7.0085\times10^3\ km$$

$$\mathcal{E} = +\mu/2|a|$$

$$\mathcal{E} = V_\infty^2/2$$

$$V_\infty = \sqrt{\mu/|a|} = 2.4784\ km/s$$

b) speed at periapsis

$$\frac{v^2}{2} - \frac{\mu}{r_p} = \frac{V_\infty^2}{2}$$

$$v^2 = V_\infty^2 + 2\mu/r_p$$

$$v = \sqrt{V_\infty^2 + 2\mu/r_p}$$

$$v = 4.5981\ km/s$$

c) It depends! Depending on the relative locations of the s/c with respect to the Martian Moons, Deimos & Phobos, their gravitational influence may be significant. Also, the irregular gravitational field of Mars may perturb the path of the s/c during a close pass.