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ASEN 5050
HW 8

**Problem 1:**

Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- 1 AU = 149,597,870.7 km

Consider a scientific satellite orbiting an unknown planet and observed to have the following orbit characteristics: periapsis radius = 7500 km, apoapsis radius = 8500 km, inclination = 105°, and orbital period =110 minutes. The radius of the planet is 6500 km. The planet is at a constant distance of 2.25 AU from the Sun, and the satellite is in a sun-synchronous orbit.

Determine the mass ($M_P$) and 2nd degree zonal gravitational coefficient ($J_{2,P}$) of the planet given this information. Be sure to list any assumptions that you use in your answer.

Given quantities:

$$r_p = 7500\ km$$
$$r_a = 8500\ km$$
$$i = 105°$$
$$\mathbb{P} = 110\ minutes$$
$$R = 6500\ km$$

Assuming the planet travels along a circular orbit around the sun:

$$a_P = 2.25\ AU = 3.3660x10^8\ km$$

The speed of the planet along this orbit:

$$v_P = \sqrt{\frac{\mu_{Sun}}{a_P}} = 19.856\ km/s$$

Time to complete one orbit revolution:

$$One\ Orbit\ Time = (2\pi a_p)\frac{1}{v_P} = 1.065089x10^8\ seconds$$

Rate of change of RAAN for a sun-synchronous orbit:

$$\dot{\Omega} = \frac{2\pi}{year} = \frac{2\pi}{1.065089x10^{8}seconds} = 5.8992x10^{-8}\frac{radians}{second}$$

Solving for the semi-major axis of the spacecraft orbit:

$$a = \frac{r_a + r_p}{2} = 8000\ km$$

Solving for the eccentricity of the spacecraft orbit:

$$e = 1 - \frac{r_p}{a} = \ 0.06250$$

Since 0 < e < 1, the orbit conic is an ellipse.

Using the period equation:

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu_P}} = 110\ minutes$$

Solving for $\mu_P$:

$$\mu_P = \left(\frac{2\pi}{110\ minutes\left(\frac{60\ seconds}{1\ minute}\right)}\right)^2 a^3 = 4.6402x10^5\ km^3/s^2$$

<u>Assuming that the spacecraft mass is much less than the mass of the unknown planet in order to calculate the unknown planet gravitational constant:</u>

$$\mu_P = Gm_P$$

$$\boldsymbol{m_P = \frac{\mu_P}{G} = \frac{4.7637x10^3\ km^3/s^2}{6.673x10^{-20}km^3/kg/s^2} = 6.9538x10^{24}km <--}$$

Using the following equation to solve for $J_{2,P}$ and <u>Assuming $J_2$ is the only perturbation effect:</u>

$$\dot{\Omega} = -\left(\frac{3}{2}\frac{\sqrt{\mu_P}J_{2,P}R^2}{(1-e^2)a^{\frac{7}{2}}}\right)\cos(i)$$

$$J_{2,P} = -\frac{2}{3}\frac{\dot{\Omega}}{\cos(i)}\frac{(1-e^2)^2 a^{\frac{7}{2}}}{\sqrt{\mu_P}R^2} = 2.39897x10^{-4}(unitless) < --$$

**Problem 2:**

<u>Assumptions:</u>

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces

During the course lectures, we covered in detail the general perturbations approach that enables a generalized analysis of the impact of perturbations on the solution space. However, recall that a special perturbation approach to studying the impact of perturbations focuses on a single point solution. One important component of the special perturbations approach (and simply generating a trajectory in a complex dynamical model) is numerical integration. Let's explore some of the challenges of numerically generating trajectories.

In this problem, we will explore the parameters associated with numerically integrating the path of a spacecraft relative to the Earth and in the two-body problem, using the following initial conditions at $t_0$:

$\bar{r}_0$ = [−6402, −1809, 1065] $km$ $\bar{v}_0$ = [0.999, −6.471, −4.302] $km/s$

expressed in an Earth-centered inertial coordinate system.

a) In Matlab, write a script to numerically integrate the equations of motion for the two-body problem, using ode45, a variable time-step Runge-Kutta Method. (I strongly recommend using Matlab for this question, but if you use a coding language other than Matlab, please identify a numerical integration scheme with similar properties to ode45 and describe the scheme you used in your writeup.) In ode45, use a relative tolerance of $1 \times 10^{-12}$ and an absolute tolerance of $1 \times 10^{-12}$ for now (we will change these values later). Include a copy of your script in your submission. Describe the setup of your script and list the differential equations you are providing to ode45 for integration. Also describe the definition of relative and absolute tolerances, as well as the approach used by the integrator to determine whether the error at each time step is acceptable using these two tolerances.

Please refer to the MATLAB code attached labeled "HW8_code.m." The ode45 function was called in the following manner:

```
state0 = [-6402,-1809,1065,0.999,-6.471,-4.302];
%1
options = odeset('Stats','off','RelTol',1*10^-12,'AbsTol',1*10^-12);
[tout1,xout1] = ode45(@EOMfile,[0 60*60],state0,options,mu_earth);
```

*Figure 1- HW8_code.m ode45 code snippet for explanation*

First an array was formulated containing the earth centered XYZ components of position and velocity given in the problem statement. This information is then fed into the ode45 function (along the given option parameters) and a time interval (in this example from 0 to 1200 seconds). The ode45 uses the system of equations stored in the function file “EOMfile.m.”

This system of equations was created by decomposing the second order equation of motion into a system of two first order differential equations as follows:

Non-vector form, second order equation of motion:

$$y(t)'' + \frac{\mu}{r^3} y(t) = 0$$

Substituting variables:

$$x_1(t) = y(t)$$
$$x_2(t) = y'(t)$$

Taking their derivatives:

$$x'_1(t) = y'(t) = x_2(t)$$
$$x'_2(t) = y''(t) = -\frac{\mu}{r^3} y(t) = -\frac{\mu}{r^3} x_1(t)$$

Using this decomposition, two first order equations were written for each XYZ component that are then used by the ode45 function to numerically propagate the orbit.

In the ode45 function the relative tolerance is a measure of the error according to the size of each solution component. It selects the equal digits for a solution except for numbers smaller than the absolute tolerance. The absolute tolerance determines the threshold at which values become unimportant. Therefore, determining the accuracy as the solution approaches zero. Both of these tolerances are used by the Runge-Kutta algorithm to optimize the integration step. If the error is above a given tolerance the step size is decreased. If the error is below a given tolerance the step size is increased to improve the algorithm’s time and find a solution.

b) Using the expressions we have developed in this class to solve the two-body problem analytically, determine the state vector of the spacecraft at the following two times: 1) $t_1 = t_0 + 1$ hr, and 2) $t_2 = t_0 + 100$ hr. At each of these two times, calculate the associated values of the specific energy and the magnitude of the specific angular momentum vector.

Calculating orbital elements using the initial state vectors at $t_0$:

$$\bar{h}_0 = \bar{r}_0 x\, \bar{v}_0 = 1.4674\text{x}10^4 \hat{X} - 2.6477\text{x}10^4 \hat{Y} + 4.3235\text{x}10^4 \hat{Z}\ km^2/s$$

$$\varepsilon_0 = \frac{{v_0}^2}{2} - \frac{\mu_{Earth}}{r_0} = -28.473\ km^2/s^2$$

Since $\varepsilon < 0$, the spacecraft is in an Earth-centric orbit that is bounded.

$$\bar{e}_0 = \frac{\bar{v}_0 x\, \bar{h}_0}{\mu_{Earth}} - \frac{\bar{r}_0}{r_0} = -0.037426\hat{X} + 0.0017723\hat{Y} + 0.013788\hat{Z}$$

$$|\bar{e}_0| = e_0 = 0.039925$$

Since 0 < $e_1$ < 1, the orbit conic is an ellipse

$$a_0 = \frac{-\mu_{Earth}}{2 \cdot \varepsilon_0} = 6.9996x10^3\ km$$

$$p_0 = \frac{h_0^2}{\mu_{Earth}} = 6.9885x10^3\ km$$

$$\theta^*{}_0 = \pm cos^{-1}\left(\frac{p_0 - r_0}{r_0 \cdot e_0}\right) = \pm 21.0239° = +21.0239°$$

Since $\bar{r}_1 \cdot \bar{v}_1 > 0$, the spacecraft at $t_0$ was heading away from periapsis and therefore $\theta^*{}_0$ will range between [0, 180] degrees.

After finding these $t_0$ orbital elements we can use Kepler's equation to evaluate $t_1$ and $t_2$:

Calculating mean motion:

$$n = \sqrt{\frac{\mu_{Earth}}{a_0^3}} = 0.001078095\ rad/s$$

Eccentric anomaly at $t_0$:

$$E_0 = 2\tan^{-1}\left(\sqrt{\frac{1 - e_0}{1 + e_0}}\tan\left(\frac{\theta^*{}_0}{2}\right)\right) = 0.3529\ radians$$

Time pass periapsis at $t_0$:

$$\left(t_0 - t_p\right) = \left(\frac{1}{n}\right)E_0 - e_0 \sin(E_0) = 314.5095\ seconds$$

Calculating time pass periapsis for $t_1$:

$$(t_1 - t_p) = (t_0 - t_p) + 1200\ seconds = \ 3.9145x10^3 seconds$$

Using the numerical solution developed for HW4 to find an eccentric anomaly value given a time pass periapsis we obtain:

$$E_1 = \ \ 4.1857\ radians$$

Calculating the position magnitude using this value:

$$r_1 = \ a_0(1 - e_0 \cos(E_1)) = \ 7.1401x10^3\ km$$

Calculating the state vectors at $t_1$ using f and g functions using eccentric anomaly equations:

$$\bar{r}_1 = f\bar{r}_0 + g\bar{v}_0$$
$$\bar{v}_1 = \ \dot{f}\bar{r}_0 + \dot{g}\bar{v}_0$$

$$f = 1 - \frac{a_0}{r_0}(1 - \cos(\Delta E)) = \ -0.83937$$

$$g = (\ t_1 - t_0) - \sqrt{\frac{a_0^3}{\mu_{Earth}}}(\Delta E - \sin(\Delta E)) = \ -546.50\ seconds$$

$$\dot{f} = \frac{-\sin(\Delta E)\sqrt{\mu_{Earth} a_0}}{r_0 r_1} = 6.99978x10^4 s^{-1}$$

$$\dot{g} = 1 - \frac{a_0}{r}(1 - \cos(\Delta E)) = \ -0.73563$$

Plugin these values into the f and g functions defined above we obtain:

$$\boldsymbol{\bar{r}_1 = f\bar{r}_0 + g\bar{v}_0 = 4.8277x10^3\hat{X} + 5.05482x10^3\hat{Y} + 1.4571x10^3\hat{Z}\ km} < --$$

$$\boldsymbol{\bar{v}_1 = \ \dot{f}\bar{r}_0 + \dot{g}\bar{v}_0 = -5.2161\ \hat{X} + 3.4940\ \hat{Y} + 3.9101\ \hat{Z}\ km/s} < --$$

Calculating specific mechanical energy:

$$\boldsymbol{\varepsilon_1 = \frac{{v_1}^2}{2} - \frac{\mu_{Earth}}{r_1} = \ -28.473\ km^2/s^2} < --$$

Magnitude of specific angular momentum:

$$\boldsymbol{h_1} = \|\bar{\boldsymbol{r}}_1 \boldsymbol{x}\, \bar{\boldsymbol{v}}_1\| = \mathbf{5.2779}\boldsymbol{x}\mathbf{10^4}\ \boldsymbol{km^2/s} < --$$

Calculating time pass periapsis for $t_2$:

$$(t_2 - t_p) = (t_0 - t_p) + 360000\ seconds = 3.6031x10^5 seconds$$

Using the numerical solution developed for HW4 to find an eccentric anomaly value given a time pass periapsis we obtain:

$$E_2 = \quad 388.4169\ radians$$

Calculating the position magnitude using this value:

$$r_2 = a_0(1 - e_0 \cos(E_2)) = \quad 6.8831x10^3\ km$$

Calculating the state vectors at $t_2$ using f and g functions using eccentric anomaly equations:

$$\bar{r}_2 = f\bar{r}_0 + g\bar{v}_0$$
$$\bar{v}_2 = \dot{f}\bar{r}_0 + \dot{g}\bar{v}_0$$

$$f = 1 - \frac{a_0}{r_0}(1 - \cos(\Delta E)) = 0.0413046$$

$$g = (t_1 - t_0) - \sqrt{\frac{a_0^3}{\mu_{Earth}}}(\Delta E - \sin(\Delta E)) = -878.34\ seconds$$

$$\dot{f} = \frac{-\sin(\Delta E)\sqrt{\mu_{Earth} a_0}}{r_0 r_1} = 0.0011356\ s^{-1}$$

$$\dot{g} = 1 - \frac{a_0}{r}(1 - \cos(\Delta E)) = 0.061598$$

Plugin these values into the f and g functions defined above we obtain:

$$\bar{\boldsymbol{r}}_2 = \boldsymbol{f}\bar{\boldsymbol{r}}_0 + \boldsymbol{g}\bar{\boldsymbol{v}}_0 = -\mathbf{1.1419x10^3}\hat{\boldsymbol{X}} + \mathbf{5.6090x10^3}\hat{\boldsymbol{Y}} + \mathbf{3.8226x10^3}\hat{\boldsymbol{Z}}\ \boldsymbol{km} < --$$

$$\bar{\boldsymbol{v}}_2 = \dot{\boldsymbol{f}}\bar{\boldsymbol{r}}_0 + \dot{\boldsymbol{g}}\bar{\boldsymbol{v}}_0 = -\mathbf{7.2087}\ \hat{\boldsymbol{X}} - \mathbf{2.4529}\ \hat{\boldsymbol{Y}} + \mathbf{0.9444}\ \hat{\boldsymbol{Z}}\ \boldsymbol{km/s} < --$$

Calculating specific mechanical energy:

$$\boldsymbol{\varepsilon_2 = \frac{v_2{}^2}{2} - \frac{\mu_{Earth}}{r_2} = -28.473\ km^2/s^2 < --}$$

Magnitude of specific angular momentum:

$$\boldsymbol{h_2 = \|\bar{r}_2 x\ \bar{v}_2\| = 5.2779x10^4\ km^2/s < --}$$

Note that as expected the specific angular momentum magnitude and the specific mechanical energy are the same for the two different times given the 2BP assumptions.

c) Using the script, you developed in part a), integrate the trajectory of the spacecraft for each of the following two time-intervals: 1) from $t_0$ to $t_1$, and 2) from $t_0$ to $t_2$. Create a table listing the state vector of the spacecraft at the end of each time interval, as well as the associated values of the specific energy and the magnitude of the specific angular momentum vector. Compare the results of your numerical integration with the state vectors identified in part b) and discuss.

Please note that the delta column represents the magnitude of the difference between the corresponding numerically iterated state end vector and the analytically calculated state vector. That is:

$$\Delta = \left\|\bar{P}_{numerical} - \bar{P}_{analytical}\right\|$$

All vectors are in the earth-centered inertial coordinate frame:

| **Vector/Time interval** | **End value X** | **End value Y** | **End value Z** | **Δ** |
|---|---|---|---|---|
| Position $t_0$ to $t_1$ (Km) | $4.8277x10^3$ | $5.0548x10^3$ | $1.4571x10^3$ | $9.8203x10^{-8}$ |
| Velocity $t_0$ to $t_1$ (Km/s) | -5.2161 | 3.4940 | 3.9101 | $5.0909x10^{-11}$ |
| Position $t_0$ to $t_2$ (Km) | $-1.1419x10^3$ | $5.6090x10^3$ | $3.8226x10^3$ | $5.4512x10^{-5}$ |
| Velocity $t_0$ to $t_2$ (Km/s) | -7.2087 | -2.4529 | 0.9444 | $5.9768x10^{-8}$ |

| **Time interval** | **Analytical Value** | **Numerical Value** | **Δ** |
|---|---|---|---|
| Specific Mechanical Energy $t_0$ to $t_1$ ($Km^2/s^2$) | -28.47299 | -28.47299 | 2.5654e-10 |
| Specific Angular Momentum Magnitude $t_0$ to $t_1$ ($Km^2/s$) | $5.27788x10^4$ | $5.27788x10^4$ | 2.5359e-07 |
| Specific Mechanical Energy $t_0$ to $t_2$ ($Km^2/s^2$) | -28.47299 | -28.47299 | 1.3963e-09 |
| Specific Angular Momentum Magnitude $t_0$ to $t_2$ ($Km^2/s$) | $5.27788x10^4$ | $5.27788x10^4$ | 1.2843e-06 |

We can observe from the delta column that all of the calculated vectors and magnitudes have a small error when compared to the analytical “true” value. This was expected since the ode45 function was set to low tolerances at the $10^{-12}$ level. This is a good iteration tolerance for numerically integrating a 2BP model.

d) In this section, we will explore the impact of the tolerances on the accuracy of the integrated solution. Set the absolute and relative tolerances to identical values. Each tolerance will be set to one of the following values: $1 \times 10^{-4}$, $1 \times 10^{-6}$, $1 \times 10^{-8}$, $1 \times 10^{-10}$, $1 \times 10^{-12}$. For each value of the tolerance, numerically integrate the trajectory of the spacecraft from $t_0$ to $t_2$ and note the state vector at the end of this time interval. Consider the “truth” value of the state vector at $t_2$ equal to the vector computed in part b). Then, for each of the five numerical integrations performed in this question, calculate the quantities $\Delta R$ and $\Delta V$, representing the magnitude of the difference in the position and velocity components between the numerical integration and the “truth” data. Also calculate $\Delta \mathcal{E}$ and $\Delta h$, representing the difference in the specific energy and specific angular momentum between the states at time $t_0$ and $t_2$ along the numerically integrated solution. Summarize this information in a table resembling the following:

| | $\Delta R$ (km) | $\Delta V$ (km/s) | $\Delta\varepsilon$ (km$^2$/s$^2$) | $\Delta h$ (km$^2$/s) |
|---|---|---|---|---|
| Tol = $1 \times 10^{-4}$ | $1.3596 \times 10^{4}$ | 15.232 | -0.88841 | $-7.94806 \times 10^{2}$ |
| Tol = $1 \times 10^{-6}$ | 9.5245 | 0.0104757 | $-1.06985 \times 10^{-4}$ | -0.0912501 |
| Tol = $1 \times 10^{-8}$ | 0.490466 | $5.3765 \times 10^{-4}$ | $6.8178 \times 10^{-8}$ | 0.0063316 |
| Tol = $1 \times 10^{-10}$ | 0.0054511 | $5.9769 \times 10^{-6}$ | $7.5181 \times 10^{-8}$ | $6.9772 \times 10^{-5}$ |
| Tol = $1 \times 10^{-12}$ | $5.4512 \times 10^{-5}$ | $5.9768 \times 10^{-8}$ | $7.510048 \times 10^{-10}$ | $6.97044 \times 10^{-7}$ |

e) Discuss the results summarized in your table. Also discuss the tradeoff you would perform to select an appropriate tolerance for numerically integrating solutions in the two-body problem via ode45.

The table shows that as the tolerance for the Ode45 function is increased the difference between values increases in magnitude. In other words, as the threshold for an acceptable calculated value for the Ode45 function becomes less stringent, the error of the values increase. The tradeoff performed by changing the tolerance values is between obtaining accurate enough results and computation time. If we are creating a large data set through the iteration time interval, the calculations can take significantly longer if we have a small tolerance. Therefore, the scientist and engineers performing these calculations need to agree on a tolerance that is reliable enough for the calculation keeping in mind the computation time of the iteration.

**Problem 3:**

a) **STK:** Open the “Earth Point Mass” propagator in the “Component Browser” and navigate to the “Numerical Integrator” tab. Report the integrator used in this propagator function as well as the absolute and relative tolerances used. Using either the summary or report function, discuss whether you think that this combination of integrator and tolerances has recovered a solution that is close to the true solution in the two-body problem. (Hint: Which quantities will you use to assess the accuracy of the solution?)

Numerical Integrator: Earth Point Mass RKF7th8th

Absolute and relative error: 1x10^-13

To assess the quantities of this solution I will use values that are expected to remain constant along the orbit assuming a relative two body problem model: $a$, $e$, $i$, $\omega$, $\Omega$

```
Time past epoch: 646402 sec   (Epoch in UTC Gregorian Date: 1 Jan 2020 00:00:00.000)

State Vector in Coordinate System: Earth J2000

Parameter Set Type:  Cartesian
         X:     1117.7709284922696042 km             Vx:       -7.2887437438203273 km/sec
         Y:     7083.2983579602323516 km             Vy:        1.0388363876797904 km/sec
         Z:     1493.9322186204922218 km             Vz:        1.2175693453506127 km/sec

Parameter Set Type:  Keplerian
       sma:     7500.0000000278951120 km           RAAN:         30.00000008454448 deg
       ecc:        0.0300000000108313                 w:         11.99999989273601 deg
       inc:          15.00000023964258 deg           TA:         39.99999989164113 deg
```

*Figure 2 - Final orbit quantities after 100 periods*

Summary Table:

| **Orbital Elements** | **Initial given state** | **After 100 orbit periods** | **Δ** |
|---|---|---|---|
| Semi-major axis | 7500 km | 7500.0000000278951120 km | $2.7895\text{x}10^{-8}$ km |
| Eccentricity | 0.03 | 0.0300000000108313 | $1.0831\text{x}10^{-11}$ |
| Inclination | 15 degrees | 15.00000023964258 degrees | $2.3964\text{x}10^{-7}$ degrees |
| AOP | 12 degrees | 11.99999989273601degrees | $1.0726\text{x}10^{-7}$ degrees |
| RAAN | 30 degrees | 30.00000008454448 degrees | $8.4544\text{x}10^{-8}$ degrees |

The table above show relatively small errors for the orbital element values calculated at the end of 100 periods. This proves that the combination of integrator and tolerances has recovered a solution that is close to the true solution in the two-body problem. Given that we would not expect these values to change as the spacecraft orbits with 2BP assumptions.

b) Describe, in as much detail as possible, the impact of the Earth J2 perturbation on each of the orbital elements for this trajectory, including your five plots of the orbital elements over 100 revolutions. Discuss, in as much detail as possible, whether these time history plots are consistent with your expectations.

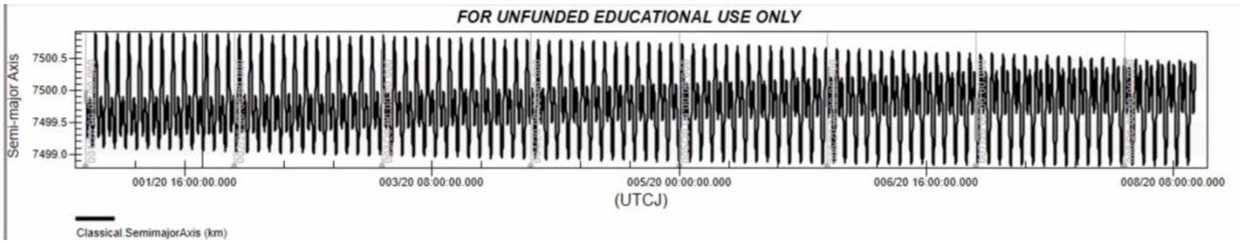


*Figure 3 - P3 b Semi-major axis vs time*

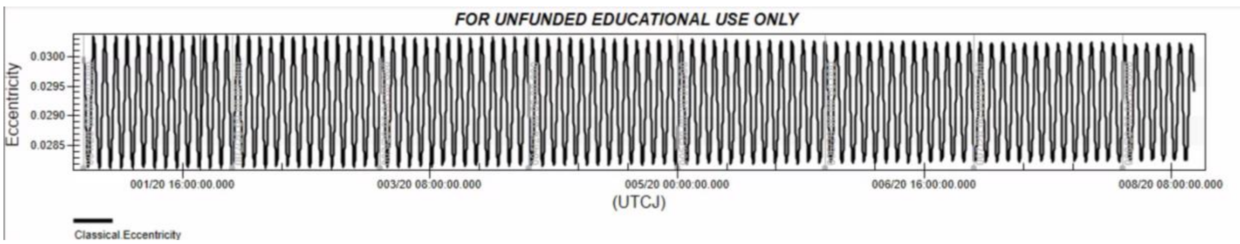


*Figure 4 - P3 b Eccentricity vs time*

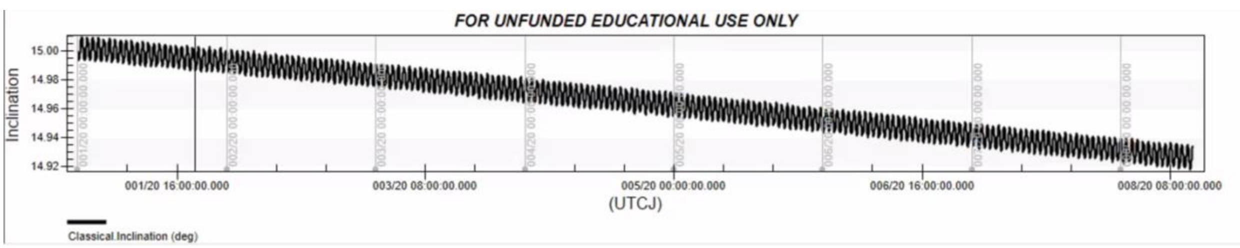


*Figure 5- P3 b Inclination vs time*

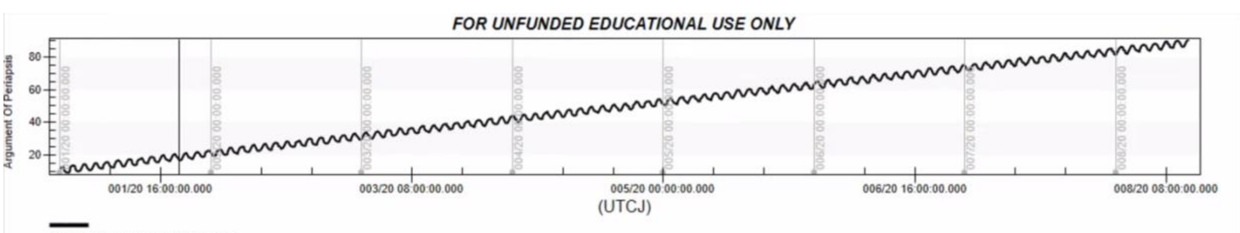


*Figure 6 - P3 b AOP vs time*

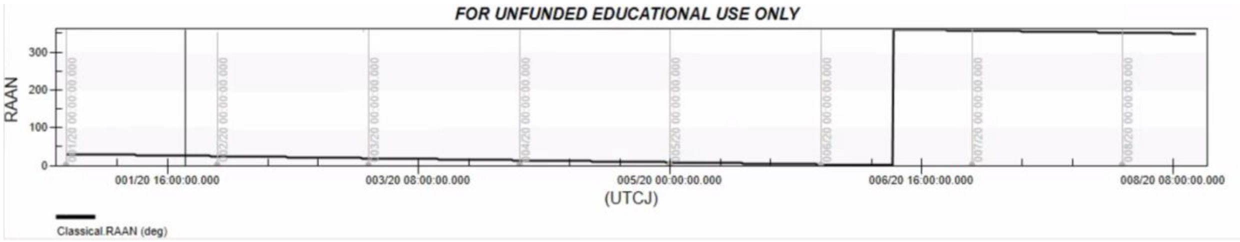


*Figure 7 - P3 b RAAN vs time*

The first observation from these graphs is the oscillation characteristics for the values that are expected to remain constant during J2 perturbations: Semi-major axis and eccentricity. Since general perturbations make values oscillate around an average as time progresses this is expected. Yet, the inclination values are drifting when they are supposed to remain constant. This might be due to human error in the creation of the simulation. Given that the initial inclination of 15 degrees we expect a prograde orbit. With prograde orbits with expect a shift west in the line of nodes as indicated by figure 7 with values decreasing slightly in quantity. Also, we would expect AOP to advance with respect to the line of nodes as seen in figure 6 with its values increasing as time progresses.

c) Include both a three-dimensional plot and a ground track plot of the satellite motion. Describe the impact of the Earth's oblateness on the entire spacecraft orbit using these plots as a reference.

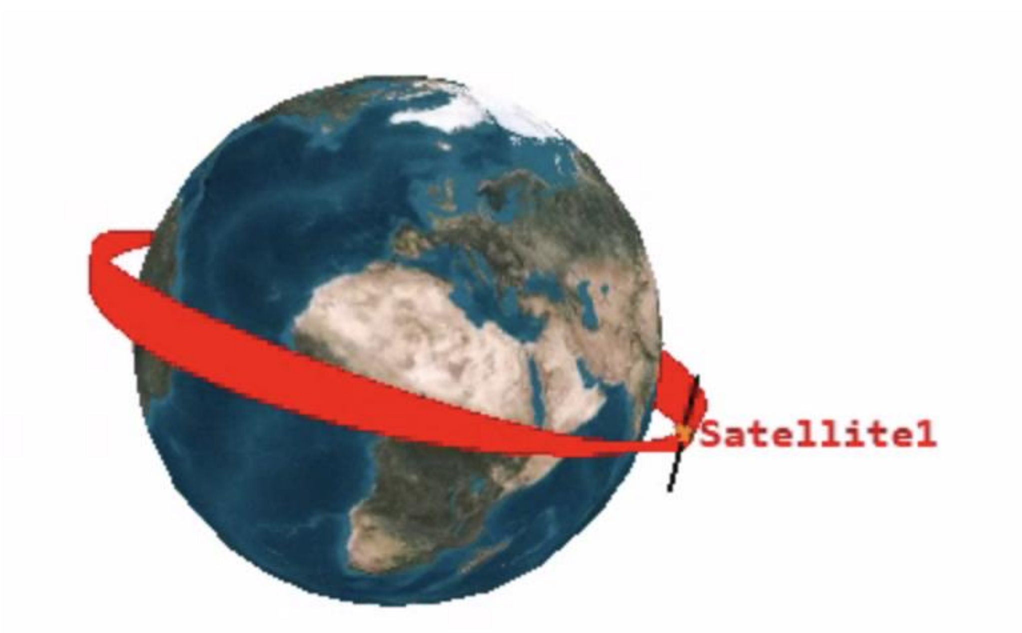


*Figure 8 - P3 C 3D Plot*

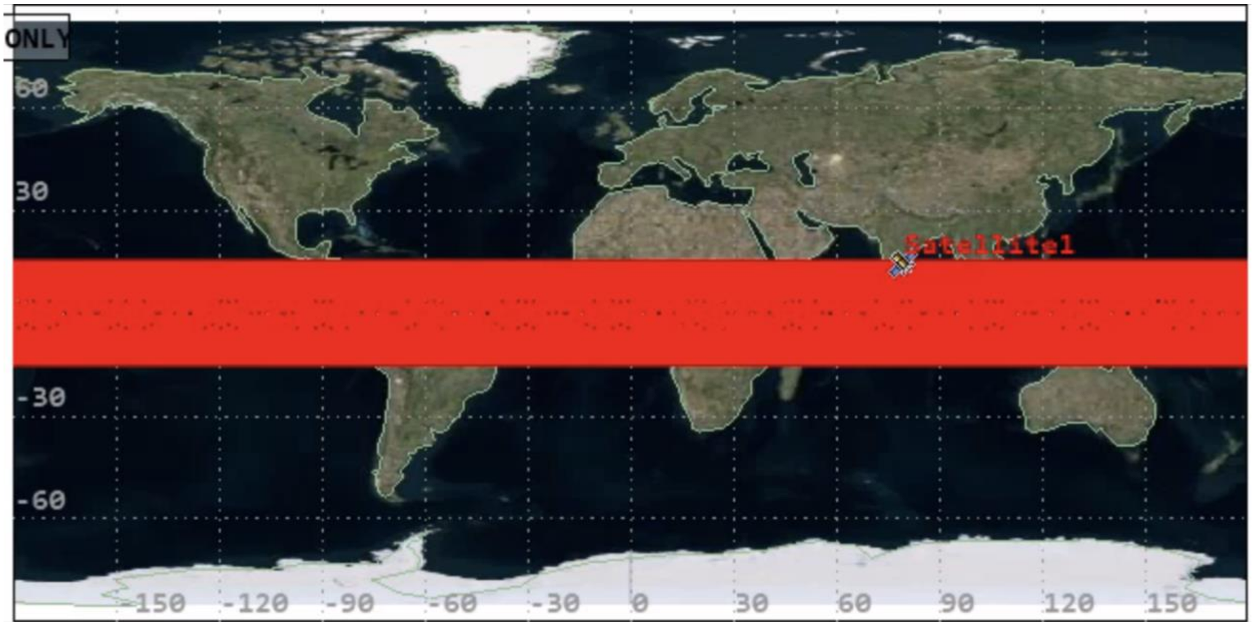


*Figure 9 - P3 C Ground tracks*

The oblateness of the earth physically creates a symmetrical drift in the orbit about the oblateness region which is at earths equator. From figure 8 this symmetry can be observed from one hemisphere of the earth to the other, with an interesting ribbon effect at the area where these two hemispheres meet. From figure 9 we can see how this perturbation does not make the satellite drift too far outside the of the equatorial region of the earth.

d) Rerun the scenario using the “Earth Higher Order” propagator. Describe, in as much detail as possible, the impact of the higher-order gravitational model on each of the orbital elements for this trajectory, including your five plots of the orbital elements over 100 revolutions. Discuss, in as much detail as possible, how the time evolution of these orbital elements differs from the time history of the orbital elements when subject only to a perturbation from J2. If you were designing a trajectory for this spacecraft, could a preliminary analysis in either the two-body problem, or the two-body problem with a J2 perturbation sufficiently predict the evolution of the orbit?

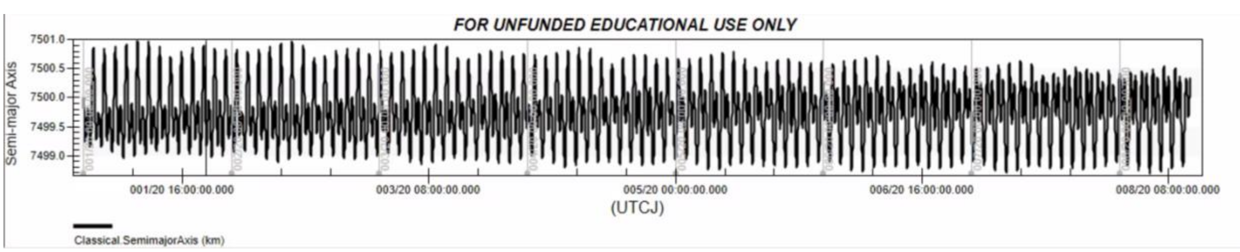


*Figure 10 - P3 d Semi-major axis vs. time*

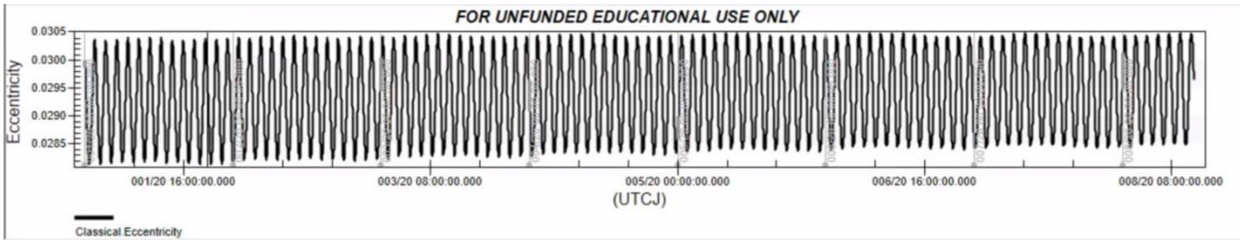


*Figure 11- P3 d Eccentricity vs. time*

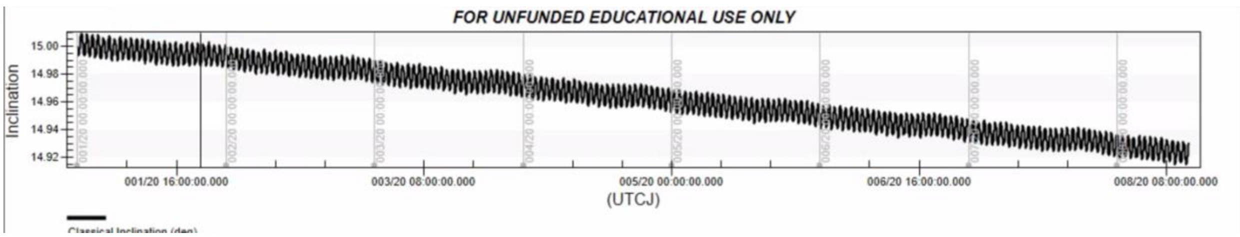


*Figure 12- P3 d Inclination vs. time*

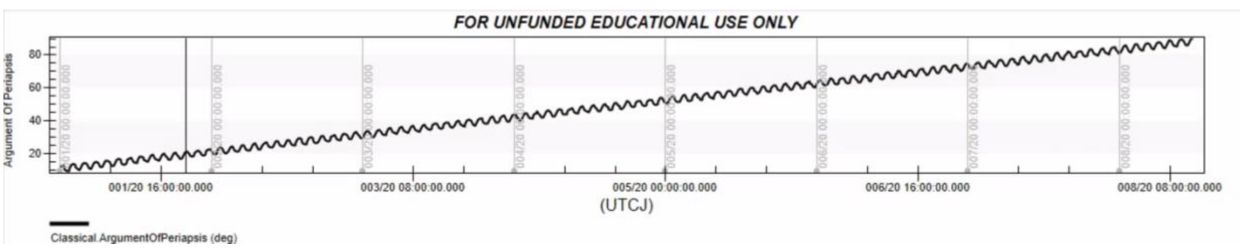


*Figure 13- P3 d AOP vs. time*

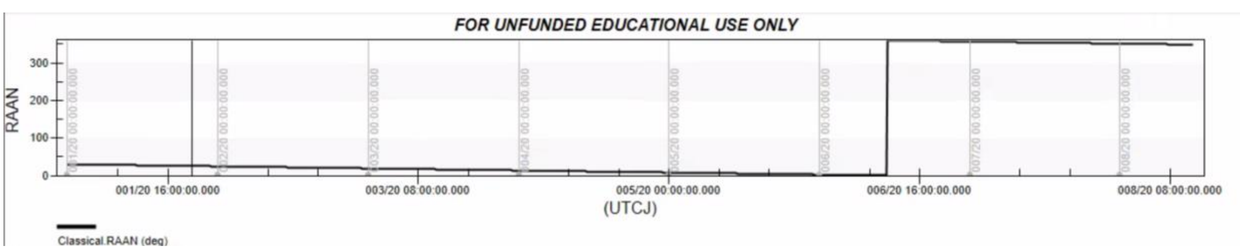


*Figure 14 - RAAN Vs. Time*

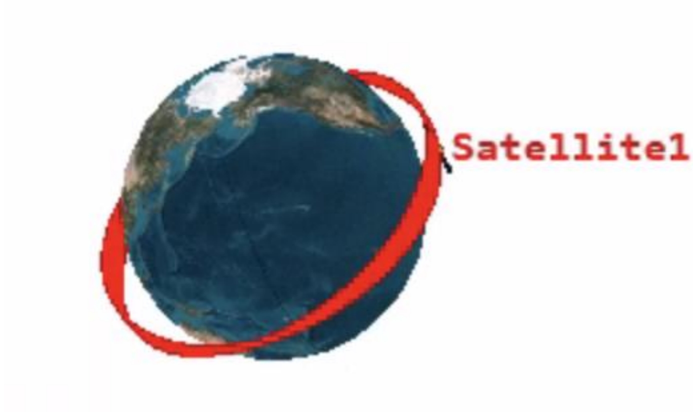


*Figure 15 - P3 d 3D representation*

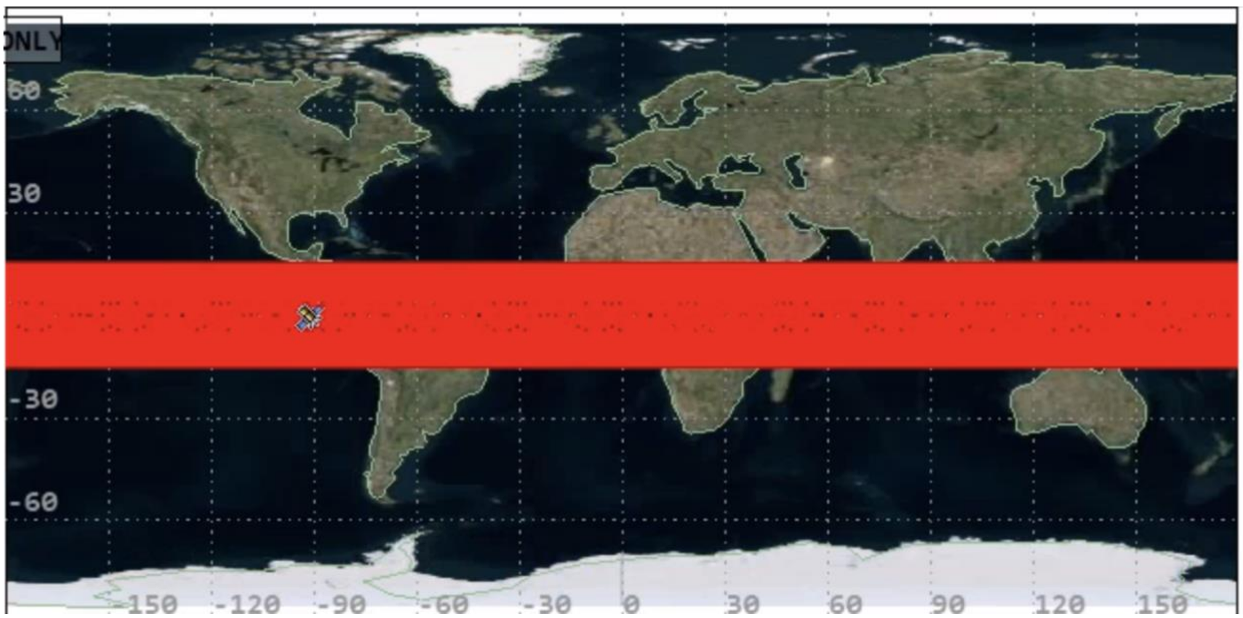


*Figure 16 - P3 d Groundtracks*

Given the information displayed from figure 10-16, it can be seen that the graphs and graphics look similar to when the model just had J2 perturbations displayed. All the values display similar behavior with barely noticeable differences. Therefore, If I was designing a trajectory for this spacecraft a preliminary analysis of the two-body problem with a J2 perturbation would sufficiently predict the evolution of the orbit. Yet, designing the orbit with just two-body problem assumptions would not be accurate because this model would not capture the secular drift of RAAN and AOP in the orbit.

e) Describe the impact of the Moon's third-body perturbation on each of the orbital elements corresponding to the reference motion, including your five plots of the orbital elements over 100 days (you may show additional zoomed-in views if you consider them necessary). Also include a three-dimensional plot of the spacecraft motion. Discuss whether you think these observations are generalizable beyond this single trajectory.

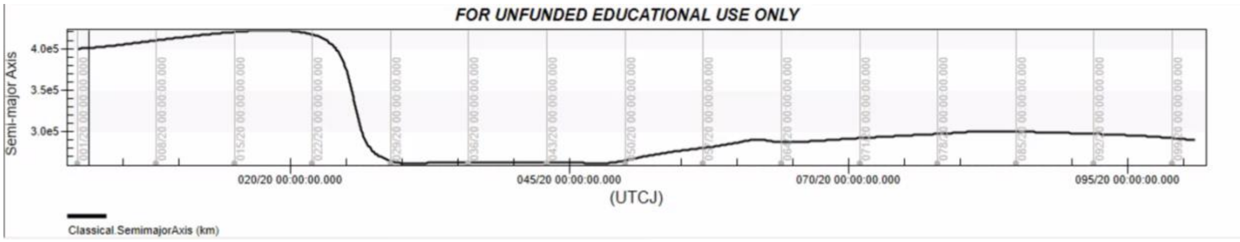


*Figure 17 - P3 e Semi-major axis vs. time*

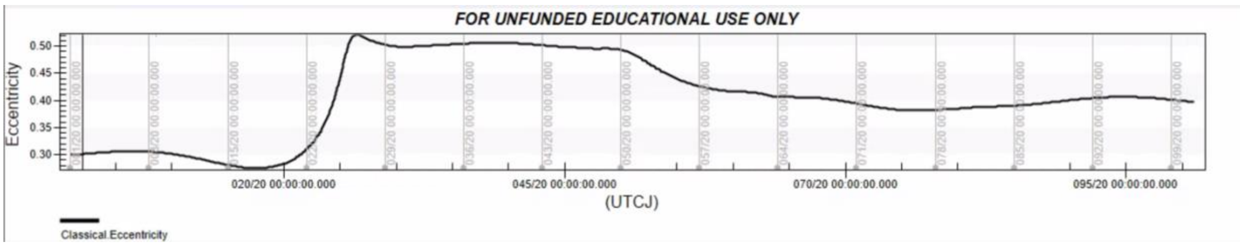


*Figure 18 - P3 e Eccentricity vs. time*

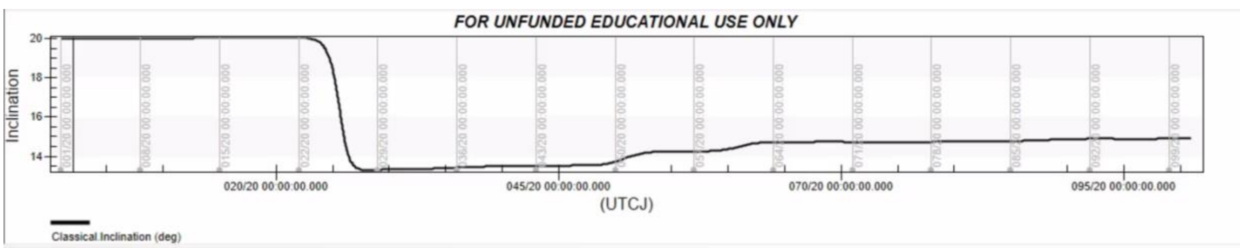


*Figure 19 - P3 e Inclination Vs. Time*

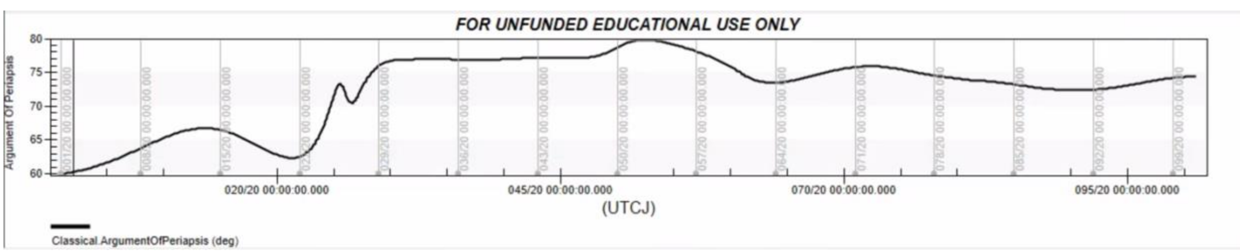


*Figure 20 - P3 e AOP Vs. Time*

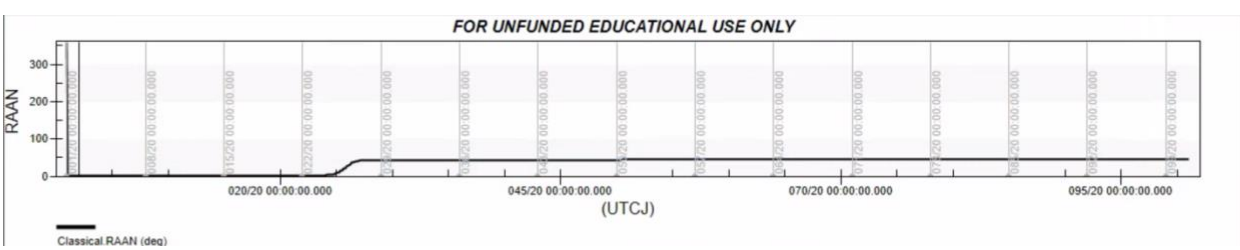


*Figure 21 - P3 e RAAN Vs. Time*

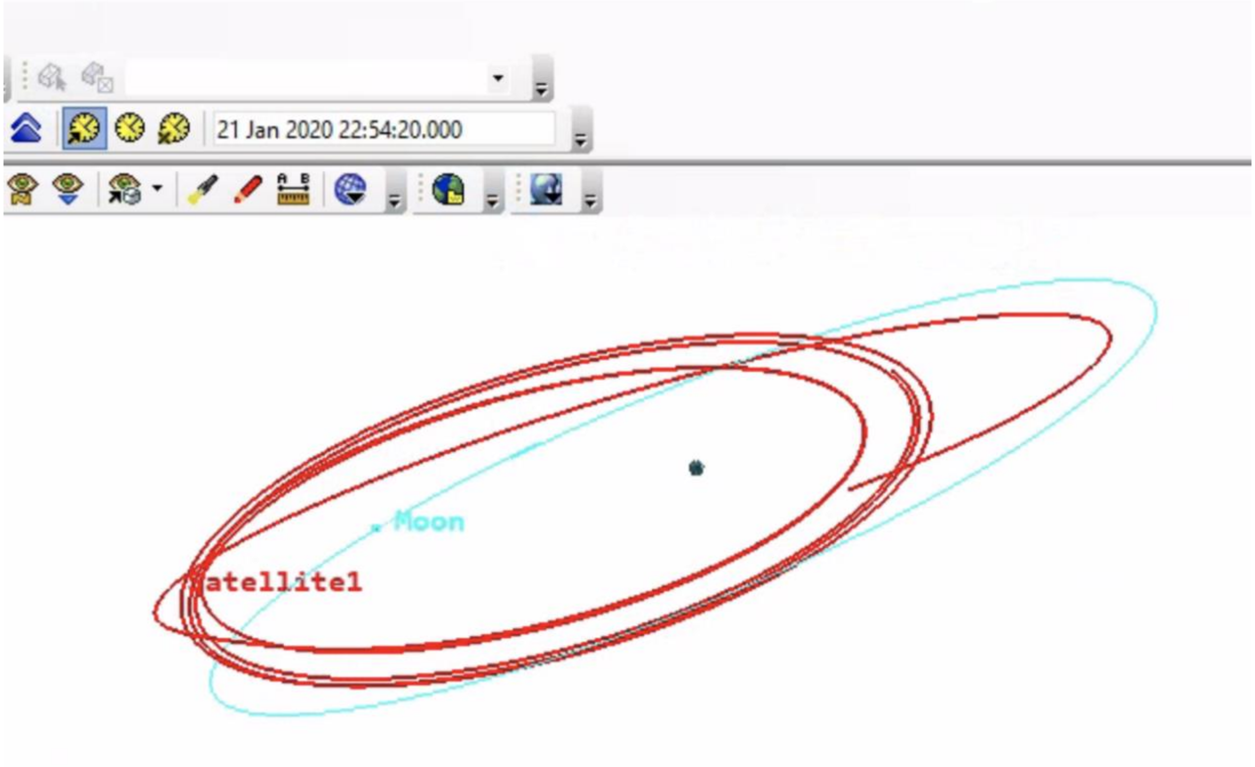


*Figure 22 - P3 e 3D Plot*

From figure 17-22 we can observe the drastic changes in orbit parameters by simulating the third body effect from the moon. Each graph shows a highly variable change of values that cannot be generalized by a single orbit. This variability can be further observed in figure 22 where the orbits for 100 days do not seem to follow a predictable pattern. To accurately represent this model the moons interaction with the spacecraft needs to be carefully analyzed and not generalized.

f) What is the change in the values of the semi-major axis, eccentricity and inclination between the beginning and end of this 100-day trajectory?

```
UTC Gregorian Date: 10 Apr 2020 00:00:00.000  UTC Julian Date: 2458949.5
Julian Ephemeris Date: 2458949.50080074
Time past epoch: 8.64e+06 sec   (Epoch in UTC Gregorian Date: 1 Jan 2020 00:00:00.000)

State Vector in Coordinate System: Earth J2000

Parameter Set Type:  Cartesian
         X:  -166153.7055405755236279 km              Vx:       -0.9534144981808292 km/sec
         Y:    69288.8695620070357108 km              Vy:       -1.4176133175222545 km/sec
         Z:    44482.5992464924347587 km              Vz:       -0.0841549662669515 km/sec

Parameter Set Type:  Keplerian
       sma:   290241.3601335162529722 km            RAAN:        45.42111272940737 deg
       ecc:        0.3974033137415480                  w:        74.41618528824945 deg
       inc:         14.91670514549459 deg             TA:        36.85340120054825 deg
```

*Figure 23 - Final orbit quantities after 100 days*

Summary Table:

| **Orbital Elements** | **Initial given state** | **After 100 days** | **Δ** |
|---|---|---|---|
| Semi-major axis | 400,000 km | 290,241.36013 km | $1.0976 \times 10^5$ km |
| Eccentricity | 0.30 | 0.3974033 | 0.0974 |
| Inclination | 20 degrees | 14.9167 degrees | 5.0833 degrees |

From the table above we can see that these values changed substantially from the initial conditions. Therefore, a 2BP model would poorly predict this environment.

g) Use the plots in part e) to identify an approximate epoch at which the closest lunar pass occurs and justify your answer.

In each one of the plots we can see an abrupt change of values around the 21st of January 2020 epoch. In figure 22 we can observe that around this time the moon and the satellite are traveling counterclockwise and coming to a close approach.

h) Describe how adding the gravitational influence of the Sun impacts the motion of the spacecraft. Use plots and the output of the reports/summary function to justify your answer.

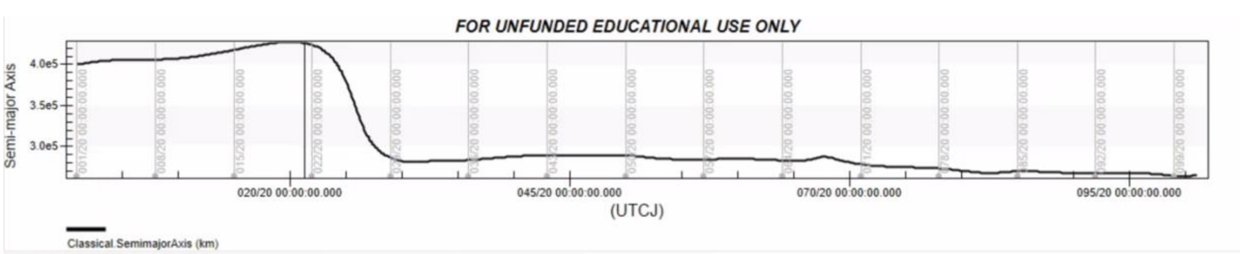


*Figure 24 - P3 h Semi-major axis vs. time*

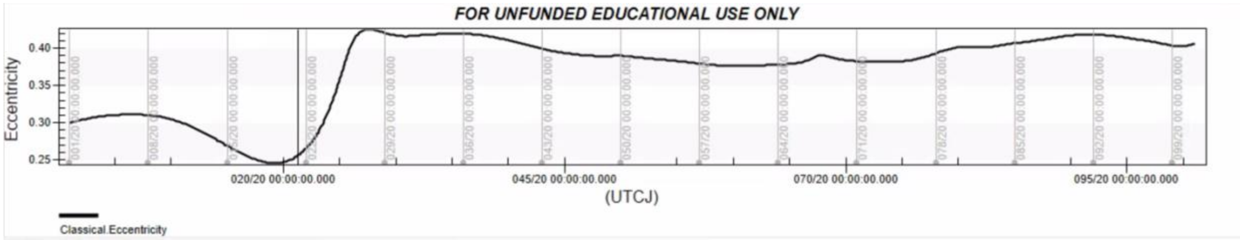


*Figure 25 - P3 h Eccentricity vs. time*

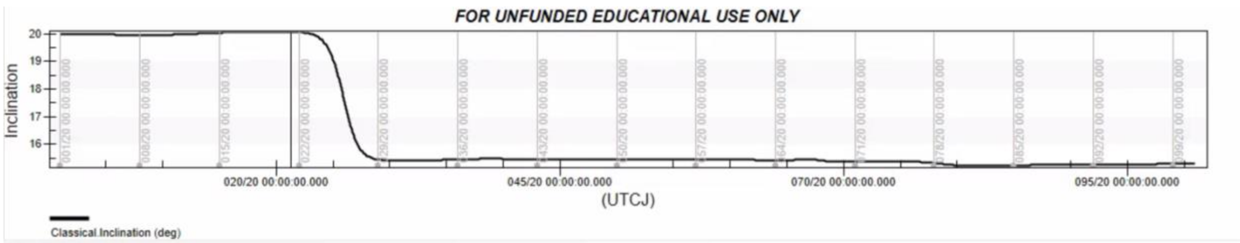


*Figure 26 - P3 h Inclination vs. time*

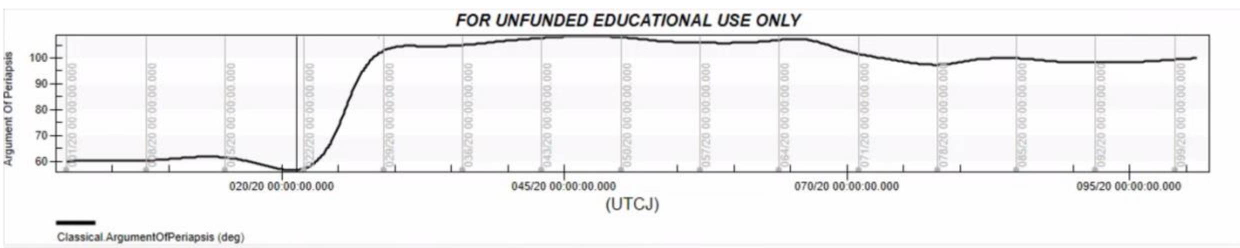


*Figure 27 - P3 h AOP vs. time*

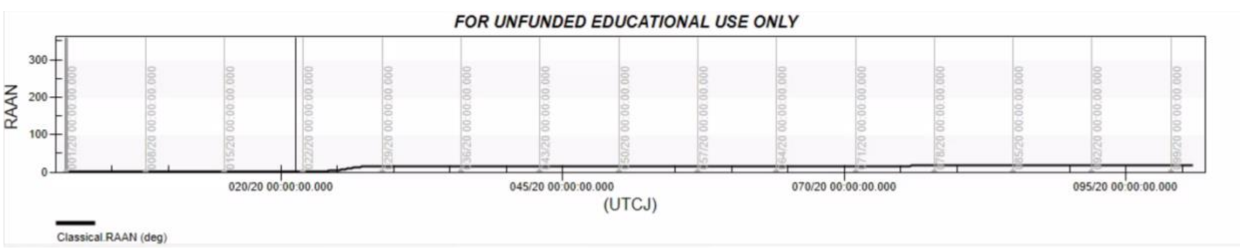


*Figure 28 - P3 h RAAN vs. time*

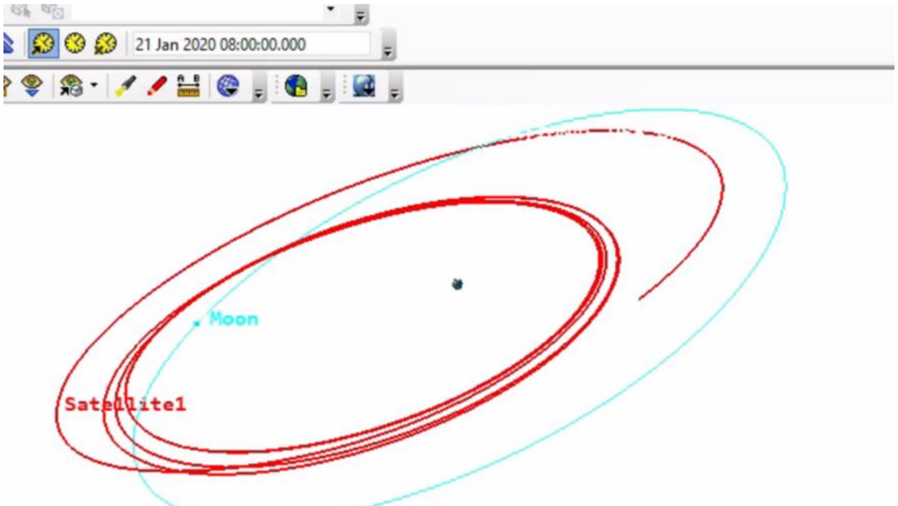


*Figure 29 - P3 h 3D Plot*

```
UTC Gregorian Date: 10 Apr 2020 00:00:00.000  UTC Julian Date: 2458949.5
Julian Ephemeris Date: 2458949.50080074
Time past epoch: 8.64e+06 sec   (Epoch in UTC Gregorian Date: 1 Jan 2020 00:00:00.000)

State Vector in Coordinate System: Earth J2000

Parameter Set Type:  Cartesian
         X:  -194587.9146588546573184 km             Vx:      -0.2374472589484996 km/sec
         Y:   -39079.4617848970447085 km             Vy:      -1.5217181997350482 km/sec
         Z:     6010.6861162189816241 km             Vz:      -0.3769465678232354 km/sec

Parameter Set Type:  Keplerian
       sma:   265604.6794774473528378 km           RAAN:        17.70919589316244 deg
       ecc:        0.4058440865300479                 w:       100.2155497061739 deg
       inc:         15.30534301906182 deg            TA:        73.19940229917923 deg
```

*Figure 30 - P3 h final values after 100 days*

Summary Table:

| **Orbital Elements** | **After 100 days (sun effect)** | **After 100 days (no sun effect)** | **Δ** |
|---|---|---|---|
| Semi-major axis | 265,604.579 km | 290,241.36013 km | 2.4637x104 km |
| Eccentricity | 0.40584 | 0.3974033 | 0.0084 |
| Inclination | 15.3053 degrees | 14.9167 degrees | 0.3886 degrees |

The graph shows that adding the sun third body effects to the simulation changes slightly the behavior of the orbit after the initial moon flyby. For example, Figure 22 show a more haphazard orbit while figure 29 shows a more stabilized orbit progression. It seems that the gravity effects of the sun add more stability to the satellite orbit after the flyby. Shown in the table above, the values at the end of 100 days for the third body effects of the sun changes the computed values. The engineers and scientist would need to figure out the fidelity of their orbit environment according to what error is acceptable due to perturbations.

i) Describe the impact of atmospheric drag on the corresponding to the integrated trajectory, including your five plots of the orbital elements over 10 days. Also include a three- dimensional plot of the spacecraft motion.

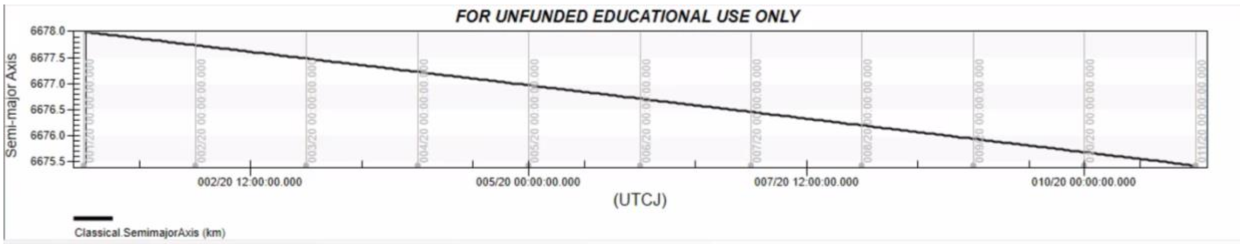


*Figure 31 - P3 i Semi-major axis vs. time*

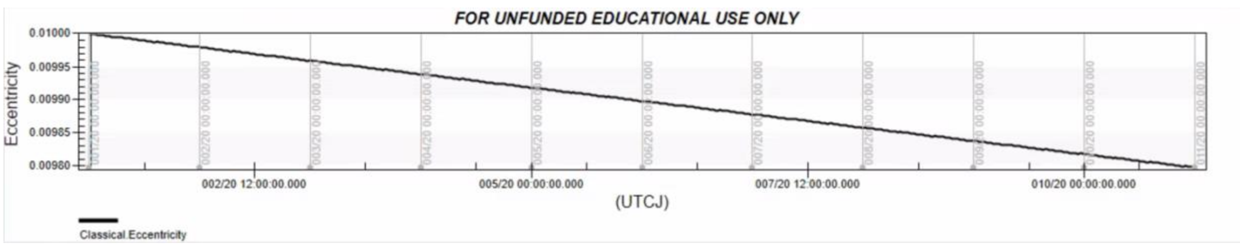


*Figure 32 - P3 i Eccentricity vs. time*

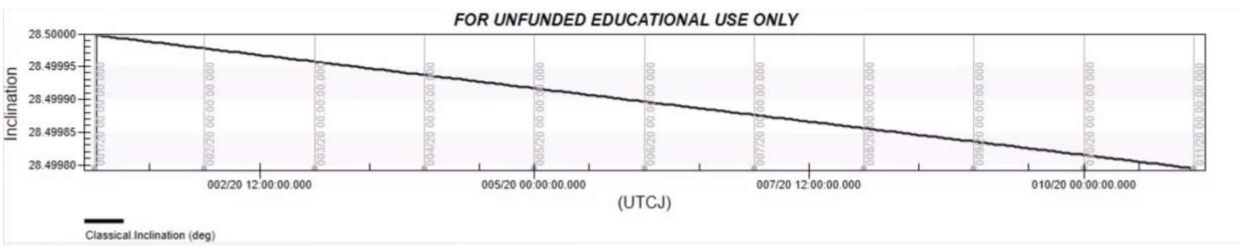


*Figure 33 - P3 i Inclination vs. time*

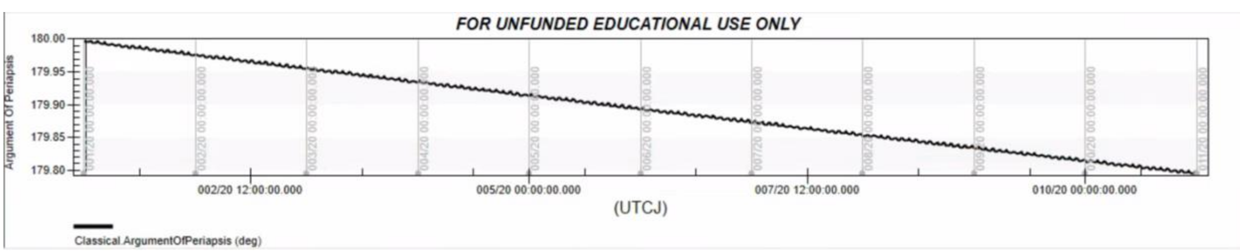


*Figure 34 - P3 i AOP vs. time*

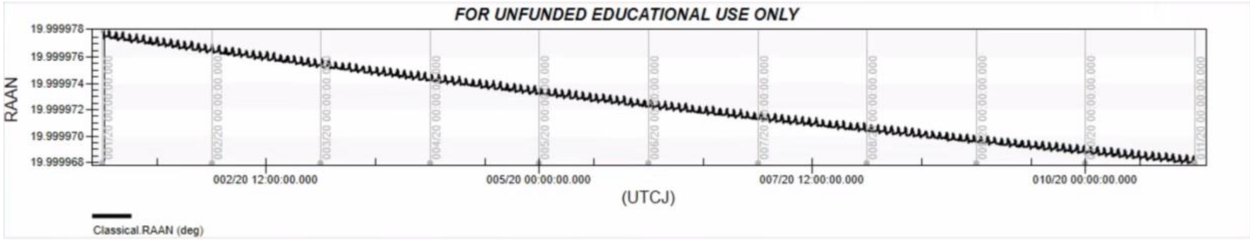


*Figure 35 - P3 i RAAN vs. time*

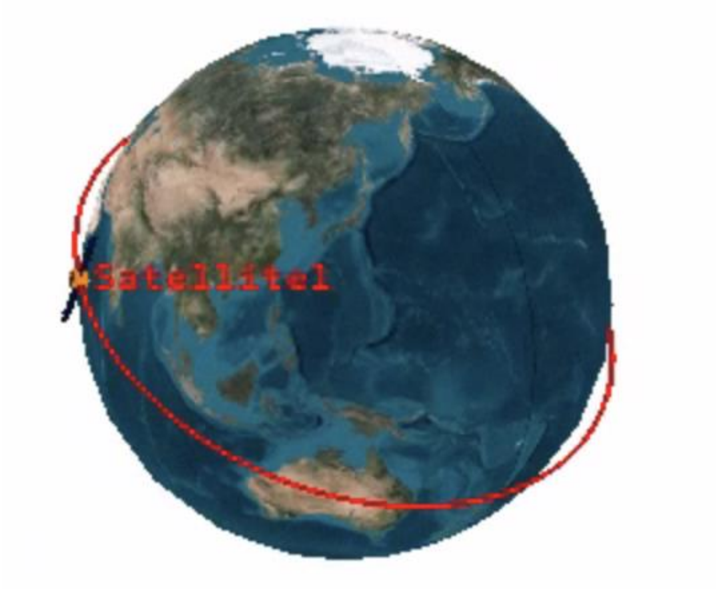


*Figure 36 - P3 i 3D Plot*

From figure 31-35 we can observe the effects of atmospheric drag by slowing down the spacecraft and decreasing the orbit size. It would be expected for the semi-major axis and the eccentricity to decrease in value. But the graphs above show RAAN, inclination, and AOP decreasing in value as well. This is unusual because one would expect these values to remain around the same. But since atmospheric drag physical effects depend on the geometric surface area of the object is possible that the friction force does not act evenly, tilting the spacecraft as it travels along the orbit.

j) What is the change in the values of the semi-major axis, eccentricity and inclination between the beginning and end of this 10-day trajectory?

```
UTC Gregorian Date: 11 Jan 2020 00:00:00.000  UTC Julian Date: 2458859.5
Julian Ephemeris Date: 2458859.50080074
Time past epoch: 864000 sec   (Epoch in UTC Gregorian Date: 1 Jan 2020 00:00:00.000)

State Vector in Coordinate System: Earth J2000

Parameter Set Type:  Cartesian
         X:    -1569.9769023013282094 km              Vx:       7.4244791436226993 km/sec
         Y:    -5861.1302038779031136 km              Vy:      -1.1556088743330302 km/sec
         Z:    -2698.8457996429674495 km              Vz:      -1.9683266633550625 km/sec

Parameter Set Type:  Keplerian
       sma:      6675.4250598320204517 km           RAAN:        19.99999011012416 deg
       ecc:         0.0097963579415510                 w:        179.7956173560071 deg
       inc:           28.4997967104126 deg            TA:        58.60250095464878 deg
```

*Figure 37 - End values after 10 days*

Summary Table:

| **Orbital Elements** | **Initial conditions** | **After 10 days** | **Δ** |
|---|---|---|---|
| Semi-major axis | 6678 km | 6675.425 km | 2.5750 km |
| Eccentricity | 0.01 | 0.00979635 | 2.0365x$10^{-4}$ |
| Inclination | 28.5 degrees | 28.4997967 degrees | 2.0330x$10^{-4}$ degrees |

From this table we can observe that the changes in the orbit remain relatively small for a 10-day period. But it is clear that each one of these values is decreasing slightly.

k) How does the inclusion of a higher-order gravitational model impact your results from part i)? Include relevant plots to justify your answer.

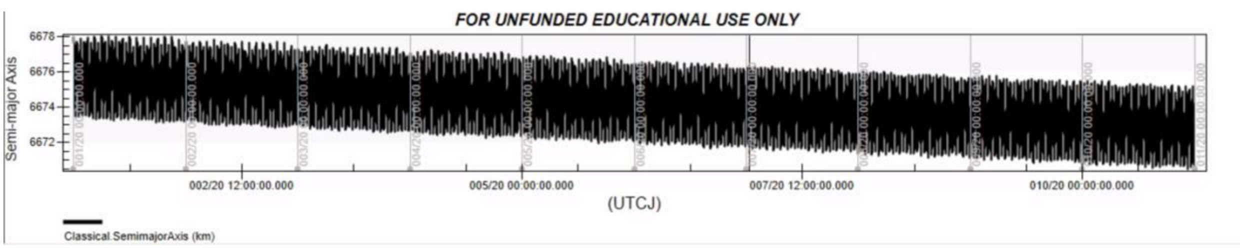


*Figure 38 - P3 k Semi-major axis vs. time*

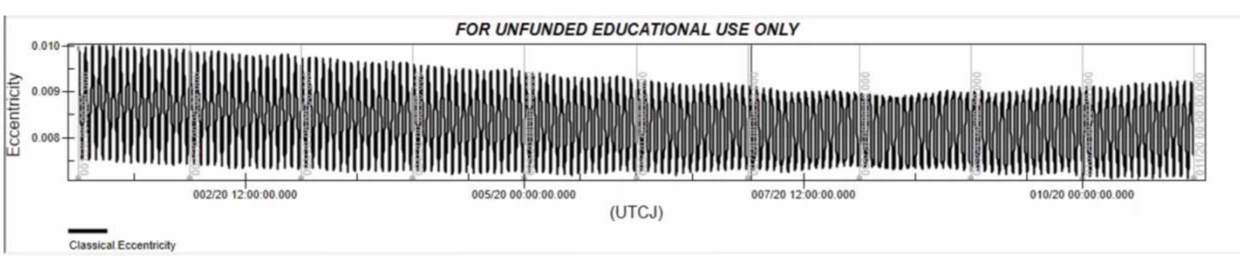


*Figure 39 - P3 k Eccentricity vs. time*

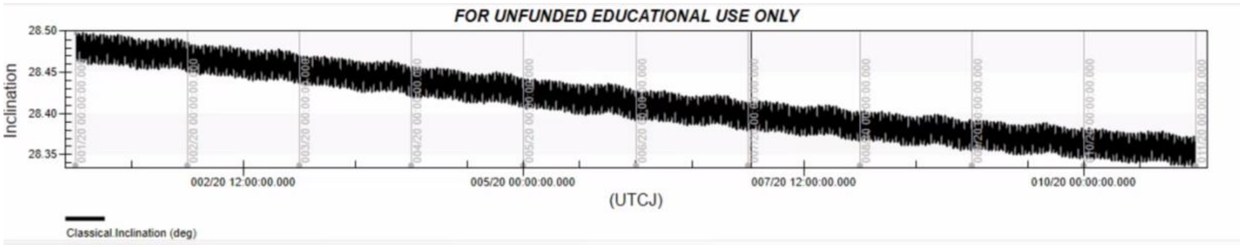


*Figure 40 - P3 k Inclination vs. time*

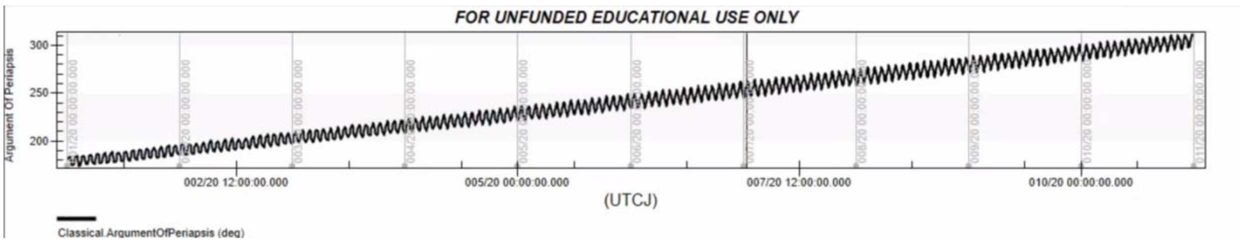


*Figure 41 - P3 k AOP vs. time*

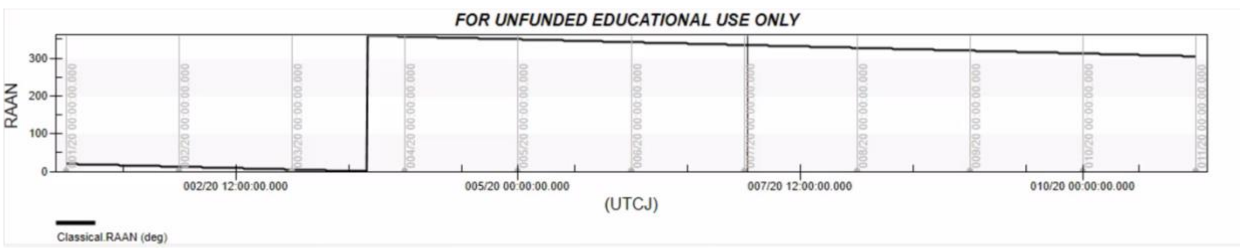


*Figure 42 - P3 k RAAN vs. time*

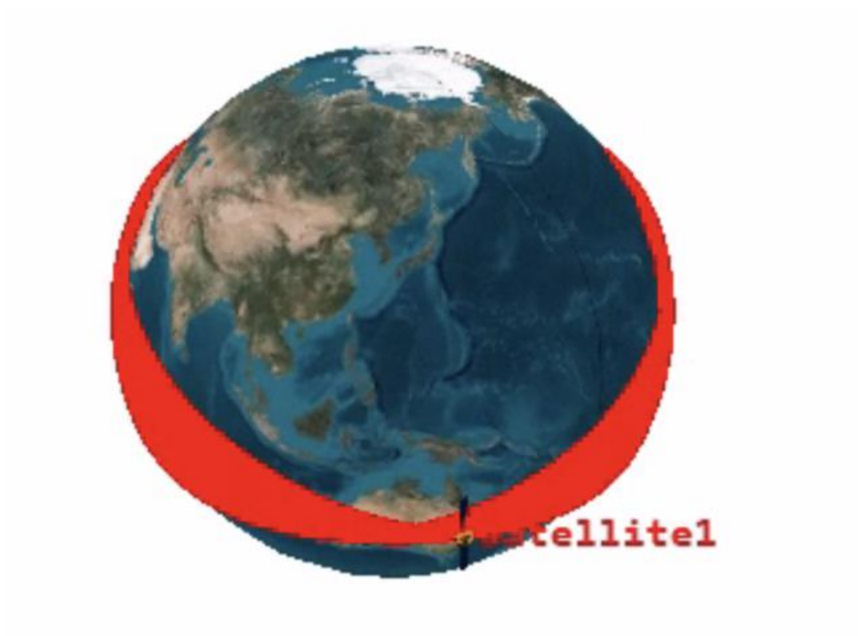

*Figure 43 - P3 k 3D plot*

From figures 38-40 we can observe the same behavior as part i with the added oscillations due to the J2 perturbation. As expected by adding the J2 perturbation and the prograde orbit, RAAN is shifting west as indicated by figure 42 with values decreasing slightly in quantity. AOP advances with respect to the line of nodes as seen in figure 41 with its values increasing as time progresses. Similar to harmonics in the sense that adding fundamental waves can create a more complex waves, adding types of perturbations sequentially to the models seems to compound the changes observed individually in each perturbation model.

```
function [statedot] = EOMfile(t,state,mu)
x = state(1);
y = state(2);
z = state(3);
R = [x,y,z];
r = norm(R);
xdot = state(4);
ydot = state(5);
zdot = state(6);
statedot(1) = xdot;
statedot(2) = ydot;
statedot(3) = zdot;
statedot(4) = -(mu/(r^3))*x;
statedot(5) = -(mu/(r^3))*y;
statedot(6) = -(mu/(r^3))*z;
statedot = statedot';
```

```
%Gustavo Grinsteins
%ASEN 5050
%HW8

%House Keeping
clc;
clear;

%% Problem 1
%given
mu_sun = 1.32712428*10^11;
G = 6.673*10^-20;
rp = 7500;
ra = 8500;
i = 105*(pi/180);
Period = 110*(60);%seconds
rP = 6500;
aP = 2.25*149597870.7;
a = (rp+ra)/2;
ecc = 1 - (rp/a);
mu_P = (a^3)*(((2*pi)/(Period))^2);
mP = mu_P/G;
vP = sqrt(mu_sun/aP);
PlanetOrbitTime = 2*pi*aP*(1/vP);
RAAN_dot = (2*pi)/(PlanetOrbitTime);
J2 = -(2/3)*(RAAN_dot/cos(i))*(((1-ecc^2)^2*a^(7/2))/(sqrt(mu_P)*rP^2));

%%  Problem 2
%given
mu_earth = 3.986004415*10^5;
a_earth = 1.0000010178*149597870.7;
R0 = [-6402,-1809,1065];
V0 = [0.999,-6.471,-4.302];

%magnitudes
r0 = norm(R0); %Km
v0 = norm(V0); %Km/s
H = cross(R0,V0); %Km^2/s
h = norm(H); %Km^2/s
p = (h^2)/mu_earth;
mech_e = ((v0)^2)/(2) - (mu_earth/r0); %Km^2/s^2
%Semi-major axis
a = -mu_earth/(2*mech_e);%Km
%Eccentricity Vector
Ecc = cross(V0,H)*(1/mu_earth) - R0/r0;%Unitless
ecc = norm(Ecc);%Unitless
theta_star0 = abs(acos((p-r0)/(r0*ecc)));
if dot(V0,R0) < 0
   theta_star0 = -theta_star0;
end
%calculatime time from tp to t0
E_0 = 2*atan(sqrt((1-ecc)/(1+ecc))*tan(theta_star0/2));
n = sqrt(mu_earth/(a^3));
t0_minus_tp = (1/n)*(E_0 - ecc*sin(E_0));
tp_to_t1 = t0_minus_tp + (60*60);
E_1 = NewtonRaphsonMethodForE(ecc,n,tp_to_t1);
r1 = a*(1-ecc*cos(E_1));
%calulate state vector with f and g func
```

```
deltaE = E_1-E_0;
f_1 = 1 - (a/r0)*(1-cos(deltaE));%unitless
g_1 = (tp_to_t1-t0_minus_tp) - sqrt((a^3)/mu_earth)*(deltaE-sin(deltaE));%seconds
fdot_1 = (-sin(deltaE)*sqrt(mu_earth*a))/(r0*r1);%1/s
gdot_1 = 1 - (a/r1)*(1-cos(deltaE));%s
R1 = f_1*R0 + g_1*V0;
V1 = fdot_1*R0 + gdot_1*V0;
h1 = norm(cross(R1,V1));
mech_e_1 = ((norm(V1))^2)/(2) - (mu_earth/norm(R1)); %Km^2/s^2
tp_to_t2 = t0_minus_tp + (100*60*60);
E_2 = NewtonRaphsonMethodForE(ecc,n,tp_to_t2);
r2 = a*(1-ecc*cos(E_2));
deltaE = E_2-E_0;
f_2 = 1 - (a/r0)*(1-cos(deltaE));%unitless
g_2 = (tp_to_t2-t0_minus_tp) - sqrt((a^3)/mu_earth)*(deltaE-sin(deltaE));%seconds
fdot_2 = (-sin(deltaE)*sqrt(mu_earth*a))/(r0*r2);%1/s
gdot_2 = 1 - (a/r2)*(1-cos(deltaE));%s
R2 = f_2*R0 + g_2*V0;
V2 = fdot_2*R0 + gdot_2*V0;
h2 = norm(cross(R2,V2));
mech_e_2 = ((norm(V2))^2)/(2) - (mu_earth/norm(R2)); %Km^2/s^2
%% ODE stuff
state0 = [-6402,-1809,1065,0.999,-6.471,-4.302];
%1
options = odeset('Stats','off','RelTol',1*10^-12,'AbsTol',1*10^-12);
[tout1,xout1] = ode45(@EOMfile,[0 60*60],state0,options,mu_earth);
output1 = xout1(end,:);
R1ode = [output1(1),output1(2),output1(3)];
V1ode = [output1(4),output1(5),output1(6)];
partCdeltaR1 = norm(R1ode-R1);
partCdeltaV1 = norm(V1ode-V1);
h1ode = norm(cross(R1ode,V1ode));
mech_e_1ode = ((norm(V1ode))^2)/(2) - (mu_earth/norm(R1ode)); %Km^2/s^2
%2
options = odeset('Stats','off','RelTol',1*10^-12,'AbsTol',1*10^-12);
[tout2,xout2] = ode45(@EOMfile,[0 100*60*60],state0,options,mu_earth);
output2 = xout2(end,:);
output2beg = xout2(1,:);
R2ode12beg = [output2beg(1),output2beg(2),output2beg(3)];
V2ode12beg = [output2beg(4),output2beg(5),output2beg(6)];
h2ode12beg = norm(cross(R2ode12beg,V2ode12beg));
mech_e_2ode12beg = ((norm(V2ode12beg))^2)/(2) - (mu_earth/norm(R2ode12beg));
R2ode12 = [output2(1),output2(2),output2(3)];
V2ode12 = [output2(4),output2(5),output2(6)];
partCdeltaR2 = norm(R2ode12-R2);
partCdeltaV2 = norm(V2ode12-V2);
h2ode12 = norm(cross(R2ode12,V2ode12));
mech_e_2ode12 = ((norm(V2ode12))^2)/(2) - (mu_earth/norm(R2ode12)); %Km^2/s^2
deltaR_12 = norm(R2ode12-R2);
deltaV_12 = norm(V2ode12-V2);
deltah_12 = h2ode12-h2ode12beg;
deltaMech_e_12 = mech_e_2ode12-mech_e_2ode12beg;
%%%%%%
options = odeset('Stats','off','RelTol',1*10^-10,'AbsTol',1*10^-10);
[tout2,xout2] = ode45(@EOMfile,[0 100*60*60],state0,options,mu_earth);
output2 = xout2(end,:);
output2beg = xout2(1,:);
R2ode10beg = [output2beg(1),output2beg(2),output2beg(3)];
```

```
V2ode10beg = [output2beg(4),output2beg(5),output2beg(6)];
h2ode10beg = norm(cross(R2ode10beg,V2ode10beg));
mech_e_2ode10beg = ((norm(V2ode10beg))^2)/(2) - (mu_earth/norm(R2ode10beg));
R2ode10 = [output2(1),output2(2),output2(3)];
V2ode10 = [output2(4),output2(5),output2(6)];
h2ode10 = norm(cross(R2ode10,V2ode10));
mech_e_2ode10 = ((norm(V2ode10))^2)/(2) - (mu_earth/norm(R2ode10)); %Km^2/s^2
deltaR_10 = norm(R2ode10-R2);
deltaV_10 = norm(V2ode10-V2);
deltah_10 = h2ode10-h2ode10beg;
deltaMech_e_10 = mech_e_2ode10-mech_e_2ode10beg;
%%%%%%%
options = odeset('Stats','off','RelTol',1*10^-8,'AbsTol',1*10^-8);
[tout2,xout2] = ode45(@EOMfile,[0 100*60*60],state0,options,mu_earth);
output2 = xout2(end,:);
output2beg = xout2(1,:);
R2ode8beg = [output2beg(1),output2beg(2),output2beg(3)];
V2ode8beg = [output2beg(4),output2beg(5),output2beg(6)];
h2ode8beg = norm(cross(R2ode8beg,V2ode8beg));
mech_e_2ode8beg = ((norm(V2ode8beg))^2)/(2) - (mu_earth/norm(R2ode8beg));
R2ode8 = [output2(1),output2(2),output2(3)];
V2ode8 = [output2(4),output2(5),output2(6)];
h2ode8 = norm(cross(R2ode8,V2ode8));
mech_e_2ode8 = ((norm(V2ode8))^2)/(2) - (mu_earth/norm(R2ode8)); %Km^2/s^2
deltaR_8 = norm(R2ode8-R2);
deltaV_8 = norm(V2ode8-V2);
deltah_8 = h2ode8-h2ode8beg;
deltaMech_e_8 = mech_e_2ode8-mech_e_2ode8beg;
%%%%%%%
options = odeset('Stats','off','RelTol',1*10^-6,'AbsTol',1*10^-6);
[tout2,xout2] = ode45(@EOMfile,[0 100*60*60],state0,options,mu_earth);
output2 = xout2(end,:);
output2beg = xout2(1,:);
R2ode6beg = [output2beg(1),output2beg(2),output2beg(3)];
V2ode6beg = [output2beg(4),output2beg(5),output2beg(6)];
h2ode6beg = norm(cross(R2ode6beg,V2ode6beg));
mech_e_2ode6beg = ((norm(V2ode6beg))^2)/(2) - (mu_earth/norm(R2ode6beg));
R2ode6 = [output2(1),output2(2),output2(3)];
V2ode6 = [output2(4),output2(5),output2(6)];
h2ode6 = norm(cross(R2ode6,V2ode6));
mech_e_2ode6 = ((norm(V2ode6))^2)/(2) - (mu_earth/norm(R2ode6)); %Km^2/s^2
deltaR_6 = norm(R2ode6-R2);
deltaV_6 = norm(V2ode6-V2);
deltah_6 = h2ode6-h2ode6beg;
deltaMech_e_6 = mech_e_2ode6-mech_e_2ode6beg;
%%%%%%%
options = odeset('Stats','off','RelTol',1*10^-4,'AbsTol',1*10^-4);
[tout2,xout2] = ode45(@EOMfile,[0 100*60*60],state0,options,mu_earth);
output2 = xout2(end,:);
output2beg = xout2(1,:);
R2ode4beg = [output2beg(1),output2beg(2),output2beg(3)];
V2ode4beg = [output2beg(4),output2beg(5),output2beg(6)];
h2ode4beg = norm(cross(R2ode4beg,V2ode4beg));
mech_e_2ode4beg = ((norm(V2ode4beg))^2)/(2) - (mu_earth/norm(R2ode4beg));
R2ode4 = [output2(1),output2(2),output2(3)];
V2ode4 = [output2(4),output2(5),output2(6)];
h2ode4 = norm(cross(R2ode4,V2ode4));
mech_e_2ode4 = ((norm(V2ode4))^2)/(2) - (mu_earth/norm(R2ode4)); %Km^2/s^2
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deltaR_4 = norm(R2ode4-R2);
deltaV_4 = norm(V2ode4-V2);
deltah_4 = h2ode4-h2ode4beg;
deltaMech_e_4 = mech_e_2ode4-mech_e_2ode4beg;
```