Gustavo Grinsteins Planchart ASEN 5050 HW4

<u>Note:</u> For details in calculations, please refer to the corresponding MATLAB script and results attached to this document. Answers are in **bold** followed by <-- symbol.

## Problem 1:

## Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- $Gm_{Saturn} = 3.794x10^7 Km^3/s^2$
- Equatorial radius of Saturn = 60,268 km

At  $t_1$  in  $\hat{X}\hat{Y}\hat{Z}$  - Saturn centered inertial frame with Saturn's equatorial plane as reference:

$$\bar{r}_1 = -720,000\hat{X} + 670,000\hat{Y} + 310,000\hat{Z} km$$
$$\bar{v}_1 = 2.160\hat{X} - 3.360\hat{Y} + 0.620\hat{Z} km/s$$

Calculate the duration of time between t<sub>1</sub> and impact.

From HW3 we know that at t<sub>1</sub>:

$$e_1 = 0.9102$$

Since  $0 < e_1 < 1$ , the orbit conic is an ellipse. Therefore, we can use Kepler's equation for ellipses

$$a_1 = \frac{-\mu_{Saturn}}{2 \cdot \varepsilon_1} = 662,781 \, km$$

$$\theta^*_1 = \pm \cos^{-1}\left(\frac{\bar{e}_1 \cdot \bar{r}_1}{|\bar{e}_1| \cdot |\bar{r}_1|}\right)\left(\frac{180}{\pi}\right) = \pm 167.83^\circ = -167.83^\circ$$

Since  $\bar{r}_1 \cdot \bar{v}_1 < 0$ , Cassini at t<sub>1</sub> was heading towards periapsis therefore  $\theta^*_1 < 0$ 

From HW3 we know that at t<sub>impact</sub>:

Given the Relative 2BP assumptions, the orbital elements shown above remain constant (except true anomaly). Therefore, they can be used to calculate the information at impact:

$$e_{impact} = e_1 = e = 0.9102$$
 $a_{impact} = a_1 = a = \frac{-\mu_{Saturn}}{2 \cdot \varepsilon_1} = 662,781 \text{ km}$ 
 $r_{impact} = 60,268 \text{ km}$ 
 $p_1 = a_1 \cdot (1 - e_1^2) = 113,681 \text{ km}$ 
 $r_{periapsis} = \frac{p_1}{1 + e_1} = 59,512 \text{ km}$ 
 $r_{apoapsis} = \frac{p_1}{1 - e_1} = 1,266,048 \text{ km}$ 
 $r_1 = 1,031,212 \text{ km}$ 

Since  $r_{impact} > r_{periapsis}$  Cassini is on a crash trajectory as it travels towards periapsis. In addition,  $r_1 < r_{apoapsis}$  therefore Cassini is pass apoapsis heading towards periapsis.

$$\theta^*_{impact} = \pm cos^{-1} \left( \frac{p_1 - r_{impact}}{r_{impact} \cdot e_1} \right) \left( \frac{180}{\pi} \right) = \pm 13.17^{\circ} = -13.17^{\circ}$$

Since  $\bar{r}_1 \cdot \bar{v}_1 < 0$ , Cassini at  $t_1$  was heading towards periapsis and therefore  $\theta^*_{impact}$  will range between [0, -180] degrees.

Using  $\theta^*_{impact}$  and  $\theta^*_{1}$ , the eccentric anomalies at  $t_1$  and  $t_{impact}$  can be calculated:

$$E_{1} = 2 \tan^{-1} \left( \sqrt{\frac{1 - e}{1 + e}} \tan \left( \frac{\theta^{*}_{1}}{2} \right) \right) \left( \frac{180}{\pi} \right) = -127.64^{\circ}$$

$$E_{impact} = 2 \tan^{-1} \left( \sqrt{\frac{1 - e}{1 + e}} \tan \left( \frac{\theta^{*}_{impact}}{2} \right) \right) \left( \frac{180}{\pi} \right) = -2.87^{\circ}$$

Using these values, we can calculate the mean anomalies M<sub>1</sub> and M<sub>impact</sub> using Kepler's equation:

$$n = \sqrt{\frac{\mu_{Saturn}}{a^3}} = 1.1415x10^{-5} rad/s$$

$$M_1 = n(t_1 - t_p) = E_1 - e \sin(E_1) = -1.5070 \ radians$$

$$M_{impact} = n(t_{impact} - t_p) = E_{impact} - e \sin(E_{impact}) = -0.004513 \ radians$$

the duration of time between t<sub>1</sub> and impact is:

$$t_{impact} - t_1 = \frac{M_{impact} - M_1}{n} \left( \frac{1 \ hour}{3600 \ seconds} \right) = 36.5616 \ Hours < --$$

## Problem 2:

## Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - o Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Gm<sub>Moon</sub> = 4902.799 Km3/s2
- Equatorial radius of Moon = 1,738 km

At a time  $t_1$ , on the date that this homework is assigned, the spacecraft is described by the following state information, measured relative to the Moon and in an inertial frame, (XYZ), that uses the Moon equatorial plane as the reference plane:

$$\begin{split} \bar{r}_1 = & \ -7.87701x10^2\hat{X} - 8.81425x10^2\hat{Y} + 1.43864x10^3\hat{Z} \ km \\ \bar{v}_1 = & \ 0.98370\hat{X} + 0.76950\hat{Y} + 1.01416\hat{Z} \ km/s \end{split}$$

Finding the orbital elements for t<sub>1</sub>:

$$\begin{split} \bar{h}_1 &= \bar{r}_1 x \; \bar{v}_1 = -2.0009 x 10^3 \hat{X} + 2.2140 x 10^3 \hat{Y} + 0.2609 x 10^3 \hat{Z} \; km^2/s \\ & \varepsilon_1 = \frac{{v_1}^2}{2} - \frac{\mu_{Moon}}{r_1} = \; -1.3389 \; km^2/s^2 \end{split}$$

Since  $\varepsilon_1 < 0$ , LRO is in a Moon-centric orbit that is bounded.

$$\bar{e}_1 = \frac{\bar{v}_1 x \bar{h}_1}{\mu_{Saturn}} - \frac{\bar{r}_1}{r_1} = 0.0060\hat{X} + 0.0071\hat{Y} - 0.0144\hat{Z}$$

$$\|\bar{e}_1\| = e_1 = 0.01711$$

Since  $0 < e_1 < 1$ , the orbit conic is an ellipse. Since  $e_1$  is close to zero we will have a near circular elliptical orbit.

$$a_1 = \frac{-\mu_{Moon}}{2 \cdot \varepsilon_1} = 1830.8853 \, km$$

$$i_1 = \pm \cos^{-1} \left( \frac{\hat{Z} \cdot \bar{h}_1}{|\hat{z}| \cdot |\bar{h}_1|} \right) \left( \frac{180}{\pi} \right) = \pm 85.00^{\circ} = 85.00^{\circ}$$

Since the angular momentum's Z component is positive, the orbit follows a prograde direction from a top-down reference view. No sign check needed for i.

$$\bar{n}_1 = \hat{Z} x \bar{h}_1 = -2.2140x10^3 \hat{X} - 2.0009x10^3 \hat{Y} + 0\hat{Z} km^2/s$$

$$\Omega_1 = \pm \cos^{-1} \left( \frac{\hat{X} \cdot \bar{n}_1}{|\hat{X}| \cdot |\bar{n}_1|} \right) \left( \frac{180}{\pi} \right) = \pm 137.89^\circ = -137.89^\circ$$

Since  $\hat{\mathbf{Y}}\cdot \bar{n}_1 < 0$  we select the negative value for RAAN.

$$\omega_1 = \pm \cos^{-1} \left( \frac{\bar{e}_1 \cdot \bar{n}_1}{|\bar{e}_1| \cdot |\bar{n}_1|} \right) \left( \frac{180}{\pi} \right) = \pm 122.65^{\circ} = -122.65^{\circ}$$

Since  $\hat{Z} \cdot \bar{e}_1 < 0$  we select the negative value for AOP. The eccentricity vector will lie below the Moon's equatorial plane.

$$\theta^*_1 = \pm \cos^{-1}\left(\frac{\bar{e}_1 \cdot \bar{r}_1}{|\bar{e}_1| \cdot |\bar{r}_1|}\right)\left(\frac{180}{\pi}\right) = \pm 173.51^\circ = 173.51^\circ$$

Since  $\bar{r}_1\cdot\bar{v}_1>0$ , LRO at  $t_1$  was heading away from periapsis therefore  $\theta^*_{\ 1}>0$ 

Given the Relative 2BP assumptions, the orbital elements above will remain constant with the exception of the true anomaly at  $t_1$ .

a) In the current orbit of the spacecraft, how long (in hours) is the spacecraft located above the Moon's equatorial plane? Label this time t<sub>pos</sub>.

The ascending and descending nodes of the orbit can be referenced to determine tpos:

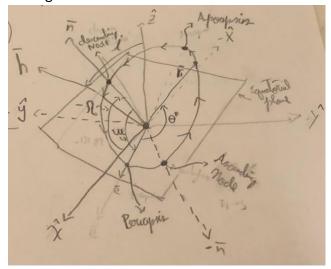


Figure 1- Problem 2: Orbit Depiction

According to the orbit drawn in figure 1, the true anomalies of the ascending and descending nodes are as follows:

$$\theta^*_{AscNode} = 180 - |\omega| = 57.35$$
 °  $\theta^*_{DescNode} = \omega = -122.65$  °

$$n = \sqrt{\frac{\mu_{Moon}}{a^3}} = 8.9378x10^{-4} rad/s$$

$$\mathbb{P}(orbit\ Period) = 2\pi \sqrt{\frac{a^3}{\mu_{Moon}}} = 7.0299x10^3 seconds$$

$$E_{AscNode} = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta^*_{AscNode}}{2}\right)\right)\left(\frac{180}{\pi}\right) = 56.52^\circ$$

$$E_{DescNode} = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta^*_{DescNode}}{2}\right)\right)\left(\frac{180}{\pi}\right) = -121.82^\circ$$

 $(t_{AscNode} - t_p) = \left(\frac{1}{n}\right) E_{AscNode} - e \sin(E_{AscNode}) = 1.0878 \times 10^3 seconds$ 

 $\left(t_{AscNode}-t_{p}
ight)$  represents the time elapsed for the traversal of LRO from periapsis to the ascending node

$$\left(t_{DescNode}-t_p\right)=\left(\frac{1}{n}\right)E_{DescNode}-e\sin(E_{DescNode})=-2.3627x10^3seconds$$
  $\left(t_{DescNode}-t_p\right)$  represents the time elapsed for the traversal of LRO from the descending node to periapsis (therefore the negative sign)

$$t_{pos} = (\mathbb{P} - |(t_{DescNode} - t_p)| - |(t_{DescNode} - t_p)|)(\frac{1 \text{ hour}}{3600 \text{ seconds}}) = 0.9943 \text{ Hours} < --$$

b) In the current orbit of the spacecraft, how long (in hours) is the spacecraft located below the Moon's equatorial plane? Label this time  $t_{\text{neg}}$ .

$$t_{neg} = \left(\left|\left(t_{DescNode} - t_p\right)\right| + \left|\left(t_{DescNode} - t_p\right)\right|\right) \left(\frac{1 \text{ hour}}{3600 \text{ seconds}}\right) = 0.9585 \text{ Hours} < --$$

c) Compare the times computed in parts a) and b) and justify, using your knowledge of orbit geometry, whether they should be similar or different. If they should be different, which time do you think should be larger and why: t<sub>pos</sub> or t<sub>neg</sub>?

The given values demonstrate that  $t_{pos} > t_{neg}$ . However, the difference between the values is small at around 0.04 hours. These results make sense given the near-circular elliptical orbit geometry (e = 0.01711). This explains the small but existent difference between the two values and that since periapsis lies below the moon's equatorial plane,  $t_{neg}$  will be smaller than  $t_{pos}$  (periapsis is closer to the moon than apoapsis making the distance traversed below the equatorial plane smaller therefore making the time to traverse smaller).<--

d) Construct an iterative numerical procedure to solve Kepler's equation for the eccentric anomaly, given a time past periapsis. Discuss the method you used, noting the equation used to update the eccentric anomaly at each iteration and justify your stopping condition as well as any tolerances. Include your code as a supplement at the end of your homework

The numerical procedure I used to create the iterative function is based on the Newton-Raphson method. The iteration uses the following equation:

$$E_1 = E_0 - \frac{f(E_0)}{f'(E_0)}$$

Where function f is defined below given the root finding approach:

$$M = E - e \sin(E) \rightarrow M - E + e \sin(E) = 0 \rightarrow f(E) = M - E + e \sin(E)$$

The derivative of f(E) is defined as:

$$f'(E) = -1 + e\cos(E)$$

 $E_0$  is the initial guess for the eccentric anomaly which is set as M in the code. The reason to set M as  $E_0$  for this problem is that given the near circular elliptic orbit the mean motion value at that specific point in time will lie close to the corresponding eccentric anomaly value.

 $E_1$  is the resulting new guess using the Newton-Raphson equation. There are two stopping conditions in the function. The first one is based on the tolerance of the difference between calculated values  $E_1$  and  $E_0$ . The code is set to have a tolerance of  $10^{-4}$  (accuracy to the  $4^{th}$  decimal digit). The second stopping condition avoids having a dividing by zero exception by checking the value of derivative function and making sure is not less than a tolerance value that is currently set to  $10^{-14}$ . For details on the function implemented in code please refer to the NewtonRaphsonForE.m code attached to this submission. <--

e) 30 minutes after t₁ in the described orbit, what is the altitude of the LRO spacecraft relative to the Moon? At this new location, is LRO moving towards or away from periapsis?

Since the function described above calculates E based on the time passed from periapsis, the time it takes to arrive at t<sub>1</sub> from periapsis needs to be determined:

From the calculated orbital elements for the given LRO position and velocity values at t1:

$$\theta^*_1 = 173.51^\circ$$

$$E_1 = 2 \tan^{-1} \left( \sqrt{\frac{1-e}{1+e}} \tan \left( \frac{\theta_1^*}{2} \right) \right) \left( \frac{180}{\pi} \right) = 173.40^\circ$$

$$(t_1 - t_p) = (\frac{1}{n})E_1 - e\sin(E_1) = 3.3839x10^3 seconds$$

Given that we are interested in the moment in time 30 minutes after t<sub>1</sub>:

$$(t_2 - t_p) = (t_1 - t_p) + 30 \text{ minutes } \left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right) = 5.1839 \text{x} 10^3 \text{ seconds}$$

Using the iterative method of part d,  $E_2$  comes to be 4.6162 radians or 264.49 degrees. Converting this into a true anomaly value:

$$\theta_2^* = 2 \tan^{-1} \left( \sqrt{\frac{1+e}{1-e}} \tan \left( \frac{E_2}{2} \right) \right) \left( \frac{180}{\pi} \right) = -96.49^\circ$$

Since  $heta^*_2 < 0$ , the spacecraft is headed towards periapsis.<--

Finding the altitude at  $\theta^*_2$ :

$$r_2 = \frac{a(1 - e^2)}{1 + e\cos(\theta^*_2)} = 1.8339x10^3 km$$

Altitude at  $t_2 = r_2 - EQR_{Moon} = 1.8339x10^3 - 1.738x10^3 = 95.8944 \ km < --$ 

```
%Gustavo Grinsteins
%ASEN 5050
%HW4 - Iterative function
function ComputedE = NewtonRaphsonMethodForE(e,n,elapsedTimeSec)
%NewtonRaphsonMethod: To solve Kepler's equation
%initial values
M = n*elapsedTimeSec;%Radians
E0 = M; %Initial value for eccentric anomaly
diff = 1000;%Initial value for tolerance stopping condition
tolerance = 10^-4; %4 digit accuracy is desired
epsilon = 10^-14; %Do not divide by a number smaller than this
%Iterate until the tolerance codition is met
    while (abs(diff) > tolerance)
    f = M - E0 + e*sin(E0);
    fprime = -1 + e*cos(E0);
   %divide by zero check
        if (abs(fprime)< epsilon)</pre>
            fprintf('divide by zero exception \n\n')
            %If f prime is getting to small break from the loop
        end
   New eccentric anomaly calc by newton-raphson method
   E1 = E0 - (f)/(fprime);
    %update values to keep iterating
    diff = abs(E1-E0);
   E0 = E1;
    end
ComputedE = E1;
fprintf('The eccentric anomaly after %4.2f minutes pass periapsis is %4.4f radians\n', ∠
elapsedTimeSec*(1/60),ComputedE)
```

```
%Gustavo Grinsteins
%ASEN 5050
%HW4
%House Keeping
clc;
clear;
%Given values
mu_moon = 4902.799; %km^3/s^2
mu_saturn = 3.794*10^7; %km^3/s^2
EQR_{moon} = 1738; %Km
EQR_saturn = 60268; %Km
%% Problem 1
fprintf('Problem 1 \n')
%Given from HW3
R_1 = [-720000;670000;310000];%km
V 1 = [2.160; -3.360; 0.620]; %km/s
H_1 = cross(R_1, V_1); %km^2/s
r_1 = norm(R_1);
v 1 = norm(V 1);
Sp_Mech_E_1 = (v_1^2/2) - (mu_saturn/r_1);
a_1 = (-mu_saturn)/(2*Sp_Mech_E_1);
Ecc_1 = (cross(V_1, H_1))/(mu_saturn)-(R_1/r_1);
ecc_1 = norm(Ecc_1);
theta_star_1 = abs(acos((dot(R_1,Ecc_1))/(norm(Ecc_1)*norm(R_1))));
if dot(R_1,V_1)<0</pre>
    theta_star_1 = (-1)*theta_star_1;
end
fprintf('True Anomaly at t1 = %4.2f deg \n',theta_star_1*(180/pi))
theta_star_impact = (-13.17)*(pi/180);
fprintf('True Anomaly at impact = %4.2f deg \n',theta_star_impact*(180/pi))
%calculating eccentric anomalies
E_1 = 2*atan(sqrt((1-ecc_1)/(1+ecc_1))*tan(theta_star_1/2));
fprintf('Eccentric Anomaly at t1 = %4.2f \text{ deg } n', E_1*(180/pi))
E_{impact} = 2*atan(sqrt((1-ecc_1)/(1+ecc_1))*tan(theta_star_impact/2));
fprintf('Eccentric Anomaly at impact = %4.2f deg \n',E_impact*(180/pi))
n = sqrt(mu_saturn/(a_1^3));
fprintf('mean motion = %4.4f rad/s \n',n)
M_1 = E_1 - ecc_1*sin(E_1);
fprintf('Mean Anomaly at t1 = %4.4f radians n',M_1)
M_impact = E_impact - ecc_1*sin(E_impact);
fprintf('Mean Anomaly at impact = %4.7f radians \n',M_impact)
T1_To_TImpact_Time = (M_impact - M_1)/n;
fprintf('Time from t1 to t_impact = %4.4f hours \n\n', T1_To_TImpact_Time/3600)
%% Problem 2 Part a b c
% Calculate the orbita elements at t1
fprintf('Problem 2 Part a b c \n')
R_1 = [-7.87701*10^2; -8.81425*10^2; 1.43864*10^3]; 
V 1 = [0.98370; 0.76950; 1.01416]; %km/s
%Calculating orbital elements at t1
%Recover Orbital Elements
r 1 = norm(R 1);
v_1 = norm(V_1);
H_1 = cross(R_1, V_1);
%fprintf('<%4.4f,%4.4f,%4.4f> \n',H_1)
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```
h_1 = norm(H_1);
Sp Mech E 1 = (v 1^2/2) - (mu moon/r 1);
%inclination
z_hat = [0,0,1];
i_1 = acos((dot(z_hat,H_1))/(norm(z_hat)*norm(H_1)));
fprintf('inclination angle i at t1 = %4.2f deg h',i_1*(180/pi))
%maior-axis
a_1 = (-mu_moon)/(2*Sp_Mech_E_1);
fprintf('semi-major axis a at t1 = %4.4f Km\n',a 1)
%eccentricity
Ecc_1 = (cross(V_1,H_1))/(mu_moon)-(R_1/r_1);
ecc_1 = norm(Ecc_1);
fprintf('e at t1 is %4.5f\n',ecc_1)
%RAAN
x_hat = [1,0,0];
y_hat = [0,1,0];
N_1 = cross(z_hat, H_1);
RAAN_1 = abs(acos((dot(x_hat,N_1))/(norm(x_hat)*norm(N_1))));
if dot(N_1,y_hat)<0</pre>
    RAAN 1 = (-1)*RAAN 1;
end
fprintf('RAAN at t1 is %4.2f deg \n', RAAN 1*(180/pi))
AOP_1 = abs(acos((dot(Ecc_1,N_1))/(norm(Ecc_1)*norm(N_1))));
if dot(Ecc_1,z_hat)<0</pre>
    AOP_1 = (-1)*AOP_1;
fprintf('Argument of Periapsis w at t1 = %4.2f \text{ deg } n',AOP_1*(180/pi))
theta star = abs(acos((dot(R 1, Ecc 1))/(norm(Ecc 1)*norm(R 1))));
if dot(R_1,V_1)<0</pre>
    theta_star = (-1)*theta_star;
end
p_1 = a_1*(1-ecc_1^2); %Km
r_p = (p_1)/(1+ecc_1*cosd(0));
r_a = (p_1)/(1+ecc_1*cosd(180));
fprintf('True Anomaly ThetaStar = %4.2f deg \n',theta_star*(180/pi))
fprintf('Moon Radius = %4.4f Km \n', EQR_moon)
fprintf('Moon orbit periapsis radius = %4.4f km \n',r_p)
fprintf('Moon orbit apoapsis radius = %4.4f km \n',r a)
%Calculating Eccentric anomalies at ascending and descending nodes
theta_star_descending = AOP_1;
theta_star_ascending = pi-((-1)*AOP_1);
n = sqrt(mu_moon/(a_1^3));
period = 2*pi*sqrt((a_1^3)/mu_moon);
E_ascending = 2*atan(sqrt((1-ecc_1)/(1+ecc_1))*tan(theta_star_ascending/2));
tasc_minus_tp = (1/n)*(E_ascending - ecc_1*sin(E_ascending));*time from tp to asc
E_descending = 2*atan(sqrt((1-ecc_1)/(1+ecc_1))*tan(theta_star_descending/2));
tdesc_minus_tp = (1/n)*(E_descending - ecc_1*sin(E_descending));%time from desc to tp tpos = period - abs(tdesc_minus_tp) - abs(tasc_minus_tp);
tneg = abs(tdesc_minus_tp) + abs(tasc_minus_tp);
fprintf('Tpos = %4.4f hours \n',tpos/3600)
fprintf('Tneg = %4.4f hours \n\n',tneg/3600)
%% Problem 2 Part d c
fprintf('Problem 2 Part d c \n')
E_1 = 2*atan(sqrt((1-ecc_1)/(1+ecc_1))*tan(theta_star/2));
fprintf('E at t1 = %4.2f degrees \n', E_1*(180/pi))
t1_{minus_tp} = (1/n)*(E_1 - ecc_1*sin(E_1));
fprintf('t1 - tp = %4.4f seconds \n',t1_minus_tp)
```

```
t2_minus_tp = t1_minus_tp + 30*(60); fprintf('t2 - tp = %4.4f seconds \n',t2_minus_tp)  
E_2 = NewtonRaphsonMethodForE(ecc_1,n,t2_minus_tp); fprintf('E at t2 = %4.2f degrees \n',E_2*(180/pi))  
theta_star_2 = 2*atan(sqrt((1+ecc_1)/(1-ecc_1))*tan((E_2)/2)); fprintf('True Anomaly at t2 = %4.2f deg \n',theta_star_2*(180/pi))  
r_2 = (p_1)/(1+ecc_1*cos(theta_star_2)); fprintf('orbit radius at t2 = %4.4f km \n',r_2)  
Altitude_t2 = r_2 - EQR_moon; fprintf('Altitude at t2 = %4.4f km \n',Altitude_t2)
```

```
Problem 1
True Anomaly at t1 = -167.83 deg
True Anomaly at impact = -13.17 deg
Eccentric Anomaly at t1 = -127.64 deg
Eccentric Anomaly at impact = -2.87 deg
mean motion = 0.0000 rad/s
Mean Anomaly at t1 = -1.5070 radians
Mean Anomaly at impact = -0.0045128 radians
Time from t1 to t_impact = 36.5616 hours
Problem 2 Part a b c
inclination angle i at t1 = 85.00 deg
semi-major axis a at t1 = 1830.8853 Km
e at t1 is 0.01711
RAAN at t1 is -137.89 deg
Argument of Periapsis w at t1 = -122.65 deg
True Anomaly ThetaStar = 173.51 deg
Moon Radius = 1738.0000 Km
Moon orbit periapsis radius = 1799.5565 km
Moon orbit apoapsis radius = 1862.2141 km
Tpos = 0.9943 hours
Tneg = 0.9585 hours
Problem 2 Part d c
E at t1 = 173.40 degrees
t1 - tp = 3383.8558 seconds
t2 - tp = 5183.8558 seconds
The eccentric anomaly after 86.40 minutes pass periapsis is 4.6162 radians
E at t2 = 264.49 degrees
True Anomaly at t2 = -96.49 deg
orbit radius at t2 = 1833.8944 km
Altitude at t2 = 95.8944 \text{ km}
```