02/09/2021

Gustavo Grinsteins Planchart
ASEN 5050
HW3

Note: For details in calculations, please refer to the corresponding MATLAB script and results attached to this document. Answers are in **bold** followed by <-- symbol.

Problem 1:

Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- $Gm_{Saturn} = 3.794 \times 10^7\ Km^3/s^2$
- Equatorial radius of Saturn = 60,268 km

At t_1 in $\hat{X}\hat{Y}\hat{Z}$ - Saturn centered inertial frame with Saturn's equatorial plane as reference:

$$\bar{r}_1 = -720{,}000\hat{X} + 670{,}000\hat{Y} + 310{,}000\hat{Z}\ km$$
$$\bar{v}_1 = 2.160\hat{X} - 3.360\hat{Y} + 0.620\hat{Z}\ km/s$$

a) Calculate the position and velocity vectors of the spacecraft at impact, expressed in the Saturn-centered inertial frame:

Calculating orbital elements at t_1:

$$\bar{h}_1 = \bar{r}_1 x\ \bar{v}_1 = 1{,}457{,}000\hat{X} + 1{,}116{,}000\hat{Y} + 972{,}000\hat{Z}\ km^2/s$$

$$\varepsilon_1 = \frac{{v_1}^2}{2} - \frac{\mu_{Saturn}}{r_1} = -28.6218\ km^2/s^2$$

Since $\varepsilon_1 < 0$, Cassini is in a Saturn-centric orbit that is bounded.

$$\bar{e}_1 = \frac{\bar{v}_1 x\ \bar{h}_1}{\mu_{Saturn}} - \frac{\bar{r}_1}{r_1} = 0.5939\hat{X} - 0.6812\hat{Y} - 0.1080\hat{Z}$$

$$|\bar{e}_1| = e_1 = 0.9102$$

Since $0 < e_1 < 1$, the orbit conic is an ellipse

$$a_1 = \frac{-\mu_{Saturn}}{2 \cdot \varepsilon_1} = 662{,}781\ km$$

$$i_1 = \pm cos^{-1}\left(\frac{\hat{Z} \cdot \bar{h}_1}{|\hat{z}| \cdot |\bar{h}_1|}\right) = \pm 62.09° = 62.09°$$

Since the angular momentum's Z component is positive, the orbit follows a prograde direction from a top-down reference view. No sign check needed for i.

$$\bar{n}_1 = \hat{Z}\ x\ \bar{h}_1 = -1{,}116{,}000\hat{X} + 1{,}457{,}000\hat{Y} + 0\hat{Z}\ km^2/s$$

$$\Omega_1 = = \pm cos^{-1}\left(\frac{\hat{X} \cdot \bar{n}_1}{|\hat{X}| \cdot |\bar{n}_1|}\right) = \pm 127.45° = 127.45°$$

Since $\hat{Y} \cdot \bar{n}_1 > 0$ we select the positive value for RAAN.

$$\omega_1 = = \pm cos^{-1}\left(\frac{\bar{e}_1 \cdot \bar{n}_1}{|\bar{e}_1| \cdot |\bar{n}_1|}\right) = \pm 172.28° = -172.28°$$

Since $\hat{Z} \cdot \bar{e}_1 < 0$ we select the negative value for AOP.

Given the Relative 2BP assumptions, the orbital elements calculated above remain constant. Therefore, they can be used to calculate the information at impact:

$$r_{impact} = 60{,}268\ km$$

$$p_1 = a_1 \cdot (1 - {e_1}^2) = 113{,}681\ km$$

$$r_{periapsis} = \frac{p_1}{1 + e_1} = 59{,}512\ km$$

$$r_{apoapsis} = \frac{p_1}{1 - e_1} = 1{,}266{,}048\ km$$

$$r_1 = 1{,}031{,}212\ km$$

Since $r_{impact} > r_{periapsis}$ Cassini is on a crash trajectory as it travels towards periapsis. In addition, $r_1 < r_{apoapsis}$ therefore Cassini is pass apoapsis heading towards periapsis.

$$\theta^*{}_{impact} = \pm cos^{-1}\left(\frac{p_1 - r_{impact}}{r_{impact} \cdot e_1}\right) = \pm 13.17° = -13.17°$$

Since $\bar{r}_1 \cdot \bar{v}_1 < 0$, Cassini at t_1 was heading towards periapsis and therefore $\theta^*{}_{impact}$ will range between [0, -180] degrees.

Using these values to calculate position and velocity in the rotating frame:

$$r_{impact} = 60{,}268\ km$$

$$v\_r_{impact} = \frac{\mu_{Saturn}}{h_1} e_1 \sin\left(\theta^*{}_{impact}\right) = -3.7882\ km/s$$

$$v\_\theta_{impact} = \frac{\mu_{Saturn}}{h_1}\left(1 + e_1 \cos\left(\theta^*{}_{impact}\right)\right) = 34.4594\ km/s$$

$$\bar{r}_{r\theta h\ impact} = \ 60{,}268\ \hat{r} + 0\ \hat{\theta} + 0\hat{h}\ km$$

$$\bar{v}_{r\theta h\ impact} = \ -3.7882\ \hat{r} + 34.4594\ \hat{\theta} + 0\hat{h}\ km/s$$

Converting these values in the Saturn-centered inertial frame through the rotation matrix:

$$C = R(\theta)_3 R(i)_1 R(\Omega)_3\ where\ \theta = \theta^* + \omega$$

C =

0.5700	-0.8173	0.0839
0.4276	0.2079	-0.8797
0.7016	0.5374	0.4680

$$\bar{r}_{XYZ\ impact} = [C]^T \bar{r}_{r\theta h\ impact}$$

$$\bar{v}_{XYZ\ impact} = [C]\bar{v}_{r\theta h\ impact}$$

$$\boldsymbol{\bar{r}_{XYZ\ Impact} = \ 34{,}356\ \hat{X} - 49{,}258\hat{Y} + 5{,}057\ \hat{Z}\ km <--}$$
$$\boldsymbol{\bar{v}_{XYZ\ Impact} = \ 12.5761\ \hat{X} + 10.2611\hat{Y} - 30.6325\ \hat{Z}\ km <--}$$

b) If the spacecraft really did impact the surface of Saturn, do you think that your prediction of the state vector at impact is accurate? Justify and explain.

The prediction at impact would not be accurate. As the spacecraft approaches Saturn, atmospheric drag forces would play an important role in the dynamics of the spacecraft. This will alter the trajectory at impact, rendering the above calculations inaccurate. <--

Problem 2:

<u>Assumptions:</u>

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses

- No other forces in the system except for gravitational forces
- Gm_{Mars} = 4.305x10^4 Km^3/s^2
- Equatorial radius of Saturn = 3,397.2 km

a) (ungraded, optional) Why did you choose to use either GMAT or STK?

I chose to work with STK since it seems to be the industry standard for professional astrodynamics. <--

b) Use the initial state information to calculate the orbital period and periapsis altitude in the two-body problem (in a useful set of units).

Given:

$$e = 0.45454$$
$$a = 6463.8\ km$$
$$i = 74.924°$$
$$\Omega = 1.2410°$$
$$\omega = 353.31° = -6.69°$$
$$\theta^* = 199.38° = -160.62°$$

$$\boldsymbol{Orbital\ Period = 2\pi\sqrt{\frac{a^3}{\mu_{Mars}}}\left(\frac{1\ hour}{3,600\ seconds}\right) = 4.37\ Hours <--}$$

$$p = a \cdot (1 - e^2) = 5{,}128.3\ km$$

$$r_{periapsis} = \frac{p}{1+e} = 3{,}525.7\ km$$

$$\boldsymbol{Periapsis\ Altitude = r_{periapsis} - EQR_{Mars} = 128.5\ km <--}$$

c) On your own and outside of STK/GMAT transform the orbital element description of the initial state to a Cartesian state vector in the Mars-centered inertial frame. Compare the values you computed to the state vector computed by STK or GMAT and, if applicable, speculate on the reason for any differences.

$$h = \sqrt{\mu_{Mars} * p} = 14{,}858\ km^2/s$$

$$r = \frac{p}{1 + e\cos(\theta^*)} = 89{,}779\ km$$

$$v\_r_{r\theta h} = \frac{\mu_{Saturn}}{h} e \sin(\theta^*{}_{impact}) = -0.437\ km/s$$

$$v\_\theta_{r\theta h} = \frac{\mu_{Saturn}}{h}(1 + e\cos(\theta^*{}_{impact})) = 1.6550\ km/s$$

$$\bar{r}_{r\theta h} = 8{,}977.9\ \hat{r} + 0\ \hat{\theta} + 0\ \hat{h}\ km$$

$$\bar{v}_{r\theta h} = -0.437\ \hat{r} + 1.6550\ \hat{\theta} + 0\ \hat{h}\ km/s$$

Converting these values in the Saturn-centered inertial frame through the rotation matrix:

$$C = R(\theta)_3 R(i)_1 R(\Omega)_3\ where\ \theta = \theta^* + \omega$$

C =

-0.9741	-0.0783	-0.2121
0.2251	-0.2489	-0.9420
0.0209	-0.9654	0.2601

$$\bar{r}_{XYZ} = [C]^T \bar{r}_{r\theta h}$$

$$\bar{v}_{XYZ} = [C] \bar{v}_{r\theta h}$$

$$\boldsymbol{\bar{r}_{XYZ} = -8745.4758\hat{X} - 702.5523\hat{Y} - 1904.3574\ \hat{Z}\ km} < --$$
$$\boldsymbol{\bar{v}_{XYZ} = 0.7983\ \hat{X} - 0.3778\hat{Y} - 1.4663\ \hat{Z}\ km} < --$$

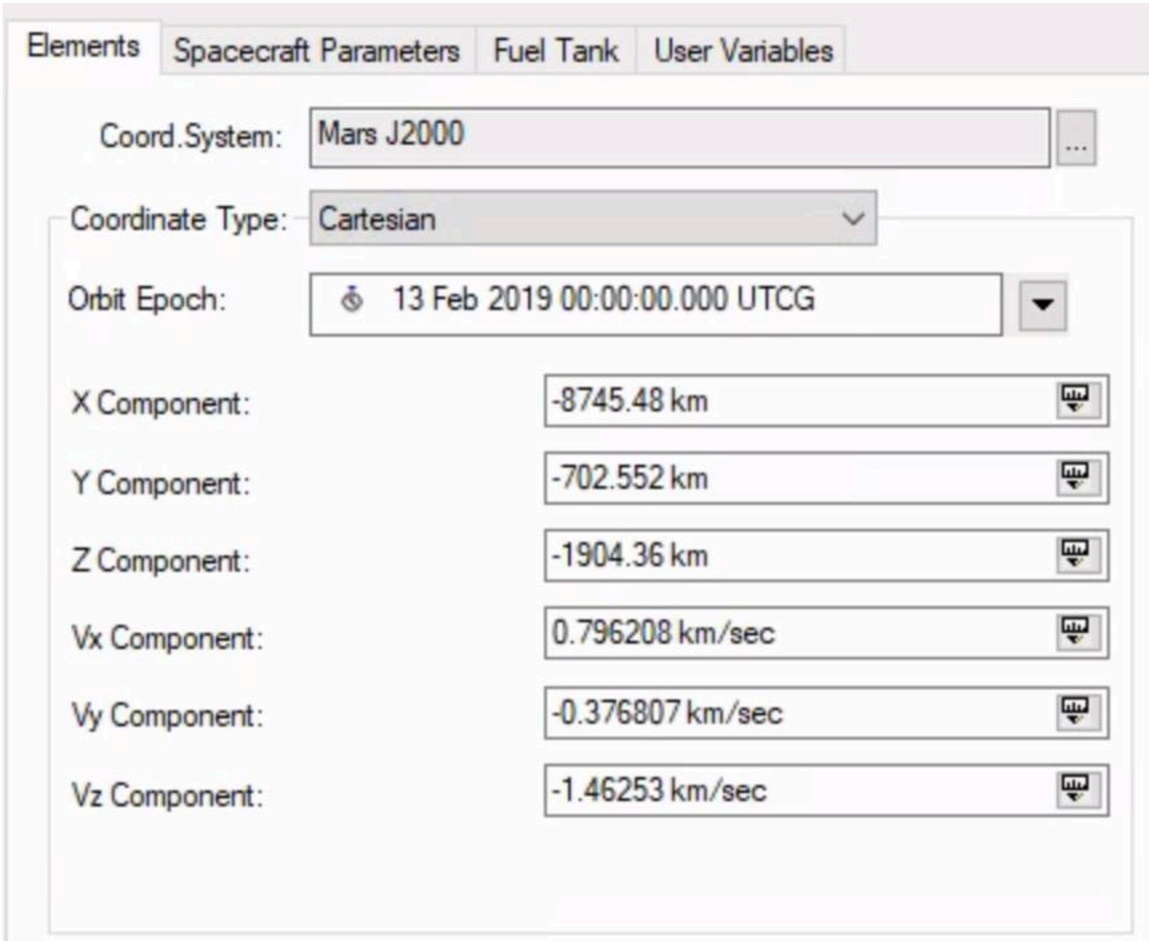

Figure 1- Cartesian Results from STK Analysis

The results from my analysis and the STK results match with the exception of some small rounding differences.<--

Recovering orbital elements:

$$\bar{h} = \bar{r}x\,\bar{v} = 311\hat{X} - 14{,}344\hat{Y} + 3{,}865\,\hat{Z}\;km^2/s$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu_{Mars}}{r} = -3.3301\;km^2/s^2$$

Since $\varepsilon < 0$ we know MAVEN its Mars-centric orbit that is bounded.

$$\bar{e} = \frac{\bar{v}x\,\bar{h}}{\mu_{Mars}} - \frac{\bar{r}}{r} = 0.4516\hat{X} - 0.0040\hat{Y} - 0.0511\hat{Z}$$

$$|\bar{e}| = \boldsymbol{e} = \mathbf{0.45454} < --$$

Since the 0 < e < 1 we know the orbit conic is an ellipse

$$a = \frac{-\mu_{Mars}}{2 \cdot \varepsilon} = \mathbf{6463.8\ km} < --$$

$$\boldsymbol{i} = \pm \boldsymbol{cos}^{-1}\left(\frac{\hat{\boldsymbol{Z}} \cdot \bar{\boldsymbol{h}}}{|\hat{\boldsymbol{z}}| \cdot |\bar{\boldsymbol{h}}|}\right) = \pm \mathbf{74.92°} = \mathbf{74.92°} < --$$

Since the angular momentum's Z component is positive, we know that the orbit follows a prograde direction from a top-down reference view. No sign check needed for i.

$$\bar{n} = \hat{Z}\ x\ \bar{h} = 14{,}344\hat{X} + 311\ \hat{Y} + 0\hat{Z}\ km^2/s$$

$$\boldsymbol{\Omega} = = \pm \boldsymbol{cos}^{-1}\left(\frac{\hat{\boldsymbol{X}} \cdot \bar{\boldsymbol{n}}}{|\hat{\boldsymbol{X}}| \cdot |\bar{\boldsymbol{n}}|}\right) = \pm \mathbf{1.24°} = \mathbf{1.24°} < --$$

Since $\hat{Y} \cdot \bar{n} > 0$ we select the positive value for RAAN.

$$\boldsymbol{\omega} = = \pm \boldsymbol{cos}^{-1}\left(\frac{\bar{\boldsymbol{e}} \cdot \bar{\boldsymbol{n}}}{|\bar{\boldsymbol{e}}| \cdot |\bar{\boldsymbol{n}}|}\right) = \pm \mathbf{6.69°} = -\mathbf{6.69°} < --$$

Since $\hat{Z} \cdot \bar{e}_1 < 0$ we select the negative value for AO

$$\boldsymbol{\theta}^* = = \pm \boldsymbol{cos}^{-1}\left(\frac{\bar{\boldsymbol{r}} \cdot \bar{\boldsymbol{e}}}{|\bar{\boldsymbol{e}}| \cdot |\bar{\boldsymbol{r}}|}\right) = \pm \mathbf{160.62°} = -\mathbf{160.62°} < --$$

Since $\bar{r} \cdot \bar{v} < 0$, MAVEN was heading towards periapsis and therefore θ^* will range between [0,-180] degrees.

d) Include two snapshots of the 3D graphics window in your report, displaying only the "MAVEN" spacecraft orbit computed in the Mars point mass dynamical model: one looking down on the orbital plane, and the other providing a useful three-dimensional perspective.

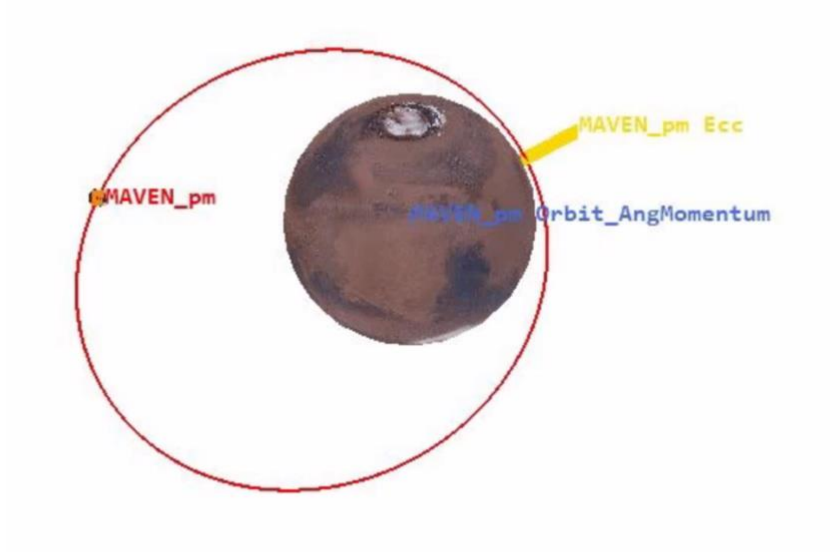

Figure 2- MAVEN_pm Looking down

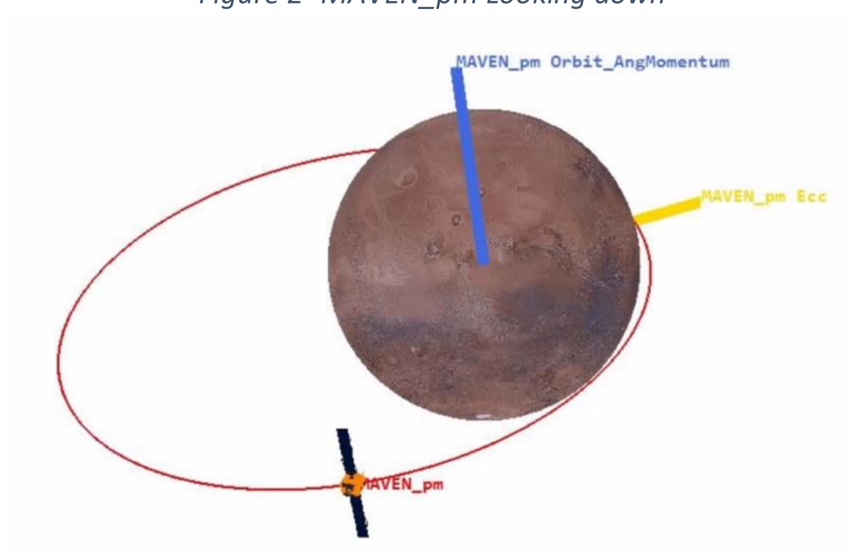

Figure 3 - MAVEN_pm 3D Perspective

e) Run the simulation for the "MAVEN" spacecraft and describe its orbit propagated in the Mars point mass dynamical model in your own words, discussing its geometry, orientation and characteristics in as much detail as possible.

The MAVEN_pm trajectory represented in Figure 2 and Figure 3 is an elliptical orbit with a close periapsis altitude (128.5 km) which makes sense given the nature of the MAVEN mission is to study the Martian upper atmosphere. The inclination (74.92 degrees) and RAAN (1.24 degrees) angles for the orbit provide ample coverage of the Martian surface given mars rotation about its axis. This would provide the MAVEN mission with a wide range of atmospheric data from several places above the Martian terrain.<--

f) How do the eccentricity and specific angular momentum evolve over time when you run the simulation for the "MAVEN" spacecraft using the Mars point mass dynamical model for 10 full orbital periods? Is this observation consistent with your expectations?

The magnitude for the angular momentum for 10 orbital periods is shown in Figure 4. This value is constant as expected from the relative 2BP assumptions and the law of conservation of angular momentum. The direction vector (Figure 3) is perpendicular to the orbital plane as expected. <--

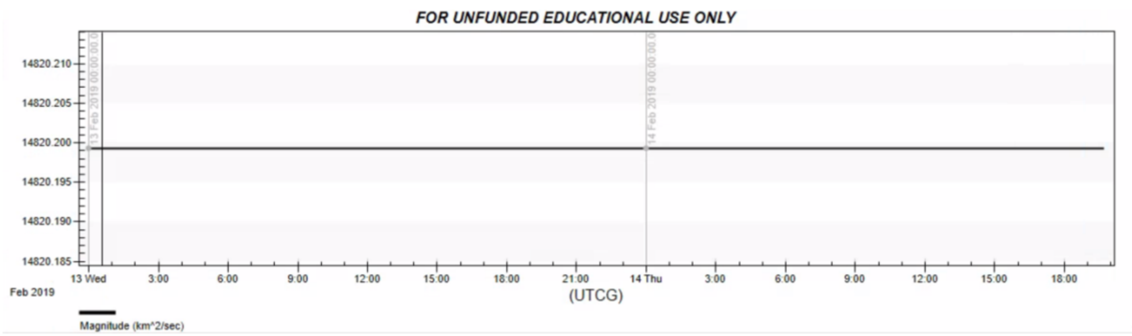

Figure 4 - Magnitude of h vs. Time MAVEN_pm

The magnitude for the orbit's eccentricity for 10 orbital periods is shown in Figure 5. This value is constant as expected from the relative 2BP assumptions. The direction vector in Figure 3 is aligned with the periapsis radius as expected.<--

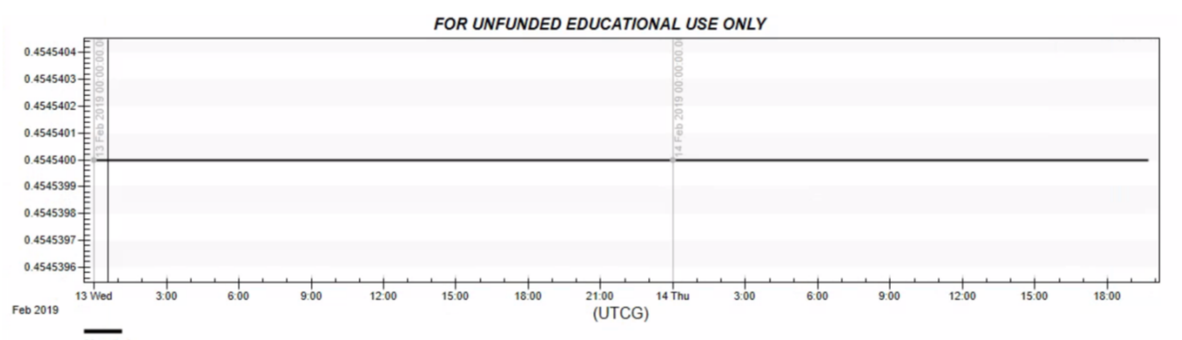

Figure 5 - Magnitude of e Vs. Time MAVEN_pm

g) Include two snapshots of the 3D graphics window in your report, displaying only the trajectory of the "MAVEN" spacecraft, propagated in a dynamical model that reflects a higher-fidelity model of Mars' dynamical environment: one looking down on the orbital plane, and the other providing a useful three-dimensional perspective.

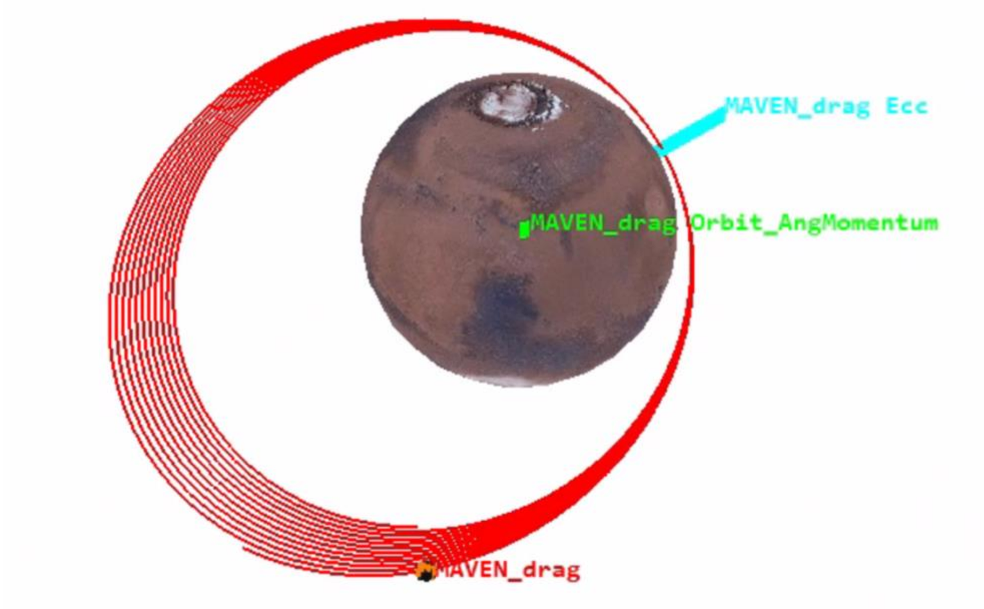

Figure 6 - MAVEN_drag Looking down

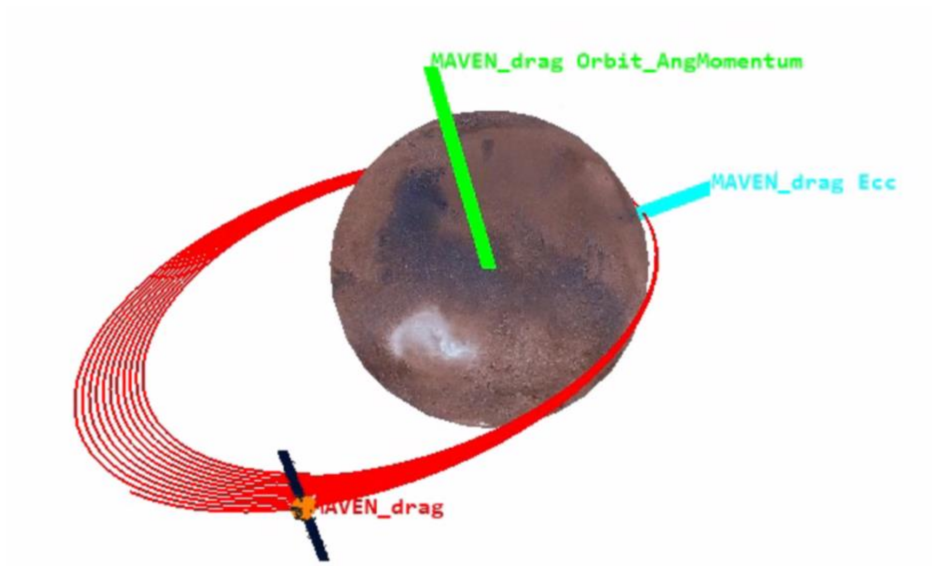

Figure 7 - MAVEN_drag 3D perspective

h) Run the simulation for the “MAVEN” spacecraft and describe its motion in your own words, discussing how the higher fidelity model of Mars’ dynamical environment has impacted the geometry, orientation and characteristics of the trajectory during the integration time period.

With the higher fidelity model, we can still observe the features discussed in part e with the exception that the orbit size is decreased every time maven traverses periapsis. Yet, this traversal does not change the inclination or RAAN values for the orbit, this maintains the

same orbit orientation as the spacecraft traverses around mars. The decrease in orbit size can be accounted by the effects of atmospheric drag when the spacecraft passes that close to mars at periapsis.<--

i) How do the eccentricity and specific angular momentum evolve over time when you run the simulation for "MAVEN" with the higher fidelity dynamical model? Is this observation consistent with your expectations? When, in the orbit, do the values change most significantly and why do you think that is?

The magnitude for the angular momentum for 10 orbital periods is shown in Figure 8. The magnitude remains constant until MAVEN crosses periapsis and slows down due to atmospheric drag therefore decreasing the angular momentum's magnitude. The direction vector (Figure 7) remains perpendicular to the orbit as expected with its size decreasing after each periapsis pass.<--

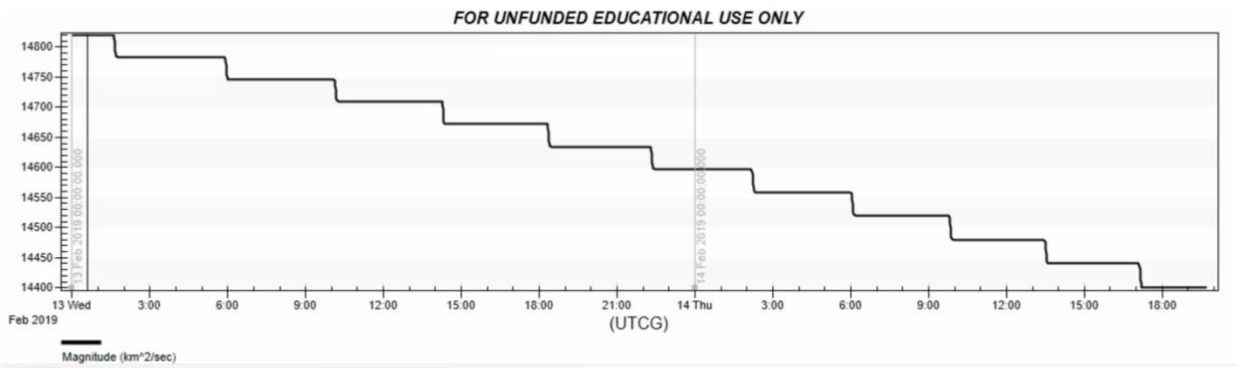

Figure 8 - Magnitude of h Vs. Time MAVEN_drag

The magnitude for the orbit's eccentricity for 10 orbital periods is shown in Figure 9. The eccentricity magnitude remains constant until MAVEN crosses periapsis, slows down, and decreases the magnitude size for the eccentricity vector. The direction vector in Figure 7 is aligned with the periapsis radius as expected but decreases in magnitude after each periapsis pass of the spacecraft.<--

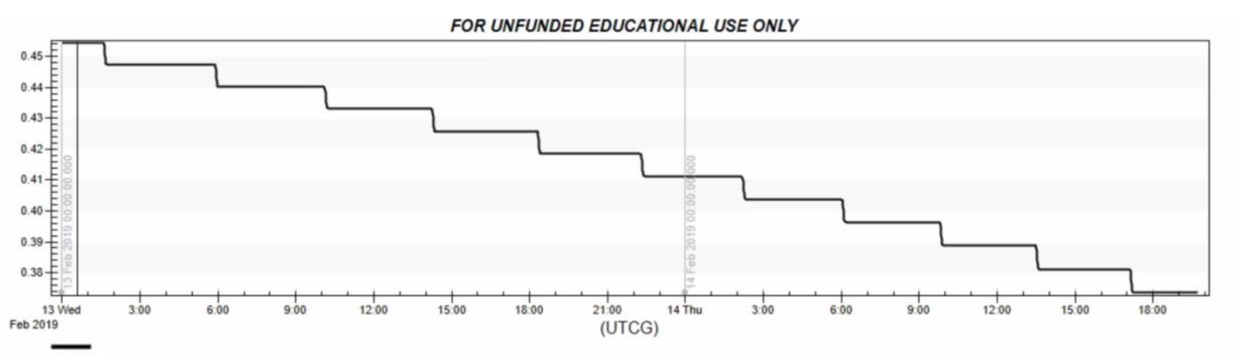

Figure 9 - Magnitude of e Vs. Time MAVEN_drag

j) Use the MAVEN website to determine the activities being conducted by the MAVEN spacecraft on the epoch corresponding to the initial condition given in this lab statement. Using your research from this website, do you think that the relative two-body problem is a sufficiently good approximation of the dynamical environment to reliably design the trajectory of the MAVEN spacecraft?

During the initial condition epoch, February 13th 2019, MAVEN entered an aerobraking campaign to reduce the orbit size and period in order to improve the communication relay between earth and balance other mission requirements. The relative two-body problem would not be a good approximation for this dynamical environment given that the aerodynamic forces of Mars upper atmosphere during aerobraking would have to be included to precisely predict the trajectory of MAVEN.<--

```
Problem 1 Part a
inclination angle i at t1 = 62.09 deg
semi-major axis a at t1 = 662780.88 Km
e at t1 is 0.91021
RAAN at t1 is 127.45 deg
Argument of Periapsis w at t1 = -172.28 deg

Given Ralative 2BP assumptions we can use t1 orbital elements to calculate impact values
True Anomaly at impact = -13.17 deg
R vector in XYZ frame at impact is <34355.5200,-49258.0066,5057.5543> Km
V vector in XYZ frame at impact is <12.5761,10.2611,-30.6325> Km/s

Problem 2 Part b
Orbital Period is 4.37 hours
Periapsis Altitude is 128.5443 Km

Problem 2 Part c
R vector in XYZ frame is <-8745.4758,-702.5523,-1904.3574> Km
V vector in XYZ frame is <0.7983,-0.3778,-1.4663> Km/s

inclination angle i = 74.92 deg
semi-major axis a = 6463.80 Km
e is 0.45454
RAAN is 1.24 deg
Argument of Periapsis w = -6.69 deg
True Anomaly ThetaStar = -160.62 deg

>>
```

```
%Gustavo Grinsteins
%ASEN 5050
%HW3

%House Keeping
clc;
clear;

%Given values
mu_mars = 4.305*10^4; %km^3/s^2
mu_saturn = 3.794*10^7; %km^3/s^2
EQR_mars = 3397.2; %Km
EQR_saturn = 60268; %Km

%% Problem 1 Part a
fprintf('Problem 1 Part a \n')
R_1 = [-720000;670000;310000];
V_1 = [2.160;-3.360;0.620];
%Calculating orbital elements at t1
%Recover Orbital Elements
r_1 = norm(R_1);
v_1 = norm(V_1);
H_1 = cross(R_1,V_1);
h_1 = norm(H_1);
Sp_Mech_E_1 = (v_1^2/2)-(mu_saturn/r_1);
%inclination
z_hat = [0,0,1];
i_1 = acos((dot(z_hat,H_1))/(norm(z_hat)*norm(H_1)));
fprintf('inclination angle i at t1 = %4.2f deg \n',i_1*(180/pi))
%major-axis
a_1 = (-mu_saturn)/(2*Sp_Mech_E_1);
fprintf('semi-major axis a at t1 = %4.2f Km\n',a_1)
%eccentricity
Ecc_1 = (cross(V_1,H_1))/(mu_saturn)-(R_1/r_1);
ecc_1 = norm(Ecc_1);
fprintf('e at t1 is %4.5f\n',ecc_1)
%RAAN
x_hat = [1,0,0];
y_hat = [0,1,0];
N_1 = cross(z_hat,H_1);
RAAN_1 = abs(acos((dot(x_hat,N_1))/(norm(x_hat)*norm(N_1))));
if dot(N_1,y_hat)<0
    RAAN_1 = (-1)*RAAN_1;
end
fprintf('RAAN at t1 is %4.2f deg \n',RAAN_1*(180/pi))
%AOP
AOP_1 = abs(acos((dot(Ecc_1,N_1))/(norm(Ecc_1)*norm(N_1))));
if dot(Ecc_1,z_hat)<0
    AOP_1 = (-1)*AOP_1;
end
fprintf('Argument of Periapsis w at t1 = %4.2f deg \n\n',AOP_1*(180/pi))

fprintf('Given Ralative 2BP assumptions we can use t1 orbital elements to calculate↙
impact values \n')
r_impact = EQR_saturn;
p_1 = a_1*(1-ecc_1^2); %Km
r_p = (p_1)/(1+ecc_1*cosd(0));%Km since r_p < r_impact we know the orbit is in collision↙
course
```

```
r_a = (p_1)/(1+ecc_1*cosd(180));
theta_star_impact = abs(acos((p_1-r_impact)/(r_impact*ecc_1)));
if dot(V_1,R_1)<0
    theta_star_impact = (-1)*theta_star_impact;
end
fprintf('True Anomaly at impact = %4.2f deg \n',theta_star_impact*(180/pi))
v_r_impact = (mu_saturn/h_1)*ecc_1*sin(theta_star_impact);%km/s
v_theta_impact = (mu_saturn/h_1)*(1 + (ecc_1*cos(theta_star_impact))); %km/s

%Calculating rotation matrix
theta_impact = theta_star_impact+AOP_1;%Degrees

R1 = [1,0,0;0,cos(i_1),sin(i_1);0,-sin(i_1),cos(i_1)];
R3_RAAN = [cos(RAAN_1),sin(RAAN_1),0;-sin(RAAN_1),cos(RAAN_1),0;0,0,1];
R3_theta = [cos(theta_impact),sin(theta_impact),0;-sin(theta_impact),cos(theta_impact),0;↙
0,0,1];

C = R3_theta*R1*R3_RAAN;

PositionRot_impact = [r_impact;0;0];
VelocityRot_impact = [v_r_impact;v_theta_impact;0];

%Transforming position from r,theta,h to XYZ
PositionXYZ_impact = C.'*PositionRot_impact;
fprintf('R vector in XYZ frame at impact is <%4.4f,%4.4f,%4.4f> Km\n',PositionXYZ_impact)

%Transforming velocity from r,theta,h to XYZ
VelocityXYZ_impact = C.'*VelocityRot_impact;
fprintf('V vector in XYZ frame at impact is <%4.4f,%4.4f,%4.4f> Km/s\n\n',↙
VelocityXYZ_impact)


%% Problem 2 Part b
fprintf('Problem 2 Part b \n')
e = 0.45454;
a = 6463.8; %Km
ideg = 74.924; %Degrees
RAANdeg = 1.2410; %Degrees
AOPdeg = 353.31; %Degrees
theta_star_deg = 199.38; %Degrees

%Orbital Period
OrbitPeriod = (((2*pi)*sqrt(a^3/mu_mars))/60)/60;%Hours
fprintf('Orbital Period is %4.2f hours \n',OrbitPeriod)

%Periapsis Altitude
p = a*(1-e^2); %Km
r_periapsis = (p)/(1+e*cosd(0));%Km
PeriapsisAltitude = r_periapsis - EQR_mars;%Km
fprintf('Periapsis Altitude is %4.4f Km \n\n',PeriapsisAltitude)

%% Problem 2 Part c
fprintf('Problem 2 Part c \n')
h = sqrt(mu_mars*p);%Km^2/s
r = (p)/(1+e*cosd(theta_star_deg)); %Km
v_r = (mu_mars/h)*e*sind(theta_star_deg);%km/s
v_theta = (mu_mars/h)*(1 + (e*cosd(theta_star_deg))); %km/s
```

```
%Calculating rotation matrix
theta = theta_star_deg+AOPdeg;%Degrees

R1 = [1,0,0;0,cosd(ideg),sind(ideg);0,-sind(ideg),cosd(ideg)];
R3_RAAN = [cosd(RAANdeg),sind(RAANdeg),0;-sind(RAANdeg),cosd(RAANdeg),0;0,0,1];
R3_theta = [cosd(theta),sind(theta),0;-sind(theta),cosd(theta),0;0,0,1];

C = R3_theta*R1*R3_RAAN;

PositionRot = [r;0;0];
VelocityRot = [v_r;v_theta;0];

%Transforming position from r,theta,h to XYZ
PositionXYZ = C.'*PositionRot;
fprintf('R vector in XYZ frame is <%4.4f,%4.4f,%4.4f> Km\n',PositionXYZ)

%Transforming velocity from r,theta,h to XYZ
VelocityXYZ = C.'*VelocityRot;
fprintf('V vector in XYZ frame is <%4.4f,%4.4f,%4.4f> Km/s\n\n',VelocityXYZ)

%Recover Orbital Elements
r_2 = norm(PositionXYZ);
v = norm(VelocityXYZ);
H = cross(PositionXYZ,VelocityXYZ);
h = norm(H);
Sp_Mech_E = (v^2/2)-(mu_mars/r_2);
%inclination
z_hat = [0,0,1];
i = acos((dot(z_hat,H))/(norm(z_hat)*norm(H)));
fprintf('inclination angle i = %4.2f deg \n',i*(180/pi))
%major-axis
a_2 = (-mu_mars)/(2*Sp_Mech_E);
fprintf('semi-major axis a = %4.2f Km\n',a_2)
%eccentricity
Ecc = (cross(VelocityXYZ,H))/(mu_mars)-(PositionXYZ/r_2);
ecc = norm(Ecc);
fprintf('e is %4.5f\n',ecc)
%RAAN
x_hat = [1,0,0];
y_hat = [0,1,0];
N = cross(z_hat,H);
RAAN = abs(acos((dot(x_hat,N))/(norm(x_hat)*norm(N))));
if dot(N,y_hat)<0
    RAAN = (-1)*RAAN;
end
fprintf('RAAN is %4.2f deg \n',RAAN*(180/pi))
%AOP
AOP = abs(acos((dot(Ecc,N))/(norm(Ecc)*norm(N))));
if dot(Ecc,z_hat)<0
    AOP = (-1)*AOP;
end
fprintf('Argument of Periapsis w = %4.2f deg \n',AOP*(180/pi))
%True Anomaly
theta_star = abs(acos((dot(PositionXYZ,Ecc))/(norm(Ecc)*norm(PositionXYZ))));
if dot(PositionXYZ,VelocityXYZ)<0
    theta_star = (-1)*theta_star;
end
fprintf('True Anomaly ThetaStar = %4.2f deg \n\n',theta_star*(180/pi))
```