ASEN 5050 – Spaceflight Dynamics

Homework #1

Assigned: Tuesday, January 19, 2021
Due: Tuesday, January 26, 2021 at 8.59pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
  - $Gm_{Mars} = 4.305 \times 10^4 km^3/s^2$
  - Equatorial radius of Mars: 3397.2 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

The Mars 2020 Perseverance Rover is currently in transit to Mars, with an expected landing date of February 18, 2021 (see https://mars.nasa.gov/mars2020/mission/overview/ to learn more about the mission). This exciting event is approximately a month away!

As of today, January 19th, 2021, the spacecraft is following its interplanetary trajectory and getting closer to Mars. At a specific instant of time on this date, the state of the spacecraft in its heliocentric orbit is partially described by the following information, relative to the Sun:

$v = 22.4346\ km/s$ $\qquad$ $v_r = 4.1219\ km/s$ $\qquad$ $h = 4.9775 \times 10^9 km^2/s$

At this instant of time:

a) Write the position and velocity vectors in the $(\hat{r}, \hat{\theta}, \hat{h})$ rotating coordinate frame.
b) Compute the following information in any order: $a, e, \mathcal{E}, p$, orbit period, type of conic
c) Calculate the true anomaly θ^* and flight path angle ϕ_{fpa} of the spacecraft

Let's confirm that some of these orbit parameters approximately match those derived from the following position and velocity vectors of the spacecraft in an inertial frame $(\hat{X}, \hat{Y}, \hat{Z})$ and relative to the Sun (to within small errors from my rounding these quantities):

$$\bar{R} = 5.3243 \times 10^7 \hat{X} + 2.1925 \times 10^8 \hat{Y} + 6.2724 \times 10^6 \hat{Z}\ km$$

$$\bar{V} = -2.0449 \times 10^1 \hat{X} + 9.2202 \hat{Y}\ -\ 3.8811 \times 10^{-1} \hat{Z}\ km/s$$

d) Use these position and velocity vectors to calculate r, v, h, e, a. (Do not use any of the information previously calculated in parts a-c in these calculations; the goal is to independently verify this information!) Are these newly-calculated values consistent with those previously calculated or given in parts a-c?

Finally, let's consider the orbit of the spacecraft relative to the Sun:

e) Draw a diagram of the orbit, indicating the following information where applicable: semi-major axis, semi-minor axis, *C*, *F*, *F'*, periapsis, apoapsis, semi-latus rectum, and direction of motion. Draw the position and velocity vectors for the object at the observation time on this diagram, also indicating the $(\hat{r}, \hat{\theta}, \hat{h})$ unit vectors, true anomaly and flight path angle. Be sure to accurately locate the spacecraft in the correct region of its orbit.

f) Given the location of the object in its orbit at the observation time, justify in as much detail as you are able to whether the combination of r and θ^* computed in this problem are likely to be correct and whether you have drawn the spacecraft in the correct location in part d) (Hint: consider any upper and lower bounds due to conic geometry)

Problem 2

When InSight launched on May 5, 2018, there were two CubeSats riding onboard as secondary payloads. This CubeSat mission, Mars Cube One (MarCO), was used for technology demonstration: flying by Mars and providing a communications relay for when InSight lands on the surface. This was also an exciting opportunity to learn more about how we can use CubeSats in deep space! The two CubeSats, MarCO-A and MarCO-B, were deployed close to the Earth and performed their own trajectory corrections maneuvers to target the desired hyperbolic orbit relative to Mars.

Let's assume a similar scenario and analyze the orbit of a CubeSat on a hyperbolic trajectory relative to Mars. The orbit is nominally described by the following parameters:

$$Periapsis\ Altitude = 2{,}342.8\ km \quad \delta = 66.7\ degrees$$

a) Calculate the values of $v_\infty, \theta^*{}_\infty$ for the CubeSat in its hyperbolic trajectory.
b) Calculate the speed of the CubeSat at periapsis.
c) Discuss whether you think the dynamical environment governing the spacecraft as it travels along this hyperbola is well-approximated by the Mars-spacecraft relative two-body problem.