ASEN 5050 – Spaceflight Dynamics
Homework #8

Assigned: Tuesday, April 13, 2021
Due: Tuesday, April 20, 2021 at 8.59pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - Gravitational parameters:
    - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
    - $Gm_{Earth} = 3.986004415 \times 10^{5} km^3/s^2$
  - Semi-major axes relative to the Sun:
    - $a_{Earth} = 1.0000010178\ AU$
    - 1 AU = 149,597,870.7 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Consider a scientific satellite orbiting an unknown planet and observed to have the following orbit characteristics: periapsis radius = 7500 km, apoapsis radius = 8500 km, inclination = 105°, and orbital period =110 minutes. The radius of the planet is 6500 km. The planet is at a constant distance of 2.25 AU from the Sun, and the satellite is in a sun-synchronous orbit.

Determine the mass (M_P) and 2nd degree zonal gravitational coefficient ($J_{2,P}$) of the planet given this information. Be sure to list <u>any</u> assumptions that you use in your answer.

Problem 2:

During the course lectures, we covered in detail the general perturbations approach that enables a generalized analysis of the impact of perturbations on the solution space. However, recall that a special perturbation approach to studying the impact of perturbations focuses on a single point solution. One important component of the special perturbations approach (and simply generating a trajectory in a complex dynamical model) is numerical integration. Let's explore some of the challenges of numerically generating trajectories.

In this problem, we will explore the parameters associated with numerically integrating the path of a spacecraft relative to the Earth and in the two-body problem, using the following initial conditions at t_0:

$$\bar{R}_0 = [-6402, -1809, 1065]\ km \qquad \bar{V}_0 = [0.999, -6.471, -4.302] km/s$$

expressed in an Earth-centered inertial coordinate system.

a) In Matlab, write a script to numerically integrate the equations of motion for the two-body problem, using ode45, a variable time-step Runge-Kutta Method. (I strongly recommend using Matlab for this question, but if you use a coding language other than Matlab, please identify a numerical integration scheme with similar properties to ode45 and describe the scheme you used in your writeup.) In ode45, use a relative tolerance of

1×10^{-12} and an absolute tolerance of 1×10^{-12} for now (we will change these values later). Include a copy of your script in your submission. Describe the setup of your script and list the differential equations you are providing to ode45 for integration. Also describe the definition of relative and absolute tolerances, as well as the approach used by the integrator to determine whether the error at each time step is acceptable using these two tolerances.

b) Using the expressions we have developed in this class to solve the two-body problem analytically, determine the state vector of the spacecraft at the following two times: 1) $t_1 = t_0 + 1$ hr, and 2) $t_2 = t_0 + 100$ hr. At each of these two times, calculate the associated values of the specific energy and the magnitude of the specific angular momentum vector.

c) Using the script you developed in part a), integrate the trajectory of the spacecraft for each of the following two time-intervals: 1) from t_0 to t_1, and 2) from t_0 to t_2. Create a table listing the state vector of the spacecraft at the end of each time interval, as well as the associated values of the specific energy and the magnitude of the specific angular momentum vector. Compare the results of your numerical integration with the state vectors identified in part b) and discuss.

d) In this section, we will explore the impact of the tolerances on the accuracy of the integrated solution. Set the absolute and relative tolerances to identical values. Each tolerance will be set to one of the following values: 1×10^{-4}, 1×10^{-6}, 1×10^{-8}, 1×10^{-10}, 1×10^{-12}. For each value of the tolerance, numerically integrate the trajectory of the spacecraft from t_0 to t_2 and note the state vector at the end of this time interval. Consider the "truth" value of the state vector at t_2 equal to the vector computed in part b). Then, for each of the five numerical integrations performed in this question, calculate the quantities ΔR and ΔV, representing the magnitude of the difference in the position and velocity components between the numerical integration and the "truth" data. Also calculate $\Delta \mathcal{E}$ and Δh, representing the difference in the specific energy and specific angular momentum between the states at time t_0 and t_2 along the numerically-integrated solution. Summarize this information in a table resembling the following:

	ΔR	ΔV	$\Delta \mathcal{E}$	Δh
Tol = 1×10^{-4}				
Tol = 1×10^{-6}				
Tol = 1×10^{-8}				
Tol = 1×10^{-10}				
Tol = 1×10^{-12}				

e) Discuss the results summarized in your table. Also discuss the tradeoff you would perform to select an appropriate tolerance for numerically integrating solutions in the two-body problem via ode45.

Problem 3:

a) **STK:** Open the "Earth Point Mass" propagator in the "Component Browser" and navigate to the "Numerical Integrator" tab. Report the integrator used in this propagator function as well as the absolute and relative tolerances used. Using either the summary or report function, discuss whether you think that this combination of integrator and tolerances has recovered a solution that is close to the true solution in the two-body problem. (Hint: Which quantities will you use to assess the accuracy of the solution?)

GMAT: Open the "EarthPM" propagator and navigate to the "Integrator" panel. Report the integrator used in this propagator function as well as the "accuracy" value used. Using either the summary or report function, discuss whether you think that this combination of integrator and tolerance has recovered a solution that is close to the true solution in the two-body problem. (Hint: Which quantities will you use to assess the accuracy of the solution?)

b) Describe, in as much detail as possible, the impact of the Earth J2 perturbation on each of the orbital elements for this trajectory, including your five plots of the orbital elements over 100 revolutions. Discuss, in as much detail as possible, whether these time history plots are consistent with your expectations.
c) Include both a three-dimensional plot and a groundtrack plot of the satellite motion. Describe the impact of the Earth's oblateness on the entire spacecraft orbit using these plots as a reference.
d) Rerun the scenario using the "Earth Higher Order" propagator. Describe, in as much detail as possible, the impact of the higher-order gravitational model on each of the orbital elements for this trajectory, including your five plots of the orbital elements over 100 revolutions. Discuss, in as much detail as possible, how the time evolution of these orbital elements differs from the time history of the orbital elements when subject only to a perturbation from J2. If you were designing a trajectory for this spacecraft, could a preliminary analysis in either the two-body problem, or the two-body problem with a J2 perturbation sufficiently predict the evolution of the orbit?
e) Describe the impact of the Moon's third-body perturbation on each of the orbital elements corresponding to the reference motion, including your five plots of the orbital elements over 100 days (you may show additional zoomed-in views if you consider them necessary). Also include a three-dimensional plot of the spacecraft motion. Discuss whether you think these observations are generalizable beyond this single trajectory.
f) What is the change in the values of the semi-major axis, eccentricity and inclination between the beginning and end of this 100-day trajectory?
g) Use the plots in part e) to identify an approximate epoch at which the closest lunar pass occurs and justify your answer.
h) Describe how adding the gravitational influence of the Sun impacts the motion of the spacecraft. Use plots and the output of the reports/summary function to justify your answer.
i) Describe the impact of atmospheric drag on the corresponding to the integrated trajectory, including your five plots of the orbital elements over 10 days. Also include a three-dimensional plot of the spacecraft motion.
j) What is the change in the values of the semi-major axis, eccentricity and inclination between the beginning and end of this 10-day trajectory?
k) How does the inclusion of a higher-order gravitational model impact your results from part i)? Include relevant plots to justify your answer.