ASEN 5050 – Spaceflight Dynamics

Homework #6

Assigned: Tuesday, March 9, 2021
Due: Thursday, March 18, 2021 at 8.59pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
  - 1 AU = 149,597,870.7 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Let's design a transfer for a spacecraft to travel from the Earth to Venus. In this problem, use the geometric approach to solving Lambert's problem as presented in class – not the universal variable formulation for solving Lambert's problem that appears in the optional textbook.

At a Julian date of 2459528.5 TDB, the Earth is described in a Sun-centered inertial coordinate system by the following position and velocity vectors:

$$\bar{R}_1 = 1.00078 \times 10^8 \hat{X} + 1.09250 \times 10^8 \hat{Y} - 5.29404 \times 10^3 \hat{Z}\ km$$
$$\bar{V}_1 = -22.46086 \hat{X} + 20.00474 \hat{Y} - 1.79921 \times 10^{-4} \hat{Z}\ km/s$$

At a later Julian date of 2459640.5 TDB, Venus is described in a Sun-centered inertial coordinate system by the following position and velocity vectors:

$$\bar{R}_2 = -1.05048 \times 10^8 \hat{X} - 2.37576 \times 10^7 \hat{Y} + 5.73539 \times 10^6 \hat{Z}\ km$$
$$\bar{V}_2 = 7.49784 \hat{X} - 34.31464 \hat{Y} - 0.90369 \hat{Z}\ km/s$$

a) Write a script to implement an iterative numerical method solving **Lambert's equation** to calculate a semi-major axis for a transfer with a specified time of flight. Write this method only for arcs along elliptical orbits. Describe the setup of your numerical method, the stopping condition/s and initial guess used – in your own words. Attach your code. (Hint: fsolve is a useful root-finding function in Matlab)
b) Use Lambert's problem to design a transfer with a transfer angle of **less than** 180 degrees, and connects these two position vectors at the specified epochs. Find the values of a, e, and $\mathcal{E}$ along the transfer relative to the Sun. Except for using the script from part a) to solve **Lambert's equation**, you must show all your working by hand or typed up with mathematical notation and discuss the procedure you use to solve this problem.
c) What is the lower bound on the semi-major axis for an elliptical transfer connecting these two position vectors?
d) At both the beginning and end of this transfer, calculate the true anomaly and velocity vectors in a Sun-centered inertial coordinate system. Use these velocities to calculate the maneuver magnitudes Δv_1 and Δv_2 required for the spacecraft to simply match the velocities of Earth and Venus at the beginning and end of the transfer.

Problem 2:

Let's increase the complexity of the transfer design problem to incorporate a variety of potential departure and arrival epochs around the single combination of epochs examined in Problem 1: when the epoch changes, so too do the states associated with each planetary body. We can recover a variety of transfers within a subset of the design space and visualize them on a porkchop plot. Let's also consider a slightly different scenario: trajectories that deliver a spacecraft from the Earth to Mars.

The two text files available in this module contain the heliocentric state vectors for Earth and Mars at various selected epochs. This data is generated from the JPL HORIZONS webpage. Import this data to create matrices containing the epoch and heliocentric state components for each body. Hint: the "Import Data" feature in Matlab simplifies this process significantly!

a) Convert the procedure that you used to solve Problem 1 into a numerical script that you can run to produce transfers with a transfer angle that is less than 180 degrees given the following inputs: the initial and final state vectors and the time of flight. Attach your code.
b) For every possible combination of the provided initial and final states and epochs for the Earth and Mars (where the final epoch occurs after the initial epoch, of course), use your script to design a transfer from the Earth to Mars with a transfer angle that is <u>less than</u> 180 degrees. For each transfer, calculate the v-infinity magnitude at each of Earth departure and Mars arrival; discuss in your writeup how you calculated this quantity. Then, create two porkchop plots: 1) displaying the v-infinity at Earth departure and 2) displaying the v-infinity at Mars arrival. For each porkchop plot, display the epoch associated with the Earth's state vector on the horizontal axis (i.e., the departure date) and display the epoch associated with Mars' state vector on the vertical axis (i.e., the arrival date).
c) Analyze and discuss the recovered subset of the transfer design space for trajectories from the Earth to Mars across the provided range of departure and arrival dates using the porkchop plots you have constructed. How could this transfer design space be expanded further in future work?

Problem 3:

The goal of this problem is to learn how to create a transfer sequence in GMAT/STK and to use the software to help you calculate these quantities. Before starting this problem, **you must have completed Problem 1**.

Follow the instructions for GMAT/STK, available on the Canvas page in the HW 6 module and answer the following questions when indicated.

a) Use the information you calculated at the beginning and end of the transfer designed in Problem 1b) to report the $\Delta\bar{v}_1$ and $\Delta\bar{v}_2$ in the Sun-centered inertial coordinate system. Also report the time of flight along the transfer.

b) Provide a screenshot of the transfer, along with the orbits of Earth and Venus, in a heliocentric view looking down on the ecliptic. Indicate the direction of motion along each arc using arrows added to the screenshot or state in words the direction of motion.

c) Use the summary or reporting function described in the lab instructions to list the state of the spacecraft in the Sun-centered inertial coordinate system after the second maneuver has been applied. Compare the state components to the $\bar{R}_2$ and $\bar{V}_2$ vectors provided in Problem 1. If there are any differences between the quantities, discuss a potential reason for this difference.

d) Provide a screenshot of the converged transfer, along with the orbits of Earth and Venus, in a heliocentric view looking down on the ecliptic. Indicate the direction of motion along each arc using arrows. Use the summary or reporting function described in the lab instructions to list the state of the spacecraft in the Sun-centered inertial coordinate system after the second maneuver has been applied – does this state vector lie within the specified tolerance of the desired state vector? Also list the $\Delta\bar{v}_1$ and $\Delta\bar{v}_2$ in the Sun-centered inertial coordinate system as computed by the targeter and compare these vectors to the quantities you reported in Problem 3a). How many iterations did the targeter require to recover this solution?