ASEN 5050 – Spaceflight Dynamics

Homework #4

Assigned: Wednesday, February 10, 2021
Due: Tuesday, February 16, 2021 at 8.59pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Saturn} = 3.794 \times 10^7 km^3/s^2$
  - $Gm_{Moon} = 4902.799 km^3/s^2$
  - Equatorial radius of Saturn: 60,268 km
  - Equatorial radius of the Moon: 1738 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Recall Problem 1 of HW 3, where you studied the conditions associated with impact for the Cassini spacecraft in a simplified model. Let's extend this analysis to explore the time to impact.

Recall that at a time t_1, during the final orbit, the position and velocity vectors of the spacecraft are expressed as follows in a Saturn-centered inertial frame $(\hat{X}, \hat{Y}, \hat{Z})$ that uses Saturn's equatorial plane as the reference plane:

$$\bar{r}_1 = -720{,}000\hat{X} + 670{,}000\hat{Y} + 310{,}000\hat{Z} km$$
$$\bar{v}_1 = 2.160\hat{X} - 3.360\hat{Y} + 0.620\hat{Z} km/s$$

Calculate the duration of time between t_1 and impact. Note: You may reference information you computed in HW 3 without repeating those specific calculations for the orbital elements and the state vector at impact.

Problem 2:

The Lunar Reconnaissance Orbiter (LRO) is a spacecraft, developed by NASA and launched on June 18, 2009, that is currently in orbit around the Moon. This mission has enabled the achievement of a variety of lunar science and exploration goals in support of extending the presence of humans beyond the Earth vicinity, with particular emphasis on studying the poles of the Moon.

At a time t_1, on the date that this homework is assigned, the spacecraft is described by the following state information, measured relative to the Moon and in an inertial frame, $(\hat{X}\hat{Y}\hat{Z})$, that uses the Moon equatorial plane as the reference plane:

$$\bar{R} = -7.87701 \times 10^2\hat{X} - 8.81425 \times 10^2\hat{Y} + 1.43864 \times 10^3\hat{Z} km$$
$$\bar{V} = 0.98370\hat{X} + 0.76950\hat{Y} + 1.01416\hat{Z} km/s$$

a) In the current orbit of the spacecraft, how long (in hours) is the spacecraft located above the Moon's equatorial plane? Label this time t_{pos}.

b) In the current orbit of the spacecraft, how long (in hours) is the spacecraft located below the Moon's equatorial plane? Label this time t_{neg}.

c) Compare the times computed in parts a) and b) and justify, using your knowledge of orbit geometry, whether they should be similar or different. If they should be different, which time do you think should be larger and why: t_{pos} or t_{neg}?

d) Construct an iterative numerical procedure to solve Kepler's equation for the eccentric anomaly, given a time past periapsis. Discuss the method you used, noting the equation used to update the eccentric anomaly at each iteration and justify your stopping condition as well as any tolerances. Include your code as a supplement at the end of your homework

e) 30 minutes after t_1 in the described orbit, what is the <u>altitude</u> of the LRO spacecraft relative to the Moon? At this new location, is LRO moving towards or away from periapsis?