ASEN 5050 – Spaceflight Dynamics
Homework #7

Assigned: Monday, March 22, 2021
Due: Thursday, April 1, 2021 at 8.59pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - Gravitational parameters:
    - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
    - $Gm_{Earth} = 3.986004415 \times 10^{5} km^3/s^2$
    - $Gm_{Mercury} = 2.2032 \times 10^{4} km^3/s^2$
    - $Gm_{Saturn} = 3.794 \times 10^{7} km^3/s^2$
    - $Gm_{Jupiter} = 1.268 \times 10^{8} km^3/s^2$
  - Semi-major axes relative to the Sun:
    - $a_{Earth} = 1.0000010178\ AU$
    - $a_{Mercury} = 0.387098309\ AU$
    - $a_{Saturn} = 9.554909595\ AU$
    - $a_{Jupiter} = 5.202603191\ AU$
    - 1 AU = 149,597,870.7 km
  - Radius of celestial bodies:
    - $r_{Mercury} = 2{,}439\ km$
    - $r_{Saturn} = 60{,}268\ km$
    - $r_{Jupiter} = 71{,}492\ km$
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Consider a mission to Mercury with the goal of placing a spacecraft into an orbit about Mercury for long-term observations of the planetary surface and environment.

a) Let's consider a Hohmann transfer from the Earth to Mercury, assuming the orbits of both bodies are circular and coplanar and modeling only solar gravity. Calculate the semi-major axis and eccentricity of the transfer as well as time of flight required.

Consider a scenario where the spacecraft propulsion system malfunctions during the heliocentric transfer and a Mercury orbit insertion maneuver does not occur. On the hyperbolic approach arc, the closest approach distance is 4,000 km from the center of Mercury, and the spacecraft passes on the 'Sun-side' of Mercury during the flyby.

b) Incorporate the gravity field of Mercury to calculate the turning angle along the hyperbolic orbit during the arrival segment and the heliocentric velocity magnitude (v_{out}) after the flyby. Did the flyby increase or decrease the spacecraft energy?
c) Draw a vector diagram of the heliocentric spacecraft velocities before and after the flyby, as well as Mercury's velocity vector and the relative velocity vectors. Also draw the hyperbolic arc in a Mercury-centered view, including: incoming and outgoing $\bar{v}_\infty$ vectors, r_p, planet velocity vector, and turning angle.

Problem 2:

Consider a spacecraft in a large orbit around Saturn, described by a periapsis radius of 700,000 km and an apoapsis radius of 2,000,000 km – and lying in the same orbit plane as Titan. Assume that Titan is in a circular orbit of radius 1,221,830 km, possesses a mass of 1.3455x10^{23} kg and is modeled as a sphere with a radius equal to the equatorial radius, 2,575 km.

a) As the spacecraft travels from periapsis to apoapsis in its orbit, what is the value of the true anomaly when it intersects Titan's orbit? At this value of the true anomaly, write the velocity vector, $\bar{v}_{in}$, relative to Saturn in the rotating frame ($\hat{r}\hat{\theta}\hat{h}$). Also calculate the velocity vector relative to Titan, i.e., $\bar{v}_{\infty,in}$. Draw a useful vector diagram of these two velocity vectors.
b) If the spacecraft performs a flyby of Titan with a closest approach distance of 2,800 km, and passes behind Titan, calculate the turning angle. Draw a diagram of the hyperbolic orbit in a Titan-centered view, including: incoming and outgoing $\bar{v}_{\infty,in}$ vectors, r_p, Titan's velocity vector, and the turning angle.
c) Add the turning angle and post-flyby relative velocity vector to your diagram in part a). Use the vector diagram to calculate the velocity vector, $\bar{v}_{out}$, relative to Saturn in the rotating coordinates ($\hat{r}\hat{\theta}\hat{h}$).
d) Determine the orbital elements (a, e, θ^*) of the Saturn-centered orbit after the flyby and describe the impact of the gravity assist in your own words. Did the flyby increase or decrease the spacecraft energy?
e) Calculate the equivalent impulsive maneuver, $\Delta\bar{v}_{eq}$, that would be required to change the heliocentric velocity of the spacecraft from $\bar{v}_{in}$ to $\bar{v}_{out}$ if a gravity assist was not used.

Problem 3:

The goal of this problem is to learn how to model a trajectory in STK/GMAT using different dynamical models along different segments. Follow the instructions for GMAT/STK available on the Canvas page in the HW 7 module and answer the following questions when indicated.

a) Take a screenshot of the MCS (STK) or mission sequence (GMAT). Also take three screenshots of the trajectory: one that is a heliocentric view; one Jupiter-centered view displaying the hyperbolic flyby from above the orbit plane (segments b and c); and one Saturn-centered view displaying the hyperbolic approach arc from above the orbit plane (segment e). Describe, in your own words, the characteristics of the trajectory in each of the heliocentric view, the Jupiter-centered view of the flyby, and the Saturn-centered view of the approach arc.
b) Instructions based on software used:
    i. STK: Use the summary function to report the periapsis altitude for the hyperbolic arc describing the Jupiter flyby, using the "Prop to Jupiter Periapsis" segment and listed as the "Rad. Peri" item. In addition, list the epoch in Gregorian UTC date format at periapsis, the inclination, and the v-infinity along the hyperbola (listed as "Excess Vel" in the summary). Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola.
    ii. GMAT: Use the report to list the periapsis altitude for the hyperbolic arc describing the Jupiter flyby. In addition, list the epoch in Gregorian UTC date

format at periapsis, the inclination, and the v-infinity along the hyperbola (compute this from the Incoming C3). Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola as well as the turning angle.

c) Instructions based on software used:
   i. STK: Use the summary function to report the periapsis altitude for the hyperbolic arc describing the Saturn approach, using the "Prop to Saturn Periapsis" segment. In addition, list the epoch in Gregorian UTC date format at periapsis, the inclination, and the v-infinity along the hyperbola (listed as "Excess Vel" in the summary).
   ii. GMAT: Use the report to determine the periapsis altitude for the hyperbolic arc describing the Saturn approach. In addition, list the epoch in Gregorian UTC date format at periapsis, the inclination, and the v-infinity along the hyperbola (compute this from the Incoming C3).

d) The true Cassini trajectory reached periapsis at Saturn on July 1, 2004 with a radius of 81,000 km. Compare this information to your answer in part c), and speculate why the simulation you constructed produces the observed differences in the trajectory at its Saturn arrival.

e) After running the targeter, list the new values of the components of the initial state vector. How much did the position and velocity vectors change to achieve the prescribed change in the periapsis radius at Saturn? Use the summary (STK) or report (GMAT) function to verify that the periapsis radius of the Saturn approach arc is, indeed, equal to 81,000 km. Also list the six orbital elements in the Saturn inertial frame at periapsis relative to Saturn. Include a screenshot of the heliocentric view of the complete trajectory, as well as the Saturn-centered view of the Saturn approach arc. Why do you think such a small change in the initial state achieved such a significant change in the periapsis radius at Saturn?

f) Include a screenshot of the Saturn-centered view, viewing from above their common orbit plane both the incoming hyperbolic arc and the elliptical orbit. What is the magnitude of the impulsive maneuver required to reach the prescribed orbit?