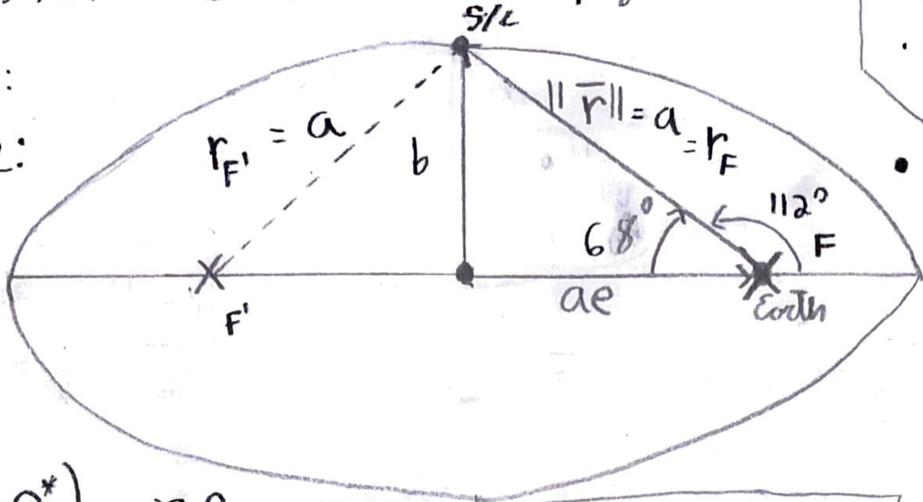
ASEN 5050 - HW 2 (Pg 1) GUSTAVO Grinsteins 02/02/2021

Problem 1: . S/C in Elliptical orbit about Earth

Assumptions: . 2BP
. No Perturbations
. $Gm_{Earth} = 3.986004415 \times 10^5 \frac{km^3}{s^2}$
. Equatorial radius of Earth: 6378.1363 km

• $P = 15{,}345$ km . $\theta^* = 112°$ when S/C top of b

P1)
a) Diagram:
Find a & e:

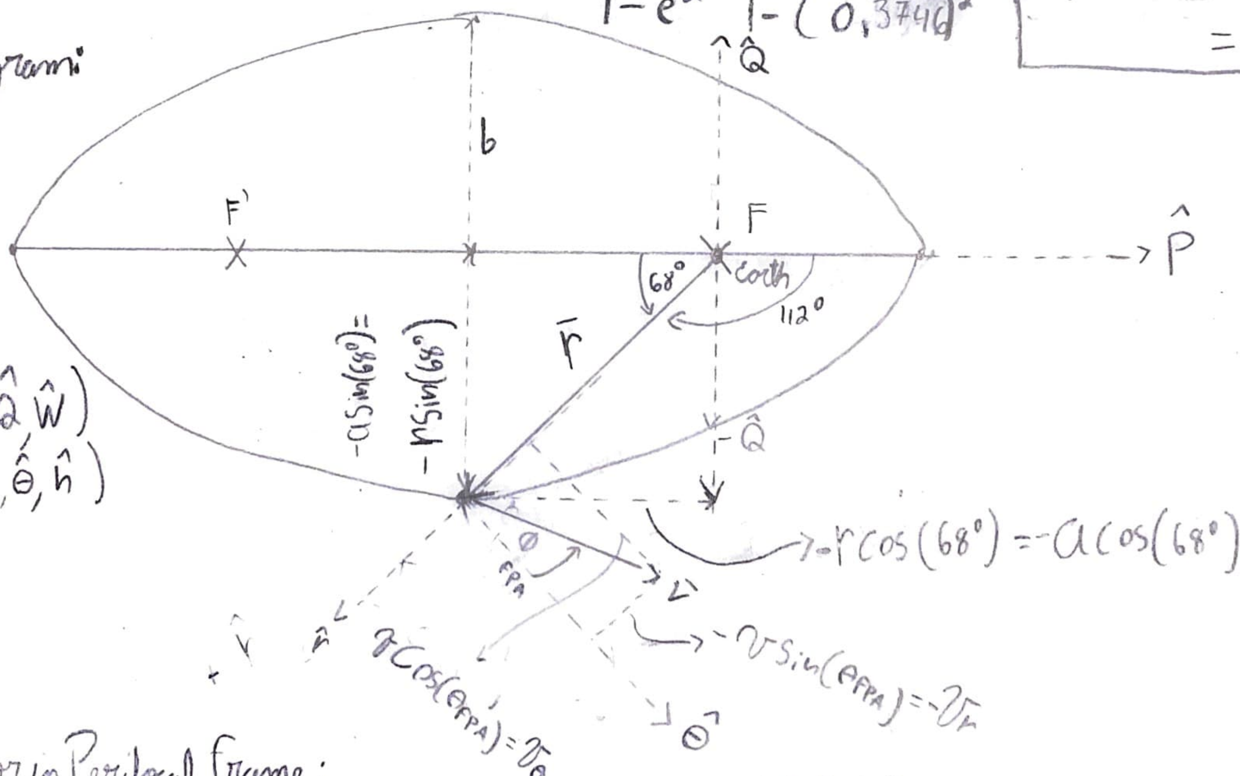


• Since:
$r_{F'} + r_F = 2a$
then $\|\bar{r}\| = a$ at top of b

$$\cos(180° - \theta^*) = \frac{\cancel{a}e}{\cancel{a}} \therefore \boxed{\cos(68°) = e = 0.3746}$$

Given that $\rightarrow P = a(1-e^2) \therefore a = \frac{P}{1-e^2} = \frac{(15{,}345)}{1-(0.3746)^2} = \boxed{17{,}849.7721 \text{ Km} = a}$

P1)
b) Diagram:
• $r = a$
• Move towards Periapsis
Find: • $\bar{r}: (\hat{P}, \hat{Q}, \hat{W})$
• $\bar{v}: (\hat{r}, \hat{\theta}, \hat{h})$



Position Vector in Perifocal Frame:

$\hat{P}: -r\cos(68°) = -a\cos(68°) = -6{,}689.6423$ km
$\hat{Q}: -r\sin(68°) = -a\sin(68°) = -16{,}550.02051$ km
$\hat{W}: 0$

$(\hat{P}, \hat{Q}, \hat{W})$
$$\boxed{\bar{r} = (-6{,}689.6423\,\hat{P} - 16{,}550.02051\,\hat{Q}) \text{ Km}}$$

P1) b) Cont. Velocity Vector in Rotating frame:

$$\rightarrow v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = \sqrt{\frac{2\mu}{a} - \frac{\mu}{a}} = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{(3.986004415\times10^5)}{(17,849.7721)}} = 4.7256 \text{ km/s}$$

$$\rightarrow p = \frac{h^2}{\mu} \therefore h = \sqrt{\mu p} = \sqrt{(3.986004415\times10^5)(15,345)} = 78,208.2079 \text{ km}^2/\text{s}$$

$$\rightarrow h = r^2\dot{\theta} \therefore r\dot{\theta} = \frac{h}{r} = \frac{h}{a} = \frac{(78,208.2079)}{(17,849.7721)} = 4.3815 \text{ km/s}$$

$$\rightarrow v\cos(\phi_{FPA}) = r\dot{\theta} \therefore \phi_{FPA} = \pm\cos^{-1}\left(\frac{r\dot{\theta}}{v}\right) = \pm\cos^{-1}\left(\frac{4.3815}{4.7256}\right) = \pm 0.3840 \text{ rad}$$

or $22.0°$

$\hat{r}: -v\sin(\phi_{FPA}) = -4.7256\sin(22.0°) = -1.7702$ km/s

$\hat{\theta}: +v\cos(\phi_{FPA}) = +r\dot{\theta} = +a\dot{\theta} = +4.3815$ km/s $\quad (\hat{r}, \hat{\theta}, \hat{h})$

$\hat{h}: 0$

$$\boxed{\bar{V} = (-1.7702\,\hat{r} + 4.3815\,\hat{\theta}) \text{ km/s}}$$

Problem 2: at a Specific Instant in time, MAVEN S/C on Martian Orbit:

$\bar{r} = 4981.75\hat{x} - 4121.90\hat{y} + 22.70\hat{z}$ km

$\bar{V} = -0.60359\hat{x} + 0.56812\hat{y} - 2.24093\hat{z}$ km/s

Assumptions: • 2B • No Perturbations

• $\mu = Gm_{mars} = 4.305\times10^4$ km^3/s^2

• Equatorial Radius of Mars: 3397.2 km

• Note: Please refer to the attached MATLAB Script + results for Some Calculations.

P2) a) Calculate: $a, e, i, \Omega, \omega, \theta^*$

$$|\bar{r}| = \sqrt{(4981.75)^2 + (-4121.90)^2 + (22.70)^2} = 6465.90 \text{ km}$$

$$|\bar{v}| = \sqrt{(-0.60359)^2 + (0.56812)^2 + (-2.24093)^2} = 2.3893 \text{ km/s}$$

$$\bar{r}\times\bar{v} = \bar{h} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4981.75 & -4121.90 & 22.70 \\ -0.60359 & 0.56812 & -2.24093 \end{vmatrix} = 9223.99\hat{x} + 11,150.05\hat{y} + 342.29\hat{z} \text{ km}^2/\text{s}$$

$$|\bar{h}| = \sqrt{(9,223.99)^2 + (11,150.05)^2 + (342.29)^2} = 14,474.90 \text{ km}^2/\text{s}$$

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{(2.3893)^2}{2} - \frac{(4.305\times10^4)}{(6465.90)} = -3.8035 \text{ km}^2/\text{s}^2$$

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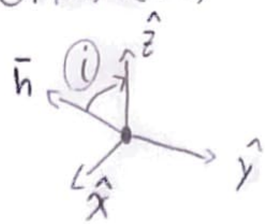
P2) a) Finding the Inclination parameter (i):



by definition $i \geq 0$

$$\hat{z} \cdot \bar{h} = |\hat{z}| \cdot |\bar{h}| \cos(i)$$

$$\cos(i) = \frac{\hat{z} \cdot \bar{h}}{|\hat{z}| \cdot |\bar{h}|} \rightarrow i = \pm \cos^{-1}\left(\frac{h_3}{h}\right) = \pm \cos^{-1}\left(\frac{342.29}{14,474.90}\right) \boxed{= +1.5471 \text{ rad}, \quad i = 88.6450^\circ}$$

Finding Semi-Major axis (a): $a = \frac{-\mu}{2\varepsilon} = \frac{(-4.305 \times 10^4)}{2(-3.8035)} = \boxed{5659.26 \text{ Km} = a}$

Finding eccentricity (e):

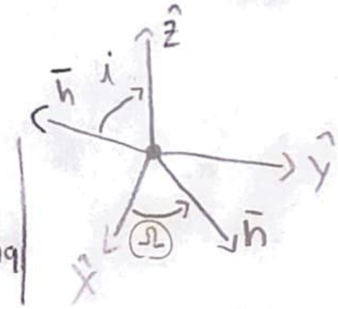
$$\bar{e} = \frac{\bar{v} \times \bar{h}}{\mu} - \frac{\bar{r}}{r} = \frac{1}{4.305 \times 10^4} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -0.60359 & 0.56812 & -2.24043 \\ 9223.99 & 11150.05 & 342.29 \end{vmatrix} - \frac{(4981.75\hat{x} - 4121.90\hat{y} + 22.70\hat{z})}{6465.90}$$

* Please refer to MATLAB Script and Results attached

$$\bar{e} = -0.1855\hat{x} + 0.1621\hat{y} - 0.2816\hat{z} \rightarrow \boxed{|\bar{e}| = 0.3742 = e}$$

$\hookrightarrow 0 < e < 1 \therefore$ Ellipse

Finding RAAN (Ω):



$$\bar{n} = \hat{z} \times \bar{h} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ 9,223.99 & 11,150.05 & 342.29 \end{vmatrix}$$

$$|\bar{h}| = \sqrt{(11,150.05)^2 + (9223.99)^2} = 14,470.8537 \text{ Km}^2/s$$

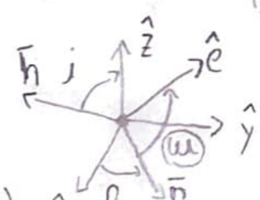
$$= -11,150.05\hat{x} + 9223.99\hat{y} + 0\hat{z} \quad \text{Km}^2/s$$

$$\cos(\Omega) = \frac{\hat{x} \cdot \bar{n}}{|\hat{x}||\bar{n}|} \rightarrow \Omega = \pm \cos^{-1}\left(\frac{n_1}{n}\right) = \pm \cos^{-1}\left(\frac{-11,150.05}{14,470.8537}\right) = \pm 2.4504 \text{ rad or } 140.4004^\circ$$

Sign check $\rightarrow \bar{n} \cdot \hat{y} = (-11,150.05\hat{x} + 9223.99\hat{y} + 0\hat{z}) \cdot (0\hat{x}, 1\hat{y}, 0\hat{z}) = 9223.99 \text{ Km}^2/s$

Since $\bar{n} \cdot \hat{y} > 0$ Then $\Omega = [0, 180^\circ] \therefore \boxed{\Omega = +2.4504 \text{ rad or } 140.4^\circ}$

Finding argument of Periapsis (ω):



$$\cos(\omega) = \frac{\bar{n} \cdot \bar{e}}{|\bar{n}||\bar{e}|}$$

$$\bar{n} \cdot \bar{e} = (-11,150.05\hat{x} + 9223.99\hat{y} + 0\hat{z}) \cdot (-0.1855\hat{x} + 0.1621\hat{y} - 0.2816\hat{z})$$

$$\bar{n} \cdot \bar{e} = 3563.5431 \text{ Km}^2/s$$

$$|\bar{n}| \cdot |\bar{e}| = 14,470.8537 \cdot 0.3742 = 5414.9934 \text{ Km}^2/s$$

$$\omega = \pm \cos^{-1}\left(\frac{3563.5431}{5414.9934}\right) = \pm 0.8525 \text{ rad or } 48.8^\circ$$

Sign Check $= \bar{e} \cdot \hat{z} = (-0.1855\hat{x} + 0.1621\hat{y} - 0.2816\hat{z}) \cdot (0\hat{x} + 0\hat{y} + 1\hat{z}) = -0.2816\hat{z}$

Since $\bar{e} \cdot \hat{z} < 0$ $\omega = [-180^\circ, 0] \therefore \boxed{\omega = -0.8525 \text{ rad or } -48.8^\circ}$

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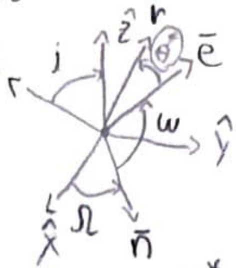
Finding true Anomaly (θ^*):



$$\cos(\theta^*) = \frac{\bar{r}\cdot\bar{e}}{|\bar{r}|\cdot|\bar{e}|}$$

$$\bar{r}\cdot\bar{e} = (4981.75\hat{x} - 4121.90\hat{y} + 22.70\hat{z}) \cdot (-.1855, 0.1621, -0.2816)$$

$$\bar{r}\cdot\bar{e} = -1598.6669 \text{ Km}$$

$$|\bar{r}|\cdot|\bar{e}| = (6465.90 \cdot 0.3742) = 2419.5398 \text{ km}$$

$$\theta^* = \pm\cos^{-1}\left(\frac{-1598.6669}{2419.5398}\right) = \pm 2.2926 \text{ rad or } 131.3557^\circ$$

Sign Check $\rightarrow \bar{r}\cdot\bar{v} = (4981.75\hat{x} - 4121.90\hat{y} + 22.70\hat{z})\cdot(-0.60359\hat{x} + 0.56812\hat{y} - 2.24093\hat{z})$

$\bar{r}\cdot\bar{v} = -5399.5374$ km²/s since $\bar{r}\cdot\bar{v} < 0$ $\theta^* = [-180^\circ, 0]$ $\therefore$ $\boxed{\theta^* = -2.2926 \text{ rad or } -131.3557^\circ}$

P2) b) Write $\bar{r}$ & $\bar{v}$ in $(\hat{r}, \hat{\theta}, \hat{h})$ 1) DCM 2) Analysis:

Position: (DCM)

*Note = Please refer to MATLAB script + Results Attached.

$$C = \begin{bmatrix} C\Omega C\theta - S\Omega C_i S\theta & -C\Omega S\theta - S\Omega C_i C\theta & S\Omega S_i \\ S\Omega C\theta + C\Omega C_i S\theta & -S\Omega C\theta + C\Omega C_i C\theta & -C\Omega S_i \\ S_i S\theta & S_i C\theta & C_i \end{bmatrix}$$

$$\bar{r}_{r\theta h} = [C]^T \cdot \bar{r}_{xyz} = [C]^T \cdot \begin{bmatrix} 4981.75\hat{x} \\ -4121.90\hat{y} \\ 22.70\hat{z} \end{bmatrix} \text{km} = (6465.9422\hat{r}) \text{ km}$$

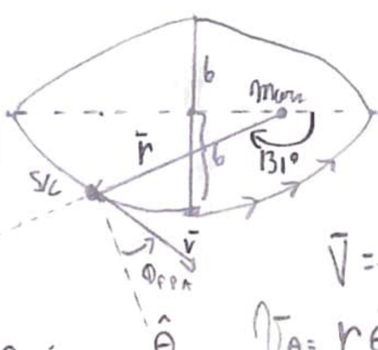
Velocity: (DCM)

$$\bar{v}_{r\theta h} = [C]^T \cdot \bar{v}_{xyz} = [C]^T \cdot \begin{bmatrix} -0.60359\hat{x} \\ 0.56812\hat{y} \\ -2.24093\hat{z} \end{bmatrix} \text{km/s} = (-0.8351\hat{r} + 2.2386\hat{\theta}) \text{ km/s}$$

Analysis: $\theta^* = 131^\circ$

$\theta^*_b = \pm\cos^{-1}(-e) = \pm 112^\circ$

Since $\theta^* > \theta^*_b$ and $\theta^* = [0, -180^\circ]$ and $r > r_b = a$; We know that the S/C is located before the top of Semi-Minor axis in the negative quadrant towards Periapsis



Position: (Analysis)

$$\boxed{\bar{r} = r\hat{r} = 6465.90\hat{r} \text{ km}}$$

Velocity: (Analysis)

$$\bar{v} = v_r\hat{r} + v_\theta\hat{\theta}$$

$$v_\theta = r\dot{\theta} \rightarrow \frac{h}{r} = r\dot{\theta} \therefore r\dot{\theta} = \frac{14{,}474.90}{6465.90} = 2.2387 \text{ km}$$

$$v\cos(\phi_{FPA}) = v_\theta \therefore \phi_{FPA} = \pm\cos^{-1}\left(\frac{2.2387}{2.3893}\right) = \pm.3569 \text{ rad or } 20.4514^\circ$$

$$v_r = v\sin(\phi_{FPA}) = (2.3893)\sin(-20.4514^\circ) = -0.8349 \text{ km/s}$$

$$\boxed{\bar{v} = (-0.8349\hat{r} + 2.2387\hat{\theta}) \text{ km/s}}$$

• The Solutions using the Analytical and DCM Approaches lead to same results

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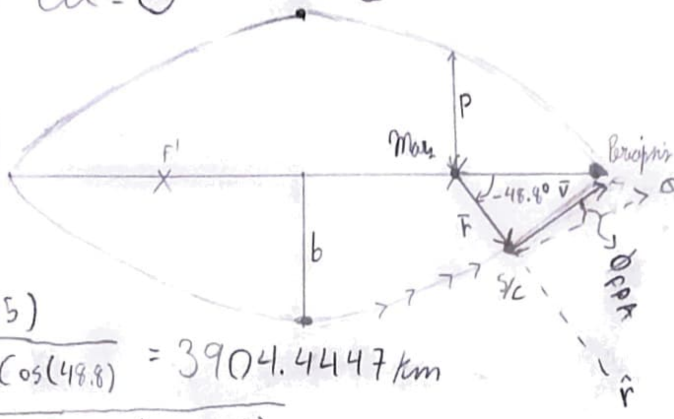
P2) c) S/C at ascending Node -> find $\bar{r}$ and $\bar{v}$ on XYZ:

• if S/C is at Ascending Node then $\omega = \theta^*$ ∴ $\theta^* = -48.8°$ Since $\theta^*_b = -112°$ then $|\theta^*| < |\theta^*_b|$ locating S/C after the top of the Semi-Minor axis towards periapsis. Since $\theta^* [-180°, 0] \rightarrow v_r < 0$



$a = 5659.26$ km $e = 0.3742$

$p = a(1-e^2) = (5659.26)(1-(0.3742)^2)$

$p = 4866.8185$ km

$$r = \frac{p}{1+e\cos(\theta^*)} = \frac{(4866.8185)}{1+(0.3742)\cos(48.8)} = 3904.4447 \text{ km}$$

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = \sqrt{\frac{2(4.305\times10^4)}{(3904.4447)} - \frac{(4.305\times10^4)}{(5659.26)}} = 3.8006 \text{ km/s}$$

Define Position and Rotation in rotating $(\hat{r}, \hat{\theta}, \hat{h})$ frame:

$\bar{r} : r\hat{r} = 3904.4447\,\hat{r}$ km

$\bar{V} : -v_r\,\hat{r} + v_\theta\,\hat{\theta}$

$\bar{V} : -3.0712\hat{r} + 2.2387\hat{\theta}$ km/s

$v_r = v\sin(\phi_{FPA})$

$v_\theta = v\cos(\phi_{FPA}) = r\dot{\theta}$

$$r\dot{\theta} = \frac{h}{r} = \frac{(14{,}474.90)}{(3904.4447)} = 3.7073 \text{ km/s}$$

$$\phi_{FPA} = \pm\cos^{-1}\left(\frac{v_\theta}{v}\right) = \pm\cos^{-1}\left(\frac{3.7073}{3.8006}\right)$$

$\phi_{FPA} = -0.2220$ rad since $v_r < 0$

$v_r = (3.8006)\sin(-0.2220) = -0.8368$ km/s

Use DCM to Convert $\bar{r}$ & $\bar{v}$ from $(\hat{r}, \hat{\theta}, \hat{h})$ to $(\hat{x}, \hat{y}, \hat{z})$

$$\bar{r}_{XYZ} = [C]\bar{r}_{r\theta h} = [C]\cdot\begin{bmatrix} 3904.4447\hat{r} \\ 0 \\ 0 \end{bmatrix} \text{km} = (3008.2186\hat{x} - 2489.0001\hat{y} + 13.7073\hat{z}) \text{ km}$$

* Please Refer to MATLAB code for solution details

$$\bar{V}_{XYZ} = [C]\bar{V}_{r\theta h} = [C]\cdot\begin{bmatrix} -0.8368\,\hat{r} \\ 3.7073\,\hat{\theta} \\ 0 \end{bmatrix} \text{km/s} = (-0.5788\hat{x} + 0.5927\hat{y} - 3.7092\hat{z}) \text{ km/s}$$

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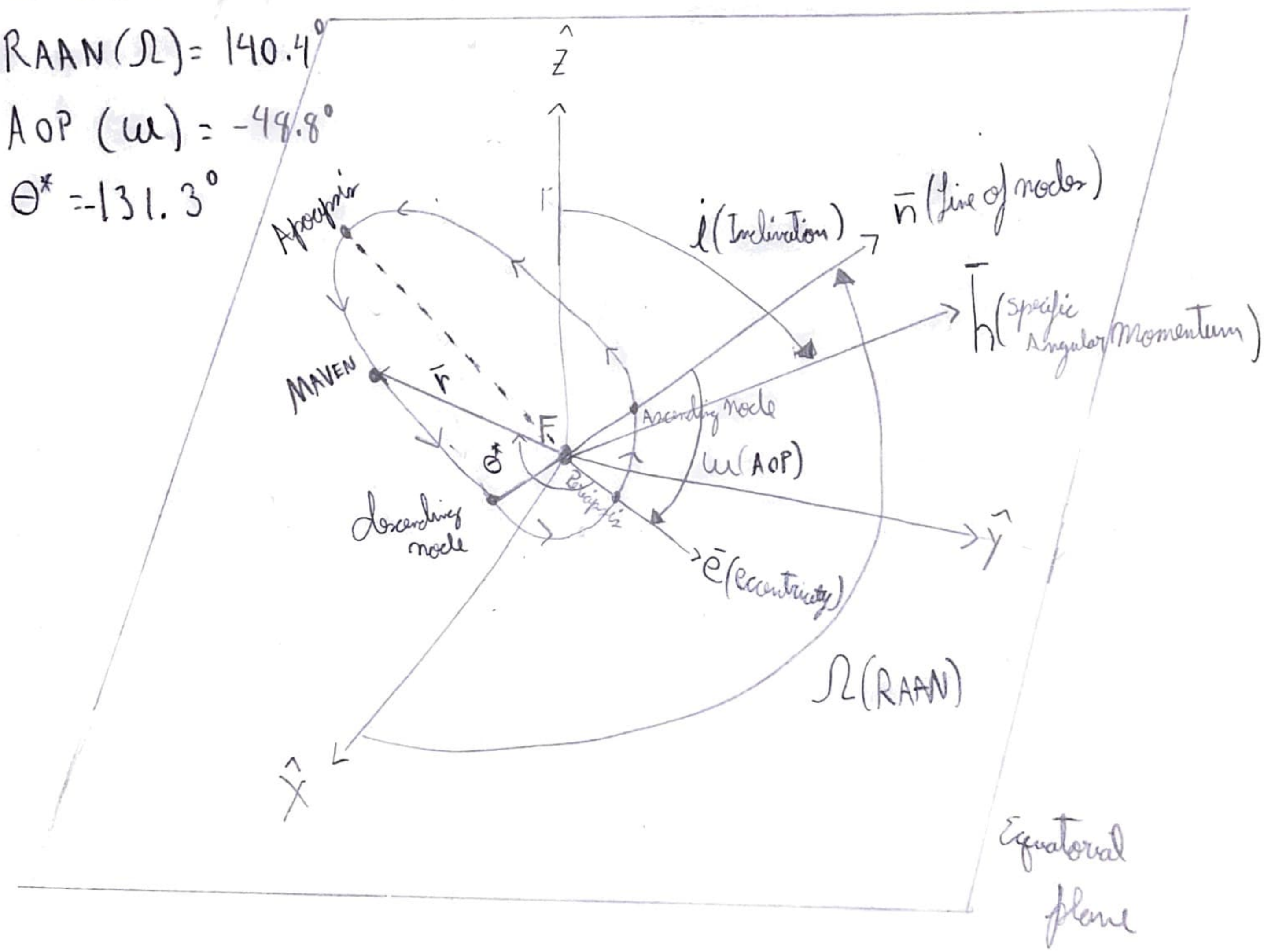
P2) d) Sketch orbit of MAVEN using $a, e, i, \Omega, \omega, \theta^*$

- $i = 88.6^\circ$
- RAAN (Ω) = 140.4°
- AOP (ω) = -48.8°
- $\theta^* = -131.3^\circ$



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Problem 2 Part A
r is 6465.94 Km, v is 2.3893 Km/s

H in XYZ frame is <9223.99,11150.05,342.29> Km^2/s
h is 14474.90 Km^2/s

Specific Energy = -3.8035 Km^2/s^2

inclination angle i = 88.6450 deg

semi-major axis a = 5659.20 Km

Eccentricity vector in XYZ frame is <-0.1855,0.1621,-0.2816>
e is 0.3742

N vector in XYZ frame is <-11150.0515,9223.9930,0.0000> Km^2/s
n is 14470.8568 Km^2/s

RAAN angle Omega = 140.4004 deg

Argument of Periapsis w = -48.8298 deg

True Anomaly ThetaStar = -131.3714 deg

Problem 2 Part B
R vector in (r,theta,h) frame is <6465.9422,-0.0000,0.0000> Km

V vector in (r,theta,h) frame is <-0.8351,2.2386,-0.0000> Km/s

Problem 2 Part C
R vector in XYZ frame is <3008.2186,-2489.0001,13.7073> Km

V vector in XYZ frame is <-0.5788,0.5927,-3.7092> Km/s

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%Gustavo Grinsteins
%ASEN 5050
%HW2 Problem 2

%House Keeping
clc;
clear;

%% Part A
fprintf('Problem 2 Part A \n')
%Position Vector
R = [4981.75,-4121.90,22.70]; %Km
%Velocity Vector
V = [-0.60359,0.56812,-2.24093]; %Km/s
%Gravitational Constant
mu = 4.305*10^4; %Km^3/s^2

%magnitudes
r = norm(R); %Km
v = norm(V); %Km/s
fprintf('r is %4.2f Km, v is %1.4f Km/s \n\n',r,v)

%Calculating specific angular momentum
H = cross(R,V); %Km^2/s
h = norm(H); %Km^2/s
fprintf('H in XYZ frame is <%4.2f,%2.2f,%4.2f> Km^2/s\n',H)
fprintf('h is %4.2f Km^2/s \n\n',h)

%Calculating Specific Energy
Sp_E = ((v)^2)/(2) - (mu/r); %Km^2/s^2
fprintf('Specific Energy = %4.4f Km^2/s^2 \n\n',Sp_E)


%Inclination angle
Zhat = [0,0,1];
i = max(min(dot(H,Zhat)/(norm(H)*norm(Zhat)),1),-1);%Radians
iDegrees = real(acosd(i));%Degrees
fprintf('inclination angle i = %4.4f deg \n\n',iDegrees)

%Semi-major axis
a = -mu/(2*Sp_E);%Km
fprintf('semi-major axis a = %4.2f Km\n\n',a)

%Eccentricity Vector
Ecc = cross(V,H)*(1/mu) - R/r;%Unitless
ecc = norm(Ecc);%Unitless

fprintf('Eccentricity vector in XYZ frame is <%4.4f,%2.4f,%4.4f> \n',Ecc)
fprintf('e is %4.4f\n\n',ecc)

%RAAN
N = cross(Zhat,H);%Km^2/s
n = norm(N);%Km^2/s
fprintf('N vector in XYZ frame is <%4.4f,%4.4f,%4.4f> Km^2/s\n',N)
fprintf('n is %4.4f Km^2/s\n\n',n)
Xhat = [1,0,0];
Om = max(min(dot(N,Xhat)/(norm(N)*norm(Xhat)),1),-1);%Radians
OmDegrees = real(acosd(Om));%Degrees
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fprintf('RAAN angle Omega = %4.4f deg \n\n',OmDegrees)

%Argument of periapsis
w = max(min(dot(N,Ecc)/(norm(N)*norm(Ecc)),1),-1);%Radians
wDegrees = -1*real(acosd(w));%Degrees
fprintf('Argument of Periapsis w = %4.4f deg \n\n',wDegrees)

%True Anomaly
ThetaStar = max(min(dot(R,Ecc)/(norm(R)*norm(Ecc)),1),-1);%Radians
TSDegrees = -1*real(acosd(ThetaStar));%Degrees
fprintf('True Anomaly ThetaStar = %4.4f deg \n\n',TSDegrees)

%Rotation matrix
theta = TSDegrees+wDegrees;%Degrees

%Rotation matrix
R1 = [1,0,0;0,cosd(iDegrees),sind(iDegrees);0,-sind(iDegrees),cosd(iDegrees)];
R3_Om = [cosd(OmDegrees),sind(OmDegrees),0;-sind(OmDegrees),cosd(OmDegrees),0;0,0,1];
R3_theta = [cosd(theta),sind(theta),0;-sind(theta),cosd(theta),0;0,0,1];

C = R3_theta*R1*R3_Om;

%% Part B
fprintf('Problem 2 Part B \n')
%Transforming position from XYZ to r,theta,h
Rrot = C*transpose(R);
fprintf('R vector in (r,theta,h) frame is <%4.4f,%4.4f,%4.4f> Km\n\n',Rrot)

%Transforming velocity from XYZ to r,theta,h
Vrot = C*transpose(V);
fprintf('V vector in (r,theta,h) frame is <%4.4f,%4.4f,%4.4f> Km/s\n\n',Vrot)

%% Part C S/C at ascending node
fprintf('Problem 2 Part C \n')
PositionRot = [3904.4447;0;0];

VelocityRot = [-0.8368;3.7073;0];

%Transforming position from r,theta,h to XYZ
PositionXYZ = C.'*PositionRot;
fprintf('R vector in XYZ frame is <%4.4f,%4.4f,%4.4f> Km\n\n',PositionXYZ)

%Transforming velocity from r,theta,h to XYZ
VelocityXYZ = C.'*VelocityRot;
fprintf('V vector in XYZ frame is <%4.4f,%4.4f,%4.4f> Km/s\n\n',VelocityXYZ)
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