ASEN 5050 Spring 2021 HW 7 Solutions

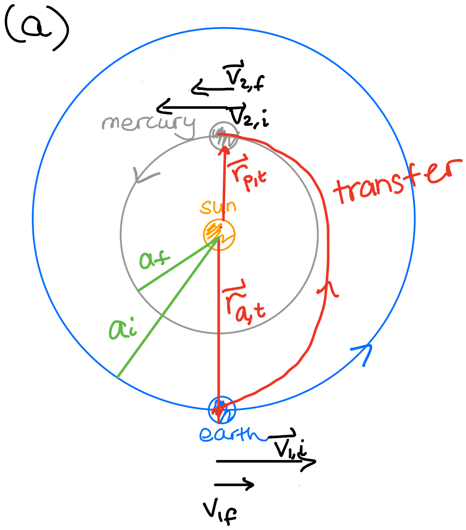
PROBLEM 1

given: $\mu_{mercury} = 2.2032 \times 10^4 \ km^3/s^2$
$\mu_{sun} = 1.32712428 \times 10^{11} \ km^3/s^2$
$\mu_{earth} = 3.986004415 \times 10^5 \ km^3/s^2$

$a_{merc} = 0.387098309 \ AU$
$a_{earth} = 1.000001078 \ AU$

$r_{merc} = 2439 \ km$

(a)



Hohmann transfer

$$a_t = \frac{1}{2}(r_{p,t} + r_{a,t}) = \frac{1}{2}(a_i + a_f)$$

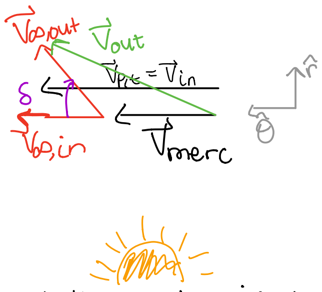
$$\boxed{a_t = 1.03754 \times 10^8 \ km}$$

$$e_t = \frac{r_{a,t} - r_{p,t}}{r_{a,t} + r_{p,t}} \quad \boxed{= 0.44186}$$

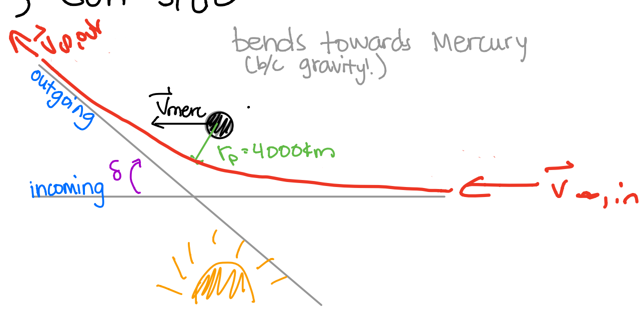
$$TOF = \frac{P}{2} = \pi \sqrt{\frac{a_t^3}{\mu_{sun}}} = 9.113780 \times 10^6 s$$

$$\boxed{TOF = 105.483 \ days}$$

(b) $r_p = 4000 \ km$, sun-side



Heliocentric view



Planet-centered view

useful eqns/notes:

$V_\infty = |\vec{V}_{\infty,in}| = |\vec{V}_{\infty,out}|$

$$\mathcal{E}_{hyp} = \frac{V_{\infty,in}^2}{2} - \frac{\mu_p}{r_\infty} = -\frac{\mu_{planet}}{2a_h}$$

$$r_{p,h} = a_h(1 - e_h)$$

$$\delta = 2\sin^{-1}\left(\frac{1}{e_h}\right) \quad \text{between } [0° - 180°]$$

taking a look at the heliocentric view:

$\vec{V}_{merc}$ = circular velocity = $\sqrt{\frac{\mu_\odot}{r_{Merc}}}$ = $47.8721\ \hat{\theta}$ km/s

$\vec{V}_{in} = \sqrt{\frac{2\mu_\odot}{r_{pt}} - \frac{\mu_\odot}{a_t}}$ with $r_{pt} = r_{merc}$

$\vec{V}_{in} = 57.4836\,\hat{\theta}$ km/s

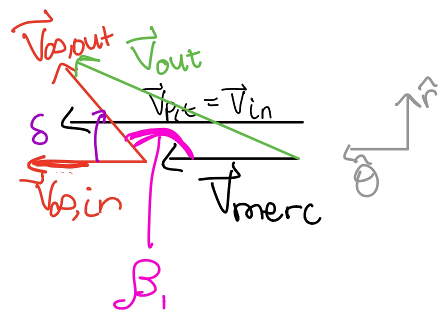
$\vec{V}_{\infty,in} = \vec{V}_{in} - \vec{V}_{merc} = 9.6115\,\hat{\theta}$ km/s

$\mathcal{E}_{hyp} = \frac{V_{\infty,in}^2}{2} = 46.1904$ km²/s²

$a_h = -\frac{\mu_{merc}}{2\mathcal{E}_h} = -238.49$ km

$e_h = 1 - \frac{r_{p,h}}{a_h} = 17.77$

$\delta = 2\sin^{-1}(1/e_h)$ $= 6.4515°$



from geometry:

$\beta_1 = 180° - \delta = 173.5488°$

$|V_{\infty,out}| = |V_{\infty,in}| = 9.6115$ km/s

cosine law: $V_{out}^2 = V_m^2 + V_{\infty,out}^2 - 2V_{\infty,out}V_m\cos(\beta_1)$

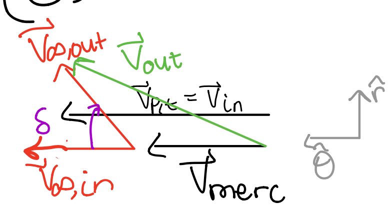
$V_{out} = 57.4329$ km/s

$\mathcal{E}_{before} = \frac{V_{in}^2}{2} - \frac{\mu_\odot}{r_{merc}} = -639.56$ km²/s²

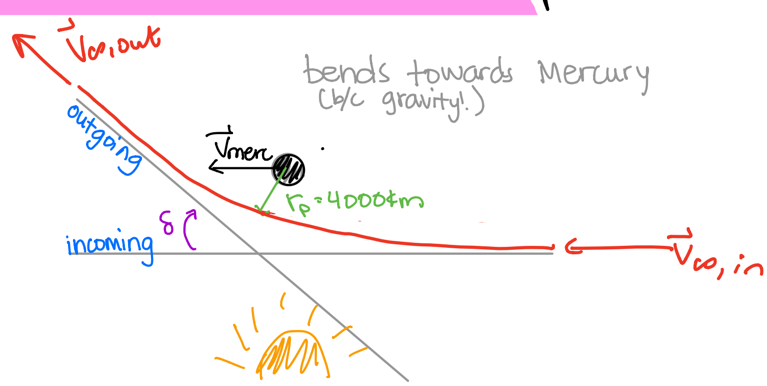
$\mathcal{E}_{after} = \frac{V_{out}^2}{2} - \frac{\mu_\odot}{r_{merc}} = -643.30$ km²/s²

specific energy has decreased in the heliocentric problem

(c)



Heliocentric view



Planet-centered view

PROBLEM 2

s/c around saturn, in same plane as Titan

$r_p = 700{,}000$ km
$r_a = 2{,}000{,}000$ km

$r_{\text{Titan orbit}} = 1{,}221{,}830$ km
$r_{\text{titan}} = 2575$ km
$m_{\text{titan}} = 1.3455 \times 10^{23}$ kg

(a) what is the true anomaly when the s/c intersects Titan's orbit?

$$a_{in} = \tfrac{1}{2}(r_{a,in} + r_{p,in}) = 1350000 \text{ km}$$
$$e_{in} = 1 - r_{p,in}/a_{in} = 0.4815$$
$$p_{in} = a_{in}(1 - e_{in}^2) = 1.037 \times 10^6 \text{ km}$$
$$\theta^*_{in} = \pm \cos^{-1}\left(\frac{p_{in} - r}{r e_{in}}\right) = \pm 1.8903 \text{ rad} = \pm 108.31°$$

travelling from periapsis to apoapsis $\therefore \theta^*_{in} > 0$

$$\theta^*_{in} = 108.31°$$

write velocity vector in $\hat{r}\hat{\theta}\hat{h}$ frame

$$h_{in} = \sqrt{\mu_{sat} p_{in}} = 6.2726 \times 10^6 \text{ km}^2/\text{s}^2$$
$$V_{r,in} = \frac{\mu_{saturn}}{h_{in}} e_{in} \sin(\theta^*_{in}) = 2.7649 \text{ km/s}$$
$$V_{theta,in} = \frac{\mu_{sat}}{h}\left(1 + e_{in}\cos(\theta^*_{in})\right) = 5.1338 \text{ km/s}$$

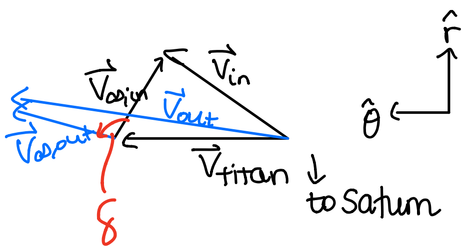
$$\vec{V}_{in} = 2.7649\hat{r} + 5.1338\hat{\theta} \text{ km/s}$$

the velocity vector relative to Titan

$$\vec{V}_{titan} = \sqrt{\frac{\mu_{saturn}}{r_{\text{titan orbit}}}} = 5.5724\hat{\theta} \text{ km/s}$$
$$\vec{V}_{\infty,in} = \vec{V}_{in} - \vec{V}_{titan}$$

$$\vec{V}_{\infty in} = 2.7649\hat{r} - 0.4387\hat{\theta} \text{ km/s}$$

$$V_{\infty in} = 2.7994 \text{ km/s}$$



Saturn-centric Frame

(b) Flyby of Titan $\mu_{Titan} \approx GM_{Titan}$ $(m_{sc} \ll m_{Titan})$

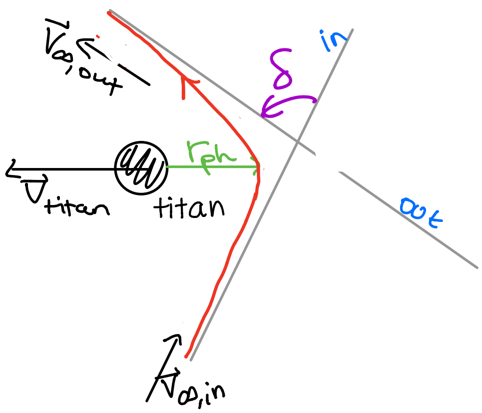
$r_{p,h} = 2800 km$, behind Titan.

$\varepsilon_h = \frac{V_{\infty,in}^2}{2} = 3.9184 \ km^2/s^2$

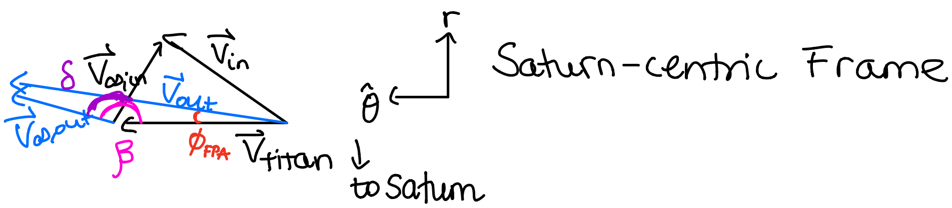
$a_h = \frac{-\mu_{titan}}{2\varepsilon_h} = -1.1452 \ km$

$e_h = 1 - \frac{r_{p,h}}{a_h} = 3.4451$

$\delta = 2\sin^{-1}(1/e_h) = 33.7483°$



(c)



$\vec{V}_{out} = \vec{V}_{titan} + \vec{V}_{\infty,out}$

solve for interior angle b/n $\vec{V}_{out}$ and $\vec{V}_{\infty,out}$

$V_{in}^2 = V_{\infty in}^2 + V_{titan}^2 - 2V_{\infty in}V_t\cos(\beta - \delta)$

$\beta = \cos^{-1}\left(\frac{V_{in}^2 - V_{\infty in}^2 - V_t^2}{-2V_{\infty in}V_t}\right) + \delta = 114.7331°$

calculate V_{out} using the law of cosines

$V_{out}^2 = V_{\infty out}^2 + V_{titan}^2 - 2V_{\infty out}V_{titan}\cos(\beta)$

$V_{out} = 7.2071 \ km/s$

find the ϕ_{FPA}

$$V_{\infty,out}^2 = V_{out}^2 + V_{titan}^2 - 2V_t V_{out}\cos(\phi_{FPA})$$

$$\phi_{FPA} = \cos^{-1}\left(\frac{(V_{\infty,out}^2 - V_{out}^2 - V_t^2)}{-2V_t V_{out}}\right) = 20.6585°$$

break $\vec{V}_{out}$ into components:

$$V_{out,r} = V_{out}\sin(\phi_{FPA})$$
$$V_{out,\theta} = V_{out}\cos(\phi_{FPA})$$

$$\vec{V}_{out} = 2.5426\,\hat{r} + 6.7437\,\hat{\theta} \text{ km/s}$$

(d) Calculate a, e, θ^* of the Saturn centered orbit after the flyby.

$$\mathcal{E}^+ = \frac{V_{out}^2}{2} - \frac{\mu_{sat}}{r}$$ where r is the distance from Saturn to Titan since we assume an instantaneous flyby.

$$= -5.0807 \text{ km}^2/\text{s}^2$$

$$a^+ = -\frac{\mu_{sat}}{2\mathcal{E}^+} = 3.7337\times10^6 \text{ km}$$

$$h^+ = r V_{out,\theta} = 8.2396\times10^6 \text{ km}^2/\text{s}$$

$$e^+ = \sqrt{1 + \frac{2h^2\mathcal{E}^+}{\mu_{sat}^2}} = 0.7216$$

$$\theta^{*+} = \pm\cos^{-1}\left(\frac{a^+(1-e^{2+}) - r}{re^+}\right) = \pm 49.93°$$

$$\theta^{*,+} = 49.93° \quad \text{b/c } \vec{V}_{out}\cdot\hat{r} > 0$$

the flyby has increased the semimajor axis and eccentricity of the orbit, thereby increasing the specific energy ($\mathcal{E} = -\mu/2a$). This makes sense for a flyby that passes behind Titan.

(e) $\Delta\vec{V}_{eq} = \vec{V}_{out} - \vec{V}_{in}$

$$\Delta\vec{V}_{eq} = -0.2222\,\hat{r} + 1.6099\,\hat{\theta} \text{ km/s}$$