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Gustavo Grinsteins Planchart
ASEN 5050
HW 9

A large spacecraft is currently in a circular orbit of radius 10,000 km around the Earth. This spacecraft is also carrying two CubeSats serving as secondary payloads. Let's explore this scenario via Problems 1 and 2.

Problem 1:
Assumptions:

- Relative 2 Body Problem Assumptions
    - Mass of satellite is negligible compared to attracting body
    - Coordinate system is inertial
    - Satellite and attracting body are treated as point masses
    - No other forces in the system except for gravitational forces
- Natural dynamics of vehicle well modeled via 2BP
- Relative distance between the spacecrafts is small compared to central body
- Target vehicle is in a circular orbit

The objective of the first CubeSat mission is to verify the performance of a new relative navigation strategy. To achieve this goal, the CubeSat must follow a trajectory that, relative to the primary spacecraft, is bounded over time.

After deployment, the CubeSat is described by an initial relative state vector $[x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]$ using the radial, along-track and cross-track unit vectors as a basis. However, only two components are known: $x_0 = -15\ m$, $\dot{x}_0 = 0$. In addition, the following characteristics of the CubeSat's trajectory relative to the primary spacecraft are known:

- The CubeSat and primary spacecraft lie in the same orbit plane
- The relative trajectory exhibits only bounded, oscillatory motion
- Along the relative trajectory, the maximum absolute value of the deviation in the along-track direction is 45 m.

a) Determine a combination of the remaining initial relative state components that produces a trajectory with these characteristics.

Analytical solution for CW equations:
Assumptions:

- Natural dynamics of vehicle well modeled via 2BP
- Relative distance between the spacecrafts is small compared to central body
- Target vehicle is in a circular orbit

$$x(t) = 4x_0 + \frac{2}{n}\dot{y}_0 + \frac{\dot{x}_0}{n}\sin(nt) - \left(3x_0 + \frac{2}{n}\dot{y}_0\right)\cos(nt)\ \ (radial)$$
$$y(t) = y_0 - \frac{2}{n}\dot{x}_0 - 3(2nx_0 + \dot{y}_0)t + 2\left(3x_0 + \frac{2}{n}\dot{y}_0\right)\sin(nt) + \frac{2}{n}\dot{x}_0\cos(nt)\ (along - track)$$
$$z(t) = \frac{1}{n}\dot{z}_0\sin(nt) + z_0\cos(nt)(cross - track)$$

Updating the equations given the known component values $x_0 = -15\ m$ and $\dot{x}_0 = 0$ m/s

$$x(t) = -60 + \frac{2}{n}\dot{y}_0 - \left(-45 + \frac{2}{n}\dot{y}_0\right)\cos(nt)\ \ m$$
$$y(t) = y_0 - 3(-30n + \dot{y}_0)t + 2\left(-45 + \frac{2}{n}\dot{y}_0\right)\sin(nt)\, m$$
$$z(t) = \frac{1}{n}\dot{z}_0\sin(nt) + z_0\cos(nt)\, m$$

Given the information that the CubeSat and primary spacecraft lie in the same orbit plane, the cross-track component will be zero. Therefore:

$$\dot{z}_0 = 0\ m/s\ \&\ z_0 = 0\ m$$

Since the relative trajectory exhibits only bounded, oscillatory motion; the drift in the along-track component needs to be suppressed. Therefore:

$$3(2nx_0 + \dot{y}_0) = 0 \therefore\ \ \dot{y}_0 = -2nx_0 = 30n$$

Updating the equations:

$$x(t) = -15\cos(nt)\, m$$
$$y(t) = y_0 + 30\sin(nt)\, m$$
$$z(t) = 0\ m$$

Because the maximum absolute value of the deviation in the along-track direction is 45 m. We can find y_0 using the following relationship:

$$\max(|y_0 + 30|) = 45\ \therefore\ y_0 + 30 = \pm 45$$

Two solutions stem from this relationship: $y_0 = 15\ m$ and $y_0 = -75\ m$. We select $y_0 = 15\ m$ since positive 45 m is defined as the value of deviation.

Updating the equations:

$$x(t) = -15\cos(nt)\, m$$
$$y(t) = 15 + 30\sin(nt)\, m$$
$$z(t) = 0\ m$$

Finding the value of the mean motion of the target vehicle:

$$n = \sqrt{\frac{\mu_{earth}}{10{,}000\ km}} = 6.3135x10^{-4}\frac{rad}{s}$$

Initial relative state component summary:

x_0	-15 m
$\dot{x}_0$	0 m/s
y_0	15 m
$\dot{y}_0$	0.0189404 m/s
z_0	0 m
$\dot{z}_0$	0 m/s

b) Plot the relative trajectory, representing the radial component on the vertical axis and the along-track component on the horizontal axis. Mark any extrema of x(t) and y(t) on the plot, along with the mean offset of the relative trajectory.

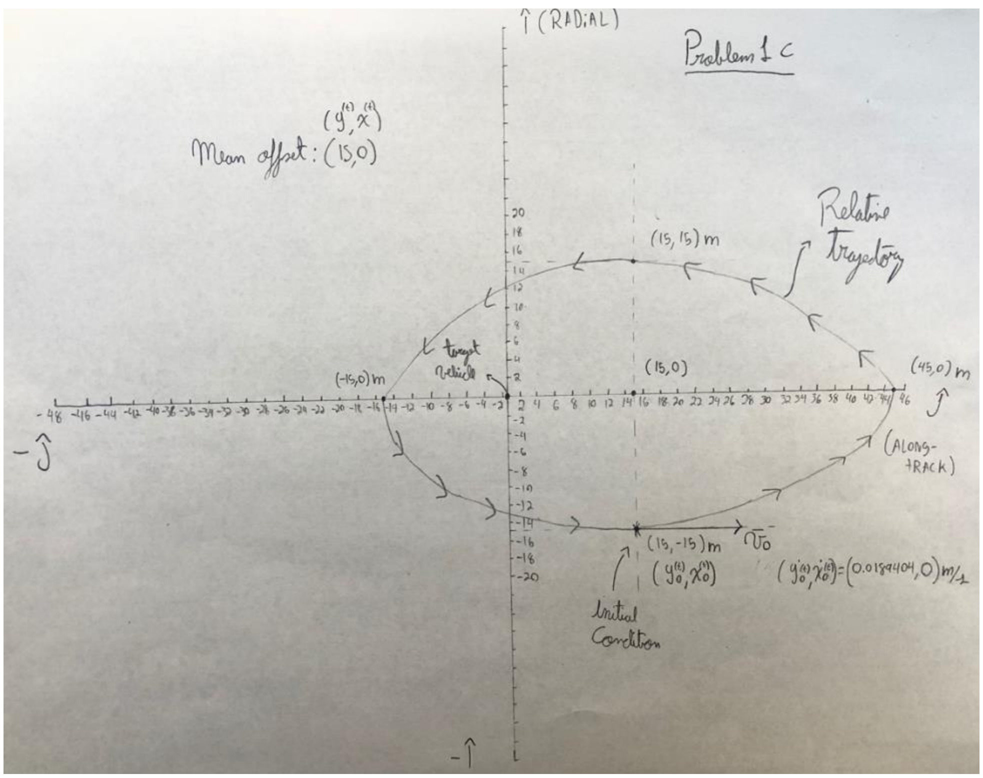

Figure 1 - Relative Trajectory P1 C

Problem 2:

Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Natural dynamics of vehicle well modeled via 2BP
- Relative distance between the spacecrafts is small compared to central body
- Target vehicle is in a circular orbit

The goal of the second CubeSat is to achieve a precise relative position with a relative velocity of zero at a specified time. In this problem, you will design a transfer to deliver the CubeSat to this location via two impulsive maneuvers.

After deployment, at a time t_0= *0 seconds*, the relative state of the CubeSat is expressed using the radial, along-track and cross-track unit vectors as:

$[x_0, y_0, z_0]$ = [−5 ,5 ,0] m & $[\dot{x}_0, \dot{y}_0, \dot{z}_0]$ = [0.04, −0.01,0.01] m/s

The goal is for the CubeSat to reach the following relative position at a time, t_1, equal to one third of the orbit period of the primary spacecraft:

$[x_1, y_1, z_1]$ = [5 ,5 ,0] m & $[\dot{x}_1, \dot{y}_1, \dot{z}_1]$ = [0, 0,0] m/s

$$t_1 = \frac{2}{3}\pi\sqrt{\frac{a^3}{\mu_{Earth}}} = 3.3173x10^3 \; seconds$$

a) To design a transfer between the two relative position vectors at time t_0 and time t_1, find the relative velocity required only the transfer at t_0. Use this relative velocity and the actual velocity of the CubeSat at t_0 to find the magnitude of the impulsive maneuver that must be applied for the CubeSat to begin the designed transfer.

Given that:

$$t_0 = 0 \; seconds$$

$$t_1 = 3.3173x10^3 \; seconds$$

$$n = \sqrt{\frac{\mu_{earth}}{10{,}000\ km}} = 6.3135x10^{-4}\frac{rad}{s}$$

$$\bar{r}(t_0) = \bar{r}_0 = -5\ \hat{x} + 5\ \hat{y} + 0\ \hat{z}\ \mathrm{m}$$

$$\bar{v}^-(t_0) = \bar{v}_0^- = [0.04, -0.01, 0.01]\ \mathrm{m/s}\ velocity\ before\ maneuver$$

$$\bar{v}_0^+ = velocity\ after\ maneuver$$

$$\bar{v}^-(t_1) = \bar{v}_1^- = velocity\ before\ maneuver$$

$$\bar{v}_1^+ = [0,0,0] m/s\ velocity\ after\ maneuver$$

$$\bar{r}(t_1) = \bar{r}_1 = 5\ \hat{x} + 5\ \hat{y} + 0\ \hat{z}\ \mathrm{m}$$

Given the CW equations in vector form:

$$\bar{r}(t) = \phi_{rr}\bar{r}(t_0) + \phi_{rv}\bar{v}(t_0)$$

Where:

ϕ_{rr} =

$$\begin{vmatrix} 4 - 3\cos(nt) & 0 & 0 \\ 6(\sin(nt) - nt) & 1 & 0 \\ 0 & 0 & cos(nt) \end{vmatrix}$$

ϕ_{rv} =

$$\begin{vmatrix} \frac{1}{n}\sin(nt) & \frac{2}{n}(1 - \cos(nt)) & 0 \\ \frac{2}{n}(\cos(nt) - 1) & \frac{4}{n}\sin(nt) - 3t & 0 \\ 0 & 0 & \frac{1}{n}sin(nt) \end{vmatrix}$$

Obtaining the time derivative:

$$\bar{v}(t) = \phi_{vr}\bar{r}(t_0) + \phi_{vv}\bar{v}(t_0)$$

Using the following derivatives:

$$\frac{d}{dt}\cos(t) = -\sin(t)$$

$$\frac{d}{dt}\sin(t) = \cos(t)$$

$$\frac{d}{dt}t = 1$$

And the chain rule property of derivatives:

$$\frac{d}{dt}\left[f\left(g(t)\right)\right] = f'\left(g(t)\right)g'(t)$$

The following matrices are created:

ϕ_{vr} =

$$\begin{vmatrix} 3nsin(nt) & 0 & 0 \\ 6(n\sin(nt) - n) & 1 & 0 \\ 0 & 0 & -nsin(nt) \end{vmatrix}$$

ϕ_{vv} =

$$\begin{vmatrix} cos(nt) & 2\sin(\mathrm{nt}) & 0 \\ -2\sin(\mathrm{nt}) & 4\cos(nt) - 3 & 0 \\ 0 & 0 & cos(nt) \end{vmatrix}$$

Finding the relative velocity after maneuver at t_0:

$$\bar{r}_1 = \phi_{rr}(t_1)\bar{r}_0 + \phi_{rv}(t_1)\bar{v}_0^+$$

$$\bar{v}_0^+ = -\phi_{rv}^{-1}(t_1)\ [\phi_{rr}(t_1)\bar{r}_0 - \bar{r}_1]$$

$$\boldsymbol{\bar{v}_0^+ = 0.0018225\,\hat{x} + 0.0063135\,\hat{y} + 0\,\hat{z}\ m/s} < --$$

Calculating $\Delta\bar{v}_1$:

$$\Delta\bar{v}_1 = \bar{v}_0^+ - \bar{v}_0^-$$

$$\Delta\bar{v}_1 = [0.0018225\,\hat{x} + 0.0063135\,\hat{y} + 0\,\hat{z}] - [0.04\,\hat{x} - 0.01\,\hat{y} + 0.01\,\hat{z}]\ m/s$$

$$\Delta\bar{v}_1 = -0.038177\,\hat{x} + 0.016313\,\hat{y} - 0.010000\,\hat{z}\ m/s$$

$$\|\Delta\boldsymbol{\bar{v}_1}\| = \boldsymbol{0.0427042\ m/s}\ < --$$

b) Find the relative velocity of the spacecraft at the end of the transfer, i.e., at time t_1. Use this relative velocity to find the magnitude of a second impulsive maneuver that must be applied for the CubeSat to achieve a relative velocity of zero at t_1.

Finding the relative before the maneuver at t_1:

$$\boldsymbol{\bar{v}_1^-} = \boldsymbol{\phi_{vr}(t_1)\bar{r}_0} + \boldsymbol{\phi_{vv}(t_1)\bar{v}_0^+} = \mathbf{0.0018225\,\hat{x} - 0.0063135\,\hat{y} + 0\,\hat{z}\;m/s} < --$$

Calculating $\Delta\bar{v}_2$:

$$\begin{aligned}\Delta\bar{v}_2 = \bar{v}_1^+ - \bar{v}_1^- &= [0\,\hat{x} + 0\,\hat{y} + 0\,\hat{z}] - [0.0018225\,\hat{x} - 0.0063135\,\hat{y} + 0\,\hat{z}\,] \\ &= [-0.0018225\,\hat{x} + 0.0063135\,\hat{y} + 0\,\hat{z}\,]\,m/s\end{aligned}$$

$$\|\Delta\boldsymbol{\bar{v}_2}\| = \mathbf{0.0065713\;m/s} < --$$

c) Plot the relative trajectory along the transfer, representing the radial component on the vertical axis and the along-track component on the horizontal axis. Add the direction of motion. Mark the initial and final CubeSat locations and the velocities before and after each maneuver.

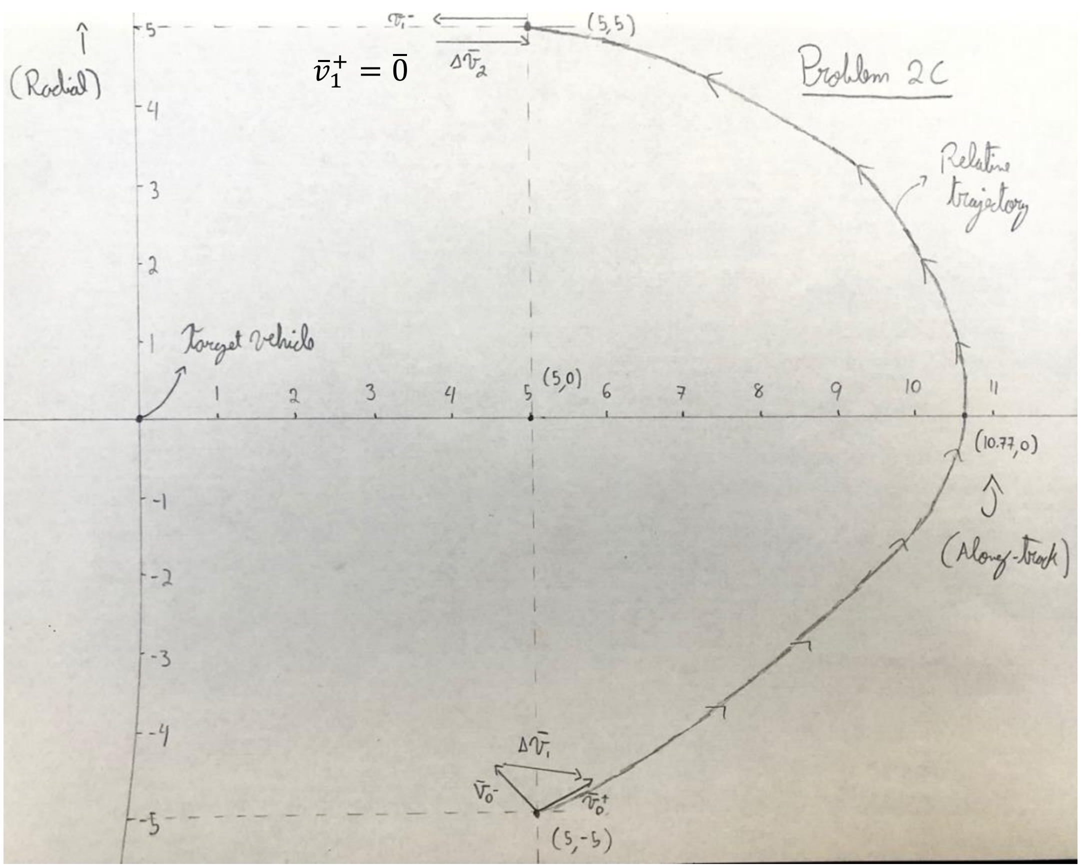

Figure 2 - Relative Trajectory P2 C

Problem 3:

Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- 1 AU = 149,597,870.7 km
- $\mu_{Mars} = 4.305x10^4\ km^3/s^2$
- $\mu_{Sun} = 1.32712428x10^{11}\ km^3/s^2$
- $a_{Mars} = 1.52367934\ AU$ = $2.2794x10^8$ km
- Mars J2: 0.001964
- Mars equatorial radius: 3397.2 km
- Mars period of rotation: 1.02595675 days

Throughout this problem, list any assumptions used in your writeup

a) Calculate the inclination required for a Sun-synchronous orbit around Mars at an altitude of 300 km.

Assuming that Mars has a circular orbit around the sun

$$v_{Mars} = \sqrt{\frac{\mu_{Sun}}{a_{Mars}}} = 24.129\ km/s$$

$$One\ Orbit\ Time = (2\pi a_{Mars})\frac{1}{v_{Mars}} = 5.9354x10^7 seconds$$

Rate of change of RAAN for a sun-synchronous orbit:

$$\dot{\Omega}_{sso} = \frac{2\pi}{year} = \frac{2\pi}{5.9354x10^7\ seconds} = 1.05859x10^{-7}\frac{radians}{second}$$

Assuming that the spacecraft has a circular orbit around mars

$$a_{sc} = Mars\ EQR + 300\ km = 3697.2\ km$$

$$e_{sc} = 0$$

Using the following equation to solve for the inclination and Assuming J_2 is the only perturbation effect:

$$Mars\ EQR = R_{Mars}$$

$$\dot{\Omega}_{sso} = -\left(\frac{3}{2}\frac{\sqrt{\mu_{Mars}}J_{2,Mars}R_{Mars}^2}{(1-e_{sc}^2)a_{sc}^{\frac{7}{2}}}\right)\cos(i)$$

$$\boldsymbol{i = \cos^{-1}\left(\frac{-\dot{\Omega}_{sso}\left(\frac{2}{3}\right)(1-e_{sc}^2)\left(a_{sc}^{\frac{7}{2}}\right)}{\sqrt{\mu_{Mars}}J_{2,Mars}R_{Mars}^2}\right)\left(\frac{180}{\pi}\right) = 92.643° <--}$$

b) Calculate the altitude required for an areostationary orbit around Mars (i.e., an orbit that possesses similar characteristics to a geostationary Earth orbit, but around Mars).

<u>Assuming that the spacecraft has a circular orbit around Mars and orbits about Mars' equator</u>

$$SC\ aerostationary\ orbit\ period = 1.02595675\ days = 8.8643x10^4 seconds$$

Using the 2BP orbit period formula to calculate the circular orbit radius:

$$a_{sc\_areost} = \left(\left(\frac{\mathbb{P}}{2\pi}\right)^2 \mu_{Mars}\right)^{\frac{1}{3}} = 2.04629x10^4\ km$$

Calculating the orbit's altitude:

$$\boldsymbol{Aerostationary\ Altitude = a_{sc\_areost} - Mars\ EQR = 1.70657x10^4\ km <--}$$

c) Is it possible to design a single orbit around Mars that is both sun-synchronous and areostationary? If so, list the relevant orbital elements that satisfy both requirements. If not, justify why.

Using the spacecraft circular orbit radius constraint calculated in part b and an inclination angle of 0 for the aerostationary orbit constraint, lets calculate the RAAN angular rate value:

$$i_{aerost} = 0°$$

$$a_{sc\_areost} = 2.04629x10^4\ km$$

$$\dot{\Omega}_{aerostat} = -\left(\frac{3}{2}\frac{\sqrt{\mu_{Mars}}J_{2,Mars}R_{Mars}^2}{(1-e_{sc}^2)a_{sc}^{\frac{7}{2}}}\right)\cos(i_{aerost}) = -5.7555x10^{-9}\frac{radians}{second}$$

Since $\dot{\Omega}_{aerostat} \neq \dot{\Omega}_{sso}$, analytically we can observe that is not possible to obtain equal RAAN angular rate values with the required constraints for each type of orbit. Furthermore, the RAAN angular rate values proceed at different directions when comparing both types of orbits given the orbit inclination parameter. Therefore, **Is NOT possible to design a single orbit around Mars that is both sun-synchronous and areostationary <--**

```
%Gustavo Grinsteins
%ASEN 5050
%HW9

%House Keeping
clc;
clear;

%problem 1
mu_earth = 3.986004415*10^5;
r = 10000;
n = sqrt(mu_earth/r^3);
for t = 0:10000
    x1(t+1) = -15*cos(n*t);
    y1(t+1) = 15 + 30*sin(n*t);
end
figure(1)
plot(y1,x1)
%Problem 2
TOF = (2/3)*pi*sqrt(r^3/mu_earth);
t_0 = 0;
t_1 = TOF;
R0 = [-5;5;0];
V_0_minus = [0.04;-0.01;0.01];
R1 = [5;5;0];
V_1_plus = [0;0;0];
%defining CW matrix eqs
phi_rr_t1 = [4-3*cos(n*t_1),0,0;6*(sin(n*t_1)-n*t_1),1,0;0,0,cos(n*t_1)];
phi_rv_t1 = [(1/n)*sin(n*t_1),(2/n)*(1-cos(n*t_1)),0;(2/n)*(cos(n*t_1)-1),(4/n)*sin↙
(n*t_1)-3*t_1,0;0,0,(1/n)*sin(n*t_1)];
phi_vr_t1 = [3*n*sin(n*t_1),0,0;6*(n*cos(n*t_1)-n),0,0;0,0,-n*sin(n*t_1)];
phi_vv_t1 = [cos(n*t_1),2*sin(n*t_1),0;-2*sin(n*t_1),4*cos(n*t_1)-3,0;0,0,cos(n*t_1)];

V_0_plus = -inv(phi_rv_t1)*(phi_rr_t1*R0-R1);
deltaV1 = V_0_plus-V_0_minus;
deltav1 = norm(deltaV1);

V_1_minus = phi_vr_t1*R0+phi_vv_t1*V_0_plus;
deltaV2 = V_1_plus-V_1_minus;
deltav2 = norm(deltaV2);

R0_2 = [-5;5];
V_0_minus_2 = [0.04;-0.01];
V_0_plus_2 = [V_0_plus(1);V_0_plus(2)];

x_0 = -5;
y_0 = 5;
xdot_0 = 0.04;
ydot_0 = -0.01;

for t_i = 0:TOF
    placeholder = [4-3*cos(n*t_i),0;6*(sin(n*t_i)-n*t_i),1]*R0_2+[(1/n)*sin(n*t_i),(2/n)*↙
(1-cos(n*t_i));(2/n)*(cos(n*t_i)-1),(4/n)*sin(n*t_i)-3*t_i]*V_0_plus_2;
    %x(t_i+1) = 4*x_0+(2/n)*(ydot_0)+(xdot_0/n)*sin(n*t_i)-(3*x_0+(2/n)*ydot_0)*cos↙
(n*t_i);
    %y(t_i+1) = y_0-(2/n)*xdot_0-3*(2*n*x_0+ydot_0)*t_i+2*(3*x_0+(2/n)*ydot_0)*sin(n*t_i)↙
+(2/n)*xdot_0*cos(n*t_i);
    x(t_i+1) = placeholder(1);
```

```
    y(t_i+1) = placeholder(2);
end
figure(2)
plot(y,x)

%% Problem 2
mu_sun = 1.32712428*10^11;
mu_mars = 4.305*10^4;
mars_EQR = 3397.2;
J2_mars = 0.001964;
a_sc = mars_EQR + 300;
a_mars = 1.52367934*149597870.7;
v_mars = sqrt(mu_sun/a_mars);
circOrbTime = (2*pi*a_mars)*(1/v_mars);
%period = 2*pi*sqrt(a_mars^3/mu_sun);
RAAN_dot = (2*pi)/circOrbTime;

incli = acos((-RAAN_dot*(2/3)*(a_sc^(7/2)))/(sqrt(mu_mars)*J2_mars*mars_EQR^2));

%b
aero_period = 1.02595675*86400;
a_sc_aero = ((aero_period/(2*pi))^(2)*mu_mars)^(1/3);
altitude_b = a_sc_aero - mars_EQR;
RAAN_dot_aero = -((3/2)*((sqrt(mu_mars)*J2_mars*mars_EQR^2)/(a_sc_aero^(7/2))));
```