03/09/2021

Gustavo Grinsteins Planchart
ASEN 5050
HW5

Problem 1:

Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Gm_{Moon} = 4902.799 Km^3/s^2
- Equatorial radius of the Moon = 1738 km
- Impulsive maneuvers act instantaneously

Note: For this problem subscript 1 will denote before maneuver quantities and subscript 2 denotes after maneuver quantities.

A spacecraft is currently in a lunar orbit described by the following orbital elements:

$$a_1 = 8500\ km \quad e_1 = 0.29$$

Since 0 < e_1 < 1, the orbit conic is an ellipse.

At a true anomaly of $\theta^*{}_1 = -21°$, an impulsive maneuver is applied.

a) Find the velocity vector at this true anomaly before the maneuver and express it in the ($\hat{r}$,$\hat{\theta}$,$\hat{h}$) rotating frame.

Calculating the semi-latus rectum with the given information:

$$p_1 = a_1(1 - e_1{}^2) = 7785.1500\ km$$

Calculating the specific angular momentum with the given information:

$$h_1 = \sqrt{\mu_{Moon}\, p_1} = 6178.1086\ km^2/s$$

Calculating the transverse and radial components of velocity in the rotating frame:

$$v_{\theta,1} = \frac{\mu_{Moon}}{h_1}\left(1 + e_1\, cos(\ \theta^*{}_1)\right) = \ 1.008427\ km/s$$
$$v_{r,1} = \frac{\mu_{Moon}}{h_1}\left(e_1\, sin(\ \theta^*{}_1)\right) = \ -0.082473\ km/s$$

Velocity vector expressed in the ($\hat{r}$,$\hat{\theta}$,$\hat{h}$) rotating frame:

$$\boldsymbol{\overline{v}_{r\theta h,1} = -0.082473\ \hat{r} + 1.008427\ \hat{\theta} + 0\hat{h}\ km/s <--}$$

b) A maneuver, $\Delta\overline{v} = 0.25\hat{r} - 0.06\hat{\theta}\ km/s$, is then applied. Draw the velocity vector before the maneuver as well as the $\Delta\overline{v}$ vector and the velocity vector after the maneuver. Also add the $\hat{r}$ and $\hat{\theta}$ unit vectors to this diagram. Calculate the velocity vector after the maneuver and express it in the ($\hat{r}$,$\hat{\theta}$,$\hat{h}$) rotating frame.

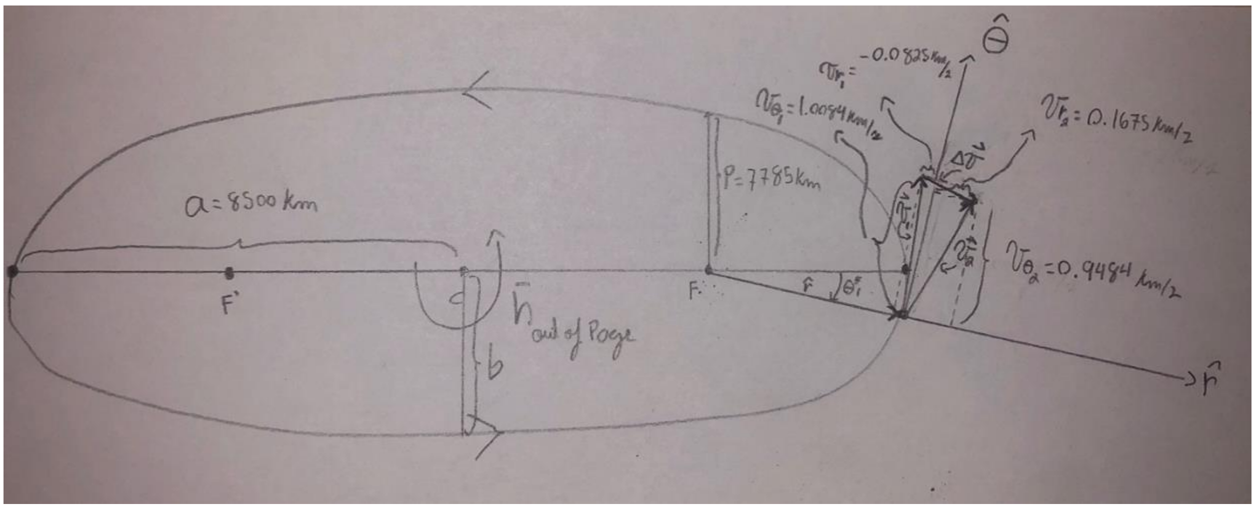

Figure 1 - Vector triangle representing the velocity vector in the rotating frame before and after the maneuver

Calculating the vector triangle after the maneuver, expressed in the ($\hat{r}$,$\hat{\theta}$,$\hat{h}$) rotating frame:

$$\boldsymbol{\overline{v}_{r\theta h,2}} = \boldsymbol{\overline{v}_{r\theta h,1}} + \boldsymbol{\Delta\overline{v}_{r\theta h}} = \boldsymbol{0.16753\ \hat{r} + 0.94843\ \hat{\theta} + 0\hat{h}\ km/s <--}$$

c) After the maneuver, calculate the orbital elements a and e of the new orbit. Also calculate the true anomaly along the new orbit.

Calculating the position vector before the maneuver:

$$r_1 = \frac{p_1}{1 + e_1 cos({\theta_1}^*)} = 6126.4777\ km$$

Since we are assuming that the impulsive maneuvers occur instantaneously, we know that the position vector for the spacecraft will remain unchanged:

$$r_2 = r_1$$

Calculating the magnitude of the velocity vector after the maneuver:

$$v_2 = \|\bar{v}_{r\theta h,2}\| = \sqrt{v_{r,2}{}^2 + v_{\theta,2}{}^2 + 0^2} = 0.963109\ km/s$$

Calculating the mechanical energy after the maneuver:

$$\varepsilon_2 = \frac{v_2^2}{2} - \frac{\mu_{Moon}}{r_2} = -0.33647\ km^2/s^2$$

since $\varepsilon < 0$ the spacecraft is in a Moon-centric orbit that is bounded after the maneuver.

Calculating the specific angular momentum after the maneuver:

$$h_2 = r_2 v_{\theta,2} = 5810.5199\ km^2/s$$

Calculating the orbit eccentricity after the maneuver:

$$\boldsymbol{e_2 = \sqrt{1 + \frac{2\varepsilon_2 h_2{}^2}{\mu_{Moon}{}^2}} = 0.23409 < --}$$

Calculating the semi-major axis after the maneuver:

$$\boldsymbol{a = -\frac{\mu_{Moon}}{2\varepsilon_2} = 7285.5534\ \ km < --}$$

Calculating the true Anomaly after the maneuver:

$$\boldsymbol{\theta_2{}^* = \pm cos^{-1}\left(\frac{v_{\theta,2} h_2 - \mu_{Moon}}{\mu_{Moon} e_2}\right)\left(\frac{180}{\pi}\right) = \pm 58.01^{\circ} = +58.01^{\circ} < --}$$

Since $v_{r,2}$> 0 we select the positive value for the true anomaly.

d) Calculate the change in the argument of periapsis due to the maneuver.

Since the delta V of this maneuver is coplanar and non-tangential there will be a coplanar shift of the orbit. This shift can be captured by the instantaneous change in value for the true anomaly which relates to the change in value of the argument of periapsis by the following expression:

$$\boldsymbol{\Delta\omega = \theta_1{}^* - \theta_2{}^* = \omega_2 - \omega_1 = -79.01^{\circ} < --}$$

Problem 2:

<u>Assumptions:</u>

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Gm_{Mars} = 4.305x10^4 Km^3/s^2
- Equatorial radius of Mars = 3397.2 km
- Impulsive maneuvers act instantaneously

Spacecraft is in orbit around mars. Needs to maintain an altitude of 400 km. Current orbit does not achieve this:

$$\varepsilon^- = -5.16187\frac{km^2}{s^2} \quad E_1^- = -1.46057\,rad \quad \theta_1^{*-} = -90°$$

a) At t_1, determine the velocity vector of the spacecraft in the ($\hat{r}$,$\hat{\theta}$,$\hat{h}$) frame.

Taking advantage of the orbit geometry at the bottom of the semi-latus rectum (figure 2):

$$e^- = \cos(E_1^-) = 0.1100$$

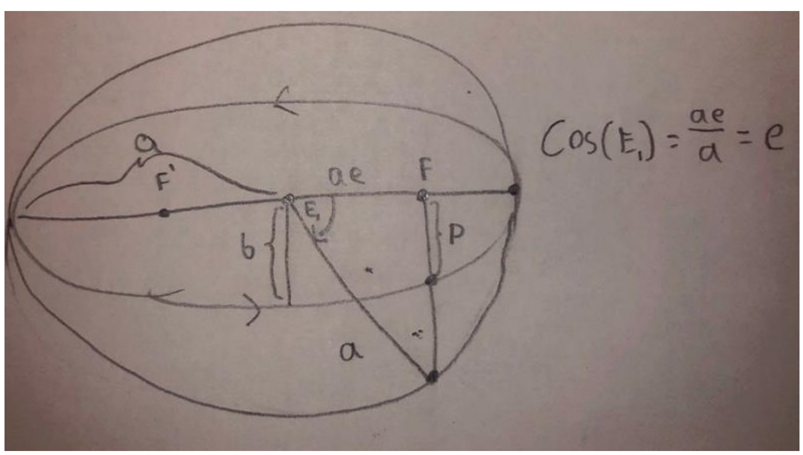

Figure 2 - Orbit Geometry at the bottom of the semi-latus rectum

Calculating semi-major axis before the maneuver:

$$a^- = -\frac{\mu_{Mars}}{2\varepsilon} = 4170.0000\ km$$

Calculating semi-latus rectum before the maneuver:

$$p^- = a^-(1-(e^-)^2) = 4119.5404\ km$$

Calculating specific angular momentum before the maneuver:

$$h^- = \sqrt{\mu_{Mars}\, p^-} = 1.33171x10^4\ km^2/s$$

Calculating the transverse and radial components of velocity in the $(\hat{r},\hat{\theta},\hat{h})$ frame:

$$v_{\theta,1-} = \frac{\mu_{Mars}}{h^-}\left(1 + e^-\ cos(\ \theta_1^{*-})\right) = 3.23268\ km/s$$
$$v_{r,1-} = \frac{\mu_{Mars}}{h^-}\left(e^-\ sin(\ \theta_1^{*-})\right) = -0.35560\ \ km/s$$

Velocity vector expressed in the $(\hat{r},\hat{\theta},\hat{h})$ rotating frame:

$$\boldsymbol{\bar{v}_{r\theta h,1-}} = \boldsymbol{-0.35560\ \ \hat{r} + 3.23268\,\hat{\theta} + 0\hat{h}\ km/s} < --$$

b) Consider applying a maneuver at t_1 to slightly adjust the velocity vector of the spacecraft and, therefore, change its orbit. The maneuver will be designed to raise the periapsis altitude to 400 km but maintain the same apoapsis radius as the original orbit. What is the magnitude of this maneuver?

Maintaining the same apoapsis:

$$r_a^- = r_a^+ = a^-(1 + e^-) = 4628.71403\ km$$

Following the altitude constraint, the periapsis radius will be:

$$r_p^+ = Mars_{EQR} + 400\ km = 3797.2\ km$$

Solving for the semi-major axis:

$$a^+ = \frac{r_a^+ + r_p^+}{2} = 4212.9570\ km$$

From the elliptical geometry we can obtain the new eccentricity:

$$r_a = a + ae\ \therefore\ e^+ = \frac{r_a^+ - a^+}{a^+} = 0.09868$$

Since $0 < e^+ < 1$, the orbit conic is an ellipse.

Calculating the new semi-latus rectum:

$$p^+ = a^+(1 - (e^+)^2) = 4171.9279\ km$$

Calculating the new specific angular momentum:

$$h^{+} = \sqrt{\mu_{Mars}\, p^{+}} = 1.34015x10^{4}\ km^{2}/s$$

Calculating the new true anomaly:

Note that since the maneuver is instantaneous:

$$r_1^{+} = r_1^{-} = p^{-}$$

$$\theta_1^{*+} = \pm cos^{-1}\left(\frac{p^{+} - r_1^{+}}{r_1^{+} \cdot e^{+}}\right)\left(\frac{180}{\pi}\right) = \pm 82.59° = -82.59°$$

Since $v_{r,1-} < 0$ we select the negative value for the true anomaly

Calculating the transverse and radial components of velocity in the ($\hat{r}$,$\hat{\theta}$,$\hat{h}$) frame after the maneuver:

$$v_{\theta,1+} = \frac{\mu_{Mars}}{h^{+}}\left(1 + e^{+}\, cos(\,\theta_1^{*+})\right) = 3.25316\ km/s$$
$$v_{r,1+} = \frac{\mu_{Mars}}{h^{+}}\left(e^{+}\, sin(\,\theta_1^{*+})\right) = -0.31436\ \ km/s$$

$$\bar{v}_{r\theta h,1+} = -0.31436\ \hat{r} + 3.25316\,\hat{\theta} + 0\hat{h}\ km/s <--$$

$$\bar{v}_{r\theta h,1+} = \bar{v}_{r\theta h,1-} + \Delta\bar{v}_{r\theta h} \therefore \Delta\bar{v}_{r\theta h} = \bar{v}_{r\theta h,1+} - \bar{v}_{r\theta h,1-}$$
$$= 0.041239\ \hat{r} + 0.020490\,\hat{\theta} + 0\hat{h}\ km/s$$

$$\boldsymbol{\Delta v} = \|\Delta\bar{\boldsymbol{v}}_{\boldsymbol{r\theta h}}\| = \sqrt{\boldsymbol{v_{r,1+}}^{2} + \boldsymbol{v_{\theta,1+}}^{2} + \boldsymbol{0}^{2}} = \boldsymbol{0.046049}\frac{\boldsymbol{km}}{\boldsymbol{s}} <--$$

c) Rather than apply the maneuver you calculated above, consider another maneuver, described by the following vector: $\Delta\bar{v}$ = 0.04$\hat{r}$ – 0.002$\hat{\theta}$ km/s. If this maneuver was applied at time t_1, would the orbit of the spacecraft possess a periapsis altitude above 400 km as required for mission success?

Calculating the resulting vector and vector magnitude after the maneuver:

$$\bar{v}_{r\theta h,1+} = \bar{v}_{r\theta h,1-} + \Delta\bar{v}_{r\theta h} = -0.31560\,\hat{r} + 3.23068\,\hat{\theta} + 0\hat{h}\ km/s$$

$$v_{1+} = \left\|\bar{v}_{r\theta h,1+}\right\| = \sqrt{v_{r,1+}{}^{2} + v_{\theta,1+}{}^{2} + 0^{2}} = 3.24605\ km/s$$

Calculating the mechanical energy after the maneuver:

$$\varepsilon^{+} = \frac{v_{1+}^{2}}{2} - \frac{\mu_{Mars}}{r_1^{+}} = -5.18176\ \ km^2/s^2$$

since $\varepsilon < 0$ the spacecraft is in a Mars-centric orbit that is bounded after the maneuver.

Calculating the specific angular momentum after the maneuver:

$$h^{+} = r_1^{+} v_{\theta,1+} = 1.33089x10^4\ km^2/s$$

Calculating the semi-major axis after the maneuver:

$$a^{+} = -\frac{\mu_{Mars}}{2\varepsilon^{+}} = 4153.9960\ km$$

Calculating the orbit eccentricity after the maneuver:

$$e^{+} = \sqrt{1 + \frac{2\varepsilon_2 {h_2}^2}{{\mu_{Mars}}^2}} = 0.097577$$

Calculating periapsis after the maneuver:

$$r_p^{+} = a^{+}(1 - e^{+}) = \ 3748.6613\ km$$

$$\boldsymbol{Altitude = \ r_p^{+} - Mars_{EQR} = 351.4613\ km} < --$$

The resulting orbit of the spacecraft falls below the 400km altitude requirement for mission success

Problem 3:

Assumptions:

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Gm_{Sun} = 1.32712428x10^{11} Km^3/s^2
- Semi-major axis of Earth's heliocentric orbit: 1.0000010178 AU
- Semi-major axis of Saturn's heliocentric orbit: 9.554909595 AU
- 1 AU = 149,597,870.7 km
- Impulsive maneuvers act instantaneously

a) Calculate the total Δv (to five significant figures) and time of flight required for a spacecraft to complete a Hohmann transfer in the Sun-spacecraft two-body problem between an approximate circular orbit for Earth and an approximate circular orbit for Saturn. Assume the semi-major axes of each orbit is equal to the value provided.

For Hohmann transfers we have the following assumptions:

- transfer happens between two circular co-planar orbits
- the delta V magnitudes are applied tangentially

First, let's define some variables that we will be using according to the following graph:

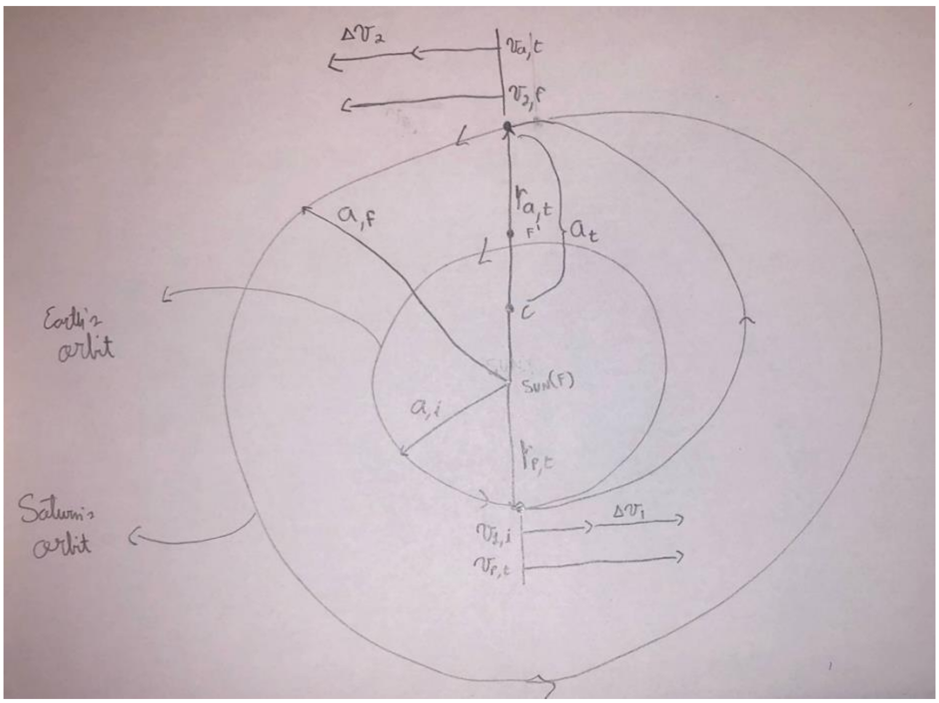

Figure 3 - graphical variable definition for Hohmann Transfer

$$a_i = Semi - major\ axis\ of\ Earth's\ heliocentric\ orbit$$
$$a_f = Semi - major\ axis\ of\ Saturn's\ heliocentric\ orbit$$
$$a_t = Hohmann\ transfer\ semi - major\ axis$$

Using the given information with the defined variables, we can calculate some orbital parameters for the transfer obit:

Transfer orbit's semi-major axis:

$$a_t = \left(\frac{1}{2}\right)(a_f + a_i) = 7.89496x10^8\ km$$

Transfer orbit's eccentricity:

$$e_t = \frac{a_f - a_i}{a_f + a_i} = 0.81051$$

Transfer orbit's specific mechanical energy:

$$\varepsilon_t = -\frac{\mu_{sun}}{2a_t} = -84.04882\ km^2/s^2$$

In order to calculate the total Δv, lets define the velocities at each stage of the transfer defined as vector magnitudes since all magnitudes are tangential.

At the initial stage, the spacecraft will be orbiting earth on its assumed circular orbit:

$$v_{1,i} = \sqrt{\frac{\mu_{sun}}{a_i}} = 29.78467\ km/s$$

At the start of the transfer orbit, the velocity will be defined as:

$$v_{p,t} = \sqrt{\frac{2\mu_{sun}}{a_i} - \frac{\mu_{sun}}{a_t}} = 40.07688\ km/s$$

Using this information, we can calculate the first change in velocity:

$$\Delta v_1 = v_{p,t} - v_{1,i} = 10.29220\ km/s$$

Moving on to the end portion of the transfer orbit:

$$v_{a,t} = \sqrt{\frac{2\mu_{sun}}{a_f} - \frac{\mu_{sun}}{a_t}} = 4.19438\ km/s$$

At the final stage, the spacecraft will be orbiting Saturn on its assumed circular orbit:

$$v_{2,f} = \sqrt{\frac{\mu_{sun}}{a_f}} = 9.63562\ km/s$$

Using this information, we can calculate the second change in velocity:

$$\Delta v_2 = v_{2,f} - v_{a,t} = 5.44124\ km/s$$

The total Δv:

$$|\Delta \boldsymbol{v_1}| + |\Delta \boldsymbol{v_2}| = \mathbf{15.73345} \frac{\boldsymbol{km}}{\boldsymbol{s}} < - -$$

The time of flight required for the spacecraft to complete the Hohmann transfer:

$$\boldsymbol{TOF} = \boldsymbol{\pi} \sqrt{\frac{\boldsymbol{a_t^3}}{\boldsymbol{\mu_{sun}}}} = \mathbf{1.91302}\boldsymbol{x}\mathbf{10^8}\boldsymbol{seconds} = \mathbf{6.06537}\ \boldsymbol{years}$$

b) Calculate the initial relative phase angle (between the spacecraft and Saturn) that is required for the spacecraft to rendezvous with Saturn after completing the Hohmann transfer.

Since this is a circular coplanar rendezvous on different orbits, the TOF of our spacecraft and the mean motion of Saturn can be used to calculate the phase angle:

$$n_{saturn} = \sqrt{\frac{\mu_{sun}}{a_f^3}} = 6.74105x10^{-9} rad/s$$

Assuming the spacecraft leads Saturn by some amount defined as the lead angle α_L:

$$\alpha_L = (TOF)(n_{saturn}) = 1.28958\, rad$$

Then to find the required phase angle ϑ from Saturn to the spacecraft knowing that this is a positive quantity:

$$\boldsymbol{\vartheta} = \boldsymbol{\pi} - \boldsymbol{\alpha_L} = \mathbf{1.85202}\, \boldsymbol{rad} = \mathbf{106.11}° < - -$$

c) Assuming an intermediate radius (r_B) of 11 AU, calculate the total Δv (to five significant figures) and time of flight required for a bi-elliptic transfer between Earth's and Saturn's approximate orbits.

For bi-elliptic transfers we have the following assumptions:

- transfer happens between two circular co-planar orbits
- the delta V magnitudes are applied tangentially

First, let's define some variables that we will be using according to the following graph:

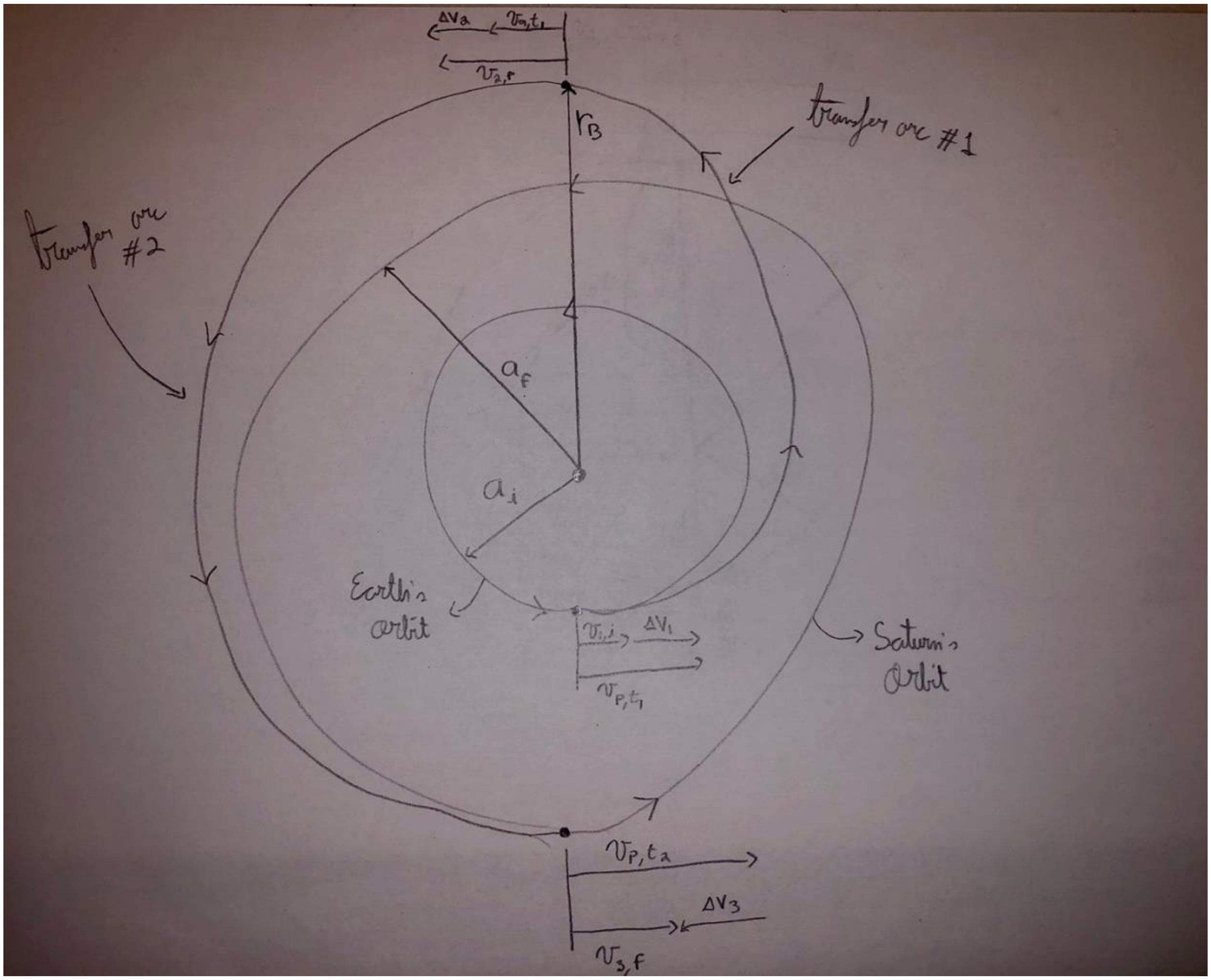

Figure 4 - graphical variable definition for Bi-Elliptic Transfer

$$a_i = Semi - major\ axis\ of\ Earth's\ heliocentric\ orbit$$
$$a_f = Semi - major\ axis\ of\ Saturn's\ heliocentric\ orbit$$
$$r_B = Intermediate\ Radius$$

Calculating the orbital parameters for the first transfer arc:

$$a_{t,1} = \left(\frac{1}{2}\right)(a_i + r_B) = 8.97587x10^8\ km$$

$$e_{t,1} = \frac{r_B - a_i}{r_B + a_i} = 0.83333$$

Calculating the orbital parameters for the second transfer arc:

$$a_{t,2} = \left(\frac{1}{2}\right)(a_f + r_B) = 8.97587x10^8\ km$$

$$e_{t,2} = \frac{r_B - a_f}{r_B + a_f} = 0.070304$$

In order to calculate the total Δv, lets define the velocities at each stage of the transfer defined as vector magnitudes since all magnitudes are tangential.

At the initial stage, the spacecraft will be orbiting earth on its assumed circular orbit:

$$v_{1,i} = \sqrt{\frac{\mu_{sun}}{a_i}} = 29.78467\ km/s$$

At the start of the transfer orbit, the velocity will be defined as:

$$v_{p,t1} = \sqrt{\frac{2\mu_{sun}}{a_i} - \frac{\mu_{sun}}{a_{t,1}}} = 40.32863\ km/s$$

Using this information, we can calculate the first change in velocity:

$$\Delta v_1 = v_{p,t1} - v_{1,i} = 10.54396\ km/s$$

At the middle stage, the spacecraft will be transition from transfer arc one to transfer arc two. Before the maneuver the velocity at the apoapsis of transfer arc 1 is:

$$v_{a,t1} = \sqrt{\frac{2\mu_{sun}}{r_B} - \frac{\mu_{sun}}{a_{t,1}}} = 3.66624\ km/s$$

The velocity at apoapsis of transfer arc one will be defined by the new semi-major axis and given intermediate radius:

$$v_{a,t2} = \sqrt{\frac{2\mu_{sun}}{r_B} - \frac{\mu_{sun}}{a_{t,2}}} = 8.65899\ km/s$$

Using this information, we can calculate the second change in velocity:

$$\Delta v_2 = v_{a,t2} - v_{a,t1} = 4.99275\ km/s$$

At the last stage, the spacecraft will be transitioning from the periapsis of the second transfer arc into the assumed circular orbit velocity for Saturn:

$$v_{p,t2} = \sqrt{\frac{2\mu_{sun}}{a_f} - \frac{\mu_{sun}}{a_{t,2}}} = 9.96858\ km/s$$

$$v_{3,f} = \sqrt{\frac{\mu_{sun}}{a_f}} = 9.63562\ km/s$$

Using this information, we can calculate the third change in velocity:

$$\Delta v_3 = v_{3,f} - v_{p,t2} = -0.33296\ km/s$$

The total Δv:

$$|\Delta \boldsymbol{v_1}| + |\Delta \boldsymbol{v_2}| + |\Delta \boldsymbol{v_3}| = \mathbf{15.86967} \frac{\boldsymbol{km}}{\boldsymbol{s}} <--$$

The time of flight required for the spacecraft to complete the Bi-Elliptic transfer:

$$\boldsymbol{TOF = \pi\sqrt{\frac{a_{t,1}^3}{\mu_{sun}}} + \pi\sqrt{\frac{a_{t,2}^3}{\mu_{sun}}} = 7.51793x10^8 seconds = 23.836\ years}$$

d) Compare the total Δv and time of flight of this bi-elliptic transfer with the total Δv and time of flight of the Hohmann transfer calculated in part a). Justify why this comparison is consistent with your expectations.

Calculating the percent change between the Hohmann and the bi-elliptic transfers:

$$\Delta v_{total\ Hohmann} = 15.73345\ km/s$$
$$\Delta v_{total\ Bi-Elliptic} = 15.86967\ km/s$$

$$\frac{\Delta v_{total\ Bi-Elliptic} - \Delta v_{total\ Hohmann}}{\Delta v_{total\ Hohmann}} x\ 100\% = 0.8658\%$$

There is an increase in delta V of about 0.8% by doing a bi-elliptic transfer orbit in comparison to the more efficient Hohmann transfer. Furthermore, comparing the given orbit geometries using efficiency comparison chart from the class lectures:

$$\frac{r_B}{r_A} = \frac{r_B}{a_i} = 11.00$$

$$\frac{r_C}{r_A} = \frac{a_f}{a_i} = 9.55$$

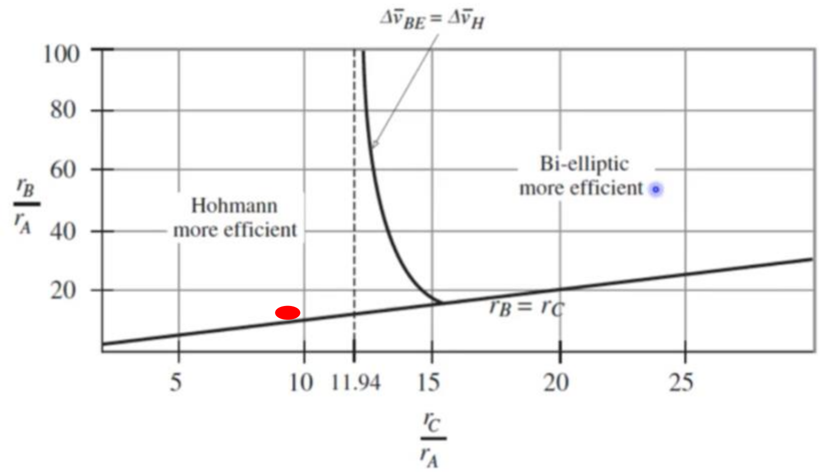

Figure 5 - Orbit transfer comparison from class lecture

It is observable, by the red dot in the graph, that given the transfer and orbit geometries the Hohmann transfer will be more efficient than the Bi-elliptic transfer.

Comparing the TOF:

$$TOF_{Bi-elliptic} = 23.836\ years$$

$$TOF_{Hohmann} = 6.06537\ years$$

$$\frac{TOF_{Bi-elliptic} - TOF_{Hohmann}}{TOF_{Hohmann}} x\ 100\% = 293\%$$

Here we can clearly see that the Bi-elliptic orbit will take more than twice the amount of time to reach the desired orbit.

e) Do you think the total Δv values you computed in 3a) and 3c) are large? Justify.

Using the ideal rocket equation to calculate weight difference ratio needed for this maneuver:

$$\frac{m_i - m_f}{m_f} = 1 - e^{\left(\frac{-\Delta v}{I_{sp} g_0}\right)}$$

Where:

m_i = is the s/c initial mass
m_f = is the s/c final mass
I_{sp} = Specific impulse = 301 sec (nominal for Cassini's bipropellant system)
g_0 = 9.81 m/s^2 → Converting to maintain same units = 9.81x10^{-3}km/s^2
Δv= 15.86967 km/s for bi-elliptic and 15.73345 km/s for Hohmann

For Hohmann:

$$\frac{m_i - m_f}{m_f} = 1 - e^{\left(\frac{-15.73345 km/s}{I_{sp} g_0}\right)} = 0.99515$$

For Bi-Elliptic:

$$\frac{m_i - m_f}{m_f} = 1 - e^{\left(\frac{-15.86967 km/s}{I_{sp} g_0}\right)} = 0.99537$$

These high weight difference ratio numbers signify that it will take most (almost all) of the spacecraft mass to achieve these impulsive maneuvers. Therefore, for each orbit transfers the total Δv is prohibitively large.<--

Problem 4:

a) At the initial state, use the reported values to record the magnitude of the specific angular momentum, specific energy, and eccentric anomaly. Also record the corresponding date in UTC modified Julian date format (recall that the modified Julian date represents the days past a reference epoch) from the initial condition input panel. Is the value of the specific energy consistent with the type of conic followed by the spacecraft?

$$\boldsymbol{h = 14,928.32485 \frac{km^2}{s} < --}$$

$$\boldsymbol{\varepsilon = -3.24457 \frac{km^2}{s^2} < --}$$

$$\boldsymbol{E = 180\,\mathrm{deg} = 2\pi\, rad < --}$$

$$\boldsymbol{UTC\ Julian\ Date = 2457113.5 < --}$$

The value of specific energy is consistent with the path followed by the spacecraft. Since $\varepsilon < 0$ the orbit is bounded following an elliptical trajectory.

b) Integrate the specified initial condition until an altitude of 500 km. At this altitude, use the report function to list the true anomaly, eccentric anomaly – and, as a quick check of your stopping condition, the altitude. Use the epoch at this altitude to determine the time of flight along the propagated trajectory segment.

$$\boldsymbol{\theta^* = 317.41\ degrees < --}$$

$$\boldsymbol{E = 333.33\ degrees < --}$$

$$\boldsymbol{Altitude = 500.000\ km < --}$$

According to the summary report:

the time past Epoch in UTC Gregorian Date: 1 Apr 2015 00:00:00.000 is 7468.46 sec.

Therefore, the TOF is 7468.46 sec or 2.07 hours. <--

c) Use the time of flight you calculated in part b) and orbit geometry to predict the time the spacecraft spends in its orbit below an altitude of 500 km. Use the eccentric anomaly at the 500 km altitude, from part b), as well as the orbital elements, to verify this flight time on your own by using Kepler's equations.

Calculating the orbit's period:

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu_{Mars}}} = 16{,}279.084223\ seconds$$

The TOF calculated in part b, starts at apoapsis (E = θ^*=180 degrees) and ends at θ^*= 317.41 degrees, this will occur twice in the orbit because of symmetry. The following expression will calculate the time the spacecraft spends in its orbit below an altitude of 500 km:

$$\boldsymbol{Time\ below\ 500\ km = \mathbb{P} - 2(TOF) = 16{,}279.084223 - 2(7468.46)}$$
$$\boldsymbol{= 1342.164223\ seconds\ or\ 22.37\ minutes < --}$$

Verifying this flight time using Kepler's equation:

$$n = \sqrt{\frac{\mu_{Mars}}{a^3}} = 3.85966x10^{-4}\ rad/sec$$

$$e = 0.460$$

$$E_{500km} = 333.33\ degrees\ or - 26.67\ degrees = 5.81770\ rad\ or - 0.46548\ rad$$

Using the negative radian value of eccentric anomaly:

$$t_{500km} - t_p = \left(\frac{1}{n}\right)(E_{500km} - e\sin(E_{500km})) = -671.06475\ seconds$$

This represents the time until periapsis at the bottom half of the ellipse starting at the 500km altitude point. Multiplying the absolute value of this quantity by two (taking advantage of symmetry) gives us a similar TOF value calculated above (with some rounding differences):

$$\boldsymbol{Time\ below\ 500\ km = 2\left|t_{500km} - t_p\right| = 1342.13\ seconds\ or\ 22.36\ minutes < - -}$$

d) Take two snapshots of your orbit in three-dimensions: one looking down on the orbital plane, and one alternate useful view. Take a snapshot of the two-dimensional view of the ground-track of the spacecraft (you may need to update the orbit colors for clarity in this 2D view). The ground-track essentially describes the projection of the spacecraft's location onto the surface. Go to https://marstrek.jpl.nasa.gov which functions a lot like Google Maps for Mars and locates named regions on Mars' surface. Assume that one of the sensors onboard the spacecraft can only take observational data at altitudes below 500 km. Refer back to the orbit you modeled in GMAT or STK – during the segment of the orbit that the spacecraft remains below an altitude of 500km, visually identify a region of the Mars surface that the spacecraft may be able to observe.

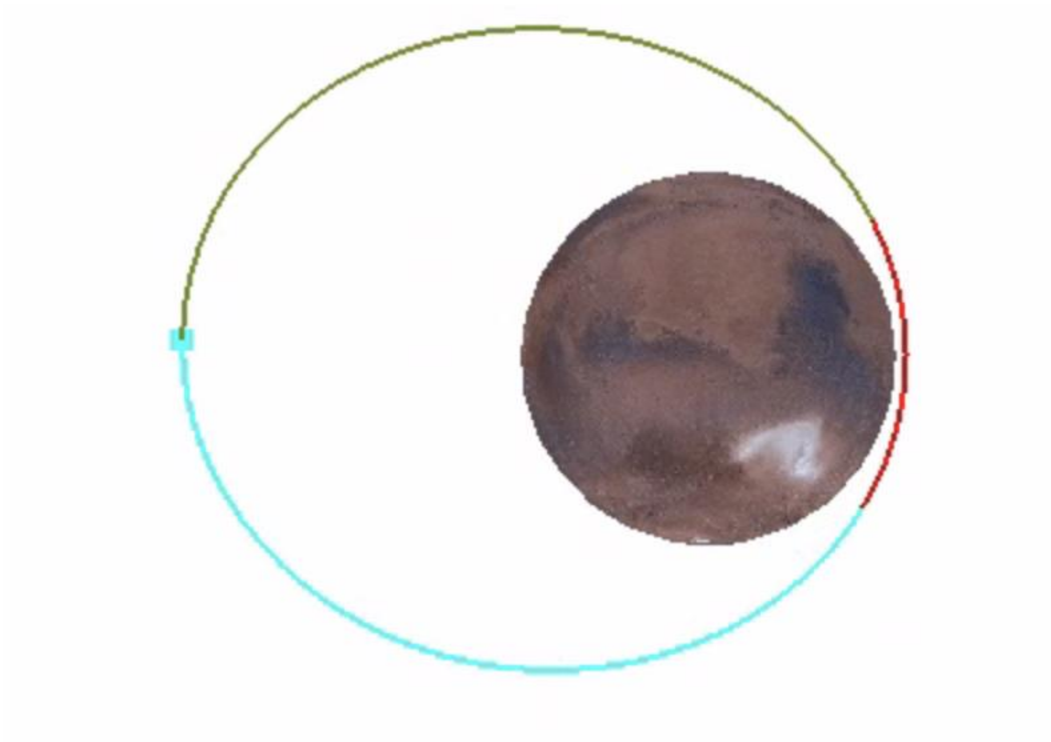

Figure 6 - 3D view, looking down

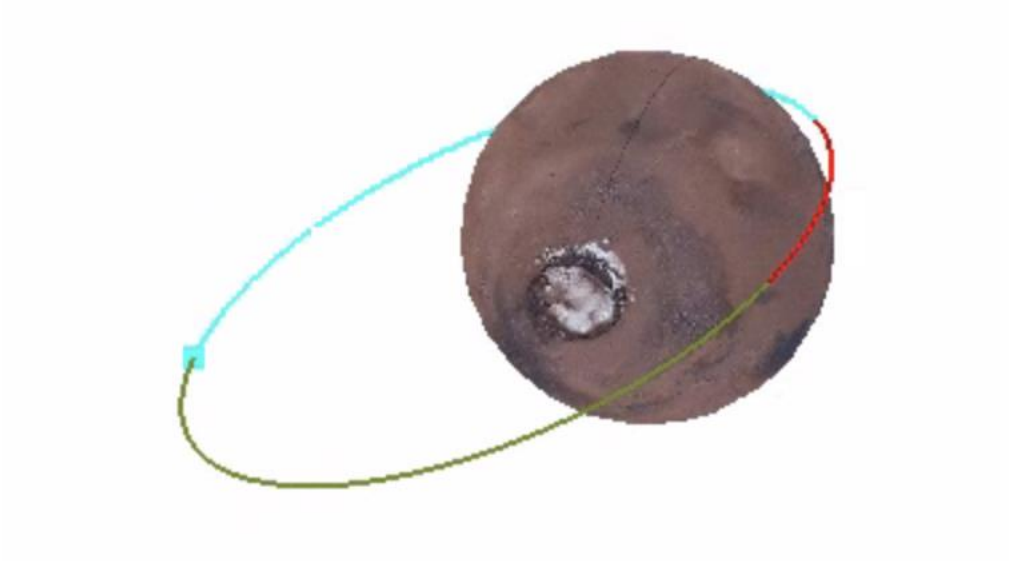

Figure 7 - 3D view, alternate useful view

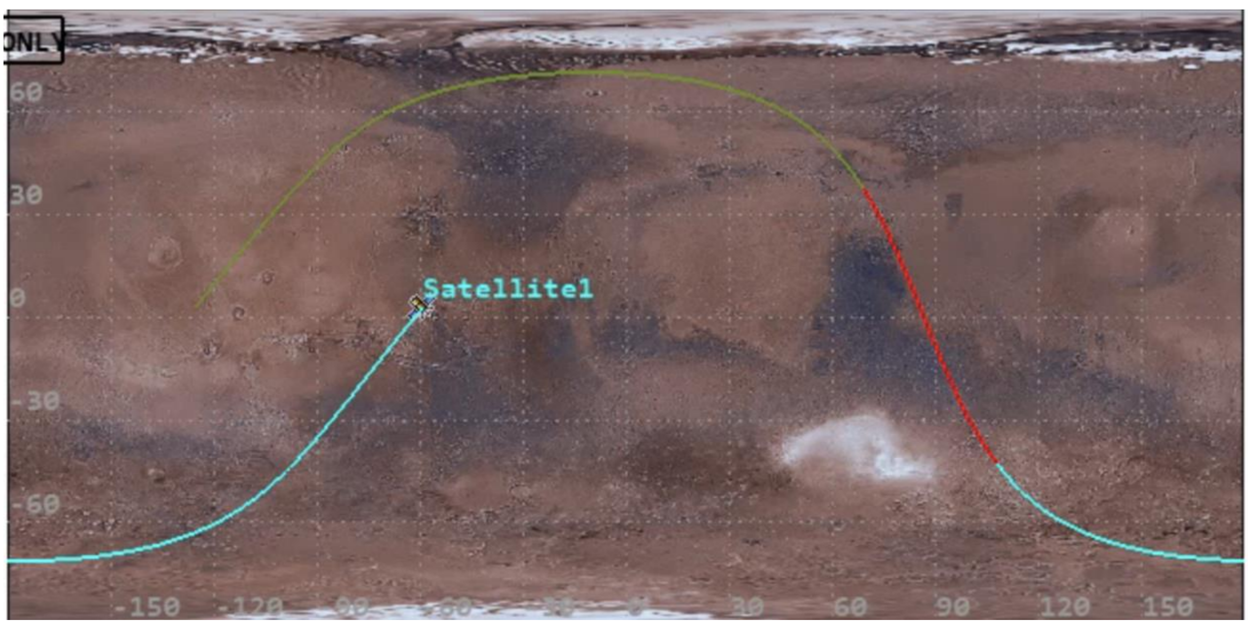

Figure 8 - 2D view, ground-track

Figure 9 – Observable Regions of mars below the 500 km altitude zone

From figure 8 and 9, we can see that the spacecraft might observe regions starting with promethea terra, Hesperia, Tyrrhena, Syrtis major and Isidis.

```
%Gustavo Grinsteins
%ASEN 5050
%HW5

%House Keeping
clc;
clear;
%% Problem 1
%Given
mu_moon = 4902.799;
EQR_moon = 1738;
a1 = 8500;
e1 = 0.29;
theta_star_1 = -21*(pi/180);
p1 = a1*(1-e1^2);
h1 = sqrt(mu_moon*p1);
vr1 = (mu_moon/h1)*e1*sin(theta_star_1);
vtheta1 = (mu_moon/h1)*(1+e1*cos(theta_star_1));
vrthetah1 = [vr1,vtheta1,0];
vdelta = [0.25,-0.06,0];
vrthetah2 = vrthetah1 + vdelta;
r1 = (p1)/(1+e1*cos(theta_star_1));
mech_e2 = (((norm(vrthetah2))^2)/2) - (mu_moon)/(r1);
h2 = r1*vrthetah2(2);
e2 = sqrt(1+((2*mech_e2*h2^2)/(mu_moon^2)));
a2 = -(mu_moon)/(2*mech_e2);
p2 = a2*(1-e2^2);
theta_star_2c = -acos((p2-r1)/(r1*e2));
theta_star_2 = -acos((vrthetah2(2)*h2 - mu_moon)/(mu_moon*e2));
%% Problem 2
%Given
mu_mars = 4.305*10^4;
mars_EQR = 3397.2;
mech_e_bef = -5.16187;
E1_bef = -1.46057;
thetas_1bef = -90*(pi/180);
e_bef = cos(E1_bef);
a_bef = -(mu_mars)/(2*mech_e_bef);
p_bef = a_bef*(1-e_bef^2);
h_bef = sqrt(mu_mars*p_bef);
vr1 = (mu_mars/h_bef)*e_bef*sin(thetas_1bef);
vtheta1 = (mu_mars/h_bef)*(1+e_bef*cos(thetas_1bef));
ra = a_bef*(1+e_bef);
rp_aft = mars_EQR + 400;
a_aft = (ra+rp_aft)/2;
e_aft = (ra-a_aft)/(a_aft);
p_aft = a_aft*(1-e_aft^2);
h_aft = sqrt(mu_mars*p_aft);
r_aft = p_bef;
theta_star_aft = -acos((p_aft-p_bef)/(p_bef*e_aft));
vr2 = (mu_mars/h_aft)*e_aft*sin(theta_star_aft);
vtheta2 = (mu_mars/h_aft)*(1+e_aft*cos(theta_star_aft));
deltaV = [vr2,vtheta2,0] - [vr1,vtheta1,0];
Vdelta_c = [0.04,-0.002,0];
V2rot = [vr1,vtheta1,0] + Vdelta_c;
mech_e_aft = (((norm(V2rot))^2)/2) - (mu_mars)/(p_bef);
a_aft_c = -(mu_mars/(2*mech_e_aft));
h_aft_c = p_bef*V2rot(2);
```

```
e_aft_c = sqrt(1-((h_aft_c ^2)/(mu_mars*a_aft_c)));
rp_aft_c = a_aft_c*(1-e_aft_c);
altitude_c = rp_aft_c - mars_EQR;
%% Problem 3
% %Given
mu_sun = 1.32712428*10^11;
AU_convert = 149597870.7;
a_i = 1.0000010178*AU_convert;%this is a for earth
a_f = 9.554909595*AU_convert;%This is a for saturn
r_pt = a_i; %transfer orbit periapsis radius
r_at = a_f; %transfer orbit apoapsis radius
v1_i = sqrt(mu_sun/a_i); %velocity at earths orbit
v2_f = sqrt(mu_sun/a_f); %velocity at saturns orbit
a_t = (1/2)*(a_i+a_f);%semi major axis of transfer orbit
e_t = (a_f-a_i)/(a_f+a_i);%eccentricity of transfer orbit
mech_e_t = -(mu_sun)/(2*a_t);%mech e of transfer orbit
vp_t = sqrt(((2*mu_sun)/(a_i))-(mu_sun/a_t));
va_t = sqrt(((2*mu_sun)/(a_f))-(mu_sun/a_t));
deltaV_1 = vp_t - v1_i;
deltaV_2 = v2_f - va_t;
totalDeltaV = deltaV_1+deltaV_2;
TOF = pi*sqrt((a_t^3)/mu_sun);
n_saturn = sqrt(mu_sun/(a_f^3));
alpha_l = TOF*n_saturn;
phase_angle = pi-alpha_l;
%PartC
rB = 11*AU_convert;
at1 = (1/2)*(a_i+rB);
et1 = (rB-a_i)/(rB+a_i);
at2 = (1/2)*(a_f+rB);
et2 = (rB-a_f)/(rB+a_f);
v1_i = sqrt(mu_sun/a_i);
vp_t1 = sqrt(((2*mu_sun)/(a_i))-(mu_sun/at1));
delta_V_1 = vp_t1 - v1_i;
va_t1 = sqrt(((2*mu_sun)/(rB))-(mu_sun/at1));
va_t2 = sqrt(((2*mu_sun)/(rB))-(mu_sun/at2));
delta_V_2 = va_t2 - va_t1;
vp_t2 = sqrt(((2*mu_sun)/(a_f))-(mu_sun/at2));
v3_f = sqrt(mu_sun/a_f);
delta_V_3 = v3_f - vp_t2;
totalDeltaV_2 = abs(delta_V_1)+abs(delta_V_2)+abs(delta_V_3);
TOF_bi = pi*sqrt((at1^3)/mu_sun)+pi*sqrt((at2^3)/mu_sun);
percentDiff = ((TOF_bi-TOF)/(TOF))*100;
WRH = 1 - exp((-totalDeltaV)/(301*.00981));
WRB = 1 - exp((-totalDeltaV_2)/(301*.00981));
```