Problem 1

Given: $a = 8500$ km
$e = 0.29$
$\theta^* = -21°$
$\mu_{moon} = 4902.799 \text{ km}^3/s^2$

part a)

$P_1 = a_1(1-e_1^2) = 7785$ km

$h_1 = \sqrt{\mu_{moon} P_1} = 6178 \text{ km}^2/s$

$\vec{r}_1 = \left(\dfrac{P_1}{1 + e_1 \cos\theta_1^*}\right)\hat{r} = 6126.5 \text{ Km } \hat{r}$

Vel vector Before maneuver

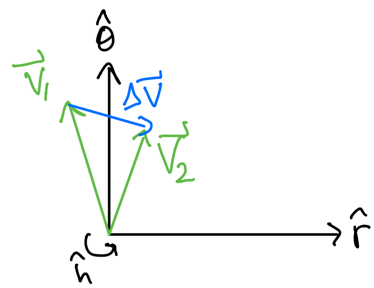
$\vec{v}_1 = v_r\hat{r} + v_\theta\hat{\theta} = \dfrac{\mu_{moon}}{h_1} e_1 \sin\theta_1^* \hat{r} + \dfrac{h_1}{r_1}\hat{\theta} = -0.0825\hat{r} + 1.0084\hat{\theta}$ Km/s

Part b)

$\Delta\vec{v} = 0.25\hat{r} - 0.06\hat{\theta}$ km/s

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$\vec{v}_2 = \Delta\vec{v} + \vec{v}_1 = 0.1675\hat{r} + 0.9484\hat{\theta}$ km/s



part c)

this is an impulsive burn, so we assume $\vec{r}_1 = \vec{r}_2$

$\vec{h}_2 = \vec{r}_2 \times \vec{v}_2 = 5810.5\,\hat{h} \text{ km}^2/s$

$\mathcal{E}_2 = \dfrac{v_2^2}{2} - \dfrac{\mu}{r_2} = -0.3365 \text{ km}^2/s^2$

$a_2 = -\dfrac{\mu}{2\mathcal{E}_2} = 7285.6$ km

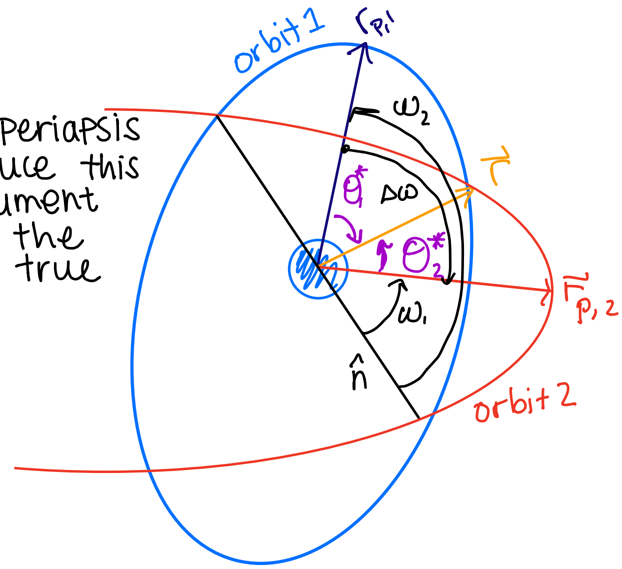
$e_2 = \sqrt{1 + \dfrac{2h_2^2\mathcal{E}_2}{\mu_{moon}^2}} = 0.2341$

$P_2 = a_2(1-e_2^2) = 6886.3$ km

$r_2 = \dfrac{P_2}{1+e_2\cos\theta_2^*} \Rightarrow \theta_2^* = +\cos^{-1}\left(\dfrac{1}{e_2}\left(\dfrac{P_2}{r_2} - 1\right)\right) = 1.024 \text{ rad} = 58.01°$, $\theta^* > 0$ b/c $V_r > 0$

Part d)

true anomaly changes, but position does not. Thus the periapsis vector must move to produce this change. Therefore, the argument of perigee must change by the negative of the change in true anomaly



$\theta_2^* - \theta_1^* = -\Delta\omega$

$\Delta\omega = -1.3790$ rads $= -79.01°$

PROBLEM 2

given: must always have alt > 400 km

$\mathcal{E}_1^- = -5.16187\ km^2/s^2$

$E_1^- = -1.46057$ rad

$\theta_1^{*-} = -90°$

$\mu_{mars} = 4.305 \times 10^4\ km^3/s^2$

$r_{mars} = 3397.2$ km

Part a)

using geometry to determine e when s/c is at bottom of semi-latus rectum, P. (known since $\theta_1^{*-} = 90°$

$\cos(E_1^-) = \frac{a_1 e_1^-}{a_1} = e_1^- = 0.11$

$a_1^- = \frac{-\mu}{2\mathcal{E}_1^-} = 4170$ km

$h_1^- = \sqrt{\mu a_1 (1 - e_1^{-2})} = 13317\ km^2/s$

$v_r^- = \mu/h\ e \sin\theta^* = -0.3556$ km/s

$v_\theta^- = \mu/h\ (1 + e_1^- \cos\theta^*) = 3.2327$ km/s

$\bar{v}_1^- = -0.3556\hat{r} + 3.2327\hat{\theta}$ km/s

Part b)

(−) denotes previous orbit, (+) denotes new orbit after maneuver.

$r_a^- = r_a^+ = a_i(1+e_i) = 4628.7$ km
$r_p^- = a_i^-(1-e_i^-) = 3711.3$ km

increase r_p alt to 400km

$r_p^+ = r_♂ + 400\text{km} = 3797.2$ km

$a^+ = \frac{1}{2}(r_p^+ + r_a^+) = 4213$ km

$e^+ = \dfrac{r_a^+ - r_p^+}{r_a^+ + r_p^+} = 0.0987$

$h^+ = \sqrt{\mu_♂ a^+(1-e^{+2})} = 13402$ km²/s

θ^* can change throughout the maneuver. Must solve for θ^{*+}

$r^- = \dfrac{a^-(1-e^{-2})}{1+e^-\cos\theta^{*-}} = 4119.5\text{ km} = r^+$

$r^+ = \dfrac{a^+(1-e^{+2})}{1+e^+\cos\theta^{*+}} \Rightarrow \theta^{*+} = \pm\cos^{-1}\left(\dfrac{a^+(1-e^{+2})-r^+}{r^+e^+}\right) = \pm 1.4416\text{rad} = \pm 82.60°$

$\theta^{*+} = -82.60°$ Choosing $\theta^{*+} < 0$ reduces $|\Delta\vec{v}|$ since it produces smaller change in velocity vector, but could choose either sign

$v_r^+ = \mu/h^+ \; e^+\sin\theta^{*+} = -0.3144$ km/s
$v_\theta^+ = \mu/h^+ \; (1+e\cos\theta^{*+}) = 3.2532$ km/s

$\vec{v}^+ = -0.3144\hat{r} + 3.2532\hat{\theta}$ km/s

$|\vec{v}^+| = 3.2683$ km/s

$\Delta\vec{v} = \vec{v}^+ - \vec{v}^- = 0.0412\hat{r} + 0.0205\hat{\theta} + 0\hat{h}$ km/s

$|\Delta\vec{v}| = 0.0460$ km/s

Part c

$\Delta\vec{v} = \vec{v}^+ - \vec{v}^- = 0.04\hat{r} - 0.002\hat{\theta}$ km/s

$\vec{v}^+ = \Delta\vec{v} + \vec{v}^- = -0.3156\hat{r} + 3.2307\hat{\theta}$ km/s

$r^+ = r^- = p^- = a^-(1-e^{-2}) = 4119.5$ km

$\mathcal{E}^+ = \dfrac{v^{+2}}{2} - \dfrac{\mu}{r^+} = -5.1818$ km²/s²

$a^+ = \frac{-\mu}{2\mathcal{E}^+} = 4154$ km

$p^+ = a^+(1-e^{+2}) = h^{+2}/\mu = (r v_\theta)^2/\mu$

$\rightarrow \frac{(r v_\theta)^2}{\mu a^+} = 1 - e^{+2}$

$e^+ = \sqrt{1 - \frac{(r v_\theta)^2}{\mu a^+}} = 0.0976$

$r_p^+ = a^+(1-e^+) = 3748.7$ km

altitude $= r_p - r_{♂} = 351.46$ km

No, this maneuver would <u>not</u> put the s/c into the required periapsis alt. of >400km.

PROBLEM 3

given: $a_\oplus = 1.00000010178$ AU   1 AU = 149597870.7 km
$a_{sat} = 9.554909595$ AU $\mu_{sun} = 1.32712428 \times 10^{11}$ km^3/s^2

Part a)

$a_t = \frac{1}{2}(r_{p,t} + r_{a,t}) = \frac{1}{2}(a_i + a_f) = 7.8950 \times 10^8$ km

$\Delta\vec{v}_1 = \left(\sqrt{\frac{2\mu_\odot}{a_i} - \frac{\mu_\odot}{a_t}} - \sqrt{\frac{\mu_\odot}{a_i}}\right)\hat{\theta} = 10.2922\hat{\theta}$ km/s

$\Delta\vec{v}_2 = \left(\sqrt{\frac{\mu_\odot}{a_f}} - \sqrt{\frac{2\mu_\odot}{a_f} - \frac{\mu_\odot}{a_t}}\right)\hat{\theta} = 5.4412\hat{\theta}$ km/s

$\Delta v_{total} = |\Delta\vec{v}_1| + |\Delta\vec{v}_2| = 15.7335$ km/s

$TOF = \frac{P_t}{2} = \pi\sqrt{\frac{a_t^3}{\mu}} = 1.913 \times 10^8$ s $= 6.0661$ yrs

Part b)

looking @ pg 16 of notes.

Saturn traces out angle α during this TOF: $\alpha = \sqrt{\frac{\mu}{a_{sat}^3}} \cdot TOF$

The relative initial phase angle is $\phi = \pi - \alpha$

$\alpha = 1.2896$ rad

$\phi = \pi - \alpha = 1.852$ rad $= 106°$

part c)

$r_B = 11$ AU

$a_{t1} = \frac{1}{2}(a_{earth} + r_B) = 8.9759 \times 10^8$ km

$$\Delta \vec{V}_1 = \left(\sqrt{\frac{2\mu_\odot}{a_\oplus} - \frac{\mu_\odot}{a_{t1}}} - \sqrt{\frac{\mu_\odot}{a_\oplus}}\right)\hat{\theta} = 10.544\,\hat{\theta} \text{ km/s}$$

$a_{t2} = \frac{1}{2}(r_B + a_{sat}) = 1.5375 \times 10^9$ km

$$\Delta \vec{V}_2 = \left(\sqrt{\frac{2\mu_\odot}{r_B} - \frac{\mu_\odot}{a_{t2}}} - \sqrt{\frac{2\mu_\odot}{r_B} - \frac{\mu_\odot}{a_{t1}}}\right)\hat{\theta} = 4.9927\,\hat{\theta} \text{ km/s}$$

$$\Delta \vec{V}_3 = \left(\sqrt{\frac{\mu_\odot}{a_{sat}}} - \sqrt{\frac{2\mu_\odot}{a_{sat}} - \frac{\mu_\odot}{a_{t2}}}\right)\hat{\theta} = -0.333\,\hat{\theta} \text{ km/s}$$

$$\Delta V_{total} = |\Delta \vec{V}_1| + |\Delta \vec{V}_2| + |\Delta \vec{V}_3| = 15.8697 \text{ km/s}$$

$$TOF = \frac{P_1}{2} + \frac{P_2}{2} = \pi\sqrt{\frac{a_{t1}^3}{\mu}} + \pi\sqrt{\frac{a_{t2}^3}{\mu}} = 23.839 \text{ yrs.}$$

part d)

	Hohmann	Bielliptic
ΔV	15.73 km/s	15.9 km/s
TOF	6 yrs	24 yrs

this is consistent w/ expectations that the Bi-elliptic transfer is less efficient in this case w/o r_B sufficiently large. This can be proved by calculating $a_f/a_i = 9.5549$, which by using the graph by Curtis in the lecture notes, places the transfer in the region where ΔV_{tot} is lowest for a Hohmann transfer w/o r_B sufficiently large, $11.94 < a_f/a_i < 15.58$, and below the solid black curve in this region.

part e)

Using data from the Cassini mission or other propulsion systems + mass combinations and the ideal rocket eqn, we can calculate the mass ratio.

$$\frac{m_i - m_f}{m_i} = 1 - e^{-\Delta V / I_{sp} \cdot g_0}$$

The larger this mass ratio, the more propellant mass required.