ASEN 5050 - HW1(Pg1) GUSTAVO Grinsteins 01/26/2020

Problem 1 : at a specific Instant in time, heliocentric, Given Values: (Relative to sun)

$v = 22.4346$ Km/s $v_r = 4.1219$ Km/s $h = 4.9775 \times 10^9$ Km²/s

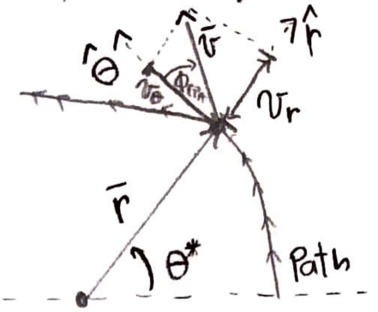
Assumptions: • 2BP • No perturbations • $\mu = Gm_{sun} = 1.32712428 \times 10^{11}$ Km³/s²

P1)
a) Write $\bar{r}$ and $\bar{v}$ in the $(\hat{r}, \hat{\theta}, \hat{h})$ rotating Coordinate system:

From Lecture & Diagram:

$\bar{r} = r\hat{r}$

$\bar{v} = v_r\hat{r} + v_\theta\hat{\theta}$



Variable Relationships:

$\sin(\phi_{FPA}) = \frac{v_r}{v}$, $\cos(\phi_{FPA}) = \frac{v_\theta}{v} = \frac{r\dot{\theta}}{v}$

solving for ϕ_{FPA}:

$\phi_{FPA} = \sin^{-1}\left(\frac{v_r}{v}\right) = \sin^{-1}\left(\frac{4.1219}{22.4346}\right) = 0.1848$ rad or $10.5871°$

Solving for r:

$r\dot{\theta} = v\cos(\phi_{FPA}) \therefore r\dot{\theta} = (22.4346)\cos(.1848) = 22.05261$ Km·rad/s

$h = r^2 \cdot \dot{\theta} = r(r \cdot \dot{\theta}) \therefore r = \frac{h}{(r \cdot \dot{\theta})} = \frac{(4.9775 \times 10^9)}{(22.05261)} = 2.2571 \times 10^8$ km

Plugin into Vector notation →
$$\boxed{\begin{aligned}\bar{r} &= 2.2571 \times 10^8\,\hat{r} \text{ Km} \\ \bar{v} &= 4.1219\,\hat{r} + 22.05261\,\hat{\theta} \text{ Km/s}\end{aligned}}$$

P1)
b) Find: $a, e, \mathcal{E}, P, \mathbb{P}$, type of conic

Solving for Specific Energy $\mathcal{E}$: $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{(22.4346)^2}{2} - \frac{(1.32712428 \times 10^{11})}{(2.2571 \times 10^8)} = \boxed{-336.3220 \text{ Km}^2/\text{s}^2 = \mathcal{E}}$

• Since $\mathcal{E} < 0$ S/C is Bounded on heliocentric orbit

Solving for Semi-latus rectum P: $P = \frac{h^2}{\mu} = \frac{(4.9775 \times 10^9)^2}{(1.32712428 \times 10^{11})} = \boxed{1.8669 \times 10^8 \text{ Km} = P}$

Solving for eccentricity e: $e = \sqrt{1 + \frac{2h^2\mathcal{E}}{\mu^2}} = \sqrt{1 + \frac{2(4.9775 \times 10^9)^2(-336.3220)}{(1.32712428 \times 10^{11})^2}} = \boxed{0.2319 = e}$

Negative Square Root term ignored

$\boxed{\text{• Since } 0 < e < 1 \text{ type of Conic = Ellipse}}$

Solving for semi-Major axis: $a = -\frac{\mu}{2\mathcal{E}} = -\frac{(1.32712428 \times 10^{11})}{2(-336.3220)} = \boxed{1.9730 \times 10^8 \text{ Km} = a}$

Solving for orbit period: $\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(1.9730 \times 10^8)^3}{(1.32712428 \times 10^{11})}} = \boxed{4.7799 \times 10^7 \text{ s} = \mathbb{P}}$

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P1) c) Calculate θ^* and Φ_{FPA} -> From P1) a) -> $\boxed{\Phi_{FPA} = \sin^{-1}\left(\frac{v_r}{v}\right) = 0.1848 \text{ rad or } 10.5871^\circ}$

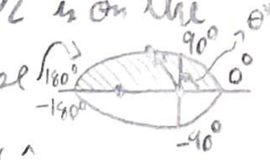
solving For θ^* -> $r = \dfrac{p}{1+e\cos(\theta^*)}$

$\rightarrow \theta^* = \pm\cos^{-1}\left(\dfrac{p-r}{re}\right) = \pm\cos^{-1}\left(\dfrac{1.8649\times10^8 - 2.2571\times10^8}{2.2571\times10^8 \cdot .2319}\right)$

$$\boxed{\theta^* = 2.4123 \text{ rad or } 138.2167^\circ}$$

↑ no sign Check Needed Since $0 \le \theta^* \le 180$ because $v_r > 0$

• Since $0 \le \theta^* \le 180^\circ$ the S/C is on the upper hemisphere of the ellipse



P1) D) Confirm orbit Parameters given: $\bar{R} = 5.3243\times10^7\hat{X} + 2.1925\times10^8\hat{Y} + 6.2724\times10^6\hat{Z}$ km

$\bar{V} = -2.0449\times10^1\hat{X} + 9.2202\hat{Y} - 3.8811\times10^{-1}\hat{Z}$ km/s

$r = \|\bar{R}\| = \sqrt{(5.3243\times10^7)^2 + (2.1925\times10^8)^2 + (6.2724\times10^6)^2} = \boxed{2.2562\times10^8 \text{ km} = r}$

$v = \|\bar{V}\| = \sqrt{(-2.0449\times10^1)^2 + (9.2202)^2 + (-3.8811\times10^{-1})^2} = \boxed{22.4274 \text{ km/s} = v}$

$h = \|\bar{R}\times\bar{V}\|$ ->

$$\bar{R}\times\bar{V} = \begin{vmatrix} \hat{X} & \hat{Y} & \hat{Z} \\ 5.3243\times10^7 & 2.1925\times10^8 & 6.2724\times10^6 \\ -2.0449\times10^1 & 9.2202 & -3.8811\times10^{-1} \end{vmatrix}$$

Formal determinant

Using Cofactor Expansion on the First Row
$$= \left[(2.1925\times10^8 \cdot -3.8811\times10^{-1}) - (6.2724\times10^6 \cdot 9.2202)\right]\hat{X}$$
$$- \left[(5.3243\times10^7 \cdot -3.8811\times10^{-1}) - (6.2724\times10^6 \cdot -2.0449\times10^1)\hat{Y}\right]$$
$$+ \left[(5.3243\times10^7 \cdot 9.2202) - (2.1925\times10^8 \cdot -2.0449\times10^1)\right]\hat{Z}$$

$$\bar{R}\times\bar{V} = -1.4293\times10^8\hat{X} - 1.0760\times10^8\hat{Y} + 4.9744\times10^9\hat{Z} \text{ km}^2/\text{s}$$

$$\|\bar{R}\times\bar{V}\| = \sqrt{(-1.4293\times10^8)^2 + (-1.0760\times10^8)^2 + (4.9744\times10^9)^2}$$

$$\boxed{h = \|\bar{R}\times\bar{V}\| = 4.9732\times10^9 \text{ km}^2/\text{s}}$$

Calculate $\mathcal{E}$ to find e & a:

$$\mathcal{E} = \frac{(22.4274)^2}{2} - \frac{(1.32712428\times10^{11})}{(2.2562\times10^8)} = -336.7180 \text{ km}^2/\text{s}^2$$

$$e = \sqrt{1 + \frac{2(4.9732\times10^9)^2(-336.7180)}{(1.32712428\times10^{11})^2}} = \boxed{0.2331 = e}$$ ← negative SQRT term ignored

$$a = -\frac{(1.32712428\times10^{11})}{2(-336.7180)} = \boxed{1.9707\times10^8 \text{ km} = a}$$

• The values for r, v, h, e, a are Consistent within Small Rounding Errors

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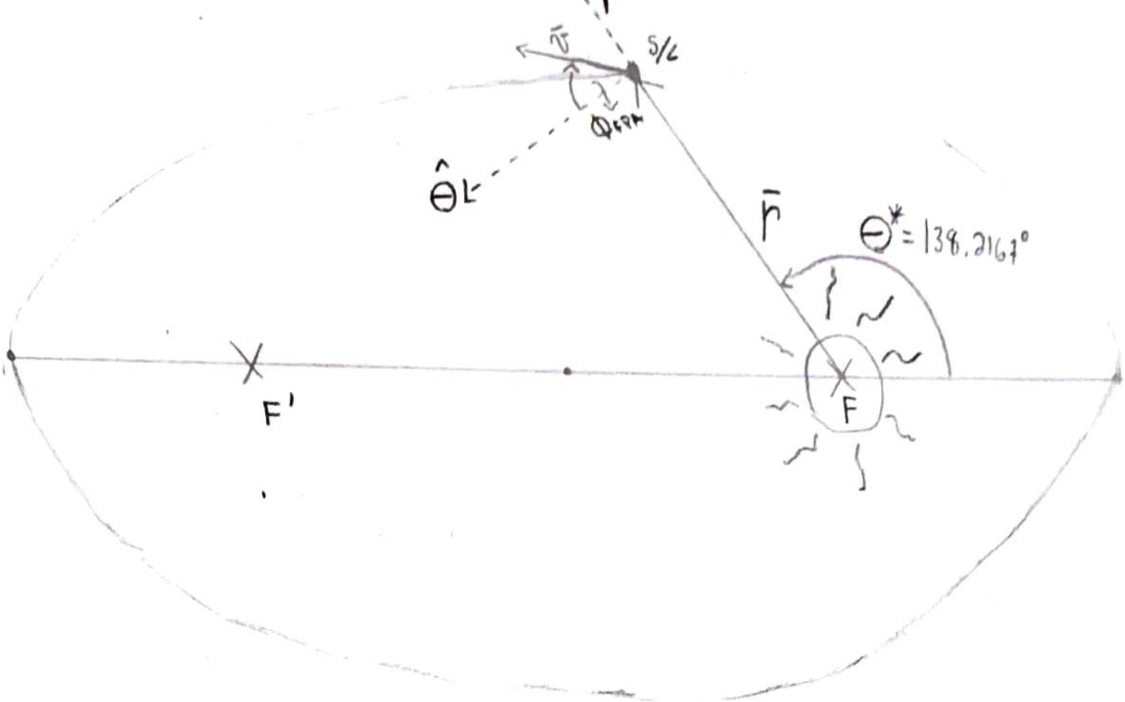
P1) e)



P1) f) Justify why r and θ^* are correct:

- Since the average distance from the Sun to Mars is about 2.28×10^9 km, a value of 2.25×10^8 km seems plausible for the S/C thus far in the trip.
- Given that $V_r > 0$ we would expect a value for $\theta^* = [0, 180°]$. A value of $\theta^* = 138°$ is plausible given that $P = 1.8669 \times 10^8$ km which represents a shorter distance than the current value for r. In other words, as θ^* becomes greater than 90° the value for r will be greater than P for Ellipses.

Problem 2: at a Specific instant in time, Mars-Centric, given values:

Periapsis Altitude = 2,342.8 km $\delta = 66.7°$

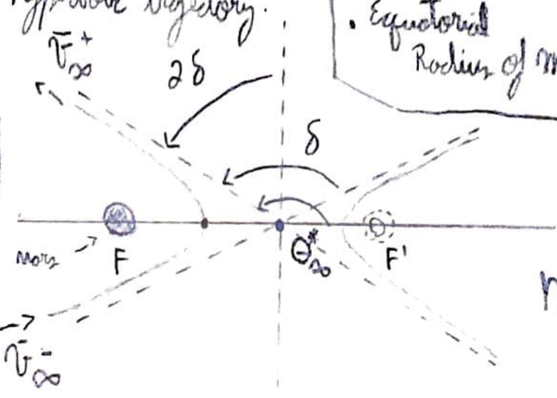
Assumptions: • 2BP • No perturbations • $\mu = Gm_{Mars} = 4.305 \times 10^4$ km³/s² • Equatorial Radius of Mars: 3397.2 km

P2) a) Find V_∞ & θ^*_∞ for S/C in its hyperbolic trajectory:

Since -> $\delta = 2\sin^{-1}\left(\frac{1}{e}\right) \therefore e = \frac{1}{\sin\left(\frac{\delta}{2}\right)} = 1.8$

$$\boxed{\theta^*_\infty = \pm \cos^{-1}\left(\frac{-1}{e}\right) = \pm 2.2 \text{ rad} \equiv \pm 123.7°}$$

$$\boxed{V_\infty = \pm\sqrt{\frac{\mu}{|a|}} = \pm 2.4 \text{ km/s}}$$



Since -> $r_p = a(1-e)$

$a = \frac{r_p}{(1-e)} = -7175$ km

$r_p = 2{,}342.8 \text{ km} + 3397.2 \text{ km} = 5740$ km

P2) b) Calculate v @ Periapsis : $v_{esc} = \sqrt{\frac{2\mu}{r_p}} = \sqrt{\frac{2(4.305\times10^4)}{(5740 \text{ km})}} = 3.9 \text{ km/s}$

Ignore negative value @ r_p $v_r > 0$

Given that, $\hookrightarrow v^2 = v_{esc_{r_p}}^2 + v_\infty^2 \Rightarrow v = \sqrt{v_{esc_{r_p}}^2 + v_\infty^2}$

$$\boxed{v_{G_{r_p}} = \sqrt{(3.9)^2 + (2.4)^2} = 4.6 \text{ km/s}}$$

P2) c) Mars moons (Phobos and Deimos) Contribute to the dynamical environment governing the S/C on its trajectory. Phobos orbits around 6,000 Km from the Martian surface. Deimos is ~24,000 Km from the surface. Since the hyperbolic orbit poses within these distances, it is possible for the S/C to get close enough to one of the Moons and have its orbit altered. If this happens, the relative 2BP would Not be suitable to describe this trajectory.