ASEN 5050 – Spaceflight Dynamics

Homework #2

Assigned: Tuesday, January 26, 2021
Due: Tuesday, February 2, 2021 at 8.59pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Earth} = 3.986004415 \times 10^5 km^3/s^2$
  - $Gm_{Mars} = 4.305 \times 10^4 km^3/s^2$
  - Equatorial radius of Earth: 6378.1363 km
  - Equatorial radius of Mars: 3397.2 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Consider a spacecraft in an elliptical orbit about the Earth. The orbit of the spacecraft possesses the following two characteristics:

- A semi-latus rectum, *p*, equal to 15,345 km
- When the spacecraft is located at the top of the minor axis, the true anomaly, θ^*, is equal to 112 degrees

a) For this orbit, calculate the semi-major axis, *a*, and the eccentricity, *e*.

b) Consider an instant of time when the spacecraft is located at a distance *r*=*a* from the focus and is moving towards periapsis. At that instant, calculate the:

- position vector in the $(\hat{P}, \hat{Q}, \hat{W})$ perifocal frame; and
- velocity vector in the $(\hat{r}, \hat{\theta}, \hat{h})$ rotating frame.

Problem 2:

The MAVEN (Mars Atmosphere and Volatile EvolutioN) mission was launched in November 2013 with a primary goal of studying the atmosphere of Mars. The MAVEN spacecraft has studied the Martian system for over six years, collecting data that has enabled a wide variety of exciting scientific insights. At a specific time instant on the day that this homework is assigned, the state of MAVEN in its Martian orbit is described by the following position and velocity vectors, expressed in a Mars-centered inertial frame $(\hat{X}, \hat{Y}, \hat{Z})$:

$$\bar{r} = 4981.75\hat{X} - 4121.90\hat{Y} + 22.70\hat{Z} km$$
$$\bar{v} = -0.60359\hat{X} + 0.56812\hat{Y} - 2.24093\hat{Z} km/s$$

a) At this instant of time, calculate the following orbital elements: $a, e, i, \Omega, \omega, \theta^*$
b) At this instant of time, write the position and velocity vectors in the $(\hat{r}, \hat{\theta}, \hat{h})$ rotating frame using two methods: 1) a transformation via the direction cosine matrix, and 2) by directly computing the components of each vector using the analytical expressions from

the first module that focused on the two-body problem. Compare the results that you recover using both of these approaches.

c) At a later instant of time, the spacecraft is located at the ascending node. Calculate the position and velocity vectors of the spacecraft in the inertial frame at this location.

d) Use the orbital elements you calculated in part a) to sketch the orbit of MAVEN in three-dimensional space. On this plot, mark the following information: the inertial frame axes $\hat{X}, \hat{Y}, \hat{Z}$, the focus F, periapsis and apoapsis, the eccentricity vector, the line of nodes, the specific angular momentum unit vector, the inclination, the right ascension of the ascending node and the argument of periapsis. Hint: while your orbit may not be to scale or look like a perfect ellipse, you can use the angular quantities in the orbital element set to sufficiently capture the orientation of the orbit in your sketch.