04/01/2021

Gustavo Grinsteins Planchart
ASEN 5050
HW7

Problem 1:

<u>Assumptions:</u>

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Orbits of Earth and Mercury are circular and coplanar
- $Gm_{Sun} = 1.32712428 \times 10^{11}\ km^3/s^2$
- $Gm_{Earth} = 3.986004415 \times 10^{5}\ km^3/s^2$
- $Gm_{Mercury} = 2.2032 \times 10^{4}\ km^3/s^2$
- a_{Earth} = 1.0000010178 AU
- $a_{Mercury}$ = 0.387098309 AU
- $r_{Mercury}$ = 2,439 km
- 1 AU = 149,597,870.7 km

Consider a mission to Mercury with the goal of placing a spacecraft into an orbit about Mercury for long-term observations of the planetary surface and environment.

a) Let's consider a Hohmann transfer from the Earth to Mercury, assuming the orbits of both bodies are circular and coplanar and modeling only solar gravity. Calculate the semi-major axis and eccentricity of the transfer as well as time of flight required.

We can graphically see this transfer in **figure 1** as well as the variable relationships and notations:

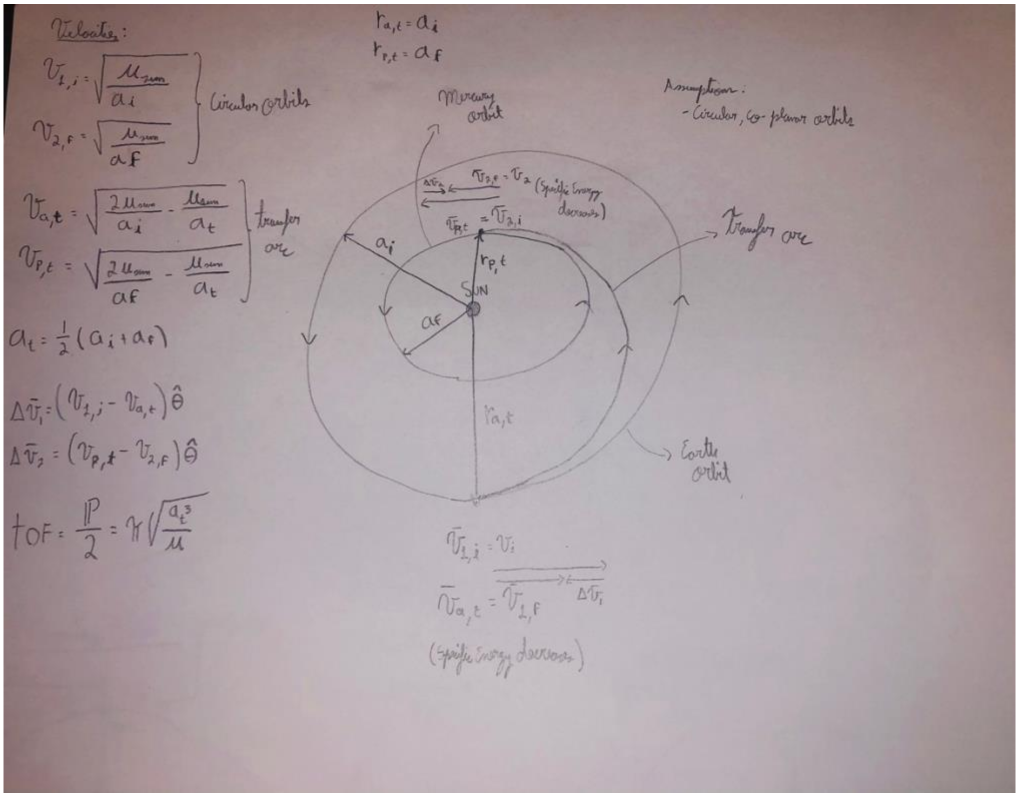

Figure 1 - Hohmann transfer between Earth and Mercury

Where:

$$a_f = final\ orbit\ semi-major\ axis\ (Mercury)$$

$$a_i = initial\ orbit\ semi-major\ axis\ (Earth)$$

$$v_{p,t} = Heliocentric\ velocity\ at\ elliptical\ transfer\ arc\ periapsis$$

$$v_{2,f} = v_{Mercury} = Mercury's\ heliocentric\ velocity$$

The semi-major axis of the transfer arc can be calculated using the following relationship:

$$\boldsymbol{a_t = \frac{r_{a,t} + r_{p,t}}{2} = 1.03754x10^8\ km < --}$$

The eccentricity for the transfer arc is:

$$\boldsymbol{e = \frac{r_{a,t}}{a_t} - 1 = 0.44186 < --}$$

Since e < 1, the transfer arc is elliptical in shape

The time of flight is:

$$\boldsymbol{TOF = \frac{\mathbb{P}}{2} = \pi\sqrt{\frac{a_t^3}{\mu_{sun}}} = 9.1138x10^6\ seconds = 105.484\ days < --}$$

Consider a scenario where the spacecraft propulsion system malfunctions during the heliocentric transfer and a Mercury orbit insertion maneuver does not occur. On the hyperbolic approach arc, the closest approach distance is 4,000 km from the center of Mercury, and the spacecraft passes on the 'Sun-side' of Mercury during the flyby.

b) Incorporate the gravity field of Mercury to calculate the turning angle along the hyperbolic orbit during the arrival segment and the heliocentric velocity magnitude (v_{Out}) after the flyby. Did the flyby increase or decrease the spacecraft energy?

Calculating Mercury's Heliocentric velocity magnitude (Given assumed circular orbit):

$$v_{Mercury} = \sqrt{\frac{\mu_{Sun}}{a_{Mercury}}} = 47.872\ km/s\ (vector\ in\ positive\ \hat{\theta}direction)$$

Calculating the spacecraft incoming Heliocentric velocity magnitude.:

$$v_{in} = \sqrt{\frac{2\mu_{sun}}{a_f} - \frac{\mu_{sun}}{a_t}} = 57.484\ km/s\ (vector\ in\ positive\ \hat{\theta}direction)$$

Calculating the spacecraft incoming excess velocity magnitude with respect to mercury:

$$v_{\infty,in} = v_{in} - v_{mercury} = 9.6115\ km/s\ (vector\ in\ positive\ \hat{\theta}direction)$$

Note that by the symmetry of the hyperbola the v-infinity magnitudes will be equal in magnitude:

$$v_{\infty,in} = v_{\infty,out}$$

Calculating the specific mechanical energy of the hyperbola:

$$\varepsilon_h = \frac{v_{\infty,in}^2}{2} = 46.1904\ km^2/s^2$$

Since $\varepsilon_h > 0$, the orbit path is not bounded by the central body

Calculating the semimajor axis of the hyperbola:

$$a_h = -\frac{\mu_{Mercury}}{2\varepsilon_h} = -238.49\ km$$

Hyperbolic eccentricity:

$$e_h = 1 - \frac{r_{periapsis}}{a_h} = 17.772$$

Since e > 1, the transfer arc is hyperbolic in shape

Calculating the hyperbola turning angle:

$$\boldsymbol{\delta = 2\sin^{-1}\left(\frac{1}{e_h}\right)\left(\frac{180}{\pi}\right) = 6.45° < --}$$

Using the law of cosines to calculate the heliocentric Vout magnitude quantity:

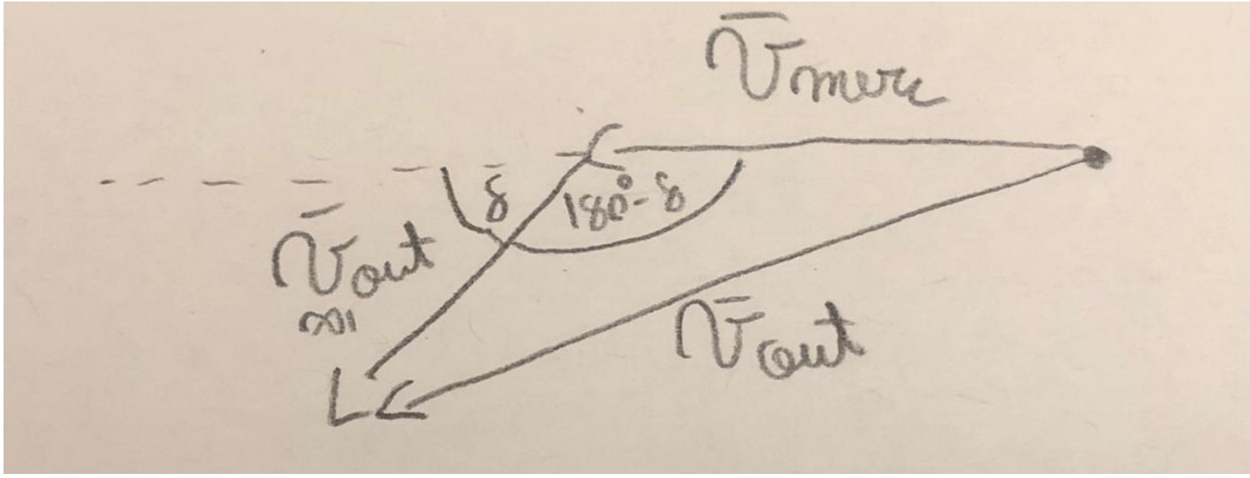

Figure 2 - Vector Triangle Used for Law of Cosine relationship

$$v_{out}^2 = v_{mercury}^2 + v_{\infty,out}^2 - 2v_{mercury}v_{\infty,out}\cos(180 - \delta)$$

$$\boldsymbol{v_{out} = 57.433\ km/s < --}$$

Since $v_{in}(57.484\frac{km}{s})$> $v_{out}(57.433\frac{km}{s})$ the flyby decreased the spacecraft trajectory energy

c) Draw a vector diagram of the heliocentric spacecraft velocities before and after the flyby, as well as Mercury's velocity vector and the relative velocity vectors. Also draw the hyperbolic arc in a Mercury-centered view, including incoming and outgoing $\bar{v}_{\text{inf}}$ vectors, r_p, planet velocity vector, and turning angle.

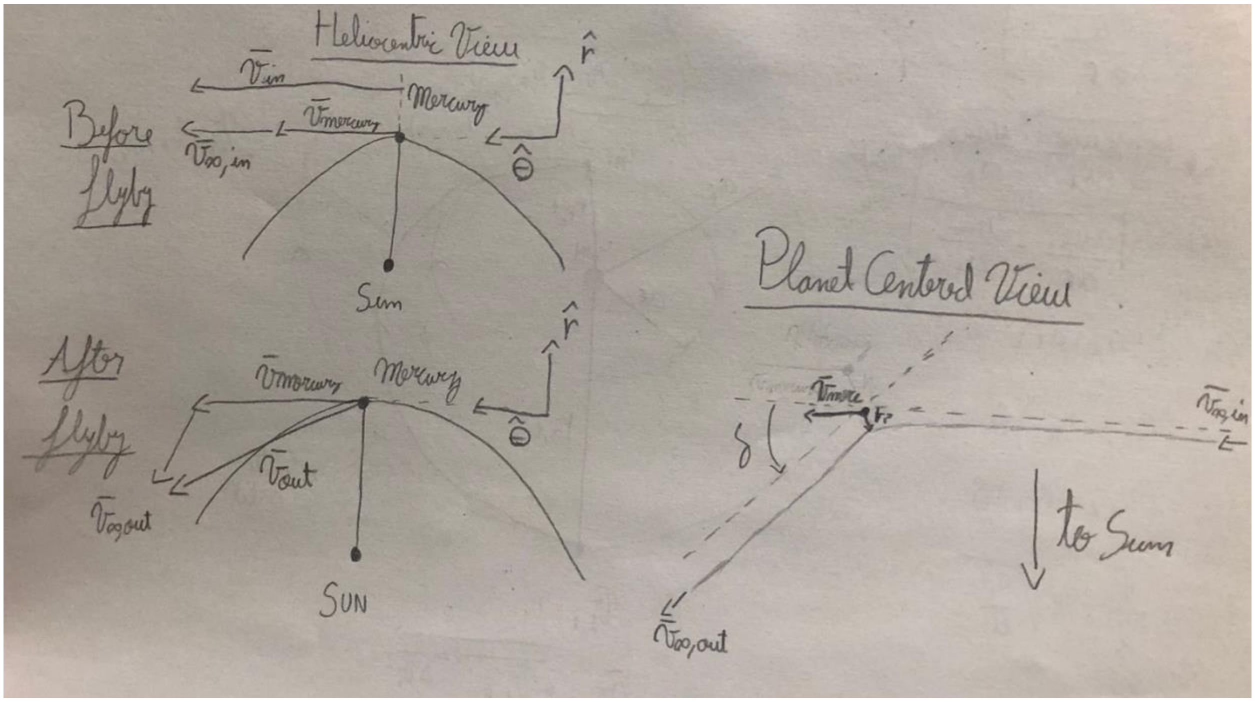

Figure 3 - Heliocentric and planet-centric vector diagrams

Problem 2

<u>Assumptions:</u>

- Relative 2 Body Problem Assumptions
  - Mass of satellite is negligible compared to attracting body
  - Coordinate system is inertial
  - Satellite and attracting body are treated as point masses
  - No other forces in the system except for gravitational forces
- Flyby velocity changes occur instantaneously
- $Gm_{Sun} = 1.32712428 \times 10^{11}\ km^3/s^2$
- $Gm_{Saturn} = 3.794 \times 10^{7}\ km^3/s^2$
- a_{Saturn} = 9.554909595 AU
- r_{Saturn}= 60,268 km
- 1 AU = 149,597,870.7 km

Consider a spacecraft in a large orbit around Saturn, described by a periapsis radius of 700,000 km and an apoapsis radius of 2,000,000 km – and lying in the same orbit plane as Titan. Assume that Titan is in a circular orbit of radius 1,221,830 km, possesses a mass of 1.3455x10^{23} kg and is modeled as a sphere with a radius equal to the equatorial radius, 2,575 km.

a) As the spacecraft travels from periapsis to apoapsis in its orbit, what is the value of the true anomaly when it intersects Titan's orbit? At this value of the true anomaly, write the velocity vector, $\bar{v}_{in}$, relative to Saturn in the rotating frame ($\hat{r}\hat{\theta}\hat{h}$). Also calculate the velocity vector relative to Titan, i.e., $\bar{v}_{inf\text{-}in}$. Draw a useful vector diagram of these two velocity vectors.

Given the quantities:

$$r_{periapsis} = 700{,}000\ km$$

$$r_{apoapsis} = 2{,}000{,}000\ km$$

$$a_{titan} = 1{,}221{,}830\ km$$

$$m_{titan} = 1.3455x10^{23} kg$$

Calculating the semi-major axis of the orbit:

$$a_{Saturn} = \frac{r_{apoapsis} + r_{periapsis}}{2} = 1{,}350{,}000\ km$$

Calculating the eccentricity of the Saturn-centered orbit:

$$e = \frac{r_{apoapsis}}{a_{Saturn}} - 1 = 0.48148$$

Semi-latus rectum:

$$p = a_{Saturn}(1 - e^2) = 1.037037x10^6\ km$$

Specific angular momentum:

$$h = \sqrt{\mu_{Saturn} p} = 6.2726x10^6\ km^2/s$$

Calculating the true anomaly at the intersection with Titan's orbit:

$$\theta^*{}_{Intersect} = \pm cos^{-1}\left(\frac{p - a_{Titan}}{a_{Titan} \cdot e}\right)\left(\frac{180}{\pi}\right) = \pm 108.31° = +108.31°$$

Since the spacecraft is traveling from periapsis to apoapsis $\theta^*{}_{Intersect} > 0$

Calculating the true anomaly at the top of the semi-minor axis:

$$\theta_b^* = cos^{-1}(-e) = 118.78°$$

Since $\theta_b^* > \theta^*{}_{Intersect}$ the tip of the radius vector will lie behind the semi-minor axis.

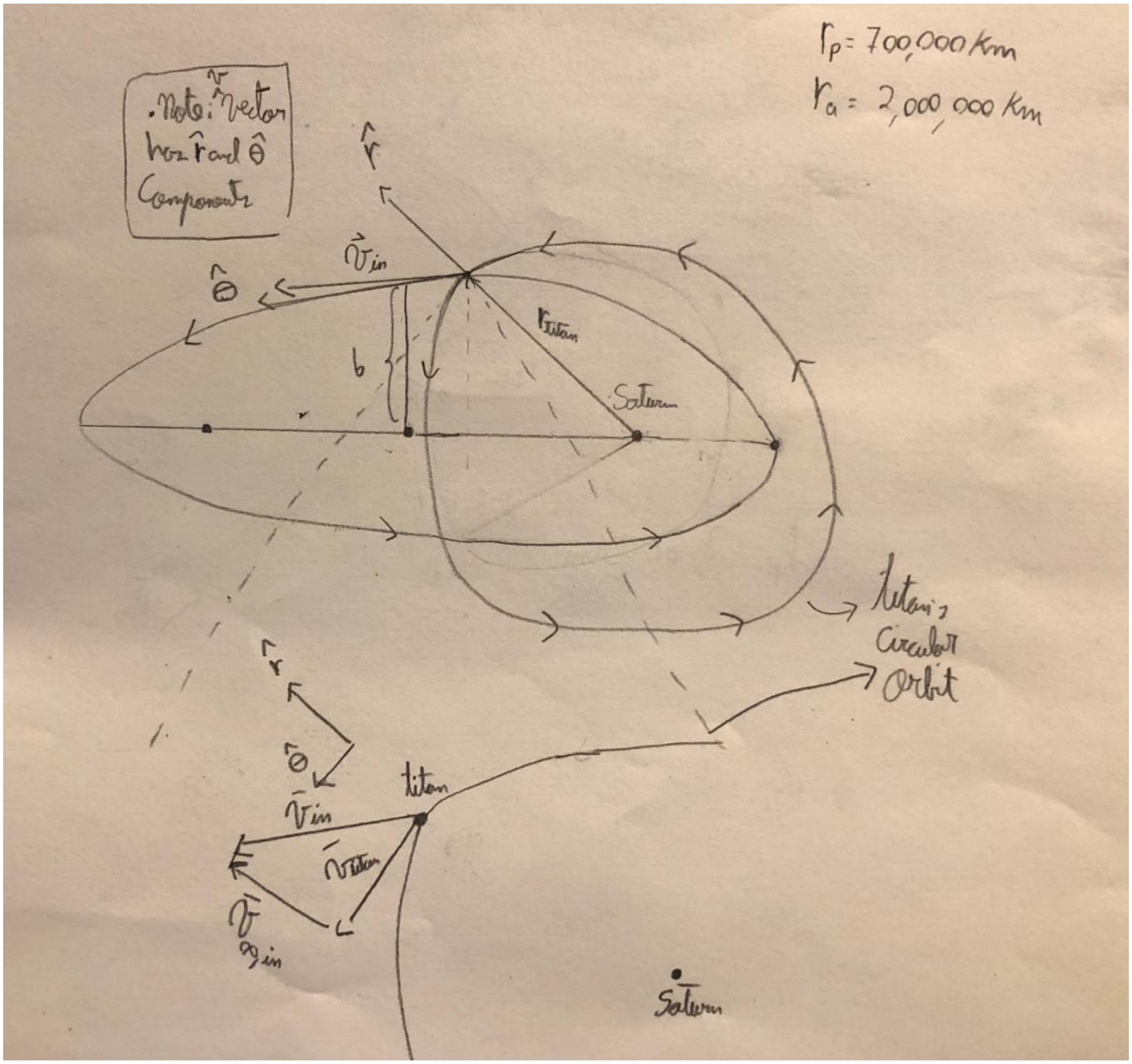

Figure 4 - Problem 2a Vector Diagram

Calculating the rotating frame velocity magnitudes:

$$v_\theta = \frac{\mu_{Saturn}}{h}\left(1 + e\, cos(\theta^*_{intersect})\right) = 5.1338\frac{km}{s}$$

$$v_r = \frac{\mu_{Saturn}}{h}\left(e\, sin(\theta^*_{intersect})\right) = 2.7649\ km/s$$

Calculating the velocity magnitude of titan in its assumed circular orbit:

$$v_{Titan} = \sqrt{\frac{\mu_{Saturn}}{a_{Titan}}} = 5.5724\ km/s$$

Formulating these quantities in vector form on the $(\hat{r}\hat{\theta}\hat{h})$ rotating frame:

$$\boldsymbol{\bar{v}_{in} = 2.7649\,\hat{r} + 5.1338\,\hat{\theta} + 0\,\hat{h}\ km/s} \ <--$$

$$\bar{v}_{Titan} == 0\,\hat{r} + 5.5724\,\hat{\theta} + 0\,\hat{h}\ km/s$$

Using these vectors:

$$\boldsymbol{\bar{v}_{\infty,in} = \bar{v}_{in} - \bar{v}_{Titan} = 2.7649\,\hat{r} + -0.43866\,\hat{\theta} + 0\,\hat{h}\ km/s} \ <--$$

b) If the spacecraft performs a flyby of Titan with a closest approach distance of 2,800 km, and passes behind Titan, calculate the turning angle. Draw a diagram of the hyperbolic orbit in a Titan-centered view, including incoming and outgoing $\bar{v}_{\text{inf-in}}$ vectors, r_p, Titan's velocity vector, and the turning angle.

Given variables:

$$r_{peri\ Titan} = 2{,}800\ km$$

$$\mu_{Titan} = Gm_{Titan} = (6.673x10^{-20}\ \text{km}^3/\text{kg/s}^2)(1.3455\text{x}10^{23}\text{kg}) = 8.9785\text{x}10^3\ \text{km}^3/\text{s}^2$$

Calculating the specific mechanical energy for the flyby hyperbola:

$$\varepsilon_{hyp} = \frac{\left\|\bar{v}_{\infty,in}\right\|^2}{2} = 3.9184\ km^2/s^2$$

Semi-major axis:

$$a_{hyp} = -\frac{\mu_{Titan}}{2\varepsilon_{hyp}} = -1.1457x10^3\ km$$

Eccentricity:

$$e_{hyp} = 1 - \frac{r_{peri\ Titan}}{a_{hyp}} = 3.4439$$

Turning Angle:

$$\boldsymbol{\delta = 2\sin^{-1}\left(\frac{1}{e_{hyp}}\right)\left(\frac{180}{\pi}\right) = 33.76° < --}$$

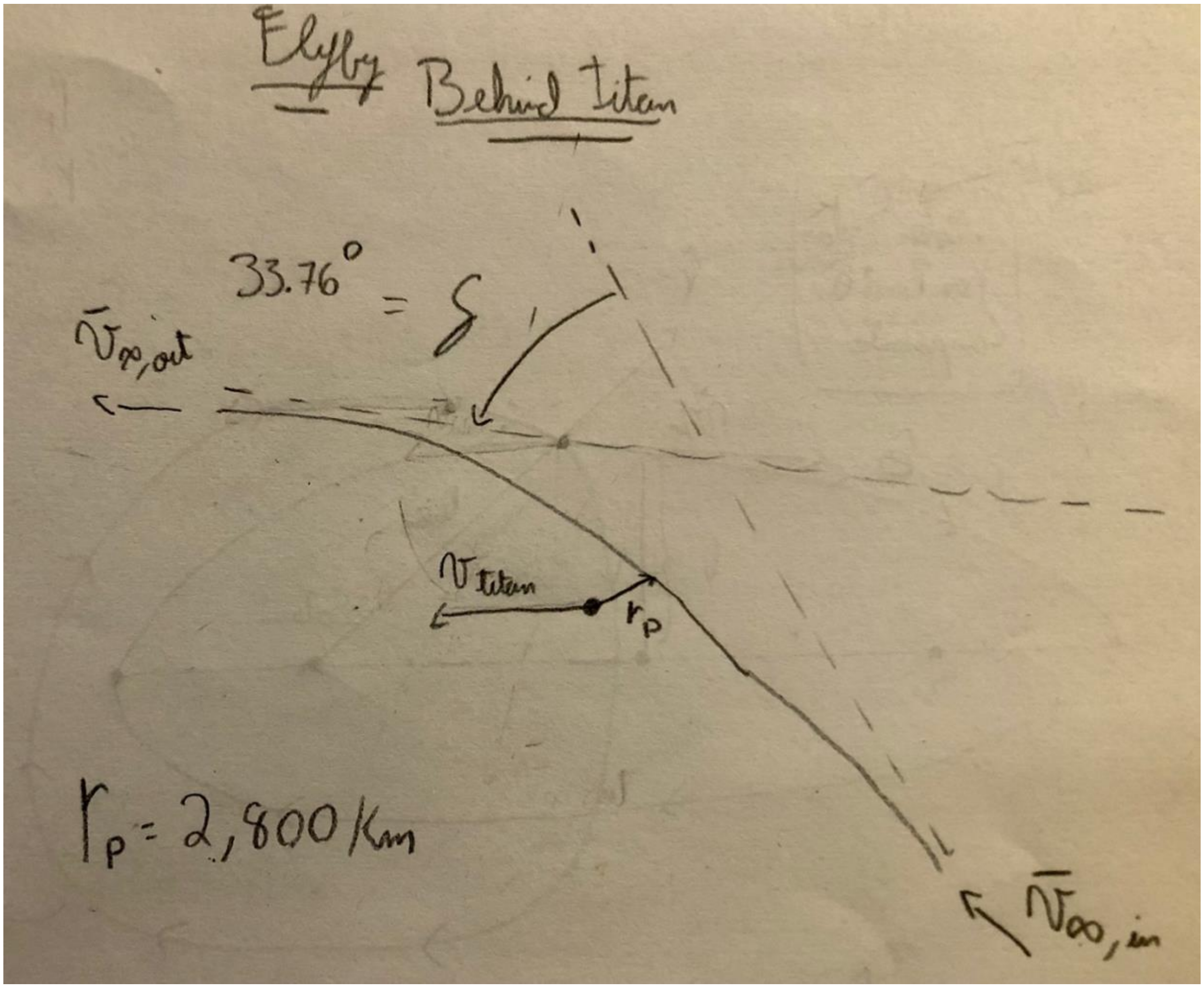

Figure 5 - Problem 2b Hyperbola diagram

c) Add the turning angle and post-flyby relative velocity vector to your diagram in part a). Use the vector diagram to calculate the velocity vector, $\bar{v}_{\text{out}}$, relative to Saturn in the rotating coordinates ($\hat{r}\hat{\theta}\hat{h}$).

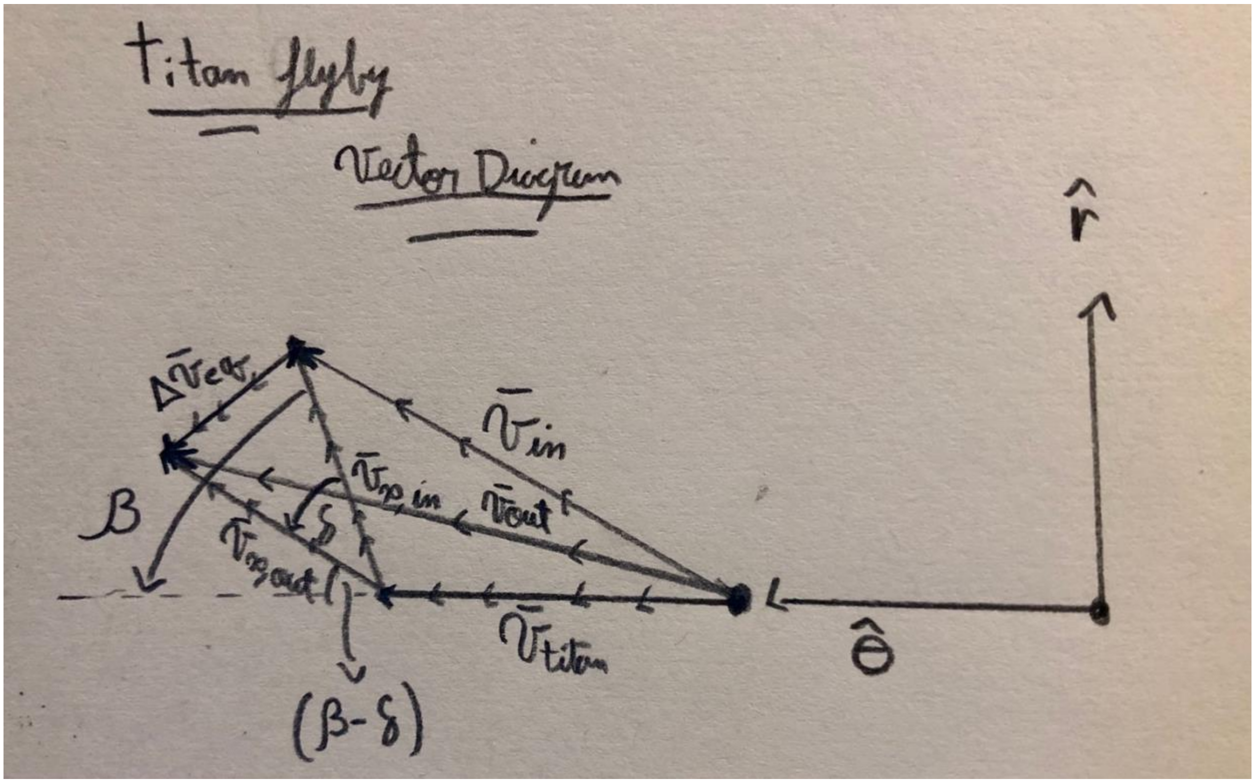

Figure 6 - Problem 2c Vector Diagram

To calculate the vector values of $\bar{v}_{\infty,out}$ lets find the angle of $\bar{v}_{\infty,in}$with respect to the $\hat{\theta}$ unit vector:

$$\beta = \pm cos^{-1}\left(\frac{\hat{\theta} \cdot \bar{v}_{\infty,in}}{\|\hat{\theta}\| \cdot \|\bar{v}_{\infty,in}\|}\right)\left(\frac{180}{\pi}\right) = \pm 91.02° = 91.02°$$

Select the positive value since we know the vector lies in the positive quadrant.

Using this angle, we can project the $\bar{v}_{\infty,in}$ vector on the $\hat{\theta}$ and $\hat{r}$ axes:

$$\bar{v}_{\infty,out} = \|\bar{v}_{\infty,in}\| \sin(\beta - \delta)\,\hat{r} + \|\bar{v}_{\infty,in}\| \cos(\beta - \delta)\,\hat{\theta} + 0\,\hat{h}\;km/s$$

$$\bar{v}_{\infty,out} = 2.5424\,\hat{r} + 1.1716\,\hat{\theta} + 0\,\hat{h}\;km/s$$

$$\boldsymbol{\bar{v}_{out} = \bar{v}_{Titan} + \bar{v}_{\infty,out} = 2.5424\,\hat{r} + 6.7441\,\hat{\theta} + 0\,\hat{h}\;km/s < --}$$

d) Determine the orbital elements (a, e, θ^*) of the Saturn-centered orbit after the flyby and describe the impact of the gravity assist in your own words. Did the flyby increase or decrease the spacecraft energy?

Assuming the flyby effects occur instantaneously:

$$\varepsilon_{aft} = \frac{\|\bar{v}_{out}\|^2}{2} - \frac{\mu_{Saturn}}{a_{titan}} = -5.07795\ km^2/s^2$$

Since $\varepsilon_{aft} < 0$, the orbit path is bounded by the central body

Semi-major axis:

$$\boldsymbol{a_{aft} = -\frac{\mu_{Saturn}}{2\varepsilon_{aft}} = 3.7357x10^6\ km <--}$$

Calculating the specific angular momentum:

$$v_{out,\hat{\theta}} = the\ \hat{\theta}\ component\ of\ velocity\ of\ \bar{v}_{out}$$

$$h_{aft} = a_{Titan} v_{out,\theta} = 8.24023x10^6\ km^2/s$$

Eccentricity:

$$\boldsymbol{e_{aft} = \sqrt{1 + \frac{2\varepsilon_{aft} h_{aft}^2}{\mu_{Saturn}^2}} = 0.72175 <--}$$

Since e < 0, the orbit is parabolic in shape

Semi-latus rectum:

$$p_{aft} = a_{aft}\left(1 - e_{aft}^2\right) = 1.7897x10^6\ km$$

True anomaly:

$$\boldsymbol{\theta^*{}_{aft} = \pm cos^{-1}\left(\frac{p_{aft} - a_{Titan}}{a_{Titan} \cdot e_{aft}}\right)\left(\frac{180}{\pi}\right) = \pm 49.91° = +49.91° <--}$$

Since $\hat{r} \cdot \bar{v}_{out} > 0$, the positive true anomaly is selected.

By comparing the semi-major axis before and after the flyby on the Saturn centered frame:

$$a_{Saturn} = 1.3500x10^6\ km$$

$$a_{aft} = 3.7357x10^6\ km$$

$$\boldsymbol{a_{Saturn} < a_{aft}}$$

After the flyby, the semi-major axis of the elliptical orbit around Saturn increased. Therefore, the energy of the s/c was increased. <--

e) Calculate the equivalent impulsive maneuver that would be required to change the heliocentric velocity of the spacecraft from $\bar{v}_{\text{in}}$ to $\bar{v}_{\text{out}}$ if a gravity assist was not used.

In the $(\hat{r}\hat{\theta}\hat{h})$ rotating frame and using the diagram in **Figure 6**:

$$\Delta\bar{\boldsymbol{v}}_{\boldsymbol{eq}} = \bar{\boldsymbol{v}}_{\boldsymbol{out}} - \bar{\boldsymbol{v}}_{\boldsymbol{in}} = \boldsymbol{-0.22245\,\hat{r} + 1.61041\,\hat{\theta} + 0\,\hat{h}\ km/s} < - -$$

Problem 3

a) Take a screenshot of the MCS (STK) or mission sequence (GMAT). Also take three screenshots of the trajectory: one that is a heliocentric view; one Jupiter-centered view displaying the hyperbolic flyby from above the orbit plane (segments b and c); and one Saturn-centered view displaying the hyperbolic approach arc from above the orbit plane (segment e). Describe, in your own words, the characteristics of the trajectory in each of the heliocentric view, the Jupiter-centered view of the flyby, and the Saturn-centered view of the approach arc.

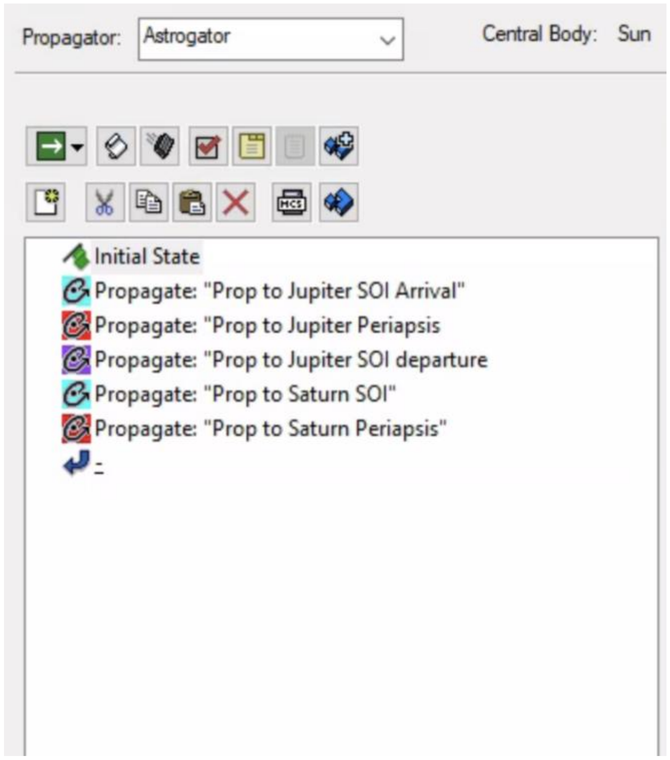

Figure 7 - HW7 STK MCS

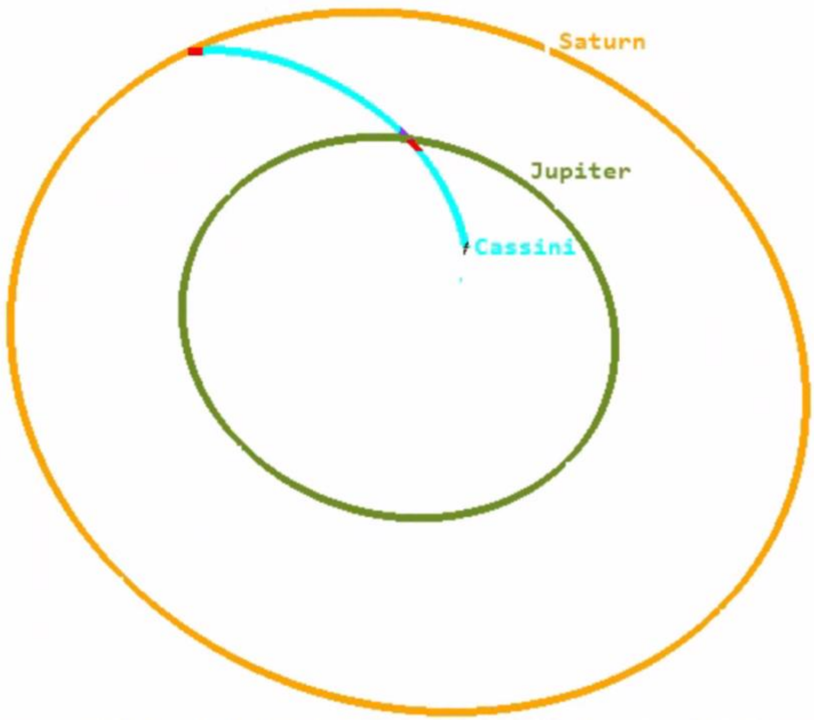

Figure 8 - Heliocentric View of Trajectory

In the heliocentric view, the spacecraft trajectory can be seen from the start of the given epoch until it approaches Saturn. The trajectory from Jupiter to Saturn is slightly altered by the gravitational effects of Jupiter. Similarly, we can see the same behavior as the spacecraft approaches Saturn. In this frame of reference, the overall geometry of the orbit appears to be more elliptical and less hyperbolic.

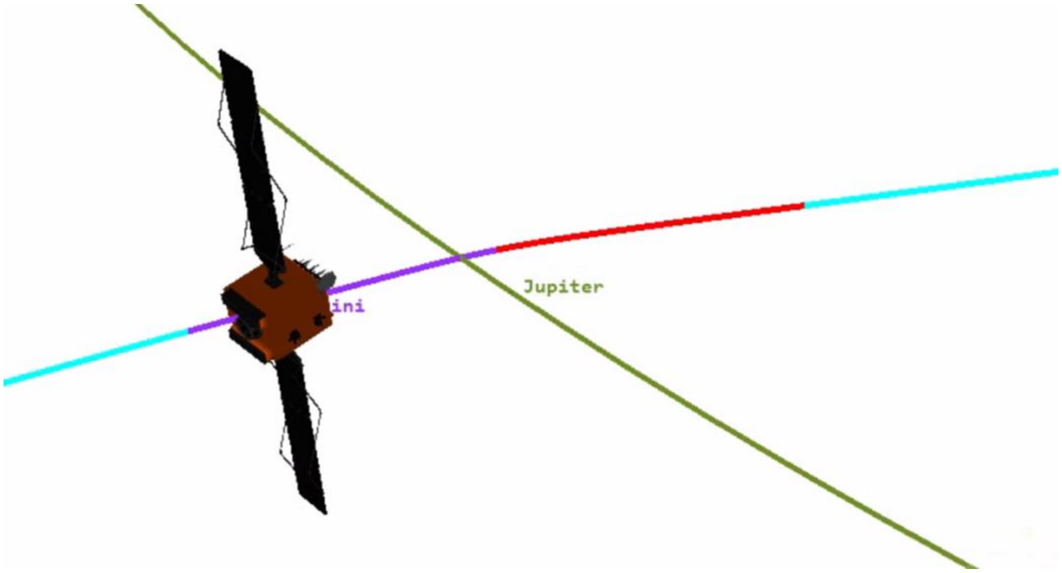

Figure 9 - Jupiter Centered View

This view depicts a trajectory that appears to be more hyperbolic than elliptical. Since the spacecraft passes in front of Jupiter as it crosses its orbit, the energy of the spacecraft will decrease for the next leg of its trajectory towards Saturn.

Figure 10 - Saturn Centered View

As the spacecraft approaches Saturn, the elliptical orbit becomes a hyperbolic orbit that would continue past Saturn.

Summary:

From the pictures attached, we can see that the spacecraft initially performs a Jupiter flyby ahead of the planet. This slows down the trajectory of the satellite. This makes sense, since from the visuals the spacecraft slows down enough to meet with Saturn at the last MCS segment. Also, the orbit shape difference is noticeable when comparing the heliocentric view (ellipse) to a planet centered view (hyperbola).

b) STK: Use the summary function to report the periapsis altitude for the hyperbolic arc describing the Jupiter flyby, using the "Prop to Jupiter Periapsis" segment and listed as the "Rad. Peri" item. In addition, list the epoch in Gregorian UTC date format at periapsis, the inclination, and the v-infinity along the hyperbola (listed as "Excess Vel" in the summary). Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola.

STK Summary Values:

- Periapsis Jupiter: 9.8999x10^6 km
- Periapsis Altitude: 9.8999x10^6 km – 71,492 km = 9,828,408 km
- UTC Gregorian date: 1 Jan 2001 11:38:01.852
- Inclination: 3.7318 deg
- V infinity: 10.27299 km/sec
- Semi-major axis: -1.200x10^6 km
- Eccentricity: 9.24702

Calculating the specific mechanical energy:

$$\varepsilon_{hyp} = \frac{\left\|\bar{v}_{\infty,in}\right\|^2}{2} = 52.767\ km^2/s^2$$

Semi-major axis:

$$a_{hyp} = -\frac{\mu_{Jupiter}}{2\varepsilon_{hyp}} = -1.201504x10^6\ km$$

Eccentricity:

$$e_{hyp} = 1 - \frac{r_{periapsis}}{a_{hyp}} = 9.2396$$

c) STK: Use the summary function to report the periapsis altitude for the hyperbolic arc describing the Saturn approach, using the "Prop to Saturn Periapsis" segment. In addition, list the epoch in Gregorian UTC date format at periapsis, the inclination, and the v-infinity along the hyperbola (listed as "Excess Vel" in the summary).

STK Summary Values:

- Periapsis Saturn flyby: 4.29053e+06 km
- Periapsis Altitude Saturn flyby: 4.29053e+06 km – 60,268 km = 4,230,262 km

- UTC Gregorian date: 13 Jul 2004 15:28:02.685
- Inclination: 24.83 deg
- V infinity: 5.1272 km/sec

d) The true Cassini trajectory reached periapsis at Saturn on July 1, 2004 with a radius of 81,000 km. Compare this information to your answer in part c) and speculate why the simulation you constructed produces the observed differences in the trajectory at its Saturn arrival.

There is a large difference between the date and the periapsis values:

- Date: 12 days of difference between actual and STK
- Periapsis: -4,290,449 km of difference between actual and STK

The large difference in these values might be caused by the initial state vector values for the simulation. As it is observed in part e, using the targeter changes the initial state vector values by minute amounts to reach the right periapsis altitude. This shows that very small changes in the initial state of the s/c can propagate to larger magnitudes as the s/c has longer orbital journeys.

e) After running the targeter, list the new values of the components of the initial state vector. How much did the position and velocity vectors change to achieve the prescribed change in the periapsis radius at Saturn? Use the summary (STK) or report (GMAT) function to verify that the periapsis radius of the Saturn approach arc is, indeed, equal to 81,000 km. Also list the six orbital elements in the Saturn inertial frame at periapsis relative to Saturn. Include a screenshot of the heliocentric view of the complete trajectory, as well as the Saturn-centered view of the Saturn approach arc. Why do you think such a small change in the initial state achieved such a significant change in the periapsis radius at Saturn?

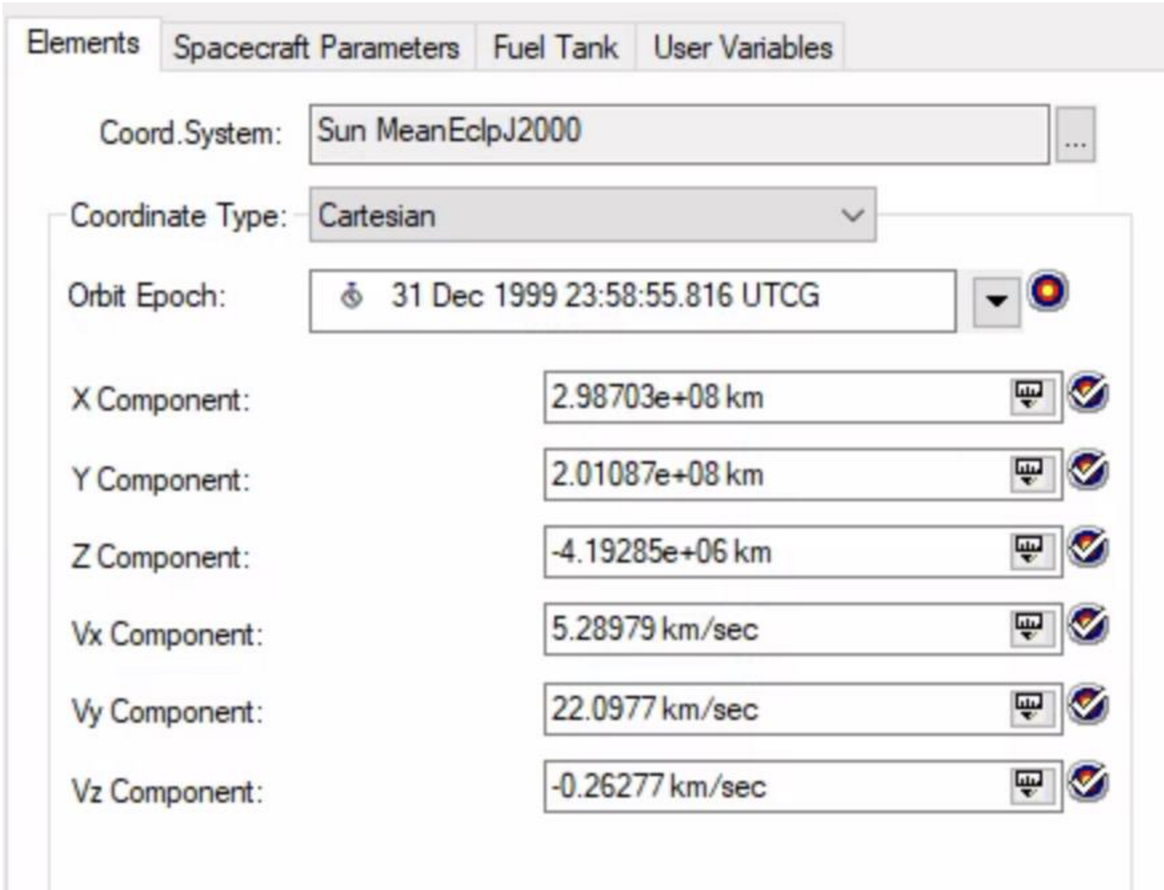

Figure 11 - New Component Values

Comparing new and old values:

- X component difference: near zero
- Y component difference: near zero
- Z component difference: near zero
- Vx component difference: near zero
- Vy component difference: near zero
- Vz component difference: near zero

Values are essentially the same by comparing STK window values. We can see the small scale of the changes by looking at last convergence summary window:

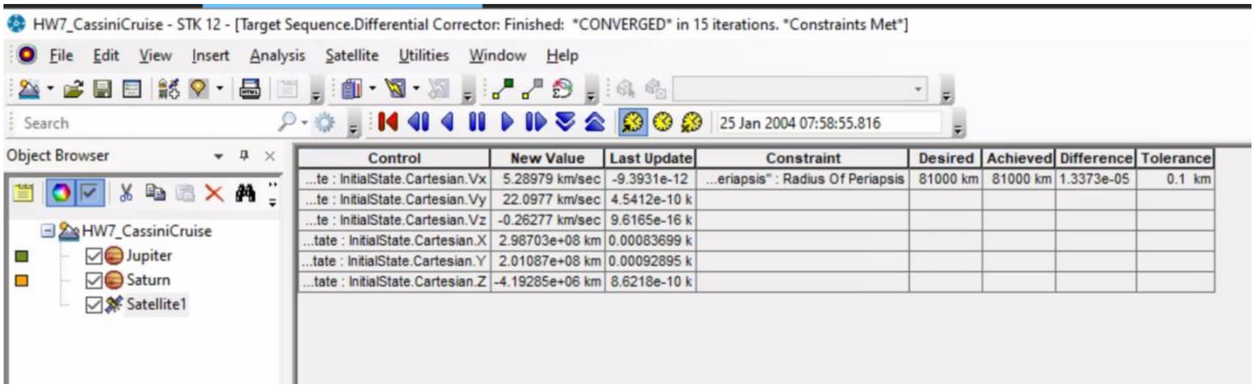

Figure 12 – Periapsis Convergence Window

The last update column shows the minute value change applied to the original values. Some values as small as the 16th decimal place.

```
State Vector in Coordinate System: Saturn Inertial

Parameter Set Type:  Cartesian
         X:    -32402.4369659275325830 km              Vx:       28.4424639337596368 km/sec
         Y:    -71226.5547797390026972 km              Vy:      -11.8954978571274328 km/sec
         Z:     20925.1039434547201381 km              Vz:        3.5521839068276324 km/sec

Parameter Set Type:  Keplerian
       sma:    -1.430304792291971 2e+06 km            RAAN:        180.3303579947081 deg
       ecc:         1.0566312861775258                    w:        66.10301101181365 deg
       inc:          16.41285731103896 deg               TA:     1.091490599902226e-07 deg

Parameter Set Type:  Spherical
 Right Asc:          245.5382273905699 deg      Horiz. FPA:     5.607728831085025e-08 deg
      Decl:          14.97126710182085 deg         Azimuth:        83.19539499213805 deg
       |R|:     81000.0000133731664391 km             |V|:       31.0337660363686183 km/sec
```

Figure 13 - Six Orbital Elements in Saturn Inertial

```
Rad. Peri:     81000.0000133731809910 km
```

Figure 14 - Periapsis Convergence Confirmation

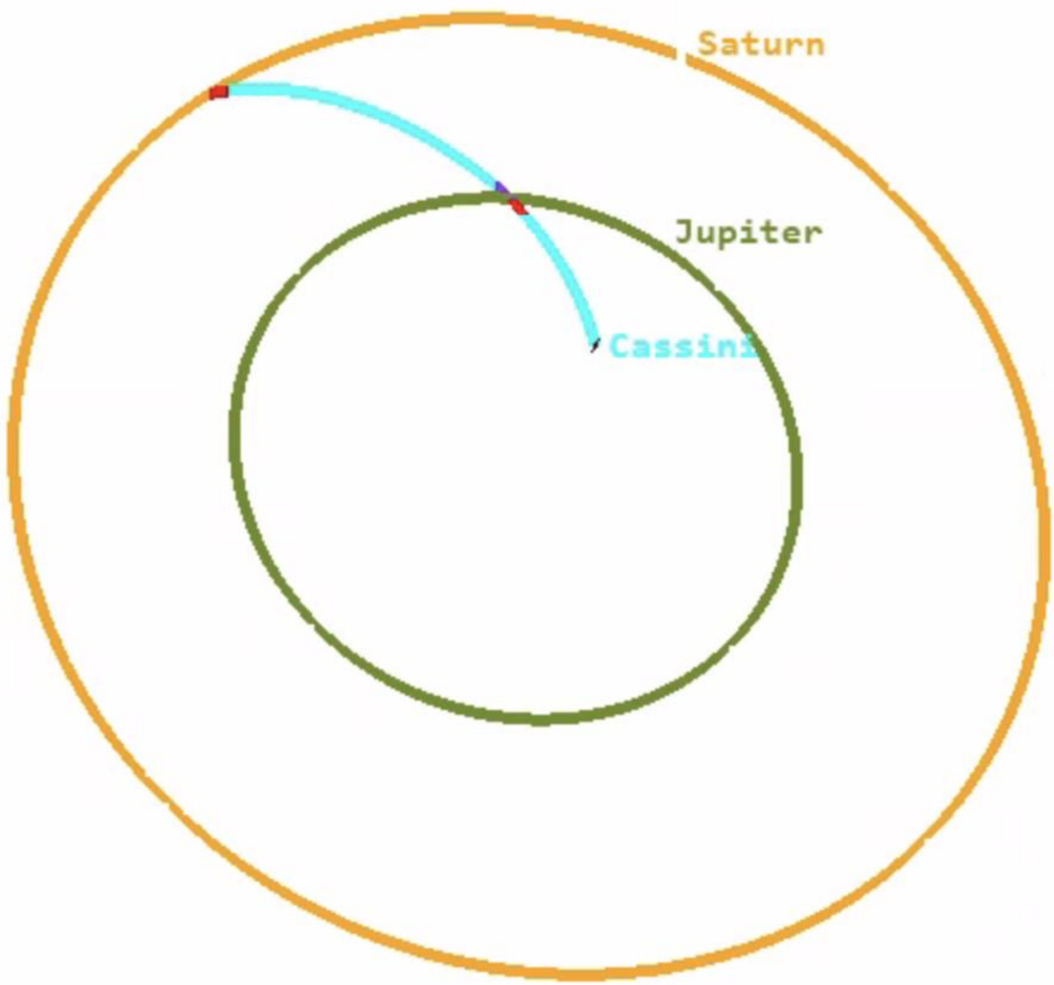

Figure 15 - Heliocentric view of new trajectory

__Why do you think such a small change in the initial state achieved such a significant change in the periapsis radius at Saturn?__

I believe these small changes at the initial state propagate as the spacecraft travels along its orbit. This propagation becomes visible as the s/c reaches its final destination along a large orbit similar to Jupiter and Saturn.

f) Include a screenshot of the Saturn-centered view, viewing from above their common orbit plane both the incoming hyperbolic arc and the elliptical orbit. What is the magnitude of the impulsive maneuver required to reach the prescribed orbit?

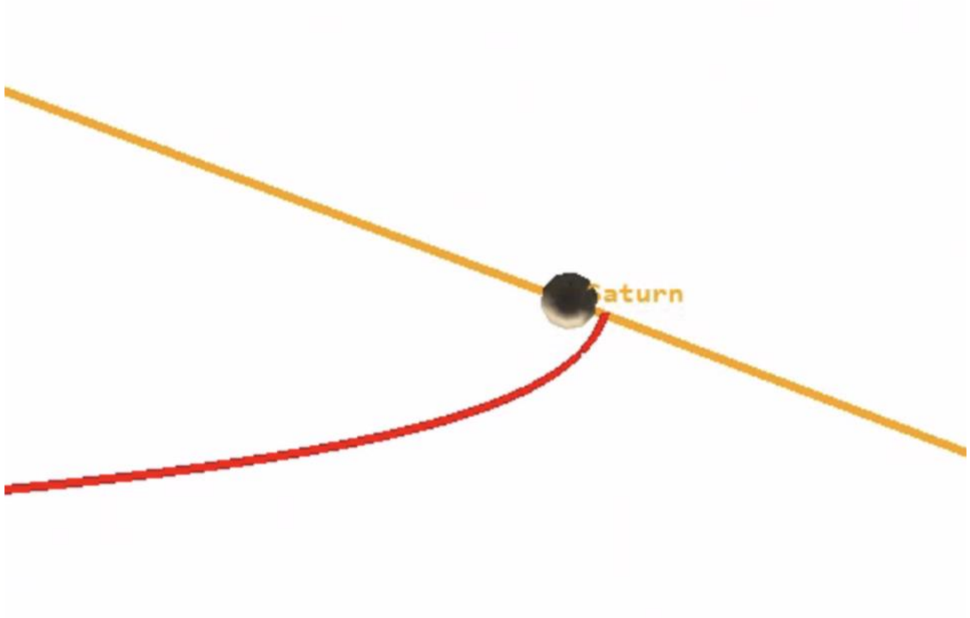

Figure 16 - Saturn centric view of new trajectory

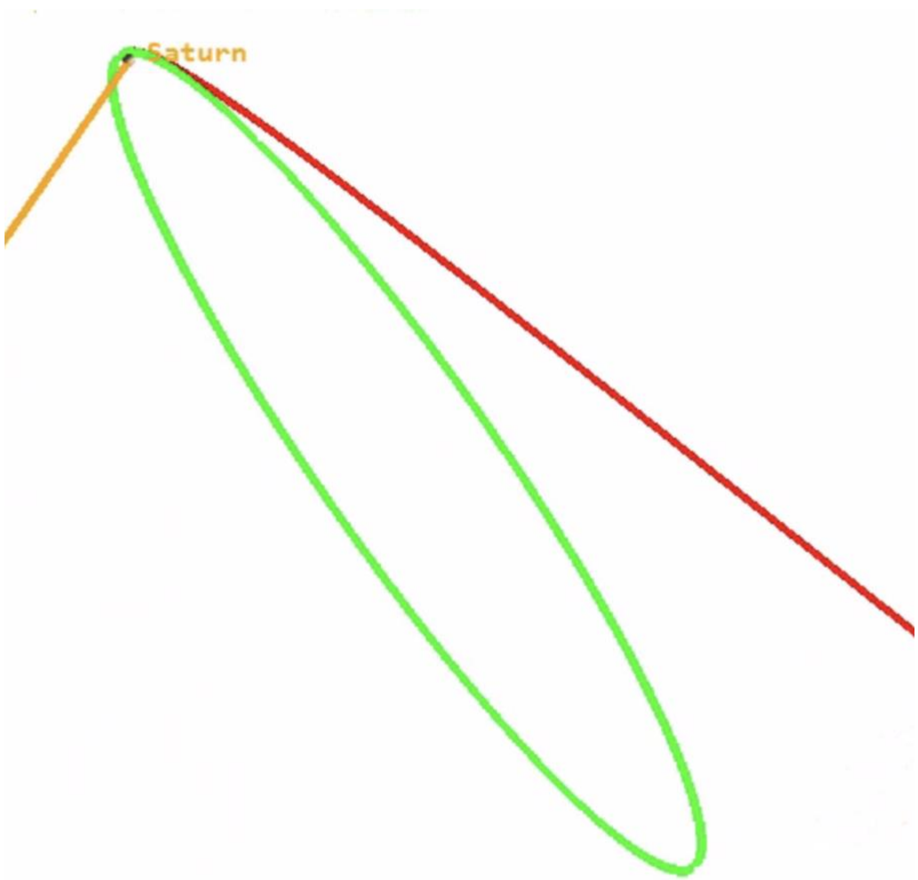

Figure 17 - Saturn centric view with resulting elliptical orbit

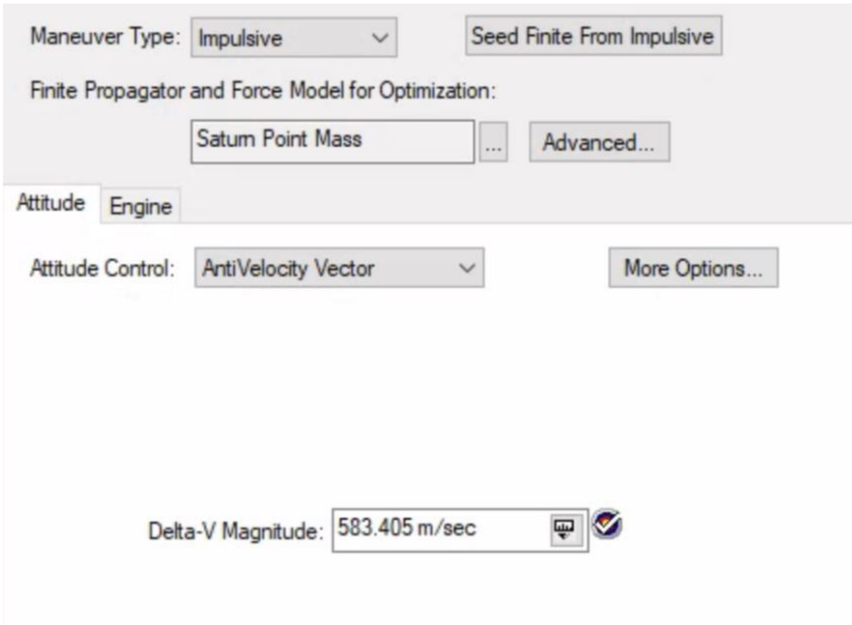

Figure 18 - Resulting maneuver magnitude

```
%Gustavo Grinsteins
%ASEN 5050
%HW6

%House Keeping
clc;
clear;

%% Problem 1
%given
mu_sun = 1.32712428*10^11; %km^3/s^2
mu_earth = 3.986004415*10^5;
mu_mercury = 2.2032*10^4;
AU_to_km = 149597870.7; %km/AU
a_Earth = 1.0000010178*AU_to_km; %km
a_mercury = 0.387098309*AU_to_km; %km
r_mercury = 2439; %km
r_periapsis = 4000; %km

%Calcs
a_t = (1/2)*(a_Earth+a_mercury);
ecc = (a_Earth/a_t)-1;
Period = pi*sqrt(((a_t)^3)/(mu_sun));
R_SOI_Merc = ((mu_mercury/mu_sun)^(2/5))*a_mercury;
v_Merc = sqrt(mu_sun/a_mercury);
v_in = sqrt(((2*mu_sun)/a_mercury)-(mu_sun/a_t));
v_inf_in = v_in - v_Merc;
mech_e_hyp = v_inf_in^2/2;
a_hyp = -(mu_mercury)/(2*mech_e_hyp);
e_hyp = 1-(r_periapsis/a_hyp);
turning_angle = 2*asin(1/e_hyp);
v_out = sqrt(v_Merc^2+v_inf_in^2-2*v_Merc*v_inf_in*cos(pi-turning_angle));

%% Problem 2
mu_saturn = 3.794*10^7;
ra = 2000000;
rp = 700000;
a_titan = 1221830;
a_saturn = (1/2)*(ra+rp);
ecc = (ra/a_saturn)-1;
p = a_saturn*(1-ecc^2);
h = sqrt(mu_saturn*p);
theta_star_intersect = acos((p-a_titan)/(a_titan*ecc));
theta_star_b = acos(-ecc);
%Rotating frame at this theta star
vtheta = (mu_saturn/h)*(1+ecc*cos(theta_star_intersect));
vr = (mu_saturn/h)*ecc*sin(theta_star_intersect);
v_titan = sqrt(mu_saturn/a_titan);
V_in = [vr,vtheta,0];
V_titan = [0,v_titan,0];
v_inf_in = V_in - V_titan;
%Part b
r_pb = 2800;
mu_titan = (6.673*10^-20)*(1.3455*10^23);
mech_e_hypb = (norm(v_inf_in)^2)/2;
a_hypb = -(mu_titan)/(2*mech_e_hypb);
e_hypb = 1-(r_pb/a_hypb);
turning_angleb = 2*asin(1/e_hypb);
```

```
%part c
%Angle between theta hat and vinf
beta = acos((dot(v_inf_in,[0,1,0]))/(norm(v_inf_in)));
v_inf_out = [norm(v_inf_in)*sin(beta-turning_angleb),norm(v_inf_in)*cos(beta-↙
turning_angleb),0];
V_out = V_titan + v_inf_out;
%Part d
%assuming instantaneous change
mech_e_aft = ((norm(V_out)^2)/2) - (mu_saturn/a_titan);
a_aft = (-mu_saturn)/(2*mech_e_aft);
h_aft = a_titan*V_out(2);
ecc_aft = sqrt(1+((2*h_aft^2*mech_e_aft)/(mu_saturn^2)));
p_after = a_aft*(1-ecc_aft^2);
theta_star_after = abs(acos((p_after-a_titan)/(a_titan*ecc_aft)));
if dot(V_out,[1,0,0]) < 0
    theta_star_after = -theta_star_after;
end
%Part e
delta_V_eq = V_out - V_in;

%% Problem 3
mu_jupiter = 1.268*10^8; %km^3/s^2
PeriapsisAlt = 9.8999*10^6 - 71492;
mech_e_hyp = (10.27299)^2/2;
a_hyp = -(mu_jupiter)/(2*mech_e_hyp);
e_hyp = 1-((9.8999*10^6)/a_hyp);
diff = 4.29053*10^6 - 60268;
```