ASEN 5050 – Spaceflight Dynamics
Homework #9

Assigned: Tuesday, April 20, 2021
Due: Thursday, April 29, 2021 at 8.59pm MT
(Since this deadline has been extended to the latest date for all students, you may **not** use one of the two opportunities for an extension to 5pm the next day for this specific homework)

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - Gravitational parameters:
    - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
    - $Gm_{Earth} = 3.986004415 \times 10^5 km^3/s^2$
    - $Gm_{Mars} = 4.305 \times 10^4 km^3/s^2$
  - Semi-major axes relative to the Sun:
    - $a_{Earth} = 1.0000010178\ AU$
    - $a_{Mars} = 1.52367934\ AU$
    - 1 AU = 149,597,870.7 km
  - Body properties:
    - Mars J2: 0.001964
    - Mars equatorial radius: 3397.2 km
    - Mars period of rotation: 1.02595675 days
- See the syllabus for a reminder of the expected components of your working.

A large spacecraft is currently in a circular orbit of radius 10,000 km around the Earth. This spacecraft is also carrying two CubeSats serving as secondary payloads. Let's explore this scenario via Problems 1 and 2.

Problem 1:

The objective of the first CubeSat mission is to verify the performance of a new relative navigation strategy. To achieve this goal, the CubeSat must follow a trajectory that, relative to the primary spacecraft, is bounded over time.

After deployment, the CubeSat is described by an initial relative state vector $[x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]$ using the radial, along-track and cross-track unit vectors as a basis. However, only two components are known: $x_0 = -15\ m,\ \dot{x}_0 = 0$. In addition, the following characteristics of the CubeSat's trajectory relative to the primary spacecraft are known:

- The CubeSat and primary spacecraft lie in the same orbit plane
- The relative trajectory exhibits only bounded, oscillatory motion
- Along the relative trajectory, the maximum absolute value of the deviation in the along-track direction is 45 m.

a) Determine a combination of the remaining initial relative state components that produces a trajectory with these characteristics.
b) Plot the relative trajectory, representing the radial component on the vertical axis and the

along-track component on the horizontal axis. Mark any extrema of *x(t)* and *y(t)* on the plot, along with the mean offset of the relative trajectory.

Problem 2:

The goal of the second CubeSat is to achieve a precise relative position with a relative velocity of zero at a specified time. In this problem, you will design a transfer to deliver the CubeSat to this location via two impulsive maneuvers.

After deployment, at a time $t_0=0$, the relative state of the CubeSat is expressed using the radial, along-track and cross-track unit vectors as:

$$[x_0, y_0, z_0] = [-5\ ,5\ ,0]m \qquad [\dot{x}_0, \dot{y}_0, \dot{z}_0] = [0.04, -0.01, 0.01]m/s$$

The goal is for the CubeSat to reach the following relative position at a time, t_1, equal to one third of the orbit period of the primary spacecraft:

$$[x_1, y_1, z_1] = [5\ ,5\ ,0]m$$

a) To design a transfer between the two relative position vectors at time t_0 and time t_1, find the relative velocity required only the transfer at t_0. Use this relative velocity and the actual velocity of the CubeSat at t_0 to find the magnitude of the impulsive maneuver that must be applied for the CubeSat to begin the designed transfer.
b) Find the relative velocity of the spacecraft at the end of the transfer, i.e., at time t_1. Use this relative velocity to find the magnitude of a second impulsive maneuver that must be applied for the CubeSat to achieve a relative velocity of zero at t_1.
c) Plot the relative trajectory along the transfer, representing the radial component on the vertical axis and the along-track component on the horizontal axis. Add the direction of motion. Mark the initial and final CubeSat locations and the velocities before and after each maneuver.

Problem 3:

Throughout this problem, list any assumptions used in your writeup

a) Calculate the inclination required for a Sun-synchronous orbit around Mars at an altitude of 300 km.
b) Calculate the altitude required for an areostationary orbit around Mars (i.e., an orbit that possesses similar characteristics to a geostationary Earth orbit, but around Mars).
c) Is it possible to design a single orbit around Mars that is both sun-synchronous and areostationary? If so, list the relevant orbital elements that satisfy both requirements. If not, justify why.

Problem 4 (optional, ungraded):

If you wish, please take a moment to reflect on and briefly share what you have learned in this class. E.g., what has been most fascinating, what skill/s have you developed or further refined, what do you expect will be most useful to you beyond this class?