**ASEN 5050 – Spaceflight Dynamics**

**Homework #5**

**Assigned: Tuesday, March 2, 2021**
**Due: Tuesday, March 9, 2021 at 8.59pm MT**

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Mars} = 4.305 \times 10^4 km^3/s^2$
  - $Gm_{Moon} = 4902.799 km^3/s^2$
  - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
  - Equatorial radius of Mars: 3397.2 km
  - Equatorial radius of the Moon: 1738 km
  - Semi-major axis of Earth's heliocentric orbit: 1.0000010178 AU
  - Semi-major axis of Saturn's heliocentric orbit: 9.554909595 AU
  - 1 AU = 149,597,870.7 km
- See the syllabus for a reminder of the expected components of your working.

**Problem 1:**

A spacecraft is currently in a lunar orbit described by the following orbital elements:

$a$ = 8500 km    $e$ = 0.29

At a true anomaly of -21 degrees, an impulsive maneuver is applied.

a) Find the velocity vector at this true anomaly before the maneuver and express it in the $(\hat{r}, \hat{\theta}, \hat{h})$ rotating frame.
b) A maneuver, $\Delta\bar{v} = 0.25\hat{r} - 0.06\hat{\theta}\ km/s$, is then applied. Draw the velocity vector before the maneuver as well as the $\Delta\bar{v}$ vector and the velocity vector after the maneuver. Also add the $\hat{r}$ and $\hat{\theta}$ unit vectors to this diagram. Calculate the velocity vector after the maneuver and express it in the $(\hat{r}, \hat{\theta}, \hat{h})$ rotating frame.
c) After the maneuver, calculate the orbital elements $a$ and $e$ of the new orbit. Also calculate the true anomaly along the new orbit.
d) Calculate the change in the argument of periapsis due to the maneuver.

**Problem 2:**

A spacecraft is currently in orbit around Mars, with the mission requirement that it must always remain above an altitude of 400 km relative to Mars. However, your colleague has discovered that it was not inserted into the correct orbit: the periapsis is currently below this lower bound. An impulsive maneuver must be implemented soon to keep the spacecraft in compliance with the mission requirement!

However, you are provided only the following information to deduce the size and shape of the current orbit as well as the location of the spacecraft at a time $t_1$:

$\mathcal{E}^- = -5.16187 km^2/s^2$    $E_1^- = -1.46057$ rad    $\theta_1^{*-} = -90$ deg where the "–" superscript denotes a quantity in the current orbit, prior to a maneuver, and the "1" subscript indicates that the quantity is evaluated at time $t_1$.

a) At $t_1$, determine the velocity vector of the spacecraft in the $(\hat{r}, \hat{\theta}, \hat{h})$ frame.
b) Consider applying a maneuver at $t_1$ to slightly adjust the velocity vector of the spacecraft and, therefore, change its orbit. The maneuver will be designed to raise the periapsis altitude to 400 km but maintain the same apoapsis radius as the original orbit. What is the magnitude of this maneuver?
c) Rather than apply the maneuver you calculated above, consider another maneuver, described by the following vector: $\Delta\bar{v} = 0.04\hat{r} - 0.002\hat{\theta}\ km/s$. If this maneuver was applied at time $t_1$, would the orbit of the spacecraft possess a periapsis altitude above 400 km as required for mission success?

**Problem 3:**

a) Calculate the total $\Delta v$ (to five significant figures) and time of flight required for a spacecraft to complete a Hohmann transfer in the Sun-spacecraft two-body problem between an approximate circular orbit for Earth and an approximate circular orbit for Saturn. Assume the semi-major axes of each orbit is equal to the value provided.
b) Calculate the initial relative phase angle (between the spacecraft and Saturn) that is required for the spacecraft to rendezvous with Saturn after completing the Hohmann transfer.
c) Assuming an intermediate radius ($r_B$) of 11 AU, calculate the total $\Delta v$ (to five significant figures) and time of flight required for a bi-elliptic transfer between Earth's and Saturn's approximate orbits.
d) Compare the total $\Delta v$ and time of flight of this bi-elliptic transfer with the total $\Delta v$ and time of flight of the Hohmann transfer calculated in part a). Justify why this comparison is consistent with your expectations.
e) Do you think the total $\Delta v$ values you computed in 3a) and 3c) are large? Justify.

**Problem 4:**

The goal of this short problem is to learn a new feature of the modeling software we use: modifying stopping conditions for numerical integration, enabling customization of the mission scenario. We will find this particularly useful when we create complex transfers next time!

**Follow the instructions for GMAT/STK, available on the Canvas page in the HW 5 module and answer the following questions when indicated.**

a) At the initial state, use the reported values to record the magnitude of the specific angular momentum, specific energy, and eccentric anomaly. Also record the corresponding date in UTC modified Julian date format (recall that the modified Julian date represents the days past a reference epoch) from the initial condition input panel. Is the value of the specific energy consistent with the type of conic followed by the spacecraft?

b) Integrate the specified initial condition until an altitude of 500 km. At this altitude, use the report function to list the true anomaly, eccentric anomaly – and, as a quick check of your stopping condition, the altitude. Use the epoch at this altitude to determine the time of flight along the propagated trajectory segment.

c) Use the time of flight you calculated in part b) and orbit geometry to predict the time the spacecraft spends in its orbit below an altitude of 500 km. Use the eccentric anomaly at the 500 km altitude, from part b), as well as the orbital elements, to verify this flight time on your own by using Kepler's equations.

d) Take two snapshots of your orbit in three-dimensions: one looking down on the orbital plane, and one alternate useful view. Take a snapshot of the two-dimensional view of the ground-track of the spacecraft (you may need to update the orbit colors for clarity in this 2D view). The ground-track essentially describes the projection of the spacecraft's location onto the surface. Go to https://marstrek.jpl.nasa.gov which functions a lot like Google Maps for Mars, and locates named regions on Mars' surface. Assume that one of the sensors onboard the spacecraft can only take observational data at altitudes below 500 km. Refer back to the orbit you modeled in GMAT or STK – during the segment of the orbit that the spacecraft remains below an altitude of 500km, visually identify a region of the Mars surface that the spacecraft may be able to observe.