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Coordination of mechanical systems

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Outline:

- Introduction
- Mutual synchronization controller
- Convergence properties
- Experiments
- Conclusions
- Future extensions

Introduction

Objective

Two or more mutually synchronized robot manipulators

Restrictions

Only position measurements

Motivation

- Synchronization tasks :
  - mobile platforms (transportation, walking robots),
  - object manipulation (manufacturing industry),
- Velocity sensor equipment
- Accessibility on the robot architecture

History

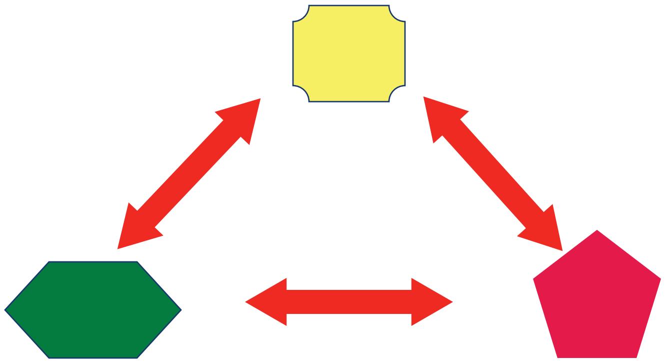
- Huygens (1673): pendulum clocks linked via (flexible) beam
- Rayleigh (1877): nearby organ tubes, tuning forks
- B. van der Pol (1920): electrical-mechanical systems

Definition

- Time conformity
- Certain relations between functionals and/or variables

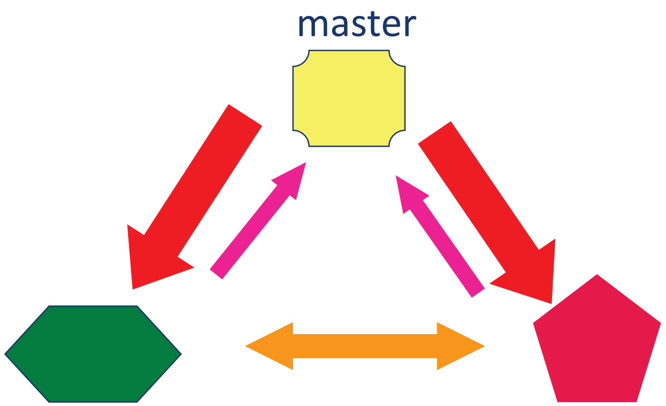
Dutch synchronization

Internal (mutual) synchronization



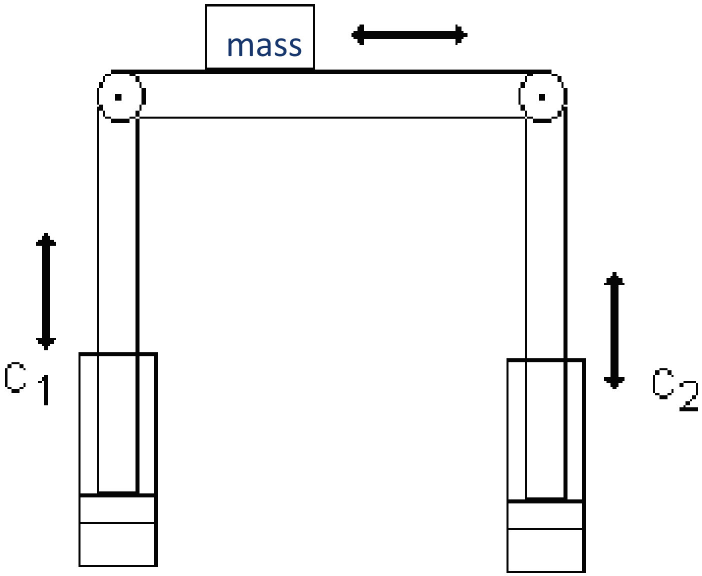
- All objects appear at equal terms
- Synchronous motion as result of interaction/coupling

External synchronization



- One object is more powerful (master)
- Synchronous motion is determined by the master


Herz-Und Kreislaufzentrum Dresden
Dresden, Germany

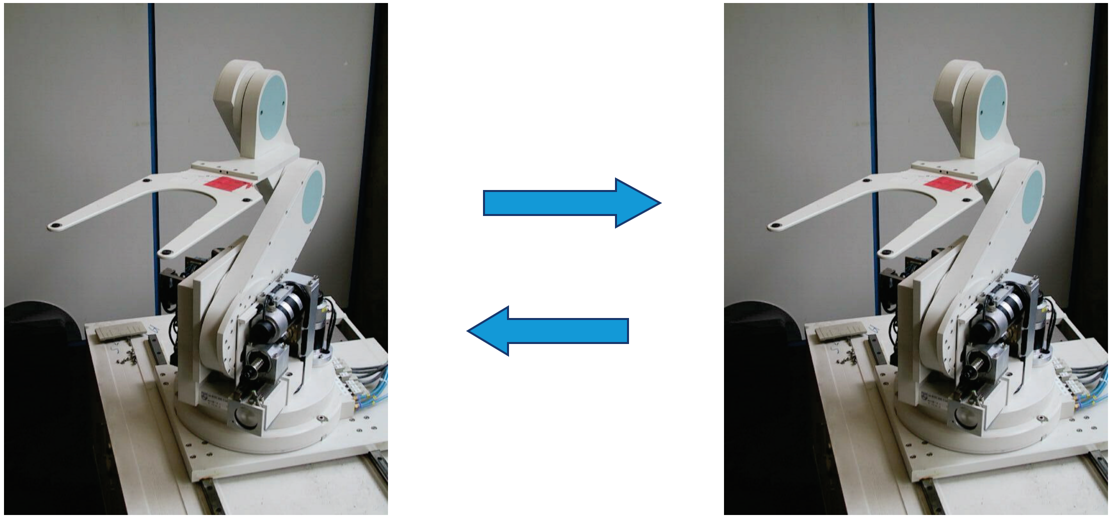


Hydraulic platform

Remote surgery

General setup

- n actuated rigid joints
- All joints are revolute



General assumptions

- Only joint position measurements
- Dynamic model and physical parameters are known for all robots
- Desired joint positions, velocities and accelerations are bounded

Synchronization index and functional

$$J_i(q_i,\dot{q}_i)=[q_i^T \quad \dot{q}_i^T]$$

$$f_{i,j}=\| \ J_i(q_i,\dot{q}_i)-J_j(q_j,\dot{q}_j) \ \|, \qquad i,j=1,\dots,p, \qquad j\neq i,$$

$$f_{i,i}=\| \ J_i(q_i,\dot{q}_i)-J_d(q_d,\dot{q}_d) \ \|, \qquad i=1,\dots,p$$

Mutual synchronization controller

Rigid joint robot dynamics

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \qquad i = 1,\dots,p$$

Ideal feedback control law

$$\tau_i = M_i(q_i)\ddot{q}_{ri} + C_i(q_i,\dot{q}_i)\dot{q}_{ri} + g_i(q_i) - K_{d,i}\dot{s}_i - K_{p,i}s_i$$

Synchronization errors

$$s_i = q_i - q_{ri}, \quad \dot{s}_i = \dot{q}_i - \dot{q}_{ri}$$

$$e_{i,i} = q_i - q_d, \qquad e_{i,j} = q_i - q_j$$

Nominal reference trajectories

$$q_{ri} = q_d - \sum_{j=1, j\neq i}^{p} K_{i,j}(q_i - q_j); \qquad \dot{q}_{ri} = \dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{i,j}(\dot{q}_i - \dot{q}_j)$$

Feedback control law with estimated variables

$$\tau_i = M_i(q_i)\hat{\ddot{q}}_{ri} + C_i(q_i,\hat{\dot{q}}_i)\hat{\dot{q}}_{ri} + g_i(q_i) - K_{d,i}\hat{\dot{s}}_i - K_{p,i}s_i$$

Synchronization errors

$$s_i = q_i - q_{ri}, \quad \hat{\dot{s}}_i = \hat{\dot{q}}_i - \hat{\dot{q}}_{ri}$$

$$e_{i,i} = q_i - q_d, \qquad e_{i,j} = q_i - q_j$$

Nominal reference trajectories

$$q_{ri} = q_d - \sum_{j=1, j\neq i}^{p} K_{i,j}(q_i - q_j); \qquad \hat{\dot{q}}_{ri} = \dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{i,j}(\hat{\dot{q}}_i - \hat{\dot{q}}_j)$$

Observer for slave joint variables

$$\frac{d}{dt}\hat{q}_i = \hat{\dot{q}}_i \qquad\qquad + \mu_{i,1}\tilde{q}_i$$

$$\frac{d}{dt}\hat{\dot{q}}_i = -M_i(q_i)^{-1}\left(C(q_i,\hat{\dot{q}}_i)\hat{\dot{q}}_i + g_i(q_i) - \tau_i \right) \qquad + \mu_{i,2}\tilde{q}_i$$

Estimation joint errors

$$\tilde{q}_i := q_i - \hat{q}_i, \quad \tilde{\dot{q}}_i := \dot{q}_i - \hat{\dot{q}}_i$$

Seemingly problem: Algebraic loop !!!

Algebraic loop

$$\frac{d}{dt}\dot{\hat{q}}_i = -M_i(q_i)^{-1}\left(C(q_i,\dot{\hat{q}}_i)\dot{\hat{q}}_i + g_i(q_i) - \tau_i \right) + \mu_{i,2}\,\tilde{q}_i$$

$$\frac{d}{dt}\dot{\hat{q}}_i = -\sum_{j=1,j\neq i}^{p} K_{i,j}\left(\frac{d}{dt}\dot{\hat{q}}_i - \frac{d}{dt}\dot{\hat{q}}_j\right) + \underbrace{\ddot{q}_d - M_i(q_i)^{-1}\left(C(q_i,\dot{\hat{q}}_i)\dot{\hat{s}}_i + K_{d,i}\dot{\hat{s}}_i + K_{p,i}s_i \right) + \mu_{i,2}\,\tilde{q}_i}_{y_i(\ddot{q}_d,\, s_i,\, \dot{\hat{s}}_i,\, q_i,\, \dot{\hat{q}}_i)}$$

For $i = 1, \ldots, p$

$$\left(I_n + \sum_{j=1, j\neq i}^{p} K_{i,j}\right)\frac{d}{dt}\dot{\hat{q}}_i - \sum_{j=1, j\neq i}^{p} K_{i,j}\frac{d}{dt}\dot{\hat{q}}_j = y_i(\ddot{q}_d, s_i, \dot{\hat{s}}_i, q_i, \dot{\hat{q}}_i)$$

Such that

$$\underbrace{\begin{bmatrix} I_n + \sum_{j=1, j\neq 1}^{p} K_{1,j} & -K_{1,2} & \cdots & -K_{1,p} \\ -K_{2,1} & I_n + \sum_{j=1, j\neq 2}^{p} K_{2,j} & \cdots & -K_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ -K_{p,1} & -K_{p,2} & \cdots & I_n + \sum_{j=1, j\neq p}^{p} K_{p,j} \end{bmatrix}}_{M_c(K_{i,j})} \begin{bmatrix} \frac{d}{dt}\dot{\hat{q}}_1 \\ \frac{d}{dt}\dot{\hat{q}}_2 \\ \vdots \\ \frac{d}{dt}\dot{\hat{q}}_p \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}$$

Nonsingular for any $K_{i,j} \geq 0$!

Main result

There exist conditions on the minimum eigenvalues of the control gains $K_{p,i}$, $K_{d,i}$ and the observer gains $\mu_{i,1}$, $\mu_{i,2}$ such that

$$s_i \rightarrow 0, \quad \dot{s}_i \rightarrow 0, \quad \tilde{q}_i \rightarrow 0, \quad \dot{\tilde{q}}_i \rightarrow 0$$

semi-globally exponentially.

Thus, the robots are semi-globally exponentially synchronized since for $i = 1, \ldots, p$, $q_i \rightarrow q_j$ and $\dot{q}_i \rightarrow \dot{q}_j$ exponentially for any initial condition in the region of convergence.

- Convergence of $s_i, \dot{s}_i, \tilde{q}_i, \dot{\tilde{q}}_i$

$$V = \frac{1}{2}\sum_{i=1}^{p}\left(\dot{s}_i^{\,T} M_i(q_i)\dot{s}_i + s_i^{\,T} K_{p,i} s_i\right) + \frac{1}{2}\sum_{i=1}^{p}\begin{bmatrix}\dot{\tilde{q}}_i^{\,T} & \tilde{q}_i^T\end{bmatrix}\begin{bmatrix} M_i(q_i) & \eta_i(\tilde{q}_i) I_n \\ \eta_i(\tilde{q}_i) I_n & \mu_{i,2} + \beta_i I_n \end{bmatrix}\begin{bmatrix}\dot{\tilde{q}}_i \\ \tilde{q}_i\end{bmatrix}$$

$$\eta_i(\tilde{q}_i) = \frac{\eta_o}{1 + \| \tilde{q}_i \|}$$

$$\beta_i = \eta_0 \mu_{i,1} + 2V_M C_{i,M}(\mu_{i,1} + \eta_0 M_{i,m}^{-1}) - \mu_{i,2}(1 - M_{i,m})$$

Convergence of $s_i, \dot{s}_i$ imply $q_i \to q_j$ and $\dot{q}_i \to \dot{q}_j$!

$s_i \rightarrow 0$ implies in the limit $t \rightarrow \infty$ that

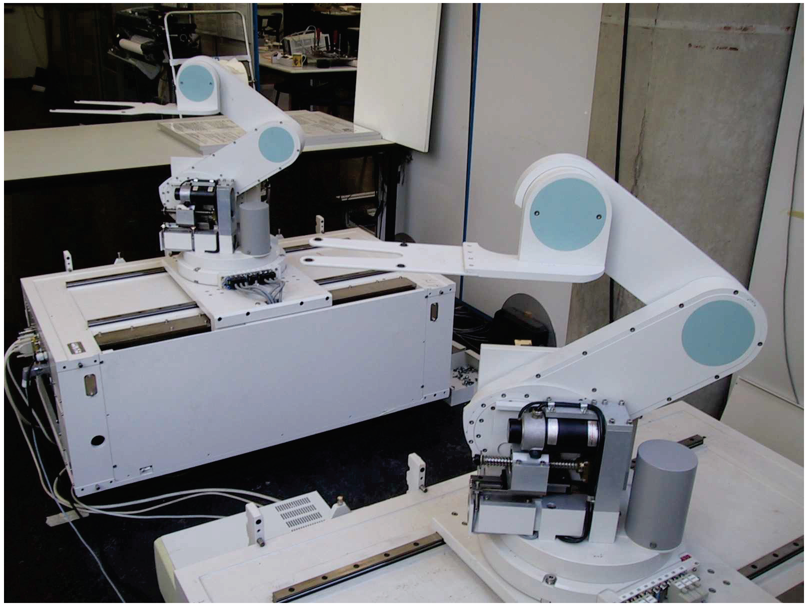
$$e_{i,i} = q_i - q_d$$

$$e_{i,j} = q_i - q_j$$

$$\begin{bmatrix} s_1 \\ \vdots \\ s_p \end{bmatrix} = \begin{bmatrix} e_{1,1} + \sum_{j=1, j\neq 1}^{p} K_{1,j} e_{1,j} \\ \vdots \\ e_{p,p} + \sum_{j=1, j\neq p}^{p} K_{p,j} e_{p,j} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} I_n + \sum_{j=1,j\neq 1}^{p} K_{1,j} & -K_{1,2} & \cdots & -K_{1,p} \\ -K_{2,1} & I_n + \sum_{j=1,j\neq 2}^{p} K_{2,j} & \cdots & -K_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ -K_{p,1} & -K_{p,2} & \cdots & I_n + \sum_{j=1,j\neq p}^{p} K_{p,j} \end{bmatrix}}_{M_c(K_{i,j})} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ q_d \\ \vdots \\ q_d \end{bmatrix}$$

Two CFT transposer robots



- 4 degrees of freedom (dof)
- sampling frequency: 2 kHz
- encoders: 2000 PPR

Robot dynamics + friction effects

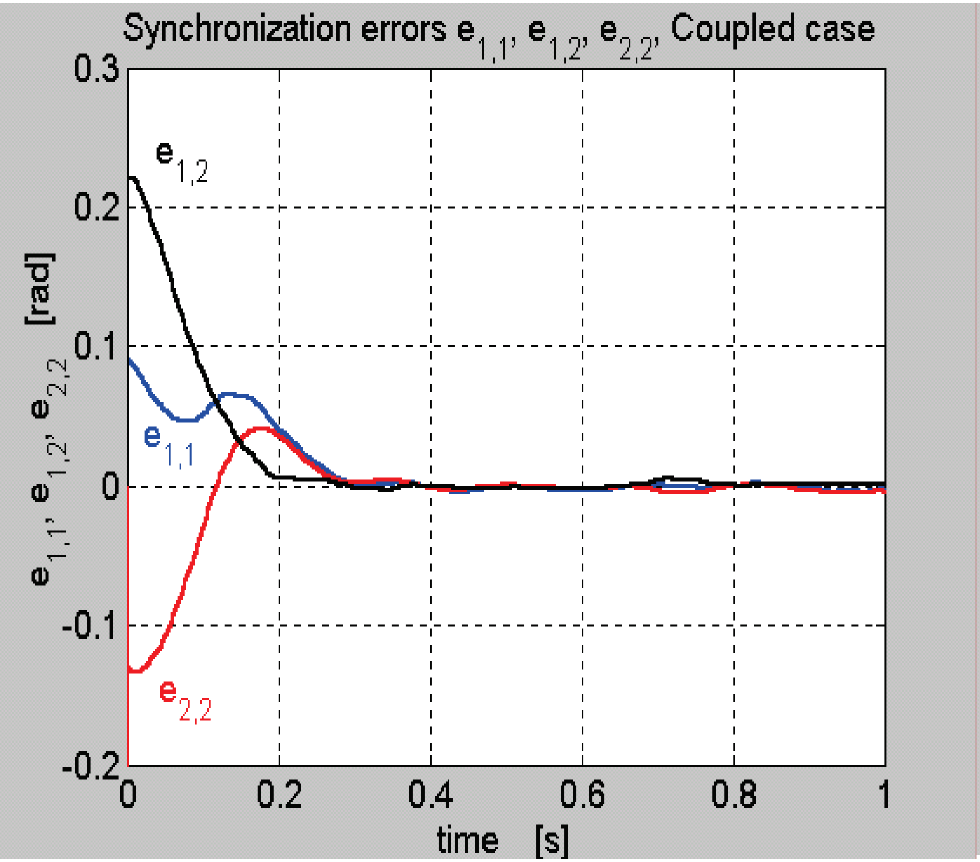
$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) + \tau_f(\dot{q}_i) = \tau_i \qquad i = 1,\dots,p$$

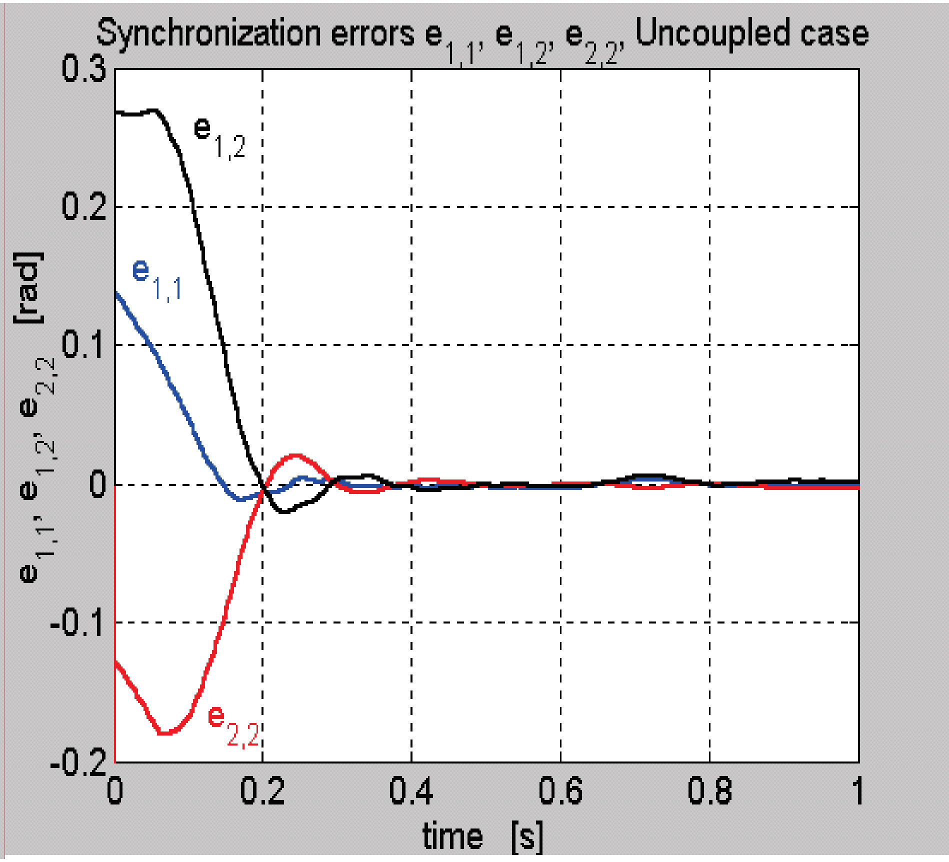
$$\tau_f(\dot{q}_i) = B_v\,\dot{q}_i + B_{f1,i}\left(1 - \frac{2}{1+e^{2w_{1,i}\dot{q}_i}}\right) + B_{f2,i}\left(1 - \frac{2}{1+e^{2w_{2,i}\dot{q}_i}}\right)$$

Feedback control law with estimated variables

$$\tau_i = M_i(q_i)\hat{\ddot{q}}_{ri} + C_i(q_i,\hat{\dot{q}}_i)\hat{\dot{q}}_{ri} + g_i(q_i) + \tau_f(\hat{\dot{q}}_i) - K_{d,i}\,\hat{\dot{s}}_i - K_{p,i}s_i$$

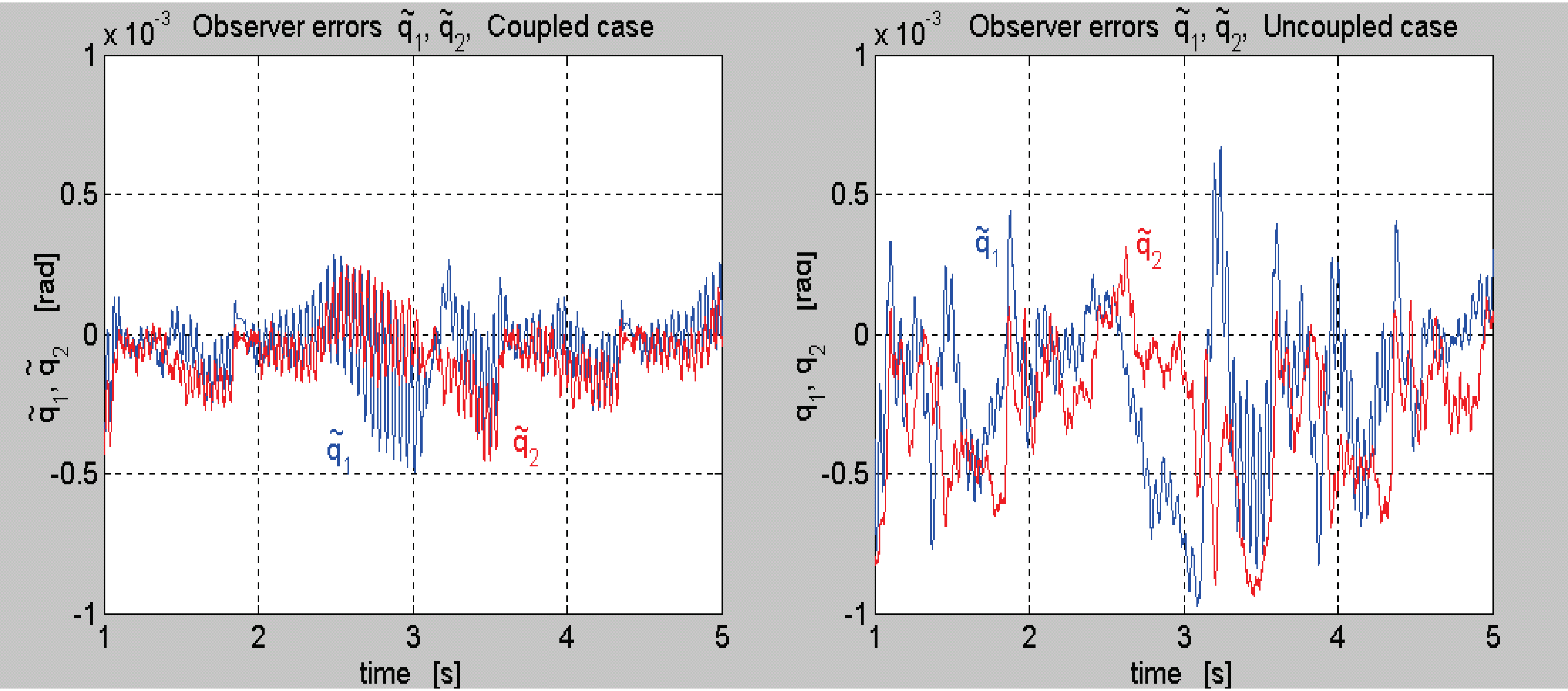
Synchronization errors





$$e_{1,1} = q_1 - q_d, \qquad e_{1,2} = q_1 - q_2, \qquad e_{2,2} = q_2 - q_d$$

Observer errors



$$\tilde{q}_1 = q_1 - \hat{q}_1, \qquad \tilde{q}_2 = q_2 - \hat{q}_2$$

Conclusions

- Semi-global exponential mutual synchronization
- Robustness against noise measurements
- Robustness against disturbances

Future extensions

- Different nominal references:
  - ✍ partial synchronization
- Other mechanical systems:
  - ✍ mobile systems
  - ✍ satellite formations