Active Disturbance Rejection Control

An Overview of a newer control technique to improve PID deficiencies

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This technique was developed by Jingqing Han (deceased) with further support by Zhiqiang Gao at Cleveland State University, see [1]. The technique aims to overcome the following four deficiencies of the PID, which is the dominant technique used in industry: (1) control effort for step set-points; (2) D-part noise sensitivity; (3) limited flexibility due to linear weight of P, I and D terms and (4) integral term with associated stability and saturation problems.

1 PID revisited

This is a simple error based control feedback scheme (the set input is compared to the reference and the difference or error is used to drive the control signal), as shown in the following figure.

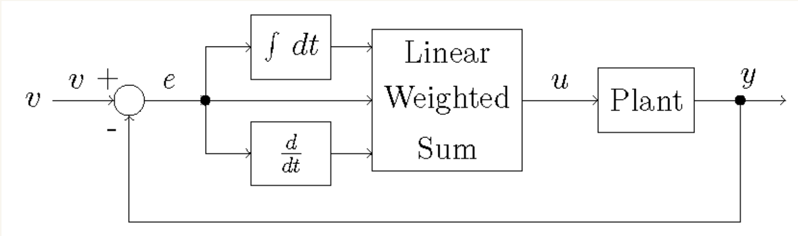


Figure 1: PID Block Diagram — the error signal is $e = v - y$

The integral term improves the tracking (smaller error) but de-stabilizes the system - hence a smaller proportional gain resulting in a lower bandwidth is the result. Similarly the derivative term increases the phase margin, resulting in better step response but lowers the low-frequency loop gain.

2 Suggested Improvements

2.1 Setpoint Steps — Generate Transient Profile

This aspect has been studied in detail and a number of standard techniques are available — most originating from the simple linear position-velocity-acceleration relationships. The profile generators we are interested in falls into the open-loop class (actual feedback position is not used), and the profile is generated either using cascaded integrators or a smooth polynomial function.

S-curve or trapezoidal profiles can be easily generated using cascaded integrators with a timed block input.

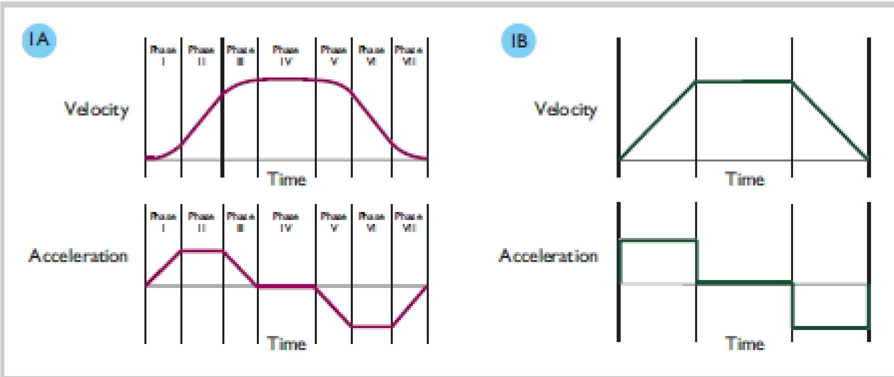


Figure 2: S-curve and Trapeziodal profiles

The equation set for a continuous trapezoidal profile would be

$$p = p_0 + v_0 t + \frac{1}{2} a t^2$$
$$v = v_0 + at$$

To use these equations, the duration of the acceleration and deceleration is pre-determined from the step size.

The fastest trajectory (limited by a hard limit on the energy/force) is obtained by using the time-optimal solution. Consider the following double-integrator plant:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

The solution for the continuous case with the command limited to $|u| \leq r$ is

$$u = -r \,\mathrm{sign}\left(x_1 - v + \frac{x_2 |x_2|}{2r}\right)$$

A suitable continuous profile generator could be generated using

$$\dot{v}_1 = v_2$$
$$\dot{v}_2 = -r \,\mathrm{sign}\left(v_1 - v + \frac{v_2 |v_2|}{2r}\right)$$

The discrete equation set is significantly more complex and will be shown later

2.2 Tracking Differentiator (Lower Noise)

Less noise sensitive solutions such as replacing the differentiator (typically approximated by $y(t) = \frac{s}{\tau s+1} v(t)$) by the following type of functions can be used

$$\dot{v}(t) \approx \frac{v(t-\tau_1) - v(t-\tau_2)}{\tau_2 - \tau_1}, \quad \tau_2 > \tau_1 > 0$$

The time-optimal solution in the previous section is an even better solution here

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -r\,\mathrm{sign}\left(x_1 - v + \frac{x_2\,|x_2|}{2r}\right) \end{aligned}$$

2.3 Non-linear Feedback

It is well known that the bang-bang control can give zero steady-state error without an integrator.

Consider the function $fal(e, \alpha, \delta)$ with $\delta > 0$ and $1 > \alpha > 0$

$$fal(e, \alpha, \delta) = \begin{cases} \frac{e}{\delta^{1-\alpha}}, & |e| \leq \delta, \alpha < 1 \\ |e|^{\alpha}\mathrm{sign}(e), & |e| > \delta, \alpha < 1 \end{cases}$$

This function provides high gain for small error signals and a lower gain for large error signals. In the extreme case ($\alpha = 0$), the control law reverts to bang-bang.

2.4 Disturbance Rejection using an Extended State Observer (ESO)

We want to estimate and reject the disturbance from both external sources as well as from model errors. This can be formulated as follows:

Assume that the continuous SISO second-order system can be described by

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x_1, x_2, w(t), t) + bu \\
y &= x_1
\end{aligned}
$$

If $F(t) = f(x_1, x_2, w(t), t)$ is not precisely known, then we would design a feedback control law to overcome any unknown part of/ or disturbance $F(t)$. To model this, treat $x_3 = F(t)$ as an additional state and let $\dot{x}_3 = G(t)$. The system equations become

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + bu \\
\dot{x}_3 &= G(t) \\
y &= x_1
\end{aligned}
$$

These equations are in the standard control canonical form and is therefore observable. We can now build a extended state observer (ESO) to estimate the states $x_1, \ldots, x_3$. The order is one higher than the plant order, to include the estimated disturbance model.

Examples of a linear and a non-linear observers are shown below.

Linear	*Non-Linear*
$e = y - z_1$	$e = z_1 - y$
	$fe = fal(e, 0.5, \delta), \quad fe_1 = fal(e, 0.25, \delta)$
$\dot{z}_1 = z_2 - L_1 e$	$\dot{z}_1 = z_2 - \beta_{01} e$
$\dot{z}_2 = z_3 - L_2 e + bu$	$\dot{z}_2 = z_3 - \beta_{02} fe + bu$
$\dot{z}_3 = -L_3 e$	$\dot{z}_3 = -\beta_{03} fe_1$

The observer states z_1, z_2 track control states x_1, x_2 respectively, and the additional state z_3 is now an estimate of the unmodeled total disturbance.

The choice of observer gains is left to the designer, and if noise is not a problem, fast observer pole positions may be used in the ESO design.

3 Active Disturbance Rejection Control (ADRC)

3.1 Control Structure

If we add the improvements on the PID discussed earlier, we get the proposed Active Disturbance Rejection Control block diagram as shown below

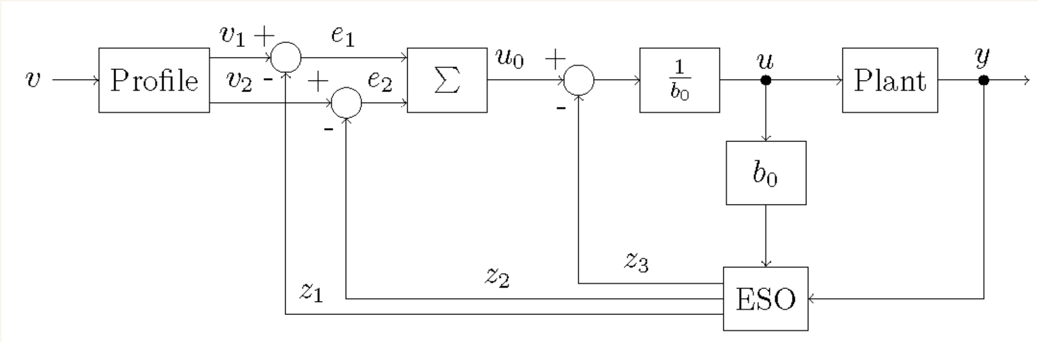


Figure 3: Active Disturbance Rejection Control Block Diagram

The control feedback resembles a PD control structure with the derivative term provided by the Profile block (input) and the ESO (feedback). The state z_3 tracks the disturbance. This disturbance correction may include tracking errors as well as modelling errors.

3.2 Disturbance Rejection Control Formulation

Transform the control law from $u = \ldots$ to $u_0 = \ldots$ as shown in the previous block diagram. This implies that

$$u = \frac{1}{b_0}\left(u_0 - x_3\right) = \frac{1}{b_0}\left(u_0 - F(t)\right)$$

If we apply this transformation to our state equations, and if $b_0 \approx b$ we get

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + b\frac{1}{b_0}\left(u_0 - x_3\right) \approx u_0 \\
y &= x_1
\end{aligned}$$

with the state x_3 hidden from the control law. The problem was now transformed to a cascade integral form that can be easily controlled using a PD controller for a zero steady-state error.

3.3 ESO

Using the ESO described earlier and setting the ESO bandwidth much faster than the plant bandwidth, will ensure that the estimated states z_1, z_2, z_3 track the two PD control states x_1, x_2 accurately as well as the disturbance state x_3.

What makes this formulation change so substantial is that by moving the error (disturbance) estimation to the observer, we have now made the tracking of this error faster and more aggressive (the normal PI integrator zero is typically more than 4$\times$ slower than the crossing frequency and the observer poles again at least 3$\times$ faster than the crossing frequency). We can expect faster tracking of the error dynamics after initialisation.

3.4 Continuous Example

To show how it works, consider the following example:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \omega_n = 1.6;\ \zeta = 0.6$$

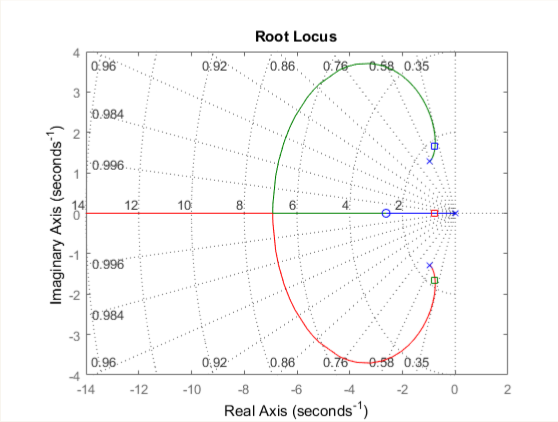
The plant pole locations is not ideal for a straightforward PID controller (on the Root-locus the poles will move to faster locations with less damping).

3.4.1 PID Design Approach

Using the Matlab pidtool with tuning parameters: response time = 1.66 and transient behaviour =0.75 results in the following controller

$$D(s) = 0.745\,\frac{(1 + 0.32s)^2}{s}$$

The root-locus plot for the loop transfer function $DG(s)$ is now



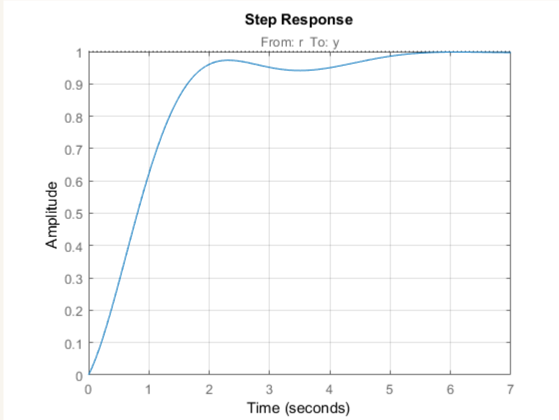


Figure 4: Root-locus and step response for example 1 PID controller

3.4.2 PD Design Approach

A PD controller approach with $D(s) = 0.66(s+1.8)$ have the following root-locus plot and step response. The root-locus plot for the loop transfer function $DG(s)$ is now

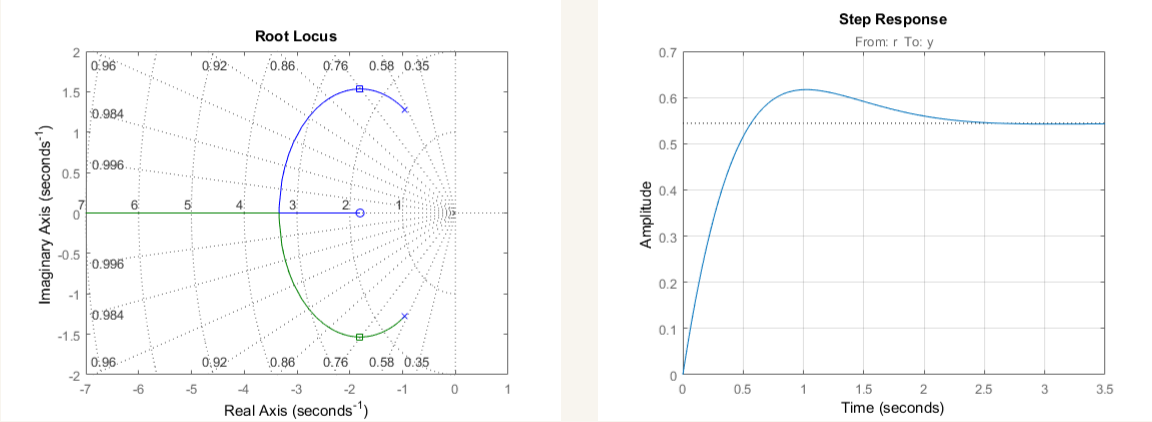
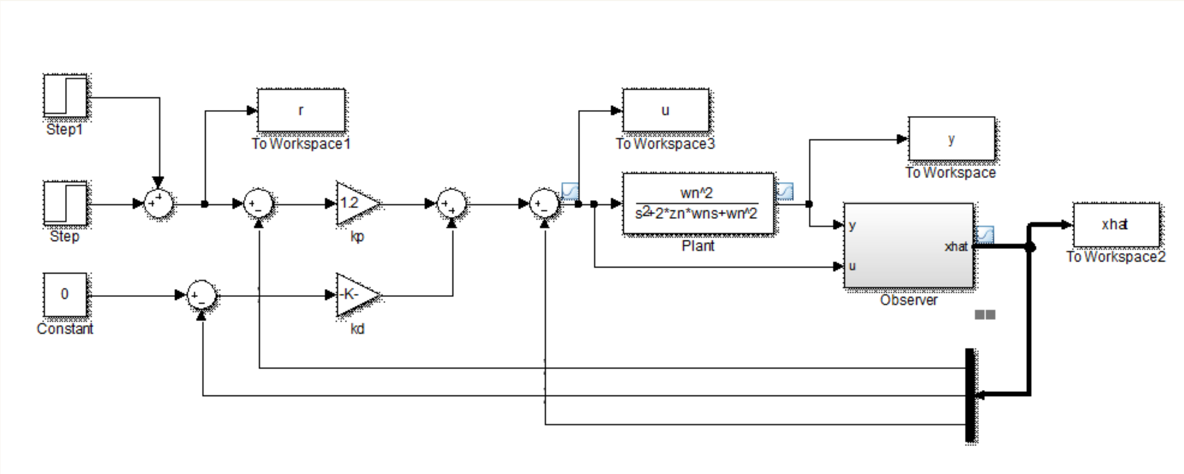


Figure 5: Root-locus and step response for example 1 PD controller

The PD controller has a fast response, but the steady-state tracking error is not really acceptable.

3.4.3 ADRC Design Approach

Let us base the ADRC control feedback on the previous PD controller. Design the ESO to have 3 poles at $s = -5$. The Simulink block diagram and step response is shown on the next page.



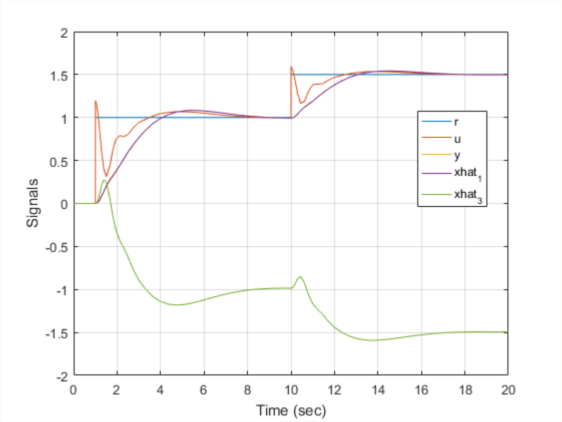


Figure 6: Simulation and step response for example 1 ADRC controller

4 Discrete Solutions

4.1 Discrete Time-Optimal Solution

Using the continuous Time-optimal solution as discussed earlier directly will lead to too much chattering if used directly. Consider the discrete-time solution for the discrete double integrator plant

$$\begin{aligned} v_1(k+1) &= v_1(k) + Tv_2(k) \\ v_2(k+1) &= v_2(k) + Tu(k), \quad |u(| \leq r \end{aligned}$$

The solution as given in [2] with sampling period T consists of

$$u = -rsat\,(a(x_1, x_2, r, T), Tr)\,, \quad y = x_1 + Tx_2, \quad |y| \geq T^2 r$$

with

$$a(x_1, x_2, r, T) = \begin{cases} x_2 + \frac{1}{2}\left(\sqrt{T^2r^2 - 8ry\,\mathrm{sign}(x_2) - Tr}\right)\mathrm{sign}(y), & |y| > T^2 r \\ x_2 + y/T, & |y| \leq T^2 r \end{cases}$$

The following computer coding is proposed: $u = fhan(x_1, x_2, r, T)$ with

$$
\begin{aligned}
d &= rT, \quad d_0 = Td \\
y &= x_1 + Tx_2 \\
a_0 &= \sqrt{d^2 + r|y|} \\
a &= \begin{cases} x_2 + \frac{a_0 - d}{2}\operatorname{sign}(y) & |y| > d_0 \\ x_2 + y/T, & |y| \leq d_0 \end{cases} \\
fhan &= \begin{cases} r\operatorname{sign}(a) & |a| > d_0 \\ r\frac{a}{d}, & |a| \leq d_0 \end{cases}
\end{aligned}
$$

This solution has the interesting and very useful property that unlike the continuous solution which always result in bang-bang operation around the origin, this solution contains a linear region (no switching), or a much smaller chattering amplitude. This is illustrated by the following example (small remaining chattering amplitude in the control signal).

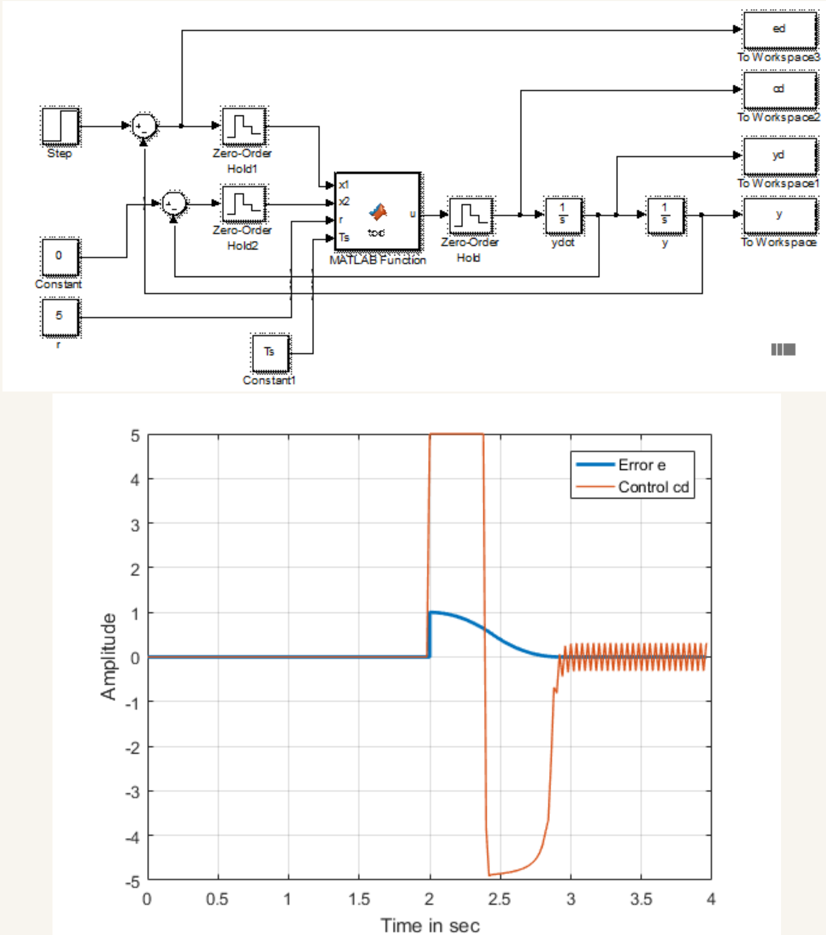


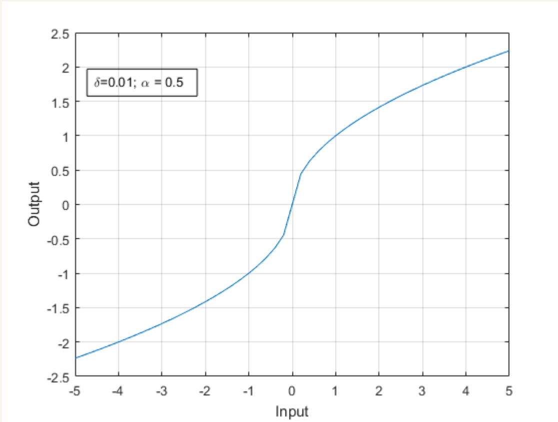
Figure 7: Discrete Time Optimal Control (DTOC) Example

4.2 Discrete Extended State Observer

The observer will typically have a higher bandwidth than the controller, so we do have some freedom to place the observer poles in very stable positions.

The proposed non-linear functions in [1] for the ESO (prediction type), called `fal(...)` is given by the following equation and typical transfer function

$$\mathsf{fal}(e, \alpha, \delta) = \begin{cases} \frac{e}{\delta^{1-\alpha}}, & |e| \le \delta \\ |e|^{\alpha}\mathsf{sign}(e), & |e| > \delta \end{cases}$$



The advantage is the potential of lower overshoot for large step inputs (lower gain for large errors).

In summary, the discrete version of the linear and non-linear type of observer (with proposed gains) is listed in the following table

Linear	Non-Linear
$e(k) = y(k) - z_1(k)$	$e = z_1(k) - y(k)$
	$fe(k) = fal(e, 0.5, \delta), \quad fe_1(k) = fal(e, 0.25, \delta)$
$z_1(k+1) = z_1(k) + Tz_2(k) - \beta_{01}e(k)$	$z_1(k+1) = z_1(k) + Tz_2(k) - \beta_{01}e(k)$
$z_2(k+1) = z_2(k) + T(z_3(k) + bu(k)) - \beta_{02}e(k)$	$z_2(k+1) = z_2(k) + T(z_3(k) + bu(k)) - \beta_{02}fe(k)$
$z_3(k+1) = z_3(k) - \beta_{03}e(k)$	$z_3(k+1) = z_3(k) - \beta_{03}fe_1(k)$
$\beta_{01} \leq 1,\ \beta_{02} \leq \frac{1}{3T},\ \beta_{03} \leq \frac{1}{32T^2}$	$\beta_{01} \leq 1,\ \beta_{02} \leq \frac{1}{2T^{0.5}},\ \beta_{03} \leq \frac{2}{25T^{1.2}}$

Another option is to use a current observer. For the second-order version (applied to a first-order plant)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \rightarrow \quad A_d = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} bT \\ 0 \end{bmatrix}$$

If we place all the ESO poles in a single location $s = -a_{\text{ESO}}$ (discrete location $z = e^{-a_{\text{ESO}}T}$), then the current observer gain are at

$$L_1 = 1 - \left(e^{-a_{\text{ESO}}T}\right)^2, \quad L_2 = \frac{1}{T}\left(1 - e^{-a_{\text{ESO}}T}\right)^2$$

Similarly for the third-order ESO (applied to the second-order plant)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad \rightarrow \quad A_d = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} bT^2/2 \\ bT \\ 0 \end{bmatrix}$$

and the current observer gains are

$$L_1 = 1 - \left(e^{-a_{ESO}T}\right)^3, \quad L_2 = \frac{3}{2T}\left(1 - e^{-a_{ESO}T}\right)^2\left(1 + e^{-a_{ESO}T}\right), \quad L_3 = \frac{1}{T^2}\left(1 - e^{-a_{ESO}T}\right)^3$$

4.3 Complete Non-Linear Discrete Example

Using the same example as before, but implementing all the non-linear elements, the proposed controller with the profile generator, controller and ESO equations are now

```
fv = fhan(v - v1, v2, r0, Ts);   % Profile Generator
v1 = v1 + Ts*v2;
v2 = v2 + Ts*fv;

e  = z1 - y;                     % ESO
fe = fal(e, 0.5, Ts);  fe1 = fal(e, 0.25, Ts);
z1 = z1 + Ts*z2 - L1*e;
z2 = z2 + Ts*(z3 + B0*u) - L2*fe;
z3 = z3 - L3*fe1;

e1 = v1 - z1; e2 = v2- z2;      % Controller
u0 = fhan(e1, c*e2, r, d*Ts);
u  = (u0 - z3)/B0;
```

A top-level Simulink model and some of the sub-blocks is shown in the following figure

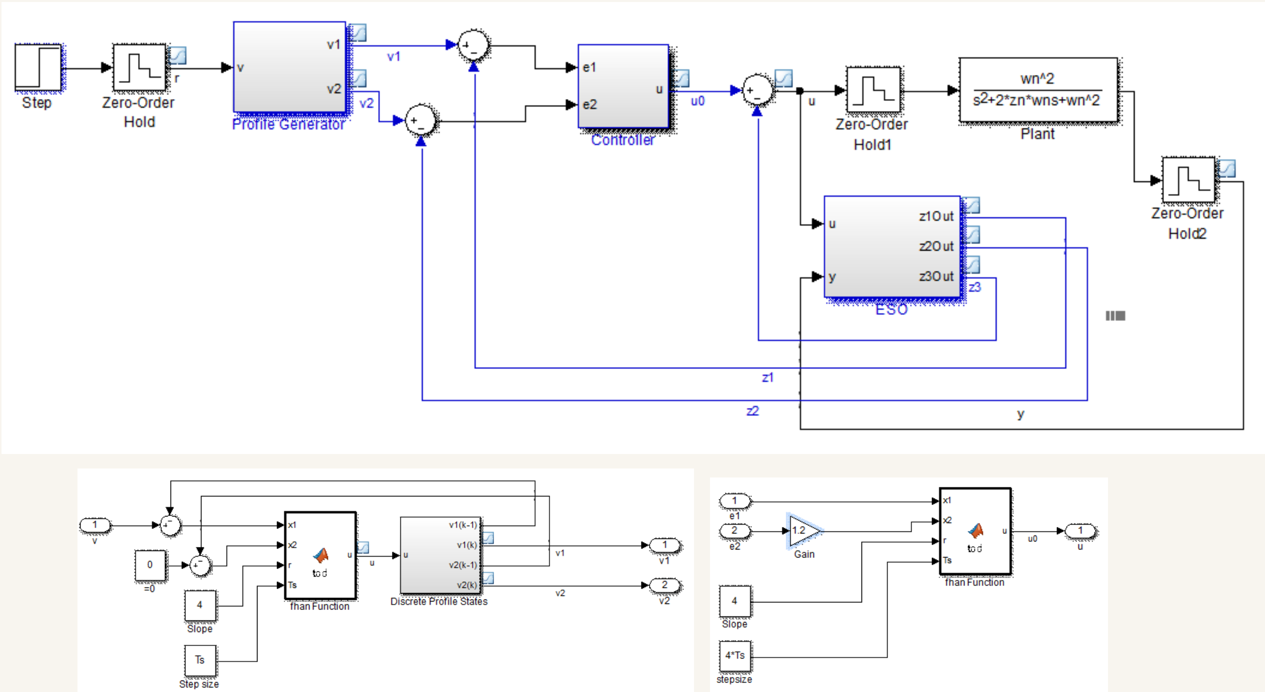


Figure 8: Discrete Non-linear Simulink example (Top-level, Profile Generator and Controller subsystems)

In the step response shown in the next figure, the output of the profile generator position v_1, speed v_2; ESO position and speed states z_1 and z_2 and the plant output is shown.

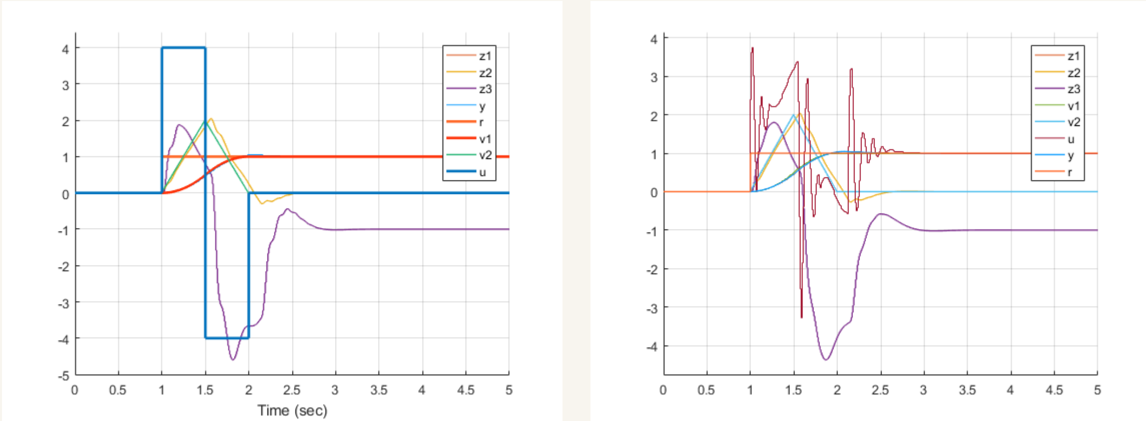


Figure 9: Step Response of the Non-Linear control system (linear ESO left, non-linear right)

The performance is greatly improved by the non-linear controller (providing optimal step response time for the limited plant dynamics). The profile generator improves the over-shoot and provides the necessary tracking command. The effect of the linear or non-linear ESO choice is less clear.

5 Summary

This technique provides an enhanced alternative to the standard PID approach, by solving some of the major shortcomings of that approach, through adding a profile generator, an extended state observer (ESO) with an explicit disturbance state, and providing PD control feedback to track the reference accurately.

The major contribution here is the generation and tracking of a disturbance error that includes both plant parameters errors and static errors, plus the structure to eliminate the stationary part of this error, without resorting to a slow control integrator.

Additional fast non-linear controller and observer techniques are also presented to improve performance without high bandwidth requirements.

A Observers: Structure and/or Dynamics

When observers are used in a design, the initial approach is to start with the best possible plant model and then use feedback to move the poles to suitable positions (typically faster than the original plant poles).

Could we start with an arbitrary model (same structure but without some of the intrinsic dynamics) and achieve the same result? For example, if we know the plant is second-order, but the parameters are not that well known, with

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

One suitable observer model would be the controller canonical form (measure or guess parameters)

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Why can we not use the double integrator model?

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

If we want to place both the observer poles at $s = -a$, then the required characteristic polynomial will be $s^2 + 2as + a^2 = 0$ in both cases.

If the observer gain is $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$, then (first case) we have

$$\begin{aligned}\det(sI - A + LC) &= \det \begin{bmatrix} s + L_1 & -1 \\ L_2 + \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix} \\ &= s^2 + s(L_1 + 2\zeta\omega_n) + L_2 + \omega_n^2 + 2\zeta\omega_n L_1\end{aligned}$$

Comparing coefficients for both models, we get

$$\begin{aligned} L_1 &= 2a - 2\zeta\omega_n & \qquad L_1 &= 2a \\ L_2 &= a^2 - \omega_n^2 - 2\zeta\omega_n L_1 & \qquad L_2 &= a^2 \end{aligned}$$

In general, as long as the observer bandwidth is much larger than the bandwidth of the plant, then the gains L_1 and L_2 will be positive. If furthermore the one state is a derivative of the other ($x_2 = \dot{x}_1$), either model will provide the same behaviour for our design. This allows us to make the observer largely independent of the intrinsic dynamics of the plant.

References

[1] *From PID to Active Disturbance Rejection Control*, Jingqing Han, IEEE Transactions on Industrial Electronics, Vol 56, No. 3, March 2009.

[2] *On Discrete Time Optimal Control: A Closed Form Solution*, Zhiqiang Gao, Proceedings of the 2004 American Control Conference, Boston, Massachusetts, June 30 – July 2, 2004.