TU/e

Quartile 4 - 2016/2017

4DM50

Dynamics and control of cooperation

# Assignment


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Eindhoven, July 5, 2017

# 1 Synchronization of mobile robots

Consider the mobile robot shown in Figure 1.1.

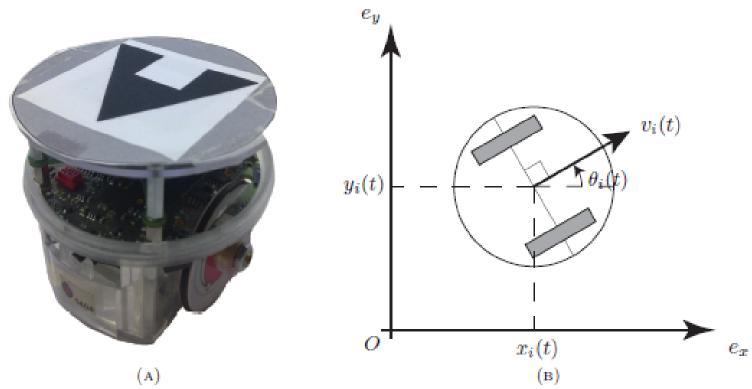


**Figure 1.1: Printer system**

The position $(x_i(t), y_i(t))$ of robot $i$ in the $(e_x, e_y)$-plane is determined by Equations 1.1 and 1.3.

$$\dot{x}_i(t) = v_i(t)cos\theta_i(t) \tag{1.1}$$
$$\dot{y}_i(t) = v_i(t)sin\theta_i(t) \tag{1.2}$$
$$\dot{\theta}_i = w_i(t) \tag{1.3}$$

where the forward velocity of the robot $v_i(t)$ and the rotational/steering velocity $w_i(t)$ are inputs. Letting $a_i(t) = \dot{v}_i(t)$, we can show that for non-zero forward velocity $v_i(t)$ the control law

$$a_i(t) = \eta_{i,1}(t)cos\theta_i(t) + \eta_{i,2}(t)sin\theta_i(t) \tag{1.4}$$
$$w_i(t) = -\eta i, 1(t)\frac{sin\theta_i(t)}{v_i(t)} + \eta_{i,2}(t)\frac{cos\theta_i(t)}{v_i(t)} \tag{1.5}$$

yields the linear dynamics

$$\ddot{x}_i(t) = \eta_{i,1}(t) \tag{1.6}$$
$$\ddot{y}_i(t) = \eta_{i,2}(t) \tag{1.7}$$

Here $\eta_1$ and $\eta_2$ are "new" inputs that are used for further controller design.

*1. Given the reference signals $x_r(t)$ and $y_r(t)$ for the leader mobile robot $i = 1$ (and their derivatives are availlable too), show that for suitable gains $k_p$ and $k_d$ the law*

$$\eta_{1,1}(t) = \ddot{x}_r(t) + k_d(\dot{x}_r(t) - \dot{x}_1(t)) + k_p(x_r(t) - x_1(t)) \tag{1.8}$$
$$\eta_{1,2}(t) = \ddot{y}_r(t) + k_d(\dot{y}_r(t) - \dot{y}_1(t)) + k_p(y_r(t) - y_1(t)) \tag{1.9}$$

*yield asymptotic tracking of the reference, i.e. $x_1(t) \to x_r(t)$ and $y_1(t) \to y_r(t)$ for $t \to \infty$.*

Suitable gains $k_p$ and $k_d$ can lead to good error behaviour. Thereby are the error dynamics derived with

$$e = \begin{bmatrix} x - x_r \\ y - y_r \end{bmatrix}, \quad \dot{e} = \begin{bmatrix} \dot{x} - \dot{x}_r \\ \dot{y} - \dot{y}_r \end{bmatrix}, \quad \ddot{e} = \begin{bmatrix} \ddot{x} - \ddot{x}_r \\ \ddot{y} - \ddot{y}_r \end{bmatrix} \tag{1.10}$$

When $\eta_{1,1}$ and $\eta_{2,2}$ are brought to the right side of Equations **??** and 1.3, the equation will be equal to zero. Use Equations 1.6 and 1.7 to substitute $\eta$ into $\ddot{x}_1$ and $\ddot{y}_1$. Now the equation is of standard form $\ddot{e} + k_d\dot{e} + k_p e = 0$, which has good tracking control if $k_p$ and $k_d$ are positive. Consequently suitable values are $k_p > 0$ and $k_d > 0$.

*2. Use numerical simulations to test your tracking controller for the reference signals*

$$x_r(t) = 10cos(0.1t), \quad y_r(t) = 10sin(0.1t) \tag{1.11}$$

*i.e., the robot drives along a circle of radius $R = 10$. Use various values of $k_d$ and $k_p$. Select the inital conditions and gains $k_d$ and $k_p$ with care such that the velocity $v_1(t)$ does never become zero! Explain why it is important that $v_1$ does not become zero?*

A numerical simulation has been used to test the tracking controller for varying $k_p$ and $k_d$. The results are depicted in Figure 1.2.

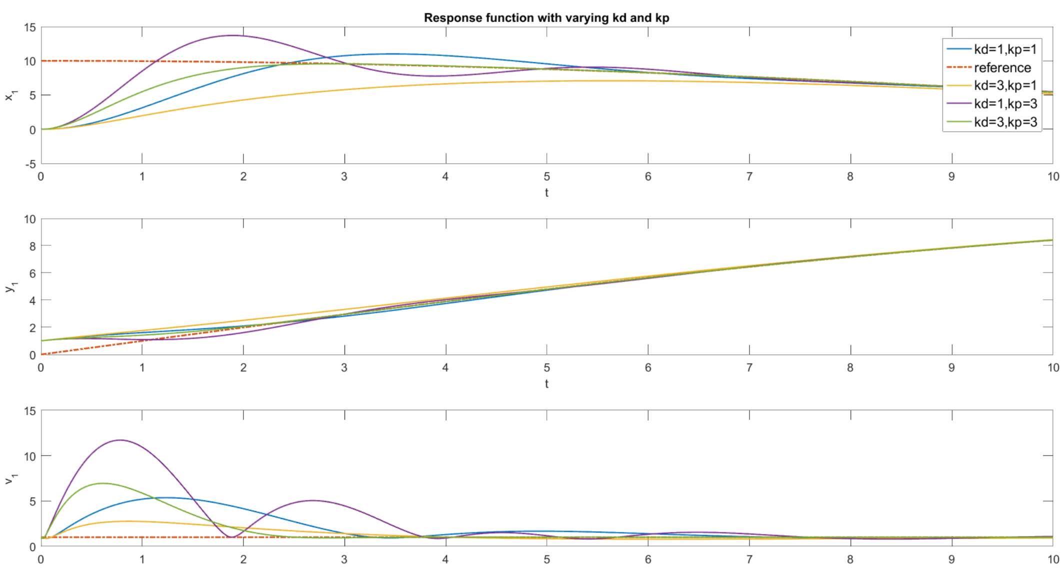


**Figure 1.2: Response functions with varying $k_d$ and $k_p$.**

As can be seen in figure 1.2, the higher the gain $k_p$, the more powerful will the position signal move towards its reference. However, it have an overshoot, because of this high power gain (see the purple line). Therefore is the $k_d$, which damps the system and thereby makes it very slow, but has minimal overshoot (yellow line). A combination of $k_p$ and $k_d$ lead to a fast convergence and low overshoot (green line). When the velocity is observed, the purple line deviates most from the reference velocity (= 1).

Note that it is important that $v = 0$ will not be reached. This will conflict with Equation 1.5, because there can never be divided by zero! The initial conditions are thereby chosen at $x_0 = \begin{bmatrix} 0 & 1 & 2 & 1 \end{bmatrix}^T$.

*3. We now let a second and third robot follow the trajectories of robot 1 and robot 2, respectively, at a fixed distance $\Delta R = R\Delta\phi = 1$ on the circle. The desired behavior can be achieved using the control laws*

$$\eta_{i,1}(t) = \ddot{x}_{c,i-1}(t) + k_D\left(\dot{x}_{c,i-1}(t) - \dot{x}_i(t)\right) + k_P\left(x_{c,i-1}(t) - x_i(t)\right) \tag{1.12}$$

$$\eta_{i,2}(t) = \ddot{y}_{c,i-1}(t) + k_D\left(\dot{y}_{c,i-1}(t) - \dot{y}_i(t)\right) + k_P\left(y_{c,i-1}(t) - y_i(t)\right) \tag{1.13}$$

*for $i = 2, 3$, where*

$$x_{c,i-1}(t) = x_{i-1}(t)cos\Delta\phi + y_{i-1}(t)sin\Delta\phi \tag{1.14}$$

$$y_{c,i-1}(t) = -x_{i-1}(t)sin\Delta\phi + y_{i-1}(t)cos\Delta\phi \tag{1.15}$$

*Perform numerical simulations to test the controller. Select the initial conditions and gains $k_D$ and $k_P$ with care such that the velocities $v_i(t)$ do never become zero!*

Again, numerical simulations are used to show the behaviour of the system. Now, there are multiple robots introduced in the simulation. The result depicted in Figure 1.3 has $k_D = k_P = 1$.

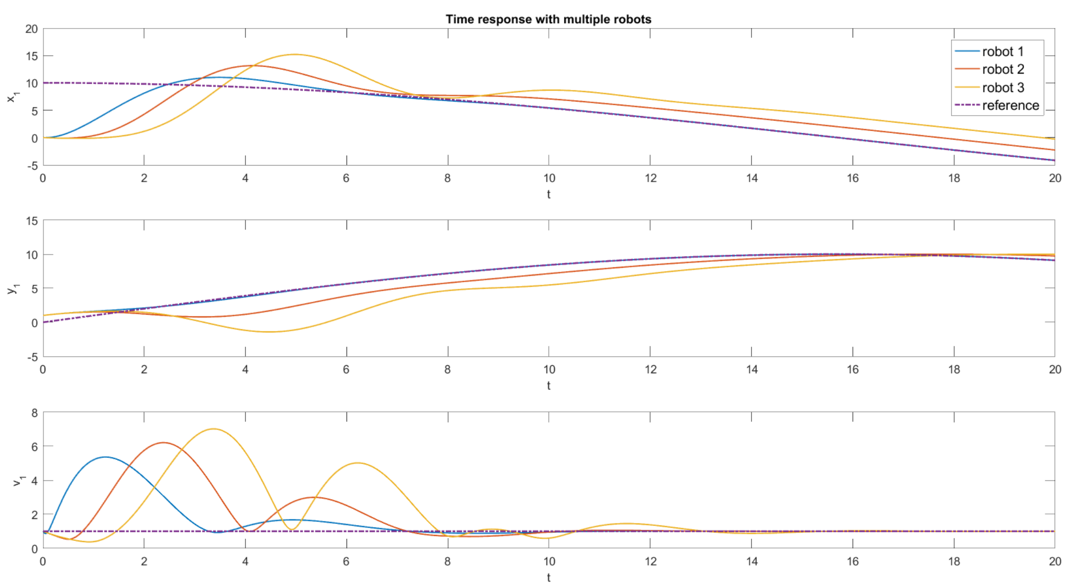


**Figure 1.3: Time response with multiple robots.**

As can be seen in Figure 1.3, not all robots seem to go to the reference. This is because the reference in this figure is the reference of the first robot only. The response of the second and third robot are converging to a delayed version of this reference. Also can be seen that the third robot has most trouble with converging. This is because it has a double delay (first to second robot and second to third robot). The third robot has the second robot as reference, which includes the errors of robot 2. Thereby it takes longer for robot 3 to obtain an equilibrium then the first or second robot.
Also the values for $k_D$ and $k_P$ are varied to show its effect. The results are shown in Figure 1.4.

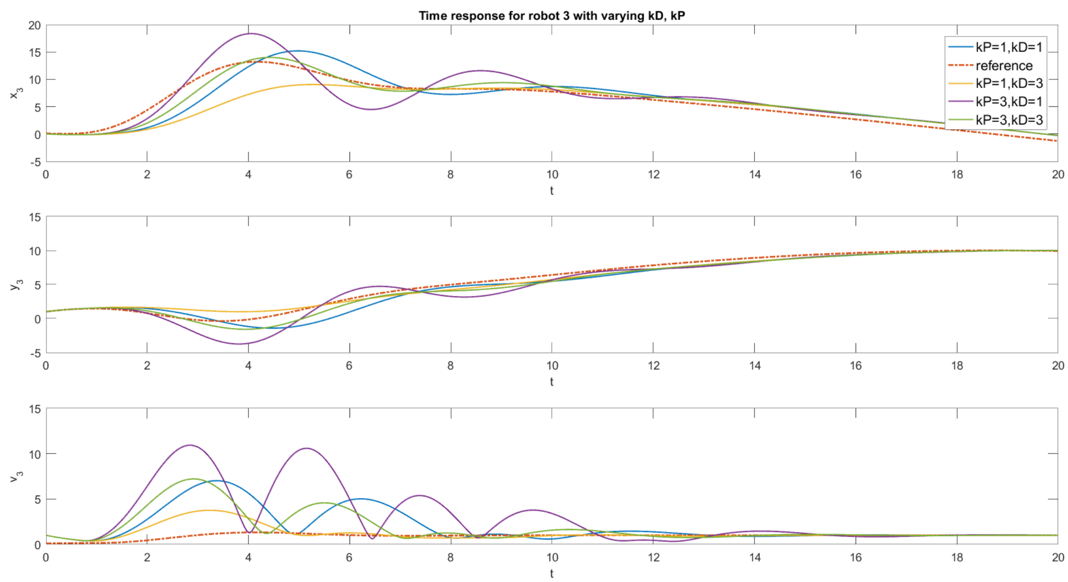


**Figure 1.4: Time response for robot 3 with varying $k_D$ and $k_P$.**

Surprisingly, another controller gives a better result than for the previous system at question 2 for robot 1. A gain of $k_P$ is to high to compensate for the damping gain $k_D$. It shows that the yellow line converges fastest, which is best visible in the velocity response. Note that the convergence is achieved much later at the third robot, excluding the delay of starting up (waiting on the first and second robot to move). Concluding, it becomes way harder to control, when increasing the amount of robots!

*4. Let the reference trajectory for robot 1 now be given by*

$$x_r(t) = 3t, \quad y_r(t) = 0, \tag{1.16}$$

*i.e., robot 1 is moving in $e_x$-direction with constant velocity. Choosing values of $k_d$ and $k_p$ such that robot 1 follows this reference. Next design control laws such that robot 2 follows robot 1 at a fixed distance $d = 1$ and robot 3 follows robot 2 at the same fixed distance $d = 1$. Prove that your control laws yield the desired behavior and perform numerical simulations to varify your results.*

Now that the references are changed from a circle to a straight reference with constant velocity, the robots need other control laws. The control laws result in the following equations

$$x_{c,i,1} = x_i - 1(t) + d \tag{1.17}$$

$$y_{c,i,1} = 0 \tag{1.18}$$

With d=1. Furthermore are the rest of the formulas equal to the ones used in question 3. The reference of the velocity and acceleration in $x$-direction become 1 and 0 respectively (equal to the derivatives of the first robot). Both values become 0 for both in $y$-direction (equal to the derivatives of the first robot). The results is shown in Figure 1.5.

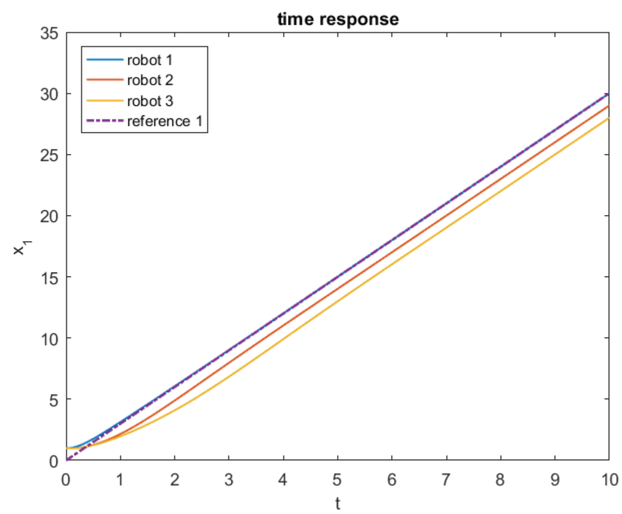


**Figure 1.5: Time response for 3 robots with constant distance $d$.**

The robots follow the references nicely after some time. The error converges relatively fast with $k_d = 5$ and $k_P = 3$. The values for $k_P$ and $k_D$ are chosen 3 and 5 respectively. The distance $d$ between the robots can clearly be seen in the different lines in the graph.

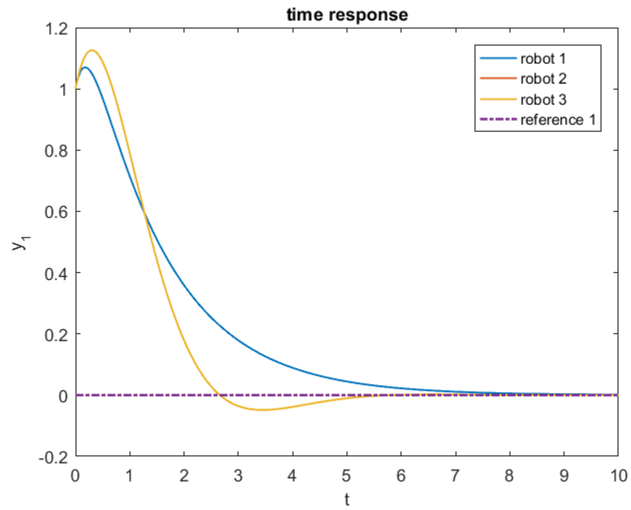


**Figure 1.6: Zoomed time response for 3 robots with constant distance $d$.**

As can be seen in Figure 1.6, the $y$ motion starts at the value 1 because of the initialy chosen starting value. After some time are the robots all located at the $y = 0$ reference line.

## 2 Synchronization Experiments

The second part has been made in collaboration with Kirill Rogov. These are added with report. The first parts were made seperately.