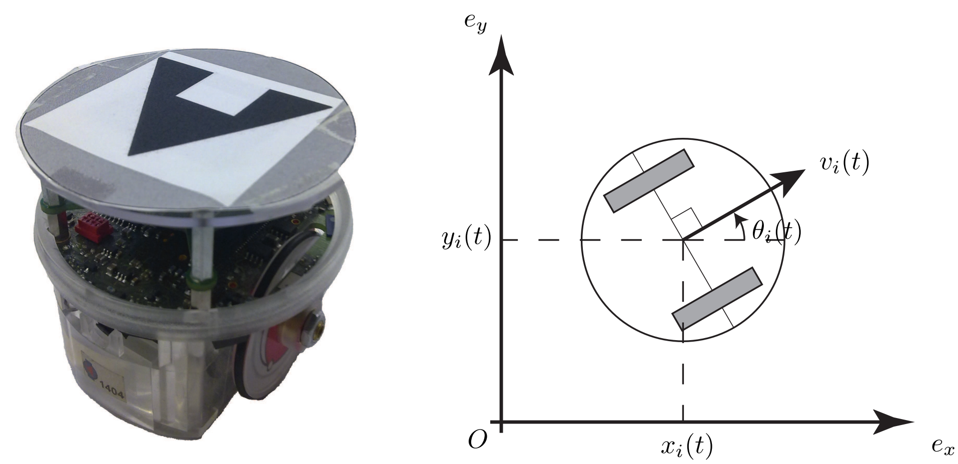
ASSIGNMENT 4DM50

Work in groups of two. The strict deadline for the written report is July 7, 2017, 12:00. You should drop the report in the mailbox of Erik Steur at the Dynamics and Control secretary at Gem-Z 0.143.

Problem 1: Synchronization of mobile robots

Consider the mobile robot shown in Figure 1. The position ($x_i(t)$,$y_i(t)$) of robot



(A) (B)

Figure 1. The mobile robot

i in the (e_x, e_y)-plane (with origin O) is determined by the equations

$$\dot{x}_i(t) = v_i(t)\cos\theta_i(t)$$
$$\dot{y}_i(t) = v_i(t)\sin\theta_i(t)$$
$$\dot{\theta}_i(t) = w_i(t)$$

where the forward velocity of the robot $v_i(t)$[1] and the rotational/steering velocity $w_i(t)$ are inputs. Letting $a_i(t) = \dot{v}_i(t)$, we can show that for non-zero forward velocity $v_i(t)$ the control law

$$a_i(t) = \eta_{i,1}(t)\cos\theta_i(t) + \eta_{i,2}(t)\sin\theta_i(t)$$
$$w_i(t) = -\eta_{i,1}(t)\frac{\sin\theta_i(t)}{v_i(t)} + \eta_{i,2}(t)\frac{\cos\theta_i(t)}{v_i(t)}$$

[1]Note that $v_i(t) = \sqrt{\dot{x}_i^2(t) + \dot{y}_i^2(t)}$.

yields the linear dynamics

$$\ddot{x}_i(t) = \eta_{i,1}(t),$$
$$\ddot{y}_i(t) = \eta_{i,2}(t).$$

Here $\eta_1(\cdot)$ and $\eta_2(\cdot)$ are "new" inputs that are used for further controller design. The goal is

- to design a tracking controller for a leader mobile robot;
- find control laws such that a second mobile robot will follow the leader, and a third mobile robot follows the second robot.

The signals $x_i(t)$ and $y_i(t)$ and all their derivatives are available for measurement. We will consider only the stability and tracking problem; practical issues such as collision avoidance and string stability need not be discussed.

Questions.

1. Given the reference signals $x_r(t)$ and $y_r(t)$ for the leader mobile robot $i = 1$ (the derivatives $\dot{x}_r(t)$, $\ddot{x}_r(t)$ and $\dot{y}_r(t)$, $\ddot{y}_r(t)$ are available too), show that for suitable gains k_p and k_d the law
$$\eta_{1,1}(t) = \ddot{x}_r(t) + k_d(\dot{x}_r(t) - \dot{x}_1(t)) + k_p(x_r(t) - x_1(t))$$
$$\eta_{1,2}(t) = \ddot{y}_r(t) + k_d(\dot{y}_r(t) - \dot{y}_1(t)) + k_p(y_r(t) - y_1(t))$$
yield asymptotic tracking of the reference, i.e. $x_1(t) \to x_r(t)$ and $y_1(t) \to y_r(t)$ for $t \to \infty$.
2. Use numerical simulations to test your tracking controller for the reference signals
$$x_r(t) = 10\cos(0.1t), \quad y_r(t) = 10\sin(0.1t),$$
i.e., the robot drives along a circle of radius $R = 10$. Use various values of k_d and k_p. Plot for each simulation (at least) the signal $x_1(t)$, $y_1(t)$ and $v_1(t)$ and discuss your results. Select the initial conditions and gains k_d and k_k with care such that the velocity $v_1(t)$ does never become zero! Explain why it is important that $v_1(\cdot)$ does not become zero?)
3. We now let a second and third robot follow the trajectories of robot 1 and robot 2, respectively, at a fixed distance $\Delta R = R\Delta\phi = 1$ on the circle. See Figure 2. The desired behavior can be achieved using the control laws
$$\eta_{i,1}(t) = \ddot{x}_{c,i-1}(t) + k_D(\dot{x}_{c,i-1}(t) - \dot{x}_i(t)) + k_P(x_{c,i-1}(t) - x_i(t))$$
$$\eta_{i,2}(t) = \ddot{y}_{c,i-1}(t) + k_D(\dot{y}_{c,i-1}(t) - \dot{y}_i(t)) + k_P(y_{c,i-1}(t) - y_i(t))$$
for $i = 2, 3$, where
$$x_{c,i-1}(t) = x_{i-1}(t)\cos\Delta\phi + y_{i-1}(t)\sin\Delta\phi$$
$$y_{c,i-1}(t) = -x_{i-1}(t)\sin\Delta\phi + y_{i-1}(t)\cos\Delta\phi.$$
Perform numerical simulations to test the controller. Select the initial conditions and gains k_D and k_P with care such that the velocities $v_i(t)$ do never become zero! Plot for each simulation (at least) the signal $x_i(t)$, $y_i(t)$ and $v_i(t)$ for $i = 1, 2, 3$ and discuss your results.
4. Let the reference trajectory for robot 1 now be given by
$$x_r(t) = 3t, \quad y_r(t) = 0,$$

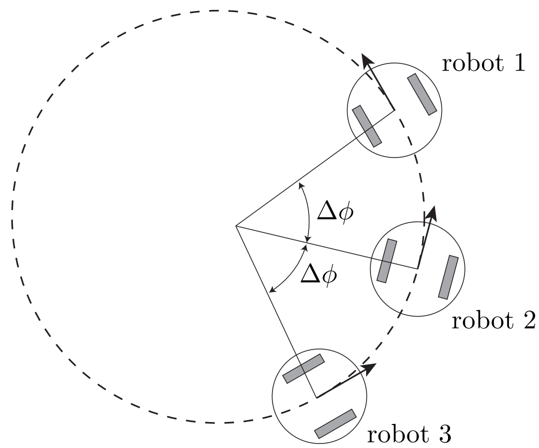


FIGURE 2. Three mobile robots on the circle.

i.e., robot 1 is moving in e_x-direction with constant velocity. Choose values of k_d and k_p such that robot 1 follows this reference. Next design control laws such that robot 2 follows robot 1 at a fixed distance $d = 1$ and robot 3 follows robot 2 at the same fixed distance $d = 1$. Prove that your control laws yield the desired behavior and perform numerical simulations to verify your results.

PROBLEM 2: SYNCHRONIZATION EXPERIMENTS

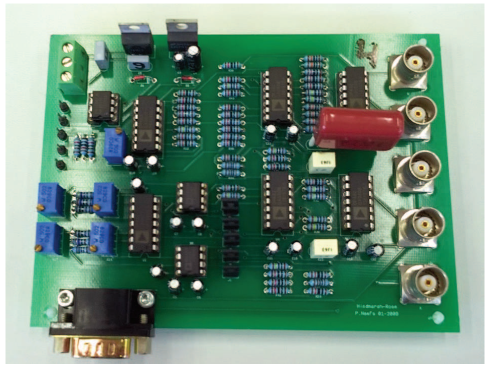
You are going to investigate synchronization using the experimental setup that is found in the DCT laboratory (GEM-Z -1.13). This experimental setup consists of a number of electronic Hindmarsh-Rose (HR) neurons, a coupling interface and data acquisition apparatus. The electronic HR neuron shown in Figure 3. This electronic HR neuron, which models the spiking or bursting activity of a neuron, satisfies the following set of differential equations:

$$\dot{y}_i(t) = 100\big(-y_i^3(t) + 3y_i(t) - 8 + 5z_{i,1}(t) - z_{i,2}(t) + E + u_i(t)\big) \tag{1a}$$

$$\dot{z}_{i,1}(t) = 100(-y_i^2(t) - 2y_i(t) - z_{i,1}(t)) \tag{1b}$$

$$\dot{z}_{i,2}(t) = 0.5(4y_i(t) + 4.472 - z_{i,2}(t)) \tag{1c}$$

where E is a parameter. We set $E = 3.3$ for which the HR neuron is operating in a so-called chaotic bursting mode. Output $y_i(t)$ represents the membrane potential of the neuron, and $z_{i,1}(t)$ and $z_{i,2}(t)$ are internal variables that are related to ionic (potassium and sodium) currents.



(A)

Figure 3. The electronic HR neuron

The HR neurons interact via coupling of the form

$$u_i(t) = \sigma \sum_j a_{ij}[y_j(t-\tau) - y_i(t-\tau)]$$

with positive constant σ being the coupling strength, non-negative constant τ denotes the time-delay, and a_{ij} is the ij^{th} entry of the adjacency matrix A. The interaction between the HR neurons is established via the coupling interface. You can define any network structure, up to eight neurons, with integer interaction weights from 0 to 5. (Obviously, 0 means there is no interaction.) Furthermore you can set the coupling strength in the range from 0 to 10 and the time-delay can be set from 0 [ms] to 5 [ms].

One can show that the HR neuron (1) is strictly semi-passive with a quadratic storage function S such that

$$\dot{S} \leq y_i u_i - H(y_i, z_{i,1}, z_{i,2})$$

with H being quartic in y_i and quadratic in $z_{i,1}$ and $z_{i,2}$. This implies that solutions of the network of HR neurons are uniformly bounded and uniformly ultimately bounded. In addition one easily verifies that the internal dynamics, i.e. the z_i-dynamics, are exponentially convergent. Thus we know that there exist values for the coupling strength σ and time-delay τ such that a network of HR neurons synchronizes.

This synchronization result is derived under the assumption that the HR neurons have identical dynamics, i.e. the dynamics of each electronic HR neuron satisfy equations (1). However, because of imperfections in the electronic components on the circuits and noise, we can not expect the HR circuits to behave completely identical; For suitable values of the coupling strength σ and time-delay τ we may only expect that the difference in outputs of the HR circuits to become sufficiently small. We say that the HR neurons *practically synchronize* if, after some transient time,

$$|y_i(t) - y_j(t)| \leq 0.25$$

holds for all t and all $i, j \in \{1, 2, \ldots, N\}$ (with $2 \leq N \leq 8$).

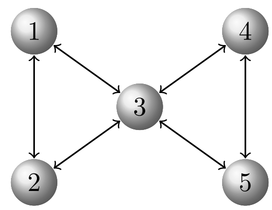


FIGURE 4. The network of question 2b. Each edge has weight 1

Questions.

1. Consider two coupled HR neurons with $a_{12} = a_{21} = 1$. For each $\tau = 0, 1, \ldots, 5$, determine a lower-bound and upper-bound for the coupling strength such that the two coupled HR neurons practically synchronize. Note that the maximum value of the coupling strength you can select is 10; In case the systems remain practically synchronous for $\sigma = 10$ you take 10 as upper-bound.
2. Consider the network shown in Figure 4 with corresponding adjacency matrix:
$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$
   a. Use your results of question 1. to predict the values of σ and τ for which network practically synchronizes. In particular, determine the lower-bounds and upper-bounds of the coupling strength for $\tau = 0, 1, 2, \ldots$. (It might be that for higher values of τ the systems do not synchronize.)
   b. Verify your results by experiments with this network.
3. Consider the network of eight coupled neurons defined by the adjacency matrix
$$A = \begin{pmatrix} 0 & 2 & 1 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 1 & 2 & 0 \end{pmatrix}.$$
   a. Take $\tau = 0$. Increase the coupling strength starting from 0 such that the whole network practically synchronizes. What coupling strength is needed for practical synchronization?
   b. Before the network becomes practically synchronous there is a mode of partial practical synchronization (for $\tau = 0$). Describe this mode of partial practical synchronization and specify what coupling strength is needed to enter this mode of partial practical synchronization.
   c. Explain your results of 3b. using the theory of partial synchronization. (Hint: draw the network and identify its symmetries.)

d. Take $\sigma = 2$ and start to increase the time-delay (starting from 0). For some value of τ you find that the practical synchronization is lost but the network exhibits partial practical synchronization. Describe this mode of partial practical synchronization and specify the value of the time-delay at the transition from practical synchronization to partial practical synchronization.

e. Explain your results of 3e. using the theory of partial synchronization. (Hint: draw the network and identify its symmetries.)