4DM50 Dynamics and Control of cooperation
Problem set 2

Problem 1. Consider a 1-DOF manipulator (pendulum with friction) controlled by torque. The system equations linearized around the stable equilibrium point looks like

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 - x_2 + u \tag{1}$$

1. Suppose that the control problem is to design a controller such that the variable x_1 tracks the desired trajectory y_r. Design such a controller under assumption that both state variables x_1, x_2 as well as the reference signals $y_r, \dot{y}_r$ and $\ddot{y}_r$ are available for measurements.

2. Now suppose the problem is to design a controller such that the second manipulator

$$\dot{z}_1 = z_2,\ \dot{z}_2 = -z_1 - z_2 + v \tag{2}$$

follows the same trajectory. In practice there are situations when the manipulators should follow the desired trajectory in a synchronous way with a restriction on a mismatch between trajectories of the two manipulators. In an ideal situation one can use the same controller for the second manipulator. In practice, this approach is not always possible because of some disturbances and/or nonidentity of the manipulators. Suppose there is a disturbance $\epsilon(t)$ due to a vibration of the second manipulator:

$$\dot{z}_1 = z_2,\ \dot{z}_2 = -z_1 - z_2 + v + \epsilon \tag{3}$$

Then, if one applies the same controller for the second manipulator there will be mismatch $x_1 - z_1 \neq 0$ and $x_2 - z_2 \neq 0$ due to the disturbance. To reduce this mismatch one can increase the control gains in the controller. The main drawback of this approach is the amplification of the measurement noise (explain why). In the situations where the synchronous motion of the pendula is crucial one can add the following synchronizing terms into the controllers:

$$\alpha(x_1 - z_1) + \beta(x_2 - z_2)$$

Via computer simulation analyze the behavior of the closed loop system with and without synchronizing terms in the controller. In particular, try to understand what the effect is of the parameters of your reference-tracking controller and the synchronization parameters α and β.

Problem 2. Consider two robot manipulators that are coupled in a master slave scheme. The robots have n rigid joints with joint coordinates $q_i \in \mathbb{R}^n$. The joints are rotational and fully actuated. Using the Euler-Lagrange formalism we obtain the dynamics of the manipulators:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = T_i$$

where $i = m, s$ (m for master and s for slave), and

- $M_i(q_i)$ is the $n \times n$ mass matrix;
- $C_i(q_i, \dot{q}_i)\dot{q}_i$ represent the Coriolis and centrifugal forces;

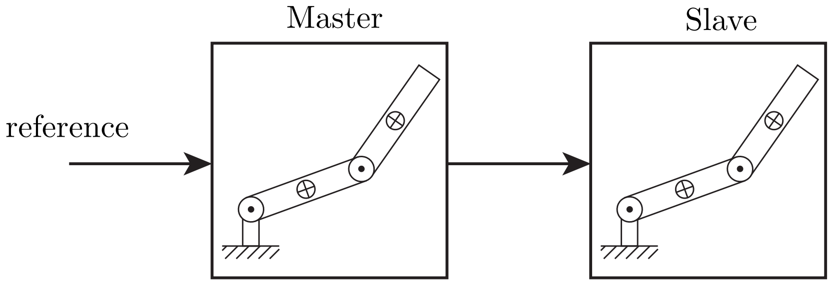

FIGURE 1. Master-Slave robot manipulator setup

- $g_i(q_i)$ are the gravity forces;
- T_i is the vector of (control) torques.

Matrices $M_i(q_i)$ are symmetric and positive definite for all $q_i \in \mathbb{R}^n$. Moreover, the matrices

$$\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$$

are such that, for all $q_i, \dot{q}_i \in \mathbb{R}^n$ and for any vector $\xi \in \mathbb{R}^n$,

$$\xi^T(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i))\xi = 0.$$

Given a reference trajectory for the master robot, the goal is to design a control law such that the slave robot synchronizes to the master robot in the sense that

$$q_s(t) \to q_m(t) \text{ and } \dot{q}_s(t) \to \dot{q}_m(t) \text{ as } t \to \infty.$$

The signals $\ddot{q}_i$, $\dot{q}_i$, q_i, $i = m, s$ can all be measured.

Let us consider the case that both the master and slave are the same *planar RR manipulator*. This planar RR manipulator has two links of length l_1 and l_2 with

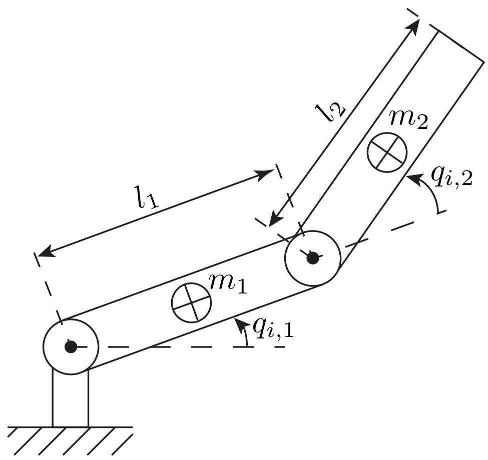

FIGURE 2. Planar RR manipulator

corresponding masses m_1 and m_2. The center of mass of link i is $\frac{1}{2}l_i$. Then we have

$$M_m(q_i) = M_s(q_i) = \begin{pmatrix} M_{11}(q_i) & M_{12}(q_i) \\ M_{21}(q_i) & M_{22}(q_i) \end{pmatrix},$$

$$C_m(q_i, \dot{q}_i) = C_s(q_i, \dot{q}_i) = \begin{pmatrix} C\dot{q}_{i,2} & C(\dot{q}_{i,1} + \dot{q}_{i,2}) \\ -C\dot{q}_{i,1} & 0 \end{pmatrix},$$

and

$$g_m(q_i) = g_s(q_i) = \begin{pmatrix} \left(\frac{1}{2}m_1 + m_2\right) l_1 g \cos q_{i,1} + \frac{1}{2} m_2 l_2 g \cos(q_{i,1} + q_{i,2}) \\ \frac{1}{2} m_2 l_2 g \cos(q_{i,1} + q_{i,2}) \end{pmatrix}$$

with

$$\begin{aligned} M_{11}(q_i) &= \tfrac{1}{4} m_1 l_1^2 + m_2 \left(l_1^2 + \tfrac{1}{4} l_2^2 + l_1 l_2 \cos q_{i,2}\right) + I_1 + I_2 \\ M_{12}(q_i) &= M_{21}(q_i) = m_2 \left(\tfrac{1}{4} l_2^2 + \tfrac{1}{2} l_1 l_2 \cos q_{i,2}\right) + I_2 \\ M_{22}(q_i) &= \tfrac{1}{4} m_2 l_2^2 + I_2 \\ C &= -\tfrac{1}{2} m_2 l_1 l_2 \sin q_{i,2} \end{aligned}$$

where I_1 and I_2 are the moments of inertia of links 1 and 2, respectively.

1. Consider the reference trajectories

$$r_1(t) = \sin(t) + 5\cos(0.2t)$$

for joint 1, and

$$r_2(t) = \sin(0.5t)$$

for joint 2. Let $m_1 = 5$, $m_2 = 6$, $I_1 = I_2 = 3$, $l_1 = l_2 = 4$ and $g = 9.81$. Apply the control law

$$\begin{aligned} T_m &= M_m(q_m)\zeta + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) \\ \zeta &= \begin{pmatrix} \ddot{r}_1(t) \\ \ddot{r}_2(t) \end{pmatrix} - \begin{pmatrix} \dot{q}_{m,1} - \dot{r}_1(t) \\ \dot{q}_{m,2} - \dot{r}_2(t) \end{pmatrix} - \begin{pmatrix} q_{m,1} - r_1(t) \\ q_{m,2} - r_2(t) \end{pmatrix} \end{aligned}$$

and perform a numerical simulation to show that the master system follows the reference trajectory.

2. Consider the control law

$$T_s = M_s(q_s)\ddot{q}_m + C_s(q_s, \dot{q}_s)\dot{q}_m + g_s(q_s) - K_d(\dot{q}_s - \dot{q}_m) - K_p(q_s - q_m).$$

Define the error $e = q_s - q_m$ and write down the error dynamics.

3. Employ the Lyapunov function

$$V(e, \dot{e}) = e^T K_p e + \dot{e}^T M_s(q_s)\dot{e}$$

and derive conditions on matrices K_p and K_d such that the master and slave synchronize. (Hint: apply LaSalle's invariance principle[1].)

4. Perform a few numerical simulations to show that for various matrices K_p and K_d the slave manipulator synchronizes to the master.

Problem 3. Consider the robot manipulators of problem 2, which are now mutually coupled:

Both manipulators are suppose to follow the same reference trajectory. The mutual interaction ensures that the robots stay synchronized, even in case one of

[1] Although the system is non-autonomous it can be shown that LaSalle's invariance principle applies to this problem.

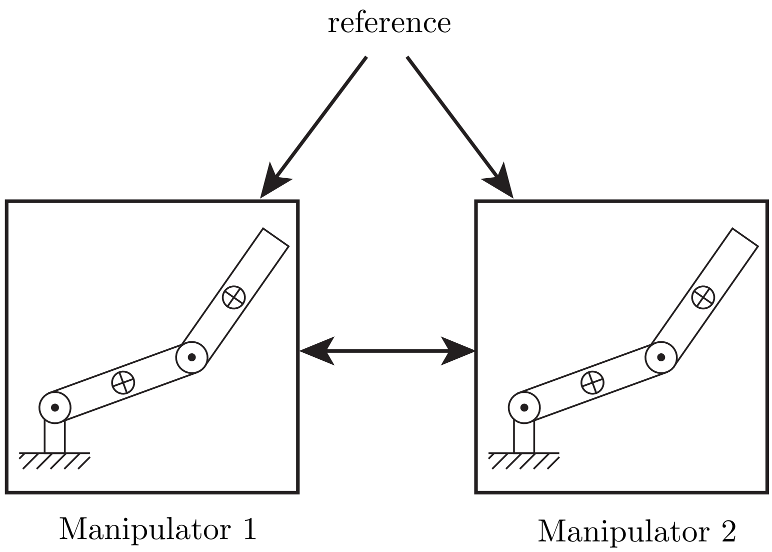

Figure 3. Mutually coupled robot manipulator setup

the manipulators is unable to follow the reference (because of e.g. a disturbance). The control law for the i-th manipulator is given by:

$$T_i = M_i(q_i)\ddot{q}_{r,i} + C_i(q_i, \dot{q}_i)\dot{q}_{r,i} + g_i(q_i) - K_d\dot{s}_i - K_p s_i, \quad i = 1, 2,$$

where K_p and K_d are symmetric and positive definite matrices. Here

$$s_i = q_i - q_{r,i}$$
$$q_{r,i} = r(t) - k_{i,j}(q_i - q_j)$$

with reference signal $r(t)$ and coupling gains $k_{1,2},\ k_{2,1} > 0$. We assume that all signals $q_i, \dot{q}_i, \ddot{q}_i, r, \dot{r}, \ddot{r}$ are available.

1. Using Lyapunov's second method and (a version of) Matrosov's theorem we can show that $s_i(t)$ and $\dot{s}_i(t)$ converge to zero for $i = 1, 2$. Let $k_{1,2} = k_{2,1} = k$ and show that $s_i = 0$ for $i = 1, 2$ implies

$$q_1(t) = q_2(t) = r(t).$$

Hint: the following matrix is non-singular for $k > 0$:

$$\begin{pmatrix} 1+k & -k \\ -k & 1+k \end{pmatrix}.$$

2. Use the matrices M_i, C_i, g_i, parameters and reference trajectories as in problem 1. Let a disturbance act on manipulator 1 and perform a number of computer simulations to investigate the influence of K_p, K_d and coupling gain $k = k_{1,2} = k_{2,1}$ on the cooperative behavior.