The Sympathy of Pendulum Clocks and Beyond

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Where innovation starts

Outline

1. **Introduction**
   - History of Synchronization, examples
   - What is synchronization?
2. **Synchronization in coupled oscillators**
   -Models, setups & experiments
   -Coordination of mechanical systems
3. **Synchronization in Networks of coupled oscillators**
   -Examples(nature, science &technology)
   -Synchronization of neural networks
   -Synchronization of traffic: cooperative driving
4. **Summary & conclusions**

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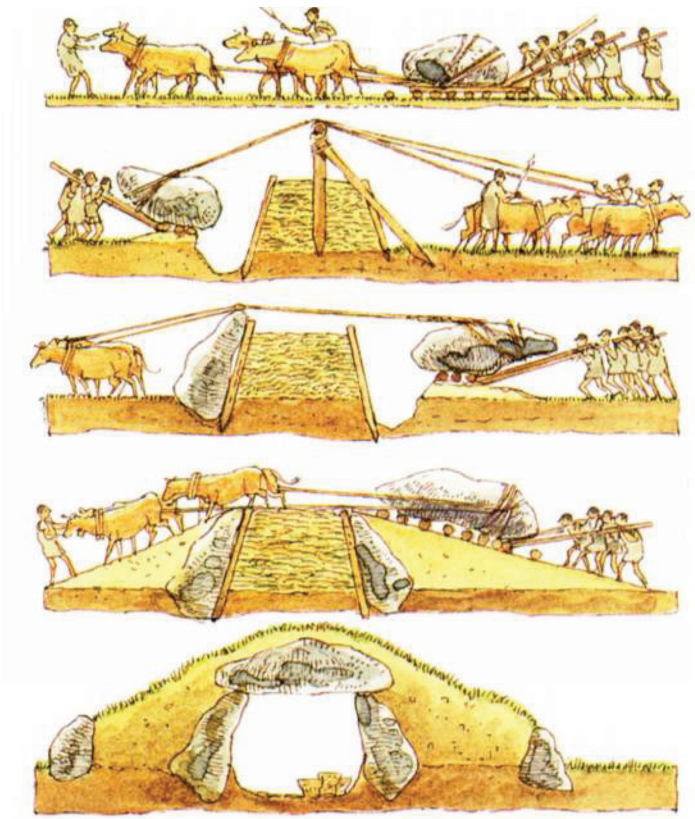
History of Synchronization



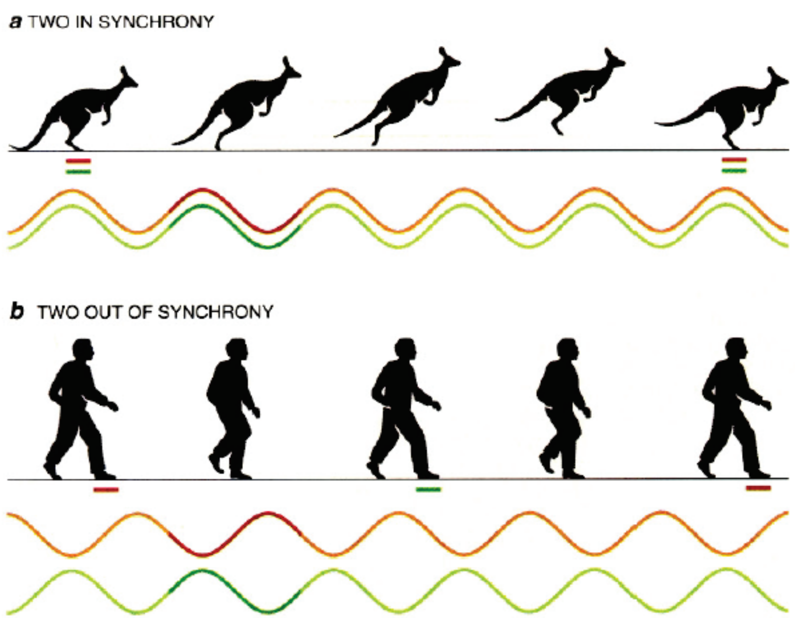
Legananny Dolmen, Ireland

Ancient times: how to built a pyramid, ‘hunnebed’, …?

History of Synchronization

Nuns in a cloister…

History of Synchronization



Synchronization:

'Simultaneous in time'
- Dynamics
- In-phase
- Anti-phase
- … other forms of partial synchrony…
- Often: two animals, systems, ..
- BUT: might be many more!

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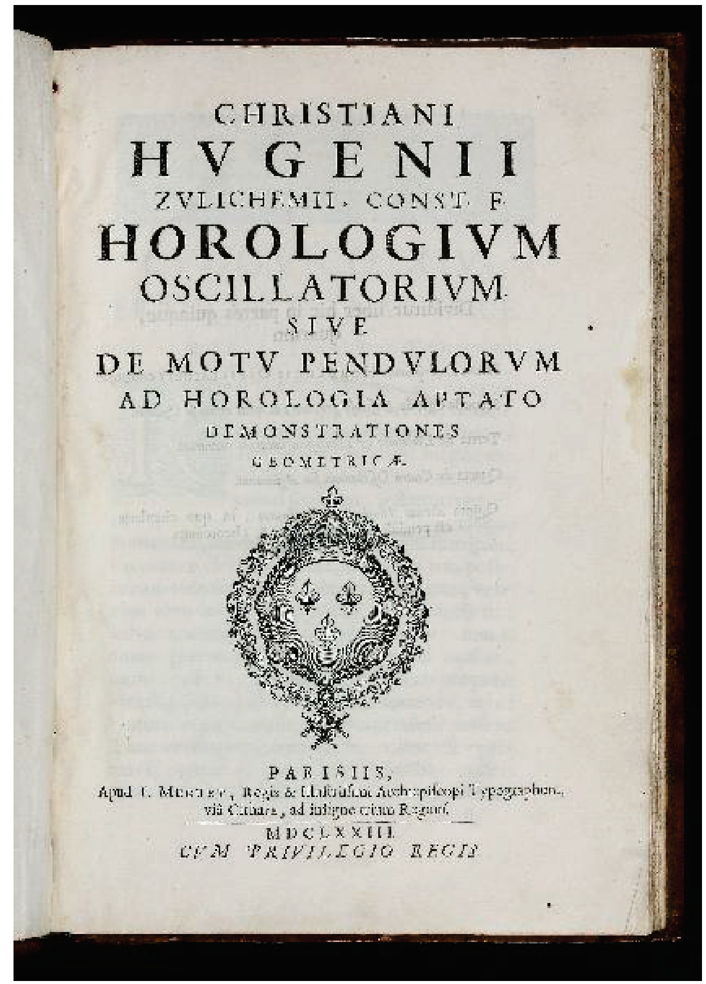
History of synchronization

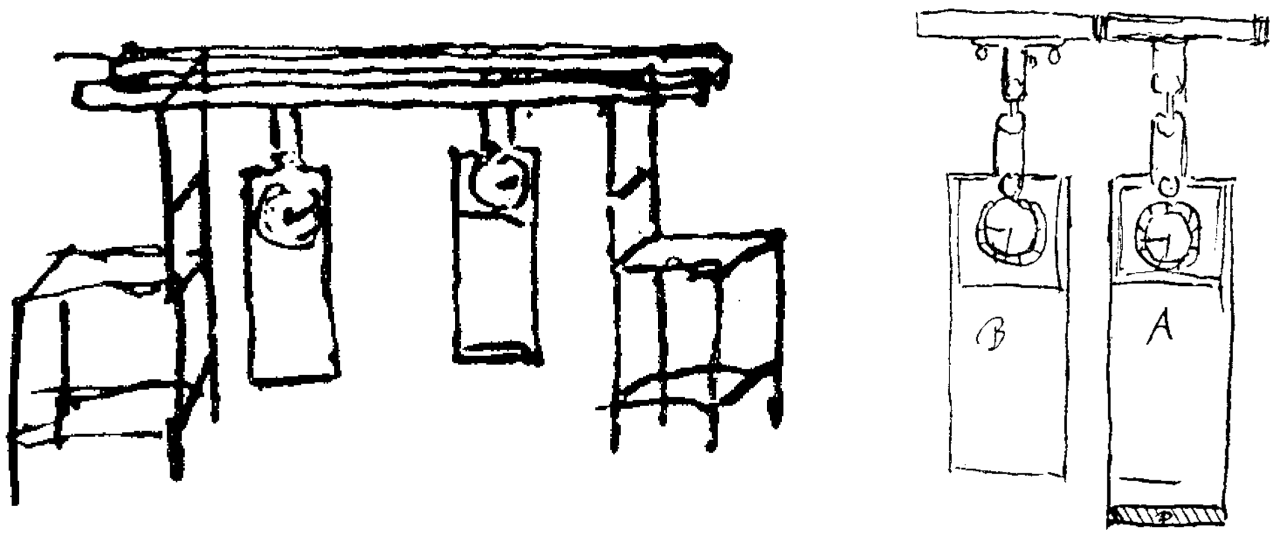


CHRISTIANI
HVGENII
ZVLICHEMII, CONST. F.
HOROLOGIVM
OSCILLATORIVM
SIVE
DE MOTV PENDVLORVM
AD HOROLOGIA APTATO
DEMONSTRATIONES
GEOMETRICÆ.

PARISIIS,
Apud F. Muguet, Regis & Illustrissimi Archiepiscopi Typographum,
via Citharæ, ad insigne trium Regum.
MDCLXXIII.
CVM PRIVILEGIO REGIS.

Christiaan Huygens
(1629-1695)

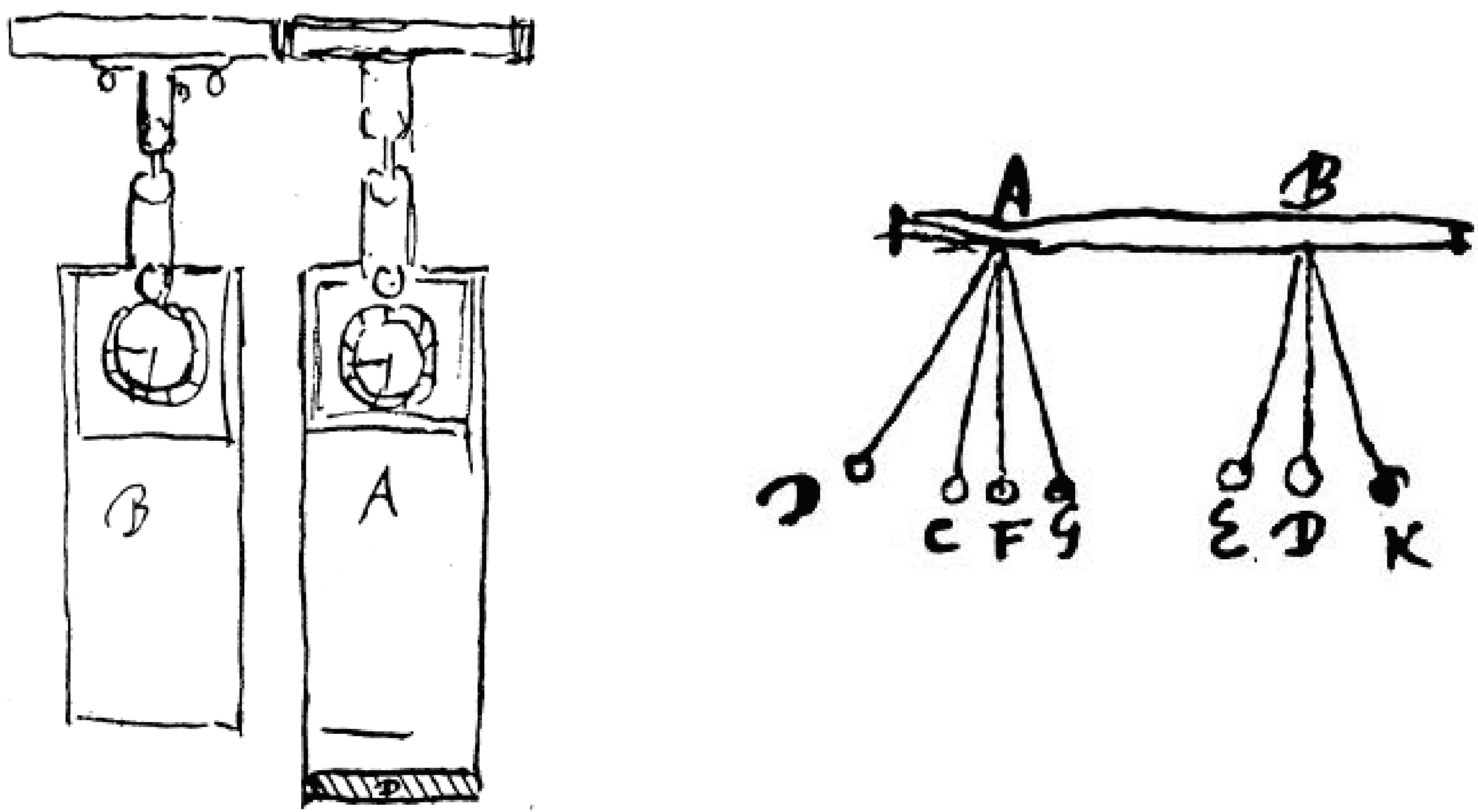
History of synchronization



Christiaan Huygens: ' ….Hence, the clockworks were at a small distance separation I began to suspect a certain **sympathy**, as if one were affected by the other…..' (letter to his father Constantyn Huygens)

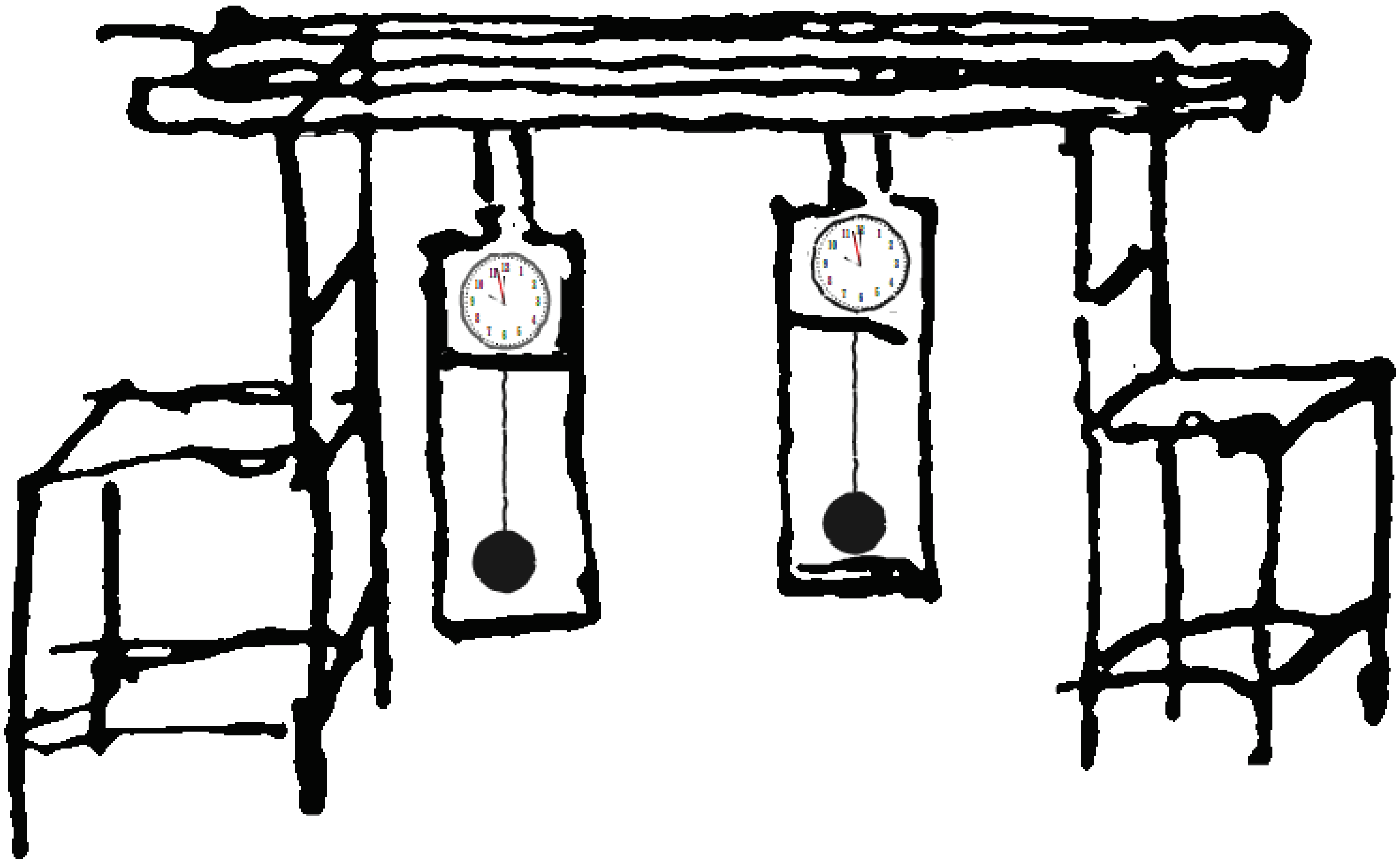
From Huygens' notebook 1665

History of synchronization



From Huygens' notebook 1665

“Sympathy of two clocks”



Beyond pendulum clocks

- The sympathy described by Huygens occurs in many other oscillators/systems/ nature.

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“Sympathy of two organ-pipes”

Sir John William Strutt
Lord Rayleigh
(1842-1919)

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“Sympathy of two organ-pipes”

- In 1877, English physicist Lord Rayleigh observed that when two almost identical organ pipes are played side by side, something strange happens. Rather than each blaring their own tone, the two pipes will barely make a whisper. But put a barrier between them, and they sing loud and clear.

Synchronization examples- Humans

Humans beings show synchronized behaviour:

- An orchestra playing in unison
- Pacemakers cells in the heart fire in unison
- Clapping, singing
- Synchronized swimmers

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Synchronization example- Humans

Synchronization examples- Animals

- The flashing of a flock of fireflies
- A starling flock in flight
- Crickets that chirp in unison
- Japanese tree frogs
- …..

Synchronization example - Animals

Japanese tree frog *Hyla japonica*

Courtesy: Ikkyu Aihara

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Synchronization example - Animals

Synchronized calling behavior

Courtesy: Ikkyu Aihara

Synchronization examples - Industry

Producing wooden shoes

Part 2: Synchronization of coupled oscillators

- What is the “secret” behind Huygens’ syncnronization in pendulum clocks?
- Why and when is anti-phase or in-phase synchronization happening?
- How to force synchronized (coordinated) motion in mechanical systems?

  - Mathematical models
  - Experimental (simplifying) platforms
  - Experiments

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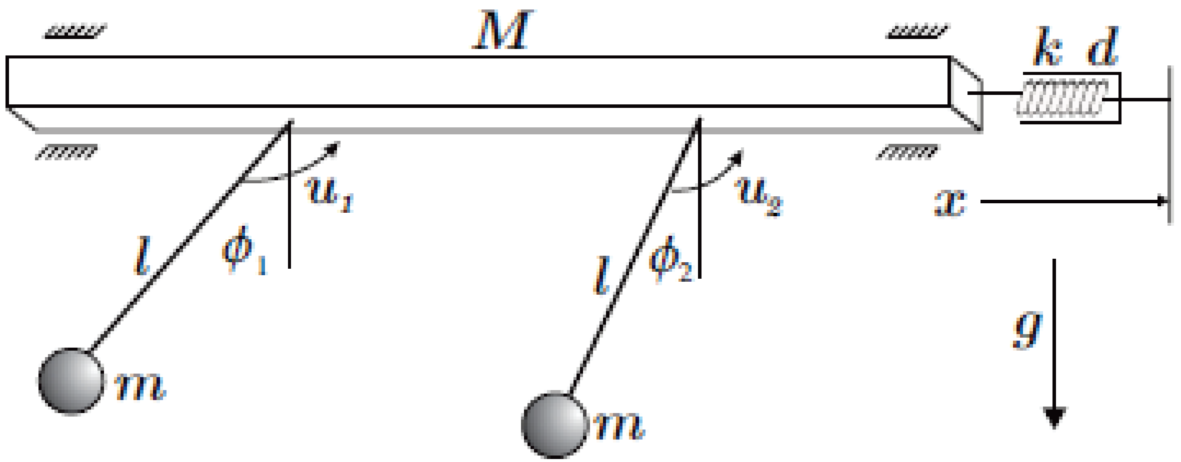
Simplified Huygens' model, no vertical motion



$$
\begin{aligned}
ml^2\ddot{\phi}_1 + ml\ddot{x}\cos\phi_1 + mgl\sin\phi_1 &= u_1 \\
ml^2\ddot{\phi}_2 + ml\ddot{x}\cos\phi_2 + mgl\sin\phi_2 &= u_2 \\
(M+2m)\ddot{x} + ml\sum_{i=1}^{2}\left(\ddot{\phi}_i\cos\phi_i - \dot{\phi}_i^2\sin\phi_i\right) &= \\
-d\dot{x} - kx, &
\end{aligned}
$$

Note: ideal world: no friction/demping!

Uncontrolled synchronization

Model:

$$\begin{aligned} ml^2\ddot{\phi}_1 + ml\ddot{x}\cos\phi_1 + mgl\sin\phi_1 &= 0 \\ ml^2\ddot{\phi}_2 + ml\ddot{x}\cos\phi_2 + mgl\sin\phi_2 &= 0 \\ (M+2m)\ddot{x} + ml\sum_{i=1}^{2}\left(\ddot{\phi}_i\cos\phi_i - \dot{\phi}_i^2\sin\phi_i\right) &= -d\dot{x} - kx, \end{aligned}$$

Energy:

$$\begin{aligned} V &= \frac{m}{2}\sum_{i=1}^{2}\left(\dot{x}^2 + l^2\dot{\phi}_i^2 + 2\dot{x}\dot{\phi}_i l\cos\phi_i\right) \\ &+ \frac{M\dot{x}^2}{2} + mgl\sum_{i=1}^{2}(1-\cos\phi_i) + \frac{kx^2}{2}. \end{aligned}$$

Note: ideal world: no friction/demping!

Dissipation:

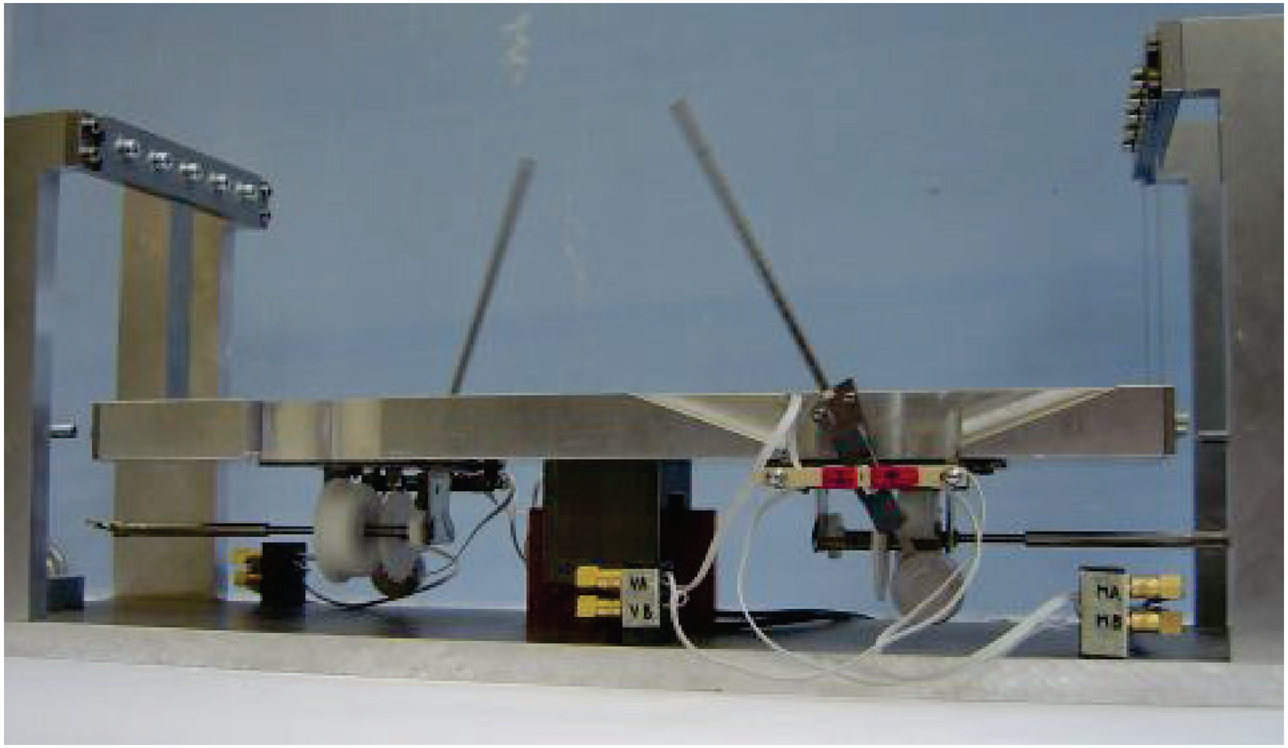
$$\dot{V} = -d\dot{x}^2 \leq 0.$$

La'Salle principle $\Longrightarrow$ all solutions tend to the set

$$\phi_1 = -\phi_2, \quad \dot{\phi}_1 = -\dot{\phi}_2, \quad x = 0, \quad \dot{x} = 0.$$

Note: ideal world: no friction/demping

Experimental setup (1)



Two metronomes supported by a platform which can translate horizontally

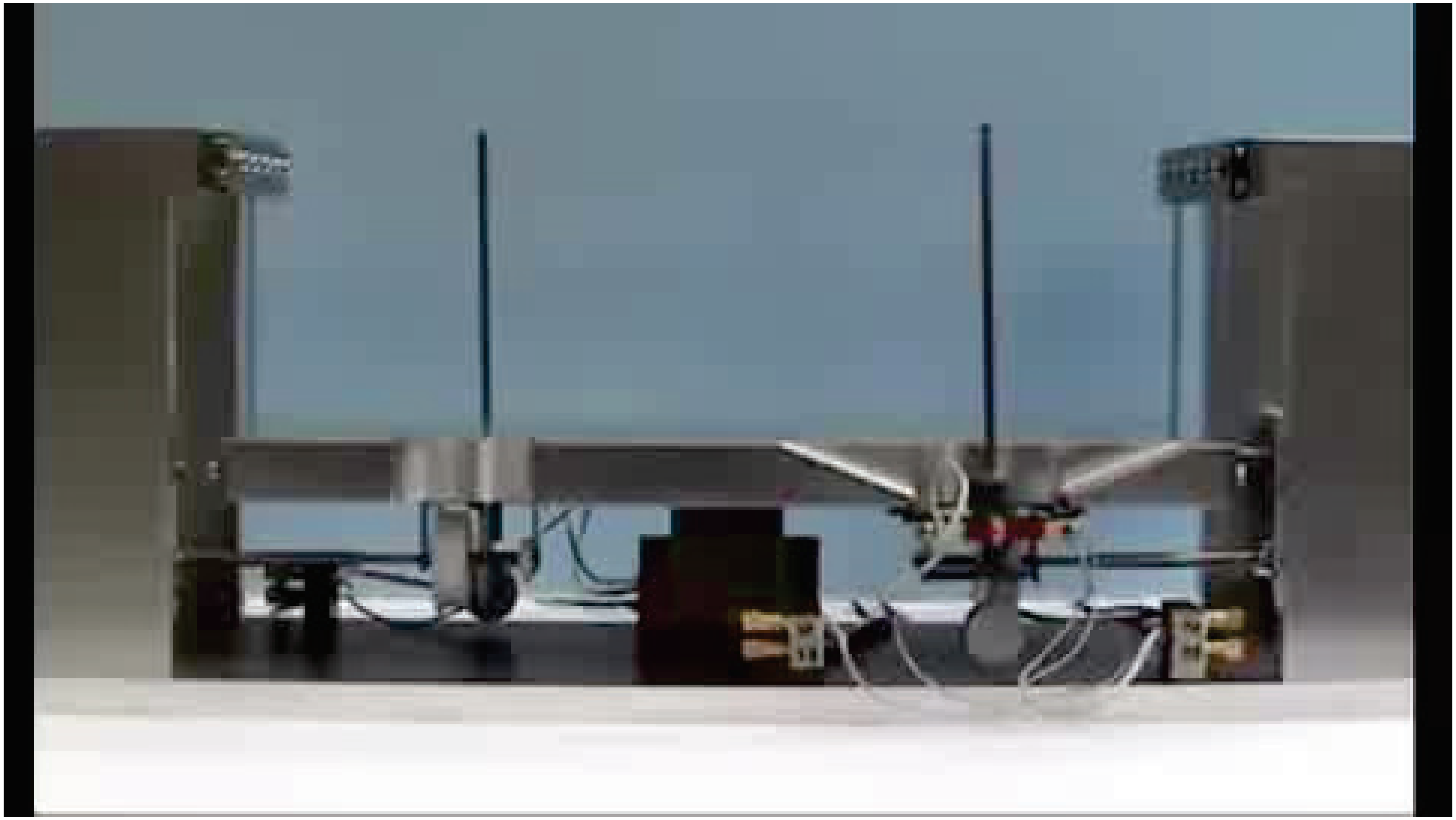
Dimensionless equations of motion

$$\theta_i'' + (1 \pm \Delta)^2 \cos\theta_i \, y'' + (1 \pm \Delta)^2 \sin\theta_i + d_i\theta_i' - \mathrm{f}(\theta_i, \theta_i') = 0$$

$$y'' + \Omega^2 y + 2\Omega\xi y' + \sum_{i=1}^{2} \beta_i (1 \pm \Delta)^{-2} (\cos\theta_i \, \theta_i'' - \sin\theta_i \, \theta_i'^2) = 0$$

θ_i	angle metronomes			
y	horizontal displacement platform			
Δ	scaled frequency difference metronomes	0	-	10^{-2}
β_i	ratio mass pendulum and platform	10^{-3}	-	10^{-1}
d_i	damping metronomes	$1 \cdot 10^{-2}$		
$\mathrm{f}(\theta_i, \theta_i')$	escapement mechanism metronomes			
Ω	eigenfrequency platform	0.9	-	1.2
ξ	damping factor platform	5%	-	12%

Anti phase synchronization



$$\Delta = -1.3 \cdot 10^{-2}, \quad \beta = 1.7 \cdot 10^{-2}, \quad \Omega = 1.0, \quad \xi = 12\%$$

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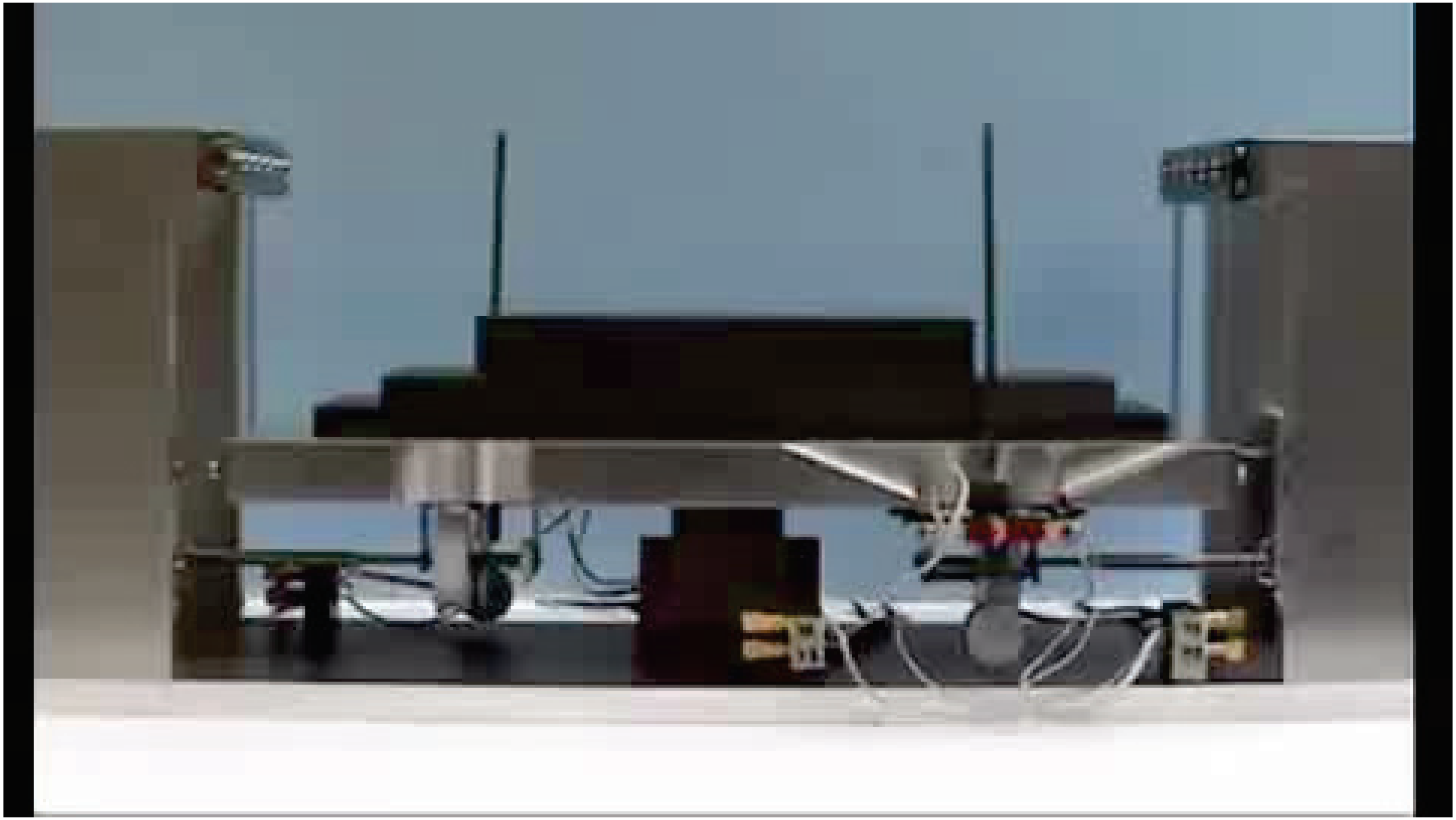
In phase synchronization



$$\Delta = -1.3 \cdot 10^{-2}, \quad \beta = 5.9 \cdot 10^{-3}, \quad \Omega = 0.95, \quad \xi = 7.6\%$$

Can Huygens’ sync occur in other oscillators?

The answer is **yes**!

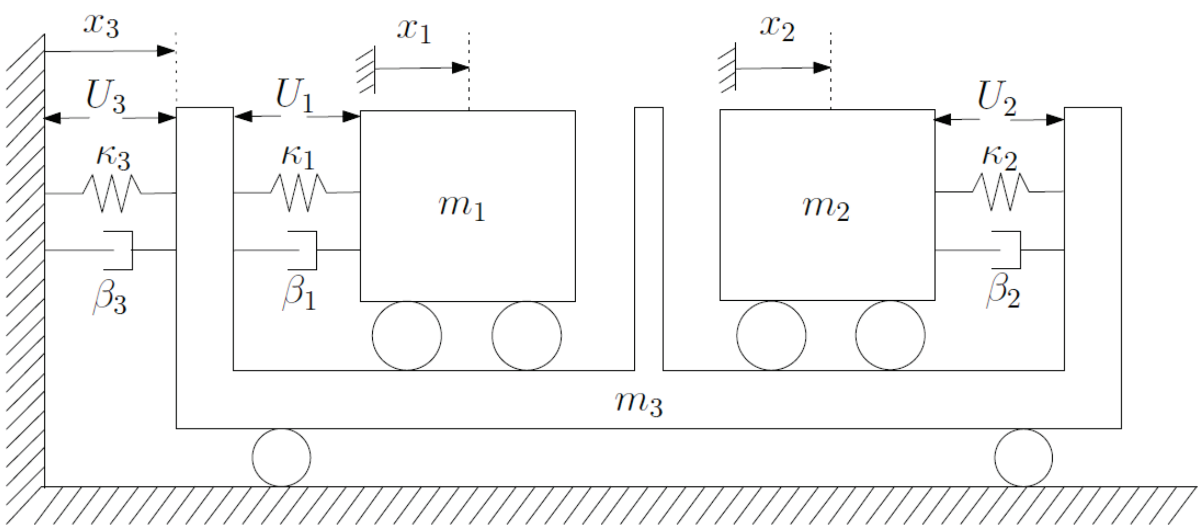
The Huygens’ experiment**:** two metronomes coupled by a platform which can translate horizontally.

However:

> The experimental results depend on the escapement mechanism of the metronomes.
>
> It is not possible to modify the inherent dynamics of the system.

A new experimental setup

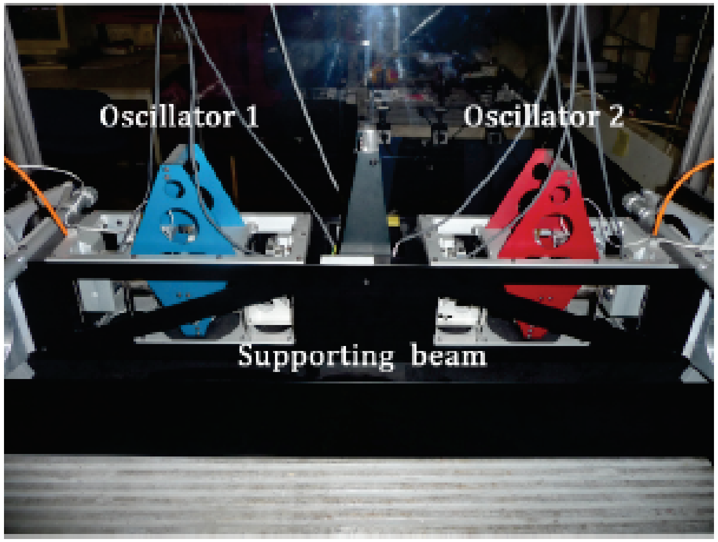
- An experimental platform (closely related with Huygens' experiment)
- Fully actuated dynamics for oscillators and platform

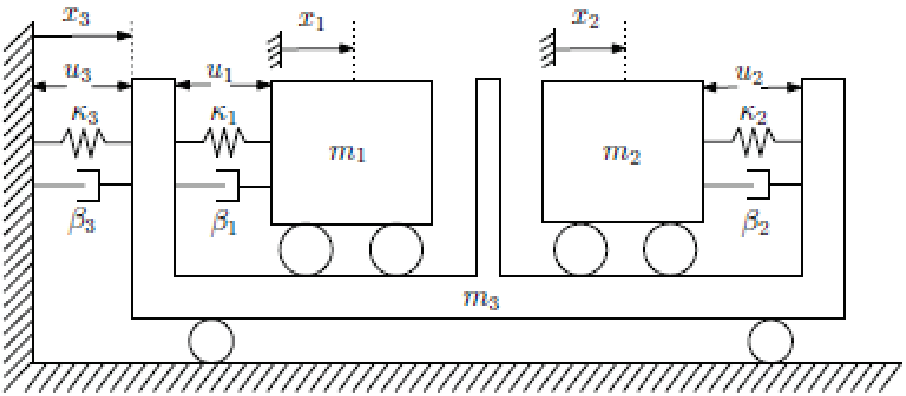


The U_i's are actuator forces

Remark: RRT (Huygens) vs TTT

The experimental setup





$$
\begin{aligned}
\ddot{x}_1 &= -\omega_1^2(x_1 - x_3) - 2\zeta_1\omega_1(\dot{x}_1 - \dot{x}_3) + \frac{1}{m_1}\mathbf{u}_1 \\
\ddot{x}_2 &= -\omega_2^2(x_2 - x_3) - 2\zeta_2\omega_2(\dot{x}_2 - \dot{x}_3) + \frac{1}{m_2}\mathbf{u}_2 \qquad (1) \\
\ddot{x}_3 &= \sum_{i=1}^{2} \mu_i \left[\omega_i^2(x_i - x_3) + 2\zeta_i\omega_i(\dot{x}_i - \dot{x}_3) - \frac{1}{m_i}\mathbf{u}_i\right] \\
&\quad -\omega_3^2 x_3 - 2\zeta_3\omega_3\dot{x}_3 + \frac{1}{m_3}\mathbf{u}_3
\end{aligned}
$$

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How to modify the dynamical behaviour of the setup?

- By using *virtual dynamics*:

  1. to cancel the original dynamics
  2. to introduce the desired dynamics

Design controls

$$
\begin{aligned}
u_i &= m_i\left[\omega_i^2(x_i - x_3) + 2\zeta_i\omega_1(\dot{x}_i - \dot{x}_3) + \ddot{x}_{id}\right] \\
u_3 &= m_3\left[\sum_{i=1}^{2}(\mu_i\ddot{x}_{id}) + \omega_3^2 x_3 + 2\zeta_3\omega_3\dot{x}_3 + \ddot{x}_{3d}\right]
\end{aligned}
$$

such that

In closed loop:

$$
\ddot{x}_i = \ddot{x}_{id}, \quad i = 1, \ldots, 3 \tag{2}
$$

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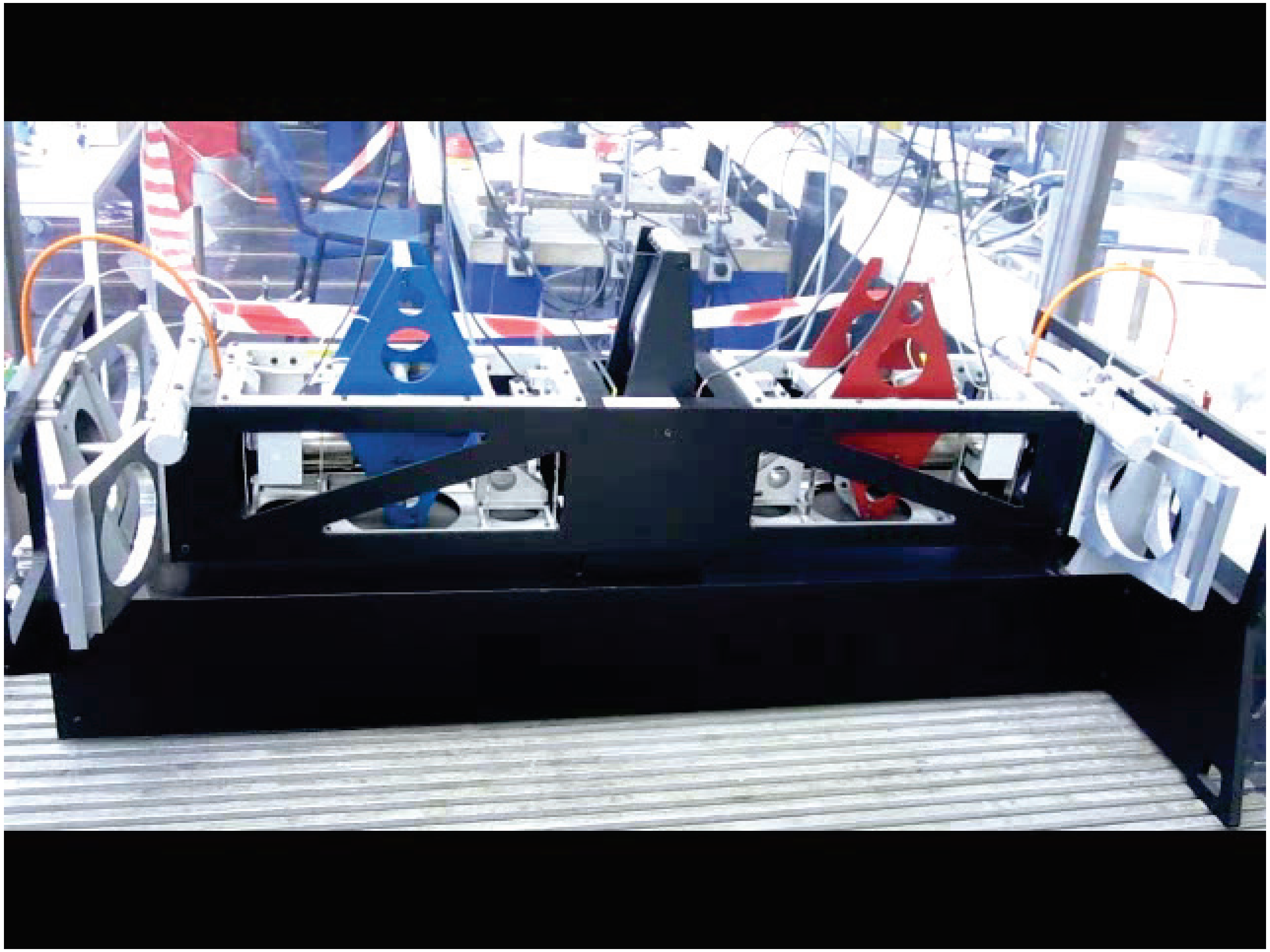
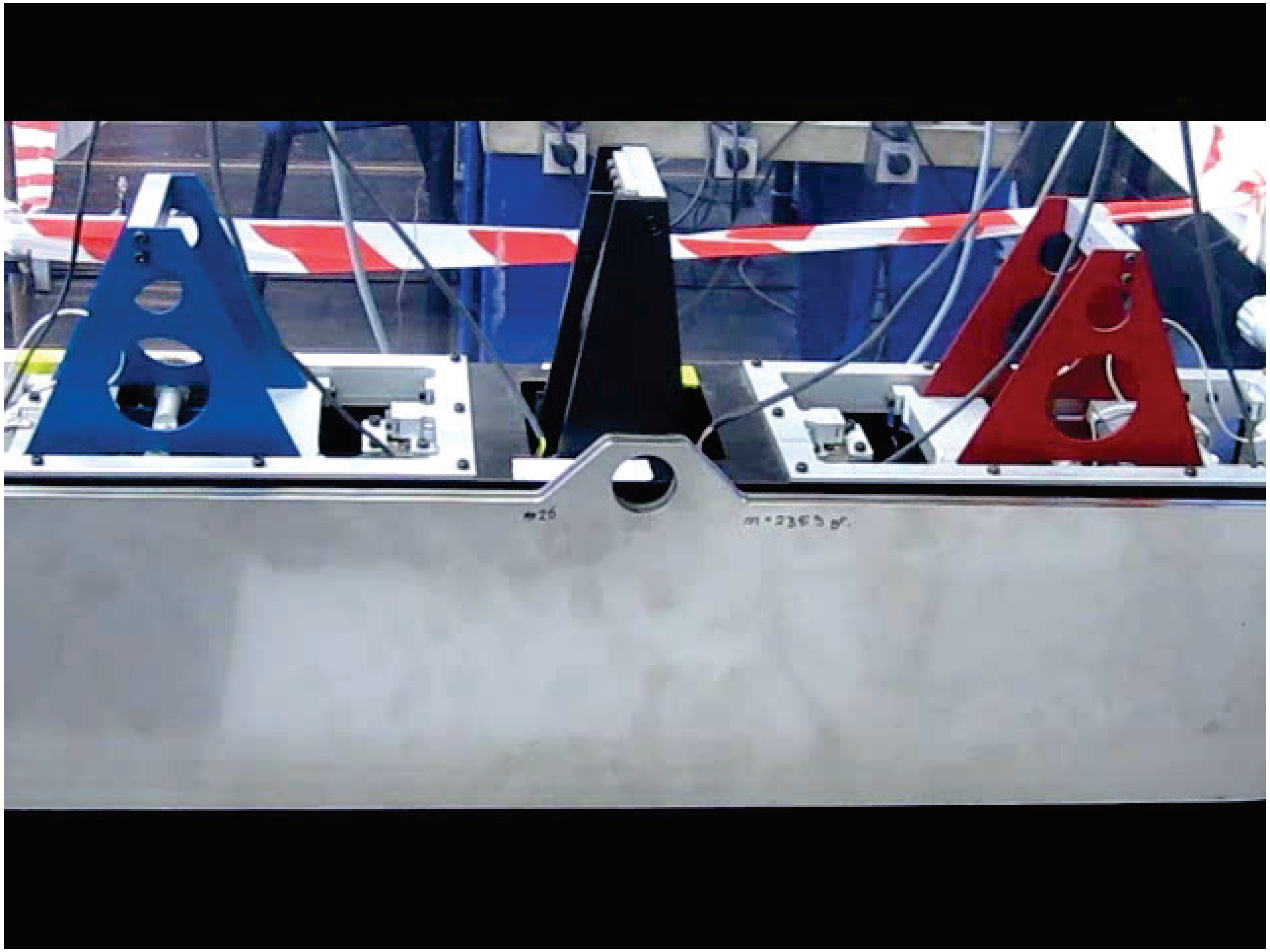
In phase synchronization



Coupled van der Pol oscillators

Anti phase synchronization



Huygens’ pendulum clocks

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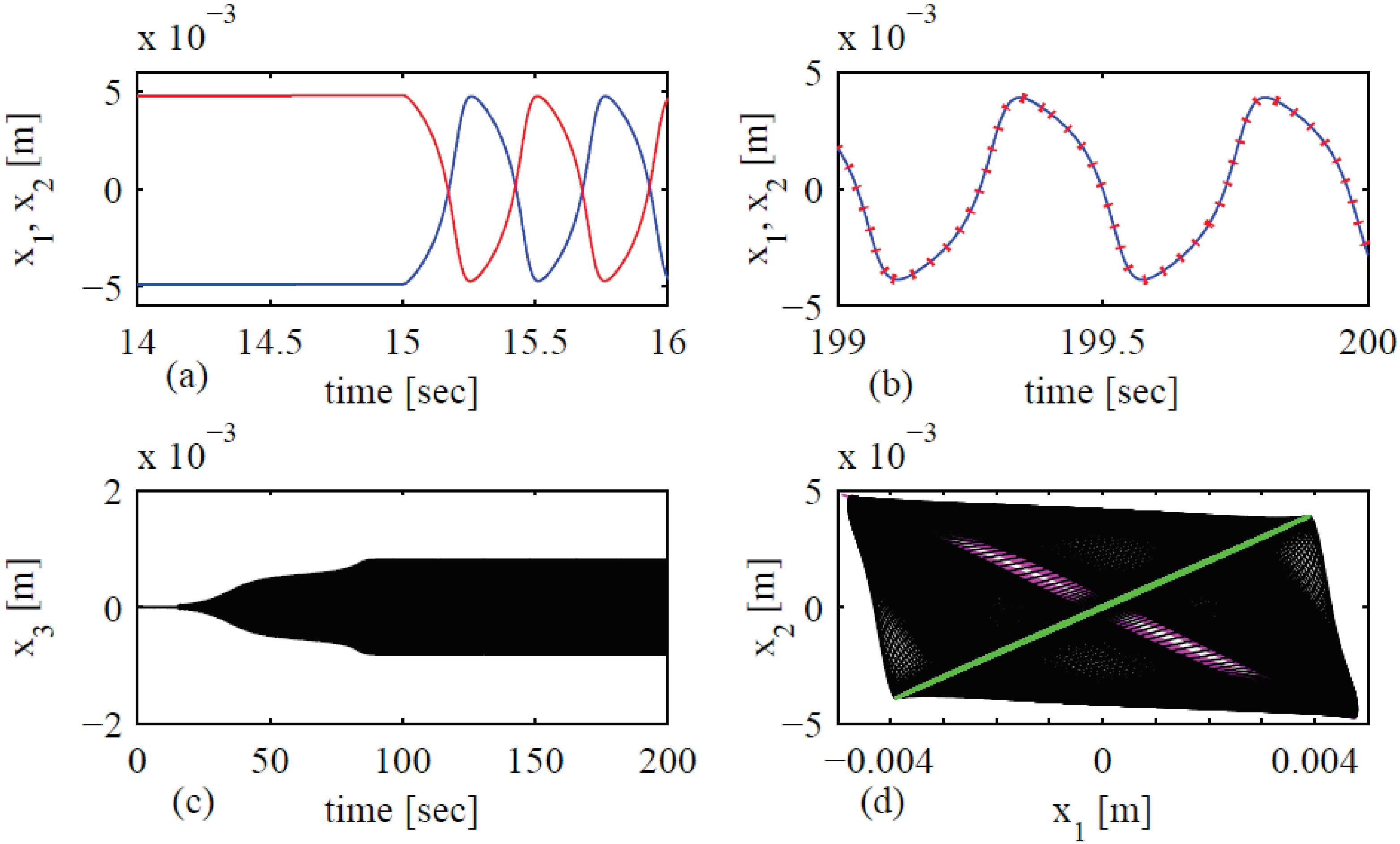
Anti-phase sync of two Van der Pol oscillators



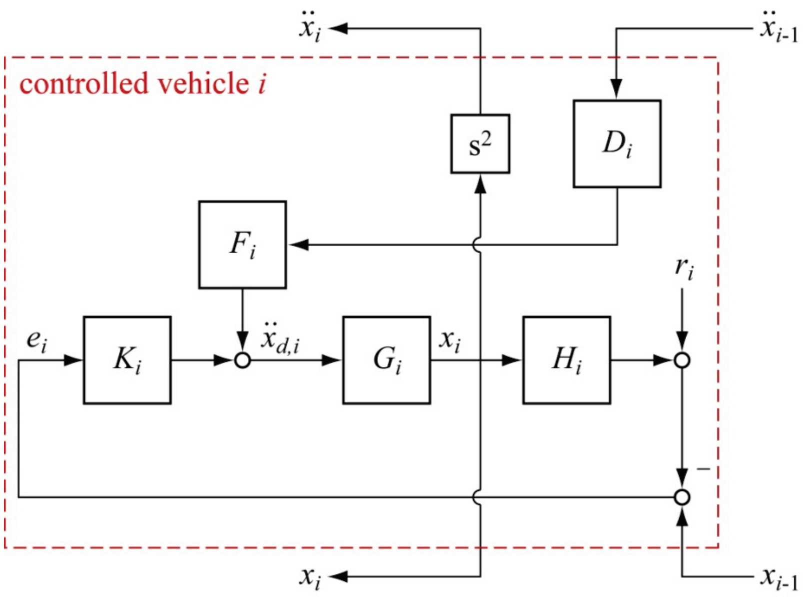
Synchronization between two cars

- Coordinated Adaptive Cruise Control (C-ACC)
- Beyond pendulum clocks- automotive
- Improve ACC using wireless communication of acceleration/deceleration

CACC: necessary hard/soft-ware

CACC: control structure

- CACC structure: add feedforward of accelaration of preceding vehicle



- CACC feedforward filter (inverse system):

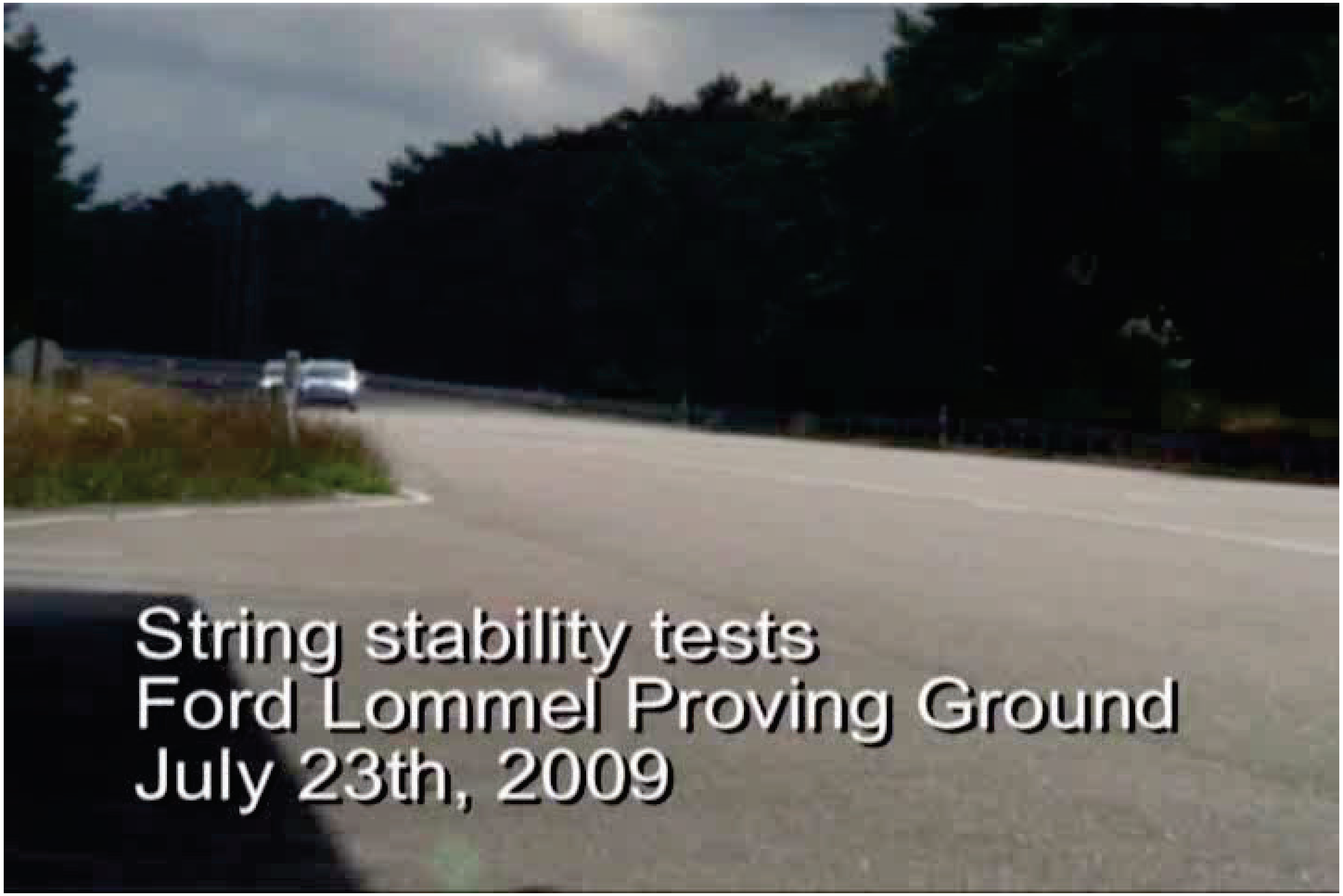
$$F_i(s) = \left(H_i(s)G_i(s)s^2\right)^{-1}$$

- Communication delay:

$$D_i(s) = e^{-\theta_{D,i}s}$$

Note: vehicle model is simply a double integrator!

Synchronization of two cars



Part 3: Synchronization in a network of oscillators

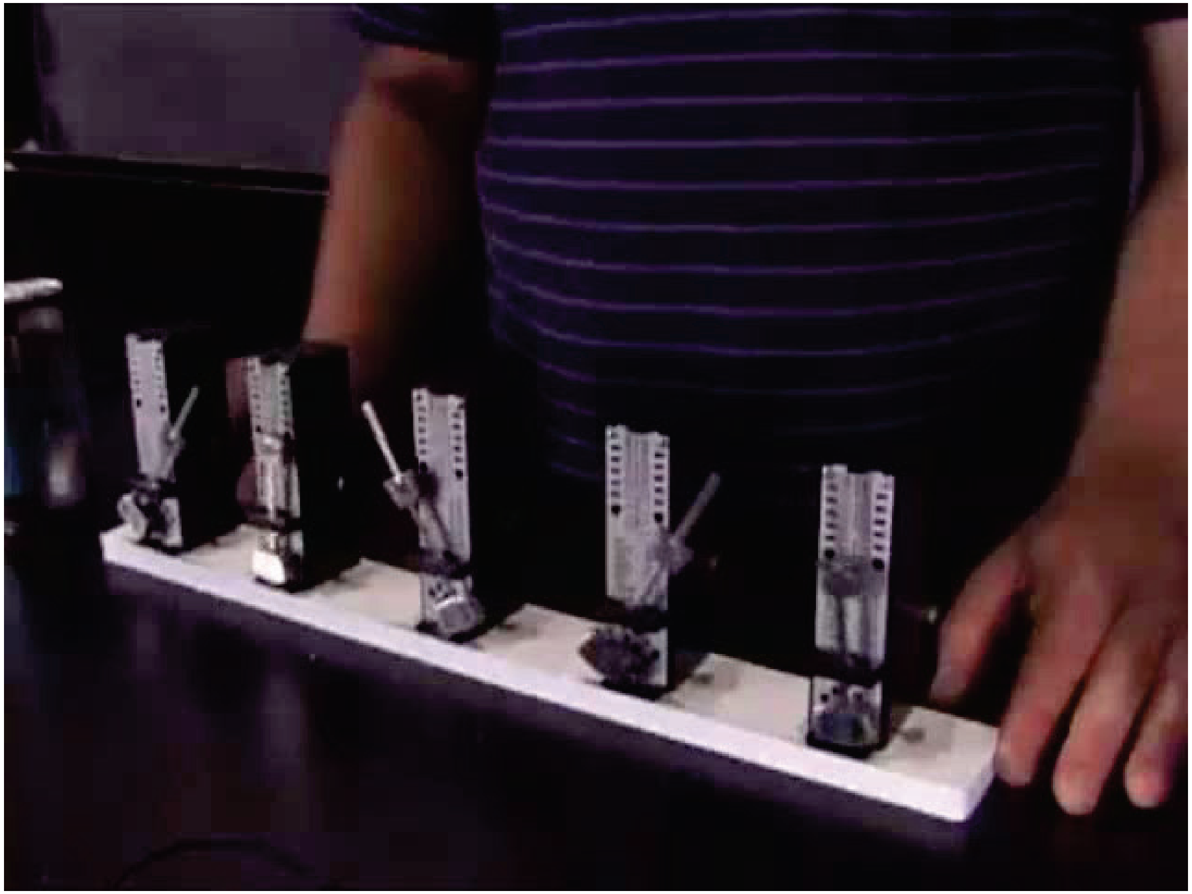
Networks are almost everywhere:

- **Traffic**
- Internet
- Biology, nature
- Medicine, **brains**, heart, diabetes
- Engineering, **robotics, master-slave systems**

Coupling: how to define this?

When will cells –**partially**- synchronize?

The “network” Huygens’ experiment



Network synchronization – Humans (1)

Network synchronization – Humans (2)

Network synchronization – Animals

Network of coupled systems - Preliminaries

- **Semipassive systems**
- **Convergent systems**
- **Why are these properties important?**
- **Types of synchronization in the network:**
  - **Partial**
  - **Full**

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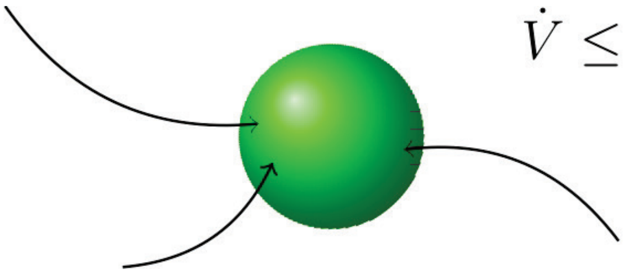
Semipassive systems: definition

- **Consider the input-output system**

(1)
$$\dot{x} = f(x,u), \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m$$
$$y = h(x), \quad y \in \mathbb{R}^m$$



- **The system (1) is (strictly) semipassive in $\mathcal{D} \in \mathbb{R}^n$ if**
  - $\exists$ **a $\mathcal{C}^1$ smooth function $V : \mathcal{D} \to \mathbb{R}_+$ such that**
  - $\dot{V} \leq y^\top u - H(x)$ **where**
  - $H(x)$ **is nonnegative (positive) outside some ball $\mathcal{B} \subset \mathbb{R}^n$**

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Semipassive systems: properties

- **A strictly semipassive system behaves similar to a passive system for large enough** $\| x \|$
- **'no free (or at least bounded) energy generation'**

- **A strictly semipassive system interconnected by any feedback** $u = \phi(y)$ **satisfying** $y^\top \phi(y) \leq 0$ **for all** y **has ultimately bounded solutions**

Convergent systems (Demidovich 1950s)

- **Consider the system**

(2) $$\dot{z} = g(z, w), \quad z \in \mathbb{R}^d, w \in \mathcal{W} \subset \mathbb{R}^m, g \in \mathcal{C}^1 : \mathbb{R}^d \times \mathcal{W} \to \mathbb{R}^d$$

The system (2) is convergent if

- **all solutions** $z(t)$ **of (2) are well-defined for**
  - **all** $t \in [t_0, +\infty)$ **,**
  - **all** $t_0 \in \mathbb{R}, z(t_0) \in \mathcal{Z}$ **,**
  - **all** $w(t) \in \mathcal{W}$ **, and**
- **there exists a unique solution** $\bar{z}_w(t)$ **which satisfies**
  - $\bar{z}_w(t)$ **is defined and bounded for all** $t \in (-\infty, +\infty)$**,**
  - $\bar{z}_w(t)$ **is asymptotically stable in** $\mathbb{R}^d$

Convergent systems (Cont'd)

- **Properties of a convergent system:**
  - **the system "forgets" its initial conditions, i.e. for any input** $w(t)$ **,** $z_1(t) - z_2(t) \to 0$ **as** $t \to \infty$
  - **for any two inputs** $w_1(t), w_2(t)$ **satisfying** $w_1(t) - w_2(t) \to 0$ **as** $t \to \infty$**, it holds that** $z_{w_1}(t) - z_{w_2}(t) \to 0$ **as** $t \to \infty$

- **Sufficient condition for a system to be convergent (Demidovich):**
  $\exists P = P^\top > 0$ **such that the matrix**

$$\frac{\partial g}{\partial z}^\top (z,w)P + P\frac{\partial g}{\partial z}(z,w)$$

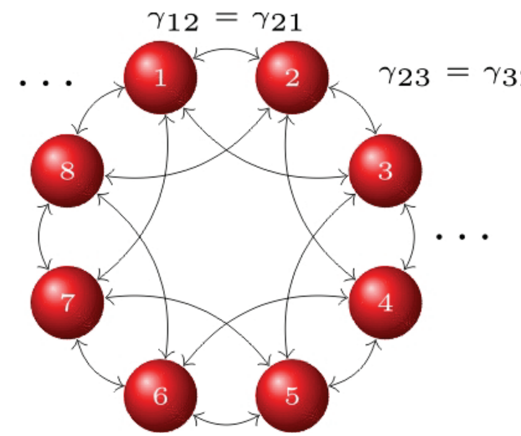
has negative eigenvalues for all $z \in \mathbb{R}^d, w \in \mathcal{W}$

Diffusively coupled systems

- **Diffusive coupling**

  $u_i = -\gamma_{i1}(y_1 - y_i) - \gamma_{i2}(y_2 - y_i) - \ldots - \gamma_{ik}(y_k - y_i)$

  where $\gamma_{ij} = \gamma_{ji} \geq 0$



- **Let** $u = col(u_1, \ldots, u_k), y = col(y_1, \ldots, y_k)$, **then**

  $u = -\Gamma y$ **with**

$$\Gamma = \begin{bmatrix} \sum_{j=2}^{k} \gamma_{1j} & -\gamma_{12} & \cdots & -\gamma_{1k} \\ -\gamma_{21} & \sum_{j=1, j\neq 2}^{k} \gamma_{2j} & \cdots & -\gamma_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_{k1} & -\gamma_{k2} & \cdots & \sum_{j=1}^{k-1} \gamma_{kj} \end{bmatrix}$$

Full synchronization

- **Consider k diffusively coupled identical systems in normal form**

$$\dot{y}_i = f(y_i, z_i) + u_i$$

(3) $$\dot{z}_i = g(z_i, y_i)$$

$$u_i = \sum_{j=1, j\neq i}^{k} \gamma_{ij}(y_j - y_i)$$

Theorem

Suppose that:

- Each system *(3)* is strictly semipassive;
- Each subsystem $\dot{z}_i = g(z_i, y_i)$ is a convergent system;
- The eigenvalues of Γ are $0 = \lambda_1 < \lambda_2 \le \lambda_3 \le \ldots \le \lambda_k$.

Then there exists a positive constant $\bar{\lambda}$ such that if $\lambda_2 \ge \bar{\lambda}$ there exists a globally asymptotically stable subset of the set

$$\mathcal{A} = \{y_i \in \mathbb{R}^m, z_i \in \mathbb{R}^{n-m} : \ y_i = y_j, z_i = z_j, \forall i, j = 1, 2, \ldots, k\}$$

Synchronization of neuronal network

- **Neurons interacting via electrical synapses, i.e. diffusive coupling**

$$I_{syn,pre} = g_{syn} \cdot (V_{post} - V_{pre})$$
$$I_{syn,post} = g_{syn} \cdot (V_{pre} - V_{post})$$

- **Neuron models that are proven to be strictly semipassive:**
  - **Hodgkin-Huxley**
  - **Morris-Lecar**
  - **FitzHugh-Nagumo**
  - **Hindmarsh-Rose**

- **All biophysically plausible models seem to be semipassive!**

Remark: no chemical interactions included

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The Hindmarsh-Rose neuron

- Braib

- **Brain synchrony, why? Epilepsy!?**
- **The Hindmarsh-Rose (H-R) model developed in the 60s**
- **equations:**

$$\begin{cases} \dot{y} = -y^3 + 3y + 5z_1 - z_2 - 8 + I + u \\ \dot{z}_1 = -y^2 - 2y - z_1 \\ \dot{z}_2 = 0.005(4(y + 1.1180) - z_2) \end{cases}$$

$y = \mathit{Membrane\ potential}$

$z_1 = \mathit{Internal\ variable}$

$z_2 = \mathit{Slowly\ varying\ current}$

$I = \mathit{Bifurcation\ parameter}$

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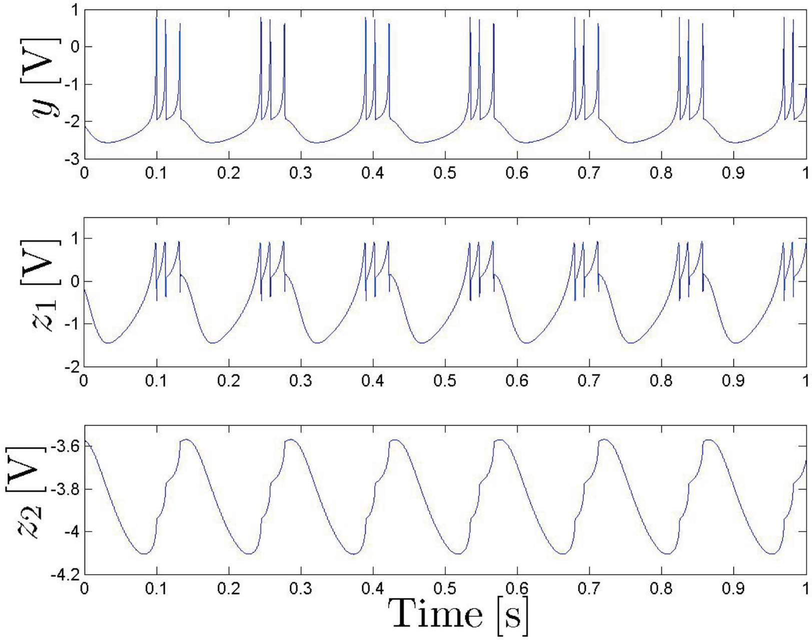
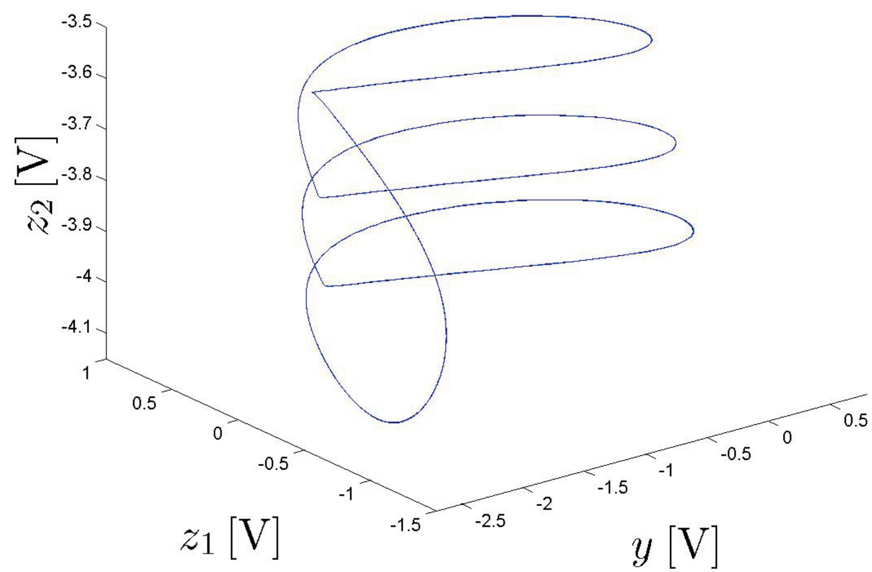
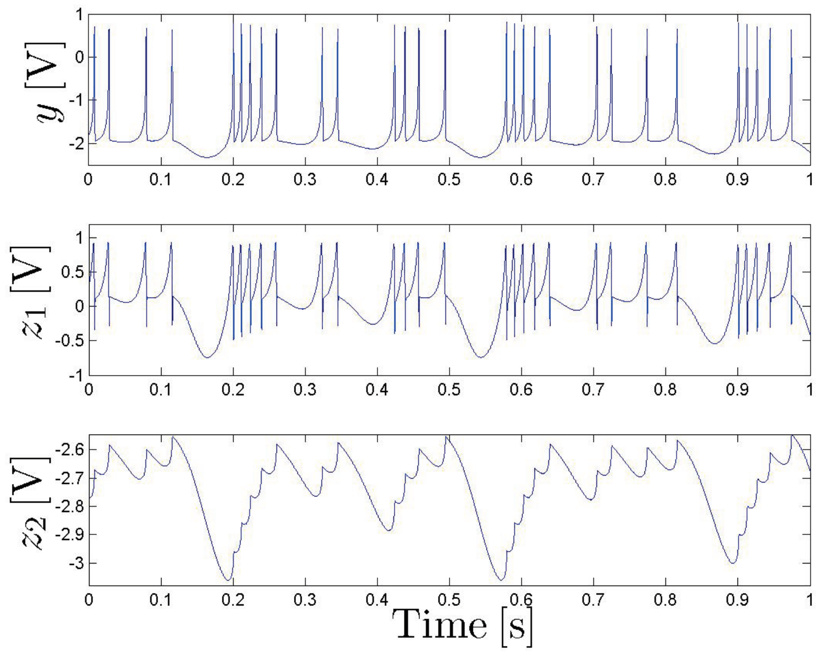
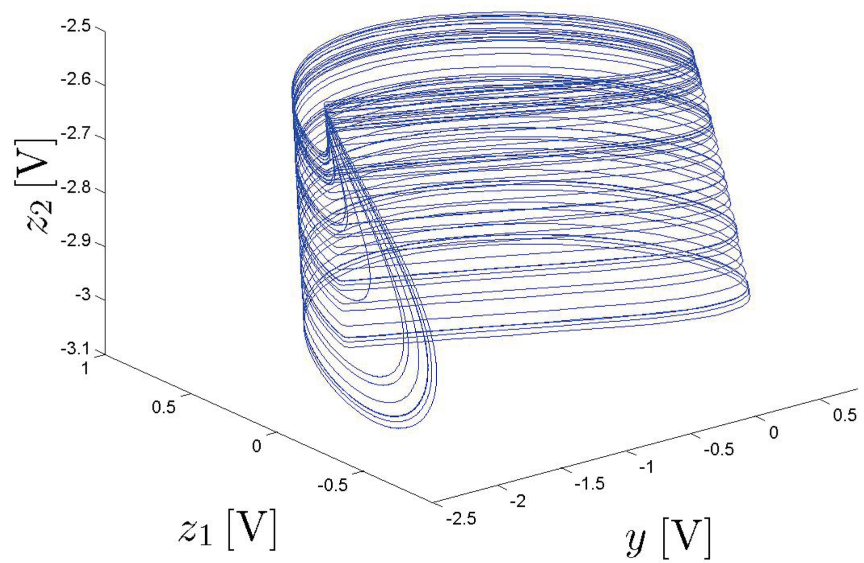
H-R dynamics: bursting



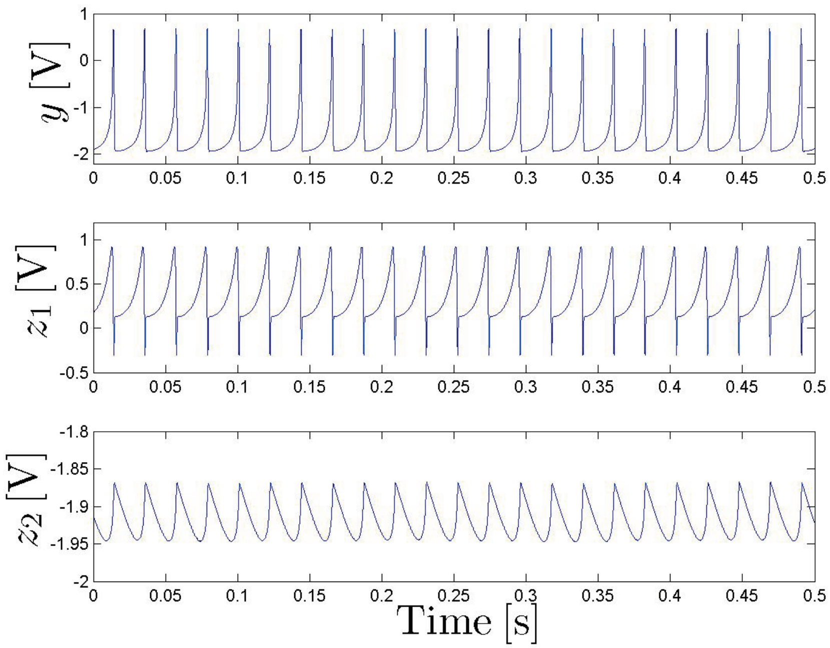


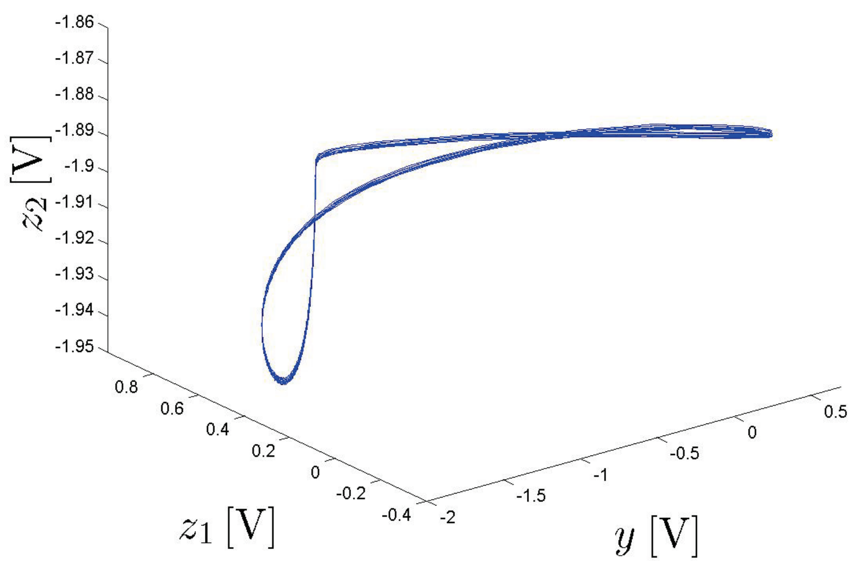
H-R dynamics: chaotic bursting





H-R dynamics: tonic spiking

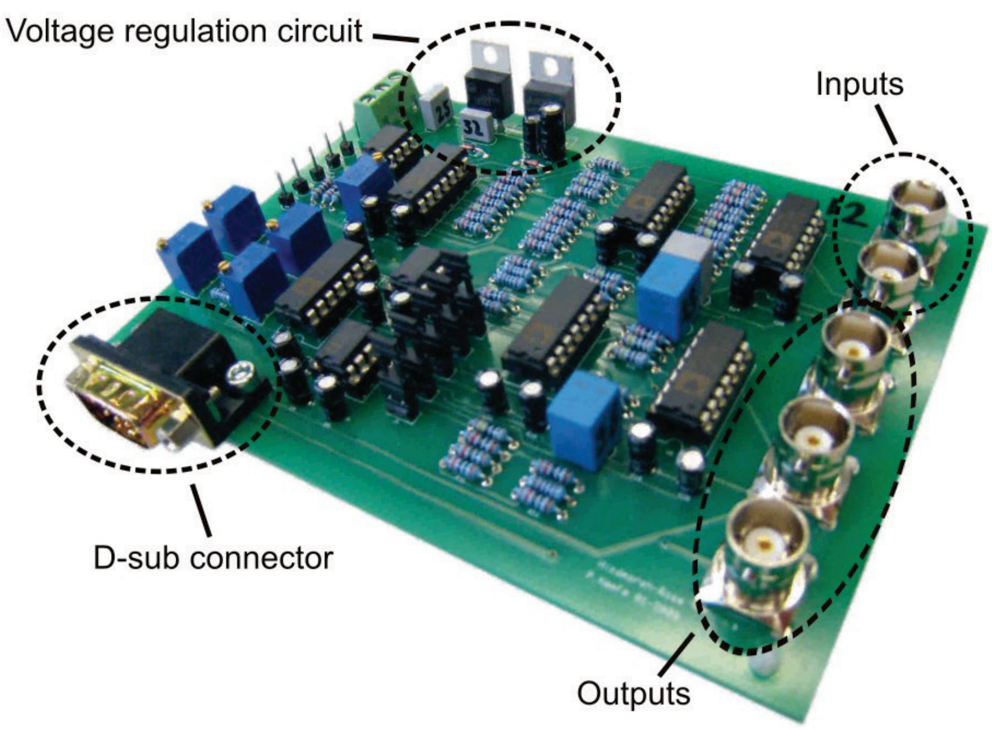




Electronic equivalent of the H-R neuron

- **3 Integrating circuits**
- **2 Multiplier circuits**
- **Voltage regulation circuit**
- **2 Inputs:**
  - **Bifurcation Parameter** I
  - **Coupling function** u
- **3 'Outputs':**
  - y, z_1 **and** z_2 **-state**

- **Components:**
  - **Operational-amplifiers:**
    6 of type (AD)OP482
    2 of type AD633
    1 of type AD587
  - **58 Metal film resistors**
  - **5 Variable resistors**
  - **23 Capacitors**
  - **2 Voltage regulators**
  - **1 D-sub connector**



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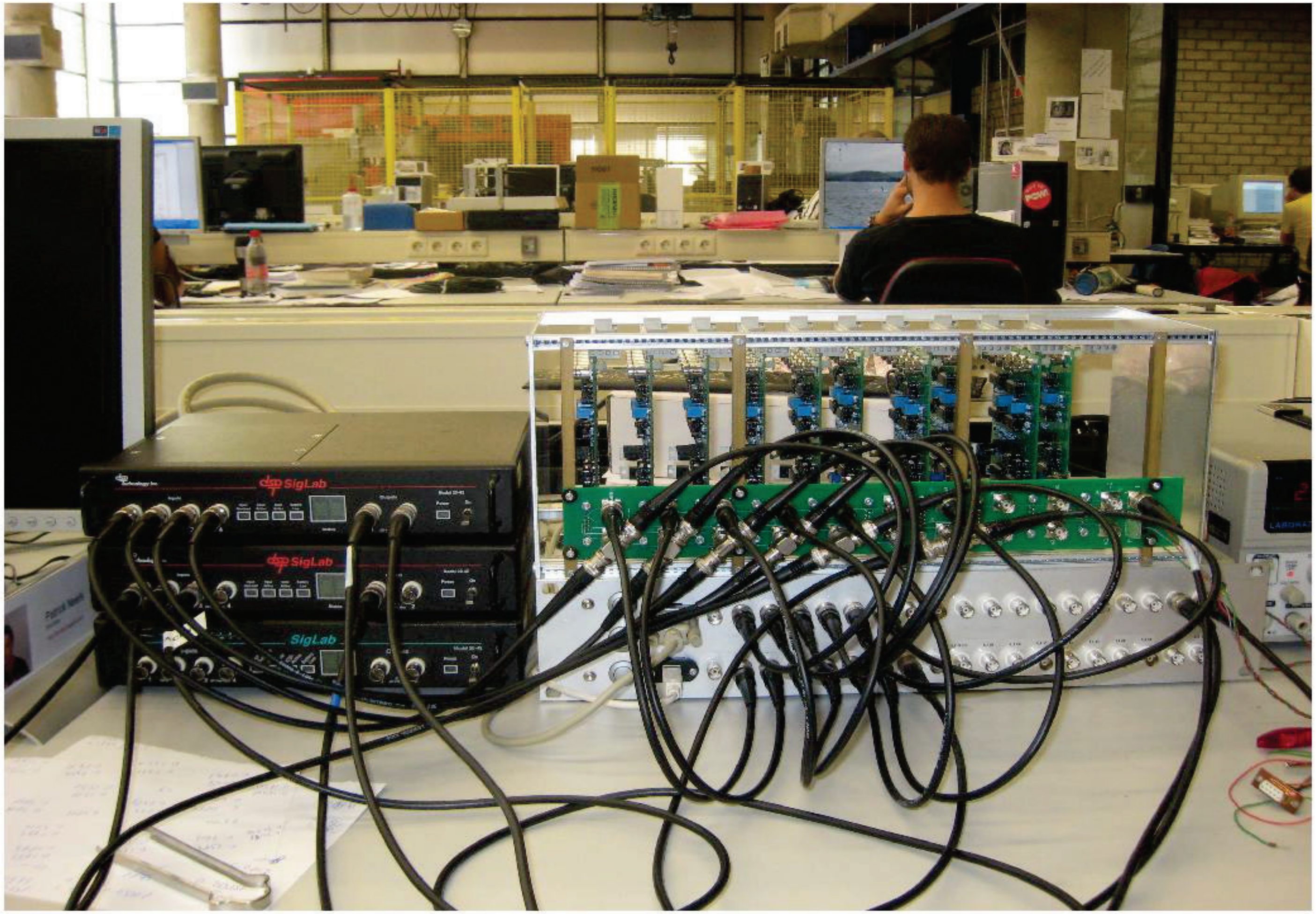
Experimental H-R neural network

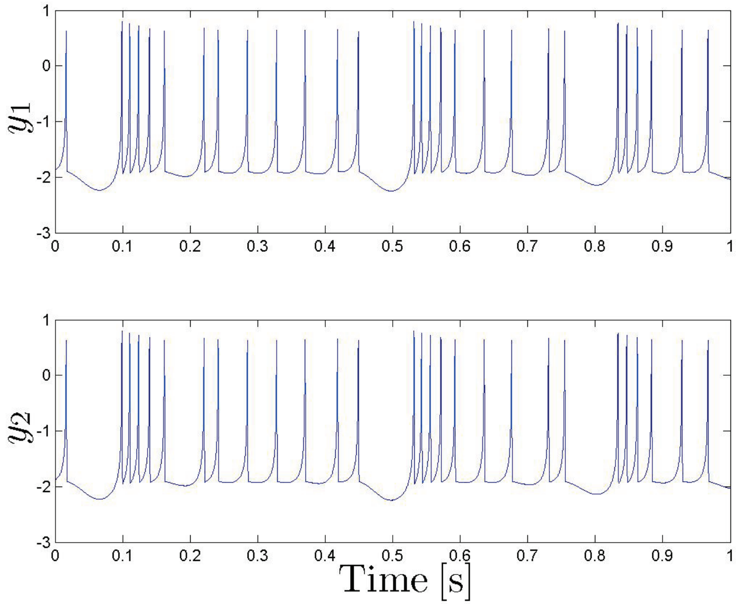


Synchronization of 2 coupled H-R neurons

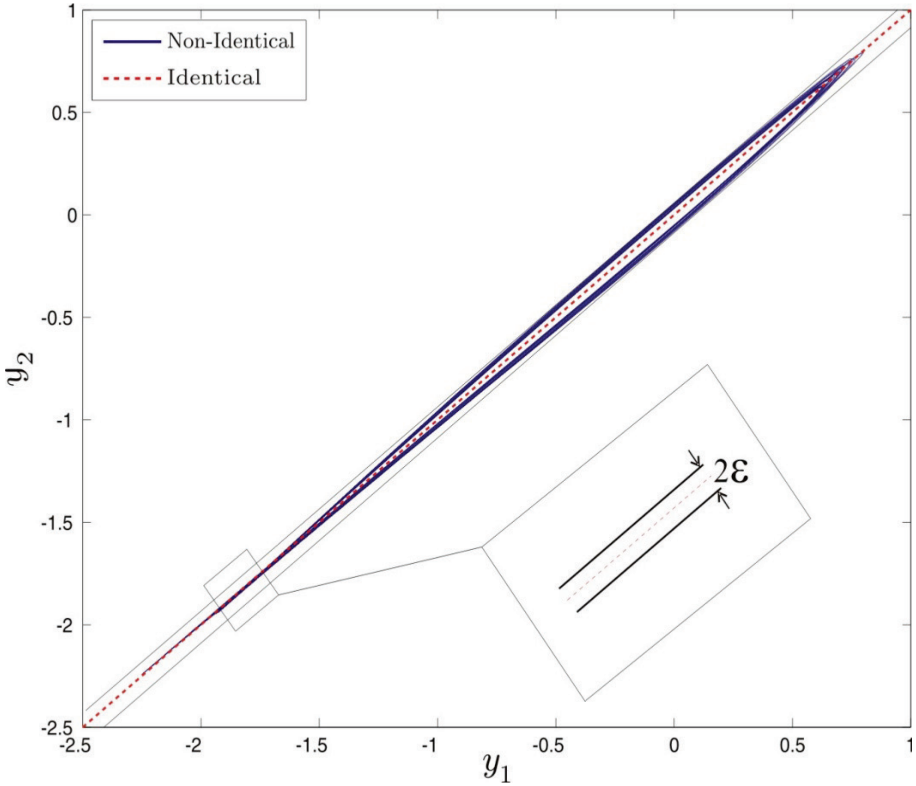
- **Non identical systems!**
- **Practical synchronization:**

$$\limsup_{t\to\infty} \| x_i(t) - x_j(t) \| \leq \varepsilon$$

- **Uniform coupling strength γ**
- **Error in phase-plane**

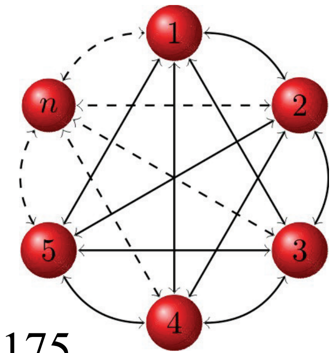


- **Synchronization:**
  - **Theoretically:** $\gamma_{\min} = 0.5$
  - **Experimentally:** $\gamma_{\min} = 0.6$



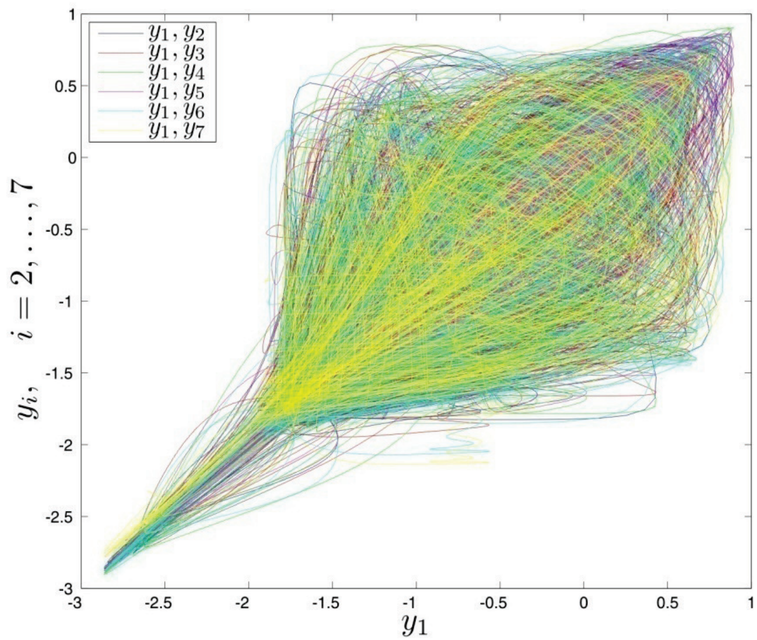
Regular topologie

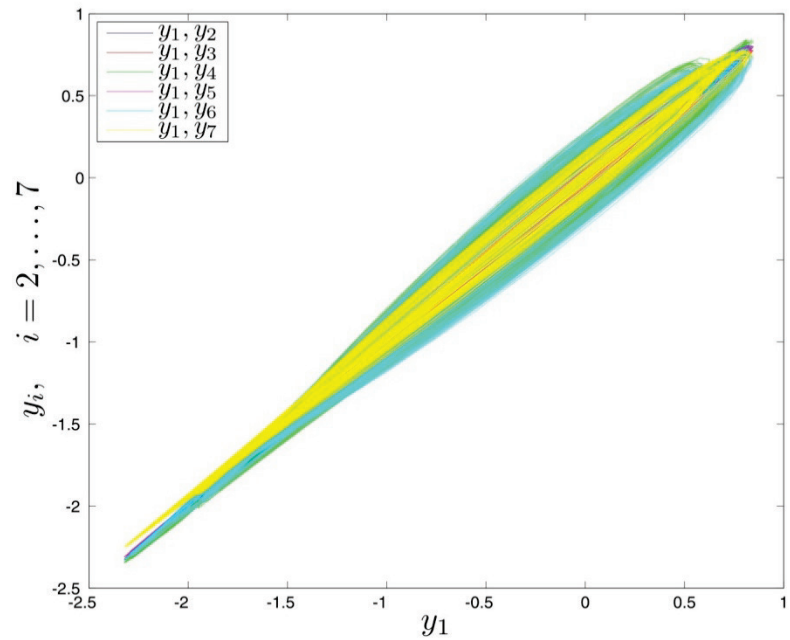
- **All-to-all (*n* = *7*):**
  - **All nodes have equal node degree**



- **No synchronization:** $\gamma = 0.1$
- **Maximum error:** $\varepsilon = 4.3681$

- **Full synchronization:** $\gamma = 0.175$
- **Maximum error:** $\varepsilon = 0.4873$

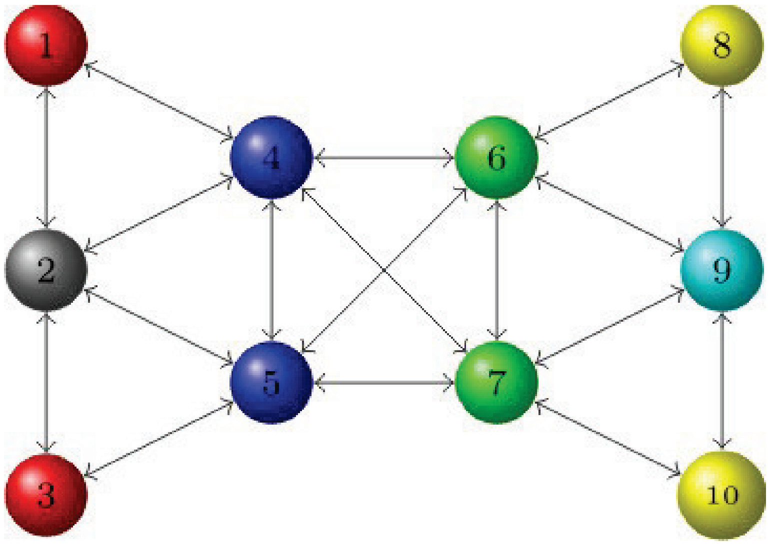




Partial synchronization (6)

- **Sandglass (*n* = 10):**
- **Partial sync. manifold:** $\gamma = 0.8$ **(full synchronization at:** $\gamma = 1.5$ **)**

$$\mathcal{A} = \left\{ x \in \mathbb{R}^{30} : x_1 = x_3 \neq x_4 = x_5 \neq x_6 = x_7 \neq x_8 = x_{10} \neq x_2 \neq x_9 \right\}$$

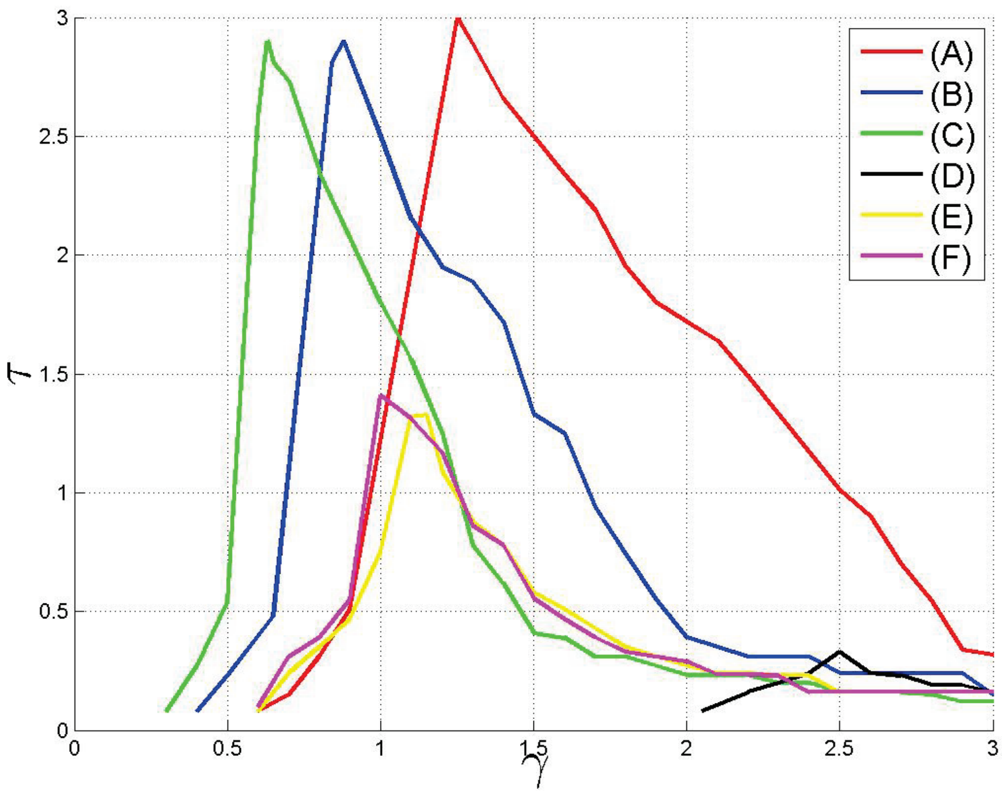


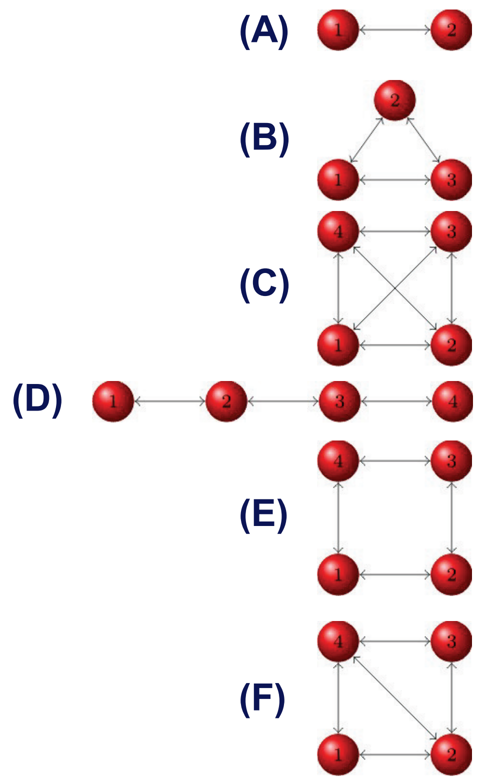
Synchronization (k - τ) diagram

- **Delayed coupling function:**

$$u_i = \sum_{j=1, j\neq i}^{k} \gamma_{ij}(y_j(t-\tau) - y_i(t-\tau)), \qquad \text{with } \tau > 0 \text{ the time-delay.}$$

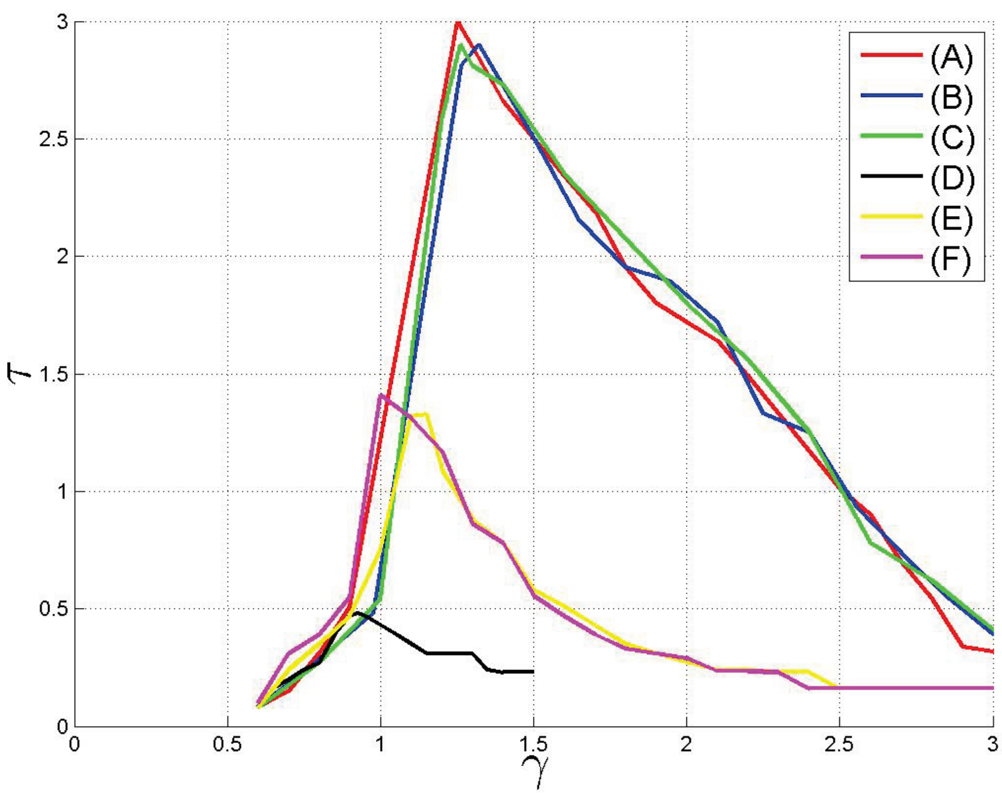
- **Synchronization regime enclosed by the lines and the γ-axis**

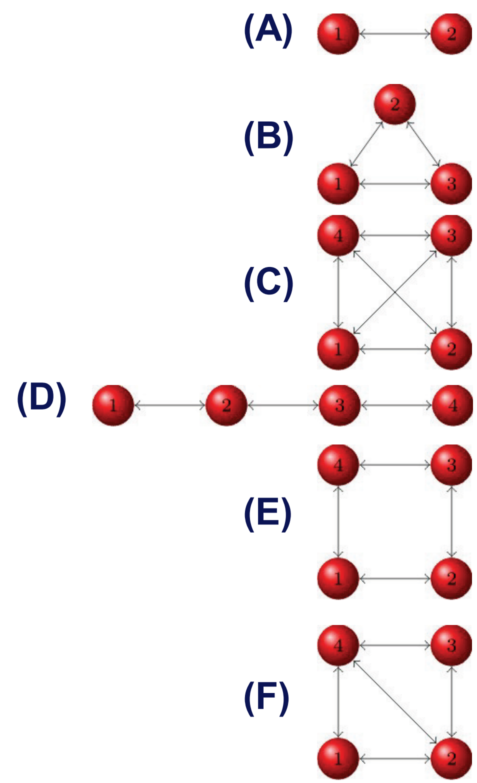




Scaled synchronization (k - τ) diagram

- **Diagram scaled by minimal coupling strength that holds for $\tau = 0$**
- **Apparent relation between the results based on path length**
  - **Networks (A), (B) and (C) have path length 1**
  - **Networks (E) and (F) have path length 2**
  - **Network (D) has path length 3**





Summary & Conclusions

I ♥

SYNC

Acknowledgements