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Control Systems 314
2018

Lecture 11: Effects of Zeroes and Additional Poles

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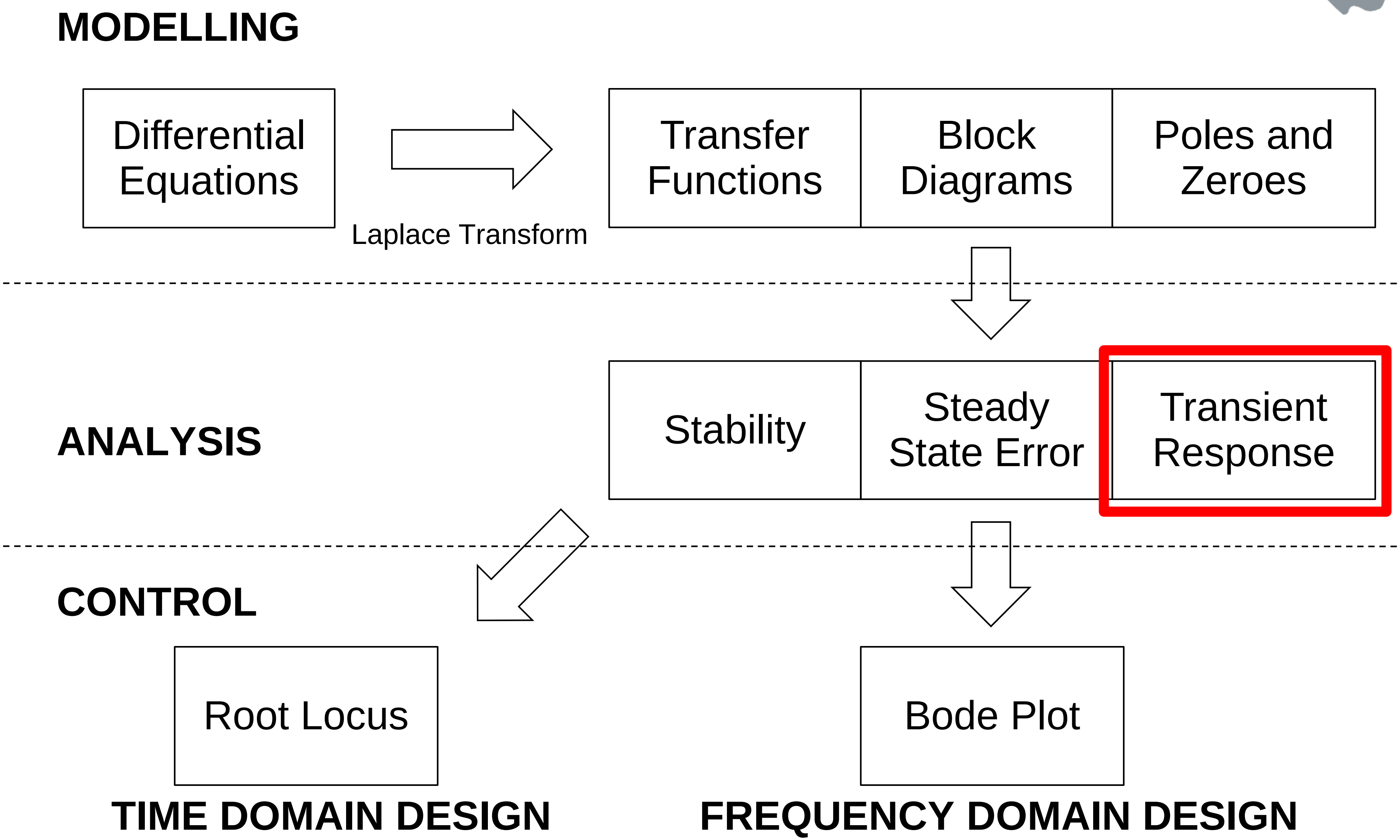
Lecture 11 Overview



Lecture 11 Overview

- Effect of Zeroes on Transient Response
  - Second-Order Transfer Function with One Zero
  - Overshoot versus Zero Location
- Effect of Additional Poles on Transient Response
  - Second-Order Transfer Function with Extra Pole
  - Rise Time versus Extra Pole Location
- Transient Response of Higher-Order Systems – Dominant Poles
  - Fast and Slow Poles
  - Effect of Zeroes

Lesing 11 Oorsig

- Effek van Zeros op Oorgangsverskynsels
  - Tweede-Orde Oordragsfunksie met Een Zero
  - Oorskiet versus Zero Ligging
- Effek van Addisionele Pool op Oorgangsverskynsels
  - Tweede-Orde Oordragsfunksie met Ekstra Pool
  - Stygtyd versus Ekstra Pool Ligging
- Oorgangsverskynsels van Hoër-Orde Stelsels – Dominante Pole
  - Vinnige en Stadige Pole
  - Effekte van Zeros

Second-Order Transfer Function with One Zero

- To plot the effect of a zero for a wide range of cases, the transfer function is written in a form with normalised time and zero locations

$$H(s) = \frac{\left(s/\alpha\zeta\omega_n\right)+1}{\left(s/\omega_n\right)^2 + 2\zeta\left(s/\omega_n\right)+1}$$

- The poles are located at $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$$= -\sigma \pm j\omega_d$$

- The zero is located at $s = -\alpha\zeta\omega_n = -\alpha\sigma$

- In other words, the real zero is α times the real part of the poles

Second-Order Transfer Function with One Zero

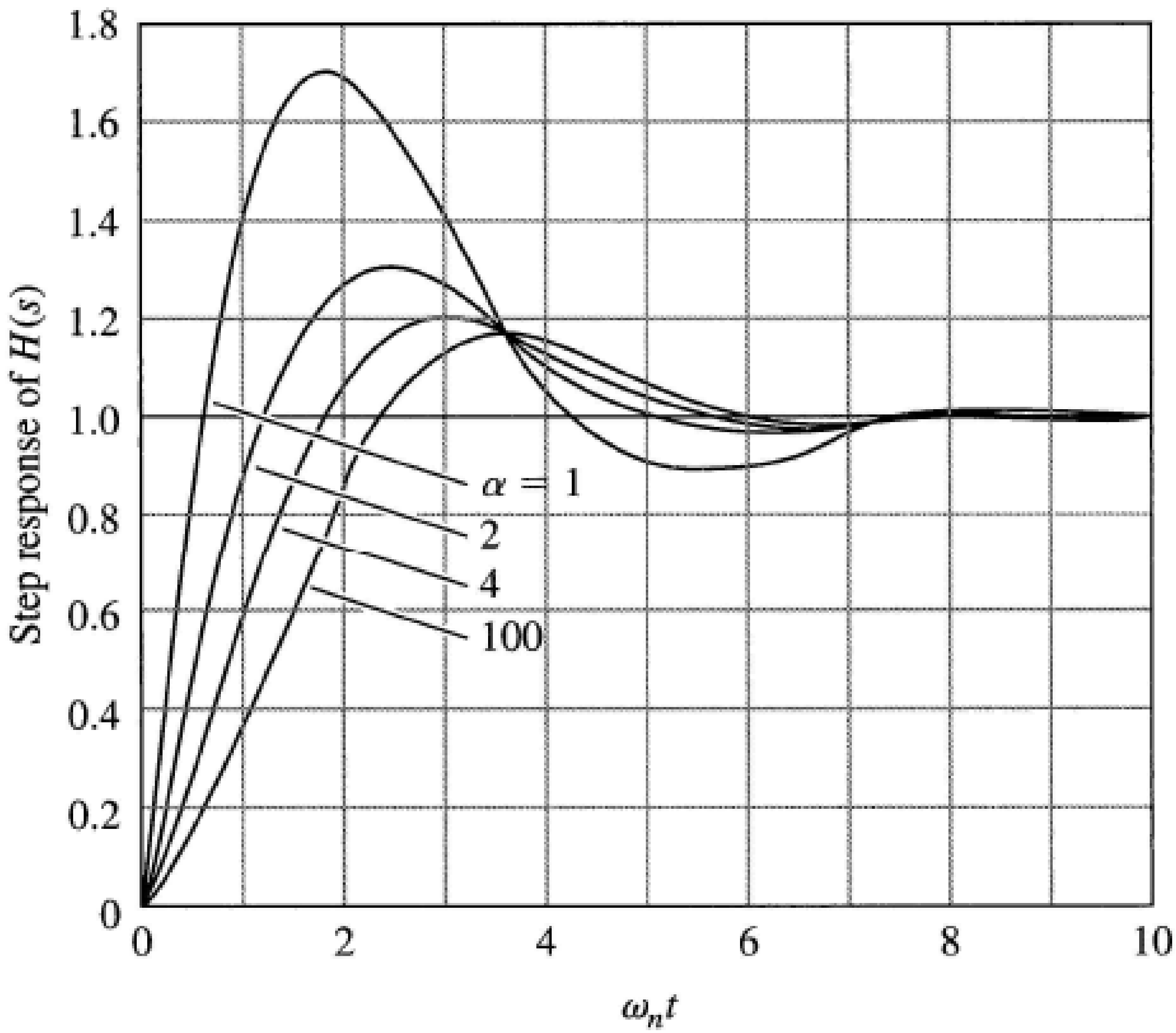
- If α is large, the zero will be far removed from the poles, and will have little effect on the transient response

- If $\alpha \approx 1$, the zero will be close to the real part of the poles and can be expected to have a substantial effect on the transient response

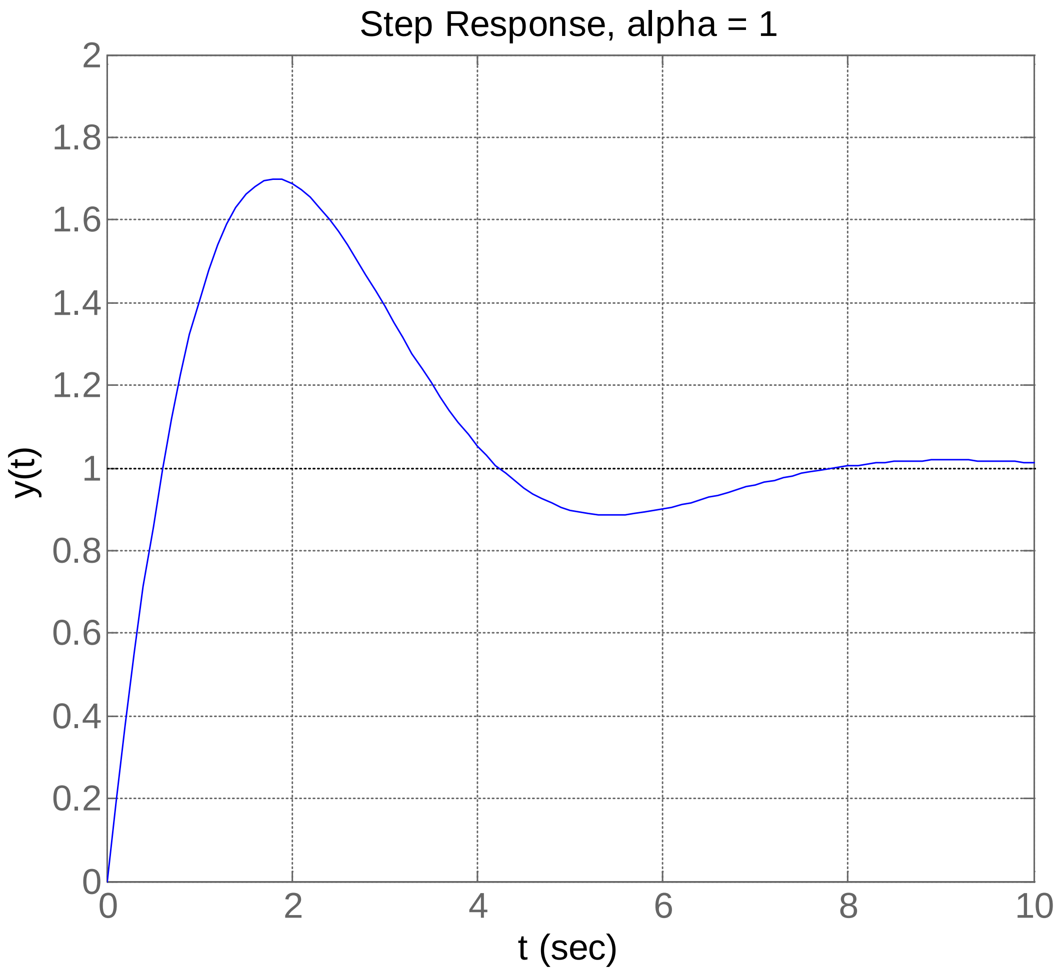
- The step response curves for $\zeta = 0.5$ and several values of α are shown on the next pages

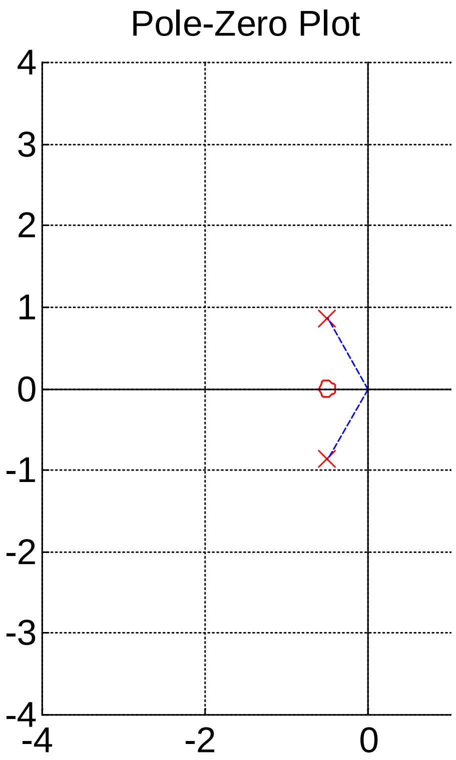
- The major effect of the zero is to increase the overshoot M_p, whereas it has little effect on the settling time

Second-Order Transfer Function with One Zero

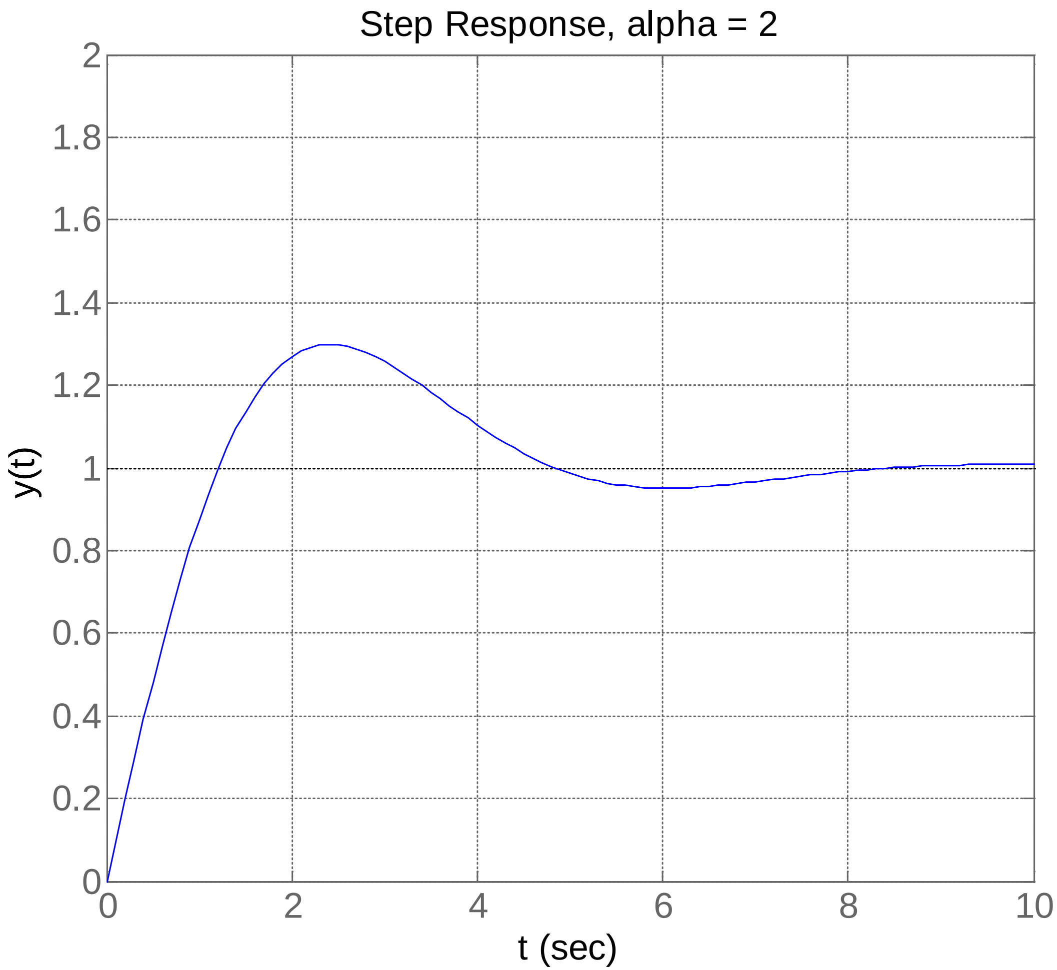


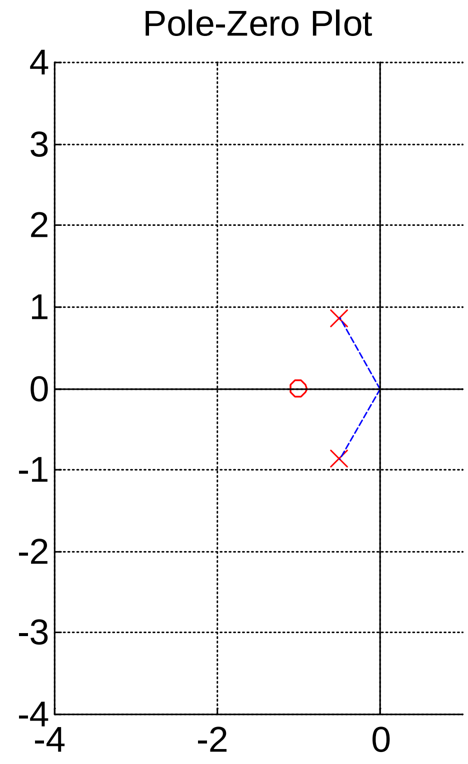
Second-Order Transfer Function with One Zero



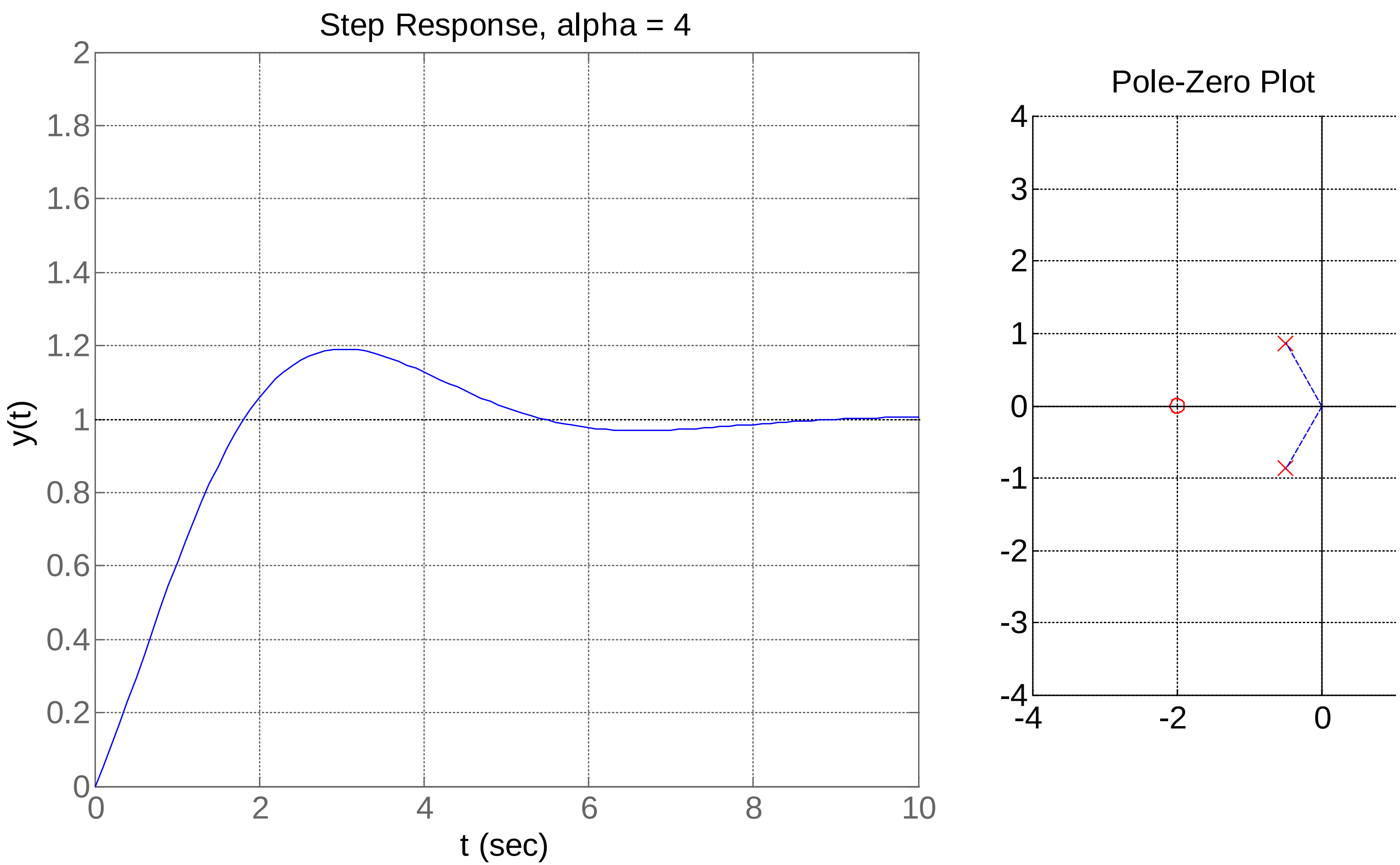


Second-Order Transfer Function with One Zero

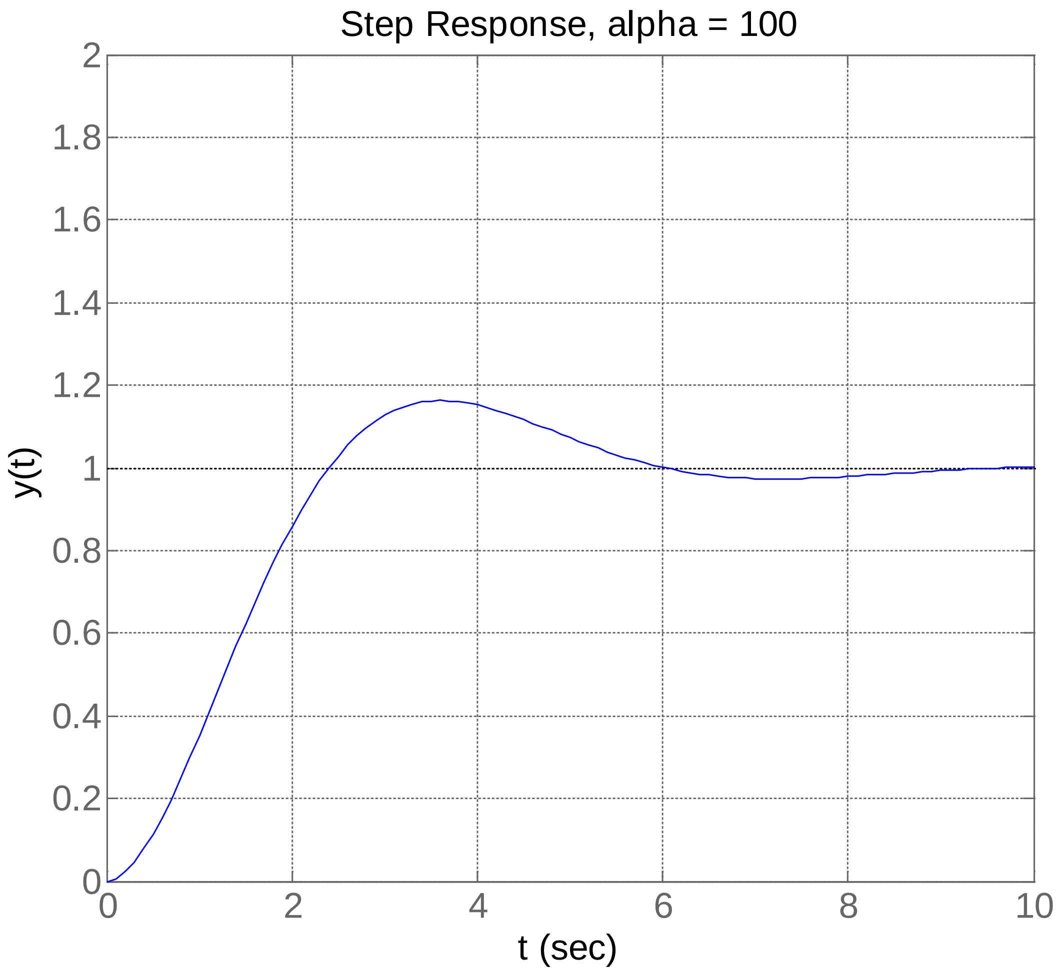


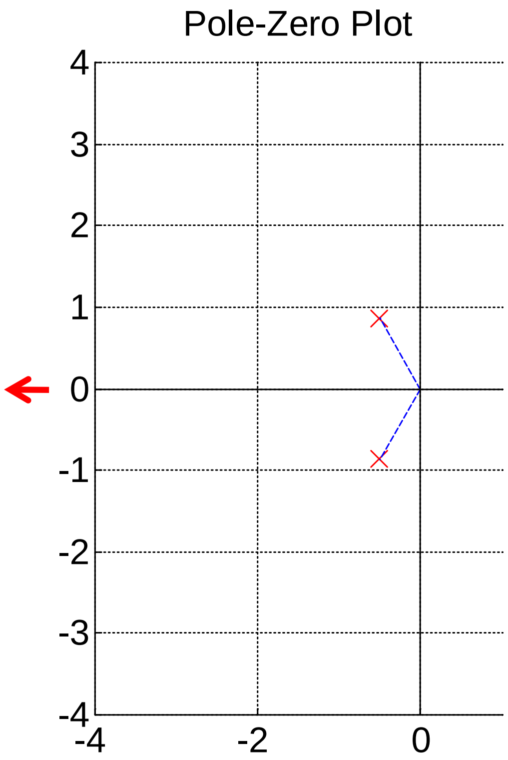


Second-Order Transfer Function with One Zero



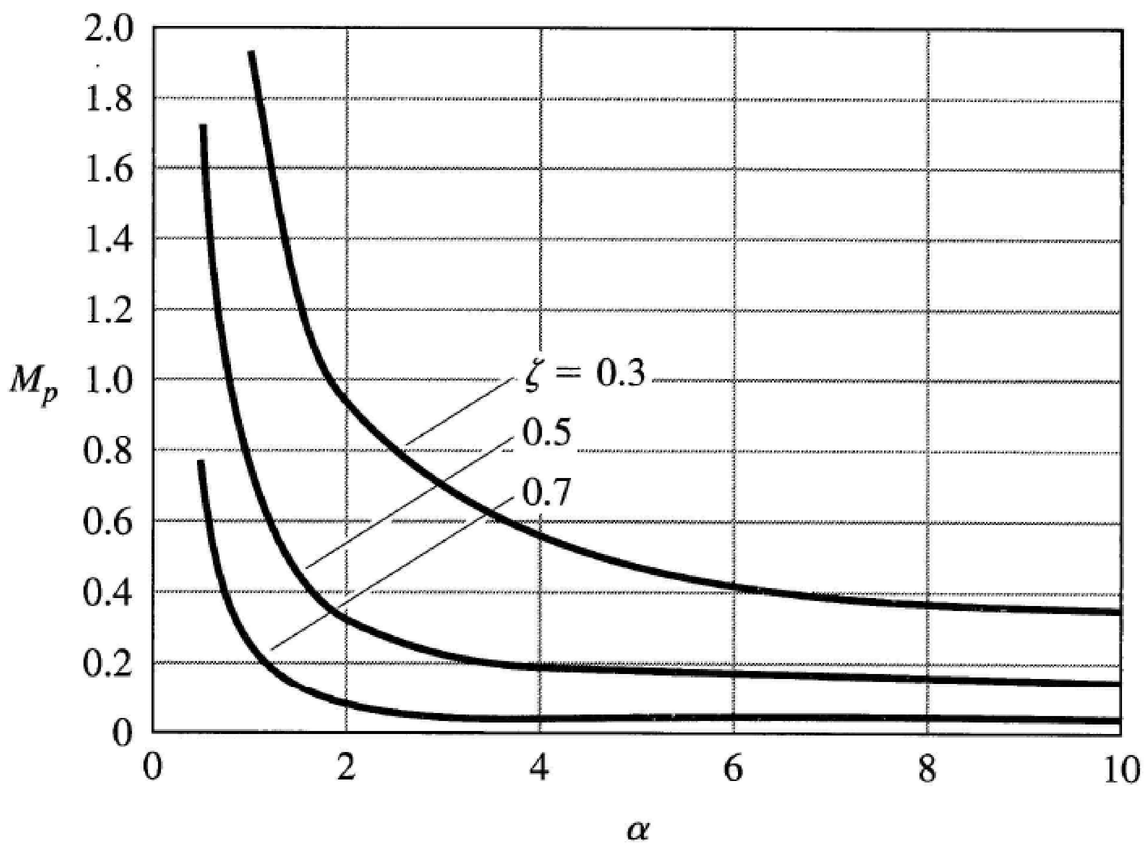
Second-Order Transfer Function with One Zero





Overshoot vs Zero Location

- A plot of the overshoot M_p versus the location of the zero relative to the real part of the poles α is shown below for $\zeta = 0.3, 0.5$ and 0.7



Overshoot vs Zero Location

- The zero has very little effect on M_p if $\alpha > 3$, but as α decreases below 3, has an increasing effect, especially when $\alpha = 1$ or less

- This can be explained in terms of Laplace transform analysis
- Without loss of generality, assume $\omega_n = 1$

$$H(s) = \frac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1}$$

- We then write the transfer function as the sum of two terms

$$H(s) = \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1}$$

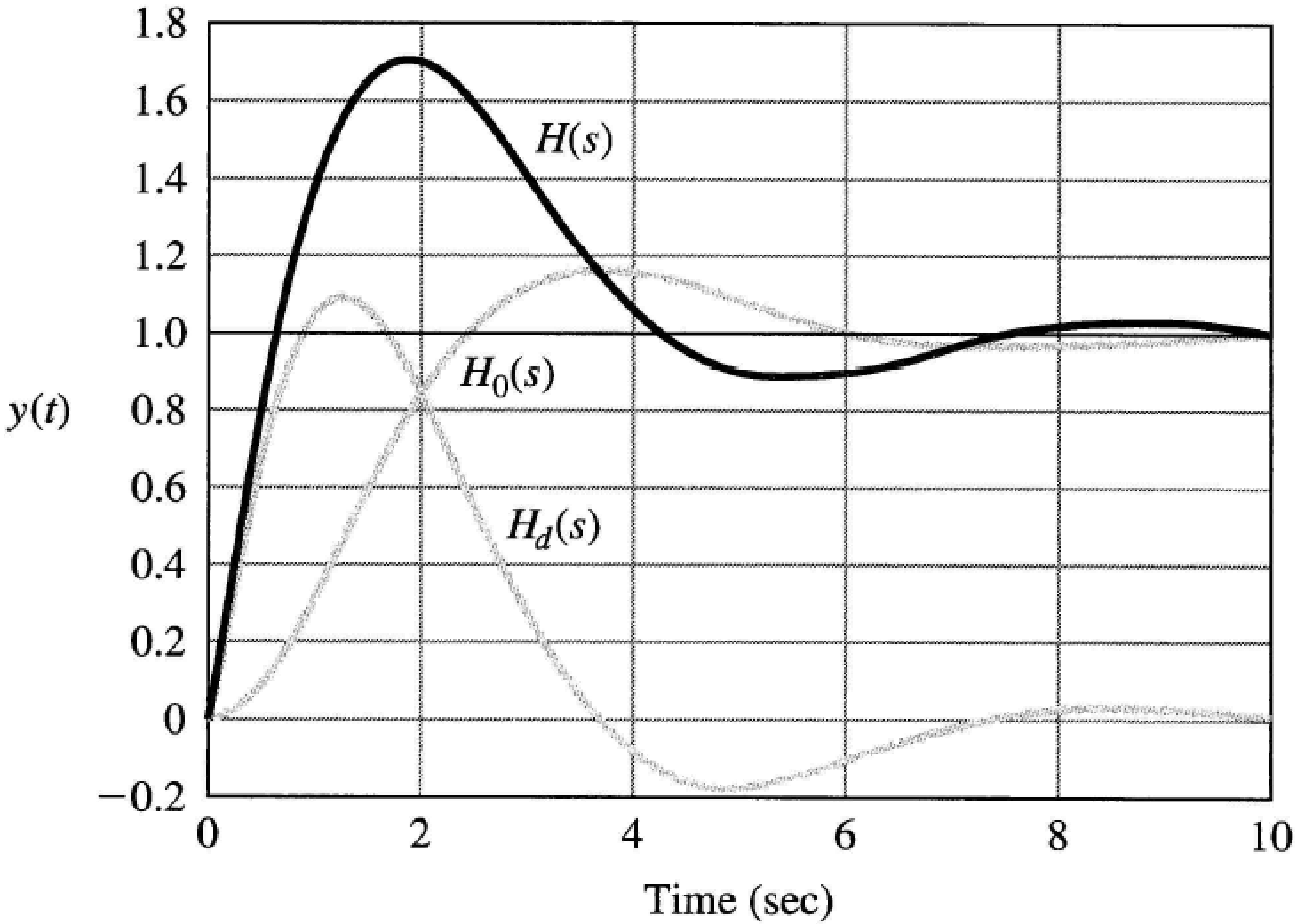
Overshoot vs Zero Location

$$
\begin{aligned}
H(s) &= \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1} \\
&= H_0(s) + H_d(s)
\end{aligned}
$$

- The first term $H_0(s)$ is the original transfer function with no zeroes
- The second term $H_d(s)$ is a constant $(1/\alpha\zeta)$ times $sH_0(s)$, which is the time derivative of $H_0(s)$

- The step response of $H_0(s)$ is therefore the weighted sum of the original second-order transfer function (with no zeroes) and its time derivative
- The larger the value of α, the less the contribution of the derivative
- The smaller the value of α, the greater the contribution of the derivative

Overshoot vs Zero Location

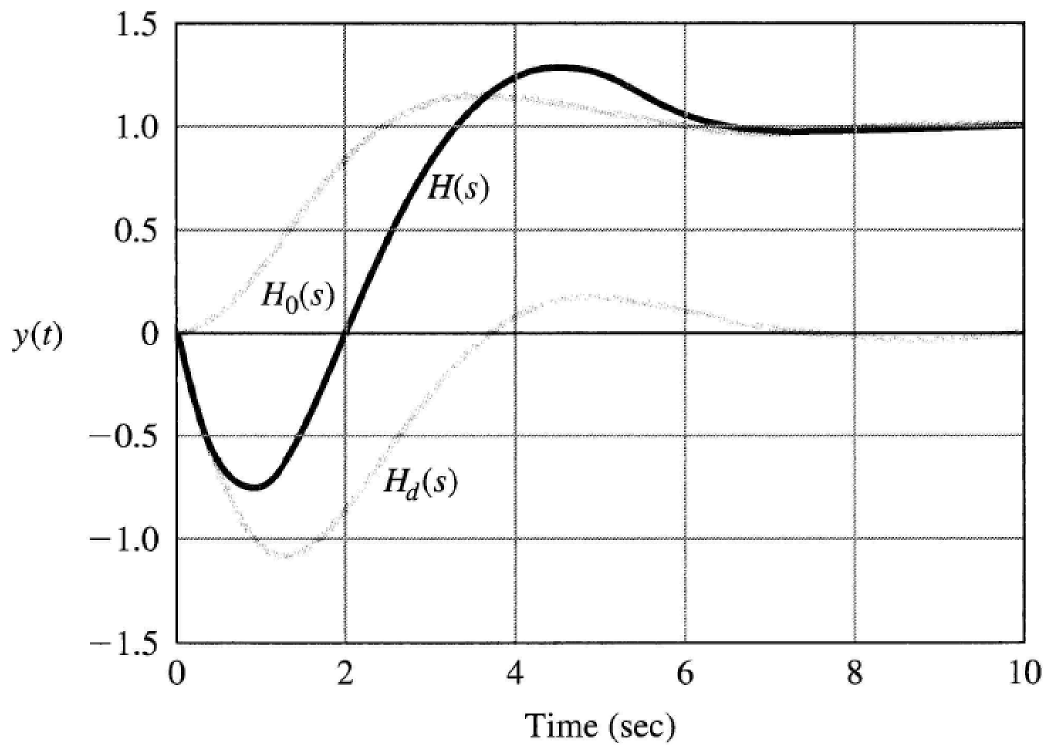


Overshoot vs Zero Location

- The step responses of $H_0(s)$ and $H_d(s)$ explain why the zero increases the overshoot
- The derivative $H_d(s)$ has a large peak in the early part of the curve
- Adding this to the original $H_0(s)$ lifts up the total response $H(s)$ to produce the overshoot
- The amount of additional overshoot is determined by the relative weight of $H_0(s)$ and $H_d(s)$ as determined by the relative location α of the zero to the poles

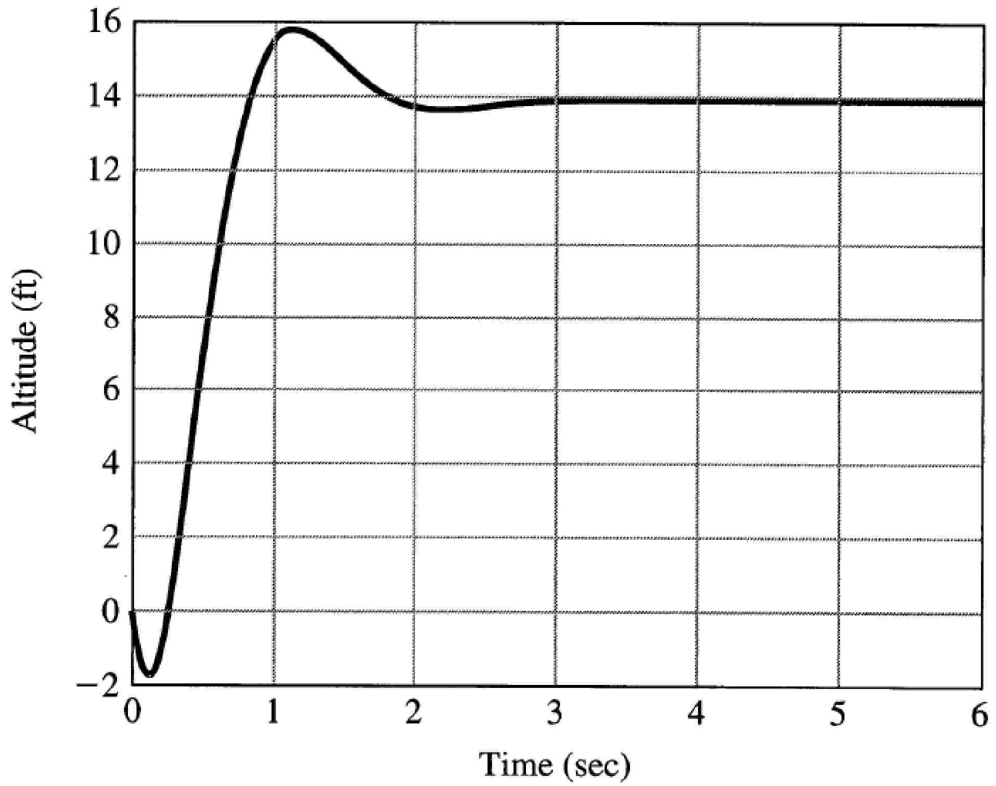
Effect of Nonminimum-Phase Zero

- When the zero is in the right half-plane, i.e. when the zero has a positive real value ($\alpha < 0$), then the derivative term is subtracted rather than added. A RHP zero is called a nonminimum phase zero.



Effect of Nonminimum-Phase Zero

- The step response of a system with a nonminimum phase zero starts of in wrong direction, e.g response of aircraft altitude to impulsive elevator input



Second-Order Transfer Function with Extra Pole

- Consider the effects of an extra pole on the standard second-order step response

$$H(s)=\frac{1}{\left(s/\alpha\zeta\omega_n+1\right)\left[\left(s/\omega_n\right)^2+2\zeta\left(s/\omega_n\right)+1\right]}$$

- The original poles are located at $s=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$

$$=-\sigma\pm j\omega_d$$

- The extra pole is located at $s=-\alpha\zeta\omega_n=-\alpha\sigma$

- In other words, the extra pole is located at α times the real part of the original poles

Second-Order Transfer Function with Extra Pole

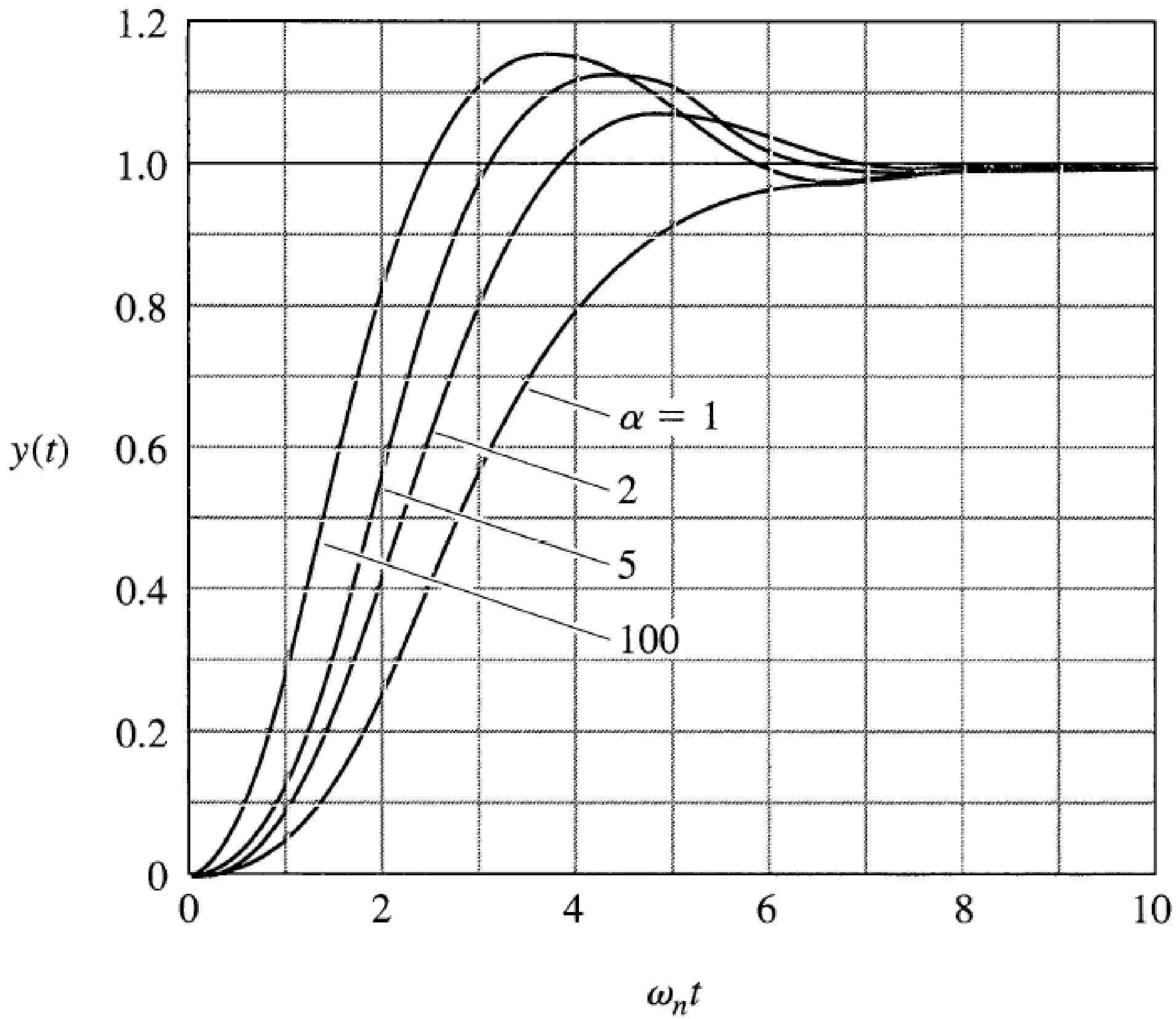
- If α is large, the extra pole will be far removed from the original poles, and will have little effect on the transient response

- If $\alpha \approx 1$, the extra pole will be close to the real part of the original poles and can be expected to have a substantial effect on the transient response

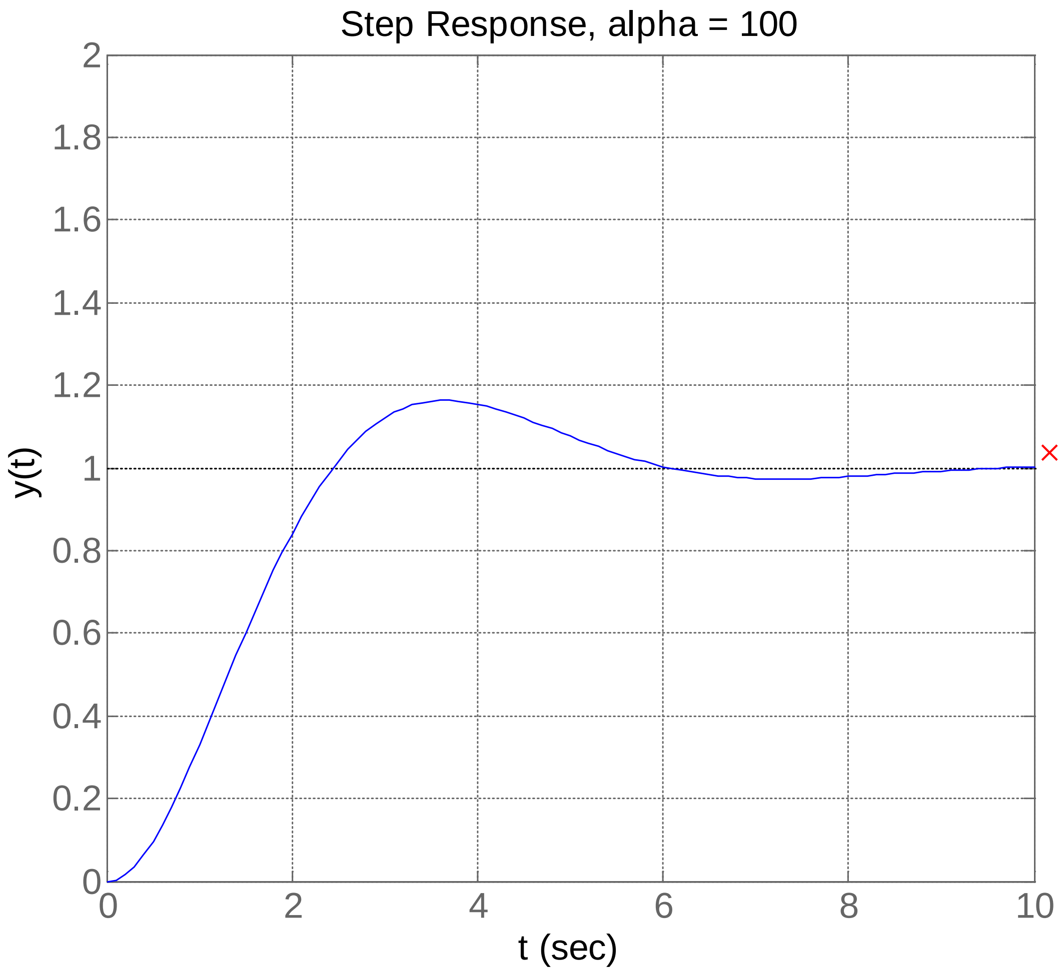
- The step response curves for $\zeta = 0.5$ and several values of α are shown on the next pages

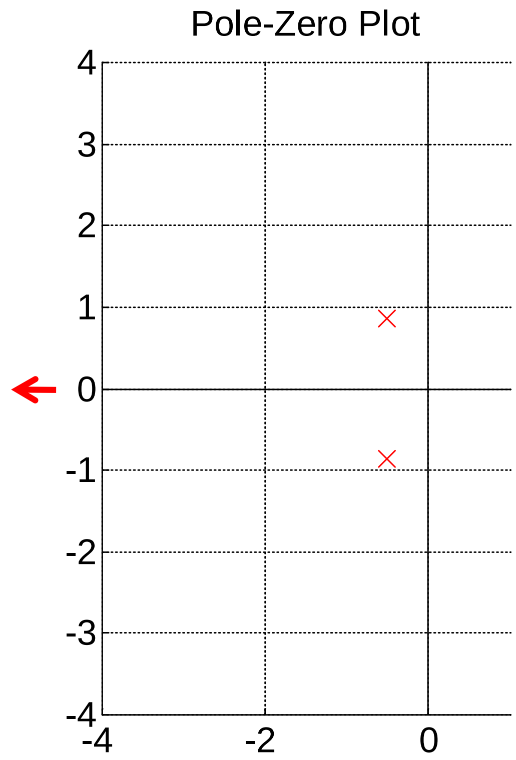
- The major effect of the extra pole is to increase the rise time t_r
- Also note how the transient response changes from a dominantly second-order response to a dominantly first-order response

Second-Order Transfer Function with Extra Pole

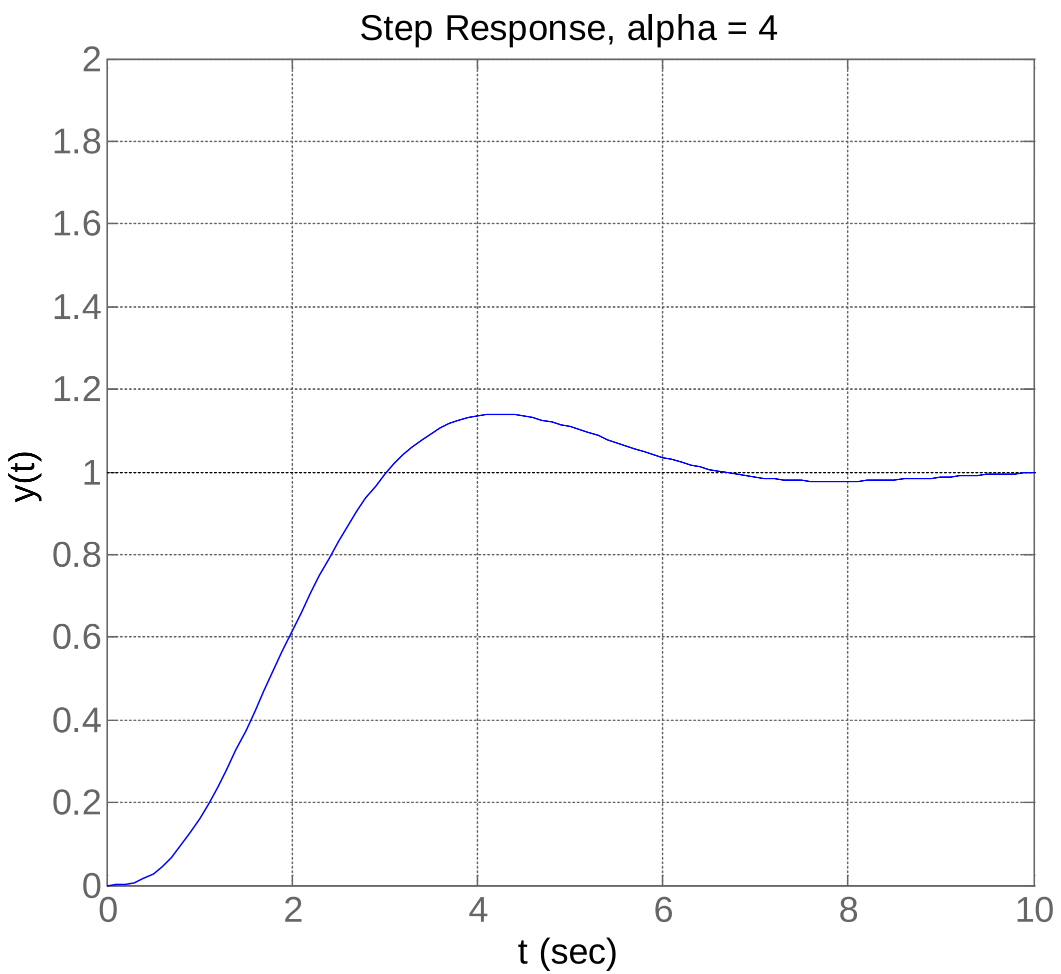


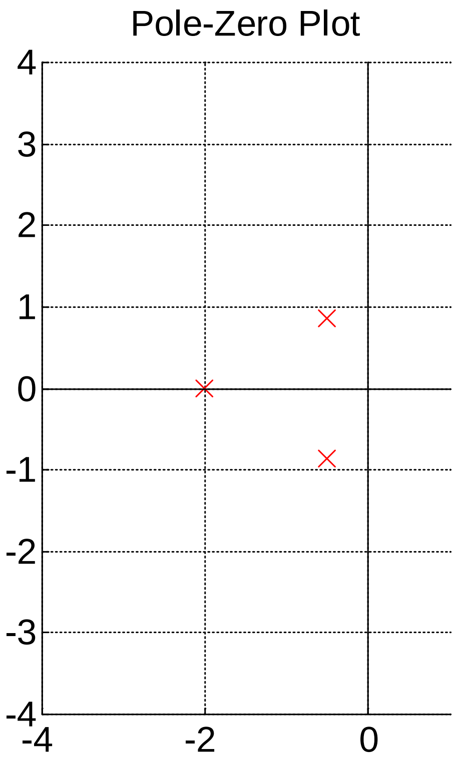
Second-Order Transfer Function with Extra Pole



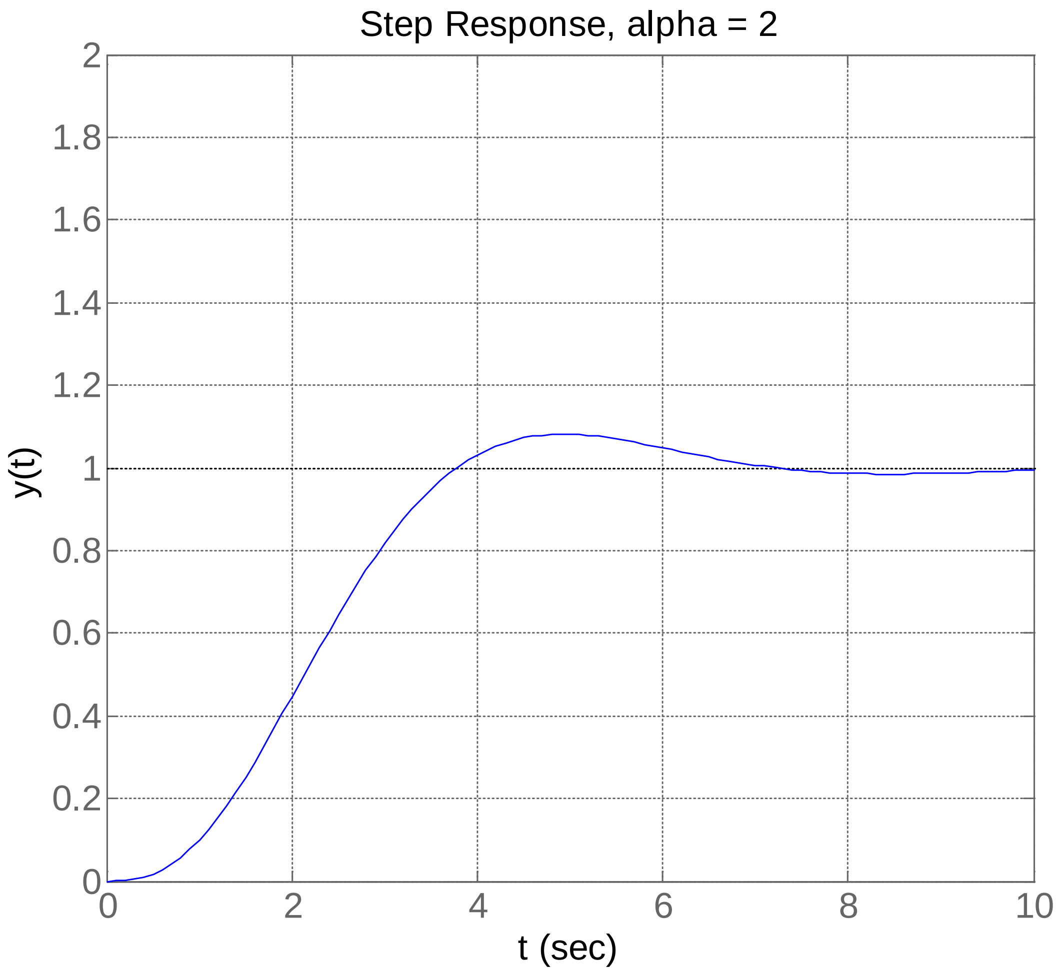


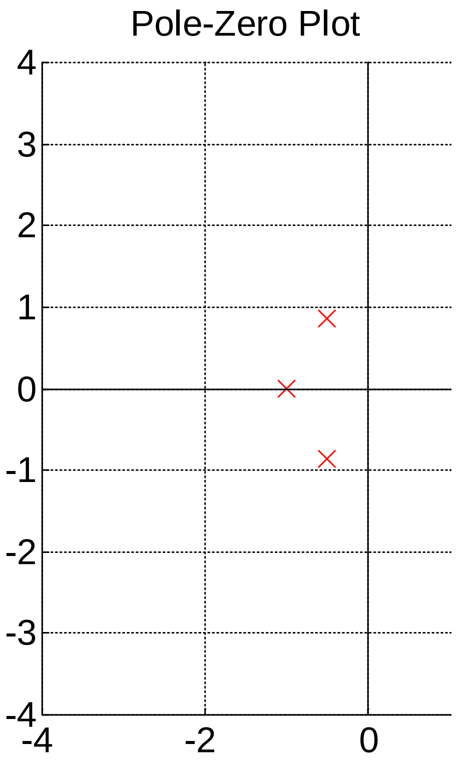
Second-Order Transfer Function with Extra Pole



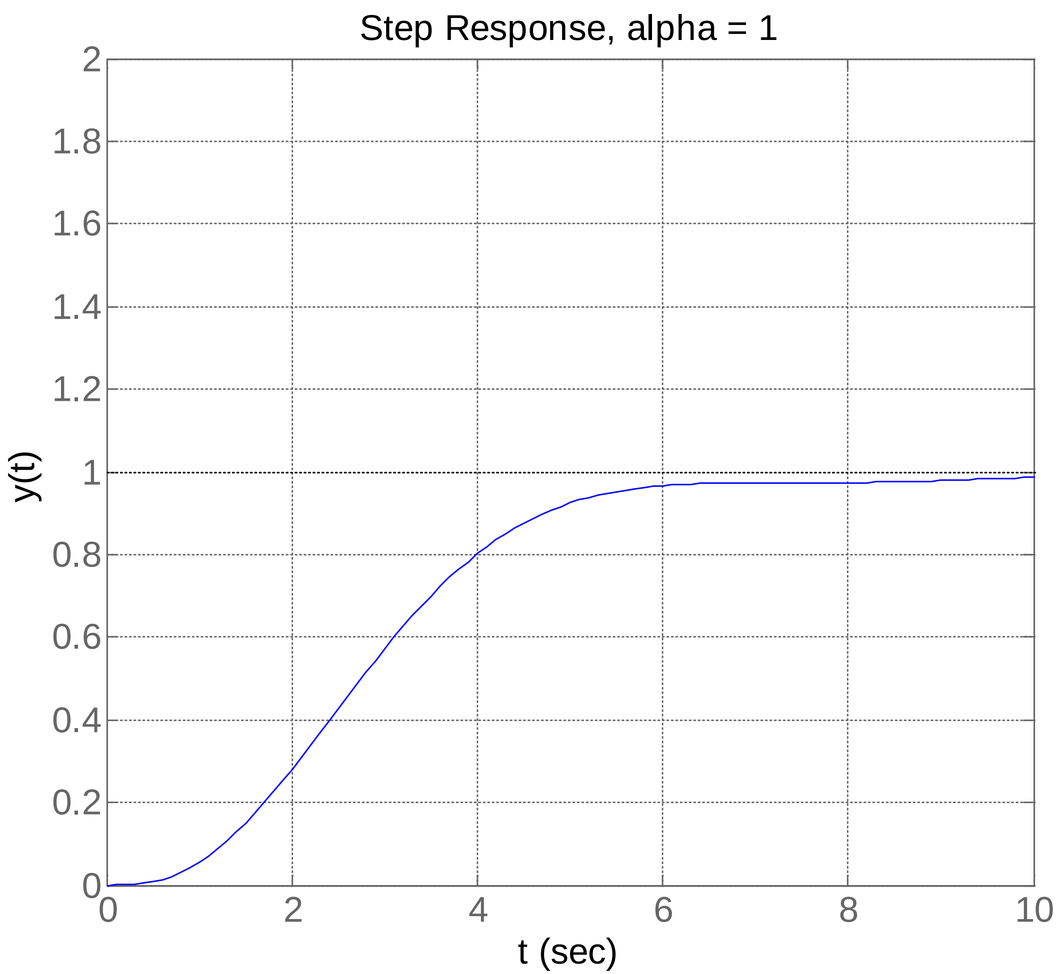


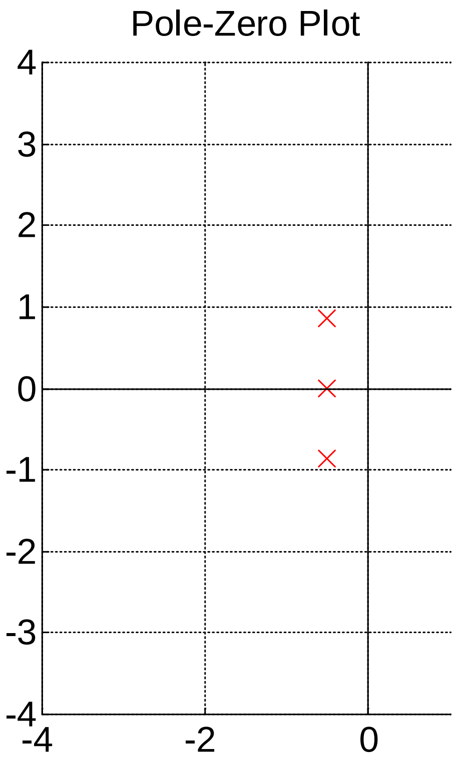
Second-Order Transfer Function with Extra Pole





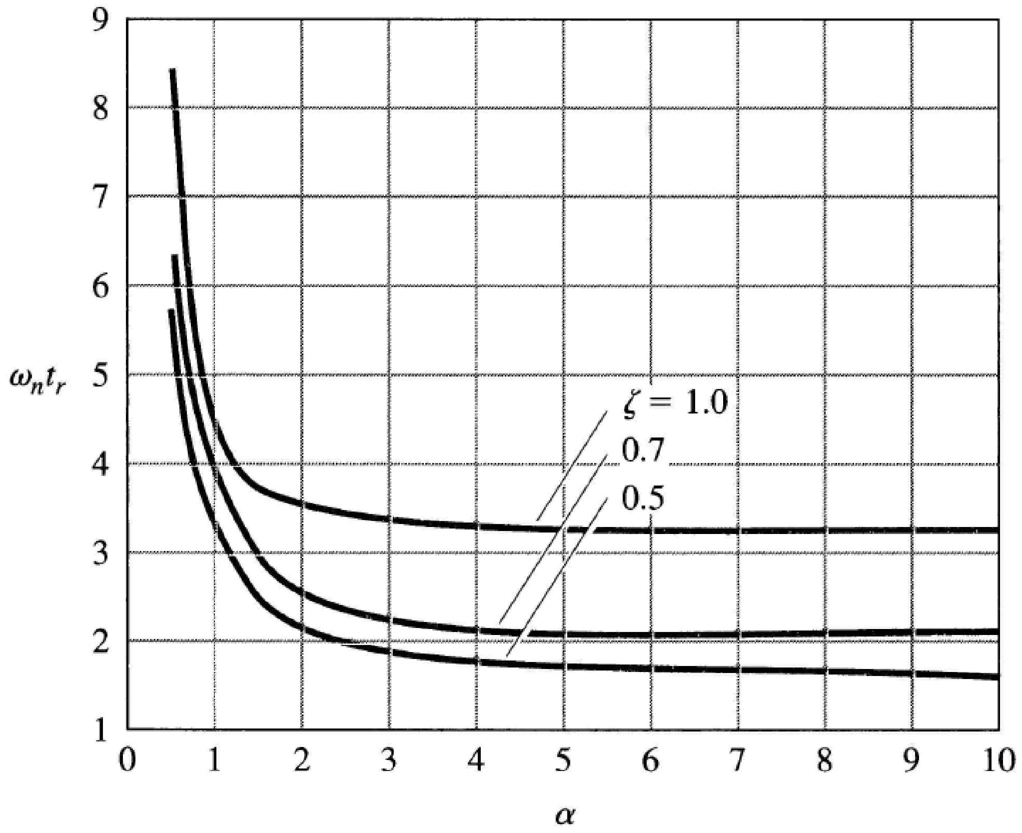
Second-Order Transfer Function with Extra Pole





Rise time vs Extra Pole Location

- A plot of the rise time t_r versus the location of the extra pole relative to the real part of the original poles α is shown for $\zeta = 1, 0.7 \text{ and } 0.5$



Summary

- For a second-order system with **no finite zeroes**

Rise time: $$t_{\mathrm{r}} \approx \frac{1.8}{\omega_n}$$

Peak time: $$t_p = \frac{\pi}{\omega_d}$$

Overshoot: $$M_p = e^{-\pi\zeta / \sqrt{1-\zeta^2}}, \forall 0 \le \zeta < 1$$

Settling time: $$t_{s\,2\%} \approx \frac{4}{\sigma}$$

Summary

- A zero in the left half-plane (LHP) will increase the overshoot if the zero is within a factor of 4 of the real part of the complex poles. *Use relevant figure in Franklin, Powell and Emami-Naeini to determine new overshoot value.*

- A zero in the right half-plane (RHP) will depress the overshoot and may cause the step response to start out in the wrong direction.

- An additional pole in the left half-plane (LHP) will increase the rise time significantly if the extra pole is within a factor of 4 of the real part of the complex poles. *Use relevant figure in Franklin, Powell and Emami-Naeini to determine new rise time.*

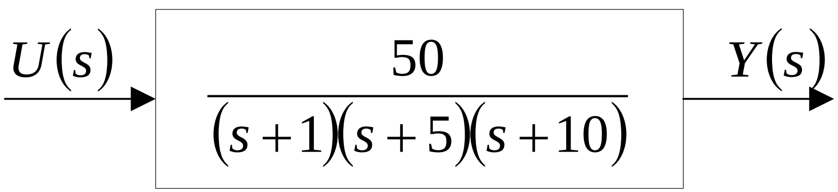
Dominant Poles

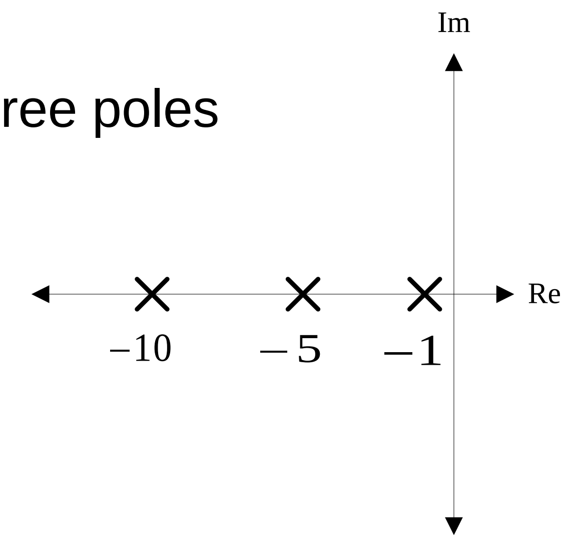
- In higher-order systems, there usually exist one or more dominant poles. These are the poles that have the most influence on the system response
- In other words, if a step input is applied to the input of the system, the system output will mostly respond as if its transfer function only contained the dominant poles

- The dominance of poles are affected by:
- Fast and slow poles
- Effect of zeroes

Dominant Poles – Fast and Slow Poles

- Consider the higher-order system with three poles





- The step response may be obtained mathematically by applying a step input and performing partial fraction expansion

$$Y(s) = H(s)U(s) = \frac{50}{(s+1)(s+5)(s+10)} \cdot \frac{1}{s}$$

$$= 1 - \frac{25}{18}\left(\frac{1}{s+1}\right) + \frac{1}{2}\left(\frac{1}{s+5}\right) - \frac{1}{9}\left(\frac{1}{s+10}\right)$$

Dominant Poles – Fast and Slow Poles

- The step response in the time domain may be obtained by taking the inverse Laplace transform

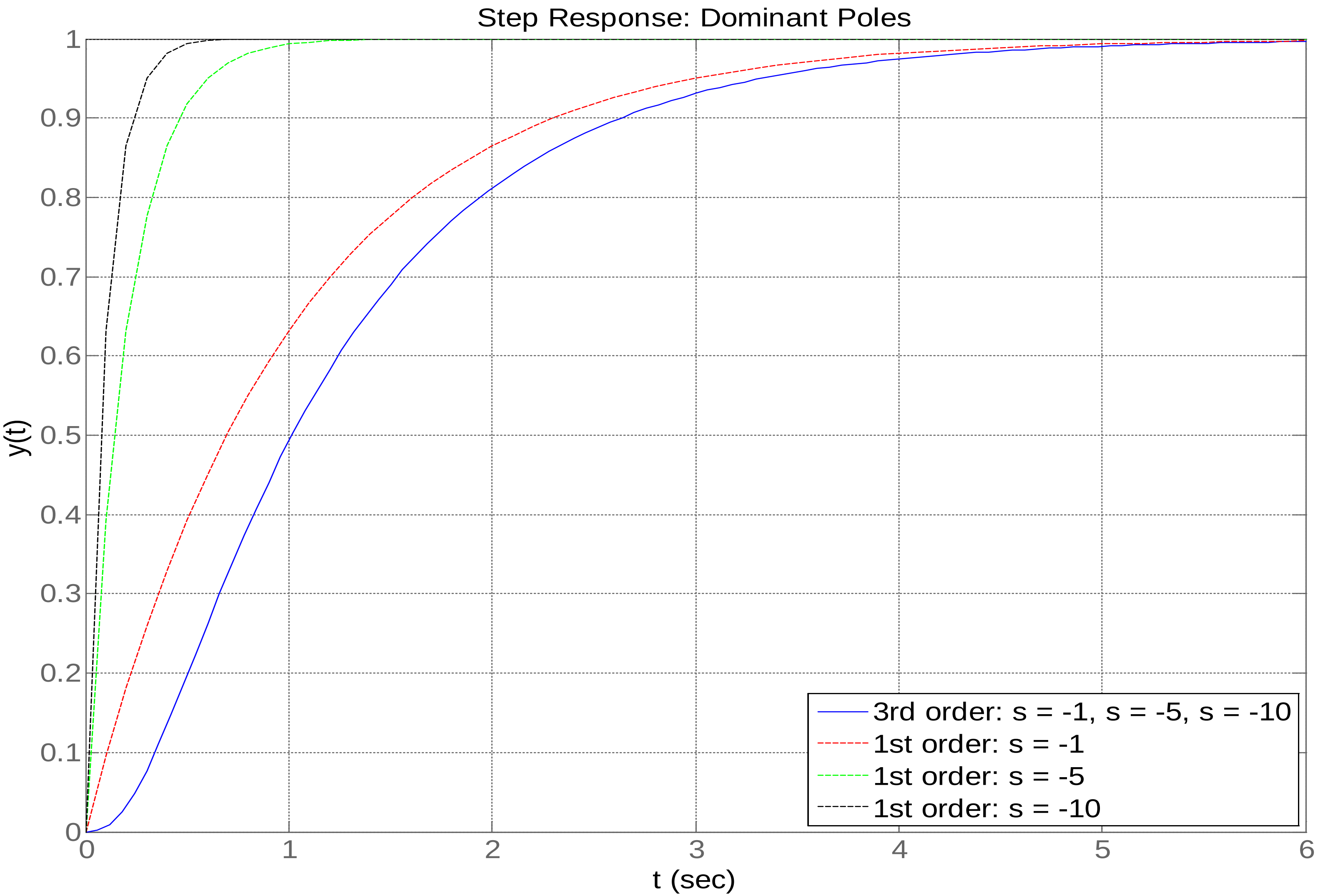
$$y(t) = 1 - \frac{25}{18}e^{-t} + \frac{1}{2}e^{-5t} - \frac{1}{9}e^{-10t}$$

- Note that:
- The coefficient of the e^{-t} term is the largest
- It takes much longer for e^{-t} to decay than either e^{-5t} or e^{-10t}

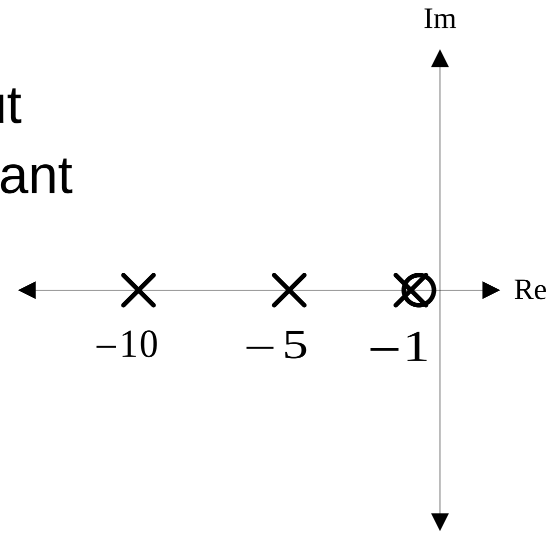
- $-\frac{25}{18}e^{-t}$ is the dominant term and that $s = -1$ is the dominant pole

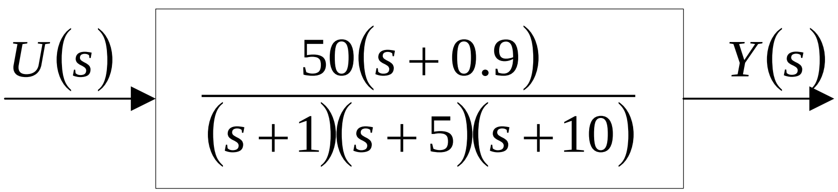
- **Conclusion:** The dynamic response of a higher-order system is dominated by the slowest poles

Dominant Poles – Fast and Slow Poles



- Consider the same third-order system, but with a zero at s = -0.9, close to the dominant pole at s = -1





- The step response may be obtained mathematically by applying a step input and performing partial fraction expansion

$$Y(s) = H(s)U(s) = \frac{50(s+0.9)}{(s+1)(s+5)(s+10)} \cdot \frac{1}{s}$$

$$= 0.9 + \frac{5}{36}\left(\frac{1}{s+1}\right) - \frac{41}{20}\left(\frac{1}{s+5}\right) + \frac{91}{90}\left(\frac{1}{s+10}\right)$$

Dominant Poles – Effect of Zeroes

- The step response in the time domain may be obtained by taking the inverse Laplace transform

$$y(t) = 0.9 + \frac{5}{36}e^{-t} - \frac{41}{20}e^{-5t} + \frac{91}{90}e^{-10t}$$

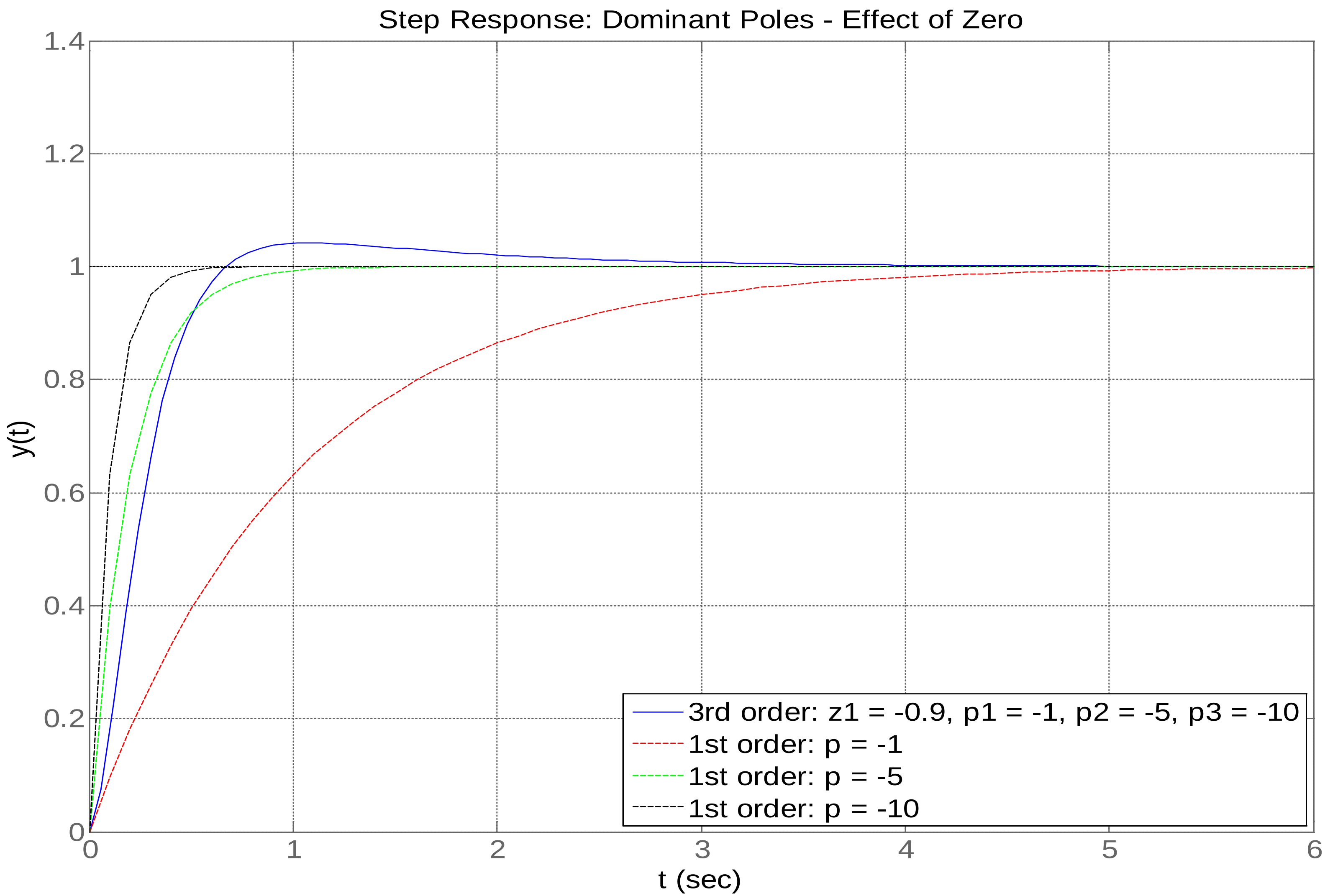
- Note that:
- Now the coefficient of the e^{-5t} term larger than that of the e^{-t} term

- Now $-\frac{41}{20}e^{-5t}$ is the dominant term and $s = -5$ is the dominant pole

Dominant Poles – Effect of Zeroes

- Let us compare the coefficients of the transfer function with and without the zero
- No zeroes: $$y(t) = 1 - \frac{25}{18}e^{-t} + \frac{1}{2}e^{-5t} + \frac{1}{9}e^{-10t}$$
- With zero at s = -0.9: $$y(t) = 0.9 + \frac{5}{36}e^{-t} - \frac{41}{20}e^{-5t} + \frac{91}{90}e^{-10t}$$
- When there are no zeroes present, the e^{-t} has the largest coefficient, and the pole at s = -1 is dominant
- With the addition of the zero at s = -0.9, the coefficient of the e^{-t} becomes significantly smaller than the coefficients of the e^{-5t} and e^{-10t} terms, and the pole at s = -1 is no longer dominant

Dominant Poles – Effect of Zeroes



Dominant Poles – Effect of Zeroes

- **Conclusion:** A zero close to a pole reduces the influence of the pole on the transient response, because it causes the pole to have a smaller residue (partial fraction constant)

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Reference:
Chapter 3.5