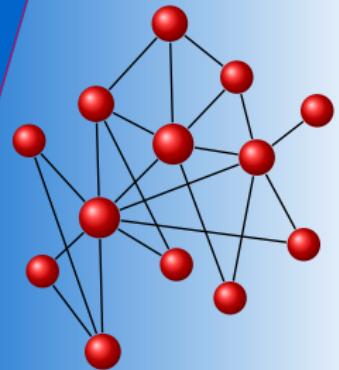


4DM50: Dynamics and Control of Cooperation

Synchronization experiments

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TU/e

Technische Universiteit
Eindhoven
University of Technology

Recap

Experimental setup

Performing experiments

- ▶ Systems of the form

$$\begin{cases} \dot{x}_i = f(x_i) + Bu_i \\ y_i = Cx_i \end{cases}$$

with $CB > 0$

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- ▶ System dynamics in new coordinates

$$\begin{cases} \dot{z}_i = q(z_i, y_i) \\ \dot{y}_i = a(z_i, y_i) + CBu_i \end{cases}$$

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- ▶ Assumption 1: systems are **strictly semi-passive**

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- ▶ Assumption 1: systems are **strictly semi-passive**
- ▶ Assumption 2: subsystem $\dot{z}_i = q(z_i, y_i)$ is **exponentially convergent** w.r.t. input y_i

- ▶ **Strict semi-passivity:** there exists $S : \mathbb{R}^n \rightarrow \mathbb{R}_+$ such that the dissipation inequality

$$\dot{S} \leq y_i^T u_i$$

is strict for all x_i outside some ball $\mathcal{B} \subset \mathbb{R}^n$

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- ▶ Condition for **exponential convergence** of subsystem $\dot{z}_i = q(z_i, y_i)$ is w.r.t. input y_i :

There exists a $(n - m) \times (n - m)$ matrix $P = P^T > 0$ such that the eigenvalues of

$$P \left(\frac{\partial q}{\partial z}(z, w) \right) + \left(\frac{\partial q}{\partial z}(z, w) \right)^T P$$

are negative and separated away from zero for all $(z, w) \in \mathbb{R}^{n-m} \times \mathcal{W}$

Networks of systems with time-delayed interaction:

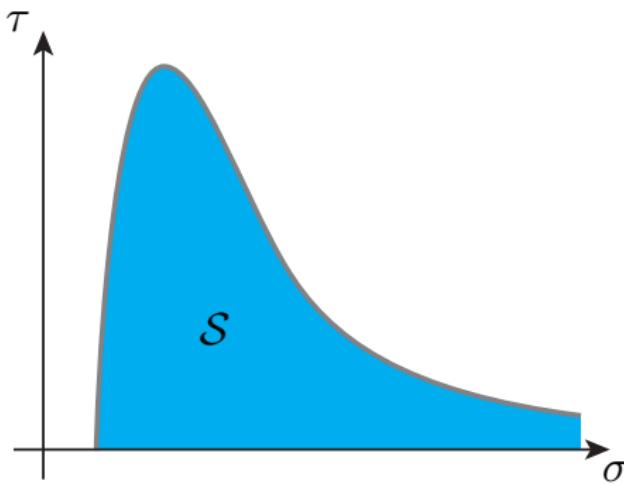
$$u_i(t) = \sigma \sum_{j \in \mathcal{N}_i} a_{ij} (y_j(t - \tau) - y_i(t - \tau))$$

with $a_{ij} = a_{ji} > 0$ and $j \in \mathcal{N}_i \Leftrightarrow i \in \mathcal{N}_j$

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Coupling matrix

$$L = \begin{pmatrix} \sum_{j=2}^N a_{1j} & -a_{12} & \cdots & -a_{1N} \\ -a_{21} & \sum_{j=1, j \neq 2}^N a_{2j} & \ddots & \vdots \\ \vdots & \ddots & \ddots & -a_{(N-1)N} \\ -a_{N1} & \cdots & -a_{N(N-1)} & \sum_{j=1}^{N-1} a_{Nj} \end{pmatrix}$$

with $a_{ij} = 0$ if (and only if) $j \notin \mathcal{N}_i$

Eigenvalues of L :

$$0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_N$$

Assumption: there exists a permutation matrix Π that commutes with L

$$\Pi L = L \Pi$$

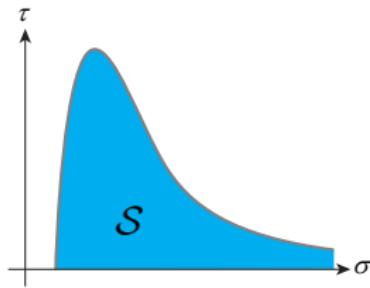
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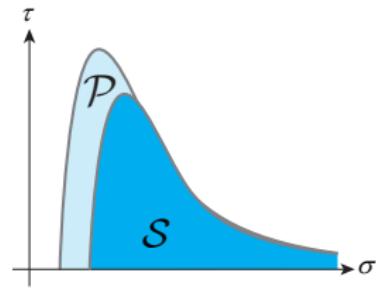
Denote

- ▶ λ' the **smallest** eigenvalue of L with eigenvector in $\text{range}(I_N - \Pi)$
- ▶ λ^* the **largest** eigenvalue of L with eigenvector in $\text{range}(I_N - \Pi)$

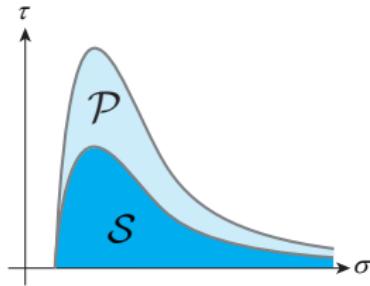
Partial synchronization



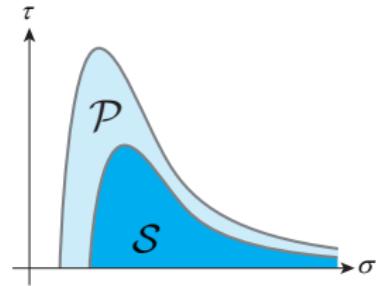
$\lambda' = \lambda_2$ and $\lambda^* = \lambda_N$



$\lambda' > \lambda_2$ and $\lambda^* = \lambda_N$

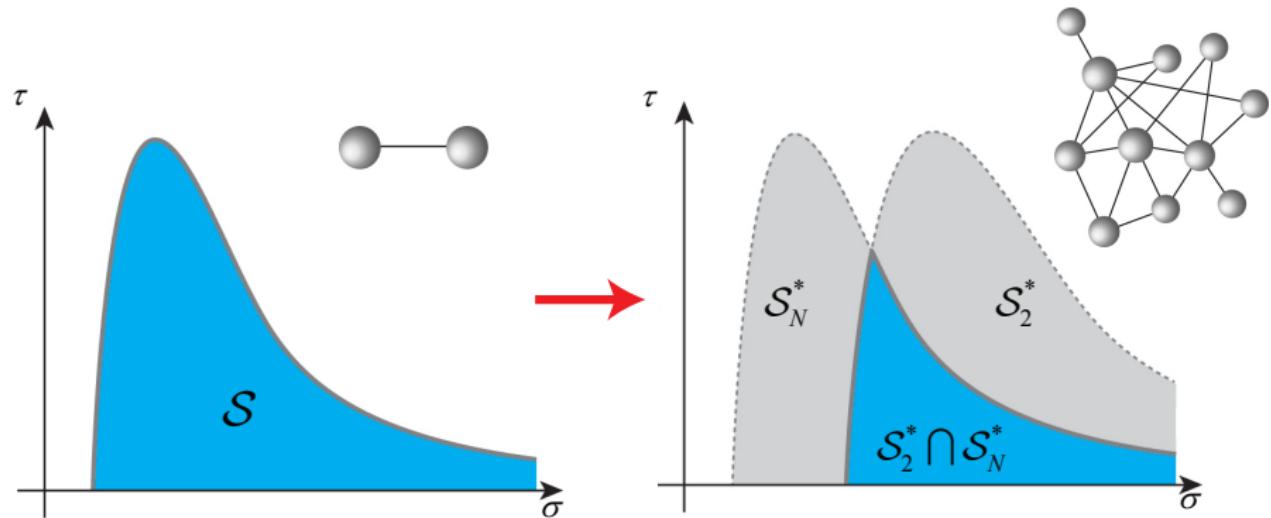


$\lambda' = \lambda_2$ and $\lambda^* < \lambda_N$



$\lambda' > \lambda_2$ and $\lambda^* < \lambda_N$

Scaling with the non-zero eigenvalues of L



$$\mathcal{S}_j^* := \left\{ (\sigma, \tau) \in \mathbb{R}_+ \times \overline{\mathbb{R}}_+ \mid \left(\frac{\lambda_j}{2} \sigma, \tau \right) \in \mathcal{S} \right\}$$

Recap

Experimental setup

Performing experiments

The Hindmarsh-Rose model neuron:

$$\dot{z}_{i,1} = 100(-y_i^2 - 2y_i - z_{i,1})$$

$$\dot{z}_{i,2} = 0.5(4y_i + 4.472 - z_{i,2})$$

$$\dot{y}_i = 100(-y_i^3 + 3y_i - 8 + 5z_{i,1} - z_{i,2} + 3.3 + u_i)$$

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Properties:

- strictly semi-passive with a quadratic storage function of the form

$$S = \frac{1}{200}y_i^2 + c_1z_{i,1}^2 + c_2z_{i,2}^2$$

for some $c_1, c_2 > 0$

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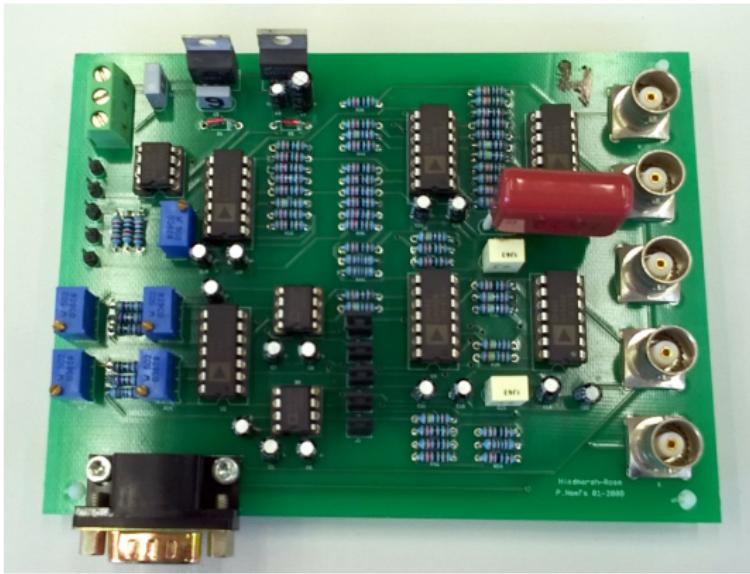
$$S = \frac{1}{200}y_i^2 + c_1z_{i,1}^2 + c_2z_{i,2}^2$$

for some $c_1, c_2 > 0$

- ▶ z_i -dynamics satisfy the test for exponential convergence with $P = I$

Hindmarsh-Rose electronic neuron

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Hindmarsh-Rose model neurons

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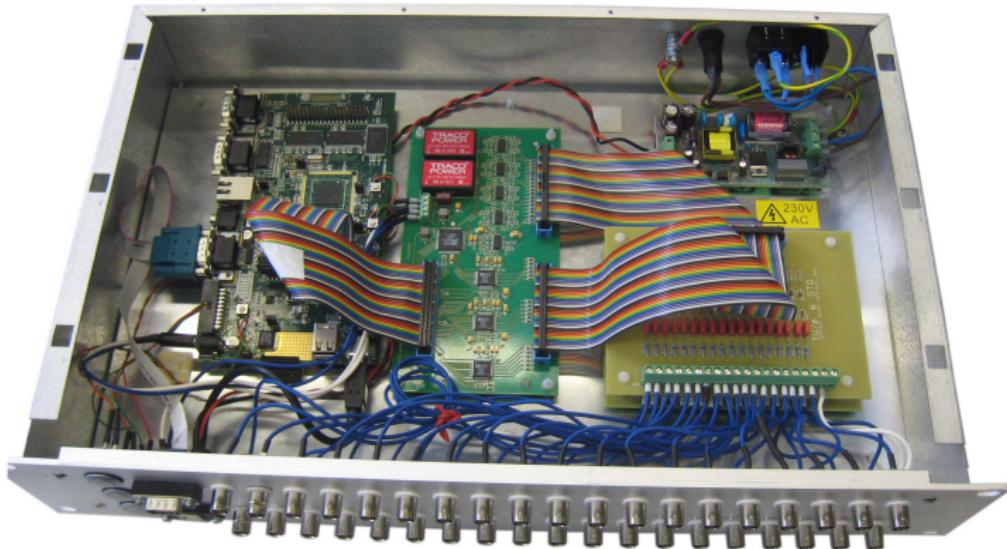
with time-delay coupling functions

$$u_i(t) = \sigma \sum_{j \in \mathcal{N}_i} a_{ij}(y_j(t - \tau) - y_i(t - \tau))$$

where $a_{ij} = a_{ji} > 0$ and $j \in \mathcal{N}_i \Leftrightarrow i \in \mathcal{N}_j$

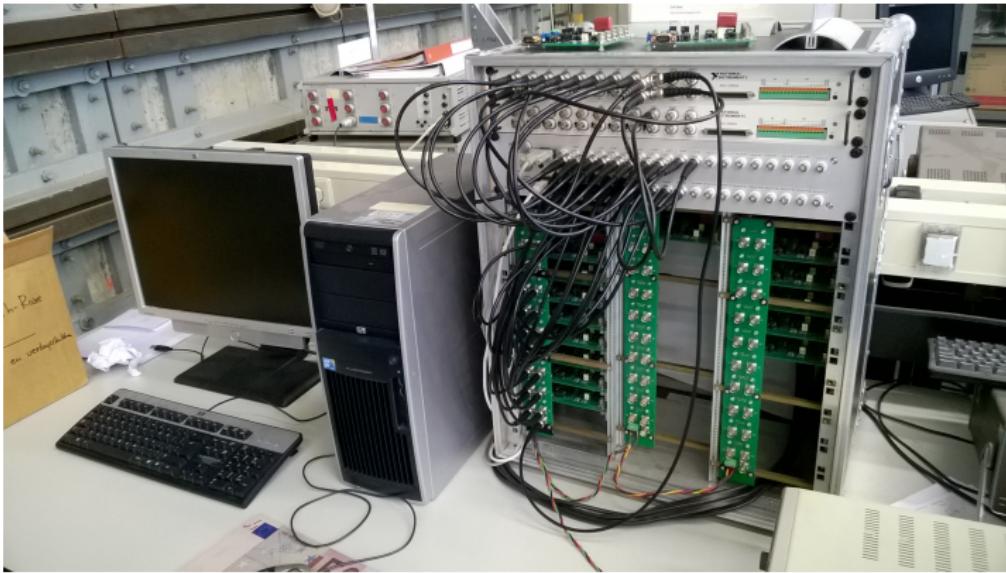
Coupled Hindmarsh-Rose electronic neurons

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Coupled Hindmarsh-Rose electronic neurons

13/28



Due to

- ▶ imperfections in the hardware realizations
- ▶ un-modeled dynamics, e.g. noise

synchronization (or partial synchronization) in the sense that

$$x_i(t) = x_j(t)$$

is not possible

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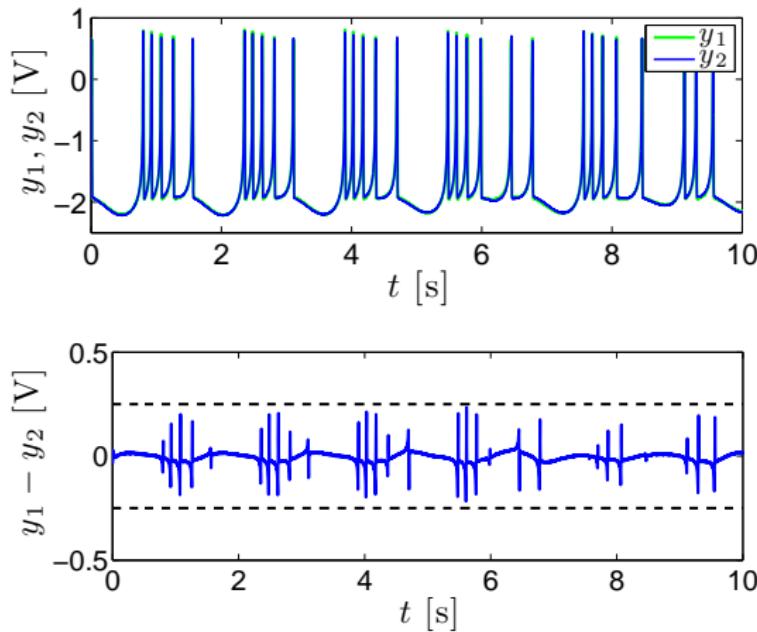
Practical (partial) synchronization: outputs $y_i(t)$ and $y_j(t)$, defined on a finite interval $t \in [t_0, t_2]$, converge towards each other within a “sufficiently small” bound ε :

$$\exists t_1 \in [t_0, t_2] \text{ such that } \|y_i(t) - y_j(t)\| \leq \varepsilon \text{ for all } t \in [t_1, t_2]$$

Practical synchronization of two HR neurons

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Practical synchronization with bound $\varepsilon = 0.25$



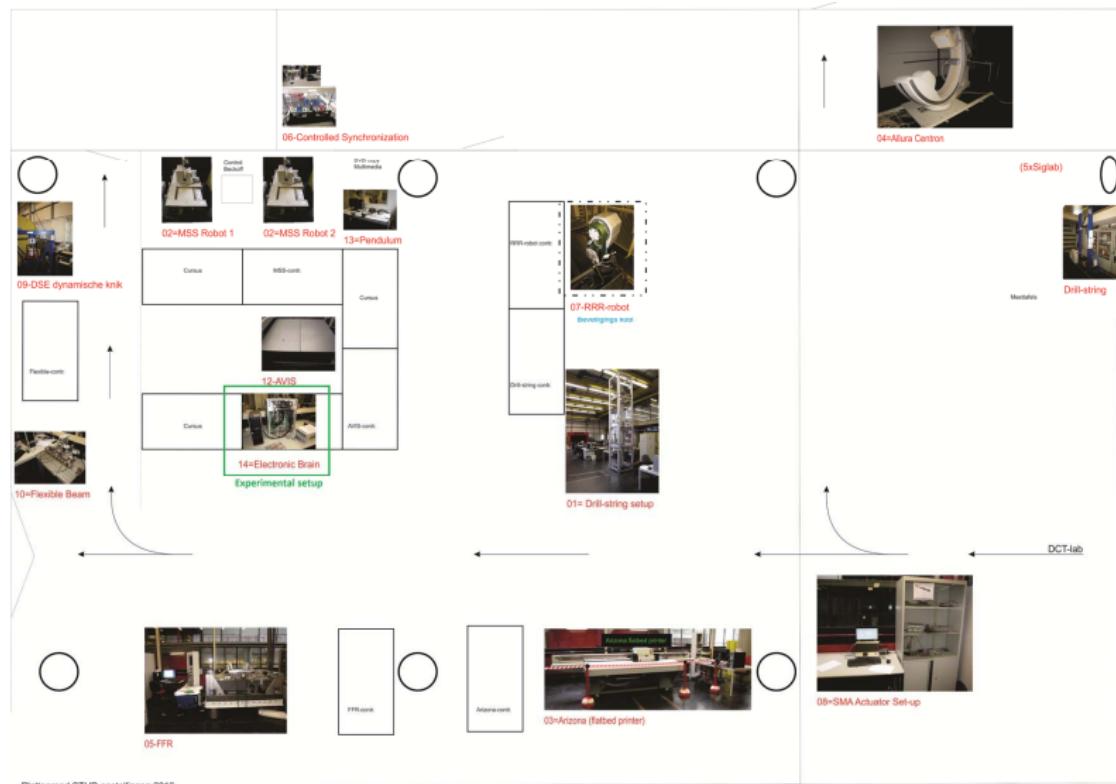
Recap

Experimental setup

Performing experiments

Location setup: DCT lab (Gem-Z -1.13)

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Dokument ID: 00000000000000000000

Powering the setup

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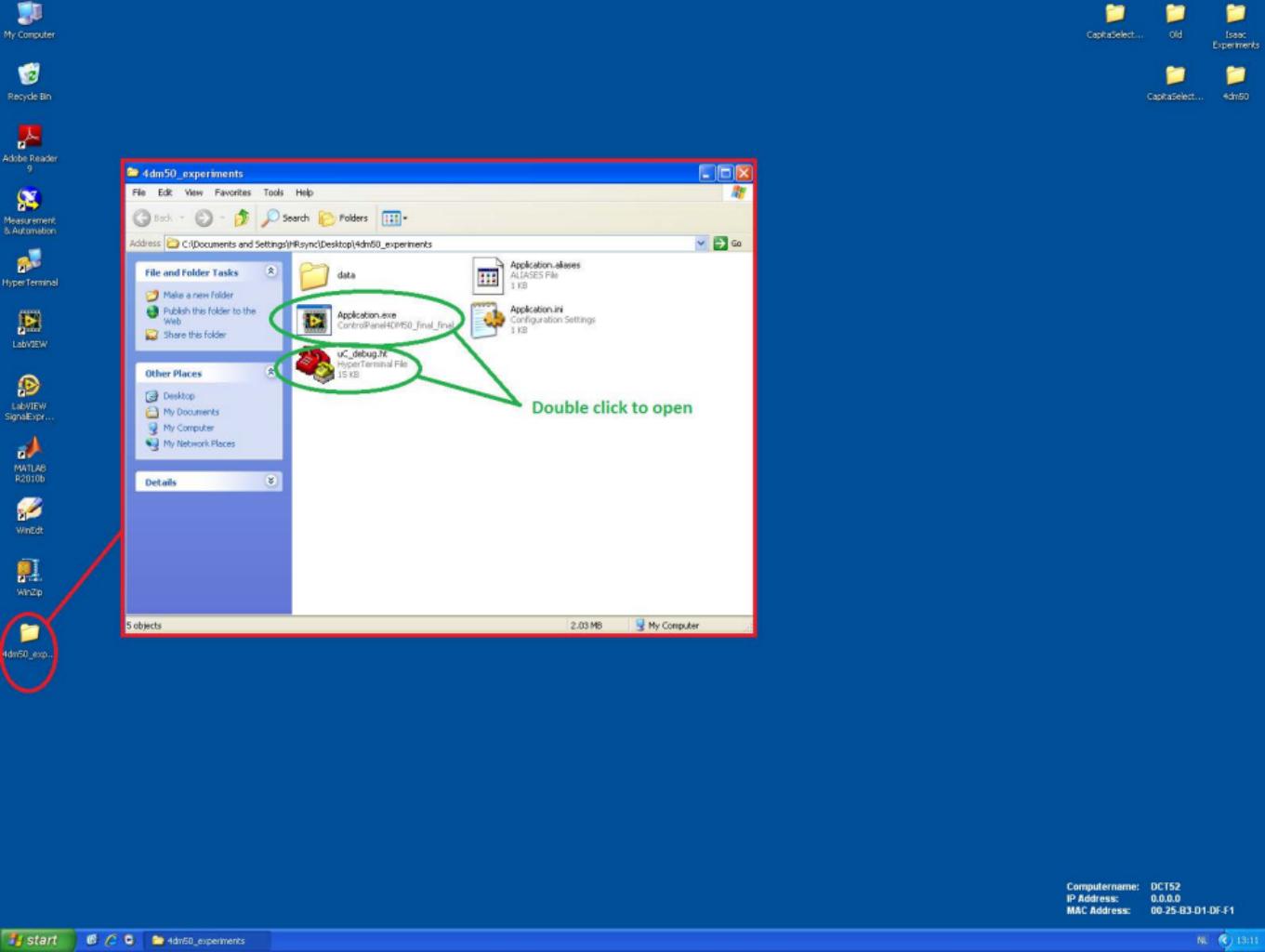


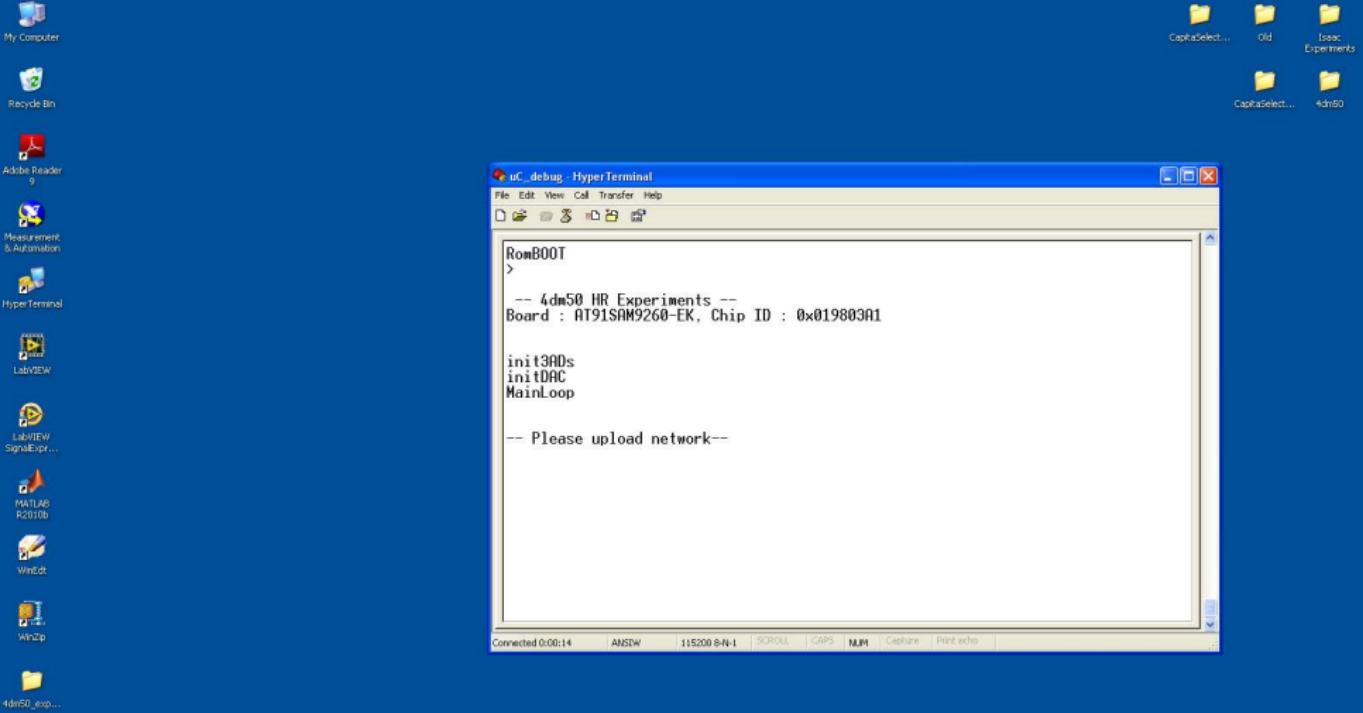
- ▶ Login PC

- User: HRsync
- Password: NI6363sync

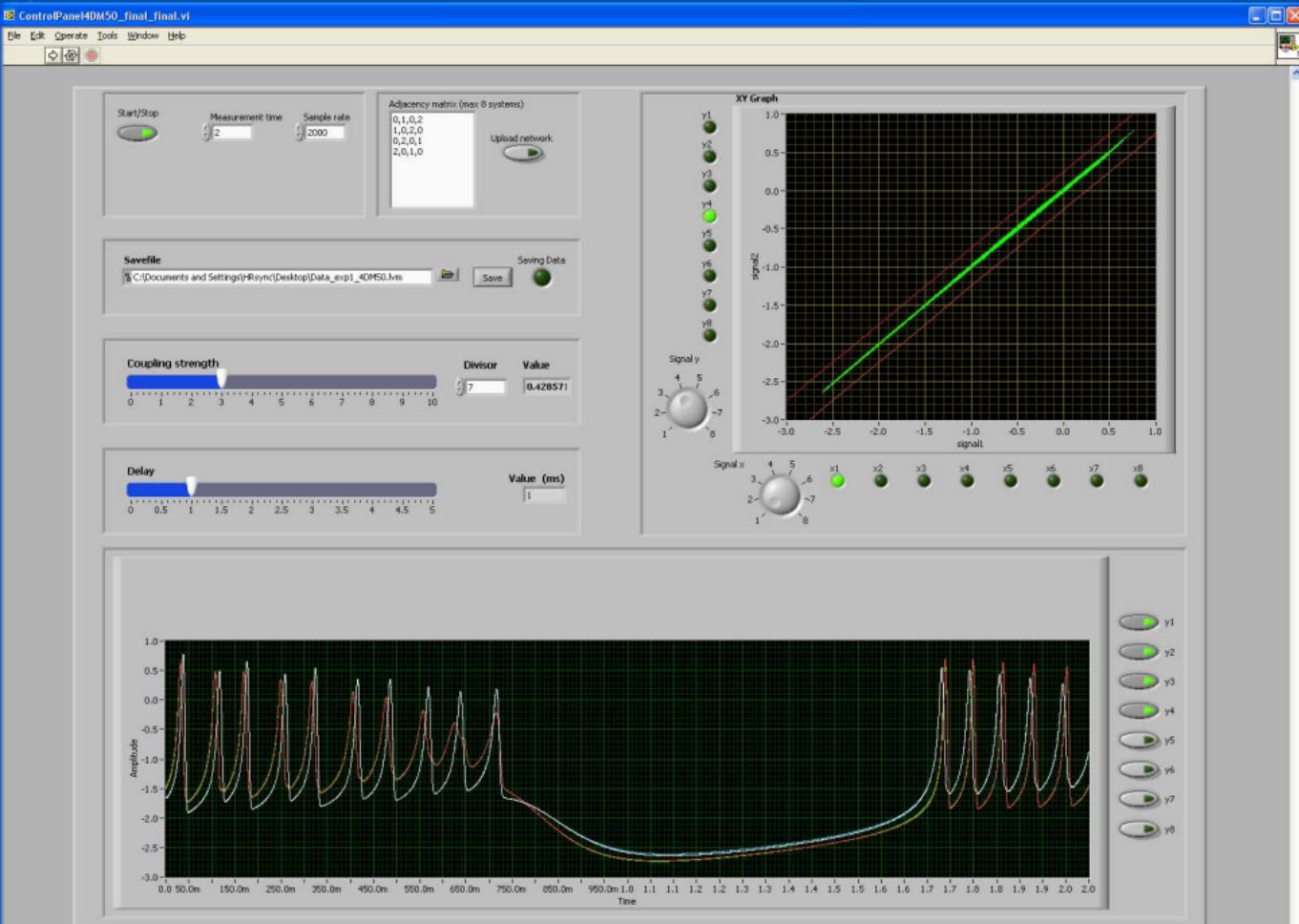
- ▶ Login PC
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 - Password: NI6363sync
- ▶ Open folder: 4dm50_experiments

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 - User: HRsync
 - Password: NI6363sync
- ▶ Open folder: 4dm50_experiments
- ▶ Open: uC_debug.ht and Application.exe





Computername: DCTS2
IP Address: 0.0.0.0
MAC Address: 00-25-B3-D1-DF-F1

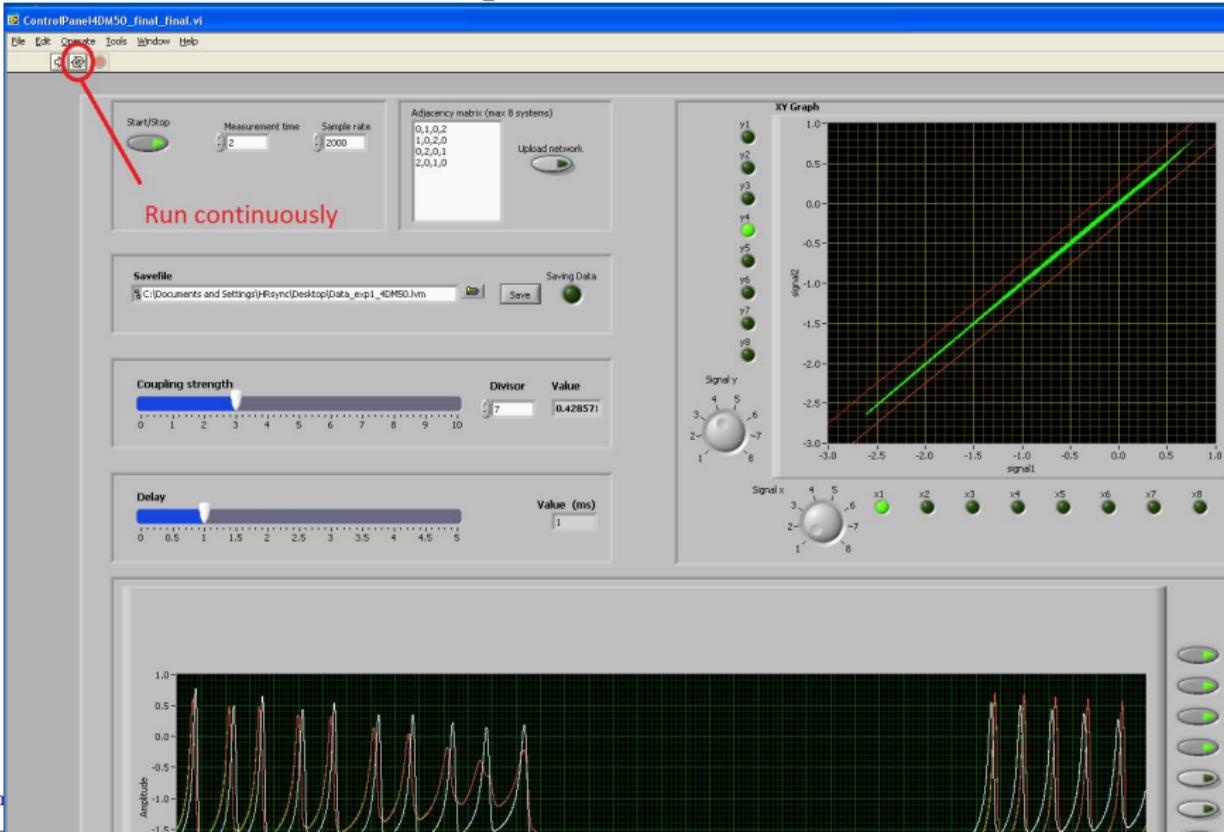


1. Press Run Continuously button

Starting the experiments

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Starting the experiments

23/28

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2. Set the value for Measurement time
(2 seconds is recommended)

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(default: 5 kHz (recommended), max: 40 kHz)

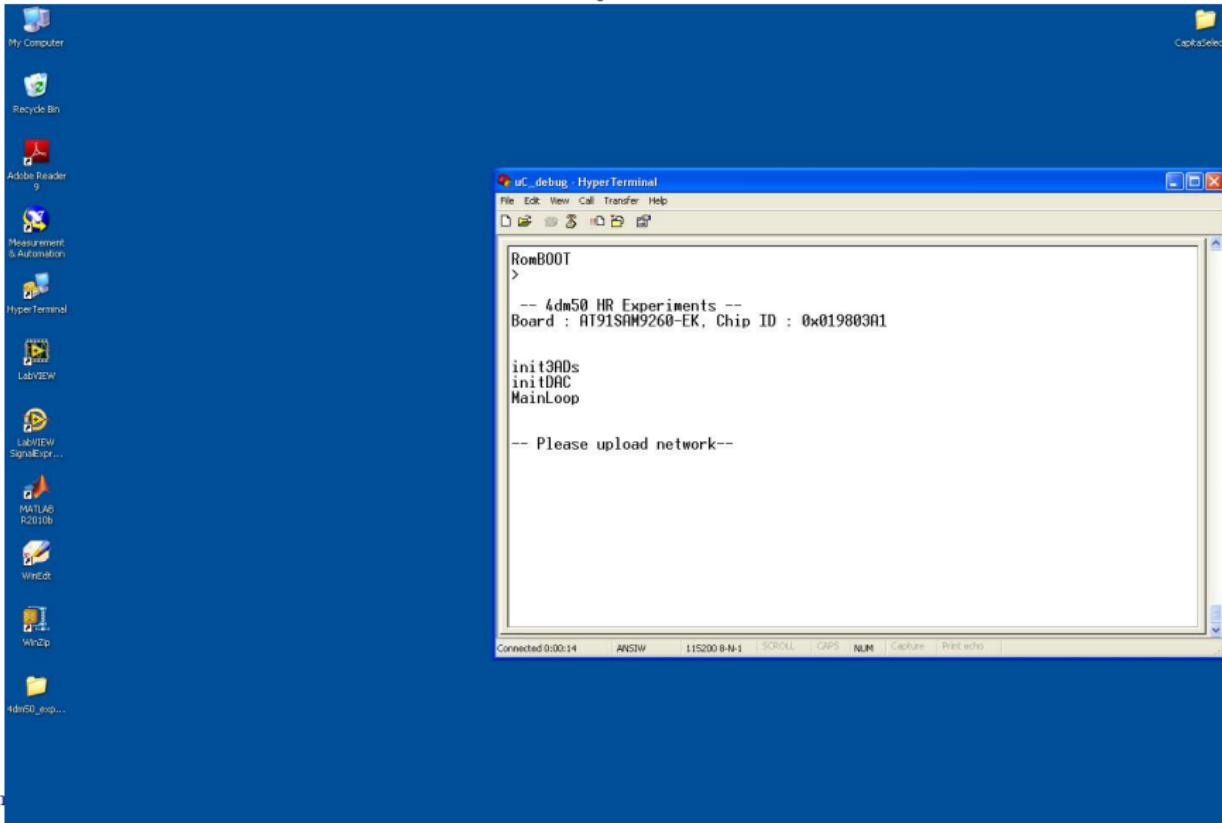
1. Press Run Continuously button
2. Set the value for Measurement time
(2 seconds is recommended)
3. Set the Sampling rate
(default: 5 kHz (recommended), max: 40 kHz)
4. Press Start/Stop button to start (or stop) the measurement

1. Press the reset button on the synchronization interface

Uploading the network

24/28

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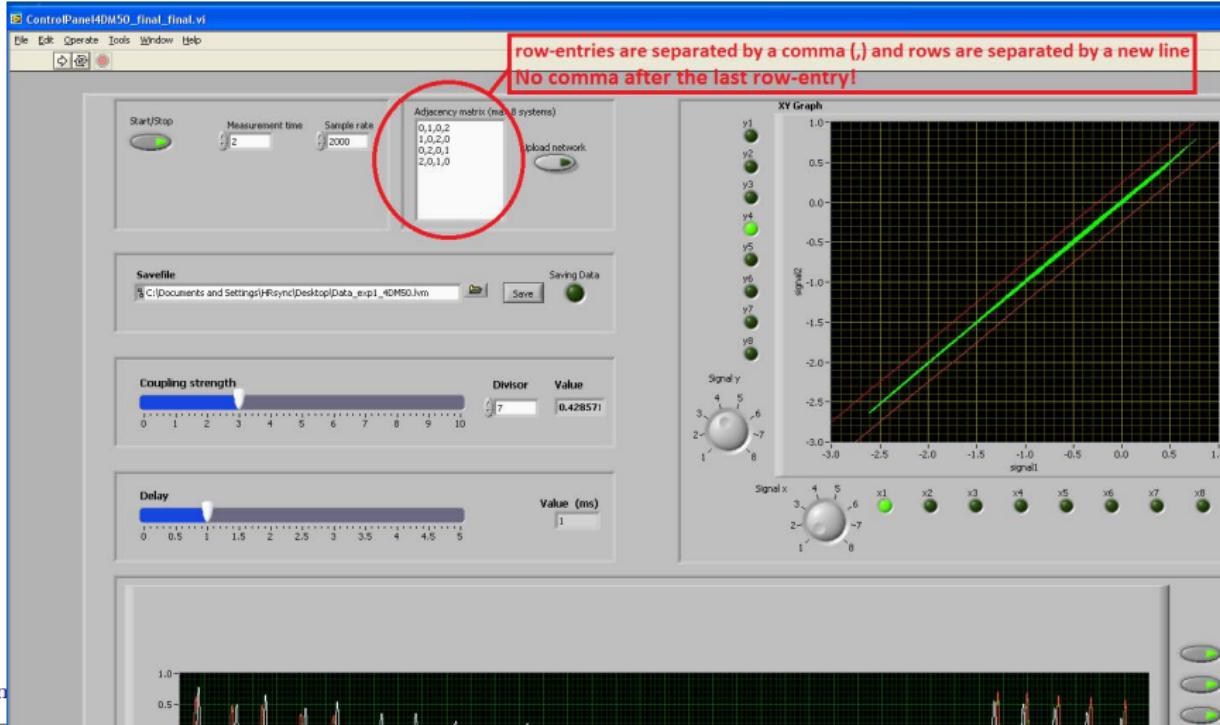


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2. Define the adjacency matrix
(max 8 systems, admissible entries $\{0, 1, 2, \dots, 5\}$)

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24/28

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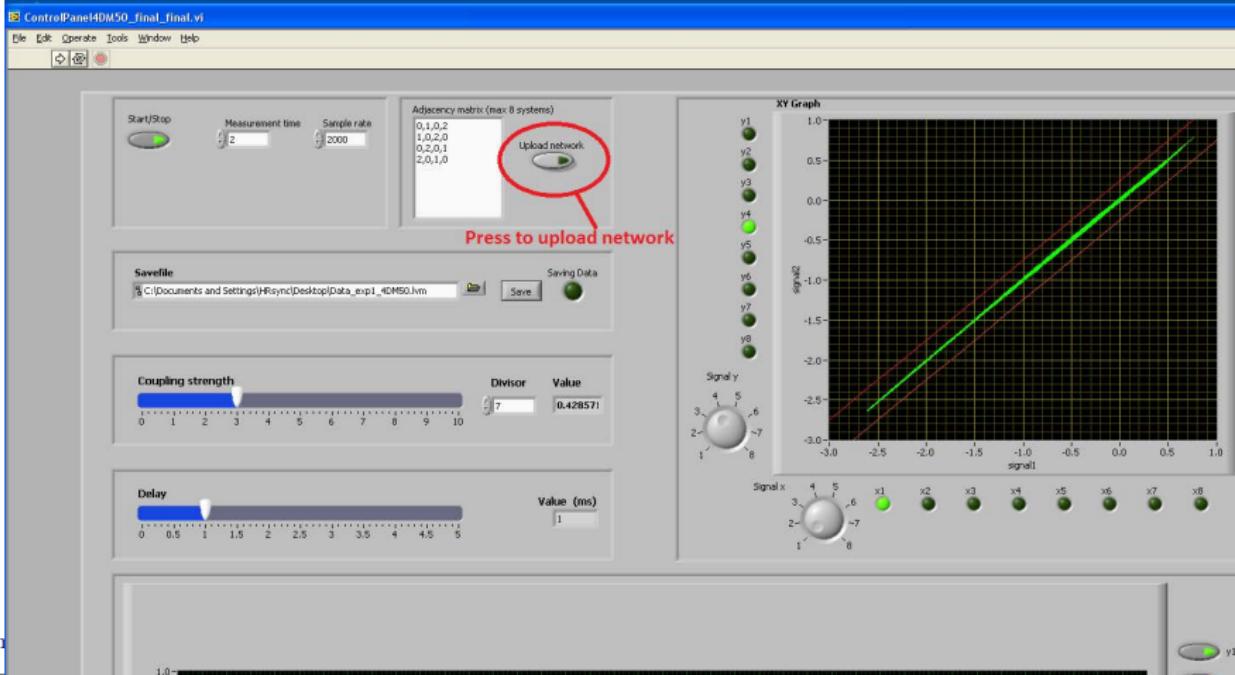


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24/28

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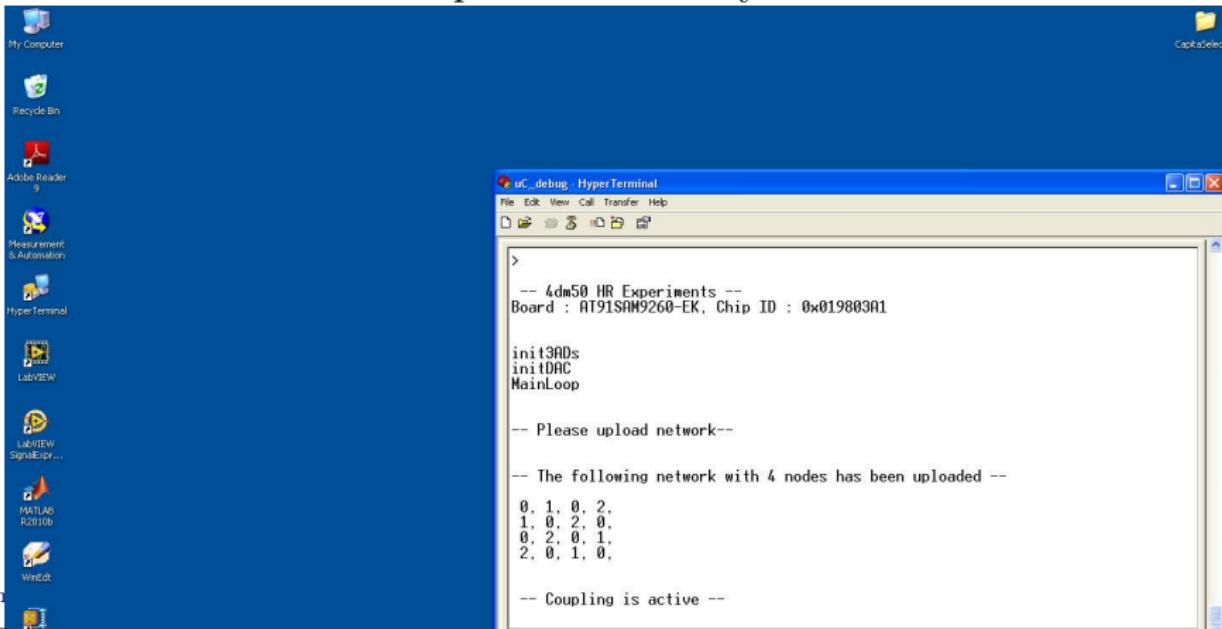


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3. Press upload network button
4. Check if the network is uploaded correctly

Uploading the network

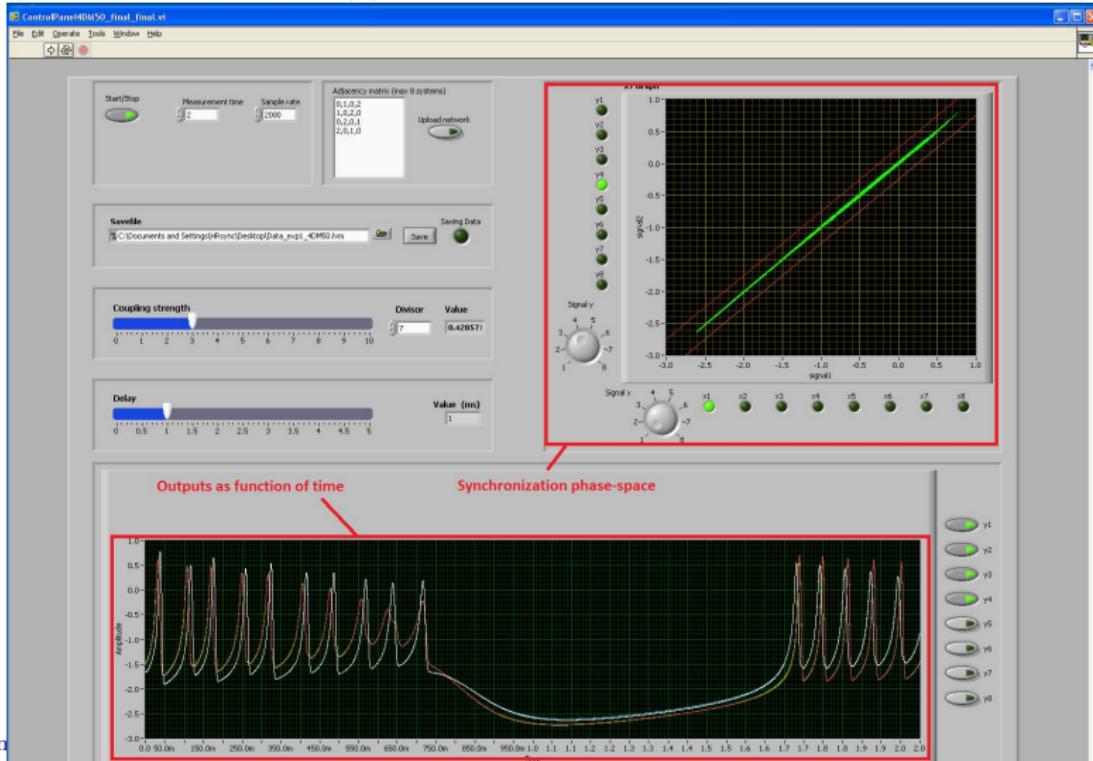
24/28

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Synchronization experiments

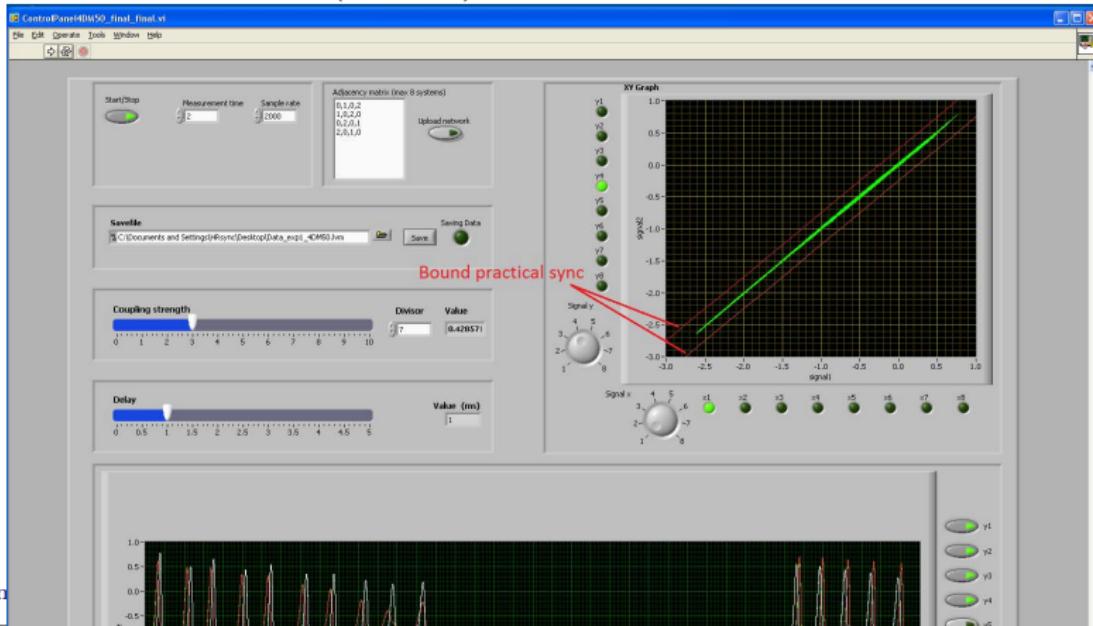
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(Visualized output(s) are indicated by the bright green “led(s)”)



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2. Select the coupling strength and time-delay using the sliders
(use a “divisor” larger than 1 to improve precision)

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(Visualized output(s) are indicated by the bright green “led(s)”)
2. Select the coupling strength and time-delay using the sliders
(use a “divisor” larger than 1 to improve precision)
3. Identify practical (partial) synchronization



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(Recommended: Measurement time: 5 s; sampling rate: 5 kHz)

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3. Press Save to start saving data
 - Saving data starts when the saving data indicator turns green
 - Saving data is completed when the saving data indicator turns off

Data file

The data files (.lvm) can be opened with a text editor or may be imported in Matlab

Time Neuron1 Neuron2 ... Neuron8

	Neuron1	Neuron2	...	Neuron8
0.000000	-2.182114	-2.188239	-2.176311	-2.186950
0.000100	-2.183081	-2.188562	-2.177578	-2.188239
0.000200	-2.182114	-2.186950	-2.175666	-2.188239
0.000300	-2.181469	-2.188239	-2.176311	-2.171153
0.000400	-2.182114	-2.191141	-2.176634	-2.187917
0.000500	-2.181792	-2.188884	-2.175989	-2.187594
0.000600	-2.182114	-2.189206	-2.176311	-2.187917
0.000700	-2.183081	-2.189851	-2.177601	-2.186950
0.000800	-2.182436	-2.188884	-2.175666	-2.187594
0.000900	-2.182436	-2.188562	-2.177923	-2.187917
0.001000	-2.184371	-2.188884	-2.175989	-2.186627
0.001100	-2.183081	-2.188562	-2.177278	-2.187272
0.001200	-2.182436	-2.191141	-2.178245	-2.187594
0.001300	-2.182759	-2.188239	-2.176311	-2.187917
0.001400	-2.182759	-2.189529	-2.176311	-2.188239
0.001500	-2.183404	-2.190818	-2.176956	-2.188239
0.001600	-2.183404	-2.190818	-2.176829	-2.171153
0.001700	-2.183404	-2.190818	-2.176311	-2.171475
0.001800	-2.185328	-2.188884	-2.170101	-2.188562
0.001900	-2.188726	-2.191018	-2.177758	-2.188627
0.002000	-2.183081	-2.188884	-2.177601	-2.188662
0.002100	-2.184693	-2.189529	-2.178568	-2.186627
0.002200	-2.183404	-2.189206	-2.177601	-2.187594
0.002300	-2.183726	-2.191141	-2.175022	-2.188562
0.002400	-2.182759	-2.187917	-2.176634	-2.188239
0.002500	-2.183404	-2.189206	-2.177923	-2.189206
0.002600	-2.185660	-2.189529	-2.177278	-2.188562
0.002700	-2.183081	-2.188884	-2.177278	-2.188239
0.002800	-2.184048	-2.190818	-2.178568	-2.189851
0.002900	-2.184371	-2.189206	-2.177923	-2.188884
0.003000	-2.184048	-2.189529	-2.177923	-2.188884
0.003100	-2.183726	-2.190174	-2.178568	-2.189529
0.003200	-2.183081	-2.190818	-2.176956	-2.188562
0.003300	-2.183404	-2.191141	-2.176101	-2.177174
0.003400	-2.184048	-2.191785	-2.177728	-2.187594
0.003500	-2.183726	-2.190174	-2.176601	-2.172443
0.003600	-2.184371	-2.190818	-2.179213	-2.188562
0.003700	-2.185983	-2.189529	-2.177278	-2.187917
0.003800	-2.184048	-2.189851	-2.178245	-2.189529
0.003900	-2.184693	-2.191141	-2.178568	-2.188239
0.004000	-2.184371	-2.190496	-2.178568	-2.189206
0.004100	-2.183404	-2.190174	-2.177601	-2.189529
0.004200	-2.184693	-2.192108	-2.179213	-2.188884
0.004300	-2.185983	-2.191141	-2.177728	-2.189529
0.004400	-2.184048	-2.191785	-2.178568	-2.190496
0.004500	-2.185983	-2.192108	-2.176956	-2.188884
0.004600	-2.185015	-2.190174	-2.177923	-2.189206
0.004700	-2.185338	-2.190818	-2.179213	-2.187272
0.004800	-2.183726	-2.190818	-2.177174	-2.186644
0.004900	-2.184048	-2.188562	-2.188562	-2.190174
0.005000	-2.184371	-2.192753	-2.177923	-2.189206
0.005100	-2.185015	-2.189529	-2.178245	-2.188884
0.005200	-2.184371	-2.190496	-2.177923	-2.190496
0.005300	-2.185983	-2.191463	-2.178245	-2.190174
0.005400	-2.185015	-2.190818	-2.177601	-2.189851

After your experiments: Power off!

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