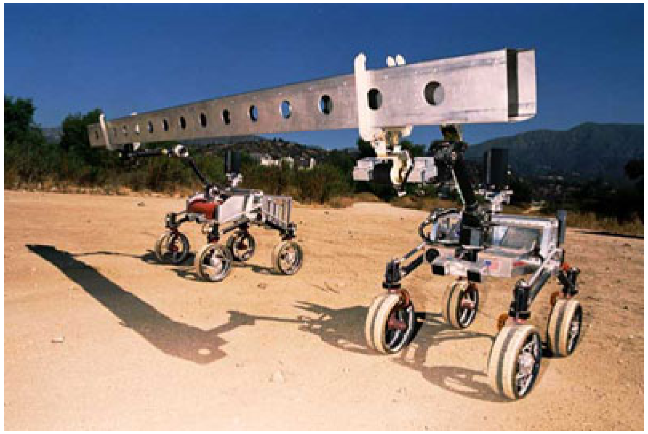
Cooperative agents

- **Applications:**
  - **simultaneous localization and mapping**
  - **automated highway systems**
  - **payload transportation**
  - **enclosing an invader**
  - **exploration of an unknown environment**
  - **robotic soccer**



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Mobile robots: research challenges

- Decentralized cooperative control
- Robustness to communication constraints
- Localization of agents
- Sensing and environment mapping
- Autonomous decision making
- Energy efficiency
- etc.

Coordinated trajectory tracking

- **A group of interacting agents moves along a desired trajectory**
- **The group is asymptotically stabilized at some geometric pattern (e.g. platoon)**
- **Inter-agent and agent-obstacle avoidance is guaranteed**
- **Standard approaches:**
  - **leader-follower**
  - **behavior-based**
  - **virtual structure**

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Coordinated control

- **Motion coordination of mobile robots**
- **Saturated control**
- **Robustness to perturbations**
- **Dynamic collision avoidance**

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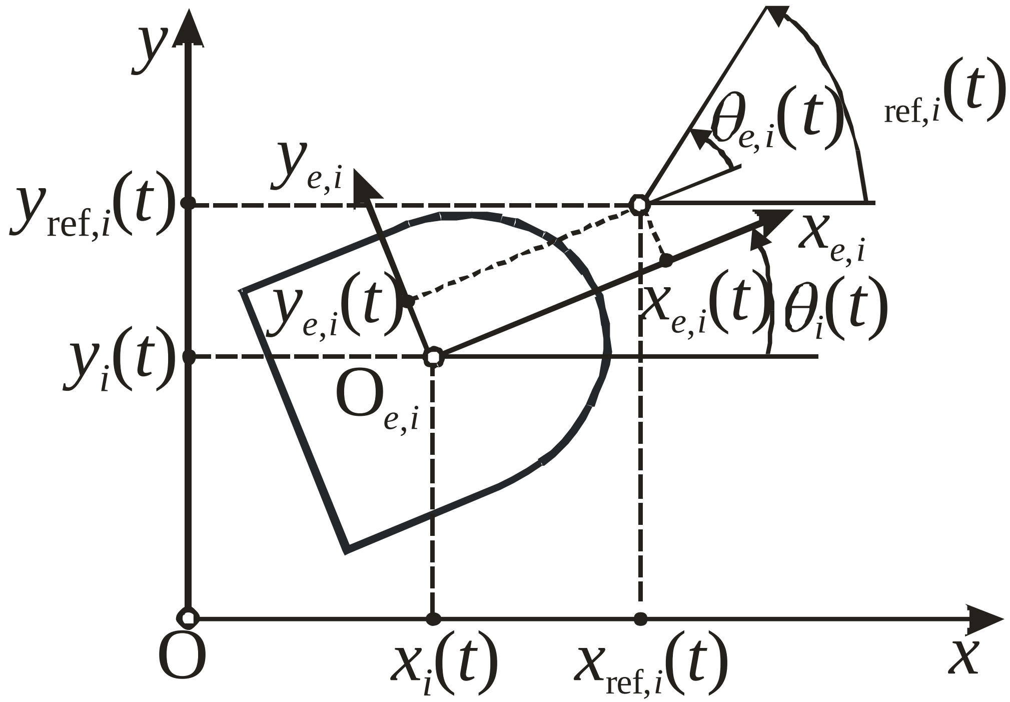
Control problem

- **Kinematics of an unicycle $i \in \{1,2,\dots,n\}$:**

$$\dot{x}_i = v_i \cos\theta_i,$$
$$\dot{y}_i = v_i \sin\theta_i,$$
$$\dot{\theta}_i = \omega_i.$$



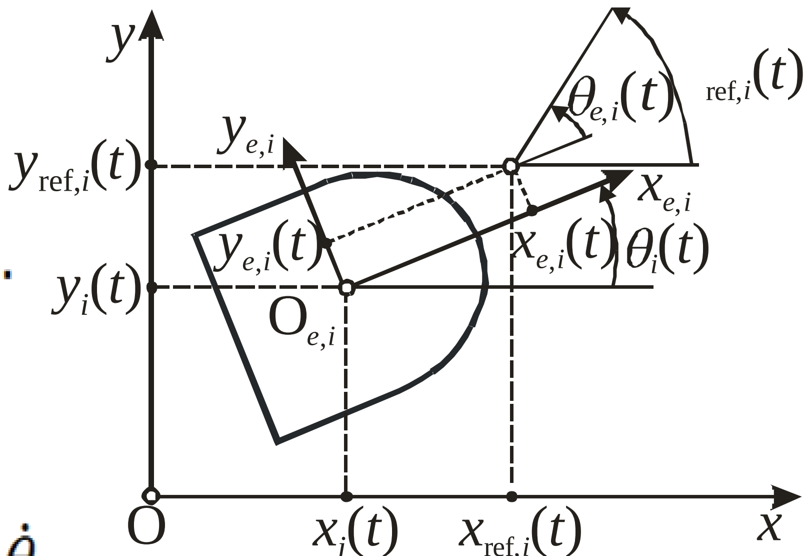
- **Reference trajectory:**

$$\mathbf{p}_{\text{ref},i}(t) = \begin{bmatrix} x_{\text{ref},i}(t) \\ y_{\text{ref},i}(t) \\ \theta_{\text{ref},i}(t) \end{bmatrix}.$$

- **Find v_i and ω_i such that the agent tracks $\mathbf{p}_{\text{ref},i}(t)$ while**

$$|v_i(t)| \le v_{\max,i}, \quad |\omega_i(t)| \le \omega_{\max,i}, \quad \forall t \ge 0.$$

Tracking error dynamics



- **Tracking errors:**

$$\begin{bmatrix} x_{e,i} \\ y_{e,i} \\ \theta_{e,i} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\mathrm{ref},i} - x_i \\ y_{\mathrm{ref},i} - y_i \\ \theta_{\mathrm{ref},i} - \theta_i \end{bmatrix}.$$

- **Define:**

$$v_{\mathrm{ref},i} = \sqrt{(\dot{x}_{\mathrm{ref},i})^2 + (\dot{y}_{\mathrm{ref},i})^2}, \quad \omega_{\mathrm{ref},i} = \dot{\theta}_{\mathrm{ref},i},$$

$$\mathbf{e}_{xy,i} = \begin{bmatrix} x_{e,i} \\ y_{e,i} \end{bmatrix}, \quad \mathbf{S}(\omega_i) = \begin{bmatrix} 0 & -\omega_i \\ \omega_i & 0 \end{bmatrix}.$$

- **Error dynamics:**

$$\dot{\mathbf{e}}_{xy,i} = -\mathbf{S}(\omega_i)\mathbf{e}_{xy,i} + \begin{bmatrix} v_{\mathrm{ref},i}\cos\theta_{e,i} - v_i \\ v_{\mathrm{ref},i}\sin\theta_{e,i} \end{bmatrix},$$

$$\dot{\theta}_{e,i} = \omega_{\mathrm{ref},i} - \omega_i.$$

Saturation functions

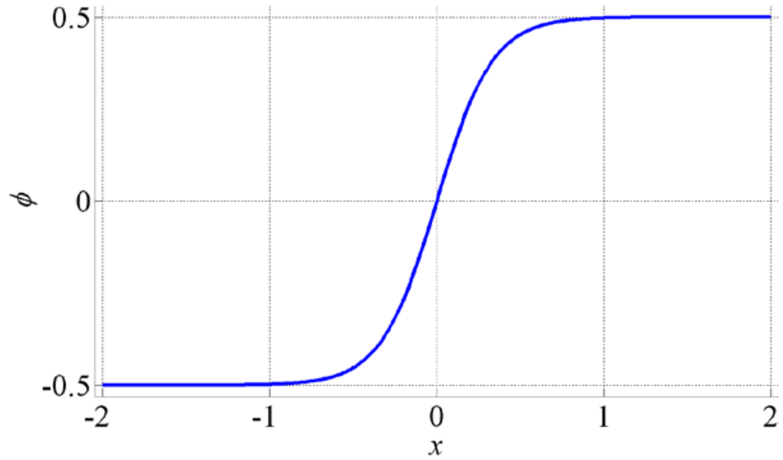
- **Set of saturation functions:**

$$\mathrm{S}_{r,k} = \{\phi_r(kx){:}\mathrm{R} \to \mathrm{R} \mid \phi_r(kx) \text{is uniformly continuous,}$$
$$-r \leq \phi_r(kx) \leq r \;\; \forall x \in \mathrm{R},\;\; \phi_r(0) \equiv 0,$$
$$x\phi_r(kx) > 0 \text{ for all } x \neq 0,\; \phi_r(kx) + \phi_r(-kx) \equiv 0\}$$

  - **Examples:**

$$\phi_{r,k}(kx) = r\frac{kx}{\sqrt{1+(kx)^2}},$$

$$\phi_{r,k}(kx) = r\tanh(kx).$$



Coordinated control with saturation

1. $v_{\mathrm{ref},i}$ **is nonzero, bounded and uniformly continuous, while** $\omega_{\mathrm{ref},i}$ **is bounded over** $t \in [0,\infty)$**;**
2. $v_{\mathrm{ref},i}$ **is bounded, uniformly continuous, and** $\lim_{t\to\infty} v_{\mathrm{ref},i} = 0$**, while** $\omega_{\mathrm{ref},i}$ **is nonzero, bounded, and uniformly continuous over** $t \in [0,\infty)$**.**

- **Control law:**

$$v_i(t) = v_{\mathrm{ref},i}(t)\cos\theta_{e,i} + \phi_{k_{x,i}(t)}\left(c_{x,i}x_{e,i}\right) + \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{l_{xx,i,j}(t)x_{e,j}\phi_{k_{xx,i,j}(t)}\left(c_{xx,i,j}(x_{e,i}-x_{e,j})\right)}{\sqrt{1+\left(l_{xx,i,j}(t)x_{e,i}x_{e,j}\right)^2}},$$

$$\omega_i(t) = \omega_{\mathrm{ref},i}(t) + \phi_{k_{\theta,i}(t)}\left(c_{\theta,i}\theta_{e,i}\right) + \frac{k y_{e,i} k_y v_{\mathrm{ref},i}(t)}{\sqrt{1+k^2\left(\mathbf{e}_{xy}\right)^T\mathbf{e}_{xy}}}\frac{\sin\theta_{e,i}}{\theta_{e,i}}$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{l_{\theta\theta,i,j}(t)\theta_{e,j}\phi_{k_{\theta\theta,i,j}(t)}\left(c_{\theta\theta,i,j}\left(\theta_{e,i}-\theta_{e,j}\right)\right)}{\sqrt{1+\left(l_{\theta\theta,i,j}(t)\theta_{e,i}\theta_{e,j}\right)^2}} + \phi_{k_{yy,i,j}(t)}\left(c_{yy,i,j}\left(y_{e,i}-y_{e,j}\right)\right)\frac{\sin\theta_{e,i}}{\theta_{e,i}}\sin\theta_{e,j}\right).$$

Asymptotic stability

- **Lyapunov direct method:** $$V = \frac{k_y}{k}\sqrt{1 + k^2(\mathbf{e}_{xy})^T\mathbf{e}_{xy}} + 0.5(\boldsymbol{\theta}_e)^T\boldsymbol{\theta}_e - \frac{k_y}{k},$$

$$\mathbf{e}_{xy} = \left[(\mathbf{e}_{xy,1})^T, \dots, (\mathbf{e}_{xy,n})^T\right]^T, \quad \boldsymbol{\theta}_e = [\theta_{e,1}, \dots, \theta_{e,n}]^T.$$

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\mathbf{e}_{xy}, \boldsymbol{\theta}_e) = -\frac{k_y k \sum_{i=1}^{n}\left[x_{e,i}\phi_{k_{x,i}(t)}(c_{x,i}x_{e,i})\right]}{\sqrt{1 + k^2(\mathbf{e}_{xy})^T\mathbf{e}_{xy}}} - \sum_{i=1}^{n}\left[\theta_{e,i}\phi_{k_{\theta,i}(t)}(c_{\theta,i}\theta_{e,i})\right] \le 0.$$

Use Barbălat’s lemma etc. resulting in asymptotic stability.

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Collision avoidance by penalizing forward velocities

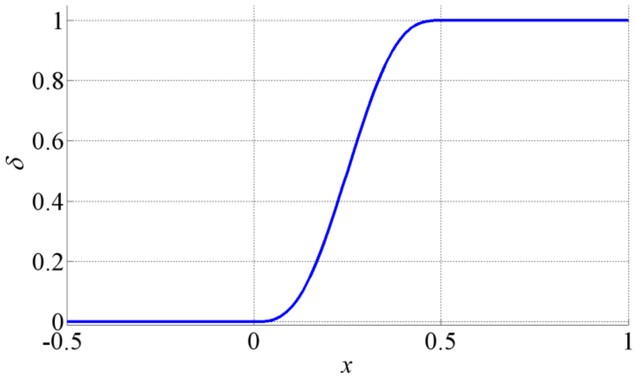
- **Distance to collision between agents i and j:**

$$\Delta_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - \mathrm{D},$$

  D **is the agent's diameter.**

- **For n agents, there are $0.5n!/(n\text{-}2)!$ different $\Delta_{i,j}$.**
- **Agents should have unequal priorities.**
- **Penalizing desired velocities:**

$$v_{\mathrm{ref},i}(t) = v_{\mathrm{des},i}(t) \prod_{\substack{j=1 \\ (j \neq i)}}^{n} \delta_{\gamma,i}(\Delta_{i,j}),$$



$v_{\mathrm{des},i}$ – **desired forward velocity,**

γ – **critical distance to collision,**

$\delta_{\gamma,i}$ – **penalty function.**

Collision avoidance based on an Artificial Potential Field

- **Artificial potential:**

$$V_{i,j}(x_i - x_j, y_i - y_j) = \begin{cases} K_i e^{-\frac{1}{p}\left(\left(\frac{x_i - x_j}{\alpha}\right)^p + \left(\frac{y_i - y_j}{\beta}\right)^p\right)} & \text{if } \left(\frac{x_i - x_j}{\alpha}\right)^p + \left(\frac{y_i - y_j}{\beta}\right)^p \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

- **Repulsive APF,** $i \in \{1,2,\dots,n\}$**:**

$$V_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} V_{i,j}(x_i - x_j, y_i - y_j),$$

- **Repulsive actions:**

$$[\delta v_{x,i}, \delta v_{y,i}] = -\mathrm{sat}\left(\left[\frac{\partial V_i}{\partial x_i}, \frac{\partial V_i}{\partial y_i}\right]\right),$$

where 'sat' is a saturation function taking care that the velocity constraints are met.

- **Collision-free trajectories:**

$$x_{\mathrm{ref},i}(t_k) = x_i(t_{k-1}) + (t_k - t_{k-1})\delta v_{x,i},$$
$$y_{\mathrm{ref},i}(t_k) = y_i(t_{k-1}) + (t_k - t_{k-1})\delta v_{y,i}.$$

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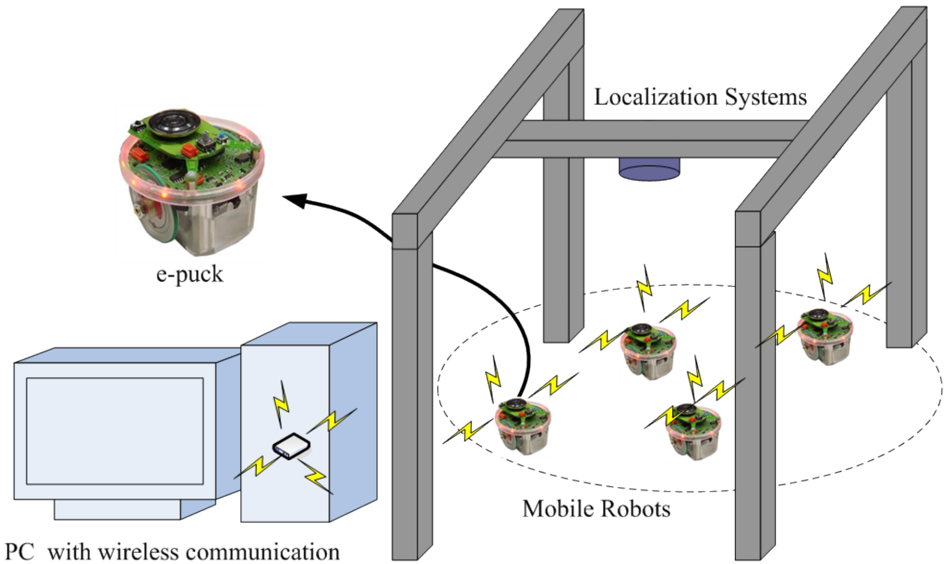
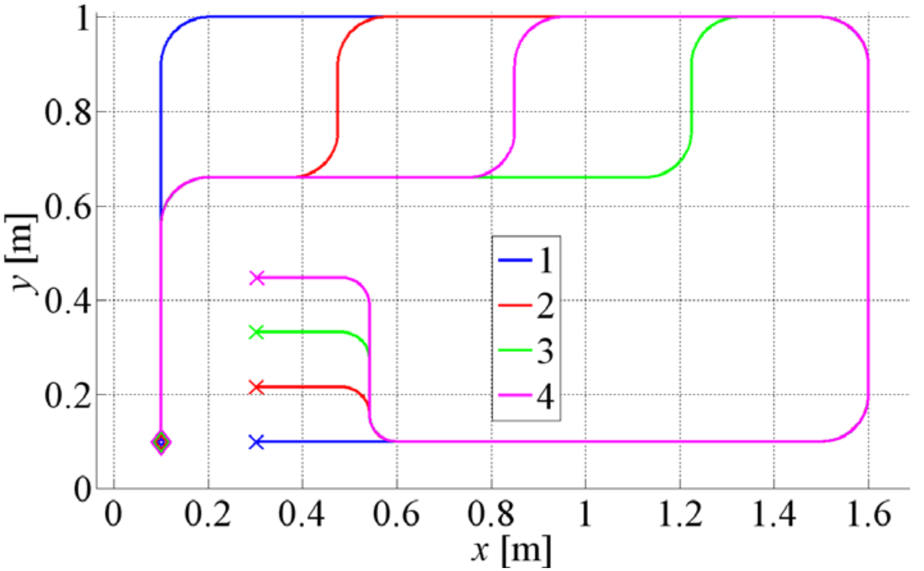
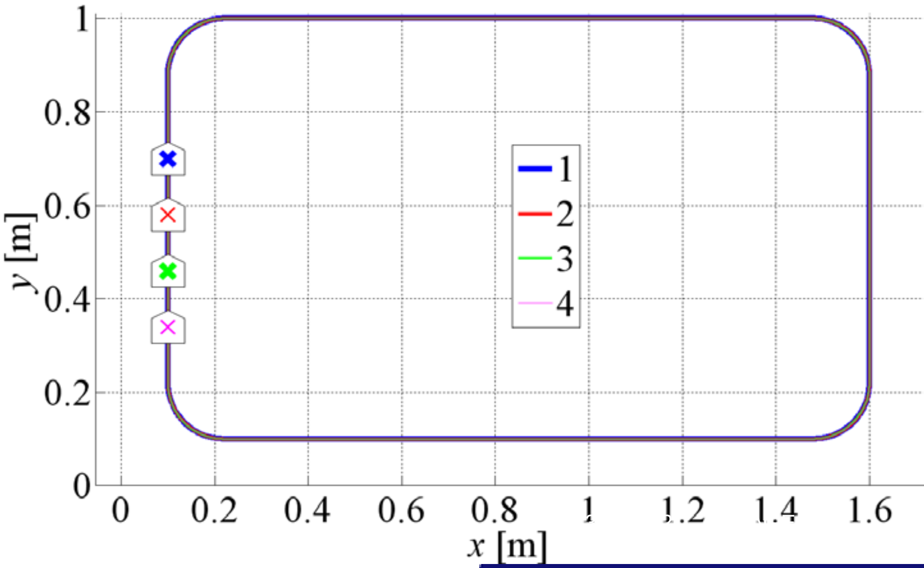
Experimental case-studies

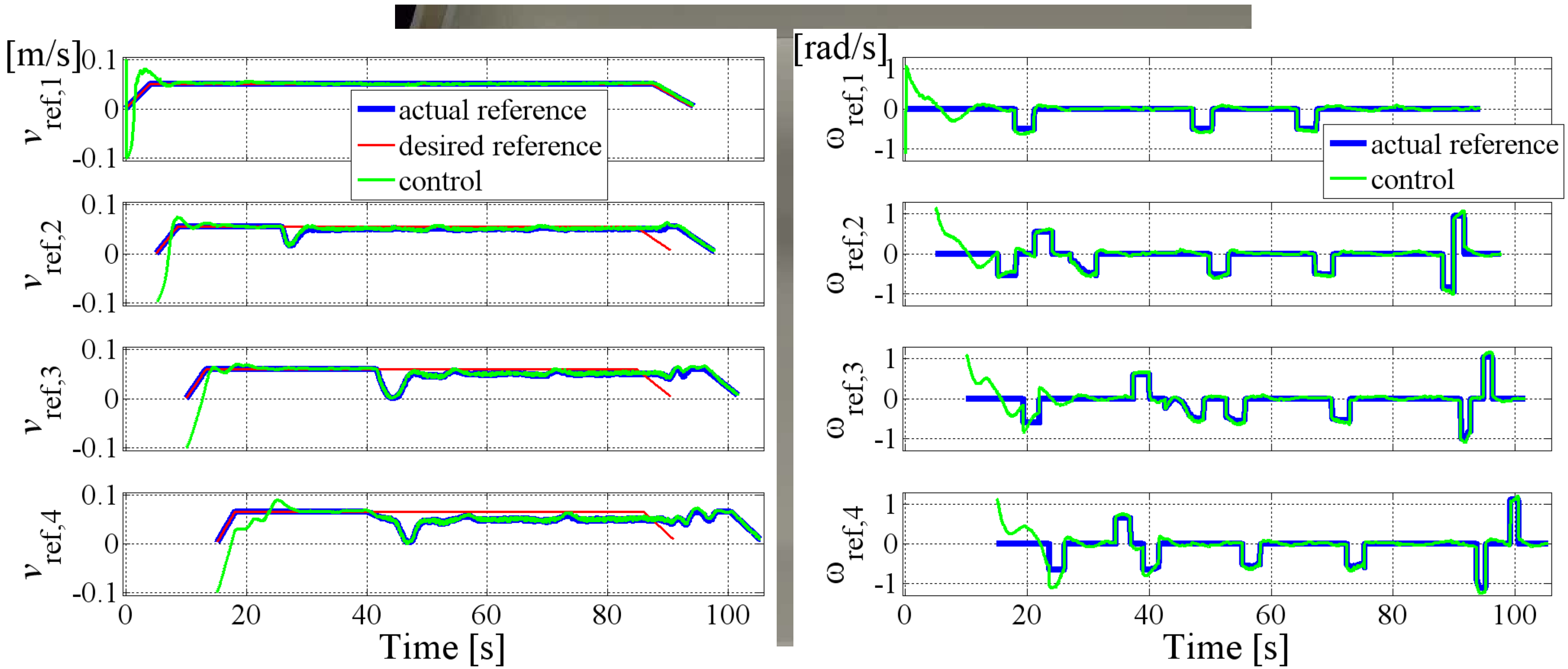






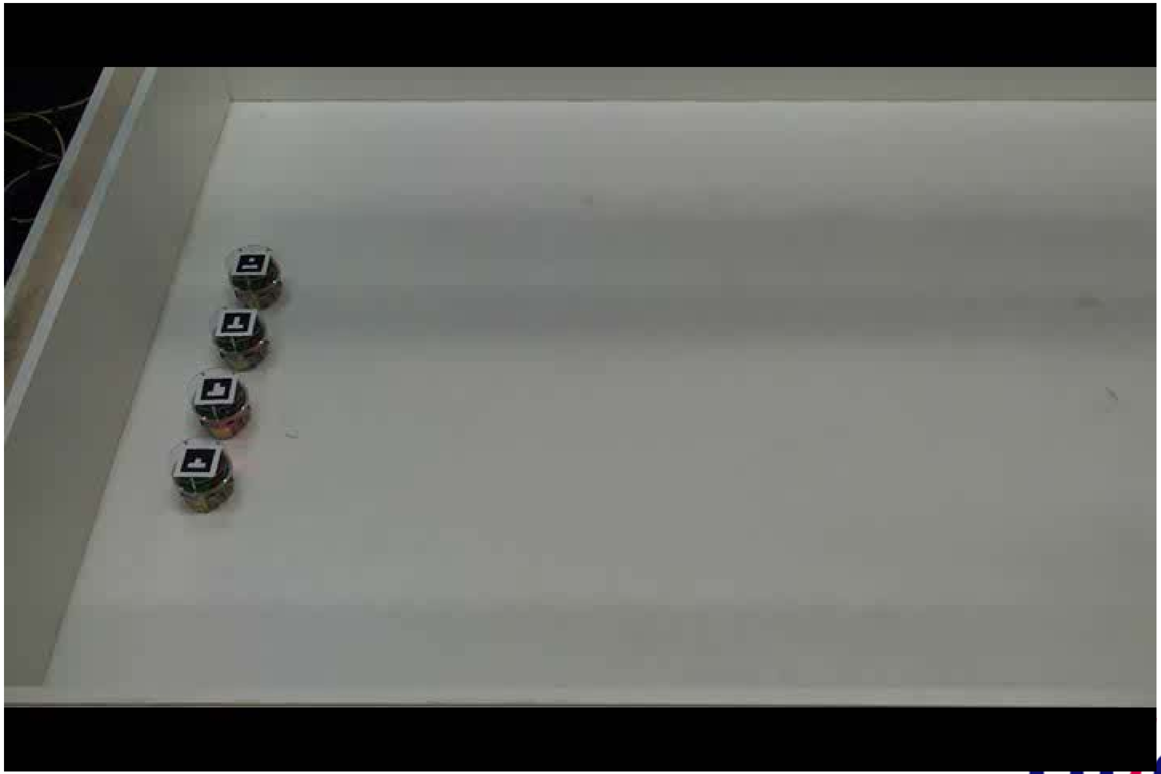
Mutual couplings disabled

- Cohistraintsoidtheecbytpehsalgnials velocities



Coordination of a platoon

- **Mutual couplings are enabled**
- **Collision avoidance based on the APF method**



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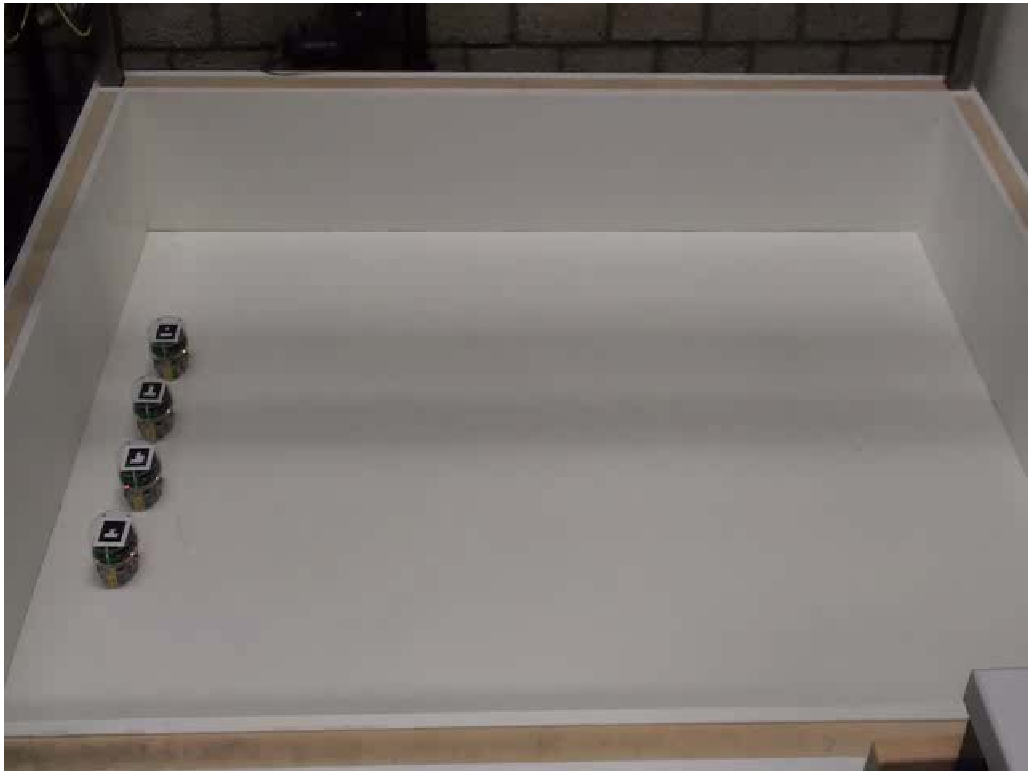
Coordination of a platoon and overtaking

- **Mutual couplings are enabled**
- **Collision avoidance based on the APF method**



Design parameters

- **Strictly positive gains:** $k_{x,i}(t), k_{\theta,i}(t), c_{x,i}, c_{\theta,i}, k_y, k \in \mathbb{R}_+$
- **Coupling gains are allowed to be zero (no coordination):**

$$k_{a,i,j}(t), c_{a,i,j}, l_{a,i,j}(t) \in \mathbb{R}_+ \cup \{0\}$$

- **Coupling gains should be symmetric:**

$$k_{a,i,j}(t) \equiv k_{a,j,i}(t), \qquad c_{a,i,j} = c_{a,j,i}, \qquad l_{a,i,j}(t) \equiv l_{a,j,i}(t).$$

- **Sufficient conditions to meet constraints on v_i and ω_i:**

$$k_{\theta,i}(t) + k_y\left|v_{\mathrm{ref},i}(t)\right| + \sum_{\substack{j=1\\ j\neq i}}^{n}\left(k_{\theta\theta,i,j}(t) + k_{yy,i,j}(t)\right) \leq \omega_{\max,i} - \left|\omega_{\mathrm{ref},i}(t)\right|,$$

$$k_{x,i}(t) + \sum_{\substack{j=1\\ j\neq i}}^{n} k_{xx,i,j}(t) \leq v_{\max,i} - \left|v_{\mathrm{ref},i}(t)\right|.$$

Mutual couplings disabled: case 2

- **Collision avoidance based on the APF method**