VEHICULAR PLATOONING
Dynamics and Controller Design

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4DM50 Dynamics and Control of Cooperation
May 1, 2017

TNO innovation for life

INTRODUCTION

- Platooning
  - Close-distance vehicle following
  - Usually automated (in longitudinal direction)
- Benefits
  - Increased road capacity
  - Decreased fuel consumption and CO/CO_2 emissions (trucks)
  - On the long run: increased traffic safety

INTRODUCTION

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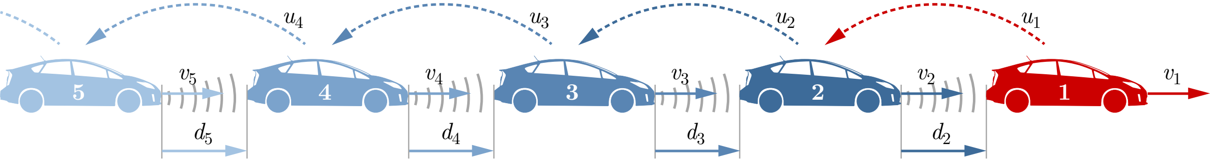
- Benefits
  - Increased road capacity
  - Decreased fuel consumption and CO/CO_2 emissions (trucks)
  - On the long run: increased traffic safety
- Cooperative Adaptive Cruise Control (CACC)
  - Platooning: A platoon leader exists, known platoon members
  - CACC: No leader, members unknown; "ad-hoc platooning"
  - Basic controller design no different

CONTROL OBJECTIVES

- Vehicle following
  - $d_i \to d_{\mathrm{r},i}$
  - Desired distance $d_{\mathrm{r},i} = r + \boldsymbol{h} v_i$



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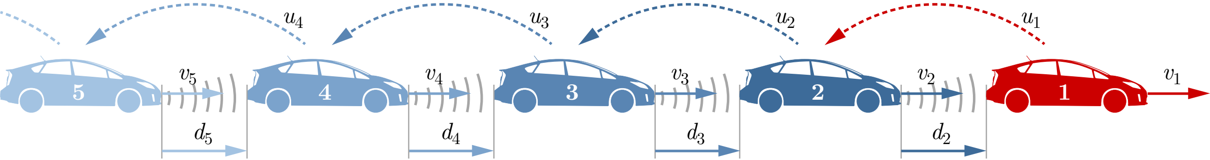
  - Attenuation of the effect of disturbances, e.g., velocity variations, over the vehicle string (in general in upstream direction)



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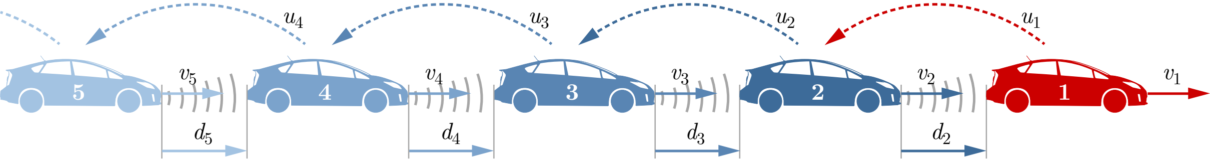
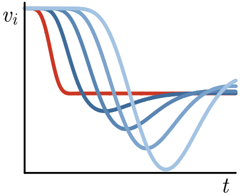
  - Attenuation of the effect of disturbances, e.g., velocity variations, over the vehicle string (in general in upstream direction)
- Safety
  - $d_i \geq 0$

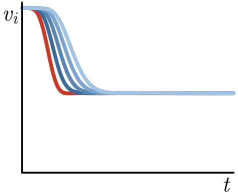


STRING (IN)STABILITY INTUITION

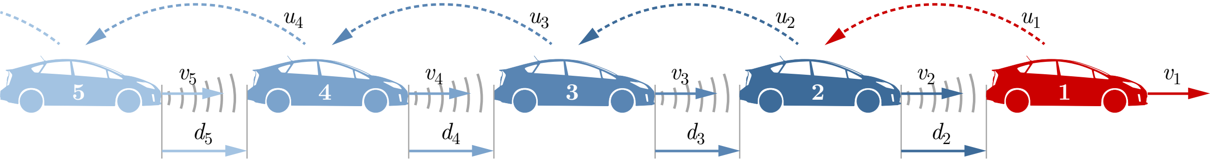
- String instability: Amplification of the effects of disturbances along the string.



"String Unstable"



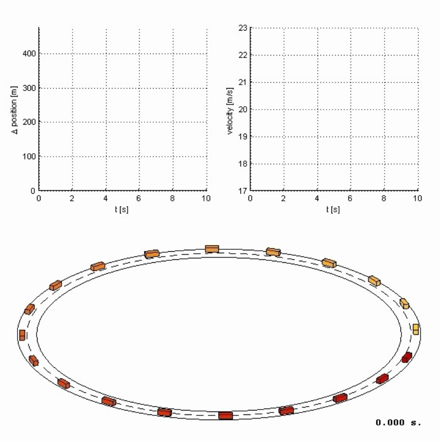
"String Stable"



STRING (IN)STABILITY INTUITION

- Circular ("infinite") string of ACC vehicles (i.e., no communication)
- 72 km/h initial velocity of all vehicles
- $h = 0.5$ s time gap (10 m) for all vehicles, except one:
- 2 m initial position error of one vehicle in the string

- Clearly string unstable



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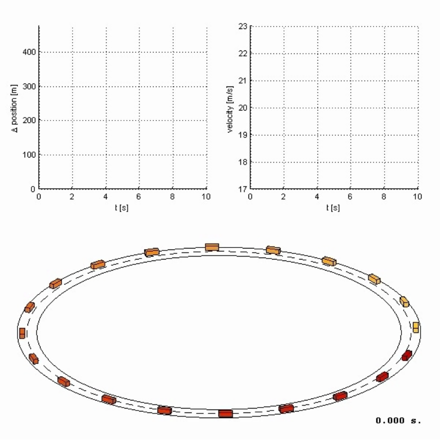
- A minimum time gap h_{min} exists, for ACC and for CACC, for which the string is string stable.



CONTROLLER DESIGN

- "Vehicle" model

$$\begin{pmatrix} \dot{d}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ a_i \\ -\frac{1}{\tau}a_i + \frac{1}{\tau}u_i \end{pmatrix}, \quad \forall\, i > 1$$

i: vehicle index (upstream increasing)
a_i: acceleration
v_i: velocity
$d_i = q_{i-1} - q_i - L_i$: distance between vehicle i and $i-1$, with position q_i and vehicle length L_i
u_i: external input (desired acceleration)
τ: time constant

- Homogeneous string of vehicles

CONTROLLER DESIGN

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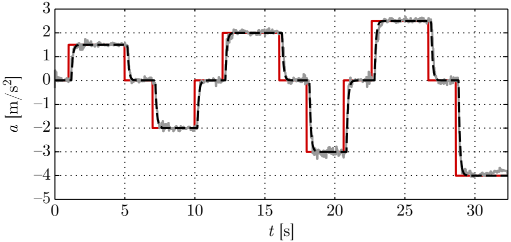
- Homogeneous string of vehicles

- 1st-order transfer function from u_i to a_i:
  - Driveline dynamics
  - Lower-level acceleration controller

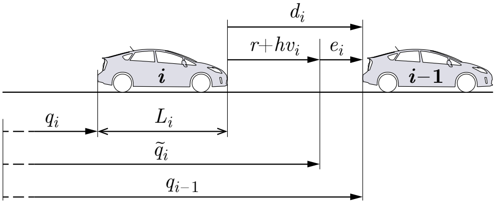


red: desired acceleration
gray: measured acceleration
dashed black: simulated acceleration

CONTROLLER DESIGN

- Control objectives: see slide 3
- Desired distance ("spacing policy"): $d_{\mathrm{r},i} = r_i + hv_i$
- Formulate **error dynamics**:

$$
\begin{aligned}
e_i &= d_i - d_{\mathrm{r},i} = (q_{i-1} - q_i - L_i) - (r + hv_i) \\
\dot{e}_i &= v_{i-1} - v_i - ha_i \\
\ddot{e}_i &= \ldots \\
\dddot{e}_i &= -\tfrac{1}{\tau}\ddot{e}_i - \tfrac{1}{\tau}\underbrace{(h\dot{u}_i + u_i)}_{:=\xi_i} + \tfrac{1}{\tau}u_{i-1}
\end{aligned}
$$



- 1st-order "input filter":

$$
\dot{u}_i = -\frac{1}{h}u_i + \frac{1}{h}\xi_i
$$

CONTROLLER DESIGN

- Choose state vector according to $\varepsilon_i^{\mathrm{T}} = \begin{pmatrix}\varepsilon_{1,i} & \varepsilon_{2,i} & \varepsilon_{3,i}\end{pmatrix}^{\mathrm{T}} = \begin{pmatrix}e_i & \dot{e}_i & \ddot{e}_i\end{pmatrix}^{\mathrm{T}}$
- Error dynamics (without input filter)

$$\begin{pmatrix}\dot{\varepsilon}_{1,i} \\ \dot{\varepsilon}_{2,i} \\ \dot{\varepsilon}_{3,i}\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau}\end{pmatrix} \begin{pmatrix}\varepsilon_{1,i} \\ \varepsilon_{2,i} \\ \varepsilon_{3,i}\end{pmatrix} - \begin{pmatrix}0 \\ 0 \\ \frac{1}{\tau}\end{pmatrix} \xi_i + \begin{pmatrix}0 \\ 0 \\ \frac{1}{\tau}\end{pmatrix} u_{i-1}$$

- Choose ξ_i as

$$\xi_i = \begin{pmatrix}k_{\mathrm{p}} & k_{\mathrm{d}} & k_{\mathrm{dd}}\end{pmatrix} \begin{pmatrix}\varepsilon_{1,i} \\ \varepsilon_{2,i} \\ \varepsilon_{3,i}\end{pmatrix} + u_{i-1}$$

with feedback gains k_{p}, k_{d}, k_{dd} such that for $u_{i-1} \equiv 0$ the origin $\varepsilon_i = 0$ is asymptotically stable

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  with feedback gains k_{p}, k_{d}, k_{dd} such that for $u_{i-1} \equiv 0$ the origin $\varepsilon_i = 0$ is asymptotically stable
- Requires wireless intervehicle communication

CONTROLLER DESIGN

- Resulting controlled error dynamics, including input filter:

$$\begin{pmatrix} \dot{\varepsilon}_{1,i} \\ \dot{\varepsilon}_{2,i} \\ \dot{\varepsilon}_{3,i} \\ \dot{u}_i \end{pmatrix} = \left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{k_\mathrm{p}}{\tau} & -\frac{k_\mathrm{d}}{\tau} & -\frac{1+k_\mathrm{dd}}{\tau} & 0 \\ \hline \frac{k_\mathrm{p}}{h} & \frac{k_\mathrm{d}}{h} & \frac{k_\mathrm{dd}}{h} & -\frac{1}{h} \end{array}\right) \begin{pmatrix} \varepsilon_{1,i} \\ \varepsilon_{2,i} \\ \varepsilon_{3,i} \\ u_i \end{pmatrix} + \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline \frac{1}{h} \end{array}\right) u_{i-1}$$

- State matrix is lower block triangular $\Rightarrow$ eigenvalues of diagonal blocks determine stability
  - (1,1) block in state matrix is in *controllable canonical form*: elements of last row are coefficients of characteristic polynomial
  - (2.2) block gives negative real eigenvalue for time gap $h \geq 0$

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- u_{i-1} perturbs u_i, but not ε_i!

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- u_{i-1} perturbs u_i, but not ε_i!
- Error dynamics do not depend on time gap h!

CONTROLLER DESIGN

- Vehicle-following control objective is satisfied
- How about string stability?

CONTROLLED SYSTEM IN LAPLACE DOMAIN

- Vehicle model $G(s)$

$$G(s) = \frac{q_i(s)}{u_i(s)} = \frac{1}{s^2\left(\tau s + 1\right)}$$

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- Spacing error, with spacing policy transfer function $H(s) = 1 + hs$ $(L_i = 0,\, r = 0)$

$$e_i(s) = q_{i-1}(s) - q_i(s) - hv_i(s) = q_{i-1} - H(s)q_i(s)$$

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- Wireless communication delay θ

$$D(s) = \frac{u^*_{i-1}(s)}{u_{i-1}(s)} = e^{-\theta s}$$

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- Wireless communication delay θ

$$D(s) = \frac{u^*_{i-1}(s)}{u_{i-1}(s)} = e^{-\theta s}$$

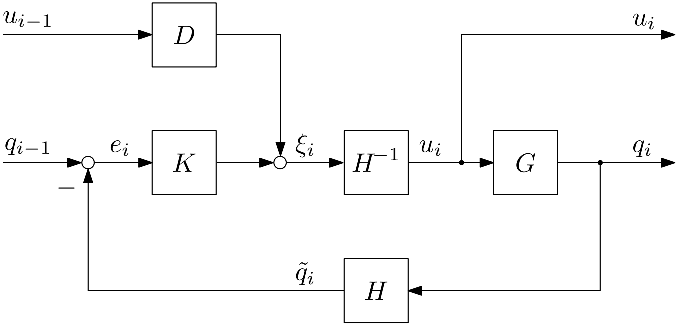
- Controller with $k_{\mathrm{dd}} = 0$ (see slide 8)

$$\xi_i(s) = K(s)e_i(s) + u^*_{i-1}(s) = \left(k_{\mathrm{p}} + k_{\mathrm{d}}s\right)e_i(s) + u^*_{i-1}(s)$$

$$u_i(s) = \frac{1}{hs+1}\xi_i(s) = H^{-1}(s)\xi_i(s)$$

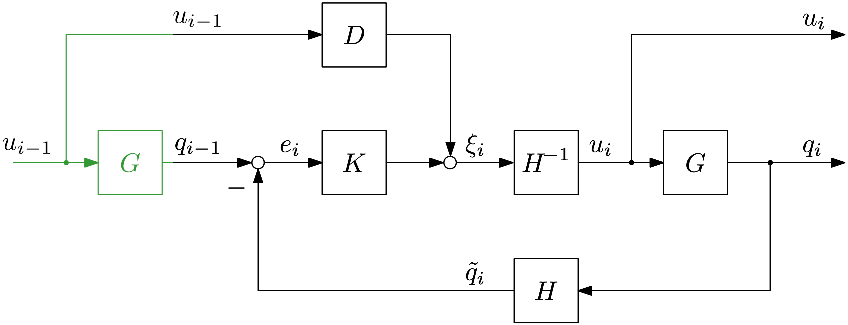
CONTROLLED SYSTEM IN LAPLACE DOMAIN

- Block scheme of controlled vehicle i



CONTROLLED SYSTEM IN LAPLACE DOMAIN

- Block scheme of controlled vehicle i
- Including preceding vehicle model (either or not controlled)



STRING STABLE?

- Choose output of interest, e.g., position q_i or velocity v_i or ...
- String Stability Complementary Sensitivity (using the block scheme)

$$\Gamma(s) = \frac{q_i(s)}{q_{i-1}(s)} = \frac{1}{H(s)} \frac{D(s) + G(s)K(s)}{1 + G(s)K(s)}$$

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$\Gamma(s)$ independent of index i; also independent of specific output (homogeneous string!)

INTERMEZZO: VECTOR, SIGNAL, AND SYSTEM NORMS

See also: Jeroen Ploeg, "A not-so-short overview of vector, matrix, signal, and system norms," May 2, 2016

Definition

The **norm** of x (which may be a vector, matrix, signal or system) is a real number, denoted by $\|x\|$, that satisfies the following properties:

1. Positive definiteness: $\|x\| \geq 0$, with $\|x\| = 0 \Leftrightarrow x = 0$
2. Homogeneity: $\|\alpha x\| = |\alpha| \cdot \|x\|$, $\forall\ \alpha \in \mathbb{C}$
3. Triangle inequality: $\|x + y\| \leq \|x\| + \|y\|$.

Definition

A norm $\|A\|$ on a matrix A is a **matrix norm** if it <u>also</u> satisfies the *submultiplicative property* $\|AB\| \leq \|A\| \cdot \|B\|$, where B is any other matrix of valid dimensions.

VECTOR NORMS

Definition

Let $u \in \mathbb{R}^n$, with elements u_i, $i = 1, \ldots, n$. Then the **vector p-norm** is defined as:

$$\|u\|_p := \left(\sum_i |u_i|^p \right)^{1/p}$$

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- Vector 2-norm:

$$\|u\|_2 = \sqrt{\sum_i |u_i|^2} = \sqrt{u^{\mathrm{H}} u}$$

- Vector ∞-norm:

$$\|u\|_\infty = \lim_{p \to \infty} \left(\sum_i |u_i|^p \right)^{1/p} = \max_i |u_i|$$

MATRIX NORMS

Definition

Let $A \in \mathbb{R}^{n \times m}$ and $u \in \mathbb{R}^{m \times 1}$. Then the **induced matrix p-norm** is defined as:

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- The induced matrix 2-norm equals the maximum singular value $\overline{\sigma}(A)$:

$$\|A\|_{i_2} = \max_{u \neq 0} \frac{\|Au\|_2}{\|u\|_2} = \max_{u \neq 0} \frac{\sqrt{u^{\mathrm{H}} A^{\mathrm{H}} A u}}{\sqrt{u^{\mathrm{H}} u}} = \max_i \sqrt{\lambda_i(A^{\mathrm{H}} A)} = \overline{\sigma}(A)$$

- Induced matrix ∞-norm (maximum row sum):

$$\|A\|_{i_\infty} = \max_{u \neq 0} \frac{\|Au\|_\infty}{\|u\|_\infty} = \max_i \sum_j |a_{ij}| = \max_i \left\| \begin{pmatrix} a_{i,1} & a_{i,2} & \dots & a_{i,m} \end{pmatrix} \right\|_1$$

SIGNAL NORMS

Definition

Let $u(t) \in \mathbb{R}^n$ be a time-dependent vector signal with elements $u_i(t)$, $i = 1, \ldots, n$. Then the **signal p-norm**, or **$\mathcal{L}_p$ norm**, is defined as:

$$\|u(t)\|_{\mathcal{L}_p} := \left(\int_{-\infty}^{\infty} \sum_i |u_i(t)|^p \, \mathrm{d}t \right)^{1/p} = \left(\int_{-\infty}^{\infty} \|u(t)\|_p^p \, \mathrm{d}t \right)^{1/p}$$

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- Parseval's theorem on the $\mathcal{L}_2$ norm: $\|u(t)\|_{\mathcal{L}_2} = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} \|\hat{u}(j\omega)\|_2^2 \, \mathrm{d}\omega} =: \|\hat{u}(j\omega)\|_{\mathcal{L}_2}$
- $\mathcal{L}_\infty$ norm:

$$\|u(t)\|_{\mathcal{L}_\infty} = \lim_{p \to \infty} \left(\int_{-\infty}^{\infty} \sum_i |u_i(t)|^p \, \mathrm{d}t \right)^{1/p} = \sup_t \|u(t)\|_\infty$$

SYSTEM NORMS: THE $\mathcal{H}_\infty$ NORM

Definition

Consider a system $y(s) = G(s)u(s)$, $s \in \mathbb{C}$, then the $\boldsymbol{\mathcal{H}_\infty}$ **norm** of the transfer function $G(s)$ is defined as:

$$\|G(s)\|_{\mathcal{H}_\infty} := \sup_{\mathrm{Re}(s)>0} \max_{u\neq 0} \frac{\|G(s)u(s)\|_2}{\|u(s)\|_2}$$

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Theorem

Let $G(s)$ be the transfer function matrix of a system with input signal $u(t)$ and output signal $y(t)$ of appropriate dimensions. The output $y(t)$ is then bounded according to

$$\|y(t)\|_{\mathcal{L}_2} \leq \|G(s)\|_{\mathcal{H}_\infty} \|u(t)\|_{\mathcal{L}_2}$$

- $\|G(s)\|_{\mathcal{H}_\infty}$ is a tight upperbound (i.e., no conservatism)
- $\mathcal{H}_\infty$ norm also known as **induced $\boldsymbol{\mathcal{L}_2}$ norm** or **$\boldsymbol{\mathcal{L}_2}$ gain**

SYSTEM NORMS: THE INDUCED $\mathcal{L}_\infty$ NORM

Definition

Consider a system $y(s) = G(s)u(s)$, $s \in \mathbb{C}$. Let $g(t)$ be the impulse response matrix of $G(s)$. Then the **induced $\mathcal{L}_\infty$ norm** (or **$\mathcal{L}_\infty$ gain**) is defined as:

$$\|g(t)\|_{\mathcal{L}_1} := \max_i \int_{-\infty}^{\infty} \sum_j |g_{ij}(t)| \, \mathrm{d}t = \max_i \|g_i^{\mathrm{T}}(t)\|_{\mathcal{L}_1}$$

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STRICT STRING STABILITY CONDITIONS

- Let the input–output behavior in the Laplace-domain of a linear state-space system be given by:

$$y_i(s) = P_i(s)u_1(s), \quad i = 1, \ldots, m, \text{ with } m \in \mathbb{N}$$

- Define the String Stability Complementary Sensitivity (SSCS) function as

$$\Gamma_i(s) = \frac{y_i(s)}{y_{i-1}(s)} = P_i(s)P_{i-1}^{-1}(s), \quad i \geq 2$$

with impulse response $\gamma_i(t)$ (assuming P_i^{-1} exists for all i)

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$$y_i(s) = P_i(s)u_1(s), \quad i = 1, \dots, m, \text{ with } m \in \mathbb{N}$$

- Define the String Stability Complementary Sensitivity (SSCS) function as

$$\Gamma_i(s) = \frac{y_i(s)}{y_{i-1}(s)} = P_i(s)P_{i-1}^{-1}(s), \quad i \geq 2$$

with impulse response $\gamma_i(t)$ (assuming P_i^{-1} exists for all i)

- Strict $\mathcal{L}_2$ string stability if and only if $\|\Gamma_i(s)\|_{\mathcal{H}_\infty} \leq 1, \quad \forall i \in \mathbb{N}$
- Strict $\mathcal{L}_\infty$ string stability if and only if $\|\gamma_i(t)\|_{\mathcal{L}_1} \leq 1, \quad \forall i \in \mathbb{N}$

Proof in Ploeg et al., "$\mathcal{L}_p$ String Stability of Cascaded Systems: Application to Vehicle Platooning," IEEE Transactions on Control Systems Technology, vol. 22, no. 2, pp. 786–793, Mar. 2014

STRING STABILITY EXAMPLE

- Take a series of 10 cascaded 2nd-order systems, i.e.,

$$\Gamma_i(s) = \frac{y_i(s)}{y_{i-1}(s)} = \frac{1}{\frac{1}{\omega_n^2}s^2 + 2\frac{\beta}{\omega_n}s + 1}, \quad i = 1, 2, \ldots, 10$$

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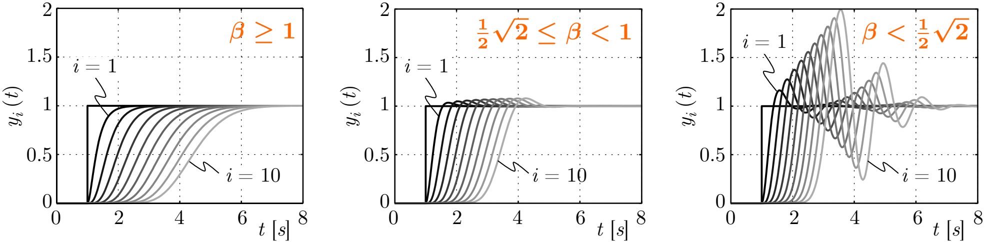

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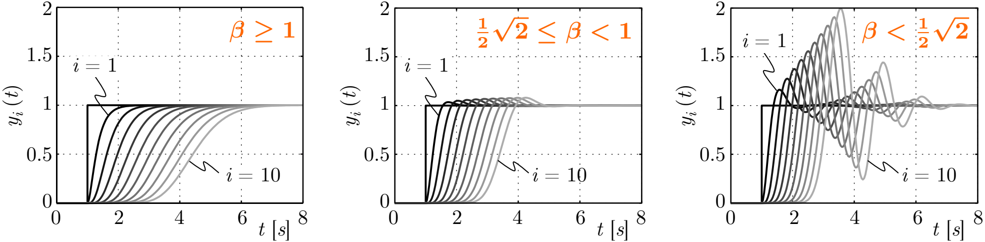

$\mathcal{L}_2$ STRING STABILITY ANALYSIS

- Choose output of interest, e.g., position q_i or velocity v_i or ...
- String Stability Complementary Sensitivity (using the block scheme)

$$\Gamma(s) = \frac{q_i(s)}{q_{i-1}(s)} = \frac{1}{H(s)} \frac{D(s) + G(s)K(s)}{1 + G(s)K(s)}$$

$\Gamma(s)$ independent of index i; also independent of specific output (homogeneous string!)

- $\mathcal{L}_2$ strict string stability: $\|\Gamma(s)\|_{\mathcal{H}_\infty} \leq 1$

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- CACC with zero communication delay $(D(s) = e^{-\phi s} = e^0 = 1)$: $\Gamma(s) = \frac{1}{H(s)} = \frac{1}{hs+1}$
  $\mathcal{L}_2$ strictly string stable for all $h \geq 0$!

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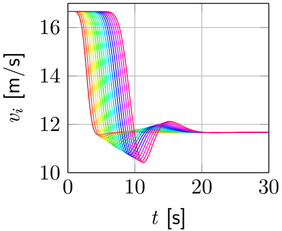
- Adaptive Cruise Control ($D(s) = 0$): $\Gamma(s) = \frac{1}{H(s)} \frac{G(s)K(s)}{1+G(s)K(s)}$
  Bad bad bad, as we will see

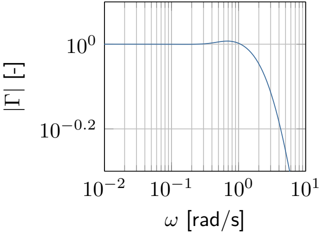
$\mathcal{L}_2$ STRING STABILITY ANALYSIS

$$\Gamma(s) = \frac{1}{H(s)} \frac{D(s) + G(s)K(s)}{1 + G(s)K(s)}$$

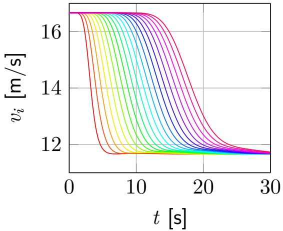
- Communication delay compromises string stability
- Delay can be counteracted by increasing h, i.e., decreasing the bandwidth of $1/H(s)$

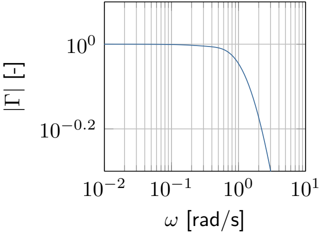
$\mathcal{L}_2$ STRING STABILITY ANALYSIS





$\theta = 0.06\,\text{s}, h = 0.3\,\text{s}$

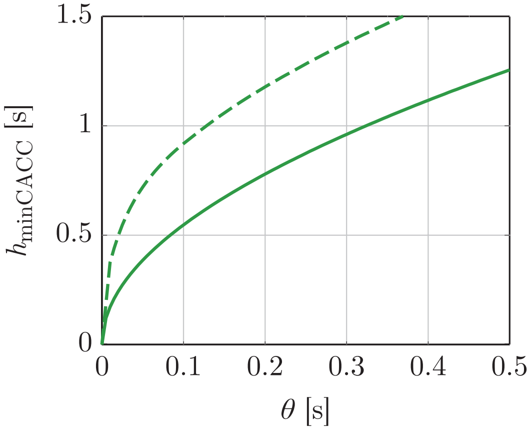




$\theta = 0.06\,\text{s}, h = 0.6\,\text{s}$

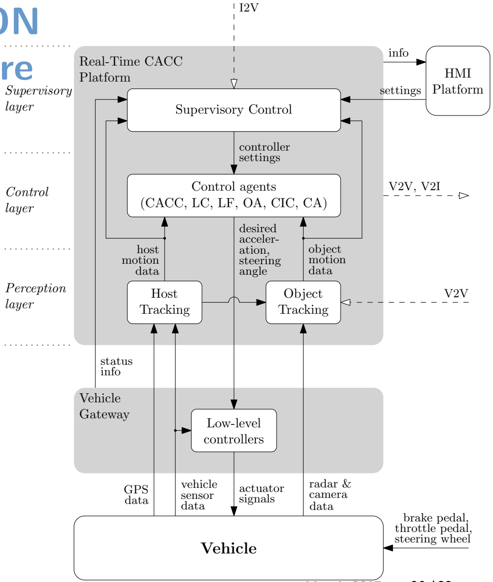
STRING STABILITY ANALYSIS
Minimum $\mathcal{L}_2$ and $\mathcal{L}_\infty$ string-stable time gap

- **ACC**
  - $\mathcal{L}_2$ string stability: $h_{\text{minACC}} = 3.16\,\text{s}$
  - $\mathcal{L}_\infty$ string stability: $h_{\text{minACC}} = 3.75\,\text{s}$
- **CACC**
  - Green solid curve: $\mathcal{L}_2$ string stability $h_{\text{minCACC}} = 0.25\,\text{s}$ for $\theta = 0.02\,\text{s}$
  - Green dashed curve: $\mathcal{L}_\infty$ string stability $h_{\text{minCACC}} = 0.50\,\text{s}$ for $\theta = 0.02\,\text{s}$
- *Assignment: determine the $\mathcal{L}_2$ curves numerically by writing a minimization/search algorithm*

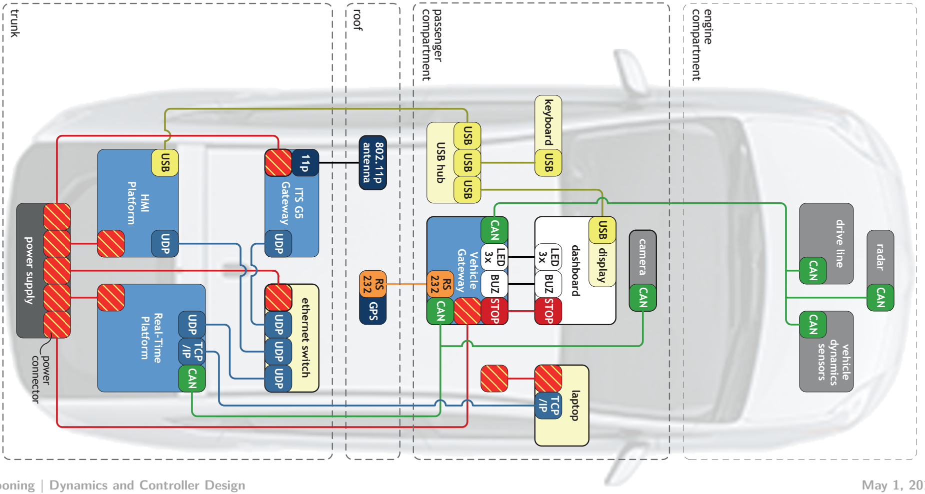


EXPERIMENTAL IMPLEMENTATION
Real-time control software architecture

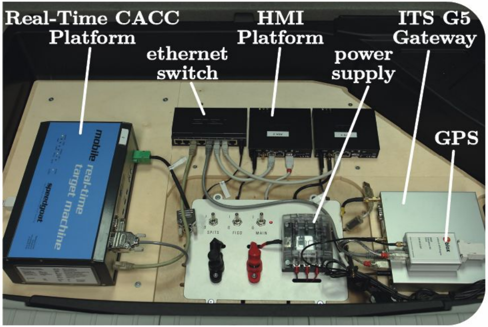
- Layered architecture
  - Supervisory layer
  - Control layer
  - Perception / information layer
- Vehicle equipped with low-level controllers
  - Acceleration
  - Steering



EXPERIMENTAL IMPLEMENTATION
Hardware architecture



EXPERIMENTAL IMPLEMENTATION
Hardware



EXPERIMENTAL IMPLEMENTATION

Hardware

EXPERIMENTAL IMPLEMENTATION
Control design

- Vehicle model $G(s)$ with actuator delay $\phi = 0.2\,\text{s}$ and $\tau = 0.1$ s.

$$G(s) = \frac{q_i(s)}{u_i(s)} = \frac{1}{s^2(\tau s + 1)} e^{-\phi s}$$

- Wireless communication delay $\theta \approx 0.15\,\text{s}$
- Controller with $k_p = 0.2$, $k_d = 0.7$, $k_{dd} = 0$ and $h = 0.6$ s

$$K(s) = k_p + k_d s + k_{dd} s^2$$

$$H(s) = \frac{1}{hs + 1}$$

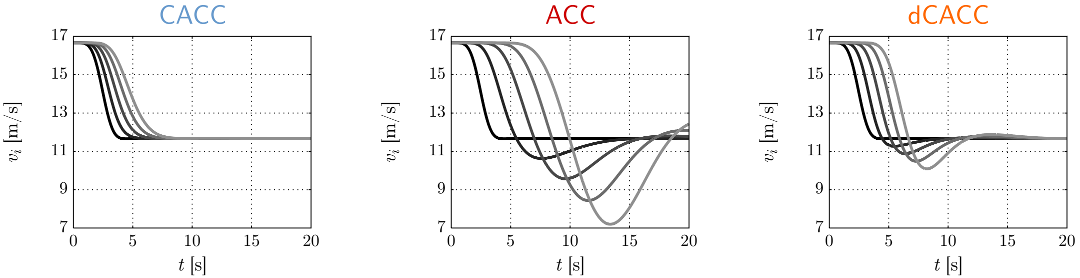
EXPERIMENTAL IMPLEMENTATION

Testing

▸ MOV

EXPERIMENTAL EVALUATION

- Controllers
  - CACC
  - ACC (i.e., CACC without wireless link)
  - dCACC (degraded CACC: acceleration estimation replaces wireless link)
- $h = 0.6\,\mathrm{s}$



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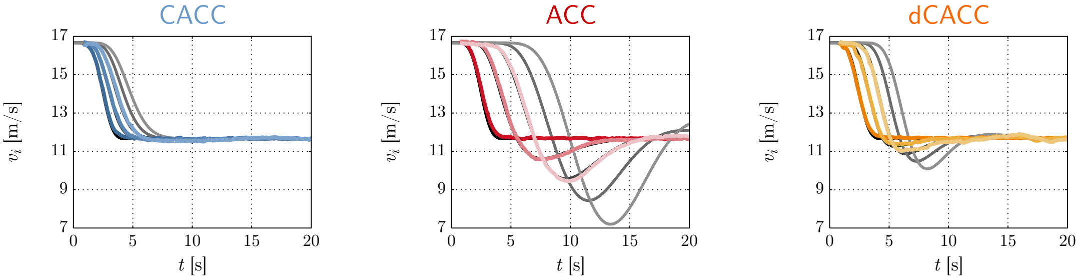

CONCLUSION

- Relatively simple controller has been designed and experimentally evaluated
- Minimum string-stable time gap heavily depends on communication delay
- At low delay, minimum time gap is $\approx 0.3\,\mathrm{s}$ (which is small enough for significant fuel savings)

- Nevertheless, further developments are required
  - Heterogeneous strings
  - System may be linear, but control problem is actually not: Driving too close is unsafe, whereas driving too far away is not $\Rightarrow$ Nonlinear controller design
  - Nonlinear vehicle behavior (brakes may react significantly faster than engine)
  - Many other aspects: Fail safety, fault tolerance, system architecture, etc.