Quadrotor Dynamics and Control

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AER1216: Fundamentals of UAVs
February 12, 2016

Institute for Aerospace Studies
UNIVERSITY OF TORONTO

DYNAMIC
SYSTEMS LAB

WELCOME!

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- Head of the *Dynamic Systems Lab* at UTIAS
- Associate Director of the *Centre for Aerial Robotics Research & Education* (CARRE)

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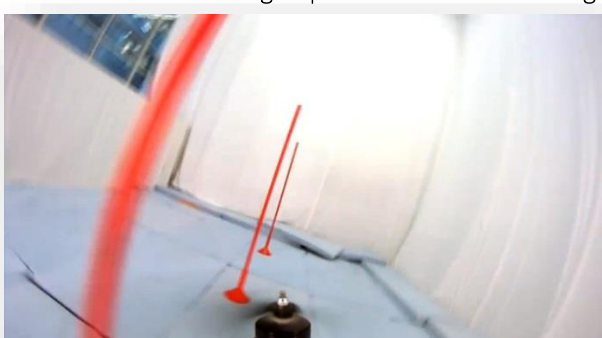
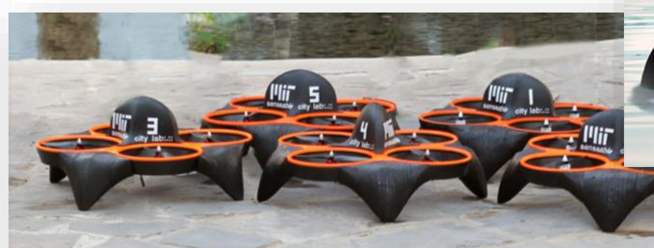
Collision-free, multi-vehicle flight.

High-speed maneuver learning.



Environmental monitoring applications.



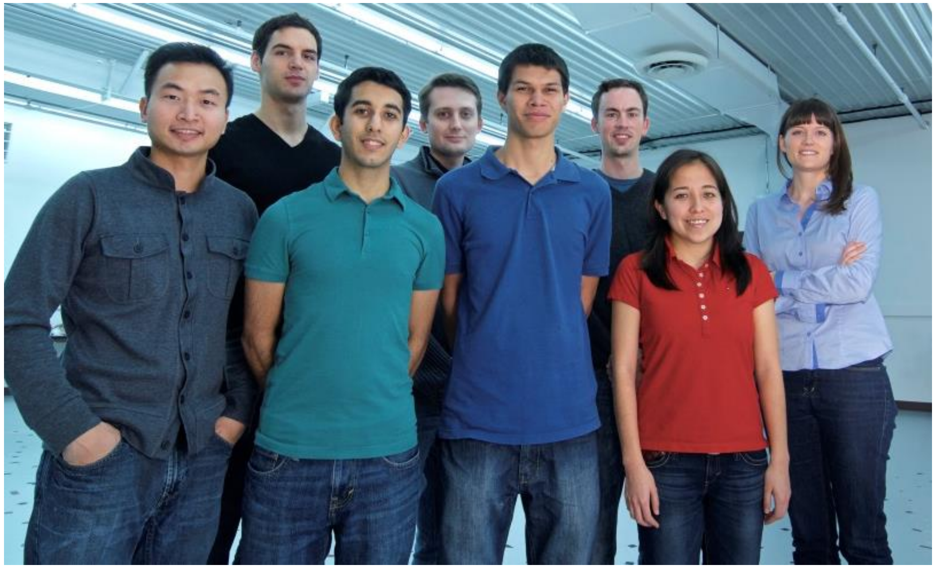
MY RESEARCH GROUP

Motion planning, control and learning for single and multiple autonomous systems/robots.

- One postdoc, five PhD students, two MASc students. *Talk to us!*
- Student office #191, lab #195.

Social media accounts found at www.dynsyslab.org.

Follow us!



Why quadrotors?

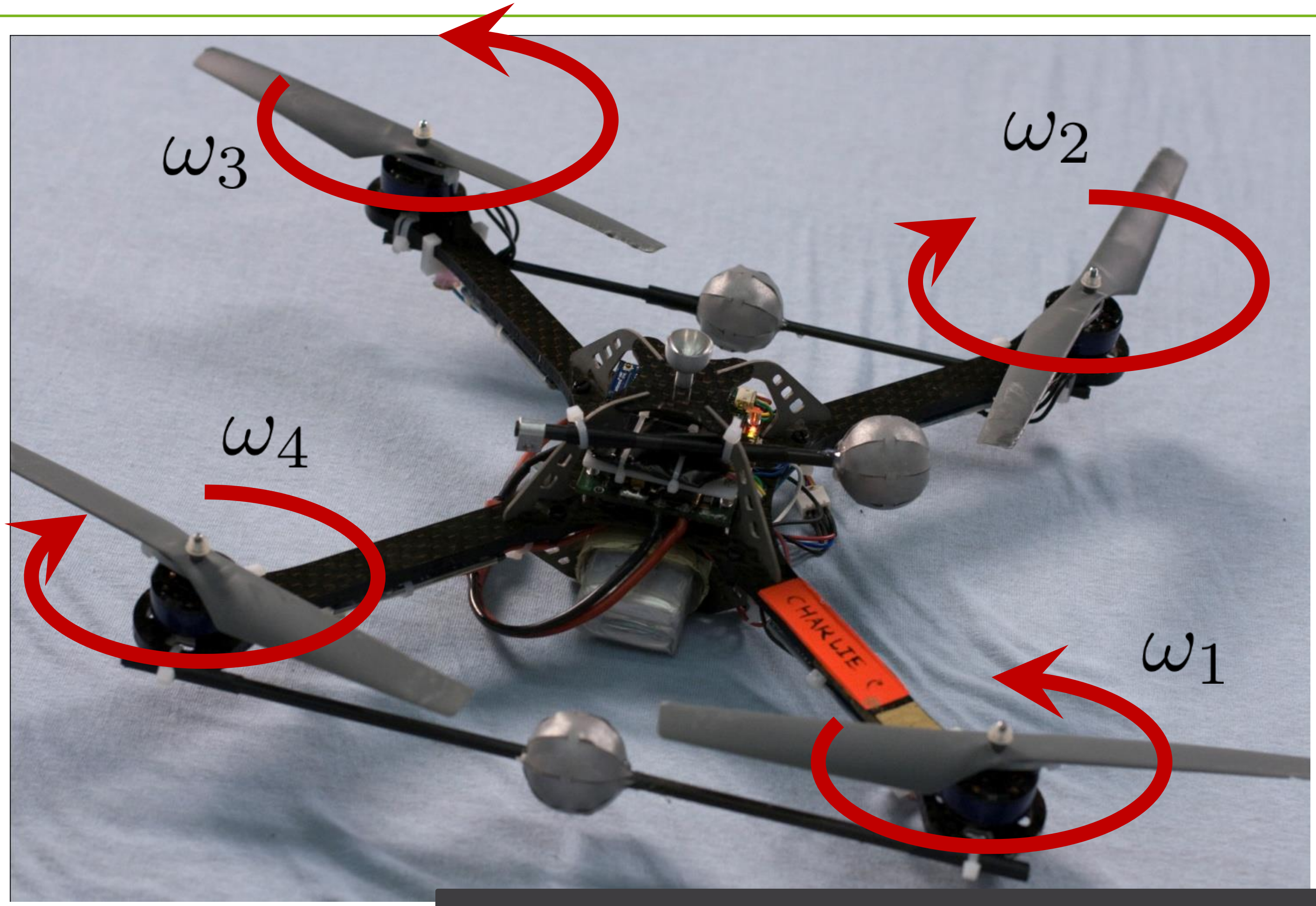
Why control?

QUADROTOR VEHICLE

- Rigid frame.
- Four independently controlled motors.
- Move by changing the motor speeds.
- Vertical take-off and landing.

Mechanically simple.
Highly maneuverable.

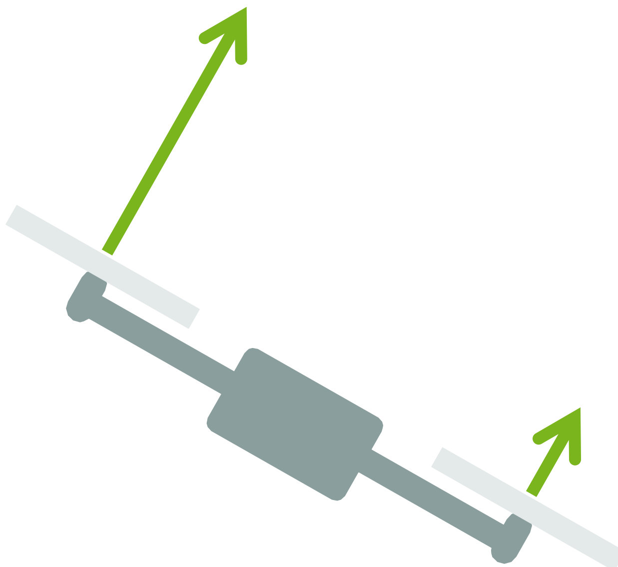
QUADROTOR VEHICLE



Vary the speeds of the rotors to control the position and orientation of the robot.

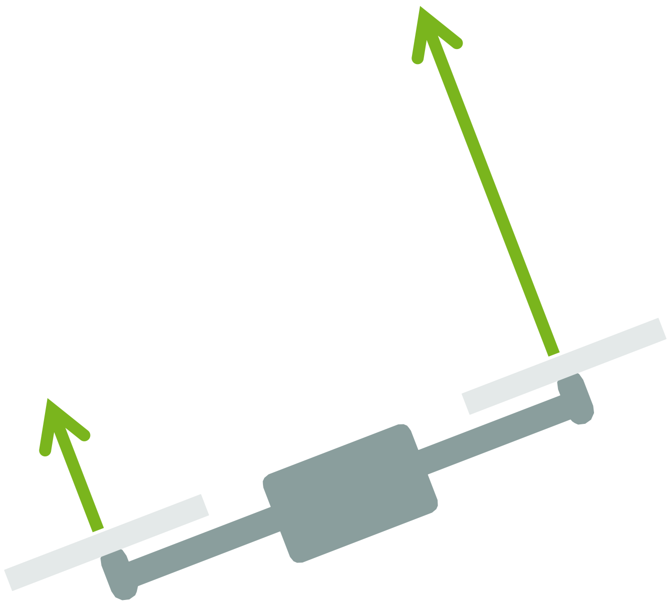
Green arrows represent the force produced by the motor, which is roughly proportional to the squared rotor speed.

ROLL AND PITCH



ROLL AND PITCH

ROLL AND PITCH

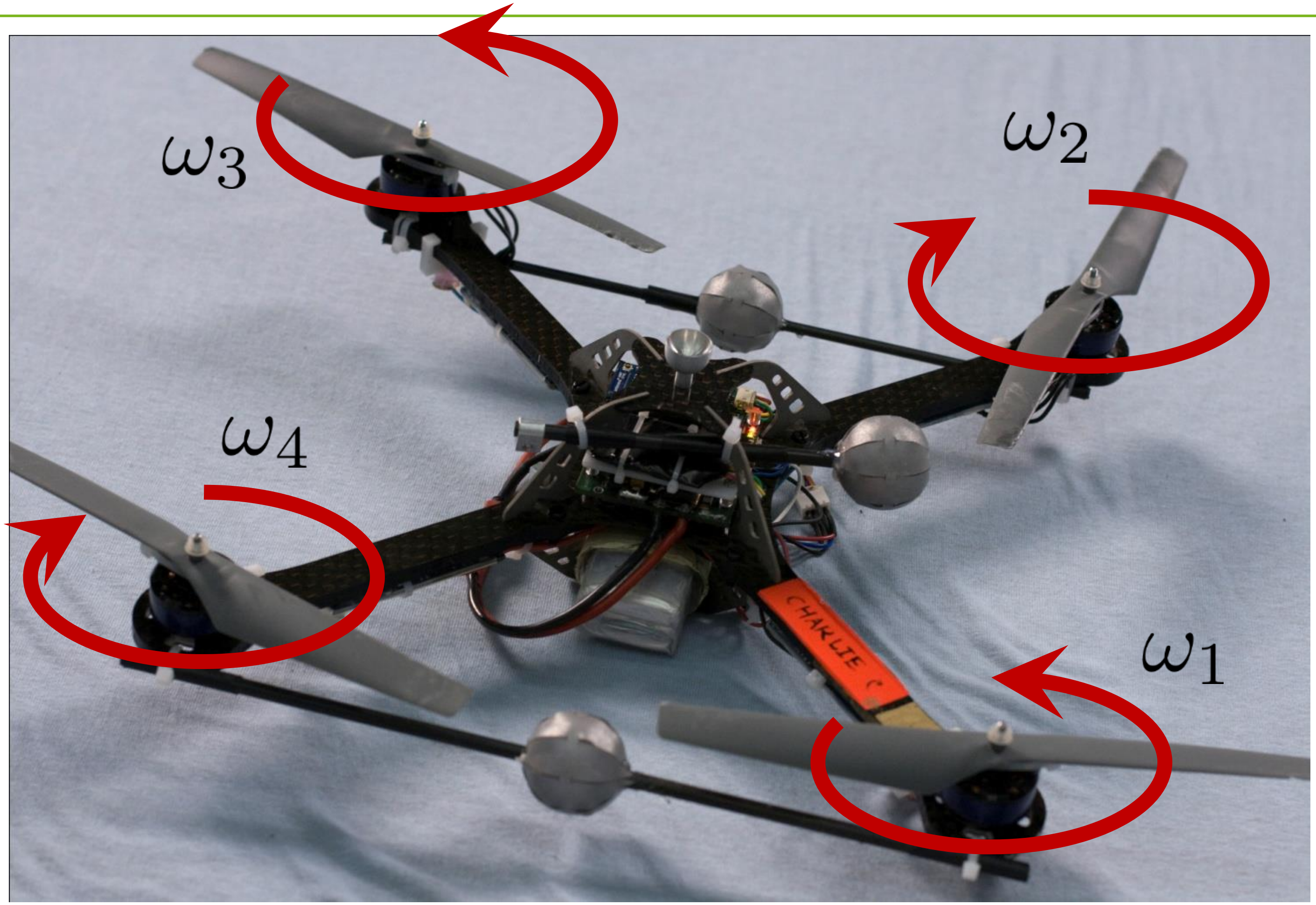


VIDEO: https://youtu.be/NPvGxIBt3Hs?list=PLD6AAACCBFFE64AC5

Up to 1800 degrees/second.

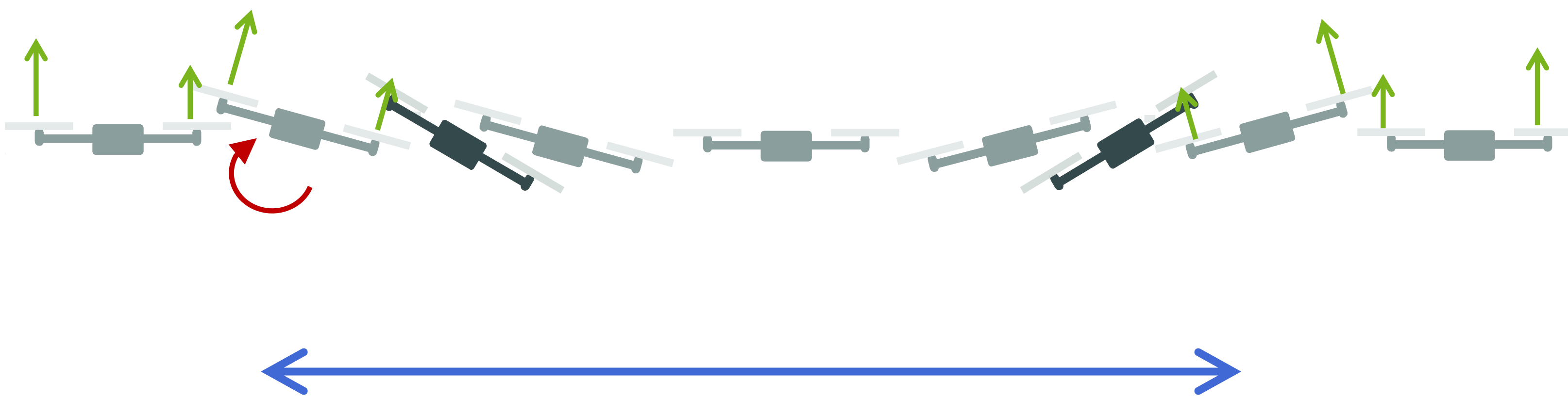
Video:
https://youtu.be/bWExDW9J9sA?list=PLC12E387419CEAFF2

YAW



How do you get the robot to steer/yaw?

TRANSLATION



The robot has six degrees of freedom!

- How many different ways can you rotate or translate the robot?
- How many of them are independent given that the robot has four motors?

Why quadrotors?

Why control?

VIDEO: https://youtu.be/nQ2ziVW6kts

What is Controls?

It's MAGIC!

Controls enables a machine to achieve a task on its own. Despite disturbances.

⇨ Self-regulating system.

VIDEO: https://youtu.be/7r281vgfotg?list=PLD6AAACCBFFE64AC5

VIDEO: https://youtu.be/6C8OJsHfmpI

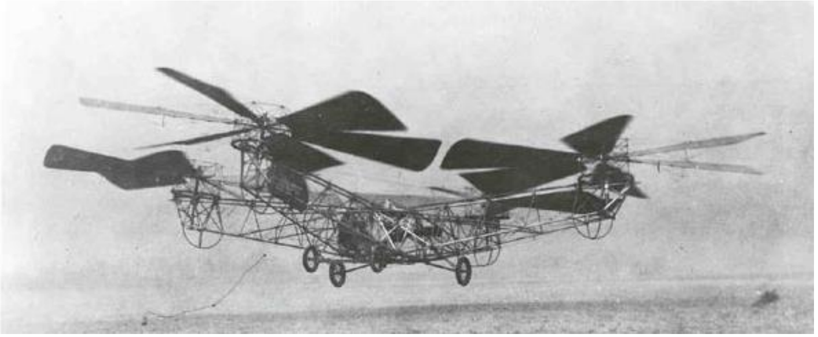
How is this relevant for flying robots?

Enable stable, (potentially) autonomous flight!

DYNAMIC SYSTEMS LAB

First successful flight: 1923

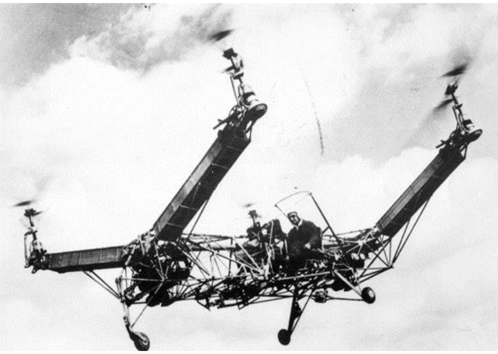
- Design: George De Bothezat
- Vertical take-off and landing (VTOL)
- Problems: stability, control



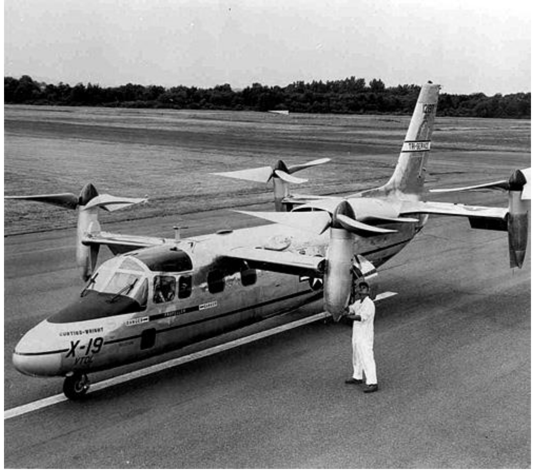
https://en.wikipedia.org/wiki/File:De_Bothezat_Quadrotor.jpg

Later attempts:

- Worked well
- Used simple feedback control



Convertawings Model "A" (1956)



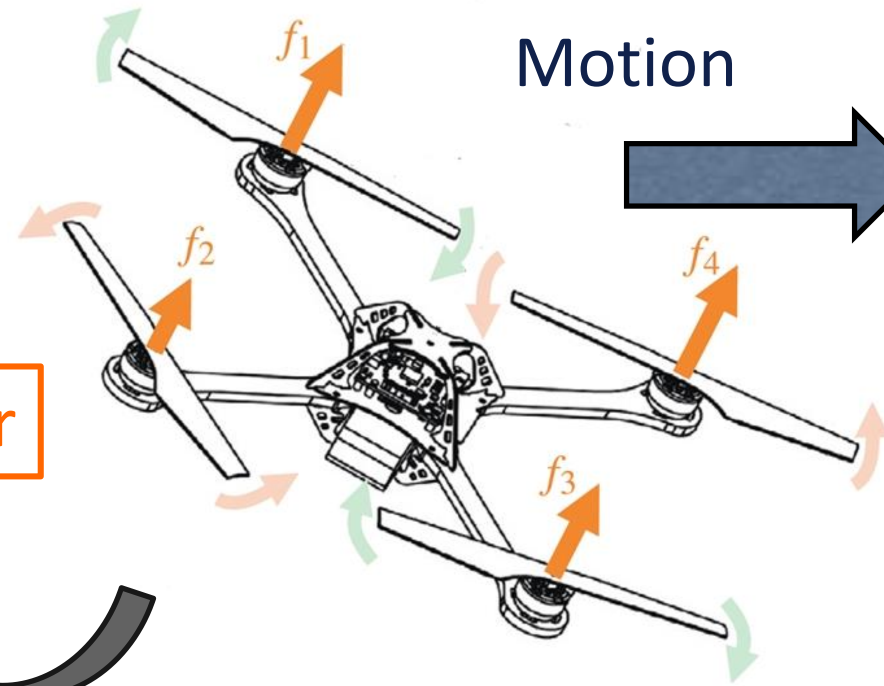
Curtiss-Wright X-19A (1960)

QUADROTOR CONTROL

Motion

Motor Turn Rates

Motor Controller



Measured Turn Rates

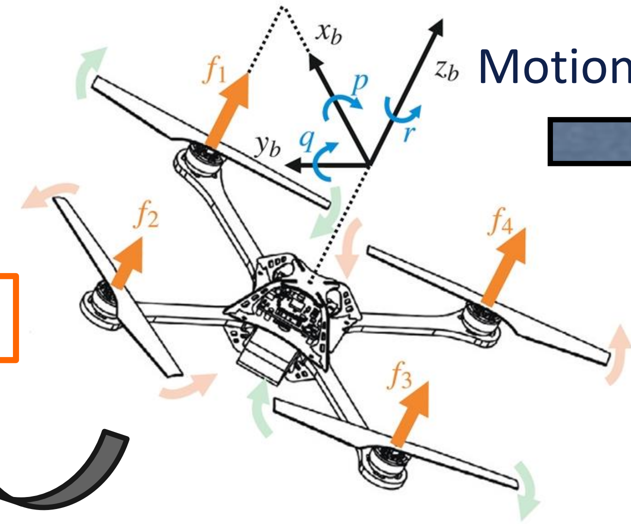
QUADROTOR CONTROL

Body Turn Rate
+ Thrust

Rate Ctrl

Motor Turn Rates

Motor Ctrl



Motion

Gyroscopes
(+Accelerometer)

Measured Turn Rates

QUADROTOR CONTROL

Angles
+ Thrust

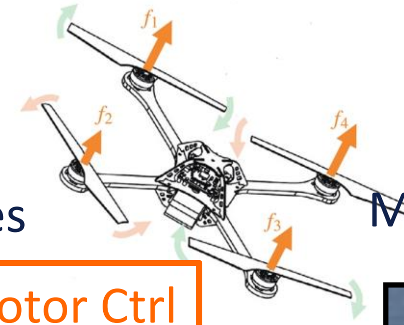
Angle Ctrl

Body Turn Rate
+ Thrust

Rate Ctrl

Motor Turn Rates

Motor Ctrl



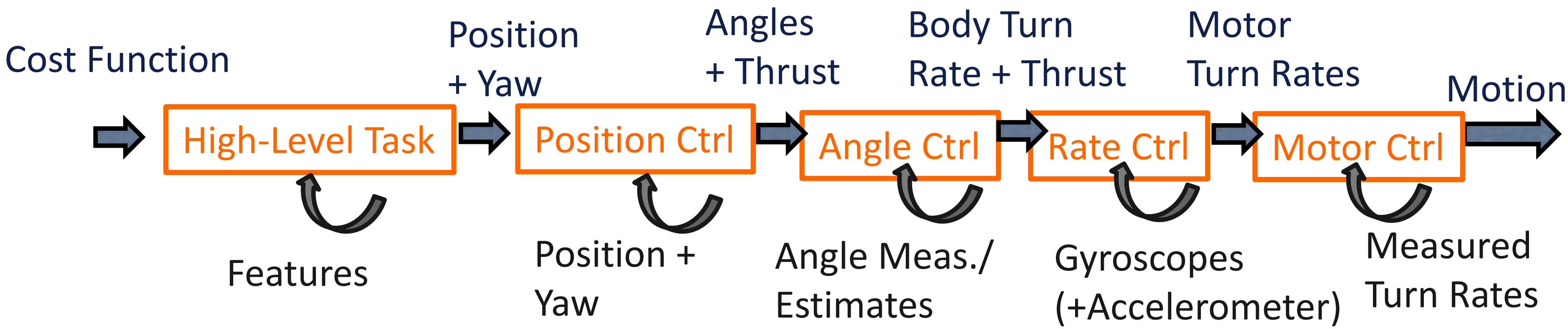
Motion

Angle Measurements/
Estimates

Gyroscopes
(+Accelerometer)

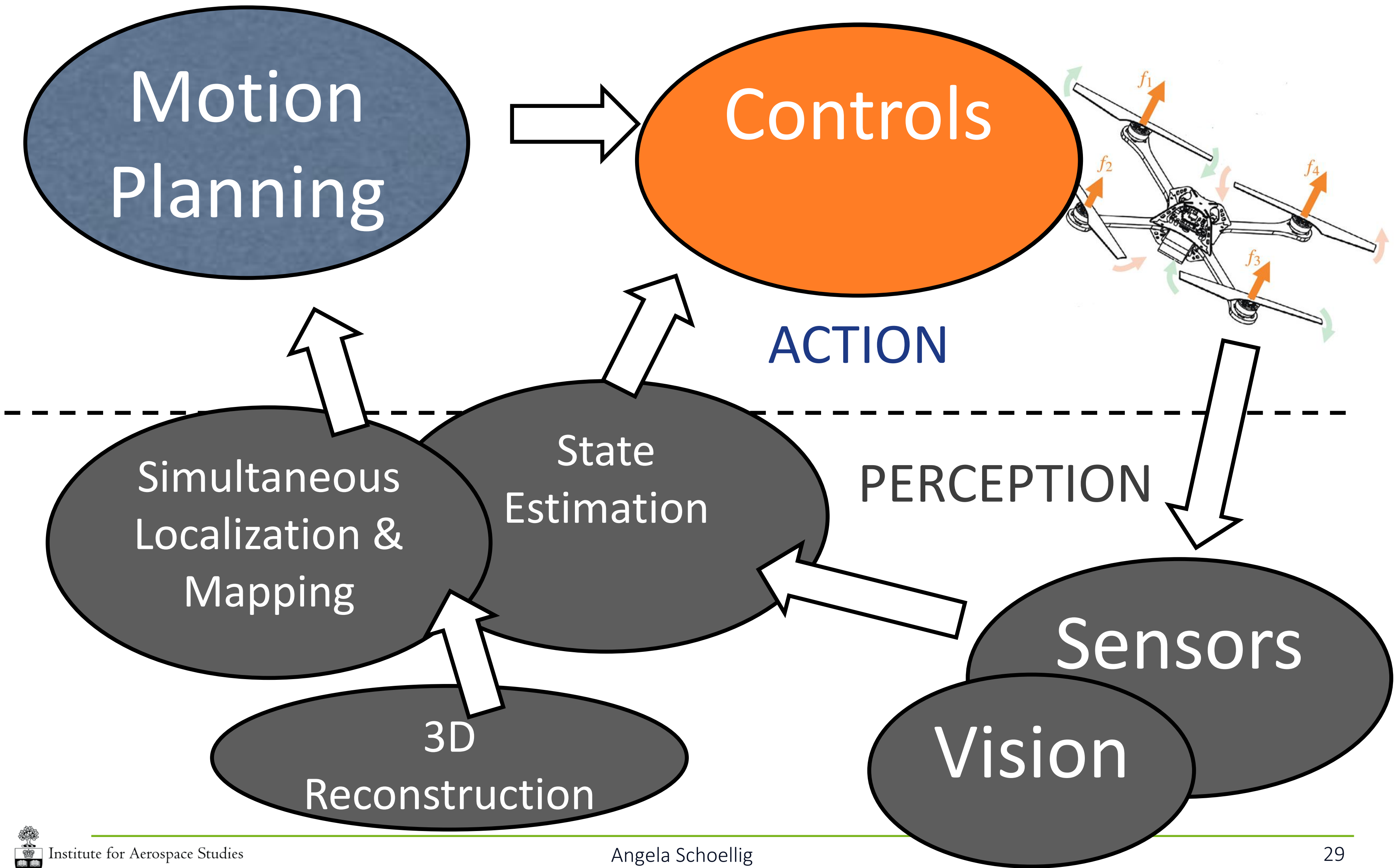
Measured Turn Rates

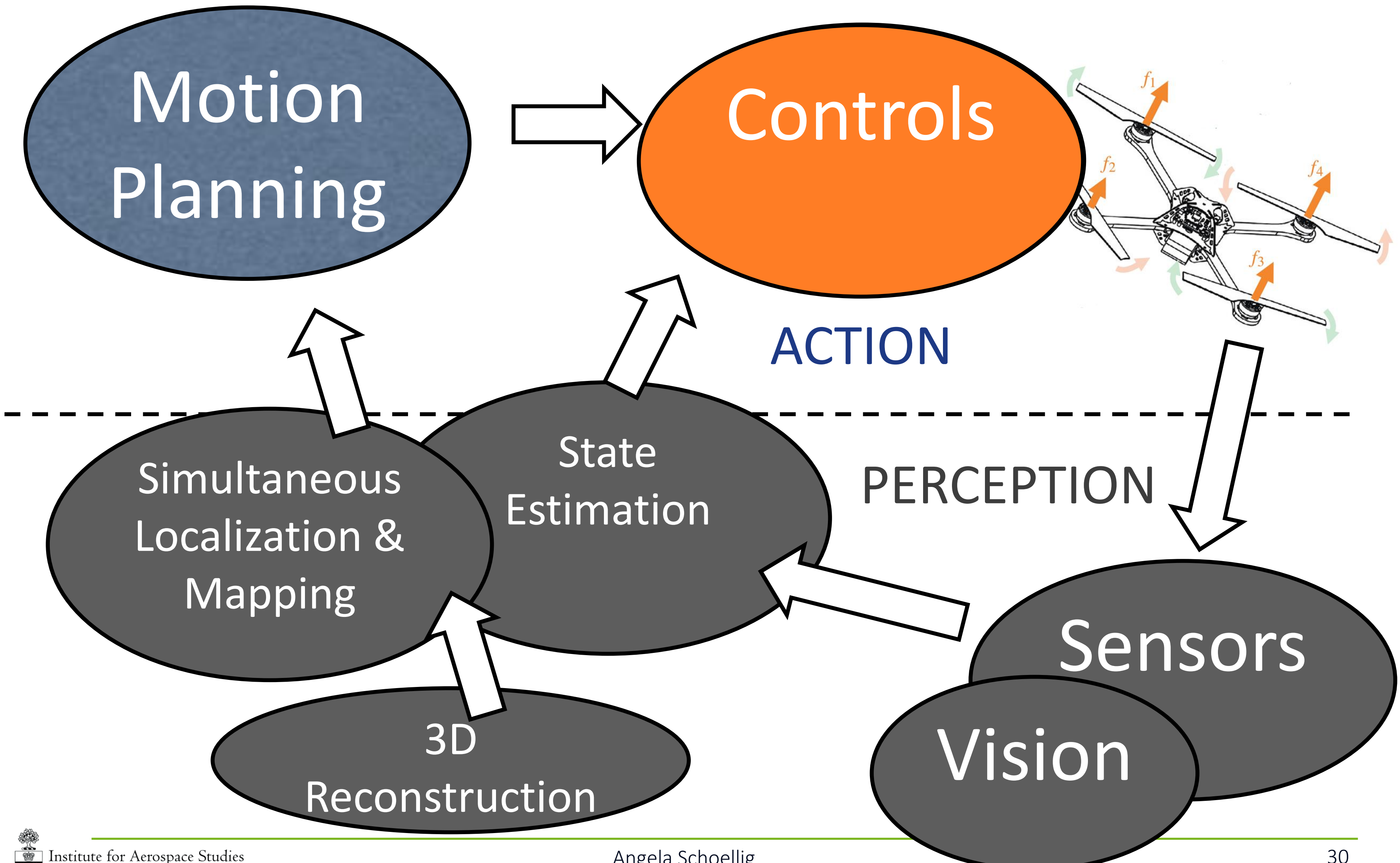
QUADROTOR CONTROL



Allows us to focus on the high-level task.

How does it fit together?

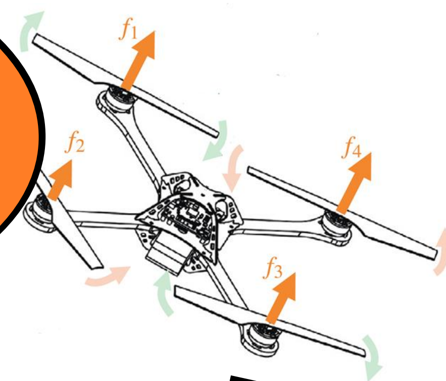

Motion Planning
Controls
ACTION
PERCEPTION
State Estimation
Simultaneous Localization & Mapping
3D Reconstruction
Sensors
Vision


Motion Planning
Controls
f_1
f_2
f_3
f_4
ACTION
PERCEPTION
Simultaneous Localization & Mapping
State Estimation
3D Reconstruction
Sensors
Vision

DYNAMIC SYSTEMS LAB

Motion Planning

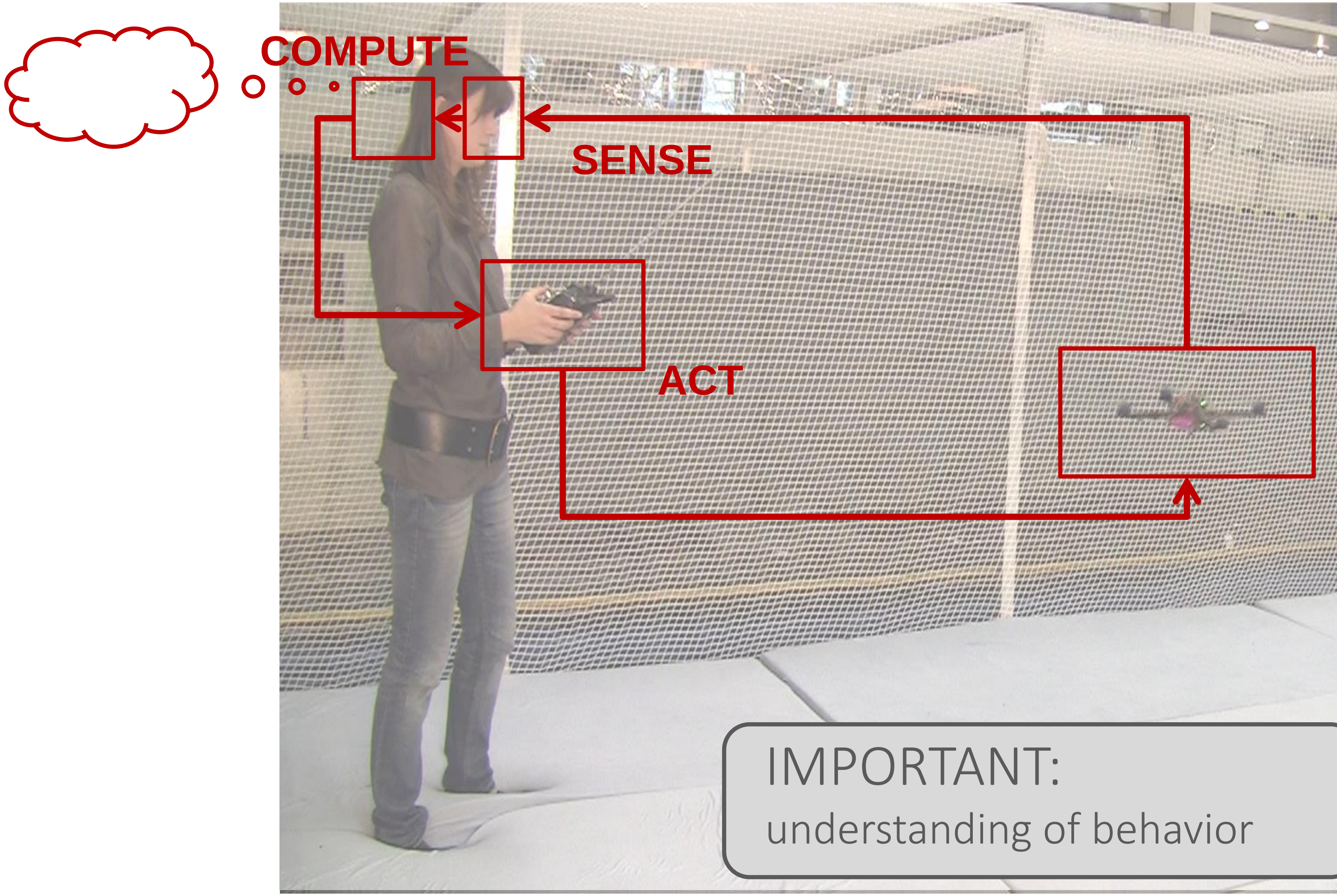
Controls



ACTION

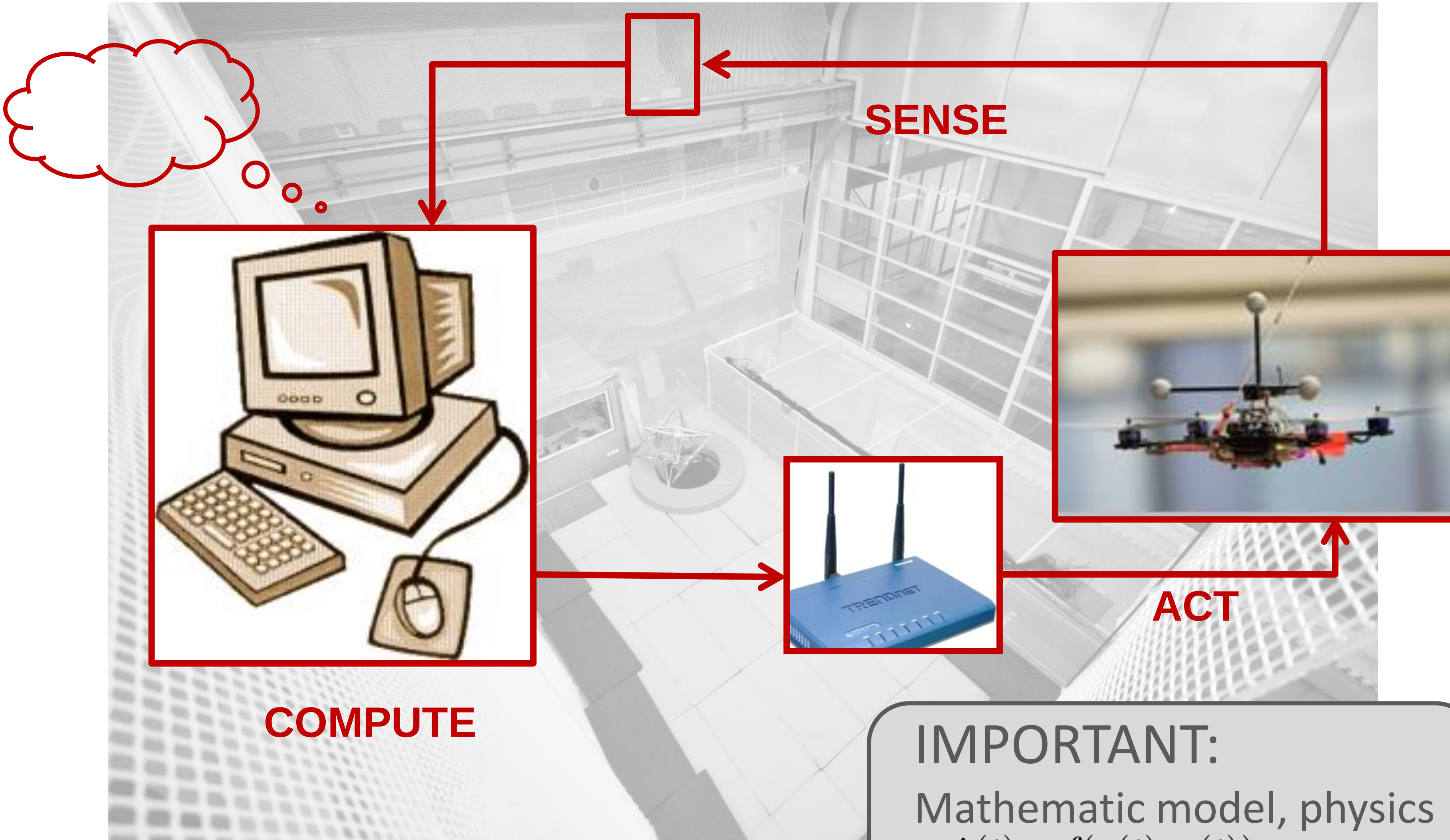
PERCEPTION

TAKING CONTROL



IMPORTANT:
understanding of behavior

AUTOMATED CONTROL



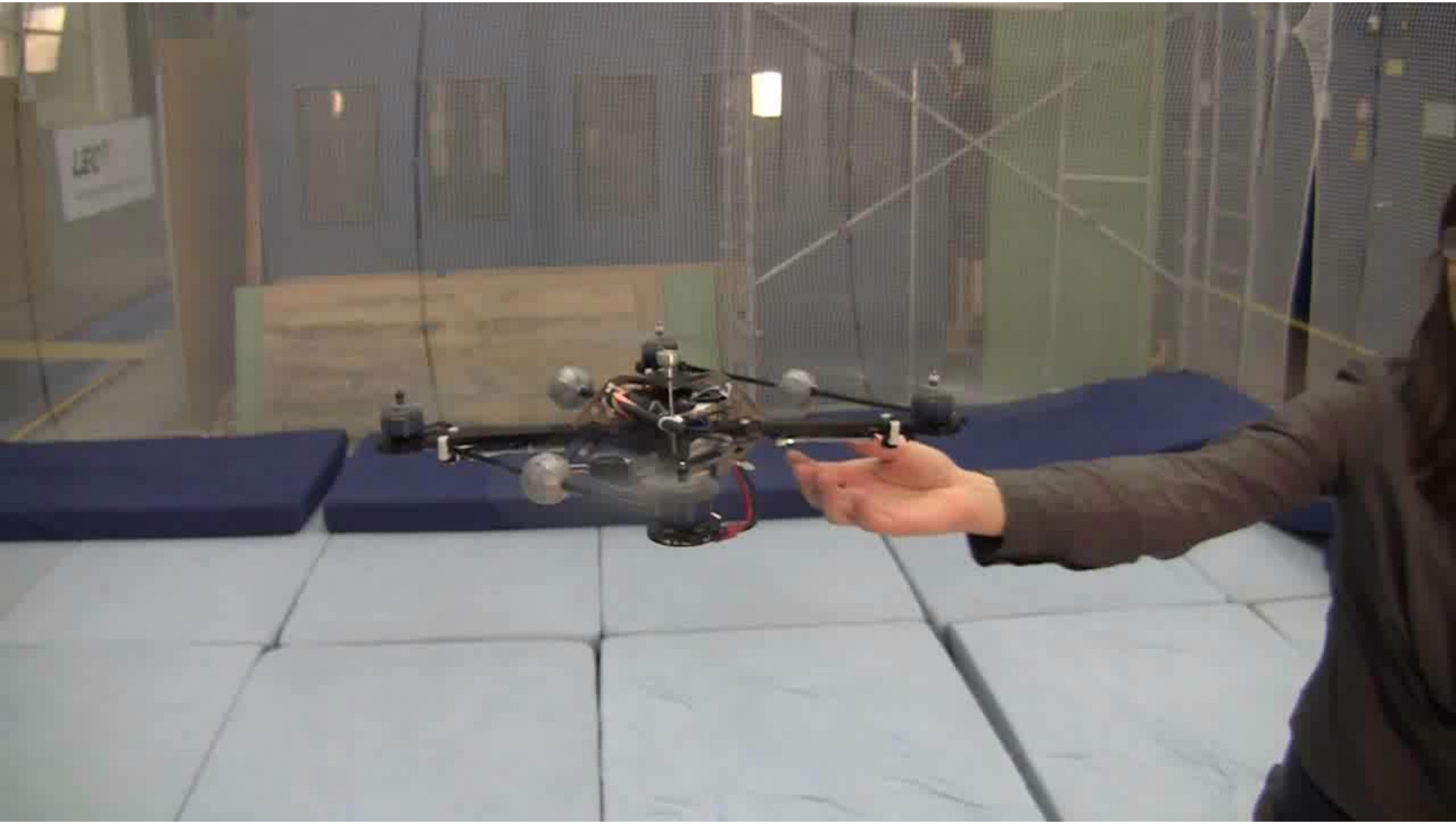
IMPORTANT:

Mathematic model, physics

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = g(x(t), u(t))$$

VIDEO: https://youtu.be/nQ2ziVW6kts



My goal for today!

Prepare you to design your own quadrotor controllers.

OUTLINE

I. Basic Mechanics

II. Dynamics & Control of the Vertical Direction (1D)

III. Dynamics & Control in the Vertical Plane (2D)

IV. Trajectory Tracking Control (3D)

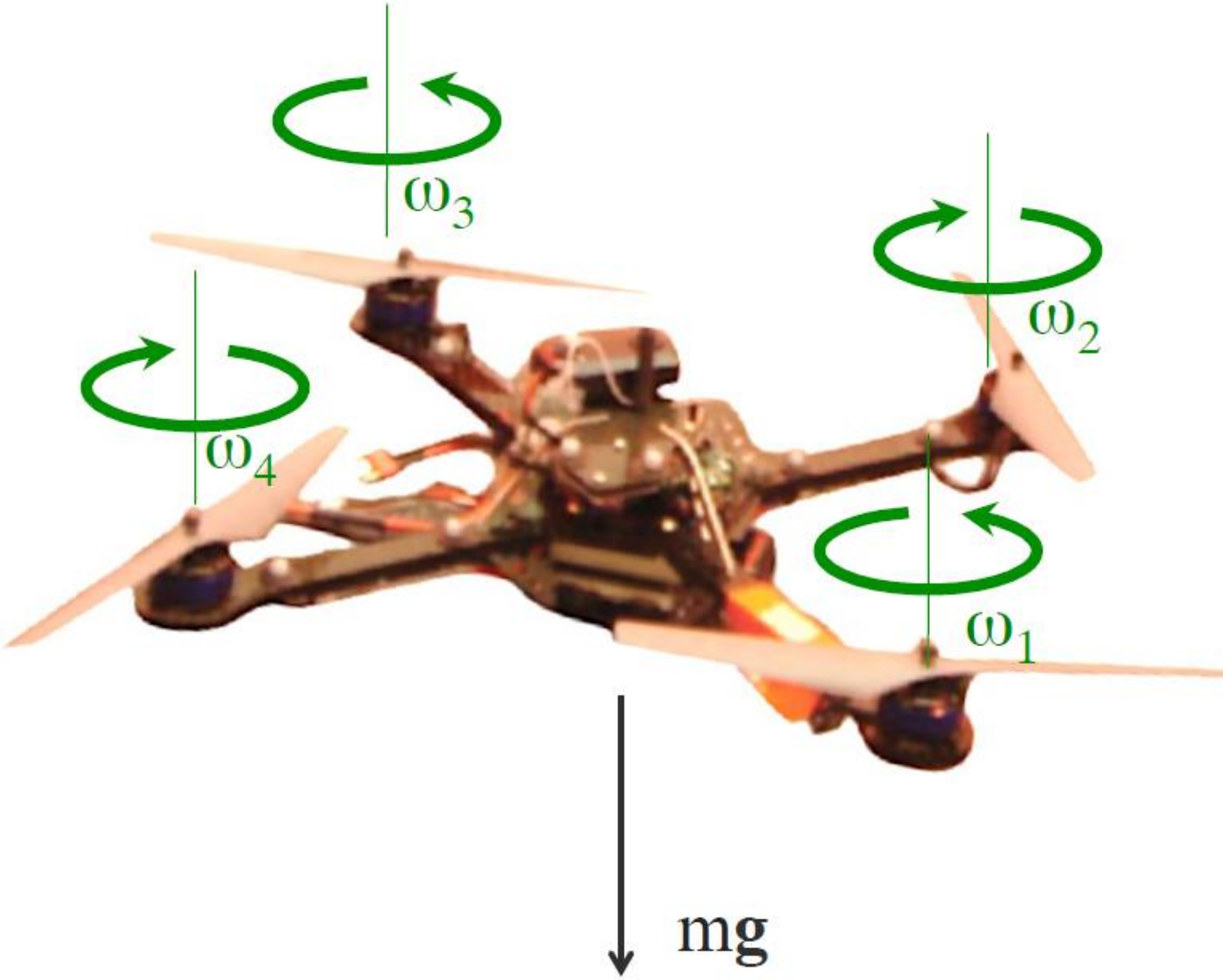
- What Can Go Wrong?

V. Learning-Enabled Control

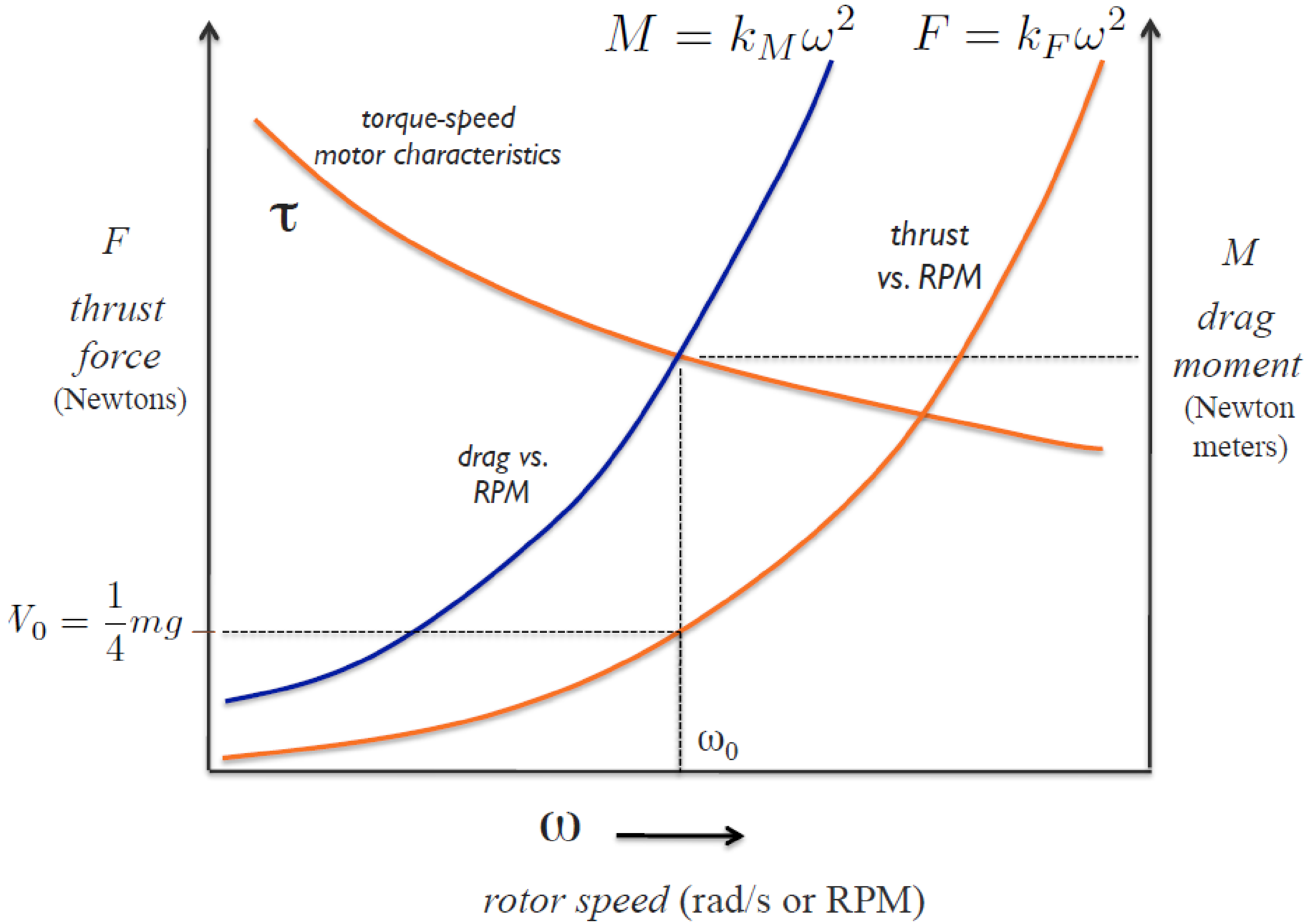
VI. Summary

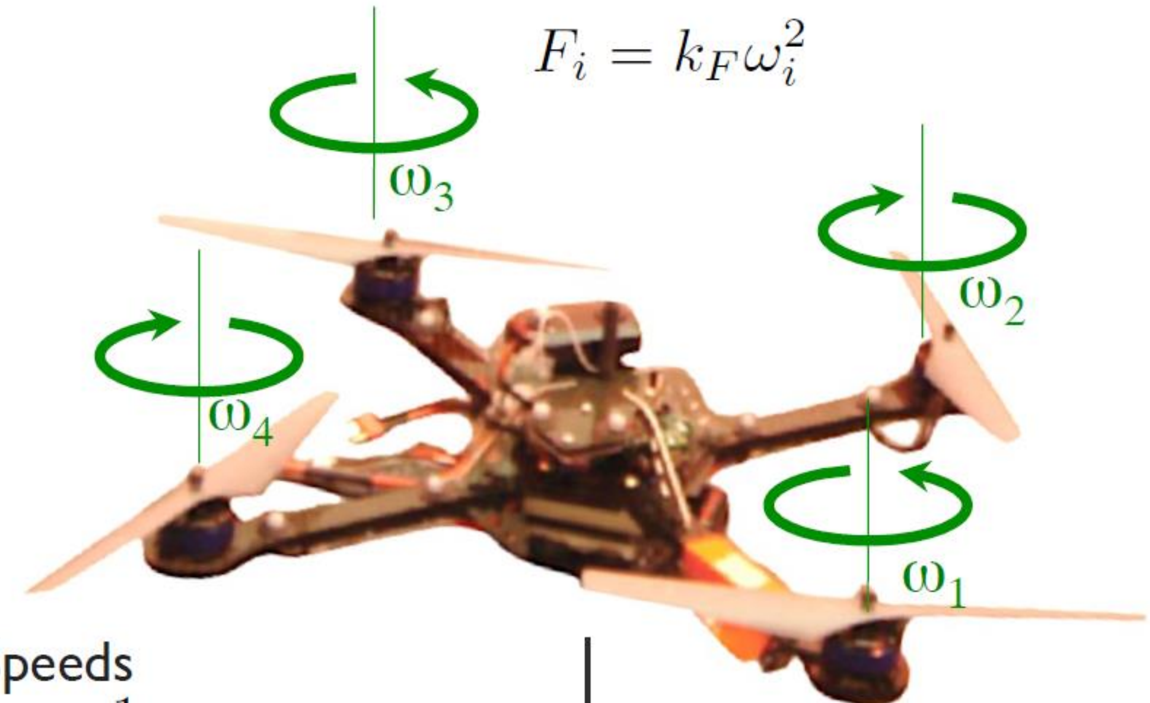
(Several figures/slides were taken from Vijay Kumar's excellent Coursera course on Aerial Robotics.)

DYNAMIC SYSTEMS LAB



ROTOR PHYSICS





Motor Speeds

$$k_F \omega_i^2 = \frac{1}{4} mg$$

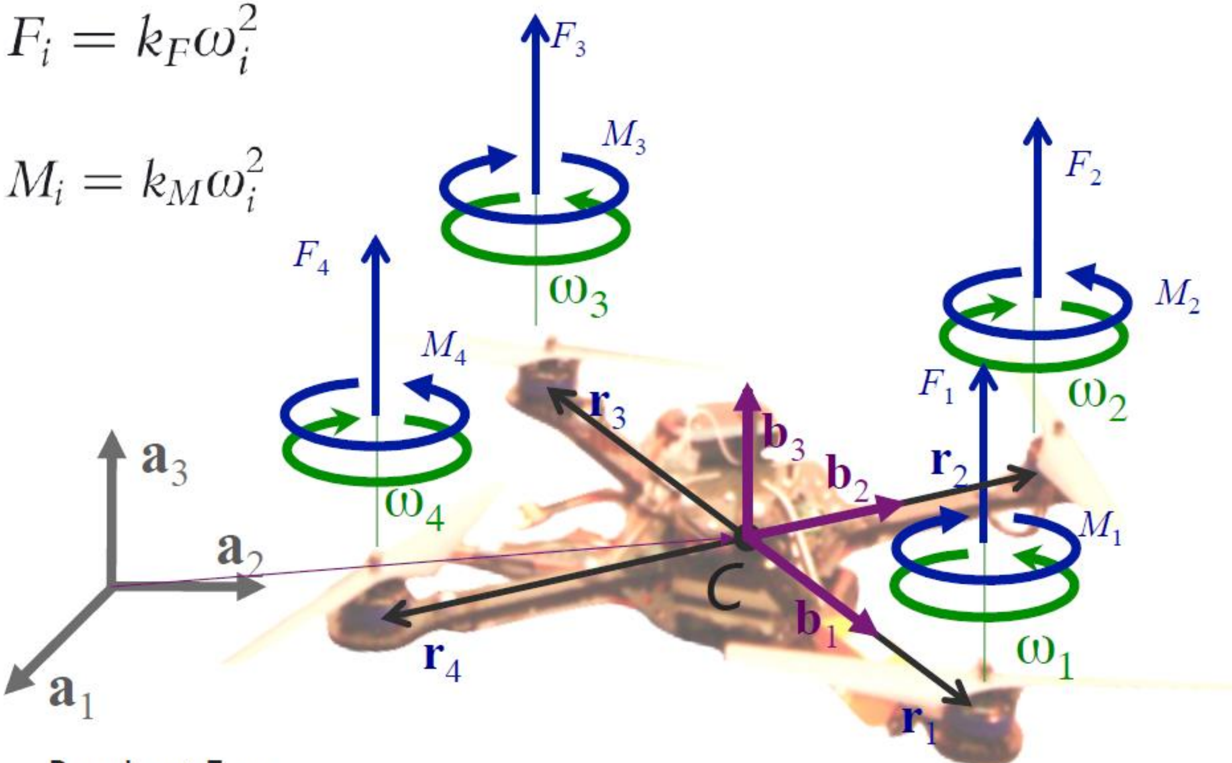
Motor Torques

$$\tau_i = k_M \omega_i^2$$

DYNAMIC SYSTEMS LAB

$$F_i = k_F \omega_i^2$$

$$M_i = k_M \omega_i^2$$



Resultant Force

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 - mg\mathbf{a}_3$$

Resultant Moment

$$\mathbf{M} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3 + \mathbf{r}_4 \times \mathbf{F}_4 + \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

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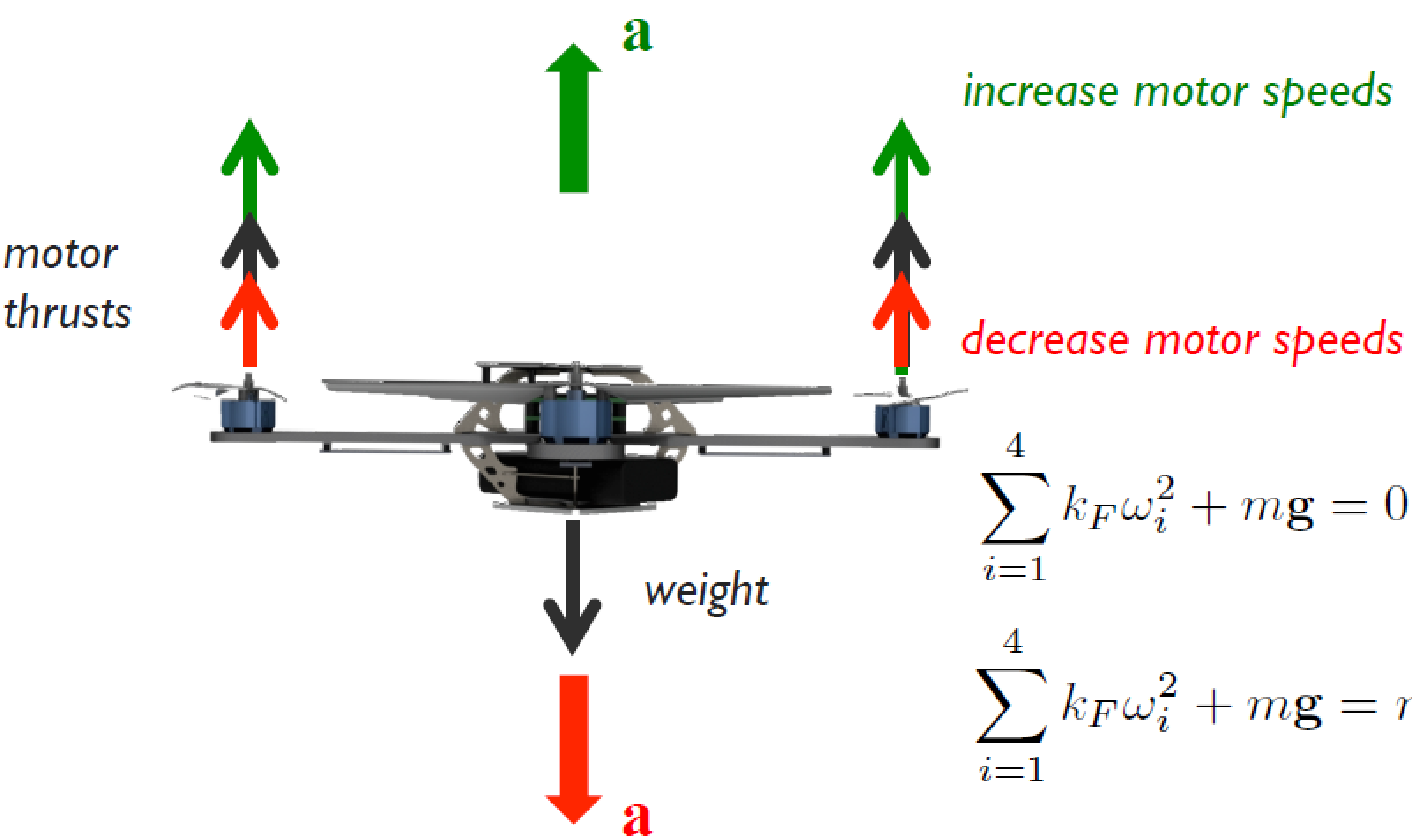
DYNAMICS IN VERTICAL DIRECTION



$$\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g} = 0$$

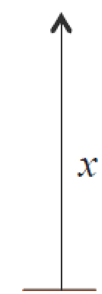
$$\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g} = m\mathbf{a}$$

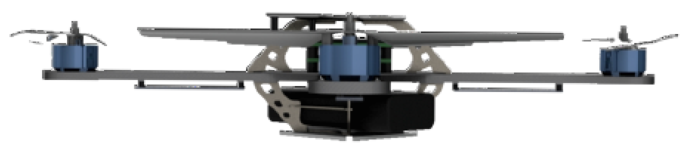
CONTROL OF HEIGHT

$$\boxed{\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}} = m\mathbf{a} \longrightarrow a = \frac{d^2x}{dt^2} = \ddot{x}$$

Input $\quad u = \frac{1}{m}\left[\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}\right]$

Second order dynamic system $\quad u = \ddot{x}$





What input drives the robot to the desired position?

CONTROL OF LINEAR, SECOND-ORDER SYSTEM

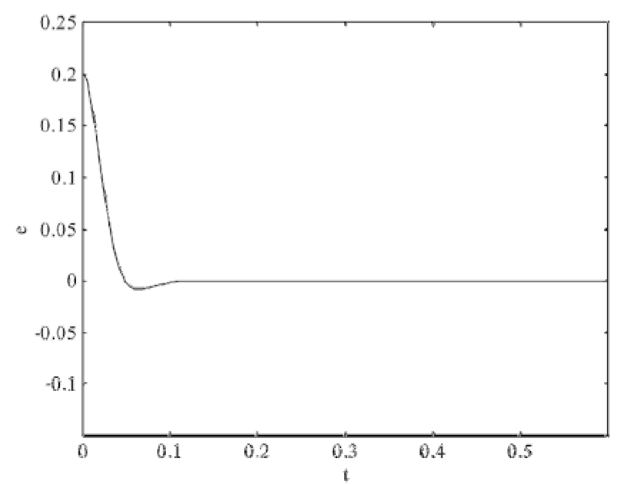
DYNAMIC SYSTEMS LAB

Problem

State, input $\quad x,\ u \in \mathbb{R}$

Plant model $\quad \ddot{x} = u$

Want x to follow the desired trajectory $x^{des}(t)$



General Approach

Define error, $e(t)=x^{des}(t)- x(t)$

Want $e(t)$ to converge exponentially to zero

Strategy

Find u such that

$$\ddot{e} + K_v\dot{e} + K_p e = 0 \qquad K_p,\ K_v\ > \ 0$$

$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v\dot{e}(t) + K_p e(t)$$

Feedforward

Derivative

Proportional

TRAJECTORY TRACKING CONTROL IN 1D

DYNAMIC SYSTEMS LAB

PD control

$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Proportional control acts like a spring (capacitance) response

Derivative control is a viscous dashpot (resistance) response

Large derivative gain makes the system overdamped and the system converges slow

PID control

In the presence of disturbances (e.g., wind) or modeling errors (e.g. unknown mass), it is often advantageous to use PID control

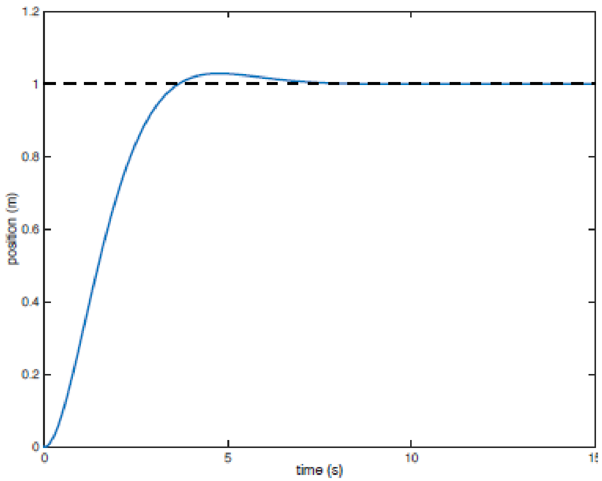
$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t) + K_i \int_0^t e(\tau) d\tau$$

Integral

PID control generates a third-order closed-loop system

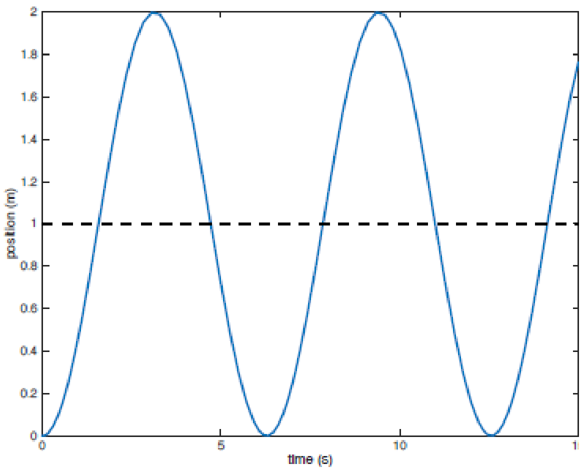
Integral control makes the steady-state error go to zero

EFFECTS OF GAINS



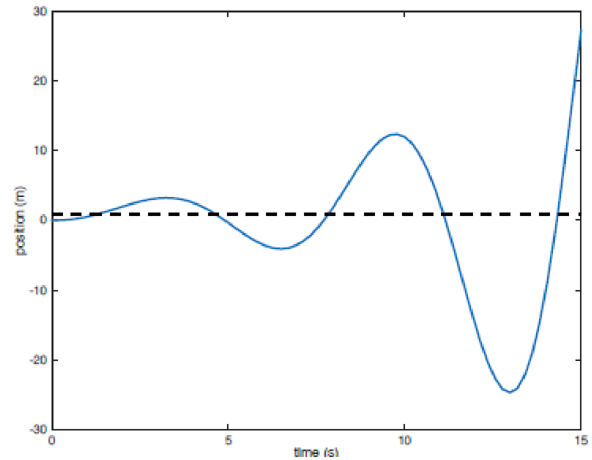
Stable

$$K_p,\ K_v\ >\ 0$$



Marginally Stable

$$K_p > 0, \quad K_v = 0$$



Unstable

$$K_p \text{ or } K_v < 0$$

DIFFERENT PARAMETRIZATION

$$\ddot{x} = u, \qquad u := K_p(x^{des} - x) + K_v(\dot{x}^{des} - \dot{x}) + \ddot{x}^{des}$$

Feed-forward term.

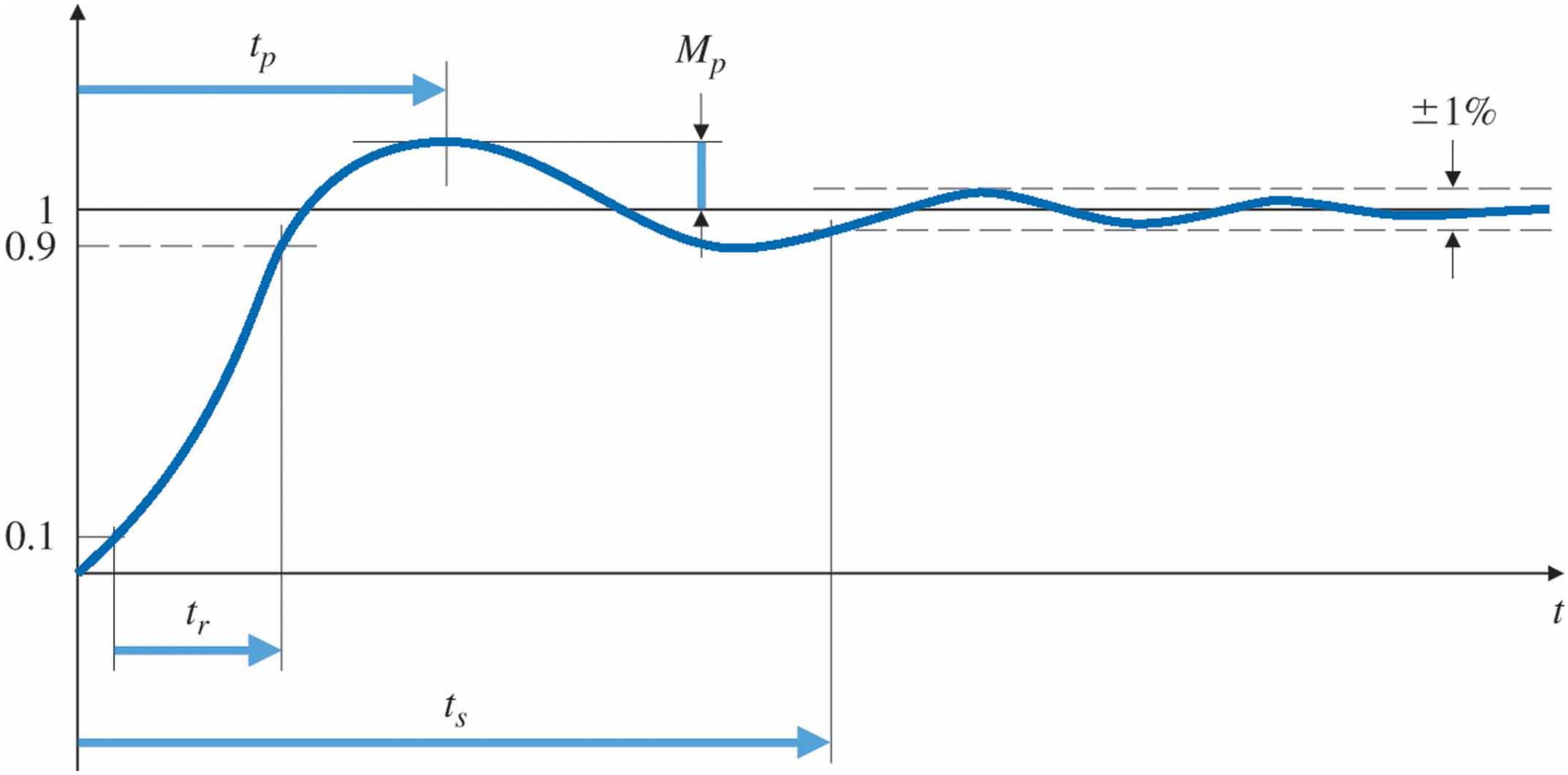
$$\Rightarrow \quad \ddot{e} + K_v\dot{e} + K_p e = 0$$

$$\Rightarrow \quad \boxed{\ddot{e} + 2\zeta\omega_n\dot{e} + \omega_n^2 e = 0}$$

Intuitive parameterization:

- Damping ratio: $\zeta \in [0.7..1]$
- Natural frequency, related to rise time (10-90%): $\omega_n \approx \frac{t_r}{1.8}$
- Settling time: $t_s \approx \frac{4.6}{\zeta\omega_n}$

Definition of rise time t_r, peak time t_p, settling time t_s, and overshoot M_p



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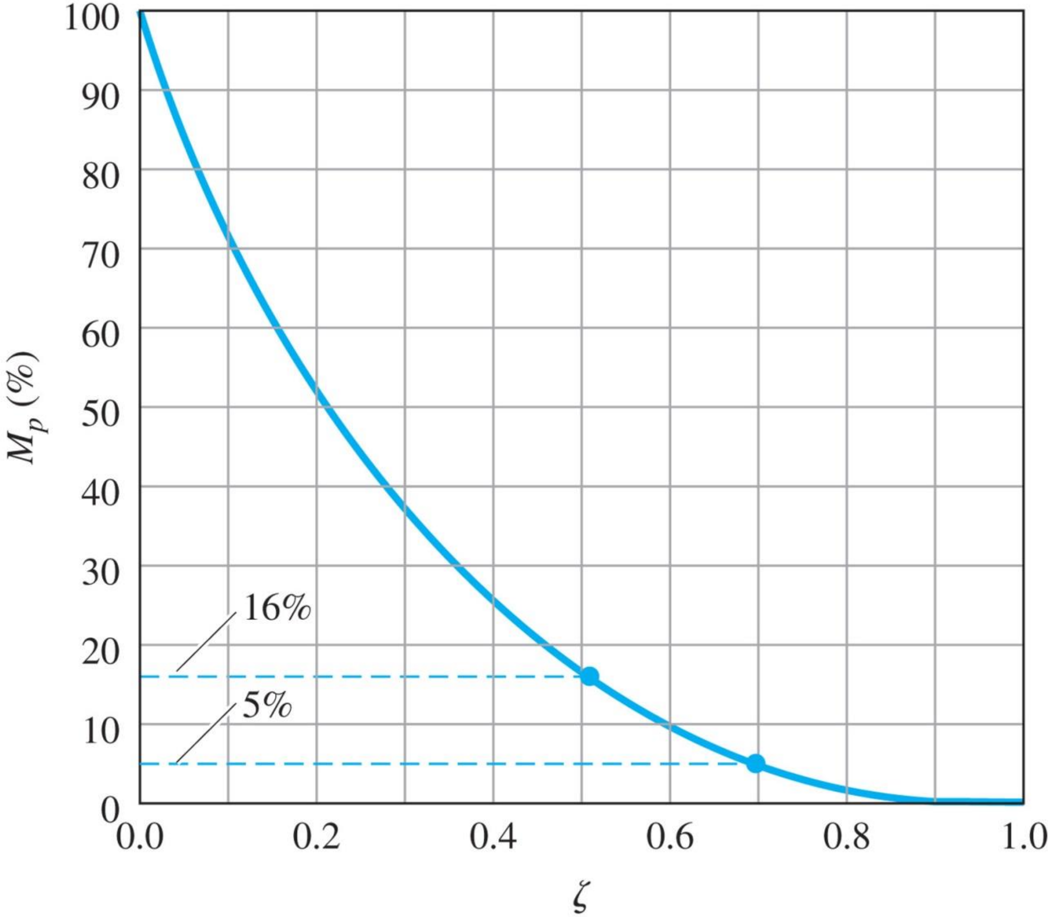
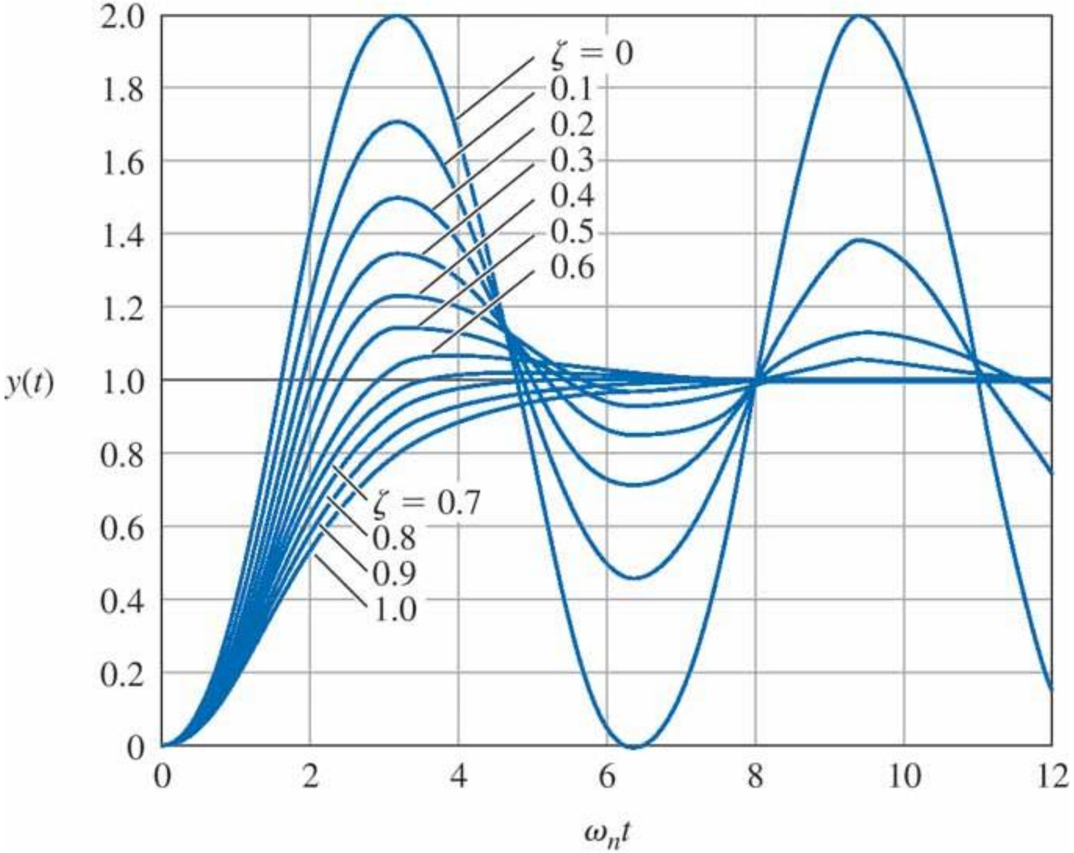
Overshoot versus damping ratio for the second-order system



Step responses of second-order systems versus ζ.



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IV. Trajectory Tracking Control (3D)

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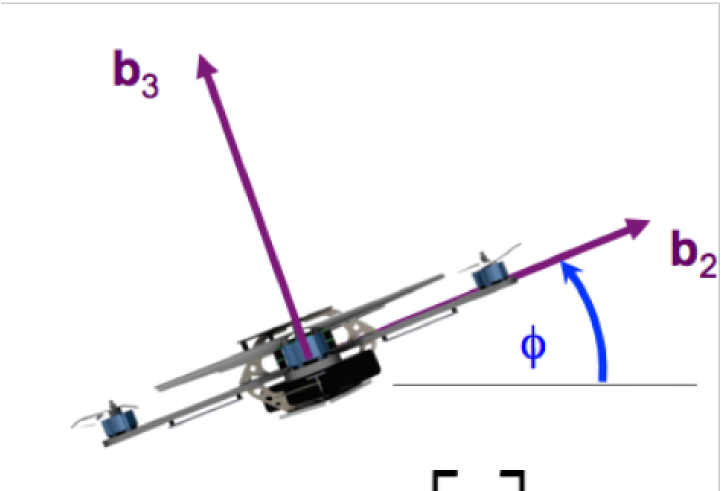
VI. Summary

PLANAR QUADROTOR MODEL



$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m}\sin\phi & 0 \\ \frac{1}{m}\cos\phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ z \\ \phi \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\phi} \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{m}\sin\phi & 0 \\ \frac{1}{m}\cos\phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

LINEARIZED DYNAMIC MODEL

Equations of motion

$$\ddot{y} = -\frac{u_1}{m}\sin(\phi)$$

$$\ddot{z} = -g + \frac{u_1}{m}\cos(\phi)$$

$$\ddot{\phi} = \frac{u_2}{I_{xx}}$$

Dynamics are nonlinear

Equilibrium hover configuration

$$y_0, z_0, \phi_0 = 0, u_{1,0} = mg, u_{2,0} = 0,$$

Linearized dynamics

$$\ddot{y} = -g\phi$$

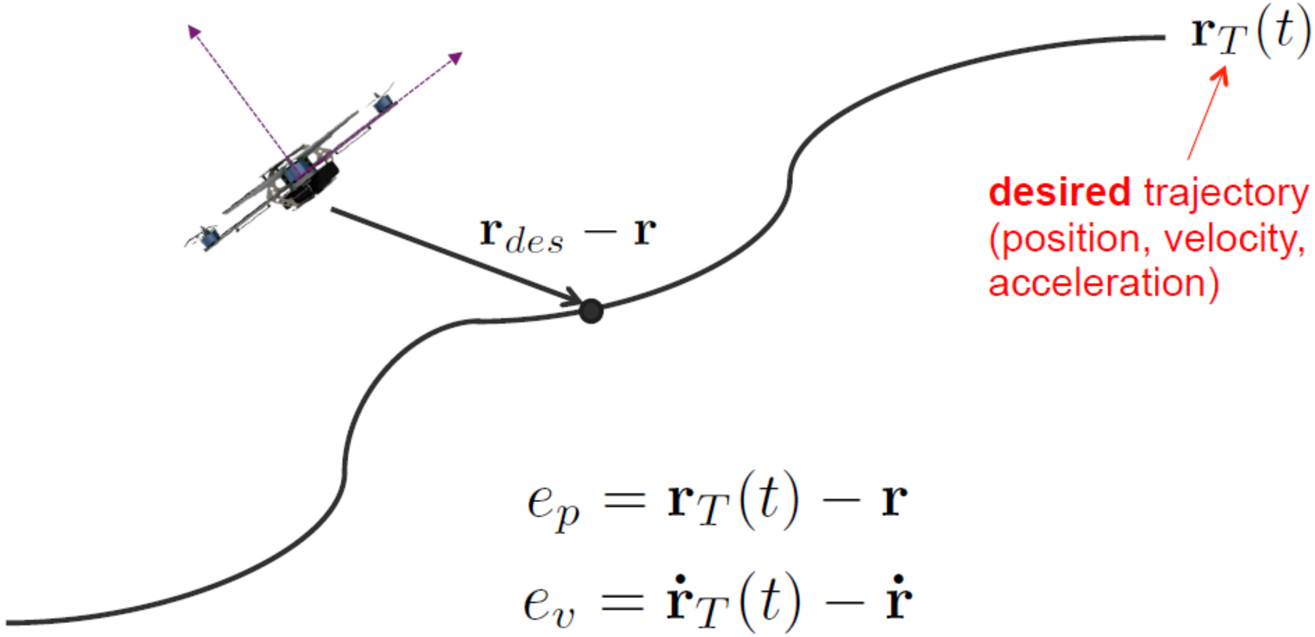
$$\ddot{z} = -g + \frac{u_1}{m}$$

$$\ddot{\phi} = \frac{u_2}{I_{xx}}$$

DYNAMIC
SYSTEMS LAB

Given $\mathbf{r}_T(t), \dot{\mathbf{r}}_T(t), \ddot{\mathbf{r}}_T(t)$

$$\boxed{\mathbf{r}_T(t) = \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}}$$



$$e_p = \mathbf{r}_T(t) - \mathbf{r}$$

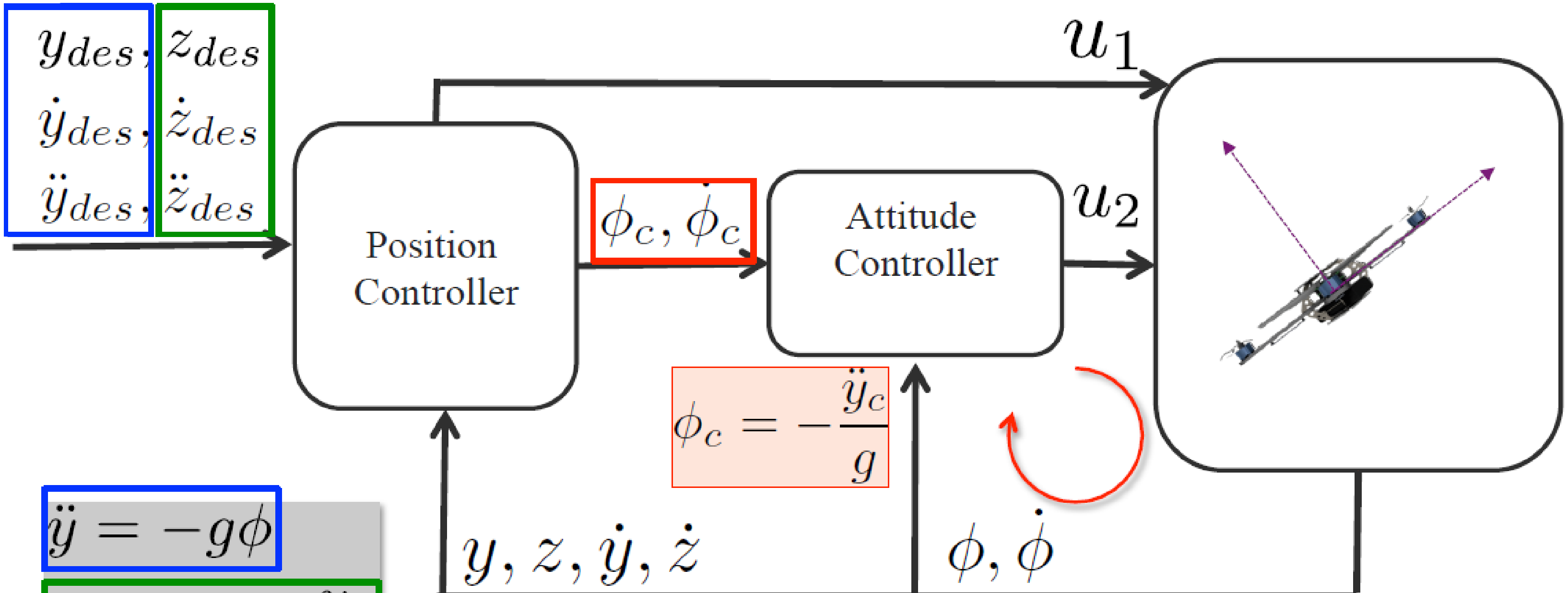
$$e_v = \dot{\mathbf{r}}_T(t) - \dot{\mathbf{r}}$$

Want $(\ddot{\mathbf{r}}_T(t) - \ddot{\mathbf{r}}_c) + k_{d,x} e_v + k_{p,x} e_p = 0$

Commanded acceleration, calculated by the controller

DYNAMIC SYSTEMS LAB

$$u_1 = m(g + \ddot{z}_{des} + k_{d,z}(\dot{z}_{des} - \dot{z}) + k_{p,z}(z_{des} - z))$$



$$\phi_c = -\frac{\ddot{y}_c}{g}$$

$$\ddot{y} = -g\phi$$

$$\ddot{z} = -g + \frac{u_1}{m}$$

$$\ddot{\phi} = \frac{u_2}{I_{xx}}$$

$$u_2 = I_{xx}(\ddot{\phi}_c + k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(\dot{\phi}_c - \dot{\phi}))$$

CONTROL EQUATIONS

$$u_1 = m(g + \ddot{z}_{des} + k_{d,z}(\dot{z}_{des} - \dot{z}) + k_{p,z}(z_{des} - z))$$

$$u_2 = k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(\dot{\phi}_c - \dot{\phi})$$

$$\phi_c = -\frac{1}{g}(\ddot{y}_{des} + k_{d,y}(\dot{y}_{des} - \dot{y}) + k_{p,y}(y_{des} - y))$$

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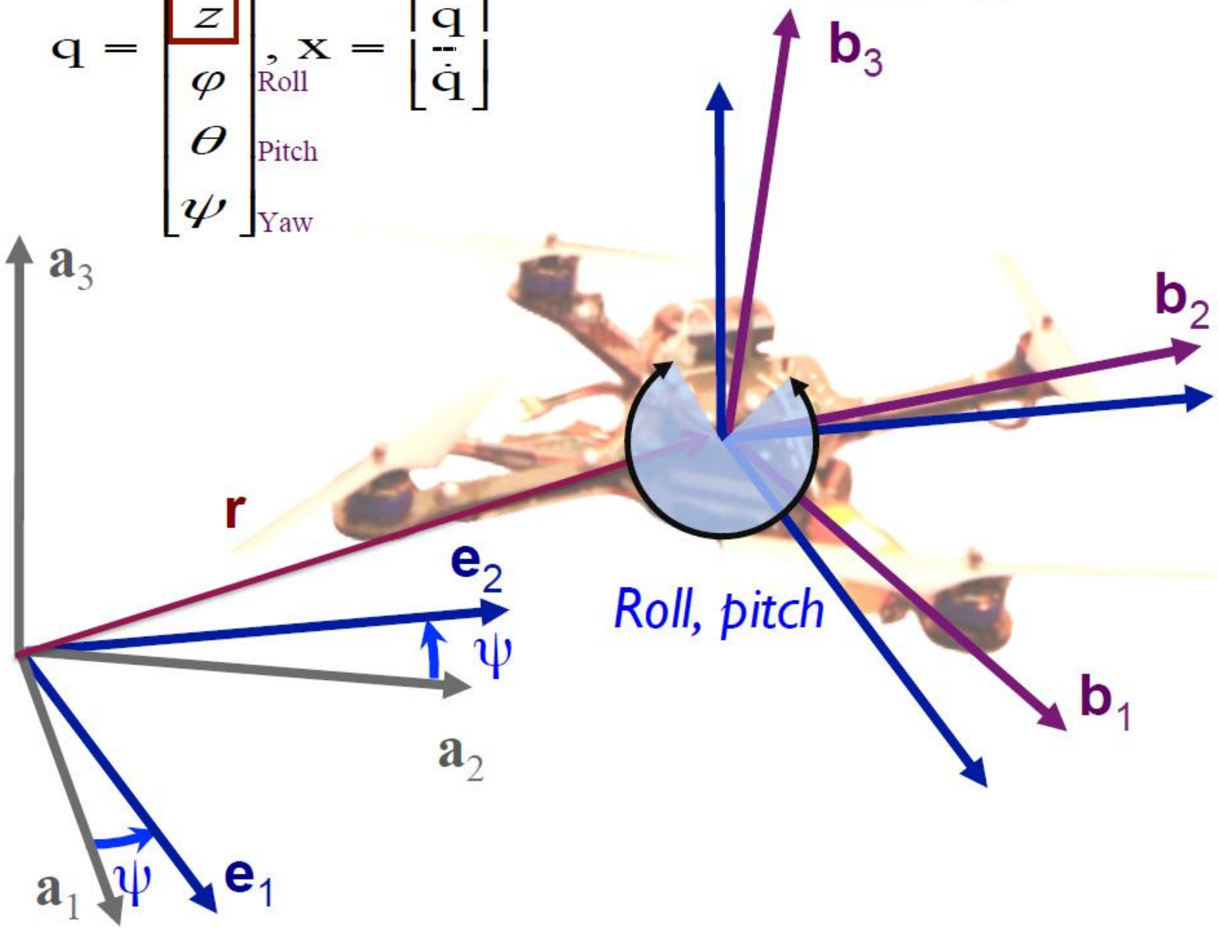
VI. Summary

$$\mathbf{q} = \left[\begin{array}{c} \mathbf{r}:\ \boxed{\begin{matrix} x \\ y \\ z \end{matrix}} \\ \varphi \\ \theta \\ \psi \end{array}\right]\begin{array}{l} \\ \\ \\ \text{Roll} \\ \text{Pitch} \\ \text{Yaw} \end{array}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$$

Angular velocity components in B

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

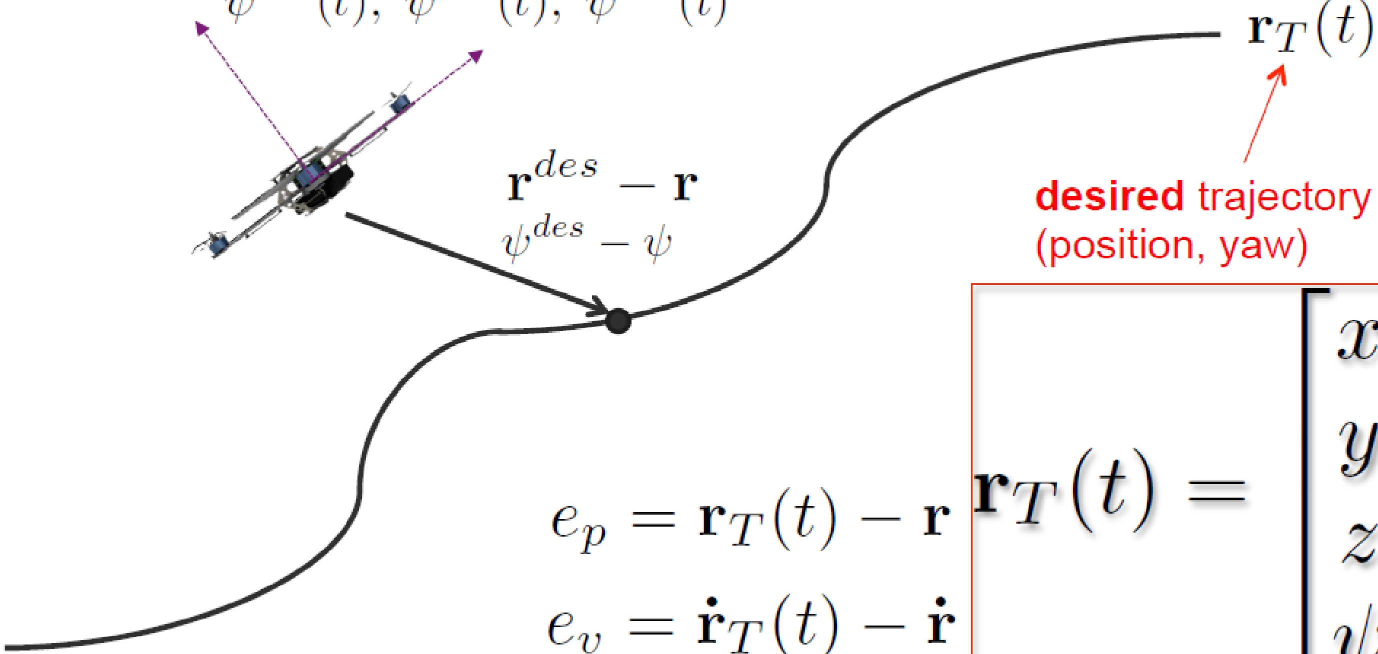


CONTROL TASK

Given $\mathbf{r}_T(t), \dot{\mathbf{r}}_T(t), \ddot{\mathbf{r}}_T(t)$

$\mathbf{r}^{des}(t),\ \dot{\mathbf{r}}^{des}(t),\ \ddot{\mathbf{r}}^{des}(t)$

$\psi^{des}(t),\ \dot{\psi}^{des}(t),\ \ddot{\psi}^{des}(t)$



$$\mathbf{r}_T(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \psi(t) \end{bmatrix}$$

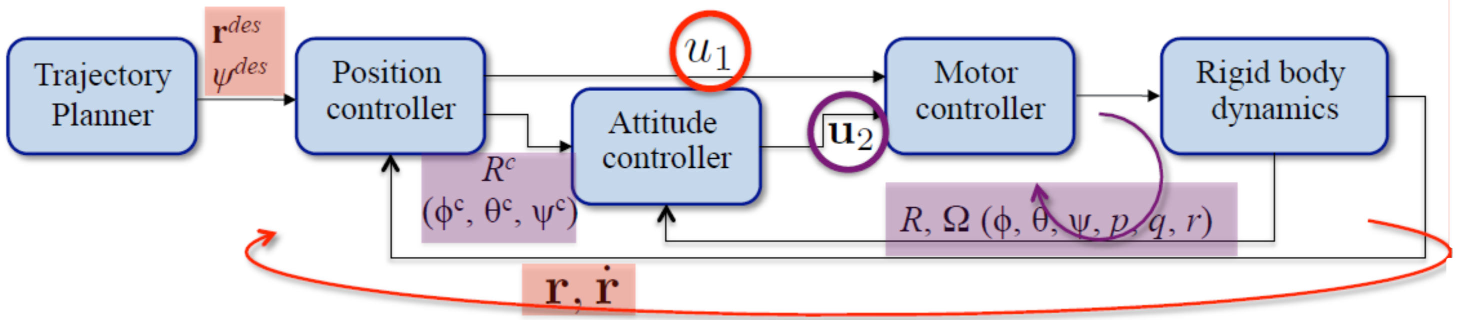
$$e_p = \mathbf{r}_T(t) - \mathbf{r}$$

$$e_v = \dot{\mathbf{r}}_T(t) - \dot{\mathbf{r}}$$

Want $(\ddot{\mathbf{r}}_T(t) - \ddot{\mathbf{r}}_c) + k_{d,x} e_v + k_{p,x} e_p = 0$

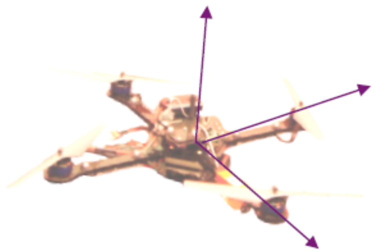
Commanded acceleration, calculated by the controller

NESTED CONTROL



$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{\boxed{F_1 + F_2 + F_3 + F_4}}_{u_1} \end{bmatrix}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}}_{\mathbf{u}_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$



Linearize the dynamics at the hover configuration

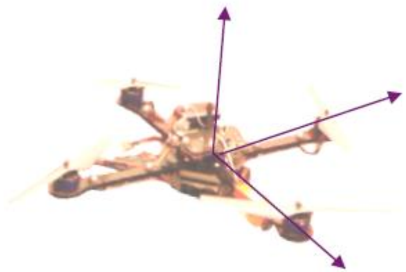
$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$

$$(u_2 \sim 0, p \sim 0, q \sim 0, r \sim 0)$$

$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{F_1 + F_2 + F_3 + F_4}_{u_1} \end{bmatrix}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}_{\mathbf{u}_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

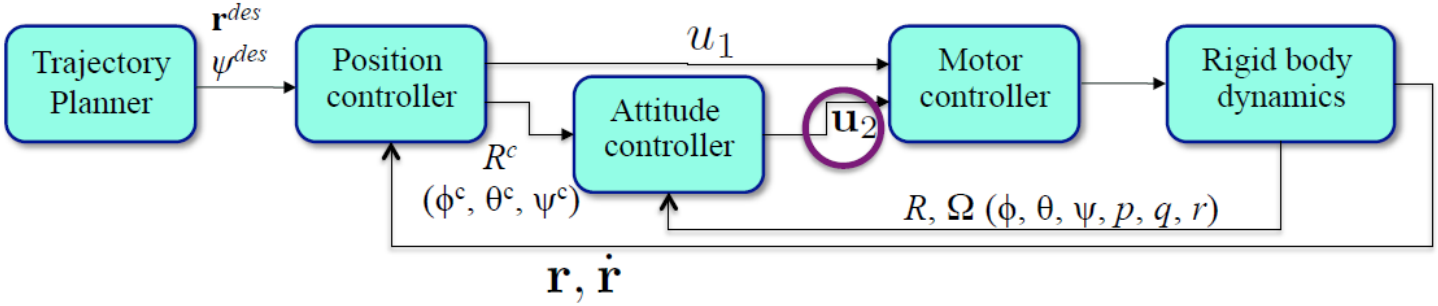
HOVER CONTROL



$$\left(u_2 \sim 0, p \sim 0, q \sim 0, r \sim 0\right)$$

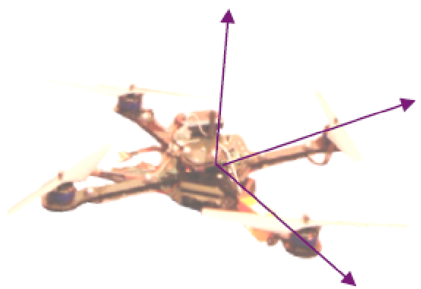
$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}}_{\mathbf{u}_2} - \cancelto{0}{\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}}$$

HOVER CONTROL



$$\boxed{\mathbf{u}_2 = \begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r) \end{bmatrix}}$$

HOVER CONTROL



$$m\ddot{\mathbf{r}} = \begin{bmatrix}0\\0\\-mg\end{bmatrix} + R\begin{bmatrix}0\\0\\ \underbrace{\boxed{F_1+F_2+F_3+F_4}}_{u_1}\end{bmatrix}$$

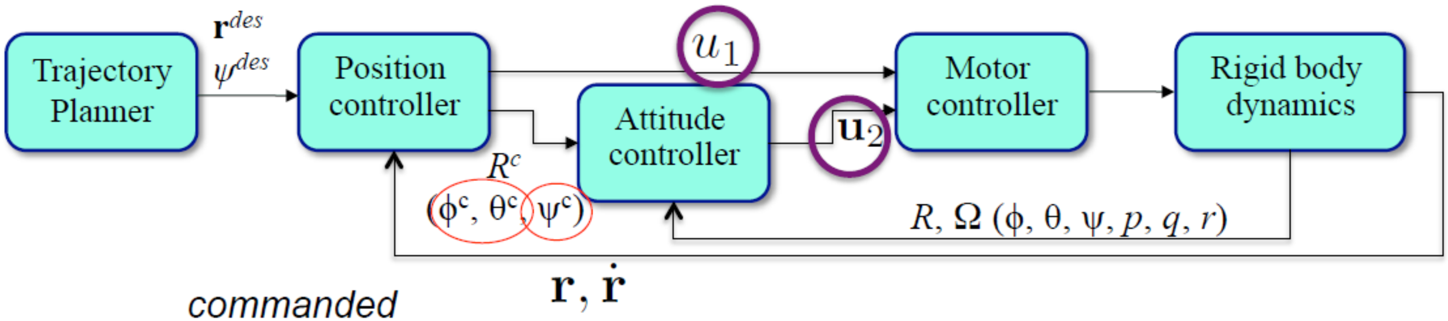
Linearization

$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$

$$\ddot{r}_1 = \ddot{x} = g(\theta\cos\psi + \phi\sin\psi)$$

$$\ddot{r}_2 = \ddot{y} = g(\theta\sin\psi - \phi\cos\psi)$$

DYNAMIC SYSTEMS LAB



commanded

actual (feedback)

$$(\ddot{r}_{i,des} - \ddot{r}_{i,c}) + k_{d,i}(\dot{r}_{i,des} - \dot{r}_i) + k_{p,i}(r_{i,des} - r_i) = 0$$

specified

$$\boxed{u_1 = m(g + \ddot{r}_{3,c})}$$

$$\phi_c = \frac{1}{g}(\ddot{r}_{1,c} \sin\psi_{des} - \ddot{r}_{2,c} \cos\psi_{des})$$

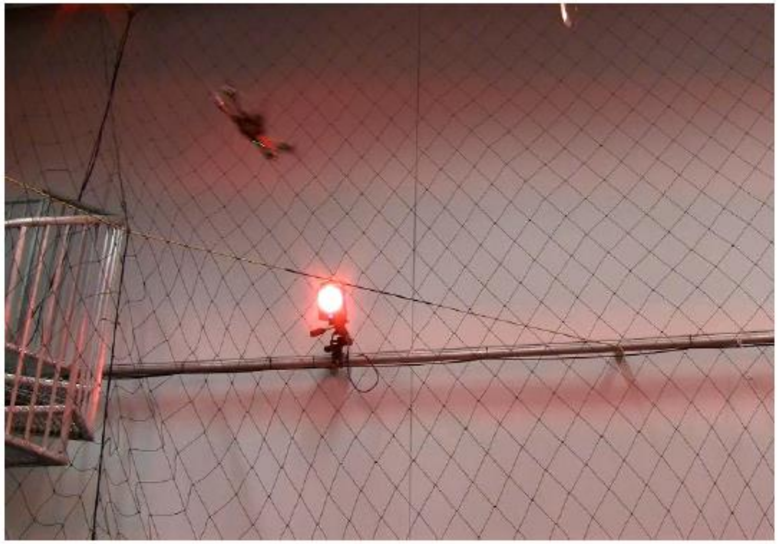
$$\theta_c = \frac{1}{g}(\ddot{r}_{1,c} \cos\psi_{des} + \ddot{r}_{2,c} \sin\psi_{des})$$

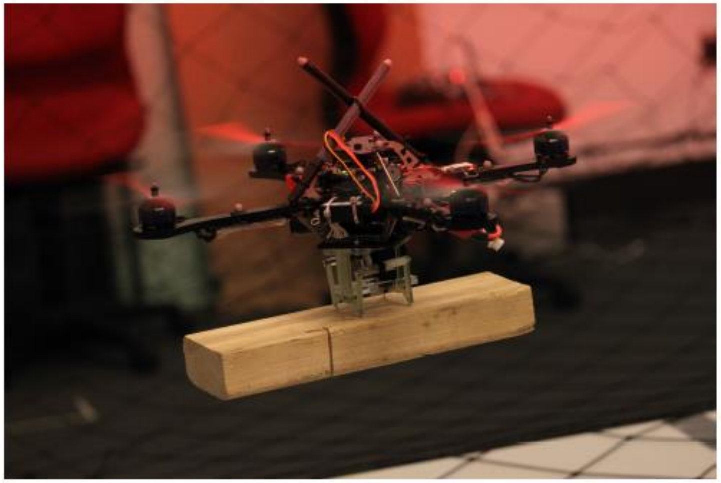
$$\psi_c = \psi^{des}$$

$$\boxed{\mathbf{u}_2 = \begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r) \end{bmatrix}}$$

LIMITATIONS OF LINEAR CONTROL

Assumption: roll and pitch angles, and all velocities are close to zero

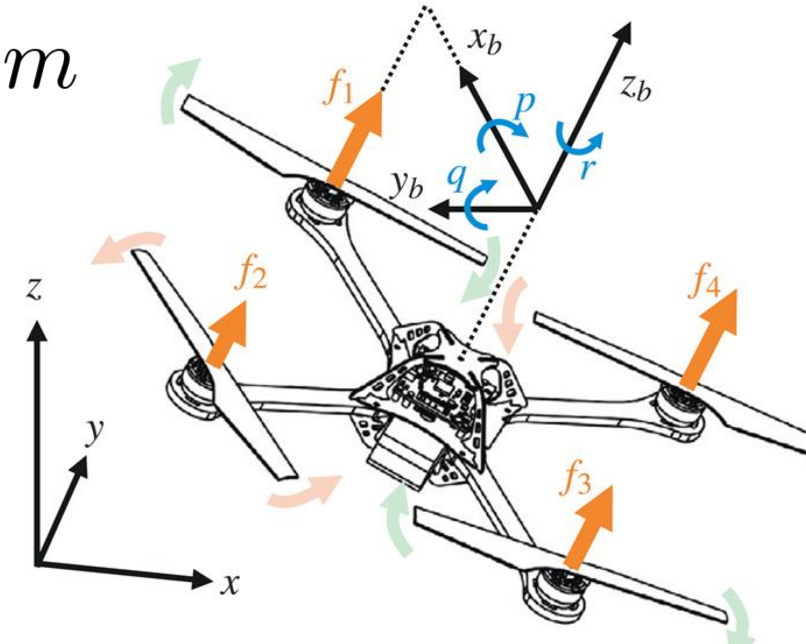




$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \boldsymbol{R} \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

Maps from body to inertial frame.

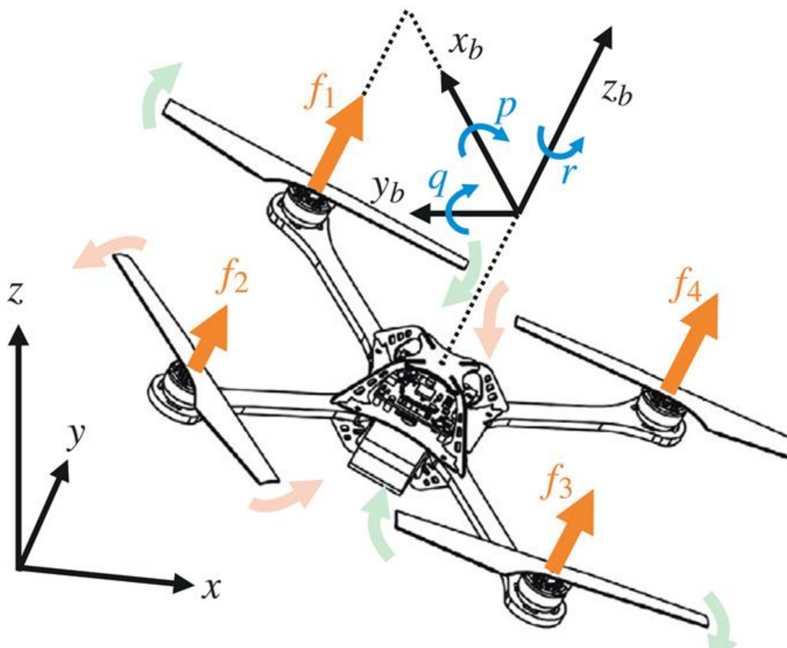
$$c = (f_1 + f_2 + f_3 + f_4)/m$$



NONLINEAR MODEL

$$\boldsymbol{J}\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix} l(f_2 - f_4) \\ l(f_3 - f_1) \\ \kappa(f_1 - f_2 + f_3 - f_4)\end{bmatrix} - \begin{bmatrix}p\\ q\\ r\end{bmatrix} \times \boldsymbol{J}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

$$\dot{\boldsymbol{R}} = \boldsymbol{R}\begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0\end{bmatrix}$$



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$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \boldsymbol{R} \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

$$c = (f_1 + f_2 + f_3 + f_4)/m$$

$$\boxed{c_d = \frac{1}{R_{33}} \left(\omega_{n,z}^2 (z_d - z) + 2\xi_z \omega_{n,z} (\dot{z}_d - \dot{z}) + \ddot{z}_d + g \right)}$$

$$a_z$$

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Define similarly for lateral direction:

$$a_x = \omega_{n,x}^2 (x_d - x) + 2\xi_x \omega_{n,x} (\dot{x}_d - \dot{x}) + \ddot{x}_d$$
$$a_y = \dots$$

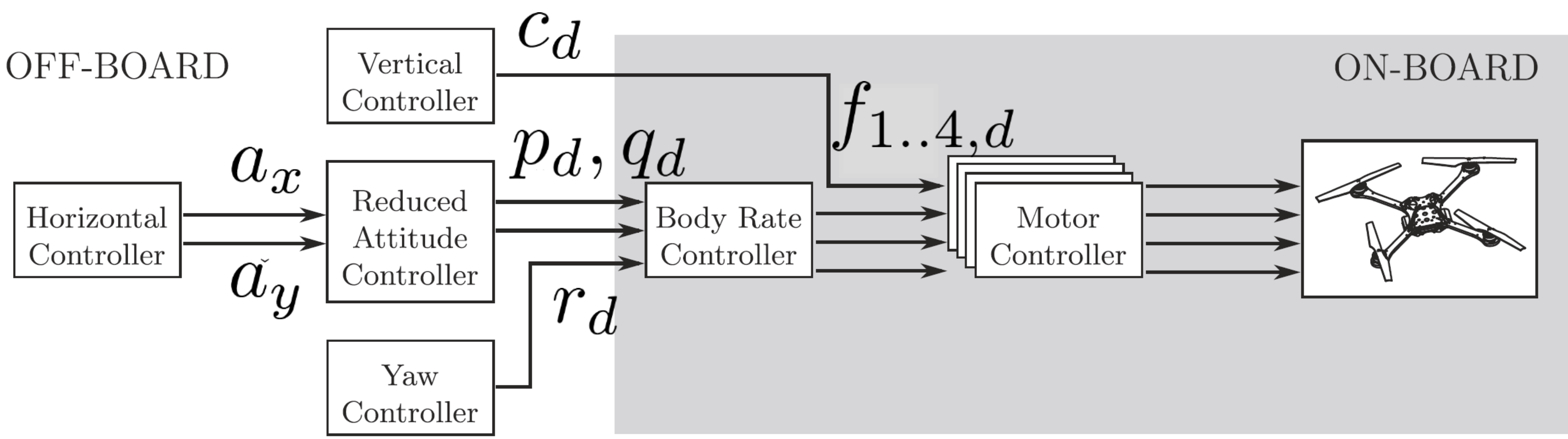
Transform into desired turn rates:

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} R_{13,d} \\ R_{23,d} \end{bmatrix} c_d$$

$$\dot{R}_{13,d} = \frac{1}{\tau_{13}} (R_{13,d} - R_{13})$$
$$\dot{R}_{23,d} = \frac{1}{\tau_{23}} (R_{23,d} - R_{23})$$

$$\begin{bmatrix} p_d \\ q_d \end{bmatrix} = \frac{1}{R_{33}} \begin{bmatrix} R_{21} & -R_{11} \\ R_{22} & -R_{12} \end{bmatrix} \begin{bmatrix} \dot{R}_{13,d} \\ \dot{R}_{23,d} \end{bmatrix}$$

OVERALL CONTROL



Body rate controller:

$$\boldsymbol{J}\begin{bmatrix}\frac{1}{\tau_p}(p_d - p)\\ \frac{1}{\tau_q}(q_d - q)\\ \frac{1}{\tau_r}(r_d - r)\end{bmatrix} + \begin{bmatrix}p\\q\\r\end{bmatrix} \times \boldsymbol{J}\begin{bmatrix}p\\q\\r\end{bmatrix} = \begin{bmatrix}l(f_{2,d} - f_{4,d})\\ l(f_{3,d} - f_{1,d})\\ \kappa(f_{1,d} - f_{2,d} + f_{3,d} - f_{4,d})\end{bmatrix}$$

$$(f_{1,d} + f_{2,d} + f_{3,d} + f_{4,d}) = mc_d$$

OTHER APPROACHES

- **Linear controller based on linearized system** [Gurdan, et al. 2007][Boubadallah, 2007][Hoffman et al., 2008]
- **LQR**
- **Backstepping**
- **Exact linearization**, differentially flat system [Mellinger]
- **L1 adaptive control**

OUTLINE

I. Basic Mechanics

II. Dynamics & Control of the Vertical Direction (1D)

III. Dynamics & Control in the Vertical Plane (2D)

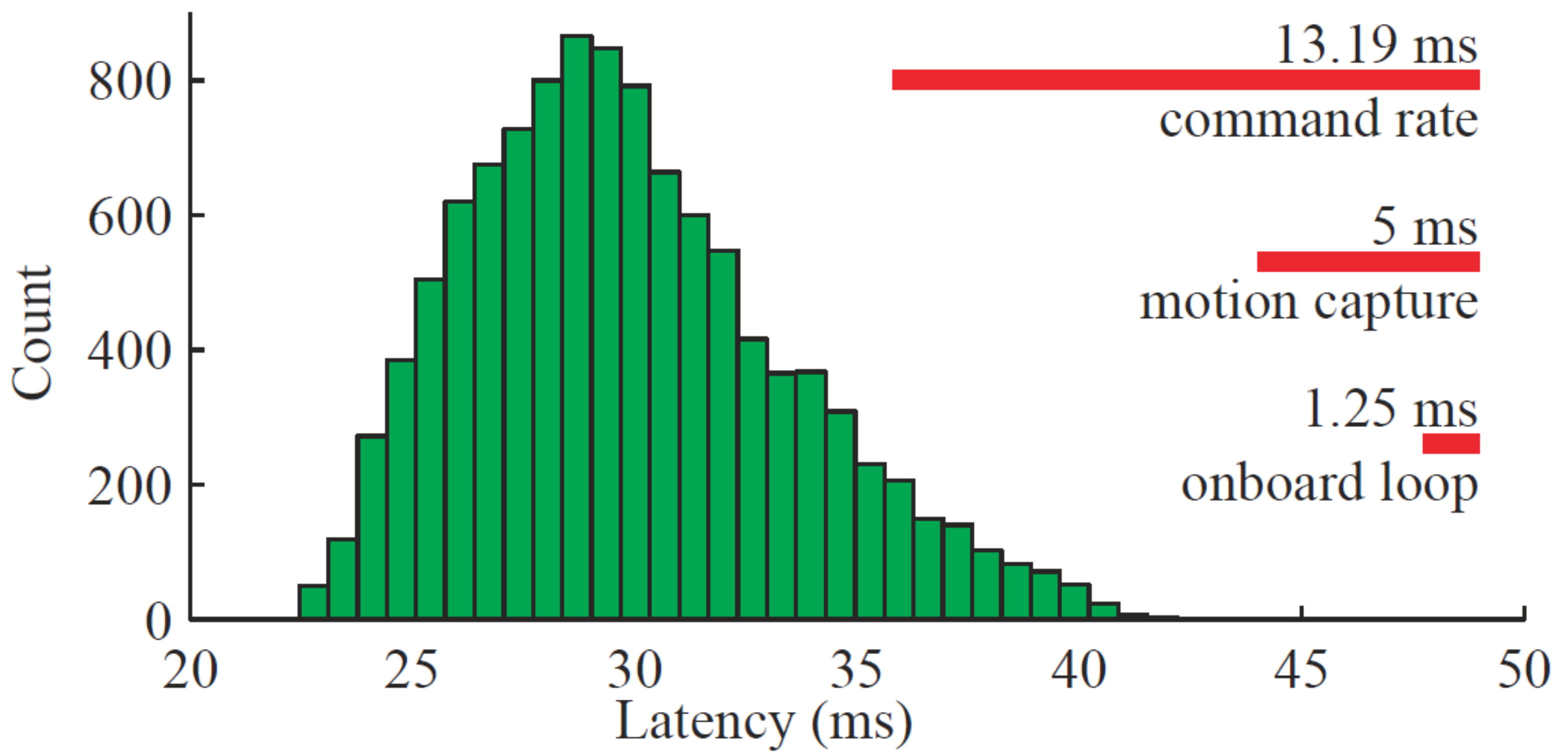
IV. Trajectory Tracking Control (3D)

- What Can Go Wrong?

V. Learning-Enabled Control

VI. Summary

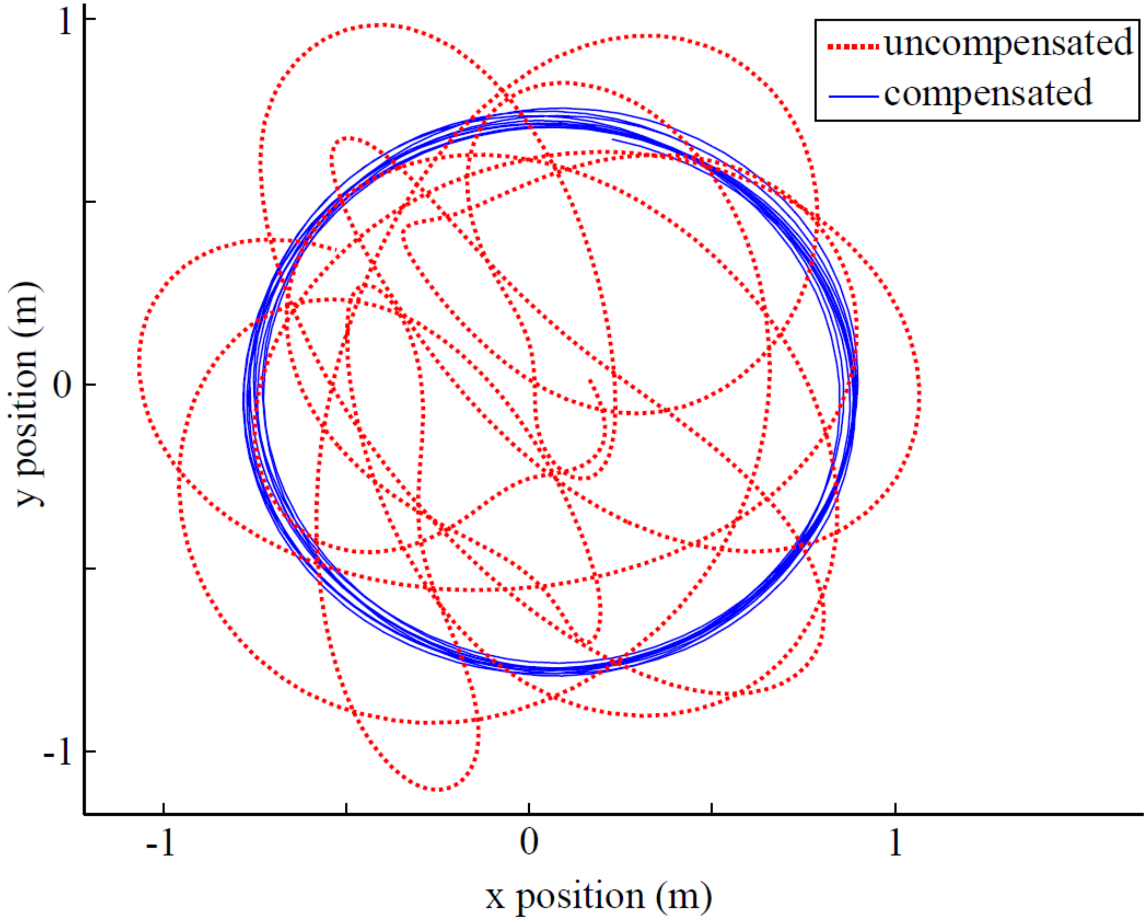
Latency


Count
800
600
400
200
0
20
25
30
35
40
45
50
Latency (ms)
13.19 ms
command rate
5 ms
motion capture
1.25 ms
onboard loop

DYNAMIC SYSTEMS LAB

Latency

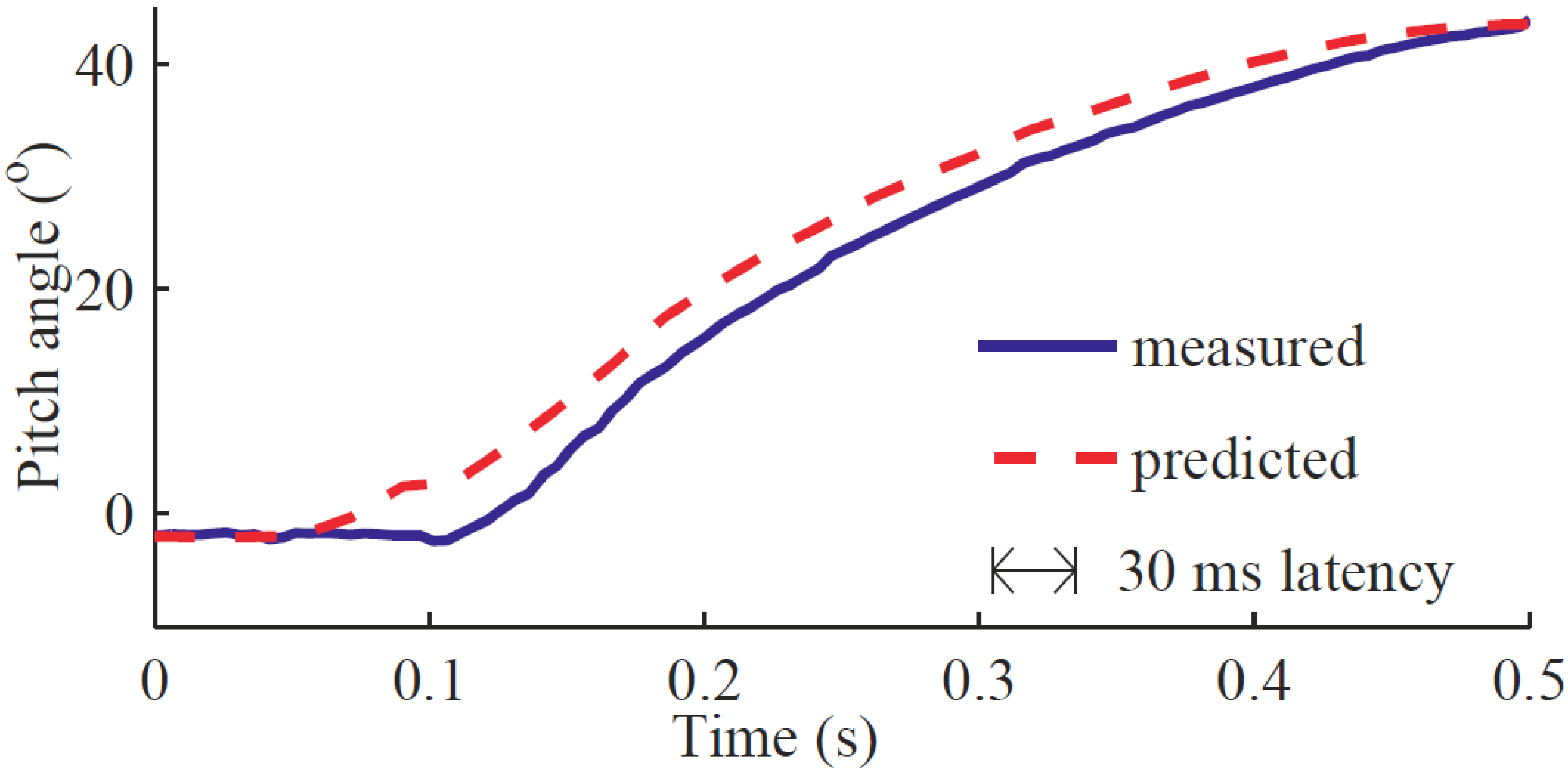
Circle motion at 4 m/s.



DYNAMIC SYSTEMS LAB

Latency

Predict the vehicle position at the time the input arrives at the vehicle.



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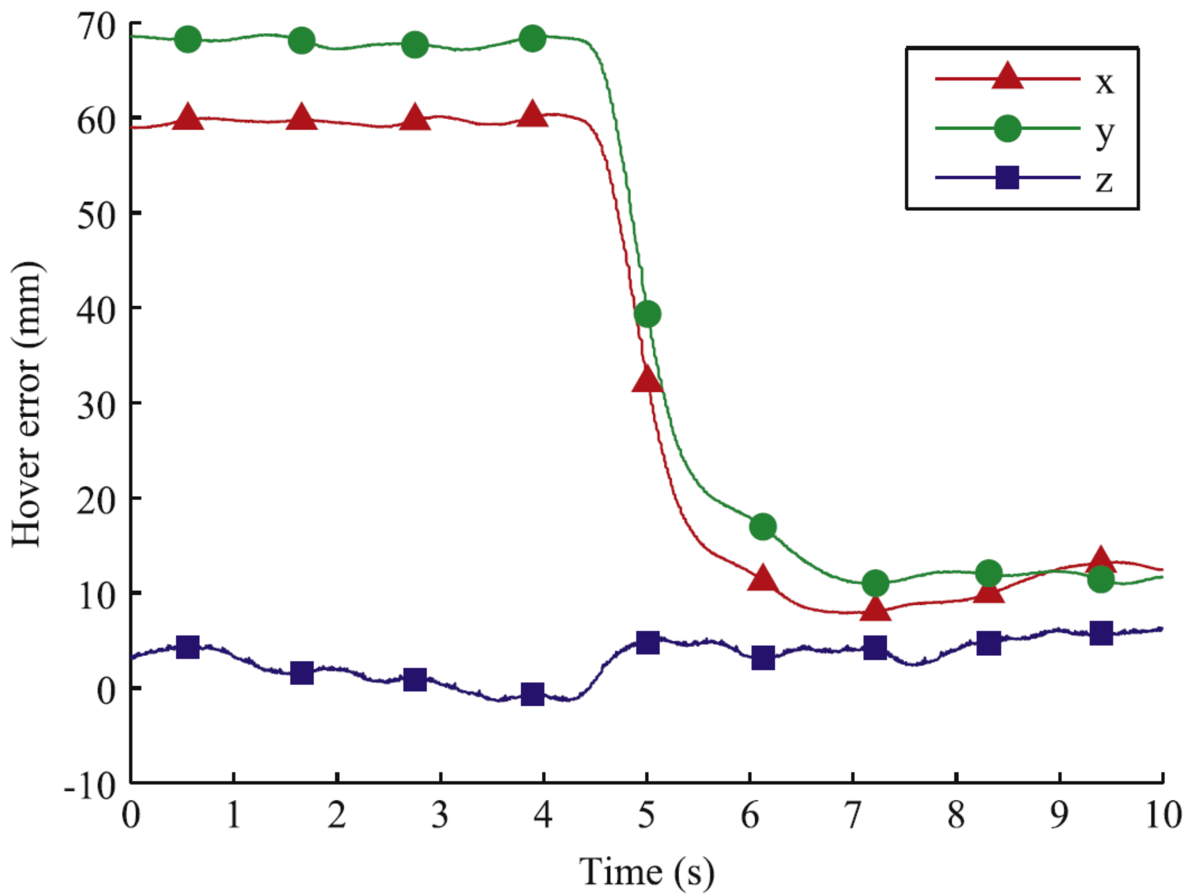
Offsets

Calibrate during hover.



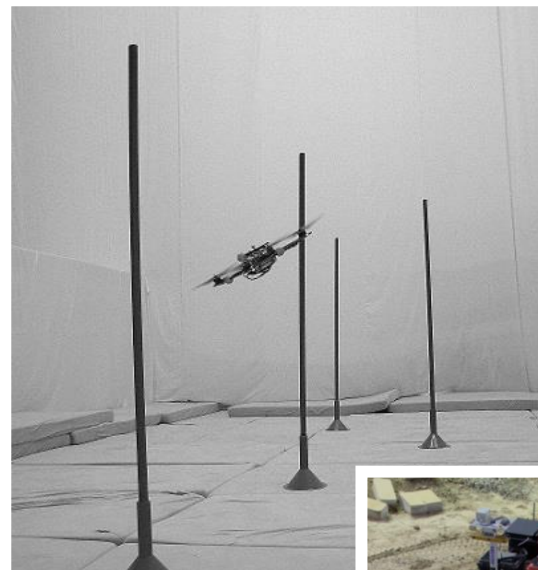
DYNAMIC SYSTEMS LAB

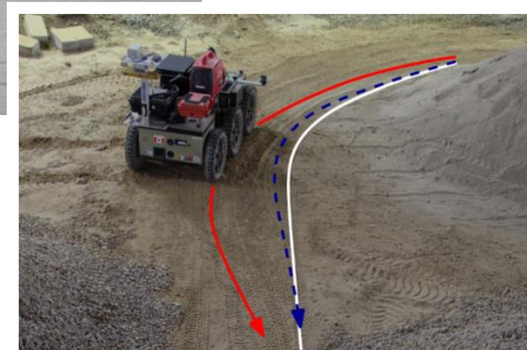
Aggressive Maneuvers

1. Triple flip with a quadrotor.

2. Time-optimized slalom.

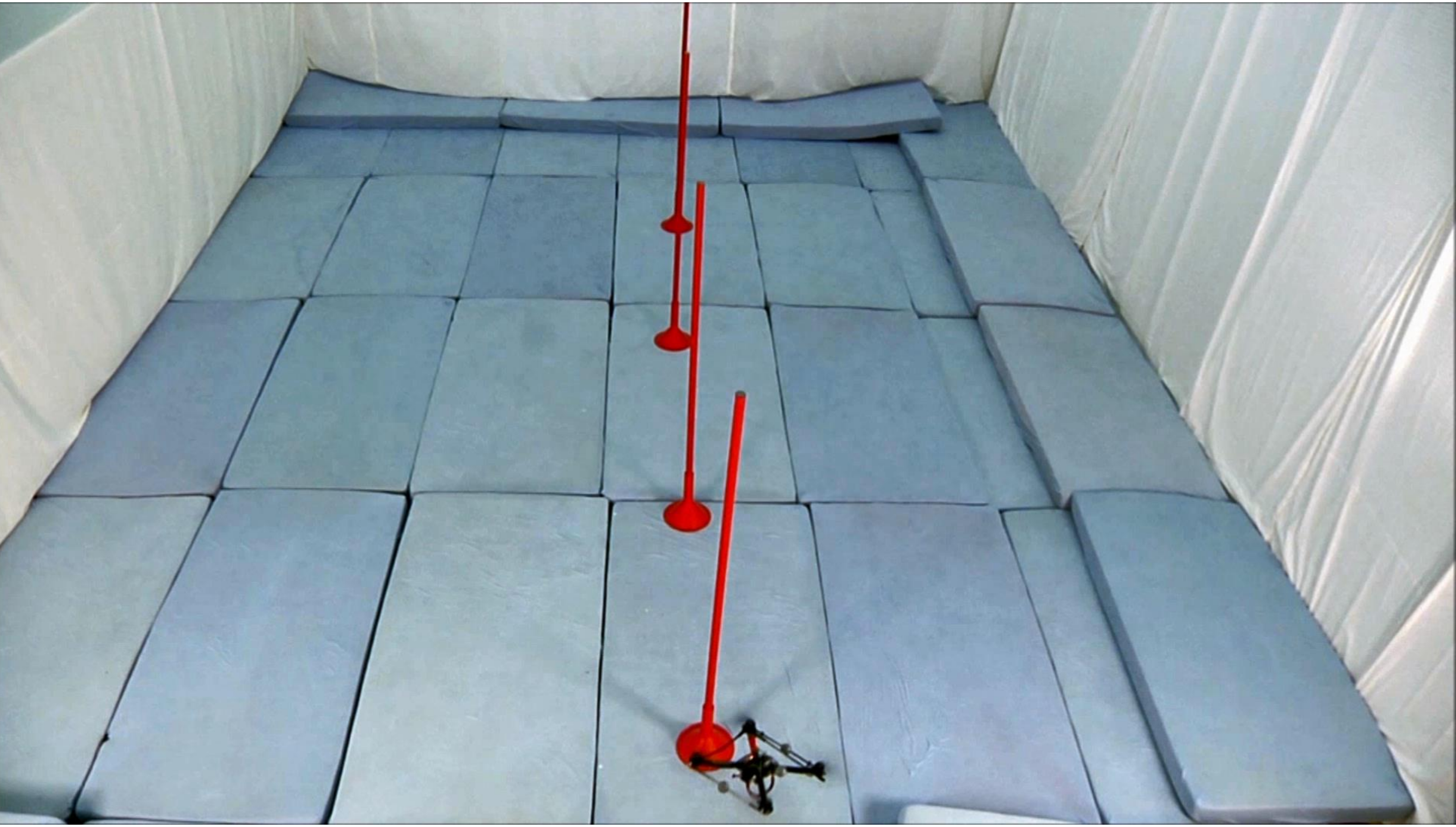
3. Fast path following with a ground vehicle.





VIDEO: https://youtu.be/bWExDW9J9sA?list=PLC12E387419CEAFF2

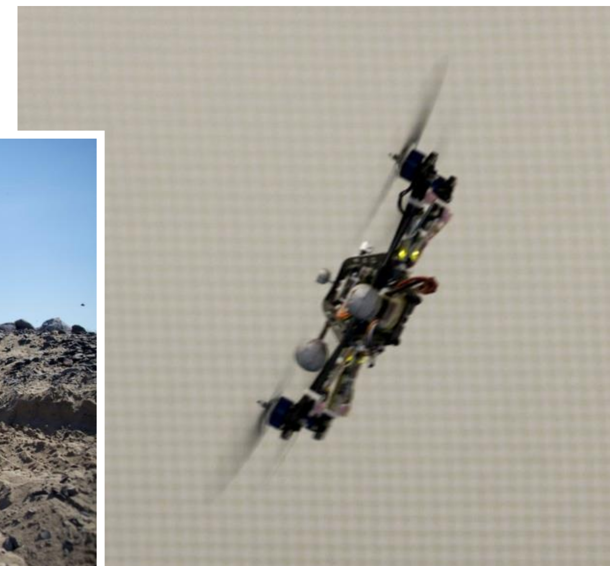
VIDEO: https://youtu.be/zHTCsSkmADo



VIDEO: https://youtu.be/08_d1HSPADA

EXPLANATION

- Unmodelled dynamics
- Unknown external disturbances (e.g., environment conditions such as wind, unknown payload, topography or weather)



Model inaccuracies limit achievable performance!

CLAIM

Learning/adaptation enables safe, high-performance motions in uncontrolled, unknown or changing environments.

RESEARCH FOCUS

DYNAMICS → CONTROLS → MACHINE LEARNING → ROBOTICS

Prior information

- Which motions are feasible?
- How to plan collision-free motions?

+ Current sensor measurement

- How to guide the vehicle along a desired path?

+ Past experiment data

- Can the performance be improved by leveraging past data?

Towards robotics applications.

VIDEO: https://youtu.be/bWExDW9J9sA?list=PLC12E387419CEAFF2

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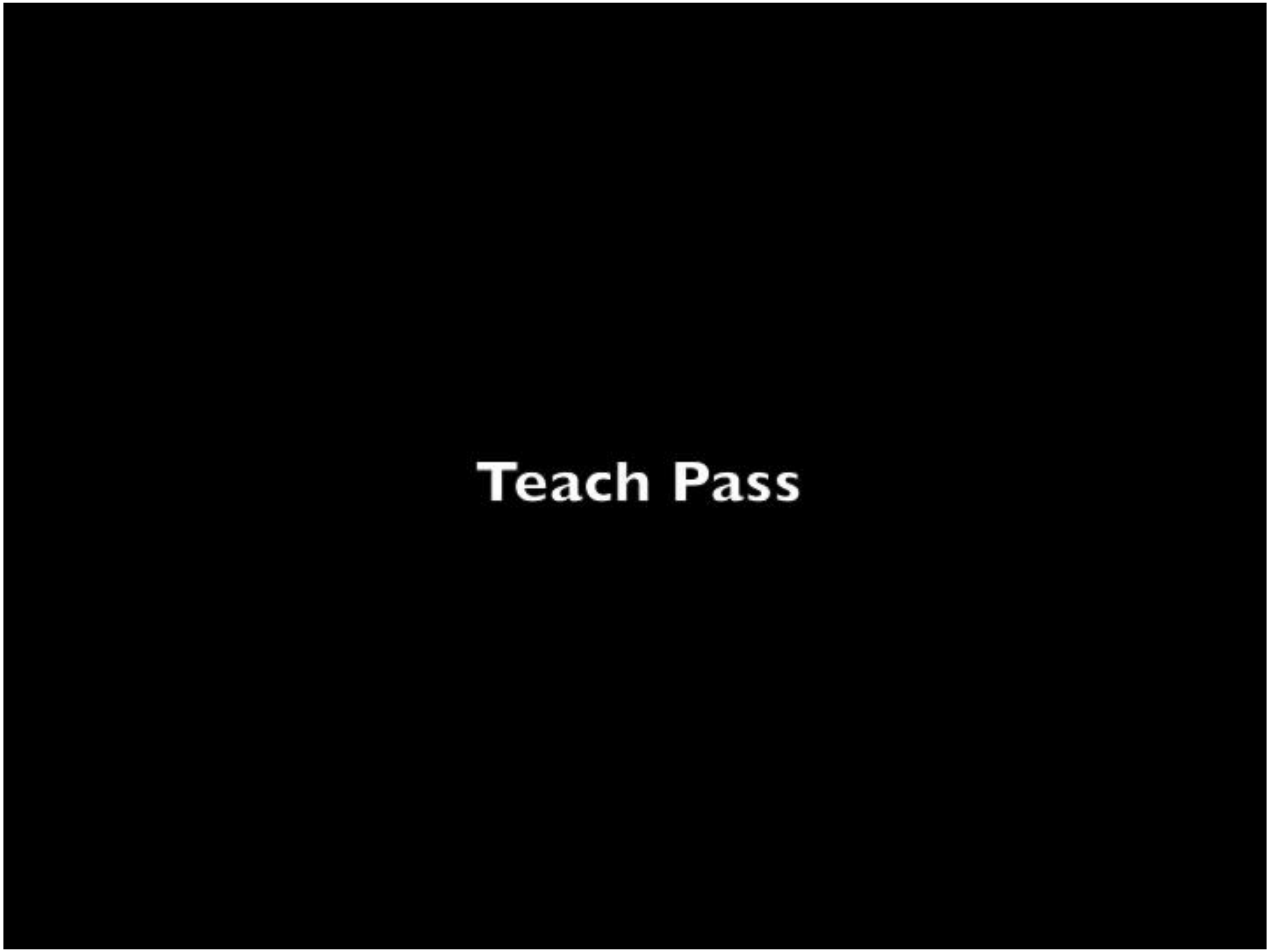
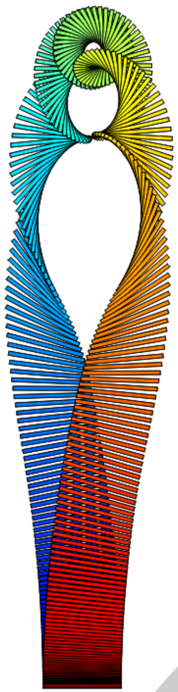
VIDEO: https://youtu.be/zHTCsSkmADo

VIDEO: https://youtu.be/08_d1HSPADA

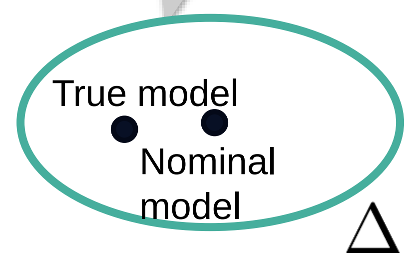


VIDEO: https://youtu.be/YqhLnCm0KXY

DEVELOPMENT



- Specific task, adaptation of a few input parameters only
- General task, full input trajectory adaptation
- Model learning, anytime learning.
- Learning with safety guarantees.



CONCLUSION

Safe, reliable, high-performance flight of single or multiple aerial vehicles in changing and unknown conditions
⇨ using a combination of a-priori models and data.

CLASS SUMMARY

- Quadrotor dynamics are nonlinear and coupled
- Four independent degrees of freedom
- Controller approach:
  - Nested controller
  - "Transform" individual loops into double integrators
  - Design the closed-loop behavior to resemble a second-order system behavior
  - Two intuitive parameters to choose for each second-order system
- There are many more advanced controllers including adaptive and learning controls

THANK YOU

DYNAMIC SYSTEMS LAB

For follow-up discussions, please contact me:

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