Proceedings of the 42nd IEEE
Conference on Decision and Control
Maui, Hawaii USA, December 2003

Comparison of PID Control and PPI Control

Zhengyun Ren[1] Hong Zhang[2] Huihe Shao[1]

1. Department of Automation, Shanghai Jiaotong University, Shanghai, China
2. School of Chemical Engineering, Shanghai University of Engineering Technology, China

Abstract

Predictive PI (PPI) control form, capable of time delay compensation, has been put forward recently. This control algorithm is essentially a PI controller with enhanced derivative action, and is not only suitable for long time delay process, but also of simple structure and excellent robust stability. The performance of PPI controller is demonstrated and compared with traditional PID controller by different tuning methods.

Keywords: PID; predictive PI; robust stability; time delay; discrete-time linear system; characteristic polynomial

1. Introduction

The PID controller is by far the most dominating form of feedback in use today. More than 90% of all control loops are PID in industry. The PID controller is used for a wide range of problems: process control, motor drives, automotive, flight control, instrumentation, etc [1]. The controller comes in many different forms, as standard single-loop controllers, as a software component in programmable logic controllers and distributed control systems.

In a few industrial processes, the presence of considerable time delay is a well common phenomenon [2]. The performance of conventional PID controllers can be severely degraded if a process has a relative large time delay compared to the dominant time constant, and PID controllers can only be detuned to retain closed loop stability resulting in sluggish performance [3].

The widespread use and success of MPC applications attests to the improved performance of MPC compared to PID for control of difficult process dynamics. However, even with these outstanding results MPC still is not used in many complex processes with difficult dynamics primarily because hardware and commissioning costs of MPC require significant economic justification. Furthermore, long-term maintenance of MPC requires a higher level of expertise and cost compared to PID maintenance.

Recently, a controller named predictive proportional plus integral (PPI) has been introduced [4]. The PPI controller is a compromise between MPC and PID, and is based on a first-order plus dead-time model of the real industrial process, which is representative of the majority of process dynamics in industry. This controller consists of two parts: a standard PI controller and a predictive term with which its dynamics depend on the system time delay. The main advantage of the PPI algorithm developed in this paper is that it is cost effective yet retains most of the advantages of MPC for single loop control. PPI preserves the same PID ideology that is familiar to operating personnel while the closed loop performance is that of MPC.

For process with time delay, the robust stability analysis of both PID control system and PPI control system is problem of considerable theoretical and practical significance that have been attracting the interest of a number of researchers [5]. In this paper, the problem of robust stability analysis for these control systems in the presence of parametric uncertainty is addressed. Disctetizing control systems into state space forms and using the state augmentation approach [6], close-loop systems are transformed to state-augmented, delay-free discrete-time systems. A simple method of computing characteristic polynomials is derived using the Laplace expansion, and the analysis of robust stability is also the analysis of the Schur stability of their characteristic polynomials. By bilinear transform, for arbitrary given parameter interval the Hurwitz stability of this kind of polynomial can be decided by edge theorem [7] and Frazer-Duncan theorem [8].

By these analysis methods, we compare robust stability among PID control systems with different tuning methods and PPI control system under process parametric uncertainty. Also, the closed-loop step responses of these control system are simulated and the performance indexes are calculated. The result shows that the performance of PPI control algorithm is supper to those of PID control algorithms, and some benifficent conclusions for chosing PID tuning methods are also derived.

2. The structure and state space form of PID control system

In our case a linear PID controller is used for control. The structure of this controller given in the form

$$G_C = K(1+\frac{1}{T_I s}+T_D s) \tag{2.1}$$

where the control parameters are the gain K, the integration time T_I and the derivation time T_D.

Consider a single-input single-output (SISO) uncertainty process with the transfer function

$$G_p = \frac{K_p}{Ts+1}e^{-Ls} \tag{2.2}$$

The time-discrete linear process counterpart of (2.2) is

$$G(z) = \frac{b}{z+a}z^{-h} \tag{2.3}$$

Let $e = y_{sp} - y$ marks the error between the reference signal y_{sp} and the actual process output y. The equivalent time domain difference equation of (2.3) is given as

$$e(k+1) = -ae(k) - bu(k-h) + (y_{sp}(k+1)+ay_{sp}(k)) \tag{2.4}$$

The relation of parameters of (2.2) and (2.3) can be expressed by the following equations

$$a = -e^{-\frac{T_s}{T}}, b = K_p(1-e^{-\frac{T_s}{T}}), h = \frac{L}{T_s} \tag{2.5}$$

where T_s is the sampling interval.

Discretization of (2.1) results in the following discrete control law

$$u(k+1) = K(1+T_D/T_s+T_s/T)e(k+1) + (KT_s/T_I)\sum_{i=0}^{k}e(i) - K(T_D/T_s)e(k-1) \tag{2.6}$$

Let

$$\theta(k+1) = \sum_{i=0}^{k}e(i) \tag{2.7}$$

then

$$\theta(k+1) = e(k) + \theta(k) \tag{2.8}$$

$$u(k+1) = K(1+T_D/T_s+T_s/T)e(k+1) + (KT_s/T_I)\theta(k+1) - K(T_D/T_s)e(k-1) \tag{2.9}$$

Formulating (2.4), (2.8) and (2.9) into a state space form, we have

$$\begin{bmatrix} e(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} -a & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e(k) \\ \theta(k) \end{bmatrix} - \begin{bmatrix} bK(1+T_D/T_s+T_s/T) & bKT_s/T_I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e(k-h) \\ \theta(k-h) \end{bmatrix} + \begin{bmatrix} bKT_D/T_s & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e(k-h-2) \\ \theta(k-h-2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}w(k)$$

$$y(k+1) = y_{sp}(k+1) - e(k+1) \tag{2.10}$$

where $w(k) = y_{sp}(k+1) + ay_{sp}(k)$.

3. The structure and state space form of PPI control system

Assume that the desired closed-loop transfer function for process (2.2) is specified as follow

$$G_0 = \frac{e^{-L_0 s}}{\lambda T_0 s + 1} \quad (3.1)$$

where λ is a changeable parameter [4]. So the controller transfer function becomes

$$G_C = \frac{T_0 s + 1}{K_{P0}(\lambda T_0 s + 1 - e^{-L_0 s})} \quad (3.2)$$

The input-output relation of the controller is

$$u(s) = \frac{1}{\lambda K_{P0}}(1 + \frac{1}{T_0 s})e(s) - \frac{1}{\lambda T_0 s}(1 - e^{-L_0 s})u(s) \quad (3.3)$$

The term $1/\lambda K_{P0}(1 + 1/T_0 s)e(s)$ is of the form of a PI controller, $-(1/\lambda T_0 s)(1 - e^{-L_0 s})u(s)$ term can be interpreted as a prediction of the process output at time t based on the values of the control signal in the time interval $(t - L_0, t)$. The controller is, therefore, called a predictive PI controller (PPI).

Discretizing (3.3), we get

$$u(k+1) = \frac{1}{\lambda K_{p0}} e(k+1) + \frac{T_s}{\lambda K_{p0} T_0} \sum_{i=0}^{k+1} e(i) - \frac{T_s}{\lambda T_0} \sum_{i=0}^{k} (u(i) - u(i - \frac{L_0}{T_s})) \quad (3.4)$$

Let $h, L_0 / T_s$ are integer by choosing a suitable T_s.

Suppose $u(i) = 0, i < 0$, then

$$u(k+1) = (\frac{1}{\lambda K_{p0}} + \frac{T_s}{\lambda K_{p0} T_0}) e(k+1) + \frac{T_s}{\lambda K_{p0} T_0} \sum_{i=0}^{k} e(i) - \frac{T_s}{\lambda T_0} \sum_{i=k-\frac{L_0}{T_s}+1}^{k} u(i) \quad (3.5)$$

Let

$$\theta(k+1) = \sum_{i=0}^{k} e(i), s(k+1) = \sum_{i=k-\frac{L_0}{T_s}+1}^{k} u(i) \quad (3.6)$$

then

$$\theta(k+1) = e(k) + \theta(k) \quad (3.7)$$

$$s(k+1) = s(k) + u(k) - u(k - \frac{L_0}{T_s}) \quad (3.8)$$

$$u(k+1) = (\frac{1}{\lambda K_{p0}} + \frac{T_s}{\lambda K_{p0} T_0}) e(k+1) + \frac{T_s}{\lambda K_{p0} T_0} \theta(k+1) - \frac{T_s}{\lambda T_0} s(k+1) \quad (3.9)$$

Formulating (2.4), (3.7), (3.8) and (3.9) into a state space form, we have

$$\begin{bmatrix} e(k+1) \\ \theta(k+1) \\ s(k+1) \end{bmatrix} = \begin{bmatrix} -a & 0 & 0 \\ 1 & 1 & 0 \\ 1/\lambda K_{p0} + T_s/\lambda K_{p0} T & T_s/\lambda K_{p0} T & 1 - T_s/\lambda T \end{bmatrix} \begin{bmatrix} e(k) \\ \theta(k) \\ s(k) \end{bmatrix}$$
$$- \frac{b}{\lambda K_{p0}} \begin{bmatrix} 1 + T_s/T_0 & T_s/T_0 & -K_{p0} T_s/T_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e(k-h) \\ \theta(k-h) \\ s(k-h) \end{bmatrix}$$
$$- \frac{1}{\lambda K_{p0}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 + T_s/T_0 & T_s/T_0 & -K_{p0} T_s/T_0 \end{bmatrix} \begin{bmatrix} e(k - L_0/T_s) \\ \theta(k - L_0/T) \\ s(k - L_0/T) \end{bmatrix}$$
$$+ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(k)$$

$$y(k+1) = y_{sp}(k+1) - e(k+1) \quad (3.10)$$

where $w(k) = y_{sp}(k+1) + a y_{sp}(k)$.

4. Robust stability analysis of PID and PPI control system

In general, the analysis of control system robust stability is very difficult under all process parameters varying simultaneously, especially for process with time delay. For control system (2.10) and (3.10), we analyze their robust stability under process parameter K_p, T, L uncertainty by a different way, by which we can judge (2.10) and (3.10) stability for arbitrary given intervals of K_p, T, L.

The characteristic polynomials of PID control system (2.10) and PPI control system (3.10) are derived respectively as

$$\begin{aligned}P_{PID}(z) &= z^{(2h+6)} + (a-1)z^{(2h+5)} - az^{(2h+4)} \\ &\quad + (Kb + T_D^{'}b)z^{(h+5)} + (T_I^{'} - T_D^{'} - K)bz^{(h+4)} \\ &\quad - T_D^{'}bz^{(h+3)} + T_D^{'}bz^{(h+2)} \\ &= z^{h+2}\{z^{h+4} + (a-1)z^{h+3} - az^{h+2} + \\ &\quad (Kb + T_D^{'}b)z^3 + (T_I^{'} - T_D^{'} - K)bz^2 \\ &\quad - T_D^{'}bz + T_D^{'}b\}\end{aligned} \tag{4.1}$$

$$\begin{aligned}P_{PPI}(z) &= z^{(3n+3)} + (a+Q-2)z^{(3n+2)} + \\ &\quad (1+Qa-2a-Q)z^{(3n+1)} + (a-Qa)z^{3n} + Mbz^{(3n-h+2)} \\ &\quad +(Nb-2Mb)z^{(3n-h+1)} + (Mb-Nb)z^{(3n-h)} - \\ &\quad Qz^{(3n-l+2)} + (Q-Qa)z^{(3n-l+1)} + Qaz^{(3n-l)} \\ &= (z-1)z^{2n}\{z^{(n+2)} + (a+Q-1)z^{(n+1)} \\ &\quad +(Qa-a)z^n + Mbz^{(n-h+1)} + (Nb-Mb)z^{(n-h)} \\ &\quad -Qz^{(n-l+1)} - Qaz^{(n-l)}\}\end{aligned} \tag{4.2}$$

where $T_I^{'} = KT_s / T_I, T_D^{'} = KT_D / T_s, M = 1/(\lambda K_{p0})$, $N = T_s/(\lambda K_{p0}T_0), Q = T_s/(\lambda T_0), l = L_0/T_s, n$ is the maximum of h, l.

Via simplification and bilinear transform, the analyses of Schur stability of $P_{PID}(z)$ and $P_{PPI}(z)$ are also the analyses of Hurwitz stability of the following polynomials correspondingly

$$\begin{aligned}P_{PID}(s) &= (s+1)^{h+4} + (a-1)(s+1)^{h+3}(s-1) - \\ &\quad a(s+1)^{h+2}(s-1)^2 + (Kb + T_D^{'}b)(s+1)^3(s-1)^{h+1} + \\ &\quad (T_I^{'} - T_D^{'} - K)b(s+1)^2(s-1)^{h+2} - T_D^{'}b(s+1)(s-1)^{h+3} \\ &\quad +T_D^{'}b(s-1)^{h+4}\end{aligned} \tag{4.3}$$

$$\begin{aligned}P_{PPI}(s) &= (s+1)^{n+2} + (a+Q-1)(s+1)^{n+1}(s-1) + \\ &\quad (Qa-a)(s+1)^n(s-1)^2 + Mb(s+1)^{n-h+1}(s-1)^{h+1} + \\ &\quad (Nb-Mb)(s+1)^{n-h}(s-1)^{h+2} - Q(s+1)^{n-l+1}(s-1)^{l+1} \\ &\quad -Qa(s+1)^{n-l}(s-1)^{l+2}\end{aligned} \tag{4.4}$$

Due to all coefficients of polynomials $P_{PID}(s)$ and $P_{PPI}(s)$ be affine functions of a, b, edge theorem [7] can be applied to analyze their Hurwitz stability.

5. Example

In this section, we compare robust stability among PID control systems with different tuning methods and PPI control system. The PID tuning methods are Cohen & Coon method [9], Clark method [10] and IMC method [11].

Suppose the nominal values of uncertainty process (2.2) are $K_{p0} = 13, T_0 = 12, L_0 = 32$, which the design of controller parameters depends on, and let $T_s = 1$, $\lambda = 1, \xi = 1$ (for Clarke method), $\xi = 12$ (for IMC method), we can get polynomial $P_{PID}(s), P_{PPI}(s)$ by (4.3) and (4.4) respectively.

In order to get stability parameters a, b, h

intervals for different control system, the following procedures are carried out:

1. Given a suitable interval of a and fixed h on a point around nominal value.
2. Set the lower limit of b to 0 and the upper limit of b to a proper value.
3. Four edge polynomials of polynomial family $P_{PID}(s)$ or $P_{PPI}(s)$ are derived, and the stability of each edge polynomial is judged by Frazer-Duncan theorem [8].
4. If all edge polynomials are stable, polynomial family $P_{PID}(s)$ or $P_{PPI}(s)$ are stable by edge theorem.
5. Increasing the upper limit of b until $P_{PID}(s)$ or $P_{PPI}(s)$ are unstable. This limit is the upper limit of stability interval of b.
6. Changing h and repeating 2-5, we get another stability interval of b.

By this way, stability a,b,h intervals are derived, i.e. stability K_p,T,L intervals are derived. The process parametric stable spaces of several PID control systems and PPI control system are plotted in Fig.1-Fig.4 respectively. The volumes of their stable spaces can be found in Table 1.

Step responses of these control systems under nominal process are simulated (Fig.5) and their performance indexes 'integral-square-error' (ISE) are summarized in Table 1 also.

Though the volume of Clarke method is the largest one, its step response is very slow and its ISE is the largest one among these control methods. Response curves of both Cohen&Coon method and IMC method fluctuate around setpoint severely. In general, PID controllers tuning by Cohen&Coon method, Clarke method and IMC method are not suitable for large time delay process. In spite of smaller volume of stable parameter space, PPI control system remain stable even when process parameters K_p,T,L deviate from nominal value as much as 150% or as less as 50% simultaneously. In the same time, step response in Fig.5 and ISE in Table 1 indicate PPI control system is of perfect dynamic performance. So we draw a conclusion PPI controller is an idea tool for large time delay process.

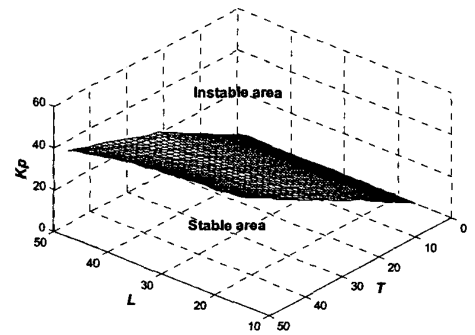

Fig. 1. Stable parameter space of K_p,T,L under PID control by Cohen&Coon tuning method

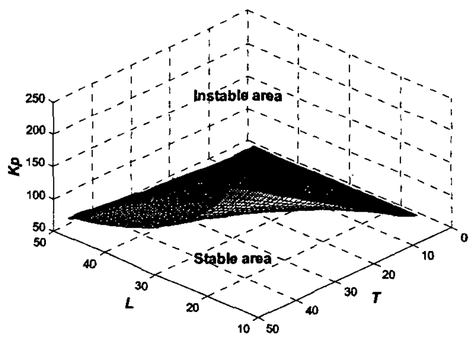

Fig. 2. Stable parameter space of K_p,T,L under PID control by Clarke tuning method

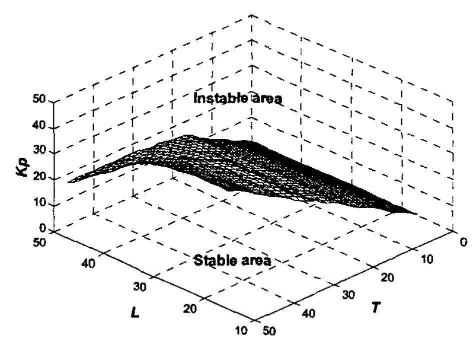

Fig. 3. Stable parameter space of K_p,T,L under PID control by IMC tuning method

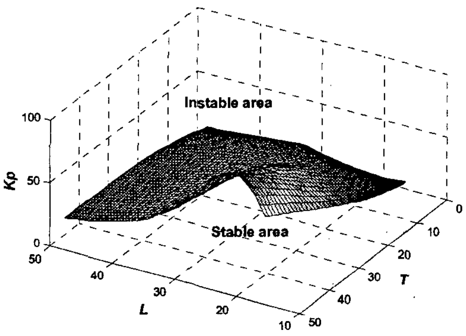

Fig. 4. Stable parameter space of K_p, T, L under PPI control

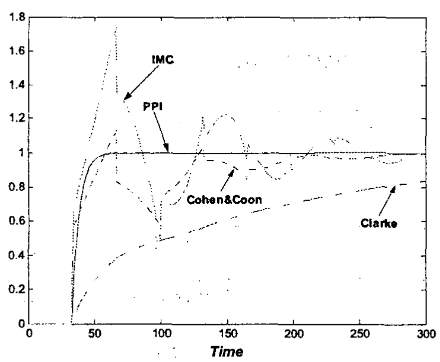

Fig. 5. Step response of different control systems

Table 1 Volume of stable parameter space and ISE of step response of different control system

Method	Volume	ISE
Cohen & Coon PID	43332	39.72
Clark PI	149778	85.55
IMC PID	36570	44.18
PPI	37032	36.27

6. Conclusion

The structures and state space forms of both PID controller and PPI controller are given firstly in this paper. Applying edge theorem and Frazer-Duncan theorem, we analyze polynomial family robust stability of these control system under process parameter uncertainty. The robust stability and performance of PPI controller are demonstrated and compared with traditional PID controller by different tuning methods, which shows PPI controller is of good robust stability and control performance.

Acknowledgements. This research was supported by "863" High-Tech Research & Development Program of China, Grant No. 2001-AA413130, and the National Key Research Project, Grant No. 2001-BA201A04.

References

[1] Åström K. and Hägglund T. The future of PID control, *Control Engineering Practice*, 2001, 9, 1163.

[2] Gorecki H., Fuska S., Garbowski P. and Korytowski A. *Analysis and Synthesis of Time-delay Systems*, New York: J. Wiley, 1989, 13

[3] Miller R., Shah S., Wood R.and Kwok E. Predictive PID. *ISA Transactions*, 1999, 38, 11.

[4] Åström K. and Hägglund T. *PID Controllers: Theory, Design, and Tuning*, N. C: Research Triangle Park, 1995, 156

[5] Rad A. B. and Lo W. L. Predictive PI controller. *International Journal of Control*, 1994, 60, 953

[6] Åström K. and Wittenmark B. *Computer Controlled Systems: Theory and Design.* NJ: Prentice-Hall Inc., Englewood Cliffs, 1984, 238

[7] Bartlett A., Hollot C. and Huang L. Root locations of an entire polytop of polynomials: it suffices to check the edges. *Mathematics of Control, Signals and Systems*, 1988, 1, 61

[8] Frazer R. and Duncan W. On the criteria for the stability of small motions. *Proc. Royal Society A*, 1929, 124, 642

[9] Cohen G. and Coon G. Theoretical considerations of retard control. *Transactions of the ASME*, 1953, 75, 827.

[10] Clark D. Automatic tuning of PID regulators. *Expert Systems and Optimization in Process Control*, 1986, 85.

[11] Riviera D., Morari M. and Skogestad S. Internal model control. 4 PID controller design. *Industrial and Engineering Chemistry, Process Design and Development*, 1986, *25*, 252.