GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 6: REGRESSION & REGULARIZATION

LAST TIME:

- INTRO TO MACHINE LEARNING
- SUPERVISED LEARNING

QUESTIONS?

I. REVIEW SUPERVISED LEARNING

II. LINEAR REGRESSION

III. REGULARIZATION

I. SUPERVISED LEARNING

Q: How does a classification problem work?

A: Data in, predicted labels out.

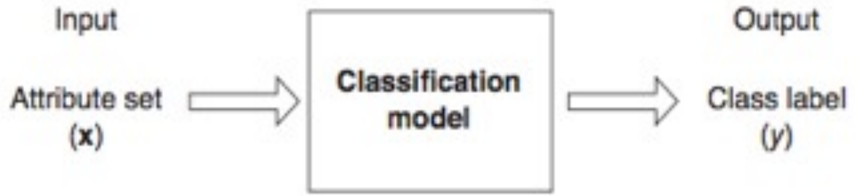
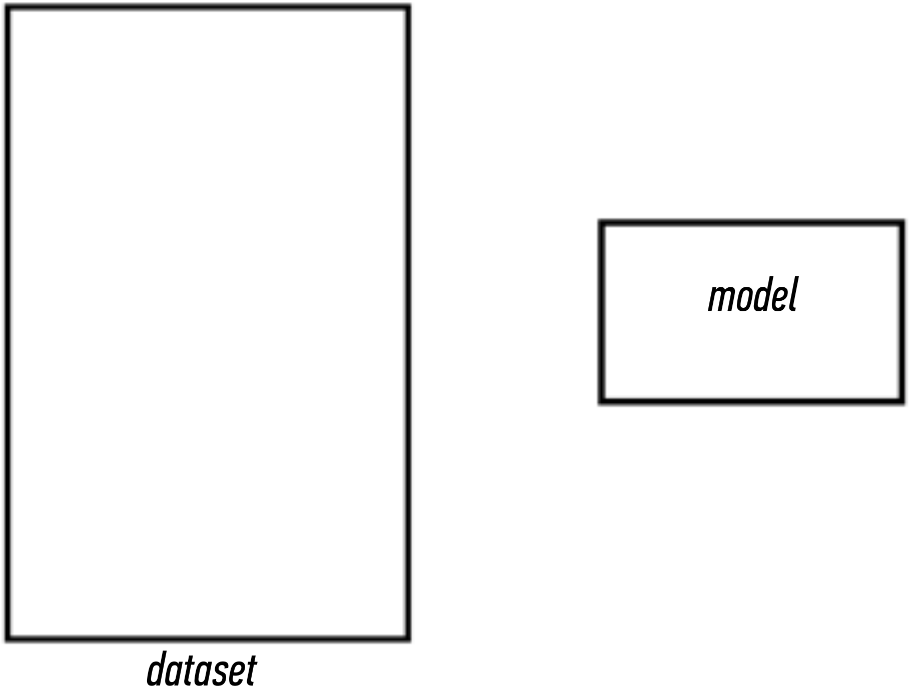


Figure 4.2. Classification as the task of mapping an input attribute set $\mathbf{x}$ into its class label y.

source: http://www-users.cs.umn.edu/~kumar/dmbook/ch4.pdf

Q: What steps does a classification problem require?



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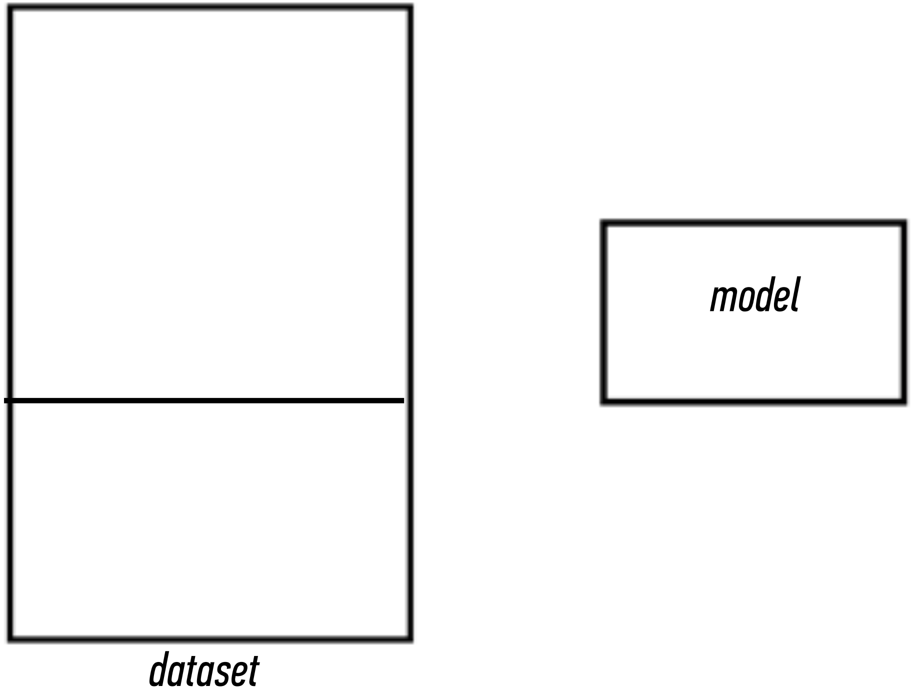
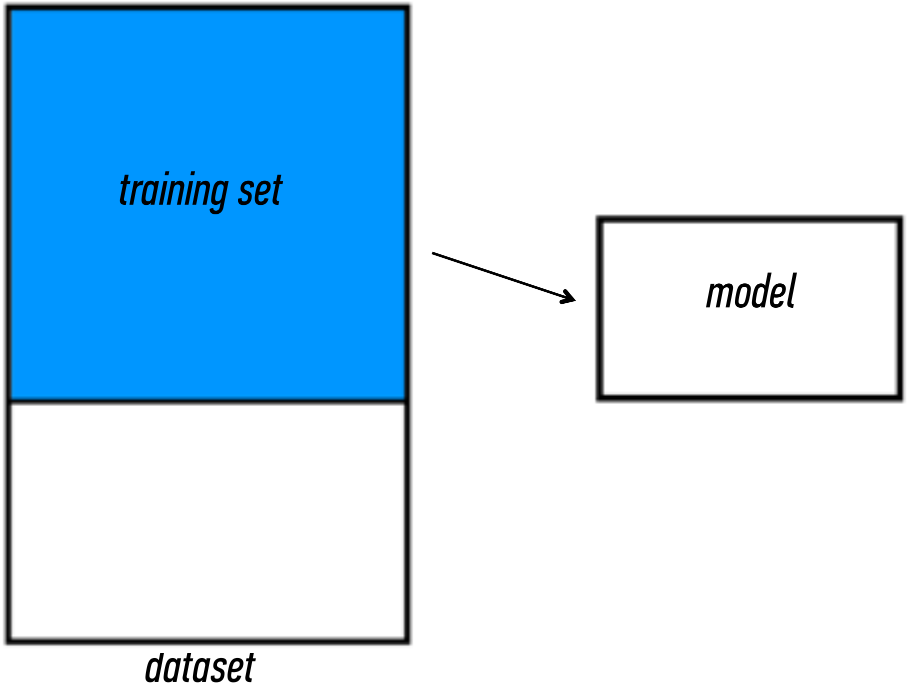
1) split dataset



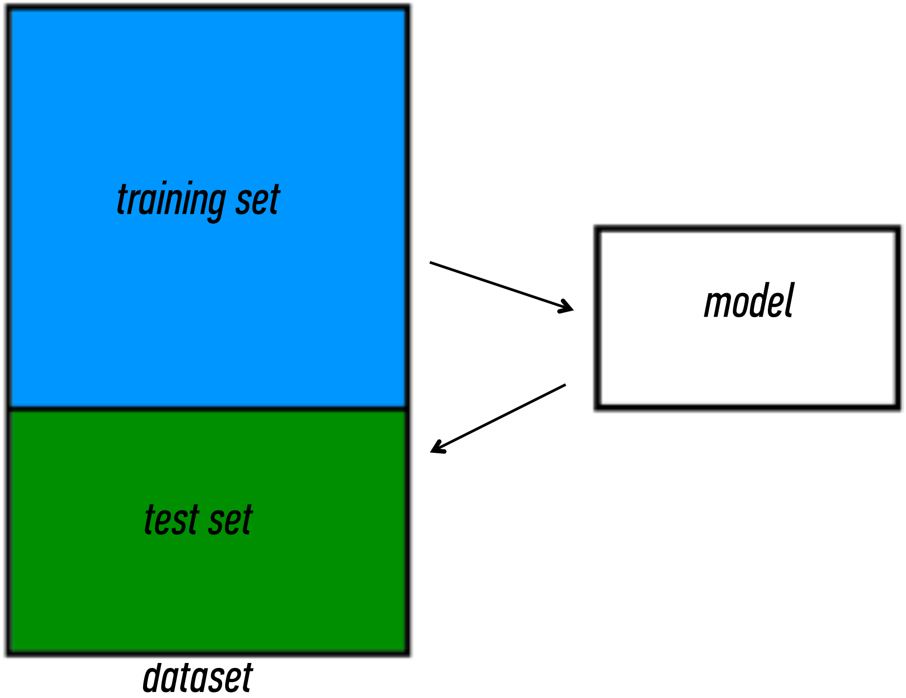
Q: What steps does a classification problem require?

1) *split dataset*
2) *train model*



Q: What steps does a classification problem require?

1) split dataset
2) train model
3) test model



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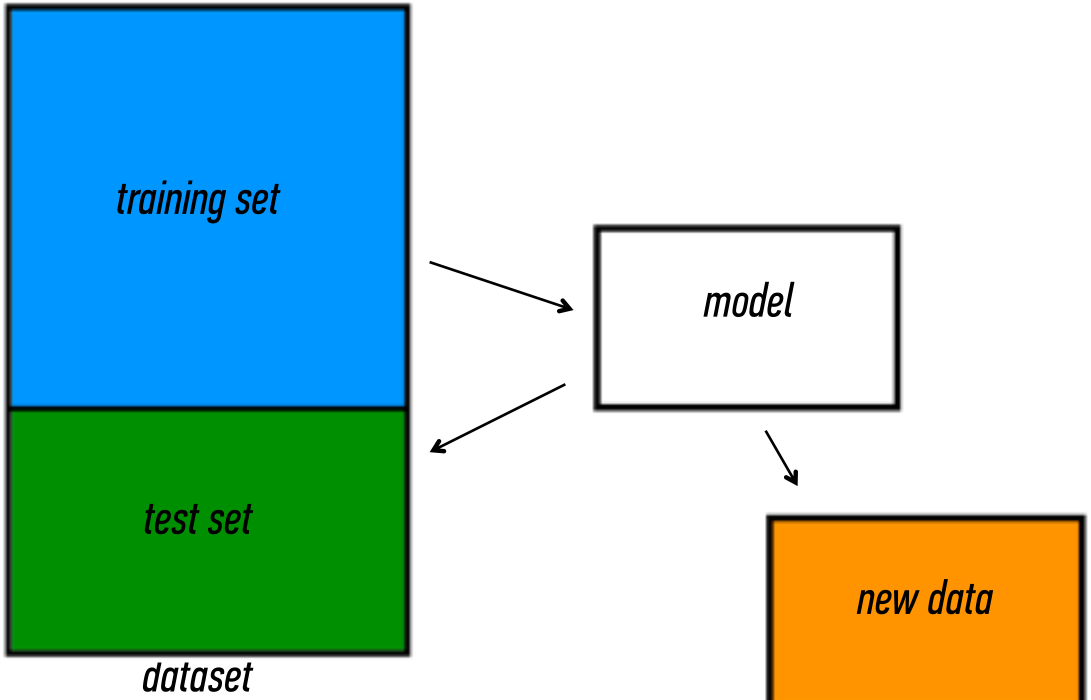
1) split dataset
2) train model
3) test model
4) make predictions



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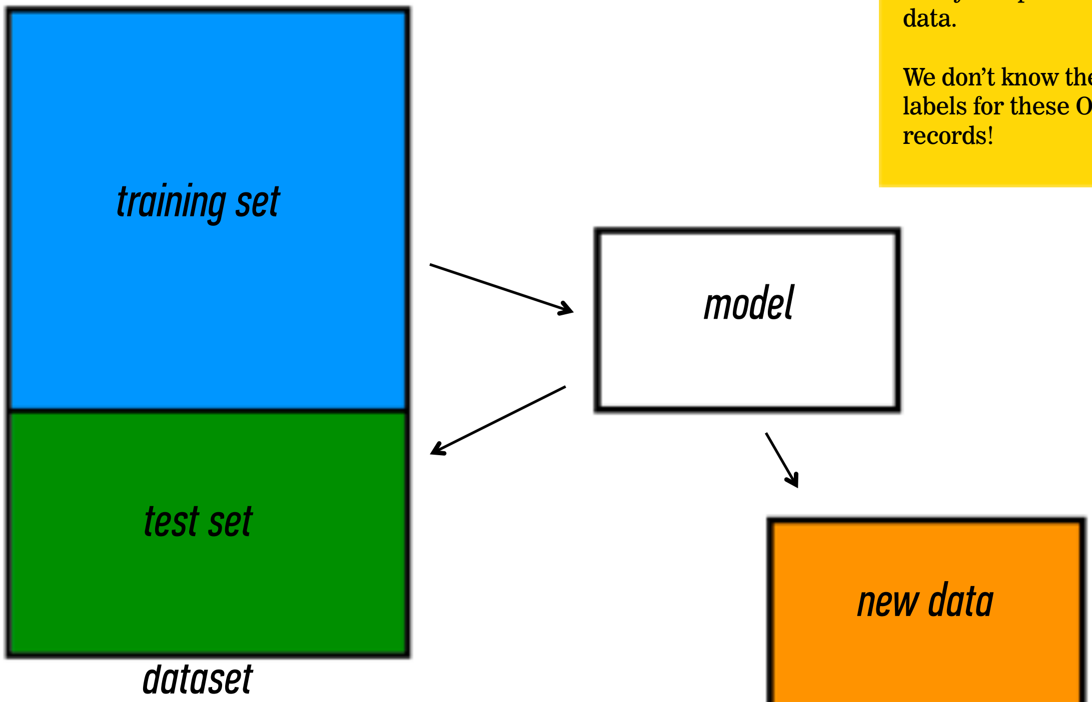
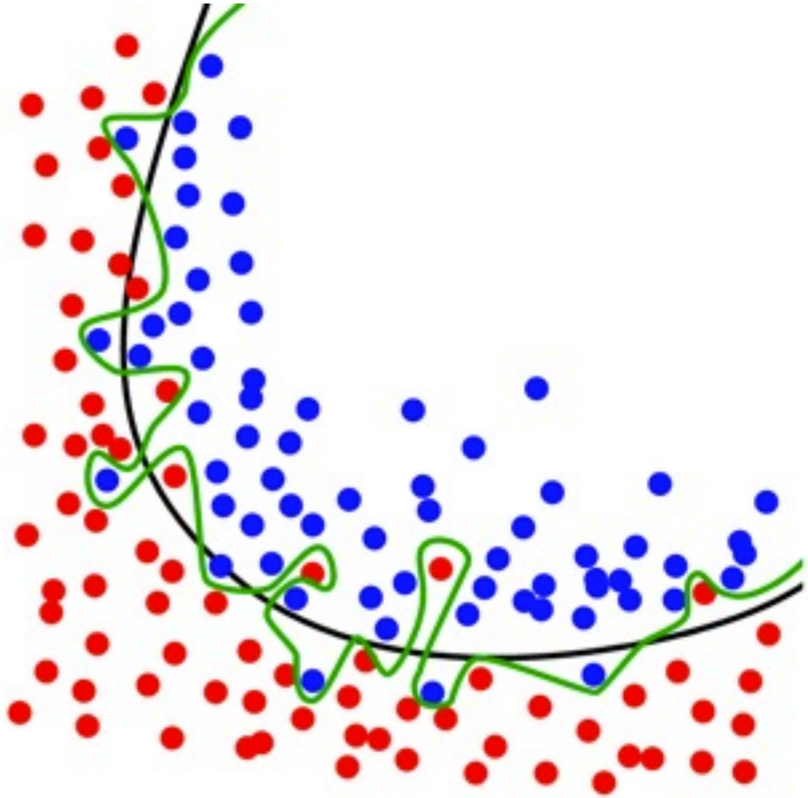


NOTE

This new data is called *out of sample* data.

We don't know the labels for these OOS records!

OVERFITTING – EXAMPLE



source: http://www.dtreg.com

II. LINEAR REGRESSION

	continuous	*categorical*
supervised	???	???
unsupervised	???	???

	continuous	categorical
supervised	regression	classification
unsupervised	dimension reduction	clustering

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$$y = \alpha + \beta x + \varepsilon$$

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β = **regression coefficient** *(the model "parameter")*

ε = **residual** *(the prediction error)*

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$$y = \alpha + \beta_1 x_1 + \ldots + \beta_n x_n + \varepsilon$$

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

The math is not very important for our purposes, but you should check it out if you get serious about solving regression problems.

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But again, if you get serious about regression, you should learn how this works!

V. POLYNOMIAL REGRESSION

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$$y = \alpha + \beta_1 x + \beta_2 x^2 + \varepsilon$$

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A: Yes, because it's linear in the β*'s!*

Polynomial regression allows us to fit very complex curves to data.

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Q: Does anyone know what it is?

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But there is one problem with the model we've written down so far.

Q: Does anyone know what it is?

A: This model violates one of the assumptions of linear regression!

This model displays **multicollinearity***, which means the predictor variables are highly correlated with each other.*

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \ldots + \beta_n x^n + \varepsilon$$

```
> x <- seq(1, 10, 0.1)
> cor(x^9, x^10)
[1] 0.9987608
```

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$$y = \alpha + \beta_1 x + \beta_2 x^2 + \ldots + \beta_n x^n + \varepsilon$$

Multicollinearity causes the linear regression model to break down, because it can't tell the predictor variables apart.

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A: Replace the correlated predictors with uncorrelated predictors.

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$$y = \alpha + \beta_1 f_1(x) + \beta_2 f_2(x^2) + \ldots + \beta_n f_n(x^n) + \varepsilon$$

So far, we've seen how polynomial regression allows us to fit complex nonlinear relationships, and even to avoid multicollinearity (by using basis functions).

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Q: Can a regression model be too complex?

V. REGULARIZATION

Recall our earlier discussion of **overfitting**.

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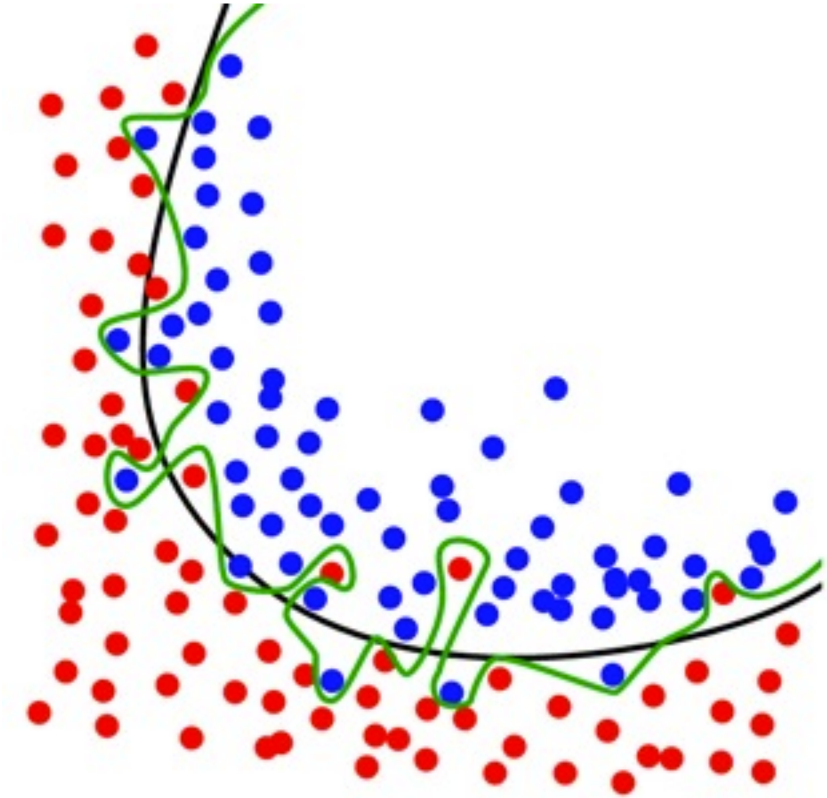
In other words, an overfit model matches the **noise** *in the dataset instead of the* **signal***.*

OVERFITTING EXAMPLE (CLASSIFICATION)

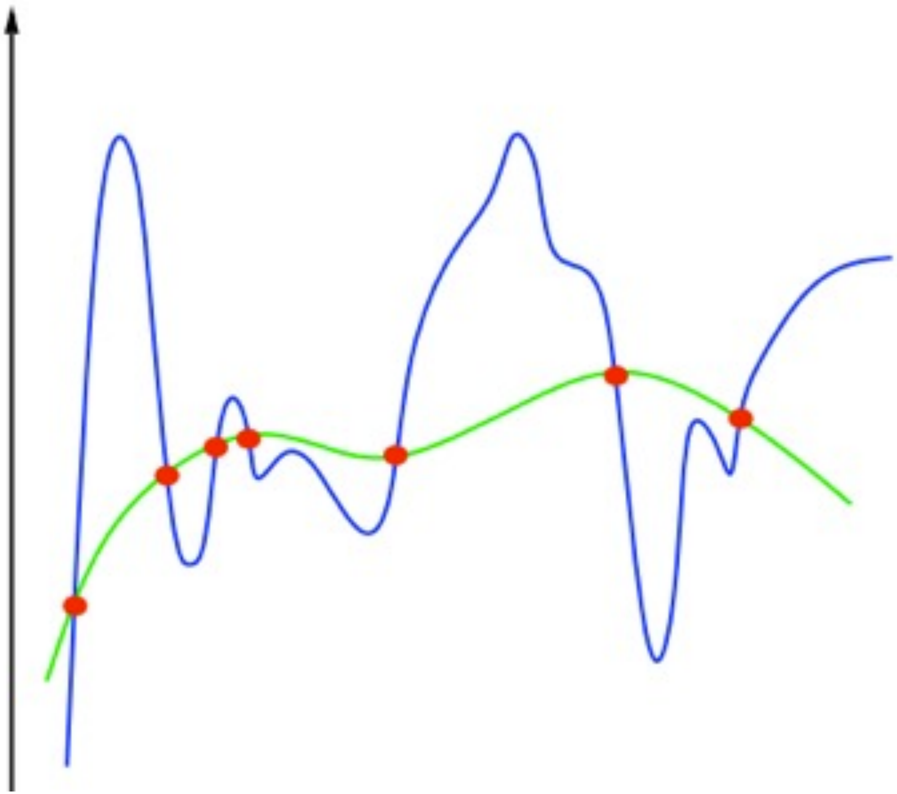


source: http://upload.wikimedia.org/wikipedia/commons/1/19/Overfitting.svg

The same thing can happen in regression.

It's possible to design a regression model that matches the noise in the data instead of the signal.

This happens when our model becomes too complex for the data to support.



source: http://www.mit.edu/~9.520/spring12/slides/class02/class02.pdf

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Ex 1: $\Sigma\,|\beta_i|$

Ex 2: $\Sigma\,\beta_i^2$

Q: How do we define the **complexity** *of a regression model?*

A: One method is to define complexity as a function of the size of the coefficients.

Ex 1: $\Sigma |\beta_i|$ *this is called the* **L1-norm**

Ex 2: $\Sigma \beta_i^2$ *this is called the* **L2-norm**

These measures of complexity lead to the following **regularization** *techniques:*

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Lasso **regularization***:* $y = \Sigma\, \beta_i x_i + \varepsilon \quad st. \quad \Sigma\, |\beta_i| < s$

Ridge **regularization***:* $y = \Sigma\, \beta_i x_i + \varepsilon \quad st. \quad \Sigma\, \beta_i^2 < s$

Regularization *refers to the method of preventing* **overfitting** *by explicitly controlling model* **complexity***.*

These regularization problems can also be expressed as:

L1 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|)$

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OLS: $min(\|y - x\beta\|^2)$

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We are no longer just minimizing error but also an additional term.

Q: What problems have we seen?

A:

1) *Correlated predictor variables*
2) *Large number of parameters allow us to overfit*

Q: What can we do about this?

A: If prediction is our only goal – nothing.

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A: If prediction is our only goal – nothing.

Otherwise,

1) Drop correlated predictors

2) Get more data