GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 13: DIMENSIONALITY REDUCTION

I. DIMENSIONALITY REDUCTION
II. PRINCIPAL COMPONENTS ANALYSIS
III. SINGULAR VALUE DECOMPOSITION
IV. OTHER METHODS

EXERCISE:
IV. DIMENSIONALITY REDUCTION IN SCIKIT-LEARN

I. DIMENSIONALITY REDUCTION

Q: What is dimensionality reduction?

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Dimensionality reduction is frequently performed as a pre-processing step before another learning algorithm is applied.

Q: What are the motivations for dimensionality reduction?

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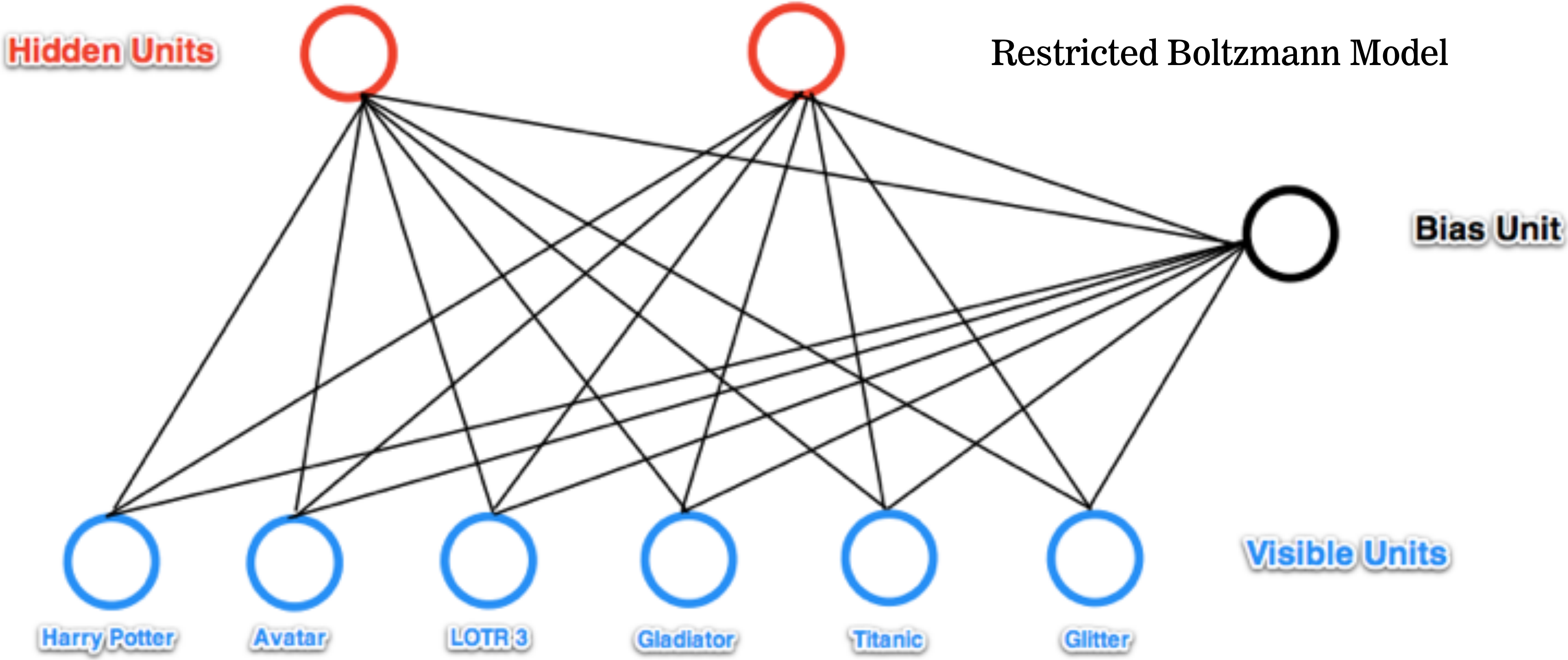
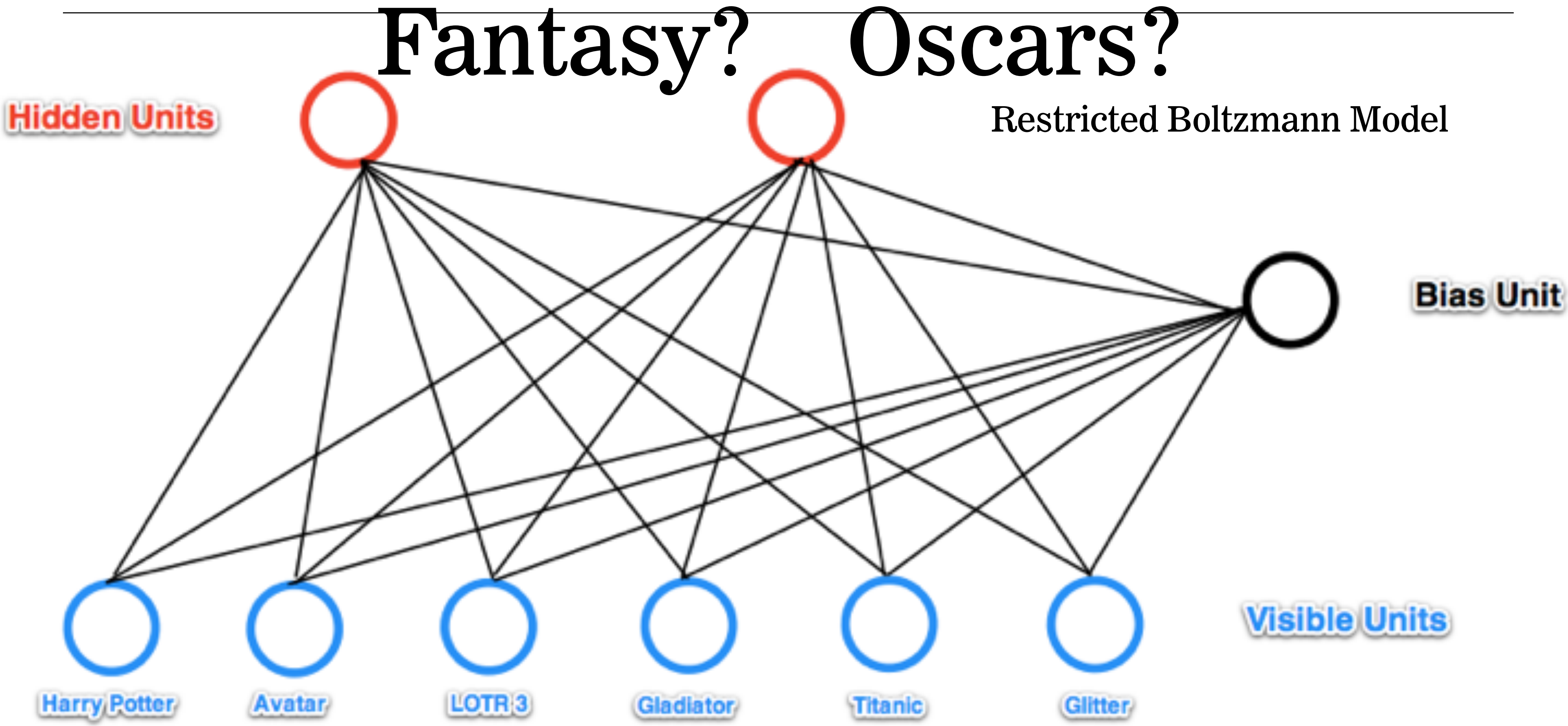
The number of features in our dataset can be difficult to manage, or even misleading (eg, if the relationships are actually simpler than they appear).

Restricted Boltzmann Model




Fantasy?
Oscars?
Hidden Units
Restricted Boltzmann Model
Bias Unit
Visible Units
Harry Potter
Avatar
LOTR 3
Gladiator
Titanic
Glitter

Q: What is the goal of dimensionality reduction?

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- *reduce computational expense*
- *reduce susceptibility to overfitting*
- *reduce noise in the dataset*
- *enhance our intuition*

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feature selection – *selecting a subset of features using an external criterion (filter) or the learning algo accuracy itself (wrapper)*

feature extraction – *mapping the features to a lower dimensional space*

Feature selection is important, but typically when people say dimensionality reduction, they are referring to feature extraction.

The goal of feature extraction is to create a new set of coordinates that simplify the representation of the data.

Q: What are some applications of dimensionality reduction?

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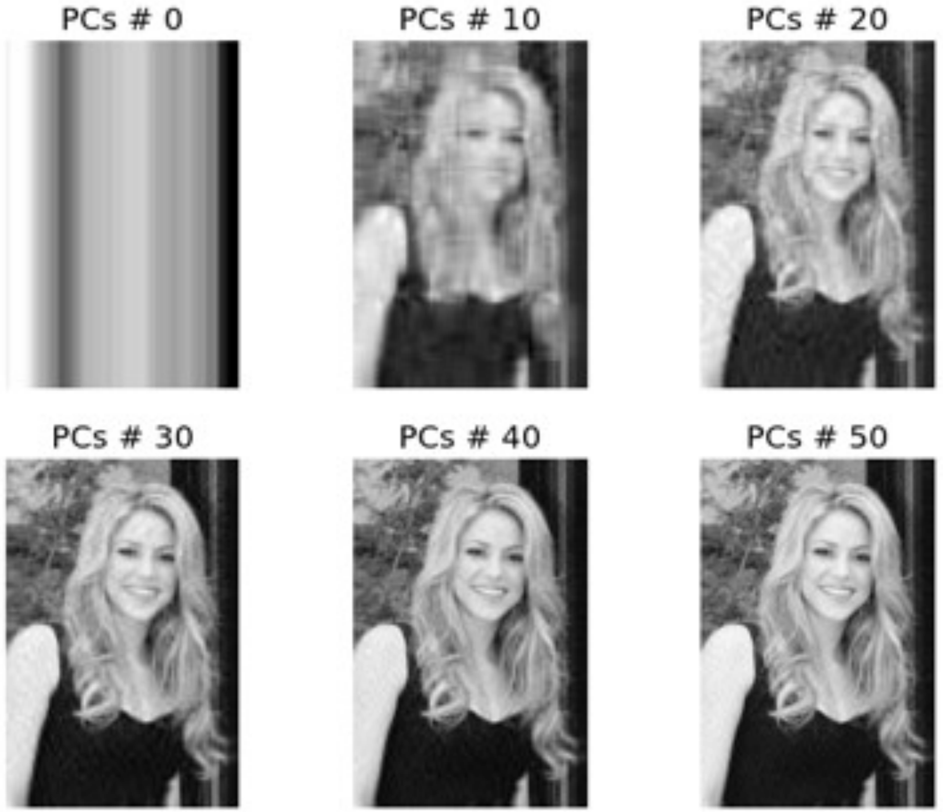
- *topic models (document clustering)*
- *image recognition/computer vision*
- *recommender systems*

II. PRINCIPAL COMPONENT ANALYSIS



source: http://glowingpython.blogspot.it/2011/07/pca-and-image-compression-with-numpy.html

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The PCA of a matrix A boils down to the **eigenvalue decomposition** *of the* **covariance matrix** *of A.*

The covariance matrix C of a matrix A is always square:

$$C = \begin{bmatrix} \mathrm{E}[(X_1-\mu_1)(X_1-\mu_1)] & \mathrm{E}[(X_1-\mu_1)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_1-\mu_1)(X_n-\mu_n)] \\ \mathrm{E}[(X_2-\mu_2)(X_1-\mu_1)] & \mathrm{E}[(X_2-\mu_2)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_2-\mu_2)(X_n-\mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{E}[(X_n-\mu_n)(X_1-\mu_1)] & \mathrm{E}[(X_n-\mu_n)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_n-\mu_n)(X_n-\mu_n)] \end{bmatrix}.$$

off-diagonal elements C_{ij} give the covariance between X_i, X_j $(i \neq j)$

diagonal elements C_{ii} give the variance of X_i

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NOTE

This relationship *defines* what it means to be an eigenvector of A.

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The eigenvectors form a basis of the vector space on which A acts (eg, they are orthogonal).

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Furthermore the basis elements are ordered by their eigenvalues (from largest to smallest), and these eigenvalues represent the amount of variance explained by each basis element.

III. SINGULAR VALUE DECOMPOSITION

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→ $UU^T = I_n, \; VV^T = I_d$ → $\Sigma_{ij} = 0 \;\; (i \neq j)$

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(n x d) (n x n) (n x d) (d x d)

NOTE

The number of singular values is equal to the *rank* of A.

The rank of a matrix measures its *non-degeneracy*.

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Q: How do you interpret the SVD?

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A: Recall that given a set of n points in d-dimensional space (eg, a matrix A), we want to find the best $k < d$ dimensional subspace to represent the data.

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For k = 1, this subspace is a line passing through the origin.

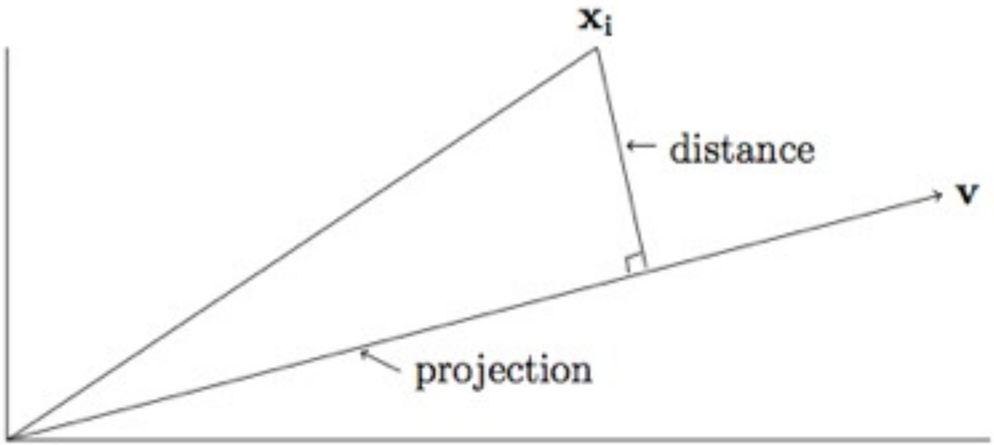

Figure 4.1: The projection of the point $\mathbf{x}_i$ onto the line through the origin in the direction of $\mathbf{v}$

source: http://www.cs.princeton.edu/courses/archive/spring12/cos598C/svdchapter.pdf

$$
\begin{array}{cc}
 & \begin{array}{ccccc}\text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung}\end{array} \\
\begin{array}{c}\text{CS}\\ \\ \\ \\ \text{MD}\\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\underbrace{\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}}_{\text{doc-to-concept similarity matrix}}
\text{x}
\underbrace{\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}}_{\text{concepts strengths}}
\text{x}
\underbrace{\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}}_{\text{term-to-concept similarity matrix}}
$$

In any case, the key difficulties with dimensionality reduction are time/space complexity, randomness (eg different results for different runs), and selecting the number of dimensions in the lower-dim subspace.