Machine Learning
Blatt 6

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Nr.1

$$
\begin{aligned}
MSE(g_T(x_0)) &= Var(g_T(x_0)) + Bias(f(x_0)) \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2\right] + (\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2 \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2\right] + 2\underbrace{\mathbb{E}_{\mathbb{T}}\left(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0))\right)}_{=\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)) = 0}(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0)) \\
&+ (\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2 \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2\right] + 2\mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))\overbrace{(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))}^{\textcircled{1}}\right] \\
&+ \mathbb{E}_{\mathbb{T}}\left[\overbrace{(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2}^{\textcircled{2}}\right] \\
&= \mathbb{E}_{\mathbb{T}}\Big[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2 + 2\left((g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))\right)(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))) \\
&+ (\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2\Big] \\
&= \mathbb{E}_{\mathbb{T}}\left[((g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0))) + \mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2\right] \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - f(x_0))^2\right]
\end{aligned}
$$

q.e.d.

[1] Konstant, kann also in den Erwartungswert gesetzt werden
[2] Konstant, somit ist der Erwartungswert er selbst

Nr.2

$$X = \begin{matrix} 3.437 & 5.791 & 3.268 & 10.649 \\ 12.801 & 4.558 & 5.751 & 14.375 \\ 6.136 & 6.223 & 15.175 & 2.811 \\ 11.685 & 3.212 & 0.639 & 0.964 \\ 5.733 & 3.22 & 0.534 & 2.052 \\ 3.021 & 4.348 & 0.839 & 2.356 \\ 1.689 & 0.634 & 0.318 & 2.209 \\ 2.339 & 1.895 & 0.61 & 0.605 \\ 1.025 & 0.834 & 0.734 & 2.825 \\ 2.936 & 1.419 & 0.331 & 0.231 \\ 5.049 & 4.195 & 1.589 & 1.957 \\ 1.693 & 3.602 & 0.837 & 1.582 \\ 1.187 & 2.679 & 0.459 & 18.837 \\ 9.73 & 3.951 & 3.78 & 0.524 \\ 14.325 & 4.3 & 10.781 & 36.863 \\ 7.737 & 9.043 & 1.394 & 1.524 \\ 7.538 & 4.538 & 2.565 & 5.109 \\ 10.211 & 4.994 & 3.081 & 3.681 \\ 8.697 & 3.005 & 1.378 & 3.338 \end{matrix} \quad Y = \begin{matrix} 27.698 \\ 57.634 \\ 47.172 \\ 49.295 \\ 24.115 \\ 33.612 \\ 9.512 \\ 14.755 \\ 10.57 \\ 15.394 \\ 27.843 \\ 17.717 \\ 20.253 \\ 37.465 \\ 101.334 \\ 47.427 \\ 35.944 \\ 45.945 \\ 46.89 \end{matrix}$$

Berechne:

$$w = (X^T \cdot X)^{-1} \cdot (X^T \cdot Y)$$

mit:

$$(X^T \cdot X)^{-1} = \begin{matrix} 0.00429129055565 & -0.00478345506973 & -0.000752238199424 & -0.000689936030722 \\ -0.00478345506973 & 0.0109403979337 & -0.00209111752592 & 0.000396499981558 \\ -0.000752238199424 & -0.00209111752592 & 0.00558456810802 & -0.00081147325675 \\ -0.000689936030722 & 0.000396499981558 & -0.00081147325675 & 0.000930021643118 \end{matrix}$$

$$(X^T \cdot Y) = \begin{matrix} 5570.426016 \\ 2944.414095 \\ 2902.209741 \\ 6296.28324 \end{matrix}$$

Es ergibt sich somit: $w = \begin{matrix} 3.29265832646 \\ 1.99479384949 \\ 0.750919342272 \\ 0.824836613555 \end{matrix}$

$Out(X) = \text{Catlle} \cdot 3.29265832646 + \text{Calves} \cdot 1.99479384949 + \text{Pigs} \cdot 0.750919342272 + \text{Lambs} \cdot 0.824836613555$