Machine Learning
Blatt 6

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7. Juni 2016

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Nr.1

$$
\begin{aligned}
MSE(g_T(x_0)) &= Var(g_T(x_0)) + Bias(f(x_0)) \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2\right] + (\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2 \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2\right] + 2\underbrace{\mathbb{E}_{\mathbb{T}}\left(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0))\right)}_{=\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)) = 0}\left(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0)\right) \\
&+ (\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2 \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2\right] + 2\mathbb{E}_{\mathbb{T}}\left[\left(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0))\right)\overbrace{\left(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0)\right)}^{①}\right] \\
&+ \mathbb{E}_{\mathbb{T}}\left[\overbrace{(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2}^{②}\right] \\
&= \mathbb{E}_{\mathbb{T}}\Big[(g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))^2 + 2\left((g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0)))\right)\left(\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0)\right) \\
&+ (\mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0))^2\Big] \\
&= \mathbb{E}_{\mathbb{T}}\left[\left((g_T(x_0) - \mathbb{E}_{\mathbb{T}}(g_T(x_0))) + \mathbb{E}_{\mathbb{T}}(g_T(x_0)) - f(x_0)\right)^2\right] \\
&= \mathbb{E}_{\mathbb{T}}\left[(g_T(x_0) - f(x_0))^2\right]
\end{aligned}
$$

q.e.d.

①Konstant, kann also in den Erwartungswert gesetzt werden
②Konstant, somit ist der Erwartungswert er selbst