

# Hash Indexing (ctd.) and Using Indexes

R&G Chapter 11, 14

(slides adapted from content by J.Gehrke, J.Shanmugasundaram, and/or C.Koch)

# Announcements

Homework 3 due tonight

No homework 4 assigned this week  
(Project 1 due 1 week from Monday)

Dr. Chomicki will be substituting Monday  
(Monday Office Hours → Wednesday)

# Recap: Hash Indexes

- As with trees: request a key  $k$  and get record(s) or record id(s) with  $k$ .
- Hash-based indexes support equality lookups
  - ... in constant time (vs  $\log(n)$  for tree)
  - ... but don't support range lookups
- Static vs Dynamic Hashing
  - Tradeoffs similar to ISAM vs B+Tree

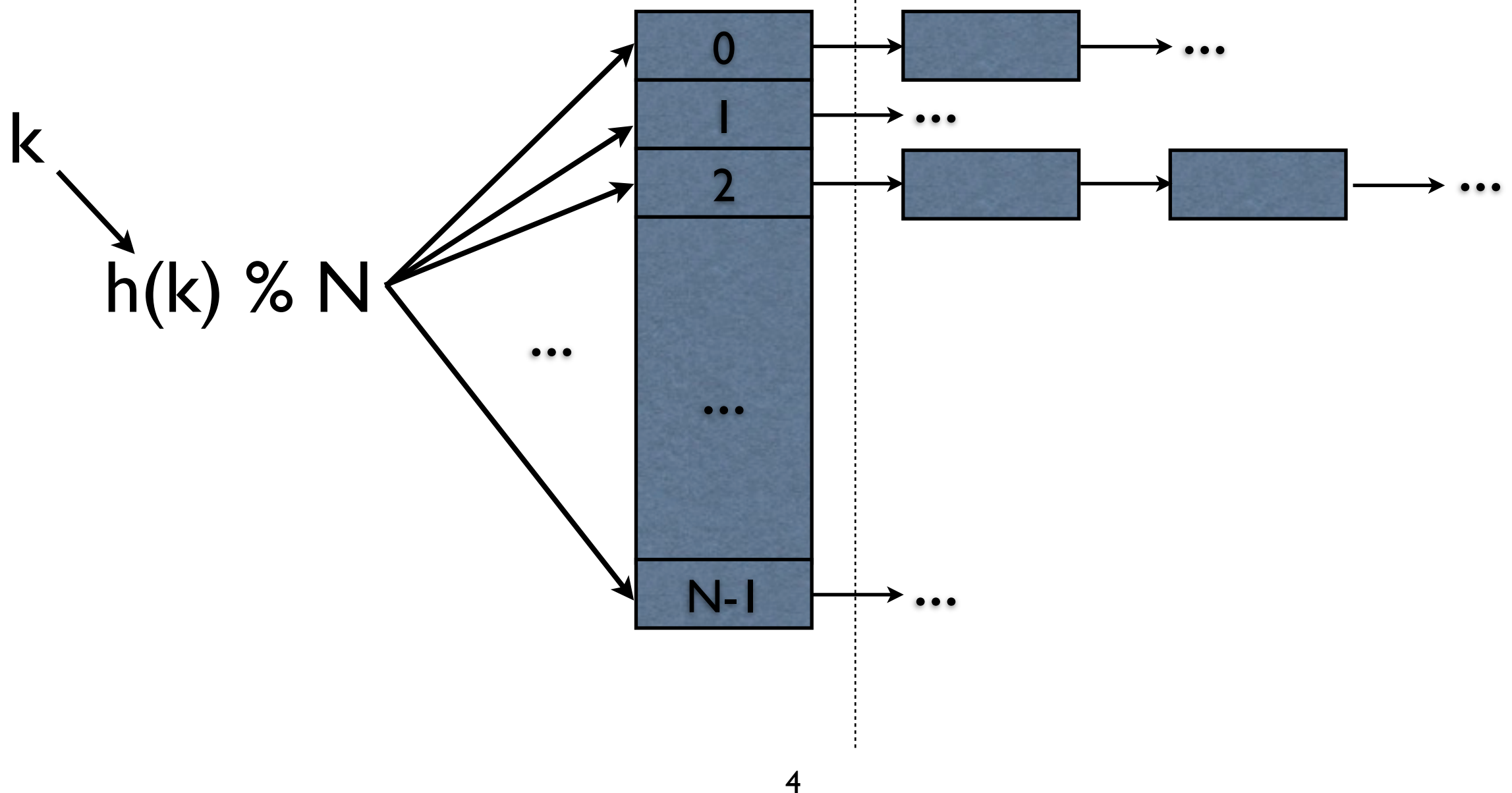
Higher fanouts mean shallower trees, and fewer pages loaded to find an entry. These trees are often quite shallow (depth 4–5), so even a small reduction is huge.

Page sizes are fixed, so the only way to get higher fanouts is to pack more keys/pointers into a page. This is difficult for fixed size keys, but consider variable-length strings.

# Recap: Static Hashing

Primary Bucket Pages  
(Contiguous)

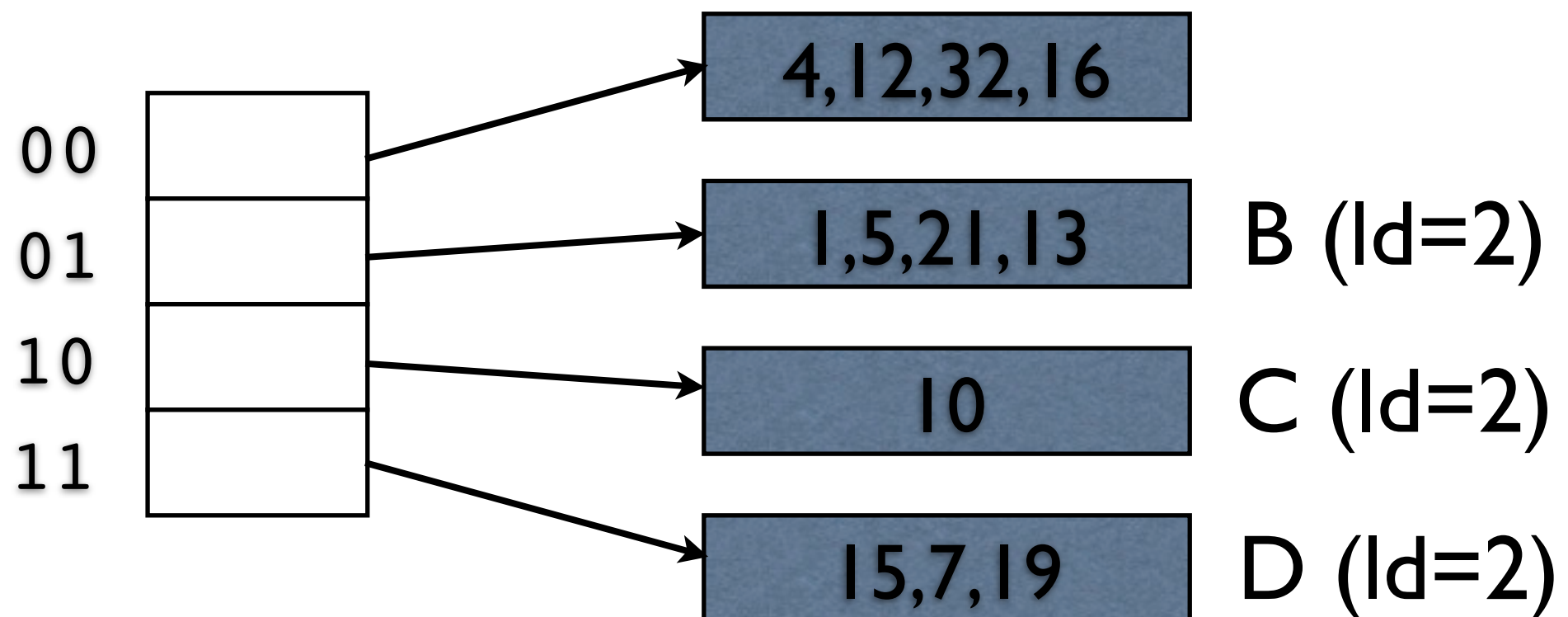
Overflow Pages  
(Linked List)



# Recap: Extendible Hashing

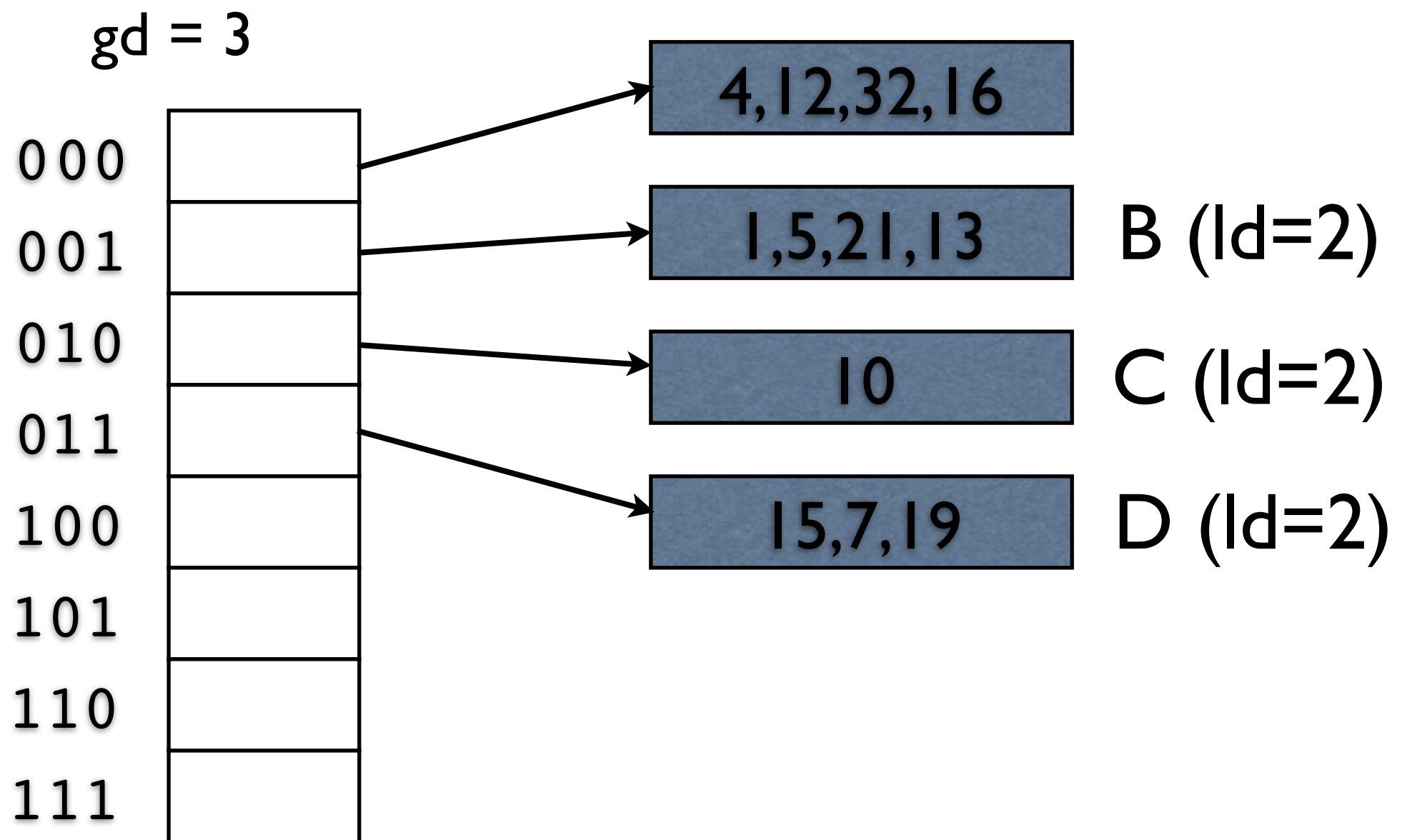
- **Situation:** A bucket becomes full
  - Solution: Double the number of buckets!
  - Expensive! ( $N$  reads,  $2N$  writes)
- **Idea:** Add one level of indirection
  - A directory of pointers to (noncontiguous) bucket pages.
  - Doubling just the directory is much cheaper.
  - Can we double only the directory?

# Extendible Hashing



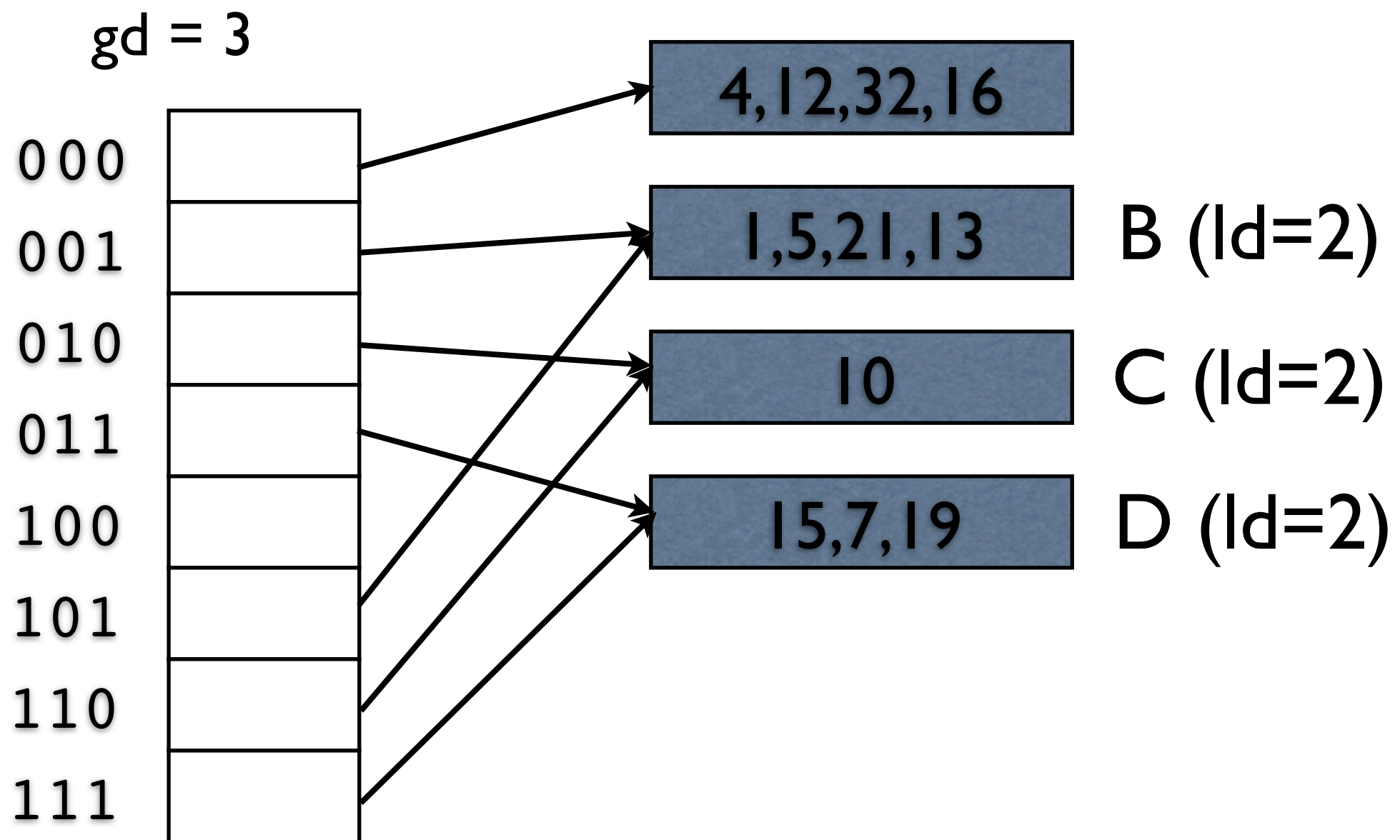
Bucket value **hashes** have the same last **ld** bits

# Extendible Hashing



Bucket value **hashes** have the same last **ld** bits

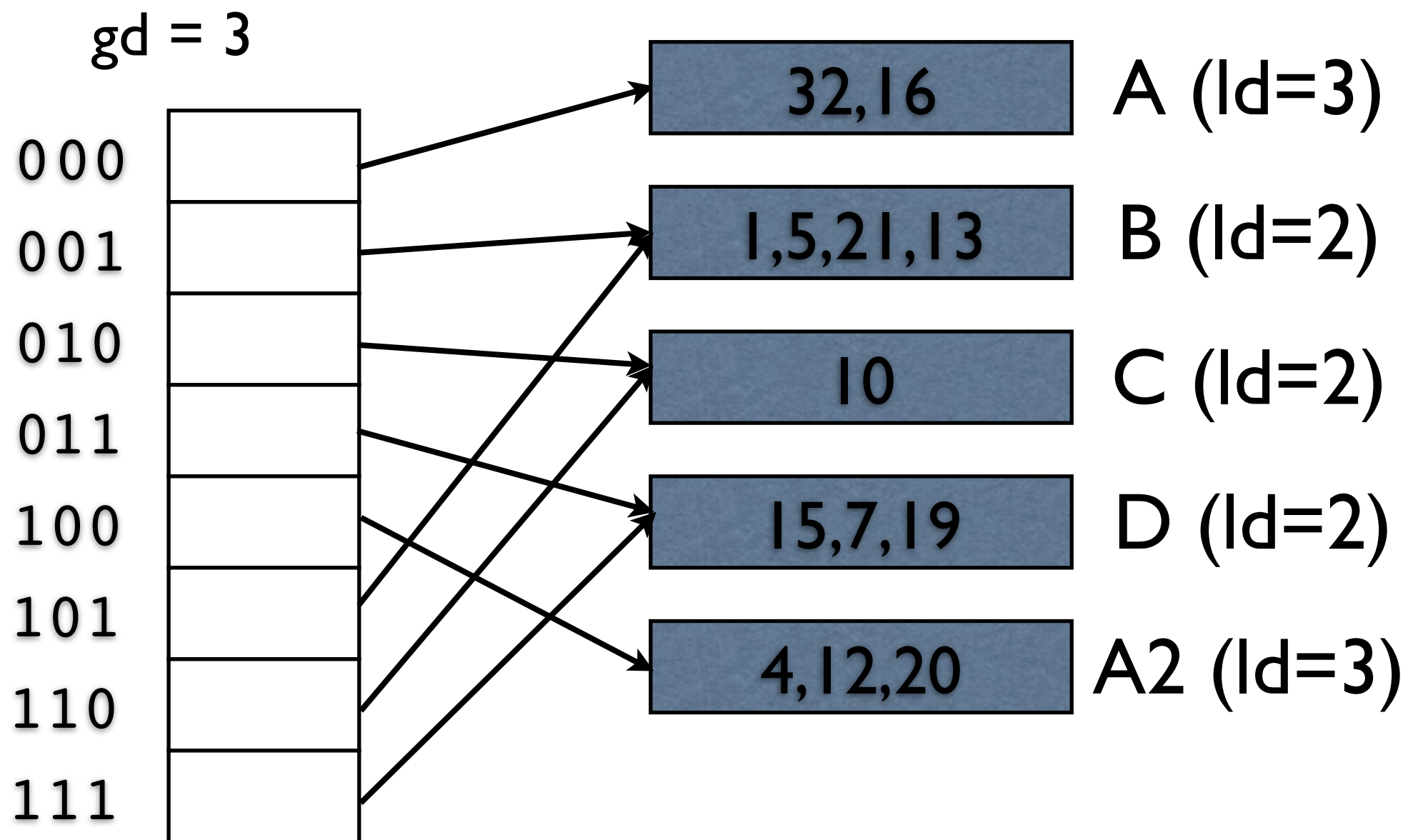
# Extendible Hashing



Bucket value **hashes** have the same last **ld** bits



# Extendible Hashing



Bucket value **hashes** have the same last **ld** bits

# Recap: Extendible Hashing

- Global depth of directory
  - **Upper bound** on # of bits required to determine the bucket of an entry.
- Local depth of a bucket
  - **Exact** # of bits required to determine if an entry belongs in this bucket.
- Using the last  $ld/gd$  bits makes it possible to double the directory size by copying entries.



Any Questions?

# Linear Hashing

- A directory page adds 1 page lookup overhead.
- Can we do similar splits without indirection?
- Linear Hashing based on similar principle.
  - Start with the last  $n$  bits of each hash fn.
  - When you decide to split, start using  $n+1$  bits.
- **Key difference:** Split incrementally
  - Part of the hash table uses  $n$  bits, rest uses  $n+1$
  - Each *round* increase  $n$  by one (1 round = 1 full split)

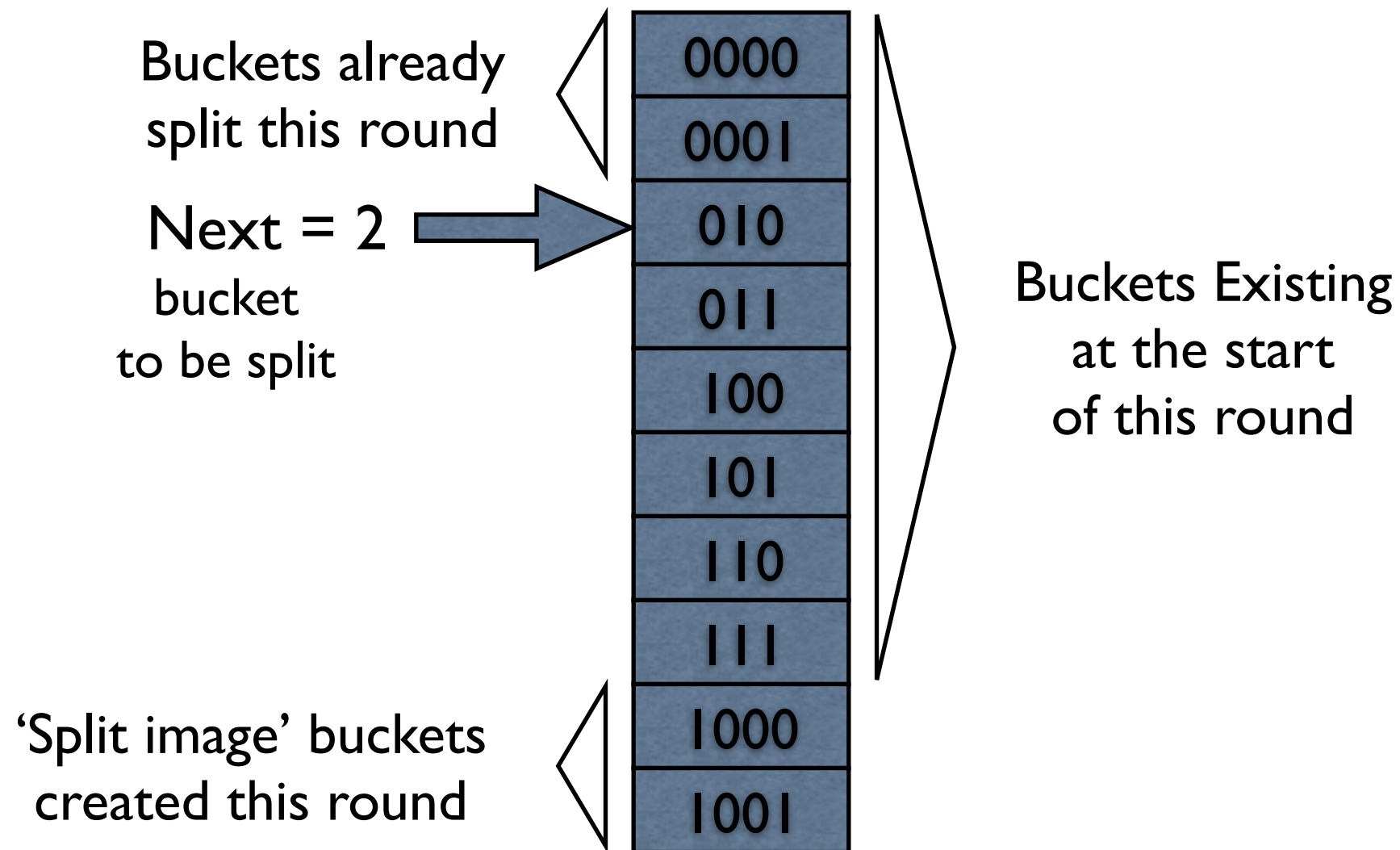
We can generalize the splitting idea a little bit: We're taking one hash function  $h(k)$ , and defining a new function:  $h'(k,n) = h(k) \% 2^n$  (2 to the  $n$ th). Another way to look at this is that we're defining a family of hash functions  $h'_1(k) = h(k) \% 2$ ,  $h'_2(k) = h(k) \% 3$ ,  $h'_4(k) = h(k) \% 8$ , ...

Any family of hash functions that satisfies the copy on split property can be switched in for this one

– That is, we can swap in any family as long as  $h'_n(k) = h'_{n+1}(k) \% 2^n$

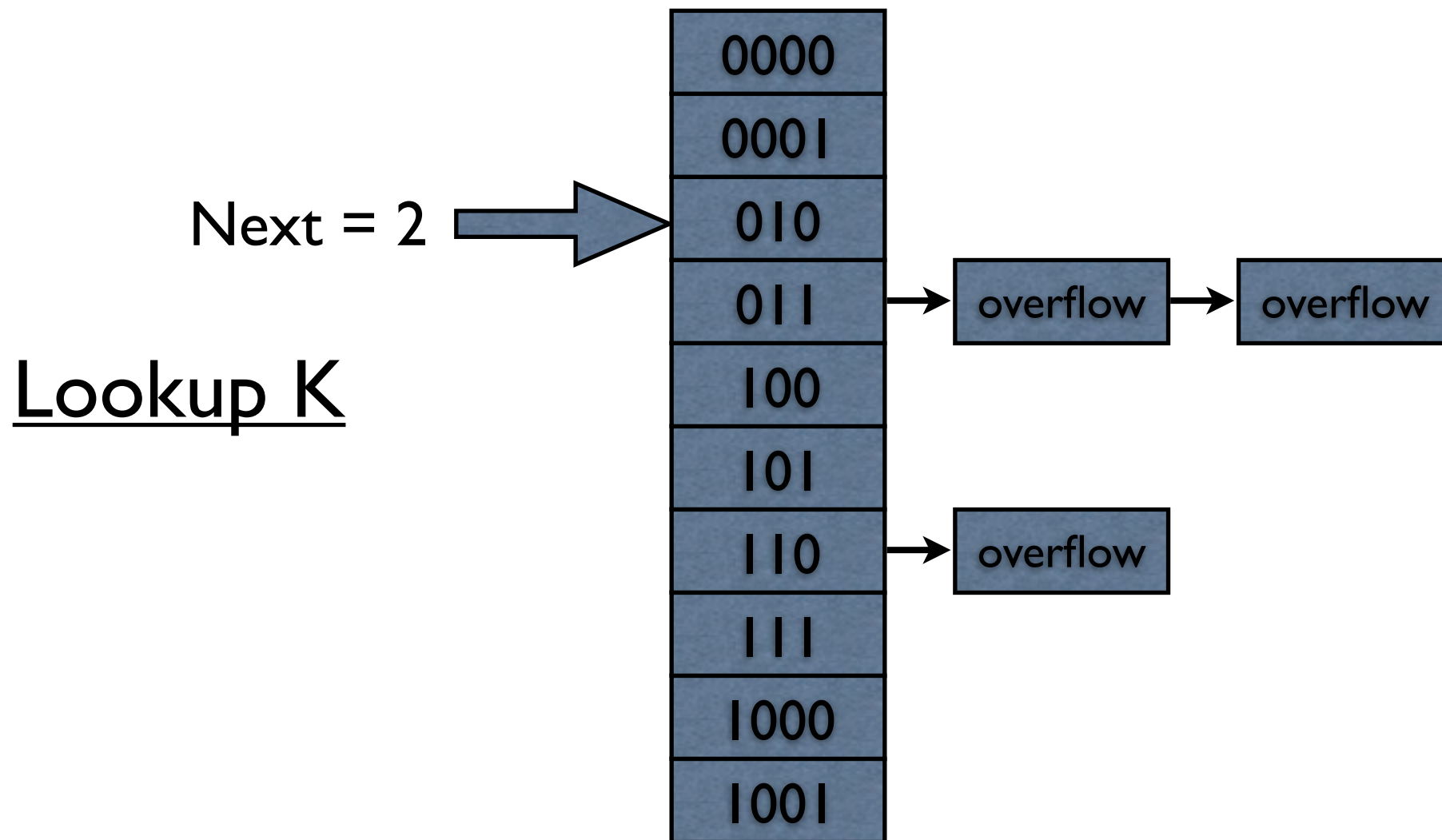
# Linear Hashing

Level = 3 ( $2^3 = 8$  Entries)



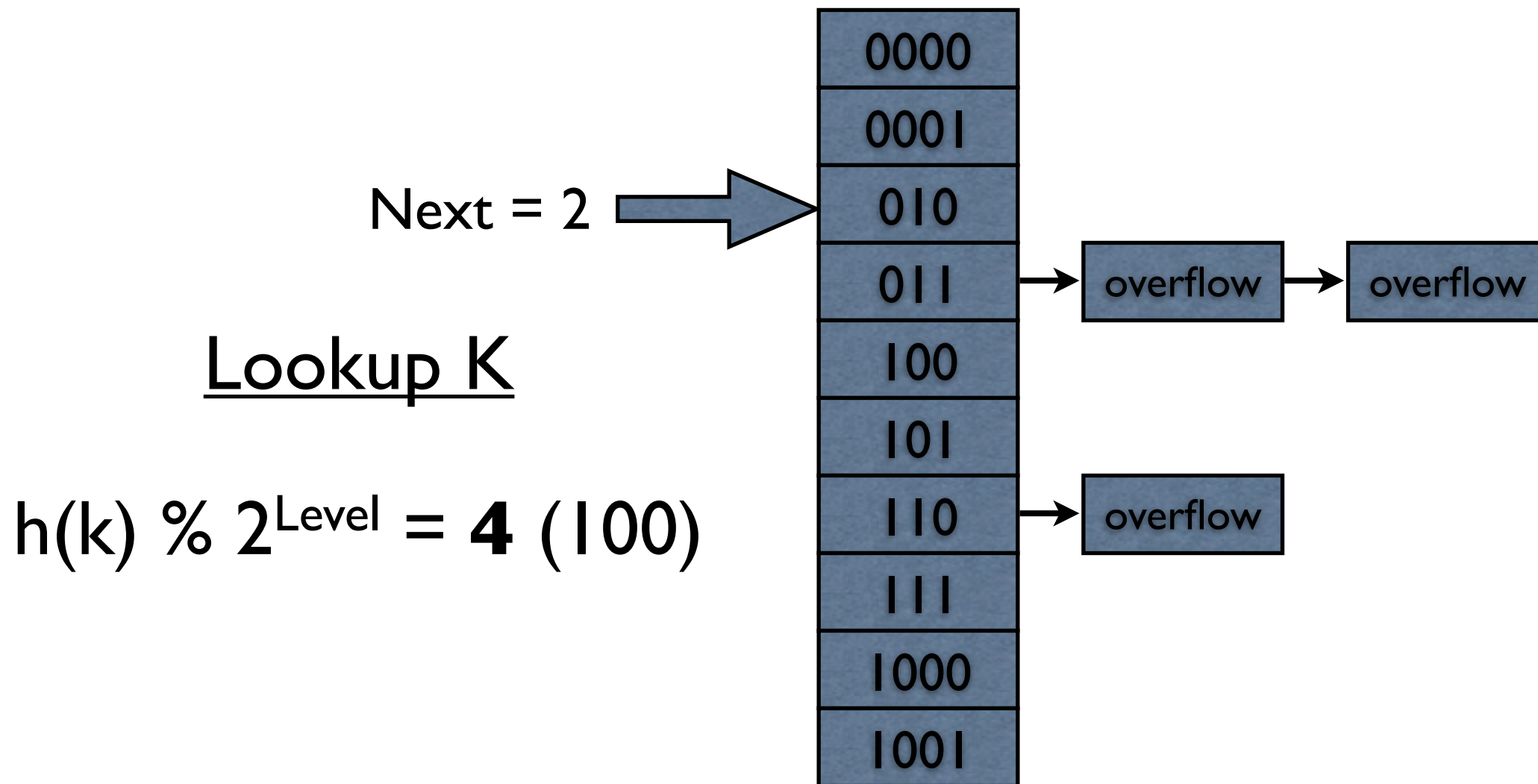
# Linear Hashing: Lookups

Level = 3 ( $2^3 = 8$  Entries)



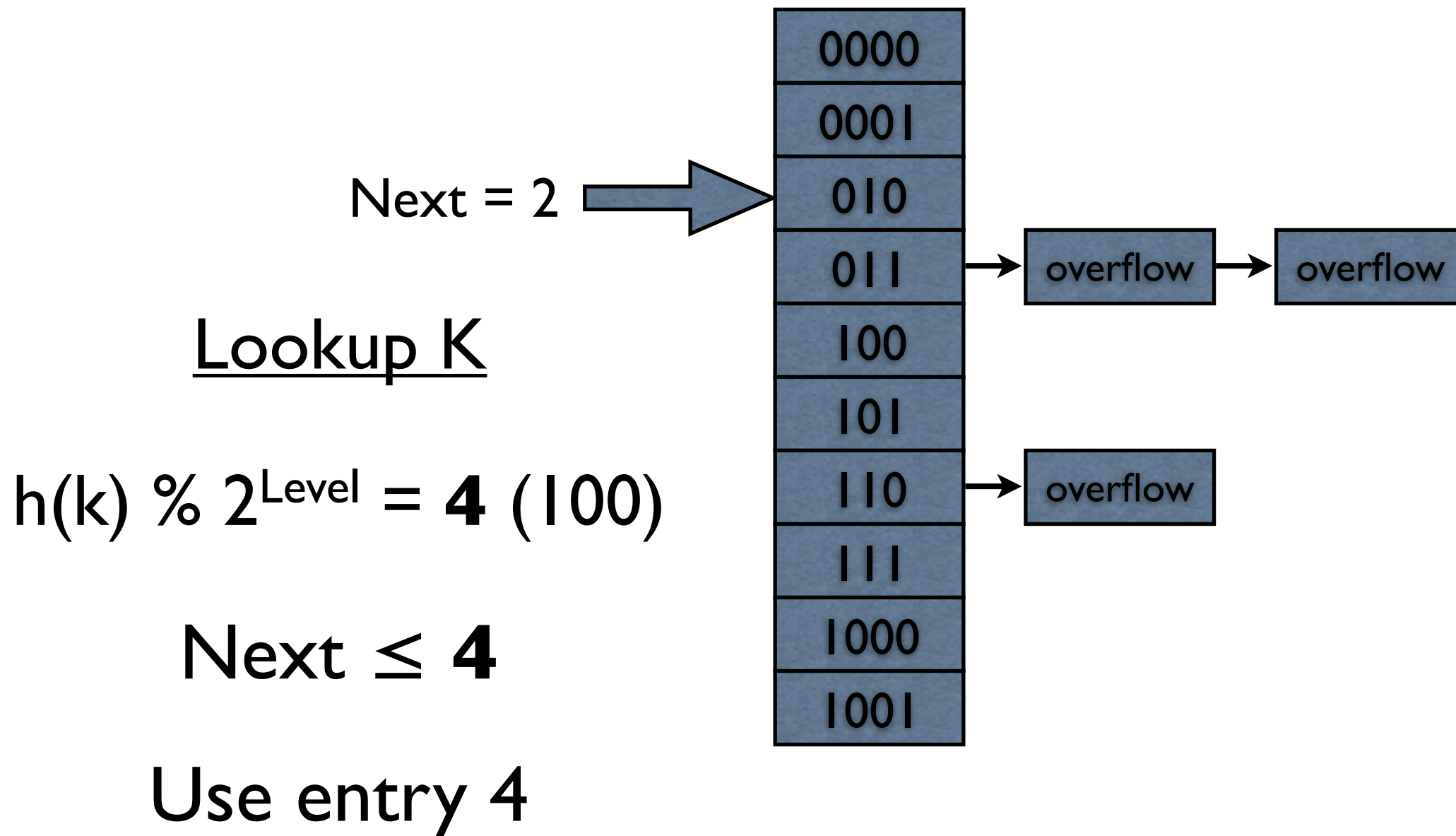
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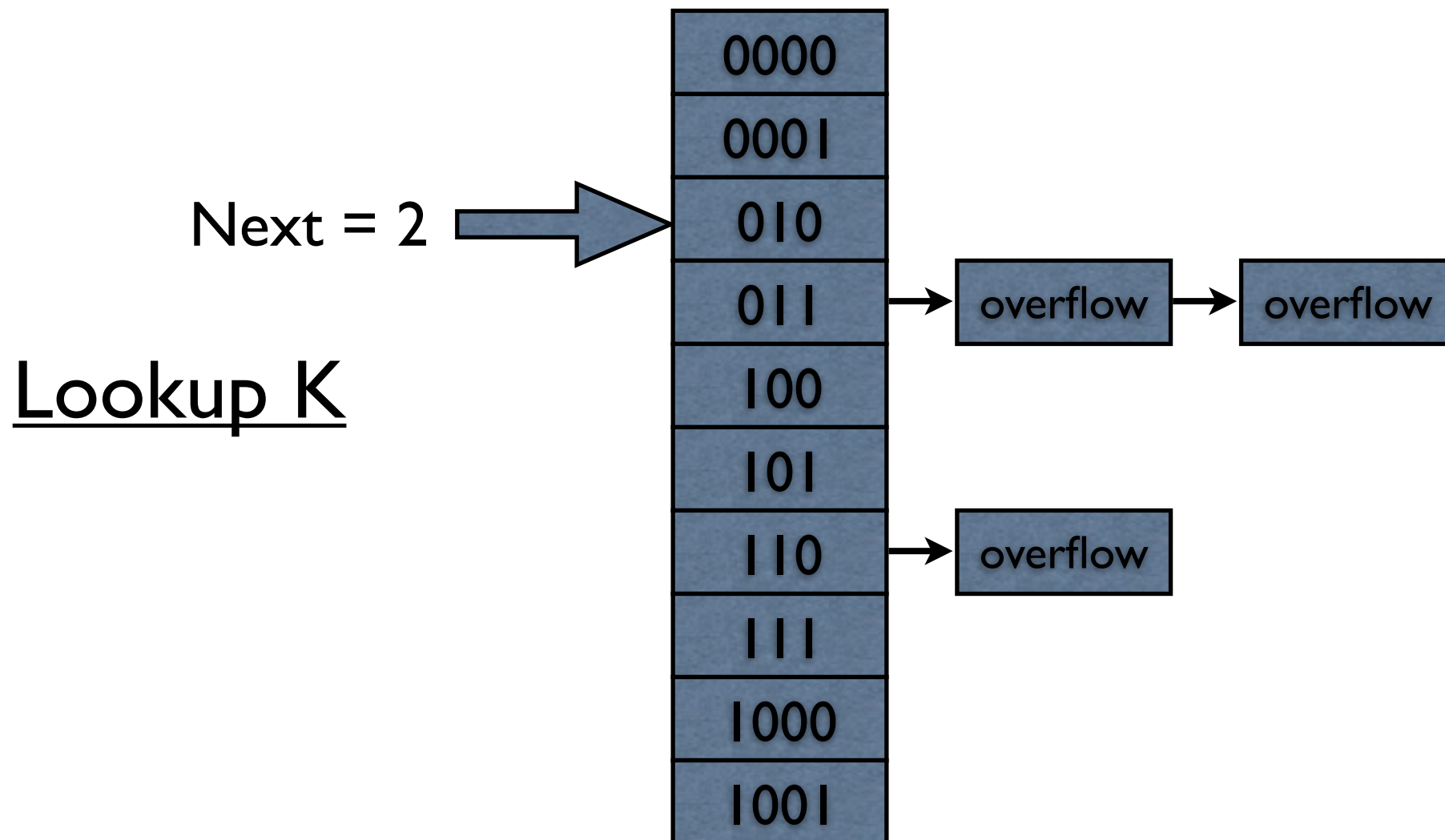
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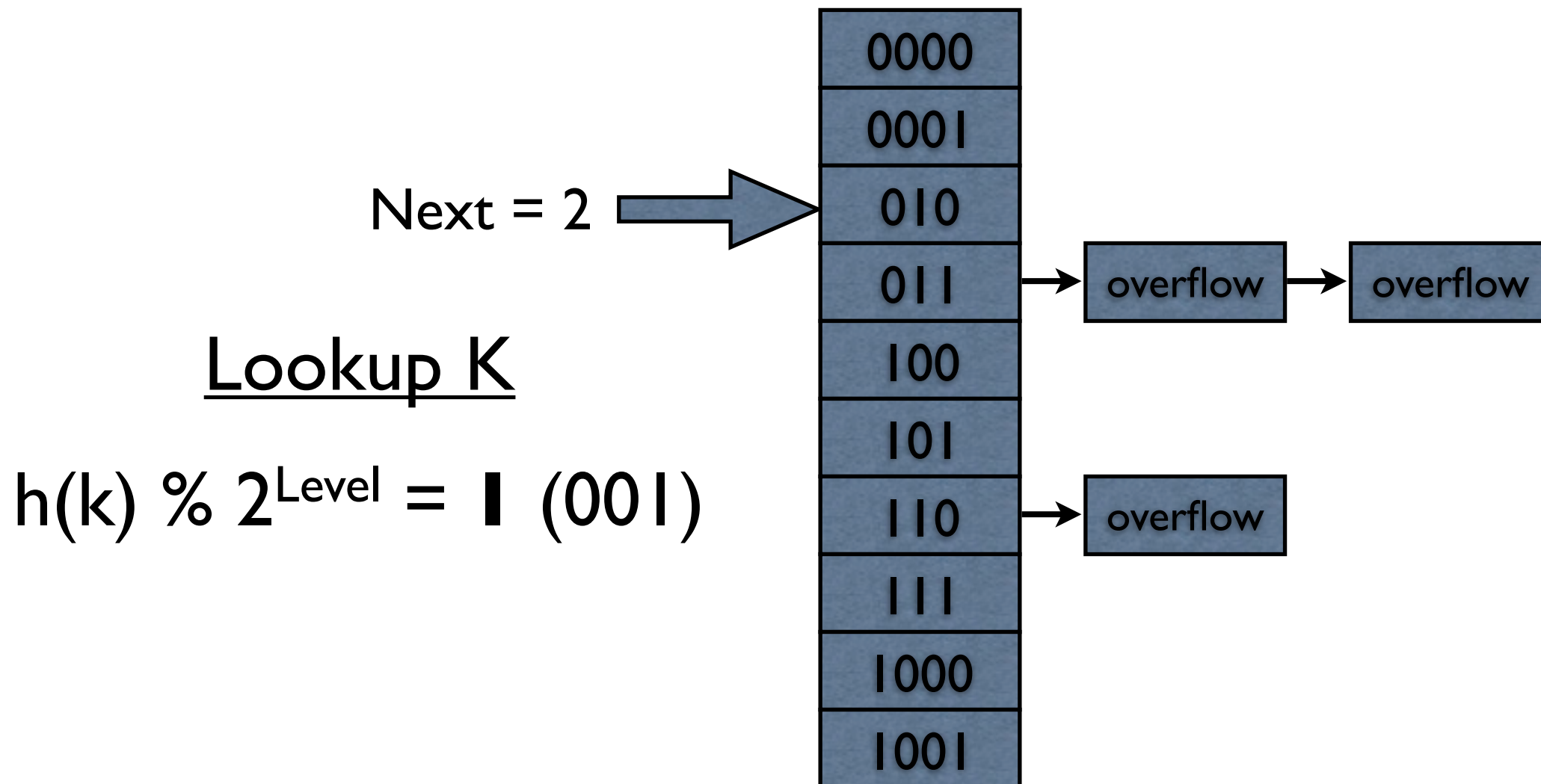
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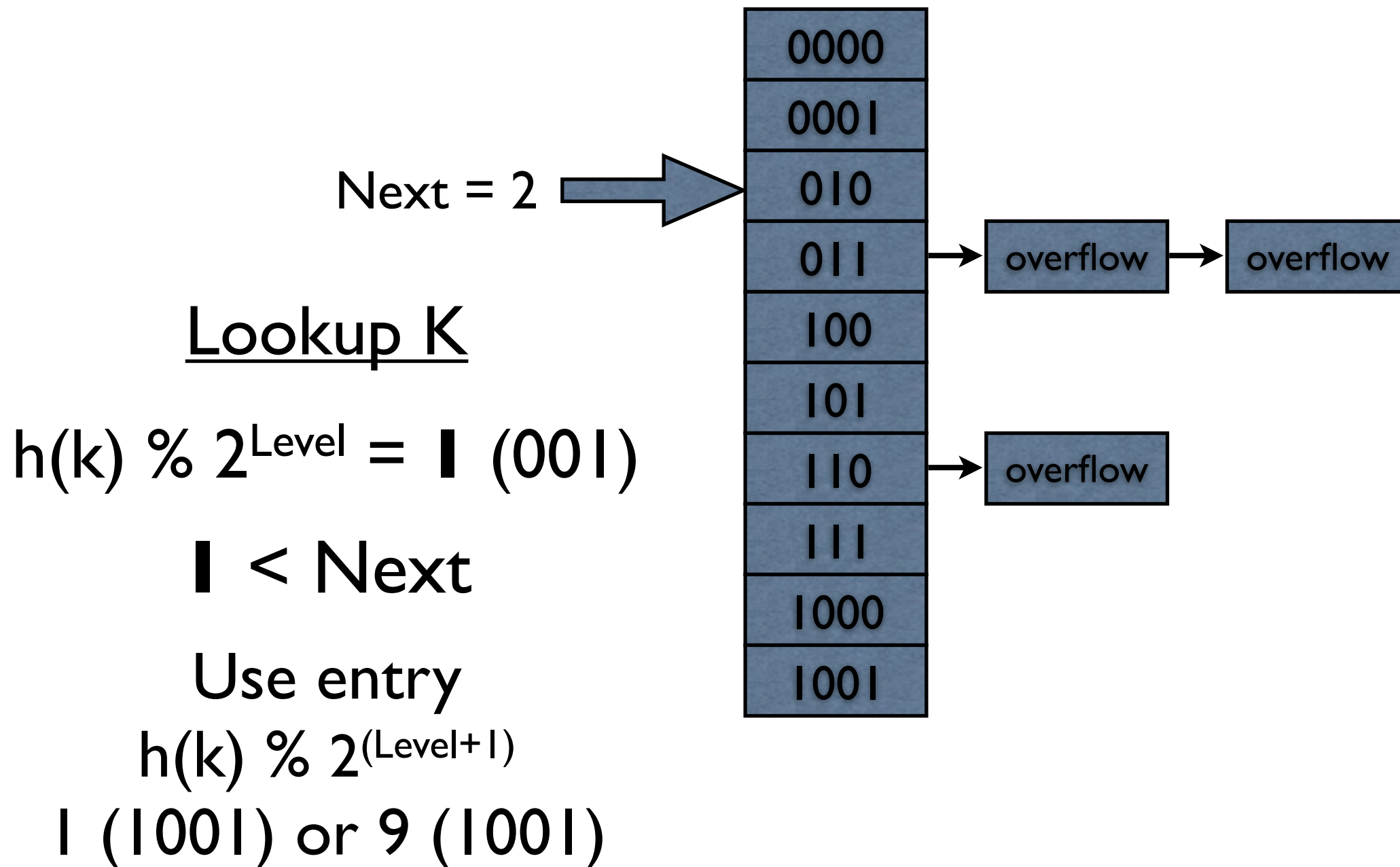
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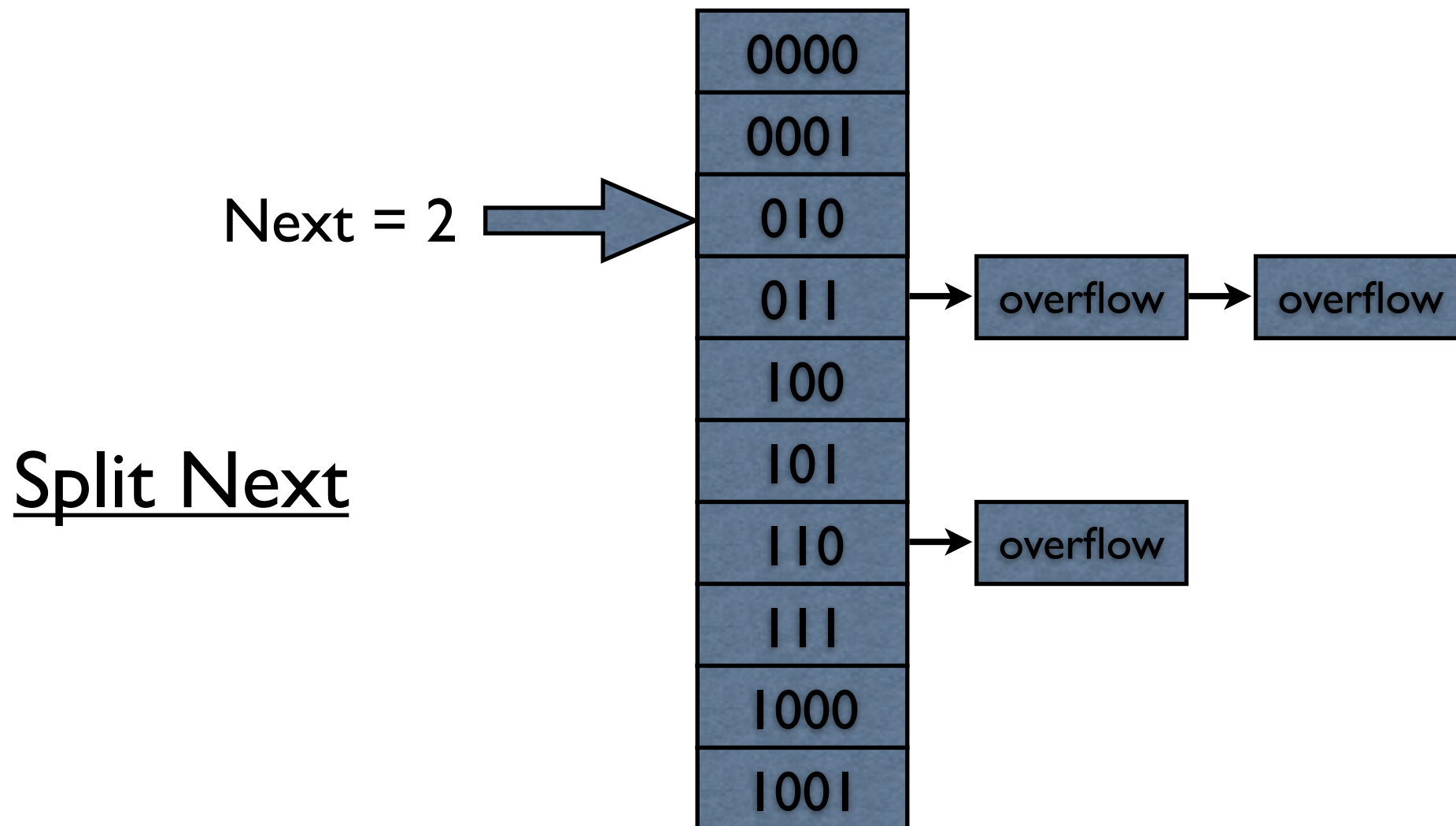
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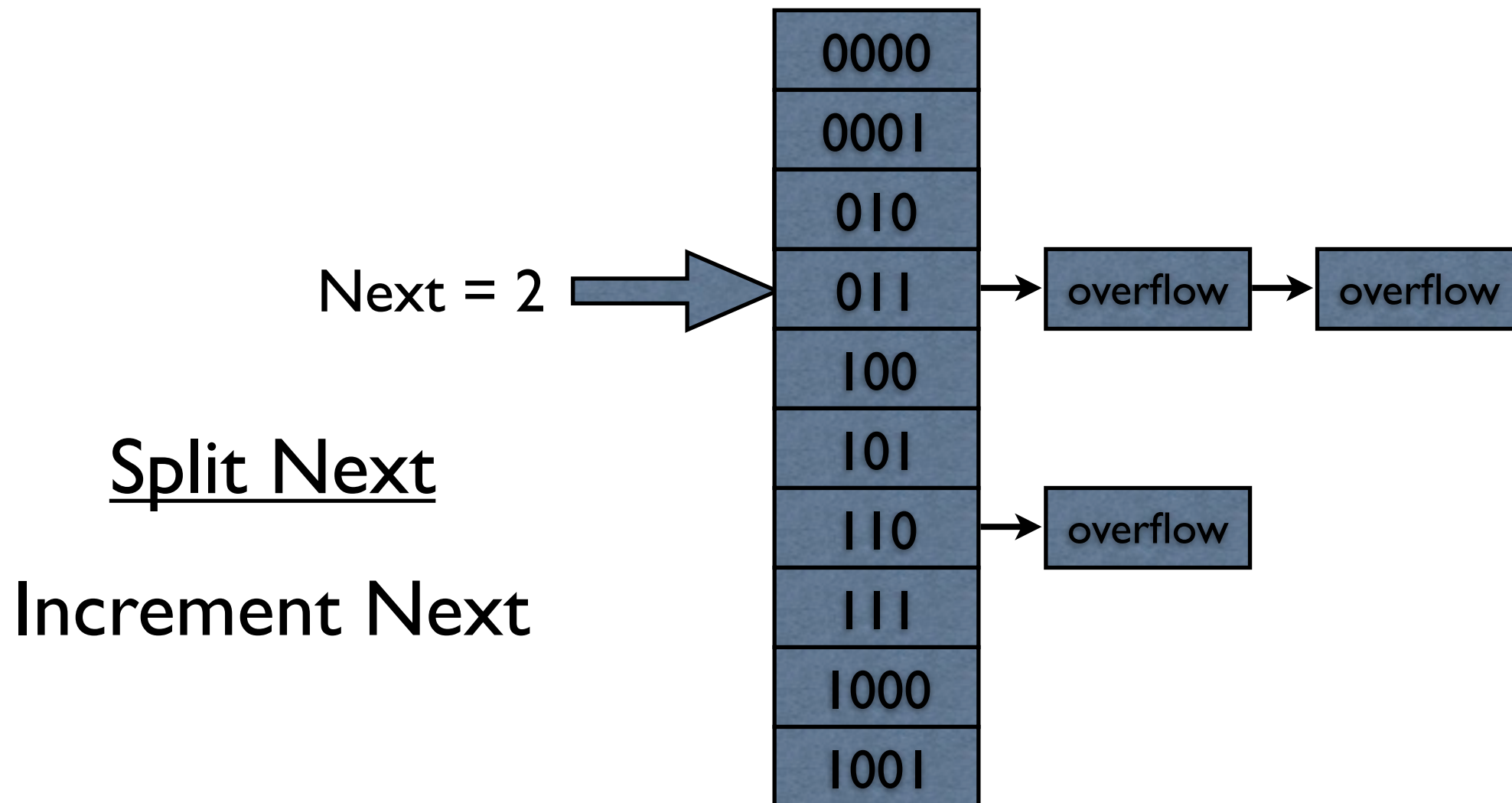
# Linear Hashing: Splitting

Level = 3 ( $2^3 = 8$  Entries)



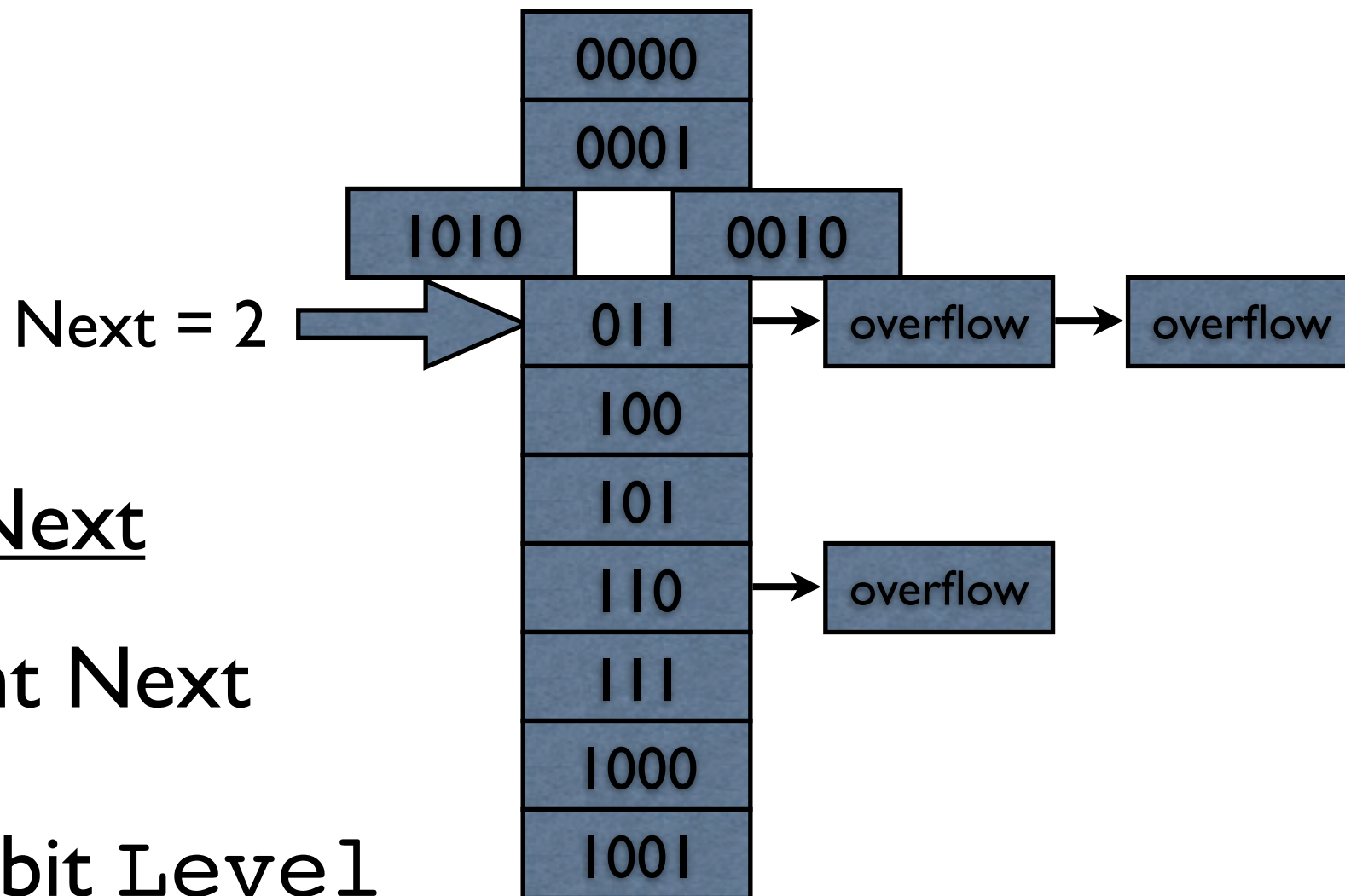
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Level = 3 ( $2^3 = 8$  Entries)



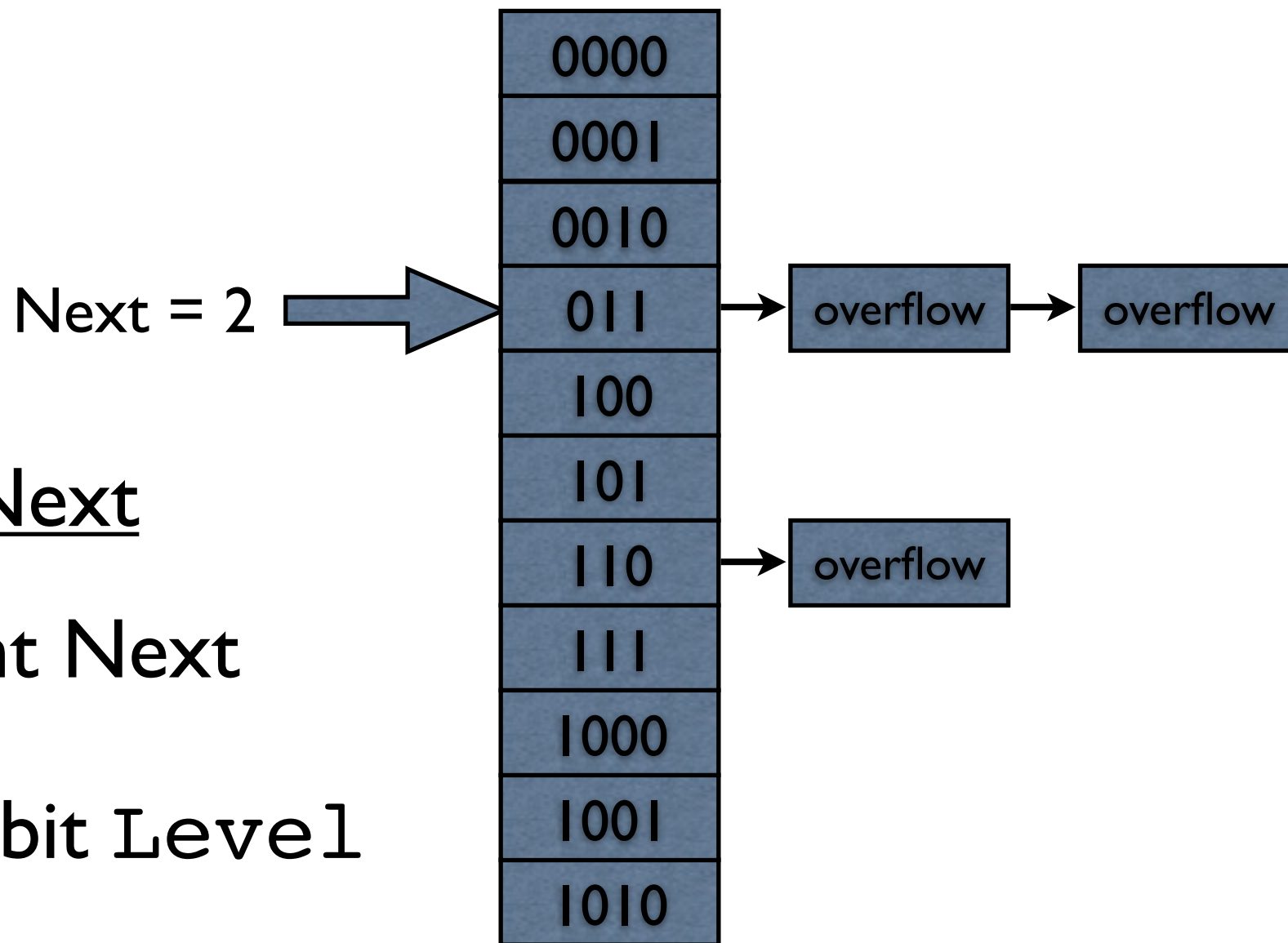
Split Next

Increment Next

Partition on bit Level

# Linear Hashing: Splitting

Level = 3 ( $2^3 = 8$  Entries)



Split Next

Increment Next

Partition on bit Level

- Entries for which the last *Level* bits  $< \text{Next}$ 
  - Split, use *Level*+1 bits to determine bucket.
- Entries for which the last *Level* bits  $\geq \text{Next}$ 
  - Unsplit, use *Level* bits to determine bucket.
- Only ever split the *Next* bucket





Any Questions?

# Linear Hashing

- When to we split?
  - It depends on the application.
- Whenever Next bucket is full
- After random insertions
- After a fixed number of insertions (size)
- Background process splits as needed.

# Extendible vs Linear

- The two algorithms are actually quite similar.
  - Keep some data pages un-split
    - Minimize repartitioning required to split.
  - Use least-significant bits to ensure that new buckets will be appended to the end.
- Linear allocates buckets in sequential order.
  - Is this helpful? When/how?



Any Questions?

# Consistent Hashing

(‘Chord: A Scalable Peer-to-peer...’, Stoica et al.)

- **Insight:** Make split/merge faster by making bin boundaries nondeterministic.
- Used mostly in distributed data-stores
  - (Amazon, Facebook, ...)
  - Minimal applications to file-based storage.

# Consistent Hashing



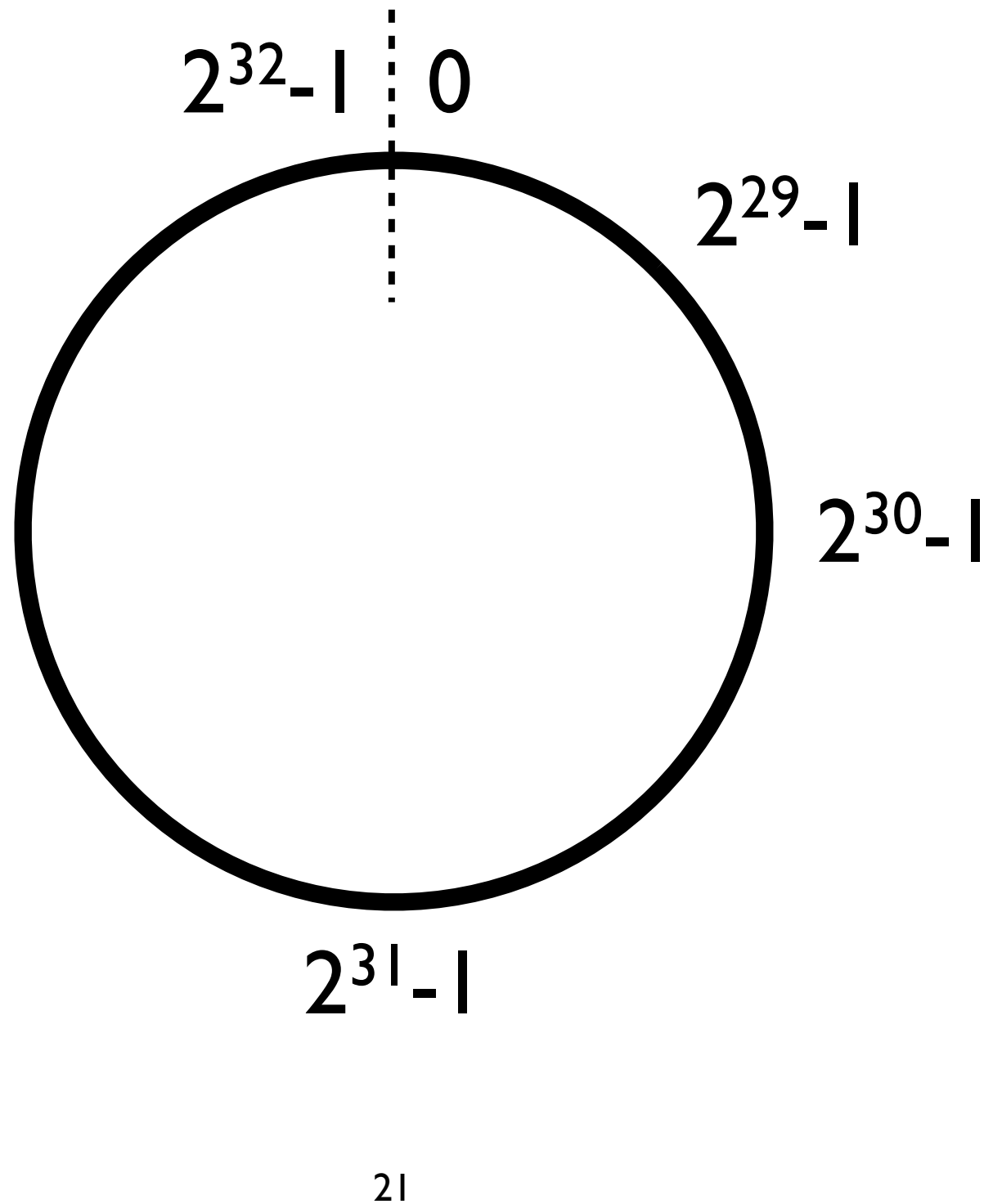
20

# Consistent Hashing

Modular  
Arithmetic  
(mod  $2^{32}$ )

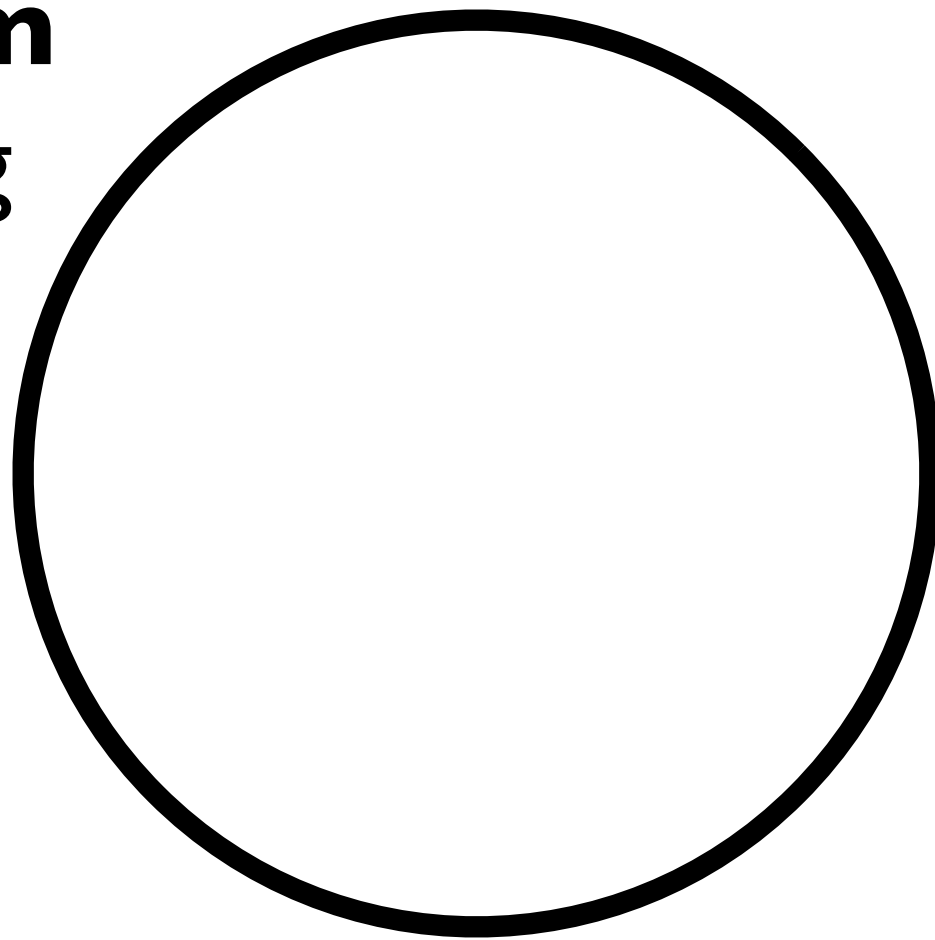
$$(2^{32}-1)+1 = 0$$

Numbers  
form a 'Ring'



# Consistent Hashing

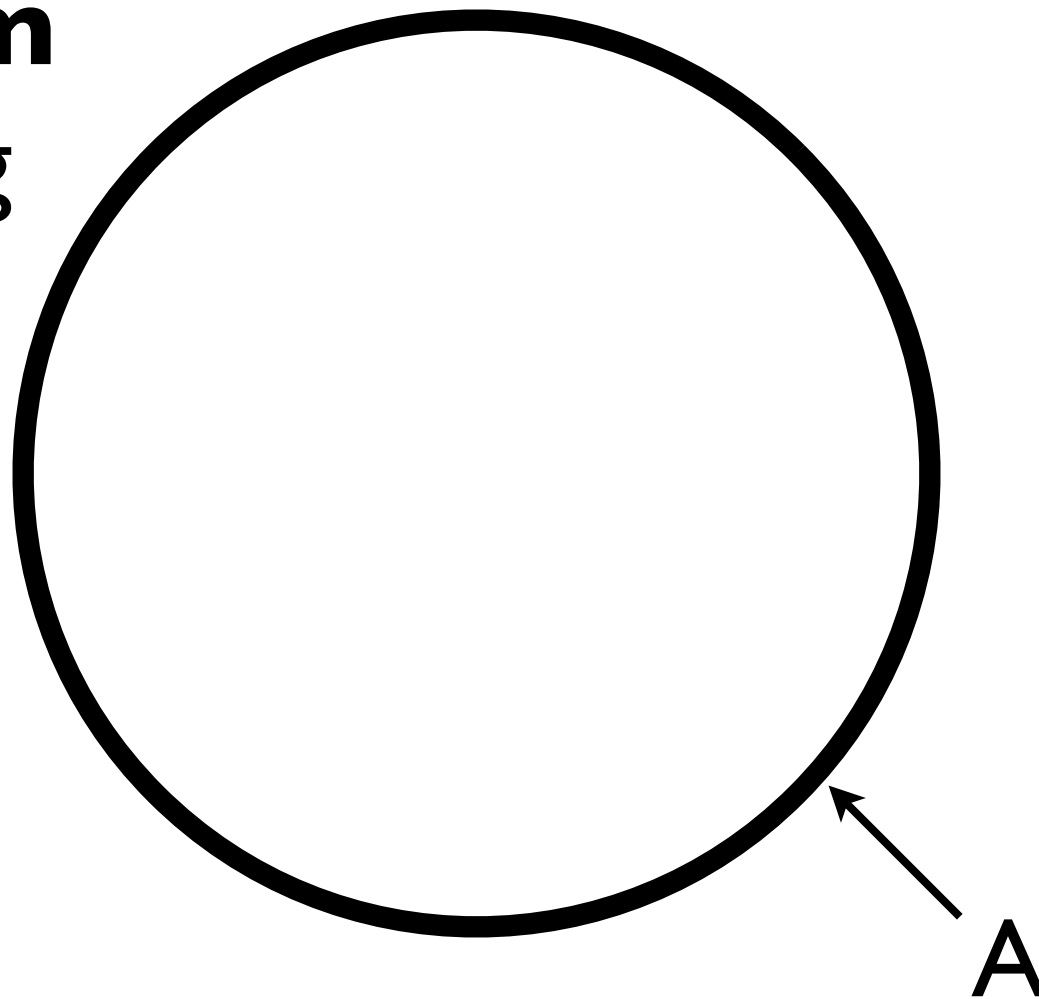
Assign each  
bucket a **random**  
point on the ring





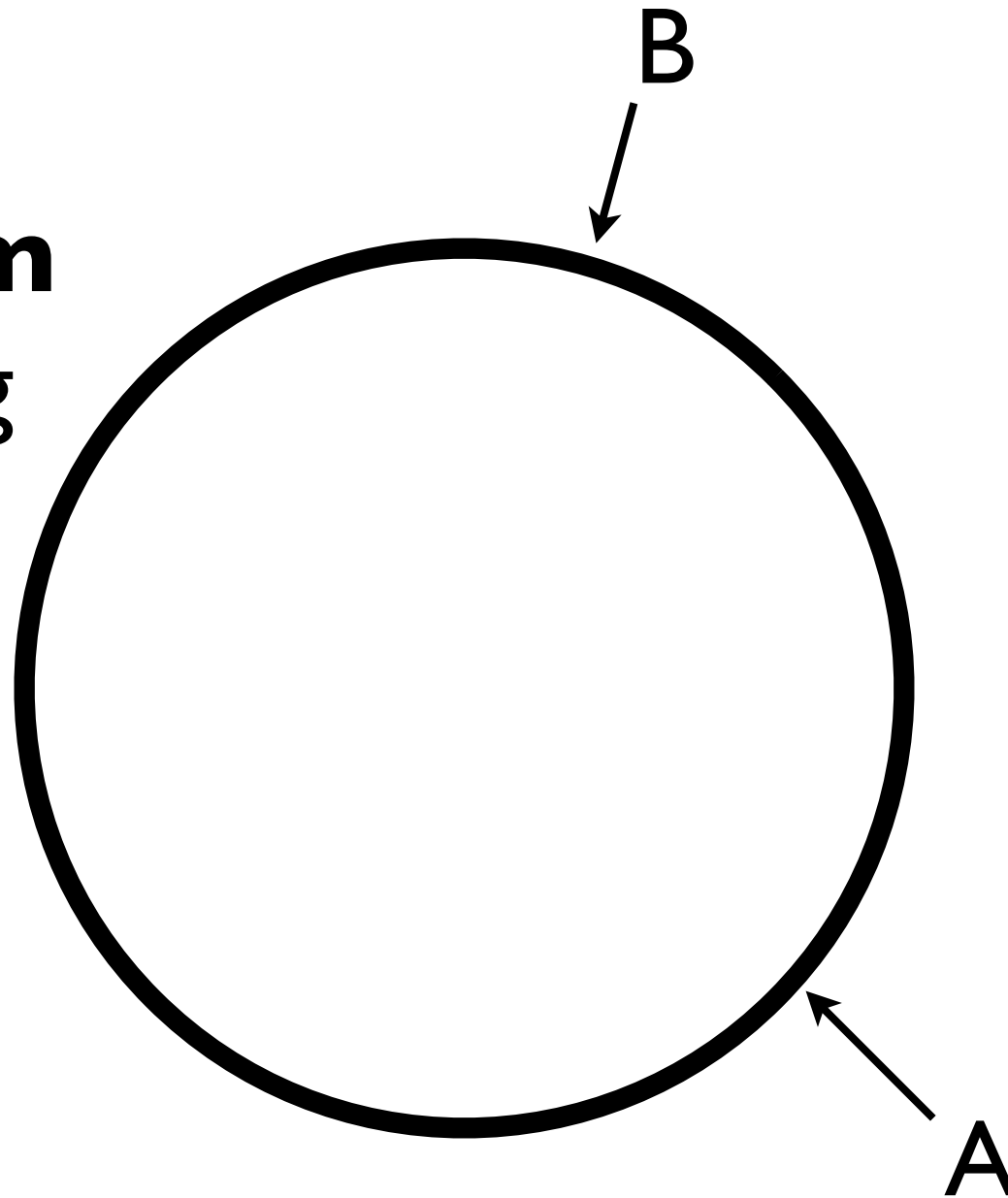
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# Consistent Hashing

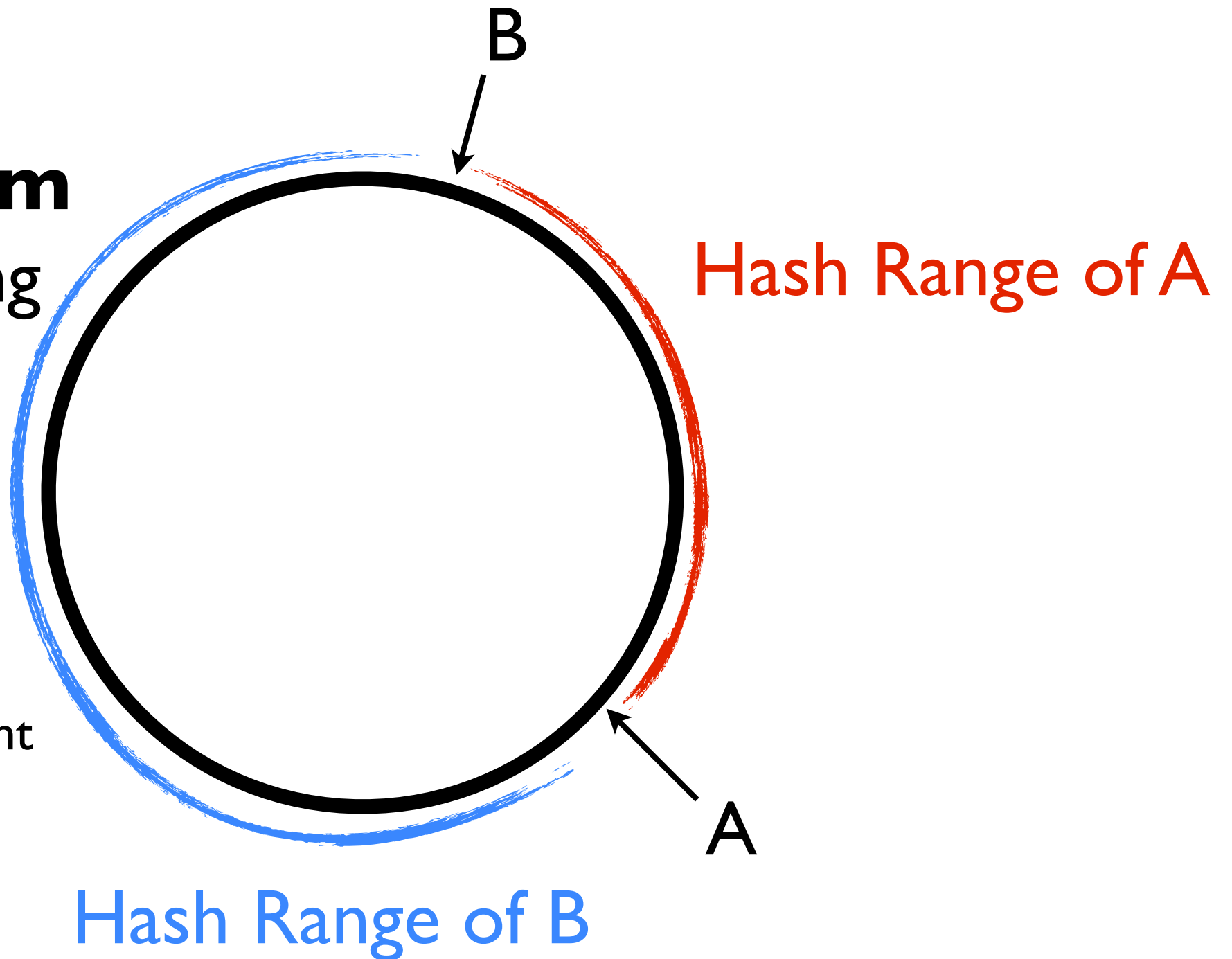
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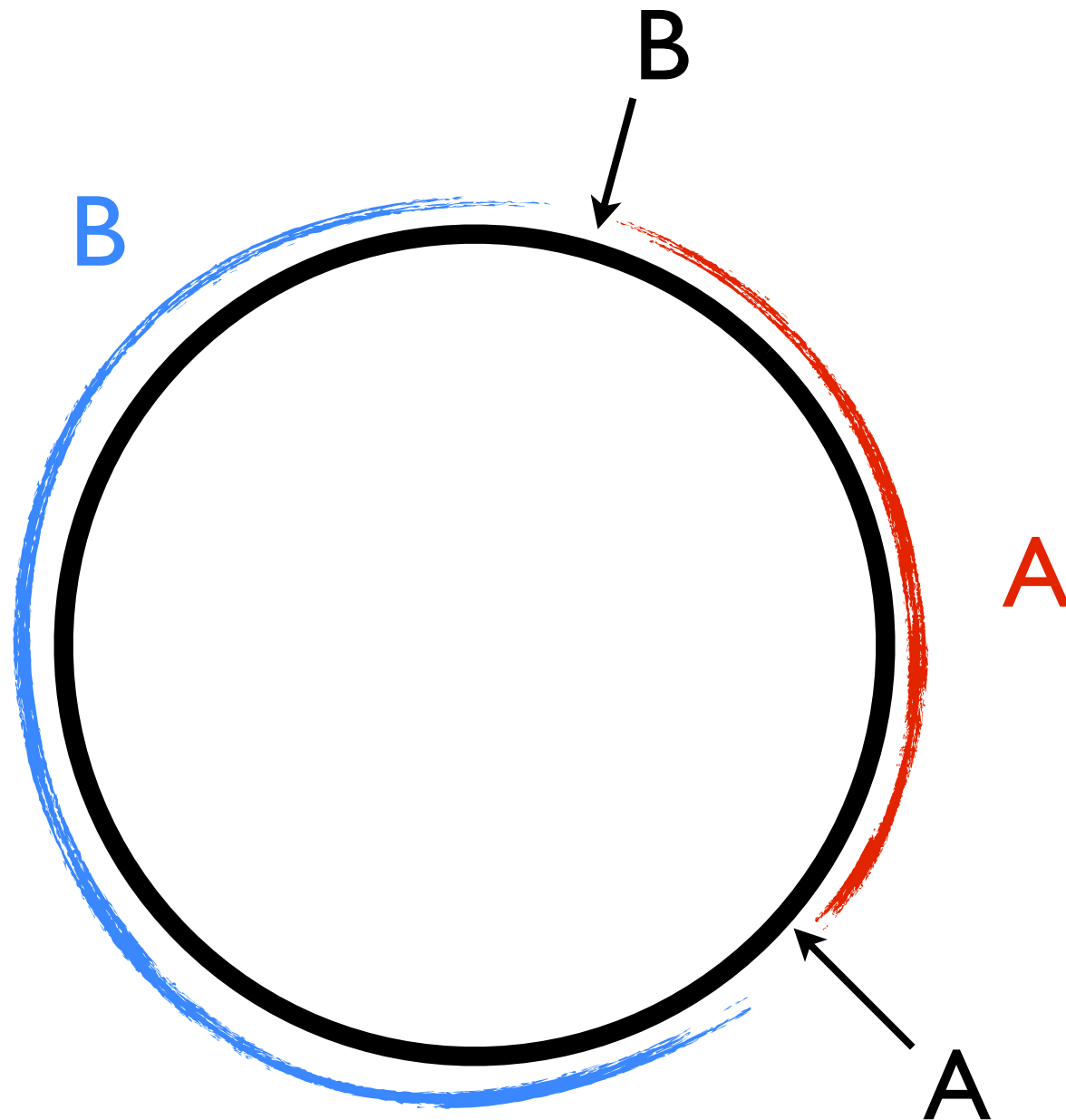
# Consistent Hashing

Assign each bucket a **random** point on the ring

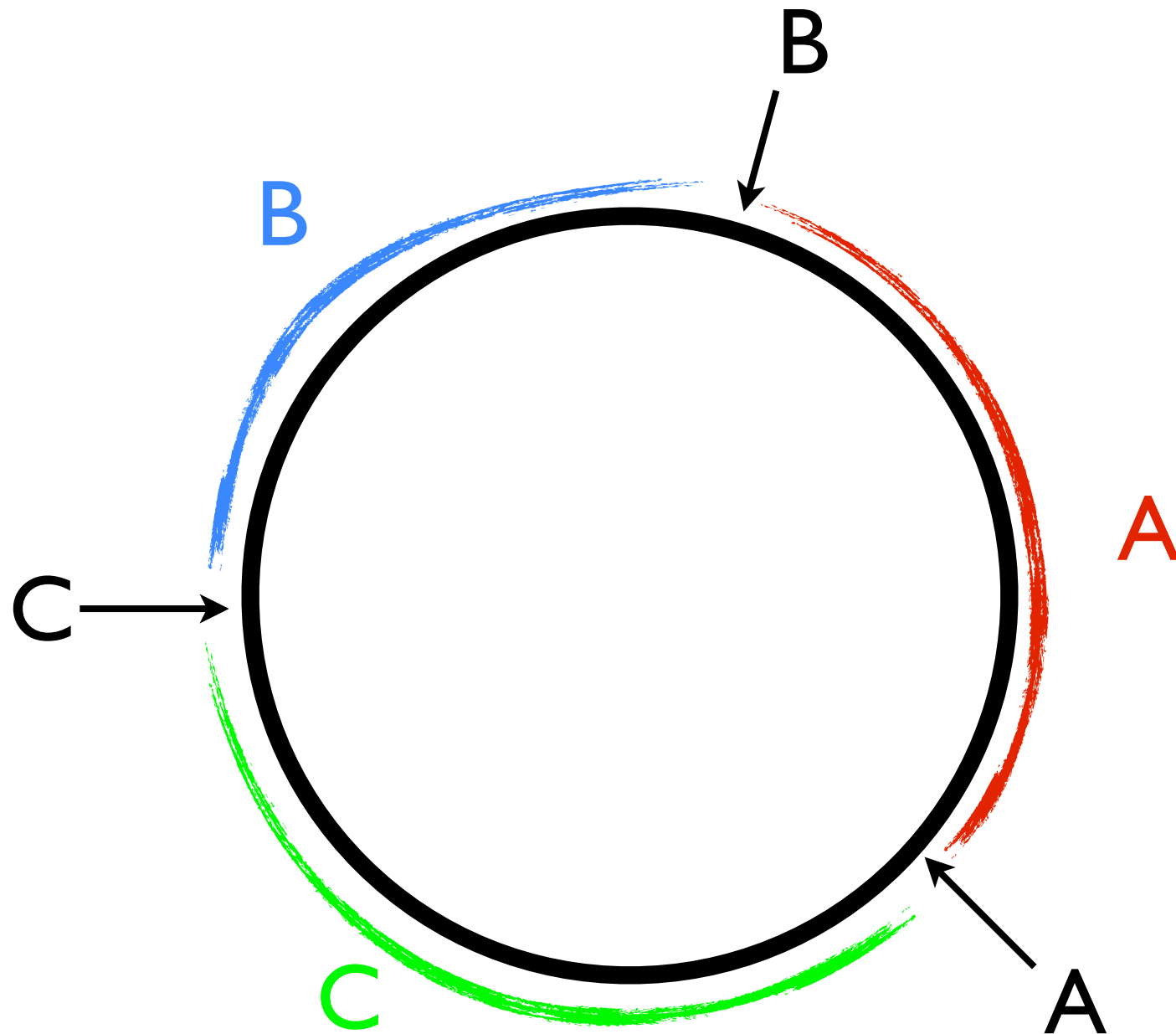
Each bucket contains values that hash to ring positions between its point and its **predecessor**



# Consistent Hashing



# Consistent Hashing



# Consistent Hashing

- Splits/Merges are cheap.
  - At most 2 buckets are affected.
  - No need for page duplication.
- Mapping hash value to bucket is expensive.
  - Need to have a lookup mechanism/directory.
- **Chord**: Decentralized lookup mechanism.



Any Questions?

# Summary

- Size of a hash table is important
  - Too big: Wasted Space/IOs
  - Too small: Collisions/Overflow Pages
- Dynamic hashing requires carefully managing how data is repartitioned.



# Index Keys

- Thus far, we've discussed single-value keys.
- We can also use multi-valued keys  $\langle A, B, C, \dots \rangle$ 
  - Equality Searches:  $A, B, C, \dots$  must all match
  - Range Searches
    - First Compare 'A's.
    - If 'A's equal, compare 'B's
    - If 'A's and 'B's equal, compare 'C's, ...

# Access Paths

- An access path is a method of retrieving tuples.
  - File Scan, **Scan of an Index** on a *Matching*  $\sigma$
- A Tree-Index matches (a conjunction of) terms that involve a **prefix** of the search key
  - Does a Tree-Index on  $\langle A, B, C \rangle$  match:
    - $A = 5$ ?
    - $A = 5 \text{ AND } B > 6$ ?
    - $A > 5 \text{ AND } B > 6$ ?
    - $A < 5 \text{ AND } A > 3$ ?
    - $B > 6$ ?

$A = 5$  is a prefix defining the range  $\langle 5, -\infty, -\infty \rangle, \langle 5, \infty, \infty \rangle$

$A = 5 \text{ and } B > 6$  is a prefix defining the range  $\langle 5, 6, -\infty \rangle, \langle 5, \infty, \infty \rangle$

$A > 5 \text{ and } B > 6$  is not a prefix, because there is no strict lower bound on the range of tuples. That is, if we used  $\langle 5, 6, -\infty \rangle$  as the lower bound for the index scan, we would still have to apply the selection predicate  $B > 6$  to eliminate tuples such as  $\langle 6, 4, 3 \rangle$  (which is greater than  $\langle 5, 6, -\infty \rangle$ , but does not fully satisfy the predicate). Note however, that  $A > 5$  is a prefix, and CAN be used as part of the access path.

# Access Paths

- An access path is a method of retrieving tuples.
  - File Scan, **Scan of an Index** on a *Matching*  $\sigma$
- A Hash Index Matches (a conjunction of) terms that have an equality for **every** attribute in the index.
- Does a Hash-Index on  $\langle A, B, C \rangle$  match:
  - $A = 5$ ?
  - $A = 5 \text{ AND } B = 6$ ?
  - $A < 5 \text{ AND } B = 6 \text{ AND } C = 4$ ?
  - $A = 5 \text{ AND } B = 6 \text{ AND } C = 4$ ?

$A = 5$  is not a match, because we have no unique key value for B or C

$A = 5 \text{ AND } B = 6$  is not a match, because we have no unique key value for C

$A < 5 \text{ AND } B = 6 \text{ AND } C = 4$  is not a match, because we have no unique key value for A

$A = 5 \text{ AND } B = 6 \text{ AND } C = 4$  is a match

# Access Path Cost

- General Strategy: Find the most **selective** access path to the data
  - The index, file, or combination of both that requires the fewest IOs to access the data.
  - Selection terms that match the index reduce the number of tuples *retrieved*.
  - The remaining terms discard tuples, but do not affect the number of pages fetched.



Any Questions?