CSE431/531, Problem Set 4 Due Monday Nov. 12, in class

Note: There are 6 problems with a total of 100 points. You are required to do all the problems.

1. (20 points) Given two circles C_1 and C_2 sharing the same center o on a plane, design a greedy algorithm to find a polygon P with the minimum number of edges to separate the two circles (i.e., the smaller circle is contained inside P and the larger circle is outside of P). State and prove the greedy choice property of this problem. State and prove the optimal substructure property of this problem.

2. (15 points) Prove that if all the weights in a graph G are distinct, then G has a unique minimal spanning tree.

3. (15 points) Let T and T' be two spanning trees of a connected graph G. Suppose that an edge e is in T but not in T'. Show that there is an edge e' in T', but not in T, such that $(T - \{e\}) \cup \{e'\}$ and $(T' - \{e'\}) \cup \{e\}$ are spanning trees of G.

4. (15 points) Suppose that n files having lengths $L_1, L_2, \cdots, L_n$ are stored on a tape. If the files are stored in the order of $i_1, i_2, \cdots, i_n$, then the time to retrieve file i_k is $T_k = \sum_{j=1}^{k} L_{i_j}$. The average retrieval time is defined as $\frac{1}{n}\sum_{k=1}^{n} T_k$. Design a greedy algorithm for determining the order of the n files on a tape so as to minimize the average retrieval time. Show that your algorithm is optimal by stating and proving the greedy choice property and the optimal substructure property.

5. (15 points) Show how to solve the fractional knapsack problem in $O(n)$ time, where n is the number of items.

6. (20 points) An n-vertex undirected graph is called a *scorpion graph* if it has a vertex of degree 1 (the sting) connected toa vertex of degree 2 (the tail) connected to a vertex of degree $n-2$ (the body) connected to the other $n-3$ vertices (the feet). Some of the feet may connect to other feet. Suppose that an adjacency matrix for an n-vertex undirected graph $G = (V, E)$ has been given. Let a *probe* be an operation that examines an entry of the adjacency matrix for the graph G. Design an $O(n)$-probe algorithm to determine whether G is a scorpion.