# Optimization: Rel. Alg. Equivalencies

R&G Chapter 14,15

(slides adapted from content by J.Gehrke, J.Shanmugasundaram, and/or C.Koch)

#### Last Class for Project Questions!

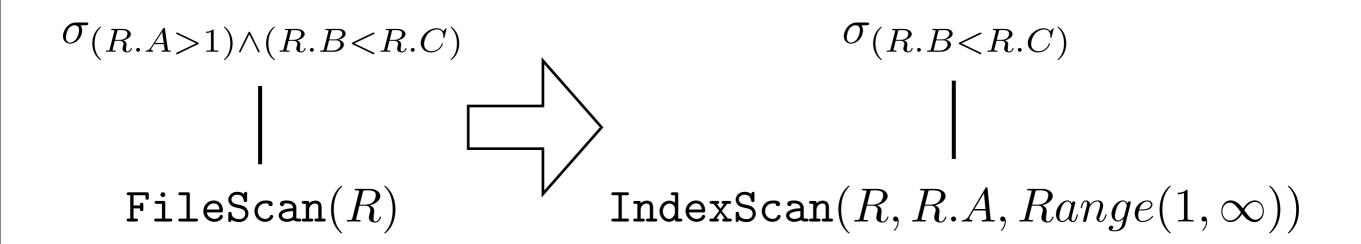
Project I due by 12:01 AM, Monday Morning (between Sunday and Monday)

(Are there any groups of 2 people looking for a 3rd?)

### Recap: Indexes

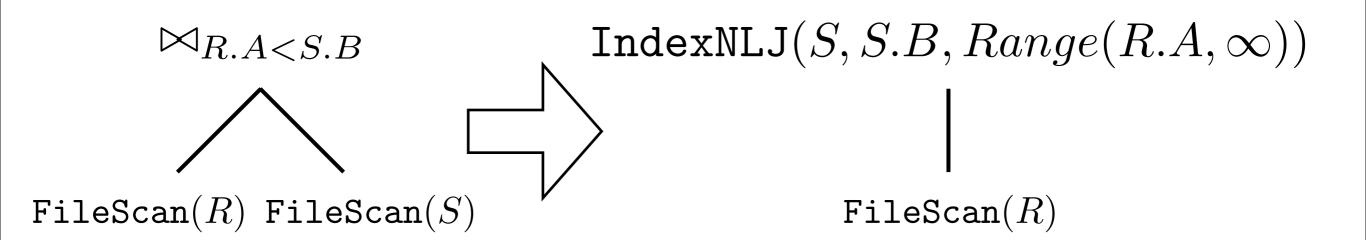
- Tree Indexes: ISAM, B+ Tree
  - Sort data, track page boundary values.
  - If over I page required for boundaries, recur.
- Hash Indexes: Static, Extendible, Linear
  - Use hash-fn to assign data to buckets
  - Buckets overflow, so use resizable hash-fns.

## Recap: Indexes



<u>Using Indexes I</u> Selection Predicate + File Scan = *Potential* Index Scan

## Recap: Indexes



Using Indexes 2

Join + 2xFile Scan = Potential Index Nested Loop

#### Index-Nested Loop

```
\begin{array}{c|c} \mathtt{IndexNLJ}(S,S.B,Range(R.A,\infty)) \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\
```

$$\begin{array}{l} \texttt{foreach} \ \langle A, \ldots \rangle \in R \\ \\ \texttt{foreach} \ \langle B, \ldots \rangle \in \texttt{IndexScan}(S, S.B, Range(A, \infty)) \\ \\ \texttt{emit} \ \langle A, \ldots, B, \ldots \rangle \end{array}$$



### Optimization

- Rewriting Queries into Equivalent Forms
- Some rewrites are always good ideas
  - Pushing down selections
  - Pushing down projections
- Some rewrites might be good ideas
  - Join reordering
- How do we get to these equivalent forms?







(No Beard, Good)

(Beard, Evil)

#### **Selection**

$$\sigma_{c_1 \wedge ... \wedge c_n}(R) \equiv \sigma_{c_1}(...(\sigma_{c_n}(R)))$$
 (Combinable) 
$$\sigma_{c_1}(\sigma_{c_2}(R)) \equiv \sigma_{c_2}(\sigma_{c_1}(R))$$
 (Commutative)

#### **Projection**

$$\pi_{a_1} \equiv \pi_{a_1}(\dots(\pi_{a_n}(R)))$$

(Combinable)

#### Cross Product (and Join)

$$R\bowtie (S\bowtie T)\equiv (R\bowtie S)\bowtie T$$
 (Associative)  $(R\bowtie S)\equiv (S\bowtie R)$  (Commutative)

#### **Selection**

$$\sigma_{c_1 \wedge \ldots \wedge c_n}(R) \equiv \sigma_{c_1}(\ldots(\sigma_{c_n}(R)))$$
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#### **Projection**

$$\pi_{a_1} \equiv \pi_{a_1}(\dots(\pi_{a_n}(R)))$$

(Combinable)

#### Cross Product (and Join)

$$R\bowtie (S\bowtie T)\equiv (R\bowtie S)\bowtie T$$
 (Associative)  $(R\bowtie S)\equiv (S\bowtie R)$  (Commutative)

Try It: Show that 
$$R\bowtie (S\bowtie T)\equiv (T\bowtie R)\bowtie S$$

#### **Selection**

$$\sigma_{c_1 \wedge ... \wedge c_n}(R) \equiv \sigma_{c_1}(...(\sigma_{c_n}(R)))$$
 (Combinable) 
$$\sigma_{c_1}(\sigma_{c_2}(R)) \equiv \sigma_{c_2}(\sigma_{c_1}(R))$$
 (Commutative)

#### **Projection**

$$\pi_{a_1} \equiv \pi_{a_1}(\dots(\pi_{a_n}(R)))$$
 (Combinable)

#### Cross Product (and Join)

$$R\bowtie (S\bowtie T)\equiv (R\bowtie S)\bowtie T$$
 (Associative) 
$$(R\bowtie S)\equiv (S\bowtie R)$$
 (Commutative)

These are general equivalencies.

We will use them to prove that our rewrites are correct.



$$\pi_a(\sigma_c(R)) \equiv \sigma_c(\pi_a(R))$$

Selection <u>commutes</u> with Projection (but only if attribute set **a** and condition **c** are *compatible*)

a must include all columns referenced by c

$$\pi_a(\sigma_c(R)) \equiv \sigma_c(\pi_a(R))$$

Selection <u>commutes</u> with Projection (but only if attribute set **a** and condition **c** are *compatible*)

a must include all columns referenced by c

$$\pi_a(\sigma_c(R)) \equiv \pi_a(\sigma_c(\pi_{a \cup cols(c)}(R)))$$



$$\sigma_c(R \times S) \equiv R \bowtie_c S$$

Selection combines with Cross Product to form a Join as per the definition of Join

(Note: This only helps if we have a join algorithm for conditions like  $\mathbf{c}$ )

$$\sigma_c(R \times S) \equiv R \bowtie_c S$$

Selection <u>combines</u> with Cross Product to form a Join as per the definition of Join (Note: This only helps if we have a join algorithm for conditions like **c**)

$$\sigma_{R.B=S.B\land R.A>3}(R\times S)\equiv\sigma_{R.A>3}(R\bowtie_{R.B=S.B}S)$$



$$\sigma_c(R \times S) \equiv (\sigma_c(R)) \times S$$

Selection <u>commutes</u> with Cross Product (but only if condition **c** references attributes of R exclusively)

$$\sigma_c(R \times S) \equiv (\sigma_c(R)) \times S$$

Selection <u>commutes</u> with Cross Product (but only if condition **c** references attributes of R exclusively)

$$\sigma_c(R \bowtie_{c'} S) \equiv (\sigma_c(R)) \bowtie_{c'} S$$

$$\sigma_c(R \times S) \equiv (\sigma_c(R)) \times S$$

Selection commutes with Cross Product (but only if condition c references attributes of R exclusively)

$$\sigma_c(R \bowtie_{c'} S) \equiv (\sigma_c(R)) \bowtie_{c'} S$$

$$\sigma_{R.B=S.B\land R.A>3}(R\times S)\equiv(\sigma_{R.A>3}(R))\bowtie_{R.B=S.B}S$$



$$\pi_a(R \times S) \equiv (\pi_{a_1}(R)) \times (\pi_{a_2}(S))$$

Projection commutes (distributes) over Cross Product (where a<sub>1</sub> and a<sub>2</sub> are the attributes in a from R and S respectively)

$$\pi_a(R \times S) \equiv (\pi_{a_1}(R)) \times (\pi_{a_2}(S))$$

Projection commutes (distributes) over Cross Product (where a<sub>1</sub> and a<sub>2</sub> are the attributes in a from R and S respectively)

$$\pi_a(R \bowtie_c S) \equiv (\pi_{a_1}(R)) \bowtie_c (\pi_{a_2}(S))$$

$$\pi_a(R \times S) \equiv (\pi_{a_1}(R)) \times (\pi_{a_2}(S))$$

Projection commutes (distributes) over Cross Product (where a<sub>1</sub> and a<sub>2</sub> are the attributes in a from R and S respectively)

#### Show that

$$\pi_a(R \bowtie_c S) \equiv (\pi_{a_1}(R)) \bowtie_c (\pi_{a_2}(S))$$

(under what condition)

How can we work around this limitation?

$$\pi_a(R \times S) \equiv (\pi_{a_1}(R)) \times (\pi_{a_2}(S))$$

Projection commutes (distributes) over Cross Product (where a<sub>1</sub> and a<sub>2</sub> are the attributes in a from R and S respectively)

#### Show that

$$\pi_a(R \bowtie_c S) \equiv (\pi_{a_1}(R)) \bowtie_c (\pi_{a_2}(S))$$

(under what condition)

How can we work around this limitation?

$$\pi_a((\pi_{a_1 \cup (\mathsf{cols}(c) \cap \mathsf{cols}(R))}(R)) \bowtie_c (\pi_{a_2 \cup (\mathsf{cols}(c) \cap \mathsf{cols}(S))}(S)))$$



Union and Intersections are Commutative and Associative

Selection and Projection both commute with both Union and Intersection

#### Enumerating Possible Plans

- There are many algorithms suitable for processing a specific data set:
  - 1. How do we identify feasible algorithms.
  - 2. How do we choose between them