CSE431/531, Problem Set 2 Due Monday Oct. 8, in class

Note: There are 6 problems with a total of 100 points. You are required to do all the problems.

1. (15 points) In the Selection algorithm discussed in class, we partition elements into groups of size 5 each. Is it possible to achieve an $O(n)$-time algorithm by partitioning elements into groups of size $3, 4, 6, 7, 9,$ and 11? Justify your answer by giving a detailed analysis of the running time of the selection algorithm for each of the 6 different sizes.

2. (20 points) Let $P = \{p_1, p_2, \cdots, p_n\}$ be n points on a 2D plane with each $p_i = (x_i, y_i)$ for $i = 1, \cdots, n$. We say that point p_i dominates p_j if $x_i \geq x_j$ and $y_i \geq y_j$. A point p_i is called a maximum point if it is not dominated by any other point in P. Design an $O(n \log n)$-time algorithm to find all maximum points. If the points in P are in 3D space (i.e., each point $p_i = (x_i, y_i, z_i)$), extend your algorithm to solve the same maximum point problem in 3D space (where a maximum point should also not be dominated by any other point in the z direction). You should make your algorithm run as fast as possible.

3. (20 points) Let $a_1, \cdots, a_n$ be n distinct real numbers, and $w_1, \cdots, w_n$ be a set of n positive weights with $w_1 + \cdots + w_n = 1$. The weighted median of the set $\{a_1, \cdots, a_n\}$ is the number a_k for which $\sum_{i:a_i<a_k} w_i < \frac{1}{2}$ and $\sum_{i:a_i>a_k} w_i \leq \frac{1}{2}$. (a) Prove that such an a_k always exists. (b) Give a $\Theta(n)$ worst-case running time algorithm computing the weighted median.

4. (15 points) Given a set $S = \{a_1, \cdots, a_n\}$ of n unsorted real numbers and a real value B, design an $O(n^2)$-time algorithm to determine whether there exist three distinct numbers a_i, a_j and a_k in S such that $a_i + a_j + a_k = B$.

5. (15 points) Given an array $A = \{a_1, \cdots, a_n\}$ of n unsorted numbers, design an $O(n \log n)$-time algorithm for reporting the number of inversions in A. An inversion in A is a pair of numbers a_i and a_j such that $i < j$ but $a_i \geq a_j$.

6. (15 points) In the Strassen's matrix multiplication algorithm, we have

$$
\begin{aligned}
p_1 = (a-c)(s+t) &= as + at - cs - ct \\
p_2 = (b-d)(u+v) &= bu + bv - du - dv \\
p_3 = (a+d)(s+v) &= as + dv + av + ds \\
p_4 = a(t-v) &= at - av \\
p_5 = (a+b)v &= av + bv \\
p_6 = (c+d)s &= cs + ds \\
p_7 = d(u-s) &= du - ds
\end{aligned}
$$

Write the followings in terms of p_i's:

$$
\begin{gathered}
as + bu = ??? \\
at + bv = ??? \\
cs + du = ??? \\
ct + dv = ???
\end{gathered}
$$