1. (15 points) In the Selection algorithm discussed in class, we partition elements into groups

of size 5 each. Is it possible to achieve an O(n)-time algorithm by partitioning elements into

groups of size 3; 4; 6; 7; 9, and 11? Justify your answer by giving a detailed analysis of the

running time of the selection algorithm for each of the 6 different sizes.

Answer:

Group size 3,suppose x is the pivot number,

the

number of elements greater than x is at least

2. (20 points) Let P = fp1; p2; \_ \_ \_ ; png be n points on a 2D plane with each pi = (xi; yi) for

i = 1; \_ \_ \_ ; n. We say that point pi dominates pj if xi \_ xj and yi \_ yj . A point pi is called

a maximum point if it is not dominated by any other point in P. Design an O(n log n)-time

algorithm to \_nd all maximum points. If the points in P are in 3D space (i.e., each point

pi = (xi; yi; zi)), extend your algorithm to solve the same maximum point problem in 3D space

(where a maximum point should also not be dominated by any other point in the z direction).

You should make your algorithm run as fast as possible.

3. (20 points) Let a1; \_ \_ \_ ; an be n distinct real numbers, and w1; \_ \_ \_ ;wn be a set of n positive

weights with w1 +\_ \_ \_+wn = 1. The weighted median of the set fa1; \_ \_ \_ ; ang is the number ak

for which

P

i:ai<ak wi < 1

2 and

P

i:ai>ak wi \_ 1

2 . (a) Prove that such an ak always exists. (b)

Give a \_(n) worst-case running time algorithm computing the weighted median.

4. (15 points) Given a set S = fa1; \_ \_ \_ ; ang of n unsorted real numbers and a real value B, design

an O(n2)-time algorithm to determine whether there exist three distinct numbers ai, aj and

ak in S such that ai + aj + ak = B.

5. (15 points) Given an array A = fa1; \_ \_ \_ ; ang of n unsorted numbers, design an O(n log n)-time

algorithm for reporting the number of inversions in A. An inversion in A is a pair of numbers

ai and aj such that i < j but ai \_ aj .

6. (15 points) In the Strassen's matrix multiplication algorithm, we have

p1 = (a 􀀀 c)(s + t) = as + at 􀀀 cs 􀀀 ct

p2 = (b 􀀀 d)(u + v) = bu + bv 􀀀 du 􀀀 dv

p3 = (a + d)(s + v) = as + dv + av + ds

p4 = a(t 􀀀 v) = at 􀀀 av

p5 = (a + b)v = av + bv

p6 = (c + d)s = cs + ds

p7 = d(u 􀀀 s) = du 􀀀 ds

Write the followings in terms of pi's:

as + bu =???

at + bv =???

cs + du =???

ct + dv =???