Regular language 正则语言

Definition 定义

Previous: A language is regular if it is recognized by some **DFA**

上一篇： 如果一种语言被某些 **DFA 识别** ，则该语言是常规的

Now: A language is regular if and only if some **NFA** recognizes it.

现在：当且仅当某些 **NFA** 识别出一种语言时，它才是常规的

Some operations on languages: Union, Concatenation and Kleene star

对语言的一些操作：Union、Concatenation 和 Kleene star

Closed under operation

A collection S of objects is **closed** under operation *f* if applying *f* to members of S always returns an object still in S.

如果对 S 的成员应用 *f* 总是返回仍在 S 中的对象，则对象的集合 S 在操作 *f* 下是 **关闭的。**

Regular languages are indeed closed under the regular operations (e.g. union, concatenation, star ...)

常规语言在常规操作下确实是封闭的（例如 union、concatenation、star ...）

Regular Languages Closed Under Union

The set of regular languages is closed under the union operation.

常规语言集在 union 操作下关闭。

- i.e. A and B are regular languages over the same alphabet Σ, then A∪B is also a regular language.

  即 A 和 B 是同一字母 Σ 上的常规语言，那么 A∪B 也是一种常规语言。

Proof:

- Since A and B are regular languages, there are finite automata M1 = (Q_1 , Σ, δ_1 ,q_1 , F_1) and M2 = (Q_2 , Σ, δ_2 , q_2 , F_2) that accept A and B, respectively.

  由于 A 和 B 是常规语言，因此存在有限自动机 M1 = （Q_1 ， Σ， δ_1 ， q_1 ， F_1 ） 和 M2 = （Q_2 ， Σ， δ_2 、q_2 、F_2 ），分别接受 A 和 B。
- In order to prove that A ∪ B is regular, we have to construct a finite automaton M that accepts A ∪ B. In other words, M must have the property that for every string w ∈ Σ* :

  为了证明 A ∪ B 是正则的，我们必须构造一个接受 A ∪ B 的有限自动机 M。换句话说， M 必须具有对于每个字符串 w ∈ Σ* 的属性：

Continue to proof

Given M1 and M2 that A = L(M1) and B = L(M2), we can define M = (Q, Σ, δ, q, F):

给定 M1 和 M2 ，其中 A = L（M1 ） 和 B = L（M2 ），我们可以定义 M = （Q， Σ， δ， q， F）：

- $Q = Q_1 \times Q_2 = \{(q_1, q_2): q_1 \in Q_1$ and $q_2 \in Q_2\}$

- Σ is same as the alphabet of A and B
- $q = (q_1, q_2)$
- $F = \{(q_1, q_2): q_1 \in Q_1 \text{ or } q_2 \in Q_2\}$
- $\delta : Q \times \Sigma \to Q$

  $\delta((q_1, q_2), a) = (\delta(q_1, a), \delta(q_2, a)), a \in \Sigma$

Continue to proof

- $\delta * ((q_1, q_2), w) = (\delta * (q_1, w), \delta * (q_2, w))$
- $\delta * ((q_1, q_2), w) \in F \Leftrightarrow \delta * (q_1, w) \in F_1$ or $\delta * (q_2, w) \in F_2$
- M accepts $w \Leftrightarrow \delta * (q_1, w) \in F_1$ or $\delta * (q_2, w) \in$ F2
- M accepts $w \Leftrightarrow$ M_1 accepts w or M_2 accepts w

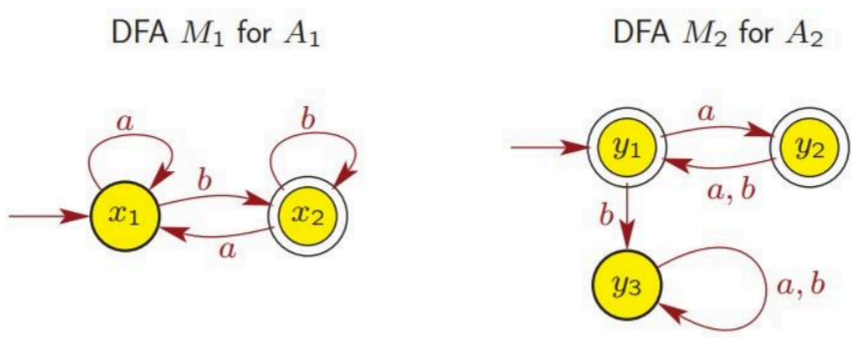
Proved

Example:

Consider the following DFAs and languages over $\Sigma = \{a, b\}$:

DFA M_1 recognizes $A_1 = L(M_1)$

DFA M_2 recognizes $A_2 = L(M_2)$



DFA M for A1 ∪ A2 ?

Example2:

How to prove this from the perspective of NFA?

如何从 NFA 的角度证明这一点?

Proof

Consider the following NFAs:

NFA M1 = $(Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes $A_1 = L(M_1)$

NFA M2 = $(Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes $A_2 = L(M_2)$

We will construct an NFA M = $(Q, \Sigma, \delta, q, F)$

- $Q = \{q_0\} \cup Q_1 \cup Q_2$
- q_0 is the start state of M

- $F = F_1 \cup F_2$
- $\delta : Q \times \Sigma\epsilon \to P(Q)$ is defined as: For any $r \in Q$ and for any $a \in \Sigma\epsilon$

$$\delta(r,a) = \begin{cases} \delta_1(r,a) & \text{if } r \in Q_1, \\ \delta_2(r,a) & \text{if } r \in Q_2, \\ \{q_1, q_2\} & \text{if } r = q_0 \text{ and } a = \epsilon, \\ \emptyset & \text{if } r = q_0 \text{ and } a \neq \epsilon. \end{cases}$$

Proof:

Regular Languages Closed Under Concatenation

The concatenation of A1 and A2 is defined as:

A1 和 A2 中的串联定义为:

- $A_1 A_2 = \{ww' : w \in A_1 \text{ and } w' \in A_2\}$

Proof

Consider the following NFAs:

NFA M1 = (Q_1 , Σ, δ_1 , q_1 , F_1) recognizes $A_1 = L(M_1)$

NFA M2 = (Q_2 , Σ, δ_2 , q_2 , F_2) recognizes $A_2 = L(M_2)$

We will construct an NFA M = (Q, Σ, δ, q, F) for $A_1 A_2$

- $Q = Q_1 \cup Q_2$
- M has the same start state as M_1 : q_1
- Set of accept states of M is same as M_2 : F_2
- $\delta : Q \times \Sigma\epsilon \to P(Q)$ is defined as: For any $r \in Q$ and for any $a \in \Sigma_\varepsilon$

$$\delta(r,a) = \begin{cases} \delta_1(r,a) & \text{if } r \in Q_1 \text{ and } r \notin F_1, \\ \delta_1(r,a) & \text{if } r \in F_1 \text{ and } a \neq \epsilon, \\ \delta_1(r,a) \cup \{q_2\} & \text{if } r \in F_1 \text{ and } a = \epsilon, \\ \delta_2(r,a) & \text{if } r \in Q_2. \end{cases}$$

Proof:

Regular Languages Closed Under Kleene star

The star of A is defined as:

- $A^* = \{u_1 u_2 \ldots u_k : k \geq 0 \text{ and } u_i \in \mathbf{A} \text{ for all } i = 1, 2, \ldots, k\}$

Proof:

Consider the following NFA:

NFA M1 = (Q_1 , Σ, δ_1 , q_1, F_1*) recognizes A = L(M_1)

We will construct an NFA M = (Q, Σ, δ, q, F) for A*

- $Q = \{q_0\} \cup Q_1$
- q_0 is the start state of M
- $F = \{q_0\} \cup F_1$
- $\delta : Q \times \Sigma\epsilon \to P(Q)$ is defined as: For any $r \in Q$ and for any $a \in \Sigma\epsilon$

$$\delta(r,a) = \begin{cases} \delta_1(r,a) & \text{if } r \in Q_1 \text{ and } r \notin F_1, \\ \delta_1(r,a) & \text{if } r \in F_1 \text{ and } a \neq \epsilon, \\ \delta_1(r,a) \cup \{q_1\} & \text{if } r \in F_1 \text{ and } a = \epsilon, \\ \{q_1\} & \text{if } r = q_0 \text{ and } a = \epsilon, \\ \emptyset & \text{if } r = q_0 \text{ and } a \neq \epsilon. \end{cases}$$

Proof

Regular Languages Closed Under Complement and Interaction

The set of regular languages is closed under the complement and interaction operations:

常规语言集在 complement 和 interaction 操作下是封闭的：

- If A is a regular language over the alphabet Σ, then the complement:
  - $\overline{A} = \{w \in \Sigma^*: w \notin A\}$ is also a regular language.
- If A1 and A2 are regular languages over the same alphabet Σ, then the interaction:

  如果 A1 和 A2 是同一字母 Σ 上的常规语言，则交互作用：
  - $A_1 \cap A_2 = \{w \in \Sigma^*: w \in A_1 \text{ and } w \in A_2\}$ is also a regular language.

Regular expression

Regular expressions are means to describe certain languages.

正则表达式是描述某些语言的手段。

Example:

Consider the expression:

- $(0 \cup 1)01^*$

The language described by this expression is the set of all binary strings satisfy:

此表达式描述的语言是满足以下条件的所有二进制字符串的集合：

- that start with either 0 or 1 (this is indicated by (0 ∪ 1)),

  以 0 或 1 开头（由 （0 ∪ 1） 表示），
- for which the second symbol is 0 (this is indicated by 0),

第二个符号为 0（用 0 表示），

- that end with zero or more 1s (this is indicated by 1^*).

  以零个或多个 1 结尾（用 $1*$ 表示）。

Further examples:

- The language {*w* : *w* contains exactly two 0s} is described by the expression:

  语言 {*w*： *w* 正好包含两个 0} 由表达式描述：
- The language {*w* : *w* contains at least two 0s} is described by the expression:

  语言 {*w*： *w* 包含至少两个 0} 由表达式描述：
- The language {*w* : 1011 is a substring of *w*} is described by the expression:

  语言 {*w*： 1011 是 *w*} 的子字符串，由表达式描述：

Formal Definition of regular expression

Let Σ be a non-empty alphabet.

设 Σ 为非空字母表。

1. ϵ is a regular expression.
2. $\emptyset$ is a regular expression.
3. For each $a \in \Sigma$, a is a regular expression.
4. If R_1 and R_2 are regular expressions, then R_1 R_2 is a regular expression.
5. If R_1 and R_2 are regular expressions, then R_1 R_2 is a regular expression.
6. If R is a regular expression, then R^* is a regular expression.

Exercise:

Given $(0\cup1)^*101(0\cup1)^*$, prove it is a regular expression (note: Σ={0, 1}).

Further Definition:

If *R* is a regular expression, then L(*R*) is the **language** generated (or described or defined) by *R*.

如果 *R* 是一个正则表达式，那么 L（*R*） 是由 *R* 生成（或描述或定义）的 **语言。**

Formal Definition of regular expressions 正则表达式的正式定义

Let Σ be a non-empty alphabet.

设 Σ 为非空字母表。

1. The regular expression ϵ describes the language $\{\epsilon\}$.
2. The regular expression $\emptyset$ describes the language $\emptyset$.
3. For each $a \in \Sigma$, the regular expression a describes the language $\{a\}$.
4. Let R_1 and R_2 be regular expressions and let L1 and L2 be the languages described by them, respectively. The regular expression $R_1 \cup R_2$ describes the language L1 ∪ L2 .
5. Let R_1 and R_2 be regular expressions and let L1 and L2 be the languages described by them, respectively. The regular expression R_1R_2 describes the language L1L2 .

6. Let R be a regular expression and let L be the language described by it. The regular expression R^* describes the language $L*$.

Example;

Given a regular expression $(0 \cup \epsilon)\, 1^*$, it describes the language:

$$\{0, 01, 011, 0111, \ldots, \epsilon, 1, 11, 111, \ldots\}.$$

- Observe that this language is also described by the regular expression $01^* \cup 1^*$