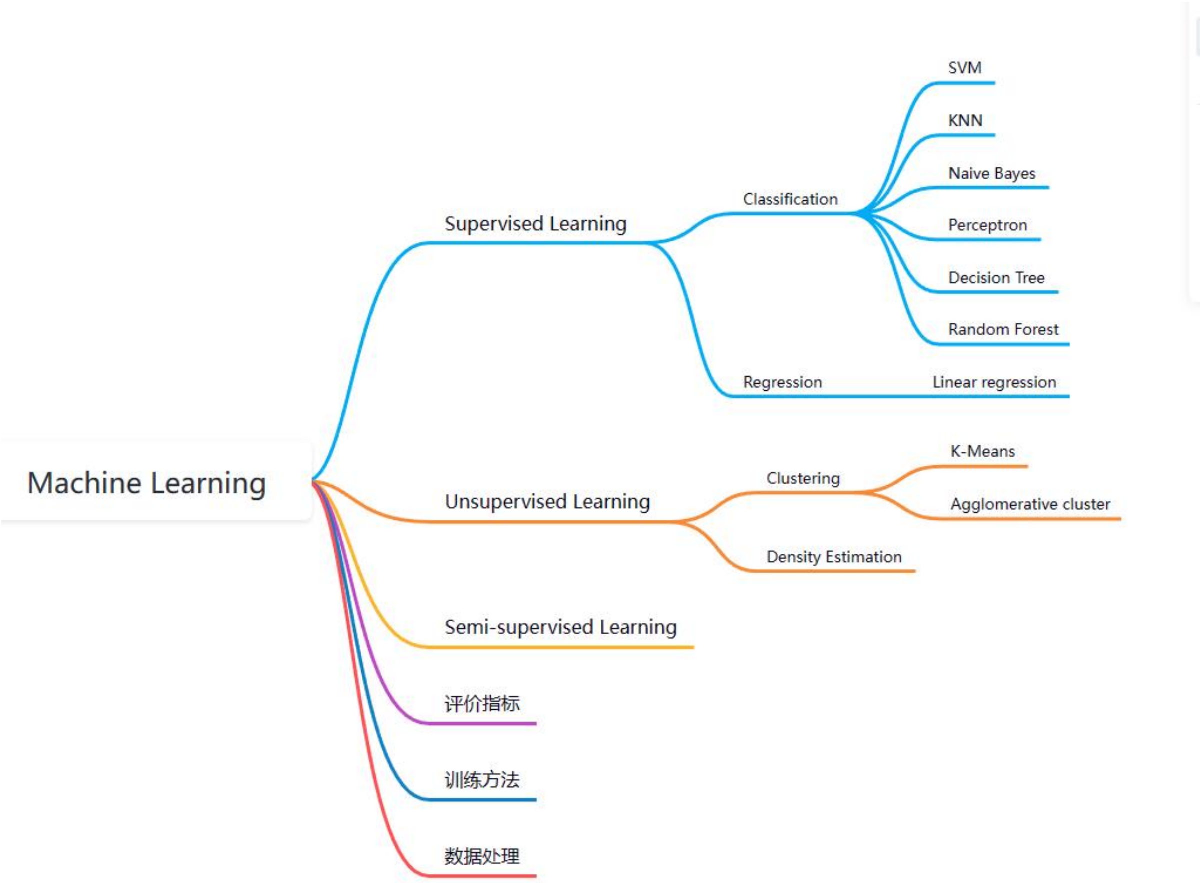
INT104 期末复习



根据往年试题，考试形式不外有监督/无监督模型算法(2021 年)。大题以模型的计算展开。

1.KNN

1. (10 points) The table below shows a training set with 10 examples that is used for training a **3-nearest-neighbors** classifier that uses Manhattan distance, i.e., the distance between two points at coordinates p and q is $|p-q|$. The only attribute, X, is real-valued, and the label Y has two possible classes, 0 and 1. The first fold contains the first 5 examples, and the second fold contains that last 5 examples. In case of ties in distance, use the example with smallest X value as the neighbor. Please compute the 2-fold cross validation accuracy (percentage correct classification).

X	0	1	2	3	4	5	6	7	8	9
Y	1	0	1	0	1	0	1	0	1	0

2.K-means

2. (10 points) You want to cluster 7 points into 3 clusters using **the k-means clustering** algorithm. Suppose after the first iteration, clusters C_1, C_2 and C_3 contain the following two-dimensional points:

   C_1 contains the 2 points: $\{(0,6),(6,0)\}$

   C_2 contains the 3 points: $\{(2,2),(4,4),(6,6)\}$

   C_3 contains the 2 points: $\{(5,5),(7,7)\}$

   Please compute the coordinates of **cluster centers** for these 3 clusters.

2. K-means

$$C_1 = \left(\frac{0+6}{2}, \frac{6+0}{2}\right) = (3,3)$$

$$C_2 = \left(\frac{2+4+6}{3}, \frac{2+4+6}{3}\right) = (4,4)$$

$$C_3 = \left(\frac{5+7}{2}, \frac{5+7}{2}\right) = (6,6)$$

3.Naive Bayes(朴素贝叶斯)

3. (20 points) The following dataset as in the table is provided to build a naive Bayes classifier, where $\{x_1, x_2, x_3, x_4\}$ and l are the features and the label, respectively. Please give the process of building the classifier and predict the label of the unknown instance $\boldsymbol{x} = [1, 0, 1, 1]^T$.

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & l \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

3、 $P(y|x) = \dfrac{P(y)P(x|y)}{P(x)}$

$$\underbrace{P(y|x)}_{\text{posterior}} \propto \underbrace{P(y)}_{\text{prior}}\underbrace{P(x|y)}_{\text{CCP}}$$

① when $y=0$

$P(y=0) = \frac{3}{4}$

直接得到

$P(x_1=0|y=0) = \frac{2}{3}$, $P(x_1=1|y=0) = \frac{1}{3}$

$P(x_2=0|y=0) = \frac{1}{3}$, $P(x_2=1|y=0) = \frac{2}{3}$

$P(x_3=0|y=0) = \frac{1}{3}$, $P(u) = \frac{2}{3}$

$P(x_4=0|y=0) = \frac{2}{3}$, $P(u) = \frac{1}{3}$

因此可以得到:

$x = [1,0,1,1]^T$

$$P(y=0|x) = \frac{P(y=0)\cdot P(x|y=0)}{P(x)}$$

$$\propto P(y=0)\cdot P(x|y=0)$$

$$= P(x_1=1, x_2=0, x_3=1, x_4=1 | y=0)$$

$$= \prod_{i=1}^{4} P(x_i|y=0) = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} = \frac{2}{3^4}$$

$$= \frac{2}{81}$$

4.感知机模型

4. (20 points) Perceptron is a function that maps input $\boldsymbol{x}$ to a label as follows

$$f(\boldsymbol{x}) = \begin{cases} 1, & w \cdot x + b > 0 \\ 0, & \text{otherwise} \end{cases}$$

Now consider solving the logical **OR** and logical **XOR** problems (as shown in two tables) with the perceptron model.

$$y = f(\boldsymbol{x}) = \begin{cases} 1, & w_1 x_1 + w_2 x_2 + b > 0 \\ 0, & \text{otherwise} \end{cases}$$

Table 1: Logical OR

x_1	x_2	y
0	1	1
1	1	1
1	0	1
0	0	0

Table 2: Logical XOR

x_1	x_2	y
0	1	1
1	1	0
1	0	1
0	0	0

1) (4 points) Please draw all datapoints of the tables in the two-dimensional space for logical OR and logical XOR problems, respectively, where different classes are marked with different shapes.

2) (16 points) Please explain seperately whether the perceptron can mimic the output of logical OR and logical XOR or not. If so, please give an example of function $f(\boldsymbol{x})$; if not, please prove that there is no such function $f(\boldsymbol{x})$.

第一题

1. (10 points) The table below shows a training set with 10 examples that is used for training a **3-nearest-neighbors** classifier that uses Manhattan distance, i.e., the distance between two points at coordinates p and q is $|p-q|$. The only attribute, X, is real-valued, and the label Y has two possible classes, 0 and 1. The first fold contains the first 5 examples, and the second fold contains that last 5 examples. In case of ties in distance, use the example with smallest X value as the neighbor. Please compute the 2-fold cross validation accuracy (percentage correct classification).

X	0	1	2	3	4	5	6	7	8	9
Y	1	0	1	0	1	0	1	0	1	0

2020-2021 -final exam

1. KNN: $\hat{x}$ 训练集，x 测试集

$\hat{y}$ 为预测值

① $\hat{x} \in [0,4]$, $x \in [5,9]$

Input	3-nearest-neighbours	Predict
$X=5$	$X=2,3,4$	$y=1$
$X=6$	$X=2,3,4$	$y=1$
$X=7$	$X=2,3,4$	$y=1$
$X=8$	$X=2,3,4$	$y=1$
$X=9$	$X=2,3,4$	$y=1$

The accuracy of this fold is

$$A_1 = \frac{1}{5}\sum_{i=0} I(y_i = \hat{y}_i)$$

$$= \frac{1}{5}(0+1+0+1+0)$$

$$= \frac{1}{5} \times 2 = \frac{2}{5} = 40\%$$

② $\hat{x} \in [5,9]$, $x \in [0,4]$

Input	3-NN	predict
$X=0$	$X=5,6,7$	$y=0$
$X=1$	$X=5,6,7$	$y=0$
$X=2$	$X=5,6,7$	$y=0$
$X=3$	$X=5,6,7$	$y=0$
$X=4$	$X=5,6,7$	$y=0$

$$A_2 = \frac{1}{5}\sum_{i=0} I(y_i = \hat{y}_i) = \frac{1}{5}(0+1+0+1+0)$$

$$= \frac{2}{5} = 40\%$$

So the total accuracy A is

$$A = \frac{\sum A_i}{n} = \frac{A_1 + A_2}{2} = \frac{\frac{2}{5} + \frac{2}{5}}{2} = \frac{2}{5}$$

$$= 40\%$$

例题

Agglomerative Clustering(层次聚类)

例 14.1 给定 5 个样本的集合，样本之间的欧氏距离由如下矩阵 D 表示：

$$D = [d_{ij}]_{5\times 5} = \begin{bmatrix} 0 & 7 & 2 & 9 & 3 \\ 7 & 0 & 5 & 4 & 6 \\ 2 & 5 & 0 & 8 & 1 \\ 9 & 4 & 8 & 0 & 5 \\ 3 & 6 & 1 & 5 & 0 \end{bmatrix}$$

G8 G7 G9

G6

其中 d_{ij} 表示第 i 个样本与第 j 个样本之间的欧氏距离。显然 D 为对称矩阵。应用聚合层次聚类法对这 5 个样本进行聚类。

解 (1) 首先用 5 个样本构建 5 个类，$G_i = \{x_i\}$，$i = 1, 2, \cdots, 5$，这样，样本之间的距离也就变成类之间的距离，所以 5 个类之间的距离矩阵亦为 D。

(2) 由矩阵 D 可以看出，$D_{35} = D_{53} = 1$ 为最小，所以把 G_3 和 G_5 合并为一个新类，记作 $G_6 = \{x_3, x_5\}$。

(3) 计算 G_6 与 G_1，G_2，G_4 之间的最短距离，有

$$D_{61} = 2, \quad D_{62} = 5, \quad D_{64} = 5$$

又注意到其余两类之间的距离是

$$D_{12} = 7, \quad D_{14} = 9, \quad D_{24} = 4$$

显然，$D_{61} = 2$ 最小，所以将 G_1 与 G_6 合并成一个新类，记作 $G_7 = \{x_1, x_3, x_5\}$。

(4) 计算 G_7 与 G_2，G_4 之间的最短距离：

$$D_{72} = 5, \quad D_{74} = 5$$

又注意到

$$D_{24} = 4$$

显然，其中 $D_{24} = 4$ 最小，所以将 G_2 与 G_4 合并成一个新类，记作 $G_8 = \{x_2, x_4\}$。

(5) 将 G_7 与 G_8 合并成一个新类，记作 $G_9 = \{x_1, x_2, x_3, x_4, x_5\}$，即将全部样本聚成一类，聚类终止。 ■

上述层次聚类过程可以用图 14.2 所示的层次聚类图表示。

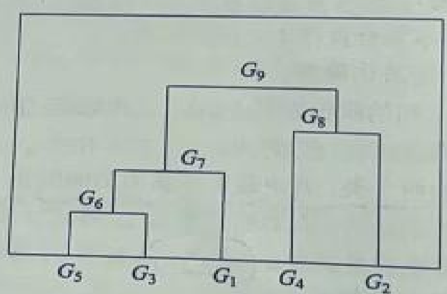


图 14.2 层次聚类图

K-Means

例 14.2 给定含有 5 个样本的集合

$$X = \begin{bmatrix} 0 & 0 & 1 & 5 & 5 \\ 2 & 0 & 0 & 0 & 2 \end{bmatrix}$$

试用 k 均值聚类算法将样本聚到两个类中。

解 按照算法 14.2：

初始化

(1) 选择两个样本点作为类的中心。假设选择 $m_1^{(0)} = x_1 = (0,2)^{\mathrm{T}}$，$m_2^{(0)} = x_2 = (0,0)^{\mathrm{T}}$。

(2) 以 $m_1^{(0)}$，$m_2^{(0)}$ 为类 $G_1^{(0)}$，$G_2^{(0)}$ 的中心，计算 $x_3 = (1,0)^{\mathrm{T}}$，$x_4 = (5,0)^{\mathrm{T}}$，$x_5 = (5,2)^{\mathrm{T}}$ 与 $m_1^{(0)} = (0,2)^{\mathrm{T}}$，$m_2^{(0)} = (0,0)^{\mathrm{T}}$ 的欧氏距离平方。

(a) 对 $x_3 = (1,0)^{\mathrm{T}}$，$d(x_3, m_1^{(0)}) = 5$，$d(x_3, m_2^{(0)}) = 1$，将 x_3 分到类 $G_2^{(0)}$。

(b) 对 $x_4 = (5,0)^{\mathrm{T}}$，$d(x_4, m_1^{(0)}) = 29$，$d(x_4, m_2^{(0)}) = 25$，将 x_4 分到类 $G_2^{(0)}$。

(c) 对 $x_5 = (5,2)^{\mathrm{T}}$，$d(x_5, m_1^{(0)}) = 25$，$d(x_5, m_2^{(0)}) = 29$，将 x_5 分到类 $G_1^{(0)}$。

(3) 得到新的类 $G_1^{(1)} = \{x_1, x_5\}$，$G_2^{(1)} = \{x_2, x_3, x_4\}$，计算类的中心 $m_1^{(1)}$，$m_2^{(1)}$：

$$m_1^{(1)} = (2.5, 2.0)^{\mathrm{T}}, \quad m_2^{(1)} = (2,0)^{\mathrm{T}}$$

(4) 重复步骤 (2) 和步骤 (3)。将 x_1 分到类 $G_1^{(1)}$，将 x_2 分到类 $G_2^{(1)}$，x_3 分到类 $G_2^{(1)}$，x_4 分到类 $G_2^{(1)}$，x_5 分到类 $G_1^{(1)}$，得到新的类 $G_1^{(2)} = \{x_1, x_5\}$，$G_2^{(2)} = \{x_2, x_3, x_4\}$。

由于得到的新的类没有改变，聚类停止。得到聚类结果：

$$G_1^* = \{x_1, x_5\}, \quad G_2^* = \{x_2, x_3, x_4\}$$

■

朴素贝叶斯

例 4.1 试由表 4.1 的训练数据学习一个朴素贝叶斯分类器并确定 $x=(2,S)^{\mathrm{T}}$ 的类标记 y。表中 $X^{(1)}$，$X^{(2)}$ 为特征，取值的集合分别为 $A_1=\{1,2,3\}$，$A_2=\{S,M,L\}$，Y 为类标记，$Y\in C=\{1,-1\}$。

表 4.1　训练数据

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$X^{(1)}$	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3
$X^{(2)}$	S	M	M	S	S	S	M	M	L	L	L	M	M	L	L
Y	-1	-1	1	1	-1	-1	-1	1	1	1	1	1	1	1	-1

解 根据算法 4.1，由表 4.1 容易计算下列概率：

$$P(Y=1)=\frac{9}{15},\quad P(Y=-1)=\frac{6}{15}$$

$$P(X^{(1)}=1|Y=1)=\frac{2}{9},\quad P(X^{(1)}=2|Y=1)=\frac{3}{9},\quad P(X^{(1)}=3|Y=1)=\frac{4}{9}$$

$$P(X^{(2)}=S|Y=1)=\frac{1}{9},\quad P(X^{(2)}=M|Y=1)=\frac{4}{9},\quad P(X^{(2)}=L|Y=1)=\frac{4}{9}$$

$$P(X^{(1)}=1|Y=-1)=\frac{3}{6},\quad P(X^{(1)}=2|Y=-1)=\frac{2}{6},\quad P(X^{(1)}=3|Y=-1)=\frac{1}{6}$$

$$P(X^{(2)}=S|Y=-1)=\frac{3}{6},\quad P(X^{(2)}=M|Y=-1)=\frac{2}{6},\quad P(X^{(2)}=L|Y=-1)=\frac{1}{6}$$

对于给定的 $x=(2,S)^{\mathrm{T}}$，计算

$$P(Y=1)P(X^{(1)}=2|Y=1)P(X^{(2)}=S|Y=1)=\frac{9}{15}\cdot\frac{3}{9}\cdot\frac{1}{9}=\frac{1}{45}$$

$$P(Y=-1)P(X^{(1)}=2|Y=-1)P(X^{(2)}=S|Y=-1)=\frac{6}{15}\cdot\frac{2}{6}\cdot\frac{3}{6}=\frac{1}{15}$$

由于 $P(Y=-1)P(X^{(1)}=2|Y=-1)P(X^{(2)}=S|Y=-1)$ 最大，所以 $y=-1$。 ■