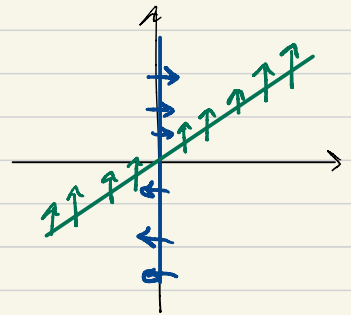


$$\begin{aligned} f &= y - x \\ g &= x^2 \end{aligned}$$

$$3.1 \ a) \quad \mathbb{J} = \begin{bmatrix} -1 & 1 \\ 2x & 0 \end{bmatrix} \Big|_{x,y=0} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1-\lambda & 0 \\ 0 & -\lambda \end{bmatrix}$$

$$\det(\mathbb{J}) = 0 \quad \tau = -1 \quad \Rightarrow \quad \lambda = -0.5 \pm 0.5i$$



$$b) \quad \begin{aligned} \dot{r} &= h(r) \\ \dot{\theta} &= 0 \end{aligned} \quad \rightarrow \quad h(r) \sim ar + O(r^2) \quad \text{for small } r$$

$\alpha \neq 0$

$$\left\{ \begin{array}{l} \text{Polar coordinates} \\ \dot{r} = f(r) \\ \dot{\theta} = \text{const.} \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \text{Cartesian dynamics} \\ \dot{x} = \frac{dr}{dt} \cos \theta - r \sin \theta \cdot \dot{\theta} \\ \dot{y} = \frac{dr}{dt} \sin \theta + r \cos \theta \cdot \dot{\theta} \end{array} \right.$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ \theta &= \arctan\left(\frac{y}{x}\right) \end{aligned}$$

$$\begin{aligned} \dot{x} &= ar \cos \theta - r \sin \theta \cdot 0 \\ \dot{y} &= ar \sin \theta + r \cos \theta \cdot 0 \end{aligned}$$

$$\begin{aligned} \cos(\arctan(\frac{y}{x})) &= \frac{x\sqrt{x^2+y^2}}{x^2+y^2} \\ \sin(\arctan(\frac{y}{x})) &= \frac{y\sqrt{x^2+y^2}}{x^2+y^2} \end{aligned}$$

$$\Rightarrow \begin{aligned} \dot{x} &= ax \\ \dot{y} &= ay \end{aligned}$$

$$c) \quad \mathbb{J} = \begin{bmatrix} 0 & 3y^2 \\ 1 & 0 \end{bmatrix} \Big|_{x,y^*} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\begin{aligned} \tau &= 0 \\ \Delta &= -1 \end{aligned} \quad = \text{Saddle-node}$$

$$d) \quad \begin{aligned} \dot{x} &= (x^2 + y^2)^{|n|/2} \cos(n \arctan(\frac{y}{x})) \\ \dot{y} &= (x^2 + y^2)^{|n|/2} \sin(n \arctan(\frac{y}{x})) \end{aligned}$$

$$\Rightarrow \begin{aligned} \dot{x} &= r^{|n|} \cos(n\theta) \\ \dot{y} &= r^{|n|} \sin(n\theta) \end{aligned}$$

$$\vartheta(\theta) = \arctan\left(\frac{\dot{y}}{\dot{x}}\right) = \arctan\left(\frac{\sin(n\theta)}{\cos(n\theta)}\right) = n\theta$$

$$\begin{aligned} \Delta\vartheta &= \vartheta(2\pi) - \vartheta(0) \\ &= 2\pi n - 0n = 2\pi n \\ I &= \frac{\Delta\vartheta}{2\pi} = n \end{aligned}$$

