Attempt number 2 of problem set 1.1

Function definition

In[39]:=

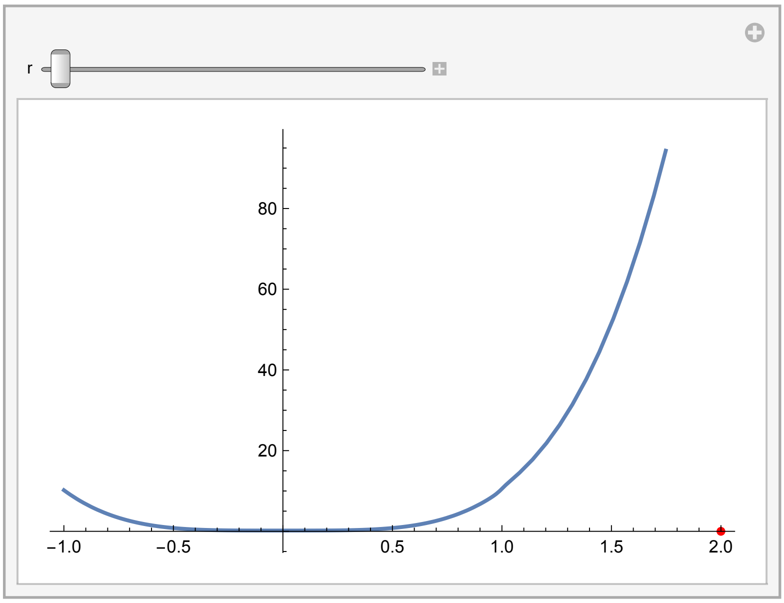
```
dxdt[x_, r_] := ( 1/5 + 7/10 * 1/(1 + Exp[80 (1 - x)]) - r*x^4)
d2xdt2[x_, r_] := N[D[dxdt[ξ, r], ξ] /. ξ → x]
```

Plot up function to study dynamics

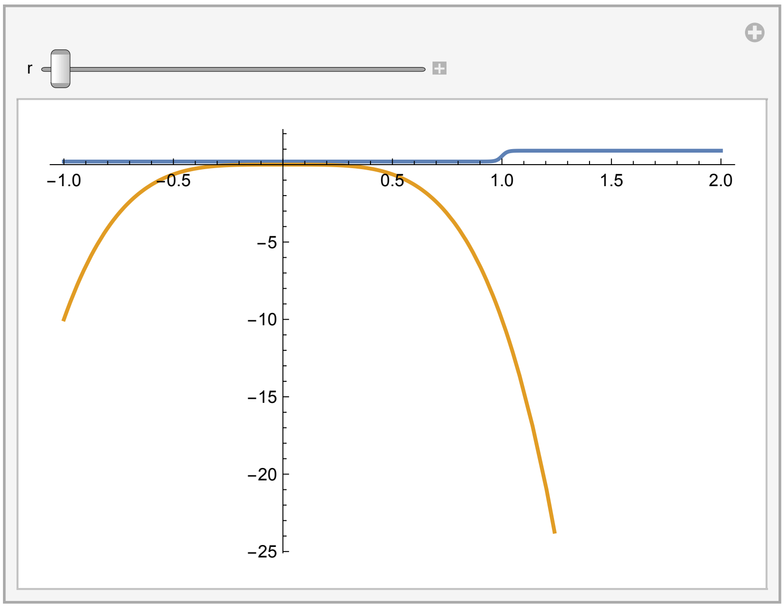
In[3]:=

```
Manipulate[
Show[
Plot[dxdt[x,r],{x,-1,2}],ListPlot[({x,0}/. NSolve[dxdt[x,r]==0,x,Reals]),PlotStyle→Red]
],{r,-10,10}]

Manipulate[Show[
Plot[{ 1/5 + 7/10 * 1/(1 + Exp[80(1-x)]),r*x^4},{x,-1,2}],ListPlot[{x,r*x^4}/.NSolve[1
```

Out[3]=



Out[4]=



Solve fixed points

In[5]:=
```
rValues=Table[r,{r,0.01,1.25, 0.001}];
fixedPoints = Flatten[
   Table[{r, x} /. NSolve[dxdt[x, r] == 0, x, Reals], {r, rValues}], 1
];
fixedPointsWithDerivative = Flatten[
  Table[
    Module[{solutions, derivatives},
      (* Solve for x such that dxdt[x, r] == 0 *)
      solutions = x /. NSolve[dxdt[x, r] == 0, x, Reals];

      (* Evaluate the derivative at each solution *)
      derivatives = d2xdt2[#, r] & /@ solutions;

      (* Pair each solution with its derivative *)
      Transpose[{ConstantArray[r, Length[solutions]], solutions, derivatives}]
    ],
    {r, rValues}
  ],
  1
]
```

Out[7]=
```
{{0.01, -2.11474, 0.378297}, {0.01, 3.08007, -1.1688}, {0.011, -2.06495, 0.387419},
 {0.011, 3.00755, -1.19699}, {0.012, -2.02052, 0.395939}, {0.012, 2.94283, -1.22331},
 {0.013, -1.98049, 0.403941}, ⋯ 3446 ⋯, {1.247, 0.632836, -1.26415},
 {1.248, -0.632709, 1.2644}, {1.248, 0.632709, -1.2644}, {1.249, -0.632582, 1.26466},
 {1.249, 0.632582, -1.26466}, {1.25, -0.632456, 1.26491}, {1.25, 0.632456, -1.26491}}
```

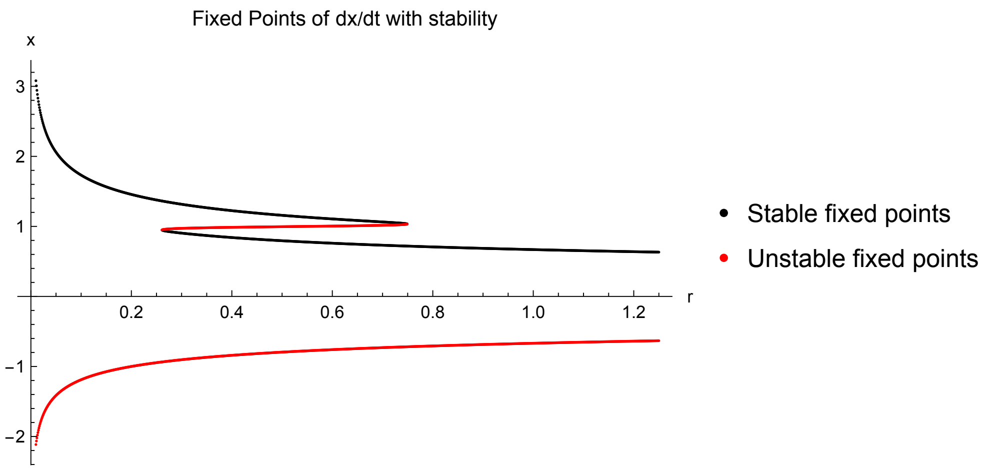
Size in memory: 416.2 kB | + Show more | Show all | Iconize | Store full expression in notebook

Plot the fixed points

In[8]:=

```
(*Unstable points*)
positiveDerivativePoints = Select[fixedPointsWithDerivative, #⟦3⟧ > 0 &];
(*Stable points*)
negativeDerivativePoints = Select[fixedPointsWithDerivative, #⟦3⟧ < 0 &];


ListPlot[
  {negativeDerivativePoints⟦All, {1, 2}⟧, positiveDerivativePoints⟦All, {1, 2}⟧},
  PlotStyle → {Directive[Black], Directive[Red]},
  AxesLabel → {"r", "x"},
  PlotRange → All,
  PlotLegends → {"Stable fixed points", "Unstable fixed points"},
  PlotLabel → "Fixed Points of dx/dt with stability"
]
```

Out[10]=



Determine critical r and it’s x*

In[11]:=
```
eqn1=dxdt[x,r]==0;
eqn2=d2xdt2[x,r]==0;

(*initial guesses from plot*)
x0=0.91;
r0=0.25;

solution=FindRoot[{eqn1,eqn2},{x,x0},{r,r0}];
{criticalX1,criticalR1}={x,r}/. solution;
Print["1: Critical x value: ",criticalX1]
Print["1: Critical r value: ",criticalR1]

x0=1.02;
r0=0.74;

solution=FindRoot[{eqn1,eqn2},{x,x0},{r,r0}];
{criticalX2,criticalR2}={x,r}/. solution;

Print["2: Critical x value: ",criticalX2]
Print["2: Critical r value: ",criticalR2]
```

```
1: Critical x value: 0.948649
1: Critical r value: 0.260928
2: Critical x value: 1.03372
2: Critical r value: 0.749498
```

Retrieve arrays of values

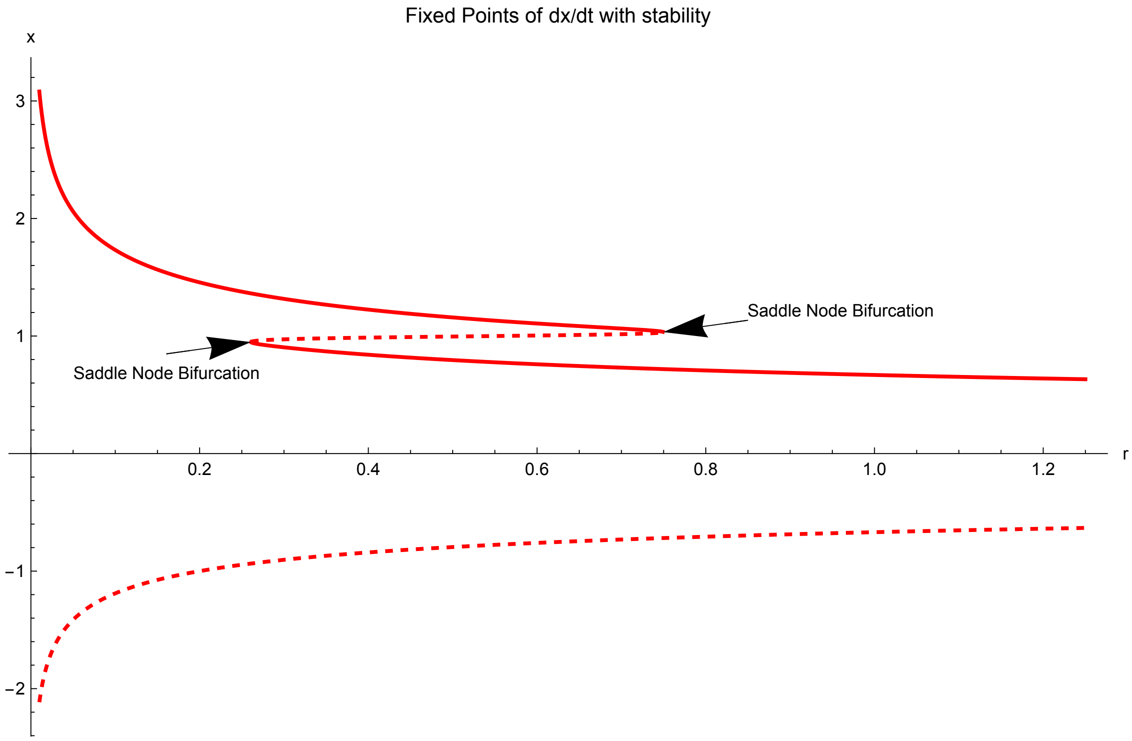
In[25]:=
```
groupedData = GatherBy[fixedPoints, First];

array1 = {};
array2 = {};
array3 = {};
array4 = {};

Do[
  rGroup = groupedData⟦i⟧;
  rValue = rGroup⟦1, 1⟧;
  xs = rGroup⟦All, 2⟧;
  n = Length[xs];

  If[n ≥ 1,
    AppendTo[array1, {rValue, xs⟦1⟧}];
  ];

  If[rValue > criticalR1,
    If[n ≥ 2, AppendTo[array2, {rValue, xs⟦2⟧}]];
    If[n ≥ 3 && rValue ≤ criticalR2,
      AppendTo[array3, {rValue, xs⟦3⟧}];
    ];
    If[n == 4,
      AppendTo[array4, {rValue, xs⟦4⟧}];
    ];
  ,
    If[n ≥ 2, AppendTo[array4, {rValue, xs⟦2⟧}]];
  ];
, {i, Length[groupedData]}];

array1;
array2;
array3;
array4;
```

Plot the final plot

In[205]:=

```
bifur1 = {criticalR1, criticalX1};
bifur2 = {criticalR2, criticalX2};

arrowStart1 = bifur1 + {-0.1, -0.1} ;
arrowStart2 = bifur2 + {0.1, 0.1};

label1 = "Saddle Node Bifurcation";
label2 = "Saddle Node Bifurcation";



plotLines = ListLinePlot[
  {array1, array2, array3, array4},
  PlotStyle → {Directive[Dashed, Red], Directive[Red], Directive[Dashed, Red], Directi
  ];

Show[plotLines, Epilog → {
        {Black, Arrow[{arrowStart1, bifur1}]},
        Text[label1, arrowStart1, {0, 2}],
        {Black, Arrow[{arrowStart2, bifur2}]},
        Text[label2, arrowStart2, {-1, -1}]
        },
      AxesLabel → {"r", "x"},
      PlotRange → All,
      PlotLegends → {"Stable fixed points", "Unstable fixed points"},
      PlotLabel → "Fixed Points of dx/dt with stability"
]
```

Out[212]=


Fixed Points of dx/dt with stability
x
3
2
1
−1
−2
0.2
0.4
0.6
0.8
1.0
1.2
r
Saddle Node Bifurcation
Saddle Node Bifurcation