Problem Set 1.2

Functions

In[13]:=
```
f[x_, h_, r_] := h + x*r - x^2
dfdx[x_, h_, r_] := D[h + x*r - x^2, x]
(*dfdx[x,h,r]/. {x→1, h→1}*)
```

Solve Systems for fixed points x*(h,r)

In[3]:=
```
fixedPoints[h_, r_] := Solve[f[x,h,r] == 0, x];
Print["Fixed points x(h,r):"]
Print[x /.fixedPoints[h,r]]
fo=fixedPoints[h,r]
```

```
Fixed points x(h,r):
```

$$\left\{\frac{1}{2}\left(r-\sqrt{4\,h+r^2}\right),\ \frac{1}{2}\left(r+\sqrt{4\,h+r^2}\right)\right\}$$

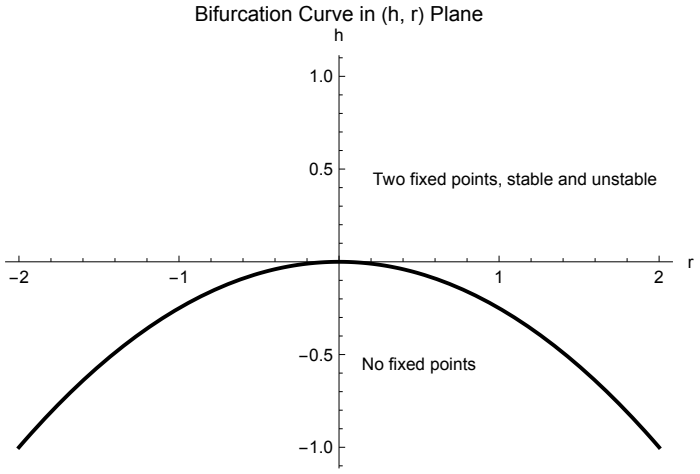
Out[6]=
$$\left\{\left\{x\to\frac{1}{2}\left(r-\sqrt{4\,h+r^2}\right)\right\},\ \left\{x\to\frac{1}{2}\left(r+\sqrt{4\,h+r^2}\right)\right\}\right\}$$

Plot plane (h,r)

In[21]:=
```
(*From fixed points above we know that the number of fixed points changes as 4h + r^2
hFunction[r_] := -r^2/4
bifurcationCurve = Plot[hFunction[r], {r, -2, 2}, PlotStyle → {Thick, Black}];

Show[
  bifurcationCurve,
  PlotRange → {{-2, 2}, {-1, 1}},
  AxesLabel → {"r", "h"},
  Epilog → {
    Text["No fixed points", {0.5, -0.5}, {0, 1}],
    Text["Two fixed points, stable and unstable", {1.1, 0.5}, {0, 1}]
  },
  PlotLabel → "Bifurcation Curve in (h, r) Plane"
]
```

Out[23]=



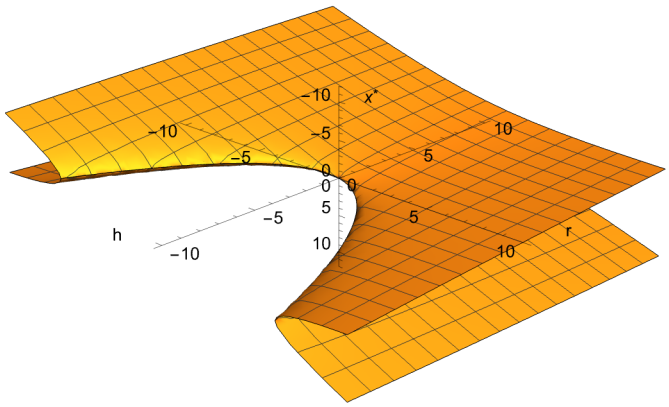
3D plot of surface over fixed points

In[100]:=

```
lowerFixedPoint[h_, r_] := x /. fixedPoints[h, r][[1]];
upperFixedPoint[h_, r_] := x /. fixedPoints[h, r][[2]];


Show[
  Plot3D[lowerFixedPoint[h, r], {h, -10, 10}, {r, -10, 10},
    AxesLabel → {"h", "r", "x*"},
    AxesOrigin→{0,0,0},
    PlotRange → {-12, 12},
    Axes → True,
    Boxed → False],
  Plot3D[upperFixedPoint[h, r], {h, -10, 10}, {r, -10, 10},
    AxesLabel → {"h", "r", "x*"},
    AxesOrigin→{0,0,0},
    PlotRange → {-12, 12},
    Axes → True,
    Boxed → False]
]
```

Out[102]=



In[103]:=

```
Simplify[Sqrt[r^2/4 + 1]]
```

Out[103]=

$$\frac{\sqrt{4 + r^2}}{2}$$