```
import numpy as np
import matplotlib.pyplot as plt
from tqdm.auto import tqdm

mu = 0.1
nu = 1
omega = 1

def F(x,y):
    Xdot = mu*x - nu*y**3 - x*y**2 - nu*x**2*y - omega*y - x**3
    Ydot = omega*x + mu*y + nu*x*y**2 + nu*x**3 - y**3 - x**2*y
    return np.array([Xdot,Ydot])

def jacobian(x,y):
    dF1dx = mu - y**2 - 2*nu*x*y - 3*x**2
    dF1dy = -3*nu*y**2 - 2*x*y - nu*x**2 - omega

    dF2dx = omega + nu*y**2 + 3*nu*x**2 - 2*x*y
    dF2dy = mu + 2*x*y - 3*y**2 - x**2
    return np.array([[dF1dx,dF1dy],[dF2dx,dF2dy]])

def calculate_Mdot(x, y, M):
    Xdot,Ydot = F(x,y)
    Mdot = np.dot(jacobian(x,y),M)
    return Xdot,Ydot,Mdot

dt = 2 * np.pi / (1.1 * 10000)
t_max = 2 * np.pi / 1.1
steps = int(t_max / dt)+1
t = np.linspace(0, t_max, steps)

x = np.zeros(steps)
y = np.zeros(steps)
M = np.zeros((steps, 2, 2))

x[0] = np.sqrt(mu)
y[0] = 0.0
M[0] = np.array([[1, 0], [0, 1]])

for i in range(steps - 1):
    Xdot, Ydot = F(x[i], y[i])
    Mdot = calculate_Mdot(x[i], y[i], M[i])[2]

    x[i + 1] = x[i] + dt * Xdot
    y[i + 1] = y[i] + dt * Ydot
    M[i + 1] = M[i] + dt * Mdot

M11 = M[:, 0, 0]
M12 = M[:, 0, 1]
M21 = M[:, 1, 0]
M22 = M[:, 1, 1]
```

```
plt.figure(figsize=(12, 6))
plt.plot(t, x, label="x(t)")
plt.plot(t, y, label="y(t)")
plt.plot(t, M11, label="M11(t)")
plt.plot(t, M12, label="M12(t)")
plt.plot(t, M21, label="M21(t)")
plt.plot(t, M22, label="M22(t)")
plt.xlabel("t")
plt.ylabel("Values")
plt.title("Evolution of x, y, and M over time")
plt.grid(True)
plt.legend()
plt.show()
```


```
print(f"M11: {M11[-1]}")
print(f"M12: {M12[-1]}")
print(f"M21: {M21[-1]}")
print(f"M22: {M22[-1]}")
```

```
M11: 0.31904900907169736
M12: -0.00016937668052115326
M21: 0.6829242139266678
M22: 1.0011770808299103
```

```
eigenvalues = np.linalg.eigvals(M[-1])

stability_exponents = np.log(np.abs(eigenvalues)) / t_max
```

```
stability_exponents = np.sort(stability_exponents)

print("Computed Stability Exponents:")
print(f"σ1 = {stability_exponents[0]:.2f}, σ2 =
{stability_exponents[1]:.2f}")
```

```
Computed Stability Exponents:
σ1 = -0.20, σ2 = 0.00
```