Problem Set 3.2

In[389]:=

```
f[x_, y_, ε_, Iext_] := 1/ε * (x - 1/3*x^3 - y + Iext)
g[x_, y_, a_, b_] := x + a - b*y

J[x_, ε_] := {{(1-x^2)/ε, -1/ε}, {1, -b}}
```

$$J[x\_,\ \varepsilon\_] := \begin{pmatrix} (1-x^2)/\varepsilon & -1/\varepsilon \\ 1 & -b \end{pmatrix}$$

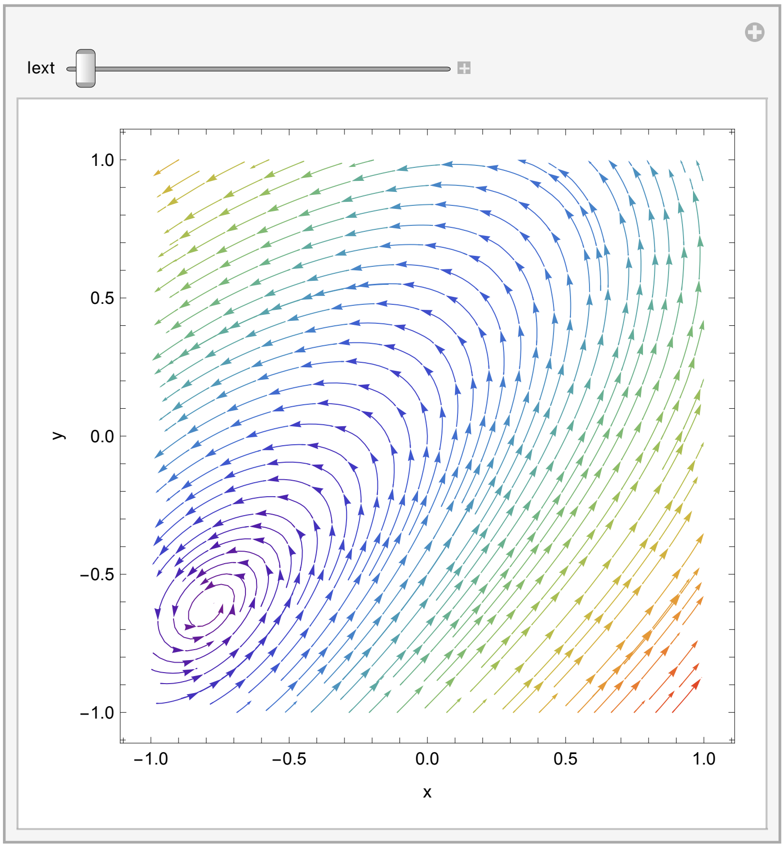
In[392]:=

```
a = 4/9;
b = 5/9;
ε = 1;
```

In[395]:=

```
xrange = 1;
yrange = 1;
Irange = 1;

Manipulate[
    Show[
        StreamPlot[{f[x,y,ε,Iext], g[x,y,a,b]}, {x, -xrange, xrange}, {y, -yrange, yra
        StreamStyle → Automatic,
        StreamColorFunction → "Rainbow",
        FrameLabel → {"x", "y"},
        StreamPoints → Fine,
        AspectRatio → 1]
    ],
{Iext,0,Irange}]
```

Out[398]=



In[399]:=

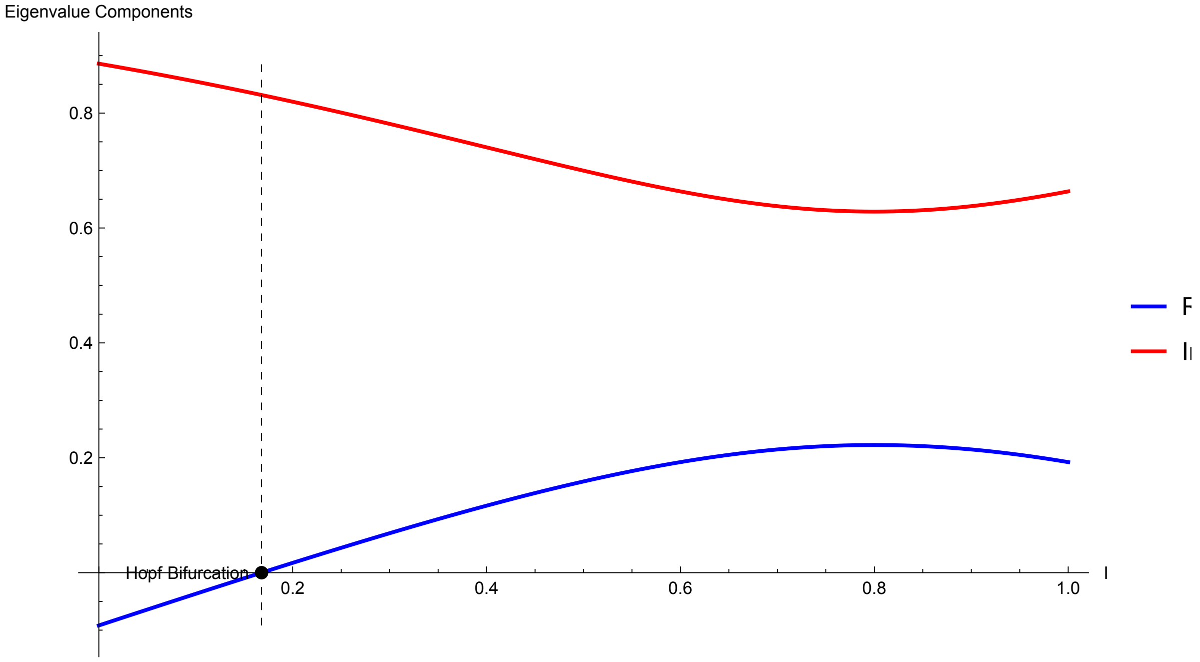
```
numPoints = 100;
Ivals = Subdivide[0, 1, numPoints];(*Table that contains a list of i values*);

ReLambda = Table[0.0, {numPoints + 1}];
ImLambda = Table[0.0, {numPoints + 1}];
fixedXs = Table[0.0, {numPoints + 1}];
fixedXprev = -0.8; (*From plot above*)

Do[
    Icurrent = Ivals[[j]];
    fixedX = x /. Quiet[
        FindRoot[
            x - (1/3) * x^3 - (x + a)/b + Icurrent == 0,
            {x, fixedXprev},
            Method → "Newton",
            MaxIterations → 100,
            AccuracyGoal → 10
        ]
    ];
    (* Compute y from fixedX *)
    fixedY = (fixedX + a)/b;

    (* Update the previous x for the next iteration *)
    fixedXprev = fixedX;
    fixedXs[[j]] = fixedX;

    (* Compute the eigenvalues of the Jacobian *)
    eig = Eigenvalues[J[fixedX, ε]];

    (* Store the real and imaginary parts *)
    ReLambda[[j]] = Re[eig[[1]]];
    ImLambda[[j]] = Im[eig[[1]]];


    ,
    {j, 1, numPoints + 1}
];
dataX = Transpose@{Ivals, fixedXs};
dataRe = Transpose@{Ivals, ReLambda}; (*Transform and combine the lists so that they c
dataIm = Transpose@{Ivals, ImLambda};
(*Plot the points (i, Re[eig]) and (i, Im[eig]) in the same plot. Don't forget to labe
```

In[409]:=

In[410]:=

```
(*Plot the points (i, Re[eig]) and (i, Im[eig]) in the same plot.
Don't forget to label the bifurcation point and add a legend
to denote which points (or line if you use ListLinePlot)
corresponds to the real part of the eigenvalue and which to the imaginary part.*)
Ihopf = 68/405;
ListLinePlot[
  {
    dataRe,
    dataIm
  },
  PlotLegends → {"Re[λ]", "Im[λ]"},
  AxesLabel → {"I", "Eigenvalue Components"},
  PlotStyle → {Blue, Red},
  PlotRange → All,
  ImageSize → Large,
  Epilog → {
        {Black, Dashed, Line[{{Ihopf, Min[ReLambda, ImLambda]}, {Ihopf, Max[ReLambda,
        {Black, PointSize[Large], Point[{Ihopf, 0}]},
        Text["Hopf Bifurcation", {Ihopf, 0}, {1.2, 0}]
        }
]
```

Out[411]=

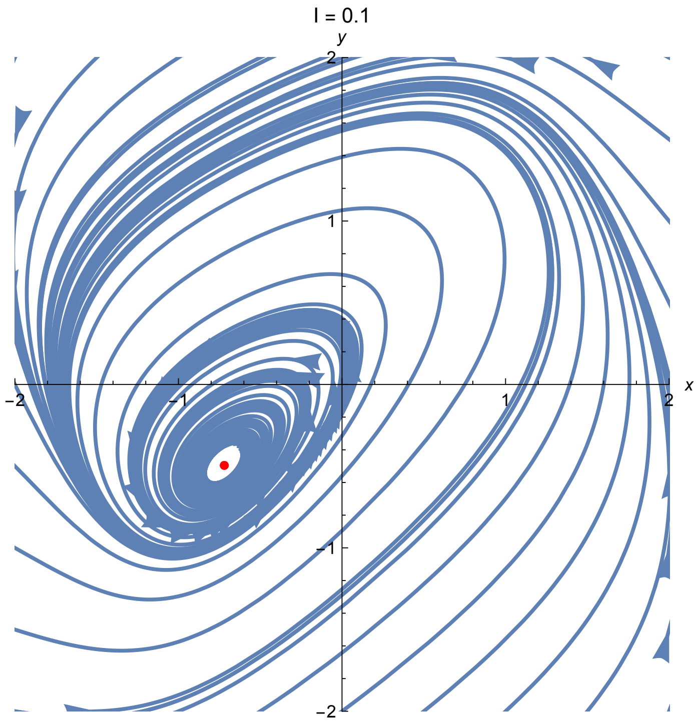


In[427]:=

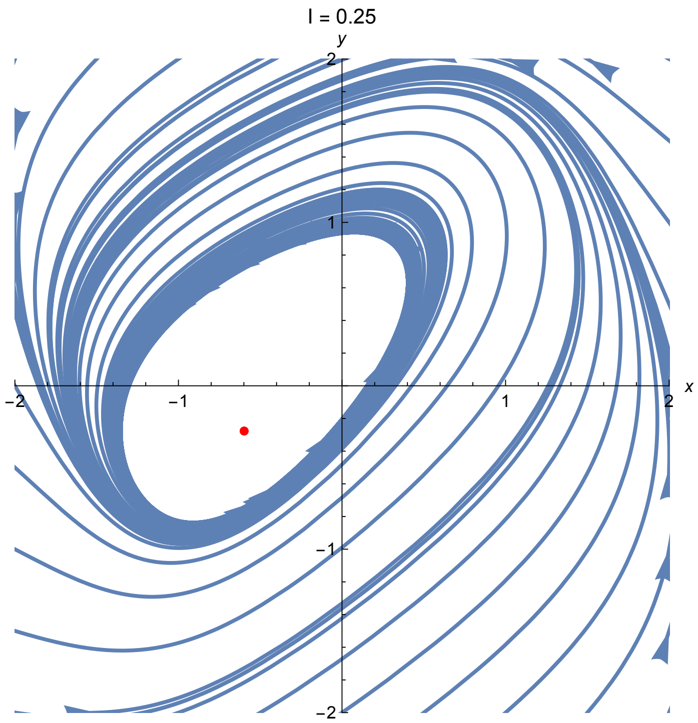
```
maxt = 50;
sol[x0_, y0_, Iext_] := NDSolve[{x'[t] == 1/ε * (x[t] - 1/3*x[t]^3 - y[t] + Iext), y'[t]
                                {x,y},
                                {t,0,maxt}]
minx=-2;
miny=-2;
maxx=2;
```

```
maxy=2;
step = 0.5;
initialC=Join[
            Table[{minx,y},{y,miny,maxy,step}],
            Table[{maxx,y},{y,miny,maxy,step}],
            Table[{x,miny},{x,minx,maxx,step}],
            Table[{x,maxy},{x,minx,maxx,step}]];
Iext = 0.1;
fixedX = -0.7196844116363366;
p1=Show[
    Table[
        ParametricPlot[
            Evaluate[{x[t],y[t]}/. sol[initialC⟦i,1⟧, initialC⟦i,2⟧, Iext]],
            {t,0,maxt},
            PlotRange→{{minx,maxx},{miny,maxy}},
            AxesLabel→{x,y}
        ]
        /. Line[x_]:→{Arrowheads[{{0.05, 0.5}, {0.05, 0.0}}],Arrow[x]},{i,1,Length[init
    ],
    ListPlot[{{fixedX, (fixedX + a)/b}},
        PlotStyle→{Red},
        PlotMarkers→{Automatic, 8}
    ],
    PlotLabel → "I = " <> ToString[Iext]
]

Iext = 0.25;
fixedX = -0.5982743232213728;
p1=Show[
    Table[
        ParametricPlot[
            Evaluate[{x[t],y[t]}/. sol[initialC⟦i,1⟧, initialC⟦i,2⟧, Iext]],
            {t,0,maxt},
            PlotRange→{{minx,maxx},{miny,maxy}},
            AxesLabel→{x,y}
        ]
        /. Line[x_]:→{Arrowheads[{{0.05, 0.5}, {0.05, 0.0}}],Arrow[x]},{i,1,Length[init
    ],
    ListPlot[{{fixedX, (fixedX + a)/b}},
        PlotStyle→{Red},
        PlotMarkers→{Automatic, 8}
    ],
    PlotLabel → "I = " <> ToString[Iext]
]
```

Out[437]=



Out[440]=



In[426]:=