2.4 $\dot{\phi} = \omega$

$\dot{\omega} = \sin(\phi)[\cos(\phi) - \tau - 1]$

IOM: constant over time

e.g. Sum of potential & kinetic energy of a ball

→ Sum is constant even if ball is moving

$\Rightarrow$ find IOM $E(x)$ if $\dot{E} = 0 \quad \forall x$

$$\Rightarrow \dot{E} = \frac{\partial E}{\partial x}\dot{x} + \frac{\partial E}{\partial y}\dot{y} = 0$$

$\rightarrow \quad E(\phi, \omega, \tau) = ?$

set $\quad E = T + V$

$$\dot{E} = 0$$

$$\dot{E} = \frac{\partial E}{\partial \phi}\dot{\phi} + \frac{\partial E}{\partial \omega}\dot{\omega} = \frac{\partial E}{\partial \phi}$$

set $\quad \dot{\omega} = -\frac{dV}{d\phi}$

$$\Rightarrow V(\phi) = -\int \sin\phi[\cos\phi - \tau - 1]\, d\phi$$

$$= -\left(\tfrac{1}{2}\sin^2\phi + (\tau+1)\cos\phi\right) + C_1$$

set $\quad \dot{\phi} = \frac{dT}{d\omega} = \omega$

$$T(\omega) = \tfrac{1}{2}\omega^2 + C_2$$

$$\Rightarrow E = \tfrac{1}{2}\omega^2 - \sin^2\phi - 2(\tau+1)\cos\phi + \underbrace{C_1 + C_2}_{= C_3}$$

cond: $\phi = \frac{\pi}{2}$ & $\omega = 0 \Rightarrow E = -1$

$$E = 0 - 1 - 0 + C_3 = -1$$

$$C_3 = 0$$

$$\Rightarrow E = \tfrac{1}{2}\omega^2 - \sin^2\phi - 2(\tau+1)\cos\phi$$