Problem set 2.1

In[1]:=
```
ClearAll["Global`*"];
Remove["Global`*"];
```

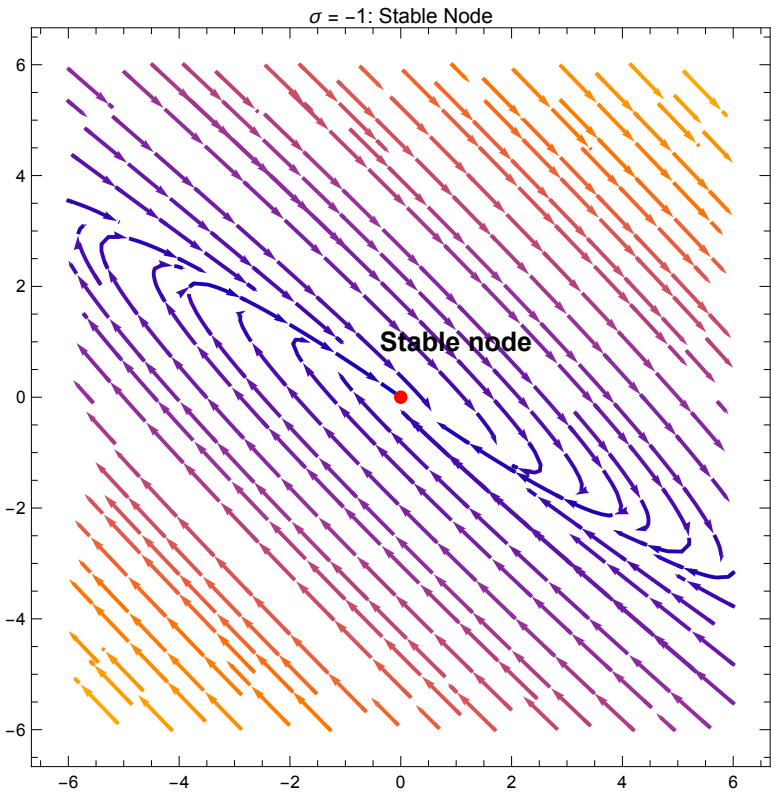
Remove : There are no symbols matching "Global`*".

a) For σ taking each of the values {−1,0,1}, plot a set of representative trajectories, for example using StreamPlot[] or NDSolve[] in Mathematica. Classify the fixed point in each of the cases and write your classification in the plots. Upload the three separate plots in one .pdf or .png file.

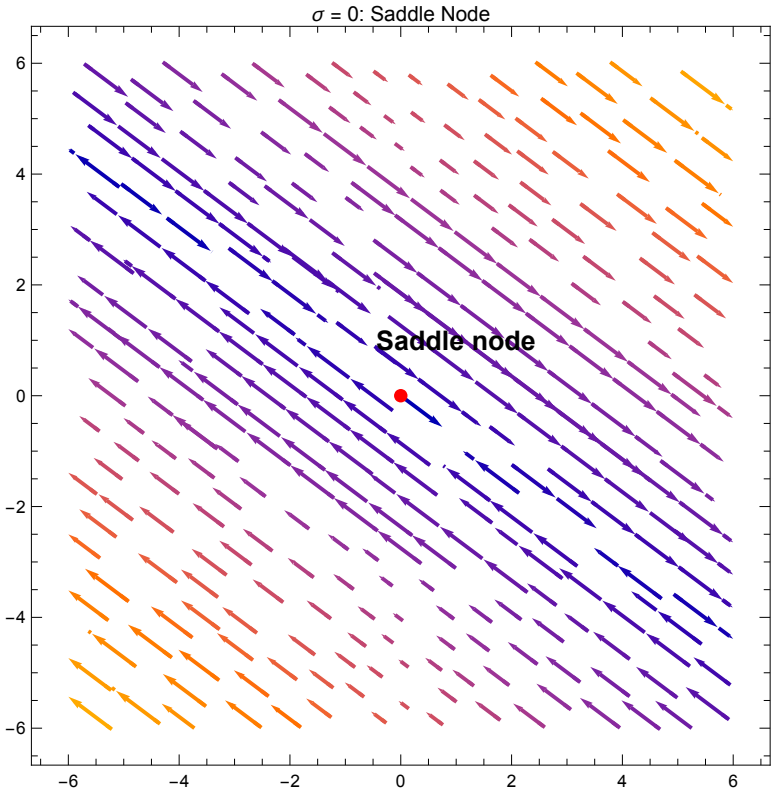
In[3]:=
```
A[sigma_] := {{sigma + 3, 4}, {-9/4, sigma - 3}};
Ax[sigma_, x_, y_] := (sigma + 3) x + 4 y;
Ay[sigma_, x_, y_] := (-9/4) x + (sigma - 3) y;

s1 = StreamPlot[
 {Ax[-1, x, y], Ay[-1, x, y]},
 {x, -6, 6}, {y, -6, 6},
 StreamStyle → Thick,
 PlotLabel → "σ = -1: Stable Node",
 Epilog → {
  Red, PointSize[Large], Point[{0, 0}],
  Black,
  Text[Style["Stable node", 14, Bold], {1, 1}]
 },
ImageSize → 400
];

s2 = StreamPlot[
{Ax[0,x,y], Ay[0,x,y]},
{x, -6, 6}, {y, -6, 6},
StreamStyle → Thick,
PlotLabel → "σ = 0: Saddle Node",
Epilog → {
  Red, PointSize[Large], Point[{0, 0}],
  Black,
  Text[Style["Saddle node", 14, Bold], {1, 1}]
 },
ImageSize → 400
];


s3 = StreamPlot[
{Ax[1,x,y], Ay[1,x,y]},
{x, -6, 6}, {y, -6, 6},
StreamStyle → Thick,
PlotLabel → "σ = 1: Stable Node",
```

```
  Epilog → {
    Red, PointSize[Large], Point[{0, 0}],
    Black,
    Text[Style["Unstable node", 14, Bold], {1, 1}]
   },
  ImageSize → 400
  ];


Show[s1]
Show[s2]
Show[s3]
```

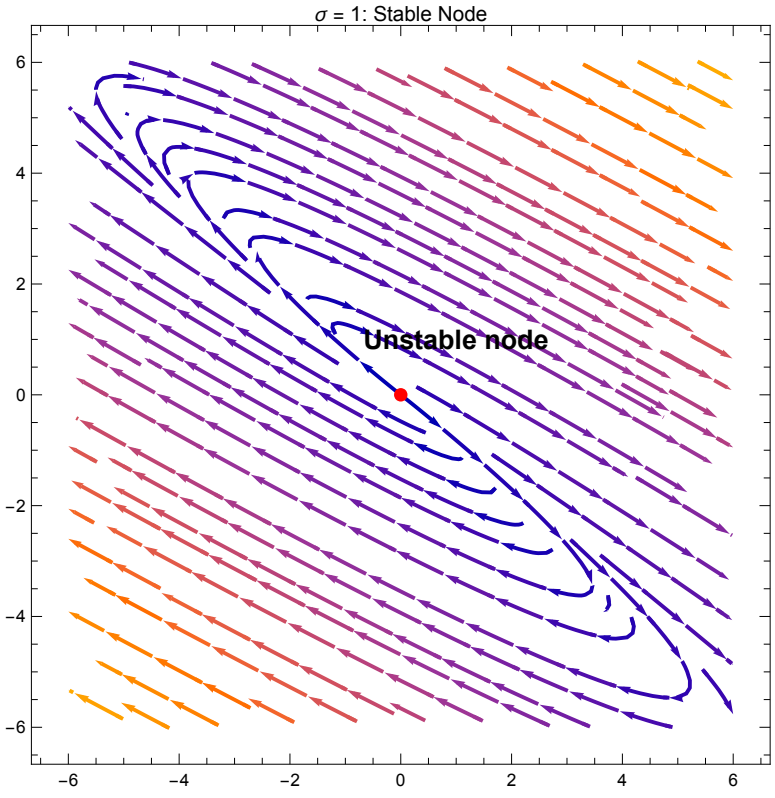
Out[9]=



Out[10]=



Out[11]=



b) Analytically solve eigenvalues for A

In[12]:=
```
eigenValues = Eigenvalues[A[sigma]]
```

Out[12]=
```
{sigma, sigma}
```

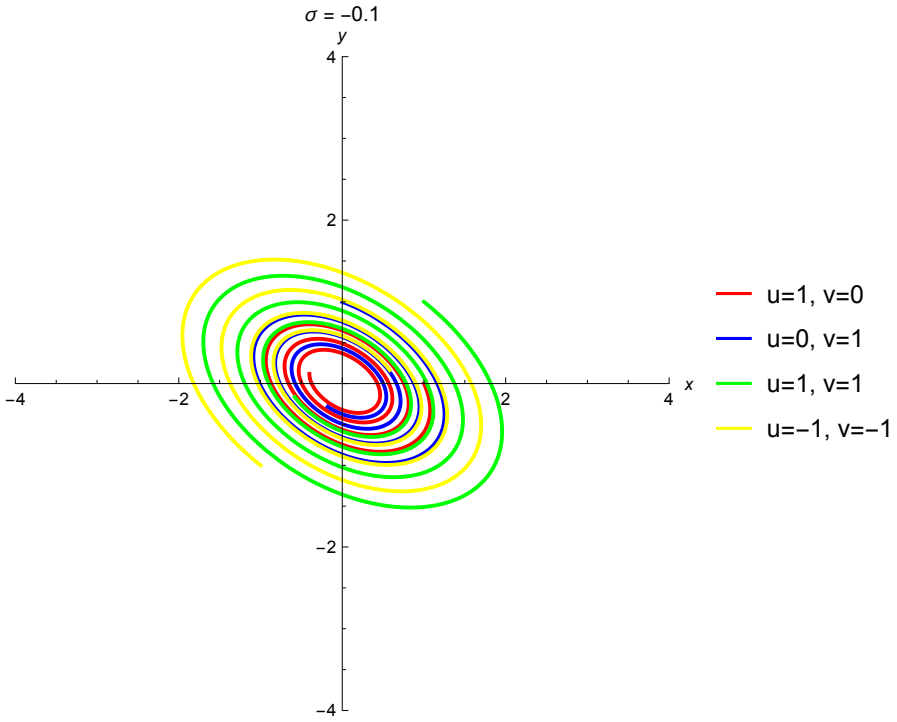
Problem set 2.2

In[13]:=
```
ClearAll["Global`*"];
Remove["Global`*"];
```

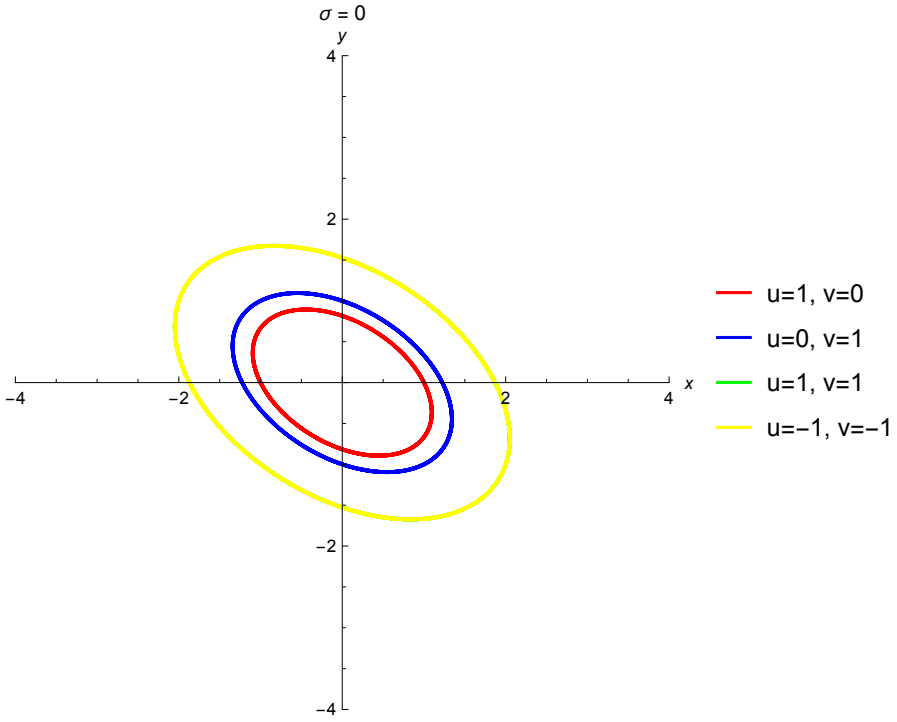
In[76]:=
```
fx[sigma_, t_, u_, v_] :=  Exp[sigma*t] * (u * Cos[Sqrt[5]*t] + ( u  + 3*v)/Sqrt[5]*Si
fy[sigma_, t_, u_, v_] :=  Exp[sigma*t] * (v * Cos[Sqrt[5]*t] - (2*u +  v )/Sqrt[5]*Si

initialConditions = {
  {1, 0},
  {0, 1},
  {1, 1},
  {-1, -1}
};
numberOfConditions = Length[initialConditions];

colors = {Red, Blue, Green, Yellow};
labels = Table[
   "u=" <> ToString[initialConditions[[i, 1]] <> ", v=" <> ToString[initialConditions[[i
   {i, 1, numberOfConditions}
];

minx=-4;
miny=-4;
maxx=4;
maxy=4;

sigma = -0.1;
p1 = ParametricPlot[
   Evaluate[
     Table[
       {
         fx[sigma, t, initialConditions[[i, 1]], initialConditions[[i, 2]]],
         fy[sigma, t, initialConditions[[i, 1]], initialConditions[[i, 2]]]
       },
       {i, 1, numberOfConditions}
     ]
   ],
   {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}},
   AxesLabel → {x, y},
```

```
   PlotStyle → colors,
   PlotLabel → "σ = " <> ToString[sigma],
   PlotLegends → LineLegend[colors, labels]
]

sigma = 0;
p2 = ParametricPlot[
   Evaluate[
     Table[
       {
         fx[sigma, t, initialConditions[[i, 1]], initialConditions[[i, 2]]],
         fy[sigma, t, initialConditions[[i, 1]], initialConditions[[i, 2]]]
       },
       {i, 1, numberOfConditions}
     ]
   ],
   {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}},
   AxesLabel → {x, y},
   PlotStyle → colors,
   PlotLabel → "σ = " <> ToString[sigma],
   PlotLegends → LineLegend[colors, labels]
]

sigma = 0.1;
p3 = ParametricPlot[
   Evaluate[
     Table[
       {
         fx[sigma, t, initialConditions[[i, 1]], initialConditions[[i, 2]]],
         fy[sigma, t, initialConditions[[i, 1]], initialConditions[[i, 2]]]
       },
       {i, 1, numberOfConditions}
     ]
   ],
   {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}},
   AxesLabel → {x, y},
   PlotStyle → colors,
   PlotLabel → "σ = " <> ToString[sigma],
   PlotLegends → LineLegend[colors, labels]
]
```

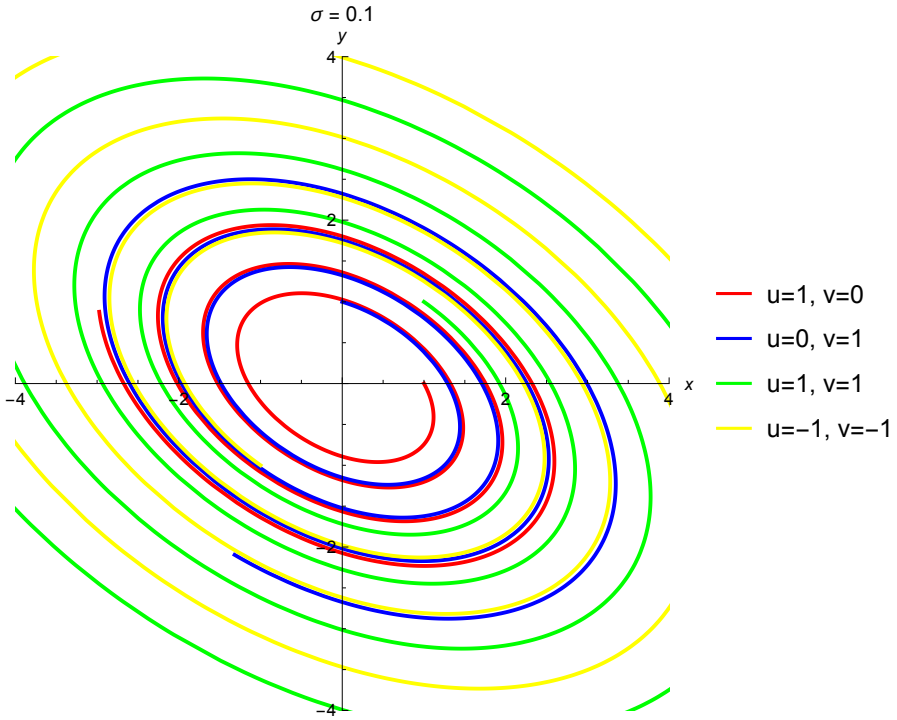
Out[87]=


σ = −0.1
y
x
4
2
−2
−4
−4
−2
2
4
u=1, v=0
u=0, v=1
u=1, v=1
u=−1, v=−1

Out[89]=

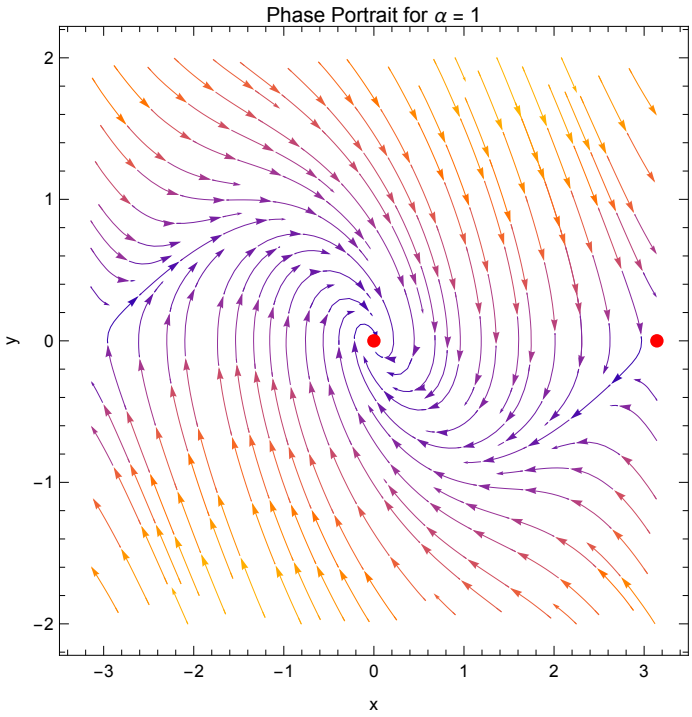

σ = 0
y
x
4
2
−2
−4
−4
−2
2
4
u=1, v=0
u=0, v=1
u=1, v=1
u=−1, v=−1

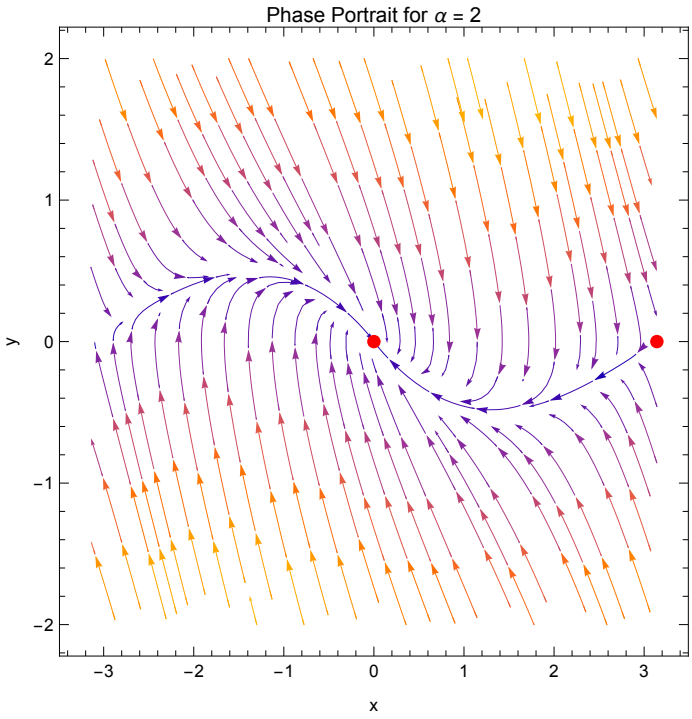
Out[91]=

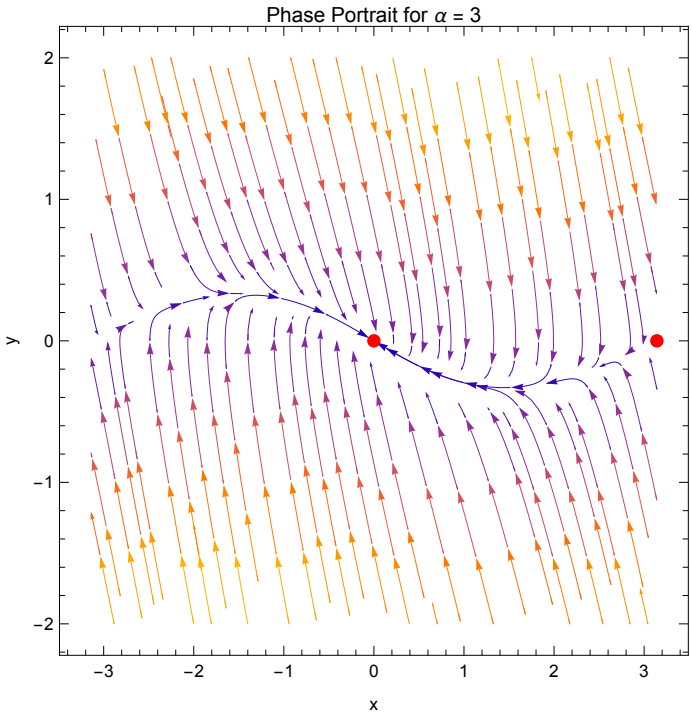


In[92]:=

```
(* Define the system *)
pendulumSystem[x_, y_, alpha_] := {y, -Sin[x] - alpha y}

(* Parameters *)
alphaValues = {1, 2, 3}; (* Example values: below, at, and above critical damping *)

(* Plot for each alpha *)
Do[
 Module[{sol, plot},
  plot = StreamPlot[
    Evaluate[{y, -Sin[x] - alpha y}],
    {x, -Pi, Pi}, {y, -2, 2},
    StreamStyle → Blue,
    FrameLabel → {"x", "y"},
    PlotLabel → Row[{"Phase Portrait for α = ", alpha}],
    Epilog → {Red, PointSize[Large], Point[{0, 0}], Point[{Pi, 0}]}
  ];
  Print[plot]
 ],
 {alpha, alphaValues}
]
```


Phase Portrait for α = 1
2
1
0
−1
−2
y
−3
−2
−1
0
1
2
3
x


Phase Portrait for α = 2
2
1
0
−1
−2
y
−3
−2
−1
0
1
2
3
x



In[189]:=

```
sol[x0_,y0_,alpha_] := NDSolve[{x'[t]== y[t], y'[t]== -Sin[x[t]] - alpha*y[t], x[0]==x0,

minx=-1;
miny=-1;
maxx=Pi + 1;
maxy=1;

step = 0.5
initialC=Join[
            Table[{minx,y},{y,miny,maxy,step}],
            Table[{maxx,y},{y,miny,maxy,step}],
            Table[{x,miny},{x,minx,maxx,step}],
            Table[{x,maxy},{x,minx,maxx,step}]];
alpha = 1
p1=Show[
    Table[
        ParametricPlot[
            Evaluate[{x[t],y[t]}/. sol[initialC[[i,1]], initialC[[i,2]], alpha]],
            {t,0,10},
            PlotRange→{{minx,maxx},{miny,maxy}},
            AxesLabel→{x,y}
        ]
        /. Line[x_]:→{Arrowheads[{{0.05, 0.5}, {0.05, 0.0}}],Arrow[x]},{i,1,Length[init
    ],
    ListPlot[{{Pi, 0}, {0, -0}},
        PlotStyle→{Red},
        PlotMarkers→{Automatic, 8}
    ],
    PlotLabel → "α = " <> ToString[alpha]
```

```
]

alpha = 2
p2=Show[
    Table[
        ParametricPlot[
            Evaluate[{x[t],y[t]}/. sol[initialC[[i,1]], initialC[[i,2]], alpha]],
            {t,0,10},
            PlotRange→{{minx,maxx},{miny,maxy}},
            AxesLabel→{x,y}
        ]
        /. Line[x_]:→{Arrowheads[{{0.05, 0.5}, {0.05, 0.0}}],Arrow[x]},{i,1,Length[init
    ],
    ListPlot[{{Pi, 0}, {0, -0}},
        PlotStyle→{Red},
        PlotMarkers→{Automatic, 8}
    ],
    PlotLabel → "α = " <> ToString[alpha]
]
alpha = 3
p3=Show[
    Table[
        ParametricPlot[
            Evaluate[{x[t],y[t]}/. sol[initialC[[i,1]], initialC[[i,2]], alpha]],
            {t,0,10},
            PlotRange→{{minx,maxx},{miny,maxy}},
            AxesLabel→{x,y}
        ]
        /. Line[x_]:→{Arrowheads[{{0.05, 0.5}, {0.05, 0.0}}],Arrow[x]},{i,1,Length[init
    ],
    ListPlot[{{Pi, 0}, {0, -0}},
        PlotStyle→{Red},
        PlotMarkers→{Automatic, 8}
    ],
    PlotLabel → "α = " <> ToString[alpha]
]
```

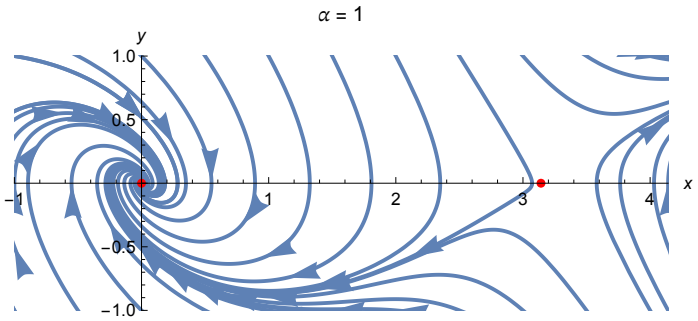
Out[194]=

0.5

Out[196]=

1

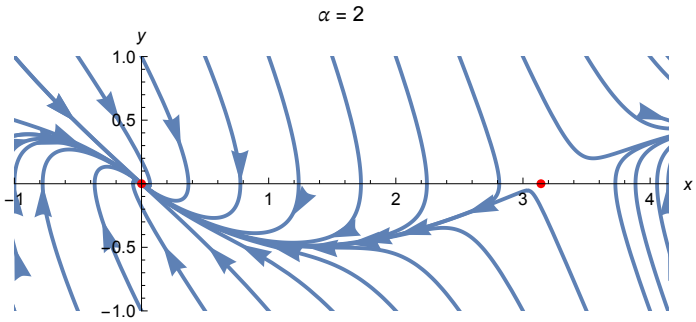
Out[197]=



Out[198]=

2

Out[199]=



Out[200]=

3

Out[201]=