Change of atmos heat

Solar irradiance

Stefan - Boltzmann Const.

$$\underbrace{c \frac{dT}{dt}}_{} = \underbrace{(1-\alpha(T))Q}_{\text{energy contribution from sun}} - \underbrace{esT^4}_{}$$

energy emitted back out of stratosphere

Earth heat capacity.

$c, Q, e, s > 0$

$0 \leq \alpha \leq 1$: Albedo, fraction of solar radiation that gets reflected

- Varies by ice, water & Land

Ice albedo effect: the warmer it gets → less ice → less reflection → warmer derivative

$$\alpha = \frac{4}{5} - \frac{7}{10}\frac{1}{1+e^{\left(-\frac{80(T-T_m)}{T_m}\right)}}$$

$\Rightarrow \alpha \approx \frac{4}{5}$ $T < 250\,K$

$\alpha \approx \frac{1}{10}$ $T > 300$

$0 < e < 1$: fraction of outgoing radiation absorbed by green house gases

Transform to dimensionless variables (x, τ)

$$x = \frac{T}{T_0}, \qquad \tau = \frac{t}{t_0}, \qquad \alpha = a - bT$$

$T = xT_0$ $\qquad t = \tau t_0$

$$\frac{dx}{d\tau} = \frac{1}{5} + \frac{7}{10}\frac{1}{1+e^{80(1-x)}} - \epsilon x^4$$

a) LHS: $c\frac{dT}{dt} = c \cdot \frac{T_0}{t_0}\frac{dx}{d\tau}$

RHS: $(1-\alpha(T))Q - esT^4 =$

$$= \left(1 - \left(\frac{4}{5} - \frac{7}{10}\frac{1}{1+e^{\left(-\frac{80(T-T_m)}{T_m}\right)}}\right)\right)Q - esT^4 =$$

$$= \left(\frac{1}{5} + \frac{7}{10}\frac{1}{1+e^{\left(-\frac{80(T-T_m)}{T_m}\right)}}\right)Q - esT^4 =$$

$$= \left(\frac{1}{5} + \frac{7}{10}\frac{1}{1+e^{-80\left(\frac{xT_0-T_m}{T_m}\right)}}\right)Q - es\,x^4T_0^4$$

set $T_0 = T_m$ $= \left(\frac{1}{5} + \frac{7}{10}\frac{1}{1+e^{80(1-x)}}\right)Q - esx^4T_m^4$

LHS = RHS: $\quad c\,\frac{T_m}{t_o}\frac{dx}{d\tau} = \left(\frac{1}{5} + \frac{7}{10}\,\frac{1}{1+e^{80(1-x)}}\right)Q - es\,x^4T_m^4$

$$\frac{dx}{d\tau} = \frac{Q\,t_o}{cT_m}\left(\frac{1}{5} + \frac{7}{10}\,\frac{1}{1+e^{80(1-x)}} - \frac{es\,x^4T_m^4}{Q}\right)$$

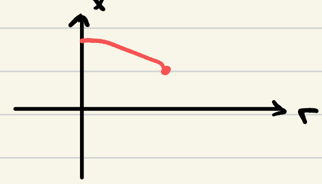
$$r = \frac{esT_m^4}{Q} \Rightarrow \frac{Q\,t_o}{cT_m}\left(\frac{1}{5} + \frac{7}{10}\,\frac{1}{1+e^{80(1-x)}} - r\,x^4\right)$$

$$t_o = \frac{cT_m}{Q} \Rightarrow \frac{dx}{d\tau} = \frac{1}{5} + \frac{7}{10}\,\frac{1}{1+e^{80(1-x)}} - r\,x^4$$

d) fixed points: set $\frac{dx}{dt} = 0$

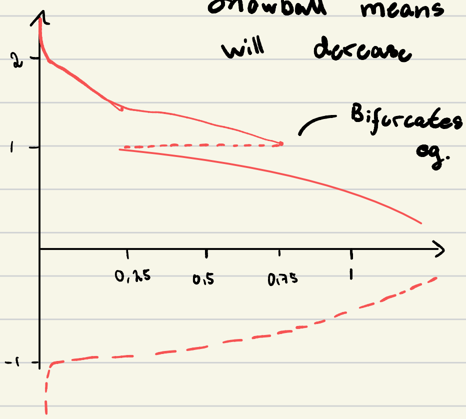
$r = \frac{esT_m^4}{Q}$ $\quad\rightarrow\quad$ at stable fixed point x^* should increase if r decreases

$x = \frac{T}{T_o}$



Sketch

Snowball means decreasing Temp will decrease



Bifurcates to a snowball eg. to decreasing x