2.3

$$\begin{cases} \frac{dx}{d\tau} = y \\ \frac{dy}{d\tau} = -\sin(x) - \alpha y \end{cases}$$

a) $(\dot{x}, \dot{y}) = 0 \Rightarrow y^* = 0$

$\sin(x^*) = 0 \Rightarrow x^* = \{0, \pi\}$

$\Rightarrow$ FP's = $(0,0)$ & $(\pi, 0)$

$\dot{x} = f(x,y)$

$\dot{y} = g(x,y)$

$$J(x^*, y^*) = \begin{pmatrix} 0 & 1 \\ -\cos(x) & -\alpha \end{pmatrix} \text{ & set } \begin{pmatrix} 0 & 1 \\ -C & -\alpha \end{pmatrix}$$

$$|(J - \lambda I)| = \left| \begin{pmatrix} -\lambda & 1 \\ -\cos(x) & -\alpha - \lambda \end{pmatrix} \right| = \lambda^2 + \alpha\lambda + \cos(x)$$

$$\lambda = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\cos(x)}}{2}$$

at $x = 0 \Rightarrow \lambda = \frac{-\alpha \pm \sqrt{\alpha^2 - 4}}{4} = (*)$

$x = \pi \Rightarrow \lambda = \frac{-\alpha \pm \alpha}{4} = \begin{pmatrix} 0 \\ \frac{\alpha}{2} \end{pmatrix} \Rightarrow$ Saddle

$(*)$ for $0 \leq \alpha < 2 \Rightarrow$ stable spiral

$\alpha = 2 \Rightarrow$ degenerate node stable

$2 < \alpha \Rightarrow$ stable node

b)