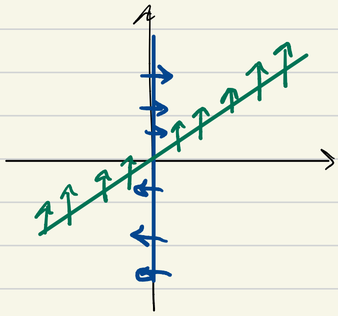
$$f = y - x$$
$$g = x^2$$

3.1 a) $\mathbb{J} = \begin{bmatrix} -1 & 1 \\ 2x & 0 \end{bmatrix}\Big|_{x,y=0} = \begin{bmatrix} -1 & 1 \\ 6 & 0 \end{bmatrix}$



$$\Rightarrow \begin{bmatrix} -1-\lambda & 0 \\ 0 & -\lambda \end{bmatrix}$$

$$\det(\mathbb{J}) = 0 \qquad \Rightarrow \qquad \lambda = -0{,}5 \pm 0{,}5$$
$$\tau = -1$$

b) $\dot{r} = h(r)$ $\quad \rightarrow h(r) \sim ar + O(r^2)$ $\quad$ for small r

$\dot{\theta} = 0$ $\quad$ & $a \neq 0$

Polar coordinates $\qquad$ Cartesian dynamics

$$\left\{ \begin{array}{l} \dot{r} = f(r) \\ \dot{\theta} = \text{const.} \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \dot{x} = \frac{dr}{dt}\cos\theta = \dot{r}\cos\theta - r\sin\theta \cdot \dot{\theta} \\ \dot{y} = \dot{r}\sin\theta + r\cos\theta \cdot \dot{\theta} \end{array} \right.$$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$\dot{x} = ar\cos\theta - r\sin\theta \cdot 0$$
$$\dot{y} = ar\sin\theta + r\cos\theta \cdot 0$$

$$\cos\left(\arctan\left(\frac{y}{x}\right)\right) = \frac{x\sqrt{x^2+y^2}}{x^2+y^2}$$
$$\sin\left(\arctan\left(\frac{y}{x}\right)\right) = \frac{y\sqrt{x^2+y^2}}{x^2+y^2}$$

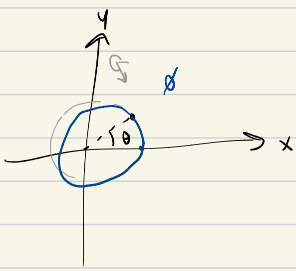
$$\Rightarrow \dot{x} = ax$$
$$\dot{y} = ay$$

c) $\mathbb{J} = \begin{bmatrix} 0 & 3y^2 \\ 1 & 0 \end{bmatrix}\Big|_{x^*,y^*} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$$\tau = 0$$
$$\Delta = -1 \qquad = \text{Saddle-node}$$

d) $\dot{x} = (x^2+y^2)^{|n|/2} \cos\left(n \arctan\left(\frac{y}{x}\right)\right)$

$\dot{y} = (x^2+y^2)^{|n|/2} \sin\left(n \arctan\left(\frac{y}{x}\right)\right)$



$$\Rightarrow \dot{x} = r^{|n|} \cos(n\theta)$$
$$\dot{y} = r^{|n|} \sin(n\theta)$$

$$\phi(\theta) = \arctan\left(\frac{\dot{y}}{\dot{x}}\right) = \arctan\left(\frac{\sin(n\theta)}{\cos(n\theta)}\right) = n\theta$$

$$\Delta\phi = \phi(2\pi) - \phi(0)$$
$$= 2\pi n - 0n = 2\pi n$$
$$I = \frac{\Delta\phi}{2\pi} = n$$