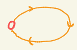
Homoclinic orbit  Conects to itself

a) $\dot{x} = \mu x + y - x^2$

$\dot{y} = -x + \mu y + 2x^2$

c) $\dot{x} = ux$

$\dot{y} = sy$

s ~ eigenvalues of unstable direction

u ~ —''— stable direction

$s < 0$

$u > 0$

$(x(0), y(0)) = (\gamma, 1)$ $\quad 0 < \gamma << 1$

find t_1 ~ time to escape saddle to $x(t_1) = 1$

$$\dot{x} = \frac{dx}{dt} = ux \Rightarrow \frac{1}{x}dx = u\,dt$$

integrate both sides $\Rightarrow \ln|x| + C_1 = ut + C_2$

$\Rightarrow \ln|x| = ut + C$

$\Rightarrow x = e^{ut+C} = e^C e^{ut}$

$x(0) = \gamma \Rightarrow \gamma = e^C$

$x(t) = \gamma e^{ut}$

$x(t_1) = 1 \Rightarrow 1 = \gamma e^{ut_1}$

$\Rightarrow \ln\left(\frac{1}{\gamma}\right) = ut_1$

$\Rightarrow t_1 = -\frac{\ln\gamma}{u}$ //

d) find expression for u

(x^*, y^*): $\dot{x} = \dot{y} = 0 \Rightarrow 0 = \mu x + y - x^2 \Rightarrow y = x^2 - \mu x$ (*)

$0 = -x + \mu y + 2x^2$ $\quad = -x + \mu x^2 - \mu^2 x + 2x^2$

$= (\mu+2)x^2 - (\mu^2+1)x \quad \Rightarrow x_1^* = 0 \Rightarrow y_2^* = 0$

$\Rightarrow x_2^* = \frac{\mu^2+1}{\mu+2}$

(*) $\Rightarrow y_2^* = \left(\frac{\mu^2+1}{\mu+2}\right)^2 - \mu\left(\frac{\mu^2+1}{\mu+2}\right)$

from plots in b) $\rightarrow (x_2^*, y_2^*)$ ~ fp for saddle

$$J = \begin{bmatrix} \mu - 2x & 1 \\ -1 + 2x & \mu \end{bmatrix}_{x^*, y^*}$$

Mathematica gives: $\lambda_{1,2} = \frac{-1 + 2\mu \pm \sqrt{\mu^4 + 4\mu^3 + 9\mu^2 + 5}}{\mu + 2}$

since $u > 0 \Rightarrow u = \frac{-1 + 2\mu + \sqrt{\mu^4 + 4\mu^3 + 9\mu^2 + 5}}{\mu + 2}$