$$\dot{x} = (\sigma+3)x + 4y$$
$$\dot{y} = -\tfrac{9}{4}x + (\sigma-3)y$$

b) $$A = \begin{pmatrix} \sigma+3 & 4 \\ -9/4 & \sigma-3 \end{pmatrix}$$

$$\lambda_{1,2} = \tfrac{1}{2}\left(\tau \pm \sqrt{\tau^2 - 4\Delta}\right)$$

$$\tau = (\sigma+3) + (\sigma-3) = 2\sigma$$
$$\Delta = (\sigma+3)(\sigma-3) - (4)\left(-\tfrac{9}{4}\right) = \sigma^2 - 9 + 9 = \sigma^2$$

$$\lambda_{1,2} = \tfrac{1}{2}\left(2\sigma \pm \sqrt{4\sigma^2 - 4\sigma^2}\right) = \sigma \pm 0$$

$$\lambda = \begin{bmatrix} \sigma \\ \sigma \end{bmatrix}$$

c) $$(A - I\lambda)v = 0$$

$$\begin{pmatrix} 3 & 4 \\ -\frac{9}{4} & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow y = -\tfrac{3}{4}x$$

$$\Rightarrow -\tfrac{9}{4}x - 3\left(-\tfrac{3}{4}x\right) = 0 \quad //$$

$$v = \begin{pmatrix} x \\ -\frac{3}{4}x \end{pmatrix}$$

$$\hat{v} = \frac{v}{\|v\|} = \frac{v}{\sqrt{x^2 + \left(-\frac{3}{4}x\right)^2}} = \begin{pmatrix} 4/5 \\ -\frac{3}{5} \end{pmatrix}$$

d) $A^{-1} = ?$

$$A^{-1} = \frac{1}{\sigma^2}\begin{pmatrix} \sigma-3 & -4 \\ \frac{9}{4} & \sigma+3 \end{pmatrix} =$$

e) $\sigma = 0$

f) —

g) $$\begin{matrix} \dot{x} \\ \dot{y} \end{matrix}\begin{cases} 3x + 4y - \frac{x^3}{100} \\ -\frac{9}{4}x - 3y \Rightarrow y = -\frac{3}{4}x \end{cases}$$

$$\dot{x} = 3x + 4(-3/4x) - \frac{x^3}{100} = -\frac{x^3}{100}$$
$$\dot{x} = 0 \Rightarrow x = 0$$

$$\Rightarrow (x^*, y^*) = (0,0)$$