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Relatório de Projeto do Mestrado Integrado em Engenharia Física, realizado sob a orientação científica dos Doutores Nome completoe Nome completo, Professores Auxiliares do Departamento de Física da Universidade de Aveiro

o júri / the jury

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agradecimentos / acknowledgements

Obrigado

Thank you

Resumo Bem resumido

Abstract Asbstract

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Chapter 1

Introduction

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Chapter 2

Torn to pieces. Glued together

This work will be using terms referring to the analysed system, which will be defined next. Sets are defined by Uppercase (A), elements by lowercase (a) and operators by ($\hat{A}$).

Goods are used and created by Agents to perform trades.

- **Shares (S)** are owned exclusively by Persons and each refer to a singular Factory. The sum of all shares from a single Factory must be 1
- **Stock ($\hat{\lambda}, \hat{\nu}$)** is owned exclusively by Factories, and are of essential or luxury type. essential Stock (λ) has a maximum consumption per Person per cycle of 1, while luxury (ν) doesn't. $\hat{\mu}$ is used to denote both types
- **Capital ($\hat{c}$)** is the currency, tradable with Goods. The total capital in the system is constant and defined by c_{tot}

Agents are objects capable of holding Goods.

- **Persons (P)** is a set of person objects that can consume stock, hold shares and work on factories.
- **Factories (F)** will hold Stock and, if it has workers ($|W(f)| > 0$), it will produce stock.

If a Factory f has leftover capital in a cycle, this capital is distributed over all Persons who hold Shares referring it.

Markets are capable of trading Goods

- **GoodsMarket ($\hat{G}$)** handles Stock consumption with Capital.
- **WorkersMarket ($\hat{W}$)** handles worker assignment to Factories.
- **SharesMarket ($\hat{S}$)** handles Capital trade with Shares, and is divided in

Primary ($\hat{S}_1$) where Factories trade with Persons

Secondary ($\hat{S}_2$) where Persons trade with Persons

Initialization Factories and Persons are created and Shares are distributed in the desired manner, these different initial distribution methods are

Egalitarianism every Person holds the same amount of value in Shares

Ownership every factory has a single share holder

Monopoly a single share holder holds all shares

Bourgeoisie shares are distributed over a percentage of People

Their consequences will be analysed further

Cycle Interactions between all these elements occur during cycles in this order:

Production ($\hat{P}$) Factories utilize workers to create stock.

GoodsMarket($\hat{G}$)

Funding ($\hat{F}$) Factories decide Stock production for next cycle and, if needed, create Shares

SharesMarket($\hat{S}$)

WorkersMarket($\hat{W}$)

Operators such as $\hat{c}$ and $\hat{\mu}$ are applied to single objects, and behave as expected, with capital and stock as their eigenvalues and p, f as their **eigenobjects**, respectively. For example, the capital of a person $p \in P$ at time step t is given by $\hat{c}_t\, p = c_p\; p = c_t(p)\; p$. Also, an operator $\hat{A}(a, b)$ signifies that this operator acts on objects a and b, acting as an identity operator $\mathbb{1}$ for all other objects.

Shares refer to both a person and a factory, so they are treated as a unique structure: $S(f, p) \in R^+,\; p \in P,\; f \in F$ refers to the share of person p in the factory f. Using $S(p)$ refers to a set of all non-zero shares held by person p, and $S(f)$ refers to a set of all non-zero shares held by person p:

$$S(p) = \big\{f \in F \;\big|\; S(f, p) \neq 0\big\}$$
$$S(f) = \big\{p \in P \;\big|\; S(f, p) \neq 0\big\}$$

This being said, all of the above will now be unfolded in a more detailed manner.

2.1 Initialization

The system is initialized given the following parameters: $(|P|, |F|, c_{max}, c_{min})$ and the initial condition. This initial condition contains information about how to distribute the available shares in the soon-to-be created factories.

The default values used are $(|P| = 20, |F| = 6, c_{max} = 1000, c_{min} = 10)$, and the default initial condition is Egalitarianism.

The first step is to create P. $p \in P$ is an object containing ID, Capital, employer, luxury and essential satisfaction and share catalog. During Initialization, a name is picked for every person and capital is assigned with a value between c_{max} and c_{min}. For all Persons, employer is set to None, luxury and essential satisfaction is set to 0 and share catalog is empty.

Next, F is created, $f \in F$ are defined by an ID, Capital, workers, share holders, product is essential (True or False) and stock.

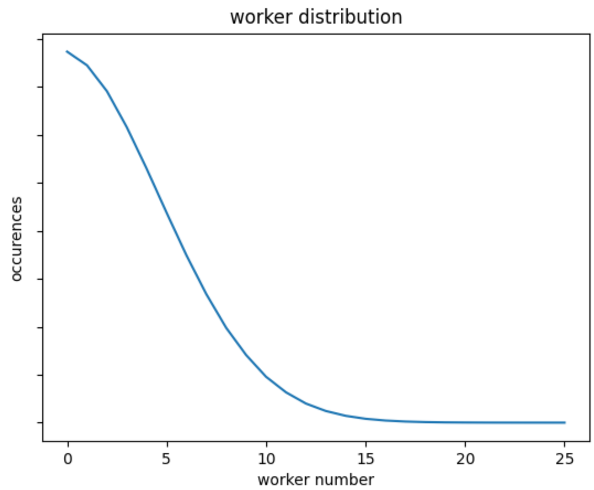


Figure 2.1: Gaussian distribution

Every factory f is assigned an ID and capital in the same way as a person. The number of workers ($|W|$) is generated by sapling a normal (Gaussian) distribution centered on one, just as depicted in figure 2.1 , with a maximum of $\frac{|P|}{2}$ and that number of unemployed Persons are picked at random, and assigned as workers. This sampling operation are translated using the sample function.

$$|W(f)| = \text{sample}(\mathbb{Z}^+, \frac{|P|}{2})$$

$$W(F) = \left\{ W(f) = \bigcup_{i=1}^{|W(f)|} \{p_i \mid p_i \in_R P \cap \overline{W(F)}\} \;\middle|\; f \in F \right\}$$

Where $\in_R$ picks a single random object from a set.

The number of share holders are picked in a similar way, without the need of filtering unemployed people, and the value of each person's stock is picked by generating a uniform random number list and normalizing it.

$$\begin{aligned}
|S(f)| &= \text{sample}(\mathbb{N}, \frac{|P|}{2}) \\
S(F) &= \left\{ S(f) = \bigcup_{i=1}^{|S(f)|} \{p_i \mid p_i \in_R P \cap \overline{S(f)}\} \;\middle|\; f \in F \right\} \\
S(f,p) \neq 0 \Leftrightarrow p &\in \bigcup_{i=1}^{|S(f)|} \{p_i \mid p_i \in_R P \cap \overline{S(f)}\} \wedge f \in F \\
1 &= \sum_{i}^{|S(f)|} S(f, p_i)
\end{aligned}$$

The first Factory's product is always essential, the second is always luxury, from there on, it's randomly picked 50% chance each. Defining F_λ and F_ν as all factories that produce essential and luxury, respectively.

$$F = \left\{ f \;\middle|\; f \in F, f_1 \in F_\lambda \wedge f_2 \in F_\nu \right\}$$

2.2 Cycle

This is the portion that is repeated to advance the simulation, represented by the time step operator $\hat{T}\,|\psi_t\rangle = |\psi_{t+1}\rangle$. This operator is simply the combination of the other cycle operators: $\hat{T} = \hat{W}\,\hat{S}\,\hat{F}\,\hat{G}\,\hat{P}$, all of these conserve total capital c_{tot}.

The capital and stock at the end of each cycle operator is written using the following short-hand notation:

$$\begin{gathered}
\hat{c}_t\,\hat{P}\,a = \hat{c}_{\hat{P}}\;a = c_{\hat{P},a}\,a \\
\hat{c}_t\,\hat{G}\,\hat{P}\,a = \hat{c}_{\hat{G}}\;a = c_{\hat{G},a}\,a \\
\dots \\
\hat{c}_t\,\hat{W}\,\hat{S}\,\hat{F}\,\hat{G}\,\hat{P}\,a = \hat{c}_{\hat{W}}\;a = c_{\hat{W},a}\,a = c_{t+1}a \\
\hat{\mu}_t\,\hat{W}\,\hat{S}\,\hat{F}\,\hat{G}\,\hat{P}\,f = \hat{\mu}_{\hat{W}}\;f = \mu_{\hat{W},f}\,f = \mu_{t+1}f \\
\forall a \in P \cup F, \forall f \in F
\end{gathered}$$

Every repetition is called a cycle, or a time step, and it acts on all agents $|\psi_t\rangle = \{\text{P,F}\}$.

During **Production** $\hat{P}$, Factories pay salaries by evenly diving all it's capital by its workers,

$$\hat{c}_{\hat{P}}\, W_t(F) = \left\{ \left\{ \hat{c}_t\, p + \frac{\hat{c}_t\, f}{|W_t(f)|} \;\middle|\; p \in W_t(f) \right\} \;\middle|\; f \in F \right\}$$
$$\hat{c}_{\hat{P}}\, F = 0$$

Each factory is left with 0 capital and produces an amount of stock defined by the production function depicted in figure 2.2

Figure 2.2: Production function

$$\hat{\mu}_{\hat{P}}\; f = \mu_{\hat{P},f}\; f = \left(\mu_f + \log(\frac{\text{salary}_t(f)}{m_{\text{wage}}}) \right)\; f$$
$$\text{salary}_t(f) = \frac{c_f}{|W_t(f)|}$$

This function also defines the minimum wage, setting $\Delta\mu = 0 \Leftrightarrow m_{wage} = e^{-1}$ is the default value for minimum wage

Lastly, the stock price is defined:

$$\Delta\text{price}(f) = \min \left\{ 2,\; 0.2 + \frac{\mu_{\hat{P}}(f) - \mu(f)}{\mu(f)} \right\}$$

$$\text{price}_t(f) = \frac{1}{\mu_{\hat{P}}(f)} \Big(\Delta\text{price}(f)\, c_f \left(\mu_{\hat{P}}(f) - \mu(f) \right) + \rho\, \text{price}_{t-1}(f)\, \mu(f) \Big)$$

$\Delta\text{price}(f)$ being the return ratio per unit of stock produced this cycle, $\Delta\text{price}(f) \in [1, 2]$ and grows proportionally to factory stock, this models the law of supply and demand, the smaller leftover stock $(\mu_{\hat{P}}(f) - \mu(f))$ a factory has, the more profit it expects, and vice versa. $price_t(f)$ is the final price per unit of stock. The price is set using a weighed median between the price of stock produced this cycle and the previous price. The price for stock produced this cycle is given by the profit margin multiplying cost of production, $\Delta\text{price}_t(f)\, c_f$, and for last cycle, a devaluation or appreciation effect is translated through ρ, if $\rho < 1$, stock devalues, if $\rho > 1$, stock appreciates. $\rho = \frac{2}{3}$ by default.

The **GoodsMarket** is divided into essential market $\hat{G}_\lambda$ and luxury market $\hat{G}_\nu$, $\hat{G} = \hat{G}_\nu\, \hat{G}_\lambda$ both markets iterate over every person the following steps: select a factory, trade with that factory, repeat. The selection step creates an ordered set of factories with non-zero stock sorted by price, $F_{G,\mu}$, and selects a factory by sampling this set using the same normal distribution depicted in figure 2.1, sample$(\mathbb{Z}^+, |F_{G,\mu}|)$. If this set is empty, that market ends, as there is no more factories to trade. Alternatively, person trades with the sampled factory: transfers capital from person to factory and destroys traded stock. In the essential market, person stops trading when it has traded one unit of essential stock or its capital reaches zero, on luxury market, person only stops trading when its luxury available capital (defined further) reaches zero. The sampling selection mechanism is mapped to

$$\text{select}(A) = a \Leftrightarrow a = i\text{'th element in } A,\ i = \text{sample}(\mathbb{Z}^+, |A|)$$

and leads to what could be called a semi-perfect market, or weighted random market, where factories with lower prices are more likely to trade, as the probability for smaller indices of the sorted factory list is bigger, but more expensive factories that would not trade on a perfect market, may also trade, creating noise proportional to the standard deviation of the distribution. This noise could translate a multitude of real world parameters, such as distance, personal preference, marketing, etc. As this market repeats a given operation for every person, it can be said that

$$\hat{G}_\mu = \widehat{\text{attemptTrade}}_\mu(p_1)\, \widehat{\text{attemptTrade}}_\mu(p_2)\, \ldots$$

Essential market tracks the capital available to person p by $c_\lambda(p)$, stock consumed by person p using $g_\lambda(p)$ and stock available for trade to factory f using $g_\lambda(f)$, these are initialized by

$$c_\lambda(p) = c_{\hat{P},f}, g_\lambda(p) = 0, \forall p \in P, \text{ and } g_\lambda(f) = \lambda_{\hat{P},f}\, \forall f \in F_\lambda.$$

$$
\hat{\text{attemptTrade}}_\lambda(p) = \begin{cases} \mathbb{1} & \text{if } |F_{G,\lambda}| = 0 \vee c_\lambda(p) = 0 \\ \hat{\text{attemptTrade}}_\lambda(p) \times \hat{\text{trade}}_\lambda(f, \alpha, p) & \text{else} \end{cases},
$$

$$
f = \text{select}(F_{G,\lambda}),
$$

$$
\alpha = \min\left\{1, \begin{cases} g_\lambda(f) - g_\lambda(p) & \text{if } c_\lambda(p) \geqslant \text{price}(f) \times g_\lambda(f) - g_\lambda(p) \\ \frac{c_\lambda(p)}{price(f)} & \text{if } g_\lambda(p) < \text{price}(f)\,(g_\lambda(f) - g_\lambda(p)) \end{cases}\right\},
$$

$$
F_{G,\lambda} = \big\{ f \in F \text{ sorted by price}(f) \bigm| g_\lambda(f) > 0 \big\}.
$$

The $\hat{\text{trade}}_\mu$ operator has the following behaviour:

$$
\begin{aligned}
\hat{\text{trade}}_\mu(f, \alpha, p) &\Rightarrow c_\mu(p) \to c_\mu(p) - \text{price}(f) \times \alpha, \\
&\Rightarrow g_\mu(p) \to g_\mu(p) + \alpha, \\
&\Rightarrow g_\mu(f) \to g_\mu(f) - \alpha
\end{aligned}
$$

At the end of essential market, the leftover capital for each person, $c_\lambda(p)$, will be allocated for luxury and shares markets in a predetermined manner,$\%_\nu = 40\%$ for luxury, $\%_s = 40\%$ for shares market and saving $\%_l = 20\%$ for next cycle.

The **luxury market** tracks the capital available to person p by $c_\nu(p)$, stock consumed by person p using $g_\nu(p)$ and stock available for trade to factory f using $g_\nu(f)$, these are initialized by

$$
c_\nu(p) = c_\lambda(p), g_\nu(p) = 0, \; \forall p \in P, \text{ and } g_\nu(f) = \nu_{\hat{P},f} \, \forall f \in F_\nu.
$$

$$
\hat{\text{attemptTrade}}_\nu(p) = \begin{cases} \mathbb{1} & \text{if } |F_{G,\nu}| = 0 \vee c_\nu(p) = 0 \\ \hat{\text{attemptTrade}}_\nu(p) \times \hat{\text{trade}}_\nu(f, \alpha, p) & \text{else} \end{cases},
$$

$$
f = \text{select}(F_{G,\nu}),
$$

$$
\alpha = \begin{cases} g_\nu(f) - g_\nu(p) & \text{if } c_\nu(p) \geqslant \text{price}(f) \times g_\nu(f) - g_\nu(p) \\ \frac{c_\nu(p)}{price(f)} & \text{if } g_\nu(p) < \text{price}(f)\,(g_\nu(f) - g_\nu(p)) \end{cases}
$$

$$
F_{G,\nu} = \big\{ f \in F \text{ sorted by price}(f) \bigm| g_\nu(f) > 0 \big\}.
$$

The final state of goods market can seen as:

$$
\begin{aligned}
\hat{c}\,\hat{G}\,\hat{P}\,p &= \Big(c_{\hat{P},p} - \big(c_\lambda(p) + c_\nu(p)\big)\Big)\,p \\
\hat{c}\,\hat{G}\,\hat{P}\,f &= \Big(c_{\hat{P},p} + \big(g_\mu(f) \times \text{price}(f)\big)\Big)\,f \\
\hat{\mu}\,\hat{G}\,\hat{P}\,f &= \Big(\mu_{\hat{P},p} - g_\mu(f)\Big)\,f
\end{aligned}
$$

During **Funding**, factories start by projecting their attempted production, μ_{proj}, for next time step, using the estimation function depicted in figure 2.3:

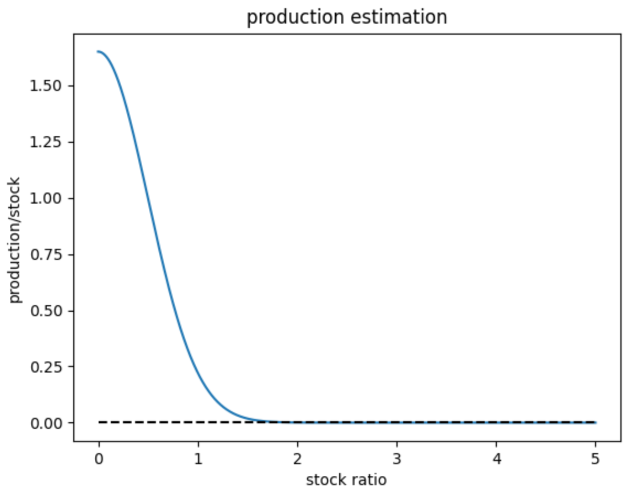


Figure 2.3: Production estimate

This function is defined by

$$
\begin{aligned}
\mu_{\text{proj}}(f) &= \mu_{\hat{P},f}\ \exp\Big(-\frac{\text{ratio}(f)^2}{k} + k\Big) + 1 \\
\text{ratio}(f) &= \frac{\mu_{\hat{G}}(f)}{\mu_{\hat{P}}(f) - \mu_{\hat{G}}(f)} \\
c_{\text{proj}}(f) &= \overline{c/\mu} \times \mu_{\text{proj}}(f) \\
\overline{c/\mu} &= \sum_{f \in F} \frac{|W(f)|\, \text{salary}(f)}{\mu_{\hat{P}}(f) - \mu(f)}
\end{aligned}
$$

Where k is a simulation constant with a default value of $k = 0.5, k \in \mathbb{R}^+$ and translates the aggressiveness of the factories in this simulation. If $k \to 0$, factories will only produce more than one unit of stock if leftover equals zero; if $k \to \infty$, factories will always produce as much stock as possible, regardless of how much stock was sold, and never returning capital to share holders. The "+1" term exists as a way to simulate ambition, avoiding production falling to zero during extreme conditions. c_{proj} is the projection of capital needed for producing this amount of stock, and $\overline{c/\mu}$ the average cost for producing a unit of stock Then, if a factory raised enough capital during goods market in order to produce the projected stock, all remaining capital is distributed over its share holders, if it did not, then it must fund itself on the share market by selling shares.

$$c_{F,p} = c_{\hat{G},p} + \sum_{f \in S(p)} \begin{cases} \left(c_{\hat{F},f} - c_{\text{proj}}\right) S(f,p) & \text{if } c_{\hat{F},f} > c_{\text{proj}}(f) \\ 0 & \text{if } c_{\hat{F},f} \leqslant c_{\text{proj}}(f) \end{cases}, \forall p \in P$$

$$\hat{F}\ f = \begin{cases} \text{distribute}(f)\, f & \text{if } c_{\hat{G},f} > c_{\text{proj}}(f) \\ \text{fund}(f)\, f & \text{if } c_{\hat{G},f} \leqslant c_{\text{proj}}(f) \end{cases}, \forall f \in F$$

For each factory, each of its share holders receives a percentage of its excess capital equal to the value of their share in that factory, and nothing if no excess capital exists. In the latter case, the factory attempts to receive the remaining capital by selling its shares on the share market, if it succeeds, every share holder will now hold a smaller share of the factory, and new share holder(s) will exist. The distribute and fund functions are defined in the following manner:

$$\text{distribute}(f) \Rightarrow \hat{c}_t\, \hat{F}\, \hat{G}\, \hat{P}\, f = c_{\text{proj}}(f)\, f$$

$$\text{fund}(f) \Rightarrow m_1(f) = \frac{c_{\text{proj}}(f) - c_{\hat{G},f}}{c_{\hat{G},f} + \text{price}_t(f)\mu_{\hat{P},f} + \text{salary}_t(f)|W(f)|}, f \in M_1$$

$M(f)$ being the share amount (percentage of f) to be sold in the shares market, the capital needed divided by the total factory value, also refered as

$$\text{val}(f) = c_{\hat{G},f} + \text{price}_t(f)\mu_{\hat{P},f} + \text{salary}_t(f)|W(f)| \text{ capital shares}^{-1}$$

The **SharesMarket** is divided into primary and secondary, $\hat{S} = 1 + \hat{S}_1\,\hat{S}_2$, during the primary shares market, factories sell shares to persons, acquiring capital, during the secondary market, persons may sell shares that they hold over factories to also acquire capital for themselves.

Much like the luxury market, shares market defines a share available capital, $c_s(p) = \%_s \times c_\lambda(p)$, factory capital $c_s(f) = c_{\hat{F},f}$ and two new share-like objects, $T_1(p)$ holds all factories that sold shares to person p, $T_1(p,f)$ holds the share amount that factory f sold to p. Now, for secondary market, T_2 must hold persons, factories and share amounts, so $T_2(p')$ holds all persons that sold shares to person p', $T_2(p',p)$ all factory from which person p sold shares to person p', and $T_2(p',p,f)$ the amount of shares sold from person p to p' on factory f. To exemplify, if only one share was sold from person p to p' in factory f, the share amount is $T_2(p,p',f) = T_2(p,p',T_2(p,p'))$, as $T_2(p,p')$ is a set of only one factory element.

Primary shares market attempts to trade shares for all factories that require funding by picking a random person with share available capital. The total capital spent on primary market by person p is defined using $c_1(p) = \sum_{f' \in T_1(p)} \text{val}(f') \times T_1(f', p)$

$$\begin{aligned}
\hat{S}_1 &= \bigcup_{f \in M_1} \text{atte}\hat{\text{mp}}\text{tSell}(f, p),\ p \in_R P_{\hat{S}_1} \\
\text{atte}\hat{\text{mp}}\text{tSell}(f, p) &= \left(\begin{cases} \mathbb{1} & \text{if } \alpha = m_1(f) \vee |P_{\hat{S}_1}| = 0| \\ \text{atte}\hat{\text{mp}}\text{tSell}(f, p') & \text{if } \alpha < m_1(f) \end{cases} \right) \times \hat{\text{sell}}_1\,(f, \alpha, p), \\
\alpha &= \begin{cases} m_1(f) & \text{if } m_1(f) \times \text{val}(f) \leqslant c_s(p) - c_1(p) \\ \frac{c_s(p) - c_1(p)}{\text{val}(f)} & \text{if } m_1(f) \times \text{val}(f) > c_s(p) - c_1(p) \end{cases}, \\
p' &\in_R P_{\hat{S}_1}, \\
P_{\hat{S}_1} &= \left\{ p \in P \;\middle|\; c_s(p) - c_1(p) > 0 \right\}
\end{aligned}$$

If $|P_{\hat{S}_1}| = 0$, no more shares will be sold during primary market nor secondary market, as all share market available capital has been exhausted The $\hat{\text{sell}}$ operator will have the following behaviour:

$$\begin{aligned}
\hat{\text{sell}}_1(f, \alpha, p) \Rightarrow\, & p \in T_1(f),\ f \in T_1(P),\ T_1(f, p) \to T_1(f, p) + \alpha, \\
& p \in S(f),\ f \in S(p),\ S(f, p) \to S(f, p) + \alpha, \\
& m_1(f) \to m_1(f) - \alpha, \\
& c_s(p) \to c_s(p) - \alpha \times \text{ val}(f), \\
& c_s(f) \to c_s(f) + \alpha \times \text{val}(f)
\end{aligned}$$

Secondary shares market attempts to trade shares for all persons that require funding by picking a random person with share available capital.

Persons will only sell shares during secondary market if they may not be capable of fully consuming essential during the next cycle, this future capital needed projection is defined by the sum of essential capital projection and luxury capital.

$$c_{\text{ess}} = \frac{1}{|F_\lambda|} \sum_{f \in F_\lambda} \text{price}(f)$$

The total capital spent on primary and secondary market by person p is defined using $c_2(p) = c_1(p) + \sum_{p' \in T_2(p)} \Big(\sum_{f \in T_2(p, p')} \text{val}(f) \times T_2(p, p', f) \Big)$. For next step it is also important to introduce the mean salary per person, $\overline{\text{salary}} = \frac{1}{|P|} \sum_{f \in F} |W(f)| \times \text{salary}(f)$

$$
\begin{aligned}
M_2 &= \left\{ p \in P \;\middle|\; \%_l \times c_\lambda(p) + \overline{\text{salary}} < c_{\text{ess}} \wedge |S(p)| > 0 \right\} \\
m_2(p) &= \min \left\{ \%_l \times c_\lambda(p) + \overline{\text{salary}} - c_{\text{ess}} \,, \sum_{f \in S(p)} S(p, f) \right\}, \, p \in M_2 \\
\hat{S}_2 &= \bigcup_{p \in M_2} \widehat{\text{attemptSell}}(p, p'), \; p' \in_R P_{\hat{S}_2} \\
\widehat{\text{attemptSell}}(p, p') &= \left(\begin{cases} \mathbb{1} & \text{if } \alpha = m_2(p) \vee |P_{\hat{S}_2}| = 0 \\ \widehat{\text{attemptSell}}(p, p'_2) & \text{if } \alpha < m_2(p) \end{cases} \right) \times \widehat{\text{sell}}_2\,(p, \alpha, p') \\
\alpha &= \begin{cases} m_2(p) & \text{if } m_2(p) \leqslant c_s(p') - c_2(p') \\ c_s(p') - c_2(p') & \text{if } m_2(p) > c_s(p') - c_2(p') \end{cases}, \\
& p'_2 \in_R P_{\hat{S}_2} \\
P_{\hat{S}_2} &= \left\{ p' \in P \;\middle|\; c_s(p') - c_2(p') > 0 \right\}
\end{aligned}
$$

It is of note that m_1 contains shares and m_2 contains needed capital, and same applies to both α's. The $\widehat{\text{sell}}_2$ operator will have a more complicated behaviour, as it will trade shares until α capital has been traded, so a second, simpler operator will handle trades, $\widehat{\text{trade}}(p, \alpha', p', f)$ trades α' shares in factory f from person p to person p', and $\alpha' \times \text{val}(f)$ capital from person p' to person p

$$
\widehat{\text{sell}}_2(p, \alpha, p') = \prod_{f \in S(p)} \begin{cases} \widehat{\text{trade}}(p, \alpha', p', f) & \text{, if } m_2(p) > 0 \\ & , \alpha' = \min\left\{ S(p, f), m_2(p)/\text{val}(f) \right\} \\ \mathbb{1} & \text{, if } m_2(p) = 0 \end{cases}
$$

$$
\begin{aligned}
\widehat{\text{trade}}(p, \alpha', p', f) \Rightarrow & \, p' \in S(f), \; f \in S(p'), \; S(f, p') \to S(f, p') + \alpha', \\
& S(f, p) \to S(f, p) - \alpha', \text{if } S(f, p) = 0, p \notin S(f), \; f \notin S(p), \\
& m_2(p) \to m_2(p) - \alpha' \times \text{val}(f), \\
& c_s(p) \to c_s(p) - \alpha' \times \, \text{val}(f), \\
& c_s(p') \to c_s(p') + \alpha' \times \text{val}(f)
\end{aligned}
$$

At the end of share market, because during primary market the normalization of shares in factories was violated, it is then recovered using a normalize operator $\widehat{\text{norm}}$, so that $\hat{S}$ becomes, in fact, $\hat{S} = \widehat{\text{norm}}\, \hat{S}_1 \, \hat{S}_2$. This could be achieved by normalizing at every trade during primary market, reaching the same end state.

WorkersMarket handles finding new workers for every factory in the next cycle. This is achieved by iterating over every person: finding the highest salary factory, if this salary is higher than minimum wage, person becomes worker in factory, update factory projected salary, repeat. There is no need to implement noise in this stage because there is more persons than factories, so noise would complicate the procedure without providing further realism.

As this market repeats a given operation for every person, $\hat{W} = \hat{\text{find}}^{|P|} = \hat{\text{find}}(p_1)\,\hat{\text{find}}(p_2)\ldots$ it also defines a new set, H, containing every person that has been hired. This $\hat{\text{find}}$ operator acts on one person only, and behaves in the following manner:

$$\hat{\text{find}}(p) = \begin{cases} \mathbb{1} & \text{if } c_{\text{proj}}(f)/\big(1+|W_{t+1}|\big) < m_{\text{wage}} \\ p \in W_{t+1}(f) \wedge p \in H & \text{if } c_{\text{proj}}(f)/\big(1+|W_{t+1}|\big) \geqslant m_{\text{wage}} \end{cases},$$
$$p \in_R \{p \in P \mid p \notin H\},$$
$$f = \max\Big\{f \in F \text{ sorted by } c_{\text{proj}}(f)/\big(1+|W_{t+1}(f)|\big)\Big\}$$

2.3 Government

A separated organ of this system is the government. It is separated because it is not essential to the system, but it does add a layer of complexity needed to explore interesting outcomes.

The government acts at the end of the cycle, and redistributes its raised capital equally over all persons, this capital can be raised in one or both ways: **Transaction** taxing and **Wealth cap** taxing.

Transaction taxing occurs every time a transaction of capital occurs during a trade in the goods market, shares market and salary payment, The government taxes a given percentage of the total capital in that transaction, 10% by default, and adds it to the raised capital.

Wealth cap taxing occurs at the end of the cycle, where the government takes all capital from every person that has capital higher than a set cap, $\frac{\text{cap}}{|P|}\sum\limits_{p\in P} c_p$. By default, cap$= 4$, a person can hold four times the hypothetical perfectly distributed capital.

This concludes all the possible operations carried out to create the system and its results, it is noteworthy that this model exists as a program, so all mentioned above is only a mathematical representation, the program source is available on TODO where do i put the code.

Chapter 3

Results

3.1 Initial conditions

The impact of initial conditions on any system is an important analysis for finding stabilization states, an economy model with a unique convergence state indicates that all systems with different initial conditions converge, modeling a behaviour that all economies eventually evolve into undifferentiated states, and a model with no convergence indicates that all economies evolve toward unique, differentiated states. The model presented, as will be discussed further, converges slowly ,at infinity, this is a desired behaviour as it mimics (even if for different reasons) real-world economy convergence [2]. Every result presented next is a median of thirty runs, this was done to mitigate outlier readings, as this system borrows from randomness and this can create edge cases.

For the analysis present in figure 3.2, the result of all different initial conditions were studied on a system with no government. A Gini coefficient shows the inequality in a system. Often used in real economies to estimate how far a country's wealth or income distribution deviates from a totally equal distribution. At a maximum of 1 (total inequality) and minimum of 0 (total equality). Real world data places inequality Gini coefficient around 40% [3]. The Gini index of any property v of an object $a \in A$, $v(a)$, is calculated by

$$G = \frac{\sum\limits_{a \in A} \sum\limits_{a' \in A} |v(a) - v(a')|}{2\,\overline{v}\,|A|^2}$$

High overall inequality compared to real world economies present in these results is attributed to a lack of government, as there is no taxation and wealth redistribution, unlike (most) real world countries.

Results show that, for any initial condition, the system evolves in a similar way, monopoly being the most differentiated condition. Inequality grows rapidly and settles, except for monopoly, where it slowly reduces. A higher inequality is to be expected on a monopoly conditioned system, but a higher production, unemployment and mean salary is not trivially assumed.

Monopoly creates more stock and lower satisfaction because there exists a larger group of people with lower capital and a very reduced group with luxury capital, therefore, this wealthier group exclusively consumes luxuries in vast quantities, and, because they attempt to consume all their capital at every trade attempt, big (factory-scope) fluctuations can occur due to the semi-perfect market mechanism, and these fluctuations lead to spikes and drops in production, resulting in a higher net production but lower overall satisfaction, as there is a much smaller group of people joining the luxury market. For all other conditions, this effect is reduced proportionally to inequality, as having a bigger group of people joining each market results in lower fluctuations, stock excess and higher satisfaction.

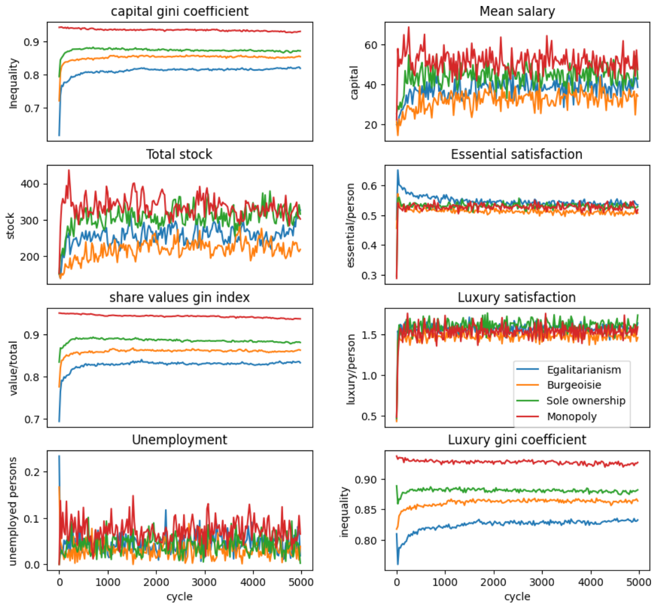


Figure 3.1

An unexpected effect is that sole ownership is considerably closer to monopoly than Bourgeoisie, this occurs because there is a small number of factories and each factory stock is not distributed over a confined group, but to a single individual; essential and luxury satisfaction is mostly equal across conditions; bourgeoisie ranks lower in production and salaries, this occurs due to smaller fluctuations in luxury markets, as more persons enter luxury market; and Egalitarianism converges more slowly, as capital is evenly distributed, inequality and its effects emerges slower. It is also interesting to note that, for all initial conditions, mean salary was, at stability, much higher than minimum salary, stabilizing around 40. This value is affected by the number of people in the system and will be explored further (section **??**).

To further study the effect of these conditions, stock production and consumption were analysed more closely on figure 3.1. Some interesting behaviours include the gap present in luxury production and salaries, with monopoly and sole ownership significantly more luxurious, and the initial fast decrease in essential employees relative to luxury.

These results further support that excess stock is behind the production increase, as consumption peaks before production, and this occurs sooner for monopoly and sole ownership, as these flood luxury market from the start, while the remaining conditions slowly reduce the group size. The delay in consumption and leftover stock is dependent on k, factory aggressiveness, and is addressed further in section **??** Also, the initial condition dependant production increase would not occur if the goods market was modeled as a perfect market, this scenario is addressed in section **??**

As mentioned previously, all initial conditions converge for a large enough number of cycles (toward infinity), but the initial state generates a different path towards it. In figure 3.3, each person is analysed for different initial conditions. The process utilized to generate these consist in distributing the initial shares using persons ID's. For example, in monopoly, all initial shares are handed to the person with an ID of 0, for sole ownership, each factory is handed to the person with the same ID as the factory, for bourgeoisie, shares are distributed over the first 10 ID's, and egalitarianism distributes randomly. The figure presented is the capital available to each person, sorted after the end of that cycle's goods market. This helps to clearly illustrate the shift in inequality, slowly declining for monopoly and greatly increasing for all others, leading to a progressively smaller group holding excess capital and consuming vast amounts of luxury, disrupting the market. Also, the shape of the curve created by these values defines the Giri coefficient, which is 0 if these values form a straight line.

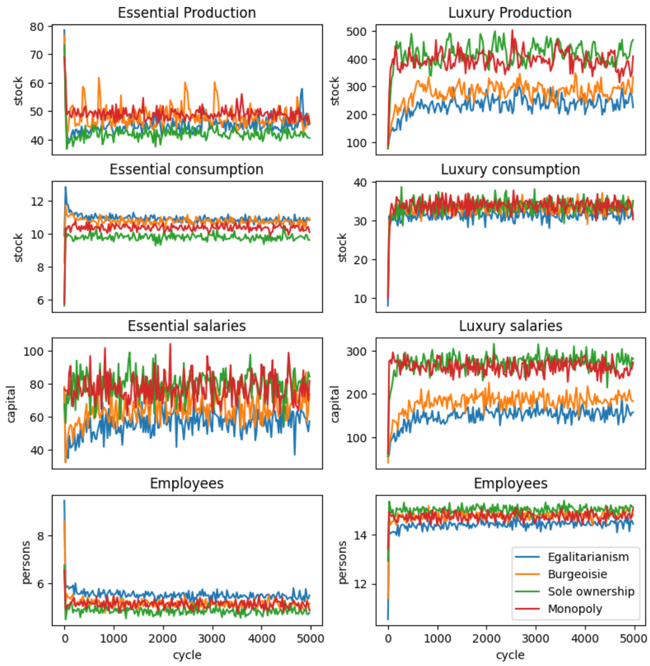


Figure 3.2

As all initial conditions stabilize on a similar state given infinite time and computational power, initial conditions can be seen as a disturbance in the system, making it indistinguishable from another system with a different initial condition. This is a desired behaviour, and occurs because of luxury, as it pushes more wealthy persons to consume more capital per cycle. This convergent behaviour would not emerge in a system without luxury market, and that (unrealistic) scenario will be addressed further (section **??**)

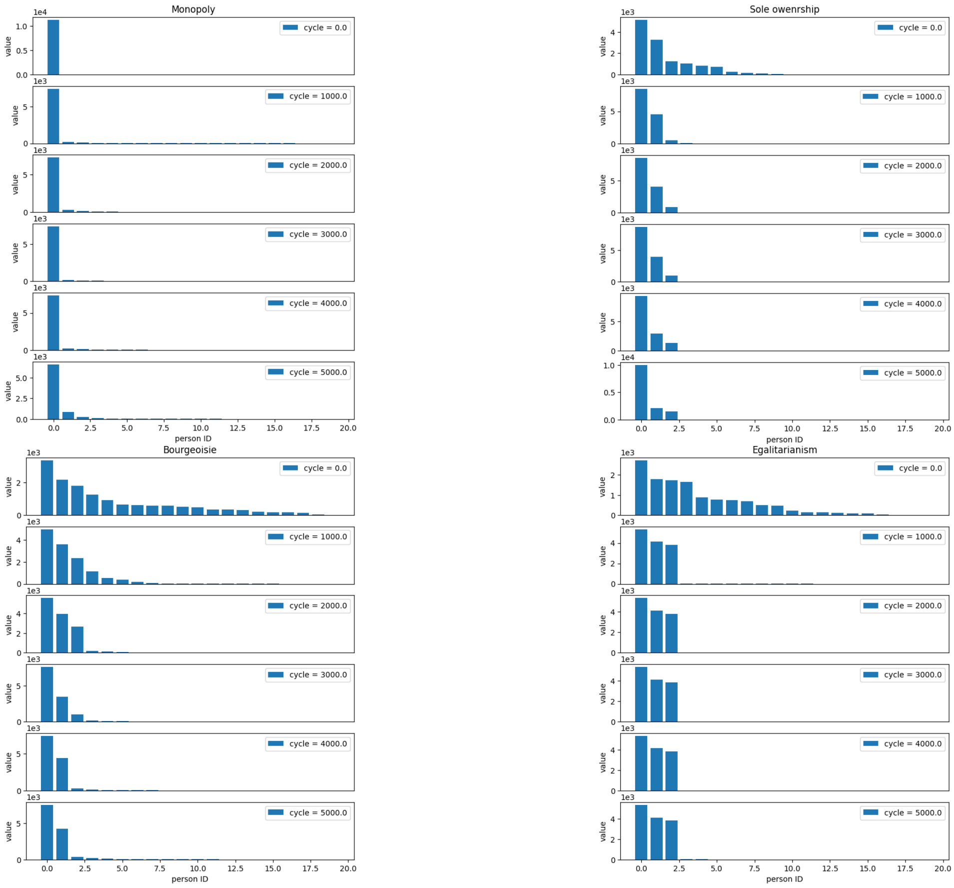


Figure 3.3: individual capital

3.2 Taxation

This section will study ways in which a government may acquire funds and redistribute wealth, redistribution is modeled here by literally redistributing capital over all persons equally, this is a simplification of much more complex real-world government wealth redistribution in the form of public services, such as security, education, health, transportation, infrastructure, etc. Two taxation methods will be implemented, not necessarily based on real-world taxation, but translate a similar effect. A transaction tax acts on every trade completed on goods market, shares market and salary payment, and adds 10% of that transaction value to the government funds and deducts from the beneficiary. A wealth cap tax acts on every person that holds more than 70% of all capital. This may sound like a very high cap, but government acts only at the end of all markets each cycle, and accounts simply for leftover capital that was neither spent on luxury nor share market. Therefore, as can be seen on 3.3, most persons will not hold any capital at that time, except for the extraordinarily wealthy, and this excess wealth will be reduced by 70% each cycle. These government funds, as previously mentioned, are then distributed equally, before salary payments (at the start of next cycle). The next analysis, presented in figure 3.4 studies whether taxation further diverges or converges systems with different initial conditions. looking specifically at capital inequality.

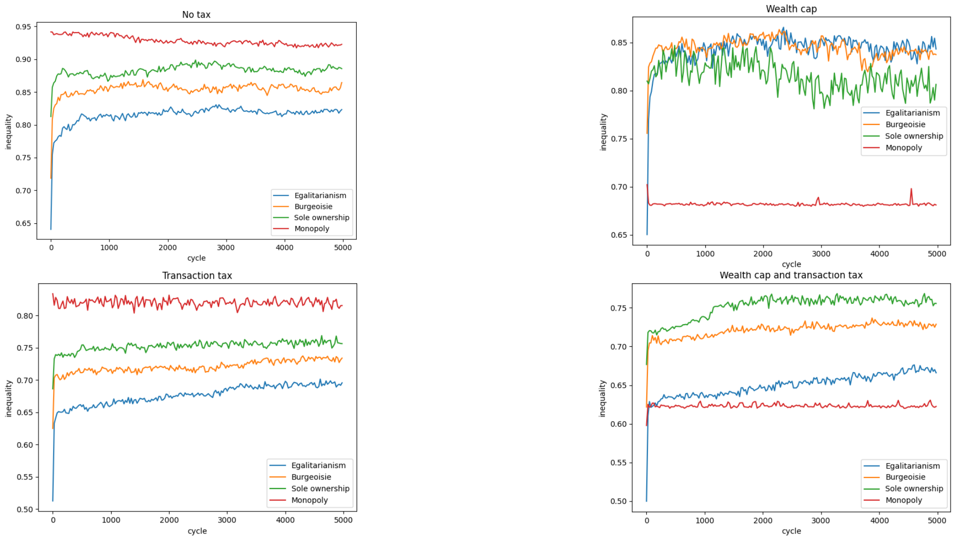
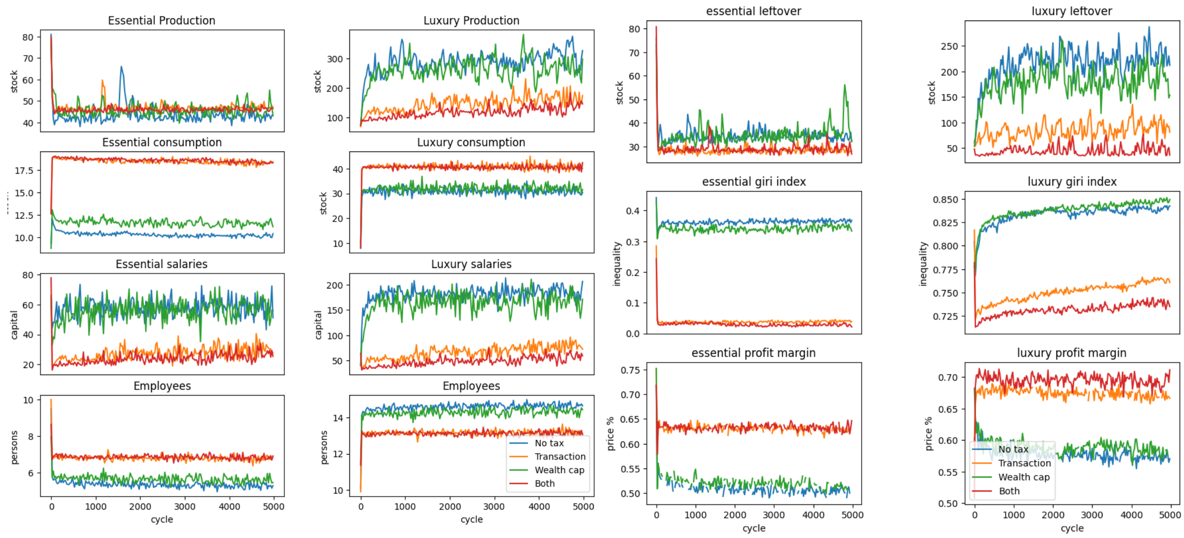


Figure 3.4: Tax effect on gini coefficient

As expected, both taxation methods lower inequality at stability, but some are more effective than others. In a system with no taxes, the stability inequality for all conditions is higher than 80%, this leads to the assumption that a libertarian capitalism leads to a much higher and quicker inequality stabilization. For a transaction tax, all inequalities lower significantly, a 15% decrease can be seen for all conditions at 5000 cycles. It is also interesting that transaction taxing lowers the speed at which the system reaches stability, with inequality still rising (even if much slower) after the 5000 cycle mark. Wealth capping leads to a very interesting behaviour, destabilizing all non-monopoly conditions and generating noise on these, some very interesting and unexpected behaviour is the reverse of inequality order, with egalitarianism being now the most unequal, with an equality around 85%, higher than the untaxed counterpart, and bourgeoisie quickly converging onto it, seemingly unaffected by wealth cap. These conditions are mostly unaffected because wealth capping raises very little or no capital. As for monopoly, wealth capping leads to a heavy reduction in inequality to around 68%, much lower than other conditions. With both taxing methods combined, their effects combine in a rather unique way. Similar to transaction taxing, inequality drops by 15% on all conditions and stability slows, except for monopoly. Monopoly instead drops even further, reaching the lowest inequality stabilization of around 62% much faster than all others, and grows much slower. It is important to note that none of these taxation methods threw the system into an unstable state, but they did induce a differentiated behaviour for each initial condition.

This inequality reduction due to taxation has an effect on production and consumption, that effect is studied for the two most extreme initial conditions, egalitarianism and monopoly.

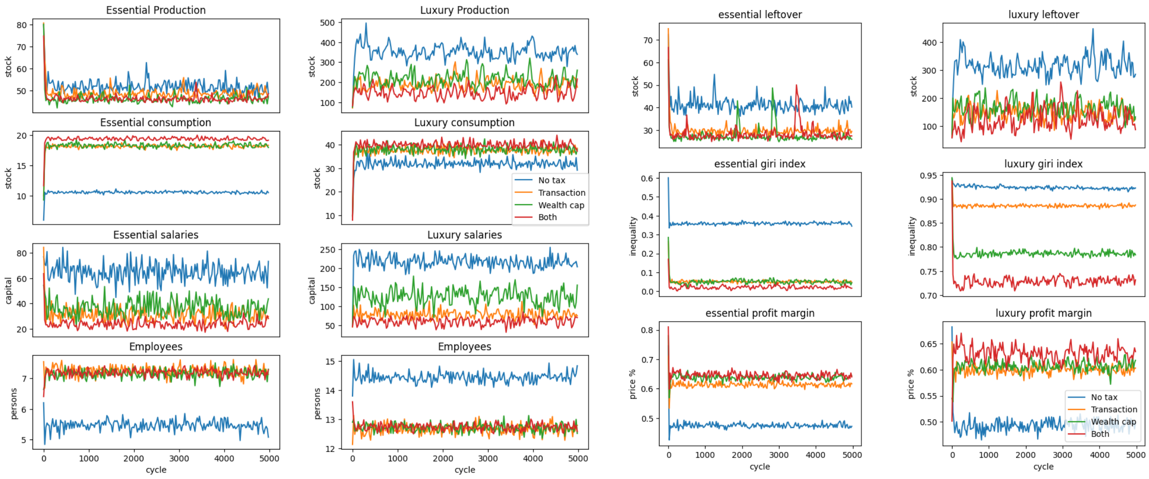


(a) (b)

Figure 3.5: Tax effects on egalitarianism production

On egalitarianism, present in figures 3.5a and 3.5b, some interesting effects can be perceived, first of all, wealth cap is much closer to no tax, this is expected given that, on egalitarianism, reaching the health cap is less likely than other initial conditions, therefore only taxing (combined and transaction tax) and no taxing (wealth cap and no tax) will be differentiated. Taxing results in lower salaries but much higher consumption for essential and a slight increase for luxury, this is achieved by maintaining a lower consumption inequality, lowering the consumption for the wealthier and raising the consumption for the less wealthy, resulting in a net increase in consumption. The mechanisms for this increase are different for essential and luxury. Essential production is maintained by lowering salaries and raising employment, producing roughly the same amount but, as capital inequality is now much lower, persons that would otherwise have little capital to spend on essentials can now consume more essentials, rising consumption greatly. As for luxury, having less employees and lower salaries leads to lower production, but, as inequality is lower, more persons enter the luxury market, causing semi-perfect market fluctuations to be less influential and lower leftover stocks, which in turn leads to lower prices and higher consumption, as stock devaluation happens more rarely. — NOT SO SURE ABOUT THAT ONE—

As for monopoly, present in figures 3.6a and 3.6b, wealth cap transitions from a zero tax system and converges onto transaction taxing, therefore, taxing groups all three forms of taxation, as their behaviour is similar. Taxing, similar to egalitarianism, results in lower salaries and higher consumption, even with lower production by maintaining lower inequality and leftover stock. Differences arise in stabilization time, the monopoly conditioned system stabilized much faster than egalitarianism, this is due to the redistribution of wealth speed being proportional to its concentration, and for luxury inequality a clear divergence of all taxation methods is observed, with transaction tax being much closer to no taxing, this is a reflection of the reversing inequality order generated by wealth cap, present in figure 3.4, where monopoly becomes the less unequal condition.



(a) (b)

Figure 3.6: Tax effects on egalitarianism production

3.3 Customization

A hand-built system's major benefit is customizability, so, for next analysis, the system will be fundamentally modified to hopefully better grasp the inner workings of its mechanisms.

Starting by k, or factory aggressiveness, this value affects the production estimation function, present in 2.3. Bigger k leads to bigger overall investments on stock production and bigger leftover stock.

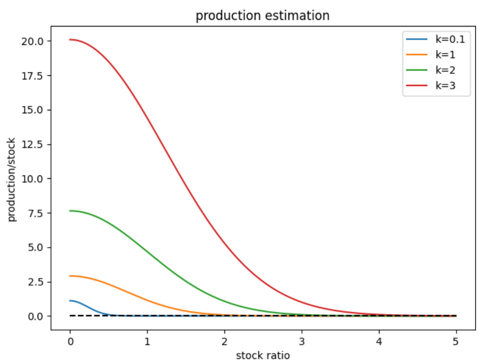


Figure 3.7: Estimation functions

Different estimation functions are depicted in a egalitarian conditioned system with no taxing in figure 3.8. It shows that more aggressiveness leads to more competition between factories, as every factory will fluctuate production wildly and offer more capital for salaries, trying to grow production even when selling less product than produced. This results in a much higher stock surplus and, due to stock value depreciation, lower prices and higher satisfaction. The default k=0.5 value was chosen because it better translates a realistic factory rationalization, meeting the point of stable production (production equals stock) for a leftover of a third stock, meaning that in a theoretical perfect stability point, factories would run with a surplus of a third of total consumption (inventory surplus of 33%), which is not very far from real-world data. [1]

These result in different stability points for the system, depicted in figure **??**

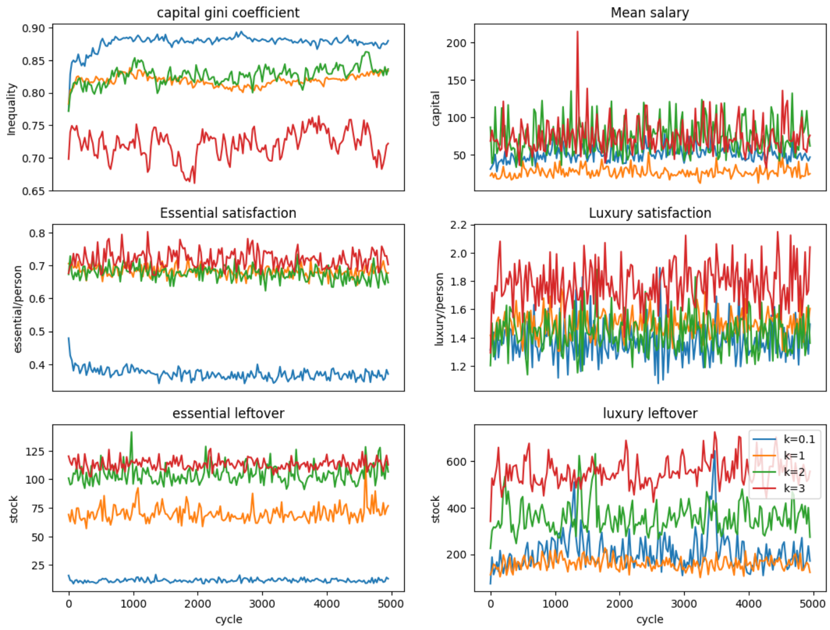


Figure 3.8: Estimation functions

- Variations of the system: - no luxury. - perfect market. - minimum wage (production function) - k - ρ - Production depends on essential satisfaction

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