Universidade de Aveiro Departamento de Física,
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Relatório de Projeto de Licenciatura em Física, realizado sob a orientação científica do Doutor Gareth John Baxter, Investigador Auxiliar do Departamento de Física da Universidade de Aveiro

o júri / the jury

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Resumo

A separação micro/macro na economia teórica está institucionalizada faz décadas e tem havido um esforço constante em juntar estes dois ramos. Uma dessas tentativas serviu de base para os modelos DSGE, utilizados em várias instituições bancárias para prever flutuações de mercado, mas fenómenos económicos recentes levantaram questões sobre a sua veracidade. Vendo como sistemas complexos foram capazes de ajudar desenvolver variadas áreas, este trabalho argumenta que modelação agencial poderá superar as limitações dos modelos DSGE. Para fundamentar esse argumento, o trabalho apresenta e implementa o seu próprio modelo, com capacidade de alcançar estabilidade económica e simular uma divisão de produtos essenciais/luxo, ajudando também a analizar os efeitos económicos, tal como desigualdade, de diferentes condições iniciais. Este encontrou que uma economia livre de impostos resulta numa grande concentração de capital e estuda a eficácia dos impostos na sua dispersão. Este modelo, mesmo que simples e com limitações, guarda lugar para subsequentes desenvolvimentos de maior complexidade capazes de servir como base para um modelo futuro capaz de feitos mais impressionantes e úteis.

Abstract

The economic micro/macro split has been institutionalized for decades and there has always been an effort in bridging these two branches. One such attempt served as basis for DSGE models, used in many banking institutions to forecast market fluctuations, but recent economic phenomena have put in question its validity. Seeing that complex systems have aided the development of many scientific areas, this work argues that agent-based modeling could remedy the shortcomings of DSGE. To support its argument, this work presents and implements an agent-based model of its own, capable of reaching market stability and simulating essential/luxury division while aiding in analysing different initial conditions and their economic behaviours like inequality. It was found that a tax-free economy results in very high capital concentration and studies the effectiveness of taxes at dispersing it. This model, even if simple and limited, holds room for further development and added complexity, and can serve as basis for future models capable of more impressive and useful feats.

Contents

Chapter 1

Introduction

Can agent based models help bridge the gap between macro and micro economics?

Macro and micro economics are two distinct economic study branches, with different theories that complement each other in overlapping issues. Microeconomics is concerned with individual markets, studying the effects of supply and demand, production theory, market competition, economic welfare, the role of imperfect information, the effects of minimum wages, taxes, etc. A very important concept in this branch is equilibrium points, studying prices and the effects of competitive versus non-competitive markets (whether single agents can significantly affect prices or not). Macroeconomics is concerned with nation-wide systems, like government taxation, inflation and social policies. It describes relationships among huge aggregates as national income, savings, and the overall price level (Fund, I.M. [1]).

This macro/micro split is institutionalized in the field since the Great Depression. Before the divide, the prevailing theory was the now called classical theory, presented in *The Wealth of Nations* [2], 1776. It postulated that markets were either in equilibrium (prices are adjusted to equalize supply and demand), or, in the event of a transient shock (financial crisis, famine, etc.), markets would quickly return to equilibrium. Leading to a notion that micro-economies (single markets) could accurately scale in size. As the classical theory could not explain the Great Depression, economists had no plausible explanation for the extreme "market failure" of the 1930s, until Keynes's *The General Theory of Employment, Interest and Money* [3], in 1936. This work studied three interrelated markets and introduced "disequilibrium economics", the explicit study of departures from general equilibrium, earning him the title of the founder of macroeconomics. Since then, there has been efforts to conjugate the macro/micro branches. The New Keynesian economics was born in an attempt to provide micro-economic foundations for Keynesian economics and created the new neoclassical synthesis, an important part of the theoretical foundation in Dynamic Stochastic General Equilibrium (DSGE) models used by the Federal Reserve and many other central banks (F.R.B. of New York [4], Sbordone [5]).

DSGE models, as all models, are based on several assumptions made to simplify a real economy. Some common assumptions are [6] .

- Perfect competition in all markets
- All prices adjust instantaneously
- Rational expectations
- No asymmetric information
- The competitive equilibrium is Pareto optimal
- Firms are identical and price takers
- Infinitely lived identical price-taking households

The arrival of the 21st century in the economic field marked by the global financial crisis of 2008 has, much like the Great Recession, put into question the effectiveness of current economic theory [7]. According to Sheng and Geng (2012) [8], Nobel laureate Ronald H. Coase stressed that microeconomic analysis consists of many unexplored "black boxes", so that it cannot provide sufficient explanations and effectively study the relationship between firms and markets. Besides, according to Solow (2010) [9], modern macroeconomics has not only failed to solve economic and financial problems, but has been "predestined" to fail. The construction of the DSGE models, which were the main approach for macroeconomics before and after the crisis of 2008 and, more generally, models based on the representative agent, did not pass the test of good forecasting. Also, Paul Krugman (2009) [10] argued that macroeconomics over the past three decades has been proved inadequate because economists have failed to identify the errors of macroeconomic analysis, mistakenly believing that "beautifully structured" theoretical models can depict reality. The modern microeconomic and macroeconomic models are not sufficient to investigate the dynamic and complex interactions between people, institutions, and the nature of the real complex economy.

A different but valid approach to understanding economy is utilizing complex systems analysis. These have been thoroughly employed in many scientific areas and may have the potential to overcome limitations in previous models like DSGE. A popular tool in complex systems is agent-based modeling, these are built from (usually) simple rules followed by agents. One of their strengths is emergence, the ability to form complex behaviour by following simple rules (examples of very elegant agent-based are flocks of birds, a bee hive and the internet [11]) [12] , they have been thoroughly studied and applied in physical sciences, including plasma physics, particle-based cancer therapies, materials science, crystallisation, magnetism and nuclear fusion[13]. Systems as these share many characteristics with the economy, granting special interest in utilizing these tools to model it [14].

Agent-based models have been gaining popularity within economists in recent years, with the field being led by economists collaborating with physicists and computer engineers. The most successful models studied housing segregation [15], shocks and price dynamics [11], vulnerability of the financial system [16] [17] and market design [18].

"The economy needs agent-based modelling", a nature article by J. Doyne Farmer & Duncan Foley [19] shares much of the argument of this work, stating "the major challenge lies in specifying how the agents behave and, in particular, in choosing the rules they use to make decisions. In many cases this is still done by common sense and guesswork, which is only sometimes sufficient to mimic real behaviour". Agreeing with this statement but not demoralized by it, the remaining content consists of common sense and guesswork applied in the creation of an agent-based complex economy model.

Therefore, this work builds and studies an agent-based model that aims to emulate a dynamic economy with a generalized lens, applying micro and macro economics concepts and studying the inner workings of the economy. During this models building process, the priority was to maximize economic features while minimizing complexity. Some economic properties studied are inequality, production, consumption, unemployment and luxury-essential division.

The next chapters in this work will first describe all model agents and their behaviours in a formal, mathematical manner while providing some justifications for the guess-work involved, then study the results provided by different initial conditions, taxation methods and the impact of some possible variations of the model, and finally discuss its shortcomings and possible further developments.

Chapter 2

Model description

This chapter will introduce and define the inner workings of the presented model. Its major (and unrealistic) assumptions are:

- No persons or factories are created or destroyed (number of agents is constant)
- Persons are detached from employers (may change employer every cycle)
- Production cost is defined by worker salaries (no base material or infrastructure cost)
- Factories and Government have no upkeep costs (100% government efficiency)

These assumptions exist mostly as a method of simplification, as removing them would increase complexity while not justifying its increase in fidelity. Figure 2.1 roughly illustrates the models steps and its cyclical nature.

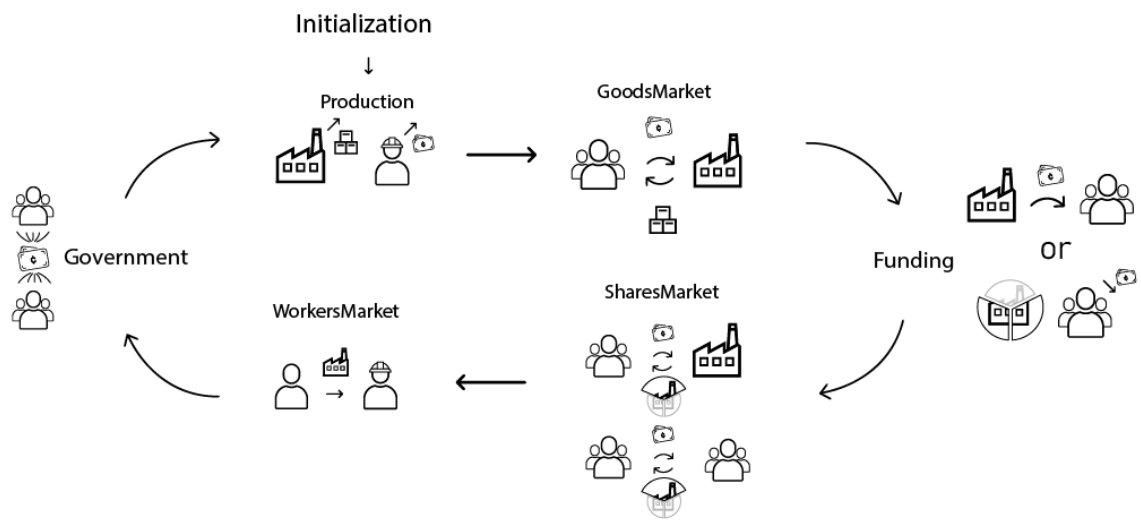


Figure 2.1: Model description simplified illustration

This work will be using terms referring to the analysed system, which will be defined next. Sets are defined by uppercase (A), elements of sets by lowercase (a) and operators are hatted ($\hat{A}$) and lowercase operators only read states of objects.

Goods are used and created by Agents to perform trades.

- **Shares (S)** are owned exclusively by Persons and each refer to a singular Factory. The sum of all shares from a single Factory must be 1.
- **Stock ($\hat{\lambda}, \hat{\nu}$)** is owned exclusively by Factories, and are of essential or luxury type. essential Stock (λ) has a maximum consumption per Person per cycle of 1, while luxury (ν) doesn't. $\hat{\mu}$ is used to denote both types.
- **Capital ($\hat{c}$)** is the currency, tradable with Goods. The total capital in the system is constant and defined by c_{tot}.

Agents are objects capable of holding Goods.

- **Persons** (P) is a set of person objects that can consume stock, hold shares and work on factories.
- **Factories (F)** will hold Stock and, if it has workers ($|W(f)| > 0$), it will produce stock.

Markets are capable of trading Goods.

- **GoodsMarket ($\hat{G}$)** handles Stock consumption with Capital.
- **WorkersMarket ($\hat{W}$)** handles worker assignment to Factories.
- **SharesMarket ($\hat{S}$)** handles Capital trade with Shares, and is divided in
  - **Primary ($\hat{S}_1$)** where Factories trade with Persons,
  - **Secondary ($\hat{S}_2$)** where Persons trade with Persons.

Initialization Factories and Persons are created and Shares are distributed in the desired manner, these different initial distribution methods are

- **Egalitarianism** every Person holds the same amount of value in Shares,
- **Ownership** every factory has a single share holder,
- **Monopoly** a single share holder holds all shares,
- **Bourgeoisie** shares are distributed over a percentage of People.

Cycle Interactions between all these elements occur during cycles in this order:

- **Production ($\hat{P}$)** Factories utilize workers to create stock,
- **GoodsMarket($\hat{G}$)** Stock trade and consumption,
- **Funding ($\hat{F}$)** Factories decide Stock production for next cycle and, if needed, create Shares,

SharesMarket($\hat{S}$) Shares trade,

WorkersMarket($\hat{W}$) Worker assignment.

Operators such as $\hat{c}$ and $\hat{\mu}$ are applied to single objects and behave as common operators returning the magnitude of capital (or stock) of the objects they operate on ($p \in P, f \in F$). For example, the capital of a person $p \in P$ at time step t is given by $c_t(p)$ or (simplified by) c_p and is obtained form the capital operator ($\hat{c}_t$) applied to p:

$$\hat{c}_t \, p = c_t(p) \, p = c_p \, p \; , c_p \in \mathbb{R}^+ .$$

Also, an operator $\hat{A}(a, b)$ signifies that this operator acts on objects a and b, acting as an identity operator $\mathbb{1}$ for all other objects.

Shares refer to both a person and a factory, so they are treated as a unique structure: $S(f, p) \in R^+, \; p \in P, \; f \in F$ refers to the share of person p in the factory f. Using $S(p)$ refers to a set of all non-zero factories where person p holds shares, and $S(f)$ refers to a set of all persons that hold shares in factory f:

$$\begin{aligned} S(p) &= \big\{ f \in F \; \big| \; S(f, p) \neq 0 \big\}, \\ S(f) &= \big\{ p \in P \; \big| \; S(f, p) \neq 0 \big\}. \end{aligned}$$

This being said, all of the above will now be unfolded in a more detailed manner.

2.1 Initialization

The system is initialized given the following parameters: $(|P|, |F|, c_{max}, c_{min})$ and the initial condition. This initial condition contains information about how to distribute the available shares in the soon-to-be created factories.

The default values used are $|P| = 20, |F| = 6, c_{max} = 1000, c_{min} = 10$. The first two were picked to balance simulation time and result fidelity. The latter were chosen as a way to introduce a reasonable initial capital inequality.

The first step is to create P. $p \in P$ is an object containing ID, Capital, employer, luxury and essential satisfaction and share catalog. During Initialization, a name is picked for every person and capital is assigned with a value between c_{max} and c_{min}. For all Persons, employer is set to None, luxury and essential satisfaction is set to 0 and share catalog is empty. Next, F is created, $f \in F$ are defined by an ID, Capital, workers, share holders, product is essential (True or False) and stock.

Every factory f is assigned an ID and capital in the same way as a person. The number of workers ($|W|$) is generated by sapling a Gaussian distribution centered on one, depicted in figure 2.2, with a maximum of $\frac{|P|}{2}$ and that number of unemployed Persons are picked at random, and assigned as workers. This sampling operation are translated using the sample function in eq. 2.1.

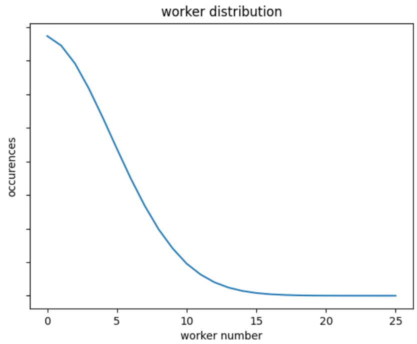


Figure 2.2: Gaussian distribution

$$
\begin{gathered}
|W(f)| = \text{sample}(\mathbb{Z}^+, \frac{|P|}{2}), \\
W(F) = \left\{W(f) \;\middle|\; f \in F\right\}, \; W(f) = \bigcup_{i=1}^{|W(f)|} \{p_i \mid p_i \in_R P \cap \overline{W(F)}\}.
\end{gathered} \tag{2.1}
$$

Where $\in_R$ picks a single random object from a set ($\overline{A}$ is defined as all objects not in set A).

The number of share holders are picked in a similar way in eq. 2.2 without the need of filtering unemployed people, and the value of each person's stock is picked by generating a uniform random number list and normalizing it.

$$
\begin{aligned}
|S(f)| &= \text{sample}(\mathbb{N}, \frac{|P|}{2}), \\
S(F) &= \left\{S(f) = \bigcup_{i=1}^{|S(f)|} \{p_i \mid p_i \in_R P \cap \overline{S(f)}\} \;\middle|\; f \in F\right\}, \\
S(f,p) \neq 0 \Leftrightarrow p &\in \bigcup_{i=1}^{|S(f)|} \{p_i \mid p_i \in_R P \cap \overline{S(f)}\} \wedge f \in F, \\
1 &= \sum_{i}^{|S(f)|} S(f, p_i).
\end{aligned} \tag{2.2}
$$

The first Factory's product is always essential, the second is always luxury, from there on, it's randomly picked 50% chance each. Defining F_λ and F_ν as all factories that produce essential and luxury, respectively ($F = \{f \in F_\lambda \cup F_\nu\}$).

2.2 Cycle

This is the portion that is repeated to advance the simulation, represented by the time step operator $\hat{T}\,|\psi_t\rangle = |\psi_{t+1}\rangle$. This operator is simply the combination of the other cycle operators: $\hat{T} = \hat{W}\,\hat{S}\,\hat{F}\,\hat{G}\,\hat{P}$, all of these conserve total capital c_{tot}.

The capital and stock at the end of each cycle operator is written using the short-hand notation in eq. 2.3:

$$
\begin{aligned}
\hat{c}_t\,\hat{P}\,a &= \hat{c}_{\hat{P}}\;a = c_{\hat{P},a}\,a,\\
&\dots\\
\hat{c}_t\,\hat{W}\,\hat{S}\,\hat{F}\,\hat{G}\,\hat{P}\,a &= \hat{c}_{\hat{W}}\;a = c_{\hat{W},a}\,a = c_{t+1}a,\\
\hat{\mu}_t\,\hat{W}\,\hat{S}\,\hat{F}\,\hat{G}\,\hat{P}\,f &= \hat{\mu}_{\hat{W}}\;f = \mu_{\hat{W},f}\,f = \mu_{t+1}f,\\
&\forall a \in P \cup F, \forall f \in F.
\end{aligned} \tag{2.3}
$$

Every repetition is called a cycle, or a time step, and it acts on all agents $|\psi_t\rangle = \big(P, F\big)$.

During **Production** $\hat{P}$, Factories pay salaries by evenly diving all it's capital by its workers, following eq. 2.4

$$
\begin{aligned}
&\hat{c}_{\hat{P}}\,W_t(F) = \Big\{\big\{\hat{c}_t\,p + \frac{\hat{c}_t\,f}{|W_t(f)|}\;\big|\;p \in W_t(f)\big\}\;\Big|\;f \in F\Big\},\\
&\hat{c}_{\hat{P}}\,F = 0.
\end{aligned} \tag{2.4}
$$

Each factory is left with 0 capital and produces an amount of stock defined by the production function in eq. 2.5, production per worker is depicted in figure 2.3

$$
\begin{aligned}
\hat{\mu}_{\hat{P}}\;f = \mu_{\hat{P},f}\;f &= \Big(\mu_f + \log(\frac{\text{salary}_t(f)}{m_{\text{wage}}})\;|W(f)|\Big)\;f,\\
\text{salary}_t(f) &= \frac{c_f}{|W_t(f)|}.
\end{aligned} \tag{2.5}
$$

This function was chosen to induce factories into hiring more people instead of raising salaries, as raising salaries always grows less production than hiring a new person, it also defines the minimum wage, for any salary lower than it, person will not produce (or be a worker) in that factory. $\Delta\mu = 0 \Leftrightarrow m_{wage} = e^{-1}$ is the default value for minimum wage.

Figure 2.3: Worker production function, shows how production per worker increases with salary

Lastly, the stock price is defined following eq. 2.6:

$$
\begin{aligned}
\Delta\text{price}(f) &= \min\Big\{2,\;0.2 + \frac{\mu_{\hat{P}}(f)}{\mu(f)}\Big\},\\
\text{price}_t(f) &= \frac{1}{\mu_{\hat{P}}(f)}\Big(\Delta\text{price}(f)\,c_f\,\big(\mu_{\hat{P}}(f) - \mu(f)\big) + \rho\,\text{price}_{t-1}(f)\,\mu(f)\Big).
\end{aligned} \tag{2.6}
$$

$\Delta\text{price}(f)$ being the return ratio per unit of stock produced this cycle, $\Delta\text{price}(f) \in [1.2, 2]$

and grows proportionally to factory stock, this models the law of supply and demand, the smaller leftover stock ($\mu(f)$) a factory has, the more profit it expects, and vice versa (if factory has excess stock, it will expect smaller profit). $price_t(f)$ is the final price per unit of stock. The price is set using a weighed median between the price of stock produced this cycle and the previous price. The price for stock produced this cycle is given by the profit margin multiplying cost of production, $\Delta\text{price}_t(f)\, c_f$, and for last cycle, a devaluation or appreciation effect is translated through ρ, if $\rho < 1$, stock devalues, if $\rho > 1$, stock appreciates. $\rho = \frac{2}{3}$ by default. This is used to translate real-world product value deprecation over time and warehouse costs.

The **GoodsMarket** is divided into essential market $\hat{G}_\lambda$ and luxury market $\hat{G}_\nu$, $\hat{G} = \hat{G}_\nu\,\hat{G}_\lambda$. Both markets iterate over every person the following steps: select a factory, trade with that factory, repeat. The selection step creates an ordered set of factories with non-zero stock sorted by price, $F_{G,\mu}$, and selects a factory by sampling this set using the same normal distribution depicted in figure 2.2, $\text{sample}(\mathbb{Z}^+, |F_{G,\mu}|)$. If this set is empty, that market ends, as there is no more factories to trade. Alternatively, person trades with the sampled factory: transfers capital from person to factory and destroys traded stock. In the essential market, person stops trading when it has traded one unit of essential stock or its capital reaches zero, on luxury market, person only stops trading when its luxury available capital (defined further) reaches zero. The sampling selection mechanism is mapped to eq. 2.7

$$\text{select}(A) = a \Leftrightarrow a = i\text{'th element in } A,\ i = \text{sample}(\mathbb{Z}^+, |A|), \tag{2.7}$$

and leads to what could be called a semi-perfect market, or weighted random market, where factories with lower prices are more likely to trade, as the probability for smaller indices of the sorted factory list is bigger, but more expensive factories that would not trade on a perfect market, may also trade, creating noise proportional to the standard deviation of the distribution. This noise could translate a multitude of real world parameters, such as distance, personal preference, marketing, etc. As this market repeats a given operation for every person, it can be defined by eq. 2.8 and eq. 2.10

$$\hat{G}_\mu = \widehat{\text{attemptTrade}}_\mu(p_1)\,\widehat{\text{attemptTrade}}_\mu(p_2)\,\cdots \tag{2.8}$$

Essential market tracks the capital available to person p by $c_\lambda(p)$, stock consumed by person p using $g_\lambda(p)$ and stock available for trade to factory f using $g_\lambda(f)$, initialized in eq. 2.9

$$c_\lambda(p) = c_{\hat{P},f}, g_\lambda(p) = 0, \forall p \in P, \text{ and } g_\lambda(f) = \lambda_{\hat{P},f}\, \forall f \in F_\lambda. \tag{2.9}$$

$$
\begin{aligned}
\hat{\text{attemptTrade}}_\lambda(p) &= \begin{cases} \mathbb{1} & \text{if } |F_{G,\lambda}| = 0 \lor c_\lambda(p) = 0 \\ \hat{\text{attemptTrade}}_\lambda(p) \times \hat{\text{trade}}_\lambda\left(f, \alpha, p\right) & \text{else} \end{cases}, \\
f &= \text{select}(F_{G,\lambda}), \\
\alpha &= \min\left\{1, \begin{cases} g_\lambda(f) - g_\lambda(p) & \text{if } c_\lambda(p) \geqslant \text{price}(f)\left(g_\lambda(f) - g_\lambda(p)\right) \\ \frac{c_\lambda(p)}{price(f)} & \text{else} \end{cases}\right\}, \\
F_{G,\lambda} &= \left\{f \in F \text{ sorted by price}(f) \;\middle|\; g_\lambda(f) > 0\right\}.
\end{aligned} \tag{2.10}
$$

The $\hat{\text{trade}}_\mu$, defined in eq. 2.11, updates the market variables

$$
\begin{aligned}
\hat{\text{trade}}_\mu\left(f, \alpha, p\right) &\Rightarrow c_\mu(p) \rightarrow c_\mu(p) - \text{price}(f) \times \alpha, \\
&\Rightarrow g_\mu(p) \rightarrow g_\mu(p) + \alpha, \\
&\Rightarrow g_\mu(f) \rightarrow g_\mu(f) - \alpha.
\end{aligned} \tag{2.11}
$$

At the end of essential market, the leftover capital for each person, $c_\lambda(p)$, will be allocated for luxury and shares markets in a predetermined manner,$\%_\nu = 40\%$ for luxury, $\%_s = 40\%$ for shares market and saving $\%_l = 20\%$ for next cycle.

Luxury market, differs in behaviour to essential only on $\hat{\text{attemptTrade}}$, defined in eq. 2.13 and tracks the capital available to person p by $c_\nu(p)$, stock consumed by person p using $g_\nu(p)$ and stock available for trade to factory f using $g_\nu(f)$, initialized in eq. 2.12

$$
c_\nu(p) = c_\lambda(p), g_\nu(p) = 0, \; \forall p \in P, \text{ and } \; g_\nu(f) = \nu_{\hat{P},f} \, \forall f \in F_\nu. \tag{2.12}
$$

$$
\begin{aligned}
\hat{\text{attemptTrade}}_\nu(p) &= \begin{cases} \mathbb{1} & \text{if } |F_{G,\nu}| = 0 \lor c_\nu(p) = 0 \\ \hat{\text{attemptTrade}}_\nu(p) \times \hat{\text{trade}}_\nu\left(f, \alpha, p\right) & \text{else} \end{cases}, \\
f &= \text{select}(F_{G,\nu}), \\
\alpha &= \begin{cases} g_\nu(f) - g_\nu(p) & \text{if } c_\nu(p) \geqslant \text{price}(f)\left(g_\nu(f) - g_\nu(p)\right) \\ \frac{c_\nu(p)}{price(f)} & \text{else} \end{cases}, \\
F_{G,\nu} &= \left\{f \in F \text{ sorted by price}(f) \;\middle|\; g_\nu(f) > 0\right\}.
\end{aligned} \tag{2.13}
$$

Eq. 2.14 sets the final state of goods market.

$$\begin{aligned}\hat{c}\,\hat{G}\,\hat{P}\,p &= \Big(c_{\hat{P},p} - \big(c_\lambda(p) + c_\nu(p)\big)\Big)\,p,\\ \hat{c}\,\hat{G}\,\hat{P}\,f &= \Big(c_{\hat{P},p} + \big(g_\mu(f) \times \text{price}(f)\big)\Big)\,f,\\ \hat{\mu}\,\hat{G}\,\hat{P}\,f &= \Big(\mu_{\hat{P},p} - g_\mu(f)\Big)\,f.\end{aligned} \tag{2.14}$$

During **Funding**, factories start by projecting their attempted production, μ_{proj}, for next time step, using the estimation function depicted in figure 2.4. This function is defined in eq. 2.15

$$\begin{aligned}\mu_{\text{proj}}(f) &= \mu_{\hat{P},f}\,\exp\Big(-\frac{\text{ratio}(f)^2}{k} + k\Big) + 1,\\ \text{ratio}(f) &= \frac{\mu_{\hat{G}}(f)}{\mu_{\hat{P}}(f) - \mu_{\hat{G}}(f)},\\ c_{\text{proj}}(f) &= \overline{c/\mu} \times \mu_{\text{proj}}(f),\\ \overline{c/\mu} &= \sum_{f\in F} \frac{|W(f)|\,\text{salary}(f)}{\mu_{\hat{P}}(f) - \mu(f)}.\end{aligned} \tag{2.15}$$

Where k is a simulation constant with a default value of $k = 0.5, k \in \mathbb{R}^+$ and translates the aggressiveness of the factories in this simulation. If $k \to 0$, factories will only produce more than one unit of stock if leftover equals zero; if $k \to \infty$, factories will always produce as much stock as possible, regardless of how much stock was sold, and never returning capital to share holders. The "+1" term exists as a way to simulate ambition, avoiding production falling to zero during extreme conditions. c_{proj} is the projection of capital needed for producing this amount of stock, and $\overline{c/\mu}$ the average cost for producing a unit of stock. Then, if a factory raised enough capital during goods market in order to produce the projected stock, all remaining capital is distributed over its share holders, if it did not, then it must fund itself on the share market by

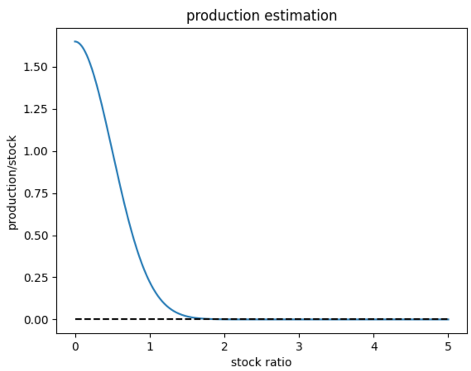


Figure 2.4: Production estimation function, shows how much stock to produce given leftover stock ratio

selling shares, this behaviour is presented in eq. 2.16

$$
\begin{aligned}
c_{F,p} &= c_{\hat{G},p} + \sum_{f \in S(p)} \begin{cases} \left(c_{\hat{F},f} - c_{\text{proj}}\right) S(f,p) & \text{if } c_{\hat{F},f} > c_{\text{proj}}(f) \\ 0 & \text{if } c_{\hat{F},f} \leqslant c_{\text{proj}}(f) \end{cases}, \forall p \in P, \\
\hat{F}\ f &= \begin{cases} \text{distribute}(f)\, f & \text{if } c_{\hat{G},f} > c_{\text{proj}}(f) \\ \text{fund}(f)\, f & \text{if } c_{\hat{G},f} \leqslant c_{\text{proj}}(f) \end{cases}, \forall f \in F.
\end{aligned}
\tag{2.16}
$$

For each factory, each of its share holders receives a percentage of its excess capital equal to the value of their share in that factory, and nothing if no excess capital exists. In the latter case, the factory attempts to receive the remaining capital by selling its shares on the share market, if it succeeds, every share holder will now lose a portion of their share and hand it to the new share holder(s). The distribute and fund functions are defined in eq. 2.17:

$$
\begin{aligned}
&\text{distribute}(f) \Rightarrow \hat{c}_t\, \hat{F}\, \hat{G}\, \hat{P}\, f = c_{\text{proj}}(f)\, f, \\
&\text{fund}(f) \Rightarrow m_1(f) = \frac{c_{\text{proj}}(f) - c_{\hat{G},f}}{c_{\hat{G},f} + \text{price}_t(f)\mu_{\hat{P},f} + \text{salary}_t(f)|W(f)|}, f \in M_1.
\end{aligned}
\tag{2.17}
$$

$M(f)$ being the share amount (percentage of f) to be sold in the shares market, the capital needed divided by the total factory value, defined in eq. 2.18

$$
\text{val}(f) = c_{\hat{G},f} + \text{price}_t(f)\mu_{\hat{P},f} + \text{salary}_t(f)|W(f)|. \tag{2.18}
$$

The **SharesMarket** is divided into primary and secondary, $\hat{S} = 1 + \hat{S}_1\, \hat{S}_2$, during the primary shares market, factories sell shares to persons, acquiring capital, during the secondary market, persons may sell shares that they hold over factories to also acquire capital for themselves.

Much like the luxury market, shares market defines a share available capital, $c_s(p) = \%_s \times c_\lambda(p)$, factory capital $c_s(f) = c_{\hat{F},f}$ and two new share-like objects, $T_1(p)$ holds all factories that sold shares to person p, $T_1(p, f)$ holds the share amount that factory f sold to p. Now, for secondary market, T_2 must hold persons, factories and share amounts, so $T_2(p')$ holds all persons that sold shares to person p', $T_2(p', p)$ all factory from which person p sold shares to person p', and $T_2(p', p, f)$ the amount of shares sold from person p to p' on factory f. To exemplify, if only one share was sold from person p to p' in factory f, the share amount is $T_2(p, p', f) = T_2(p, p', T_2(p, p'))$, as $T_2(p, p')$ is a set of only one factory element.

Primary shares market attempts to trade shares for all factories that require funding by picking a random person with share available capital, defined in eq. 2.19. The total capital spent on primary market by person p is defined using $c_1(p) = \sum\limits_{f' \in T_1(p)} \text{val}(f') \times T_1(f', p)$.

$$
\begin{aligned}
\hat{S}_1 &= \bigcup_{f \in M_1} \text{attem\hat{p}tSell}(f, p),\ p \in_R P_{\hat{S}_1}, \\
\text{attem\hat{p}tSell}(f, p) &= \left(\begin{cases} \mathbb{1} & \text{if } \alpha = m_1(f) \vee |P_{\hat{S}_1}| = 0| \\ \text{attem\hat{p}tSell}(f, p') & \text{if } \alpha < m_1(f) \end{cases} \right) \times \hat{\text{sell}}_1\,(f, \alpha, p), \\
\alpha &= \begin{cases} m_1(f) & \text{if } m_1(f) \times \text{val}(f) \leqslant c_s(p) - c_1(p) \\ \frac{c_s(p) - c_1(p)}{\text{val}(f)} & \text{if } m_1(f) \times \text{val}(f) > c_s(p) - c_1(p) \end{cases}, \\
p' &\in_R P_{\hat{S}_1}, \\
P_{\hat{S}_1} &= \left\{ p \in P \;\middle|\; c_s(p) - c_1(p) > 0 \right\}.
\end{aligned}
\tag{2.19}
$$

If $|P_{\hat{S}_1}| = 0$, no more shares will be sold during primary market nor secondary market, as all share market available capital has been exhausted. The $\hat{\text{sell}}$ operator is defined in eq. 2.20:

$$
\begin{aligned}
\hat{\text{sell}}_1(f, \alpha, p) \Rightarrow\ & p \in T_1(f),\ f \in T_1(P),\ T_1(f, p) \to T_1(f, p) + \alpha, \\
& p \in S(f),\ f \in S(p),\ S(f, p) \to S(f, p) + \alpha, \\
& m_1(f) \to m_1(f) - \alpha, \\
& c_s(p) \to c_s(p) - \alpha \times \text{ val}(f), \\
& c_s(f) \to c_s(f) + \alpha \times \text{val}(f).
\end{aligned}
\tag{2.20}
$$

Secondary shares market attempts to trade shares for all persons that require funding by picking a random person with share available capital, defined in eq. 2.21.

Persons will only sell shares during secondary market if they may not be capable of fully consuming essential during the next cycle, this future capital needed projection is defined by the sum of essential capital projection and luxury capital.

$$
c_{\text{ess}} = \frac{1}{|F_\lambda|} \sum_{f \in F_\lambda} \text{price}(f).
$$

The total capital spent on primary and secondary market by person p is defined using $c_2(p) = c_1(p) + \sum\limits_{p' \in T_2(p)} \Big(\sum\limits_{f \in T_2(p, p')} \text{val}(f) \times T_2(p, p', f) \Big)$. For next step it is also important to introduce the mean salary per person, $\overline{\text{salary}} = \frac{1}{|P|} \sum_{f \in F} |W(f)| \times \text{salary}(f)$.

$$
\begin{aligned}
M_2 &= \left\{ p \in P \;\middle|\; \%_l \times c_\lambda(p) + \overline{\text{salary}} < c_{\text{ess}} \wedge |S(p)| > 0 \right\}, \\
m_2(p) &= \min \left\{ \%_l \times c_\lambda(p) + \overline{\text{salary}} - c_{\text{ess}} \,, \sum_{f \in S(p)} S(p,f) \right\}, \; p \in M_2, \\
\hat{S}_2 &= \bigcup_{p \in M_2} \text{atte}\hat{\text{m}}\text{ptSell}(p, p'), \; p' \in_R P_{\hat{S}_2}, \\
\text{atte}\hat{\text{m}}\text{ptSell}(p,p') &= \left(\begin{cases} \mathbb{1} & \text{if } \alpha = m_2(p) \vee |P_{\hat{S}_2}| = 0 \\ \text{atte}\hat{\text{m}}\text{ptSell}(p, p'_2) & \text{if } \alpha < m_2(p) \end{cases} \right) \times \text{se}\hat{\text{l}}\text{l}_2\,(p, \alpha, p'), \\
\alpha &= \begin{cases} m_2(p) & \text{if } m_2(p) \leqslant c_s(p') - c_2(p') \\ c_s(p') - c_2(p') & \text{if } m_2(p) > c_s(p') - c_2(p') \end{cases}, \\
& p'_2 \in_R P_{\hat{S}_2}, \\
P_{\hat{S}_2} &= \left\{ p' \in P \;\middle|\; c_s(p') - c_2(p') > 0 \right\}.
\end{aligned}
\tag{2.21}
$$

It is of note that m_1 contains shares and m_2 contains needed capital, and same applies to both α's. The $\text{se}\hat{\text{l}}\text{l}_2$ operator, defined in eq. 2.22 will have a more complicated behaviour, as it will trade shares until α capital has been traded, so a second, simpler operator will handle trades, $\text{tr}\hat{\text{a}}\text{de}(p, \alpha', p', f)$ trades α' shares in factory f from person p to person p', and $\alpha' \times \text{val}(f)$ capital from person p' to person p

$$
\begin{aligned}
\text{se}\hat{\text{l}}\text{l}_2(p, \alpha, p') &= \prod_{f \in S(p)} \begin{cases} \text{tr}\hat{\text{a}}\text{de}(p, \alpha', p', f) & \text{, if } m_2(p) > 0 \\ & \text{, } \alpha' = \min\left\{ S(p,f), m_2(p)/\text{val}(f) \right\} \\ \mathbb{1} & \text{, if } m_2(p) = 0 \end{cases}, \\
\text{tr}\hat{\text{a}}\text{de}(p, \alpha', p', f) \Rightarrow & \, p' \in S(f), \; f \in S(p'), \; S(f,p') \to S(f,p') + \alpha', \\
& S(f,p) \to S(f,p) - \alpha', \text{if } S(f,p) = 0, p \notin S(f), \; f \notin S(p), \\
& m_2(p) \to m_2(p) - \alpha' \times \text{val}(f), \\
& c_s(p) \to c_s(p) - \alpha' \times \; \text{val}(f), \\
& c_s(p') \to c_s(p') + \alpha' \times \text{val}(f).
\end{aligned}
\tag{2.22}
$$

At the end of share market, because during primary market the normalization of shares in factories was violated, it is then recovered using a normalize operator $\text{no}\hat{\text{r}}\text{m}$, so that $\hat{S}$ becomes, in fact, $\hat{S} = \text{no}\hat{\text{r}}\text{m}\, \hat{S}_1 \hat{S}_2$. This could be achieved by normalizing at every trade during primary market, reaching the same end state.

WorkersMarket handles finding new workers for every factory in the next cycle. This is achieved by iterating over every person: finding the highest salary factory, if this salary is higher than minimum wage, person becomes worker in factory, update factory projected salary, repeat. There is no need to implement noise in this stage because there is more persons than factories, so noise would complicate the procedure without providing further realism.

As this market repeats a given operation for every person, $\hat{W} = \hat{\text{find}}^{|P|} = \hat{\text{find}}(p_1)\,\hat{\text{find}}(p_2)\ldots$ it also defines a new set, H, containing every person that has been hired. This $\hat{\text{find}}$ operator, defined in eq. 2.23, acts on one person only.

$$\begin{aligned}\hat{\text{find}}(p) &= \begin{cases}\mathbb{1} & \text{if } c_{\text{proj}}(f)/\big(1+|W_{t+1}|\big) < m_{\text{wage}} \\ p \in W_{t+1}(f) \wedge p \in H & \text{if } c_{\text{proj}}(f)/\big(1+|W_{t+1}|\big) \geqslant m_{\text{wage}}\end{cases}, \\ & p \in_R \{p \in P \mid p \notin H\}, \\ & f = \max\Big\{f \in F \text{ sorted by } c_{\text{proj}}(f)/\big(1+|W_{t+1}(f)|\big)\Big\}.\end{aligned} \tag{2.23}$$

2.3 Government

A separated organ of this system is the government. It is separated because it is not essential to the system, but it does add a layer of complexity needed to explore interesting outcomes.

The government acts at the end of the cycle, and redistributes its raised capital equally over all persons, this capital can be raised in one or both ways: **Transaction** taxing and **Wealth cap** taxing.

Transaction taxing occurs every time a transaction of capital occurs during a trade in the goods market, shares market and salary payment; the government taxes a given percentage of the total capital in that transaction, 10% by default, and adds it to the raised capital.

Wealth cap taxing occurs at the end of the cycle, where the government takes all capital from every person that has capital higher than a set cap, $\frac{\text{cap}}{|P|}\sum\limits_{p\in P} c_p$. By default, cap$= 14$, a person can hold fourteen times the hypothetical perfectly distributed capital.

This concludes all the possible operations carried out to create the system and its results, it is noteworthy that this model exists as a program, so all mentioned above is only a mathematical representation.

Chapter 3

Results

This model was implemented using python programming language utilizing object oriented programming, mostly because of its accessibility options, some libraries utilized were numpy for computation and matplotlib for data collection. The implementation was a slow and iterative process, with long and hard stretches of deliberation over all kinds of behaviours. The two most difficult parts were finding a set of rules that created a stable, closed system and debugging, as the system is, just as a real economy, fully interconnected. [20]

3.1 Initial conditions

The impact of initial conditions on any system is an important analysis for finding stabilization states. An economy model with a unique convergence state indicates that all systems with different initial conditions converge, modeling a behaviour that all economies eventually evolve into undifferentiated states, and a model with no convergence indicates that all economies evolve toward unique, differentiated states. The model presented converges slowly, meeting at infinity, this is a desired behaviour as it mimics (even if for different reasons) real-world economy convergence [21]. Every result presented next is an average of thirty runs, this was done to mitigate outlier readings, as this system borrows from randomness and thus can create edge scenarios.

For the analysis present in figure 3.1, the result of all different initial conditions were studied on a system with no government. A Gini coefficient shows the inequality in a system. Often used in real economies to estimate how far a country's wealth or income distribution deviates from a totally equal distribution. At a maximum of 1 (total inequality) and minimum of 0 (total equality). Real world data places inequality Gini coefficient around 40% [22]. The

Gini index of any property v of an object $a \in A$, $v(a)$, is calculated by [23]

$$g = \frac{\sum\limits_{a \in A} \sum\limits_{a' \in A} |v(a) - v(a')|}{2\,\overline{v}\,|A|^2}.$$

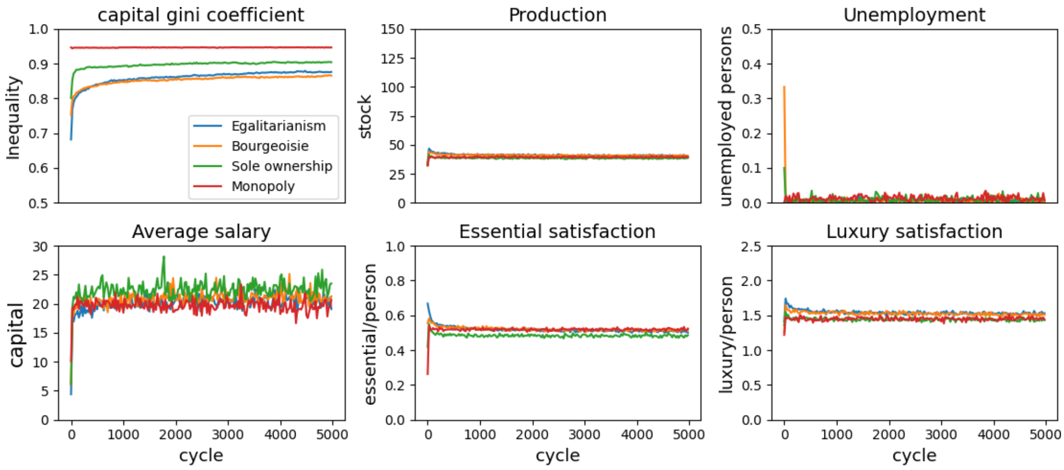


Figure 3.1: Tax-free time evolution of general indicators for all initial conditions

High overall inequality compared to real world economies present in these results is attributed to a lack of government, as there is no taxation and wealth redistribution, unlike (most) real world countries. Production is the total stock produced that cycle, including essential and luxury, unemployment is a number of persons, not a proportion (it appears as a fraction because each point is an average of thirty runs), satisfaction is the amount of stock consumed that cycle and share values are calculated as in 2.18.

As desired, results show stability for all initial conditions, evolving in a similar way with monopoly being the most differentiated condition, where inequality slowly reduces instead of growing. A higher inequality is to be expected on a monopoly conditioned system, but lower production and higher mean salary is not trivially assumed.

Monopoly produces less stock per cycle and has lower essential satisfaction because there exists a larger group of people with very small capital and a single person with enormous amounts of capital, therefore, this wealthier person (almost) exclusively consumes luxuries in vast quantities, and, because they attempt to consume all their capital at every trade attempt, (factory-scope) fluctuations (or shocks) occur due to the semi-perfect market mechanism, and these fluctuations lead to spikes and drops in production, resulting in lower factory profits and overall satisfaction, as there is a much smaller group of people joining the luxury market. For all other conditions, this effect is reduced proportionally to inequality, as having a bigger group of people joining each market results in lower fluctuations, stock excess and higher satisfaction. This effect would not occur if the goods market was modeled as a perfect market. This scenario is addressed in section 3.2.

An unexpected effect is that sole ownership is considerably closer to monopoly than bourgeoisie. This occurs because there is a small number of factories and each factory stock is not distributed over a confined group, but to a single individual; essential and luxury satisfaction is mostly equal across conditions; bourgeoisie ranks lower in production and salaries (this occurs due to smaller fluctuations in luxury markets, as more persons enter luxury market) and Egalitarianism converges more slowly, as capital is evenly distributed, inequality and its effects emerges slower. It is also interesting to note that, for all initial conditions, mean salary was, at stability, much higher than minimum salary, stabilizing around 20. This value is affected by the number of people in the system and will be explored further (section 3.2).

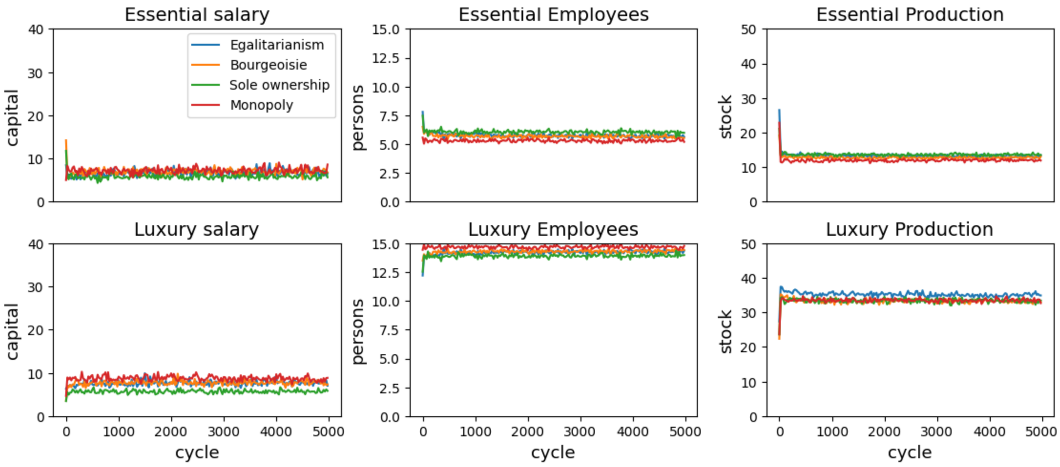


Figure 3.2: Tax-free time evolution of production indicators for all initial conditions

To further study the effect of these conditions, stock production and consumption were analysed more closely on figure 3.2. Some interesting behaviours include the quick decrease in essential production, salary and employees and their increase in its luxury counterpart. This occurs due to growing inequality, as the total capital in the system is constant.

These results further support that excess stock is behind the production increase, as consumption peaks before production. This occurs sooner for monopoly and sole ownership, as these restrict luxury market from the start, while the remaining conditions slowly reduce the group size, adding to the fact that, under said conditions, shares market capital is also more available to rapidly grow production. The delay in consumption and leftover stock is dependent on k, factory aggressiveness, and is addressed further in section 3.2.

As mentioned previously, all initial conditions converge for a large enough number of cycles (toward infinity), but the initial state generates a different path towards it. In figure 3.3, each person is analysed for different initial conditions, presented is the capital available to each person, sorted after the end of that cycle's goods market. This helps to clearly illustrate the shift in inequality, slowly declining for monopoly and greatly increasing for all others, with

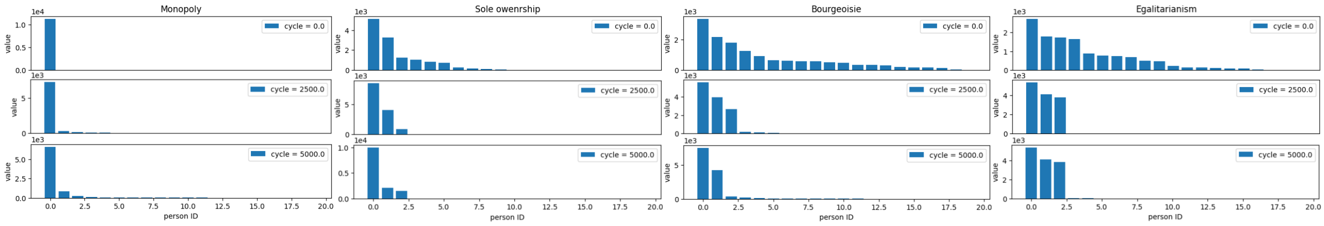


Figure 3.3: Time evolution of individual persons capitals for all initial conditions.

capital concentrating on a progressively smaller group that consumes vast amounts of luxury, disrupting the market. Also, the shape of the curve created by these values defines the Giri coefficient, which is 0 if these values form a straight line.

This inequality behaviour is affected by luxury, as it pushes wealthy persons to consume more capital per cycle, inequality convergence would emerge much slower in a system without luxury market. That (unrealistic) scenario will be addressed further (section 3.2).

3.2 Customization

A hand-built system's major benefit is customizability, so, for next analysis, the system will be fundamentally modified to hopefully better grasp the inner workings of its mechanisms.

Starting by k, or **factory aggressiveness**, this value affects the production estimation function, present in 2.4. A bigger k leads to bigger overall investments on stock production and bigger leftover stock. Different estimation functions are depicted in a egalitarian conditioned system with no taxing in figure 3.4. It shows that more aggressiveness leads to more competition between factories, as every factory will fluctuate production wildly and offer more capital for salaries, trying to grow production even when selling less product than produced. This effect is depicted on an egalitarian system with no taxation in figure 3.5.

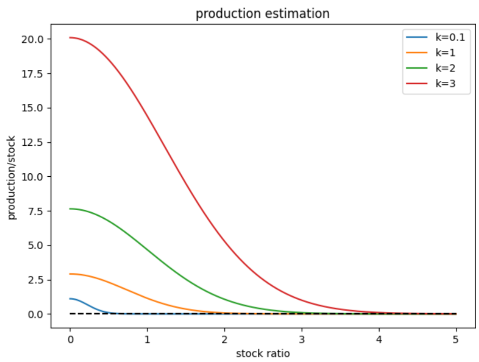


Figure 3.4: Stock estimation function for different k values

This results in a much higher stock surplus and, due to stock value depreciation, lower prices and higher satisfaction. As mentioned previously, this reflects in a much faster adaptation to rising consumption, and, at higher values, a clear oversensitization (or overshooting) can be seen, with bigger peaks and valleys in consumption, production and inequality. The default k=0.5 value was chosen because it better translates a realistic factory rationalization, meeting the point of stable production (production equals stock) for a leftover of a third stock, meaning that in a theoretical perfect stability point, factories would run with a surplus of a third of total consumption (inventory surplus of 33%),

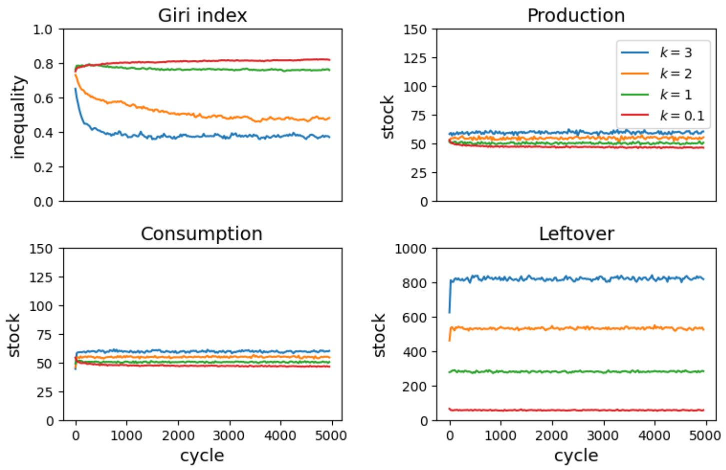


Figure 3.5: Egalitarianism, tax-free time evolution of various indicators for different k values

which is not very far from real-world data [24].

The other model constant is ρ, or **stock depreciation**, that affects how leftover stock value evolves each cycle, with a default value of $\rho = \frac{2}{3}$. Lower values mean that excess stock loses a lot of value, values bigger than one mean that excess stock, sitting in a warehouse, is gaining value. Results are presented in figure 3.6, the cycle count was lowered to 1000 because leftover grows continuously. Bigger depreciation ($\rho \ll 1$) leads to lower excess stock, as much of its value is lost every cycle, making it affordable and rising satisfaction at the expense of factories losing a portion of their value each cycle. Approaching $\rho = 1$, excess stock maintains value and is sold next cycle at same price. When $\rho > 1$, three regimes exist, leftover skyrocket,

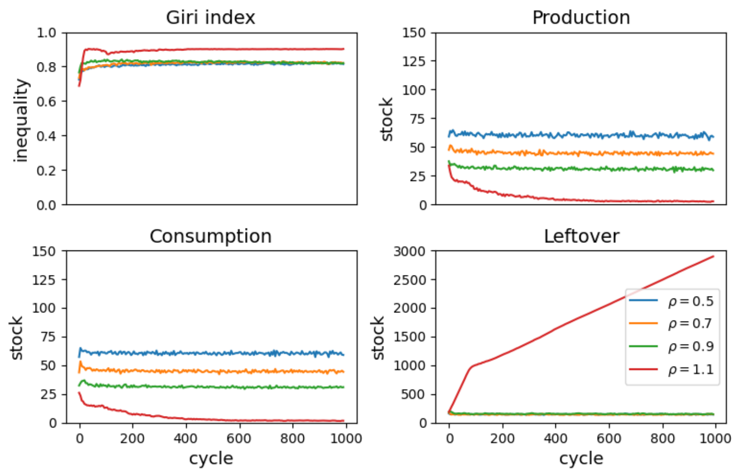


Figure 3.6: Egalitarianism, tax-free time evolution of various indicators for different ρ values

plateaus, and slow rise. This is due to essential and luxury, during the initial skyrocket, the luxury and essential market function normally with products progressively more expensive to the point a plateau is reached, wealthier persons can no longer consume luxury but still fully consume essentials, causing luxury factories to cease production entirely. The slow rise occurs because essential factories can still trade recently produced stock to those wealthier people, creating a weak stimulation. $\rho = 1$ can be seen as a phase transition in the model, resulting in vastly different outcomes.

A system with **no luxury** has a consumption cap, so wealthier people cannot consume massive amounts of luxury, which aided in evolving inequality, resulting in a much slower evolution. Results from that system are presented in figure 3.7 and comparing to figure 3.1, mean salary is now much closer to minimum salary, as there is now a consumption cap, making consumption limited, leading to massive competition and reducing production.

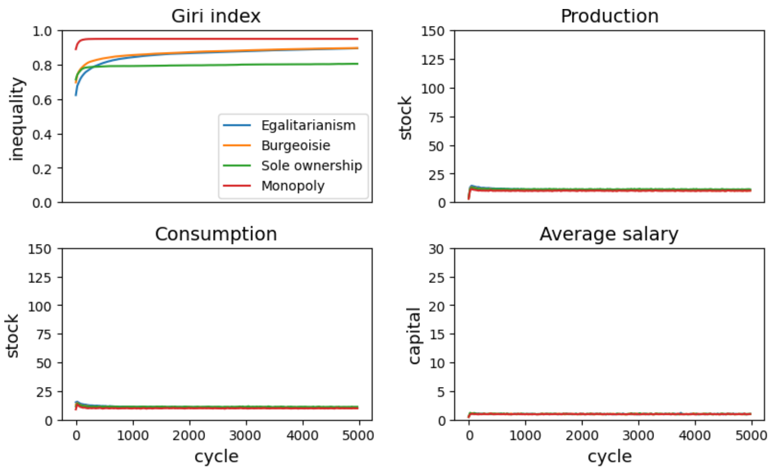


Figure 3.7: Egalitarianism, tax-free time evolution of various indicators with no luxury market

Under these conditions, factories are not producing as much capital for share holders and shares are sold more often. This explains why sole ownership creates lower inequality, as each factory is owned by only one person and when selling shares, these will be cheaper. Monopoly is different because one person holds the total share values of all factories, so when a factory requires capital and sells shares, these will be bought by the same person, as at least one of the factories will always produce profit. This effect is an indirect form of capital injection in factories and occurs much less frequently on all other conditions, as capital is more dispersed.

A **perfect market** was built by changing factory select behaviour to always choose the ideal factory for that person, one with the lowest price (and non-zero stock available). It was mentioned before that a perfect market would reduce the inequality effect on factory-scope fluctuations. Those results are present in figure 3.8, and compared to 3.1, the system now stabilizes much faster on all conditions and the effects of destabilization created by capital concentration are greatly reduced. This destabilization occurs because the semi-perfect market forces persons to make non-optimal trades, which artificially pushed stock prices and excess stock up. On a perfect market persons now always choose the most beneficial trade and this effect no longer occurs.

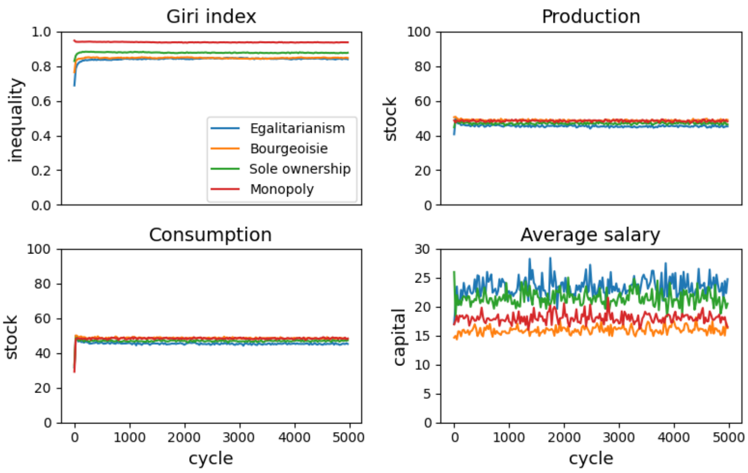


Figure 3.8: Egalitarianism, tax-free time evolution of various indicators with perfect market

As for **median salary** analysis for different number of persons in the system, the computation was made for ten factories and a variable number of persons. The results are present in figure 3.9 and, as mentioned previously, salary decreases with persons density $\frac{|P|}{|F|}$, but presenting two behaviours, decrease occurs very rapidly for low densities and slower for greater densities, reaching a linear relationship closer to minimum wage. Production grows logarithmically, similar to production function on figure 2.2.

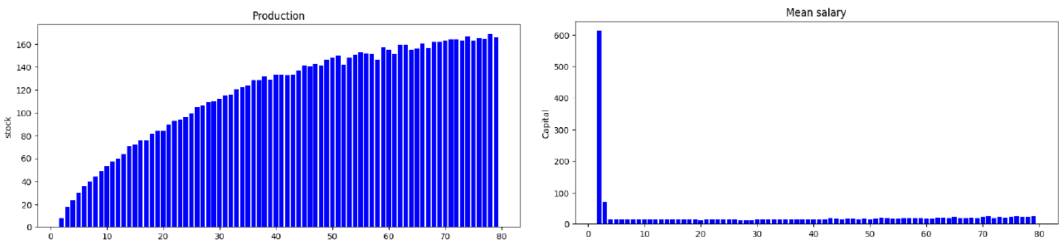


Figure 3.9: Person salary and production

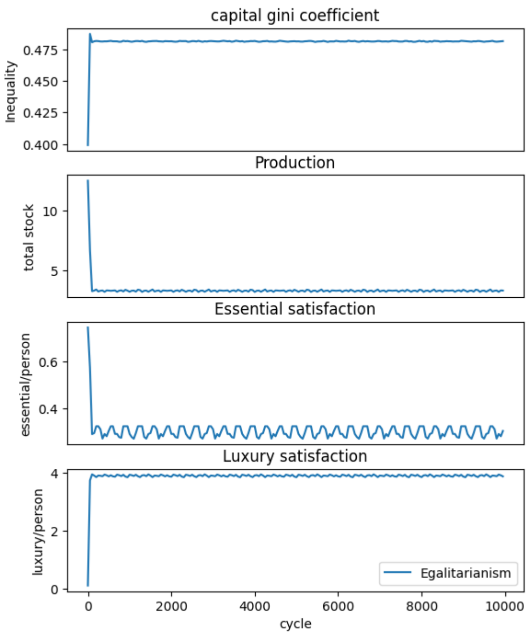
For an **extremely simple system**, with just two persons and two factories, an interesting effect emerges, where luxury satisfaction grows and stabilizes at a plateau but essential falls towards zero. This occurs because the luxury factory can aggregate more capital than its essential counterpart, as its consumption isn't capped, resulting in an essential domination with high inequality and low production. This is an undesired behaviour, because a real economy of that size would prioritize essential satisfaction over luxury. To remedy this behaviour, a new rule was implemented for the production function in figure 2.3, where

$$\mu_{\hat{P},f}\, f = \left(\mu_f + \log(\frac{\text{salary}_t(f)}{m_{\text{wage}}})\ |W(f)|\right)\, f$$

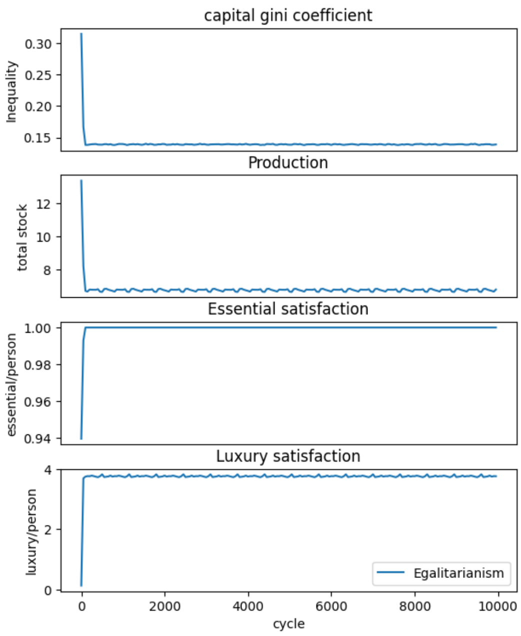
becomes

$$\mu_{\hat{P},f}\, f = \left(\mu_f + \log(\frac{\text{salary}_t(f)}{m_{\text{wage}}}) \sum_{p \in P} g_\lambda(p)\right) f.$$

So that **lower essential satisfaction leads to lower production**. The results are presented in figure 3.10a and 3.10b, where essential satisfaction was raised. This occurs because the luxury factory loses investment when essential satisfaction decreases, becoming less profitable and allowing the essential factory to regain its market. reaching stability, essential satisfaction is very close to one and luxury is practically maintained. Upon testing for bigger systems, this multiplier plays close to no effect and introduces added complexity, so it was not introduced in the default production function.



(a) No essential multiplier



(b) With essential multiplier

3.3 Taxation

Seeing all initial conditions converge towards great inequality, and that real economies diffuse capital with taxation, this section will study ways in which a government may acquire funds and redistribute wealth. Redistribution is modeled here by literally redistributing capital over all persons equally. This is a simplification of much more complex real-world government wealth redistribution in the form of public services, such as security, education, health, transportation, infrastructure, etc. Two taxation methods will be implemented, not necessarily based on real-world taxation, but translate a similar effect. A transaction tax acts on every trade completed on goods market, shares market and salary payment, and adds 10% of that transaction value to the government funds and deducts from the beneficiary. A wealth cap tax acts on every person that holds more than 70% of all capital. This may sound like a very high cap, but government acts only at the end of all markets each cycle and accounts simply for leftover capital that was neither spent on luxury nor share market. Therefore, as can be seen on fig. 3.3, most persons will not hold any capital at that time, except for the extraordinarily wealthy, and this excess wealth will be reduced each cycle. These government funds, as previously mentioned, are then distributed equally, before salary payments (at the start of the next cycle). The next analysis, presented in figure 3.11 studies whether taxation further diverges or converges systems with different initial conditions. looking specifically at capital inequality.

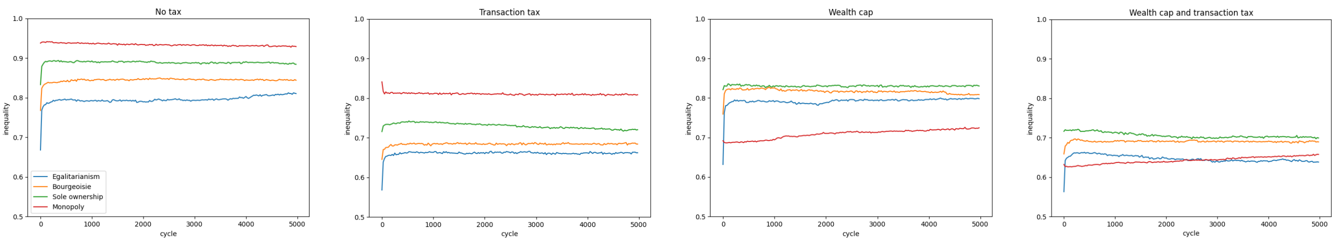


Figure 3.11: time evolution of inequality for different taxation methods and initial conditions

As expected, both taxation methods lower inequality at stability, but some are more effective than others. In a system with no taxes, the stability inequality for all conditions is higher than 80%, this leads to the assumption that a libertarian capitalism leads to a much higher and quicker inequality stabilization. For a transaction tax, all inequalities lower significantly, a 15% decrease can be seen for all conditions at 5000 cycles. It is also interesting that transaction taxing lowers the speed at which the system reaches stability and generates a larger gap between monopoly and other initial conditions. Wealth capping leads to a very interesting behaviour, hardly affecting egalitarianism and bourgeoisie conditions, slightly lowering sole ownership inequality (about half a percentage) and greatly reducing it beyond all others for monopoly. An unexpected behaviour is the quick convergence of all non-monopoly conditions. Egalitarianism and bourgeoisie are mostly unaffected because wealth capping raises very little or no capital, in contrast to monopoly. With both taxing methods combined, their effects combine in a rather unique way. Inequality drops even further. Monopoly reaches the lowest inequality at around 62% and then grows steadily converging towards sole ownership and bourgeoisie at 70%, egalitarianism instead steadily decreases, diverging from all others toward 60%.

An analysis similar to the one performed for figure 3.3 is done in figure 3.12 for a monopoly

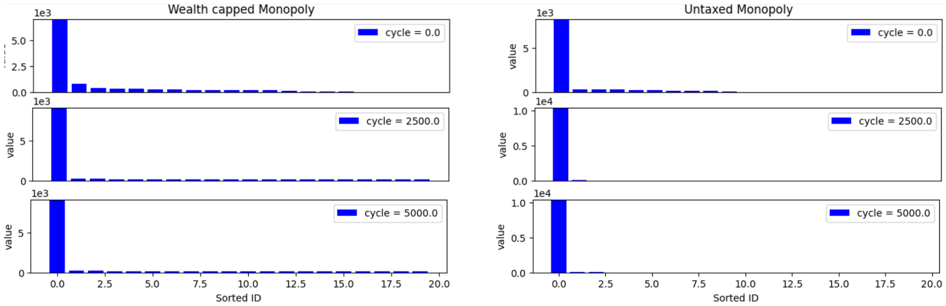


Figure 3.12: Individual persons capital for wealth cap taxation on Monopoly condition

system with and without wealth capping, it can be seen that the monopolist indeed caps its capital and cannot gain any further, and its capital is distributed equally trough every person.

It is important to note that none of these taxation methods threw the system into an unstable state, but they did induce a differentiated behaviour for each initial condition, it is therefore fair to assume these systems also differentiate their other properties. Egalitarianism and monopoly, the two most differentiated initial conditions, were more closely analysed in figures 3.13 and 3.14.

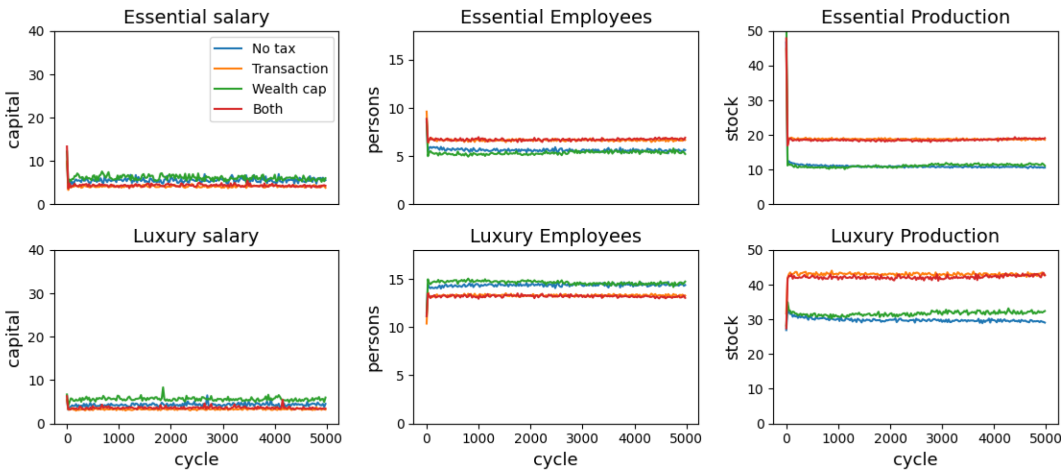


Figure 3.13: Egalitarianism time evolution of various indicators for different taxation methods

On egalitarianism, figure 3.13, some interesting effects can be perceived, first of all, wealth cap is much closer to no tax, this is expected given that, on egalitarianism, reaching the health cap is less likely than other initial conditions, therefore only taxing (combined and transaction tax) and no taxing (wealth cap and no tax) will be differentiated. Taxing results in lower salaries but higher production (and consumption) for essential and luxury, this is achieved by maintaining a lower inequality, lowering the consumption for the wealthier and raising the consumption for the less wealthy, that results in a net increase in consumption (leading to an increase in production). The mechanisms for this increase are different for essential and luxury. Essential production is maintained by lowering salaries and raising employment, producing roughly the same amount but, as capital inequality is now much lower, persons that would otherwise have little capital to spend on essentials can now consume, rising consumption greatly and giving essential factories more influence over the market. As for luxury, because many persons now satisfy all essential needs, they enter the luxury market with a very diffuse capital. Due to a loss of market influence, luxury factories have less employees, lower salaries, production and prices, but, as more persons enter the luxury market, these cause the semi-perfect market fluctuations to be less influential and lower leftover stocks, which in turn leads to higher (overall) profit (as stock devaluation happens more rarely) counteracting lower prices and allowing a rise in consumption.

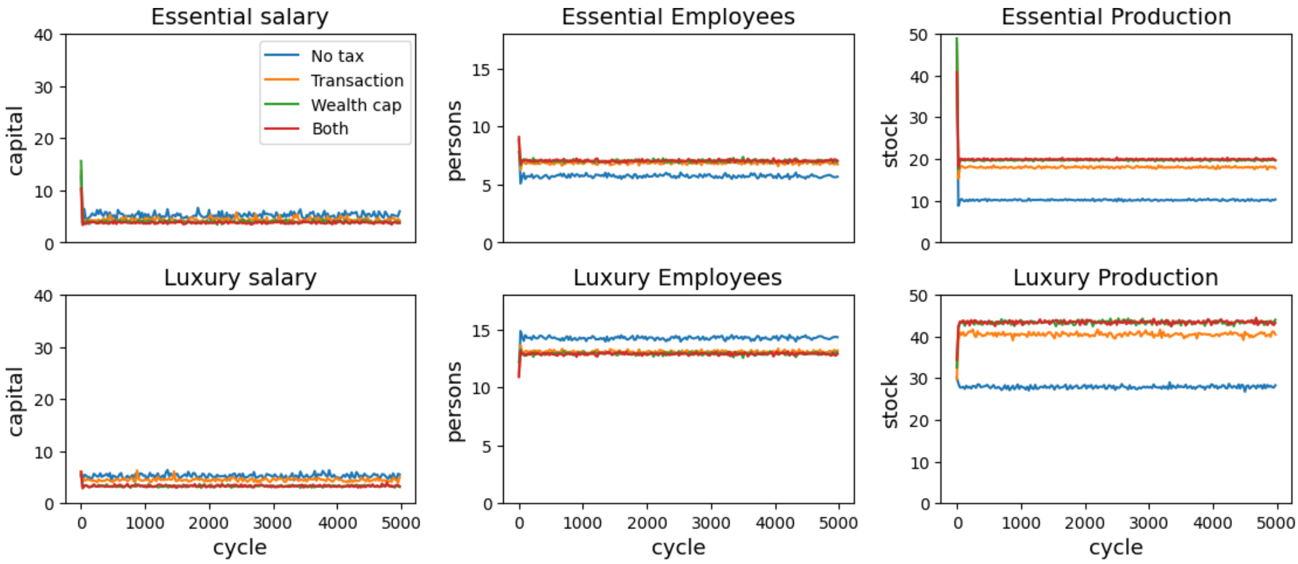


Figure 3.14: Monopoly time evolution of various indicators for different taxation methods

As for monopoly, figure 3.14, wealth cap transitions from a zero tax system and converges onto transaction taxing, therefore, taxing groups all three forms of taxation. Taxing, similar to egalitarianism, results in lower salaries and higher production by maintaining lower inequality and leftover stock. Differences arise in stabilization time, the monopoly conditioned system stabilized much faster than egalitarianism, this is due to the redistribution of wealth speed being proportional to its concentration.

Chapter 4

Conclusions

This work intended to present a viable agent-based general economy model. Realizing that such a model holds a serious science-wide challenge, the content presented was indeed a sketch model capable of studying various economic relationships and behaviours.

Some of the major behaviours studied were that a tax-free economy evolves toward great inequality and different initial conditions lead to different paths toward it. Taxation presents a form of combating inequality and rising consumption and different taxation methods vary in effectiveness for different initial conditions.

For the sake of exploring it, effects of different variations of the model were also studied. Factory aggressiveness lowers inequality and rises consumption at the expense of excess stock, stock depreciation plays a major role in the model stability, luxury acts as a catalyst for inequality behaviours, a semi-perfect market influences greatly the inequality behaviour, the relationship between the number of persons and factories is the leading factor behind mean salaries and for small enough systems, essential-dependant production output greatly affects the system.

This models limitations lie in its stiff, convergent nature, lacking the macroeconomic characteristic shocks (points of economic imbalance) and therefore being unable to predict crisis, the primary function of a main-stream general economy model.

Despite this limitation, this general economy model sketch holds much room for improvement and could be the basis toward developing a future much more complete and general model, capable of dethroning the currently predominant DSGE model.

There are many possible further development ideas. For example, the introduction of a smarter government with dynamic taxation that could attempt to maintain inflation by enabling, for example, counter-cyclical fiscal measures, creating government debt toward central or world banks. People could also be capable of borrowing capital from these banks, allowing for the study of a global banking crisis. The introduction of other forms of investment, outside of the shares market, like, for example, housing market, creating an oportunity towards more complex crisis. Another simpler form of studying crisis is by artificially creating them, and studying the model evolution.

Returning to the original question, can agent based models help bridge the gap between macroeconomics and microeconomics? The answer is: not yet, but existing models such as this one produce hope that one day economics becomes one more field that complex system analysis has successfully helped revolutionize.

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