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Título provisório



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Thank you

Resumo

Bem resumido

Abstract

Asbstract

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Chapter 1

Introduction + State of the art

TODO: REWRITE EVERYTHING, FORMAT, ORGANIZE

KEYNESIAN ECONOMICS:

Keynesian economists generally argue that aggregate demand is volatile and unstable and that, consequently, a market economy often experiences inefficient macroeconomic outcomes – a recession, when demand is low, or inflation, when demand is high. Further, they argue that these economic fluctuations can be mitigated by economic policy responses coordinated between government and central bank. In particular, fiscal policy actions (taken by the government) and monetary policy actions (taken by the central bank), can help stabilize economic output, inflation, and unemployment over the business cycle.[3] Keynesian economists generally advocate a regulated market economy – predominantly private sector, but with an active role for government intervention during recessions and depressions. He saw the economy as unable to maintain itself at full employment automatically, and believed that it was necessary for the government to step in and put purchasing power into the hands of the working population through government spending.

”The General Theory of Employment, Interest and Money”

In contrast to classical theory (Economy balances itself):

Say’s law: Production is the source of demand, because everything that is produced would eventually be consumed once the appropriate price was found for it, rests upon the assumption that if a surplus of goods or services exists, they would naturally drop in price to the point where they would be consumed.

the New Keynesian economics was born in an attempt to provide microeconomic foundations for Keynesian economics,

In contrast with classical approach, New Keynesian analysis usually assumes a variety of market failures (non-optimal allocation of goods). In particular, New Keynesians assume that there is imperfect competition (can occur if firms can charge higher prices as their goods and services are perceived as better, or there are barriers to market entry and exit.) in price and

wage setting to help explain why prices and wages can become "sticky", which means they do not adjust instantaneously to changes in economic conditions, as classical models would assume.

A merge of these two major ideas created the *new neoclassical synthesis*, an important part of the theoretical foundation in Dynamic stochastic general equilibrium (DSGE) models used by the Federal Reserve and many other central banks.

These have the following assumptions:

- Perfect competition in all markets
- All prices adjust instantaneously
- Rational expectations
- No asymmetric information
- The competitive equilibrium is Pareto optimal
- Firms are identical and price takers
- Infinitely lived identical price-taking households

to which the following frictions are added:

- Distortionary taxes (Labour taxes) – to account for not lump-sum taxation
- Habit persistence (the period utility function depends on a quasi-difference of consumption)
- Adjustment costs on investments – to make investments less volatile
- Labour adjustment costs – to account for costs firms face when changing the level of employment

Assumptions made, other models, justify model type

Important points: - Macroeconomics are concerned with nation-wide growth, government taxation, inflation, (aggregate values). It describes relationships among aggregates so big as to be hard to apprehend—such as national income, savings, and the overall price level. The field is conventionally divided into the study of national economic growth in the long run, the analysis of short-run departures from equilibrium, and the formulation of policies to stabilize the national economy—that is, to minimize fluctuations in growth and prices. Those policies can include spending and taxing actions by the government

- Microeconomics study individual markets, supply and demand, production theory (or theory of the firm), market competition, economic welfare, the role of imperfect information

in economic outcomes, the effects of minimum wages, taxes, price supports, or monopoly on individual markets and is filled with concepts that are recognizable in the real world. It has applications in trade, industrial organization and market structure, labor economics, public finance, and welfare economics. Microeconomic analysis offers insights into such disparate efforts as making business decisions or formulating public policies.

- Econometrics have the objective to convert qualitative statements (such as “the relationship between two or more variables is positive”) into quantitative statements (such as “consumption expenditure increases by 95 cents for every one dollar increase in disposable income”). Econometricians—practitioners of econometrics—transform models developed by economic theorists into versions that can be estimated. As Stock and Watson (2007) put it, “econometric methods are used in many branches of economics, including finance, labor economics, macroeconomics, microeconomics, and economic policy.” Economic policy decisions are rarely made without econometric analysis to assess their impact.

The macro/micro split is institutionalized in economics Created because of the great depression

Chapter 2

Model description

This work will be using terms referring to the analysed system, which will be defined next. Sets are defined by Uppercase (A), elements by lowercase (a) and operators by (\hat{A}).

Goods are used and created by Agents to perform trades.

Shares (S) are owned exclusively by Persons and each refer to a singular Factory. The sum of all shares from a single Factory must be 1

Stock ($\hat{\lambda}, \hat{\nu}$) is owned exclusively by Factories, and are of essential or luxury type. essential Stock (λ) has a maximum consumption per Person per cycle of 1, while luxury (ν) doesn't. $\hat{\mu}$ is used to denote both types

Capital (\hat{c}) is the currency, tradable with Goods. The total capital in the system is constant and defined by c_{tot}

Agents are objects capable of holding Goods.

Persons (P) is a set of person objects that can consume stock, hold shares and work on factories.

Factories (F) will hold Stock and, if it has workers ($|W(f)| > 0$), it will produce stock.

If a Factory f has leftover capital in a cycle, this capital is distributed over all Persons who hold Shares referring it.

Markets are capable of trading Goods

GoodsMarket (\hat{G}) handles Stock consumption with Capital.

WorkersMarket (\hat{W}) handles worker assignment to Factories.

SharesMarket (\hat{S}) handles Capital trade with Shares, and is divided in

Primary (\hat{S}_1) where Factories trade with Persons

Secondary (\hat{S}_2) where Persons trade with Persons

Initialization Factories and Persons are created and Shares are distributed in the desired manner, these different initial distribution methods are

Egalitarianism every Person holds the same amount of value in Shares

Ownership every factory has a single share holder

Monopoly a single share holder holds all shares

Bourgeoisie shares are distributed over a percentage of People

Their consequences will be analysed further

Cycle Interactions between all these elements occur during cycles in this order:

Production (\hat{P}) Factories utilize workers to create stock.

GoodsMarket(\hat{G})

Funding (\hat{F}) Factories decide Stock production for next cycle and, if needed, create Shares

SharesMarket(\hat{S})

WorkersMarket(\hat{W})

Operators such as \hat{c} and $\hat{\mu}$ are applied to single objects, and behave as expected, with capital and stock as their eigenvalues and p, f as their **eigenobjects**, respectively. For example, the capital of a person $p \in P$ at time step t is given by $\hat{c}_t p = c_p p = c_t(p) p$. Also, an operator $\hat{A}(a, b)$ signifies that this operator acts on objects a and b , acting as an identity operator $\mathbb{1}$ for all other objects.

Shares refer to both a person and a factory, so they are treated as a unique structure: $S(f, p) \in R^+$, $p \in P$, $f \in F$ refers to the share of person p in the factory f . Using $S(p)$ refers to a set of all non-zero shares held by person p , and $S(f)$ refers to a set of all non-zero shares held by person p :

$$S(p) = \{f \in F \mid S(f, p) \neq 0\}$$

$$S(f) = \{p \in P \mid S(f, p) \neq 0\}$$

This being said, all of the above will now be unfolded in a more detailed manner.

2.1 Initialization

The system is initialized given the following parameters: $(|P|, |F|, c_{max}, c_{min})$ and the initial condition. This initial condition contains information about how to distribute the available shares in the soon-to-be created factories.

The default values used are $(|P| = 20, |F| = 6, c_{max} = 1000, c_{min} = 10)$, and the default initial condition is Egalitarianism.

The first step is to create P . $p \in P$ is an object containing ID, Capital, employer, luxury and essential satisfaction and share catalog. During Initialization, a name is picked for every person and capital is assigned with a value between c_{max} and c_{min} . For all Persons, employer is set to None, luxury and essential satisfaction is set to 0 and share catalog is empty.

Next, F is created, $f \in F$ are defined by an ID, Capital, workers, share holders, product is essential (True or False) and stock.

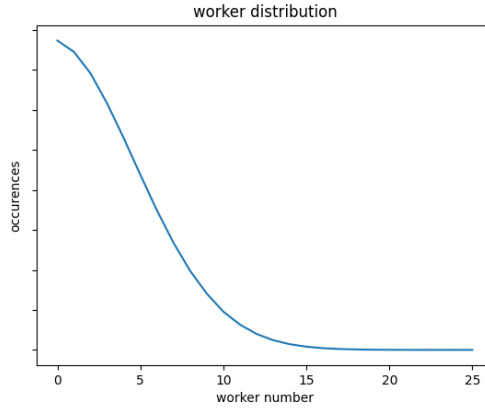


Figure 2.1: Gaussian distribution

Every factory f is assigned an ID and capital in the same way as a person. The number of workers ($|W|$) is generated by sampling a normal (Gaussian) distribution centered on one, just as depicted in figure 2.1, with a maximum of $\frac{|P|}{2}$ and that number of unemployed Persons are picked at random, and assigned as workers. This sampling operation are translated using the sample function in eq. 2.1.

$$\begin{aligned}
 |W(f)| &= \text{sample}(\mathbb{Z}^+, \frac{|P|}{2}) \\
 W(F) &= \left\{ W(f) = \bigcup_{i=1}^{|W(f)|} \{p_i \mid p_i \in {}_R P \cap \overline{W(F)}\} \mid f \in F \right\}
 \end{aligned} \tag{2.1}$$

Where \in_R picks a single random object from a set.

The number of share holders are picked in a similar way in eq. 2.2 without the need of

filtering unemployed people, and the value of each person's stock is picked by generating a uniform random number list and normalizing it.

$$\begin{aligned}
|S(f)| &= \text{sample}(\mathbb{N}, \frac{|P|}{2}) \\
S(F) &= \left\{ S(f) = \bigcup_{i=1}^{|S(f)|} \{p_i \mid p_i \in_R P \cap \overline{S(f)}\} \mid f \in F \right\} \\
S(f, p) \neq 0 &\Leftrightarrow p \in \bigcup_{i=1}^{|S(f)|} \{p_i \mid p_i \in_R P \cap \overline{S(f)}\} \wedge f \in F \\
1 &= \sum_i^{|S(f)|} S(f, p_i)
\end{aligned} \tag{2.2}$$

The first Factory's product is always essential, the second is always luxury, from there on, it's randomly picked 50% chance each. Defining F_λ and F_ν as all factories that produce essential and luxury, respectively.

$$F = \{f \in F_\lambda \cup F_\nu\}$$

2.2 Cycle

This is the portion that is repeated to advance the simulation, represented by the time step operator $\hat{T}|\psi_t\rangle = |\psi_{t+1}\rangle$. This operator is simply the combination of the other cycle operators: $\hat{T} = \hat{W} \hat{S} \hat{F} \hat{G} \hat{P}$, all of these conserve total capital c_{tot} .

The capital and stock at the end of each cycle operator is written using the short-hand notation in eq. 2.3:

$$\begin{aligned}
\hat{c}_t \hat{P} a &= \hat{c}_{\hat{P}} a = c_{\hat{P},a} a \\
\hat{c}_t \hat{G} \hat{P} a &= \hat{c}_{\hat{G}} a = c_{\hat{G},a} a \\
&\dots \\
\hat{c}_t \hat{W} \hat{S} \hat{F} \hat{G} \hat{P} a &= \hat{c}_{\hat{W}} a = c_{\hat{W},a} a = c_{t+1} a \\
\hat{\mu}_t \hat{W} \hat{S} \hat{F} \hat{G} \hat{P} f &= \hat{\mu}_{\hat{W}} f = \mu_{\hat{W},f} f = \mu_{t+1} f \\
\forall a \in P \cup F, \forall f \in F
\end{aligned} \tag{2.3}$$

Every repetition is called a cycle, or a time step, and it acts on all agents $|\psi_t\rangle = (P, F)$.

During **Production** \hat{P} , Factories pay salaries by evenly diving all it's capital by its

workers, following eq. 2.4

$$\begin{aligned}\hat{c}_{\hat{P}} W_t(F) &= \left\{ \left\{ \hat{c}_t p + \frac{\hat{c}_t f}{|W_t(f)|} \mid p \in W_t(f) \right\} \mid f \in F \right\} \\ \hat{c}_{\hat{P}} F &= 0\end{aligned}\tag{2.4}$$

Each factory is left with 0 capital and produces an amount of stock defined by the production function in eq. 2.5, production per worker is depicted in figure 2.2

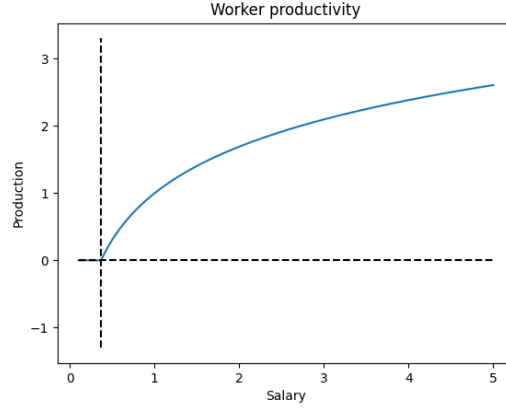


Figure 2.2: Production function

$$\begin{aligned}\hat{\mu}_{\hat{P}} f &= \mu_{\hat{P},f} f = \left(\mu_f + \log\left(\frac{\text{salary}_t(f)}{m_{\text{wage}}}\right) |W(f)| \right) f \\ \text{salary}_t(f) &= \frac{c_f}{|W_t(f)|}\end{aligned}\tag{2.5}$$

This function also defines the minimum wage, setting $\Delta\mu = 0 \Leftrightarrow m_{\text{wage}} = e^{-1}$ is the default value for minimum wage

Lastly, the stock price is defined following eq. 2.6

$$\begin{aligned}\Delta\text{price}(f) &= \min \left\{ 2, 0.2 + \frac{\mu_{\hat{P}}(f) - \mu(f)}{\mu(f)} \right\} \\ \text{price}_t(f) &= \frac{1}{\mu_{\hat{P}}(f)} \left(\Delta\text{price}(f) c_f (\mu_{\hat{P}}(f) - \mu(f)) + \rho \text{price}_{t-1}(f) \mu(f) \right)\end{aligned}\tag{2.6}$$

$\Delta\text{price}(f)$ being the return ratio per unit of stock produced this cycle, $\Delta\text{price}(f) \in [1, 2]$ and grows proportionally to factory stock, this models the law of supply and demand, the smaller leftover stock $(\mu_{\hat{P}}(f) - \mu(f))$ a factory has, the more profit it expects, and vice versa. $\text{price}_t(f)$ is the final price per unit of stock. The price is set using a weighed median between the price of stock produced this cycle and the previous price. The price for stock produced this cycle is given by the profit margin multiplying cost of production, $\Delta\text{price}_t(f) c_f$, and for last

cycle, a devaluation or appreciation effect is translated through ρ , if $\rho < 1$, stock devalues, if $\rho > 1$, stock appreciates. $\rho = \frac{2}{3}$ by default. This is used to translate both real-world product value depreciation over time but also warehouse costs.

The **GoodsMarket** is divided into essential market \hat{G}_λ and luxury market \hat{G}_ν , $\hat{G} = \hat{G}_\nu \hat{G}_\lambda$ both markets iterate over every person the following steps: select a factory, trade with that factory, repeat. The selection step creates an ordered set of factories with non-zero stock sorted by price, $F_{G,\mu}$, and selects a factory by sampling this set using the same normal distribution depicted in figure 2.1, $\text{sample}(\mathbb{Z}^+, |F_{G,\mu}|)$. If this set is empty, that market ends, as there is no more factories to trade. Alternatively, person trades with the sampled factory: transfers capital from person to factory and destroys traded stock. In the essential market, person stops trading when it has traded one unit of essential stock or its capital reaches zero, on luxury market, person only stops trading when its luxury available capital (defined further) reaches zero. The sampling selection mechanism is mapped to eq. 2.7

$$\text{select}(A) = a \Leftrightarrow a = i\text{'th element in } A, i = \text{sample}(\mathbb{Z}^+, |A|) \quad (2.7)$$

and leads to what could be called a semi-perfect market, or weighted random market, where factories with lower prices are more likely to trade, as the probability for smaller indices of the sorted factory list is bigger, but more expensive factories that would not trade on a perfect market, may also trade, creating noise proportional to the standard deviation of the distribution. This noise could translate a multitude of real world parameters, such as distance, personal preference, marketing, etc. As this market repeats a given operation for every person, it can be defined by eq. 2.8 and eq. 2.10

$$\hat{G}_\mu = \text{attemptTrade}_\mu(p_1) \text{attemptTrade}_\mu(p_2) \dots \quad (2.8)$$

Essential market tracks the capital available to person p by $c_\lambda(p)$, stock consumed by person p using $g_\lambda(p)$ and stock available for trade to factory f using $g_\lambda(f)$, initialized in eq. 2.9

$$c_\lambda(p) = c_{\hat{P},f}, g_\lambda(p) = 0, \forall p \in P, \text{ and } g_\lambda(f) = \lambda_{\hat{P},f} \forall f \in F_\lambda. \quad (2.9)$$

$$\begin{aligned}
\text{attemptTrade}_\lambda(p) &= \begin{cases} \mathbb{1} & \text{if } |F_{G,\lambda}| = 0 \vee c_\lambda(p) = 0 \\ \text{attemptTrade}_\lambda(p) \times \text{trade}_\lambda(f, \alpha, p) & \text{else} \end{cases}, \\
f &= \text{select}(F_{G,\lambda}), \\
\alpha &= \min \left\{ 1, \begin{cases} g_\lambda(f) - g_\lambda(p) & \text{if } c_\lambda(p) \geq \text{price}(f) \times g_\lambda(f) - g_\lambda(p) \\ \frac{c_\lambda(p)}{\text{price}(f)} & \text{if } g_\lambda(p) < \text{price}(f) (g_\lambda(f) - g_\lambda(p)) \end{cases} \right\}, \\
F_{G,\lambda} &= \{f \in F \text{ sorted by } \text{price}(f) \mid g_\lambda(f) > 0\}.
\end{aligned} \tag{2.10}$$

The trade_μ , defined in eq. 2.11, updates the market variables

$$\begin{aligned}
\text{trade}_\mu(f, \alpha, p) &\Rightarrow c_\mu(p) \rightarrow c_\mu(p) - \text{price}(f) \times \alpha, \\
&\Rightarrow g_\mu(p) \rightarrow g_\mu(p) + \alpha, \\
&\Rightarrow g_\mu(f) \rightarrow g_\mu(f) - \alpha
\end{aligned} \tag{2.11}$$

At the end of essential market, the leftover capital for each person, $c_\lambda(p)$, will be allocated for luxury and shares markets in a predetermined manner, $\%_\nu = 40\%$ for luxury, $\%_s = 40\%$ for shares market and saving $\%_l = 20\%$ for next cycle.

Luxury market, differs in behaviour to essential only on attemptTrade , defined in eq. 2.13. It tracks the capital available to person p by $c_\nu(p)$, stock consumed by person p using $g_\nu(p)$ and stock available for trade to factory f using $g_\nu(f)$, initialized in eq. 2.12

$$c_\nu(p) = c_\lambda(p), g_\nu(p) = 0, \forall p \in P, \text{ and } g_\nu(f) = \nu_{\hat{P},f} \forall f \in F_\nu. \tag{2.12}$$

$$\begin{aligned}
\text{attemptTrade}_\nu(p) &= \begin{cases} \mathbb{1} & \text{if } |F_{G,\nu}| = 0 \vee c_\nu(p) = 0 \\ \text{attemptTrade}_\nu(p) \times \text{trade}_\nu(f, \alpha, p) & \text{else} \end{cases}, \\
f &= \text{select}(F_{G,\nu}), \\
\alpha &= \begin{cases} g_\nu(f) - g_\nu(p) & \text{if } c_\nu(p) \geq \text{price}(f) \times g_\nu(f) - g_\nu(p) \\ \frac{c_\nu(p)}{\text{price}(f)} & \text{if } g_\nu(p) < \text{price}(f) (g_\nu(f) - g_\nu(p)) \end{cases} \\
F_{G,\nu} &= \{f \in F \text{ sorted by } \text{price}(f) \mid g_\nu(f) > 0\}.
\end{aligned} \tag{2.13}$$

Eq. 2.14 sets the final state of goods market.

$$\begin{aligned}
\hat{c} \hat{G} \hat{P} p &= \left(c_{\hat{P},p} - (c_{\lambda}(p) + c_{\nu}(p)) \right) p \\
\hat{c} \hat{G} \hat{P} f &= \left(c_{\hat{P},p} + (g_{\mu}(f) \times \text{price}(f)) \right) f \\
\hat{\mu} \hat{G} \hat{P} f &= \left(\mu_{\hat{P},p} - g_{\mu}(f) \right) f
\end{aligned} \tag{2.14}$$

During **Funding**, factories start by projecting their attempted production, μ_{proj} , for next time step, using the estimation function depicted in figure 2.3:

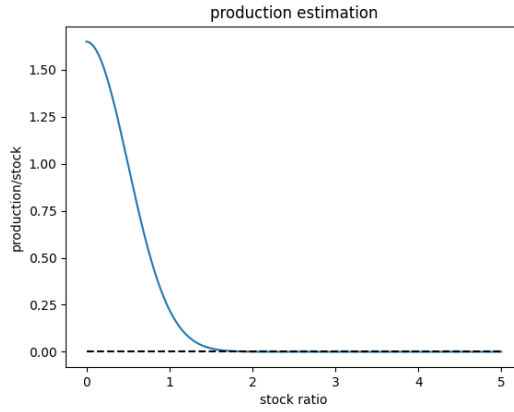


Figure 2.3: Production estimate

This function is defined in eq. 2.15

$$\begin{aligned}
\mu_{\text{proj}}(f) &= \mu_{\hat{P},f} \exp\left(-\frac{\text{ratio}(f)^2}{k} + k\right) + 1 \\
\text{ratio}(f) &= \frac{\mu_{\hat{G}}(f)}{\mu_{\hat{P}}(f) - \mu_{\hat{G}}(f)} \\
c_{\text{proj}}(f) &= \overline{c/\mu} \times \mu_{\text{proj}}(f) \\
\overline{c/\mu} &= \sum_{f \in F} \frac{|W(f)| \text{salary}(f)}{\mu_{\hat{P}}(f) - \mu(f)}
\end{aligned} \tag{2.15}$$

Where k is a simulation constant with a default value of $k = 0.5, k \in \mathbb{R}^+$ and translates the aggressiveness of the factories in this simulation. If $k \rightarrow 0$, factories will only produce more than one unit of stock if leftover equals zero; if $k \rightarrow \infty$, factories will always produce as much stock as possible, regardless of how much stock was sold, and never returning capital to share holders. The "+1" term exists as a way to simulate ambition, avoiding production falling to zero during extreme conditions. c_{proj} is the projection of capital needed for producing this amount of stock, and $\overline{c/\mu}$ the average cost for producing a unit of stock. Then, if a factory raised enough capital during goods market in order to produce the projected stock,

all remaining capital is distributed over its share holders, if it did not, then it must fund itself on the share market by selling shares, this behaviour is presented in eq. 2.16

$$c_{Ep} = c_{\hat{G},p} + \sum_{f \in S(p)} \begin{cases} (c_{\hat{F},f} - c_{\text{proj}}) S(f, p) & \text{if } c_{\hat{F},f} > c_{\text{proj}}(f) \\ 0 & \text{if } c_{\hat{F},f} \leq c_{\text{proj}}(f) \end{cases}, \forall p \in P \quad (2.16)$$

$$\hat{F} f = \begin{cases} \text{distribute}(f) f & \text{if } c_{\hat{G},f} > c_{\text{proj}}(f) \\ \text{fund}(f) f & \text{if } c_{\hat{G},f} \leq c_{\text{proj}}(f) \end{cases}, \forall f \in F$$

For each factory, each of its share holders receives a percentage of its excess capital equal to the value of their share in that factory, and nothing if no excess capital exists. In the latter case, the factory attempts to receive the remaining capital by selling its shares on the share market, if it succeeds, every share holder will now lose a portion of their share and hand it to the new share holder(s). The distribute and fund functions are defined in eq. 2.17:

$$\text{distribute}(f) \Rightarrow \hat{c}_t \hat{F} \hat{G} \hat{P} f = c_{\text{proj}}(f) f \quad (2.17)$$

$$\text{fund}(f) \Rightarrow m_1(f) = \frac{c_{\text{proj}}(f) - c_{\hat{G},f}}{c_{\hat{G},f} + \text{price}_t(f) \mu_{\hat{P},f} + \text{salary}_t(f) |W(f)|}, f \in M_1$$

$M(f)$ being the share amount (percentage of f) to be sold in the shares market, the capital needed divided by the total factory value, defined in eq. 2.18

$$\text{val}(f) = c_{\hat{G},f} + \text{price}_t(f) \mu_{\hat{P},f} + \text{salary}_t(f) |W(f)| \quad (2.18)$$

The **SharesMarket** is divided into primary and secondary, $\hat{S} = 1 + \hat{S}_1 \hat{S}_2$, during the primary shares market, factories sell shares to persons, acquiring capital, during the secondary market, persons may sell shares that they hold over factories to also acquire capital for themselves.

Much like the luxury market, shares market defines a share available capital, $c_s(p) = \%_s \times c_\lambda(p)$, factory capital $c_s(f) = c_{\hat{F},f}$ and two new share-like objects, $T_1(p)$ holds all factories that sold shares to person p , $T_1(p, f)$ holds the share amount that factory f sold to p . Now, for secondary market, T_2 must hold persons, factories and share amounts, so $T_2(p')$ holds all persons that sold shares to person p' , $T_2(p', p)$ all factory from which person p sold shares to person p' , and $T_2(p', p, f)$ the amount of shares sold from person p to p' on factory f . To exemplify, if only one share was sold from person p to p' in factory f , the share amount is $T_2(p, p', f) = T_2(p, p', T_2(p, p'))$, as $T_2(p, p')$ is a set of only one factory element.

Primary shares market attempts to trade shares for all factories that require funding by picking a random person with share available capital, defined in eq. 2.19. The total capital

spent on primary market by person p is defined using $c_1(p) = \sum_{f' \in T_1(p)} \text{val}(f') \times T_1(f', p)$

$$\begin{aligned}
\hat{S}_1 &= \bigcup_{f \in M_1} \text{attemptSell}(f, p), p \in_R P_{\hat{S}_1} \\
\text{attemptSell}(f, p) &= \left(\begin{cases} 1 & \text{if } \alpha = m_1(f) \vee |P_{\hat{S}_1}| = 0 \\ \text{attemptSell}(f, p') & \text{if } \alpha < m_1(f) \end{cases} \right) \times \text{sell}_1(f, \alpha, p), \\
\alpha &= \begin{cases} m_1(f) & \text{if } m_1(f) \times \text{val}(f) \leq c_s(p) - c_1(p) \\ \frac{c_s(p) - c_1(p)}{\text{val}(f)} & \text{if } m_1(f) \times \text{val}(f) > c_s(p) - c_1(p) \end{cases}, \\
p' &\in_R P_{\hat{S}_1}, \\
P_{\hat{S}_1} &= \left\{ p \in P \mid c_s(p) - c_1(p) > 0 \right\}
\end{aligned} \tag{2.19}$$

If $|P_{\hat{S}_1}| = 0$, no more shares will be sold during primary market nor secondary market, as all share market available capital has been exhausted. The sell operator is defined in eq. 2.20:

$$\begin{aligned}
\text{sell}_1(f, \alpha, p) &\Rightarrow p \in T_1(f), f \in T_1(P), T_1(f, p) \rightarrow T_1(f, p) + \alpha, \\
p &\in S(f), f \in S(p), S(f, p) \rightarrow S(f, p) + \alpha, \\
m_1(f) &\rightarrow m_1(f) - \alpha, \\
c_s(p) &\rightarrow c_s(p) - \alpha \times \text{val}(f), \\
c_s(f) &\rightarrow c_s(f) + \alpha \times \text{val}(f)
\end{aligned} \tag{2.20}$$

Secondary shares market attempts to trade shares for all persons that require funding by picking a random person with share available capital, defined in eq. 2.21

Persons will only sell shares during secondary market if they may not be capable of fully consuming essential during the next cycle, this future capital needed projection is defined by the sum of essential capital projection and luxury capital.

$$c_{\text{ess}} = \frac{1}{|F_\lambda|} \sum_{f \in F_\lambda} \text{price}(f)$$

The total capital spent on primary and secondary market by person p is defined using $c_2(p) = c_1(p) + \sum_{p' \in T_2(p)} \left(\sum_{f \in T_2(p, p')} \text{val}(f) \times T_2(p, p', f) \right)$. For next step it is also important to

introduce the mean salary per person, $\overline{\text{salary}} = \frac{1}{|\bar{P}|} \sum_{f \in F} |W(f)| \times \text{salary}(f)$

$$\begin{aligned}
M_2 &= \{p \in P \mid \%_l \times c_\lambda(p) + \overline{\text{salary}} < c_{\text{ess}} \wedge |S(p)| > 0\} \\
m_2(p) &= \min \left\{ \%_l \times c_\lambda(p) + \overline{\text{salary}} - c_{\text{ess}}, \sum_{f \in S(p)} S(p, f) \right\}, p \in M_2 \\
\hat{S}_2 &= \bigcup_{p \in M_2} \text{attemptSell}(p, p'), p' \in_R P_{\hat{S}_2} \\
\text{attemptSell}(p, p') &= \left(\begin{cases} 1 & \text{if } \alpha = m_2(p) \vee |P_{\hat{S}_2}| = 0 \\ \text{attemptSell}(p, p'_2) & \text{if } \alpha < m_2(p) \end{cases} \right) \times \text{sell}_2(p, \alpha, p') \\
\alpha &= \begin{cases} m_2(p) & \text{if } m_2(p) \leq c_s(p') - c_2(p') \\ c_s(p') - c_2(p') & \text{if } m_2(p) > c_s(p') - c_2(p') \end{cases}, \\
&\quad p'_2 \in_R P_{\hat{S}_2} \\
P_{\hat{S}_2} &= \{p' \in P \mid c_s(p') - c_2(p') > 0\}
\end{aligned} \tag{2.21}$$

It is of note that m_1 contains shares and m_2 contains needed capital, and same applies to both α 's. The sell_2 operator, defined in eq. 2.22 will have a more complicated behaviour, as it will trade shares until α capital has been traded, so a second, simpler operator will handle trades, $\text{trade}(p, \alpha', p', f)$ trades α' shares in factory f from person p to person p' , and $\alpha' \times \text{val}(f)$ capital from person p' to person p

$$\begin{aligned}
\text{sell}_2(p, \alpha, p') &= \prod_{f \in S(p)} \begin{cases} \text{trade}(p, \alpha', p', f) & , \text{if } m_2(p) > 0 \\ & , \alpha' = \min \{S(p, f), m_2(p)/\text{val}(f)\} \\ 1 & , \text{if } m_2(p) = 0 \end{cases} \\
\text{trade}(p, \alpha', p', f) &\Rightarrow p' \in S(f), f \in S(p'), S(f, p') \rightarrow S(f, p') + \alpha', \\
&\quad S(f, p) \rightarrow S(f, p) - \alpha', \text{if } S(f, p) = 0, p \notin S(f), f \notin S(p), \\
&\quad m_2(p) \rightarrow m_2(p) - \alpha' \times \text{val}(f), \\
&\quad c_s(p) \rightarrow c_s(p) - \alpha' \times \text{val}(f), \\
&\quad c_s(p') \rightarrow c_s(p') + \alpha' \times \text{val}(f)
\end{aligned} \tag{2.22}$$

At the end of share market, because during primary market the normalization of shares in factories was violated, it is then recovered using a normalize operator norm , so that \hat{S} becomes, in fact, $\hat{S} = \text{norm} \hat{S}_1 \hat{S}_2$. This could be achieved by normalizing at every trade during primary market, reaching the same end state.

WorkersMarket handles finding new workers for every factory in the next cycle. This

is achieved by iterating over every person: finding the highest salary factory, if this salary is higher than minimum wage, person becomes worker in factory, update factory projected salary, repeat. There is no need to implement noise in this stage because there is more persons than factories, so noise would complicate the procedure without providing further realism.

As this market repeats a given operation for every person, $\hat{W} = \hat{\text{find}}^{|P|} = \hat{\text{find}}(p_1) \hat{\text{find}}(p_2) \dots$ it also defines a new set, H , containing every person that has been hired. This $\hat{\text{find}}$ operator, defined in eq. 2.23, acts on one person only.

$$\begin{aligned} \hat{\text{find}}(p) &= \begin{cases} \mathbb{1} & \text{if } c_{\text{proj}}(f)/(1 + |W_{t+1}|) < m_{\text{wage}} \\ p \in W_{t+1}(f) \wedge p \in H & \text{if } c_{\text{proj}}(f)/(1 + |W_{t+1}|) \geq m_{\text{wage}} \end{cases}, \\ p &\in_R \{p \in P \mid p \notin H\}, \\ f &= \max \left\{ f \in F \text{ sorted by } c_{\text{proj}}(f)/(1 + |W_{t+1}(f)|) \right\} \end{aligned} \quad (2.23)$$

2.3 Government

A separated organ of this system is the government. It is separated because it is not essential to the system, but it does add a layer of complexity needed to explore interesting outcomes.

The government acts at the end of the cycle, and redistributes its raised capital equally over all persons, this capital can be raised in one or both ways: **Transaction** taxing and **Wealth cap** taxing.

Transaction taxing occurs every time a transaction of capital occurs during a trade in the goods market, shares market and salary payment, The government taxes a given percentage of the total capital in that transaction, 10% by default, and adds it to the raised capital.

Wealth cap taxing occurs at the end of the cycle, where the government takes all capital from every person that has capital higher than a set cap, $\frac{\text{cap}}{|P|} \sum_{p \in P} c_p$. By default, cap= 14, a person can hold fourteen times the hypothetical perfectly distributed capital.

This concludes all the possible operations carried out to create the system and its results, it is noteworthy that this model exists as a program, so all mentioned above is only a mathematical representation, the program source is available on TODO where do i put the code.

Chapter 3

Results

3.1 Initial conditions

The impact of initial conditions on any system is an important analysis for finding stabilization states, an economy model with a unique convergence state indicates that all systems with different initial conditions converge, modeling a behaviour that all economies eventually evolve into undifferentiated states, and a model with no convergence indicates that all economies evolve toward unique, differentiated states. The model presented converges slowly, meeting at infinity, this is a desired behaviour as it mimics (even if for different reasons) real-world economy convergence [2]. The model convergence is detailed further (section ??) Every result presented next is a median of thirty runs, this was done to mitigate outlier readings, as this system borrows from randomness and thus can create edge scenarios.

For the analysis present in figure 3.1, the result of all different initial conditions were studied on a system with no government. A Gini coefficient shows the inequality in a system. Often used in real economies to estimate how far a country's wealth or income distribution deviates from a totally equal distribution. At a maximum of 1 (total inequality) and minimum of 0 (total equality). Real world data places inequality Gini coefficient around 40% [3]. The Gini index of any property v of an object $a \in A$, $v(a)$, is calculated by

$$g = \frac{\sum_{a \in A} \sum_{a' \in A} |v(a) - v(a')|}{2 \bar{v} |A|^2}$$

High overall inequality compared to real world economies present in these results is attributed to a lack of government, as there is no taxation and wealth redistribution, unlike (most) real world countries.

Results show that, for any initial condition, the system evolves in a similar way, monopoly being the most differentiated condition. Inequality grows rapidly and settles, except for

monopoly, where it slowly reduces. A higher inequality is to be expected on a monopoly conditioned system, but lower production and higher mean salary is not trivially assumed. Monopoly produces less stock per cycle and has lower essential satisfaction because there exists a larger group of people with very small capital and a single person with enormous amounts of capital, therefore, this wealthier person (almost) exclusively consumes luxuries in vast quantities, and, because they attempt to consume all their capital at every trade attempt, big (factory-scope) fluctuations (or shocks) occur due to the semi-perfect market mechanism, and these fluctuations lead to spikes and drops in production, re-

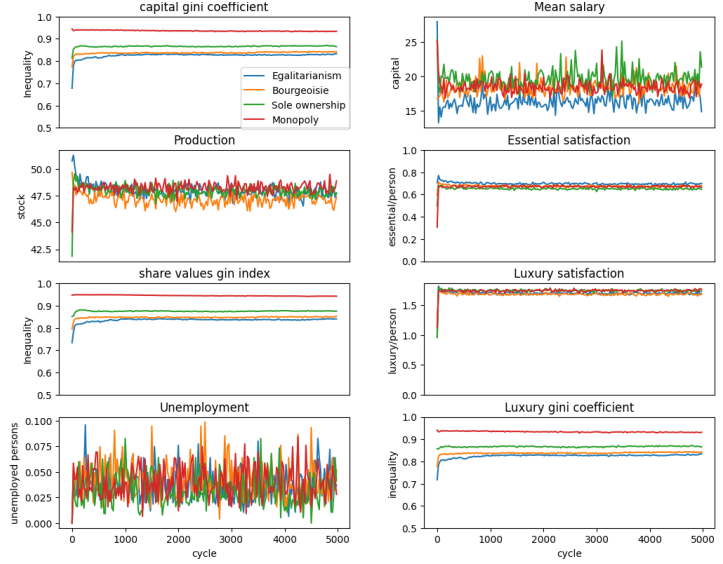


Figure 3.1

sulting in lower factory profits and overall satisfaction, as there is a much smaller group of people joining the luxury market. For all other conditions, this effect is reduced proportionally to inequality, as having a bigger group of people joining each market results in lower fluctuations, stock excess and higher satisfaction. This effect would not occur if the goods market was modeled as a perfect market, this scenario is addressed in section 3.3

An unexpected effect is that sole ownership is considerably closer to monopoly than Bourgeoisie, this occurs because there is a small number of factories and each factory stock is not distributed over a confined group, but to a single individual; essential and luxury satisfaction is mostly equal across conditions; bourgeoisie ranks lower in production and salaries, this occurs due to smaller fluctuations in luxury markets, as more persons enter luxury market; and Egalitarianism converges more slowly, as capital is evenly distributed, inequality and its effects emerges slower. It is also interesting to note that, for all initial conditions, mean salary was, at stability, much higher than minimum salary, stabilizing around 40. This value is affected by the number of people in the system and will be explored further (section 3.3).

To further study the effect of these conditions, stock production and consumption were analysed more closely on figure 3.2. Some interesting behaviours include the gap present in luxury production and salaries, with monopoly and sole ownership significantly more luxurious, and the initial fast decrease in essential employees relative to luxury.

These results further support that excess stock is behind the production increase, as con-

sumption peaks before production, and this occurs sooner for monopoly and sole ownership, as these restrict luxury market from the start, while the remaining conditions slowly reduce the group size. The delay in consumption and leftover stock is dependent on k , factory aggressiveness, and is addressed further in section 3.3.

As mentioned previously, all initial conditions converge for a large enough number of cycles (toward infinity), but the initial state generates a different path towards it. In figure 3.3, each person is analysed for different initial conditions. The process utilized to generate these consist in distributing the initial shares using persons ID's. For example, in monopoly, all initial shares are handed to the person with an ID of 0, for sole ownership, each factory is handed to the person with the same ID as the factory, for bourgeoisie, shares are distributed over the first 10 ID's, and egalitarianism distributes randomly. The figure presented is the

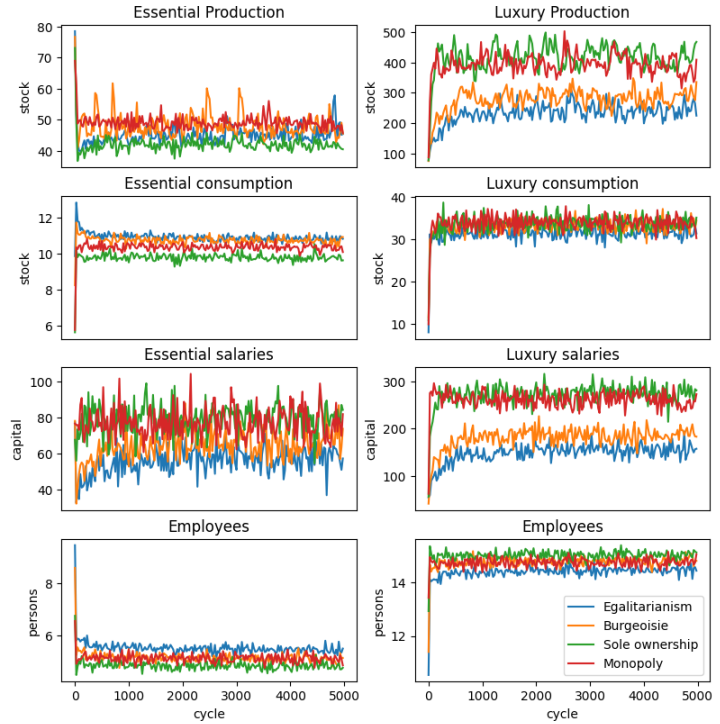


Figure 3.2

capital available to each person, sorted after the end of that cycle's goods market. This helps to clearly illustrate the shift in inequality, slowly declining for monopoly and greatly increasing for all others, leading to a progressively smaller group holding excess capital and consuming vast amounts of luxury, disrupting the market. Also, the shape of the curve created by these values defines the Gini coefficient, which is 0 if these values form a straight line.

As all initial conditions stabilize on a similar state given infinite time and computational power, initial conditions can be seen as a disturbance in the system, making it indistinguishable from another system with a different initial condition. This is a desired behaviour, and occurs because of luxury, as it pushes more wealthy persons to consume more capital per cycle. This convergent behaviour would emerge much slower in a system without luxury market, and that (unrealistic) scenario will be addressed further (section 3.3)

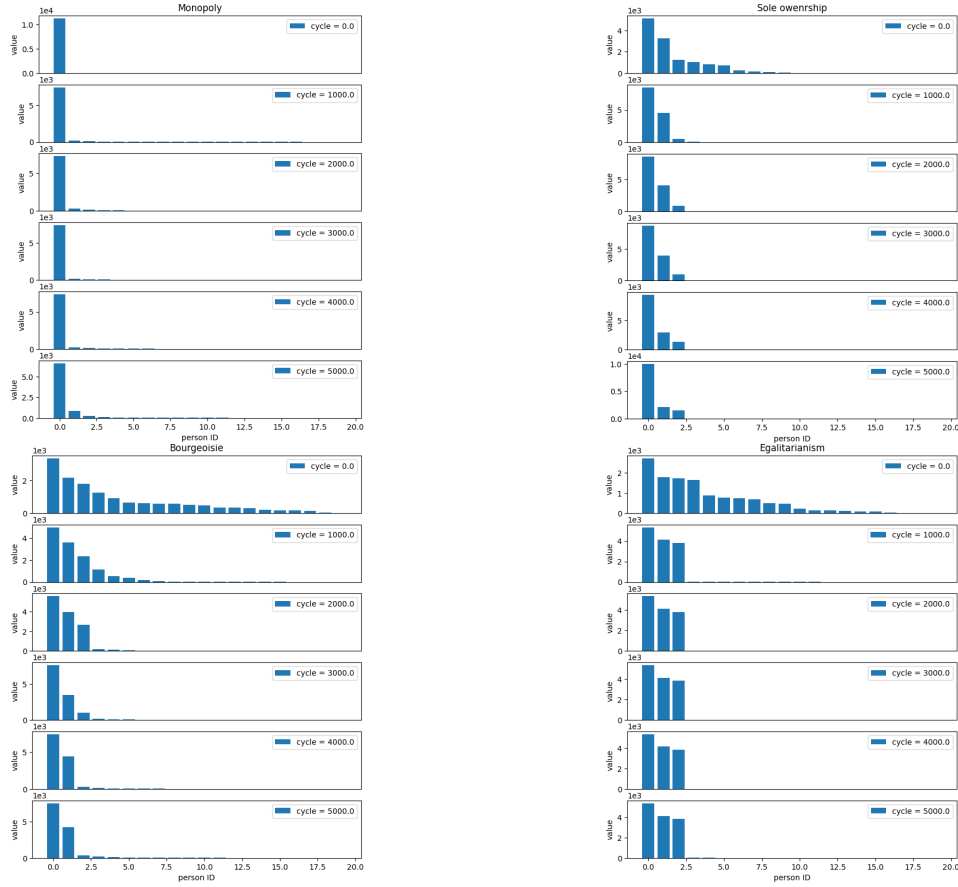


Figure 3.3: individual capital

3.2 Taxation

This section will study ways in which a government may acquire funds and redistribute wealth, redistribution is modeled here by literally redistributing capital over all persons equally, this is a simplification of much more complex real-world government wealth redistribution in the form of public services, such as security, education, health, transportation, infrastructure, etc. Two taxation methods will be implemented, not necessarily based on real-world taxation, but translate a similar effect. A transaction tax acts on every trade completed on goods market, shares market and salary payment, and adds 10% of that transaction value to the government funds and deducts from the beneficiary. A wealth cap tax acts on every person that holds more than 70% of all capital. This may sound like a very high cap, but government acts only at the end of all markets each cycle, and accounts simply for leftover capital that was neither spent on luxury nor share market. Therefore, as can be seen on 3.3, most persons will not hold any capital at that time, except for the extraordinarily wealthy, and this excess wealth will be reduced each cycle. These government funds, as previously mentioned, are then distributed equally, before salary payments (at the start of next

cycle). The next analysis, presented in figure 3.4 studies whether taxation further diverges or converges systems with different initial conditions. looking specifically at capital inequality.

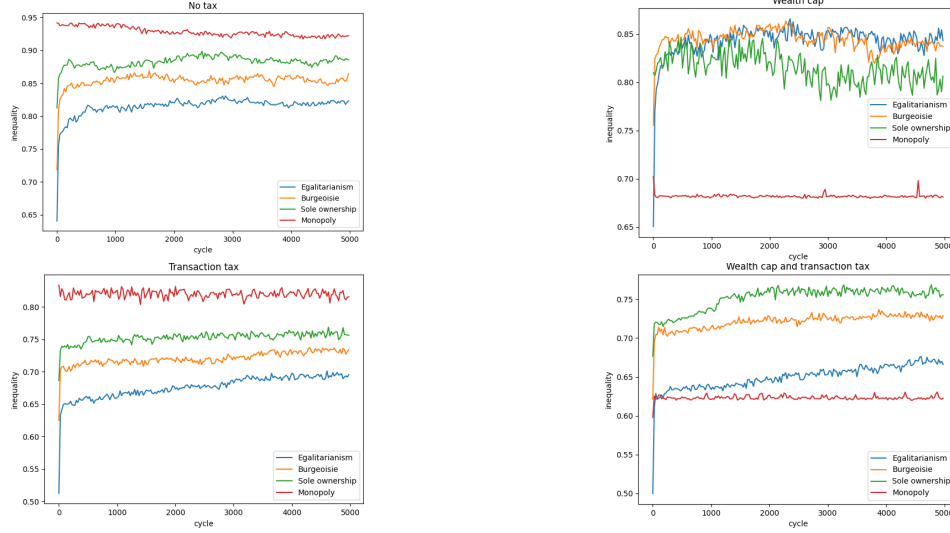


Figure 3.4: Tax effect on gini coefficient

As expected, both taxation methods lower inequality at stability, but some are more effective than others. In a system with no taxes, the stability inequality for all conditions is higher than 80%, this leads to the assumption that a libertarian capitalism leads to a much higher and quicker inequality stabilization. For a transaction tax, all inequalities lower significantly, a 15% decrease can be seen for all conditions at 5000 cycles. It is also interesting that transaction taxing lowers the speed at which the system reaches stability, with inequality still rising (even if much slower) after the 5000 cycle mark.

Wealth capping leads to a very interesting behaviour, destabilizing all non-monopoly conditions and generating noise on these, some very interesting and unexpected behaviour is the reverse of inequality order, with egalitarianism being now the most unequal, with an equality around 85%, higher than the untaxed counterpart, and bourgeoisie quickly converging onto it, seemingly unaffected by wealth cap. These conditions are mostly unaffected because wealth capping

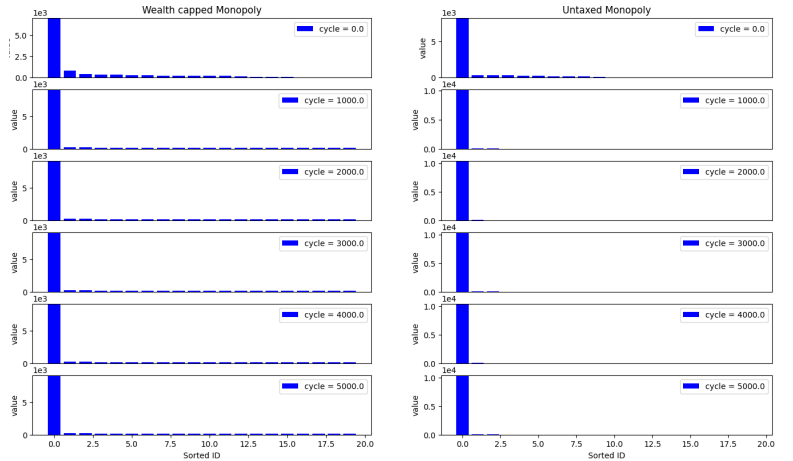


Figure 3.5: Wealth cap effect on Monopoly inequality

raises very little or no capital. As for monopoly, wealth capping leads to a heavy reduction in inequality to around 68%, much lower than other conditions. With both taxing methods combined, their effects combine in a rather unique way. Similar to transaction taxing, inequality drops by 15% on all conditions and stability slows, except for monopoly. Monopoly instead drops even further, reaching the lowest inequality stabilization of around 62% much faster than all others, and grows much slower. It is important to note that none of these taxation methods threw the system into an unstable state, but they did induce a differentiated behaviour for each initial condition.

An analysis similar to the one performed for figure 3.3 is done in figure 3.5 for a monopoly system with and without wealth capping, it can be seen that the monopolist indeed caps its capital and cannot gain any further, and its capital is distributed equally trough every person.

This inequality reduction due to taxation has an effect on production and consumption, that effect is studied for the two most extreme initial conditions, egalitarianism and monopoly.

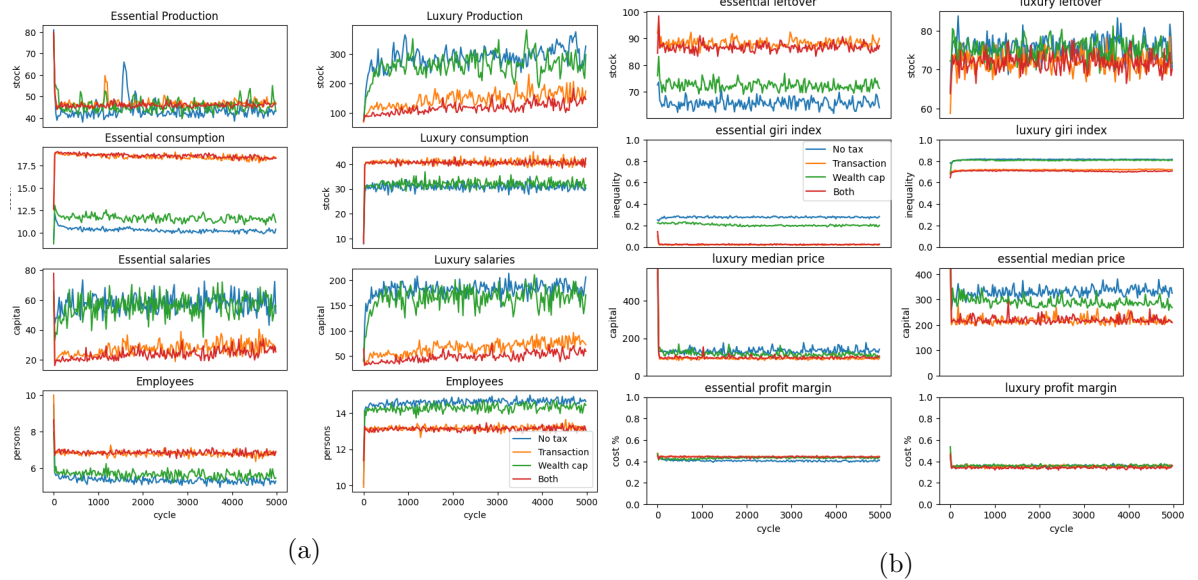


Figure 3.6: Tax effects on egalitarianism production

On egalitarianism, present in figures 3.6a and 3.6b, some interesting effects can be perceived, first of all, wealth cap is much closer to no tax, this is expected given that, on egalitarianism, reaching the health cap is less likely than other initial conditions, therefore only taxing (combined and transaction tax) and no taxing (wealth cap and no tax) will be differentiated. Taxing results in lower salaries but higher consumption for essential and luxury, this is achieved by maintaining a lower consumption inequality, lowering the consumption for the wealthier and raising the consumption for the less wealthy, resulting in a net increase in consumption. The mechanisms for this increase are different for essential and luxury. Essential production is maintained by lowering salaries and raising employment, producing roughly

the same amount but, as capital inequality is now much lower, persons that would otherwise have little capital to spend on essentials can now consume more essentials, rising consumption greatly. As for luxury, having less employees and lower salaries leads to lower production, but, as inequality is lower, more persons enter the luxury market, causing semi-perfect market fluctuations to be less influential and lower leftover stocks, which in turn leads to higher consumption and profit, as stock devaluation happens more rarely.

As for monopoly, present in figures 3.7a and 3.7b, wealth cap transitions from a zero tax system and converges onto transaction taxing, therefore, taxing groups all three forms of taxation, as their behaviour is similar. Taxing, similar to egalitarianism, results in lower salaries and higher consumption, even with lower production by maintaining lower inequality and leftover stock. Differences arise in stabilization time, the monopoly conditioned system stabilized much faster than egalitarianism, this is due to the redistribution of wealth speed being proportional to its concentration, and for luxury inequality a clear divergence of all taxation methods is observed, with transaction tax being much closer to no taxing, this is a reflection of the reversing inequality order generated by wealth cap, present in figure 3.4, where monopoly becomes the less unequal condition.

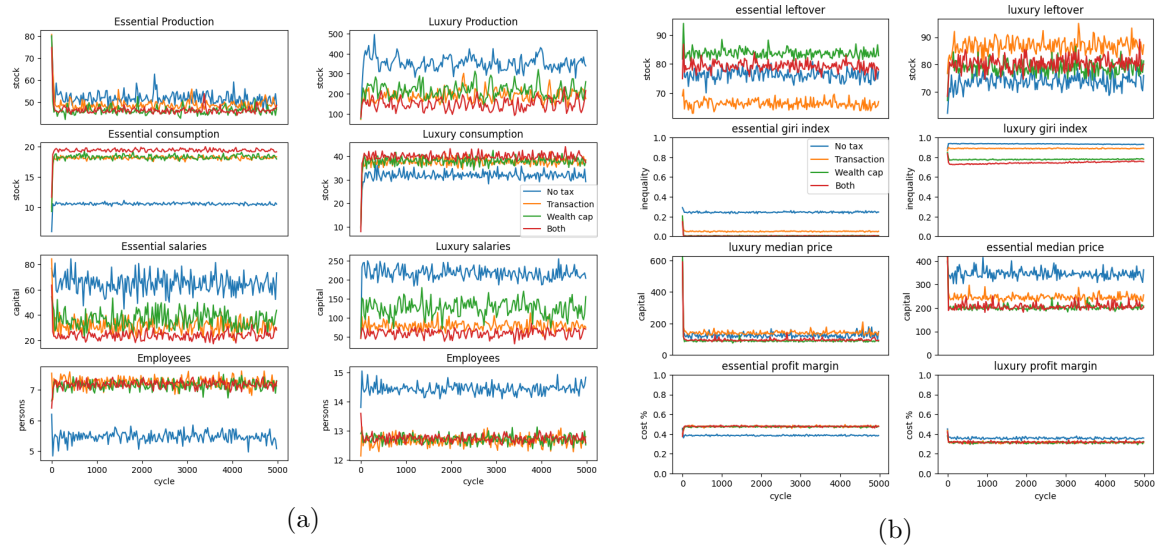


Figure 3.7: Tax effects on monopoly production

3.3 Customization

A hand-built system's major benefit is customizability, so, for next analysis, the system will be fundamentally modified to hopefully better grasp the inner workings of its mechanisms.

Starting by k , or **factory aggressiveness**, this value affects the production estimation function, present in 2.3. A bigger k leads to bigger overall investments on stock production and bigger leftover stock. Different estimation functions are depicted in a egalitarian conditioned system with no taxing in figure 3.8. It shows that more aggressiveness leads to more competition between factories, as every factory will fluctuate production wildly and offer more capital for salaries, trying to grow production even when selling less product than produced. This effect is depicted on an egalitarian system with no taxation in figure 3.10

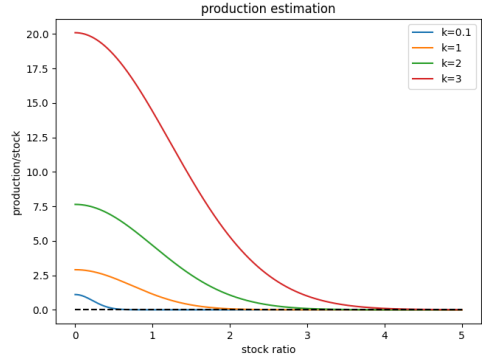


Figure 3.8: Estimation functions

This results in a much higher stock surplus and, due to stock value depreciation, lower prices and higher satisfaction. As mentioned previously, this reflects in a much faster adaptation to rising consumption, and, at higher values, a clear over-sensitization (or overshooting) can be seen, with bigger peaks and valleys in satisfaction, production, inequality and salary. The default $k=0.5$ value was chosen because it better translates a realistic factory rationalization, meeting the point of stable production (production equals stock) for a leftover of a third stock, meaning that in a theoretical perfect stability point, factories would run with a surplus of a third of total consumption (inventory surplus of 33%), which is not very far from real-world data. [1]

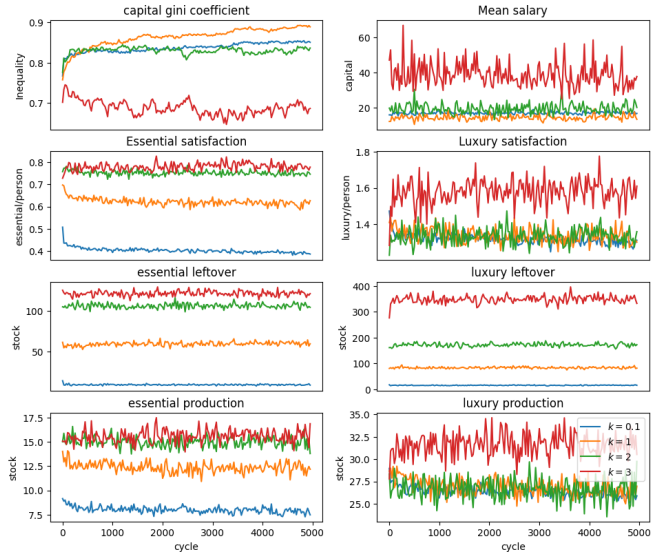


Figure 3.9: aggressiveness systems

The other model constant is ρ , or **stock depreciation**, that affects how leftover stock value evolves each cycle, with a default value of $\rho = \frac{2}{3}$. Lower values mean that excess stock loses a lot of value, values bigger than one mean that excess stock, sitting in a warehouse, is gaining value. Bigger depreciation ($\rho \ll 1$) leads to lower excess stock, as much of its value is lost every cycle, making it affordable and rising satisfaction at the expense of factories losing a big portion of their value each cycle. To make up for this, production increases to make up for loses and regaining value. Approaching $\rho = 1$, excess stock maintains value

and is sold next cycle at same price, this generates lower satisfaction, production and salary. When $\rho > 1$, excess luxury stock sky-rockets and plateaus, as essential is too expensive and no person enters the luxury market, This is translated into satisfaction dropping to zero. Essential factories retain some production due to the newly produced stock being somewhat affordable for most wealthy persons, but satisfaction also falls to practically zero values.

A system with **no luxury** has a consumption cap, so wealthier people cannot consume massive amounts of luxury, which aided in evolving inequality, resulting in a much slower evolution. Results from that system are presented in

figure 3.11, comparing to figure 3.1, mean salary is now much closer to minimum salary, as there is now double the number of essential factories and consumption is very limited, leading to massive competition, reducing production, profit margins, and salaries.

Under these conditions, factories are not producing as much capital for share holders, so shares are sold more often. This explains why sole ownership creates lower inequality, as each factory is owned by only one person and when selling shares, these will be cheaper and easily bought. Monopoly is different because one person holds the total share values of all factories, so when a factory requires capital and sells shares, these will be bought by the same person, as at least one of the factories will always produce profit. This effect is an indirect form of capital injection in factories and occurs much less frequently on all other conditions, as capital is more dispersed.

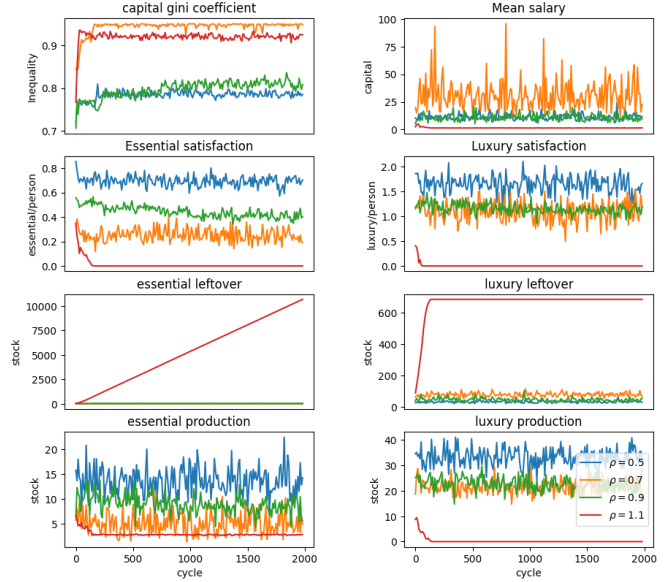


Figure 3.10: Depreciation systems

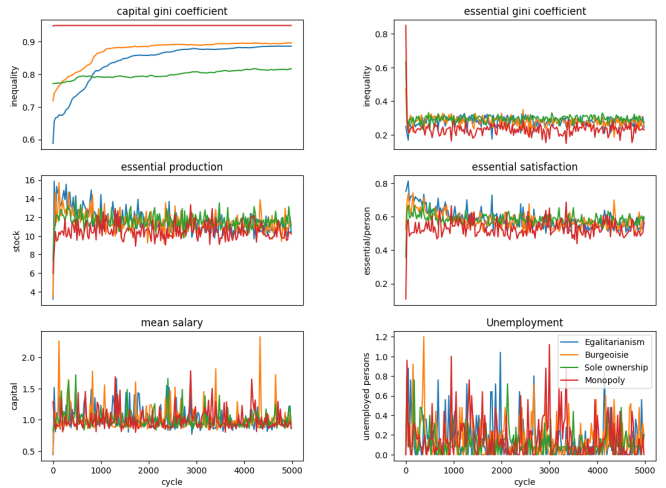


Figure 3.11: No luxury

A **perfect market** was built by changing factory select behaviour to always choose the ideal factory for that person, one with the lowest price (and non-zero stock available). It was mentioned before that a perfect market would reduce the inequality effect on factory-scope fluctuations, lower factory profits and overall satisfaction. Those results are present in figure 3.12, and compared to 3.1, monopoly effects are greatly reduced, as a smaller amount of people entering a market no longer destabilizes said market, so satisfaction, salary and produced stock homogenizes without necessarily lowering inequality.

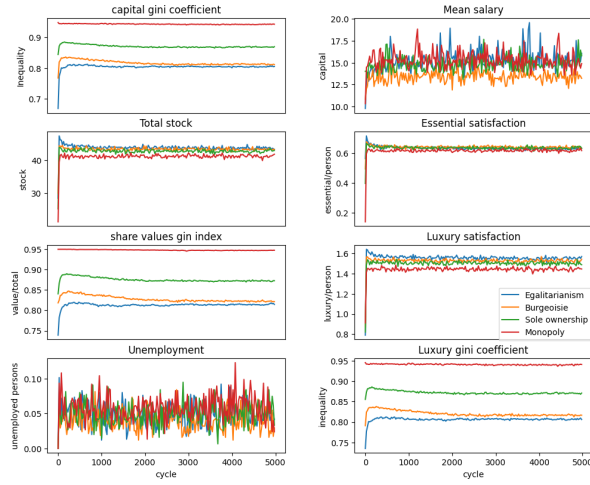


Figure 3.12: Perfect market

As for **median salary** analysis for different number of persons in the system, the computation was made for ten factories and a variable number of persons. The results are present in figure 3.13 and, as mentioned previously, salary decreases with persons density $\frac{|P|}{|F|}$, but presenting two behaviours, decrease occurs very rapidly for low densities and slower for greater densities, reaching a linear relationship closer to minimum wage. Production grows logarithmically, similar to production function on figure 2.2.

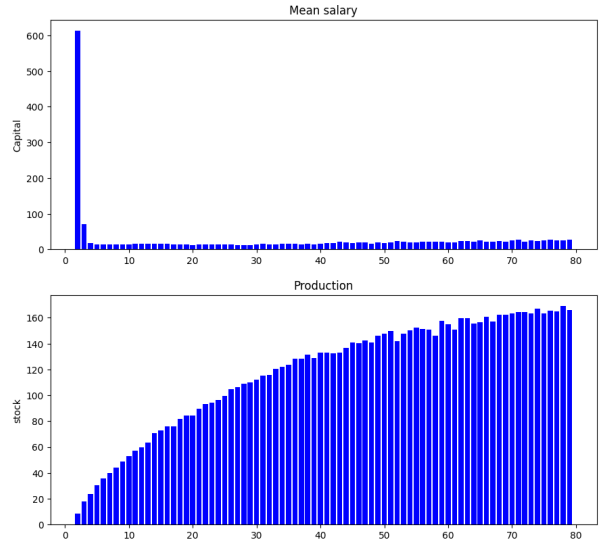


Figure 3.13: Person salary and production

For an **extremely simple system**, with just two persons and two factories, an interesting effect emerges, where luxury satisfaction grows and stabilizes at a plateau but essential falls towards zero. This occurs because the luxury factory can aggregate more capital than its essential counterpart, as its consumption isn't capped, resulting in an essential domination with high inequality and low production. This is an undesired behaviour, because a real economy of that size would prioritize essential satisfaction over luxury. To remedy this behaviour, a new rule was implemented for the

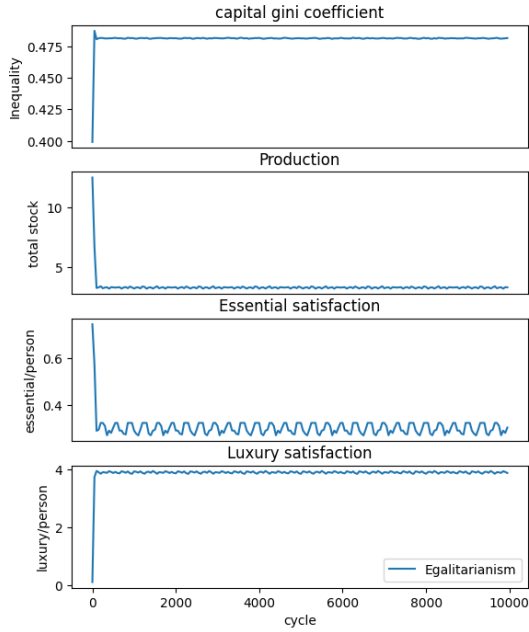
production function in figure 2.2, where

$$\mu_{\hat{P},f} f = \left(\mu_f + \log\left(\frac{\text{salary}_t(f)}{m_{\text{wage}}}\right) |W(f)| \right) f$$

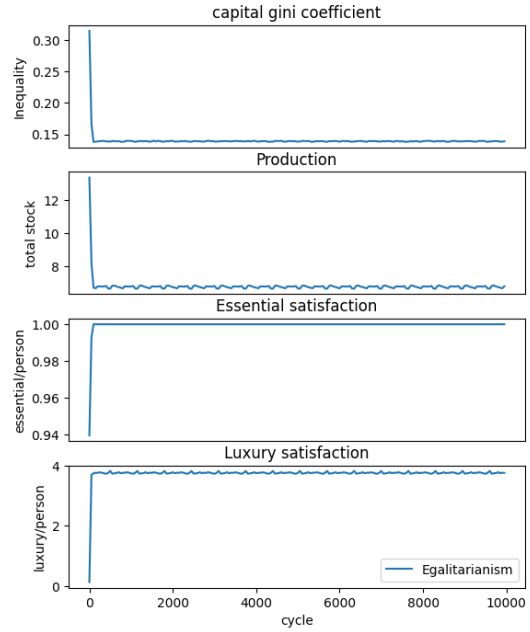
becomes

$$\mu_{\hat{P},f} f = \left(\mu_f + \log\left(\frac{\text{salary}_t(f)}{m_{\text{wage}}}\right) \sum_{p \in P} g_{\lambda}(p) \right) f$$

So that lower essential satisfaction leads to lower production. The results are presented in figure ??, where essential satisfaction was raised. This occurs because the luxury factory loses investment when essential satisfaction decreases, becoming less profitable and allowing the essential factory to regain its market. reaching stability, essential is very close to one and luxury is practically maintained. Upon testing for bigger systems, this multiplier plays close to no effect and introduces added complexity, so it was not introduced in the default production function.



(a) No essential multiplier



(b) With essential multiplier

Todo (maybe) - convergence analysis - minimum wage

Chapter 4

Conclusion

Chapter 5

Future Work

Smarter government with dynamic taxation, (attempting to maintain inflation?) by enabling, for example, Counter-cyclical fiscal measures, government debt, central banks...

Studying consequences of mass withdrawal (if shareholders attempt to sell most of their shares, what is the consequence for the economy?)

Different fiscal policies, such as stimulus checks, negative interest rates, etc.
explain global crisis?

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