$z = 2 - x^2 - 2y^2$

$2 - x^2 - 2y^2 = 0$

$2y^2 = 2 - x^2$

$y^2 = \dfrac{2 - x^2}{2} \rightarrow y = \pm\sqrt{\dfrac{2 - x^2}{2}}$

Tomando la raíz positiva:

$\sqrt{\dfrac{2 - x^2}{2}} = 0$

$\dfrac{2 - x^2}{2} = 0 \rightarrow \begin{array}{l} 2 - x^2 = 0 \\ -x^2 = -2 \\ x = \pm\sqrt{2} \end{array}$

$V = \displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{\frac{2-x^2}{2}}}^{\sqrt{\frac{2-x^2}{2}}} (2 - x^2 - 2y^2)\,dy\,dx$

$V = \displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{\frac{2-x^2}{2}}}^{\sqrt{\frac{2-x^2}{2}}} \left[(2 - x^2) - 2y^2\right]dy\,dx = 4\int_{0}^{\sqrt{2}}\int_{0}^{\sqrt{\frac{2-x^2}{2}}}(2 - x^2) - 2y^2\,dy\,dx$

$\dfrac{V}{4} = \displaystyle\int_{0}^{\sqrt{2}}\left[(2-x^2)y - \dfrac{2y^3}{3}\right]\Bigg|_{0}^{\sqrt{\frac{2-x^2}{2}}}dx = 4\int_{0}^{\sqrt{2}}(2-x^2)\sqrt{\dfrac{2-x^2}{2}} - \dfrac{2}{3}\left[\sqrt{\dfrac{2-x^2}{3}}\right]^3 dx$

$V = 4\displaystyle\int_{0}^{\sqrt{2}}\dfrac{(2-x^2)^{3/2}}{\sqrt{2}^3} - \dfrac{2}{3}\left(\dfrac{2-x^2}{2}\right)^{3/2}dx = 4\int_{0}^{\sqrt{2}}\dfrac{(2-x^2)^{3/2}}{\sqrt{2}^3} - \dfrac{2(2-x^2)^{3/2}}{3(\sqrt{2})^3}dx =$

$$4\int_0^{\sqrt{2}} \frac{(2-x^2)^{3/2}}{\sqrt{2}} - \frac{2(2-x^2)^{3/2}}{3(x\sqrt{2})}dx \quad = 4\int_0^{\sqrt{2}} \frac{(2-x^2)^{3/2}}{\sqrt{2}} - \frac{(2-x^2)^{3/2}}{3\sqrt{2}}dx$$

$$= 4\int_0^{\sqrt{2}} \frac{3(2-x^2)^{3/2} - (2-x^2)^{3/2}}{3\sqrt{2}}dx = 4\int_0^{\sqrt{2}} \frac{2(2-x^2)^{3/2}}{3\sqrt{2}}dx$$

$$= \frac{8}{3\sqrt{2}}\int_0^{\sqrt{2}} (2-x^2)^{3/2}dx$$

$$\rightarrow \int (a^2-u^2)^{3/2}du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\left(\frac{u}{a}\right)+C$$

$u = x$
$a = \sqrt{2}$

$$V = \frac{8}{3\sqrt{2}}\int_0^{\sqrt{2}} (2-x^2)^{3/2}dx = \frac{8}{3\sqrt{2}}\left[\frac{-x}{8}(2x^2-10)\sqrt{2-x^2} + \frac{3}{2}\operatorname{sen}^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]\Big|_0^{\sqrt{2}}$$

$$V = \frac{8}{3\sqrt{2}}\left[\frac{-\sqrt{2}}{8}(2(\sqrt{2})^2-10)\sqrt{2-(\sqrt{2})^2} + \frac{3}{2}\operatorname{sen}^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\right]$$

$$V = \frac{8}{3\sqrt{2}}\left[\frac{3}{4}\pi\right] = \sqrt{2}\,\pi$$