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Exfinal - Matemáticas P-CIMAT

1. Inversa de matriz $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix} = A$

$\det A = 8 - 3 = 5$

$\operatorname{adj} A = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$

$\Rightarrow A^{-1} = \dfrac{1}{\det A} \operatorname{adj} A = \dfrac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 2/5 & -3/5 \\ -1/5 & 4/5 \end{pmatrix}$

$\therefore A^{-1} = \begin{pmatrix} 2/5 & -3/5 \\ -1/5 & 4/5 \end{pmatrix}$

2. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3a - b & -5a + 2b \\ 3c - d & -5c + 2d \end{pmatrix}$

$\Rightarrow \begin{matrix} 3a - b = 1 \\ -5a + 2b = -1 \end{matrix} \quad (1) \qquad \begin{matrix} 3c - d = 2 \\ -5c + 2d = 0 \end{matrix} \quad (2)$

Resolviendo (1)

$b = 3a - 1$

$\Rightarrow -5a + 6a - 2 = -1$

$\boxed{a = 1}$

$\Rightarrow \boxed{b = 2}$

Resolviendo (2)

$d = 3c - 2$

$\Rightarrow -5c + 6c - 4 = 0$

$\boxed{c = 4}$

$d = 12 - 2 = 10$

$\therefore \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 4 & 10 \end{pmatrix}$

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(4) $A(4,6)$, $B(1,0)$, $C(0,2)$

$$\Rightarrow \begin{aligned} a + 4b + 16c &= 6 \\ a + b + c &= 0 \\ a + 0b + 0c &= 2 \end{aligned}$$

$$\Rightarrow \boxed{a = 2} \Rightarrow \begin{aligned} 4b + 16c &= 4 \\ b + c &= -2 \end{aligned}$$

$$\left(\begin{array}{cc|c} 1 & 1 & -2 \\ 4 & 16 & 4 \end{array}\right) \begin{matrix}(-4) \\ \lrcorner\end{matrix} \sim \left(\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & 12 & 12 \end{array}\right)$$

$$\Rightarrow \boxed{c = 1} \;, \; b = -2 - 1 = -3 \quad \boxed{b = -3}$$

$$\therefore \; y = 2 - 3x + x^2$$

(5) Calcular det. $\begin{pmatrix} 1 & 0 & -3 & 3 \\ 0 & 0 & 0 & 7 \\ 0 & 4 & 3 & 1 \\ 0 & 0 & 5 & -1 \end{pmatrix}$

$$\begin{vmatrix} 1 & 0 & -3 & 3 \\ 0 & 0 & 0 & 7 \\ 0 & 4 & 3 & 1 \\ 0 & 0 & 5 & -1 \end{vmatrix} = 1 \begin{vmatrix} 0 & 0 & 7 \\ 4 & 3 & 1 \\ 0 & 5 & -1 \end{vmatrix} = 7 \begin{vmatrix} 4 & 3 \\ 0 & 5 \end{vmatrix}$$

$$= 7 \cdot 4 \cdot 5 = 20 \cdot 7 = 140$$

$$\therefore \det \begin{pmatrix} 1 & 0 & -3 & 3 \\ 0 & 0 & 0 & 7 \\ 0 & 4 & 3 & -1 \\ 0 & 0 & 5 & -1 \end{pmatrix} = 140$$

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6. $f(x) = \begin{cases} \dfrac{x^2+x-6}{x^2-4} & \text{si } x \neq 2 \\ kx^2 & \text{si } x = 2 \end{cases}$

$$\frac{x^2+x-6}{x^2-4} = \frac{\cancel{(x-2)}(x+3)}{(x+2)\cancel{(x-2)}} = \frac{x+3}{x+2}$$

$$\Rightarrow \lim_{x\to 2^-} \frac{x^2+x-6}{x^2-4} = \lim_{x\to 2^-} \frac{x+3}{x+2} = \frac{5}{4} = \lim_{x\to 2^+} \frac{x+3}{x+2}$$

$$= \lim_{x\to 2} kx^2 = 4k$$

$$\Rightarrow 4k = \frac{5}{4}$$

$$\therefore \boxed{k = 5/16}$$

7. $f(x) = \dfrac{5x^2}{x+47}$

$$f'(x) = \frac{1}{(x+47)^2}\left[(x+47)(10x) - 5x^2(1)\right]$$

$$= \left[10x^2 + 470x - 5x^2\right]\frac{1}{(x+47)^2}$$

$$\boxed{f'(x) = \frac{5x^2+470}{(x+47)^2}}$$

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8) $$\lim_{x \to -2} \frac{x+7}{x+2}$$

Si hacemos límite por la derecha.

$$\lim_{x \to -2^+} \frac{x+7}{x+2} = \infty$$

Si hacemos límite por la izq.

$$\lim_{x \to -2^-} \frac{x+7}{x+2} = -\infty$$

$$\Rightarrow \lim_{x \to -2^-} \frac{x+7}{x+2} \neq \lim_{x \to -2^+} \frac{x+7}{x+2}$$

$\therefore$ El límite no existe

9) $$3x^3 + 4xy^2 = 1$$

$$\Rightarrow 9x^2 \quad 4y^2 + 8xyy' = 0$$

$$\Rightarrow 8xyy' = -9x^2 - 4y^2$$

$$\therefore \boxed{y' = \frac{-9x^2 - 4y^2}{8xy}}$$

(10) $f(x) = \begin{cases} x^3+1 & , x < 1 \\ ax^2+bx & , x \geq 1 \end{cases}$

$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$ (1)

$\wedge$

$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$ (2)

$\Rightarrow$ De (1) $2 = a + b$ (3)

De (2) $f'(x) = \begin{cases} 3x^2 & , x < 1 \\ 2ax+b & , x \geq 1 \end{cases}$

$\Rightarrow 3 = 2a + b$ (4)

$$\Rightarrow \begin{array}{r} a + b = 2 \\ 2a + b = 3 \\ \hline -a = -1 \end{array} \qquad \Rightarrow b = 2 - a$$

$\boxed{a = 1}$ $\qquad$ $\boxed{b = 1}$

$\boxed{\therefore\ a = b = 1}$