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Tarea 2 - Matemáticas

1.

a)
$$\begin{aligned} x_1 + 4x_2 + (-2)x_3 + 8x_4 &= 12 \\ x_2 + (-7)x_3 + 2x_4 &= -4 \\ 5x_3 - x_4 &= 7 \\ x_3 + 3x_4 &= -5 \end{aligned}$$

En este ejemplo podemos multiplicar al tercer renglón $(-\frac{1}{5})$ y sumarselo al cuarto renglón. Con esto obtendríamos una matriz triangular superior.

b)
$$\begin{aligned} x_1 - 3x_2 + 5x_3 - 3x_4 &= 1 \\ x_2 - 7x_3 &= -3 \\ x_3 &= 3 \\ 3x_4 &= -3 \end{aligned}$$

Aquí podemos multiplicar por $(1/3)$ el último renglón y obtendríamos la matriz triangular con 1's en la diagonal.

2.

a) $\left(\begin{array}{ccc|c} 2 & 1 & 0 & 5 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$

Sí, es consistente y tiene infinitas soluciones.

b) $\left(\begin{array}{ccc|c} 3 & 2 & 2 & -6 \\ 0 & 2 & -7 & 2 \\ 0 & 0 & 3 & 0 \end{array}\right)$

Es consistente con una única solución.

c) $\left(\begin{array}{ccc|c} 1 & 3 & 1 & 6 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow 0 = 1$ 'y' Es inconsistente.

d) $\left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Consistente con inf. sol.

3. ¿(3, 4, −2) es una solución del sistema?

$$\begin{aligned} 5x_1 - x_2 + 2x_3 &= 7 \\ -2x_1 + 6x_2 + 9x_3 &= 0 \\ -7x_1 + 5x_2 - 3x_3 &= -7 \end{aligned}$$

$$\Rightarrow \begin{pmatrix} 5 & -1 & 2 \\ -2 & 6 & 9 \\ -7 & 5 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 15-4-4 \\ -6+24-18 \\ -21+20+6 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 0 \\ 5 \end{pmatrix} \neq \begin{pmatrix} 7 \\ 0 \\ -7 \end{pmatrix}$$

∴ No es solución. //

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4.

$$\left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right) \overset{(2)}{\sim} \left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{array}\right) (1)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & h+k+2g \end{array}\right)$$

$\Rightarrow h + k + 2g = 0$ // Y así, el sistema es consistente.

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5.

a) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}$ ; $\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 5 & 5 & 1 & 0 & 0 & 1 \end{array}\right) \overset{(-5)}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array}\right) (-1/4)$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{5}{4} & 0 & -1/4 \end{array}\right) (-3)(-1) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -1/4 & 0 & 1/4 \\ 0 & 2 & 0 & -15/4 & 1 & 3/4 \\ 0 & 0 & 1 & 5/4 & 0 & -1/4 \end{array}\right) (1/2)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -1/4 & 0 & 1/4 \\ 0 & 1 & 0 & -15/8 & 1/2 & 3/8 \\ 0 & 0 & 1 & 5/4 & 0 & -1/4 \end{array}\right) (-1) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 13/8 & -1/2 & -1/8 \\ 0 & 1 & 0 & -15/8 & 1/2 & 3/8 \\ 0 & 0 & 1 & 5/4 & 0 & -1/4 \end{array}\right)$$

$$\therefore \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 13/8 & -1/2 & -1/8 \\ -15/8 & 1/2 & 3/8 \\ 5/4 & 0 & -1/4 \end{pmatrix}$$

b) $\begin{pmatrix}1&1&1&1\\1&2&-1&2\\1&-1&2&1\\1&3&3&2\end{pmatrix} : \left(\begin{array}{cccc}1&1&1&1\\1&2&-1&2\\1&-1&2&1\\1&3&3&2\end{array}\right)(-1) \sim \left(\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\0&1&-2&1&-1&1&0&0\\0&-2&1&0&-1&0&1&0\\0&2&2&1&-1&0&0&1\end{array}\right)(2)\,(-2)\,(-1)$

$\sim \left(\begin{array}{cccc|cccc}1&0&3&0&2&-1&0&0\\0&1&-2&1&-1&1&0&0\\0&0&-3&2&-3&2&1&0\\0&0&6&-1&1&-2&0&1\end{array}\right)(-1/3) \sim \left(\begin{array}{cccc|cccc}1&0&3&0&2&-1&0&0\\0&1&-2&1&-1&1&0&0\\0&0&1&-\frac{2}{3}&1&-\frac{2}{3}&-\frac{1}{3}&0\\0&0&6&-1&1&-2&0&1\end{array}\right)(-6)\,(2)\,(-3)$

$\sim \left(\begin{array}{cccc|cccc}1&0&0&2&-1&1&1&0\\0&1&0&-1/3&1&-1/3&-2/3&0\\0&0&1&-2/3&1&-2/3&-1/3&0\\0&0&0&3&-5&2&2&1\end{array}\right)(1/3) \sim \left(\begin{array}{cccc|cccc}1&0&0&2&-1&1&1&0\\0&1&0&-1/3&1&-1/3&-2/3&0\\0&0&1&-2/3&1&-2/3&-1/3&0\\0&0&0&1&-5/3&2/3&2/3&1/3\end{array}\right)$

$\sim \left(\begin{array}{cccc|cccc}1&0&0&0&7/3&-1/3&-1/3&-2/3\\0&1&0&0&4/9&-1/9&-4/9&1/9\\0&0&1&0&-1/9&-2/9&1/9&2/9\\0&0&0&1&-5/3&2/3&2/3&1/3\end{array}\right)$

$\therefore \begin{pmatrix}1&1&1&1\\1&2&-1&2\\1&-1&2&1\\1&3&3&2\end{pmatrix}^{-1} = \begin{pmatrix}7/3&-1/3&-1/3&-2/3\\4/9&-1/9&-4/9&1/9\\-1/9&-2/9&1/9&2/9\\-5/3&2/3&2/3&1/3\end{pmatrix}$

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6.

a) $\begin{pmatrix}2&3&1&4\\1&-1&6&4\\0&-1&5&8\\1&2&-1&4\end{pmatrix}(-1/2) \sim \begin{pmatrix}2&3&1&4\\0&-5/2&11/2&2\\0&-1&5&8\\0&1/2&-3/2&2\end{pmatrix}(-2/5)\,(1/5) \sim$

$\sim \begin{pmatrix}2&3&1&4\\0&-5/2&11/2&2\\0&0&14/5&36/5\\0&0&-2/5&12/5\end{pmatrix}(1/7) \sim \begin{pmatrix}2&3&1&4\\0&-5/2&11/2&2\\0&0&14/5&36/5\\0&0&0&24/7\end{pmatrix}$

$\Rightarrow U = \begin{pmatrix}2&3&1&4\\0&-5/2&11/2&2\\0&0&14/5&36/5\\0&0&0&24/7\end{pmatrix}$

$$\Rightarrow L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ +1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +1/2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2/5 & 1 & 0 \\ 0 & -1/5 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/7 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 2/5 & 1 & 0 \\ 1/2 & -1/5 & -1/7 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & -1 & 6 & 4 \\ 0 & -1 & 5 & 8 \\ 1 & 2 & -1 & 4 \end{pmatrix} = LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 2/5 & 1 & 0 \\ 1/2 & -1/5 & -1/9 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/5 & 36/5 \\ 0 & 0 & 0 & 24/7 \end{pmatrix}$$

b) $$\begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ -2 & 5 & -2 & 0 \\ 0 & -4 & 5 & 2 \end{pmatrix} (-2)(1) \sim \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 8 & -3 & 6 \\ 0 & -4 & 5 & 2 \end{pmatrix} (-8)(4) \sim$$

$$\sim \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 21 & -42 \end{pmatrix} (3/5) \sim \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 0 & 72/5 \end{pmatrix} = U$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3/5 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 8 & 1 & 0 \\ 0 & -4 & -3/5 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ -2 & 5 & -2 & 0 \\ 0 & -4 & 5 & 2 \end{pmatrix} = LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 8 & 1 & 0 \\ 0 & -4 & -3/5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 0 & 72/5 \end{pmatrix}$$

7.

a) $f(x) = \sqrt{x+1}$

$x+1 \geq 0$
$\Rightarrow \quad x \geq -1 \quad \Rightarrow \quad D: \{x \in \mathbb{R} / x \geq -1\}$

$Im: \{y \in \mathbb{R} / y \in [0; \infty)\}$

b) $f(x) = x\sqrt{2x+3}$

$2x+3 \geq 0$
$x \geq -3/2 \quad \Rightarrow \quad D: \{x \in \mathbb{R} / x \geq -3/2\}$

$Im: \{y \in \mathbb{R} / y \in [0; \infty)\}$

c) $f(x) = \dfrac{x^2-16}{x+4}$

$\dfrac{x^2-16}{x+4} = x-4$

$D: x \in \mathbb{R}$
$Im: y \in \mathbb{R}$

d) $f(x) = \dfrac{x}{1-1/x}$

$\dfrac{x}{1-\frac{1}{x}} = \dfrac{x}{\frac{x-1}{x}} = \dfrac{x^2}{x-1}$

$\Rightarrow \quad x \neq 1 \quad \Rightarrow \quad D: x \in (\mathbb{R} - \{1\})$

Sea $y \in \mathbb{R} \Rightarrow y = \dfrac{x^2}{x-1}$

$x^2 = y(x-1)$
$x^2 - yx + y = 0$

$x = \dfrac{y \pm \sqrt{y^2-4y}}{2a}$

$\Rightarrow y^2 - 4y \geq 0$
$y(y-4) \geq 0$

$+ \quad - \quad +$
$0 \quad 4$

$\Rightarrow Im: \{y \in \mathbb{R} / y \leq 0 \vee y \geq 4\}$

8.

a) $\lim_{x \to 5} 3x + 1 = 16$

Sean $\varepsilon > 0$, $\delta > 0$, probamos:

$$0 < |x - 5| < \delta \to |3x + 1 - 16| < \varepsilon$$

$$|3x - 15| < \varepsilon$$
$$|3(x - 5)| < \varepsilon$$
$$3|x - 5| < \varepsilon$$
$$\Rightarrow |x - 5| < \varepsilon/3$$

$\bullet$ $\delta = \varepsilon/3$

$$|x - 5| < \delta$$
$$3|x - 5| < 3\delta$$
$$|3x - 5| < 3\delta/3$$
$$|f(x) - L| < \varepsilon$$

LQQD.

b) $\lim_{x \to 2} x^2 = 4$

Con un $\delta < 1$; $0 < |x - 2| < \delta$

$$\Rightarrow -1 < 0 < x - 2 < 1$$
$$3 < x + 2 < 5 \qquad (1)$$

Retomando

$$|x^2 - 4| < \varepsilon$$
$$|(x + 2)(x - 2)| < \varepsilon$$
$$5|x - 2| < \varepsilon$$
$$|x - 2| < \varepsilon/5$$

Con $\delta = \varepsilon/5$ $\quad |x - 2| < \delta$

$$5|x - 2| < 5\delta$$
$$|x + 2||x - 2| < 5\delta$$
$$|x^2 - 4| < 5\delta = \varepsilon$$

$\therefore |x^2 - 4| < \varepsilon$ LQQD.