Tarea 4 - Algebra Matricial
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1. Considerar la sig. matriz $A = \begin{pmatrix} 5 & -10 & -5 \\ 2 & 14 & 2 \\ -4 & -8 & 6 \end{pmatrix}$

Comprobar o refutar si:

a) $\lambda = 5$ es un valor propio de A

b) $\lambda = -1$ es un valor propio de A.

c) $(1, 1, 1)^T$ es un vector propio de A.

d) $(-1, 0, 1)^T$ es un vector propio de A.

a) Comprobamos $\det(A - \lambda I) = 0$ para $\lambda = 5$.

$$\det(A-\lambda I) = \begin{vmatrix} 5-5 & -10 & -5 \\ 2 & 14-5 & 2 \\ -4 & -8 & 6-5 \end{vmatrix} = \begin{vmatrix} 0 & -10 & -5 \\ 2 & 9 & 2 \\ -4 & -8 & 1 \end{vmatrix} \begin{matrix} (2) \\ \sim \end{matrix}$$

$$= \begin{vmatrix} 0 & -10 & -5 \\ 2 & 9 & 2 \\ 0 & 10 & 5 \end{vmatrix} = -2 \begin{vmatrix} -10 & -5 \\ 10 & 5 \end{vmatrix} = 0$$

$$\boxed{\therefore \lambda = 5 \text{ es valor propio de } A.}$$

b) Comprobamos si $\lambda = -1$ es valor propio de A

$$\det(A-\lambda I) = \begin{vmatrix} 5+1 & -10 & -5 \\ 2 & 14+1 & 2 \\ -4 & -8 & 6+1 \end{vmatrix} = \begin{vmatrix} 6 & -10 & -5 \\ 2 & 15 & 2 \\ -4 & -8 & 7 \end{vmatrix} \begin{matrix} \sim \\ (-3)\ (2) \end{matrix}$$

$$= \begin{vmatrix} 0 & -55 & -11 \\ 2 & 15 & 2 \\ 0 & 32 & 11 \end{vmatrix} = -2 \begin{vmatrix} -55 & -11 \\ 32 & 11 \end{vmatrix} = -2 \cdot 11(-55+32) \neq 0$$

$$\boxed{\therefore \lambda = -1 \text{ no es valor propio de } A}$$

c) Comprobaremos que $A\vec{v} = \lambda\vec{v}$ para algún $\lambda \in \mathbb{R}$ y con $\vec{v} = (1,1,1)^T$.

$$A\vec{v} = \begin{pmatrix} 5 & -10 & -5 \\ 2 & 14 & 2 \\ -4 & -8 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5-10-5 \\ 2+14+2 \\ -4-8+6 \end{pmatrix} = \begin{pmatrix} -10 \\ 18 \\ -6 \end{pmatrix}$$

No existe λ : $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} -10 \\ 18 \\ -6 \end{pmatrix}$

$\boxed{\therefore (1,1,1)^T \text{ no es vector propio de } A.}$

d) Comprobaremos si $\vec{v} = (-1,0,1)$ es vector propio de A.

$$A\vec{v} = \begin{pmatrix} 5 & -10 & -5 \\ 2 & 14 & 2 \\ -4 & -8 & 6 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -5-5 \\ -2+2 \\ 4+6 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 10 \end{pmatrix} = 10\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Válido para $\lambda = 10$. $\boxed{\therefore (-1,0,1) \text{ es vector propio de } A.}$

2. Calcular los valores y vectores propios de las siguientes matrices.

a) $B = \begin{pmatrix} -5 & 2 \\ -7 & 4 \end{pmatrix}$; b) $C = \begin{pmatrix} 2 & 2 & -2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix}$

a) Resolvemos $\det(B - \lambda I) = 0$

$$\det(B - \lambda I) = \begin{vmatrix} -5-\lambda & 2 \\ -7 & 4-\lambda \end{vmatrix} = (-5-\lambda)(4-\lambda) + 14 =$$

$$= -20 - \lambda + \lambda^2 + 14 = \lambda^2 + \lambda - 6 = (\lambda - 2)(\lambda + 3) = 0$$

$$\Rightarrow \boxed{\lambda_1 = 2 \ ; \ \lambda_2 = -3}$$

• Para λ_1: Resolvemos $(B - \lambda_1 I)x = 0$ para algún $x \in \mathbb{R}^2$.

$$\left(\begin{array}{cc|c} -5-2 & 2 & 0 \\ -7 & 4-2 & 0 \end{array}\right) = \left(\begin{array}{cc|c} -7 & 2 & 0 \\ -7 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -7 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{7}x_2 \\ x_2 \end{pmatrix} = \frac{1}{7}x_2 \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$\boxed{\therefore \begin{pmatrix} 2 \\ 7 \end{pmatrix} \text{ es eigenvector de } A}$$

• Para λ_2: Resolvemos $(B - \lambda_2 I)x = 0$ para algún $x \in \mathbb{R}^2$.

$$\left(\begin{array}{cc|c} -5+3 & 2 & 0 \\ -7 & 4+3 & 0 \end{array}\right) = \left(\begin{array}{cc|c} -2 & 2 & 0 \\ -7 & 7 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_1 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\boxed{\therefore \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ es un eigenvector de } A.}$$

b) Resolvemos $\det(C-\lambda I)=0$

$$\det(C-\lambda I)=\begin{vmatrix} 2-\lambda & 2 & -2 \\ 1 & 3-\lambda & -1 \\ -1 & 1 & 1-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix}$$

$$-\begin{vmatrix} 2 & -2 \\ 1 & 1-\lambda \end{vmatrix} - \begin{vmatrix} 2 & -2 \\ 3-\lambda & -1 \end{vmatrix} = (2-\lambda)\left[(3-\lambda)(1-\lambda)+1\right]$$

$$-\left[2(1-\lambda)+2\right]-\left[-2+2(3-\lambda)\right]=$$

$$=(2-\lambda)(3-\lambda)(1-\lambda)\cancel{+2}-\lambda\cancel{-2}+2\lambda+$$

$$\cancel{-2}+\cancel{2}-6+2\lambda=$$

$$=(6-6\lambda-2\lambda+2\lambda^2-3\lambda+3\lambda^2+\lambda^2-\lambda^3)+$$

$$+3\lambda\cancel{-6}=-\lambda^3+6\lambda^2-8\lambda=0$$

$$\Rightarrow -\lambda(\lambda^2-6\lambda+8)=0$$

$$-\lambda(\lambda-4)(\lambda-2)=0$$

$$\boxed{\therefore \lambda_1=0,\ \lambda_2=4,\ \lambda_3=2}$$

- Para λ_1:

Comprobaremos igual que el caso anterior.

$$\left(\begin{array}{ccc|c} 2 & 2 & -2 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right)\begin{matrix} \\ (-2)\ (1) \\ \end{matrix} \sim \left(\begin{array}{ccc|c} 0 & -4 & 0 & 0 \\ 1 & 3 & -1 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right)\begin{matrix} \\ \\ (1) \end{matrix} \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right)\begin{matrix} \\ \\ (\frac{1}{4})(-3) \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} x_3 \\ 0 \\ x_3 \end{pmatrix}=x_3\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\boxed{\therefore \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ es vector propio de } A \text{ con } \lambda_1}$$

• Para λ_2 comprobamos

$$\left(\begin{array}{ccc|c} 2-4 & 2 & -2 & 0 \\ 1 & 3-4 & -1 & 0 \\ -1 & 1 & 1-4 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -2 & 2 & -2 & 0 \\ 1 & -1 & -1 & 0 \\ -1 & 1 & -3 & 0 \end{array}\right) \begin{array}{l} (2)\ (1) \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & -4 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \\ 0 \end{pmatrix} = x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\boxed{\therefore \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \text{ es vector propio de } A \text{ con } \lambda_2}$$

• Para λ_3 comprobamos

$$\left(\begin{array}{ccc|c} 2-2 & 2 & -2 & 0 \\ 1 & 3-2 & -1 & 0 \\ -1 & 1 & 1-2 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 0 & 2 & -2 & 0 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & -1 & 0 \end{array}\right) \begin{array}{l} (1) \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 0 & 2 & -2 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} (-1) \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\boxed{\therefore \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ es vector propio de } A \text{ con } \lambda_3}$$

3. Diagonalizar las siguientes matrices.

a) $D = \begin{pmatrix} -4 & -1 \\ 0 & -4 \end{pmatrix}$

- Encontramos eigenvalores

$$\det(D - \lambda I) = \begin{vmatrix} -4-\lambda & -1 \\ 0 & -4-\lambda \end{vmatrix} = (-4-\lambda)^2 = 0$$

$$\Rightarrow \lambda_1 = -4 \quad , \lambda_2 = -4$$

- Encontramos eigenvectores para λ_1

$$\left(\begin{array}{cc|c} -4-(-4) & -1 & 0 \\ 0 & -4-(-4) & 0 \end{array}\right) = \left(\begin{array}{cc|c} 0 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\Rightarrow \lambda_1$ con multiplicidad algebraica 2 y de multiplicidad geométrica 1.

$\boxed{\therefore D \text{ no es diagonalizable.}}$

b) $E = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$

- Encontramos eigenvalores

$$\det(E - \lambda I) = \begin{vmatrix} 1-\lambda & -1 & 4 \\ 3 & 2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{vmatrix}$$

$$= (1-\lambda)[(2-\lambda)(-1-\lambda)+1] - 3[(\lambda+1)-4] + 2[1-4(2-\lambda)]$$

$$= -\lambda^3 \cancel{-\lambda^2} + 2\lambda^2 + \cancel{\lambda^2} + \cancel{2\lambda} + \lambda \cancel{-2\lambda} - 2 - 3\lambda + 9 + 8\lambda - 14$$

$$= -\lambda^3 + 2\lambda^2 + 5\lambda - 6 = 0$$

$$= -(\lambda-1)(\lambda+2)(\lambda-3) = 0$$

$\Rightarrow \lambda_1 = 1, \ \lambda_2 = -2, \ \lambda_3 = 3$

- Para λ_1

$$\left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 1 & -1 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right)(-1) \sim \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right)(-2) \sim \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right)(1)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_3 \\ 4x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{v}_1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$$

- Para λ_2.

$$\left(\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right)(-1) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right)(-3)\ (-2)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 10 & -10 & 0 \\ 0 & 5 & -5 & 0 \end{array}\right)(-1/2) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)(2)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_3 \\ x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{v}_2 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

- Para λ_3.

$$\left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 3 & -1 & -1 & 0 \\ 2 & 1 & -4 & 0 \end{array}\right)(1) \sim \left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)(2) \sim \left(\begin{array}{ccc|c} 0 & -5 & 10 & 0 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)\left(-\tfrac{2}{5}\right)$$

$$\sim \left(\begin{array}{ccc|c} 0 & -1 & 2 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ 2x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Como E tiene 3 valores propios diferentes.
$\Rightarrow E$ es diagonalizable

$$E = Q P Q^{-1}$$

Con $Q = (\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{pmatrix} -1 & -1 & 1 \\ 4 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

y $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Obtenemos Q^{-1}:

$$Q^{-1} = \frac{1}{\det Q} \begin{pmatrix} -1 & -2 & 3 \\ 2 & -2 & 0 \\ -3 & 6 & 3 \end{pmatrix}^T$$

$$\det Q = \begin{vmatrix} -1 & -1 & 1 \\ 4 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} \neg \\ (-4)(1) \end{matrix} \sim \begin{vmatrix} 0 & 0 & 2 \\ 0 & -3 & -2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ -3 & -2 \end{vmatrix} = 6$$

$$\Rightarrow Q^{-1} = \begin{pmatrix} -1 & 2 & -3 \\ -2 & -2 & 6 \\ 3 & 0 & 3 \end{pmatrix} \cdot \frac{1}{6}$$

$$\boxed{\therefore E = \begin{pmatrix} -1 & -1 & 1 \\ 4 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}}$$

c) $F = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$

• Obtenemos λ.

$$\det(F - \lambda I) = \begin{vmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)(-3\lambda + \lambda^2 - 4) - 2(2(3-\lambda) - 8) + 4(4 + 4\lambda)$$

$$= -\lambda^3 + 3\lambda^2 - 5\lambda - 12 + 4 + 4\lambda + 16 + 16\lambda + 3\lambda^2$$

$$= -\lambda^3 + 6\lambda^2 + 15\lambda + 8 = 0$$

$$-(\lambda+1)(\lambda+1)(\lambda-8) = 0$$

$\lambda_{1,2} = -1$; $\lambda_3 = 8$

• Para $\lambda_{1,2}$:

$$\left(\begin{array}{ccc|c} 4 & 2 & 4 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 2 & 4 & 0 \end{array}\right) \begin{matrix} \\ (-2) \\ \downarrow \end{matrix} \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ -2x_1 - 2x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ -2x_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -2x_3 \\ x_3 \end{pmatrix}$$

$$= x_1 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} ; \vec{v}_2 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$\lambda_{1,2}$ tiene multiplicidad algebraica de 2, y tiene multiplicidad geométrica de 2.

• Para λ_3.

$$\left(\begin{array}{ccc|c} -5 & 2 & 4 & 0 \\ 2 & -8 & 2 & 0 \\ 4 & 2 & -5 & 0 \end{array}\right)\begin{matrix} \\ \\ (1) \end{matrix} \sim \left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ 2 & -8 & 2 & 0 \\ 4 & 2 & -5 & 0 \end{array}\right)\begin{matrix} (2)\ (4) \\ \\ \\ \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 18 & -9 & 0 \end{array}\right)\begin{matrix} \\ \\ (1/18) \end{matrix} \sim \left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & 0 \end{array}\right)\begin{matrix} \\ \\ (-4) \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & 0 \end{array}\right) \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ \frac{1}{2}x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 1/2 \\ 1 \end{pmatrix} = \frac{x_3}{2}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \vec{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

la multiplic. algebr. de todos los eigen valores es igual a cada una de sus multiplic. geo.
$\Rightarrow F$ es diagonalizable.

$$F = QPQ^{-1}$$

Con $Q = \begin{pmatrix} 1 & 0 & 2 \\ -2 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$; $P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

Determinamos Q^{-1}.

$$Q^{-1} = \frac{1}{\det Q}\begin{pmatrix} -5 & 4 & -2 \\ 2 & 2 & -1 \\ 4 & -5 & -2 \end{pmatrix}^T$$

$$\det Q = \begin{vmatrix} 1 & 0 & 2 \\ -2 & -2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = -5 + 2\cdot(-2) = -9$$

$$\Rightarrow Q^{-1} = \begin{pmatrix} -5 & 2 & 4 \\ 4 & 2 & -5 \\ -2 & -1 & -2 \end{pmatrix}\cdot\frac{-1}{9} = \begin{pmatrix} 5/9 & -2/9 & -4/9 \\ -4/9 & -2/9 & 5/9 \\ 2/9 & 1/9 & 2/9 \end{pmatrix}$$

$$\therefore \boxed{F = \begin{pmatrix} 1 & 0 & 2 \\ -2 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} 5/9 & -2/9 & -4/9 \\ -4/9 & -2/9 & 5/9 \\ 2/9 & 1/9 & 2/9 \end{pmatrix}}$$

4. Determinar la evolución del sistema dinámico $x_{k+1} = G x_k$ cuando $k \to \infty$

$$G = \begin{pmatrix} 4/5 & 0 \\ 0 & 16/25 \end{pmatrix}$$

El estado al tiempo k:

$$x_k = G^k x_0$$

Como G es diagonal:

$$G^k = \begin{pmatrix} (4/5)^k & 0 \\ 0 & (16/25)^k \end{pmatrix}$$

Notemos que $\frac{4}{5} < 1$ y $\frac{16}{25} < 1$, si $k \to \infty$

$$\Rightarrow \lim_{k \to \infty} (4/5)^k = \lim_{k \to \infty} \left(\frac{16}{25}\right)^k = 0$$

$$\Rightarrow G_\infty = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad y \quad x_\infty = G_\infty x_0$$

$$\therefore \boxed{x_\infty = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

$\therefore$ dos objetos de estudio eventualmente desaparecerán.