Álgebra Matricial. Tarea 3.

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1.- Obtenga la descomposición LU/PLU de las siguientes matrices.

$$a)\ \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & -1 & 6 & 4 \\ 0 & -1 & 5 & 8 \\ 1 & 2 & -1 & 4 \end{pmatrix} \left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)$$

$$\sim \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & -1 & 5 & 8 \\ 0 & 1/2 & -3/2 & 2 \end{pmatrix} \left(-\tfrac{2}{5}\right)\left(\tfrac{2}{5}\cdot\tfrac{1}{2}\right) \sim$$

$$\sim \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/5 & \frac{36}{5} \\ 0 & 0 & -2/5 & \frac{12}{5} \end{pmatrix} \left(\tfrac{1}{7}\right) \sim \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/5 & \frac{36}{5} \\ 0 & 0 & 0 & 24/7 \end{pmatrix} = U$$

Para hallar L, tomamos las inversas de las operaciones elementales que hicimos, en el mismo orden.

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1/2 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{5} & 0 & 1 \end{pmatrix}^{-1}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{7} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{2}{5} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{5} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{7} & 1 \end{pmatrix} =$$

Iremos multiplicando las últimas dos matrices en cada renglón.

$$L=\begin{pmatrix}1&0&0&0\\ \frac{1}{2}&1&0&0\\ 0&0&1&0\\ 0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ \frac{1}{2}&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\ 0&1&0&0\\ 0&\frac{2}{5}&1&0\\ 0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&-\frac{1}{5}&-\frac{1}{7}&1\end{pmatrix}=$$

$$=\begin{pmatrix}1&0&0&0\\ \frac{1}{2}&1&0&0\\ 0&0&1&0\\ 0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ \frac{1}{2}&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\ 0&1&0&0\\ 0&\frac{2}{5}&1&0\\ 0&-1/5&-\frac{1}{7}&1\end{pmatrix}=$$

$$=\begin{pmatrix}1&0&0&0\\ \frac{1}{2}&1&0&0\\ 0&0&1&0\\ 0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\ 0&1&0&0\\ 0&\frac{2}{5}&1&0\\ \frac{1}{2}&-\frac{1}{5}&-\frac{1}{7}&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\ 1/2&1&0&0\\ 0&2/5&1&0\\ 1/2&-1/5&-\frac{1}{7}&1\end{pmatrix}$$

$$\therefore\ \begin{pmatrix}2&3&1&4\\ 1&-1&6&4\\ 0&-1&5&8\\ 1&2&-1&4\end{pmatrix}=LU=\begin{pmatrix}1&0&0&0\\ \frac{1}{2}&1&0&0\\ 0&2/5&1&0\\ 1/2&-1/5&-\frac{1}{7}&1\end{pmatrix}\begin{pmatrix}2&3&1&4\\ 0&-5/2&\frac{11}{2}&2\\ 0&0&14/5&36/5\\ 0&0&0&24/7\end{pmatrix}$$

b) $A = \begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ -2 & 5 & -2 & 0 \\ 0 & -4 & 5 & 2 \end{pmatrix}$

$$\begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ -2 & 5 & -2 & 0 \\ 0 & -4 & 5 & 2 \end{pmatrix} \begin{matrix}(-2) & (1)\end{matrix} \sim \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 8 & -3 & 6 \\ 0 & -4 & 5 & 2 \end{pmatrix} \begin{matrix}(-8) & (4)\end{matrix}$$

$$\sim \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 21 & -42 \end{pmatrix} \left(\frac{21}{35}\right) \sim \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 0 & 72/5 \end{pmatrix} = U$$

Para obtener L, con el mismo procedimiento anterior:

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -8 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{21}{35} & 1 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{21}{35} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -4 & -\frac{21}{35} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 8 & 1 & 0 \\ 0 & -4 & -\frac{21}{35} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 8 & 1 & 0 \\ 0 & -4 & -\frac{21}{35} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 8 & 1 & 0 \\ 0 & -4 & -\frac{21}{35} & 1 \end{pmatrix}$$

$$\therefore A = LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 8 & 1 & 0 \\ 0 & -4 & -21/35 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 0 & 72/5 \end{pmatrix}$$

2. Calcule la descomposición Cholesky de las siguientes matrices.

a) $A = \begin{pmatrix} 4 & 10 & 8 \\ 10 & 26 & 26 \\ 8 & 26 & 61 \end{pmatrix}$

Hallamos U:

$$\begin{pmatrix} 4 & 10 & 8 \\ 10 & 26 & 26 \\ 8 & 26 & 61 \end{pmatrix} \begin{matrix} (-\frac{5}{2}) & (-2) \end{matrix} \sim \begin{pmatrix} 4 & 10 & 8 \\ 0 & 1 & 6 \\ 0 & 6 & 45 \end{pmatrix} (-6) \sim \begin{pmatrix} 4 & 10 & 8 \\ 0 & 1 & 6 \\ 0 & 0 & 9 \end{pmatrix} \begin{matrix} (1/4) \\ \\ (1/9) \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 5/2 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix} = U$$

$$\Rightarrow L = U^T = \begin{pmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 2 & 6 & 1 \end{pmatrix}$$

Para D; multiplicamos las inversas de las operaciones de escalamiento:

$$D^2 = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/9 \end{pmatrix}^{-1} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$\Rightarrow D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Formamos la matriz $T = LD$; $T^T = D^T L^T = DU$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 2 & 6 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 5 & 1 & 0 \\ 4 & 6 & 3 \end{pmatrix}$$

$$\therefore A = TT^T = \begin{pmatrix} 2 & 0 & 0 \\ 5 & 1 & 0 \\ 4 & 6 & 3 \end{pmatrix} \begin{pmatrix} 2 & 5 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{pmatrix}$$

b) $A = \begin{array}{c} \begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix} \end{array} (-\tfrac{3}{5})(\tfrac{1}{5})$

Para U:

$$\sim \begin{pmatrix} 25 & 15 & -5 \\ 0 & 9 & 3 \\ 0 & 3 & 10 \end{pmatrix} \begin{array}{c} \\ (-1/3) \\ \end{array} \sim \begin{pmatrix} 25 & 15 & -5 \\ 0 & 9 & 3 \\ 0 & 0 & 9 \end{pmatrix} \begin{array}{c} (1/25) \\ (1/9) \\ (1/9) \end{array}$$

$$\sim \begin{pmatrix} 1 & 3/5 & -1/5 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{pmatrix} = U$$

$$y \quad D^2 = \begin{pmatrix} 1/25 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/9 \end{pmatrix}^{-1} = \begin{pmatrix} 25 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$\Rightarrow D = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\text{Con } T^T = DU = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 3/5 & -1/5 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 3 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\therefore A = TT^T = \begin{pmatrix} 5 & 0 & 0 \\ 3 & 3 & 0 \\ -1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 5 & 3 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$