Basics of Statistical Learning

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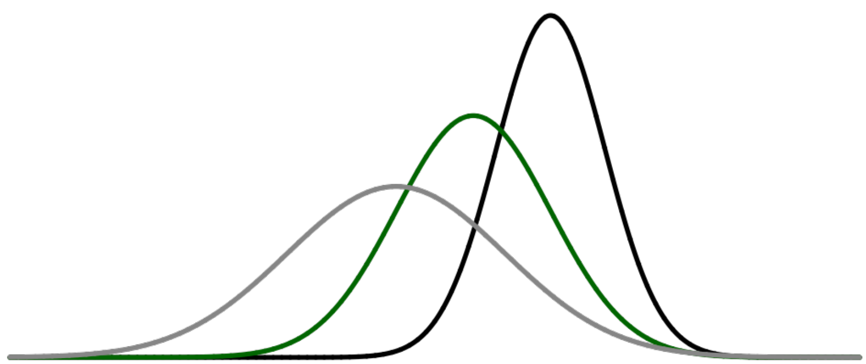
2019-09-09

Contents

Preface



Welcome to Basics of Statistical Learning!

- TODO: Warning about development.
- TODO: Warning about PDF version.
- TODO: Transfer acknowledgements.
- TODO: discuss https://daviddalpiaz.github.io/r4sl/
- TODO: course vs book
- TODO: stat432.org
- TODO: https://yihui.name/en/2013/06/fix-typo-in-documentation/
- TODO: http://varianceexplained.org/r/ds-ml-ai/

License

Chapter 1

Introduction

```
library(readr)
library(tibble)
library(dplyr)
library(purrr)
library(ggplot2)
library(ggridges)
library(lubridate)
library(randomForest)
library(rpart)
library(rpart.plot)
library(cluster)
library(caret)
library(factoextra)
library(rsample)
library(janitor)
library(rvest)
library(dendextend)
library(knitr)
library(kableExtra)
library(ggthemes)
```

- TODO: Show package messaging? check conflicts!
- TODO: Should this be split into three analyses with different packages?

1.1 Regression: Powerlifting

1.1.1 Background

- TODO: https://www.openpowerlifting.org/
- TODO: https://en.wikipedia.org/wiki/Powerlifting

1.1.2 Data

- TODO: Why readr::col_factor() and not just col_factor()?
- TODO: Characters should be character and "categories" should be factors.
- TODO: Is `na.omit()` actually a good idea?

```
pl = read_csv("data/pl.csv", col_types = cols(Sex = readr::col_factor()))
```

```
pl
```

```
## # A tibble: 3,604 x 8
##    Name              Sex   Bodyweight   Age Squat Bench Deadlift Total
##    <chr>             <fct>      <dbl> <dbl> <dbl> <dbl>    <dbl> <dbl>
##  1 Ariel Stier       F           60      32 128.   72.5     150   350
##  2 Nicole Bueno      F           60      26 110    60       135   305
##  3 Lisa Peterson     F           67.5    28 118.   67.5     138.  322.
##  4 Shelby Bandula    F           67.5    26  92.5  67.5     140   300
##  5 Lisa Lindhorst    F           67.5    28  92.5  62.5     132.  288.
##  6 Laura Burnett     F           67.5    30  90    45       108.  242.
##  7 Suzette Bradley   F           75      38 125    75       158.  358.
##  8 Norma Romero      F           75      20  92.5  42.5     125   260
##  9 Georgia Andrews   F           82.5    29 108.   52.5     120   280
## 10 Christal Bundang F           90      30 100    55       125   280
## # ... with 3,594 more rows
```

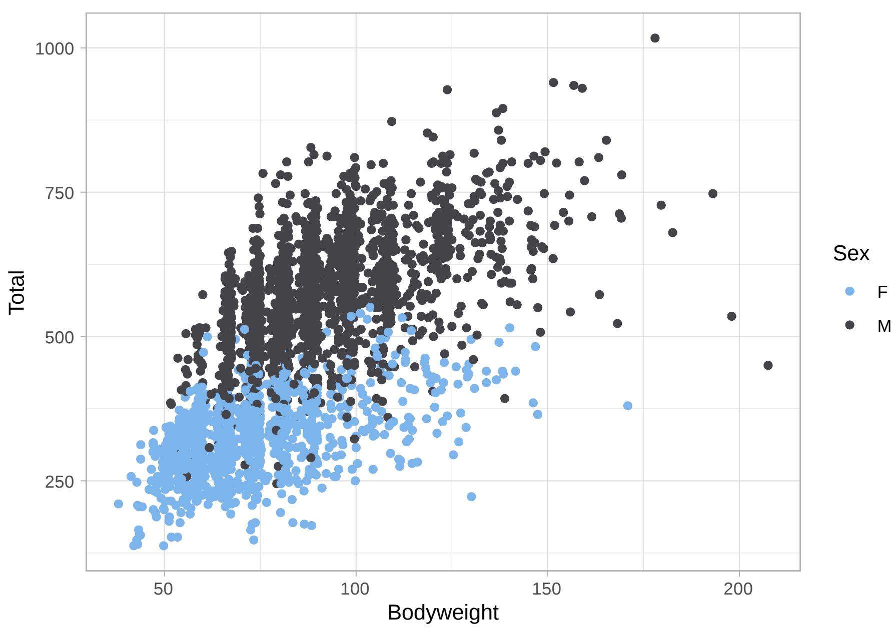
1.1.3 EDA

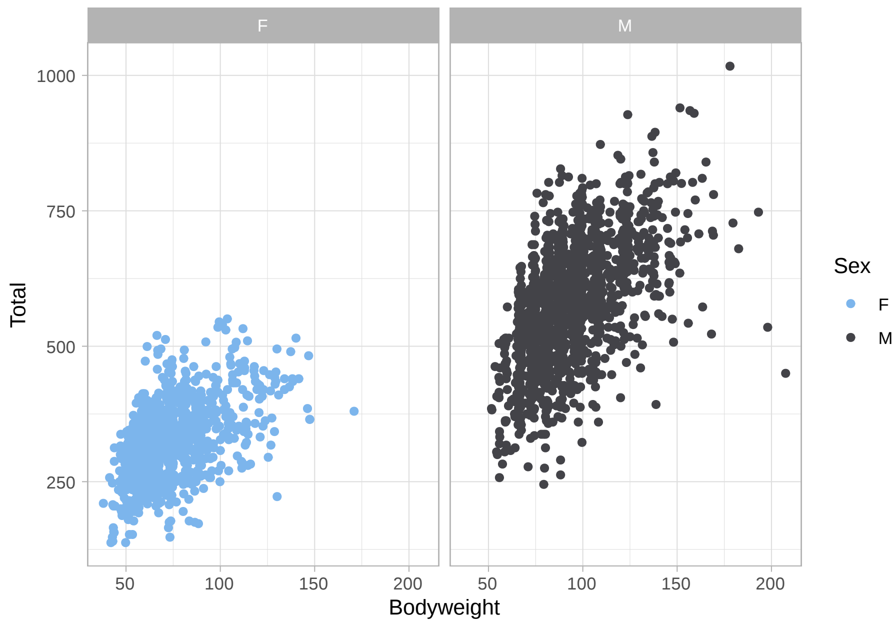
```
set.seed(1)

# test-train split
pl_tst_trn_split = initial_split(pl, prop = 0.80)
pl_trn = training(pl_tst_trn_split)
pl_tst = testing(pl_tst_trn_split)

# estimation-validation split
pl_est_val_split = initial_split(pl_trn, prop = 0.80)
```

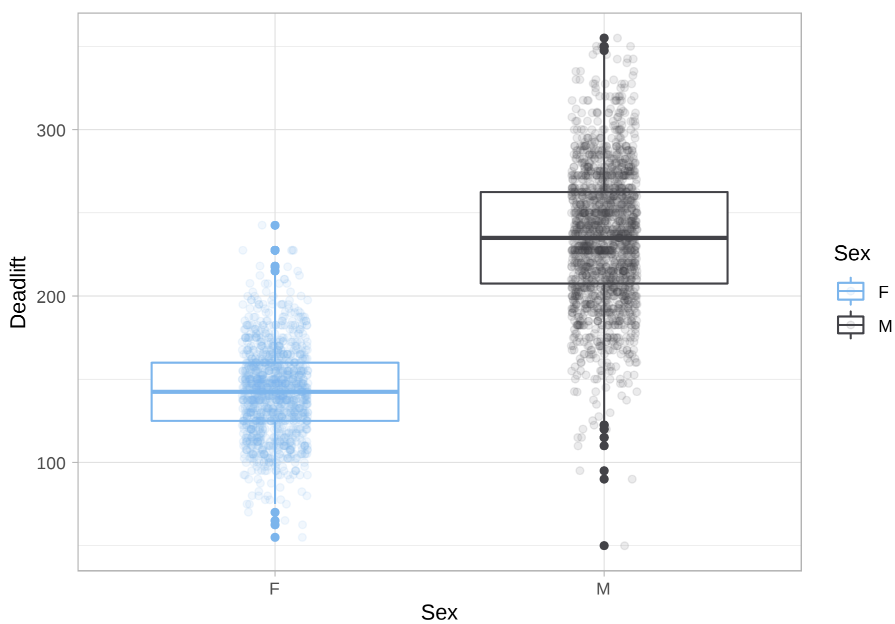
```
pl_est = training(pl_est_val_split)
pl_val = testing(pl_est_val_split)
```

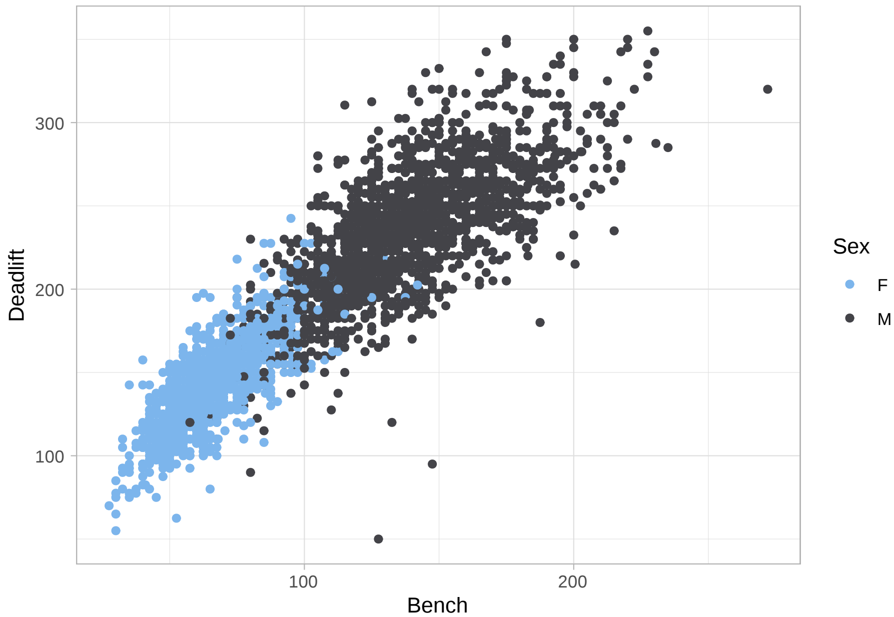
```
rm(pl)
```

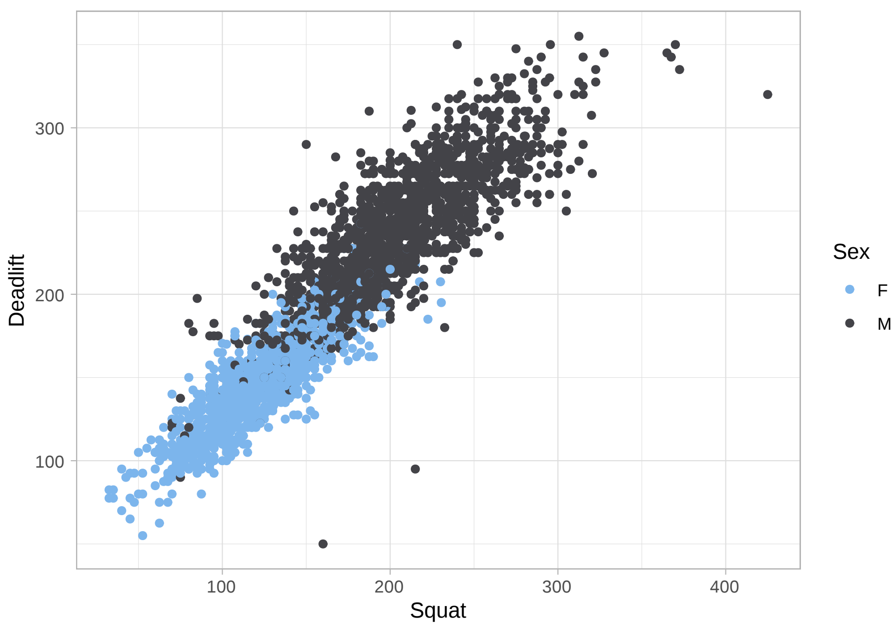
- TODO: Train can be used however you want. (Including EDA.)
- TODO: Test can only be used after all model decisions have been made!




F
M
1000
750
500
250
Total
50
100
150
200
50
100
150
200
Bodyweight
Sex
F
M


300
200
100
Deadlift
F
M
Sex
Sex
F
M

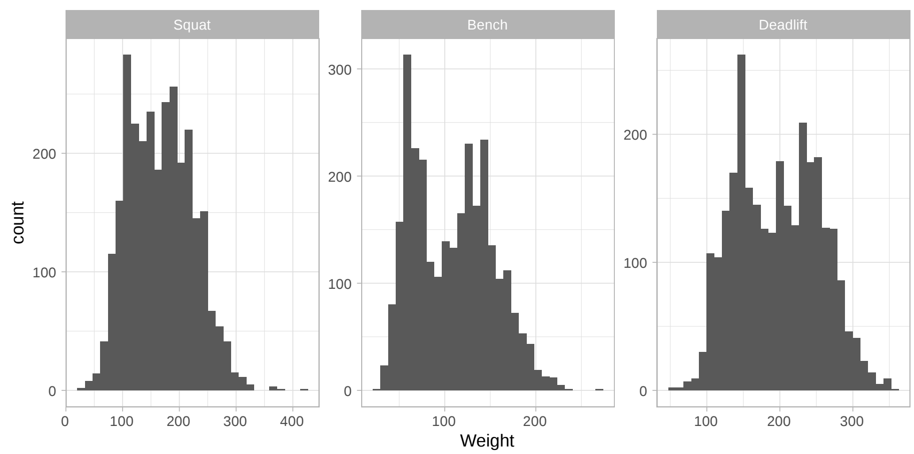


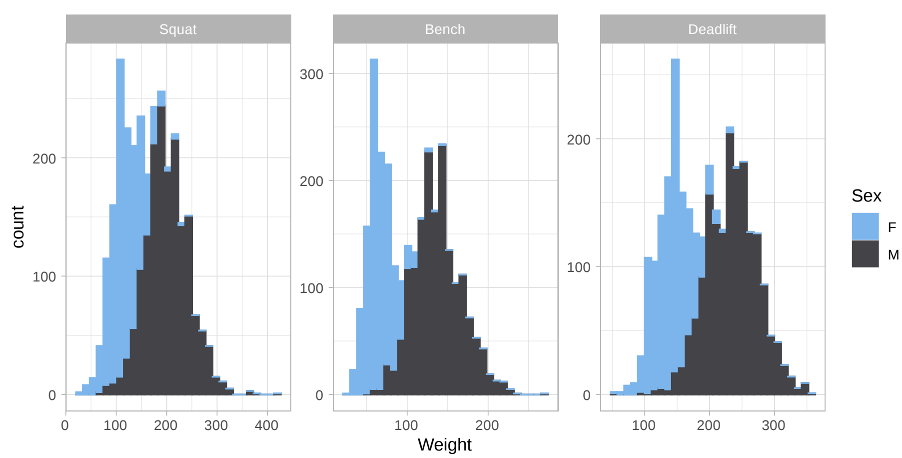


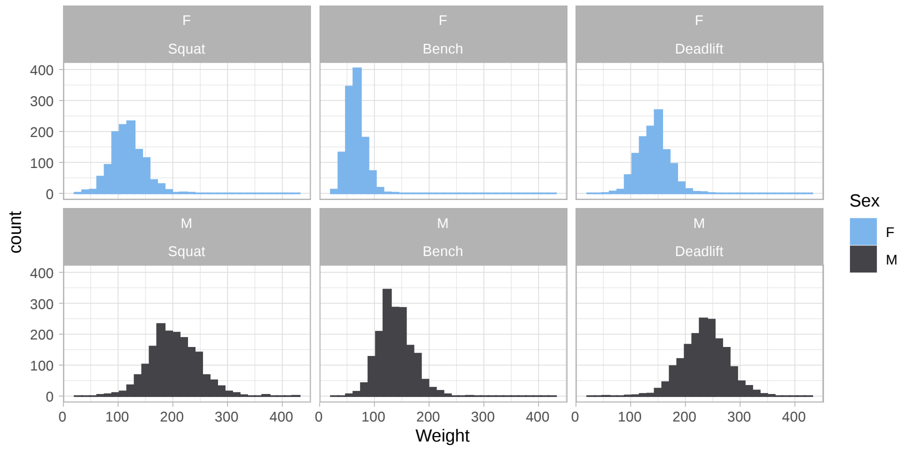

```
pl_trn_tidy = gather(pl_trn, key = "Lift", value = "Weight",
                     Squat, Bench, Deadlift)
```

```
pl_trn_tidy$Lift = factor(pl_trn_tidy$Lift, levels = c("Squat", "Bench", "Deadlift"))
```

- TODO: https://www.tidyverse.org/
- TODO: https://en.wikipedia.org/wiki/Tidy_data
- TODO: http://vita.had.co.nz/papers/tidy-data.pdf





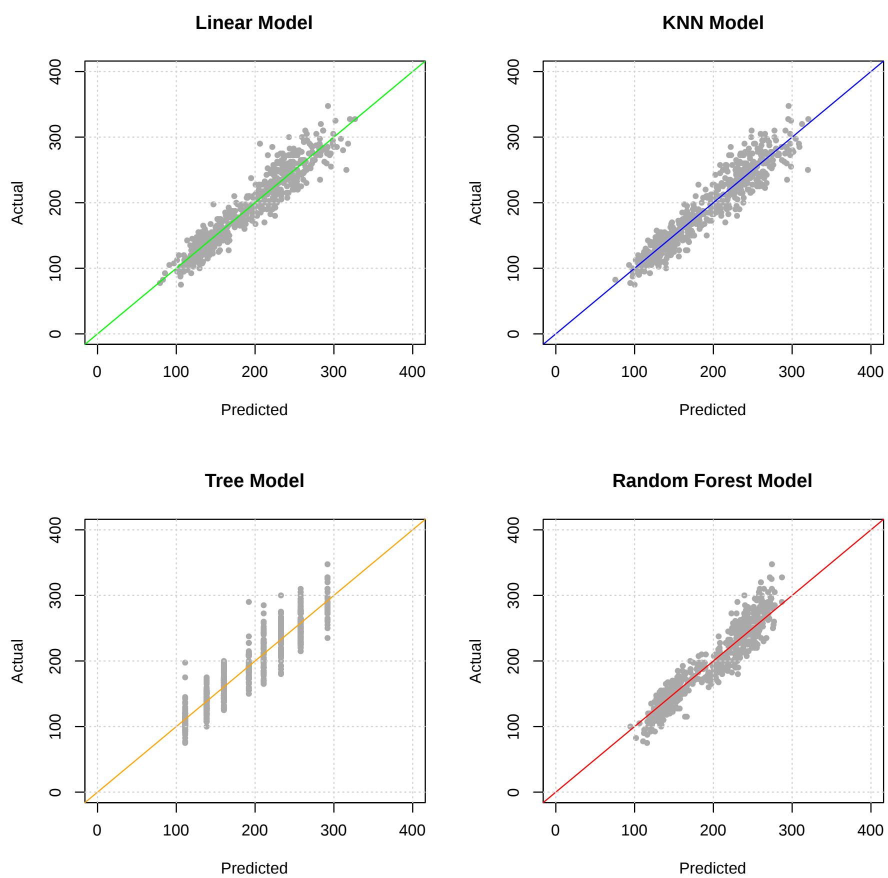


1.1.4 Modeling

```
dl_mod_form = formula(Deadlift ~ Sex + Bodyweight + Age + Squat + Bench)

set.seed(1)
lm_mod  = lm(dl_mod_form, data = pl_est)
knn_mod = caret::knnreg(dl_mod_form, data = pl_est)
rf_mod  = randomForest(dl_mod_form, data = pl_est)
rp_mod = rpart(dl_mod_form, data = pl_est)
```

- TODO: Note: we are not using `Name`. Why? We are not using `Total`. Why?
- TODO: look what happens with `Total`! You'll see it with `lm()`, you'll be optimistic with `randomForest()`.
- TODO: What variables are allowed? (With respect to real world problem.)
- TODO: What variables lead to the best predictions?

1.1.5 Model Evaluation




```
calc_rmse = function(actual, predicted) {
  sqrt(mean( (actual - predicted) ^ 2) )
}
```

```
c(calc_rmse(actual = pl_val$Deadlift, predicted = predict(lm_mod, pl_val)),
  calc_rmse(actual = pl_val$Deadlift, predicted = predict(knn_mod, pl_val)),
  calc_rmse(actual = pl_val$Deadlift, predicted = predict(rp_mod, pl_val)),
  calc_rmse(actual = pl_val$Deadlift, predicted = predict(rf_mod, pl_val)))
```

```
## [1] 18.26654 19.19625 21.68142 19.23643
```

```
reg_preds = map(list(lm_mod, knn_mod, rp_mod, rf_mod), predict, pl_val)
map_dbl(reg_preds, calc_rmse, actual = pl_val$Deadlift)
```

```
## [1] 18.26654 19.19625 21.68142 19.23643
```

- TODO: **Never** supply `data = df` to `predict()`. You have been warned.

```
knitr::include_graphics("img/sim-city.jpg")
```


```
calc_mae = function(actual, predicted) {
  mean(abs(actual - predicted))
}
```

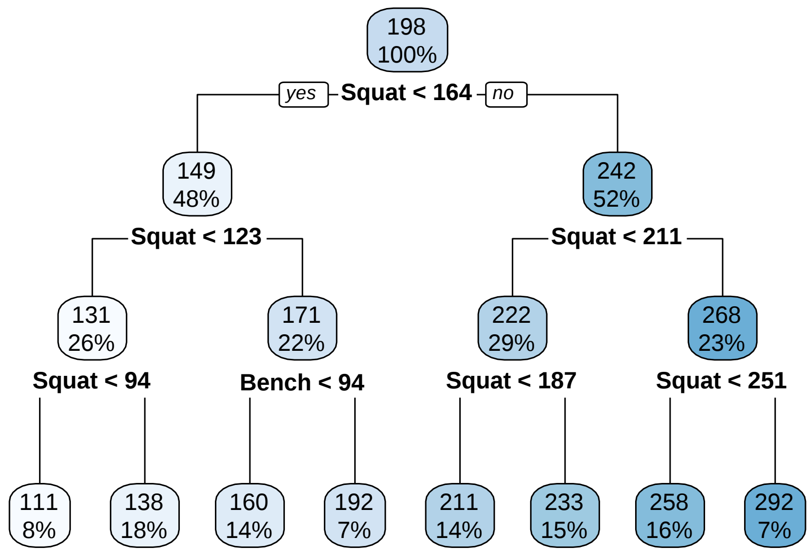
```
map_dbl(reg_preds, calc_mae,  actual = pl_val$Deadlift)
```

```
## [1] 14.38953 14.99748 17.14823 15.28626
```

```
reg_results = tibble(
  Model = c("Linear", "KNN", "Tree", "Forest"),
  RMSE = map_dbl(reg_preds, calc_rmse, actual = pl_val$Deadlift),
  MAE = map_dbl(reg_preds, calc_mae,  actual = pl_val$Deadlift))
```

Model	RMSE	MAE
Linear	18.26654	14.38953
KNN	19.19625	14.99748
Tree	21.68142	17.14823
Forest	19.23643	15.28626

1.1.6 Discussion




```
lm_mod_final = lm(dl_mod_form, data = pl_trn)
```

```
calc_rmse(actual = pl_tst$Deadlift,
          predicted = predict(lm_mod_final, pl_tst))
```

```
## [1] 22.29668
```

- TODO: Is this a good model?
- TODO: Is this model useful?

```
william_biscarri = tibble(
  Name = "William Biscarri",
  Age = 28,
  Sex = "M",
  Bodyweight = 83,
  Squat = 130,
  Bench = 90
)
```

```
predict(lm_mod_final, william_biscarri)
```

```
##       1
## 175.495
```

1.2 Classification: Handwritten Digits

1.2.1 Background

- TODO: https://en.wikipedia.org/wiki/MNIST_database
- TODO: http://yann.lecun.com/exdb/mnist/

1.2.2 Data

- TODO: How is this data pre-processed?
- TODO: https://gist.github.com/daviddalpiaz/ae62ae5ccd0bada4b9acd6dbc9008706
- TODO: https://github.com/itsrainingdata/mnistR
- TODO: https://pjreddie.com/projects/mnist-in-csv/
- TODO: http://varianceexplained.org/r/digit-eda/

```
mnist_trn = read_csv(file = "data/mnist_train_subest.csv")
mnist_tst = read_csv(file = "data/mnist_test.csv")
```

```
mnist_trn_y = as.factor(mnist_trn$X1)
mnist_tst_y = as.factor(mnist_tst$X1)

mnist_trn_x = mnist_trn[, -1]
mnist_tst_x = mnist_tst[, -1]
```

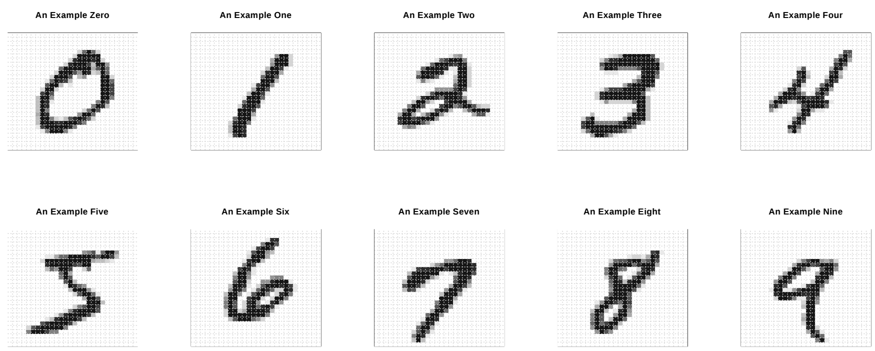
- TODO: If we were going to tune a model, we would need a validation split as well. We're going to be lazy and just fit a single random forest.
- TODO: This is an agreed upon split.

1.2.3 EDA

```
pixel_positions = expand.grid(j = sprintf("%02.0f", 1:28),
                              i = sprintf("%02.0f", 1:28))
pixel_names = paste("pixel", pixel_positions$i, pixel_positions$j, sep = "-")
```

```
colnames(mnist_trn_x) = pixel_names
colnames(mnist_tst_x) = pixel_names
```

```
show_digit = function(arr784, col = gray(12:1 / 12), ...) {
  image(matrix(as.matrix(arr784), nrow = 28)[, 28:1],
        col = col, xaxt = "n", yaxt = "n", ...)
  grid(nx = 28, ny = 28)
}
```



1.2.4 Modeling

```
set.seed(42)
mnist_rf = randomForest(x = mnist_trn_x, y = mnist_trn_y, ntree = 100)
```

1.2.5 Model Evaluation

```
mnist_tst_pred = predict(mnist_rf, mnist_tst_x)
mean(mnist_tst_pred == mnist_tst_y)
```

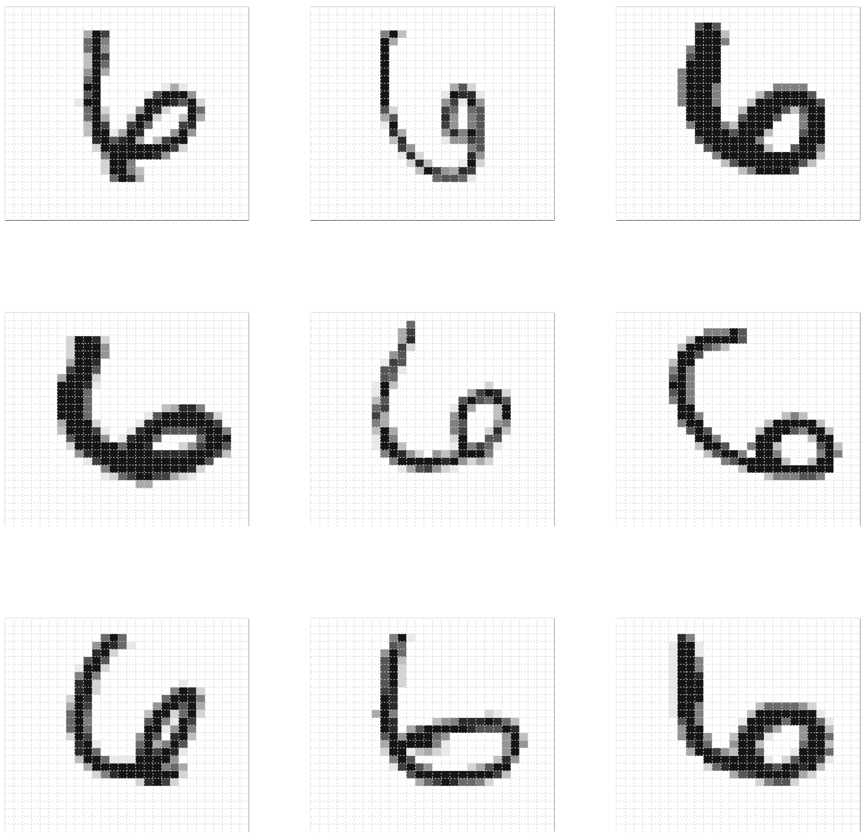
```
## [1] 0.8839
```

```
table(predicted = mnist_tst_pred, actual = mnist_tst_y)
```

```
##          actual
## predicted    0    1    2    3    4    5    6    7    8    9
##         0  959    0   14    6    1   15   22    1   10   10
##         1    0 1112    5    5    1   16    5    9    5    6
##         2    1    2  928   31    3    5   19   24   17    8
##         3    0    2   11  820    1   24    0    1   13   13
##         4    4    0   13    1  839   21   39   11   18   40
##         5    3    1    1   88    3  720   18    1   25    9
##         6    7    2   15    3   25   15  848    0   18    2
##         7    2    1   29   24    1   14    2  928   15   30
##         8    4   14   13   22    5   19    5    4  797    3
##         9    0    1    3   10  103   43    0   49   56  888
```

1.2.6 Discussion

```
par(mfrow = c(3, 3))
plot_mistake(actual = 6, predicted = 4)
```



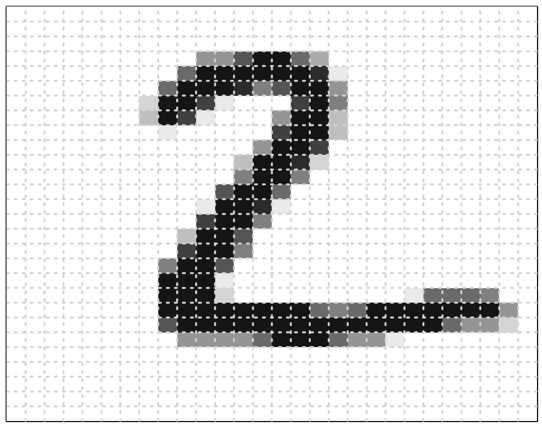
```
mnist_obs_to_check = 2
predict(mnist_rf, mnist_tst_x[mnist_obs_to_check, ], type = "prob")[1, ]
```

```
##    0    1    2    3    4    5    6    7    8    9
## 0.09 0.03 0.25 0.14 0.02 0.14 0.25 0.01 0.05 0.02
```

```
mnist_tst_y[mnist_obs_to_check]
```

```
## [1] 2
## Levels: 0 1 2 3 4 5 6 7 8 9
```

```
show_digit(mnist_tst_x[mnist_obs_to_check, ])
```



1.3 Clustering: NBA Players

1.3.1 Background

- https://www.youtube.com/watch?v=cuLprHh_BRg
- https://www.youtube.com/watch?v=1FBwSO_1Mb8
- https://www.basketball-reference.com/leagues/NBA_2019.html

1.3.2 Data

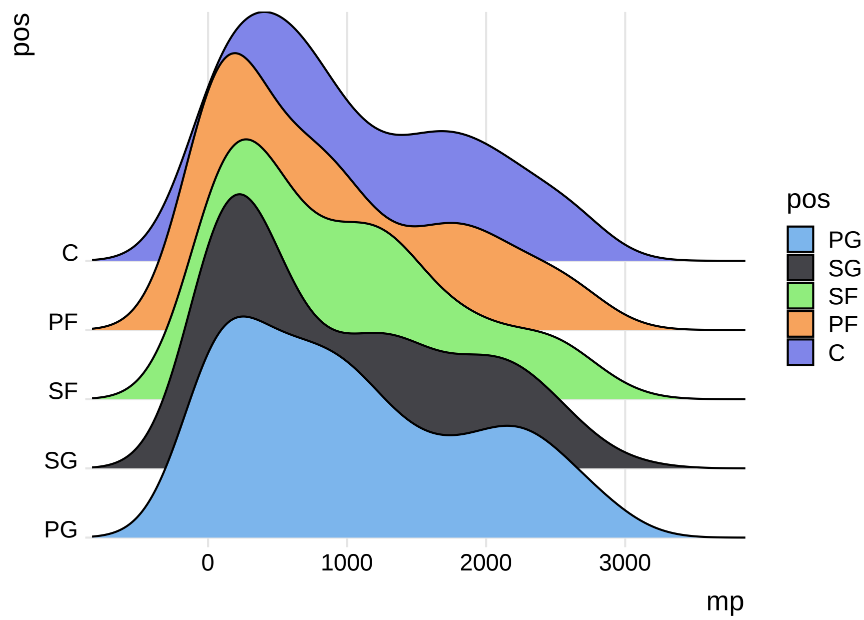
- https://www.basketball-reference.com/leagues/NBA_2019_totals.html
- https://www.basketball-reference.com/leagues/NBA_2019_per_minute.html

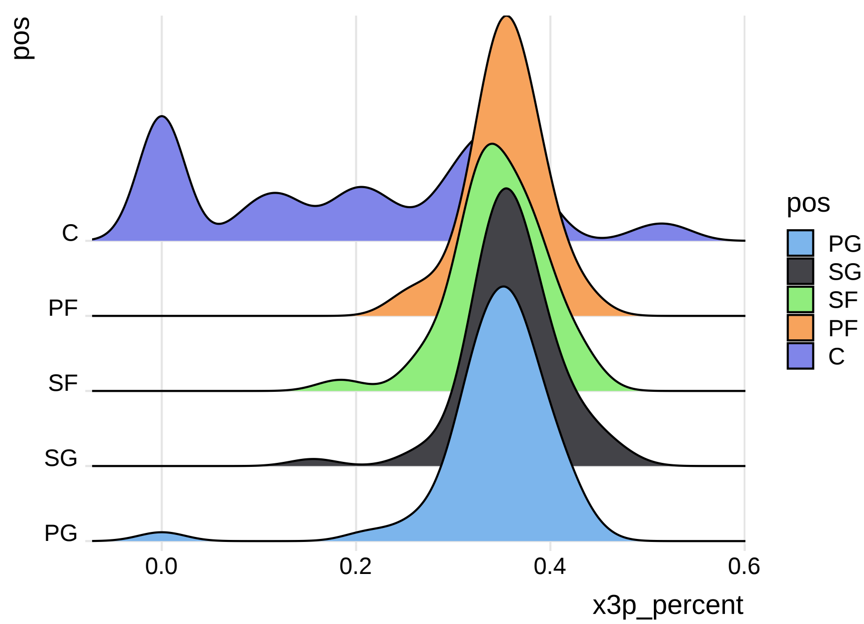
- https://www.basketball-reference.com/leagues/NBA_2019_per_poss.html
- https://www.basketball-reference.com/leagues/NBA_2019_advanced.html

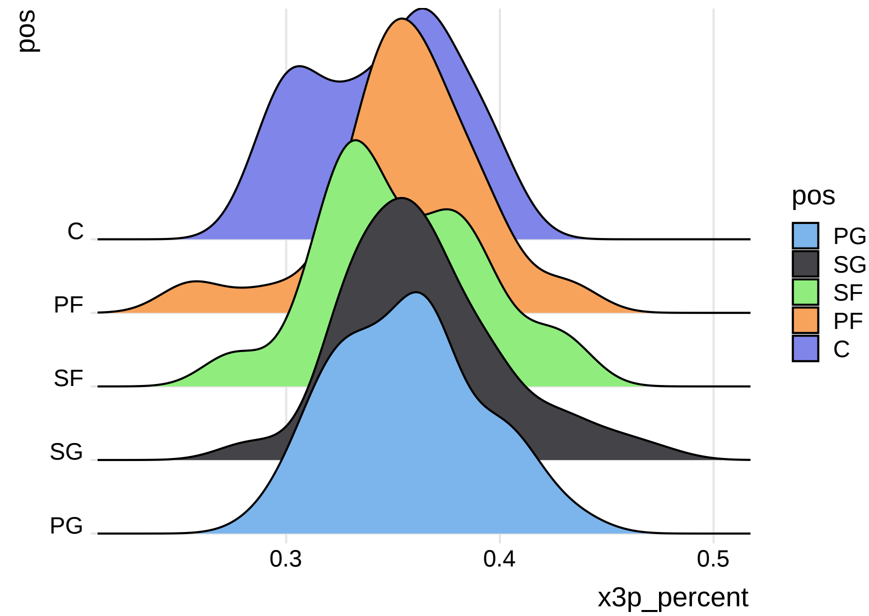
```
nba = scrape_nba_season_player_stats()
nba$pos = factor(nba$pos, levels = c("PG", "SG", "SF", "PF", "C"))
```

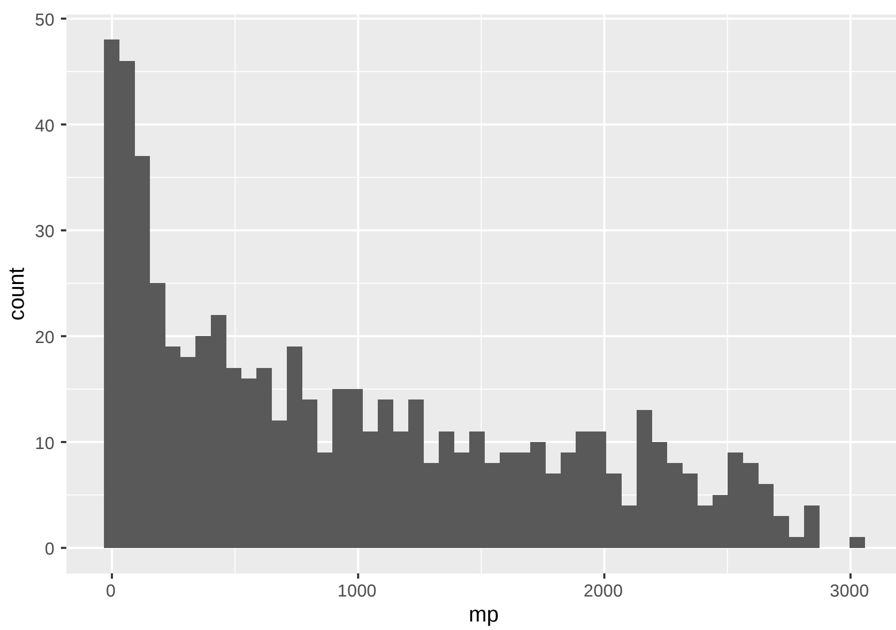
```
## # A tibble: 100 x 93
##    player_team pos     age tm        g    gs    mp    fg   fga fg_percent
##    <chr>       <fct> <dbl> <chr> <dbl> <dbl> <dbl> <dbl> <dbl>      <dbl>
##  1 Álex Abrin~ SG       25 OKC      31     2   588    56   157      0.357
##  2 Quincy Acy~ PF       28 PHO      10     0   123     4    18      0.222
##  3 Jaylen Ada~ PG       22 ATL      34     1   428    38   110      0.345
##  4 Steven Ada~ C        25 OKC      80    80  2669   481   809      0.595
##  5 Bam Adebay~ C        21 MIA      82    28  1913   280   486      0.576
##  6 Deng Adel ~ SF       21 CLE      19     3   194    11    36      0.306
##  7 DeVaughn A~ SG       25 DEN       7     0    22     3    10      0.3
##  8 LaMarcus A~ C        33 SAS      81    81  2687   684  1319      0.519
##  9 Rawle Alki~ SG       21 CHI      10     1   120    13    39      0.333
## 10 Grayson Al~ SG       23 UTA      38     2   416    67   178      0.376
## # ... with 90 more rows, and 83 more variables: x3p <dbl>, x3pa <dbl>,
## #   x3p_percent <dbl>, x2p <dbl>, x2pa <dbl>, x2p_percent <dbl>,
## #   e_fg_percent <dbl>, ft <dbl>, fta <dbl>, ft_percent <dbl>, orb <dbl>,
## #   drb <dbl>, trb <dbl>, ast <dbl>, stl <dbl>, blk <dbl>, tov <dbl>,
## #   pf <dbl>, pts <dbl>, fg_pm <dbl>, fga_pm <dbl>, fg_percent_pm <dbl>,
## #   x3p_pm <dbl>, x3pa_pm <dbl>, x3p_percent_pm <dbl>, x2p_pm <dbl>,
## #   x2pa_pm <dbl>, x2p_percent_pm <dbl>, ft_pm <dbl>, fta_pm <dbl>,
## #   ft_percent_pm <dbl>, orb_pm <dbl>, drb_pm <dbl>, trb_pm <dbl>,
## #   ast_pm <dbl>, stl_pm <dbl>, blk_pm <dbl>, tov_pm <dbl>, pf_pm <dbl>,
## #   pts_pm <dbl>, fg_pp <dbl>, fga_pp <dbl>, fg_percent_pp <dbl>,
## #   x3p_pp <dbl>, x3pa_pp <dbl>, x3p_percent_pp <dbl>, x2p_pp <dbl>,
## #   x2pa_pp <dbl>, x2p_percent_pp <dbl>, ft_pp <dbl>, fta_pp <dbl>,
## #   ft_percent_pp <dbl>, orb_pp <dbl>, drb_pp <dbl>, trb_pp <dbl>,
## #   ast_pp <dbl>, stl_pp <dbl>, blk_pp <dbl>, tov_pp <dbl>, pf_pp <dbl>,
## #   pts_pp <dbl>, o_rtg_pp <dbl>, d_rtg_pp <dbl>, per <dbl>,
## #   ts_percent <dbl>, x3p_ar <dbl>, f_tr <dbl>, orb_percent <dbl>,
## #   drb_percent <dbl>, trb_percent <dbl>, ast_percent <dbl>,
## #   stl_percent <dbl>, blk_percent <dbl>, tov_percent <dbl>,
## #   usg_percent <dbl>, ows <dbl>, dws <dbl>, ws <dbl>, ws_48 <dbl>,
## #   obpm <dbl>, dbpm <dbl>, bpm <dbl>, vorp <dbl>
```

1.3.3 EDA


pos
C
PF
SF
SG
PG
0
1000
2000
3000
mp
pos
PG
SG
SF
PF
C


pos
C
PF
SF
SG
PG
0.0
0.2
0.4
0.6
x3p_percent
pos
PG
SG
SF
PF
C

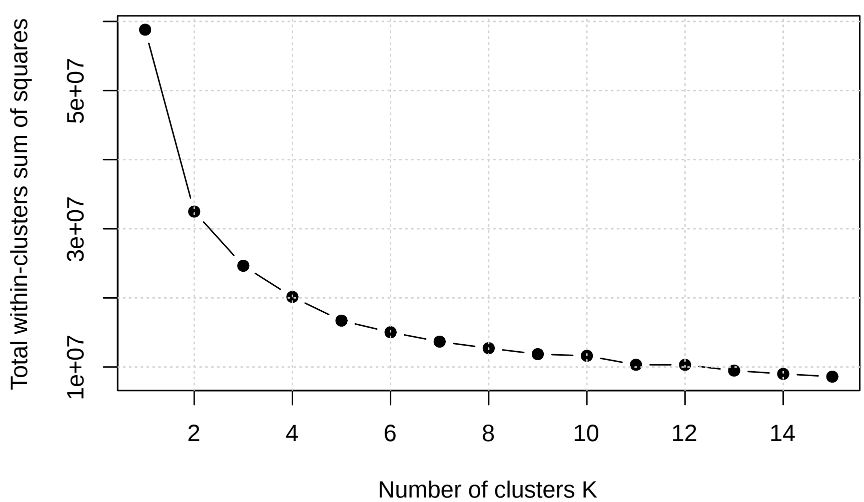





```
nba_for_clustering = nba %>%
  filter(mp > 2000) %>%
  column_to_rownames("player_team") %>%
  select(-pos, -tm)
```

1.3.4 Modeling

```
set.seed(42)

# function to compute total within-cluster sum of square
wss = function(k, data) {
  kmeans(x = data, centers = k, nstart = 10)$tot.withinss
}

# Compute and plot wss for k = 1 to k = 15
k_values = 1:15

# extract wss for 2-15 clusters
wss_values = map_dbl(k_values, wss, data = nba_for_clustering)

plot(k_values, wss_values,
       type = "b", pch = 19, frame = TRUE,
       xlab = "Number of clusters K",
       ylab = "Total within-clusters sum of squares")
grid()
```

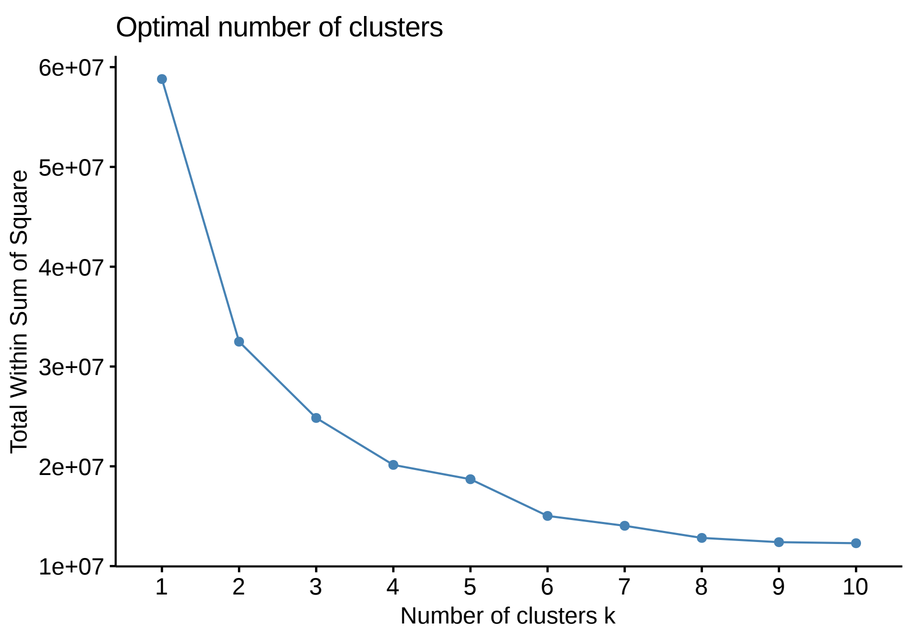


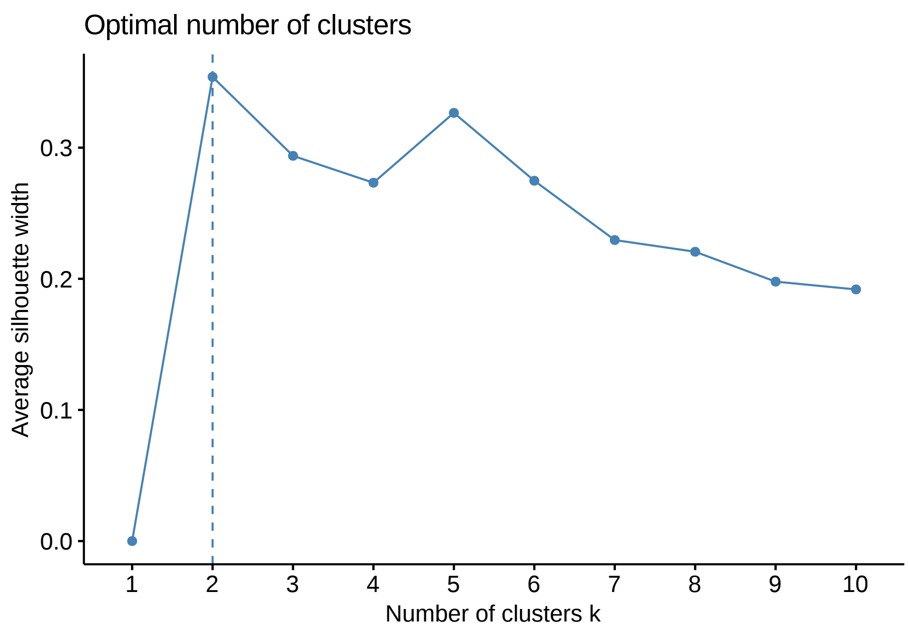
- TODO: K-Means likes clusters of roughly equal size.
- TODO: http://varianceexplained.org/r/kmeans-free-lunch/

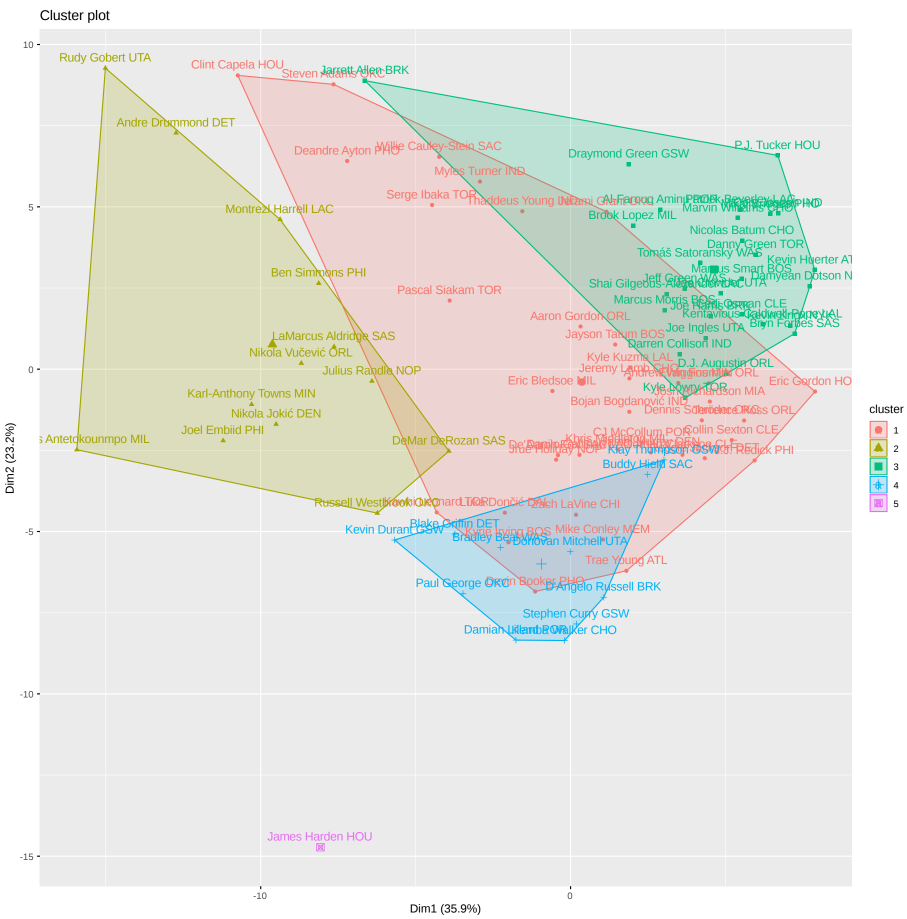
```r
nba_hc = hclust(dist(nba_for_clustering))
nba_hc_clust = cutree(nba_hc, k = 5)
table(nba_hc_clust)
```

```
## nba_hc_clust
##  1  2  3  4  5
## 38 13 28 11  1
```

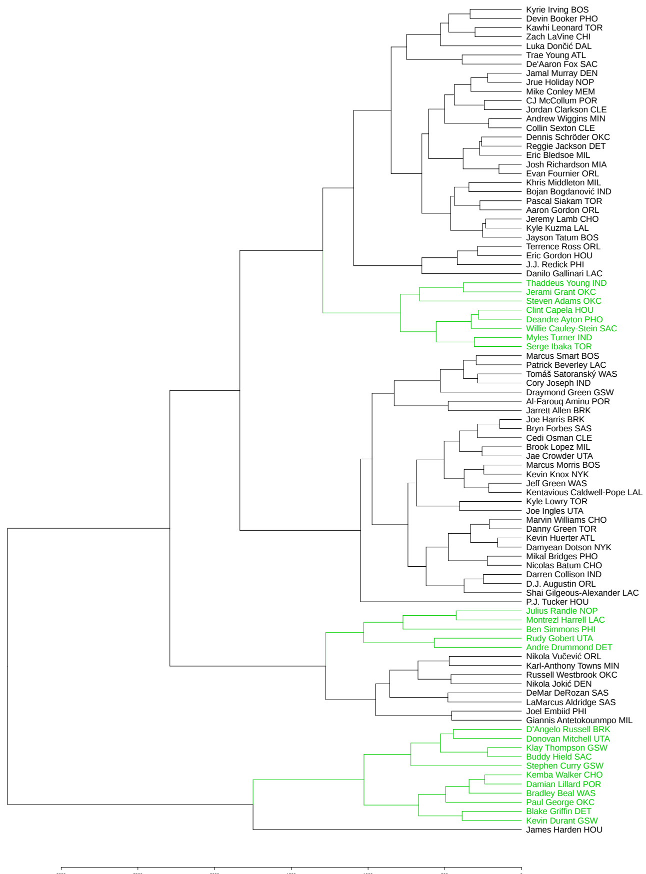
1.3.5 Model Evaluation




Optimal number of clusters
Average silhouette width
0.0
0.1
0.2
0.3
1
2
3
4
5
6
7
8
9
10
Number of clusters k


Cluster plot
Dim2 (23.2%)
Dim1 (35.9%)
10
5
0
-5
-10
-15
-10
0
cluster
1
2
3
4
5
Rudy Gobert UTA
Clint Capela HOU
Jarrett Allen BRK
Andre Drummond DET
Deandre Ayton PHO
Draymond Green GSW
P.J. Tucker HOU
Mylés Turner IND
Serge Ibaka TOR
Montrezl Harrell LAC
Brook Lopez MIL
Nicolas Batum CHO
Ben Simmons PHI
Pascal Siakam TOR
Aaron Gordon ORL
Jayson Tatum BOS
Joe Ingles UTA
Darren Collison IND
D.J. Augustin ORL
Julius Randle NOP
Karl-Anthony Towns MIN
Nikola Jokić DEN
Joel Embiid PHI
DeMar DeRozan SAS
Eric Bledsoe MIL
Bojan Bogdanović IND
Eric Gordon HOU
Collin Sexton CLE
Buddy Hield SAC
Zach LaVine CHI
Kevin Durant GSW
Blake Griffin DET
Mike Conley MEM
Trae Young ATL
Paul George OKC
D'Angelo Russell BRK
Stephen Curry GSW
James Harden HOU

1.3.6 Discussion



Chapter 2

Computing

This is not a book about R. It is however, a book that uses R. Because of this, you will need to be familiar with R. The text will point out some thing about R along the way, but some previous study of R is necessary.

The following (freely availible) readings are highly recommended:

- Hands-On Programming with R - *Garrett Grolemund*
  - If you have never used R or RStudio before, Part 1, Chapters 1 - 3, will be useful.
- R for Data Science - *Garrett Grolemund, Hadley Wickham*
  - This book helps getting you up to speed working with data in R. While it is a lot of reading, Chapters 1 - 21 are highly recommended.
- Advanced R - *Hadley Wickham*
  - Part I, Chapters 1 - 8, of this book will help create a mental model for working with R. These chapters are not an easy read, so they should be returned to often. (Chapter 2 could be safely skipped for our purposes, but is important if you will use R in the long term.)

If you are a UIUC student who took the course STAT 420, the first six chapters of that book could serve as a nice refresher.

- Applied Statistics with R - *David Dalpiaz*

2.1 Resources

The following resources are more specific or more advanced, but could still prove to be useful.

2.1.1 R

- Efficient R programming
- R Programming for Data Science
- R Graphics Cookbook
- Modern Dive
- The `tidyverse` Website
  - `dplyr` Website
  - `readr` Website
  - `tibble` Website
  - `forcats` Website

2.1.2 RStudio

- RStudio IDE Cheatsheet
- RStudio Resources

2.1.3 R Markdown

- R Markdown Cheatsheet
- R Markdown: The Definitive Guide - *Yihui Xie, J. J. Allaire, Garrett Grolemund*
- R4DS R Markdown Chapter

2.1.3.1 Markdown

- Daring Fireball - Markdown: Basics
- GitHub - Mastering Markdown
- CommonMark

2.2 BSL Idioms

Things here supercede everythign above.

2.2.1 Reference Style

- `tidyverse` Style Guide

2.2.2 BSL Style Overrides

- TODO: `=` instead of `<-`
  - http://thecoatlessprofessor.com/programming/an-opinionated-tale-of-why-you-should-replace---with-/
- TODO: never use `T` or `F`, only `TRUE` or `FALSE`

```
FALSE == TRUE
```

```
## [1] FALSE
```

```
F     == TRUE
```

```
## [1] FALSE
```

```
F     =  TRUE
F     == TRUE
```

```
## [1] TRUE
```

- TODO: never ever ever use `attach()`
- TODO: never ever ever use `<<-`
- TODO: never ever ever use `setwd()` or set a working directory some other way
- TODO: a newline before and after any chunk
- TODO: use headers appropriately! (short names, good structure)
- TODO: never ever ever put spaces in filenames. use `-`. (others will use `_`.)
- TODO: load all needed packages at the beginning of an analysis in a single chunk (TODO: pros and cons of this approach)
- TODO: one plot per chunk! no other printed output

Be consistent…

- with yourself!
- with your group!
- with your organization!

```
set.seed(1337);mu=10;sample_size=50;samples=100000;
x_bars=rep(0, samples)
for(i in 1:samples)
{
x_bars[i]=mean(rpois(sample_size,lambda = mu))}
x_bar_hist=hist(x_bars,breaks=50,main="Histogram of Sample Means",xlab="Sample Means",col="darkor
mean(x_bars>mu-2*sqrt(mu)/sqrt(sample_size)&x_bars<mu+2*sqrt(mu)/sqrt(sample_size))
```

2.2.3 Objects and Functions

To understand computations in R, two slogans are helpful:

- Everything that exists is an object.
- Everything tha thappens is a function call.

— John Chambers

2.2.4 Print versus Return

```
cars_mod = lm(dist ~ speed, data = cars)
```

```
summary(cars_mod)
```

```
##
## Call:
## lm(formula = dist ~ speed, data = cars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -29.069  -9.525  -2.272   9.215  43.201
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -17.5791     6.7584  -2.601   0.0123 *
## speed         3.9324     0.4155   9.464 1.49e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.38 on 48 degrees of freedom
## Multiple R-squared:  0.6511, Adjusted R-squared:  0.6438
## F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

```
is.list(summary(cars_mod))
```

```
## [1] TRUE
```

```
names(summary(cars_mod))
```

```
##  [1] "call"          "terms"         "residuals"     "coefficients"
##  [5] "aliased"       "sigma"         "df"            "r.squared"
##  [9] "adj.r.squared" "fstatistic"    "cov.unscaled"
```

```
str(summary(cars_mod))
```

```
## List of 11
##  $ call         : language lm(formula = dist ~ speed, data = cars)
##  $ terms        :Classes 'terms', 'formula'  language dist ~ speed
##   .. ..- attr(*, "variables")= language list(dist, speed)
##   .. ..- attr(*, "factors")= int [1:2, 1] 0 1
##   .. .. ..- attr(*, "dimnames")=List of 2
##   .. .. .. ..$ : chr [1:2] "dist" "speed"
##   .. .. .. ..$ : chr "speed"
```

```
##   .. ..- attr(*, "term.labels")= chr "speed"
##   .. ..- attr(*, "order")= int 1
##   .. ..- attr(*, "intercept")= int 1
##   .. ..- attr(*, "response")= int 1
##   .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
##   .. ..- attr(*, "predvars")= language list(dist, speed)
##   .. ..- attr(*, "dataClasses")= Named chr [1:2] "numeric" "numeric"
##   .. .. ..- attr(*, "names")= chr [1:2] "dist" "speed"
##  $ residuals    : Named num [1:50] 3.85 11.85 -5.95 12.05 2.12 ...
##   ..- attr(*, "names")= chr [1:50] "1" "2" "3" "4" ...
##  $ coefficients : num [1:2, 1:4] -17.579 3.932 6.758 0.416 -2.601 ...
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : chr [1:2] "(Intercept)" "speed"
##   .. ..$ : chr [1:4] "Estimate" "Std. Error" "t value" "Pr(>|t|)"
##  $ aliased      : Named logi [1:2] FALSE FALSE
##   ..- attr(*, "names")= chr [1:2] "(Intercept)" "speed"
##  $ sigma        : num 15.4
##  $ df           : int [1:3] 2 48 2
##  $ r.squared    : num 0.651
##  $ adj.r.squared: num 0.644
##  $ fstatistic   : Named num [1:3] 89.6 1 48
##   ..- attr(*, "names")= chr [1:3] "value" "numdf" "dendf"
##  $ cov.unscaled : num [1:2, 1:2] 0.19311 -0.01124 -0.01124 0.00073
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : chr [1:2] "(Intercept)" "speed"
##   .. ..$ : chr [1:2] "(Intercept)" "speed"
##  - attr(*, "class")= chr "summary.lm"
```

```
# RStudio only
View(summary(cars_mod))
```

2.2.5 Help

- TODO: `?`, google, stack overflow, (office hours, course forums)

2.2.6 Keyboard Shortcuts

- TODO: copy-paste, switch program, switch tab, etc…
- TODO: TAB!!!
- TODO: new chunk!
- TODO: style!
- TODO: keyboard shortcut for keyboard shortcut

2.3 Common Issues

- TODO: cannot find function called ""

Chapter 3

Estimation

- TODO: Where we are going, estimating conditional means and distributions.
- TODO: estimation = learning. "learning from data." what are we learning about? often parameters.
- TODO: http://stat400.org
- TODO: http://stat420.org

3.1 Probability

- TODO: See Appendix A
- TODO: In R, `d*()`, `p*()`, `q*()`, `r*()`

3.2 Statistics

- TODO: parameters are a function of the population distribution
- TODO: statistics are a function of data.
- TODO: parameters:population::statistics::data
- TODO: statistic vs value of a statistic

3.3 Estimators

- TODO: estimator vs estimate
- TODO: Why such a foucs on the mean, E[X]? Because E[(X - a)^2] is minimized by E[X]
  - https://www.benkuhn.net/squared

- https://news.ycombinator.com/item?id=9556459

3.3.1 Properties

3.3.1.1 Bias

$$\text{bias}\left[\hat{\theta}\right] \triangleq \mathbb{E}\left[\hat{\theta}\right] - \theta$$

3.3.1.2 Variance

$$\text{var}\left[\hat{\theta}\right] \triangleq \mathbb{E}\left[\left(\hat{\theta} - \mathbb{E}\left[\hat{\theta}\right]\right)^2\right]$$

3.3.1.3 Mean Squared Error

$$\text{MSE}\left[\hat{\theta}\right] \triangleq \mathbb{E}\left[\left(\hat{\theta} - \theta\right)^2\right] = \text{var}\left[\hat{\theta}\right] + \left(\text{Bias}\left[\hat{\theta}\right]\right)^2$$

3.3.1.4 Consistency

An estimator $\hat{\theta}_n$ is said to be a **consistent estimator** of θ if, for any positive ϵ,

$$\lim_{n\to\infty} P\left(\left|\hat{\theta}_n - \theta\right| \leq \epsilon\right) = 1$$

or, equivalently,

$$\lim_{n\to\infty} P\left(\left|\hat{\theta}_n - \theta\right| > \epsilon\right) = 0$$

We say that $\hat{\theta}_n$ **converges in probability** to θ and we write $\hat{\theta}_n \xrightarrow{P} \theta$.

3.3.2 Methods

- TODO: MLE

Given a random sample $X_1, X_2, \ldots, X_n$ from a population with parameter θ and density or mass $f(x \mid \theta)$, we have:

The Likelihood, $L(\theta)$,

$$L(\theta) = f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(x_i \mid \theta)$$

The **Maximum Likelihood Estimator**, $\hat{\theta}$

$$\hat{\theta} = \underset{\theta}{\text{argmax}} \ L(\theta) = \underset{\theta}{\text{argmax}} \ \log L(\theta)$$

- TODO: Invariance Principle

If $\hat{\theta}$ is the MLE of θ and the function $h(\theta)$ is continuous, then $h(\hat{\theta})$ is the MLE of $h(\theta)$.

- TODO: MOM
- TODO: https://daviddalpiaz.github.io/stat3202-sp19/notes/fitting.html
- TODO: ECDF: https://en.wikipedia.org/wiki/Empirical_distribution_function

Appendix A

Probability

- TODO: Note! This is copy-pasted from R4SL.

We give a very brief review of some necessary probability concepts. As the treatment is less than complete, a list of references is given at the end of the chapter. For example, we ignore the usual recap of basic set theory and omit proofs and examples.

A.1 Probability Models

When discussing probability models, we speak of random **experiments** that produce one of a number of possible **outcomes**.

A **probability model** that describes the uncertainty of an experiment consists of two elements:

- The **sample space**, often denoted as Ω, which is a set that contains all possible outcomes.
- A **probability function** that assigns to an event A a nonnegative number, $P[A]$, that represents how likely it is that event A occurs as a result of the experiment.

We call $P[A]$ the **probability** of event A. An **event** A could be any subset of the sample space, not necessarily a single possible outcome. The probability law must follow a number of rules, which are the result of a set of axioms that we introduce now.

A.2 Probability Axioms

Given a sample space Ω for a particular experiment, the **probability function** associated with the experiment must satisfy the following axioms.

1. *Nonnegativity*: $P[A] \geq 0$ for any event $A \subset \Omega$.
2. *Normalization*: $P[\Omega] = 1$. That is, the probability of the entire space is 1.
3. *Additivity*: For mutually exclusive events $E_1, E_2, \ldots$

$$P\left[\bigcup_{i=1}^{\infty} E_i\right] = \sum_{i=1}^{\infty} P[E_i]$$

Using these axioms, many additional probability rules can easily be derived.

A.3 Probability Rules

Given an event A, and its complement, A^c, that is, the outcomes in Ω which are not in A, we have the **complement rule**:

$$P[A^c] = 1 - P[A]$$

In general, for two events A and B, we have the **addition rule**:

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

If A and B are also *disjoint*, then we have:

$$P[A \cup B] = P[A] + P[B]$$

If we have n mutually exclusive events, $E_1, E_2, \ldots E_n$, then we have:

$$P\left[\bigcup_{i=1}^{n} E_i\right] = \sum_{i=1}^{n} P[E_i]$$

Often, we would like to understand the probability of an event A, given some information about the outcome of event B. In that case, we have the **conditional probability rule** provided $P[B] > 0$.

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]}$$

Rearranging the conditional probability rule, we obtain the **multiplication rule**:

$$P[A \cap B] = P[B] \cdot P[A \mid B] \cdot$$

For a number of events $E_1, E_2, \dots E_n$, the multiplication rule can be expanded into the **chain rule**:

$$P\left[\bigcap_{i=1}^{n} E_i\right] = P[E_1] \cdot P[E_2 \mid E_1] \cdot P[E_3 \mid E_1 \cap E_2] \cdots P\left[E_n \mid \bigcap_{i=1}^{n-1} E_i\right]$$

Define a **partition** of a sample space Ω to be a set of disjoint events $A_1, A_2, \dots, A_n$ whose union is the sample space Ω. That is

$$A_i \cap A_j = \emptyset$$

for all $i \neq j$, and

$$\bigcup_{i=1}^{n} A_i = \Omega.$$

Now, let $A_1, A_2, \dots, A_n$ form a partition of the sample space where $P[A_i] > 0$ for all i. Then for any event B with $P[B] > 0$ we have **Bayes' Rule**:

$$P[A_i|B] = \frac{P[A_i]P[B|A_i]}{P[B]} = \frac{P[A_i]P[B|A_i]}{\sum_{i=1}^{n} P[A_i]P[B|A_i]}$$

The denominator of the latter equality is often called the **law of total probability**:

$$P[B] = \sum_{i=1}^{n} P[A_i]P[B|A_i]$$

Two events A and B are said to be **independent** if they satisfy

$$P[A \cap B] = P[A] \cdot P[B]$$

This becomes the new multiplication rule for independent events.

A collection of events $E_1, E_2, \dots E_n$ is said to be independent if

$$P\left[\bigcap_{i \in S} E_i\right] = \prod_{i \in S} P[E_i]$$

for every subset S of $\{1, 2, \ldots n\}$.

If this is the case, then the chain rule is greatly simplified to:

$$P\left[\bigcap_{i=1}^{n} E_i\right] = \prod_{i=1}^{n} P[E_i]$$

A.4 Random Variables

A **random variable** is simply a *function* which maps outcomes in the sample space to real numbers.

A.4.1 Distributions

We often talk about the **distribution** of a random variable, which can be thought of as:

$$\text{distribution} = \text{list of possible } \textbf{values} + \text{associated } \textbf{probabilities}$$

This is not a strict mathematical definition, but is useful for conveying the idea.

If the possible values of a random variables are *discrete*, it is called a *discrete random variable*. If the possible values of a random variables are *continuous*, it is called a *continuous random variable*.

A.4.2 Discrete Random Variables

The distribution of a discrete random variable X is most often specified by a list of possible values and a probability **mass** function, $p(x)$. The mass function directly gives probabilities, that is,

$$p(x) = p_X(x) = P[X = x].$$

Note we almost always drop the subscript from the more correct $p_X(x)$ and simply refer to $p(x)$. The relevant random variable is discerned from context

The most common example of a discrete random variable is a **binomial** random variable. The mass function of a binomial random variable X, is given by

$$p(x|n,p) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n, \ n \in \mathbb{N}, \ 0 < p < 1.$$

This line conveys a large amount of information.

- The function $p(x|n,p)$ is the mass function. It is a function of x, the possible values of the random variable X. It is conditional on the **parameters** n and p. Different values of these parameters specify different binomial distributions.
- $x = 0, 1, \ldots, n$ indicates the **sample space**, that is, the possible values of the random variable.
- $n \in \mathbb{N}$ and $0 < p < 1$ specify the **parameter spaces**. These are the possible values of the parameters that give a valid binomial distribution.

Often all of this information is simply encoded by writing

$$X \sim \text{bin}(n,p).$$

A.4.3 Continuous Random Variables

The distribution of a continuous random variable X is most often specified by a set of possible values and a probability **density** function, $f(x)$. (A cumulative density or moment generating function would also suffice.)

The probability of the event $a < X < b$ is calculated as

$$P[a < X < b] = \int_a^b f(x)dx.$$

Note that densities are **not** probabilities.

The most common example of a continuous random variable is a **normal** random variable. The density of a normal random variable X, is given by

$$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad -\infty < x < \infty, \ -\infty < \mu < \infty, \ \sigma > 0.$$

- The function $f(x|\mu,\sigma^2)$ is the density function. It is a function of x, the possible values of the random variable X. It is conditional on the **paramters** μ and σ^2. Different values of these parameters specify different normal distributions.
- $-\infty < x < \infty$ indicates the sample space. In this case, the random variable may take any value on the real line.

- $-\infty < \mu < \infty$ and $\sigma > 0$ specify the parameter space. These are the possible values of the parameters that give a valid normal distribution.

Often all of this information is simply encoded by writing

$$X \sim N(\mu, \sigma^2)$$

A.4.4 Several Random Variables

Consider two random variables X and Y. We say they are independent if

$$f(x, y) = f(x) \cdot f(y)$$

for all x and y. Here $f(x, y)$ is the **joint** density (mass) function of X and Y. We call $f(x)$ the **marginal** density (mass) function of X. Then $f(y)$ the marginal density (mass) function of Y. The joint density (mass) function $f(x, y)$ together with the possible (x, y) values specify the joint distribution of X and Y.

Similar notions exist for more than two variables.

A.5 Expectations

For discrete random variables, we define the **expectation** of the function of a random variable X as follows.

$$\mathbb{E}[g(X)] \triangleq \sum_x g(x)p(x)$$

For continuous random variables we have a similar definition.

$$\mathbb{E}[g(X)] \triangleq \int g(x)f(x)dx$$

For specific functions g, expectations are given names.

The **mean** of a random variable X is given by

$$\mu_X = \text{mean}[X] \triangleq \mathbb{E}[X].$$

So for a discrete random variable, we would have

$$\text{mean}[X] = \sum_x x \cdot p(x)$$

For a continuous random variable we would simply replace the sum by an integral.

The **variance** of a random variable X is given by

$$\sigma_X^2 = \text{var}[X] \triangleq \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2.$$

The **standard deviation** of a random variable X is given by

$$\sigma_X = \text{sd}[X] \triangleq \sqrt{\sigma_X^2} = \sqrt{\text{var}[X]}.$$

The **covariance** of random variables X and Y is given by

$$\text{cov}[X, Y] \triangleq \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X] \cdot \mathbb{E}[Y].$$

A.6 Likelihood

Consider n iid random variables $X_1, X_2, \ldots X_n$. We can then write their **likelihood** as

$$\mathcal{L}(\theta \mid x_1, x_2, \ldots x_n) = \prod_{i=i}^{n} f(x_i; \theta)$$

where $f(x_i; \theta)$ is the density (or mass) function of random variable X_i evaluated at x_i with parameter θ.

Whereas a probability is a function of a possible observed value given a particular parameter value, a likelihood is the opposite. It is a function of a possible parameter value given observed data.

Maximumizing likelihood is a common techinque for fitting a model to data.

A.7 Videos

The YouTube channel mathematicalmonk has a great Probability Primer playlist containing lectures on many fundamental probability concepts. Some of the more important concepts are covered in the following videos:

- Conditional Probability
- Independence
- More Independence
- Bayes Rule

A.8 References

Any of the following are either dedicated to, or contain a good coverage of the details of the topics above.

- Probability Texts
  - Introduction to Probability by Dimitri P. Bertsekas and John N. Tsitsiklis
  - A First Course in Probability by Sheldon Ross
- Machine Learning Texts with Probability Focus
  - Probability for Statistics and Machine Learning by Anirban DasGupta
  - Machine Learning: A Probabilistic Perspective by Kevin P. Murphy
- Statistics Texts with Introduction to Probability
  - Probability and Statistical Inference by Robert V. Hogg, Elliot Tanis, and Dale Zimmerman
  - Introduction to Mathematical Statistics by Robert V. Hogg, Joseph McKean, and Allen T. Craig