Pairs Trading: Optimizing via Mixed Copula versus Distance Method for S&P 500 Assets

Fernando A. B. Sabino da Silva[1], Flavio A. Ziegelmann[1,2], Joao F. Caldeira[2]

[1]Department of Statistics/UFRGS, [2]PPGE/UFRGS

May 18, 2018

- Gerry Bamberger and Nunzio Tartaglia's quantitative group at Morgan Stanley in the early 1980s.
  - David Shaw founded one of the most successfull statistical arbitrage hedge funds to this day (D. E. Shaw & Co).
  - Foundation of statistical arbitrage and consequently, algorithmic trading.
- Pairs Trading is a contrarian strategy designed to generate abnormal profits from the mean-reverting behavior between a pair of stocks.
  - It is well-planned assault on the Efficient Market Hypothesis.

History

- Gerry Bamberger and Nunzio Tartaglia's quantitative group at Morgan Stanley in the early 1980s.
    - David Shaw founded one of the most successfull statistical arbitrage hedge funds to this day (D. E. Shaw & Co).
    - Foundation of statistical arbitrage and consequently, algorithmic trading.
- Pairs Trading is a contrarian strategy designed to generate abnormal profits from the mean-reverting behavior between a pair of stocks.
    - It is well-planned assault on the Efficient Market Hypothesis.

- Gerry Bamberger and Nunzio Tartaglia's quantitative group at Morgan Stanley in the early 1980s.
  - David Shaw founded one of the most successfull statistical arbitrage hedge funds to this day (D. E. Shaw & Co).
  - Foundation of statistical arbitrage and consequently, algorithmic trading.
- Pairs Trading is a contrarian strategy designed to generate abnormal profits from the mean-reverting behavior between a pair of stocks.
  - It is well-planned assault on the Efficient Market Hypothesis.

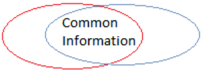
History

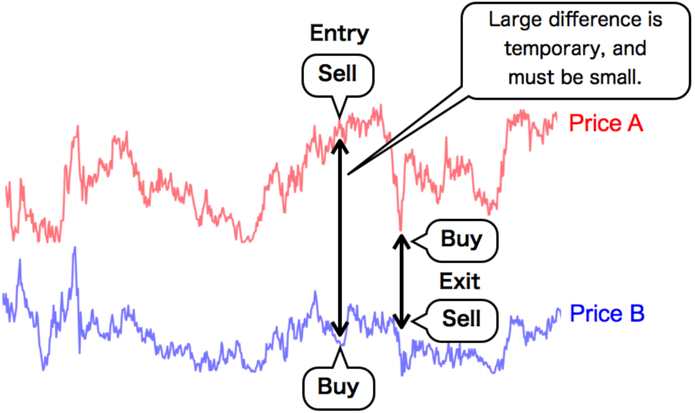
- Gerry Bamberger and Nunzio Tartaglia's quantitative group at Morgan Stanley in the early 1980s.
  - David Shaw founded one of the most successfull statistical arbitrage hedge funds to this day (D. E. Shaw & Co).
  - Foundation of statistical arbitrage and consequently, algorithmic trading.
- Pairs Trading is a contrarian strategy designed to generate abnormal profits from the mean-reverting behavior between a pair of stocks.
  - It is well-planned assault on the Efficient Market Hypothesis.

History

- Gerry Bamberger and Nunzio Tartaglia's quantitative group at Morgan Stanley in the early 1980s.
  - David Shaw founded one of the most successfull statistical arbitrage hedge funds to this day (D. E. Shaw & Co).
  - Foundation of statistical arbitrage and consequently, algorithmic trading.
- Pairs Trading is a contrarian strategy designed to generate abnormal profits from the mean-reverting behavior between a pair of stocks.
  - It is well-planned assault on the Efficient Market Hypothesis.

Two-Dimensional Pairs Trading





What is Pairs Trading?

- Pairs Trading is statistical arbitrage that involves the simultaneous long and short positions of two relatively mispriced stocks which have strong historical co-movements.
  - Market-neutral strategy.
  - Self-financing.
  - Exploit the mean reverting behaviour of co-integrated pairs.
- However, pairs trading strategy is by no means risk free.
  - Trend rather than mean-reverting.

What is Pairs Trading?

- Pairs Trading is statistical arbitrage that involves the simultaneous long and short positions of two relatively mispriced stocks which have strong historical co-movements.
  - Market-neutral strategy.
  - Self-financing.
  - Exploit the mean reverting behaviour of co-integrated pairs.
- However, pairs trading strategy is by no means risk free.
  - Trend rather than mean-reverting.

What is Pairs Trading?

- Pairs Trading is statistical arbitrage that involves the simultaneous long and short positions of two relatively mispriced stocks which have strong historical co-movements.
  - Market-neutral strategy.
  - Self-financing.
  - Exploit the mean reverting behaviour of co-integrated pairs.
- However, pairs trading strategy is by no means risk free.
  - Trend rather than mean-reverting.

What is Pairs Trading?

- Pairs Trading is statistical arbitrage that involves the simultaneous long and short positions of two relatively mispriced stocks which have strong historical co-movements.
    - Market-neutral strategy.
    - Self-financing.
    - Exploit the mean reverting behaviour of co-integrated pairs.
- However, pairs trading strategy is by no means risk free.
    - Trend rather than mean-reverting.

What is Pairs Trading?

- Pairs Trading is statistical arbitrage that involves the simultaneous long and short positions of two relatively mispriced stocks which have strong historical co-movements.
  - Market-neutral strategy.
  - Self-financing.
  - Exploit the mean reverting behaviour of co-integrated pairs.
- However, pairs trading strategy is by no means risk free.
  - Trend rather than mean-reverting.

Distance Method

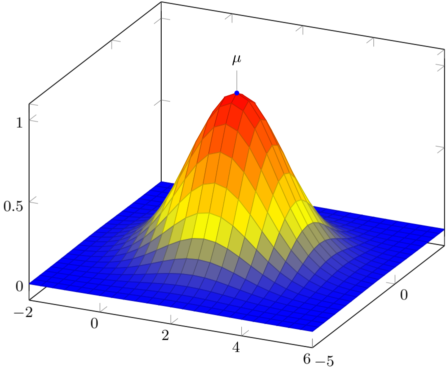
- The most popular strategy is known as distance method (see Gatev *et al.*, 2006).
    - It uses the distance between normalized prices to capture the degree of mispricing stocks.
    - According to Xie *et al.* (2014) the distance method has a multivariate normal nature.
        - Alternative measurement of the linear association.

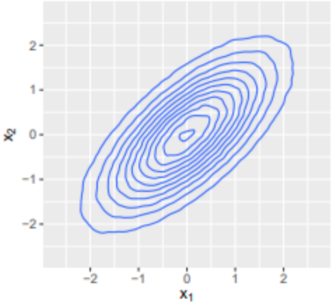
Distance Method

- The most popular strategy is known as distance method (see Gatev *et al.*, 2006).
  - It uses the distance between normalized prices to capture the degree of mispricing stocks.
  - According to Xie *et al.* (2014) the distance method has a multivariate normal nature.
    - Alternative measurement of the linear association.

Bivariate Normal Distribution

$$f(x,y) = \frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}$$





Motivation

- Linear correlation (ρ) fully describes the dependence between securities if the series have joint normal distribution.
- Tail dependence
  - Heavy tails.
  - Possibly Asymmetric.
- A single distance measure may fail to catch the dynamics of the spread between a pair of securities.
  - Initiate and close the trades at non-optimal positions.
- Lie and Wu (2013) proposed a pairs trading strategy based on 2-dimensional copulas to overcome the limitations of the distance method.

Motivation

- Linear correlation (ρ) fully describes the dependence between securities if the series have joint normal distribution.
- Tail dependence
  - Heavy tails.
  - Possibly Asymmetric.
- A single distance measure may fail to catch the dynamics of the spread between a pair of securities.
  - Initiate and close the trades at non-optimal positions.
- Lie and Wu (2013) proposed a pairs trading strategy based on 2-dimensional copulas to overcome the limitations of the distance method.

Motivation

- Linear correlation (ρ) fully describes the dependence between securities if the series have joint normal distribution.
- Tail dependence
  - Heavy tails.
  - Possibly Asymmetric.
- A single distance measure may fail to catch the dynamics of the spread between a pair of securities.
  - Initiate and close the trades at non-optimal positions.
- Lie and Wu (2013) proposed a pairs trading strategy based on 2-dimensional copulas to overcome the limitations of the distance method.

Motivation

- Linear correlation (ρ) fully describes the dependence between securities if the series have joint normal distribution.
- Tail dependence
  - Heavy tails.
  - Possibly Asymmetric.
- A single distance measure may fail to catch the dynamics of the spread between a pair of securities.
  - Initiate and close the trades at non-optimal positions.
- Lie and Wu (2013) proposed a pairs trading strategy based on 2-dimensional copulas to overcome the limitations of the distance method.

Motivation

- Linear correlation (ρ) fully describes the dependence between securities if the series have joint normal distribution.
- Tail dependence
  - Heavy tails.
  - Possibly Asymmetric.
- A single distance measure may fail to catch the dynamics of the spread between a pair of securities.
  - Initiate and close the trades at non-optimal positions.
- Lie and Wu (2013) proposed a pairs trading strategy based on 2-dimensional copulas to overcome the limitations of the distance method.

Theorem 1

Let $X_1, ..., X_d$ *be random variables with distribution functions* $F_1, ..., F_d$*, respectively. Then, there exists an d-copula* C *such that,*

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)), \tag{1}$$

for all $\mathbf{x} = (x_1, ..., x_d) \in \mathbb{R}^d$*. If* $F_1, ..., F_d$ *are all continuous, then the function* C *is unique; otherwise* C *is determined only on* $\operatorname{Im} F_1 \times ... \times \operatorname{Im} F_d$*.*

Why should we care about copulas?

- Assuming that $F(\cdot)$ and $C(\cdot)$ are differentiable, by (1) we have

$$\frac{\partial^d F(x_1,...,x_d)}{\partial x_1 ... \partial x_d} \equiv f(x_1,...,x_d) = \frac{\partial^d C(F_1(x_1),...,F_d(x_d))}{\partial x_1 ... \partial x_d} \tag{2}$$

$$= c(u_1,...,u_d) \prod_{i=1}^{d} f_i(x_i), \tag{3}$$

where $u_i = F_i(x_i)$, $i = 1,...,d$.

- Estimate the multivariate distribution in two parts: (i) finding the marginal distributions; (ii) finding the dependency between the filtered data from (i).

Why should we care about copulas?

- Assuming that $F(\cdot)$ and $C(\cdot)$ are differentiable, by (1) we have

$$\frac{\partial^d F(x_1, ..., x_d)}{\partial x_1 ... \partial x_d} \equiv f(x_1, ..., x_d) = \frac{\partial^d C(F_1(x_1), ..., F_d(x_d))}{\partial x_1 ... \partial x_d} \quad (2)$$

$$= c(u_1, ..., u_d) \prod_{i=1}^{d} f_i(x_i), \quad (3)$$

where $u_i = F_i(x_i)$, $i = 1, ..., d$.

- Estimate the multivariate distribution in two parts: (i) finding the marginal distributions; (ii) finding the dependency between the filtered data from (i).

Copula

- Any multivariate joint distribution can be written in terms of their univariate marginal distribution function and the dependence structure (represented in C) between the variables.

- Copulas accommodate various forms of dependence through suitable choice of the copula "correlation matrix" since they conveniently separate marginals from dependence component.

Strategy	Associations Captured	Required Marginal Distributions
Distance	Linear	Gaussian
Copula	Linear and Nonlinear	No assumption

Copula

- Any multivariate joint distribution can be written in terms of their univariate marginal distribution function and the dependence structure (represented in C) between the variables.
- Copulas accommodate various forms of dependence through suitable choice of the copula "correlation matrix" since they conveniently separate marginals from dependence component.

Strategy	Associations Captured	Required Marginal Distributions
Distance	Linear	Gaussian
Copula	Linear and Nonlinear	No assumption

Copula

- Any multivariate joint distribution can be written in terms of their univariate marginal distribution function and the dependence structure (represented in C) between the variables.
- Copulas accommodate various forms of dependence through suitable choice of the copula "correlation matrix" since they conveniently se parate marginals from dependence component.

Strategy	Associations Captured	Required Marginal Distributions
Distance	Linear	Gaussian
Copula	Linear and Nonlinear	No assumption

Data

- Adjusted closing prices of all shares that belongs to the S&P 500 market index from July 2nd, 1990 to December 31st, 2015.
  - 6,426 days
  - 1100 stocks over all periods
  - Eliminate survivorship bias
  - Bloomberg
  - Fama and French factors

Distance: Methodology

- The matching partner for each stock is found by looking for the security that minimizes the sum of squared deviations between two normalized price series over a twelve-month period (formation period).
  - January to December or from July to June.
  - Adjust them by dividends, stock splits and other corporate actions.

- Select the top 5, 10, . . ., 35 of those combinations that have the smallest sum of squared spreads, allowing re-selection of a specific pair, during the formation period.
  - These pairs are then traded in the next six-month period (trading period).

- In Gatev *et al.* (2006), the long-short position is opened when pair prices have diverged by 2σ and the position is closed when prices revert back.

Distance: Methodology

- The matching partner for each stock is found by looking for the security that minimizes the sum of squared deviations between two normalized price series over a twelve-month period (formation period).
  - January to December or from July to June.
  - Adjust them by dividends, stock splits and other corporate actions.

- Select the top 5, 10, ..., 35 of those combinations that have the smallest sum of squared spreads, allowing re-selection of a specific pair, during the formation period.
  - These pairs are then traded in the next six-month period (trading period).

- In Gatev *et al.* (2006), the long-short position is opened when pair prices have diverged by 2σ and the position is closed when prices revert back.

Distance: Methodology

- The matching partner for each stock is found by looking for the security that minimizes the sum of squared deviations between two normalized price series over a twelve-month period (formation period).
  - January to December or from July to June.
  - Adjust them by dividends, stock splits and other corporate actions.

- Select the top 5, 10, ..., 35 of those combinations that have the smallest sum of squared spreads, allowing re-selection of a specific pair, during the formation period.
  - These pairs are then traded in the next six-month period (trading period).

- In Gatev *et al.* (2006), the long-short position is opened when pair prices have diverged by 2σ and the position is closed when prices revert back.

- The matching partner for each stock is found by looking for the security that minimizes the sum of squared deviations between two normalized price series over a twelve-month period (formation period).
  - January to December or from July to June.
  - Adjust them by dividends, stock splits and other corporate actions.
- Select the top 5, 10, ..., 35 of those combinations that have the smallest sum of squared spreads, allowing re-selection of a specific pair, during the formation period.
  - These pairs are then traded in the next six-month period (trading period).
- In Gatev *et al.* (2006), the long-short position is opened when pair prices have diverged by 2σ and the position is closed when prices revert back.

- To calculate the daily percentage excess returns for a pair, we compute

$$r_{pt} = w_{1t} r_t^L - w_{2t} r_t^S, \tag{4}$$

where L and S stands for long and short, respectively.

- The weights w_{1t} and w_{2t} are initially assumed to be one. After that, they change according to the changes in the value of the stocks, *i.e.*, $w_{it} = w_{it-1}(1 + r_{it-1})$.

- Committed capital and fully invested capital.

Pairs Methodology

- To calculate the daily percentage excess returns for a pair, we compute

$$r_{pt} = w_{1t} r_t^L - w_{2t} r_t^S, \tag{4}$$

where L and S stands for long and short, respectively.

- The weights w_{1t} and w_{2t} are initially assumed to be one. After that, they change according to the changes in the value of the stocks, *i.e.*, $w_{it} = w_{it-1}(1 + r_{it-1})$.

- Committed capital and fully invested capital.

- Trades that do not converge can result in a loss if they are still open at the end of the trading period when they are automatically closed.
  - Fat left tails.
- It may be inappropriate to use constant trigger points because the volatility differs at different price levels.

Distance

- Trades that do not converge can result in a loss if they are still open at the end of the trading period when they are automatically closed.
  - Fat left tails.

- It may be inappropriate to use constant trigger points because the volatility differs at different price levels.

Distance

- Trades that do not converge can result in a loss if they are still open at the end of the trading period when they are automatically closed.
  - Fat left tails.

- It may be inappropriate to use constant trigger points because the volatility differs at different price levels.

Copula: Methodology

- By using the fact that the partial derivative of the copula function gives the conditional distribution function, i.e.,

$$
\begin{aligned}
P\left(U_1 \leq u_1 \,|U_2 = u_2\right) &= \frac{\partial C\left(u_1, u_2\right)}{\partial u_2} = P\left(X_1 \leq x_1 \,|X_2 = x_2\right), \\
P\left(U_2 \leq u_2 \,|U_2 = u_1\right) &= \frac{\partial C\left(u_1, u_2\right)}{\partial u_1} = P\left(X_2 \leq x_2 \,|X_1 = x_1\right),
\end{aligned}
$$

Xie *et al*. (2014) define a measure to denote the degree of mispricing.

Definition 2

- Let R_t^X and R_t^Y represent the random variables of the daily returns of stocks X and Y on time t, and the realizations of those returns on time t are r_t^X and r_t^Y, we have

$$
MI^t_{X|Y} = P(R_t^X < r_t^X \mid R_t^Y = r_t^Y)
$$

and

$$
MI^t_{Y|X} = P(R_t^Y < r_t^Y \mid R_t^X = r_t^X),
$$

- By using the fact that the partial derivative of the copula function gives the conditional distribution function, i.e.,

$$
\begin{aligned}
P\left(U_1 \leq u_1 \left| U_2 = u_2 \right.\right) &= \frac{\partial C\left(u_1, u_2\right)}{\partial u_2} = P\left(X_1 \leq x_1 \left| X_2 = x_2 \right.\right), \\
P\left(U_2 \leq u_2 \left| U_2 = u_1 \right.\right) &= \frac{\partial C\left(u_1, u_2\right)}{\partial u_1} = P\left(X_2 \leq x_2 \left| X_1 = x_1 \right.\right),
\end{aligned}
$$

Xie *et al*. (2014) define a measure to denote the degree of mispricing.

Definition 2

- Let R_t^X and R_t^Y represent the random variables of the daily returns of stocks X and Y on time t, and the realizations of those returns on time t are r_t^X and r_t^Y, we have

$$
\begin{aligned}
MI_{X|Y}^t &= P(R_t^X < r_t^X \mid R_t^Y = r_t^Y) \\
&\text{and} \\
MI_{Y|X}^t &= P(R_t^Y < r_t^Y \mid R_t^X = r_t^X),
\end{aligned}
$$

$$
\begin{aligned}
MI^t_{X|Y} = \quad & \frac{\partial C(u_1,u_2)}{\partial u_2} = \quad P(R^X_t < r^X_t \mid R^Y_t = r^Y_t) \\
\text{and} & \\
MI^t_{Y|X} = \quad & \frac{\partial C(u_1,u_2)}{\partial u_1} = \quad P(R^X_t < r^X_t \mid R^Y_t = r^Y_t).
\end{aligned} \tag{5}
$$

- A conditional value of 0.5 means that the two underlying stocks are considered fairly-valued.

Copula: Methodology

- The conditional probabilities, $M_t^{X|Y}$ and $M_t^{Y|X}$, only measure the degrees of relative mispricing for a single day.
  - To determine an overall degree of relative mispricing we follow Rad *et al*. (2016).
- Let $m_{1,t}$ and $m_{2,t}$ be the overall mispricing indexes of stocks X_1 and X_2, defined by $\left(M_t^{X|Y} - 0.5\right)$ and $\left(M_t^{Y|X} - 0.5\right)$, respectively. At beggining of each trading period two cumulative mispriced indexed M_1 and M_2 are set to zero and then evolve for each day through

$$\begin{aligned} M_{1,t} &= M_{1,t-1} + m_{1,t} \\ M_{2,t} &= M_{2,t-1} + m_{2,t} \end{aligned}$$

- The conditional probabilities, $M_t^{X|Y}$ and $M_t^{Y|X}$, only measure the degrees of relative mispricing for a single day.
  - To determine an overall degree of relative mispricing we follow Rad *et al*. (2016).

- Let $m_{1,t}$ and $m_{2,t}$ be the overall mispricing indexes of stocks X_1 and X_2, defined by $\left(M_t^{X|Y} - 0.5\right)$ and $\left(M_t^{Y|X} - 0.5\right)$, respectively. At beggining of each trading period two cumulative mispriced indexed M_1 and M_2 are set to zero and then evolve for each day through

$$
\begin{aligned}
M_{1,t} &= M_{1,t-1} + m_{1,t} \\
M_{2,t} &= M_{2,t-1} + m_{2,t}
\end{aligned}
$$

- Sensitivity analysis to open a long-short position once one of the cumulative indexes is above 0.05, 0.10, ..., 0.55 and the other one is below -0.05, -0.10, ..., -0.55 at the same time for Top 5, 10, ..., 35 pairs.
- The positions are unwound when both cumulative mispriced indexes return to zero.

- Sensitivity analysis to open a long-short position once one of the cumulative indexes is above 0.05, 0.10, . . ., 0.55 and the other one is below -0.05, -0.10, . . ., -0.55 at the same time for Top 5, 10, . . ., 35 pairs.
- The positions are unwound when both cumulative mispriced indexes return to zero.

Pairs Methodology: Copula

1. Calculate daily returns for each stock during the formation period and estimate the marginal distributions of returns separately.
   - ARMA(p,q)-GARCH(1,1).
2. Estimate the two-dimensional copula model to data that has been transformed to [0,1] margins, i.e.,

$$H\left(r_t^X, r_t^Y\right) = C\left(F_X\left(r_t^X\right), F_Y\left(r_t^Y\right)\right),$$

   where H is the joint distribution, r_t^X e r_t^Y are stock returns and C is the copula.
   - Gaussian, t, Clayton, Frank, Gumbel.
   - Archimedean mixture copula consisting of the optimal linear combination of Clayton, Frank and Gumbel copulas.
   - Mixture copula consisting of the optimal linear combination of Clayton, t and Gumbel copulas.

Pairs Methodology: Copula

1. Calculate daily returns for each stock during the formation period and estimate the marginal distributions of returns separately.

   - ARMA(p,q)-GARCH(1,1).

2. Estimate the two-dimensional copula model to data that has been transformed to [0,1] margins, i.e.,

$$H\left(r_t^X, r_t^Y\right) = C\left(F_X\left(r_t^X\right), F_Y\left(r_t^Y\right)\right),$$

   where H is the joint distribution, r_t^X e r_t^Y are stock returns and C is the copula.

   - Gaussian, t, Clayton, Frank, Gumbel.
   - Archimedean mixture copula consisting of the optimal linear combination of Clayton, Frank and Gumbel copulas.
   - Mixture copula consisting of the optimal linear combination of Clayton, t and Gumbel copulas.

1. Calculate daily returns for each stock during the formation period and estimate the marginal distributions of returns separately.
   - ARMA(p,q)-GARCH(1,1).
2. Estimate the two-dimensional copula model to data that has been transformed to [0,1] margins, i.e.,

$$H\left(r_t^X, r_t^Y\right) = C\left(F_X\left(r_t^X\right), F_Y\left(r_t^Y\right)\right),$$

where H is the joint distribution, r_t^X e r_t^Y are stock returns and C is the copula.

   - Gaussian, t, Clayton, Frank, Gumbel.
   - Archimedean mixture copula consisting of the optimal linear combination of Clayton, Frank and Gumbel copulas.
   - Mixture copula consisting of the optimal linear combination of Clayton, t and Gumbel copulas.

Methodology: Mixed Copula

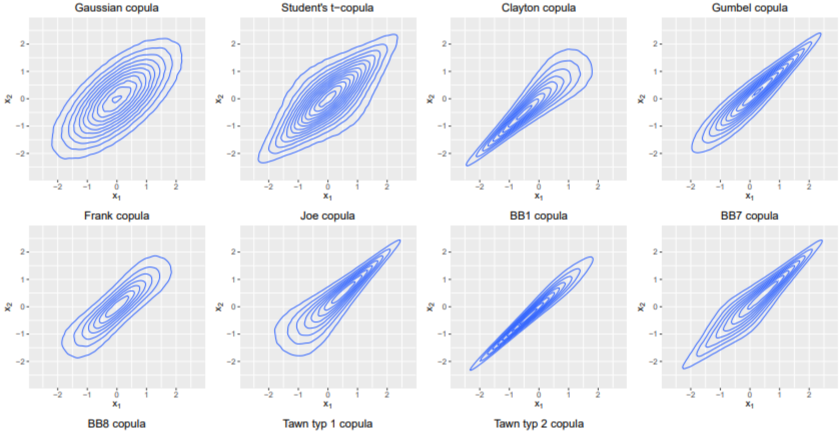
$$\mathcal{C}_{\theta}^{CFG}(u_1, u_2) = \pi_1 \mathcal{C}_{\alpha}^{C}(u_1, u_2) + \pi_2 \mathcal{C}_{\beta}^{F}(u_1, u_2) + (1 - \pi_1 - \pi_2)\, \mathcal{C}_{\delta}^{G}(u_1, u_2),$$

and

$$\mathcal{C}_{\xi}^{CtG}(u_1, u_2) = \pi_1 \mathcal{C}_{\alpha}^{C}(u_1, u_2) + \pi_2 \mathcal{C}_{\Sigma,\nu}^{t}(u_1, u_2) + (1 - \pi_1 - \pi_2)\, \mathcal{C}_{\delta}^{G}(u_1, u_2),$$

where $\theta = (\alpha, \beta, \delta)'$ are the Clayton, Frank and Gumbel copula (dependence) parameters and $\xi = (\alpha, (\Sigma, \nu), \delta)'$ are the Clayton, t and Gumbel copula parameters, respectively, and π_1, $\pi_2 \in [0, 1]$.

Tail Dependence



3. Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day.

3. Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day.

Methodology: Mixed Copula

(3) Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day.

(4) Build long and short positions of Y and X on the days that $M_{1,t} > \Delta_1$ and $M_{2,t} < \Delta_2$ if there is no positions in X or Y.

- Conversely, build positions long/short of X and Y on the day that $M_{1,t} < \Delta_2$ and $M_{2,t} > \Delta_1$ if there is no positions in X or Y.

Methodology: Mixed Copula

(3) Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day.

(4) Build long and short positions of Y and X on the days that $M_{1,t} > \Delta_1$ and $M_{2,t} < \Delta_2$ if there is no positions in X or Y.

- Conversely, build positions long/short of X and Y on the day that $M_{1,t} < \Delta_2$ and $M_{2,t} > \Delta_1$ if there is no positions in X or Y.

- All positions are closed if $M_{1,t}$ reaches Δ_3 or $M_{2,t}$ reaches Δ_4, where $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are predetermined thresholds or are automatically closed out on the last day of the trading period if they do not reach the thresholds.

(3) Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day.

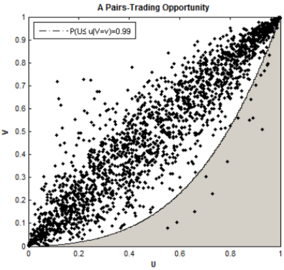
(4) Build long and short positions of Y and X on the days that $M_{1,t} > \Delta_1$ and $M_{2,t} < \Delta_2$ if there is no positions in X or Y.

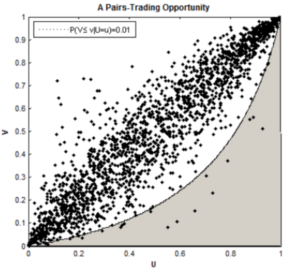
- Conversely, build positions long/short of X and Y on the day that $M_{1,t} < \Delta_2$ and $M_{2,t} > \Delta_1$ if there is no positions in X or Y.

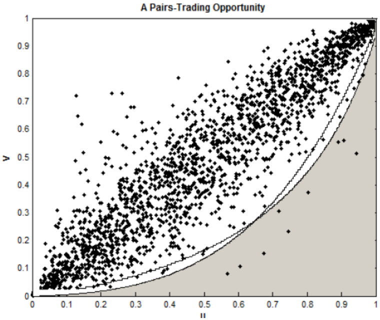
- All positions are closed if $M_{1,t}$ reaches Δ_3 or $M_{2,t}$ reaches Δ_4, where $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are predetermined thresholds or are automatically closed out on the last day of the trading period if they do not reach the thresholds.

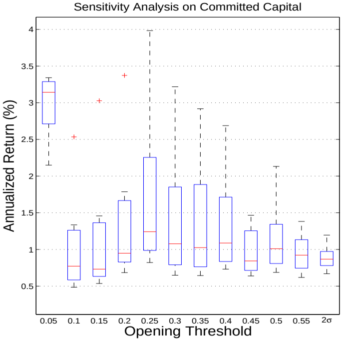
- Here we use $\Delta_1 = 0.2, \Delta_2 = -0.2$ and $\Delta_3 = \Delta_4 = 0$.

Illustration









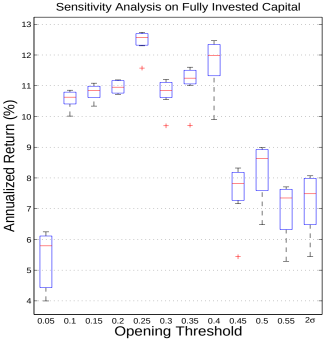


Figure 1: **Annualized returns of pairs trading strategies after costs on committed and fully invested capital**

These boxplots show annualized returns on committed (left) and fully invested (right) capital after transaction cost to different opening thresholds from July 1991 to December 2015 for Top 5 to Top 35 pairs.

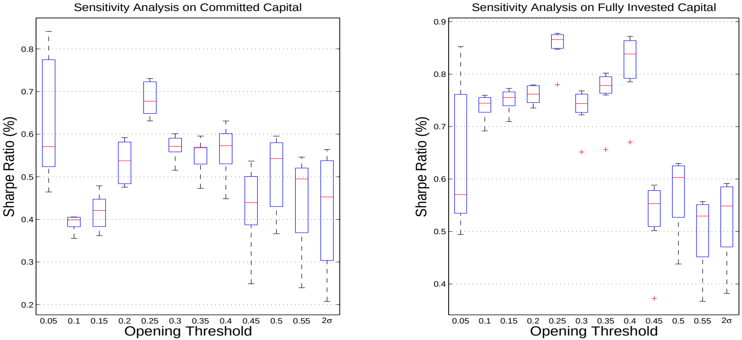


Figure 2: **Sharpe ratio of pairs trading strategies after costs on committed and fully invested capital**

Trading Statistics

Table 1: Trading statistics.

Strategy	Distance	Mixed Copula
	Panel A: Top 5	
Average price deviation trigger for opening pairs	0.0594	0.0665
Total number of pairs opened	352	348
Average number of pairs traded per six-month period	7.18	7.10
Average number of round-trip trades per pair	1.44	1.42
Standard Deviation	1.0128	1.33
Average time pairs are open in days	50.70	37.70
Standard Deviation	39.24	38.93
Median time pairs are open in days	38.5	19
	Panel B: Top20	
Average price deviation trigger for opening pairs	0.0681	0.0821
Total number of pairs opened	1312	749
Average number of pairs traded per six-month period	26.78	15.29
Average number of round-trip trades per pair	1.34	0.76
Standard Deviation	0.99	0.99
Average time pairs are open in days	51.65	23.60
Standard Deviation	39.62	32.90
Median time pairs are open in days	41	9

Trading Statistics

Table 2: Trading statistics.

Strategy	Distance	Mixed Copula
	Panel C: Top 35	
Average price deviation trigger for opening pairs	0.0729	0.0893
Total number of pairs opened	2238	941
Average number of pairs traded per six-month period	45.68	19.20
Average number of round-trip trades per pair	1.30	0.55
Standard Deviation	1.02	0.84
Average time pairs are open in days	52.72	19.35
Standard Deviation	40.48	30.56
Median time pairs are open in days	42	6

Note: Trading statistics for portfolio of top 5, 20 and 35 pairs between July 1991 and December 2015 (49 periods). Pairs are formed over a 12-month period according to a minimum-distance (sum of squared deviations) criterion and then traded over the subsequent 6-month period. Average price deviation trigger for opening a pair is calculated as the price difference divided by the average of the prices.

Empirical Results

Table 3: Excess returns on committed capital of pairs trading strategies on portfolios of Top 5, 20 and 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio	t-stat	% of negative trades	MDD1	MDD2
			Return on Committed Capital				
			Panel A - Top 5 pairs				
Distance	2.60	0.31	0.58	1.86*	46.98	6.73	19.62
Mixed Copula	3.98	0.63	1.08	3.49***	41.79	4.36	9.29
			Panel B - Top 20 pairs				
Distance	3.14	0.65	1.13	3.32***	48.02	3.88	9.69
Mixed Copula	1.24	0.64	1.04	3.52***	41.33	2.07	3.43
			Panel C - Top 35 pairs				
Distance	3.12	0.77	1.36	3.92***	47.97	2.70	7.52
Mixed Copula	0.82	0.73	1.19	3.95***	41.31	1.18	1.98
S&P 500	4.36	0.23	0.52	1.79*	47.45	12.42	46.74

Note: Summary statistics of the annualized excess returns, annualized Sharpe and Sortino ratios on portfolios of top 5, 20 and 35 pairs between July 1991 and December 2015 (6,173 observations). The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively.

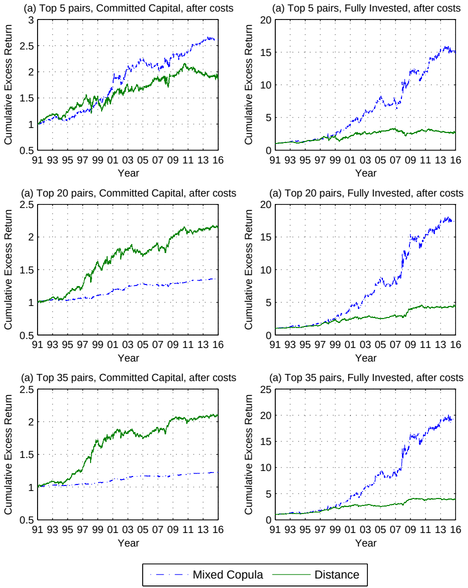
***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 4: Excess returns on fully invested capital of pairs trading strategies on portfolios of Top 5, 20 and 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio	t-stat	% of negative trades	MDD1	MDD2
Return on Fully Invested Capital							
Panel A - Top 5 pairs							
Distance	4.01	0.28	0.57	1.81*	46.98	8.70	38.36
Mixed Copula	11.58	0.78	1.43	4.26***	41.79	9.00	25.68
Panel B - Top 20 pairs							
Distance	6.07	0.66	1.19	3.55***	48.06	5.43	20.03
Mixed Copula	12.30	0.85	1.54	4.60***	41.31	9.00	25.68
Panel C - Top 35 pairs							
Distance	5.76	0.76	1.38	4.05***	47.97	4.24	15.07
Mixed Copula	12.73	0.88	1.59	4.73***	41.28	9.00	25.68

***, **, * significant at 1%, 5% and 10% levels, respectively.

Cumulative excess returns of pairs trading strategies after costs



Kernel density estimation of 5-year rolling window Sharpe ratio after costs

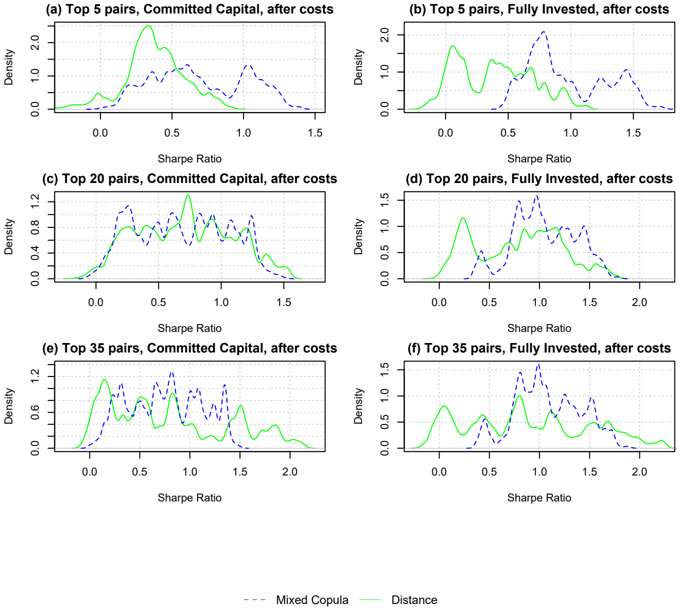


Table 5: Monthly risk profile of Top 5 pairs: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal.

Strategy	Intercept	Rm-Rf	SMB	HML	RMW	CMA	Mom	LRev	R^2	R^2_{adj}
Section 1: Return on Committed Capital										
Distance	0.0025	0.0091	-0.0032	0.0113	0.0003	-0.0029	-0.0107	-0.0084	0.028	0.027
	(1.89)*	(4.22)***	(-0.71)	(2.05)**	(0.25)	(-0.18)	(−4.80)***	(−1.96)**		
Mixed Copula	0.0035	0.0052	-0.0043	0.0039	-0.0035	0.0027	-0.0054	-0.0057	0.015	0.014
	(3.55)***	(3.68)***	(−1.83)*	(1.20)	(-0.99)	(0.63)	(−2.99)***	(-1.57)		
Section 2: Return on Fully Invested Capital										
Distance	0.0040	0.0170	-0.0031	0.0185	0.0049	-0.0018	-0.0161	-0.0150	0.025	0.024
	(1.75)*	(4.88)***	(-0.45)	(2.22)**	(0.76)	(0.05)	(−4.30)***	(−1.97)**		
Mixed Copula	0.0098	0.0148	-0.0084	0.0152	-0.0053	0.0087	-0.0082	-0.0222	0.018	0.017
	(4.17)***	(3.51)***	-1.45	1.6355	-0.60	0.75	(−2.19)**	(−2.08)**		

***, **, * significant at 1%, 5% and 10% levels, respectively.

- Alphas are significantly positive and higher than the raw excess returns by about 2-7 bps per month.
  - Only a small part of the excess returns can be attributed to their exposures to the seven risk determinants.

Table 5: Monthly risk profile of Top 5 pairs: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal.

Strategy	Intercept	Rm-Rf	SMB	HML	RMW	CMA	Mom	LRev	R^2	R^2_{adj}
Section 1: Return on Committed Capital										
Distance	0.0025	0.0091	-0.0032	0.0113	0.0003	-0.0029	-0.0107	-0.0084	0.028	0.027
	(1.89)*	(4.22)***	(-0.71)	(2.05)**	(0.25)	(-0.18)	(−4.80)***	(−1.96)**		
Mixed Copula	0.0035	0.0052	-0.0043	0.0039	-0.0035	0.0027	-0.0054	-0.0057	0.015	0.014
	(3.55)***	(3.68)***	(−1.83)*	(1.20)	(-0.99)	(0.63)	(−2.99)***	(-1.57)		
Section 2: Return on Fully Invested Capital										
Distance	0.0040	0.0170	-0.0031	0.0185	0.0049	-0.0018	-0.0161	-0.0150	0.025	0.024
	(1.75)*	(4.88)***	(-0.45)	(2.22)**	(0.76)	(0.05)	(−4.30)***	(−1.97)**		
Mixed Copula	0.0098	0.0148	-0.0084	0.0152	-0.0053	0.0087	-0.0082	-0.0222	0.018	0.017
	(4.17)***	(3.51)***	-1.45	1.6355	-0.60	0.75	(−2.19)**	(−2.08)**		

***, **, * significant at 1%, 5% and 10% levels, respectively.

- Alphas are significantly positive and higher than the raw excess returns by about 2-7 bps per month.
  - Only a small part of the excess returns can be attributed to their exposures to the seven risk determinants.

Fama-French

Table 5: Monthly risk profile of Top 5 pairs: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal.

Strategy	Intercept	Rm-Rf	SMB	HML	RMW	CMA	Mom	LRev	R^2	R^2_{adj}
Section 1: Return on Committed Capital										
Distance	0.0025	0.0091	-0.0032	0.0113	0.0003	-0.0029	-0.0107	-0.0084	0.028	0.027
	(1.89)*	(4.22)***	(-0.71)	(2.05)**	(0.25)	(-0.18)	(−4.80)***	(−1.96)**		
Mixed Copula	0.0035	0.0052	-0.0043	0.0039	-0.0035	0.0027	-0.0054	-0.0057	0.015	0.014
	(3.55)***	(3.68)***	(−1.83)*	(1.20)	(-0.99)	(0.63)	(−2.99)***	(-1.57)		
Section 2: Return on Fully Invested Capital										
Distance	0.0040	0.0170	-0.0031	0.0185	0.0049	-0.0018	-0.0161	-0.0150	0.025	0.024
	(1.75)*	(4.88)***	(-0.45)	(2.22)**	(0.76)	(0.05)	(−4.30)***	(−1.97)**		
Mixed Copula	0.0098	0.0148	-0.0084	0.0152	-0.0053	0.0087	-0.0082	-0.0222	0.018	0.017
	(4.17)***	(3.51)***	-1.45	1.6355	-0.60	0.75	(−2.19)**	(−2.08)**		

***, **, * significant at 1%, 5% and 10% levels, respectively.

- Alphas are significantly positive and higher than the raw excess returns by about 2-7 bps per month.
  - Only a small part of the excess returns can be attributed to their exposures to the seven risk determinants.

1. By capturing linear/nonlinear associations and covering a wider range of possible dependencies structures, the mixed copula strategy outperforms the distance method when the number of trading signals is equiparable, especially after the subprime mortgage crisis.

2. We show that the mixed copula pairs trading strategy generates large and significant (at 1%) abnormal returns.
    - Only a small part of the pairs trading profits can be explained by market portfolio (beta), size (SMB), value (HML), investment (CMA), profitability (RMW), momentum (Mom) and reversal (LRev) based factors.

Conclusions

1. By capturing linear/nonlinear associations and covering a wider range of possible dependencies structures, the mixed copula strategy outperforms the distance method when the number of trading signals is equiparable, especially after the subprime mortgage crisis.

2. We show that the mixed copula pairs trading strategy generates large and significant (at 1%) abnormal returns.
   - Only a small part of the pairs trading profits can be explained by market portfolio (beta), size (SMB), value (HML), investment (CMA), profitability (RMW), momentum (Mom) and reversal (LRev) based factors.

Conclusions

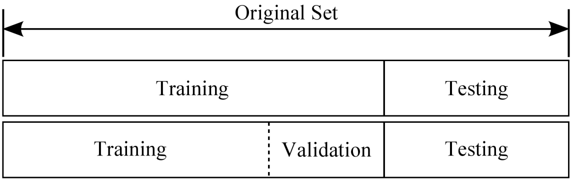
1. By capturing linear/nonlinear associations and covering a wider range of possible dependencies structures, the mixed copula strategy outperforms the distance method when the number of trading signals is equiparable, especially after the subprime mortgage crisis.

2. We show that the mixed copula pairs trading strategy generates large and significant (at 1%) abnormal returns.
   - Only a small part of the pairs trading profits can be explained by market portfolio (beta), size (SMB), value (HML), investment (CMA), profitability (RMW), momentum (Mom) and reversal (LRev) based factors.

- Varying Threshold
- Copula-based arbitrage for triplets to increase information dependency information and measure relative pricing more comprehensively.

- Vine Copulas (Pair-Copula Constructions)
  - Superior flexibility

Future Research



- Machine Learning and AI-based methodologies
  - Explore non-linear approaches

- News Sentiment Factor
  - Contain relevant information about the economic activity
  - Can provide increased risk-adjusted returns

Thanks! Any questions?

Subperiod Analysis

Table 6: Excess returns on committed capital on portfolios of Top 5 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio
Return on Committed Capital			
Panel A: 1991-1995			
S&P 500	7.17	0.72	1.30
Mixed Copula	2.66	0.45	0.74
Panel B: 1996-2000			
S&P 500	10.03	0.51	1.01
Mixed Copula	6.90	1.05	1.77
Panel C: 2001-2005			
S&P 500	-2.28	-0.13	-0.06
Mixed Copula	6.84	0.83	1.44
Panel D: 2006:2010			
S&P 500	-1.71	-0.07	0.09
Mixed Copula	1.56	0.24	0.46
Panel E: 2011:2015			
S&P 500	9.91	0.61	1.09
Mixed Copula	2.01	0.61	1.08

***, **, * significant at 1%, 5% and 10% levels, respectively.

Subperiod Analysis

Table 7: Excess returns on fully invested capital on portfolios of Top 5 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio
Return on Fully Invested Capital			
Panel A: 1991-1995			
S&P 500	7.17	0.72	1.30
Mixed Copula	7.69	0.56	1.02
Panel B: 1996-2000			
S&P 500	10.03	0.51	1.01
Mixed Copula	19.61	1.13	1.96
Panel C: 2001-2005			
S&P 500	-2.28	-0.13	-0.06
Mixed Copula	18.07	1.14	2.07
Panel D: 2006:2010			
S&P 500	-1.71	-0.07	0.09
Mixed Copula	9.42	0.57	1.16
Panel E: 2011:2015			
S&P 500	9.91	0.61	1.09
Mixed Copula	3.62	0.37	0.69

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 8: Excess returns on committed capital on portfolios of Top 20 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio
	Return on Committed Capital		
	Panel A: 1991-1995		
S&P 500	7.17	0.72	1.30
Mixed Copula	0.93	0.46	0.70
	Panel B: 1996-2000		
S&P 500	10.03	0.51	1.01
Mixed Copula	1.67	0.84	1.37
	Panel C: 2001-2005		
S&P 500	-2.28	-0.13	-0.06
Mixed Copula	2.43	1.09	1.86
	Panel D: 2006:2010		
S&P 500	-1.71	-0.07	0.09
Mixed Copula	0.49	0.22	0.38
	Panel E: 2011:2015		
S&P 500	9.91	0.61	1.09
Mixed Copula	0.70	0.77	1.30

***, **, * significant at 1%, 5% and 10% levels, respectively.

Subperiod Analysis

Table 9: Excess returns on fully invested capital on portfolios of Top 20 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio
Return on Fully Invested Capital			
Panel A: 1991-1995			
S&P 500	7.17	0.72	1.30
Mixed Copula	8.18	0.63	1.10
Panel B: 1996-2000			
S&P 500	10.03	0.51	1.01
Mixed Copula	18.48	1.08	1.85
Panel C: 2001-2005			
S&P 500	-2.28	-0.13	-0.06
Mixed Copula	21.07	1.34	2.42
Panel D: 2006:2010			
S&P 500	-1.71	-0.07	0.09
Mixed Copula	12.09	0.74	1.48
Panel E: 2011:2015			
S&P 500	9.91	0.61	1.09
Mixed Copula	2.33	0.25	0.49

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 10: Excess returns on committed capital on portfolios of Top 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio
	Return on Committed Capital		
	Panel A: 1991-1995		
S&P 500	7.17	0.72	1.30
Mixed Copula	0.70	0.60	0.93
	Panel B: 1996-2000		
S&P 500	10.03	0.51	1.01
Mixed Copula	0.99	0.84	1.37
	Panel C: 2001-2005		
S&P 500	-2.28	-0.13	-0.06
Mixed Copula	1.59	1.23	2.11
	Panel D: 2006:2010		
S&P 500	-1.71	-0.07	0.09
Mixed Copula	0.35	0.28	0.46
	Panel E: 2011:2015		
S&P 500	9.91	0.61	1.09
Mixed Copula	0.50	0.86	1.56

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 11: Excess returns on fully invested capital on portfolios of Top 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio
Return on Fully Invested Capital			
Panel A: 1991-1995			
S&P 500	7.17	0.72	1.30
Mixed Copula	8.50	0.65	1.14
Panel B: 1996-2000			
S&P 500	10.03	0.51	1.01
Mixed Copula	19.10	1.12	1.93
Panel C: 2001-2005			
S&P 500	-2.28	-0.13	-0.06
Mixed Copula	21.81	1.38	2.50
Panel D: 2006:2010			
S&P 500	-1.71	-0.07	0.09
Mixed Copula	12.39	0.76	1.51
Panel E: 2011:2015			
S&P 500	9.91	0.61	1.09
Mixed Copula	2.56	0.27	0.53

***, **, * significant at 1%, 5% and 10% levels, respectively.