Mixed Copula Pairs Trading Strategy on the S&P 500

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Abstract

We propose an alternative investment strategy for pairs trading using Archimedean copulas in order to cover a wider range of tail dependence patterns and apply it to the S&P 500 stocks from 1990 to 2015. Empirical results show that our mixed copula approach generates higher average and risk adjusted excess returns and lower drawdown risk than the traditional distance method under different weighting structures when we control for trading costs. Particularly, the mixed copula and distance methods show a mean annualized value-weighted excess returns after costs on committed capital of 3.68% and 2.30% for top 5 pairs, with annual Sharpe ratios of 0.58 and 0.28, respectively. In addition, the mixed copula method shows a higher probability of yielding positive returns than the distance approach in different scenarios. The high returns can only partially be explained by common sources of systematic risk. The proposed mixture copulas have been found to be a superior fitting model over different market states, especially during subperiods of a stronger joint tail dependence.

Keywords: Pairs Trading; Copula; Distance; Long-Short; Quantitative Trading Strategies; S&P 500; Statistical Arbitrage.

JEL Codes: C51, G10, G14.

1 Introduction

During the last decade, we have experienced two deep bear markets (results of the high technology bubble and the subprime mortgage crisis) that have challenged conventional wisdom in finance. Many investors lost large amounts of their capital during the financial crashes, and thereby, some of them had to put their retirement plans on hold. The market has proven challenging, and traditional portfolio management such as the mean-variance portfolio optimization theory of Markowitz (1952) has been shown to be inadequate in helping investors to protect their investments against the financial crises. As a result, market participants are questioning these wide-ranging theoretical frameworks and are particularly interested in a subset of long-short equity strategies that are market-neutral.

Pairs trading is a statistical arbitrage strategy based solely on past stock prices and simple contrarian principles. An investor has to find two stocks whose prices have strong historical co-movements. When their prices are deviating from the equilibrium abnormally, i.e., when the spread between them widens, the investor should, simultaneously, take a short position in the asset with the higher price and a long position in the other asset. Thus, the strategy bets on prices convergence to the equilibrium. The performance of the strategy has attracted considerable recent interest in numerous subfields of finance, since they have potential to generate positive, low-volatility returns that are uncorrelated with market returns. The strategy has a long history on

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Wall Street. In the mid-1980s, the onetime astrophysicist Nunzio Tartaglia assembled a group of physicists, computer scientists and mathematicians to develop market-neutral or long-short strategies using the most sophisticated statistical models and automated trading systems. However, it became popular through the study carried out by Gatev et al. (2006), named distance method.

The traditional distance method has been widely researched and tested throughout the pairs trading literature. The approach is a distribution-free method and thus does not subject the stock prices to follow any particular distribution. However, this approach only captures dependencies well in the case of elliptically distributed random variables. This assumption is generally not met in practice, motivating the utilization of copula-based models to address the univariate and multivariate stylized facts for multivariate financial return stocks. Nevertheless, the use of copulas in this context is still recent and needs more comprehensive and profound studies.

The performance of the distance method has been measured thoroughly using different data sets and financial markets (Gatev et al., 2006; Perlin, 2009; Do and Faff, 2010, 2012; Broussard and Vaihekoski, 2012; Caldeira and Moura, 2013; Rad et al., 2016). In an efficient market, strategies based on mean-reversion concepts should not generate consistent profits. However, Gatev et al. (2006) find that pairs trading generates consistent statistical arbitrage profits in the U.S. equity market during 1962-2002 using CSRP data, although the profitability declines over the period. They obtain a mean excess return above 11% a year during the reported period. The authors attribute the abnormal returns to a non-identified systematic risk factor. They support their view showing that there is a high degree of correlation between the excess returns of no overlapping top pairs even after accounting for risk factors from an augmented version of Fama and French (1993)'s three factors. Do and Faff (2010) extend their work expanding the data sample and also find a declining trend - 33 basis points (bps) mean excess return per month for 2003-09 versus 124 basis points mean excess return per month for 1962-88. Do and Faff (2012) show that the distance method is unprofitable after 2002 when trading costs are considered. Broussard and Vaihekoski (2012) test the profitability of pairs trading under different weighting structures and trade initiation conditions using data from the Finnish stock market. They also find that their proposed strategy is profitable even after initiating the positions one day after the signal. Rad et al. (2016) evaluates distance, cointegration and copula methods using a long-term comprehensive data set spanning over five decades. They find that the copula method has a weaker performance than the distance and cointegration methods in terms of excess returns and various risk-adjusted metrics.

The distance strategy (Gatev et al., 2006) uses the distance between normalized security prices to capture the degree of mispricing between stocks. According to Xie et al. (2016) the distance method has a multivariate normal nature since it assumes a symmetric distribution of the spread between the normalized prices of the stocks within a pair and it uses a single distance measure, which can be seen as an alternative measurement of the linear association, to describe the relationship between two stocks. We know that if the random variables have joint gaussian distribution, then the correlation fully describes the dependence structure (Embrechts et al., 2001). However, it is well known that the dependence between two securities is rarely jointly normal and thus the traditional hypothesis of (multivariate) gaussianity is completely inadequate (Campbell et al., 1997; Artzner et al., 1999; Cont, 2001; Ane and Kharoubi, 2003; Szegö, 2005; McNeil et al., 2015). Therefore, a single distance measure may fail to catch the dynamics of the spread between a pair of securities, and thus initiate and close the trades at non-optimal positions. Throughout the years, different approaches have been devised to capture the mean-reverting properties of the spread in order to improve the profitability of the distance method (see, among others, Elliott et al. (2005); Do et al. (2006); Mudchanatongsuk et al. (2008); Montana et al. (2009); Avellaneda and Lee (2010); Triantafyllopoulos and Montana (2011); Bogomolov (2013); Murota and Inoue (2015); de Moura et al. (2016); Focardi et al. (2016); Stübinger et al. (2016); Krauss (2017); Liu et al. (2017); Ramos-Requena et al. (2017); Stübinger and Endres (2018); Wen et al. (2018)).

Due to the very complex dependence patterns of financial markets, a multivariate approach that allow for differente degrees of tail dependence is surely more insightful than assuming multivariate normal returns. Given its flexibility, copulas are able to model better the empirically verified regularities normally attributed to multivariate financial returns: (1) asymmetric conditional variance with higher volatility for negative returns than for positive returns (Hafner, 1998); (2) conditional skewness (Ait-Sahalia and Brandt, 2001; Chen et al., 2001; Patton, 2001); (3) Leptokurdicity (Tauchen, 2001; Andreou et al., 2001); and (4) nonlinear temporal dependence (Cont, 2001; Campbell et al., 1997). Thus, to best capture the stylized facts and thus give a more complete description of the joint distribution, Liew and Wu (2013) propose a pairs trading strategy based on two-dimensional copulas. However, they evaluate its performance using only three pre-selected pairs over a period of less than three years. Xie et al. (2016) employ a similar methodology over a ten-year period with 89 stocks. Both studies show that the copula approach is a powerful alternative compared to the distance method. Rad et al. (2016) use a more comprehensive data set consisting of all stocks in the US market from 1962 to 2014. However, they find opposite results. Particularly, the distance, cointegration and Copula-GARCH strategies show an average monthly excess return of 36, 33, and 5 bps after transaction costs and 88, 83, and 43 basis points before transaction costs. Thus, a copula-based approach may sound plausible but it may not lead to a viable standalone trading quantitative strategy due to overfitting issues, hence not justifying the marginal performance improvement given by a more complex model.

In this paper, we extend on Xie et al. (2016) and Rad et al. (2016) strategies on the highly liquid stocks of the S&P 500. Rad et al. (2016) suggests a relatively poor performance of the copula method. However, they estimate only copulas that model either lower or upper tail dependence, but not both. Our contribution to the literature is twofold. First, we make a methodological contribution in order to improve the modeling of dependencies in practice. To highlight this shortcoming and illustrate how one may circumvent it, we propose a mixed copula-based model to capture linear and nonlinear associations and at the same time cover a wider range of tail dependence patterns. In particular, our alternative method provides a flexible environment for the search of dependence measures that are better suited for capturing extreme co-movement asymmetries. Second, we employ a method that is particularly suited to make the models equiparable. We show that our formulation worked well even in a highly efficient market and in recent years, generating higher average and risk-adjusted excess returns and lower drawdown risk than the distance approach when the number of tradable signals and consequently transaction costs are comparable. We therefore view the paper as a persuasive message to pairs trading literature, as well as offering new perspectives for capturing the dynamics of the dependence structure. Furthermore, abnormal returns are not fully explained by common sources of systematic risk, thereby providing additional support to potential profits of relative value arbitrage rules. Finally, compatible with the Law of One Price (LOP)[1], the mixed copula strategy excess returns have not diminished consistently over time. To sum it up, we believe that the presented experiments are sufficient rich to suggest that copula-based pairs trading has a practical value and it is an important tool in the quantitative management of funds.

We compare the performance out-of-sample of the strategies using a variety of criteria to see whether the results are robust, all of which are computed using a rolling period procedure similar to that used by Gatev et al. (2006) with the exception that the time horizon of formation and trading periods are rolled forward by six months as in Broussard and Vaihekoski (2012). The main criteria we focus on are: (1) mean and cumulative excess return, (2) non-dimensional risk-adjusted metrics as Sharpe and Sortino[2] ratios, (3) percentage of negative

[1]The process of asset pricing can be seen in absolute or relative terms. For example, in absolute terms, asset pricing is made by way of fundamentals, such as discounted cash flow (DCF). Relative pricing means that two assets that are close substitutes for each other should sell for similar prices - it does not say what the price should be. Reversion to the mean requires a driving mechanism; pairs trading would not work if prices were truly random. The Law of One Price (LOP) is the proposition that two investments with the same payoff in every state of nature must have the same current value. Hence, the price spread between close substitute assets should have a long term stable equilibrium over time. Any short deviations from these equivalent pricing conditions are coming from a temporary shock or reaction from the market and, as a result, can create arbitrage opportunities (Bogomolov, 2013).

[2]The Sharpe ratio is the average realized return in excess of the risk-free rate r_f over the ex-post standard deviation of returns

trades, (4) t-values for various risk factors, and (5) maximum drawdown between two consecutive days and between two days within a maximum period of six months. We find that the mixture copula strategy presents better results than the distance method when the number of trading signals is equiparable (top 5 pairs).

To evaluate if pairs trading profitability is associated to exposure to different systematic risk factors, we regress daily excess returns on daily Fama and French (2015)'s five research factors. Due to endogeneity issues, we add momentum, short and long-term reversal as controls. All the data used to fit the above regressions are described in and obtained from Professor Kenneth French's data library[3]. We find that the intercept is statistically greater than zero for all regressions at 1% level when considering the mixed copula strategy, showing that our results are robust to the augmented Fama and French (2015)'s risk adjustment factors. In addition, the share of observations with negative excess returns is smaller for the mixed copula than for the distance strategy.

To test for differences in returns and Sharpe ratios we use the stationary bootstrap of Politis and Romano (1994) with the automatic block-length selection of Politis and White (2004) and 10,000 bootstrap resamples. To compute the bootstrap p-values we employ the methodology proposed by Ledoit and Wolf (2008). We aim to compare the results on a statistical basis to mitigate potential data snooping issues.

The remainder of the paper is organized as follows. In Section 2, we present a brief review of the distance and copula models as well as the methodologies and strategies we employed. Section 3 summarizes the data and empirical results, followed by our conclusions in Section 4. Additional results are reported in the Appendix.

2 Methodology

In this Section we describe the strategies employed in our paper. The distance approach is described in Section 2.1, whereas the copula method is outlined in Section 2.2. We generalize the existing copula method by employing a mixture copula model. We want to evaluate if we can improve the profitability of pairs trading by capturing a wider variety of dependence structures.

$$\mathbb{SR}\{Y\} = \frac{\mathbb{E}\{R - r_f\}}{\mathbb{S}d\{R\}}$$

, where we used that $\mathbb{S}d\{R - r_f\} = \mathbb{S}d\{R\}$ due to the affine equivalence. The standard deviation $\mathbb{S}d\{R\}$ is symmetric because it measures both the upside and downside risk. Practitioners are more focused on downside risk. Therefore, following Kaplan and Knowles (2004) and Van Dyk, van Vuuren, and Heymans (2014), for a suitable level of minimum acceptable return (usually zero or the risk-free rate) y and an integer $\theta > 0$, we define the kappa ratio

$$\mathbb{K}appa_{\theta,y}\{Y\} = \frac{\mathbb{E}\{Y\} - y}{\left(\mathbb{LPM}_y^\theta\{Y\}\right)^{\frac{1}{\theta}}},$$

where $\mathbb{LPM}_y^\theta\{Y\}$ are the lower partial moments (Bawa, 1975; Jean, 1975), with respect to the performance target y. When $\theta = 2$ the kappa ratio is the Sortino ratio

$$\mathbb{S}oR_y\{Y\} = \frac{\mathbb{E}\{Y\} - y}{\sqrt{\mathbb{LPM}_y^2\{Y\}}},$$

i.e., the ratio of the mean excess return to the standard deviation of negative asset returns. Sortino ratio is an improvement on the Sharpe ratio since it is more sensitive to extreme risks or downside than measures Sharpe ratio. Sortino contends that risk should be measured in terms of not meeting the investment goal. The Sharpe ratio penalizes financial instruments that have a lot of upward jumps, which investors usually view as a good thing.

[3] http://http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

2.1 Distance Framework

Our implementation of the distance strategy is similar to Broussard and Vaihekoski (2012). First, we calculate the sum of Euclidean squared distances[4] of all n(n+1)/2 combination of pairs during the next 12 months, named formation period, adjusting them by dividends, stock splits and other corporate actions. Specifically, the pairs are formed using data from January to December or from July to June. Prices are scaled to $1 at the beginning of each formation period and then evolve using the return series[5]. We then select the top 5, 10, 15, 20, 25, 30 and 35 of those pairs with minimum distance. These pairs are then traded in the next six months (trading period).

The advantages of this methodology are relatively clear. First, functionally misspecified models and misestimations are avoided since the approach uses a non-parametric model. Second, the procedure is straightforward and easy to implement. However, the choice of Euclidean squared distance for identifying pairs is analytically suboptimal (Krauss, 2017; Ramos-Requena et al., 2017).

In Gatev et al. (2006), when the spread diverges by two or more standard deviations (which is calculated in the formation period) from the mean, the stocks are assumed to be mispriced in terms of their relative value to each other and thus one opens a short position in the outperforming stock and a long in the underperforming one.

The price divergence is expected to be temporary, i.e., the prices are expected to converge to its long-term mean value (mean-reverting behavior). Hence, the positions are closed once the normalized prices cross. The pair is then monitored for another divergence and thus a pair can complete multiple round-trip trades. Nonconvergent trades can result in a loss if they are still open at the end of the trading period when they are automatically closed. This results in fat left tails. Therefore, since the conditional variance is empirically higher for large negative returns and smaller for positive returns, it may be inappropriate to use constant trigger points because the volatility differs at different price levels.

To calculate the daily percentage returns for a pair, we compute

$$r_{pt} = w_{1t} r_t^L - w_{2t} r_t^S, \tag{1}$$

where L and S stands for long and short, respectively. Returns r_{pt} can be interpreted as excess returns since in (1) the riskless rate is canceled out when one calculates the long and short excess returns. The weights w_{1t} and w_{2t} are initially assumed to be one. After that, they change according to the changes in the value of the stocks, i.e., $w_{it} = w_{it-1}(1 + r_{it-1})$.

2.2 Copulas

Copulas are often defined as multivariate distribution functions whose marginals are uniformly distributed on $[0, 1]$. In other words, a copula C is a function such that

$$C(u_1, ..., u_d) = P(U_1 \leq u_1, ..., U_d \leq u_d), \tag{2}$$

where $U_i \sim U[0, 1]$ and u_i are realizations of U_i, $i = 1, ..., d$. The margins u_i can be replaced by $F_i(x_i)$, where x_i, $i = 1, ..., d$ is a realization of a (continuous) random variable, since they both belong to the domain $[0, 1]$ and

[4] Spread between the normalized daily closing prices.
[5] Missing values have been interpolated.

are uniformly distributed by its probability integral transform (note that $P(F(x) \leq u) = P\left(x \leq F^{-1}(u)\right) = F\left(F^{-1}(u)\right) = u$). In general, the entries of $\mathbf{U} \equiv (U_1, ..., U_d)'$ are not independent.

Formally, we can define a copula function C as follows.

Definition 1. *An d-dimensional copula (or simply d-copula) is a function C with domain $[0,1]^d$, such that:*

1. C is grounded an d-increasing;
2. C has marginal distributions C_k, $k = 1, ..., d$, where $C_k(\mathbf{u}) = u$ for every $\mathbf{u} = (u_1, ..., u_d)$ in $[0,1]^d$.

Equivalently, an d-copula is a function

$$C : [0,1]^d \to [0,1]$$

with the following properties:

(i) (grounded) For all $\mathbf{u}$ in $[0,1]^d$, $C(\mathbf{u}) = 0$, if at least one coordinate of $\mathbf{u}$ is 0 and $C(\mathbf{u}) = u_k$, if all the coordinates of $\mathbf{u}$ are 1 except u_k;

(ii) (d-increasing) For all $\mathbf{a}$ and $\mathbf{b}$ in $[0,1]^d$ such that $a_i \leq b_i$, for every i, $V_C([a,b]) \geq 0$, where V_c is called $C-$volume.

A representation theorem due to Sklar (1959) has defined the copula approach to dependence modeling.

Theorem 1. *(Sklar's Theorem) Let $X_1, ..., X_d$ be random variables with distribution functions $F_1, ..., F_d$, respectively. Then, there exists an d-copula C such that,*

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)), \tag{3}$$

for all $\mathbf{x} = (x_1, ..., x_d) \in \mathbb{R}^d$. If $F_1, ..., F_d$ are all continuous, then the function C is unique; otherwise C is determined only on $\operatorname{Im} F_1 \times ... \times \operatorname{Im} F_d$. Conversely, if C is an n-copula and $F_1, ..., F_d$ are distribution functions, then the function F defined above is an $d-$dimensional distribution function with marginals $F_1, ..., F_d$.

Corollary 1.1. *Let F be an d-dimensional distribution function with continuous marginals $F_1, ..., F_d$, and copula C. Therefore, for any $\mathbf{u} = (u_1, ..., u_d)$ in $[0,1]^d$,*

$$C(u_1, ..., u_d) = F\left(F_1^{-1}(u_1), ..., F_d^{-1}(u_d)\right), \tag{4}$$

where F_i^{-1}, $i = 1, ..., d$ are the quasi-inverses of the marginals.

Using Sklar's theorem we can derive an important relation between the marginal distributions and a copula. Let f be a joint density function (derived from the d-dimensional distribution function F) and $f_1, \ldots, f_d$ univariate density functions of the margins $F_1, \ldots, F_d$. Assuming that $F(\cdot)$ and $C(\cdot)$ are differentiable, by (3) we have

$$\frac{\partial^d F(x_1, \ldots, x_d)}{\partial x_1 \ldots \partial x_d} \equiv f(x_1, \ldots, x_d) = \frac{\partial^d C(F_1(x_1), \ldots, F_d(x_d))}{\partial x_1 \ldots \partial x_d} \tag{5}$$

$$= c(u_1, \ldots, u_d) \prod_{i=1}^{d} f_i(x_i), \tag{6}$$

where $u_i = F_i(x_i)$, $i = 1, \ldots, d$. Thus, we can clearly see that copulas characterize the dependence structure among the variables. Loosely speaking, copulas accommodate various forms of dependence through suitable choice of the copula "correlation matrix" since they conveniently disantangle the dependence structure between random variables from their marginals, even when these are not normally distributed. In fact, copulas allow the marginal distributions to be modeled independently from each other, and no assumption on the joint behavior of the marginals is required, which provides a high degree of flexibility in modeling joint distributions[6]. From a modeling perspective, Sklar's Theorem laid the theoretical foundation that allows us to estimate the multivariate distribution in two stages so-called inference functions for margins or copula-marginalization: (i) modeling stochastic marginal distributions; (ii) modeling dependency structure between the filtered data from (i). The choice of the copula function is also not dependent on the marginal distributions. Thus, by using copulas, different dependence structures can be modeled to allow for any non-linear dependences in a much finer way and thus identify more reliable trading opportunities[7].

A further important property of copulas concerns the partial derivatives of a copula with respect to its variables. Let now H be a bivariate function with marginal distribution functions F and G. According to Sklar (1959) there exists a copula $C : [0,1]^2 \to [0,1]$ such that $H(x_1, x_2) = C(F(x_1), G(x_2))$ for all $x_1, x_2 \in \mathbb{R}^2$. If F and G are continuous, then C is unique; otherwise, C is uniquely determined in $\operatorname{Im} F \times \operatorname{Im} G$. Conversely, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with marginals F and G and we can write

$$C(u_1, u_2) = H(F^{-1}(u_1), G^{-1}(u_2)), \tag{7}$$

where $u_1 = F(x_1) \Rightarrow x_1 = F^{-1}(u_1)$, $u_2 = G(x_2)) \Rightarrow x_2 = G^{-1}(u_2)$ and F^{-1} and G^{-1} are the quasi-inverses of F and G, respectively. For any copula C, $\dfrac{\partial C(u_1, u_2)}{\partial u_1}$ and $\dfrac{\partial C(u_1, u_2)}{\partial u_2}$ exist almost everywhere. The proposition below states that the partial derivatives of a copula function corresponds to the conditional probabilities of the random variables (see Cherubini et al., 2004; Nelsen, 2006).

Proposition 1. *Let U_1 and U_2 be two random variables with distribution $U(0,1)$. Then,*

$$\begin{aligned} P(U_1 \le u_1 \,|U_2 = u_2) &= \frac{\partial C(u_1, u_2)}{\partial u_2} = P(X_1 \le x_1 \,|X_2 = x_2), \\ P(U_2 \le u_2 \,|U_2 = u_1) &= \frac{\partial C(u_1, u_2)}{\partial u_1} = P(X_2 \le x_2 \,|X_1 = x_1) \end{aligned}$$

where

$$\frac{\partial C(u_1, u_2)}{\partial u_2} = \lim_{h \to 0} P(U_1 \le u_1 \,|u_2 \le U_2 \le u_2 + h) \tag{8}$$

and

$$\frac{\partial C(u_1, u_2)}{\partial u_1} = \lim_{h \to 0} P(U_2 \le u_2 \,|u_1 \le U_1 \le u_1 + h). \tag{9}$$

By using the fact that the partial derivative of the copula function gives the conditional distribution function, Xie et al. (2016) define a measure to denote the degree of mispricing:

[6] Note that the information contained in each marginal distribution $f_i(x_i)$ is swept away by feeding each random variable X_i into its own cdf. Then, what is left is the pure joint information amongst the X_i's, i.e., the copula C. Intuitively, the copula is the information missing from the marginals to complete the joint distribution, i.e., "Joint = Copula + Marginals" (Meucci, 2011). Hence, in practical terms, the copula allows us to create flexible multivariate distributions by gluing arbitrary marginal distributions with a give copula, and provides an effective tool to monitor (detect misalignment or opportunities) and hedge the risks in the markets.

[7] Copulas admit lower or upper tail dependencies, or both, in a non-symmetric way. They are also invariant under strictly monotonic transformations (Cherubini et al., 2004; Nelsen, 2006) and hence the same copula is obtained if we use price or return series, for example.

Definition 2. *(Mispricing Index) Let R_t^X and R_t^Y represent the random daily returns of stocks X and Y at time t, and r_t^X and r_t^Y represent the realizations of those returns at time t. Then define*

$$\begin{aligned} MI_{X|Y}^t = & \quad \frac{\partial C(u_1,u_2)}{\partial u_2} = \quad P(R_t^X < r_t^X \mid R_t^Y = r_t^Y) \\ & \textit{and} \\ MI_{Y|X}^t = & \quad \frac{\partial C(u_1,u_2)}{\partial u_1} = \quad P(R_t^X < r_t^X \mid R_t^Y = r_t^Y). \end{aligned} \tag{10}$$

where $u_1 = F_X\left(r_t^X\right)$ and $u_2 = F_Y\left(r_t^Y\right)$.

Therefore, the conditional probabilities $MI_{X|Y}^t$ and $MI_{Y|X}^t$ indicate whether the return of X is considered high or low at time t, given the information on the return of Y on the time t and the historical relation between the two stocks' returns, and vice-versa. When we observe a value of $MI_{X|Y}^t$ equal to 0.5, r_t^X is neither too high nor too low given r_t^Y and their historical relation, i.e., we can say that this reflects no mispricing. In other words, the historical data indicates that, on average, there is an equal number of observations of the return of X being larger or smaller than r_t^X if the return of stock Y is equal to r_t^Y and therefore, a conditional value of 0.5 means that the stock X is fairly priced relative to stock Y at day t.

Note that the conditional probabilities, $MI_t^{X|Y}$ and $MI_t^{Y|X}$, only measure the degrees of relative mispricing for a single day. Following Xie et al. (2016) and Rad et al. (2016), we determine an overall degree of relative mispricing. Initially, let $m_{1,t}$ and $m_{2,t}$ be the overall mispricing indexes of stocks X and Y, defined by $\left(MI_t^{X|Y} - 0.5\right)$ and $\left(MI_t^{Y|X} - 0.5\right)$, respectively. At the beggining of each trading period two cumulative mispricing indexes $M_{1,t}$ and $M_{2,t}$ are set to zero and then evolve for each day through

$$\begin{aligned} M_{1,t} &= M_{1,t-1} + m_{1,t}, \\ M_{2,t} &= M_{2,t-1} + m_{2,t} \end{aligned}$$

for $t = 1, ..., T$. By construction $M_{1,t}$ and $M_{2,t}$ are not stationary time series. The properties of $M_{i,t}$, $i = 1, 2$, depend on the correlation between $m_{i,t}$ and $M_{i,t-1}$. If the correlation is zero, $M_{i,t}$ follows a pure random walk and (statistical) arbitrage opportunities should be absent. If the correlation is positive $M_{i,t}$ has a tendency to diverge, which generally results in a loss. Finally, if the correlation is negative $M_{i,t}$ has a tendency to converge when it moves away from zero significantly, which generally results in profits. Empirically, $M_{i,t}$ alternates across the three mechanisms and as long as the convergent mechanism dominates we can profit from our strategy.

This approach is attractive as it reflects the mispricings over multiple periods, thus reflecting how farther away the prices are out from their equilibrium. In contrast to the mispricing indices, it can lead to optimal trading strategies, since it results in a more stable strategy. Positive (negative) $M_{1,t}$ and negative (positive) $M_{2,t}$ are interpreted as stock 1 (stock 2) being overvalued relative to stock 2 (stock 1).

We perform a sensitivity analysis to open a long-short position once one of the cumulative indexes is above $0.05, 0.10, \ldots, 0.55$ and the other one is below -0.05, -0.10,…,-0.55 at the same time for top $5, 10, \ldots, 35$ pairs. The positions are closed when both cumulative mispricing indexes return to zero. The pairs are then monitored for other possible trades throughout the remainder of the trading period. Similar to Rad et al. (2016), we propose the following steps to obtain $M_{1,t}$ and $M_{2,t}$ using copula-based models:

1. First, we calculate daily returns for each stock during the formation period and estimate the marginal distributions of these returns separately by fitting an appropriate ARMA(p,q)-GARCH(1,1) model[8] to

[8] We look for the best ARMA(p,q) model up to order (1,1).

each univariate time series by obtaining the estimates $\widehat{\mu}_t$ and $\widehat{\sigma}_t$ of the conditional mean and standard deviation of these processes, respectively. Moreover, using the estimated parametric models, we construct the standardized residuals vectors given, for each $t = 1, ..., T$, by

$$\widehat{\varepsilon}_t = \frac{x_t - \widehat{\mu}_t}{\widehat{\sigma}_t}. \tag{11}$$

The estimated standardized residuals vectors are then converted to the pseudo-observations $u_t = \frac{T}{T+1} F_t\left(\widehat{\varepsilon}_t\right)$, where F_t is estimated by using their empirical distribution function[9];

2. After obtaining the estimated marginal distributions from the previous step, we estimate the two-dimensional copula model to data that has been transformed to [0,1] margins to connect the joint distributions with the marginals F_X and F_Y, i.e.,

$$H\left(r_t^X, r_t^Y\right) = C\left(F_X\left(r_t^X\right), F_Y\left(r_t^Y\right)\right),$$

where H is the joint distribution, r_t^X e r_t^Y are stock returns and C is the copula. Copulas that are tested in this step are the elliptical copula functions Gaussian and Student t, and the Archimedean[10] copula functions Clayton, Frank and Gumbel. Moreover, we consider the case where the probability distribution π is only known to belong to a set of distributions consisting of all mixtures of some possible copula functions, say $\mathcal{C}_M$, i.e.,

$$C\left(\cdot\right) \in \mathcal{C}_M \equiv \left\{ \sum_{i=1}^{d} \pi_i C^i\left(\cdot\right) : \sum_{i=1}^{d} \pi_i = 1,\ \pi_i \geq 0,\ i = 1, ..., d \right\}, \tag{13}$$

where $C^i\left(\cdot\right)$ denotes the j-th likelihood distribution and build two flexible mixed copula models: one Archimedean mixture copula consisting of the optimal linear combination of Clayton, Frank and Gumbel copulas and one mixture copula consisting of the optimal linear combination of Clayton, Student t and Gumbel copulas. Specifically, mixtures of Clayton, Frank and Gumbel copulas and Clayton, Student t and Gumbel copulas can be written, respectively, as

$$\mathcal{C}_\theta^{CFG}\left(u_1, u_2\right) = \pi_1 \mathcal{C}_\alpha^C\left(u_1, u_2\right) + \pi_2 \mathcal{C}_\beta^F\left(u_1, u_2\right) + \left(1 - \pi_1 - \pi_2\right) \mathcal{C}_\delta^G\left(u_1, u_2\right), \tag{14}$$

and

$$\mathcal{C}_\xi^{CtG}\left(u_1, u_2\right) = \pi_1 \mathcal{C}_\alpha^C\left(u_1, u_2\right) + \pi_2 \mathcal{C}_{\Sigma,\nu}^t\left(u_1, u_2\right) + \left(1 - \pi_1 - \pi_2\right) \mathcal{C}_\delta^G\left(u_1, u_2\right), \tag{15}$$

where $\theta = (\alpha, \beta, \delta)'$ are the Clayton, Frank and Gumbel copula (dependence) parameters and $\xi = (\alpha, (\Sigma, \nu), \delta)'$ are the Clayton, Student t and Gumbel copula parameters, respectively, and π_1, $\pi_2 \in [0, 1]$. The estimates are obtained by the minimization of the negative log-likelihood consisting of the weighted densities of the copulas;

[9] The asymptotically negligible scaling factor, $\frac{T}{T+1}$, is used to force the variates to fall inside the open unit hypercube to avoid, for example, problems with density evaluation at the boundaries.

[10] An Archimedean copula has the form

$$C\left(u_1, \ldots, u_n\right) = \psi\left(\psi^{-1}\left(u_1\right) + \psi^{-1}\left(u_2\right) + \ldots + \psi^{-1}\left(u_n\right)\right), \tag{12}$$

for an appropriate generator $\psi\left(\cdot\right)$, where $\psi : [0, \infty] \to [0, 1]$ and satisfies (i) $\psi(0) = 1$ and $\psi(\infty) = 0$; (ii) ψ is d-monotone, i.e., ψ has continuous derivatives on $(0, \infty)$ up to the order $d - 2$, $(-1)^k \psi^{(k)}(x) \geq 0$ for any $k \in \{1, \ldots, d - 2\}$ and $(-1)^{d-2} \psi^{(d-2)}$ is non-negative, non-increasing and convex on $(0, \infty)$. The most known copulas from this class of copulas have a closed form expression. Moreover, each member has a single parameter that controls the degree of dependence, which allows modeling dependence in arbitrarily high dimensions with only one parameter.

3. Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day in the trading period using the copula and estimated parameters;

4. Build long and short positions of Y and X on the days that $M_{1,t} > \Delta_1$ and $M_{2,t} < \Delta_2$ if there are no positions in X or Y. Conversely, build positions long/short of X and Y on the day that $M_{1,t} < \Delta_2$ and $M_{2,t} > \Delta_1$ if there are no positions in X or Y;

5. All positions are closed if $M_{1,t}$ reaches Δ_3 or $M_{2,t}$ reaches Δ_4, where $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are predetermined thresholds or are automatically closed out on the last day of the trading period if they do not reach the thresholds. Practitioners are free to choose the threshold trigger values. However, instead of choosing an arbitrary value, we set the thresholds for initiating a trade based on the number of trading signals that makes the models comparable, which occurs only for the top 5 pairs in this study. This enables a fair comparison between the performance of both measures. Here we use $\Delta_1 = 0.2, \Delta_2 = -0.2$ and $\Delta_3 = \Delta_4 = 0$.

Following Gatev et al. (2006), two measures of excess returns for each portfolio are computed: the return on committed capital (CC) and on fully invested capital (FI). The former commits[11] equal amounts of capital to each one of the pairs even if the pair has not been traded[12], whereas the latter divides all capital among the pairs that are open during the trading period. For example, if, in the top 20 pairs trading portfolio, only ten pairs are open based on the historical two standard deviation trigger or cumulative mispricing indexes criteria, then the FI portfolio returns are scaled by 10. Hence, CC portfolio returns are more conservative.

3 Data and Empirical Results

Our data set consists of daily data of adjusted closing prices of all stocks that belongs to the Standard and Poor's 500 index from July 02, 1990 to December 31, 2015, a period that covers several market upturns and downturns, as well as relatively calm periods. We obtain adjusted closing prices from Bloomberg, whereas daily factor returns are pulled from Professor Kenneth French's website. The data set spans 6,426 trading days and includes a total of 1100 stock price series that appeared at least once as constituents of the S&P 500. To remove survivorship bias only stocks that are listed durng the formation period are included in the analysis, i.e., around the 500 largest U.S. publicly traded companies by market value, accounting for approximately 80 percent of total U.S. market capitalization, in each trading period. We assume that all trades occur at the closing price of that day. It should be emphasized that all backtesting is rigorously out-of-sample.

Using data from the Center for Research in Security Prices (CRSP) from 1980 to 2006, French (2008) estimates that the cost of active investing, including total commissions, bid-ask spreads, and other cost investors pay for trading services, has dropped from 146 basis points in 1980 to 11 basis points in 2006. Do and Faff (2012) estimate institucional one-way commissions to be 7-9 basis points for 2007-2009. Avellaneda and Lee (2010); Stübinger et al. (2016); Liu et al. (2017); Stübinger and Endres (2018) assume transaction costs of 5 basis points per share half-turn, thus 10 basis points for the round-trip transaction cost. Given that our sample extends until 2015 and trading costs have become even lower with the rise of dark pools and rebates to supply

[11] We assume zero return for non-open pairs, although in practice one could earn returns on idle capital.

[12] The commited capital is considered more realistic as it takes into account the opportunity cost of the capital that has been allocated for trading.

liquidity, we follow these studies and assume trading costs of 0.20% (20 basis points) as proxy for the S&P 500 constituents per round-trip pair trade.

3.1 Copula Probability Distributions

To decide which copula best reproduces the joint distribution of the data (the ”optimal“ copula), we fit two single elliptical copulas, three single Archimedean copulas, and two mixture copulas already mentioned to our data set and compare the competing models using their negative log-likelihood functions.

Figure 1 shows the optimal copula distribution for the top 5, top 6-20, and top 21-35 pairs. Our proposed mixture copulas models are identified as the best fit to the data in 92.3%, 91.7% and 89.3% for the top 5, top 6-20, and top 21-35 pairs, respectively. In other words, it reveals a devastating superiority of mixture copula models over the simple ones, with the mixture copula Clayton - Student's t – Gumbel (CtG) clearly being the best fitting dependence model overall, although relatively a little less when including the 15 pairs after the top 5, and the 15 pairs after the top 20. This indicates that most pairs have a dependency structure in their residual process that cannot be captured by the traditional distance method. This is evidence that a single distance measure fails to catch the dynamics of the spread between a pair, and thus a pair position may be forced to close at a sub-optimal position and therefore the arbitrageur may not capture the profit from the pair convergence.

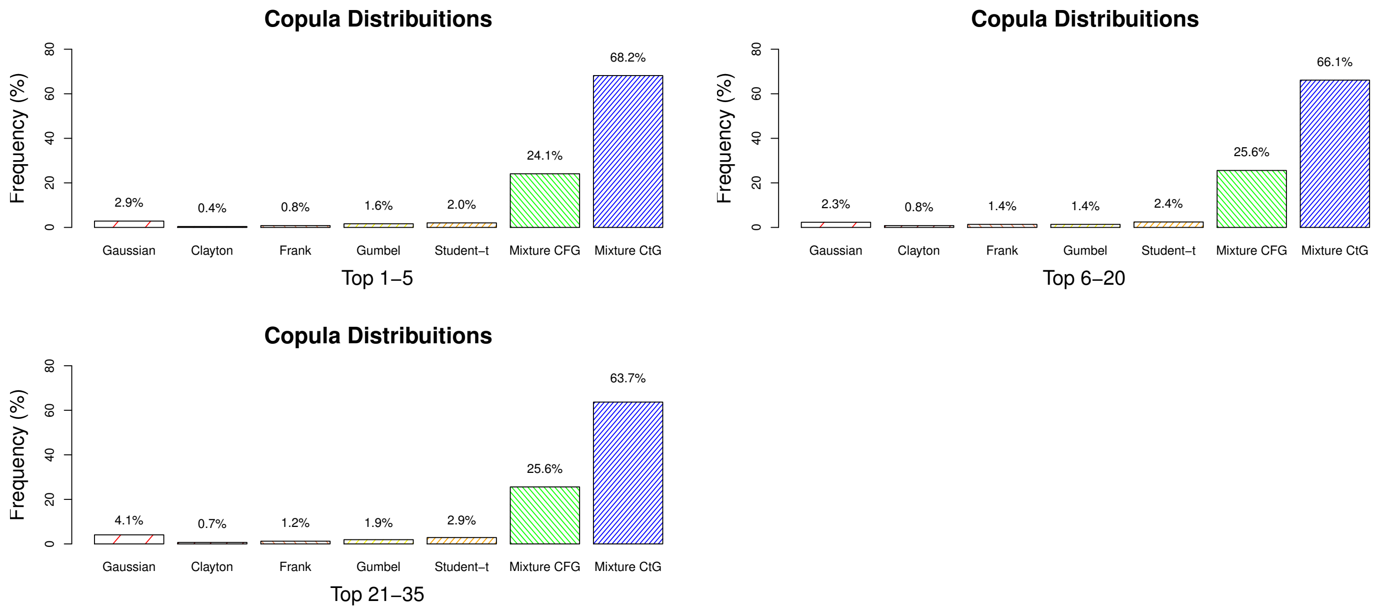

Figure 1: Copula probability distribution for the top 5, top 6-20, and top 21-35 pairs

Bar heights are proportional to the fraction each copula-based model is the best fitting model to available data based on the log-likelihood criterion.

3.2 Profitability of the Strategies

First, we provide multiple boxplots in Figures 2 and 3 to analyze the sensitivity of the annualized excess returns (Figure 2) and annualized Sharpe ratios (Figure 3) when the opening thresholds Δ_1 and Δ_2 are changed to seven evenly spaced number of top pairs (from top 5 to top 35 mean-reversion pairs) for each of the strategies from 1991/2-2015 on commited capital and on fully invested capital after costs. The last boxplot (from left to right) shows the performance for the distance strategy (2.0σ), while the others report the outcomes using multiple opening trigger points for the cumulative mispricing indexes $M_{1,t}$ and $M_{2,t}$. Based on Sharpe ratios and mainly on the number of trading signals that approximately equalizes the transaction costs (see results in table 2) we perform the subsequent analyses considering 0.2 and -0.2 as the opening thresholds for the mixed copula strategy.

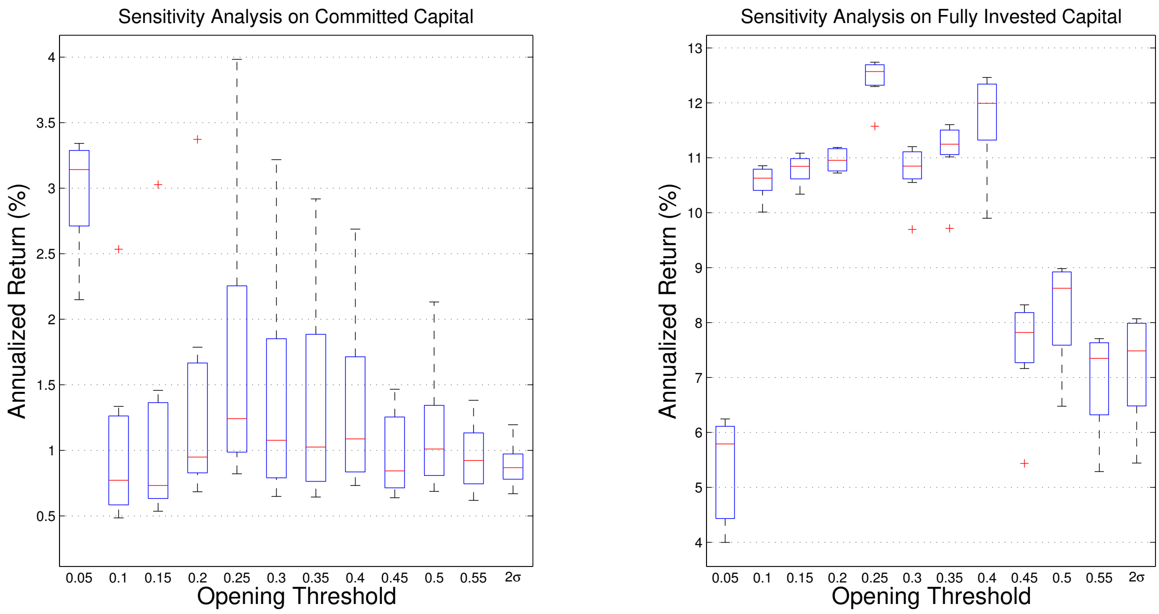

Figure 2: Annualized returns of pairs trading strategies after costs on committed and fully invested capital

These boxplots show annualized returns on committed (left) and fully invested (right) capital after transaction cost to different opening thresholds over the period between July 1991 and December 2015 (from top 5 to top 35 pairs). Pairs are formed based on the smallest sum of squared deviations.

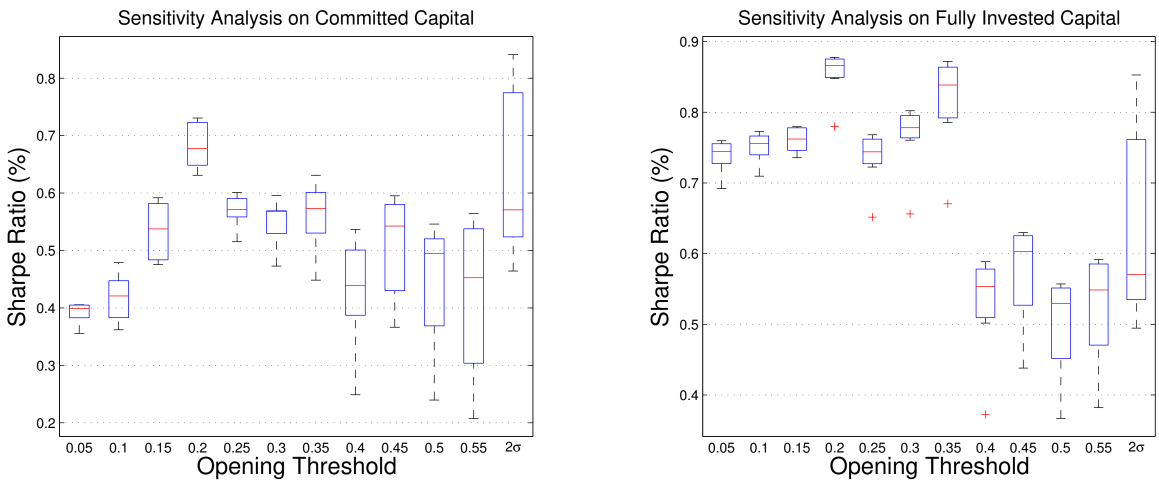

Figure 3: Sharpe ratio of pairs trading strategies after costs on committed and fully invested capital

These boxplots show Sharpe ratios on committed (left) and fully invested (right) capital after transaction cost to different opening thresholds over the period between July 1991 and December 2015 (from top 5 to top 35 pairs). Pairs are formed based on the smallest sum of squared deviations.

Table 1 reports annualized mean excess returns, annualized Sharpe and Sortino ratios, Newey and West (1987) adjusted t-statistics, share of negative observations, the maximum drawdown in terms of maximum percentage drop between two consecutive days (MDD1) and between two days within a period of maximum six months (MDD2), annualized standard deviation, minimum and maximum daily return for both strategies from 1991/2-2015, for top 5 (Panel A), top 20 (Panel B), and top 35 (panel C) pairs after costs[13]. Furthermore, Section 1 shows the return on Committed Capital and Section 2 on Fully Invested Capital.

By analyzing Table 1, it is possible to observe a series of important facts. First, note that the copula-based pairs strategy consistently outperforms the distance method for top 5 pairs and committed capital. The mixed copula strategy yields the highest average excess returns (3.68%), the lowest annualized standard deviation (6.30%) and reaches a Sharpe ratio of 0.58 after costs. As the Sharpe ratio penalizes excessively the investment for good risk, we also consider the Sortino ratio which only considers the downside risk performance. The mixed copula approach reaches a Sortino ratio of 1.00 after costs, about twice as much as what we get from investing in the traditional distance method. This suggests that the copula-based method not only offers better risk-adjusted returns, but also improve the quality of trades. The statistics also indicate that the mixed copula model offers the highest t-statistics (statistically significant at 1%) and a lower probability of a negative trade, where the share of days with negative returns (41.97%) is consistently smaller than the delivered by the tradicional distance method and the market performance (47.09% and 47.45% of negative returns over the period, respectively). This suggests that by employing our copula-based strategy we obtain a higher probability of yielding positive returns. In addition, the summary statistics also show that mixed copula method offers better hedges against losses than the distance strategy for top 5 pairs on committed capital when considering the drawdowns MDD1 and MDD2. We find that the number of tradable signals along the competitive strategies is only equiparable in this study for the case of top 5 pairs. We will explore this point further in the next subsection.

The listed results for top 20 and top 35 pairs on committed capital show that the distance strategy is more profitable than the mixed copula method, although the Sharpe ratios are similar, indicating that returns are alike when we take into account the risks taken. All profits are statistically significant at 1%. Overall, the

[13]The outcomes are also robust for other number of pairs considered. Since the results are very much alike they are not presented here and are available under request.

copula method is again a less risky strategy regarding the drawdown measures.

Table 1: Excess returns of pairs trading strategies on portfolios of top 5, 20 and 35 pairs after costs.

	Distance			Mixed Copula		
	top 5	top 20	top 35	top 5	top 20	top 35
			Section 1: Return on Committed Capital			
			Panel A: after transaction costs			
Annualized Excess Returns (%)	2.30	2.86*	2.78***	3.68	1.08	0.71
Sharpe ratio	0.28	0.59	0.69	0.58	0.56	0.63
Sortino ratio	0.52	1.03	1.22	1.00	0.91	1.02
t-stat	1.67*	3.04***	3.54***	3.25***	3.08***	3.43***
% of negative trades	47.09	48.18	48.13	41.97	41.63	41.54
MDD1	6.73	3.88	2.70	4.36	2.06	1.18
MDD2	19.63	9.82	7.63	9.33	3.49	2.01
Annualized Std. Dev. (%)	8.23	4.85	4.03	6.30***	1.92***	1.11***
Minimum Daily Return (%)	-4.43	-2.76	-1.51	-4.16	-1.47	-0.84
Maximum Daily Return (%)	5.37	2.80	1.75	3.47	0.87	0.68
			Panel B: before transaction costs			
Annualized Excess Returns (%)	2.90	3.43**	3.39***	4.29	1.40	0.93
Sharpe ratio	0.35	0.70	0.83	0.68	0.73	0.83
Sortino ratio	0.64	1.23	1.48	1.16	1.18	1.36
t-stat	2.04**	3.59***	4.25***	3.73***	3.95***	4.46***
% of negative trades	46.95	47.87	47.77	41.65	41.24	41.20
MDD1	6.73	3.89	2.69	4.36	2.07	1.18
MDD2	19.61	9.55	7.43	9.25	3.37	1.94
Annualized Std. Dev. (%)	8.27	4.88	4.07	6.33***	1.93***	1.13***
Minimum Daily Return (%)	-4.43	-2.77	-1.50	-4.16	-1.47	-0.84
Maximum Daily Return (%)	5.41	2.81	1.77	3.47	0.87	0.68
			Section 2: Return on Fully Invested Capital			
			Panel A: after transaction costs			
Annualized Excess Returns (%)	3.52	5.63	5.23	10.86*	11.51**	11.93**
Sharpe ratio	0.24	0.61	0.69	0.73**	0.80	0.83
Sortino ratio	0.52	1.11	1.26	1.35	1.45	1.50
t-stat	1.65*	3.32***	3.70***	4.05***	4.35***	4.49***
% of negative trades	47.09	48.17	48.13	41.97	41.55	41.49
MDD1	8.70	5.43	4.25	9.00	9.00	9.00
MDD2	38.42	20.17	15.21	25.75	25.75	25.75
Annualized Std. Dev. (%)	14.47	9.17***	7.55***	14.77	14.44	14.43
Minimum Daily Return (%)	-8.34	-4.71	-3.09	-10.19	-10.19	-10.19
Maximum Daily Return (%)	10.07	3.73	3.16	10.16	10.16	10.16
			Panel B: before transaction costs			
Annualized Excess Returns (%)	4.49	6.56	6.24	12.30**	13.10**	13.53**
Sharpe ratio	0.31	0.71	0.82	0.82**	0.90	0.93
Sortino ratio	0.63	1.28	1.49	1.51	1.62	1.68
t-stat	1.98**	3.81***	4.35***	4.48***	4.84***	4.98***
% of negative trades	46.95	47.87	47.77	41.65	41.24	41.20
MDD1	8.71	5.43	4.23	9.00	9.00	9.00
MDD2	38.30	19.89	14.93	25.60	25.60	25.60
Annualized Std. Dev. (%)	14.56	9.23***	7.60***	14.91	14.59	0.15
Minimum Daily Return (%)	-8.34	-4.71	-3.10	-10.19	-10.19	-10.19
Maximum Daily Return (%)	10.07	3.75	3.18	10.16	10.16	10.16

Note: Summary statistics of the annualized excess returns, standard devations, Sharpe and Sortino ratios on portfolios of top 5, 20 and 35 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of squared deviations. Excess Returns, Sharpe ratios, and Standard Deviations are compared applying the stationary bootstrap procedure of Politis and Romano (1994) with the automatic block-length selection of Politis and White (2004) and 10,000 bootstrap resamples. All bootstrap p-values are computed using the method proposed by Ledoit and Wolf (2008). The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively.

***, **, * significant at 1%, 5% and 10% levels, respectively.

Section 2 of Table 1 shows results on fully invested capital scheme. The copula-based strategy largely outperforms the distance strategy for all pairs considered. It delivers an average portfolio excess return of

10.86% a year, more than three times as large as the return of the distance method, with large and significant Newey-West adjusted t-statistic of 4.05 for top 5 pairs after costs. The Sharpe and Sortino ratios confirm that the mixed copula method offers better returns per unit of risk.

One possible criticism might be that the conclusions are based on only one realization of the stochastic process of asset returns computed from the observed series of prices, since among thousands of different strategies is very likely that we find some that show superior performance in terms of excess returns or Sharpe ratio for this specific realization. In order to mitigate data-snooping criticisms, we use the stationary bootstrap of Politis and Romano (1994) to compute the bootstrap p-values using the methodology proposed by Ledoit and Wolf (2008).

Our bootstrapped null distributions result from Theorem 2 of Politis and Romano (1994). We select the optimal block length for the stationary bootstrap following Politis and White (2004). As the optimal bootstrap block-length is different for each strategy, we average[14] the block-lengths found to proceed the comparisons between the mixed copula and the distance strategies.

To test the hypotheses that the average excess returns, standard deviations and Sharpe ratios of the copula-based strategy are equal to that of distance method, that is,

$$H_0 : \mu_c = \mu_d, \quad H_0 : \sigma_c = \sigma_d, \quad \text{and} \quad H_0 : \frac{\mu_c}{\sigma_c} = \frac{\mu_d}{\sigma_d}, \tag{16}$$

we compute, following Davison and Hinkley (1997), a two-sided p-value using $B = 10,000$ (stationary) bootstrap re-samples as follows:

$$p_{sboot} = \begin{cases} 2\frac{\sum_{b=1}^{B} \mathbb{I}\{0<t^{*(b)}\}+1}{B+1}, & \text{if } \ median\left\{t^{*(1)}, ..., t^{*(B)}\right\} > 0, \\ 2\frac{\sum_{b=1}^{B} \mathbb{I}\{0\geq t^{*(b)}\}+1}{B+1}, & \text{otherwise}, \end{cases} \tag{17}$$

where $\mathbb{I}$ is the indicator function, $t^{*(b)}$ are the values in each block stationary bootstrap replication, and B denotes the number of bootstrap replications.

Overall, these results reinforce the ones previously obtained. As it can be observed, the distance approach is more profitable than the copula method, at least at 10%, for top 20 and top 35 pairs on committed capital. On the other hand, the copula approach significantly outperforms the distance strategy in terms of mean excess returns and risk-adjusted returns (Sharpe ratio) when the number of tradable signals is comparable (top 5 pairs) on fully invested weighting structure.

Figure 4 shows cumulative excess returns through the full dataset for both strategies for top 5 (top), top 20 (center) and top 35 (bottom) pairs. The left panels display cumulative returns on committed capital, whereas the right panels on fully invested capital. The patterns found in the figure strengthen the mean returns and t-statistics displayed in Table 1. The mixed copula strategy shows a superior out-of-sample performance relative to the distance approach, especially after the subprime mortgage crisis for top 5 pairs (when the number of trades is comparable) on committed capital. Note that the cumulative excess return of the traditional distance method falls significantly after 2010, while our model increases the profits.

Figure 5 shows five-year rolling window Sharpe ratio after costs. It reveals mixed results over the long-term period. However, when the number of tradable signals is similar (top 5 pairs), the copula-based approach yields the highest five-year Sharpe ratio (up to 1.41) on committed capital in 68.94% of the days. In 26.22% of the days over the period the copula method delivers a rolling Sharpe ratio above 1.0, whereas the distance strategy never attains 1.0. In fact, the distance approach produces a five-year Sharpe ratio above 0.5 (and most often

[14] We also use the optimal block size for each strategy. We find that the results are robust to the optimal block size, and therefore, we do not report them here.

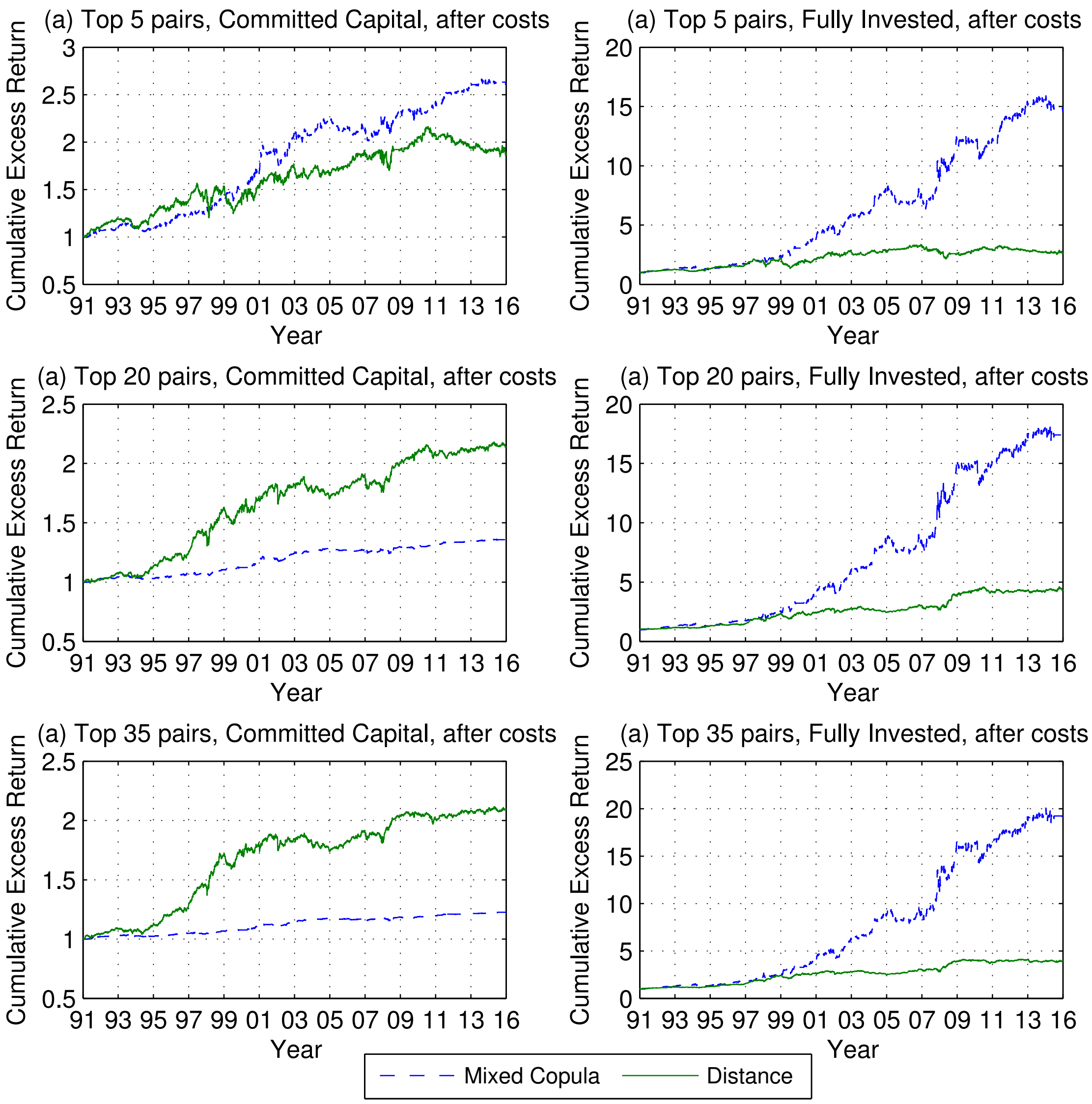

Figure 4: Cumulative excess returns of pairs trading strategies after costs

This figure shows how an investment of $1 evolves over the period between July 1991 and December 2015 for each of the strategies.

below zero after 2014) in only 25.4% of the full period, indicating that the strategy does not reward the risk taken.

For top 20 and top 35 pairs on committed capital the strategies show a more competitive pattern. The distance approach presents a greater rolling window Sharpe ratio in 53.37% and 50.99% of the days for top 20 and top 35 pairs, respectively. However, as we will explore further, the distance approach is a more volatile strategy, identifying a greater number of trading opportunities (more opportunities to make profit) than the copula approach, making the comparison less reliable for a larger number of pairs. The 5-year Sharpe ratios for distance and mixed copula methods are greater than one in 24.97% and 24.04% for top 20 pairs, and 31.92% and 30.33% for top 35 pairs, respectively.

For fully invested weighting scheme the mixed copula approach achieves the highest five-year risk-adjusted statistic over the long-term period in 88.5%, 69.14% and 61.8% of the data sample for top 5, top 20 and top 35 pairs, respectively.

Finally, Figure 6 shows the densities of the five-year Sharpe ratios after costs estimated by means of Sheather and Jones (1991) bandwidth. It should be noted that the densities reinforce our findings from Figure 5, showing that the mass of the distribution of the copula-based strategy has a concentration after 1.0 for top 5 pairs.

3.3 Trading statistics

Table 2 reports trading statistics. Panel A, B and C report results for top 5, top 20 and top 35 pairs, respectively. The average price deviation trigger for opening pairs is listed in the first row of each panel. We can observe that, in average, we initiate the positions before when using the distance approach. The positions are initiated when prices have diverged by 5.94%, 6.81%, and 7.29% for top 5, top 20, and top 35 pairs, respectively. Similar to Gatev et al. (2006), the trigger spread increases with the number of pairs for all approaches.

The table reveals that the average number of pairs traded per six-month period is only equiparable among the strategies for top 5 pairs. For top 20 and top 35 pairs the total number of pairs opened is about 75% and 138% greater when starting positions based on the distance approach. This suggests that a two standard deviation trigger as opening criterion (Gatev et al., 2006) is less conservative than the opening threshold suggested by Rad et al. (2016) using the cumulative mispricing indexes $M_{1,t}$ and $M_{2,t}$. Thus, the distance approach will be able to identify more trading opportunities to profit making the comparison less meaningful, although in practice the benefits are partly offset by the trading costs.

Finally, note that each pair is held open, in average, by 50.7 and 37.7 trading days (2.4 and 1.8 months) under the distance and copula approaches, respectively, for top 5 pairs, which indicates that they are a medium-term

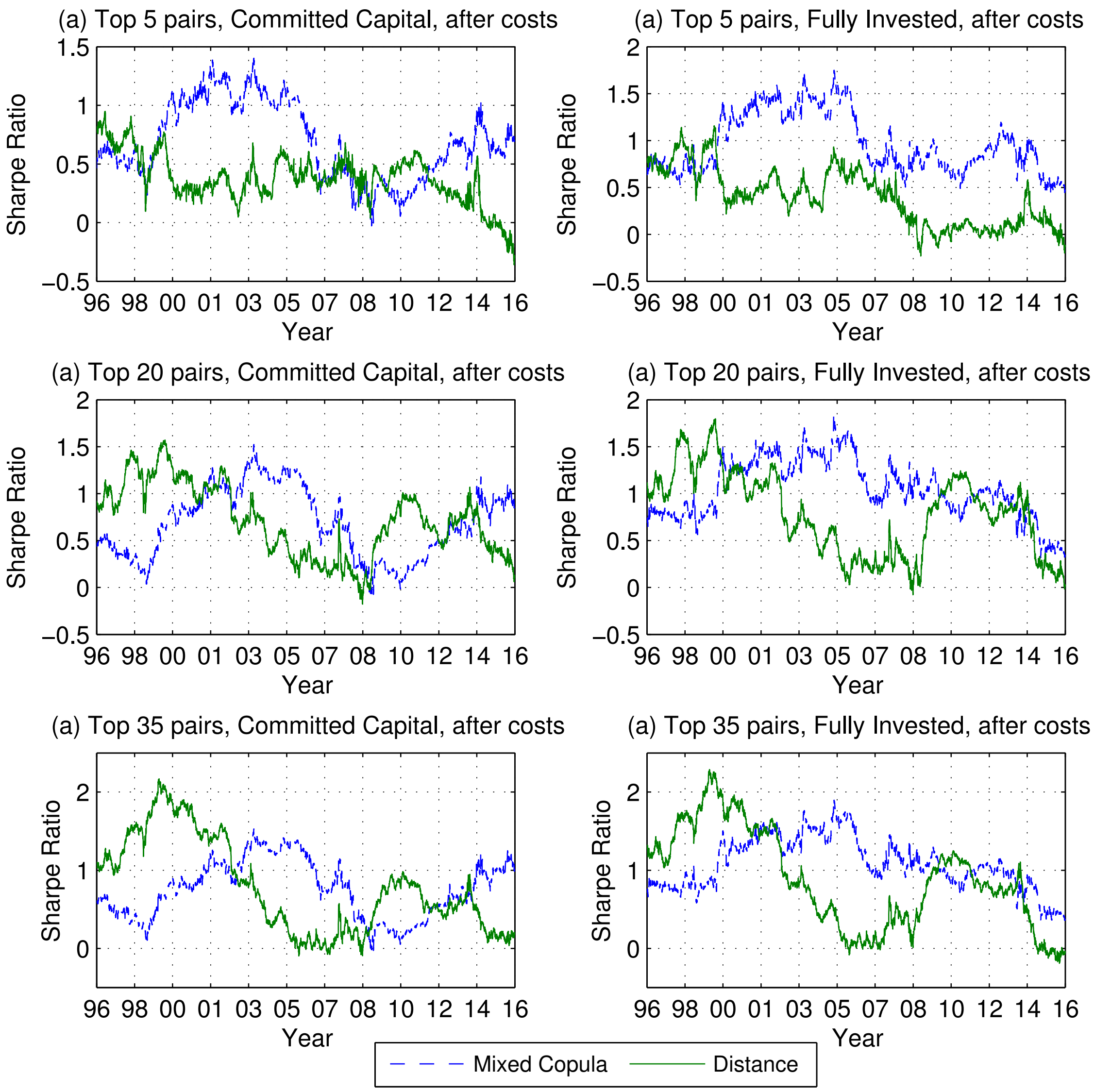

Figure 5: Five-year rolling window Sharpe ratio after costs

This figure shows how the 5-year rolling window Sharpe ratio evolves from July 1996 to December 2015 for each of the strategies.

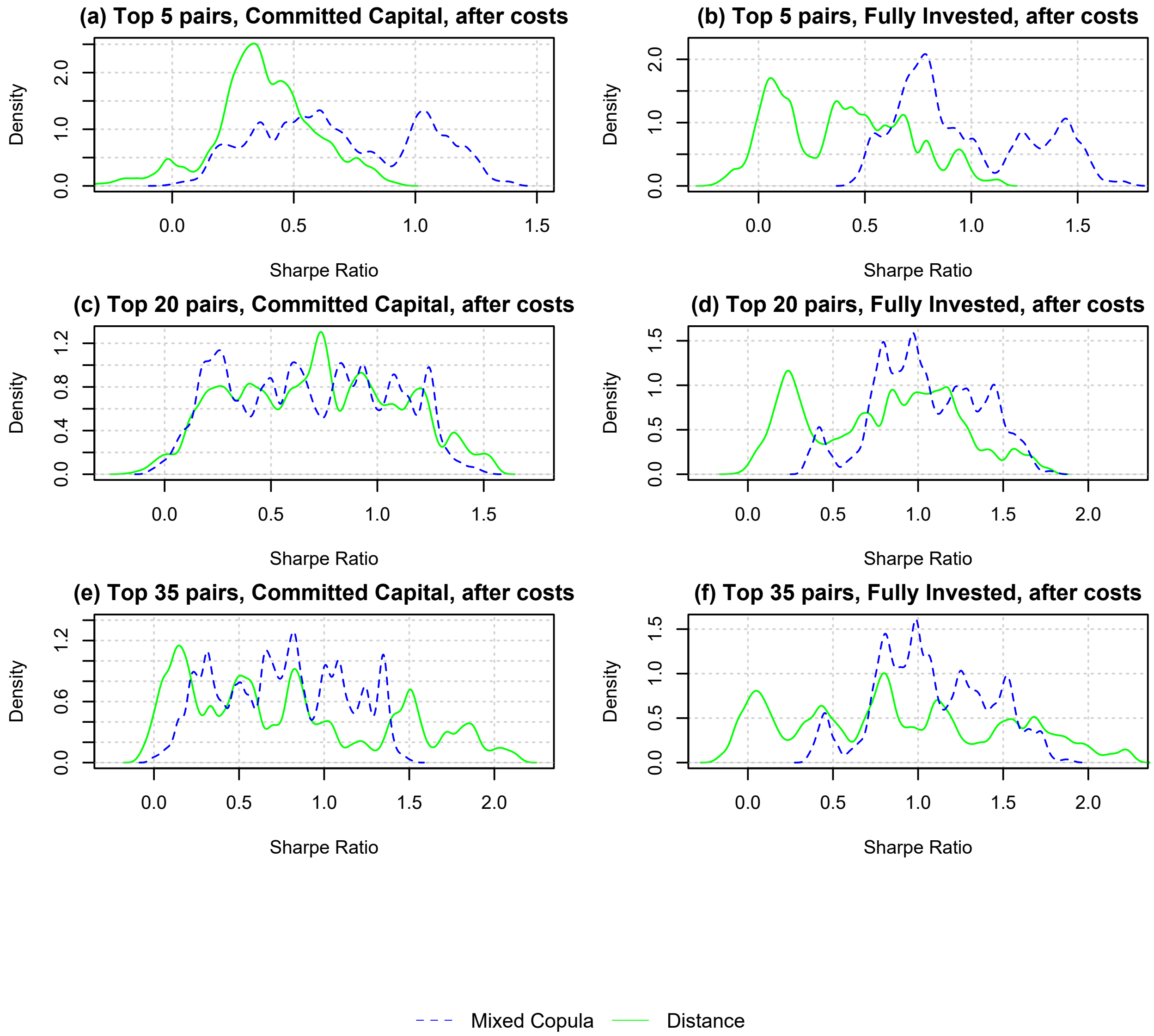

Figure 6: Kernel density estimation of 5-year rolling window Sharpe ratio after costs

This figure shows how the 5-year rolling window Sharpe ratio densities evolve from July 1996 to December 2015 for each of the strategies with Sheather and Jones (1991)'s bandwidths.

investment under these strategies.

Table 2: Trading statistics.

Strategy	Distance	Mixed Copula
	Panel A: top 5	
Average price deviation trigger for opening pairs	0.0594	0.0665
Total number of pairs opened	352	348
Average number of pairs traded per six-month period	7.18	7.10
Average number of round-trip trades per pair	1.44	1.42
Standard Deviation	1.0128	1.33
Average time pairs are open in days	50.70	37.70
Standard Deviation	39.24	38.93
Median time pairs are open in days	38.5	19
	Panel B: top20	
Average price deviation trigger for opening pairs	0.0681	0.0821
Total number of pairs opened	1312	749
Average number of pairs traded per six-month period	26.78	15.29
Average number of round-trip trades per pair	1.34	0.76
Standard Deviation	0.99	0.99
Average time pairs are open in days	51.65	23.60
Standard Deviation	39.62	32.90
Median time pairs are open in days	41	9
	Panel C: top 35	
Average price deviation trigger for opening pairs	0.0729	0.0893
Total number of pairs opened	2238	941
Average number of pairs traded per six-month period	45.68	19.20
Average number of round-trip trades per pair	1.30	0.55
Standard Deviation	1.02	0.84
Average time pairs are open in days	52.72	19.35
Standard Deviation	40.48	30.56
Median time pairs are open in days	42	6

Note: Trading statistics for portfolio of top 5, 20 and 35 pairs between July 1991 and December 2015 (49 periods). Pairs are formed over a 12-month period according to a minimum-distance (sum of squared deviations) criterion and then traded over the subsequent 6-month period. Average price deviation trigger for opening a pair is calculated as the price difference divided by the average of the prices.

3.4 Regression on Fama-French asset pricing factors

In an attempt to understand the economic drivers behind our data as well as to evaluate whether abnormal returns are a compensation for the risk, we regress daily excess returns onto various risk factors: daily Fama and French (2015)'s five research factors, i.e., the excess returns on a broad market portfolio, (the market factor $R_m - R_f$), the difference between the returns on a portfolio of small stocks and the returns on a portfolio of large stocks (the size factor (SMB), small minus big), the difference between the returns on a portfolio of high book-to-market stocks and the returns on a portfolio of low book-to-market stocks (the value factor (HML), high minus low), the difference between the returns of companies with high operating profitability and the returns of those with low operating profitability (the profitability factor (RMW), robust minus weak), the difference between the returns of companies with aggressive investments and the returns of those who are more conservative (the investment factor (CMA), conservative minus aggressive) plus momentum (Mom), short-term reversal (SRev), and long-term reversal (LRev) factors, i.e.,

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i \left(R_{m,t} - R_{f,t}\right) + s_i SMB_t + h_i HML_t + r_i RMW_t \\ + c_i CMA_t + m_i Mom_t + v_i SRev_t + l_i LRev_t + \varepsilon_{i,t}, \tag{18}$$

with $E(\varepsilon_{i,t}) = 0$, $Var(\varepsilon_{i,t}) = \sigma^2_{\varepsilon_i}$ and $E(\varepsilon_{i,t}\varepsilon_{i,s}) = 0, t \neq s$, where i and t stands for portfolio and time index, respectively.

We select the model in terms of an approximation to the mean squared prediction error using Bayesian Information Criterion (BIC) (Schwarz et al., 1978). Based on this variable selection procedure we remove the short-term reversal factor from the model.

The main purpose of these regressions is to estimate the intercept alpha, the average excess return not explained after controlling for these factors, as a measure of risk-adjusted performance. The errors have been adjusted for heteroskedasticity and autocorrelation by using Newey-West adjustment with six lags.

Tables 3, A.1 and A.2 report factor exposures, the respective Newey-West t-statistics and p-values as well as the R-squared and adjusted R-squared for each of the strategies from 1991/2-2015, after transaction costs, for top 5, top 20 and top 35 pairs. For each table, Section 1 lists the Return on Committed Capital and Section 2 on Fully Invested Capital. Panel A provides results after transaction costs and Panel B before transaction costs. Tables A.1 and A.2 are provided in Appendix since we want to focus on the case where the number of tradable signals is comparable. As expected, one could observe that the seven-factor adjusted alphas for top 20 and top 35 pairs agree with the patterns found in the center and bottom of Figure 4. Table 3 shows the results for top 5 pairs. The mixed copula approach produces higher and statistically more significant adjusted alphas than the distance method for both weighting schemes, especially on fully invested capital (98 basis points with a t-value of 3.95). It should also be noted that the risk-adjusted returns provided by copula and distance strategies are positive and significant at 1% and 10% on committed capital, respectively, after accounting for all the previously mentioned factors. In addition, we find that the regression alphas are significantly positive and higher than the raw excess returns by about 2-7 bps per month, indicating that the results are robust and only a small part of the excess returns can be attributed to their exposures to the seven risk determinants.

From Table 3 one could also observe that the magnitude of the loadings on the market and momentum factors are larger and with higher t-values for the distance method on committed capital (significant at 1% for both strategies), thus contributing to the pairs trading profitability. However, these factors are not sufficient to explain all profits of these strategies. Among the other factors, the portfolios load positively on the HML (significant at 5% for the distance method) and load negatively on the SMB and long-term reversal. Furthermore, the correlation of the excess returns with other traditional equity risk premia factors (RMA and CMW) is nearly zero. Finally, it should be noted that the results show that the exposures to the various sources of systematic profile risk provide a very low explanation of the average excess returns for any strategy, with adjusted R^2 ranging from 1.4% to 2.7%, particularly for the copula-based pairs strategy, indicating that the method is nearly factor-neutral over the whole sample period.

The regression on asset pricing factors for top 5 pairs strengthen the patterns found in the Figure 4, indicating that the mixed copula strategy is able to produce relatively economically larger profits after costs than the distance approach when the number of tradable signals is similar.

Table 3: Daily risk profile of top 5 pairs 1991-2015: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal factors.

Strategy	Intercept	Rm-Rf	SMB	HML	RMW	CMA	Mom	LRev	R^2	R^2_{adj}
					Section 1: Return on Committed Capital					
Distance	1.05	0.0432	-0.0149	0.0536	0.0015	-0.0139	-0.0510	-0.0408	0.0282	0.0272
	(1.72)*	(4.40)***	(-0.69)	(2.27)**	(0.05)	(-0.43)	$(-4.92)^{***}$	$(-1.91)^{*}$		
Mixed Copula	1.54	0.0245	-0.0204	0.0188	-0.0167	0.0128	-0.0255	-0.0277	0.0153	0.0141
	(3.30)***	(3.65)***	$(-1.81)^{*}$	(1.21)	(-1.00)	(0.62)	$(-2.96)^{***}$	(-1.59)		
					Section 2: Return on Fully Invested Capital					
Distance	1.70	0.0803	-0.0145	0.0873	0.0231	-0.0088	-0.0770	-0.0729	0.0252	0.0240
	(1.60)	(5.06)***	(-0.47)	(2.49)**	(0.52)	(-0.15)	$(-4.32)^{***}$	$(-1.96)^{**}$		
Mixed Copula	4.41	0.0699	-0.0399	0.0719	-0.0253	0.0411	-0.0388	-0.0107	0.0179	0.0168
	(3.95)***	(3.49)***	-1.44	1.63	-0.59	0.74	$(-2.18)^{**}$	$(-2.08)^{**}$		

Note: This table shows results of regressing daily portfolio return series onto Fama and French (2016)'s five factors factors plus momentum and long-term reversal factors over the period between July 1991 and December 2015 (6173 observations). Section 1 shows the Return on Committed Capital and Section 2 on Fully Invested Capital after transaction costs. Pairs are formed based on the smallest sum of squared deviations. The t-statistics (shown in parentheses) are computed using Newey-West standard errors with six lags. Intercepts have been multiplied by 100 to aid interpretation.
***, **, * significant at 1%, 5% and 10% levels, respectively.

3.5 Subperiod analysis

The existing literature on trading strategies provides evidence of the sensitivity of performance over different market conditions. To identify how robust these results are to changes in the market state, we split the full sample period into five subperiods: (1) July 1991 to December 1995, (2) January 1996 to December 2000, (3) January 2001 to December 2005, (4) January 2006 to December 2010, and (5) January 2011 to December 2015. The third subperiod corresponds to the bear market that comprises the dotcom crisis and the September 11th terrorist attack, whereas the fourth subperiod corresponds to the subprime mortgage financial crisis period.

Figures 7 and 8 show the profitability and risk-adjusted patterns of both strategies for top 5 (top), top 20 (center) and top 35 (bottom) pairs after costs, respectively, for each subperiod on commited capital (left) and fully invested capital (right).

Overall, the mixed copula strategy yields a superior out-of-sample performance relative to the distance approach in the second and third subperiods (1996-2000 and 2001-2005) and after the subprime mortgage crisis (2011-2015), whereas the distance method delivers a significant better performance in the first (1991-1995) and fourth subperiods (2006-2010) when the number of trades (top 5 pairs) are similar on committed capital. Particularly, during the periods of higher volatility, the distance and mixed copula strategies generate a 3.16% and 6.50% excess return per year with a Sharpe ratio of 0.36 and 0.79 (from 2001 to 2005), and 3.36% and 1.22% with a Sharpe ratio of 0.42 and 0.19 (from 2006 to 2010), respectively, which far exceed the market's average excess return of -2.28% and -1.71% with Sharpe ratios of -0.13 and -0.07 over the same subperiods. For fully invested weighting scheme the results are consistent with those we have found in the full period analysis (see the right panels in Figure 4). Specifically, for top 5 pairs during the most volatile periods, the average excess returns per annum is 8.53% and -0.68% with Sharpe ratios of 0.53 and -0.05 using the distance approach, and a solid performance of 17.34% and 8.51% with Sharpe ratios of 1.10 and 0.52 for the copula-based method.

Figure 9 and Table 4 display the "optimal" distribution of the copulas over subperiods. There are 9 trading periods for the first subperiod and 10 for the four further subperiods. For top 5 pairs this means 45 pairs to be fitted to the data during 1991-1995 and 50 pairs for the next subperiods. The importance of the mixture copula models is stressed, particularly during the subperiods of higher volatility (including the dotcom bubble of 2001-2002 and the financial crisis period of 2008-2009). The proposed mixture copulas have been found to be a superior fitting model in 48 (96%) and 49 (98%) pairs (out of 50) in these subperiods (2001-2005 and

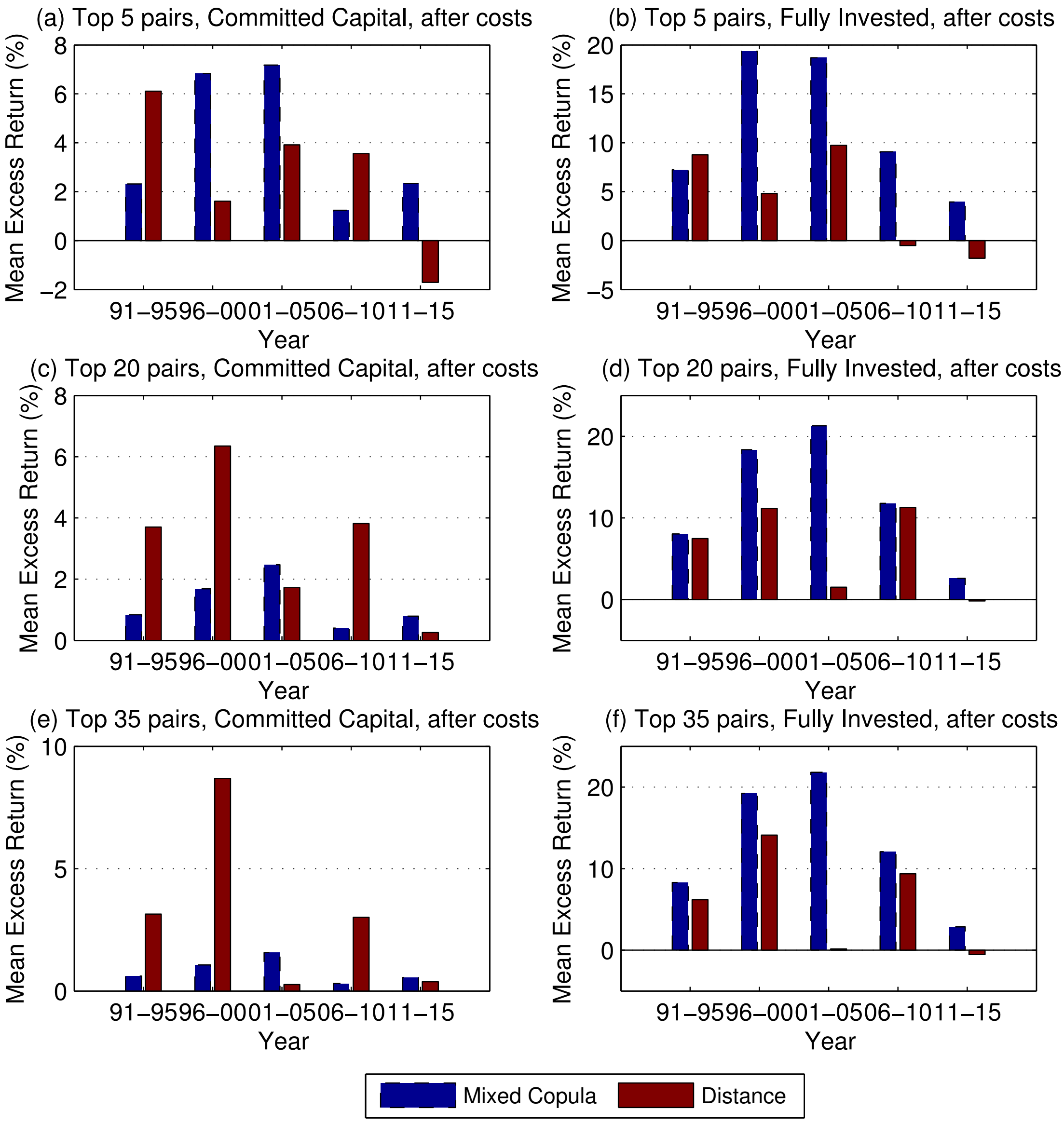

Figure 7: Periodic Profitability of Pairs Trading Strategies

This figure shows the annualized mean excess return performance of each strategy on five different subperiods after costs.

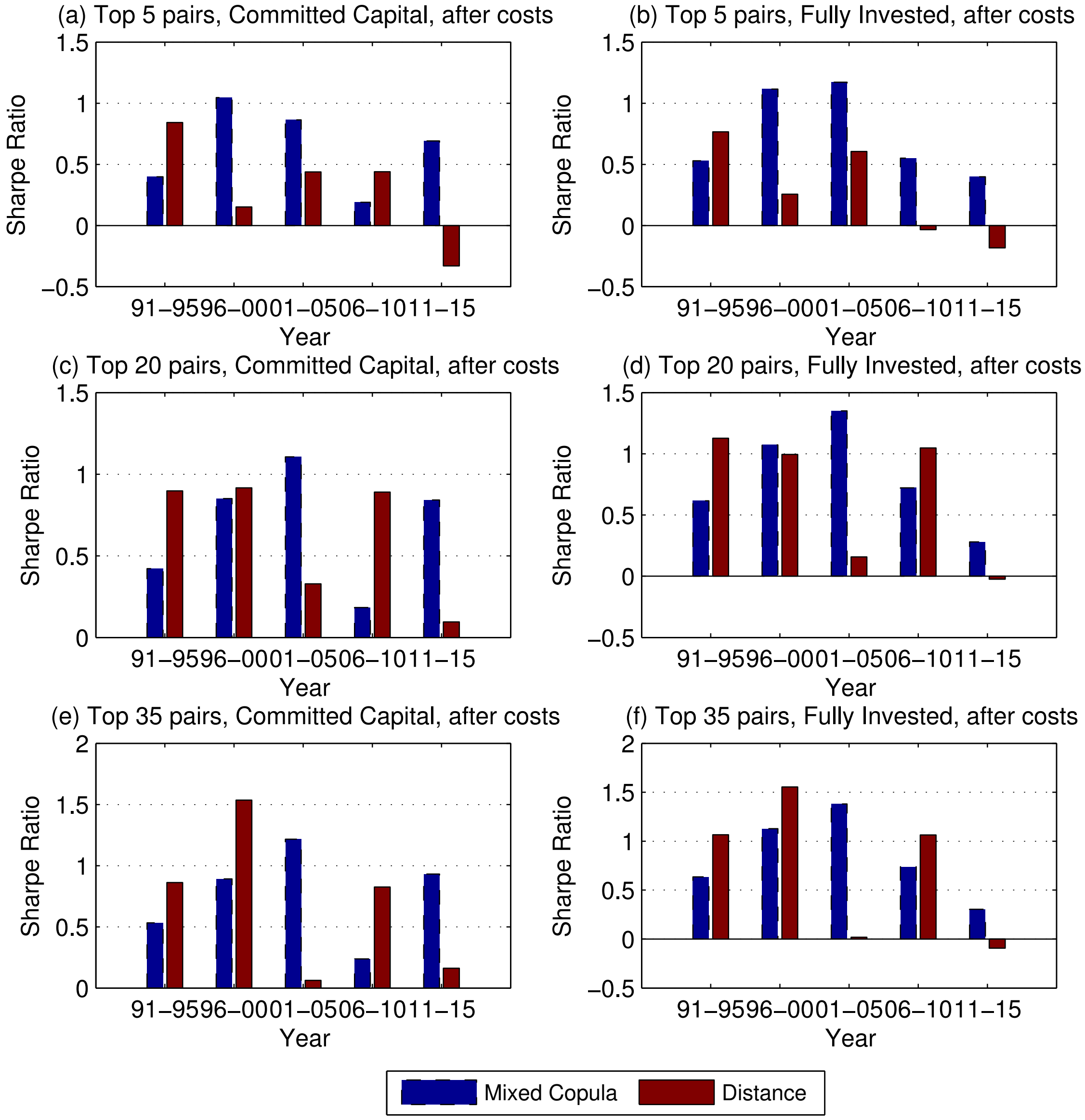

Figure 8: Periodic Risk-Adjusted Return of Pairs Trading Strategies

This figure shows the Sharpe ratio performance of each strategy on five different subperiods after costs.

2006-2010), respectively, for top 5 pairs. In other words, our approach based on mixed copulas better captures the dependency structure as compared to the other five single distributions especially in periods of a stronger joint tail dependence.

For pairs ranked between 6-20 and 21-35 the order of importance among the copula-based models remains similar, although the magnitude tends to be somewhat lower for the mixture copula consisting of the optimal linear combination of Clayton, Student-t and Gumbel (CtG) copulas, whereas for the Gaussian copula the effect is in the opposite direction. This is not surprisingly if the process of matching up the pairs according to a minimum distance criterion between normalized historical prices performs well since the joint dependency patterns in the tails of the joint distribution are not so strong as compared to the top 5 pairs.

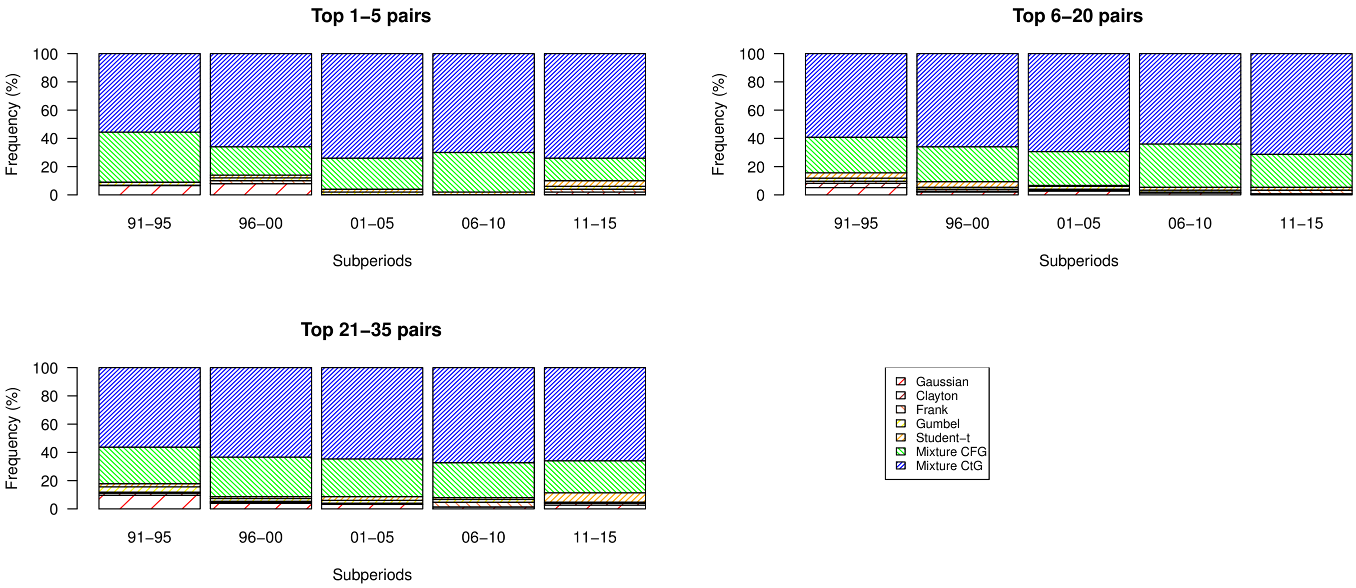

Figure 9: Periodic Optimal Copula Distribution

Bar heights are proportional to the fraction each copula-based model is the best fitting model on five different subperiods for the top 1-5, top 6-20, and top 21-35 pairs.

Table 4: Optimal Copula Distribution over the subperiods (%).

Copula	top 1-5	top 6-20	top 21-35
		Panel A: 1991-1995	
Gaussian	6.67	5.19	9.63
Clayton	0.00	2.96	1.48
Frank	0.00	1.48	0.74
Gumbel	2.22	2.22	3.70
Student-t	0.00	3.70	2.22
Mixture CFG	35.56	25.19	25.93
Mixture CtG	55.55	59.26	56.30
		Panel B: 1996-2000	
Gaussian	8.00	2.00	4.00
Clayton	0.00	0.67	0.67
Frank	2.00	1.33	0.67
Gumbel	2.00	1.33	2.00
Student-t	2.00	4.00	1.33
Mixture CFG	20.00	24.67	28.00
Mixture CtG	66.00	66.00	63.33
		Panel C: 2001-2005	
Gaussian	0.00	2.66	3.33
Clayton	0.00	0.67	0.00
Frank	0.00	0.67	0.67
Gumbel	2.00	2.00	2.00
Student-t	2.00	0.67	2.67
Mixture CFG	22.00	24.00	26.66
Mixture CtG	74.00	69.33	64.67
		Panel D: 2006-2010	
Gaussian	0.00	1.33	1.33
Clayton	0.00	0.00	0.00
Frank	0.00	0.67	3.34
Gumbel	0.00	1.33	2.00
Student-t	2.00	2.00	1.33
Mixture CFG	28.00	30.67	24.67
Mixture CtG	70.00	64.00	67.33
		Panel E: 2011-2015	
Gaussian	0.00	0.67	2.67
Clayton	2.00	0.00	1.33
Frank	2.00	2.67	0.67
Gumbel	2.00	0.00	0.00
Student-t	4.00	2.00	6.66
Mixture CFG	16.00	23.33	22.67
Mixture CtG	74.00	71.33	66.00

Note: This table shows the optimal copula probability distributions (%) over all estimation subperiods for the top 1-5, top 6-20, and top 21-35 pairs. The copulas are selected based on their negative log-likelihood functions.

4 Conclusions

Pairs trading fall under the class of statistical arbitrage strategies. It involves a portfolio consisting of long one stock and short the other betting on the empirical fact that the spread among stocks which have strong co-movements tend to return to their historical level.

The main goal of this paper is to verify in what extent pairs trading benefit from applying an alternative copula-based investment strategy compared to the more traditional method. We present results for different measures of performance, weighting structures, and market states to see whether they persist. We are also interested in understanding better the factors that affect their profitability.

Using a long-term comprehensive data set spanning 25 years in a highly liquid market, our empirical analysis suggests that the mixed copula strategy has a superior performance than the distance approach when the number of trades and consequently transaction costs are comparable, which occurs for the case of top 5 pairs. The level of profit associated with a medium-frequency strategy bring new perspectives on asset pricing theory.

The main findings when we control by trading costs are summarized below.

1. Empirical findings show that the mixture copula models are able to provide a more accurate estimate of dependence structures than the other five single distributions, especially in periods of high volatility and consequently strong tail dependence structure. This is evidence that a single distance measure cannot capture the dynamics of the spread between most pairs, and thus a position may be forced to open or close at sub-optimal positions.

2. The results on committed capital show that our mixed copula strategy generates a superior out-of-sample mean excess return and the Sharpe and Sortino ratios about twice as much as we get from investing in the traditional distance method after costs. This suggests that our copula-based approach not only offers better risk-adjusted returns, but also improve the quality of the trades. In addition, the performance of the proposed method is consistently better than that of the benchmark after the subprime mortgage crisis. The maximum drawdown measures also show that our strategy offers better downside protection against the worst market states.

3. Over the 25-year period of the backtest, our copula-based approach produces higher and statistically more significant alphas after accounting for common systematic risk factors such as momentum, liquidity, profitability and investment than the distance method for both weighting schemes, especially on fully invested capital. Thus, the results show that the asset pricing factors are unable to explain our portfolio returns.

We found that the average number of pairs traded per six-month period is only comparable among the strategies for top 5 pairs in this study. This suggests that a constant two standard deviation threshold (Gatev et al., 2006) is less conservative than the opening trigger point suggested by Rad et al. (2016) using the cumulative mispricing indexes $M_{1,t}$ and $M_{2,t}$. The results obtained are somehow expected since the distance between normalized security prices is by construction time dependent, i.e., the spread is not a stationary time series. Therefore, a fixed volatility measure may be misleading. Further studies in the application of copulas in pairs trading should investigate the optimal points of entry and exit based on the standard deviation of the mispricing indexes, which are stationary series, $MI^t_{X|Y}$ and $MI^t_{Y|X}$ to allow for a more accurate comparison of these two competing alternatives.

Appendix - Regressions on asset pricing factors

This appendix contains the regressions on asset pricing factors for top 20 and top 35 pairs for both strategies.

Table A.1: Daily risk profile of top 20 pairs 1991-2015: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal factors.

Strategy	Intercept	Rm-Rf	SMB	HML	RMW	CMA	Mom	LRev	R^2	R^2_{adj}
					Section 1: Return on Committed Capital					
Distance	1.20	0.0265	-0.0078	0.0062	-0.0148	0.0281	-0.0334	-0.0330	0.0275	0.0264
	$(3.20)^{***}$	$(5.04)^{***}$	(-0.59)	(0.48)	(-0.79)	(1.64)	$(-4.78)^{***}$	$(-2.53)^{**}$		
Mixed Copula	0.44	0.0070	-0.0026	0.0010	-0.0025	0.0034	-0.0059	-0.0061	0.0088	0.0077
	$(3.10)^{***}$	$(3.45)^{***}$	(-0.69)	(0.22)	(-0.51)	(0.55)	$(-2.13)^{**}$	(-1.17)		
					Section 2: Return on Fully Invested Capital					
Distance	2.37	0.0487	-0.0161	0.0324	-0.0243	0.0503	-0.0677	-0.0586	0.0309	0.0298
	$(3.43)^{***}$	$(5.20)^{***}$	(-0.67)	(1.32)	(-0.73)	(1.55)	$(-4.68)^{***}$	$(-2.30)^{**}$		
Mixed Copula	4.61	0.0671	-0.0417	0.0646	-0.0295	0.0407	-0.0231	-0.1187	0.0162	0.0151
	$(4.25)^{***}$	$(3.38)^{***}$	(-1.53)	(1.47)	(-0.70)	(0.74)	(-1.35)	$(-2.36)^{**}$		

Note: This table shows results of regressing daily portfolio return series onto Fama and French (2016)'s five factors factors plus momentum and long-term reversal factors over the period between July 1991 and December 2015 (6173 observations). Section 1 shows the Return on Committed Capital and Section 2 on Fully Invested Capital after transaction costs. Pairs are formed based on the smallest sum of squared deviations. The t-statistics (shown in parentheses) are computed using Newey-West standard errors with six lags. Intercepts have been multiplied by 100 to aid interpretation.
***, **, * significant at 1%, 5% and 10% levels, respectively.

Table A.2: Daily risk profile of top 35 pairs 1991-2015: Fama and French (2016)'s five factors plus Momentum and Long-Term Reversal factors.

Strategy	Intercept	Rm-Rf	SMB	HML	RMW	CMA	Mom	LRev	R^2	R^2_{adj}
					Section 1: Return on Committed Capital					
Distance	1.13	0.0283	-0.0041	-0.0050	0.0065	0.0282	-0.0316	-0.0255	0.0342	0.0331
	$(3.65)^{***}$	$(5.54)^{***}$	(-0.46)	(-0.46)	(0.46)	$(2.00)^{**}$	$(-5.35)^{***}$	$(-2.10)^{**}$		
Mixed Copula	0.28	0.0043	-0.0013	-0.0001	-0.0021	0.0033	-0.0034	-0.0037	0.0091	0.0080
	$(3.96)^{***}$	$(3.81)^{***}$	(-0.69)	(-0.04)	(-0.69)	(0.98)	$(-2.17)^{**}$	(-1.14)		
					Section 2: Return on Fully Invested Capital					
Distance	2.14	0.0524	-0.0151	-0.0002	0.0067	0.0657	-0.0605	-0.0552	0.0371	0.0360
	$(3.80)^{***}$	$(5.32)^{***}$	(-0.94)	(-0.01)	(0.26)	$(2.51)^{**}$	$(-5.35)^{***}$	$(-2.23)^{**}$		
Mixed Copula	4.76	0.0685	-0.0407	0.0616	-0.0334	0.0473	-0.0231	-0.1211	0.0166	0.0155
	$(4.38)^{***}$	$(3.46)^{***}$	(-1.50)	(1.40)	(-0.79)	(0.86)	(-1.35)	$(-2.42)^{**}$		

Note: This table shows results of regressing daily portfolio return series onto Fama and French (2016)'s five factors factors plus momentum and long-term reversal factors over the period between July 1991 and December 2015 (6173 observations). Section 1 shows the Return on Committed Capital and Section 2 on Fully Invested Capital after transaction costs. Pairs are formed based on the smallest sum of squared deviations. The t-statistics (shown in parentheses) are computed using Newey-West standard errors with six lags. Intercepts have been multiplied by 100 to aid interpretation.
***, **, * significant at 1%, 5% and 10% levels, respectively.

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