

Performance of Pairs Trading on the S&P 500 using Distance and Mixed Copula models

Fernando A. Boeira Sabino da Silva^{1,2}, Flavio A. Ziegelmann^{1,2}, Joao
F. Caldeira²

¹Department of Statistics/UFRGS, ²PPGE/UFRGS

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- Pairs trading is a statistical arbitrage strategy that involves the simultaneous long/short of two relatively mispriced stocks which have strong historical co-movements.
- The most popular strategy is known as distance method (see [Gatev et al., 2006](#)).
- The distance strategy ([Gatev et al.](#), 2006) uses the distance between normalized prices to capture the degree of mispricing stocks. According to [Xie et al., 2014](#) the distance method has a multivariate normal nature since it assumes a symmetric distribution of the spread between normalized prices of the stocks within a pair and it uses a single distance measure, which can be seen as an alternative measurement of the linear association, to describe the relationship between two stocks.

- Linear correlation fully describes the dependence between securities if the series have joint normal distribution.
- However, a main feature of joint distributions characterized by tail dependence is the presence of heavy and possibly asymmetric tails. Therefore, a single distance measure may fail to catch the dynamics of the spread between a pair of securities, and thus initiate and close the trades at non-optimal positions.
- [Lie and Wu., 2013](#) proposed a pairs trading strategy based on 2-dimensional copulas to overcome the limitations of the distance method.
- Copulas are able to model better the empirically verified regularities normally attributed to multivariate financial returns.

- (1) Conduct an empirical investigation to understand any differences in performances of the distance and copula strategies. We want to assess if a more sophisticated approach than the distance method can take advantage of any market frictions.
- (2) Investigate how trading costs and different measures for pairs selection affect the profitability of these strategies.
- (3) Investigate the sensitivity of copula's method to different opening thresholds.
- (4) Find if pairs trading profitability is associated to exposure to different systematic risk factors.

Two-Dimensional Pairs Trading



Theorem 1

(Sklar's Theorem) Let X_1, \dots, X_d be random variables with distribution functions F_1, \dots, F_d , respectively. Then, there exists an d -copula C such that,

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)), \quad (1)$$

for all $\mathbf{x} = (x_1, \dots, x_d) \in \mathbb{R}^d$.

■ Assuming that $F(\cdot)$ and $C(\cdot)$ are differentiable, by (1) we have

$$\frac{\partial^d F(x_1, \dots, x_d)}{\partial x_1 \dots \partial x_d} \equiv f(x_1, \dots, x_d) = \frac{\partial^d C(F_1(x_1), \dots, F_d(x_d))}{\partial x_1 \dots \partial x_d} \quad (2)$$

$$= c(u_1, \dots, u_d) \prod_{i=1}^d f_i(x_i), \quad (3)$$

where $u_i = F_i(x_i)$, $i = 1, \dots, d$.

- Sklar's theorem states that any multivariate joint distribution can be written in terms of their univariate marginal distribution function and the dependence structure (represented in C) between the variables.
- Copulas accommodate various forms of dependence through suitable choice of the copula “correlation matrix” since they conveniently separate marginals from dependence component.

Strategy	Associations Captured	Required Marginal Distributions
Distance	Linear	Gaussian
Copula	Linear and Nonlinear	No assumption

- From a modelling perspective, Sklar's Theorem allow us to estimate the multivariate distribution in two parts: (i) finding the marginal distributions; (ii) finding the dependency between the filtered data from (i).

- Daily data of adjusted closing prices of all shares that belongs to the S&P500 market index from July 2nd, 1990 to December 31st, 2015. We obtain the adjusted closing prices from Bloomberg and the returns on the Fama and French factors from French's website.
- The data set sample period is made up of 6,426 days and includes a total of 1100 stocks over all periods. Only stocks that are listed during the formation period are included in the analysis, *i.e.*, around 500 stocks in each trading period.

- Pairs are sorted based on the sum of squared and absolute differences between their normalized prices during the next 12 months (formation period) adjusting them by dividends, stock splits and other corporate actions. Specifically, the pairs are formed using data from January to December or from July to June.
- Prices are scaled to \$1 at the beginning of each formation period and then evolve using the return series. We then select the top 5, 10,...,35 of those combinations that have the smallest sum of squared/absolute spreads, allowing re-selection of a specific pair, during the formation period. These pairs are then traded in the next six months (trading period).
- In [Gatev *et al.*, 2006](#) , when the spread diverges by two or more standard deviations (which is calculated in the formation period) from the mean, the stocks are assumed to be mispriced in terms of their relative value to each other and thus we open a short position in the outperforming stock and a long in the underperforming one.

- The price divergence is expected to be temporary, i.e., the prices are expected to converge to its long-term mean value of 0 (mean-reverting behavior). Any value of spread above (below) two positive (negative) standard deviation indicates overvaluation (undervaluation) (see [Gatev et al., 2006](#)). The trading positions are closed when the spread reverts to the mean value of 0.
- Trades that do not converge can result in a loss if they are still open at the end of the trading period when they are automatically closed. This results in fat left tails.
- Therefore, since the conditional variance is empirically higher for large negative returns and smaller for positive returns, it may be inappropriate to use constant trigger points because the volatility differs at different price levels.

- To calculate the daily percentage excess returns for a pair, we compute

$$r_{pt} = w_{1t}r_t^L - w_{2t}r_t^S, \quad (4)$$

where L and S stands for long and short, respectively.

- The weights w_{1t} and w_{2t} are initially assumed to be one. After that, they change according to the changes in the value of the stocks, *i.e.*, $w_{it} = w_{it-1}(1 + r_{it-1})$.
- Following [Gatev *et al.*, 2006](#), we calculate returns using two weighting schemes: the return on committed capital and on fully invested capital.

- By using the fact that the partial derivative of the copula function gives the conditional distribution function, i.e.,

$$P(U_1 \leq u_1 | U_2 = u_2) = \frac{\partial C(u_1, u_2)}{\partial u_2} = P(X_1 \leq x_1 | X_2 = x_2),$$

$$P(U_2 \leq u_2 | U_1 = u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1} = P(X_2 \leq x_2 | X_1 = x_1)$$

Xie *et al.*, 2014 define a measure to denote the degree of mispricing.

Definition 2

- Let R_t^X and R_t^Y represent the random variables of the daily returns of stocks X and Y on time t , and the realizations of those returns on time t are r_t^X and r_t^Y , we have

$$MI_{X|Y}^t = P(R_t^X < r_t^X | R_t^Y = r_t^Y)$$

and

$$MI_{Y|X}^t = P(R_t^Y < r_t^Y | R_t^X = r_t^X),$$

- Given current realizations r_t^X and r_t^Y , if F_X and F_Y are the marginal distribution functions of R_t^X and R_t^Y and C is the copula connecting F_X and F_Y , we define $u_1 = F_X(r_t^X)$ and $u_2 = F_Y(r_t^Y)$, and have

$$MI_{X|Y}^t = \frac{\partial C(u_1, u_2)}{\partial u_2} = P(R_t^X < r_t^X \mid R_t^Y = r_t^Y)$$

and

$$MI_{Y|X}^t = \frac{\partial C(u_1, u_2)}{\partial u_1} = P(R_t^Y < r_t^Y \mid R_t^X = r_t^X). \quad (5)$$

- A conditional value of 0.5 means that the two underlying stocks are considered fairly-valued.

- The conditional probabilities, $M_t^{X|Y}$ and $M_t^{Y|X}$, only measure the degrees of relative mispricing for a single day. To determine an overall degree of relative mispricing we follow [Rad et al., 2016](#).
- Let $m_{1,t}$ and $m_{2,t}$ be the overall mispricing indexes of stocks X_1 and X_2 , defined by $\left(M_t^{X|Y} - 0.5\right)$ and $\left(M_t^{Y|X} - 0.5\right)$, respectively. At beginning of each trading period two cumulative mispriced indexed M_1 and M_2 are set to zero and then evolve for each day through

$$\begin{aligned} M_{1,t} &= M_{1,t-1} + m_{1,t} \\ M_{2,t} &= M_{2,t-1} + m_{2,t} \end{aligned}$$

- Positive (negative) M_1 and negative (positive) M_2 are interpreted as stock 1 (stock 2) being overvalued relative to stock 2 (stock 1).
- We perform a sensitivity analysis to open a long-short position once one of the cumulative indexes is above 0.05, 0.10,..., 0.55 and the other one is below -0.05, -0.10,...,-0.55 at the same time for Top 5, 10,...,35 pairs.
- The positions are unwound when both cumulative mispriced indexes return to zero. The pairs are then monitored for other possible trades throughout the remainder of the trading period.

- (1) Calculate daily returns for each stock during the formation period and estimate the marginal distributions of returns separately.

- We fit an appropriate ARMA(p,q)-GARCH(1,1) model to each univariate time series.

- (2) Estimate the two-dimensional copula model to data that has been transformed to $[0,1]$ margins, i.e.,

$$H(r_t^X, r_t^Y) = C(F_X(r_t^X), F_Y(r_t^Y)),$$

where H is the joint distribution, r_t^X e r_t^Y are stock returns and C is the copula.

- Copulas that are tested in this step are Gaussian, t, Clayton, Frank, Gumbel, one Archimedean mixture copula consisting of the optimal linear combination of Clayton, Frank and Gumbel copulas and one mixture copula consisting of the optimal linear combination of Clayton, t and Gumbel copulas.

- (3) Take the first derivative of the copula function to compute conditional probabilities and measure mispricing degrees $MI_{X|Y}$ and $MI_{Y|X}$ for each day in the trading period using the copula and estimated parameters.

- (4) Build long and short positions of Y and X on the days that $M_{1,t} > \Delta_1$ and $M_{2,t} < \Delta_2$ if there is no positions in X or Y . Conversely, build positions long/short of X and Y on the day that $M_{1,t} < \Delta_2$ and $M_{2,t} > \Delta_1$ if there is no positions in X or Y .
- All positions are closed if $M_{1,t}$ reaches Δ_3 or $M_{2,t}$ reaches Δ_4 , where $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are predetermined thresholds or are automatically closed out on the last day of the trading period if they do not reach the thresholds.
 - Here we use $\Delta_1 = 0.2, \Delta_2 = -0.2$ and $\Delta_3 = \Delta_4 = 0$.

Empirical Results

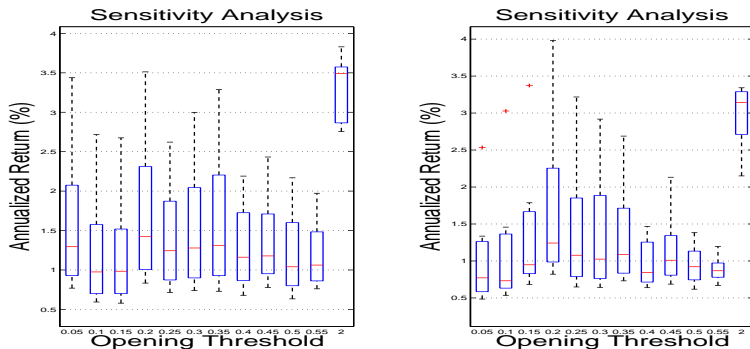


Figure 1: Annualized returns of pairs trading strategies after costs on committed capital

These boxplots show annualized returns on committed capital after transaction cost to different opening thresholds from July 1991 to December 2015 for Top 5 to Top 35 pairs. Pairs are formed based on the smallest sum of absolute (left) and squared (right) deviations.

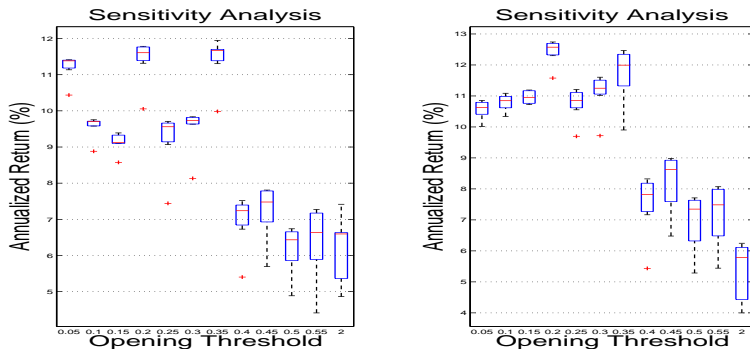


Figure 2: Annualized returns of pairs trading strategies after costs on fully invested capital

These boxplots show annualized returns on gully invested capital after transaction cost to different opening thresholds from July 1991 to December 2015 for Top 5 to Top 35 pairs. Pairs are formed based on the smallest sum of absolute (left) and squared (right) deviations.

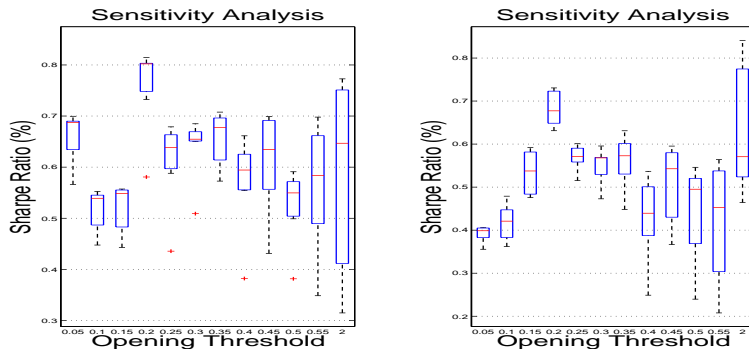


Figure 3: Sharpe ratio of pairs trading strategies after costs on committed capital

These boxplots show Sharpe ratios on committed capital after transaction cost to different opening thresholds from July 1991 to December 2015 for Top 5 to Top 35 pairs. Pairs are formed based on the smallest sum of absolute (left) and squared (right) deviations.

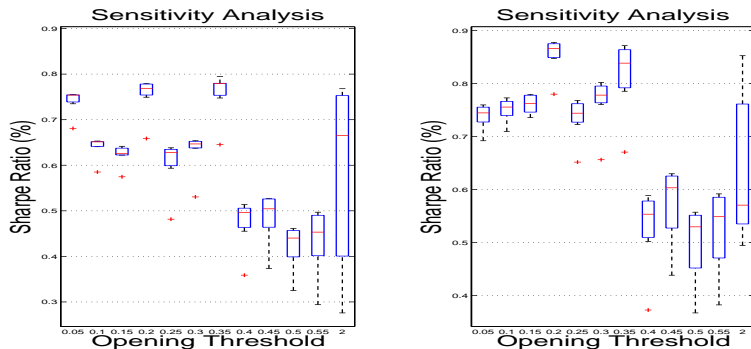


Figure 4: Sharpe ratio of pairs trading strategies after costs on fully invested capital

These boxplots show Sharpe ratios on fully invested capital after transaction cost to different opening thresholds from July 1991 to December 2015 for Top 5 to Top 35 pairs. Pairs are formed based on the smallest sum of absolute (left) and squared (right) deviations.

Table 1: Excess returns on committed capital of pairs trading strategies on portfolios of Top 5, 20 and 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio	t-stat	% of negative trades	MDD1	MDD2
Return on Committed Capital							
<i>Panel A - Top 5 pairs</i>							
Distance	2.60	0.31	0.58	1.86*	46.98	-6.73	-19.62
Mixed Copula	3.98	0.63	1.08	3.49***	41.79	-4.36	-9.29
<i>Panel B - Top 20 pairs</i>							
Distance	3.14	0.65	1.13	3.32***	48.02	-3.88	-9.69
Mixed Copula	1.24	0.64	1.04	3.52***	41.33	-2.07	-3.43
<i>Panel C - Top 35 pairs</i>							
Distance	3.06	0.76	1.34	3.87***	48.02	-2.70	-7.52
Mixed Copula	0.82	0.73	1.19	3.95***	41.31	-1.18	-1.98

Note: Summary statistics of the annualized excess returns, annualized Sharpe and Sortino ratios on portfolios of top 5, 20 and 35 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of squared deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively.

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 2: Excess returns on committed capital of pairs trading strategies on portfolios of Top 5, 20 and 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio	t-stat	% of negative trades	MDD1	MDD2
Return on Committed Capital							
<i>Panel A - Top 5 pairs</i>							
Distance	3.83	0.46	0.85	2.64***	46.83	-6.12	-18.85
Mixed Copula	3.51	0.58	1.00	3.27***	40.74	-3.73	-11.12
<i>Panel B - Top 20 pairs</i>							
Distance	2.85	0.57	1.00	2.94***	47.85	-4.27	-11.38
Mixed Copula	1.42	0.80	1.35	4.42***	40.34	-1.29	-3.02
<i>Panel C - Top 35 pairs</i>							
Distance	3.49	0.84	1.49	4.30***	47.76	-2.35	-7.90
Mixed Copula	0.83	0.80	1.35	4.40***	40.34	-0.74	-1.82

Note: Summary statistics of the annualized excess returns, annualized Sharpe and Sortino ratios on portfolios of top 5, 20 and 35 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of absolute deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively.

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 3: Excess returns on fully invested capital of pairs trading strategies on portfolios of Top 5, 20 and 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio	t-stat	% of negative trades	MDD1	MDD2
Return on Fully Invested Capital							
<i>Panel A - Top 5 pairs</i>							
Distance	4.01	0.28	0.57	1.81*	46.98	-8.70	-38.36
Mixed Copula	11.58	0.78	1.43	4.26***	41.79	-9.00	-25.68
<i>Panel B - Top 20 pairs</i>							
Distance	6.12	0.67	1.20	3.58***	48.02	-5.43	-20.03
Mixed Copula	12.30	0.85	1.54	4.60***	41.31	-9.00	-25.68
<i>Panel C - Top 35 pairs</i>							
Distance	5.71	0.75	1.37	4.01***	48.02	-4.23	-15.07
Mixed Copula	12.73	0.88	1.59	4.73***	41.28	-9.00	-25.68

Note: Summary statistics of the annualized excess returns, annualized Sharpe and Sortino ratios on portfolios of top 5, 20 and 35 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of squared deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively.

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 4: Excess returns on fully invested capital of pairs trading strategies on portfolios of Top 5, 20 and 35 pairs after costs.

Strategy	Mean Return (%)	Sharpe ratio	Sortino ratio	t-stat	% of negative trades	MDD1	MDD2
Return on Fully Invested Capital							
<i>Panel A - Top 5 pairs</i>							
Distance	7.42	0.49	1.00	2.97***	46.83	-7.87	-28.40
Mixed Copula	10.06	0.66	1.18	3.69***	40.73	-12.93	-43.71
<i>Panel B - Top 20 pairs</i>							
Distance	5.49	0.59	1.05	3.15***	47.85	-5.51	-24.74
Mixed Copula	11.78	0.78	1.37	4.26***	40.32	-12.93	-42.81
<i>Panel C - Top 35 pairs</i>							
Distance	6.62	0.85	1.55	4.49***	47.76	-3.56	-15.38
Mixed Copula	0.83	0.80	1.35	4.40***	40.34	-0.74	-1.82

Note: Summary statistics of the annualized excess returns, annualized Sharpe and Sortino ratios on portfolios of top 5, 20 and 35 pairs between July 1991 and December 2015 (6,173 observations). Pairs are formed based on the smallest sum of absolute deviations. The t-statistics are computed using Newey-West standard errors with a six-lag correction. The columns labeled MDD1 and MDD2 compute the largest drawdown in terms of maximum percentage drop between two consecutive days and between two days within a period of maximum six months, respectively.

***, **, * significant at 1%, 5% and 10% levels, respectively.

Table 5: Trading statistics.

Strategy	Distance	Mixed Copula
<i>Panel A: Top 5</i>		
Average price deviation trigger for opening pairs	0.0594	0.0665
Total number of pairs opened	352	348
Average number of pairs traded per six-month period	7.18	7.10
Average number of round-trip trades per pair	1.44	1.42
Standard Deviation	1.0128	1.33
Average time pairs are open in days	50.70	37.70
Standard Deviation	39.24	38.93
Median time pairs are open in days	38.5	19
<i>Panel B: Top20</i>		
Average price deviation trigger for opening pairs	0.0681	0.0821
Total number of pairs opened	1312	749
Average number of pairs traded per six-month period	26.78	15.29
Average number of round-trip trades per pair	1.34	0.76
Standard Deviation	0.99	0.99
Average time pairs are open in days	51.65	23;60
Standard Deviation	39.62	32.90
Median time pairs are open in days	41	9

Table 6: Trading statistics.

Strategy	Distance	Mixed Copula
<i>Panel A: Top 5</i>		
Average price deviation trigger for opening pairs	0.0622	0.0605
Total number of pairs opened	346	345
Average number of pairs traded per six-month period	7.06	7.04
Average number of round-trip trades per pair	1.41	1.41
Standard Deviation	1.04	1.40
Average time pairs are open in days	50.84	36.37
Standard Deviation	38.74	38.93
Median time pairs are open in days	39.5	19
<i>Panel B: Top20</i>		
Average price deviation trigger for opening pairs	0.0692	0.0353
Total number of pairs opened	1302	733
Average number of pairs traded per six-month period	26.58	14.96
Average number of round-trip trades per pair	1.33	0.75
Standard Deviation	0.99	1.02
Average time pairs are open in days	52.74	22.09
Standard Deviation	40.20	32.34
Median time pairs are open in days	42	8

Cumulative Excess Returns

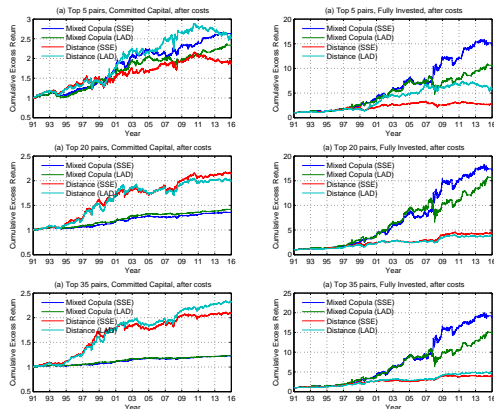


Figure 5: Cumulative excess returns of pairs trading strategies after costs

This figure shows how an investment of \$1 evolves from July 1991 to December 2015 for each of the strategies.

Sharpe ratio

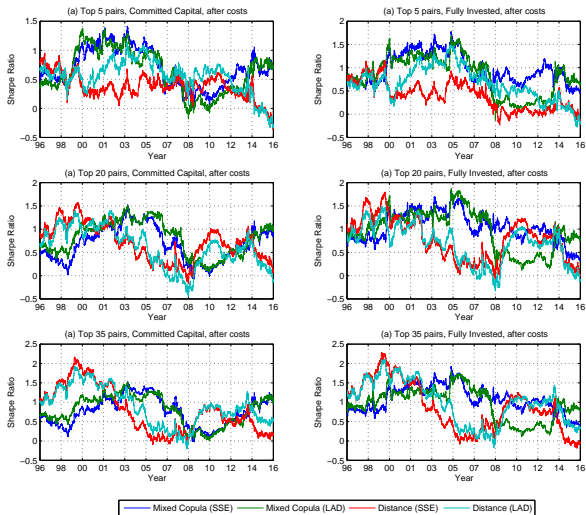
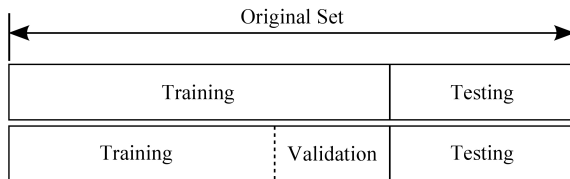


Figure 6: Five-year rolling window Sharpe ratio after costs



- Create similar copula-based arbitrage for triplets to increase information dependency information and measure relative pricing more comprehensively.



- Randomized Dependence Coefficient (RDC): measure of nonlinear dependence between random variables of arbitrary dimension. RDC is defined in terms of correlation of random non-linear copula projections (Lopez-Paz *et al.*, 2013).

Thanks! Any questions?