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Experimental Particle Physics
Understanding the measurements and searches at the Large Hadron Collider
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Chapter 1

Groundwork

Physics is really nothing more than a search for ultimate simplicity, but so far all we have is a kind of elegant messiness.

—Bill Bryson

We start by briefly reviewing natural units in section 1.1, followed by a discussion of the particles discovered so far and open questions in section 1.2. Relativistic kinematics is used extensively in particle physics, so we include a brief overview and some examples in section 1.3.

A full theoretical overview of Quantum Field Theory (QFT), Quantum Chromodynamics (QCD) and the Standard Model (SM) of particle physics is beyond the scope of this book. The interested reader is referred to books [1–7], which the author has found helpful. Application of statistical methods to interpret the results is also of critical importance. Again, for a comprehensive overview, dedicated resources [8–10] should be consulted. These references are by no means complete, and additionally there are excellent lecture notes and slides from numerous summer schools, which the author always finds extremely helpful.

1.1 Natural units

We are familiar with the three base units, [Length], [Mass] and [Time]. Along with them, [Electric Current], [Temperature], [Luminous intensity], and [Amount of substance] complete the basic units (square brackets is the usual way of denoting units or dimensions). In the most commonly used *Systéme International* (SI) units, the first three are represented by meter, kilogram and second. That is sufficient for everyday instances, but when dealing with quantities which are orders-of-magnitude larger or smaller, as encountered in particle physics, they prove inadequate. Let us see how we can derive units suitable for our purpose.

In electromagnetism, we have the force $\vec{F}$ experienced by a charge q, moving with a velocity $\vec{v}$, in an electric field $\vec{E}$ and magnetic field $\vec{B}$, given by:

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$$\vec{F} = q(\vec{E} + (\vec{v}/c) \times \vec{B}),$$

where c is the speed of light. In Quantum Mechanics, Planck's constant h (actually the reduced Planck's constant, $\hbar = h/2\pi$, which has the dimension of energy multiplied by time) is the fundamental constant. Energy, E, is expressed as:

$$E = \hbar\omega$$

with ω being the angular frequency which is quantised.

We adopt the convention where $c = \hbar = 1$. That implies that the velocities can be expressed as dimensionless fractions of c, and the energies (and masses, which now have the same unit) can be expressed as reciprocals of time. This is termed *natural units*. The advantage is that we end up with reasonable numbers for the quantities of interest in the subatomic domain, as c and $\hbar$ are respectively very large and small.

This imposes two constraints on three of the basic units, leaving us the freedom to define one of them as per our convenience. That is usually taken to be energy, and expressed in terms of electron-volts (eV, with prefixes as needed), which is defined in terms of the rest mass of the electron (0.511 keV or 9.11×10^{-31} kg). This choice is arbitrary. To give a sense of this measure, one TeV is about the energy caused by the motion of a flying mosquito. At the Large Hadron Collider (LHC), energy of the order of TeV is produced at a much smaller length scale in each collision, essentially within the radius of a proton.

> In particle physics, the quantities are almost always presented in natural units. Mass, momentum and energies will all be expressed in terms of energy, using some prefix of eV. The conversion to ordinary units merely involves multiplying or dividing the formula or the value by some combination(s) of c and $\hbar$.

For completeness, the values of second, meter and kg in natural unit are given by:

$1 \text{ m} = 5.076 \times 10^{15} \text{ GeV}^{-1}$.
$1 \text{ s} = 1.519 \times 10^{24} \text{ GeV}^{-1}$.
$1 \text{ kg} = 5.625 \times 10^{26} \text{ GeV}$.

When we refer to the mass of the Higgs boson as 125 GeV, this actually means the mass is $125 \times 10^{9}\ c^{-2}\ \text{eV} = 2.228 \times 10^{-25}$ kg.

1.2 Particle content

1.2.1 What we have

The elementary particles discovered so far are depicted in figure 1.1, which is almost like the periodic table of elements, but with elementary particles instead. The Standard Model [11] of particle physics is the theoretical framework which encapsulates our best understanding of the interactions of the elementary particles discovered so far.

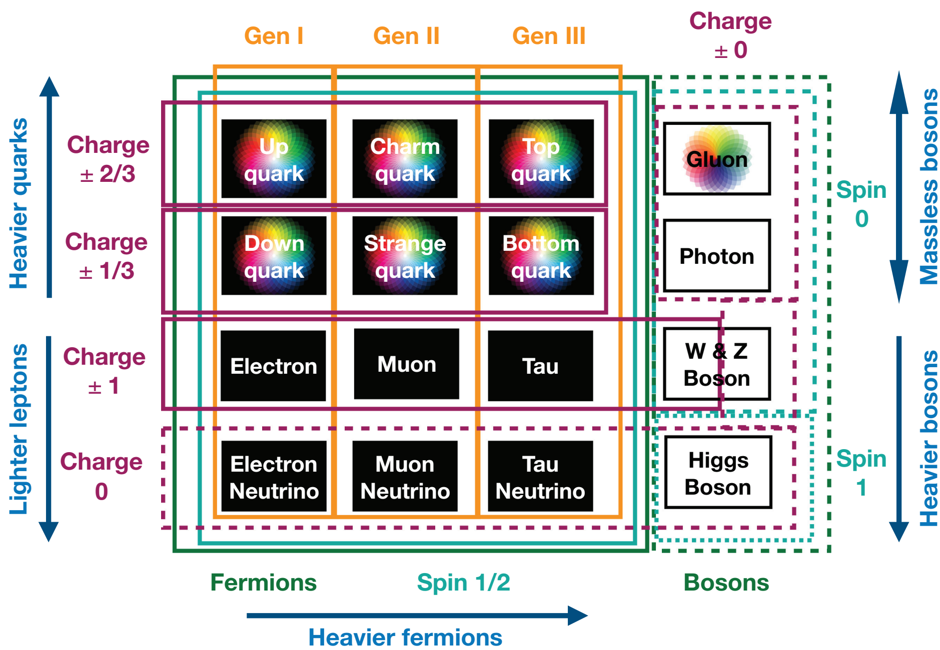

Figure 1.1. The 'periodic table' of elementary particles, adapted from [12]. The fermions are in the black boxes, while the bosons are in the white boxes. The colour wheels in the boxes indicate that the corresponding particles have colour charge, and the absence of those indicate that the corresponding particles do not have colour charge. The green lines group together particles by spin. The purple lines group together particles by charge. The orange lines group together fermions by generation.

The elementary particles are classified as *fermions* (named after Enrico Fermi) or *bosons* (named after Satyendranath Bose), depending on whether they have half integral or integral spin quantum numbers. The so-called matter particles are fermions, while the bosons are the force carriers. Fermions are divided into leptons and quarks, based on how they interact. Three of the leptons: electrons, muons and the tau leptons (τ) carry electromagnetic charge, and each has a corresponding neutral neutrino. The electron and muon are stable at the LHC detectors, while the τ leptons decay before reaching the detector. They can decay to an electron or a muon with the corresponding anti-neutrino or to one or two pions. Therefore, experimentally the term lepton refers only to electrons and muons in most cases, and we will follow the same convention here. Their (electromagnetic and weak) interactions are described by quantum electrodynamics (QED).

The quarks in addition carry so-called colour charge, and their (strong) interactions are described by quantum chromodynamics (QCD) (the name quark comes from James Joyce's novel *Finnegans Wake*). They can also interact with the leptons via electromagnetic and weak interactions. The six quarks are named up (u), down (d), charm (c), strange (s), top (t) and bottom (b). Each of the quarks can have three possible colour charges, which are termed as red (R), blue (B) and green (G), with corresponding anti-colours. A combination of red, blue and green quarks, as well as of one coloured quark and its anti-coloured quark results in a white (or uncoloured, alternatively termed colour-neutral) particle. It must be kept in mind that the colours are just labels for us to visualise abstract quantum numbers, and have no connection to the everyday colours from the light spectrum. Gluons carry a combination of colour and anti-colour charge in order to participate in strong interactions (i.e. the glue that holds the quarks together). That can lead to nine possible combinations. However, combinations like $R\bar{R}$, $G\bar{G}$ or $B\bar{B}$, that correspond to colourless states cannot exist, neither can a linear combination $(R\bar{R} + G\bar{G} + B\bar{B})$, as that cannot change the colour of a quark in an interaction. So colour-neutral states are not possible for gluons. However, unbalanced linear combinations are possible, but only two combinations can be shown to be linearly independent from QCD. So it leads to eight gluons, designated as $R\bar{G}$, $R\bar{B}$, $G\bar{B}$, $G\bar{R}$, $B\bar{R}$, $B\bar{G}$, $\frac{1}{\sqrt{2}}(R\bar{R} - G\bar{G})$, and $\frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B})$, where the last two combinations can be formed differently as well. This is referred to as the colour *octet*[1].

Each fermion has a corresponding anti-particle, which is identical to the specific fermion, except it has opposite charge. For the leptons, the anti-particles carry opposite electromagnetic charge, and the anti-quarks (denoted by, for example, $\bar{t}$, corresponding to t), carry anti-colour charge as discussed above (red and anti-red, for example, can also combine to form a white). The fermions are divided into three generations (or flavours), with members of subsequent generations having higher masses than the preceding ones. Also, only the first generation is considered stable, the rest can decay with different lifetimes. In experiments, sometimes the second or third generation quarks are referred to as *heavy flavours*. The top quark is the heaviest known elementary particle (roughly as heavy as a tungsten atom), and it has a lifetime of 10^{-23} s, which means it does not even get to decay to hadrons directly.

QCD dictates that the colour-charged particles cannot be free in nature, and the properties of strong forces results in their *confinement*. Quarks can create colour-neutral objects in two ways, either by forming a quark and anti-quark pair, termed mesons, (integral spin of zero and $\pm$ unity) or by the appropriate colour (or anti-colour) combination of three quarks (or anti-quarks), termed baryons (half integral spin of $\pm$ 1/2 or $\pm$ 3/2). The baryons and mesons are collectively called the *hadrons*.

Some examples of mesons are pions, kaons, ρ, Ω etc. They all include two different types of quarks and anti-quarks (like neutral pion is made from u and $\bar{d}$, and the charged pions from combinations of $u\bar{u}$ and $d\bar{d}$). The exceptions are ϕ, J/ψ (it is believed that the name was chosen to look like the Chinese name of Samuel Ting, the discoverer) and Υ mesons, that are composed of $s\bar{s}$, $b\bar{b}$ and $c\bar{c}$ combinations, respectively. When a meson includes the strange quark s, it is sometimes termed as a strange meson. Protons and neutrons are examples of baryons (consisting of u, u, d and u, d, d respectively). That explains the naming of the LHC as well, with the word large characterising the collider. Tetraquarks containing two heavy (bottom) quarks and two light anti-quarks have been

[1] Formally QCD is expressed in terms of non-abelian symmetry group SU(3), where quarks are in fundamental representation of SU(3) while gluons are in adjoint representation of SU(3), which leads to the colour octet gluons, but such a discussion is beyond the scope of this book.

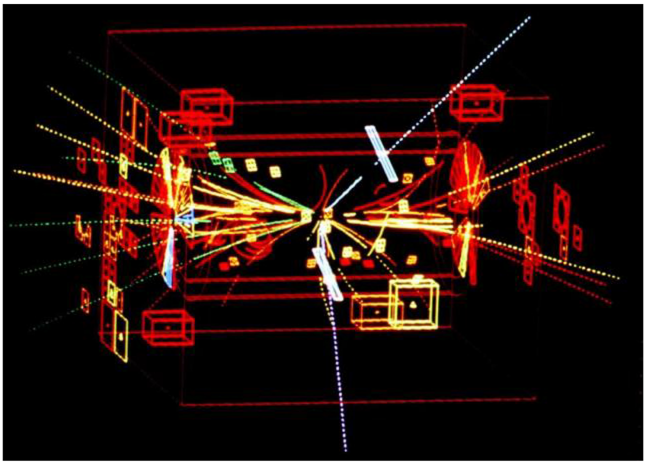

Figure 1.2. An actual event display from one of the first events recorded with a Z boson [31]. The white dashed lines indicate the two muons coming from the decay of the Z boson (© 2018 CERN).

proposed, and pentaquarks have been observed by the LHCb experiment [13] in Λ_b^0 baryon decay (the Λ baryons contain one up quark, one down quark, and a third quark from a higher generation, in this case a b-quark).

The bosons mediate in all the interactions. The bosons with non-zero spin are termed *vector or gauge* bosons, which include photon, W/Z bosons, and gluons. The latter participates in the strong interaction. The only *scalar* boson, i.e. a boson with a zero spin, is the Higgs boson. The Higgs boson was postulated in order to theoretically reconcile the existence of the massless photon, which carries the long-range electromagnetic force, and the massive W/Z bosons, which carry the short-range weak force. It was based on the independent work of Peter Higgs [14, 15], Francois Englert and Robert Brout [16], and Gerald Guralnik, Carl Hagen and Tom Kibble [17]. It is also closely related to the concept of superconductivity generated by the Anderson mechanism [18]. It must be noted that the Higgs mechanism only gives mass to fermions (quarks and leptons) when they interact with the Higgs field generated by so-called spontaneous symmetry breaking. The mass of everyday objects comes from protons and neutrons, which get their masses from the binding energy of quarks and gluons, governed by QCD. The Higgs boson is often referred to as the ‘God particle’ in popular culture, which came about when Leon Lederman apparently referred to it as the ‘Goddamn Particle’, which was (unfortunately) shortened [19]!

A brief comment about the discovery of new particles in colliders is in order here. The W and Z bosons were discovered in 1983 at the CERN Super Proton Synchrotron (SPS) collider, by the UA1 and UA2 experiments [20–23]. The discoveries were made essentially by looking at event displays (figure 1.2 shows an example) where the Z boson decayed to an electron–positron pair. Interested readers can refer to the book *Nobel Dreams* [24] to read the fascinating background stories about this discovery. The Υ (and, hence, the bottom quark) was discovered earlier at Fermilab in 1977, but it was at a fixed target experiment [25]. The top quark was discovered in 1995 at the Tevatron [26, 27] by CDF (Collider Detector at Fermilab) and D ∅ (which is just named after the quadrant position of the detector in the ring; in fact CDF was supposed to be named B∅ before it chose its name) experiments. The Higgs boson was discovered in 2012 by the ATLAS (A Toroidal Lhc ApparatuS) and CMS (Compact Muon Solenoid) experiments at the LHC [28, 29], and the book *Smashing Physics* [30] gives behind the scene stories from the effort.

1.2.2 What we do not have

It is impossible to say what we do not have, in fact that is why the field of particle physics is so exciting. We are always searching for the *known–unknowns*, based on our best guess of what should be there, but there always remains the possibility that we can find a true *unknown–unknown*.

The Standard Model (SM) has been remarkably successful in predicting and explaining most experimental observations so far, but it can be a *low-energy* approximation of a broader framework, just like classical mechanics is perfectly suitable to explain most everyday phenomena, but it is valid at a limit of the underlying quantum mechanical principles governing the world. The SM contains 24 free parameters that are simply not calculable and have to be taken from experiment. Some of the significant open questions are:

- Why do particles have these particular mass values? Why are there three generations of fermions, with increasing masses? Why are the neutrinos not massless (or so light)? How do neutrinos transform between the flavours (which is known as *neutrino oscillation*)? Is there a fourth type of neutrino, called a *sterile neutrino* that we have not discovered yet?
- Planck (after Max Planck) mass is defined to be $\sqrt{c\hbar/G}$, where G is the gravitational constant. It roughly corresponds to the mass of the smallest possible black hole. An obvious question arises, why is the mass of everything we see, including that of elementary particles, so small compared to Planck mass? In other words, why is the gravitational mass scale so large? This is called the *hierarchy problem*. This is tied to the mass of the Higgs boson, which determines the electroweak symmetry breaking. The quantum corrections to the Higgs field almost magically cancel each other out, making the Higgs boson stable. This is called the little hierarchy problem, with an explanation still eluding us. I find an analogy from reference [32] particularly helpful. *This is a bit like a bank account which has huge inputs and outputs during the month, but balances at exactly zero on the last day of every month. Looking at this behaviour, an accountant would be unlikely to think this was a coincidence. There is probably some principle underlying the operation of the account which forces this to happen.*
- Why is gravity so weak, and why is it the only force that has not been explained by a quantum field theory? The hypothetical particle *graviton* has been proposed as the force carrier of gravity, without any experimental

evidence so far. This leads to the idea that all forces will be unified at some scale (termed grand unification).

- The observation in 1998 by the Hubble Space Telescope, that the Universe is expanding now at a much faster rate than before, raised an important question. Current models predicted a slowing down of the expansion, and the proposed solution was to postulate the existence of *dark energy*. Other cosmological observations indicated that the galaxies are spinning much faster than they should be, based on the gravitational pull of their visible matter. The existence of *dark matter* was postulated to explain this. The Universe is estimated to consist of 68% dark energy and about 27% dark matter. Not much is known about the origin and composition of dark energy and dark matter.
- The Universe is dominated by matter, whereas during its creation in the Big Bang, matter and anti-matter should have been produced in equal parts. The SM has no mechanism to explain the imbalance. The combination of charge conjugation (particles and anti-particles behave the same way) and parity symmetry is referred to as CP. Strong and electromagnetic interactions obey CP symmetry, while CP violation is seen to occur in some weak decays. The first observation was from Fermilab's KTeV collaboration [33], where kaons were seen to preferentially decay to electrons rather than to positrons (along with a charged pion and the corresponding neutrino). The kaons consist of strange quarks, and later CP violation was also observed in neutral B mesons, containing bottom quarks. Recently, the LHCb collaboration at the LHC has observed CP violation in the D^0 meson, which contains a pair of charm quarks [34]. However, the observed CP violation is not enough to explain the observed matter–anti-matter asymmetry in the Universe.

So far experimental particle physics has been mostly driven by theoretical predictions, in the same way that predictions of the Higgs boson in the middle of the last century dictated experimental programmes. Now we are perhaps at a unique position, as we do not have a specific answer to any of the unanswered questions, so in some sense experimental results have to show the way forward.

1.3 Relativistic kinematics

1.3.1 Basic ideas

Particles participating in or emerging from collisions at colliders move at speeds close to the speed of light. Therefore, they follow relativistic kinematics. We can define the usual dimensionless quantities, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$, where v denotes the velocity and γ is referred to as the *boost* factor, as it represents the (relativistic) increase of the energy of the colliding particle. In natural units, $\beta = v$.

The collision is best visualised at the centre-of-mass (c.m.) frame, where colliding particles have zero combined momentum, and the collision energy is simply twice the energy of one proton. However, the result of the collision is measured in the inertial frame of LHC detectors (which will be referred to as the *detector frame*), in which we would see two equal energy particles of equal masses colliding. We will discuss how to connect these two frames.

In relativistic kinematics, three-spatial-component vectors need to be expanded with a time-like component, and they are termed four-vectors. A generic four-vector can be denoted by $X = (x_0, \vec{X}) = (x_0, x_1, x_2, x_3)$, where the convention of having the 0th component as time-like is chosen, and $\vec{X}$ is the ordinary three-vector. A four-vector (and in general a tensor), which is represented with lower indices or subscripts as above, is called a covariant vector. It will have a corresponding contravariant vector, which inverts the sign of the space-like components, and will be denoted by upper indices or superscripts, such as $(x^0, -x^1, -x^2, -x^3)$. Equivalently, the covariant and the contravariant vectors can be denoted by X_μ and X^μ, where μ runs from 0 to 3.

Then a dot product (mathematically called an inner product), analogous to the three-vector dot product is defined, which must be between a covariant and a contravariant vector. So for two four-vectors X and Y, the dot product is:

$$X \cdot Y \equiv X_\mu Y^\mu = x_0 y^0 - x_1 y^1 - x_2 y^2 - x_3 y^3$$

In three-vector notation, the dot product represents the projection of one vector in the direction of another, multiplied by the length of the second vector. In four-vector notation, the notion of distance or direction does not apply, but the dot product still serves an important role.

A Lorentz transformation of a four-vector X to X', to a frame moving with velocity β along x-axis between is given by:

$$\begin{aligned}
x_0' &= \frac{x_0 - \beta x_1}{\sqrt{1 - \beta^2}} \\
x_1' &= \frac{x_1 - \beta x_0}{\sqrt{1 - \beta^2}} \\
x_2' &= x_2 \\
x_3' &= x_3
\end{aligned}$$

This can be rewritten as:

$$\begin{aligned}
x_0' &= \frac{x_0}{\sqrt{1 - \beta^2}} - \frac{\beta x_1}{\sqrt{1 - \beta^2}} = x_0\gamma - \beta x_1\gamma \\
x_1' &= \frac{x_1}{\sqrt{1 - \beta^2}} - \frac{\beta x_0}{\sqrt{1 - \beta^2}} = x_1\gamma - \beta x_0\gamma
\end{aligned}$$

In general, this can be written in matrix form:

$$\begin{bmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The transformation of another vector Y to Y' can be written similarly. Now it is straightforward to show:

$$X \cdot Y = X' \cdot Y'$$

This is a very important result, which means that the dot products of two four-vectors is frame-independent, or as it is generally called, *Lorentz invariant*. Therefore, any quantity which can be expressed as a four-vector dot product, can be calculated in any frame, yielding the same result. In collider physics, the most commonly encountered four-vectors are the position and momentum four-vectors (often termed *four-momentum*). The former can be represented by $(t, x, y \cdot z)$, and the latter as (E, p_x, p_y, p_z), where the spatial components are the usual three-vectors.

The dot products of the position and momentum four-vectors are Lorentz invariant. For position vectors, it is designated as the space–time interval:

$$s^2 = X' \cdot X' = x_0'^2 - x_1'^2 = x_0^2 - x_1^2 = X \cdot X$$

In general terms, for a spatial separation of Δx_1 and a temporal separation of Δx_0:

$$(\Delta s)^2 = (\Delta x_0)^2 - (\Delta x_1)^2$$

is constant for all observers. The sign of $(\Delta s)^2$ is not positive definite. For space-like intervals, $(\Delta s)^2 < 0$, while for time-like intervals, $(\Delta s)^2 > 0$. Similarly, for momentum four-vectors, the dot product with itself is invariant, which is given by:

$$P \cdot P = E^2 - p^2,$$

where p is the magnitude of three-momentum. This is defined as the *invariant mass* (as this quantity will be same in any inertial frame). For a single particle, invariant mass is equivalent to its rest mass (the mass it has at a frame that is at rest, which is a measure of inertia), however, for a system of particles that participates in an interaction, the appropriate term to use is the invariant mass, which can be thought of as the effective rest mass of the system, accounting for their motion[2]. In the limiting case of a particle at rest, $p = 0$, this can also be thought of as equivalent to Einstein's famous equation: $E = mc^2$ in terms of the rest mass $m = P \cdot P$ (and not using natural units). The original equation involved relativistic mass, a concept which is deprecated now.

[2] Occasionally the term relativistic mass is used to denote $m_{rel} = \gamma m$, where m is the rest mass. This definition is useful only to keep the standard form of momentum $p = m_{rel}v$, but it is not used otherwise.

For a particle decaying to multiple particles, then the outgoing daughter energies and momenta can be added to find the invariant mass. As this is the same in all frames, including the centre-of-mass frame, then the particle can be identified in any frame from its decay product energies and momenta.

The momentum four-vector can be obtained from the position four-vector by dividing it by a quantity analogous to time. Proper time, $d\tau = dt\sqrt{(1-\beta^2)}$, which is equivalent to the invariant space–time interval (this can be considered as the time measured by a clock moving with the particle) is used, so:

$$P \equiv m(dx_0/d\tau,\ dx_1/d\tau,\ dx_2/d\tau,\ dx_3/d\tau),$$

where m is the mass. For $\beta \ll 1$, it reduces to $P_1 = mv$, the usual three-momentum, as can be seen from the first two components, $P = (m/\sqrt{1-\beta^2},\ mv/\sqrt{1-\beta^2})$. The first term then stands for the rest energy.

1.3.2 Specific examples

- Fixed target experiments: this is equivalent to a situation where particle of mass m_1 and energy E_1 hits a stationary particle of mass m_2. The available energy (at the c.m. frame) to create a new particle (or a particle pair) of certain invariant mass can be expressed as (from the definition of invariant mass):

  $$s = (E_1 + E_2)^2 - (p_1 + p_2)^2,$$

  where p_1 and p_2 denote the (three) momentum of the particles, and E_2 denotes the energy of the target particle. As the target particle is at rest, $p_2 = 0$, and $E_2 = m_2$. This leads to:

  $$s = E_1^2 + m_2^2 + 2E_1m_2 - p_1^2 = m_1^2 + m_2^2 + 2E_1m_2$$

  Alternatively, this can be rearranged to:

  $$E_1 = \left(s - m_2^2 - m_1^2\right)\big/2m_2,$$

  which gives the required energy of the incoming particle to reach a certain target c.m. energy at the collision.

  If the two particles are protons, and a proton anti-proton pair is to be created (each having a mass m_p), then $s = (4m_p)^2$, where $2m_p$ comes from the beam and target protons and other $2m_p$ from a newly created pair. The total momentum is zero at the c.m. frame. That gives: $E_1 \approx 7m_p = 6.6$ GeV. If the target is a nucleus instead of a bare proton, then protons due to their uncertainty in position within the nucleus, will necessarily have some momentum as governed by the uncertainty principle. That will make the three-momentum in the detector frame somewhat smaller.

- Collider experiment: when colliding particles with masses m_1 and m_2 have energies E_1 and E_2, the c.m. energy is:

$$\sqrt{s} = E_1 + E_2$$

Equating the momentum magnitudes in the c.m. frame,

$$E_1 - m_1^2 = E_2 - m_2^2$$

or

$$(E_1 + E_2)(E_1 - E_2) = m_1^2 - m_2^2$$

it follows that:

$$E_1^2 - E_2^2 = \left(m_1^2 - m_2^2\right)/\sqrt{s}$$

Adding the two,

$$E_1 = \sqrt{s}/2 + \left(m_1^2 - m_2^2\right)/2\sqrt{s}$$

Now for the fixed target case before, if $\sqrt{s} \gg m_1, m_2$, we get $E_1 \approx s/2m_2$, and for the colliding case here, we get, $E_1 \approx \sqrt{s}/2$. That means the available energy in the collider set-up is more, as energy is wasted as the kinetic energy of the created particle in the c.m. frame for fixed target, where it is at rest in the c.m. frame.
- Decays:

For a heavy particle $(M, 0)$ (in the c.m. frame, therefore the momentum is zero) decaying into two particles with energies and three-momenta given by $(E_{a/b}, \pm\vec{p})$:

$$M^2 = E_a + E_b$$

$$E_a = \sqrt{m_a^2 + p^2} = \frac{M^2 - \left(m_a^2 - m_b^2\right)}{2M}$$

$$E_b = \sqrt{m_b^2 + p^2} = \frac{M^2 - \left(m_b^2 - m_b^2\right)}{2M}$$

From this, we can get:

$$p = \frac{\sqrt{M^2 - \left(m_a^2 - m_b^2\right)}\sqrt{M^2 - \left(m_a^2 + m_b^2\right)}}{2M}$$

Which shows, $M > m_a + m_b$. A heavy particle can decay only if the mass exceeds the sum of decay product's masses. In other words, if a particle has mass exceeding the masses of two other particles, that particle is unstable and decays unless decay is forbidden by some conservation law, i.e. conservation of charge, momentum, or angular momentum.

Here we work out an example, the decay of neutral pion to two photons. In the c.m. frame of the pion, the photons are back-to-back. We work in the x–z

plane, and assume the photons are at angles of ϕ and $-\phi$ with the z-axis. As the photons are massless, their four-momentum can be written as: $(M_\pi/2, (M_\pi/2)\sin\phi, 0, (M_\pi/2)\cos\phi)$ and $(M_\pi/2, -(M_\pi/2)\sin\phi, 0, -(M_\pi/2)\cos\phi)$, where M_π is the mass of the pion, and the energy of each photon is just half of it.

Now in the detector frame, the pion can be considered to be moving with a velocity β along the $+z$-direction. The two reference frames are shown in figure 1.3, where θ_1 and θ_2 are the angles the photons make with the z-axis in the detector frame.

If in the detector frame, the photons have energies E_1 and E_2, then their four-momentum can be written as: $(E_1, E_1\sin\theta_1, 0, E_1\cos\theta_1)$ and $(E_2, -E_2\sin\theta_2, 0, E_2\cos\theta_2)$.

Then the four-momentum in the detector frame and in c.m. frame are related by:

$$\begin{bmatrix} E_1 \\ E_1\sin\theta_1 \\ 0 \\ E_1\cos\theta_1 \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} M_\pi/2 \\ (M_\pi/2)\sin\phi \\ 0 \\ (M_\pi/2)\cos\phi \end{bmatrix}$$

and

$$\begin{bmatrix} E_2 \\ -E_2\sin\theta_2 \\ 0 \\ E_2\cos\theta_2 \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} M_\pi/2 \\ -(M_\pi/2)\sin\phi \\ 0 \\ -(M_\pi/2)\cos\phi \end{bmatrix}$$

corresponding to the boost of β along the z-axis, with $1 - \beta^2 = 1/\gamma^2$.

A few useful results can be obtained, derivations of which are left to the reader as an exercise. The invariant mass can be calculated in either frame of reference, and they must be identical. The invariant mass, m in the detector frame can be shown to be:

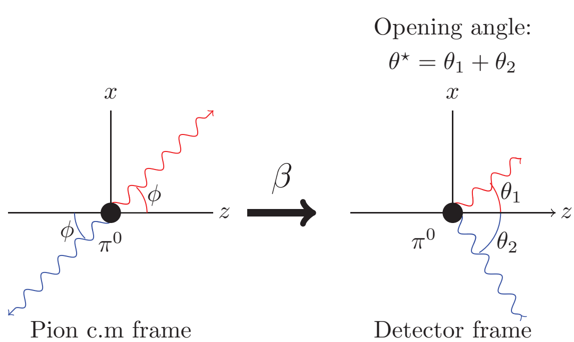

Figure 1.3. The center-of-mass and the detector reference frames for the neutral pion to diphoton decay.

$$m^2 = 2E_1E_2(1 - \cos(\theta_1 + \theta_2)) = E_1E_2 \sin^2(\theta^\star/2)$$

where $\theta^\star = \theta_1 + \theta_2$ is the opening angle between the two photons.

For large boost, and using Taylor expansion in powers of $(2/\gamma^2)$, we can show that $\theta^\star \approx (2/\gamma)\sin\phi$. Since the pion has zero spin, the angle ϕ is isotropic, resulting in no preferred direction for decay products in the detector frame. This is an important general relationship. It shows that the decay products of a highly boosted particle tend to be collimated with an opening angle that decreases as the boost factor γ increases, as shown in figure 1.4. The minimum opening angle occurs for $\phi = \pi/2$ corresponding to a symmetric decay in the detector frame and the maximum opening angle is π, corresponding to $\phi = 0$ (one photon going forward, the other going backward). The distribution of the opening angle can be seen to peak at the minimum corresponding to $\sin(\theta^\star/2) = 1/\gamma$, and vanish at the maximum value of π.

The distribution of the energy, E_γ of one of photons, $\mathrm{d}N/\mathrm{d}E_\gamma$ can be expressed as:

$$\mathrm{d}N/\mathrm{d}E_\gamma = (\mathrm{d}N/\mathrm{d}\cos\theta)(\mathrm{d}\cos\theta/\mathrm{d}E_\gamma) = (1/2)(\mathrm{d}\cos\theta/\mathrm{d}E_\gamma)$$

where θ is taken as the angle of the photon under consideration with the z-axis in the detector frame, and the first term is half by normalising it to unity over $-1 \geqslant \cos\theta \leqslant 1$. Again, using the above mentioned Lorentz transformation, it can be shown that $\mathrm{d}E_\gamma/\mathrm{d}\cos\theta = (M_\pi/2)\gamma\beta$, which leads to:

$$\mathrm{d}N/\mathrm{d}E_\gamma = 1/P_\pi$$

where P_π is the momentum of the pion in the detector frame. This distribution is flat, with limiting values of $(E_\pi \pm P_\pi)$, where E_π is the energy of the pion in the detector frame.

The decay rate at the c.m. frame for a general two-body decay is given by $W = 1/\tau$, where τ is the mean lifetime. For short-lived particles, a related quantity is the width Γ. It can be interpreted from quantum field theory (QFT), or from experimental observations. In QFT, the probability amplitude for the decay of a particle with energy E and mass M is given by the so-called Breit–Wigner relation [35]:

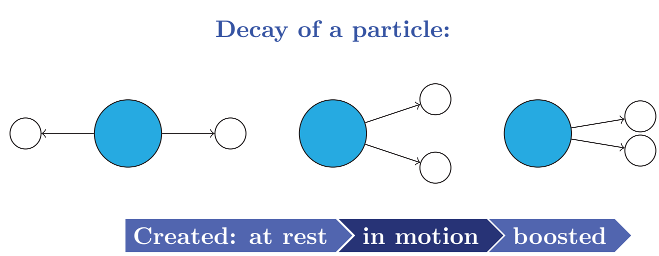

Figure 1.4. Illustration of orientation of the decay products from a two-body decay. The more the energy of the initial particle is, the less the angular separation of the decay products will be. Boost refers to the transverse energy of the initial particle.

$$P = \frac{1}{(E^2 - M^2)^2 - M^2\Gamma^2}$$

This means that we expect to see a narrow spike in the mass distribution of a particle constructed from the invariant mass of the decay products. Experimentally, however, a bell-shaped curve is obtained, as shown in figure 1.5. This distribution is often referred to as the *lineshape*. The width not only comes from potential experimental mis-measurements, but from the Breit–Wigner width of the decay. Another way of saying this is: it is the manifestation of uncertainty principle, $\Delta E \Delta t > \hbar$, where mass is related to the energy E, and the width is related to the time t. The width of the distribution at half height is Γ, which also represents the uncertainty in mass. Measuring the lineshape of a resonance therefore corresponds to measuring the mass (i.e. the value for which the production rate is maximum), the width of the Breit–Wigner around that mass (according to the above formula), and the production rate at the peak itself.

> Probability of a particle to decay to a particular channel is called the branching fraction.

For a particle with several different modes of decay, one can also define partial decay rates or widths or individual modes, termed *branching fraction*, BF = $\Gamma_{\text{process}}/\Gamma$. The total width is the sum of all partial widths.

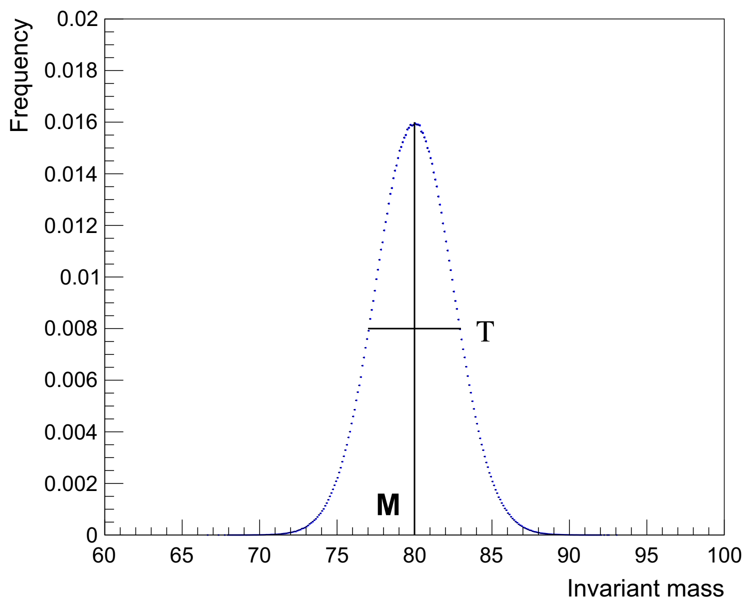

Figure 1.5. Illustration of the finite width of the invariant mass distribution around the actual mass M of a particle, constructed from its decay products. The width of the distribution at half the height is denoted by Γ.

Exercises

1. While defining natural units, we expressed energy in terms of electron-volt. What would the problem be if we tried to express energy in terms of $\hbar$, and derive the other units? Express length and mass in terms of natural units.
2. It can be seen from section 1.2, that all the colliders mentioned had (at least) two experiments (corresponding to recoding data independently in two detectors). SPS at CERN had UA1 and U1, Tevatron at Fermilab had CDF and D∅, and LHC has ATLAS and CMS (focussing on general purpose ones). The HERA experiment at DESY, which we have not talked about, made fundamental contributions to the field as well, and the main experiments were H1 and ZEUS. The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) in the USA also has STAR and PHENIX experiments. The question is, why?
3. What prevents anti-matter from being dark matter?
4. We have two photons with (E, p_x, p_y, p_z) given by:
   Photon1: 109.38, −41.85, −9.29, −100.74 GeV.
   Photon2: 40.21, 34.10, 8.35, 19.49 GeV.
   Find the invariant mass of the combined object. What do you think was the original particle that decayed to these two photons?
5. If the energy of a proton when it undergoes a collision at the LHC is 6.5 TeV, what is its speed in terms of the speed of light? What is the energy seen by one proton for another?
6. If the Z boson mass and decay width are measured to be 90.1 GeV and 2.5 GeV, what is the lifetime in seconds?
7. Why can a single photon not produce an electron–positron pair? How does this process, termed pair production, take place?
8. Show that the decay $e^- \rightarrow e^- + \gamma$ is kinematically not allowed.
9. The Ω^{-1} baryon was predicted by Murray Gell-Mann. It is produced by the strong interaction process:

$$K^- p \rightarrow \Omega^{-1} K^+ K^0$$

   The *strangeness* of a particle can be defined by number of strange quarks making up a hadron, with anti-strange quarks resulting in negative strangeness. In order for this process not to violate strangeness conservation, what is the strangeness of Ω^{-1}? Can it decay via strong interaction to the following?

$$\Omega^{-1} \rightarrow \overline{K^0} + \Xi$$

   Check all the conservation laws you can think of. PDG [36] is your friend.
10. Is the decay of a neutral pion to one or three photons possible?
    Is the decay: $\Lambda^0 \rightarrow p + \pi^-$ possible (where the p stands for proton)?

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