Using a Genetic Algorithm to Tune First-Person Shooter Bots

Nicholas Cole, Sushil J. Louis, and Chris Miles
Evolutionary Computing Systems Laboratory
Department of Computer Science
University of Nevada
Reno, Nevada 89557
Email: {ncole, sushil, miles}@cs.unr.edu

Abstract— **First-person shooter robot controllers (bots) are generally rule-based expert systems written in C/C++. As such, many of the rules are parameterized with values, which are set by the software designer and finalized at compile time. The effectiveness of parameter values is dependent on the knowledge the programmer has about the game. Furthermore, parameters are non-linearly dependent on each other. This paper presents an efficient method for using a genetic algorithm to evolve sets of parameters for bots which lead to their playing as well as bots whose parameters have been tuned by a human with expert knowledge about the game's strategy. This indicates genetic algorithms as being a potentially useful method for tuning bots.**

I. Introduction

Commercial game developers are faced with the challenge of creating realistic, human-like artificial intelligence robot controllers (game AI) within tight hardware constraints [1]. In terms of first-person shooters, rule-based expert system robot controllers which play the game are called "bots." Due to computational constraints, most bots are written in C/C++, taking the form of state-based machines [2].

In order to save both computation and the programmer's time, the game AI uses many hard-coded parameters to complete the bot's logic. Authors of bots spend an enormous amount of time setting parameters. Parameters can be thought of as values which act as thresholds in the bot's rule-based logic. Figure 1 shows an example of such parameters in terms of Counter Strike gameplay. Counter Strike is a popular first-person shooter game [3].

By adding more parameterized rules, the bots become more realistic. Consequently, the development time increases since the programmer has more parameters to tune using trial-and-error. The tuning of these parameters becomes increasingly complicated even if the programmer is an expert in the game's strategy.

While there exists work in applying genetic algorithms to board games such as checkers and game theoretic problems like the iterated prisoner's dilemma, little work has been done within the scientific community in applying a genetic algorithm to a popular, 3-D, first-person shooter game like Counter Strike [4], [5].

We propose a technique which applies a genetic algorithm to the task of tuning these parameters, such as the bold face ones in Figure 1. This will help game developers write game AI that is efficient, realistic, and easy to develop.

```
if( enemy.distance ≤ 5 )
{
        ATTACK-WITH-KNIFE()
}
else if( enemy.distance ≥ 5 AND enemy.distance ≤ 30 )
{
        ATTACK-WITH-SUBMACHINE-GUN()
}
else
{
        ATTACK-WITH-RIFLE()
}
```

Fig. 1. An example of parameterized rule-based AI. The parameters we are concerned with appear in **bold face**.

The process of finding an acceptable set of parameters is referred to as *tweaking* the bot. In Counter Strike, this would be the time spent tuning the bot's weapon preference, initial *aggressivity*, path preference, and style of gameplay. If a bot selected the same weapon round after round, then opponents would easily exploit the bot since no single weapon is perfect for every situation. Therefore, programmers give each weapon a preference of selection. This leads to a biased randomness in the bot's weapon selection behavior. Table I shows an example of the weapon preferences in the column labelled *WP*. The higher the preference for a weapon, the more likely it is to be purchased by the bot. Using relative preferences for weapon selection allowed for fast tuning of the bots, since a preference for a single weapon can be updated without having to update the preferences for other weapons. Weapon preferences are normalized to weapon selection probabilities during evaluation.

Choosing a correct set of parameters is not always a straightforward process and requires a great deal of trial-and-error testing. An acceptable parameter set can be thought of as the code to a safe, and in the case of Counter Strike bots, there are many combinations which will unlock the safe. In terms

0-7803-8515-2/04/$20.00 ©2004 IEEE

of search space, there are a number of acceptable parameter optima that will lead to good gameplay. Determining the correct set of parameters is a tedious and time-consuming one, since a slight change to one parameter will often have a negative impact on other parameters, i.e. they are non-linearly dependent. This is our motivation for applying a genetic algorithm to find a good set of these rule-based parameters. We ask, "Is it possible to use a genetic algorithm to tune the parameters as well as a human can?"

Recently, the scientific community has taken an interest in using commercial 3-D engines such as Quake, Unreal, and Half-Life as a testbed for advanced AI research [1], [6], [7], [8]. We also believe Counter Strike, a game which runs inside the Half-Life engine, to a be a good testbed for AI research. John Laird uses the Soar AI Engine, a rule-based expert system, to design bots that play Quake II, another popular first-person shooter. The Soar engine is re-useable from game to game within a specific genre, requiring only changes to the engine calls and not to the AI logic inside of the Soar Engine [2]. Rogelio Adobatti et al. have developed a framework inside the Unreal Tournament engine which allows them to study AI behavior within the virtual environment provided by the engine [6]. Adobatti et al. have also developed a non-violent game in order to attract more researchers, who would otherwise be turned away by the extreme violence found in most first-person shooters.

Realizing the fact that computer game AI is constrained by hardware limitations, Khoo et al. proposed inexpensive yet effective methods for improving game AI [1]. Their work included adding an Elisa-based chat program to an existing Counter Strike bot in order to make the human players believe they were not playing against bots but rather other humans [1].

Many existing works present tutorials and background information and provide excellent fundamental information for commercial game developers [9], [10], [11].

Work by Fogel with Blondie24 shows that coevolutionary computation can lead to a ranked AI checker player [4]. Axelrod's work with the iterated prisoner's dilemma problem has also shown that coevolution techniques can lead to the discovery of successful game strategies [5]. Since Counter Strike is not a turn-based game like most board and card games, it may not lend itself well to currently established coevolution techniques. Since coevolution may not be directly applicable, we are simply applying a genetic algorithm to the tuning of parameters to ascertain how effective evolutionary computation techniques are at first-person shooter games. If the genetic algorithm can make progress, then we will continue with our plans to coevolve Counter Strike bots. Our eventual goal is to investigate evolutionary computing techniques for knowledge acquisition, human modelling, and teamplay based on this platform. We chose Counter Strike because it is extremely popular. Last month, players spent over 1.5 billion minutes playing Counter Strike online [12]. Gamers connect from across the globe. This popularity will make it easier to collect more data for human modelling from human players as they connect to our server.

A. Genetic Algorithms

Genetic algorithms (GAs) are stochastic, parallel search algorithms based on the mechanics of natural selection and evolution [13], [14]. GAs were designed to efficiently search large, non-linear, poorly-understood search spaces where expert knowledge is scarce or difficult to encode and where traditional optimization techniques fail. Robust and flexible, GAs exhibit the adaptiveness of biological systems. As such, GAs appear well-suited for searching the large, poorly-understood spaces that arise in tuning problems, specifically, tuning parameterized rule-based bots for Counter Strike.

II. Counter Strike

Counter Strike is a popular first-person shooter game in which counter terrorists try to neutralize terrorists. The game has a strong emphasis on tactics, decision-making, and teamplay, and we believe it serves as a good testbed for our work. Figure 2 shows an in-game screenshot.

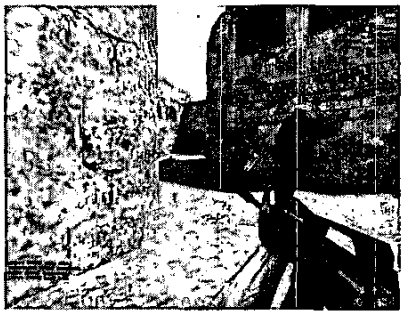

Fig. 2. In-game screenshot of Counter Strike

There are a number of variant types of gameplay for Counter Strike, but we focused on one subset of gameplay inside Counter Strike called the *defuse mission*. In the *defuse mission*, counter terrorists are tasked with preventing the terrorists from planting a bomb at one of two locations on the map. A map can be thought of as an environment in which the game takes place. Figure 3 shows an overhead view of a typical Counter Strike map. The counter terrorists may win by defusing a planted bomb. The terrorists may win by planting a bomb and protecting it from defusal until its detonation. Either side may win by eliminating all members of the opposite team, since either side's goal could be trivially accomplished without interference from the other team.

To constrain things further, each round of gameplay is limited to five minutes with a six-second planning phase called *freeze time*. During *freeze time*, each side may purchase new weapons and equipment. Each item has an associated cost. Table I shows the cost of each primary weapon. A primary weapon, as the name suggests, is the player's weapon of choice. Secondary weapons, pistols, can also be purchased in case the primary weapon fails.

The money to purchase these items is earned by the result of the previous round. There are penalties and rewards for certain actions during the round as shown in Table II. The

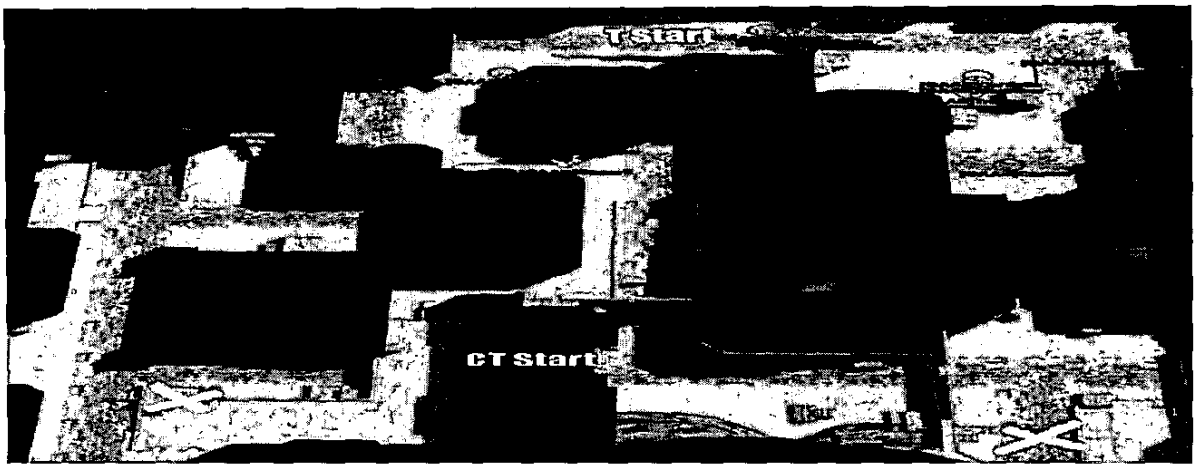

Fig. 3. This is an overhead view of the Counter Strike map "de-dust2." The locations where the bomb may be planted by the terrorists are denoted by a large X. Counter terrorist begin at the location marked *CT Start*. Terrorists begin each round at the location marked *T Start*.

TABLE I
PRIMARY WEAPON COSTS IN COUNTER STRIKE[3]

Name	Cost	WP	Notes
Benneli M3 Super90	$1,700	15	Shotgun
Benneli XM1014	$3,000	5	Automatic Shotgun
Hechler and Koch MP5-Navy	$1,500	30	Sub-Machine Gun
Steyr Tactical	$1,250	10	Machine-Pistol
FN P90	$2,350	5	Sub-Machine Gun
Ingram MAC-10	$1,400	15	Sub-Machine Gun
Hechler and Koch UMP	$1,700	10	Sub-Machine Gun
AK-47	$2,500	20	Assault Rifle
Colt M4A1 Carbine	$3,100	20	Assault Rifle
Steyr AUG	$3,500	30	Assault Rifle
Sig SG-552 Commando	$3,500	30	Assault Rifle
Steyr Scout	$2,750	10	Sniper Rifle
A1 Arctic Warfare/Magnum	$4,750	25	Sniper Rifle
Hechler and Koch G3/SG-1	$5,000	5	Sniper Rifle
Sig SG-550 Sniper	$4,200	5	Sniper Rifle
FN M249 Para	$5,750	5	Machine Gun

Payoff/Fine table will serve as a useful gradient for our GA in determining fitness.

During this planning time, teammates communicate with one another to determine which path they wish to take to neutralize the opposite team. Planning is a key element of gameplay as each player should purchase a weapon which is effective for the chosen path. For example, if a player has decided to take a path which follows a narrow series of hallways and air vents, it is desirable to use a close-quarters automatic weapon such as the MP5-Navy. On the other hand, if the player decides to head towards a large, open plaza, then perhaps a long-range, bolt-action sniper rifle such as the Steyr Scout would be more desirable. The individual tactics of the player is paramount to the weapon selection. Long-range, bolt-action rifles can only be effective when fired from a fixed position where the player is not moving and has a clear line of sight towards the enemy. An *aggressivity* parameter determines the bot's style of play. A less aggressive player tends to locate easily-defended positions and wait for the enemy to enter their kill-zone, whereas an aggressive player relies on the effectiveness of a fast-attack to try and catch the enemy off-guard. A bot should have an aggressivity parameter that is appropriate for its weapon selection. Guarding a large open plaza from long distance is nearly impossible with a shotgun. Likewise, charging into a room with a sniper rifle that needs significant time to aim properly is not a prudent tactic. By the time the bot can aim the weapon, its opponents will have already eliminated it from the round.

TABLE II
PAYOFF/FINE TABLE FROM COUNTER STRIKE[3]

Action	Payoff / Fine
Kill Opponent	$300 for individual
Kill Team Mate	-$3,300 for individual
Terrorists Win by Bombing Target	$2,750 for team members
Win by Elimination (Defuse Mission)	$2,500 for team members
Counter Terrorists Defuse Bomb	$2,750 for team members
Losing a Round	$1,400 for team members
Losing over 2 Consecutive Rounds	$1,400 + $500 per round over 2 (maximum $2,900) for team members

The remainder of this paper will discuss our architecture, methodology, results, and, finally, our conclusions and future work. For convenience, we shall now refer to bots which use genetic algorithm chosen parameters as GAABs which stands for *Genetic Algorithm Assisted Bot.*

III. ARCHITECTURE

Working with a commercial 3-D engine is not an easy task, and Counter Strike is a game which runs inside Half-Life, a popular commercial 3-D engine. The Half-Life engine only makes a Software Developer Kit (SDK) available for public use. The SDK allows for engine calls to be made, such as

requesting the number of kills a certain player has, but the SDK does not allow one to view or modify the actual code inside the engine. Half-Life, like many other commercial 3-D engines, is frame-driven. This means if a task takes up too much processing time, the frame-rate will drop, clients will lose their connections, and the engine will become non-responsive.

The Half-Life engine itself handles physics, rendering 3-D geometry, drawing textures, playing sound effects, and managing client connections. Counter Strike has its own Dynamic Link Library (DLL) to manage Counter Strike specific tasks, which includes weapon profiles (rate of fire and reload time), user interface, and gameplay code (win/loss conditions). Since Counter Strike does not have any of its own AI code, it became necessary to employ the use of yet another DLL which contained our bot code. Figure 4 shows how the DLLs and the engine interact. All components enclosed inside of the Half-Life Dedicated Server communicate by means of function calls. As mentioned previously, we wanted to keep the frame-rate inside of Half-Life up, so we developed a GA Server which manages all GA-related operations outside of the Half-Life engine. The GA Server communicates directly with the Bot DLL via TCP/IP. The bots receive new parameters from the GA Server each round. At the end of the round, they report their score, which becomes the fitness for that individual (parameter set). The GA Server is written in Java, so it is platform independent. Since the Half-Life engine is real-time dependent for its physics calculations, it is infeasible to speed up the engine artificially, while maintaining a reliable simulation. Realizing this, we are working towards upgrading the GA Server so it can manage multiple Half-Life Dedicated Servers, all of which are running simulations of our bots in parallel. This will significantly reduce our simulation time. Currently, it takes over two hours to run 50 generations with a population size of 30.

Ideally, commercial 3-D engines could be more modular. At the moment, the physics, the graphics, and multiplayer logic, run interdependently. For the purpose of tuning bots, it is only necessary for us to have information about the physics and geometry of the world – rendering graphics for a bot is wasteful.

IV. Methodology

In order to have a genetic algorithm tune the parameterized rule-based AI, we had to: (1) select parameters to tune, (2) allow the GA to evolve parameters values while GAABs play only against other GAABs, and (3) pit the best GAABs against bots which were tuned by us in order to evaluate the GA-selected parameters. As we have many years of Counter Strike playing experience as well as a solid understanding of the elements of which a good bot is comprised, the bots we tuned are challenging to most veteran Counter Strike players.

Although many commercial bots cheat to play well, our bots, however, do not cheat. Cheating game AI destroys the game experience [15]. For example, imagine a human player is hiding behind a car. Cheating game AI would be able to detect this player just as easily as if they were standing in the middle of the street. Our bots, however, use sensor information gathered from their environment much like human players. For example, if they detect another player it is only because the bot has a line of sight to the player, or it has heard the player's footsteps.

The rest of this section will describe the selection of parameters, encoding, the evaluation function, the GA's parameters, and Counter Strike game settings.

A. Parameter Selection

First, we identified the parameters to optimize. The two sets of parameters we focused on were: (1) weapon selection parameters and (2) an *aggressivity* parameter (which ultimately affects path preference). The weapon selection and *aggressivity* of the bot are closely-related in playing Counter Strike. Previously, we described a bot which used a sniper rifle to play more defensively, waiting for the enemy to come to it – this would be associated with a low aggressivity value. One the other hand, a bot that uses small automatic weapons should be highly aggressive to be effective against its enemies because its weapons have limited range. There exists no one correct strategy to Counter Strike, but it is generally accepted that following these styles of play will lead to a better score. Since weapon selection and *aggressivity* are somewhat dependent sets of parameters, we chose to allow the GA to optimize these bot parameter sets in the hope that the GA would find a player who fit either one of these predominant styles. It was also quite possible that the GA would arrive at an strategy that is not easily understood yet effective.

B. Encoding

The encoding was straightforward. Each parameter was encoded into a binary string consisting of 178 bits. Figure 5 shows the chromosome's layout. The number bits to encode weapon preferences and aggressivity was higher than necessary. At the time, we believed such resolution was required and in future work we plan to use fewer bits for each parameter in order to decrease the size of the search space.

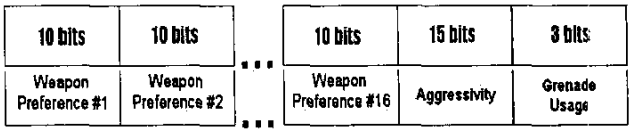

Fig. 5. Basic layout a of chromosome. Each parameter was represented in binary.

C. Evaluation Function

The evaluation function was an approximation of the standard Counter Strike Payoff/Fine table found in Table II and provided a straightforward method for measuring fitness. For example, if a bot is tuned with parameters that would lead to excessive friendly fire (such as high *aggressivity* and a strong weapon preference for automatic shotguns), then it would receive a low fitness per the Payoff/Fine table since team killing is severely punished.

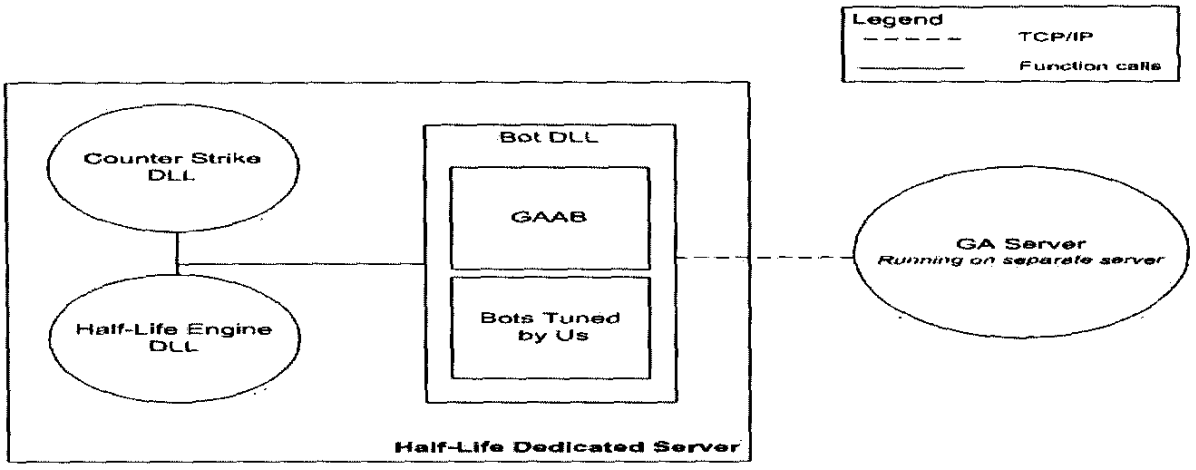

Fig. 4. The Half-Life engine's architecture relies on communication between DLLs to operate the game. This figure shows how the various DLLs interact with each other inside of the Half-Life Dedicated Server (HLDS) and how our GA Server communicates with the HLDS.

D. GA Parameters

The GA used the parameters in Table III during the training phase for the GAABs. Since we considered our search space rather large, we wanted the GA to move quickly away from poor parameter selections. We used an elitist selection method. In this method, candidates for mating are selected proportional to fitness, and the offspring double the population size to $2n$. We then select the n best individuals from this combined population for further processing [16]. Since our crossover probability was so high, elitism was necessary to prevent the high fitness individuals from being destroyed during crossover.

TABLE III
GA PARAMETERS

Parameter	Setting
Generations	50
Individuals	30
Crossover Points	2
Probability of Crossover	0.95
Probability of Mutation	0.1
Selection Method	Elitist
Chromosome Length	178

E. Game Settings

Counter Strike has settings which determine the gameplay constraints and style. We changed the round time from its default 5:00 minutes per round to 3:00 minutes. This reduced the evaluation time per generation by a minimum of 2:00 minutes, which was a significant gain. During the training phase, when the GA is still tuning parameters, GAABs only play other GAABs. The teams were even, 15 counter terrorists versus 15 terrorists. The map chosen for the match was "de-dust2," a well-rounded map, which offers a good mix of both indoor and outdoor combat as well as long-range and short-range combat. An overhead view of "de-dust2" can be seen in Figure 3. It is also popular among Counter Strike players.

V. RESULTS

The results of the GA can be seen in Figure 6. The GA made steady progress during the 50 generations.

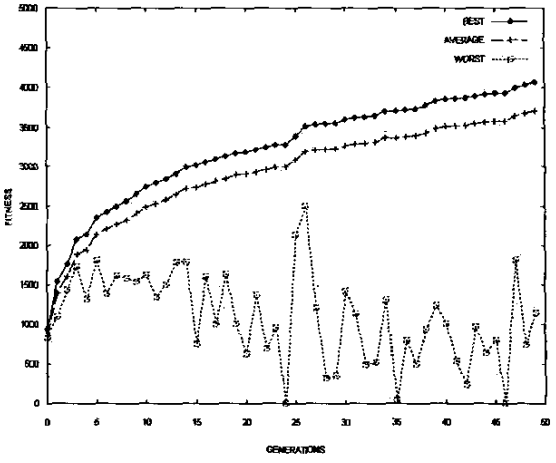

Fig. 6. The results of the GA selecting the optimal parameters for the bots averaged over 15 runs of the GA

At the end of the training phase, the best individual's phenotype from generation 50 was saved to file. Then, this phenotype was shared among 15 GAABs. These 15 GAABs then played over 100 rounds of Counter Strike against bots whose parameters we tuned. The average time per round was 2 minutes and 4 seconds. The results of the match can be found in Table IV. The first column shows the names of the two teams. The second column shows the average skill for each team. The third column shows the standard deviation of each team's average skill. Finally, the fourth column shows the median skill on each team.

The statistics found in Table IV are based on standard tournament ranking of Counter Strike players [17]. The *skill* of a bot is a calculated by the following formula:

TABLE IV
MATCH RESULTS BETWEEN BOTS

Team	Average Skill	Standard Deviation	Median
GAABs	1005	55.2	987
Bots tuned by us	999	40.3	991

$$NewSkill = Skill + \frac{K}{1 + 10^{(k-v)/1000}}$$

Skill is calculated using the ELO Method, a standard chess player rating system created by Arpad Elo [18]. The system takes into consideration two important factors when rewarding skill points to a player: (1) the difficulty of the kill and (2) the experience of the current player [17]. Each player begins with skill 1000 by default. When someone is killed, the resulting skill of each player is calculated by the formula above, where K represents the experience of the player and k and v represent the skills of the killer and victim respectively. K is a simple coefficient which begins at 20 and after the player has 100 kills or more, K is reduced to 15 [17].

We chose to only let the GAABs play against other GAABs during the training phase for two reasons. First, we did not want the GAABs to be annihilated by near-optimal, hand-tuned bots. As in real-life, it is much easier to learn from players who are at or slightly above your level of play. Since GAABs begin with completely random parameters, there will generally be a large range of playstyles present in the first few rounds. Second, Counter Strike only supports 30 players (client connections) per server. If we were to give half of those client connections to hand-tuned bots, it would have doubled our already long training time, which was undesirable.

At first glance, when observing the bots in game, the GAABs and the bots tuned by us play about the same. This is expected since they share the same rule-based logic. We noticed that GAABs would generally begin with no particular preference for a single weapon or even single type of weapon (shotgun, sub-machine gun, sniper rifle, or assault rifle). By the last few generations, they would converge strongly on at least one weapon from each of the various types. In some cases, however, when the GAABs only converged on a single weapon preference, they chose one inexpensive weapon and were highly aggressive. This is an unexpected but not unheard of strategy. The GAABs could manage to inflict enough damage before dying to barely earn just enough money to purchase another inexpensive sub-machine gun for the next round to do it over again.

Our results indicate that GAABs play statistically the same as bots tuned by us indicating that genetic algorithm tunes a bot's parameters as well as a human. With game engine or compiler support, this method would be far superior to tuning the bots manually. Programmers would only need to define a range of values for each parameter. The rest of the work would be performed by the GA. Depending on the constraints the GA evaluation function exerts on the bots, the AI could be universally shaped to favor a certain play style or tactic. A good mix of different types of bots makes a game more interesting.

VI. CONCLUSIONS AND FUTURE WORK

This paper presented a method for tuning the parameterized rules of rule-based, expert system robot controllers (bots) using a genetic algorithm. Our results show that using this method results in bots that play as well as bots tuned by a human with expert knowledge about the game. We believe this method is generalizable not only to other first-person shooter bots but to other games as well. Genetic algorithms should significantly reduces the development time for such systems, moreover, with compiler or game engine support, the tweaking process could be automated, allowing the programmer to focus on other tasks. Finally, the programmer need no longer be be an expert in the game's strategy. We also believe this method could lead to more sophisticated bots since more rules can be added to the bot's logic without the consequence of tuning a growing number of parameters using trial-and-error.

This work, like Khoo and Zubek's work in employing computationally inexpensive methods to improve AI in computer games, is useful in that all of the extra computation performed to determine the correct parameters is done during a training phase which happens before human players ever meet the bots in game [1]. This saves developer and testing time while yielding similar bots.

For future work, we would like to move toward having people evaluate the bots as they play them to determine their fitness. Play-testing is nothing new for game development. It would be interesting to see the results of play-testers evaluating how human-like a bot is and then using their evaluation as the individual's fitness. This would hopefully move the bots from being more efficient to playing more like human players.

Finally, we will look at using a genetic algorithm to determine the "optimal" configuration for a team. Bots can be coded with a personality which drives their behavior. A *Leader* bot will give radio commands and signals to other bots instead of trying to hunt down opponents. On the other hand, a *Psycho* bot will simply try to score the most kills in the round regardless of how it affects the team and their accomplishing of the goal. A genetic algorithm could be used to determine the optimal configuration of personalities, given some time to train. This work represents a start in a promising new area of research.

ACKNOWLEDGEMENT

This material is based in part upon work supported by the Office of Naval Research under contract number N00014-03-1-0104. We would like to thank Jeffrey "Botman" Broome, Johannes Lampel, and the rest of the Half-Life coding community.

REFERENCES

[1] A. Khoo and R. Zubek. Applying inexpensive techniques to computer games. In *IEEE Intelligent Systems*, pages 2–7, 2002.

[2] M. van Lent and J. Laird. Developing an artifi cial intelligence engine. In *GDC Proceedings*, 1999.

[3] *Counter Strike v1.6 Manual*. http://www.counter-strike.net/manual.html, 1.6 edition.

[4] D. B. Fogel. *Blondie24: Playing at the Edge of AI*. Morgan Kaufmann, 2002.

[5] R. Axelrod. The evolution of strategies in the iterated prisoner's dilemma. In L. Davis, editor, *Genetic Algorithms and Simulated Annealing*, pages 32–41. Morgan Kaufman, 1987.

[6] R. Adobbati, A. Marshall, A. Scholer, S. Tejada, G. Kaminka, S. Schaffer, and C. Sollitto. Gamebots: A 3d virtual world test-bed for multi-agent research. In *Communications of the ACM*, 2002.

[7] J. E. Laird. Research in human-level ai using computer games. In *Communications of the ACM*, pages 32–35, 2002.

[8] M. Lewis and J. Jacobson. Game engines in scientifi c research. In *Communications of the ACM*, pages 27–31, 2002.

[9] M. Buckland, editor. *AI Techniques for Game Programming*. Premier Press, 2002.

[10] F.D. Laramee. Genetic algorithms: Evolving the perfect troll. In S. Rabin, editor, *AI Programming Wisdom*, pages 629–639. Charles River Media, 2003.

[11] A.J. Champandard. *AI Game Development*. Pearson Education, 2003.

[12] Valve. Steam network status. Web page, February 2004. http://www.steampowered.com/status/game_stats.html.

[13] J. Holland. *Adaptation In Natural and Artificial Systems*. The University of Michigan Press, Ann Arbor, 1975.

[14] D. E. Goldberg. *Genetic Algorithm in Search, Optimization, and Machine Learning*. Addison-Wesley, Reading, MA, 1989.

[15] J. Laird and M. van Lent. Human-level ai's killer application: Interactive computer games. In *AAAI*, 2000.

[16] Larry J. Eshelman. The CHC adaptive search algorithm: How to have safe search when engaging in nontraditional genetic recombination. In Gregory J. E. Rawlins, editor, *Foundations of Genetic Algorithms-1*, pages 265–283. Morgan Kauffman, 1991.

[17] *Psychostats FAQ*. http://www.psychostats.com/faq.php, 1.9.1 edition.

[18] A. Elo. *The rating of chessplayers, past and present*. Arco, 1978.