

## Apresentação do Aproximador

# Esperanaça da duração de um passeio aleatório simples

## Fórmula deduzida.

Sabemos que a esperança da duração de um passeio aleatório simples com início em  $S_0 = a$  e barreira superior em  $N$  é dada por

$$E(T_a) = \begin{cases} a(N-a), & p = q = 1/2 \\ \frac{a}{q-p} - \frac{N}{q-p} \left( \frac{1 - (q/p)^a}{1 - (q/p)^N} \right), & p \neq q \end{cases}$$

## Fórmula da aproximação

Seja  $X$  a variável aleatória que representa os passos possíveis em determinado instante de um passeio aleatório. Para um passeio simples, tomaria a seguinte forma:

$$X = \begin{cases} 1, & \text{Com probabilidade } p \\ -1, & \text{Com probabilidade } q = 1 - p \end{cases}$$

Então, podemos aproximar a esperança de duração de um passeio aleatório iniciado em  $a$  com teto  $N$  ( $E(T_a)$ ) por:

$$E(T_a) \approx \begin{cases} \frac{N-a}{E(X)}, & E(X) > 0 \\ \frac{a}{|E(X)|}, & E(X) < 0 \end{cases}$$

## Simulações

Vamos gerar 10 mil passeios em inícios diferentes (10, 30, 50, 70, 90) e confirmar a acurácia da fórmula quando comparada à Esperança real:

5×6 DataFrame

Row	Inicio Int64	Teto Int64	Probabilidade Float64	MediaDuracao Float64	Esperanca Float64	Aproximado Float64
1	10	100	0.6	440.262	450.0	450.0
2	30	100	0.6	351.069	350.0	350.0

3	50	100	0.6	251.378	250.0	250.0
4	70	100	0.6	149.159	150.0	150.0
5	90	100	0.6	49.8668	50.0	50.0

## Justificativa

Diferentemente da esperança teórica dessa duração, podemos usar a aproximação para estimarmos durações de passeios mais complexos, como os com os passos:

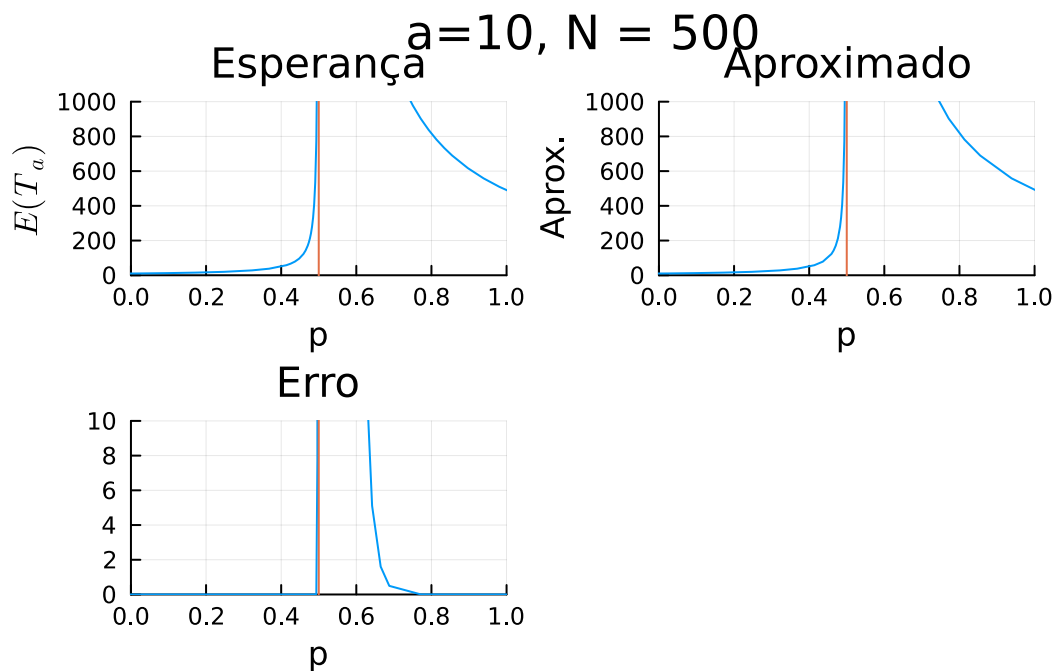
$$X = \begin{cases} 2, p = 0.37 \\ -9, q = 0.63 \end{cases}$$

Ou até com pausas e atrasos

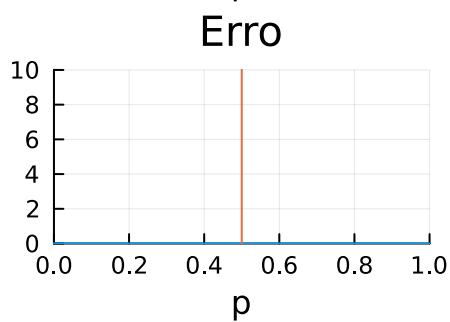
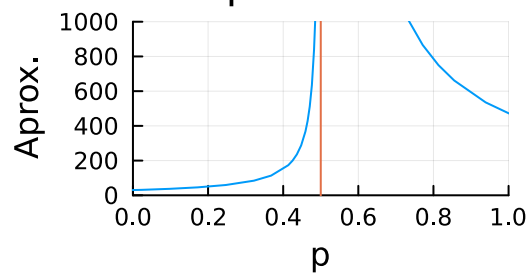
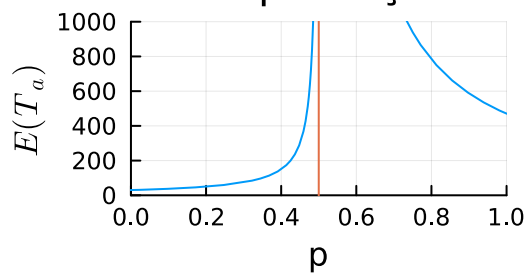
$$X = \begin{cases} 2, p = 0.2 \\ 0, p = 0.5 \\ -1, q = 0.3 \end{cases}$$

O que não é possível com a fórmula da esperança original.

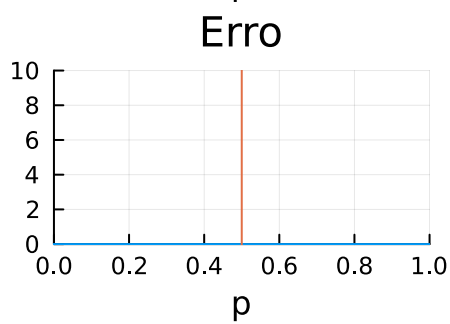
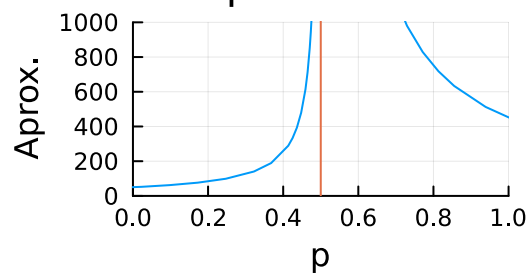
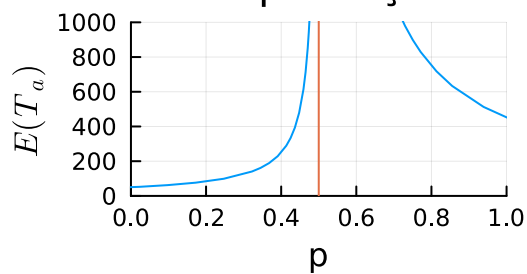
## Gráficos dos erros



$a=30, N = 500$



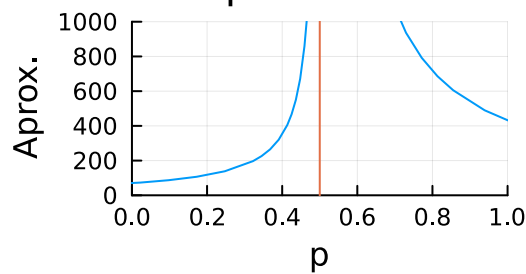
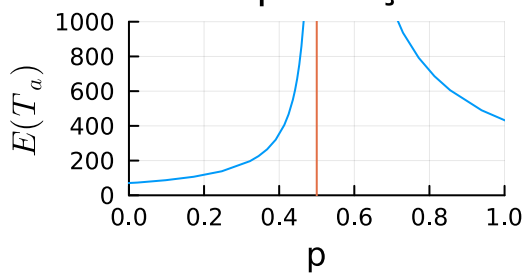
$a=50, N = 500$



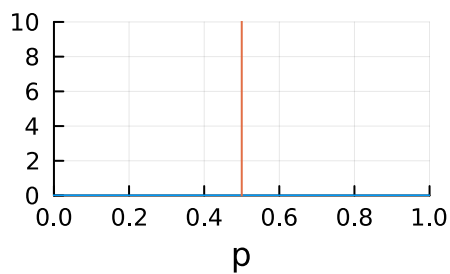
$a=70, N = 500$

Esperança

Aproximado



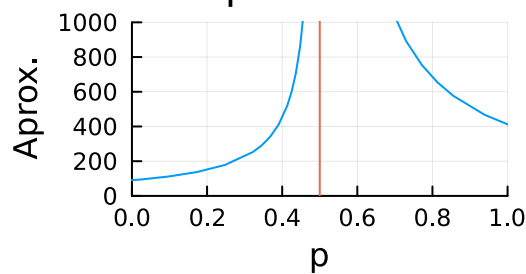
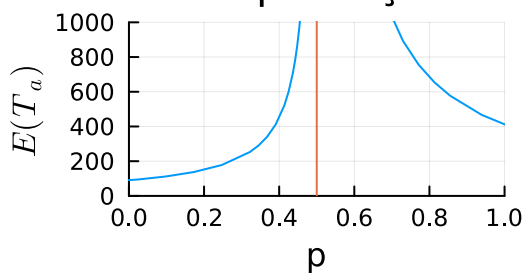
Erro



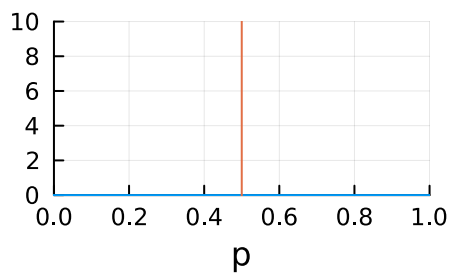
$a=90, N = 500$

Esperança

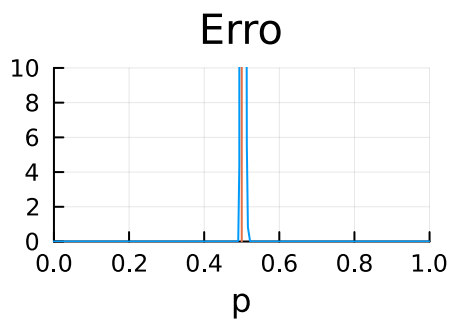
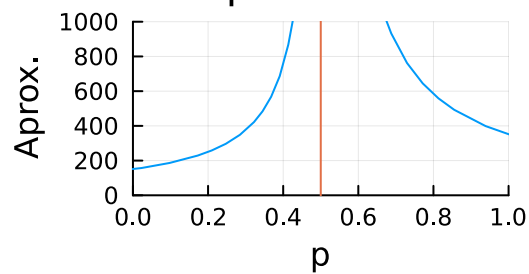
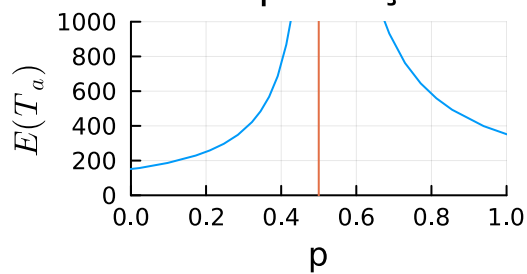
Aproximado



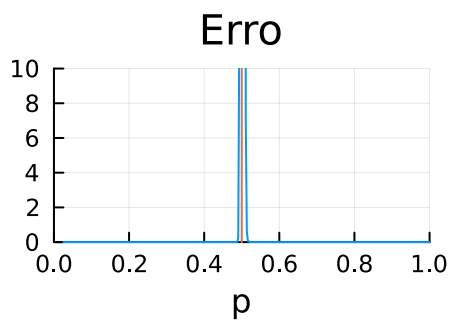
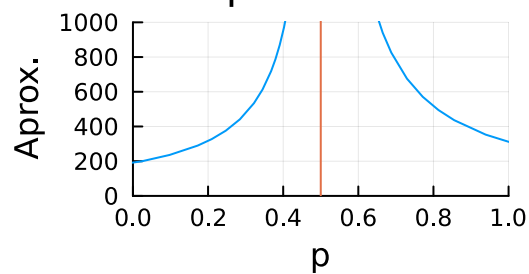
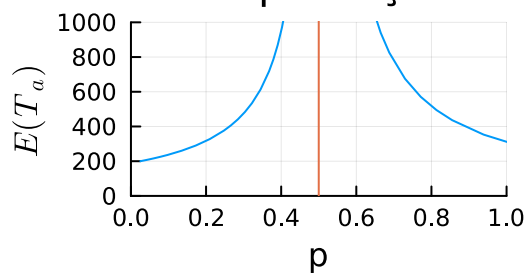
Erro



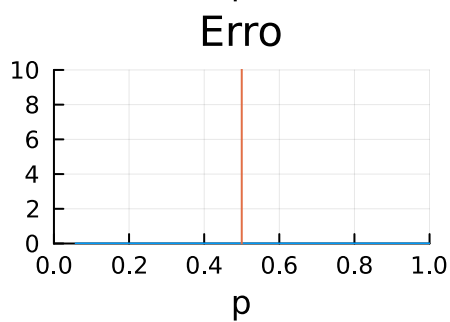
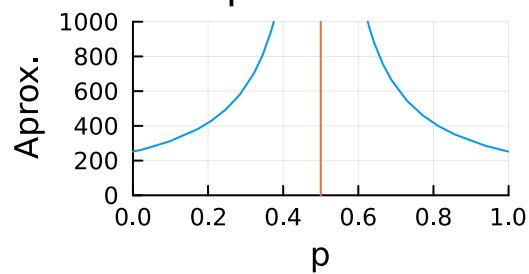
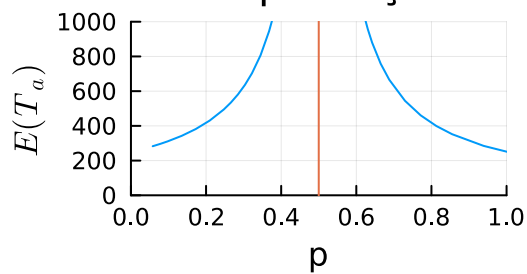
a=150, N = 500



a=190, N = 500



a=250, N = 500



a=490, N = 500

