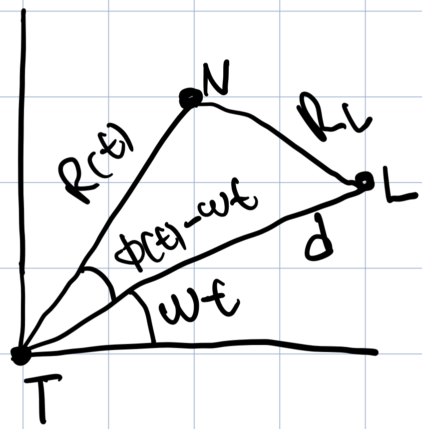
Metodos Computacionales II: Cohete Lunar



Por el teorema del coseno:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a = R_L \quad b = R(t) \quad C = d \quad A = \phi_{(t)} - \omega t$$

$$\therefore R_L^2 = R^2_{(t)} + d^2 - 2R(t)\,d\cos(\phi_{(t)} - \omega t)$$

$$\Rightarrow R_L = \sqrt{R^2(t) + d^2 - 2R(t)\,d\cos(\phi(t) - \omega t)}$$

$$L = K - V \qquad K = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$x(t) = r(t)\cos(\phi(t)) \qquad \dot{x}(t) = \dot{r}\cos\phi - r\sin\phi\cdot\dot{\phi}$$

$$y(t) = r(t)\sin\phi(t) \qquad \dot{y}(t) = \dot{r}\sin\phi + r\cos\phi\cdot\dot{\phi}$$

$$\dot{x}^2(t) + \dot{y}^2(t) = \dot{r}^2\sin^2\phi + r^2\cos^2\phi\,\dot{\phi}^2 + 2\dot{r}\sin\phi\, r\cos\phi\,\dot{\phi}$$
$$+ \dot{r}^2\cos^2\phi + r^2\sin^2\phi\,\dot{\phi}^2 - 2\dot{r}\cos\phi\, r\sin\phi\,\dot{\phi}$$
$$= \dot{r}^2\sin^2\phi + r^2\cos^2\phi\,\dot{\phi}^2 + \dot{r}^2\cos^2\phi + r^2\sin^2\phi\,\dot{\phi}^2$$
$$= \dot{r}^2 + r^2\dot{\phi}^2$$

$$\therefore\ K=\frac{1}{2}m(\dot{r}^2+r^2\dot{\phi}^2)$$

$$V=U_T+U_L=-G\frac{mM_T}{r}-G\frac{mM_L}{r_L}$$

$$L=\frac{1}{2}m(\dot{r}^2+r^2\dot{\phi}^2)+Gm\left(\frac{M_T}{r}+\frac{M_L}{r_L}\right)$$

$$H=p_r\dot{r}+p_\phi\dot{\phi}-L \qquad p_r=\frac{\partial L}{\partial \dot{r}}=m\dot{r} \quad p_\phi=\frac{\partial L}{\partial \dot{\phi}}=mr^2\dot{\phi}$$

$$\Rightarrow H=\frac{p_r^2}{m}+\frac{p_\phi^2}{mr^2}-L$$

$$H=\frac{p_r^2}{m}+\frac{p_\phi^2}{mr^2}-\frac{m\dot{r}^2}{2}-\frac{mr^2\dot{\phi}^2}{2}-G\frac{mM_T}{r}-G\frac{mM_L}{r_L}$$

$$H=\frac{p_r^2}{m}+\frac{p_\phi^2}{mr^2}-\frac{p_r\dot{r}}{2}-\frac{p_\phi\dot{\phi}}{2}-Gm\left(\frac{M_T}{r}+\frac{M_L}{r_L}\right)$$

$$H=\frac{p_r^2}{m}+\frac{p_\phi^2}{mr^2}-\frac{p_r^2}{2m}-\frac{p_\phi^2}{2mr^2}-Gm\left(\frac{M_T}{r}+\frac{M_L}{r_L}\right)$$

$$\underline{H=\frac{p_r^2}{2m}+\frac{p_\phi^2}{2mr^2}-G\frac{mM_T}{r}-G\frac{mM_L}{r_L}}$$

$$\underline{\dot{r}=\frac{\partial H}{\partial p_r}=\frac{p_r}{m}} \qquad \underline{\dot{\phi}=\frac{\partial H}{\partial p_\phi}=\frac{p_\phi}{mr^2}}$$

$$\dot{p}_r = -\frac{\partial H}{\partial r} = -\left(-\frac{p_\phi^2}{mr^3} + G\frac{mM_T}{r^2} + G\frac{mM_L}{r_L^2}\cdot\frac{\partial R_L}{\partial r}\right)$$

$$\frac{\partial R_L}{\partial r} = \frac{r - d\cos(\phi-\omega t)}{R_L}$$

$$\underline{\dot{p}_r = \frac{p_\phi^2}{mr^3} - G\frac{mM_T}{r^2} - G\frac{mM_L}{r_L^3}\left(r - \cos(\phi-\omega t)\right)}$$

$$\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = G\frac{mM_L}{r_L^2}\cdot\frac{\partial r_L}{\partial \phi}$$

$$\frac{\partial r_L}{\partial \phi} = -\frac{rd\sin(\phi-\omega t)}{r_L}$$

$$\Rightarrow \underline{\dot{p}_\phi = \frac{-GmM_L rd\sin(\phi-\omega t)}{r_L^3}}$$

$$\tilde{r} = \frac{r}{d} \qquad \tilde{p}_r = \frac{p_r}{md} \qquad \tilde{p}_\phi = \frac{p_\phi}{md^2} \qquad \mu = \frac{M_L}{M_T} \qquad \Delta = G\frac{M_T}{d^3}$$

$$\frac{\dot{r}}{d} = \frac{p_r}{md} \Rightarrow \underline{\dot{\tilde{r}} = \tilde{p}_r}$$

$$\dot{\phi} = \frac{p_\phi}{mr^2} = \frac{p_\phi}{md^2\frac{r^2}{d^2}} \Rightarrow \underline{\dot{\phi} = \frac{\tilde{p}_\phi}{\tilde{r}^2}}$$

$$\frac{\dot{p}_r}{md} = \frac{p^2_\phi}{m^2 d r^3} - \underbrace{\frac{Gm}{md}}\left(\frac{M_T}{r^2} + \frac{M_L}{r_L^3}(r - \cos(\phi - \omega t)\right)$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}^2_\phi}{\tilde{r}^3} - \frac{GM_T}{d}\left(\frac{1}{\tilde{r}^2 d^2} + \frac{\mu}{r_L^3}(r - \cos(\phi - \omega t))\right)$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}^2_\phi}{\tilde{r}^3} - \Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu d^2}{r_L^3}(r - \cos(\phi - \omega t))\right]$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}_L^3}(\tilde{r} - \cos(\phi - \omega t))\right)$$

$$\dot{p}_\phi = \frac{-GmM_L r d \sin(\phi - \omega t)}{r_L^3}$$

$$\frac{\dot{p}_\phi}{md^2} = \frac{-G \quad M_L r \sin(\phi - \omega t)}{d\, r_L^3}$$

$$\dot{\tilde{p}} = -\frac{GM_T M_L \tilde{r} \sin(\phi - \omega t) d^3}{M_T r_L^3 d^3}$$

$$\dot{\tilde{p}} = -\frac{\Delta \mu \tilde{r}}{\tilde{r}_L^3} \sin(\phi - \omega t)$$
