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Tarefa Básica - Matrizes Inversas

1- $B = \begin{pmatrix} 3 & -1 \\ y & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 \\ -y & 3 \end{pmatrix} = \begin{pmatrix} x & 1 \\ 5 & 3 \end{pmatrix} = A$ $\begin{cases} x = 2 \\ y = -5 \end{cases}$

$2 - 5 = -3$ (C)

2- $\begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix}$ $(3+k^2)-(1+3k)$
$k^2 - 3k + 2 = 0$

Girard: $1 + 2 = 3$
$1 \cdot 2 = 2$

Para det $A = 0$, então k deve valer 1 ou 2. (C)

3- $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix} = B$ → (C)

$\det A = 12 - 10 = 2$

4- $\begin{vmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{vmatrix}$ $(x^2+6+20)-(20+2x+3x)$
$x^2 - 5x + 6 \neq 0$

Girard: $2 + 3 = 5$
$2 \cdot 3 = 6$

Para det $A \neq 0$, então $\{x \neq 3$ e $x \neq 2\}$. (A).

5- $A = \begin{pmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix}$ $(1+2+4)-(2+2+2)$
$7 - 6$
$\det A = 1$

$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$
↳ A^{-1}

$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$ (B)

6- $(X\cdot A)^t = B \rightarrow X\cdot A = B^t$

$((X\cdot A)^t)^t = B^t \quad X\cdot A\cdot A^{-1} = B^t\cdot A^{-1} \rightarrow X = B^t\cdot A^{-1}$ (B)

7- $A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \cdot B = \begin{bmatrix} x \\ y \end{bmatrix} = C = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix}$ $\det A = (4\cdot 6)-(5\cdot 5) = 24-25 = -1$

$A' = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \xrightarrow{(A')^t} \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \underset{\det A}{\div -1} \rightarrow \boxed{A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}}$ (D)

8- $\det A^{-1} = \dfrac{1}{\det A}$ $\det A = \begin{vmatrix} 2 & K \\ -2 & 1 \end{vmatrix} = 2-2K$

$2-2K = \dfrac{1}{2-2K}$

$2-2K\cdot(2-2K) = 1$

$4+4K+4K+4K^2 = 1$

$4K^2+8K+3 = 0$

$\Delta = 8^2-4\cdot4\cdot3$

$\Delta = 64-48 = 16$

$\dfrac{-8\pm\sqrt{16}}{2\cdot 4} = \dfrac{-8\pm4}{8}$

$K1 = -1/2$

$K2 = -3/2$

$K1+K2 = -1/2+-3/2 = -4/2 = -2$ (B)

9- a)

$(A+B)\cdot(A-B) = \boxed{A^2-AB+BA-B^2}$

b) $A^2+AB+BA+B^2 = A^2+2AB+B^2$ → Em matrizes a ordem dos fatores influência o produto, então se $\boxed{AB=BA}$, satisfaz a equação.

$AB+BA = 2AB$ →

c) $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $ad-bc$ $-A = \begin{vmatrix} -a & -b \\ -c & -d \end{vmatrix}$ $ad-bc$ $\dfrac{\det(A)}{\det(-A)} = \boxed{\dfrac{ad-bc}{ad-bc} = 1}$

d) $B = A^{-1}$ então $\det B = \det A^{-1}$

$\det A^{-1} = \boxed{\dfrac{1}{\det A} = \det B}$